A fast calibrating volatility model for option pricing

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Abstract

In this paper, we propose a new random volatility model, where the volatility has a deterministic term structure modified by a scalar random variable. Closed-form approximation is derived for European option price using higher order Greeks with respect to volatility. We show that the calibration of our model is often more than two orders of magnitude faster than the calibration of commonly used stochastic volatility models. such as the Heston model or Bates model. On fifteen different index option data-sets, we show that our model achieves accuracy comparable with the aforementioned models, at a small fraction of the computational cost for calibration. Further, our model yields prices for certain exotic options in the same range as these two models. Lastly, the model yields delta and gamma values for options in the same range as the other commonly used models, over most of the data-sets considered. Our model has a significant potential for use in high frequency derivative trading.

Keywords: stochastic volatility models, option pricing

1. Introduction

The central assumption of the celebrated Black-Scholes formula for European option pricing is that the volatility of the underlying asset is constant [4]. This is known to be untrue in practice. The observed prices of liquid options on the same underlying, for a given set of maturities and strikes, imply differ-

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ent volatilities under Black-Scholes formulation. Modelling the future evolution of the volatility of the underlying asset, which is consistent with the observed option prices, is obviously essential to price illiquid securities on the same underlying asset. The topic of suitable volatility models which provide a consistent match with the observed prices has resulted in extensive literature over the past few decades.

There are two broad classes of volatility models: local volatility models and stochastic volatility models. Note that this is a rather imprecise taxonomy, but it will be sufficient for our purpose. The former class of models does not have an additional source of uncertainty (apart from the sources of uncertainty in the underlying) incorporated in the volatility model and the volatility is assumed to be a deterministic function of the current underlying price and time. Examples of this type of models include the models proposed by Dupire [9], Derman and Kani [7] and Alexander [1]. In contrast, stochastic volatility models include an extra source (or sources) of randomness and provide more flexibility in modelling the dynamics of volatility surface. Significant models in this class, with an emphasis on option pricing, include those proposed by Hull and White [12], Merton [15], Heston [11], Bates [3], Kou [14], Duffie et al [8] and Carr et al [6]. Bakshi et al [2] have compared a variety of stochastic volatility models in terms of their pricing and hedging performance. Heston as well as Bates model yields semi-closed form solutions in terms of Fourier transform of European option price and are hence amenable to relatively easy calibration to market data. Gatheral [10] and Javaheri [13] provide comprehensive reviews of development of volatility models.

In this work, we propose a new method for modelling the volatility as implied by option prices. In our model, volatility is represented as a deterministic function of time, with its *level* being a random variable on positive support. The proposed volatility model offers the following benefits:

- It provides a very simple approximate pricing function for calibrating the model from option price data. In the experiments performed, we demonstrate that the proposed model requires only around 1% of the computational time as the Heston model or the Bates model for calibration, on the same hardware.

- In fifteen different data sets tested for three different indices and using two different methods of measuring the pricing error, the proposed model is shown to be extremely competitive in terms of accuracy with the popular existing stochastic volatility models.

- When calibrated from the same data-set, the proposed model also yields prices for path-dependent payoffs which are in the same range as the Heston model and Bates model. This is important since the prices of illiquid payoffs are non-unique under stochastic volatility and any new model which gives significantly different prices from the established models is unlikely to be accepted by the industrial community.

- When calibrated from the same data-set and using the same numerical method, the proposed model yields option price sensitivity parameters which are very close to those found for the Heston model, for most data-sets. Option sensitivities (or Greeks) are important for risk monitoring and hedging purposes and our experiments show that hedging using our model is unlikely to provide significantly different results than hedging using the Heston model.

Note that, apart from Bates model and Heston model, several other analytically tractable options exist for modelling volatility (as mentioned earlier). Our purpose is simply to establish that our new model yields accuracy comparable to some of the popular existing models, while being significantly easier to calibrate, and easier to simulate from, than those models. Hence we have restricted our benchmark comparison to the two aforementioned models.

The rest of the paper is organized as follows. In the next section, we will briefly outline the two main stochastic volatility models to which our model will later be compared. In section 3, we will present our new model. Section

4 on numerical experiments is split into three subsections: section 4.1 outlines the data used, section 4.2 explains the methodology employed in comparing the performance of different models and lastly section 4.3 provides the results and a discussion. Finally, section 5 summarizes the contributions of the paper and outlines the directions of future research.

2. Heston model and Bates (SVJ) model

We will first outline the formulae for pricing European options using Heston and Bates (SVJ) models, since we will later use these two models as benchmarks. All the subsequent discussion is in a (non-unique) equivalent martingale measure and we will omit explicit mention of measure for simplicity. For Heston model, the asset price dynamics is assumed to be governed by:

$$dS_t = rS_t dt + \sqrt{v_t} S_t dW_t^1, \tag{1}$$

$$dv_t = -\theta(\bar{v} - v_t)dt + \sigma_v \sqrt{v_t} dW_t^2, \tag{2}$$

where r is the risk-free rate, W_t^1 and W_t^1 are standard Wiener processes with a given correlation $< W_t^1, W_t^2 >= \rho$ and $\rho, \sigma_v, \theta, v_0, \bar{v}$ are known constants. The price of European call option with strike price K is given by:

$$C_{EUR} = S_t P_1 - Ke^{-r(T-t)} P_2, \tag{3}$$

where S_t is a spot price at time t, T is a the expiration time and $P_j, j = 1, 2$ are called the pseudo-probabilities:

$$P_j = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty Re \left[\frac{e^{ix \log(\frac{S_t}{K})} e^{\phi_j(v_t, \tau, x)}}{ix} \right] dx. \tag{4}$$

