

Should I Stay or Should I Go?
Migration under Uncertainty: A Real Options Approach

by

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Abstract

This paper considers migration as an investment decision. It develops a continuous-time stochastic model to explain the optimal timing of migration, in the presence of ongoing uncertainty over wage differentials. The results show that individuals prefer to wait before migrating, even if the present value of the wage differential is positive, because of the uncertainty and the sunk costs associated with migration. An increased degree of risk aversion discourages migration, and interacts with the other variables and parameters affecting migration by exacerbating their effects.

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1. Introduction

The conventional theories of migration for developing countries (Lewis, 1954, Todaro, 1969, and Harris and Todaro, 1970) require the existence of a dual labour market as fundamental to the industrialisation process, which was seen as the main vehicle for rapid growth. These theories rest on the assumption of a clearly defined duality in the labour market. The agricultural sector is defined as inefficient with low wages, while the industrial sector, usually urban based, is viewed as modern and technologically advanced with typically higher wages. Migration is therefore the mechanism by which wages in the two sectors eventually equalize. Once the rural-urban wage gap is eroded, migration ceases.

However, the very presence of a wage gap, central to the commonly accepted theories of migration, has been called into question (see for example Kanappan, 1985). Migration does occur in the absence of a positive income differential suggesting that income differences in themselves do not constitute a *necessary condition* for migration.

In addition, numerous studies have shown that the expected income hypothesis for rural-urban migration is not supported by empirical evidence (Banerjee and Kanbur, 1981; Salvatore, 1981; Garrison, 1982). The main economic argument for rural-urban migration is that individuals migrate to take advantage of a positive wage differential but the evidence shows that, even in the presence of a positive wage differential, people do not always migrate. It would appear then that income differences in themselves do not constitute a *sufficient condition* to motivate migration.

More recently, Stark (1984, 1991) understood that an alternative explanation was needed to analyse migration behaviour in terms other than just the inter-sectoral wage gap. The application of relative deprivation to the migration decision seems to fit the stylised facts of observed migration behaviour. However, the difficulty with relative deprivation is that whilst migration may be motivated by a desire to feel relatively better off, there is the real possibility that migrants may end up being worse-off in

absolute terms. In the limit, the relative deprivation approach would imply that migration only ceases when there is no intra-village inequality.

Additionally, in many developing countries, e.g. India, it is rural-rural migration that accounts for a significant proportion of total migration flows (see Skeldon, 1986, and Sundaram, 1989). Given that, conventionally, no duality is assumed between rural labour markets, an important question is why rural-rural migration occurs on the scale it does. The model developed in this paper does not require the presence of a dual labour market to explain the migration decision.

Most of the work on migration tends to consider migration returns in one direction, even though there is the possibility that things in the destination area might go wrong. Thus, the consideration of return migration in the initial outward migration decision is very important. Most theories of return migration show that the marginal utility of consumption in the area of origin is greater than that in the destination area (see Stark *et al.*, 1997, and Dustmann, 1997). Several studies show that there can be considerable returns to returning (Co, Gang and Yun, 2000, for instance, look at return migration to Hungary).

If the rural-urban wage differential is subject to fluctuations, then reverse migration flows are likely following a severe economic downturn associated with one particular area. Hatton and Williamson (1992) noted, in their study of US economic history, that reductions in the urban-rural wage ratio in the late 1920s was marked by migration back into agriculture. The very fact that return migration is possible, albeit at a cost, could make it easier for individuals to bring forward their decision to migrate in the first place.

This paper models migration behaviour in a general framework as an investment decision under uncertainty (see Burda, 1995). The individual migration decision is analysed in the rural-rural context of developing countries. The motivation for this stems from seeking to explain why large numbers of people move from one rural area to another, given that there may be no significant differences in their underlying economic structures. Simply, if wages in two rural areas display a similar trend, then why migrate?

The paper adopts a general approach to uncertainty. It can be considered that migrants will be reluctant to move, given prolonged uncertainty over the wage differential. Initially, the information set of the migrant is limited to uncertainty over the wage differential. The migrant does not know the primary source of uncertainty. However, the analysis is extended so that the migrant's information set identifies the source of uncertainty, by applying a neutral spread to the stochastic process for the destination wage, thus allowing for a more general approach than the traditional mean-variance analysis.

A key result of this model is that, even if agents are risk neutral, uncertainty plays a critical role in the migration decision. An increased degree of risk aversion interacts with the other parameters that influence the migration decision by exacerbating their effects. This is in contrast with the predictions of the Todaro-type framework, where it is only under risk aversion that migrants incorporate uncertainty into their decision-making.

The next section motivates and presents a model of the migration decision under uncertainty. Section 3 applies a neutral spread to the stochastic process describing the wage in the area of origin. Section 4 extends the model to consider risk aversion. Section 5 concludes.

2. The model

2.1. Motivation

The analytical root to this paper derives principally from work done on real options (see Dixit, 1991, and Dixit and Pindyck, 1994). Migration is modelled as an investment that can therefore be compared to most other forms of investment. The investment-making unit, in this case the potential migrant, must consider future expected payoffs and balance these against the costs incurred in undertaking the investment. Given the irreversibility of the sunk costs associated with the investment, future payoffs must offset these costs. Under migration, the future expected payoffs derive from the future income streams in the destination area while the sunk costs derive from the initial cost of migration, including for example travel and accommodation costs, and the search costs involved in finding a job.

For the individual migrant, the decision involved in migration is not just of *whether* to migrate, but *when*. Individuals may choose to wait before moving. There is a value to waiting that arises from two aspects. First, waiting allows the migrant to protect herself from ‘bad news’, that is, the wage differential could decline in the future. Second, the sunk cost may be lower if the migrant waits. The individual information set of the migrant may change as a result of established networks that would ultimately impact on search costs, accommodation and living costs, etc.

