Use of Global Positioning System velocity outputs for determining airspeed measurement error

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Abstract

Several methods have been derived since the advent of GPS (Global Positioning System) receivers in aircraft cockpits by which these receivers may be used to calibrate these aircraft's other instrumentation; in particular the pitot-static system. This paper presents the four most suitable methods, two of which have been developed by the author. These methods are shown with a common symbology, and their strengths, weaknesses, analysis and operational use are compared.

Introduction

It has been accepted since the earliest days of formalised aircraft design, testing and operations that calibration of aircraft instruments, and in particular pitot-static (airspeed and height) instruments is important for both certification testing, and for navigation purposes. The differences between actuality and indication are referred to as PEC (Pressure Error Corrections). It has never proved possible to accurately predict the PEC for an airspeed indicator system, and even if such a method were developed, it would still be essential to check the results experimentally. PEC may be broken into three parts: TPEC (Total Pressure Error), SPEC (Static Pressure Error), and PPEC (Pitot Pressure Error). The most important is TPEC, since except at high angles of attack it can usually be assumed that PPEC are trivial, and thus TPEC≈SPEC, whilst TPEC itself determines airspeed indication corrections, which are the most important for the observance of structural limitations. Determination of TPEC can be performed either by finding a means of accurately measuring wind vector and groundspeed, or by comparing to an airspeed indicating system of sufficiently known accuracy.

Although to some extent radio other radio navigation aids could be used, until the advent of GPS (the Global Positioning System), most methods of PEC determination required certain expensive complexities which could include: modification to the test aircraft, an external calibrated pacer aircraft, external ground observers and possibly flight close to the ground. All of these added cost and complexity to a test and certification programme. With the availability of GPS however, it is possible to a large extent to conduct all testing at safe, turbulence free, altitudes, with all measurement conducted internally and without modification to the aircraft. The technology therefore presents substantial cost and time advantages to the flight test organisation.

This paper sets out to show the available methods by which receiver groundspeed output can be used as the base for determination of TPEC (and thus potentially estimation of SPEC and PPEC, depending upon system design). Even simple GPS receivers now can be assumed to offer an accuracy of better than ±0.1 knots[1] accompanied by similar precision, which should provide sufficient accuracy for total system calibration, so long as: (a) the calibration method itself is adequate, (b) sufficient precision is available both for the GPS velocity output and the aircraft's own Airspeed Indicator (ASI).

The primary interest in the work that led to this paper was in the calibration of airspeed indication systems in manned aeroplanes. It is however anticipated that these methods may also potentially be adapted for use with autonomous or remotely controlled Unmanned Aerial Vehicles (UAV); specific methods of doing so however not described.

So far as reasonably possible, a common terminology has been used throughout this paper – this means that terminology will in many cases vary from that of source documents, which have used several alternate nomenclatures.

Nomenclature

σ	Air density, relative to ISA sea-level value.
δ_n	Difference between magnetic heading and magnetic track during test segment (leg) *n*
Ψ	Wind direction
ASI	Air Speed Indicator
BMAA	British Microlight Aircraft Association
CAA	(United Kingdom) Civil Aviation Authority
CAS	Calibrated Airspeed (may be considered the same as EAS below 0.5Mach and 10,0000ft). Also known as RAS – Rectified Airspeed.

EAS	Equivalent Airspeed
IAS	Indicated Air Speed
NTPS	National Test Pilots School (based at Mojave, California, USA)
OAT	Outside Air Temperature
PEC	Pressure Error Corrections
PPEC	Pitot Pressure Error Corrections
RAS	Rectified Air Speed, alternative term to CAS.
RoD	Rate of descent
SETP	Society of Experimental Test Pilots
sHp	Standard Pressure Altitude (altimeter reading with 1013.25 hPa set on subscale)
SPEC	Static Pressure Error Corrections
TAS	True Air Speed
TP	Test Pilot
TPEC	Total (pitot-static system) Pressure Error Corrections
V_A	Manoeuvre speed
V_{AT}	Target approach speed
V_D	Maximum design speed
V_H	Maximum achievable airspeed in level flight.

VLA Very Light Aeroplane: an artificial aircraft category defined by V_{S0}<= 45 kn CAS and Maximum All Up Mass<=750 kg.

V_n Ground speed during test segment (leg) *n*

V_{NE} Never Exceed Speed

V_S Stalling speed

V_{S0} Stalling speed in the landing configuration

V_T True Air Speed

V_W Wind speed

Throughout this paper knots (nautical miles per hour) have been used when referring to speed measurement, and feet have been used when referring to height or altitude. Whilst not standard scientific units, these are the units most commonly used when recording aircraft operations. To convert knots to metres per second multiply by 0.5144. To convert feet to metres, multiply by 0.3048.

Several working variables without physical significance are also used within this paper; these are not included in this nomenclature.