Here, $\tau = T - t$ and $\phi_j(v_t, \tau, x) = \exp\{C_j(\tau, x)\bar{v} + D_j(\tau, x)v_t\}$ is the characteristic function, with

$$
\begin{aligned}
C_j(\tau, x) &= rxi\tau + \frac{\theta}{\sigma_v^2}\left[(b_j - \rho\sigma_v xi + d_j)\tau - 2\log\frac{1 - d_j e^{d_j\tau}}{1 - g_j}\right], \\
D_j(\tau, x) &= \frac{b_j - \rho\sigma_v xi + d_j}{\sigma_v^2}\left[\frac{1 - e^{d_j\tau}}{1 - g_j e^{d_j\tau}}\right], \\
g_j &= \frac{b_j - \rho\sigma_v xi + d_j}{b_j - \rho\sigma_v xi - d_j}, \quad d_j = \sqrt{(\rho\sigma_v xi)^2 - \sigma_v^2(2u_j xi - x^2)}, \\
u_1 &= \frac{1}{2}, u_2 = -\frac{1}{2}, \text{ and } b_j = \kappa + \theta - (\mathbb{1}_{j=1})\rho\sigma_v.
\end{aligned}
$$

Bates in [3] proposed adding a compound Poisson process in the underlying for the above model, which leads to a modification of (1):

$$
\frac{dS_t}{S_t} = rdt + \sqrt{v_t}dW_t^1 + (e^{\alpha+\beta\epsilon} - 1)dJ_t, \tag{5}
$$

where J_t is Poisson process with a known jump intensity λ_p, α, β are known constants and $\epsilon \sim N(0,1)$. The process J_t is uncorrelated with $W_t^i, (i = 1, 2)$. The volatility dynamics is described by equation (2). The solution for price of a European call option is given by modifying the characteristic function in the Heston model above:

$$
\phi_j(v_t, \tau, x) = exp\{C_j(\tau, x)\bar{v} + D_j(\tau, x)v_t + E(x)\tau\},
$$

where

$$
E(x) = -\lambda_p ix(e^{\alpha+\beta^2/2} - 1) + \lambda_p(e^{ix\alpha - x^2\beta^2/2} - 1).
$$

While both these models have proved popular and are known to provide good fits to option prices, they have a few shortcomings. Some of these are discussed in [16]. In particular, it was shown that Heston model usually fails to fit to a short term market skew while the SVJ model usually fails to fit an inverse yield curve. In addition, the option price is given through a fairly involved numerical integral with several parameters, which presents significant difficulties in calibration.

3. High order Moments based Stochastic Volatility model

We will now introduce the basic idea of our model. Recall that, by definition, European call option is a right to buy an asset at maturity time T for a strike price K. For a non-divident paying stock, its price at time t is given by discounted expectation of terminal pay-off:

$$C_t = e^{-r(T-t)}\mathbb{E}[(S_T - K, 0)^+].$$

Under Black-Scholes framework with constant volatility, this discounted expected value is given by

$$\begin{aligned} C_{BS} &= S_t N(d_1) - e^{-r\tau} K N(d_2), \\ d_1 &= (\sigma\sqrt{\tau})^{-1}[\log(S_t/K) + (r + \sigma^2/2)\tau], \\ d_2 &= d_1 - (\sigma\sqrt{\tau}), \end{aligned}$$

where r is the constant risk-free rate, σ is the volatility, $N(x)$ is the standard normal cumulative distribution function and $\tau = T - t$ is the time to maturity. The derivation of Black-Scholes price also assumes that short-selling as well as trading in continuous time is possible. One of the simplest frameworks to introduce a stochastic component in the volatility is to consider a Hull-White type model of the asset price process [12]:

$$dS_t = rS_t dt + \sqrt{v_t} S_t dW_t^1, \tag{6}$$

$$dv_t = f_1(t, v_t)dt + f_2(t, v_t)dW_t^2, \tag{7}$$

where W_t^1 and W_t^2 are uncorrelated Wiener processes and $f_1,\ f_2$ are smooth functions bounded by linear growth such as v_t remains non-negative almost surely. [12] shows that the price of European vanilla call option at time 0, for a time to maturity τ can be derived as expectation of Black-Scholes price with respect to the variance rate:

$$C_{EUR} = \mathbb{E}\left[C_{BS}\left(\frac{1}{\tau}\int_0^\tau v_t dt\right)\right] \tag{8}$$

where $C_{BS}(x)$ denotes Black-Scholes price evaluated at variance x. The above formula is independent of the exact process followed by v_t (under normal assumptions about $t-$ continuity and uniqueness). Denoting the variance rate $\frac{1}{\tau}\int_0^\tau v_t dt$ by $\bar{V}_\tau$ and assuming that the moments in question exist, we can expand the right hand side of (8) around $\mathbb{E}(\bar{V}_\tau)$ in Taylor series as