The timing of migration is influenced by two aspects: (1) the value of waiting for the outward migration decision, and (2) the possibility of return migration. For example, an individual migrant may be more willing to leave his or her village in the knowledge that she can always return to her home village if things go wrong, provided the cost of return migration can be covered. While sunk costs are irreversible, the act of migration is not.

The inclusion of uncertainty in the evaluation of future expected pay-offs means that migration may be regarded as a strategy to limit the ongoing uncertainties associated with a particular wage, in which case the presence of a wage differential in itself is *not* therefore necessary to induce migration. Greater uncertainty associated with, say,

the destination wage compared to the wage in the area of origin means that outward migration is subject to delay while return migration may be hastened. The uncertainty must be prolonged over time for migration decisions to be influenced. Transitory uncertainties are unlikely to result in migration in either direction, outward or return. Groote and Tassenaar (2000), for instance, show in their study of nineteenth century rural Netherlands that short-run stresses did not result in out-migration, because of the existence of buffer mechanisms to lessen the effects of temporary adverse shocks.

Although migration in this paper is modelled as an individual decision, in practice migration decisions are not always entirely individual. Whilst the act of migration may be undertaken on an individual basis, the actual decision to migrate could be made at the household level. How this decision is arrived at, and who actually migrates, depend on the interactions among the family members. Even when the unit of analysis is the selfish individual, it can be considered that this decision too is the outcome of household interactions. Although the process of internal decision-making within the household is not explored in this paper, the analysis would still apply for a household characterised by both income pooling and no divergence of preferences of individual household members.

2.2. Structure of the model

This section presents a continuous-time model of migration where the wage differential evolves in a stochastic manner over time and where there is ongoing uncertainty. The model assumes symmetry in the wage profile between the area of origin and the area of destination. This is done to consider rural-rural migration, where there may not be a systematic divergence in the wage trends in two areas. The optimal decision rule for an individual to migrate in either direction is derived.

It is shown that the predictions of standard Marshallian microeconomic analysis do not hold when the net present value of the wage differential is positive. Migration only occurs when the net present value of the wage differential is sufficiently large to compensate for the irreversibility of the sunk costs. A correct measure of the net

present value must include both the sunk costs and the value of holding the option to migrate.

The cost of initial migration is I , and E is the cost of return migration. W^o is the wage in the village of origin, W^D the destination wage. The exponential of the wage differential, $V \equiv e^{W^D - W^o}$, is assumed to follow a geometric Brownian motion:

$$(1) \quad dV = \sigma_v V dz_v$$

where $\sigma_v > 0$ measures the variability of the wage differential. Equation (1) implies that dV is proportional to the existing level of the wage differential, rather than independent of it as would be the case with a simple Brownian motion. Moreover, equation (1) excludes the possibility that the stochastic process for the wage differential might have the origin as an absorbing state. In economic terms, this means that there can be negative values of the wage differentials, and that zero is not an absorbing state. This is particularly important for modelling return migration, where the wage differential may well be negative.

Equation (1) implies that today's wage differential is the best predictor for tomorrow's wage differential. Hence, the stochastic trend in wages is the same for both the home village and destination village. The component dz_v in (1) is a Wiener disturbance defined as $dz_v(t) = \varepsilon_v(t) \cdot \sqrt{dt}$, where $\varepsilon_v(t) \sim N(0,1)$ is a white noise stochastic process (see Cox and Miller, 1965). The Wiener component dz_v is therefore normally distributed with zero expected value and variance equal to dt : $dz_v \sim N(0, dt)$.

For analytical simplicity, it is assumed that there is no uncertainty over I and E ¹.

Two migration rules can be identified for the individual:

- (a). *Optimal migration rule for an individual in the village of origin.*
- (b). *Optimal migration rule for an individual the village of destination.*

Solution to (a).

The individual will remain in the village of origin as long as the wage differential is less than a critical value, and will migrate as soon as this critical value is reached. The relevant range for V is $V \in (0, V^H)$. As the wage differential tends to minus infinity, V approaches zero. The individual will remain in the village of origin for $V < V^H$, and will migrate when $V = V^H$.

The value to the individual of holding the option to migrate is F^O :

$$(2) \quad F^O = F^O(e^{w^D - w^O}) = F^O(V) \quad V \in (0, V^H)$$

The Bellman equation (or the asset equation) for the dynamic programme of the individual is defined as:

$$rF^O(V)dt = W^O dt + E[dF^O(V)]$$

or:

$$(3) \quad rF^O dt = W^O dt + E(dF^O)$$

where r is the instantaneous rate of interest. Equation (3) requires that the annuity value of the asset (*LHS*) be equal to the sum of the instantaneous benefit and the expected capital gain or loss from the asset (*RHS*).

From the Appendix, the solution for $F^O(V)$ is:

$$(4) \quad F^O(V) = A_1 V^{\beta_1} + \frac{W^O}{r} \quad V \in (0, V^H)$$

where β_1 is given by (see Appendix):

$$\beta_1 = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2r}{\sigma_v^2}} > 1$$

¹ Uncertainty over I and E is examined in Khwaja (2001).

and where A_1 is a constant. Equation (4) shows that the value of remaining in the village of origin is the sum of the value of the option to migrate, $A_1V^{\beta_1}$ and the present value of the expected future income streams, W^O/r . The value of the option to migrate is an increasing function of V . As the wage differential increases, holding the option to migrate becomes increasingly valuable until such time when $V=V^H$, when the option is exercised. The critical parameter β_1 is an increasing function of the rate of interest r and a decreasing function of the variability of the wage differential, σ_v^2 .

Solution to (b).

The individual will remain in the destination village as long as the wage differential is greater than a critical value, and will migrate as soon as this critical value is reached. The relevant range for V is $V \in (V^L, \infty)$. The individual will remain in the village of destination for $V > V^L$ and will migrate when $V = V^L$.