The Racetrack method

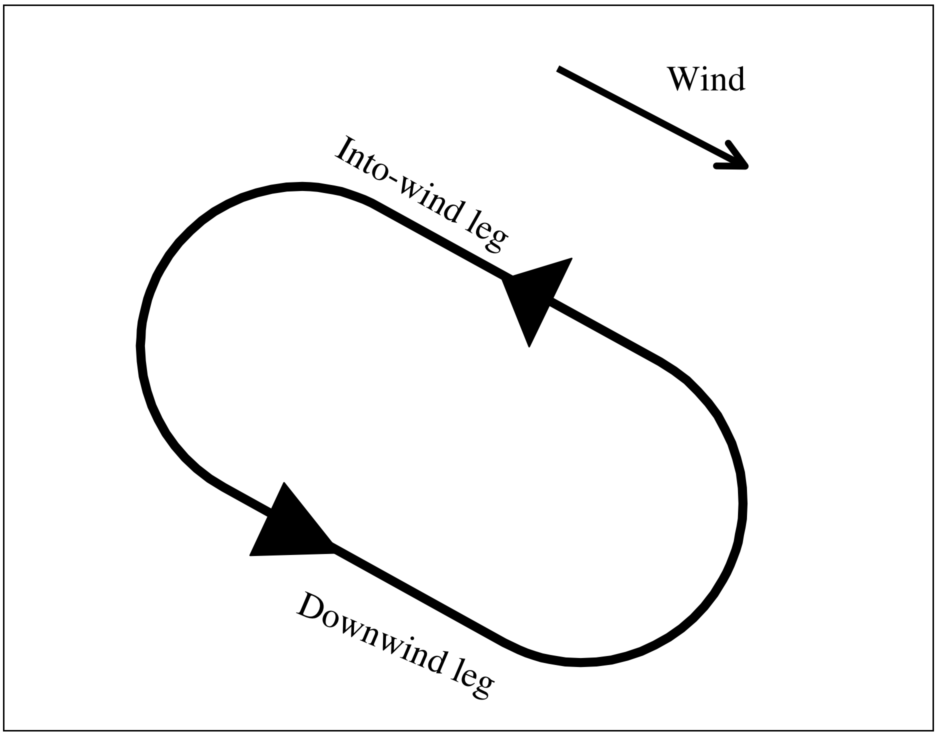
The racetrack method was developed for use by the BMAA initially around 1999 although then refined over several years[2, 3]; it has been used to good effect on a number of projects since for both certified and uncertified aeroplanes, particularly for tasks related to approval by the CAA. Required are turbulence-free conditions (an essential for any ASI calibration task), accurate knowledge of outside air temperature, a GPS unit, and approximate wind heading data.

The aircraft is pointed as accurately into wind as the forecast will allow. Precise wind heading is then obtained by varying heading slightly whilst maintaining constant speed and altitude. The aircraft is known to be exactly into wind when the lowest indication is obtained of GPS groundspeed. This heading is noted. [Note: NTPS reported in 1997[4] using a similar technique, except that they aimed to identify wind heading by matching ground track to aircraft heading: this method was found insufficiently accurate and its use was abandoned.]

The aircraft is flown at a range of speeds from just above the stall, to at-least V_H (often to V_{NE}) with GPS groundspeed being noted against indicated airspeed at each increment.

The aircraft, maintaining a constant nominal altitude, is then turned (using GPS heading so as to not be affected by any magnetic anomalies) onto a reciprocal heading, and this exercise repeated. If necessary (limitations of available airspace tend to control the flightpath) multiple turns are flown in a “racetrack” method as indicated below.

Figure 1, Illustration of racetrack method flightpath



For each IAS value, the corresponding TAS value is then determined as the mean of into-wind and downwind groundspeeds.

The 2-heading method

The 2-heading method was developed by the author in 2005 although has not yet had extensive use. The method is based upon the assumption that the aeroplane will be fitted (as most are) with a calibrated magnetic compass, again at constant altitude in still air. Two substantially different headings are flown at each speed, such that on each of the two heading, the following data is recorded:

Track (from GPS) relative to magnetic north

Heading (from calibrated compass)

GPS groundspeed

For each pair of groundspeeds (at the same IAS) then, TAS may be determined by:

$$V_T = \frac{V_1^2 - V_2^2}{2(V_1 \cos\delta_1 - V_2 \cos\delta_2)} \tag{1}$$

Where V_1,V_2 are the two groundspeeds, and δ_1, δ_2 are the differences between GPS (magnetic North referenced) ground track and magnetic heading for the two legs (i.e. δ_n = $track_n$ – $heading_n$).

If required, the wind velocity may then also be determined from any data point as:

$$V_W = \sqrt{V_n^2 + V_T{}^2 - 2V_T.V_n.\cos(\delta_n)} \tag{2}$$

Where *n* is the number of the leg being flown.

A derivation of this is shown in Appendix A.

The 3-heading triangular method

The 3-heading triangular method was published in reference[5] and in turn appears to be based upon reference[6]. This uses a similar means for groundspeed determination to that described for the racetrack method above, but instead uses three legs, separated by 120° magnetic heading A particular consideration is that continuously flying a triangular course with 120° between legs is an internationally accepted procedure by which an aircraft which has suffered a failure of radio and navigation equipment, indicates its need for assistance from a "shepherd" aircraft. So, to fly a course which might unnecessarily indicate distress to a radar controller, could potentially be embarrassing. However, from a purely engineering viewpoint, the method is perfectly valid, it simply imposes a greater communication and airmanship requirement upon the Test Pilot. The formulae for determining wind vector and airspeed are given below without proof ; a full derivation of this method is shown in Appendix B.

Groundspeed must be measured, using GPS, whilst flying the aircraft on three headings (not tracks – so heading <u>must</u> be measured using an error corrected compass, not GPS) that differ by 120 degrees (eg 50, 170 and 290 degrees). These speeds will be termed V_1, V_2 and V_3.