$$C_{EUR} \approx C_{BS}(\mathbb{E}(\bar{V}_\tau)) + \sum_{i=2}^{M} \frac{\partial^i C_{BS}}{\partial \bar{V}_\tau{}^i} \frac{\mathbb{E}(\bar{V}_\tau - \mathbb{E}(\bar{V}_\tau))^i}{i!}, \tag{9}$$

where the partial derivatives are evaluated at $\mathbb{E}(\bar{V}_\tau)$. Our aim is to construct a process for v_t for which the right hand side of the above equation is easy to evaluate (for a reasonably large M), while remaining sufficiently flexible to fit the observed option prices. Note that truncating after the first term will mean that prices of options with *all* strikes for a *fixed* time to maturity should be the same, which is obviously nonsense. This illustrates the need for non-zero higher moments for $\bar{V}_\tau$ (and hence the need for randomness in volatility) in an intuitively simple fashion.

Without loss of generality, let $t = 0$ be the current time and let $t_0 > 0$ be an arbitrary time which is less than the shortest time to maturity of any derivative product which we want to price using our model. We will allow the diffusion term in the volatility process of (7) to be non-zero only within $[0, t_0)$. This will allow us to use a *single random variable*, rather than an evolving random process, to model the randomness in volatility when pricing securities at time $t = 0$, whose payoffs are beyond t_0. Note that option pricing models are always used for pricing securities with finite, rather than infinitesimal, time to maturity. Further, t_0 itself does not appear in the pricing formulae (only an integrated variance term does, as we shall see) and can be assumed to be arbitrarily small. Next, we assume that v_t in (7) is governed by the following, specific stochastic process:

$$dv_t = (\mu_t dt + \gamma_t dW_t^2) v_t, \tag{10}$$

where μ_t is a positive deterministic and integrable function, γ_t is a positive deterministic function which is piecewise continuous, with $\gamma_t = 0,\ t > t_0$ and

W_t^2 is a standard Wiener process uncorrelated with W_t^1. Using Itô's lemma, it is straightforward to show that

$$v_t = \exp\left(\int_0^t \mu_s ds\right) \zeta_t,$$

where ζ_t is a log-normal process with unit mean and a constant variance for $t > t_0$. In particular,

$$\mathbb{V}ar(\zeta_t) = \left(\exp\left\{\int_0^{t_0} \gamma_s^2 ds\right\} - 1\right), \ t > t_0.$$

We will henceforth assume that $t > t_0$ holds. Let $k = \sqrt{\mathbb{V}ar(\zeta_t)}$. Then the third and the forth centered moments of ζ_t, m_3 and m_4 respectively, can be expressed as:

$$m_3 = k^4(3 + k^2), \tag{11}$$

$$m_4 = k^4\{(1 + k^2)^4 + 2(1 + k^2)^3 + 3(1 + k^2)^2 - 3\}. \tag{12}$$

We will parameterize the standard deviation k of the lognormal random variable ζ_t directly, with no reference to γ_t or t_0. Finally, we parameterize $\exp(\int_0^t \mu_s ds)$ as

$$\exp\left(\int_0^t \mu_s ds\right) = \hat{\sigma}_0^2 e^{-\lambda t} + \hat{\sigma}_1^2 \lambda t e^{-\lambda t} + \hat{\sigma}_2^2,$$

where $\hat{\sigma}_0, \hat{\sigma}_1, \hat{\sigma}_2, \lambda$ are scalar parameters. This gives our variance model parameterization as

$$v_t = \zeta_t(\hat{\sigma}_0^2 e^{-\lambda t} + \hat{\sigma}_1^2 \lambda t e^{-\lambda t} + \hat{\sigma}_2^2), \ \zeta_t \sim LN(1, k^2), \ t > t_0. \tag{13}$$

Along with (6), (13) completely specifies our pricing model within the chosen pricing measure, which is implicitly specified by the data used for calibration. We will call our model as high order Moments-based Stochastic Volatility (MSV) model, since it is based on the use of higher order moments of the aforementioned random variable. With this definition of v_t, we have

$$\bar{V}_\tau := \frac{1}{\tau}\int_0^\tau v_t dt = \zeta_t \underbrace{\left(\frac{\hat{\sigma}_0^2 + \hat{\sigma}_1^2}{\lambda\tau} + \hat{\sigma}_1^2 + \frac{\hat{\sigma}_2^2 - \hat{\sigma}_1^2}{1 - e^{-\lambda\tau}}\right)(1 - e^{-\lambda\tau})}_{Q_\tau}, \tag{14}$$

where Q_τ is a deterministic function. As we can see in the equation (14) Q_τ is actually the equation for Nelson-Siegel [17] spot rate curve used in interest rate modelling. While our application is unrelated to modelling interest rates, we chose this parametrization for its known ability to represent a variety of relevant shapes of term structure (both concave and convex), with a suitable choice of parameters. Since, European option price for any $\tau > 0$ is a smooth function with respect to $\bar{V}_\tau$, one can apply Taylor series expansion to the Black-Scholes option price C_{BS} around a point $\mathbb{E}(\bar{V}_\tau) = Q_\tau$:

$$\begin{aligned} C_{EUR}(\bar{V}_\tau) \approx\, & C_{BS} + \frac{\partial^2 C_{BS}}{\partial \bar{V}_\tau^2} \frac{\mathbb{E}(\bar{V}_\tau - Q_\tau)^2}{2} + \frac{\partial^3 C_{BS}}{\partial \overline{V}_\tau^3} \frac{\mathbb{E}(\bar{V}_\tau - Q_\tau)^3}{6} \\ & + \frac{\partial^4 C_{BS}}{\partial \bar{V}_\tau^4} \frac{\mathbb{E}(\bar{V}_\tau - Q_\tau)^4}{24}, \end{aligned} \tag{15}$$

where C_{BS} and its partial derivatives are evaluated at $\bar{V}_\tau = Q_\tau$. These partial derivatives for a European call option can be evaluated as:

$$\begin{aligned} \nu := \frac{\partial C_{BS}}{\partial \bar{V}_\tau} &= K e^{-r\tau} \phi(-d_2)\sqrt{\tau}, \\ \frac{\partial^2 C_{BS}}{\partial \bar{V}_\tau^2} &= \nu \frac{d_1 d_2}{Q_\tau}, \\ \frac{\partial^3 C_{BS}}{\partial \bar{V}_\tau^3} &= \frac{-\nu}{Q_\tau^2} \left[d_1 d_2 (1 - d_1 d_2) + d_1^2 + d_2^2 \right], \\ \frac{\partial^4 C_{BS}}{\partial \bar{V}_\tau^4} &= \nu \frac{12 d_1 d_2 + 3\tau Q_\tau^2 (1 - d_1 d_2) - d_1^2 d_2^2 (9 - d_1 d_2)}{Q_\tau^3}, \end{aligned} \tag{16}$$

with $d_1 = \frac{\log(S_0/K) + (r + Q_\tau^2/2)\tau}{Q_\tau \sqrt{\tau}}$, $d_2 = d_1 - Q_\tau \sqrt{\tau}$ and
$\phi(x) = (\sqrt{2\pi})^{-1} \int_0^x e^{-0.5u^2} du$. Similar expressions can easily be derived for an approximation to the price of a European put option.

We can now re-write the first four moments of $\bar{V}$ as the following:

$$\begin{aligned} \mathbb{E}(\bar{V}_\tau) &= Q_\tau, \\ \mathbb{E}(\bar{V}_\tau - Q_\tau)^2 &= k^2 Q_\tau^2, \\ \mathbb{E}(\bar{V}_\tau - Q_\tau)^3 &= k^4 (3 + k^2) Q_\tau^3, \\ \mathbb{E}(\bar{V}_\tau - Q_\tau)^4 &= k^4 \{(1 + k^2)^4 + 2(1 + k^2)^3 + 3(1 + k^2)^2 - 3\} Q_\tau^4. \end{aligned} \tag{17}$$

Equations (13)-(15) together with equations (16)-(17) define our approximate option pricing model. Along with the parameters $\hat{\sigma}_0, \hat{\sigma}_1, \hat{\sigma}_2, \lambda$ which appear in

$Q(\tau)$, the parameter k which characterises the distribution of ζ_t completes the set of parameters for our volatility model specification.

A few remarks on this model are in order.

- In empirical experiments which follow in the next section, we found that a third or a fifth order Taylor series approximation, in place of the fourth order approximation used here, makes very little difference. However, using $k = 0$ leads to very poor fits on calibration, again indicating that randomness is necessary to model the volatility dynamics adequately.

- There is zero correlation between the sources of randomness in the underlying and the volatility, and there is no risk premium attached to the randomness in volatility. However, our choice of simpler volatility model seems to provide a fit which is quite competitive in terms of accuracy when compared to models with non-zero correlation, at a small fraction of calibration cost, over a large number of data sets. Our admittedly limited evidence indicates that choosing a sufficiently flexible parameterized function of time can compensate at least partially for *not* modeling the correlation between the volatility and the price of the underlying.

4. Numerical Experiments

4.1. Data Specification

For calibration and validation of our model, we used option price data {Strike price, Maturity, Implied Volatility, Bid, Ask and underlying values on the date of reading} obtained from Bloomberg Option Monitor (OMON). Implied risk free rates were calculated using implied volatilities and option prices by simple nonlinear least squares, for each maturity. We chose European call options with a minimum of 30 days to maturity and up to 3 years to maturity, with strike prices to be both in-the-money and out-of-the money values. The total data consisted of closing option prices on 3 different stock indices {S&P500, FTSE 100 and DAX} on five different, arbitrarily chosen days {01 November 2012, 26

November 2012, 25 July 2013, 26 July 2013, 29 July 2013}, with 100 options for each index and day. This gave a total of 15 data sets (one for each index and each day), from two different years, with 100 prices in each data set. [1]