The Bellman equation is:

$$(5) \quad rF^D dt = W^D dt + E(dF^D)$$

A general solution of the form:

$$(6) \quad F^D(V) = C_2V^{\beta_2} + \frac{W^D}{r} \quad V \in (V^L, +\infty)$$

is obtained, where C_2 is a constant and where

$$\beta_2 = \frac{1}{2} - \sqrt{\frac{1}{4} + \frac{2r}{\sigma_v^2}} < 0$$

(see Appendix). Equation (6) shows that the value of remaining in the village of destination is the sum of the value of the option to return migrate, $C_2V^{\beta_2}$ and the present value of the expected future income streams, W^D/r . The value of the option to return migrate is a decreasing function of V . The critical parameter β_2 is a

decreasing function of the rate of interest r and an increasing function of the variability of the wage differential, σ_v^2 .

The value matching and smooth pasting conditions are used to determine A_1 and C_2 (see Dixit and Pindyck, 1994). The value matching conditions equate the values of the alternative options, open to the decision maker at each critical boundary. Thus:

$$(7a) \quad F^O(V^H) = F^D(V^H) - I$$

$$(7b) \quad F^D(V^L) = F^O(V^L) - E$$

Equation (7a) shows that, for a given V^H , an individual in the village of origin must be indifferent between remaining in the village and migrating to the destination village, whereby it will incur a cost I . Similarly, equation (7b) shows that, for a given V^L , an individual in the destination village must be indifferent between remaining in the village and return migrating, whereby it will incur a cost E .

The smooth pasting conditions equate the marginal changes of the individual value functions, at each one the critical boundaries. Equations (8a) and (8b) show that, at the critical boundaries, the individual value functions for the village of origin and for the village of destination must be tangential to each other:

$$(8a) \quad F_v^O(V^H) = F_v^D(V^H)$$

$$(8b) \quad F_v^D(V^L) = F_v^O(V^L)$$

2.3. Results

The optimal migration strategy for an individual involves the identification of a critical value of the wage differential at which it becomes optimal to migrate. This value is, in general dependent, on the location of the migrant. The value V^H marks the

critical threshold level for out-migration, whereas V^L marks the critical threshold level for return migration. The following proposition is demonstrated in the Appendix.

Proposition 1.

There exists a non-singular interval (V^L, V^H) of the logarithm of the wage differential in which migration does not occur in either direction. The ratio V^H/V^L is an increasing function of the variance of the wage differential, σ_v^2 , and of the migration costs I and E .

There is a range of values of the wage differential, $V^L < V < V^H$, in which it is optimal for an individual to maintain the *status quo*. Individuals do not migrate in either direction and are reluctant to respond to small changes in the wage differential, preferring to wait until the wage gap is large enough for migration to be an optimal strategy. The presence of the non-singular interval (V^L, V^H) corroborates the empirical evidence that a positive wage gap is not a sufficient condition for migration to occur.

This version of the model assumes that the trends in wages in the region of origin and in the region of destination are symmetrical (i.e. there is no systematic divergence in the evolution of wages over time). However, if the model is extended so that there is an asymmetry in the trends in wages, then migration could occur even in the absence of a positive wage gap (Khwaja, 2001). Therefore, a positive wage gap would not be a necessary condition for migration.

Increased uncertainty associated with the wage differential will effectively delay migration in either direction. The analytical results in the Appendix show that z is an increasing function of σ_v^2 . Faced with increased uncertainty, migrants prefer to adopt a strategy of “wait and see”. The numerical calibrations in Table 1 show that increases in uncertainty result in very large increases in the region of inertia, which suggests that migration behaviour is extremely sensitive to uncertainty.

The critical variable z is also an increasing function of I and E , the costs of initial and return migration respectively. The interval (V^L, V^H) describing the region of inertial

behaviour widens. It should be noted that I and E exert a symmetrical effect on the region of inertia.

The rate of interest r does not exert a monotonic influence on the size of the region of inertia, as the numerical calibrations in Table 1 illustrate. For small values of r the region of inertia narrows, whereas for larger values of r migrants are more reluctant to move. The reason for this result can be explained in terms of equations (4) and (6). The individual value function consists of two components: the present value of future wage earnings in the current location and the value of the option of waiting. The first component is always a declining function of r . The second component can be an increasing function of r . For smaller values of the rate of interest the first component dominates, whereas for larger values of the interest rate it is the second component that dominates.

The relationship between interest rates and migration is complex. Rose (2000) shows that investment is a hump-shaped function of r . From Table 1 it can be seen that the size of the region of inertia in migration decisions is a U-shaped function of the rate of interest. It should be noted, however, that the quantitative effect of increased uncertainty impacts more heavily on migration behaviour than the rate of interest.

Uncertainty matters. The information set of the migrant is limited: the source of uncertainty is not identified. There may be uncertainty about the origin wage, the destination wage, or both. The migrant is not able to ascertain which wage is associated with the greatest uncertainty. The net effect is that the migrant displays caution given this limited information set. Section 3 provides a more precise characterisation of the precise source of uncertainty.

The results on inertial behaviour in migration decisions bear close similarity to the predictions of labour demand theory, where increases in hiring and firing costs make the firm more reluctant to hire or fire labour in response to fluctuations in demand for its output (see Bentolila and Bertola, 1990). During a recession, firms are reluctant to fire workers if the decline in demand is perceived to be temporary. Firing workers would increase the costs of the firm, as redundancy obligations in the form of severance pay would need to be met. Moreover, once the recession is over the firm

would incur hiring costs. However, if firms believe the recession to be long term, workers are fired.