The mean sum of squared speeds, V'[2] is calculated as

$$V'^2 = \frac{V_1^2 + V_2^2 + V_3^2}{3}$$

(3)

We now non-dimensionalise the three groundspeeds and term them each a, so that

$a_n = \frac{V_n^2}{V'^2} - 1$, and also define a working variable $\mu = \frac{a_1^2 + a_2^2 + a_3^2}{6}$

(4, 5)

True Airspeed is now given by $V = V'^2 \sqrt{½ + \sqrt{\frac{1}{4 - \mu}}}$

(6)

And windspeed is given by $V_W = V'^2 \sqrt{\frac{\mu}{½ + \sqrt{\frac{1}{4 - \mu}}}}$

(7)

The 3-track method

The 3-track method which was first published at reference[7], and was probably the first published method for PEC determination using GPS. The aircraft is initially established onto a fixed track (not heading as with most other methods), which may be adhered to by following GPS display directions. The method is not reproduced here, since it was rapidly superseded by methods using aircraft heading (rather than GPS track) as the primary flying reference – this is believed to be because aircraft heading instruments are generally more conveniently designed for a pilot to follow than GPS ground-track displays of any common unit.

The box-pattern method

A variant upon the triangle method above has been published separately by Lowry[8] who referred to as the "Box Pattern" method, and G V Lewis[4] (who offered no title for the technique).

This technique requires the aircraft to fly three legs at 90° spaced magnetic headings, and then by trigonometry (reproduced below) without proof, which may be found in reference 8: V_T is determined at each speed.

Three groundspeeds (V_1, V_2, V_3) are recorded for each IAS value, each flown on an orthogonal cardinal heading (e.g. North, East then South), from these

Wind direction, relative to initial heading:

$$\Psi = \tan^{-1}\left(\frac{2V_2^2 - V_1^2 - V_3^2}{V_3^2 - V_1^2}\right) \tag{8}$$

Note: Lowry[8] recommends that the first heading flown is due North, and thus Ψ becomes actual wind direction.

Wind velocity

$$V_W = \frac{1}{2}\left[V_3^2 + V_1^2 \pm \sqrt[2]{\left(V_3^2 + V_1^2\right)^2 - \left(\frac{2V_2^2 - V_1^2 - V_3^2}{\sin\Psi}\right)}\right]^{\frac{1}{2}} \tag{9}$$

(selecting the "±" so that the value within the square brackets is positive)

True airspeed:

$$V_T = \sqrt[2]{\frac{V_3^2 + V_1^2}{2} - V_W} \tag{10}$$

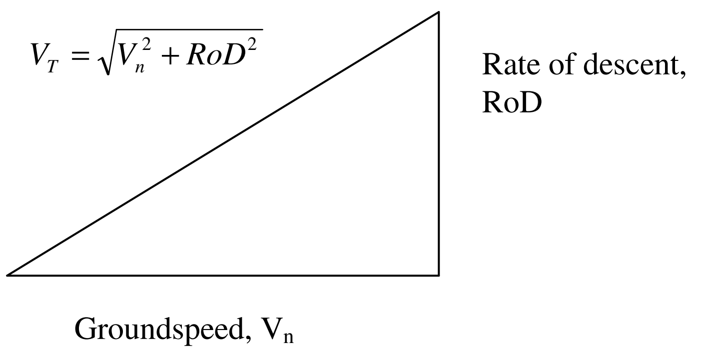
This is again a valid method (with the advantage of avoiding the risk of embarrassment with air traffic control which may occur with the 3-leg method), the box-pattern method uses three rather than two speeds (giving greater opportunity for error in an individual datum to be reduced by calculation) and also does not present the risk of inadvertently appearing to declare an emergency posed by the triangular method, although requiring similar time to fly.

Testing at speeds above V_H

Most previously published explanations of the use of GPS for TPEC determination have disregarded the fact that almost all aeroplanes have a significant operating range above V_H (indeed, most fixed wing airworthiness standards include requirements that V_{NE} must be a significant margin above V_H: typically between 1.13 and 1.26 depending upon class of aeroplane): being the maximum achievable speed in level flight. Whilst experience has indeed shown that in most cases, the pattern of PEC displayed immediately below PEC may be extrapolated up to V_{NE} or above with a good degree of confidence – nonetheless such extrapolation of test data, particularly where it will be used to determine operating limitations is a poor practice, and one unlikely to be accepted by any competent authority. Similarly, a few aeroplanes may also be unable to sustain level flight due to the power requirements as the stall speed is approached (although this is rare).

When the aeroplane is descending, it is straightforward to correct for this, although formal inclusion of this descent path in data reduction tables is then essential. Normal practice is to record the aircraft's time to descend between two altitudes close to the nominal test altitude (so, for example, if the level flight test altitude has been 5,000ft, then it may be appropriate to climb the aeroplane above this if it is known that V_H is exceeded; then for example time can be measured to descend between 5,100ft and 4,900ft in a constant speed descent, with the GPS groundspeed recorded at 5,000 ft during the descent). Descent rate is measured using an altimeter; vertical speed indicators (VSI) rarely possess the precision, and sometimes nor the accuracy, for sufficiently accurate RoD (Rate of Descent) determination. Since both rate of descent and GPS groundspeed can be considered geometrically accurate, this can then be used to determine the aeroplane's TAS, V_T thus:

Figure 2, triangle of velocities for descending aircraft



(Remembering of-course to ensure that V_n and RoD are expressed in identical units).