4.2. Methodology

To calibrate and validate the models (Heston model, Bates model and our MSV model), we randomly separated the option prices with proportion 80 and 20 percent for in-sample and out-of-sample model evaluation respectively, within each of the fifteen data-sets. Changing this proportion to $90\% - 10\%$ or $70\% - 30\%$ does not make any qualitative difference to the results. The in-sample data was used for calibration as well as validation and the out-of-sample data was used for validation only. For calibration, we solve the following minimization problem for each of the three models:

$$\min_{\Theta} \sum_{i=1}^{N} \frac{|C_i^{market} - C_i^{model}(\Theta)|^2}{|Bid_i - Ask_i|^4},$$

where Θ is the vector of parameters, $C_i^{model}(\Theta)$ is the price given by the model parametrised by Θ, N is the number of options in the in-sample data and Bid_i, Ask_i are closing bid and ask prices of the i^{th} option, respectively. C_i^{market} is the market price of the i^{th} option which is obtained as an arithmetic average of Bid_i, Ask_i for each option. The choice of weight, which is inverse of (option price spread)^4, under-emphasizes any illiquid options during calibration. Three different powers of bid-offer spread were tried $(1, 2, 4)$ for the choice of weight and 4 seems to offer the best fit for all of the models. Calibration was done using Matlab 2012b on a Windows 8 laptop, with Intel i7 processor and 8 Gb memory. As mentioned earlier, Heston stochastic volatility model and Bates, i.e. stochastic volatility with jumps model (SVJ) [3] are used as benchmarks for option pricing models. For Heston and Bates models, 8192 point FFT was used in approximating the option price evaluation integral.

[1] Note that the authors have carried out numerical experiments over more data-sets and the results presented here are deemed to be representative.

The calibrated models are compared with each other in three different ways:

1. For each in-sample and out-of-sample data set after calibration (30 data-sets in all - with each of 15 data-sets split into in-sample and out-of-sample subsets), we will use two commonly used error metrics, *viz* Mean Relative Absolute Error (MRAE) and Root Mean Square Error (RMSE). Further, since computational speed is one of the main selling points of our method, we will also compare the models on the computational time for model calibration. The two error metrics are defined below:

$$MRAE = \frac{1}{N}\sum_{i=1}^{N}\frac{|C_i^{market} - C_i^{model}|}{C_i^{market}},$$
$$RMSE = \sqrt{\sum_{i=1}^{N}\frac{(C_i^{market} - C_i^{model})^2}{N}},$$

   where N is the number of data points. These two error metrics and the computation time will be reported for all the data-sets.

2. Since we are treating Heston and SVJ models as 'benchmark' models, one expects that any new, *sensible* model calibrated from the same data-set as one of these models will yield similar prices for illiquid or non-traded payoffs. We test whether this is the case for our model by pricing down-and-out-call barrier options for a range of strikes, barriers and expiration, using the three models calibrated from the same data-set. We repeat the experiments with floating strike, arithmetic average Asian calls. Note that in both these cases, there are no 'true' or unique prices and we are simply expecting the models calibrated from the same data to yield similar prices for illiquid securities.

3. Finally, one also expects the models calibrated from the same data to yield similar option price sensitivity parameters, which are crucial in risk monitoring and hedging purposes. This fact is tested by numerically calculating $\Delta = \frac{\partial C}{\partial S}$ and $\Gamma = \frac{\partial \Delta}{\partial S}$ for options for each of the models, over all the data sets.

The next subsection and the accompanying tables and figures in the Appendix provide representative results to support our arguments.

4.3. Results

The application of out model to the real market data is now discussed. As mentioned above, we consider three different sets of results: the accuracy in matching the traded option prices, comparison of illiquid option prices via simulation and comparison of the sensitivity parameters via numerical approximation.

- The in-sample and the out-of-sample errors (as measured by MRAE and RMSE in both the cases) of all the data-sets are presented in the Appendix. The in-sample errors are denoted by MRAE-I, RMSE-I and the out-of-sample errors are denoted by MRAE-O, RMSE-O. In particular, tables 1-5 provide the achieved errors for data on five different days, with each table reporting in-sample as well as out-of-sample error metrics for the three indices for that day. Boldface numbers in each column indicate the worst value for the error metric obtained for that data subset (in-sample or out-of-sample subset, for each data-set). With three indices, five days, two data subsets for each index on each day and two error metrics, we have a total of 60 error columns to compare the three models (Heston, Bates and MSV) with. From the tables 1-5, MSV has the worst performance (out of the three models) only 9 out of 60 times, with one of the two benchmark models being the worst performer in all of the remaining 51 cases. This supports our modest claim that our model is very competitive in terms of accuracy with our benchmark models. The other important set of numbers is the calibration times. As the tables 1-5 show, MSV can be calibrated within 1.25 seconds in all the fifteen cases, while the lowest calibration time for the other two models is 41.32 seconds. In summary, tables 1-5 indicate that we can obtain a very good fit to option prices with our model at a fraction of the calibration cost, as compared to some of the existing popular models.

- Next, we compared the three models for prices of illiquid options, when calibrated from the same data set. Table 6 outlines the prices obtained for down and out barrier call options, priced using each of three models calibrated from the 1st November 2012 DAX and FTSE options data-set. It may be recalled that down-and-out call barrier option with strike K and barrier B has a payoff $\max(S_T - K, 0)$ at expiration time T unless $S_t < B$ at any point between $t = 0$ and $t = T$, in which case the option ceases to exist. Two choices of barriers and strikes for DAX and one choice for FTSE are considered for demonstrating performance. We simulated the option prices using Euler discretisation for all the models with 10000 steps for each sample path and with 10000 sample paths. The obtained prices and confidence intervals (denoted as CI) for various values of expiration times T, interest rates r, barriers and strike prices are reported in table 6. As can be seen, the prices given by our model are within 10% (in the worst case) of *either* Heston price *or* SVJ price. As there is no unique option price in this case, our aim is simply to establish that our model gives *believable* prices, which are not too far from those given by benchmark models. Moreover, the prices by Heston and SVJ models can themselves differ by 10% or more. It should also be noted that simulation using our model is computationally somewhat cheaper than that with either of the other two models.