The costs of migration and of return migration can be considered as analogous to the hiring and firing costs of the firm. A small wage differential that is expected to be permanent will generate a large present value. In this case, an individual is willing to take part in migration and incur the cost I of leaving the village of origin, or the cost E of returning. If individuals observe a large positive wage differential but expect it to be transitory they are unlikely to migrate. Equally, a large negative differential that is expected to be transitory will not prompt return migration.

3. Rural-rural migration and uncertainty on the wage differential

How well individual income can be maintained in the face of adverse shocks can be an important determinant of migration. Rural areas that are predominantly agricultural can suffer from huge variations in income. The adoption of new technology and/or state-funded rural investment in non-agricultural activities may be a means for reducing the instability of agricultural income, and can also result in fundamental changes in the structure of the rural area. This is certainly been the case in China, where Fujian Province has experienced an urbanization of its rural areas through the development of village enterprises and investment (see Zhu, 2000).

Large-scale investment may therefore serve as a means to reduce wage differentials and so stem the rural-urban flow, which has reached critical levels in many urban centres in developing countries. The effect of any investment in the rural area is twofold. First, wages may be raised over the long run. Second, there may be a reduction in the uncertainty associated with the rural wage. Both these effects may change the underlying structure of a particular rural area, and could stimulate intra-rural migration. This of course suggests that there may be an asymmetry in the wage levels of two rural areas. However, intra-rural migration flows may also occur between two structurally similar areas, in which case there is no such asymmetry. Migration therefore is motivated by something other than the wage gap. Reduced

uncertainty, as part of a strategy to stabilise long-term income, could account for some intra-rural flows.

Rural-rural migration is analysed by considering a *neutral spread* of the stochastic process of the destination wage (see Ingersoll and Ross, 1992, for an application to finance). Consider an initial stochastic process and transform it by adding uncorrelated random noise, which has the effect of increasing the variability of the destination wage. The initial destination wage and the transformed destination wage both yield the same net present value. A migrant will prefer to move to the destination that gives greater security in terms of future expected incomes. It is shown that the addition of a neutral spread to the destination wage process raises the critical threshold value of the wage differential at which it is optimal to migrate. Increased uncertainty in the destination wage has the effect of delaying the time at which it is optimal to migrate.

It should be noted that the migrant now associates the destination wage with greater uncertainty relative to the wage in the area of origin. It is assumed that the only source of uncertainty lies with the destination wage. By contrast, in the previous section individuals are unaware of the source of uncertainty, and are thus more reluctant to undertake migration in either direction. However, if uncertainty can be traced to a particular location, then this could result in out-migration away from the area and reduced levels of migration into the area.

The increase in the uncertainty associated with the wage in the destination area can be modelled as a neutral spread of the stochastic process describing the destination wage, W^D . A neutral spread can be regarded as the dynamic extension to stochastic processes of the mean-preserving spread for static random variables. The analysis of a mean-preserving spread is known to lead to a more satisfactory examination of risk than the mean-variance approach, since the latter only applies to statistical distributions that are fully characterised by the by their first and second moments (Rothschild and Stiglitz, 1970; see also Laffont, 1991, chapter 2).

The stochastic process describing the destination wage $\{W^D(t)\}_{t=0}^{\infty}$ is augmented by an uncorrelated white noise stochastic process, $\{h(t)\}_{t=0}^{\infty}$. The destination wage after the neutral spread becomes:

$$(9) \quad W^{hD}(t) = W^D(t) + h(t)$$

where the neutral spread is such that its expectation is equal to zero and its increments are uncorrelated with the increments of the process W^D : $E[h] = 0$, $E[dW^D, dh] = 0$.

In order to assess the impact of the neutral spread on the decision to migrate, the value to the individual of having the migration opportunity before and after the neutral spread is computed. For analytical simplicity, and without essential loss of generality, the option to return migrate is not considered in the present section. It is possible to show that a neutral spread increases the value to the individual of keeping open the option to migrate in the future.

Let F_t be the value at time $t = 0$ of the opportunity to migrate at time $t \geq 0$ before the spread:

$$(10) \quad F_t = E \left\{ e^{-\int_0^t r(s) ds} [\text{PV}_t(W^D) - I] \right\}$$

and let F be the value at time $t = 0$ of the opportunity to migrate at *any* time $t \geq 0$ before the spread:

$$(11) \quad F = \sup_{t \geq 0} F_t$$

Let now F_t^h be the value at time $t = 0$ of the opportunity to migrate at time $t \geq 0$ after the spread:

$$(12) \quad F_t^h = E \left\{ e^{-\int_0^t r(s) ds} [\text{PV}_t(W^{hD}) - I] \right\}$$

and let F^h be the value at time $t = 0$ of the opportunity to migrate at *any* time $t \geq 0$ after the spread:

$$F^h = \sup_{t \geq 0} F_t^h$$

The following definition is made: $t^* = \arg \max(F_t)$, that is, $F_{t^*} \geq F_t$, $\forall t \geq 0$. It follows that:

$$\begin{aligned}
(14) \quad F^h &\geq F_{t^*}^h \\
&= E \left\{ e^{-\int_0^{t^*} r(s) ds} [\text{PV}_{t^*}(W^{hD}) - I] \right\} \\
&= E \left\{ e^{-\int_0^{t^*} r(s) ds} [\text{PV}_{t^*}(W^D) + \text{PV}_{t^*}(h) - I] \right\} \\
&= E \left\{ e^{-\int_0^{t^*} r(s) ds} [\text{PV}_{t^*}(W^D) - I] \right\} + e^{-\int_0^{t^*} r(s) ds} E\{\text{PV}_{t^*}(h)\} \\
&= E \left\{ e^{-\int_0^{t^*} r(s) ds} [\text{PV}_{t^*}(W^D) - I] \right\} \\
&= F
\end{aligned}$$

where the second line of (14) follows from equation (12), the third line from the definition of neutral spread (9), the fourth line from the additivity property of the expectation operator, the fifth line from $E[h] = 0$, and finally the last line follows from the definitions (10) and (11).