So, the groundspeed V_n (that is, the value which was determined for TAS using formulae derived for testing in level flight) may be modified to an actual value of TAS, V_T useable for subsequent system calibration.

Theoretically, it may be possible to use GPS geometric height (or rate of change thereof) for these calculations; however the author is unaware neither of this being used in practice to date, nor of any commercially available GPS receiver which will output rate of climb or descent without modification. However, for small changes in height at constant airspeed, the relationship between differences of barometric pressure altitude, and changes in geopotential altitude is sufficiently close to 1:1 that RoC results may reasonably be regarded as identical.

Further reduction from knowledge of True Air Speed to operating data

Considering as an example the racetrack method the data is recorded and reduced, using a table such as that given below (or more commonly, a similarly configured spreadsheet):-

Table 1, ASI calibration data reduction table [Based upon reference [3]].

IAS (any unit)	V_1 (Into wind) (knots)	time per 200 ft (s)	Adjusted V_1 (knots)	V_2 (downwind) (knots)	time per 200 ft (s)	Adjusted V_2 (knots)	V_T (knots)	EAS (knots)
(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
	from GPS	from stopwatch	$\sqrt{(b)^2+\left(\frac{118}{(c)}\right)^2}$	from GPS	from stopwatch	$\sqrt{(e)^2+\left(\frac{118}{(f)}\right)^2}$	$\frac{(d)+(g)}{2}$	$(h)\times\sqrt{\sigma}$
			or (b) if not descending			or (e) if not descending		
30								
40								
etc.								

This data is then plotted to produce an ASI calibration (TPEC) chart of IAS versus EAS (which may be considered identical to CAS for lower speed aircraft), such as that in Figure 3 below which was produced as part of the approval process for a prototype amateur-built aeroplane. In this case, the data presentation was performed with a commonly available office spreadsheet (Microsoft Excel™) and the curve fitted through the points is a quadratic, showing a correlation coefficient (R^2) better than 0.99.

Figure 3, Sample PEC chart for amateur built aeroplane

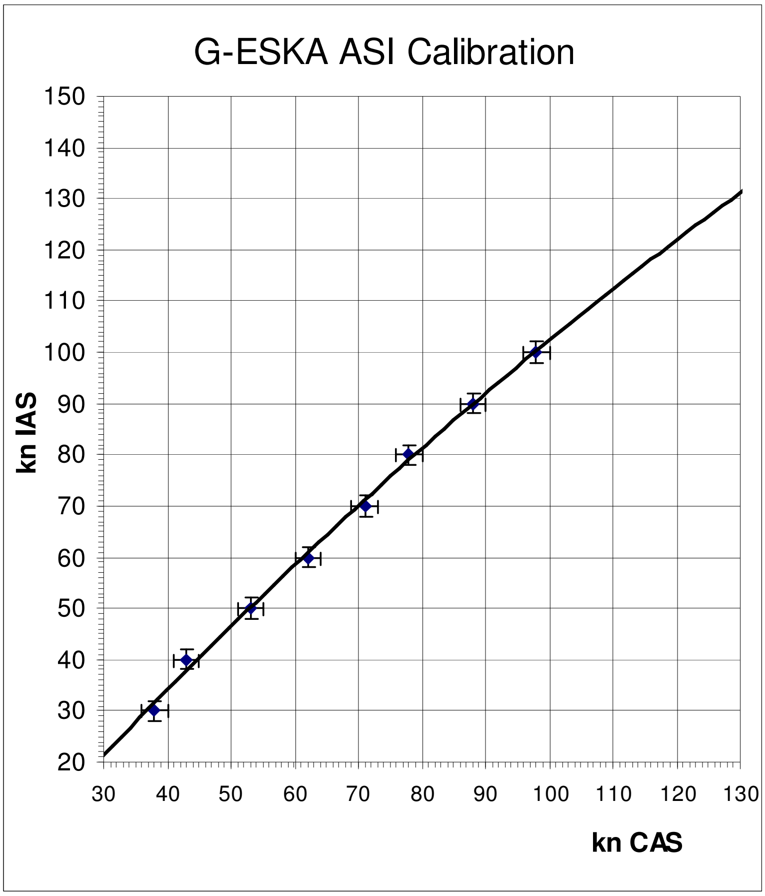


Figure 3 above it may be noted uses standard error bars of ±2kn. This issue of error analysis can be problematic, since whilst it is possible to create a classical error analysis of the experimental data, invariably (or at-least for the light aircraft testing where GPS calibration methods have mostly been used to date) it will be found when comparing this statistical analysis to Test Pilots' or Flight Test Engineers' estimates of the consistency within which they were able to maintain conditions, the test crews estimates show a potential for error substantially greater than that indicated by error analysis. Therefore common practice, at-least in UK use of GPS methods, has been for the pilot's estimate of the accuracy with which they were able to fly a steady and planned IAS, and the steadiness of GPS groundspeed reading in the air, to determine the magnitude of assumed experimental error. Typically ±1kn or ±2kn is a typical value. A degree of judgement must then be applied to curve fitting: most common methods are to use a proprietary graph-plotting program such as within Microsoft Excel™, and depending upon operators judgment to either use the lowest order curve which fits within all the error-bounds, or to use the function that offers a correlation coefficient (R^2) closest to 1. Fortunately, with a well flown test in calm conditions (such as is illustrated in Figure 3 above), frequently these coincide with a linear or quadratic function.