  We also priced floating strike, arithmetic average Asian call options with the three models, calibrated from the 1st November 2012 data-sets (for all the three indices). This generally illiquid option has a payoff $\max(0, S_T - S_{av})$ at expiration, where S_{av} represents the time average of the underlying price between $t = 0$ and $t = T$, T being the expiration. In this case as well, we simulated the option prices using Euler discretisation, 10000 steps for each sample path and 10000 sample paths. The results are reported in table 7, along with 95% confidence intervals. As can be seen, the prices obtained by our model are close to those obtained by SVJ model.

Similar experiments were performed with other data-sets with the same qualitative conclusions; hence results are omitted for brevity.

- As a final measure of performance, we compare the three models in terms of the sensitivity parameters delta and gamma for the options. We compare these parameters over all the fifteen data-sets. For all the models, approximate values of these parameters are obtained using a central difference approximation scheme as follows:

$$\Delta \approx \frac{C(S+\delta)-C(S-\delta)}{2\delta} \text{ and}$$
$$\Gamma \approx \frac{C(S+\delta)-2C(S)+2C(S-\delta)}{\delta^2},$$

  where $C(x)$ indicates option price evaluated at the price of underlying equal to x, S is the price of the underlying and δ is a small increment. While more sophisticated methods to calculate these parameters exist (and it is trivial to find these analytically for our model by differentiation), our purpose is to compare whether the values given by our method are in the same range as the values given by the other two methods. A selection of results is presented in figures 1-3. The remaining results are qualitatively similar, and are omitted for brevity. Note that the apparent periodicity is simply a result of the same set of strikes being repeated for different expirations. For FTSE and S & P data-sets, the sensitivity parameter estimates from MSV tends to be close to one of the other two models, except at short maturities. The deviation of MSV delta and gamma from those given by the other two models is the highest for 25 July 2013 DAX data set. This is also the only data-set when the RMSE and MRAE errors for MSV model are the worst among the three models; please see table 3. Gamma values of all the three models at short maturities vary quite significantly and it is not immediately obvious which values should serve as benchmark values.

It is also worth mentioning that we did not find any evidence whether MSV model performs consistently better or consistently worse at short or long time maturities, or for in-the-money or out-of-the money options.

5. Conclusion and future research

The contribution of this paper are threefold. First and the main contribution is that we have proposed a new random volatility model, called high order moments-based stochastic volatility model (or MSV model), in which the volatility is a function of time with its level being modulated by a random variable. By using a Taylor series expansion of the option price, we have shown that the model yields an easy formula for approximate option prices and hence can be calibrated extremely fast. The proposed model can even be implemented on a spreadsheet. Secondly, we have demonstrated through comprehensive numerical experiments that MSV model is very competitive in terms of accuracy with Heston model and SVJ model, while being computationally significantly cheaper to calibrate. Lastly, we have backed up our claims for the usefulness of our model with simulation experiments for comparison of exotic option prices as well as comparison of numerically evaluated option price sensitivity parameters. MSV model thus provides a competitive alternative to the existing option pricing models; it is particularly suitable for high frequency financial trading due to its speed of calibration.

Note that it is conceptually straightforward to use a semi-parametric model, by using a piecewise linear γ_t in (10) which is non-zero for $t > t_0$, to match the observed option prices even more accurately. The use of such semi-parametric models with piecewise constant volatilty parameters is quite common in financial modelling, *e.g.* it is used in calibrating a LIBOR forward model to observed caplet prices (see [5] and references therein, for example). Exploring calibration of such model as well as experiments with derivatives in other markets such as currencies is the topic of current research.

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References

[1] Alexander, C., 2004. Normal mixture diffusion with uncertain volatility: Modelling short-and long-term smile effects. Journal of Banking & Finance 28 (12), 2957–2980.

[2] Bakshi, G., Cao, C., Chen, Z., 1997. Empirical performance of alternative option pricing models. The Journal of Finance 52 (5), 2003–2049.

[3] Bates, D. S., 1996. Jumps and stochastic volatility: Exchange rate processes implicit in deutsche mark options. Review of Financial Studies 9 (1), 69–107.

[4] Black, F., Scholes, M., 1973. The pricing of options and corporate liabilities. The Journal of Political Economy, 637–654.

[5] Brigo, D., Mercurio, F., 2006. Interest rate models-theory and practice: with smile, inflation and credit. Springerverlag Berlin Heidelberg.

[6] Carr, P., Geman, H., Madan, D. B., Yor, M., 2003. Stochastic volatility for lévy processes. Mathematical Finance 13 (3), 345–382.

[7] Derman, E., 1994. Kani, 1., riding on a smile. Risk 7 (2), 32.

[8] Duffie, D., Pan, J., Singleton, K., 2000. Transform analysis and asset pricing for affine jump-diffusions. Econometrica 68 (6), 1343–1376.

[9] Dupire, B., 1994. Pricing with a smile. Risk 7 (1), 18–20.