The following proposition has therefore been proved.

Proposition 2.

A neutral spread to the stochastic distribution of the wage in the destination area increases the value to the individual of keeping the option open to migrate in the future.

The proof of Proposition 2 shows that a neutral spread increases the value to the individual of keeping open the option to migrate. It should be noted that this result would be enhanced under risk aversion. This issue is explored in the next section.

4. Migration and risk aversion

For a risk averse individual, the opportunity cost attached to migrating now is higher than under risk neutrality. Thus, it is expected that risk averse individuals display a greater degree of inertial behaviour in migration.

Under risk aversion, instantaneous individual utility can be modelled as:

$$(15) \quad U = U(W)$$

where U is increasing and concave, such that $U' > 0$, $U'' < 0$. Proceeding as in section 2, the optimal migration strategy must have the form: do not migrate if $V \in (0, V^*)$, migrate if $V \in [V^*, \infty)$. The solution for the individual value function is (see appendix):

$$(16) \quad F(V) = A_1 V^{\beta_1} + \frac{U(W^o)}{r} \quad V \in (0, V^*)$$

The values of the coefficient A_1 and of the critical threshold V^* are obtained from the value-matching and the smooth-pasting condition. It is possible to show that (see appendix):

$$(17) \quad \frac{dV^*}{dI} = -\frac{r\beta_1}{[U''(W^D) - \beta_1 U'(W^D)]/V^*} > 0$$

Equation (17) shows that an increase in the cost of migration, I , will increase the critical value V^* and thus delay the decision to migrate. This is because the region where it is optimal not to migrate has widened. Similarly,

$$(18) \quad \frac{dV^*}{dr} = -\frac{\beta_1 I + rI \cdot \partial\beta_1 / \partial r}{[U''(W^D) - \beta_1 U'(W^D)]/V^*} > 0$$

Equation (18) shows that an increase in the rate of interest, r , also has the effect of delaying migration. This finding is consistent with intuition, in that one would expect high interest rates to act as a deterrent in the migration decision.

To evaluate the response of V^* to changes in the variance of the instantaneous shocks, σ_v^2 , note that (see appendix):

$$(19) \quad \frac{dV^*}{d\sigma_v^2} > 0 \quad \Leftrightarrow \quad U(W^D) - U(W^O) > rI$$

Suppose $W^O > W^D$: in the absence of stochastic shocks, it would never be profitable to migrate since $U(W^D) - U(W^O) < rI$. With positive shocks, as the variance σ_v^2 increases there is an increased probability that the wage of destination W^D will climb above the wage of origin W^O , and therefore migration would be more attractive. This would result in a decline of the critical value V^* (the set of values of V for which migration is not optimal will be smaller). Conversely, when $U(W^D) - U(W^O) > rI$ an increase in the variance of the stochastic shocks will make it more likely for the destination wage to fall below the wage of origin, thereby discouraging migration.

The intuition for this result can be explained by analogy to an American-type financial option. If $W^O > W^D$, individuals would own an option that is currently out-of-the-money. Under certainty (*i.e.*, $\sigma_v^2 = 0$), the option would be worthless. Under

uncertainty (*i.e.*, when $\sigma_v^2 > 0$), there is a positive probability that the option would eventually be in-the-money (net of the costs), in which case it can be profitably exercised.

Consider now the effect of an increase in the degree of risk aversion on the decision to migrate. The coefficient of relative risk aversion is defined as usual as (see e.g. Laffont, 1991, page 24):

$$\gamma(W) = -\frac{W \cdot U''(W)}{U'(W)}$$

where $\gamma > 0$ for a risk-averse individual. The appendix proves the following result:

Proposition 3.

The effects on V^* of changes in the parameters I , r and σ_v^2 are magnified by the presence of positive risk aversion.

The importance of this result is twofold. Firstly, it allows us to establish the role of risk aversion in the decision-making process. For instance, section 2 showed that an increase in migration cost I makes the individual more reluctant to migrate, by raising the critical threshold V^* . In the presence of risk aversion, the critical value V^* is raised even further by increases in I . Risk aversion therefore exacerbates the effects of those parameters that affect migration. In other words, the qualitative effects are unchanged, but the quantitative effects are stronger.

Secondly, the degree of risk aversion implicitly captures a source of heterogeneity across individuals. This could explain why some individuals are more likely to migrate than others, even if their income and cost profiles are the same. That is, for any particular wage differential, a more risk averse individual will display a greater degree of inertial behaviour.

5. Conclusions

This paper develops a migration model where individuals incorporate uncertainty into their decision. It is shown that increased uncertainty over the wage differential reduces the propensity to migrate. Individuals require a net present value of the expected wage differential to more than offset the cost of migration. The possibility of return is explicitly included in the initial migration decision. There exists a region of inertia over which no migration takes place. Whilst wage differentials are important, they are neither necessary nor sufficient to motivate migration. Individuals may consider migration as a strategy to overcome the uncertainties of the wage profile in a particular location. Under risk aversion, the effects of the parameters that influence migration are magnified.

While the analysis presented here considers only intra-rural migration flows, the model may be extended to consider rural-urban flows where the wage profiles can be asymmetric.

At an individual level, migration may be seen as a means of acquiring an income that is higher, or more predictable, or both. The gains from migration accrue to the individual. However, in the context of household migration, the gains from migration also include the benefits of income pooling. In this sense, even though only one individual member migrates, it is the household attitude to risk that dominates. Thus, migration may be brought forward precisely because it is part of the risk diversifying strategy of the household. The region of inertia at household level could be expected to be narrower than at the individual level. This may be because access to the funds required for migration is already available, or because the household may have access to some form of credit. Moreover, in the event that migration turns out to be an unsuccessful investment, returning is always a feasible option, provided the costs of return can be met. In many developing countries, it is likely that individual migration tends to be the outcome of a household strategy to pool income.