A caution about testing at high angles of attack

All of the GPS methods described here have been shown to work well, so long as their use is understood, and test crews take care with precision in their flying, and in ensuring that all testing is flown in turbulence-free conditions. However, it is commonly observed that PPEC and TPEC curves will commonly show discontinuities as the stall is approached. This is believed to be partly because the pitot-head becomes less efficient (developing greater losses) at higher angles of attack, and partly because of the inaccuracy of the ASI itself at low pressures. However, PEC testing to these low speeds can be hazardous, since this involves attempting stable flight very close to the stall condition.

To some extent this may be compensated for by two strategies. Firstly an aeroplane may be flown at very light weights, allowing it to be flown stably to speeds below the normal stalling speed; this allows calibration at low pressures, and determination of the form of discontinuity, but only if low pressure rather than angle of attack is the principle source of error. Secondly, it is possible to add a second airspeed measuring system, with a pitot set significantly more nose-down than the usual system (or possibly a more complex device such as a Kiel probe), and to calibrate this at normal weights and lower speeds, eliminating any AoA discontinuities. This second method is particularly useful when trying to accurately determine V_{S0} values for certification purposes, although is unlikely to be useful as an operational system, since it would be unacceptable to present a pilot with two separate ASIs with different calibrations and indicated stall speeds. Additionally, the complexity and thus cost of more than a relatively simple airspeed measurement system is unlikely to be justifiable on the majority of aircraft.

No perfect solution has yet been found to the determination of the form of the low-speed discontinuity commonly seen in PPEC or TPEC curves. Generally this is not a problem, so long as it is ensured that flying limitations such as V_A or V_{AT} are, in cases of uncertainty, are set at the lower bounds of their predicted range of values (thus providing structural conservatism). Difficulty is most commonly encountered when compliance with a certification standard is dependent upon meeting a particular stall speed requirement (e.g. 35 knots CAS for approval as a microlight aeroplane, or 45 knots CAS for approval in the VLA category), and that the aeroplane is sufficiently close to this limit that precise knowledge of the value becomes critical. It is likely that where this occurs, certification engineers from company and authority will need to agree between them an acceptable solution for the particular project.

A comparison of the methods

With the exception of the still new 2-heading method, all of the methods described in this paper have been used by various organisations in the UK, Australia, USA and almost certainly elsewhere – in all cases the methods have been found satisfactory for their purposes. It would be useful eventually to perform parallel calibrations upon the same aeroplane, in order to identify the most efficient method in terms of flight time. However, pending such a trial, it is at-least possible to compare the methods for their specific characteristics, so that potential users of GPS for TPEC determination may select the most appropriate method for their own purposes. Such a comparison is presented in Table 2 below.

Table 2, Comparison of known methods for GPS airspeed based determination of TPEC

	Characteristics				Additional issues
	Number of legs	Main Error sources			
Method		Precision in flying	GPS	Compass calibration	
Racetrack	2	X	X	-	Further flying requirement to establish wind heading
2-heading	2	-	X	X	
3-heading	3	X	X	X	Flightpath may inadvertently indicate lost aircraft

3-track	3	X	X	-	Requirement to follow GPS track rather than aircraft heading
Box pattern	3	X	X	X	

Expressing a personal view, the author maintains a slight preference for the racetrack method, since it appears to require slightly less flying than most other methods, whilst also avoiding any errors that may occur due to magnetic compass calibration. However, clearly it offers no monopoly upon quality or efficiency as has been shown by numerous organisations using other methods to good effect.

Acknowledgments: Whilst this work is primarily that of the author, in addition to the authors of referenced documents, he would like to acknowledge the work and assistance of: Mr. Paul Dewhurst, Flylight Airsports and BMAA; Mr. John Hamer, PFA and BMAA; Dr. Simon Newman, University of Southampton; Capt. Tom Porteous, Thruster Air Services and BMAA; Dr. Rosi Sexton, SBG-UK, formerly Manchester University

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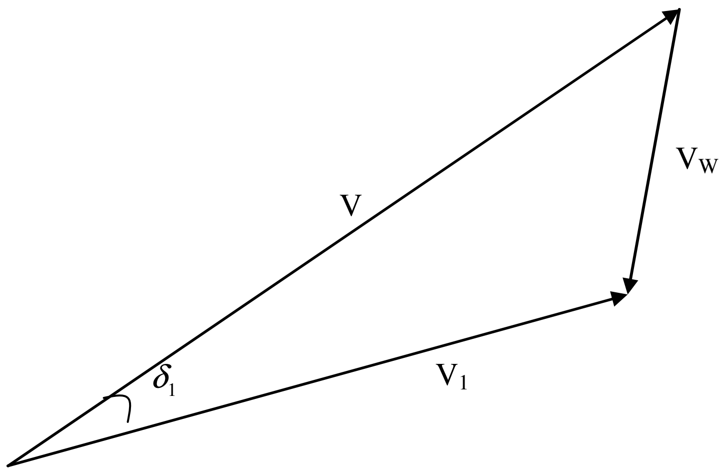
Appendix A, derivation of two-heading method

Whilst the magnetic compass will give aircraft heading, the GPS will give aircraft track, so considering a single leg as shown in Figure 4 below, the Groundspeed V_1 is a function of the true airspeed V_T, and the wind V_W.