[10] Gatheral, J., 2006. The volatility surface: a practitioner's guide. Wiley.

[11] Heston, S., 1993. A closed-form solution for options with stochastic volatility with applications to bond and currency options. Review of Financial Studies 6 (2), 327–343.

[12] Hull, J., White, A., 1987. The pricing of options on assets with stochastic volatilities. Journal of Finance 42 (2), 281–300.

[13] Javaheri, A., 2011. Inside volatility arbitrage: the secrets of skewness. Wiley.

[14] Kou, S. G., 2002. A jump-diffusion model for option pricing. Management Science 48 (8), 1086–1101.

[15] Merton, R., 1976. Option pricing when underlying stock returns are discontinuous. Journal of Financial Economics 3 (1), 125–144.

[16] Mikhailov, S., Nögel, U., 2004. Hestons stochastic volatility model implementation, calibration and some extensions. The Best of Wilmott 1: Incorporating the Quantitative Finance Review, Volume 1 Wilmott magazine, 401–411.

[17] Nelson, C. R., Siegel, A. F., 1987. Parsimonious modeling of yield curves. Journal of Business, 473–489.

6. Appendix

Table 1: 01 November 2012

	MRAE-I	RMSE-I	Time (sec.)	MRAE-O	RMSE-O
			FTSE		
Heston	**4.79**	10.43	148	**4.97**	**14.85**
SVJ	3.33	**11.29**	407.70	3.53	3.38
MSV	3.30	9.62	1.24	2.08	5.77
			S&P500		
Heston	**8.20**	**5.99**	605.72	**7.13**	**6.9**
SVJ	1.23	1.38	1419	1.28	0.54
MSV	4.75	3.30	0.44	4.73	3.27
			DAX		
Heston	2.45	9.74	77.35	3.72	9.51
SVJ	**4.75**	**35.38**	771.17	4.38	2.80
MSV	4.37	20.51	0.24	**5.60**	**23.42**

Table 2: 26 November 2012

	MRAE-I	RMSE-I	Time (sec)	MRAE-O	RMSE-O
			FTSE 100		
Heston	**6.36**	12.04	109	**6.21**	**10.45**
SVJ	3.09	**12.97**	378.7	2.98	1.73
MSV	3.33	7.82	0.76	4.32	8.63
			S&P 500		
Heston	**4.68**	**4.42**	193	**4.83**	**5.46**
SVJ	3.31	3.13	1115.62	3.2	0.65
MSV	3.81	2.32	0.36	3.84	3.15
			DAX		
Heston	6.32	55.19	95.87	5.78	**46.05**
SVJ	**7.25**	**61.94**	910.39	**6.78**	45.85
MSV	4.79	42.80	0.28	4.42	40.38

Table 3: 25 July 2013

	MRAE-I	RMSE-I	Time	MRAE-O	RMSE-O
			FTSE 100		
Heston	**7.34**	**22.26**	1332.88	**6.20**	**14.31**
SVJ	4.50	10.87	671.12	5.15	5.23
MSV	4.15	9.80	0.81	5.27	11.11
			S&P 500		
Heston	**4.78**	**3.26**	252.93	**6.28**	**3.45**
SVJ	3.59	2.63	1156.91	3.80	1.79
MSV	3.97	3.11	1.01	4.30	2.34
			DAX		
Heston	10.32	88.39	145.26	12.56	102.16
SVJ	8.20	89.12	600.71	11.89	34.93
MSV	**13.81**	**110.51**	0.71	**17.18**	**114.35**

Table 4: 26 July 2013

	MRAE-I	RMSE-I	Time	MRAE-O	RMSE-O
			FTSE 100		
Heston	**8.32**	**23.82**	1146.46	**7.07**	**14.75**
SVJ	4.50	10.87	671.12	5.15	5.23
MSV	4.45	9.37	0.62	5.83	10.80
			S&P 500		
Heston	**5.00**	**3.34**	257.44	**6.30**	**3.43**
SVJ	3.01	2.65	1337.72	3.06	1.79
MSV	3.86	3.06	1.09	1.00	2.43
			DAX		
Heston	**4.33**	**34.34**	286.69	4.19	**22.75**
SVJ	2.70	26.32	418.64	2.99	4.59
MSV	4.24	17.55	0.65	**8.01**	18.17

Table 5: 29 July 2013

	MRAE-I	RMSE-I	Time	MRAE-O	RMSE-O
			FTSE 100		
Heston	**8.66**	**23.58**	667.25	**7.70**	**14.53**
SVJ	4.78	12.38	590.72	4.40	4.65
MSV	4.52	9.35	0.85	5.74	10.73
			S&P 500		
Heston	**5.78**	3.50	41.32	**6.01**	3.20
SVJ	2.57	**19.43**	291.20	3.81	**5.87**
MSV	4.71	2.99	1.00	5.29	2.30
			DAX		
Heston	4.26	**28.49**	134.29	4.47	**19.44**
SVJ	2.70	26.32	418.64	2.99	4.59
MSV	**4.34**	18.35	0.64	**8.03**	18.75