The generality of the approach allows for an examination of international migration. The costs of migrating abroad are higher than domestically, but the issue of uncertainty is just as relevant. Moreover, the real options model can explain episodes

of large-scale forced migration, such as Ireland during the nineteenth century famine, which triggered mass emigration to the United States (see O'Rourke, 1991, 1995). There is ample evidence of forced migration on a large scale as a result of war, famine or both. Distress migration can be analysed in terms of the extremely large and very prolonged uncertainty over the wage in the area of origin. Alternatively, income in the area of origin can be expected to assume very low values with near certainty. In either case, the optimal strategy is to move away from the source of distress.

Uneven distribution of income is a feature common to many poor countries. For many people access to formal institutions for the purposes of obtaining funds for migration is limited. However, the presence of informal credit markets may provide individuals with access to some form of credit. Often, the labour and credit markets are interlinked. Future research intends to consider the funding of migration in the informal credit market. This would also permit an analysis of the heterogeneity of potential migrants, the type of credit they obtain being a function of the assets they own and of other household characteristics.

A further area of research is to analyse the general equilibrium consequences of migration, both in the area of origin and in the area of destination. This implies that the stochastic processes defining the wage differentials become endogenous.

This paper shows that uncertainty is a fundamental component of the migration decision. Whilst wage differentials are important, they are neither a necessary nor a sufficient condition for migration to take place. Uncertainty can in some instances dominate the propensity to migrate more than the presence of a positive wage differential.

Table 1. Interest rate, uncertainty and the wage gap.

	<i>r</i>										
	0.01	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
0.1	1.476	1.467	1.458	1.458	1.468	1.492	1.529	1.580	1.645	1.725	1.821
0.2	1.857	1.844	1.831	1.821	1.817	1.821	1.834	1.857	1.892	1.937	1.994
0.3	2.249	2.234	2.217	2.202	2.191	2.187	2.189	2.199	2.219	2.248	2.287
0.4	2.668	2.650	2.629	2.610	2.595	2.585	2.580	2.582	2.593	2.611	2.639
σ, 0.5	3.119	3.098	3.073	3.051	3.031	3.017	3.007	3.003	3.007	3.019	3.039
0.6	3.608	3.584	3.555	3.528	3.505	3.486	3.472	3.464	3.462	3.468	3.482
0.7	4.138	4.111	4.078	4.047	4.020	3.997	3.979	3.967	3.961	3.962	3.971
0.8	4.714	4.682	4.645	4.610	4.579	4.552	4.530	4.514	4.504	4.502	4.507
0.9	5.338	5.302	5.261	5.222	5.186	5.156	5.130	5.110	5.097	5.091	5.093
1	6.014	5.974	5.928	5.884	5.845	5.810	5.781	5.757	5.741	5.732	5.731

Note: $I = 0.5$, $E = 0.5$.

Appendix

From Section 2.

The Bellman equation (3) from the text can be expanded by using Itô's Lemma:

$$(A1) \quad dF^o = \frac{\partial F^o}{\partial t} dt + \frac{\partial F^o}{\partial V} dV + \frac{1}{2} \frac{\partial^2 F^o}{\partial V^2} (dV)^2 \\ = \frac{1}{2} \sigma_v^2 V^2 F_{vv}^o dt + \sigma_v V F_v^o \varepsilon_v^2 dz_v$$

Taking expectations of (A1) we have:

$$(A2) \quad E(dF^o) = \frac{1}{2} \sigma_v^2 V^2 F_{vv}^o dt$$

Replacing (A2) into (3) the Bellman equation becomes:

$$(A3) \quad rF^o dt = W^o dt + \frac{1}{2} \sigma_v^2 V^2 F_{vv}^o dt$$

Dividing (A3) by dt a 2nd-order differential equation in $F^o(V)$ is obtained:

$$(A4) \quad \frac{1}{2} \sigma_v^2 V^2 F_{vv}^o - rF^o = -W^o$$

The solution to (A4) is given by the sum of the general solution for the homogeneous equation and of a particular solution for the inhomogeneous equation. Therefore, a solution for the homogeneous equation must first be found:

$$(A5) \quad \frac{1}{2} \sigma_v^2 V^2 F_{vv}^o - rF^o = 0$$

Using a guess solution of the form:

$$(A6) \quad F^o = AV^\beta$$

implies

$$(A6a) \quad F_v^o = \beta AV^{\beta-1}$$

$$(A6b) \quad F_{vv}^o = \beta(\beta-1)AV^{\beta-2}$$

Substituting (A6) and (A6b) into the homogeneous equation (A5) and dividing by AV^β leads to:

$$(A7) \quad \frac{1}{2}\sigma_v^2\beta(\beta-1) - r = 0$$

The roots of the quadratic equation (A7) are:

$$(A8a) \quad \beta_1 = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2r}{\sigma_v^2}} > 1$$

$$(A8b) \quad \beta_2 = \frac{1}{2} - \sqrt{\frac{1}{4} + \frac{2r}{\sigma_v^2}} < 0$$

The general solution for the homogeneous equation (A4) is:

$$(A9) \quad F^O(V) = A_1V^{\beta_1} + A_2V^{\beta_2}$$

A particular solution for the inhomogeneous equation (A4) takes the form:

$$(A10) \quad F^O(V) = K$$

where K is a constant. Replacing (A10) into the differential equation (A4) gives:

$$(A11) \quad K = \frac{W^O}{r}$$

The general solution for the second-order inhomogeneous differential equation (A4) is thus given by:

$$(A12) \quad F^O(V) = A_1V^{\beta_1} + A_2V^{\beta_2} + \frac{W^O}{r} \quad V \in (0, V^H)$$

Consider A_2 . As $V \rightarrow 0$, $W^D - W^O \rightarrow -\infty$ and therefore the option to migrate should be worthless. Since $\beta_2 < 0$, in order to avoid $F^O(V) \rightarrow \infty$ as $V \rightarrow 0$, A_2 must be set to equal zero, i.e. $A_2=0$. Hence,

$$(A13) \quad F^O(V) = A_1V^{\beta_1} + \frac{W^O}{r} \quad V \in (0, V^H)$$

Problem b can be solved similarly to obtain:

$$(A14) \quad F^D(V) = C_2 V^{\beta_2} + \frac{W^D}{r} \quad V \in (V^L, +\infty)$$

To determine A_1 and C_2 the value matching and smooth pasting conditions are used.