Figure 4, Triangle of velocities



The difference between magnetic heading and GPS track, is available, and can be termed δ_1. By applying the cosine rule, we know that $a^2 = b^2 + c^2 - 2.b.c.\cos A$. In the context of this problem, that equates to:

$$V_W^2 = V_1^2 + V_T^2 - 2V_T.V_1.\cos(\delta_1) \tag{A1}$$

And by symmetry, for a second leg,

$$V_W^2 = V_2^2 + V_T{}^2 - 2V_T.V_2.\cos(\delta_2)$$

(A2)

It will be seen that there is in-fact no requirement for a third leg, since we have two simultaneous equations with two unknowns (and we are not interested in the value of windspeed in any case).

So, since wind must be considered constant, we can equate these two formulae, giving:

$$V_1{}^2 + V_T{}^2 - 2V_T.V_1.\cos\delta_1 = V_2^2 + V_T{}^2 - 2V_T.V_2.\cos\delta_2$$

(A3)

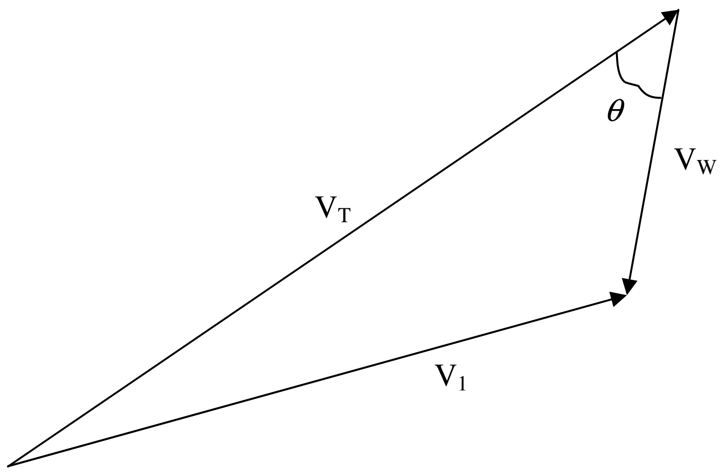
Which re-arranges to:

$$V_T = \frac{V_1^2 - V_2^2}{2(V_1\cos\delta_1 - V_2\cos\delta_2)}$$

(A4)

Appendix B, derivation of triangle method

Taking the example of three legs, flown at 120° heading to each other, these can be considered again in terms…



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aking true airspeed as V_T, the wind strength as V_W for all legs. For the three legs the groundspeeds are V_1, V_2. V_3; for the first leg the angle between the heading and wind is given by θ, so for the second leg it is θ+120°, and for the third it is θ+240°.

The cosine rule states that:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

So, for the three legs, it can be written that:

$$V_1^2 = V_W^2 + V_T^2 - 2V_W V_T \cos\theta$$
$$V_2^2 = V_W^2 + V_T^2 - 2V_W V_T \cos(\theta + 120)$$
$$V_3^2 = V_W^2 + V_T^2 - 2V_W V_T \cos(\theta - 120)$$

(B1a, B1b, B1c)

Adding these three relationships together, we get:

$$V_1^2 + V_{21}^2 + V_3^2 = 3V_W^2 + 3V_t^2 + 2V_T V_W \left[\cos\theta + \cos(\theta + 120) + \cos(\theta - 120)\right]$$
$$\therefore V_1^2 + V_{21}^2 + V_3^2 = 3V_W^2 + 3V_t^2 + 2V_T V_W \left[\cos\theta + \cos(\theta + 120) + \cos(\theta - 120)\right]$$

(B2)

Looking at the terms in the square brackets on the right hand side of this last:-

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$
$$\therefore \cos(\theta + 120) = \cos\theta \cos 120 - \sin\theta \sin 120$$
$$and$$
$$\cos(\theta - 120) = \cos\theta \cos 120 + \sin\theta \sin 120$$
$$\therefore \cos(\theta + 120) + \cos(\theta - 120) = 2\cos\theta \cos 120 = -\cos\theta$$
$$\therefore \cos\theta + \cos(\theta + 120) + \cos(\theta - 120) = 0$$

(B3)

So,

$$V_1^2 + V_2^2 + V_3^2 = 3V_W^2 + 3V_T^2$$

(B4)

Hence, we know the relationship between true airspeed and windspeed, in terms of the three measured groundspeeds, so long as the three aircraft headings were 120° apart, specifically:

$$V_T^2 = \frac{V_1^2 + V_2^2 + V_3^2}{3} - V_W^2$$

(B5)

Or, if we define that:

$$\frac{V_1^{\,2} + V_2^{\,2} + V_3^{\,2}}{3} = V_{RMS}^{\,2}$$
$$\Rightarrow$$
$$V_T^{\,2} = V_{RMS}^{\,2} - V_W^{\,2}$$
$$or\ ,$$
$$V_W^{\,2} = V_{RMS}^{\,2} - V_T^{\,2}$$
$$or\ ,$$
$$\frac{V_W^{\,2} + V_T^{\,2}}{V_{RMS}^{\,2}} = 1$$