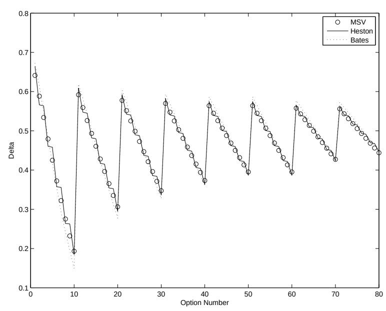
Table 6: Down-and-out Call Barrier option prices (models calibrated from 1st November 2012 DAX and FTSE data)

DAX S	7281.18			Barrier	7100.00			Strike	7250.00	
		SVJ			Heston			MSV		
T	r	Price	CI		Price	CI		Price	CI	
0.10	0.0051	92.93	90.18	95.67	83.13	80.03	86.24	94.66	91.59	97.72
0.21	0.0049	132.92	122.56	143.27	117.24	112.83	121.65	135.96	130.93	140.99
0.32	0.0052	137.97	132.14	143.80	131.89	126.48	137.30	143.25	137.10	149.41
0.43	0.0056	144.88	138.38	151.39	141.61	135.36	147.86	153.09	145.96	160.23
DAX S	7281.18			Barrier	7200.00			Strike	7300.00	
		SVJ			Heston			MSV		
T	r	Price	CI		Price	CI		Price	CI	
0.10	0.0051	55.16	52.92	57.41	53.24	50.75	55.73	60.75	58.00	63.50
0.21	0.0049	63.16	60.17	66.16	60.67	57.36	63.98	68.46	64.58	72.35
0.32	0.0052	69.14	65.52	72.76	67.40	63.43	71.37	73.65	68.99	78.32
0.43	0.0056	77.26	60.28	94.24	73.40	68.67	78.14	78.96	73.48	84.45
FTSE S	5812.06			Barrier	5750.00			Strike	5820.00	
		SVJ			Heston			MSV		
T	r	Price	CI		Price	CI		Price	CI	
0.06	0.0051	61.63	55.40	67.86	40.07	38.56	41.58	54.88	51.50	58.25
0.20	0.0049	61.26	53.02	69.49	53.11	50.57	55.66	58.66	53.77	63.54
0.31	0.0052	65.69	55.56	75.82	55.68	52.72	58.65	64.78	58.73	70.83
0.42	0.0056	59.17	49.33	69.01	57.67	54.35	60.98	66.69	60.03	73.35

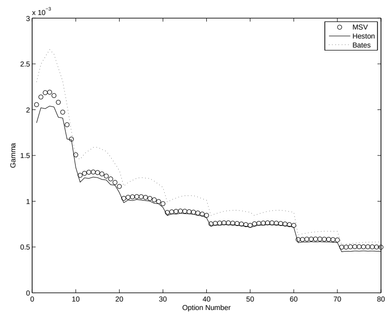
Table 7: Arithmetic average Asian option with floating strike (1st November 2012, all indices)

				SVJ			Heston			MSV		
Index	S0	T	r	Price	CI		Price	CI		Price	CI	
FTSE	5812.06	0.42	0.0056	89.31	86.02	92.59	84.94	82.49	87.38	88.41	85.82	91.00
S&P	1412.16	0.31	0.003	21.56	21.04	22.08	14.88	13.84	15.91	20.44	19.84	21.04
DAX	7281.18	0.10	0.001	60.88	59.18	62.58	56.04	53.93	58.15	61.73	59.91	63.55

Figure 1: FTSE100: Delta and Gamma on 25.07.2013

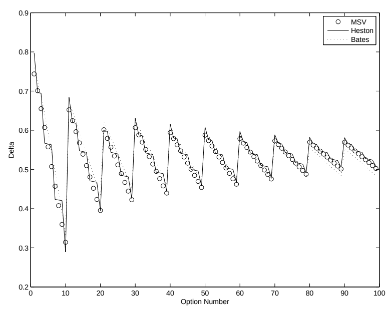


Delta for 25.07.2013



Gamma for 25.07.2013

Figure 2: S&P 500:Delta and Gamma on 25.07.2013



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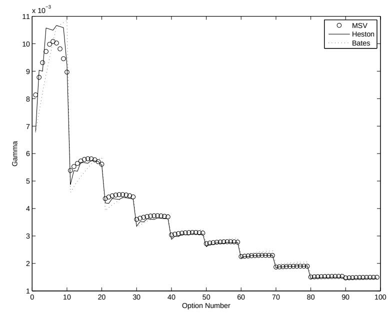
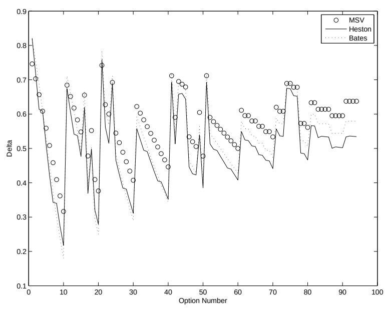


Gamma for 25.07.2013

Figure 3: DAX:Delta and Gamma on 25.07.2013



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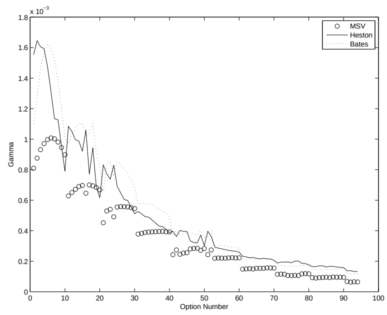

Gamma for 25.07.2013