The value matching conditions for the problems indicated above are:

$$(A15) \quad F^O(V^H) = F^D(V^H) - I$$

$$(A16) \quad F^D(V^L) = F^O(V^L) - E$$

Equations (A15) and (A16) say that, a household in a particular location must be indifferent between remaining and migrating minus the associated cost.

The smooth pasting conditions are:

$$(A17) \quad F_v^O(V^H) = F_v^D(V^H)$$

$$(A18) \quad F_v^D(V^L) = F_v^O(V^L)$$

Equations (A17) and (A18) say that, at the critical boundaries, the value functions for the household in the village of origin and for the household in the village of destination must be tangential to each other.

Since $W^D = \ln V + W^O$ and by using (A13) and (A14), one obtains:

$$(A19) \quad F_v^O(V) = A_1 \beta_1 V^{\beta_1 - 1}$$

$$(A20) \quad F_v^D(V) = C_2 \beta_2 V^{\beta_2 - 1} + \frac{1}{rV}$$

By replacing (A13), (A14), (A19) and (A20) into (A15)-(A18) the following system of equations for A_1 , C_2 , V^L and V^H is obtained:

$$(A21) \quad A_1 V^{H\beta_1} + \frac{W^O}{r} = C_2 V^{H\beta_2} + \frac{\ln V^H}{r} + \frac{W^O}{r} - I$$

$$(A22) \quad C_2 V^{L\beta_2} + \frac{\ln V^L}{r} + \frac{W^O}{r} = A_1 V^{L\beta_1} + \frac{W^O}{r} - E$$

$$(A23) \quad A_1 \beta_1 V^{H\beta_1 - 1} = C_2 \beta_2 V^{H\beta_2 - 1} + \frac{1}{rV^H}$$

$$(A24) \quad A_1 \beta_1 V^{L\beta_1-1} = C_2 \beta_2 V^{L\beta_2-1} + \frac{1}{rV^L}$$

The system (A21)-(A24) is non-linear in the variables A_1 , C_2 , V^L and V^H . In order to solve it, the methods illustrated in Dixit (1991) are adapted. Using (A23) and (A24) to solve for A_1 and C_2 gives:

$$(A25) \quad A_1 = \frac{V^{H\beta_2} - V^{L\beta_2}}{r\beta_1(V^{H\beta_2}V^{L\beta_1} - V^{H\beta_1}V^{L\beta_2})}$$

$$(A26) \quad C_2 = \frac{V^{H\beta_1} - V^{L\beta_1}}{r\beta_2(V^{H\beta_2}V^{L\beta_1} - V^{H\beta_1}V^{L\beta_2})}$$

Let $K \equiv (V^{H\beta_2}V^{L\beta_1} - V^{H\beta_1}V^{L\beta_2})$. Replace A_1 and C_2 into equations (A21) and (A22) and adding gives:

$$(A27) \quad (V^{H\beta_2} - V^{L\beta_2})(V^{H\beta_1} - V^{L\beta_1})(\beta_2 - \beta_1) - \beta_1\beta_2 K \ln(V^H / V^L) = -r\beta_1\beta_2 K(I + E)$$

Define:

$$(A28) \quad M \equiv (V^H \cdot V^L)^{1/2}$$

$$(A29) \quad z \equiv \frac{1}{2} \cdot \ln\left(\frac{V^H}{V^L}\right)$$

Then:

$$(A30) \quad e^z = \left(\frac{V^H}{V^L}\right)^{1/2}$$

$$(A31) \quad V^H = Me^z$$

$$(A32) \quad V^L = Me^{-z}$$

Replacing (A29)-(A32) into (A27) and simplifying obtains:

$$(A33) \quad (\beta_2 - \beta_1)(e^{z\beta_2} - e^{-z\beta_2})(e^{z\beta_1} - e^{-z\beta_1}) - 2\beta_1\beta_2 z(e^{z\beta_2}e^{-z\beta_1} - e^{z\beta_1}e^{-z\beta_2}) = \\ = -r\beta_1\beta_2(e^{z\beta_2}e^{-z\beta_1} - e^{z\beta_1}e^{-z\beta_2})(I + E)$$

Use $\sinh(x) = (e^x - e^{-x})/2$ (see e.g. Smirnov, chapter 17) to obtain:

$$(A34) \quad 2(\beta_2 - \beta_1) \cdot \sinh(z\beta_2) \cdot \sinh(z\beta_1) - 2\beta_1\beta_2 z \cdot \sinh(z(\beta_2 - \beta_1)) = \\ = -r\beta_1\beta_2 \cdot \sinh(z(\beta_2 - \beta_1)) \cdot (I + E)$$

Equation (A34) can be evaluated by using a fourth-order Taylor expansion about the point $z=0$, noting that $d \sinh(x)/dx = \cosh(x)$ and $d \cosh(x)/dx = \sinh(x)$ where $\cosh(x) = (e^x + e^{-x})/2$ (Smirnov, chapter 17), to obtain:

$$(A35) \quad z^3 + \frac{r(I+e)(\beta_2 - \beta_1)^2}{4\beta_1\beta_2} \cdot z^2 + \frac{3r(I+E)}{2\beta_1\beta_2} = 0$$