(B6)

Now, from previous:

$$V_1^2 = V_W^2 + V_T^2 - 2V_W V_T \cos\theta$$
$$\therefore \frac{V_1^2}{V_{RMS}^2} = \frac{V_W^2 + V_T^2 - 2V_W V_T \cos\theta}{V_{RMS}^2}$$
$$\therefore \frac{V_1^2}{V_{RMS}^2} = \frac{V_W^2 + V_T^2}{V_{RMS}^2} - \frac{2V_W V_T \cos\theta}{V_{RMS}^2}$$
$$\therefore \frac{V_1^2}{V_{RMS}^2} = 1 - \frac{2V_W V_T \cos\theta}{V_{RMS}^2}$$
$$or,$$
$$\frac{V_1^2}{V_{RMS}^2} - 1 = -\frac{2V_W V_T \cos\theta}{V_{RMS}^2}$$

(B7a)

And by symmetry:

$$\frac{V_2^2}{V_{RMS}^2} - 1 = \frac{-2V_W V_T \cos(\theta + 120°)}{V_{RMS}^2}$$

$$and,$$

$$\frac{V_3^2}{V_{RMS}^2} - 1 = \frac{-2V_W V_T \cos(\theta - 120°)}{V_{RMS}^2}$$

(B7b, B7c)

This can be simplified slightly by writing:

$$\alpha_1 = \frac{V_1^2}{V_{RMS}^2} - 1, \alpha_2 = \frac{V_2^2}{V_{RMS}^2} - 1, \alpha_3 = \frac{V_3^2}{V_{RMS}^2} - 1$$

$$So,$$

$$\alpha_1 = \frac{-2V_W V_T \cos\theta}{V_{RMS}^2},$$

$$\alpha_2 = \frac{-2V_W V_T \cos(\theta + 120°)}{V_{RMS}^2}$$

$$and,$$

$$\alpha_3 = \frac{-2V_W V_T \cos(\theta - 120°)}{V_{RMS}^2}.$$

(B8a, B8b, B8c)

So, if these three terms are squared and added together:

$$\alpha_1{}^2 + \alpha_2{}^2 + \alpha_3{}^2 = \frac{4V_T^2 V_W^2}{V_{RMS}^4}\left(\cos^2\theta + \cos^2(\theta + 120°) + \cos^2(\theta - 120°)\right)$$

(B9)

To simplify the terms in the right hand brackets:

$$\cos^2\theta = \cos^2\theta$$

$$\cos^2(\theta+120°) = (\cos\theta\cos 120° - \sin\theta\sin 120°)^2$$
$$= \frac{\cos^2\theta}{4} + \frac{3}{4}\sin^2\theta - \cos\theta\sin\theta\cos 120°\sin 120°$$

$$\cos^2(\theta-120) = (\cos\theta\cos 120° + \sin\theta\sin 120°)^2$$
$$= \frac{\cos^2\theta}{4} + \frac{3}{4}\sin^2\theta + \cos\theta\sin\theta\cos 120°\sin 120°$$

So,

$$\cos^2\theta + \cos^2(\theta+120°) + \cos^2(\theta-120) = \frac{3}{2}(\cos^2\theta + \sin^2\theta) = \frac{3}{2}$$

(remembering that $cos^2+sin^2=1$)

And hence,

$$\alpha_1{}^2 + \alpha_2{}^2 + \alpha_3{}^2 = 4\frac{V_T^2 V_W^2}{V_{RMS}^2}.\frac{3}{2} = 6\frac{V_T^2 V_W^2}{V_{RMS}^2}$$

or,

$$\frac{\alpha_1{}^2 + \alpha_2{}^2 + \alpha_3{}^2}{6} = \frac{V_T^2 V_W^2}{V_{RMS}^2} = \mu$$

(B10)

So, given that

$$V_T^2 = \mu \frac{V_{RMS}^2}{V_W^2}$$

and,

$$V_W^2 = V_{RMS}^2 - V_T^2$$

then,

$$V_T^2 = \frac{\mu . V_{RMS}^2}{V_{RMS}^2 - V_T^2}$$

or,

$$V_T^2 \left(V_{RMS}^2 - V_T^2 \right) = \mu . V_{RMS}^2$$

$$\therefore (-1) V_T^4 + V_{RMS}^2 V_T^2 - \mu . V_{RMS}^2 = 0$$

(B11)

Which is a quadratic of the form, ax²+bx+c=0, so taking the roots of the quadratic, we can see that the solution for True Air Speed, V_T is:

$$V_T^2 = \frac{-V_{RMS}^2 \pm \sqrt[2]{V_{RMS}^4 - 4\mu V_{RMS}^2}}{-2}$$

(B12)

(Remembering that $V_{RMS}^2 = \frac{V_1^2 + V_2^2 + V_3^2}{3}, \mu = \frac{\alpha_1^{\ 2} + \alpha_2^{\ 2} + \alpha_3^{\ 2}}{6}$, and

$$\alpha_1 = \frac{V_1^2}{V_{RMS}^2}, \alpha_2 = \frac{V_2^2}{V_{RMS}^2}, \alpha_3 = \frac{V_3^2}{V_{RMS}^2}.$$

By symmetry, the larger root of this will be V_T, and the smaller will be V_W

Appendix C, Conversion between airspeeds

Whilst the specialist reader will be familiar with the different definitions of airspeed, as used within aircraft testing and operations, some may not.

There is not a single term which one may measure and term “airspeed”, there are a number of different speeds, which are used in different applications. These are:-

(a) **Groundspeed (G/S)**: The speed which an aircraft is travelling relative to a fixed point on the ground.

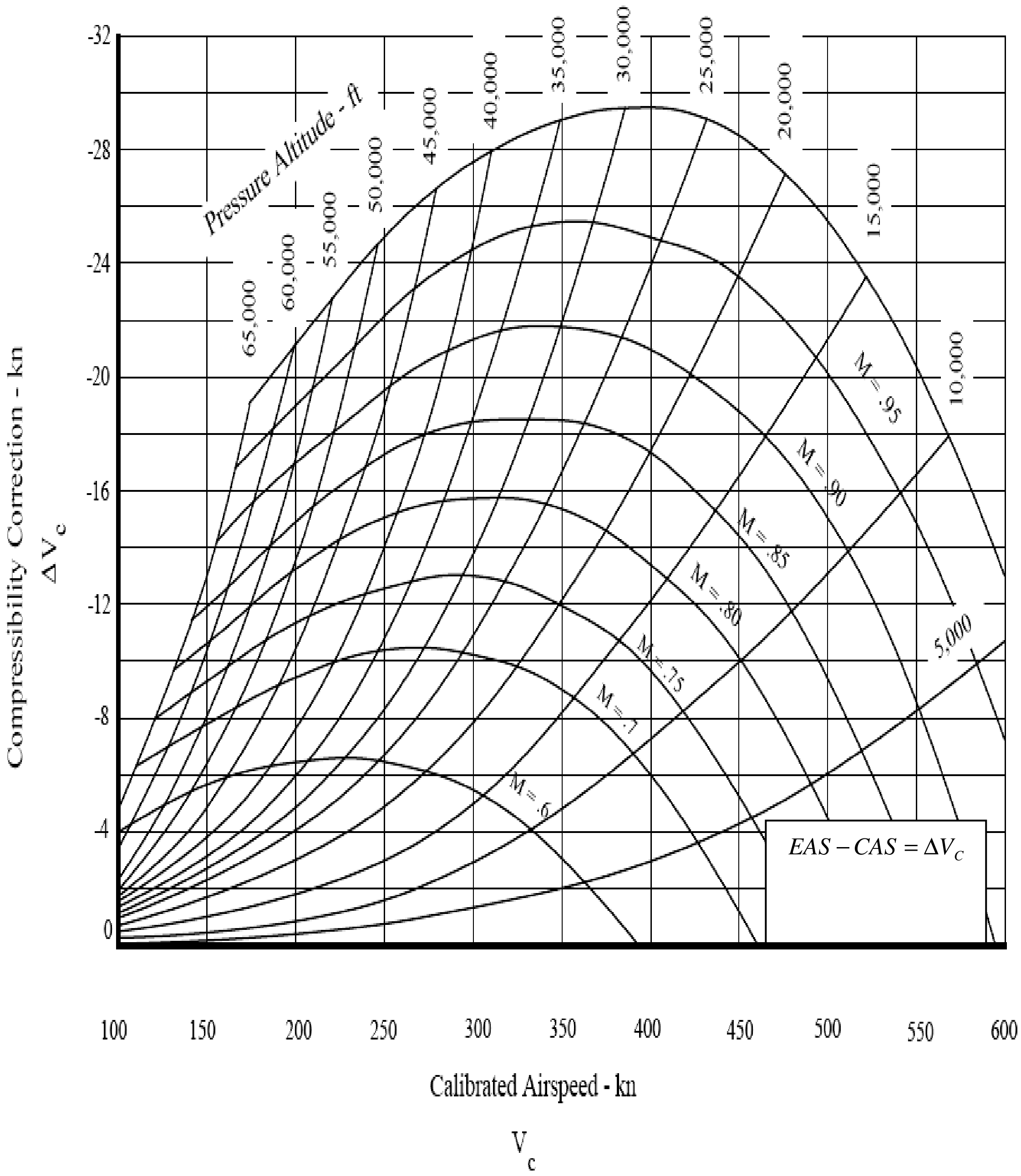
(b) **True Airspeed (TAS)**: The speed at which an aircraft is travelling through the air surrounding it. In level flight this is simply G/S adjusted for wind; in climbing or descending flight, it is G/S adjusted for wind and slope. Alternatively, TAS is obtained from EAS (or vice-versa) by correcting for altitude errors. Specifically, $V_T = \frac{EAS}{\sqrt{\sigma}}$

(c) **Indicated Air Speed (IAS):** This is the readout of an Airspeed Indicator (ASI).

(d) **Calibrated Air Speed (CAS):** This is the IAS, corrected for known position and instrument errors. CAS is sometimes also called **Rectified Air Speed (RAS).**

(e) **Equivalent Air Speed (EAS):** This is the CAS, corrected for compressibility (not generally necessary in operational flying below about M=0.6 and 10,000 ft, where it can be assumed that EAS=CAS, although still usually advisable during calibration exercises). This is the value most commonly used for structural calculations. Figure 5 below shows without proof the corrections made between CAS and EAS.

Figure 5, Compressibility corrections between CAS and EAS



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