Using Cardano's formula (see Kurosh, chapter 9), the cubic equation (A35) has one real root and two complex conjugate roots. The real root of the equation is:

$$(A36) \quad z = \sqrt[3]{-\frac{q}{2} + \sqrt{-\frac{D}{108}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{-\frac{D}{108}}} - \frac{r(I+E)(\beta_2 - \beta_1)^2}{12\beta_1\beta_2}$$

where

$$(A37) \quad q = \frac{r^3(I+E)^3(\beta_2 - \beta_1)^6}{864\beta_1^3\beta_2^3} + \frac{3r(I+E)}{2\beta_1\beta_2}$$

$$(A38) \quad D = -\frac{243r^2(I+E)^2}{4\beta_1^2\beta_2^2} - \frac{9}{32} \frac{r^4(I+E)^4(\beta_2 - \beta_1)^6}{\beta_1^4\beta_2^4} < 0$$

Since $D < 0$, the cubic equation has one real root and two complex conjugate roots. Let

$$(A39) \quad \alpha = \sqrt[3]{-\frac{q}{2} + \sqrt{-\frac{D}{108}}}$$

$$(A40) \quad \gamma = \sqrt[3]{-\frac{q}{2} - \sqrt{-\frac{D}{108}}}$$

The real root for y is:

$$(A41) \quad y = \alpha + \gamma$$

The real root for z can therefore be written as:

$$(A42) \quad z = \sqrt[3]{h_1^3 + h_2 + h_3} + \sqrt[3]{h_1^3 + h_2 - h_3} + h_1 > 0$$

where

$$(A43) \quad h_1 = -\frac{r(I+E)(\beta_2 - \beta_1)^2}{12\beta_1\beta_2} > 0$$

$$(A44) \quad h_2 = -\frac{3r(I+E)}{4\beta_1\beta_2} > 0$$

$$(A45) \quad h_3 = \frac{r(I+E)}{96\beta_1^2\beta_2^2} \sqrt{648 + 3r^2(I+E)^2(\beta_2 - \beta_1)^6} > 0$$

It can be shown that:

$$\frac{\partial z}{\partial h_1} > 0, \quad \frac{\partial z}{\partial h_2} > 0, \quad \frac{\partial z}{\partial h_3} > 0,$$

$$\frac{\partial h_1}{\partial \beta_1} - \frac{\partial h_1}{\partial \beta_2} < 0, \quad \frac{\partial h_2}{\partial \beta_1} < 0, \quad \frac{\partial h_2}{\partial \beta_2} > 0, \quad \frac{\partial h_3}{\partial \beta_1} < 0, \quad \frac{\partial h_3}{\partial \beta_2} > 0,$$

$$\frac{\partial h_1}{\partial r} > 0, \quad \frac{\partial h_2}{\partial r} > 0, \quad \frac{\partial h_3}{\partial r} > 0,$$

$$\frac{\partial h_1}{\partial I} = \frac{\partial h_1}{\partial E} > 0, \quad \frac{\partial h_2}{\partial I} = \frac{\partial h_2}{\partial E} > 0, \quad \frac{\partial h_3}{\partial I} = \frac{\partial h_3}{\partial E} > 0,$$

$$\frac{\partial \beta_1}{\partial \sigma_v^2} < 0, \quad \frac{\partial \beta_1}{\partial r} > 0, \quad \frac{\partial \beta_2}{\partial \sigma_v^2} > 0, \quad \frac{\partial \beta_3}{\partial r} < 0$$

By the chain rule,

$$(A46) \quad \frac{\partial z}{\partial \sigma_v^2} > 0, \quad \frac{\partial z}{\partial I} > 0, \quad \frac{\partial z}{\partial E} > 0$$

From Section 4.

The Bellman equation is:

$$(A47) \quad rF(V)dt = U(W^0)dt + E[dF(V)]$$

Using Itô's Lemma, taking expectations, replacing into the Bellman equation and rearranging the following second-order differential equation in the value function $F(V)$ is obtained:

$$(A48) \quad \frac{1}{2}\sigma_v^2 V^2 F_{vv} - rF = -U(W^O)$$

Proceeding as in section 2, the general solution for the inhomogeneous equation (A48) is:

$$(A49) \quad F(V) = A_1 V^{\beta_1} + A_2 V^{\beta_2} + \frac{U(W^O)}{r} \quad V \in (0, V^*)$$

where $\beta_1 > 1$ and $\beta_2 < 0$ are defined in equations (A5a) and (A5b) respectively, and where V^* is the critical threshold of the wage differential.

The optimal migration strategy must have the form: do not migrate if $V \in (0, V^*)$, migrate if $V \in [V^*, \infty)$. The general solution for the differential equation (A48) is:

$$(A50) \quad F(V) = A_1 V^{\beta_1} + \frac{U(W^O)}{r} \quad V \in (0, V^*)$$

The values of the coefficient A_1 and of the critical threshold V^* are obtained from the value-matching and the smooth-pasting condition. Under risk aversion, the value-matching condition is:

$$(A51) \quad F(V^*) = \frac{U(W^D)}{r} - I$$

and the smooth-pasting condition is:

$$(A52) \quad F_v(V^*) = \frac{U'(W^D)}{r}$$

Using (A50) and the definition $V = e^{W^D - W^O}$ the following system is obtained:

$$(A53) \quad A_1 V^{*\beta_1} + \frac{U(W^O)}{r} = \frac{U(\ln V^* + W^O)}{r} - I$$

$$(A54) \quad A_1 \beta_1 V^{*(\beta_1-1)} = \frac{U'(\ln V^* + W^O)}{r V^*}$$

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