Long-Range Dependence in Daily Interest Rates

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Abstract

We employ a number of parametric and non-parametric techniques to establish the existence of long-range dependence in daily interbank offer rates for four countries. We test for long memory using classical R/S analysis, variance-time plots and Lo's (1991) modified R/S statistic. In addition we estimate the fractional differencing parameter using Whittle's (1951) maximum likelihood estimator and we shuffle the data to destroy long and short memory in turn, and we repeat our non-parametric tests. From our non-parametric tests we find strong evidence of the presence of long memory in all four series independently of the chosen statistic. We find evidence that supports the assertion of Willinger *et al* (1999) that Lo's statistic is biased towards non-rejection of the null hypothesis of no long-range dependence. The parametric estimation concurs with these results. Our results suggest that conventional tests for capital market integration and other similar hypotheses involving nominal interest rates should be treated with caution.

JEL classification: C13, C14, C22, E4, G10

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1 Long-Range Dependence in Time Series

The question of whether economic and financial time series display long-range dependence is one that has a long history and has remained a topic of active research. The notion that observations in the distant past are nontrivially correlated with current observations has received ample confirmation in natural phenomena of hydrology, meteorology and geophysics (see, for example, Mandelbrot (1982)). Motivated by the cyclical patterns that typified plots of economic aggregates over time, some early theories of business cycles argued that

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economic time series are long-range dependent. Indeed, these features were common enough for Granger (1966) to describe them as the "typical spectral shape of an economic variable".

Mandelbrot (1967,1971) was among the first to identify long-range dependence in asset markets. The statistical vehicle for Mandelbrot's analysis was the *rescaled range* or R/S statistic (defined precisely below), first introduced by Hurst (1951). In several seminal papers Mandelbrot and Wallis (1968, 1969a–c), and Mandelbrot and Taqqu (1979) demonstrated the superiority of R/S analysis to more conventional methods of detecting long-range dependence, such as analysing autocorrelations. Greene and Fielitz (1977) lent further support to Mandelbrot's findings, using classical R/S analysis on 200 time series of security prices listed on the New York Stock Exchange, and they found that many of the series were characterised by long-term memory.

Baillie (1996) provides evidence that major economic time series exhibit mean reversion, strong persistence and near non-stationarity. In the same spirit Hassler and Wolters (1995) produce econometric results suggesting that the rate of inflation in a number of countries can best be described by an I(d) process with the estimate of the parameter d, the fractional differencing parameter, in the neighbourhood of 1/2.

1.1 Long-Range Dependence in Finance

The presence of long-term dependence in asset returns has serious implications for some of the paradigms used in financial economics. For example, it may be inconsistent with efficient market hypotheses, and the conclusions of recent tests of such hypotheses and of stock market rationality rely heavily on the absence of long-range dependence (see LeRoy (1989) and Merton (1987) for reviews of these areas). Long memory is also at odds with much of the literature on derivative security pricing, which utilises martingale methods and stochastic processes that are usually inconsistent with long memory (see Maheswaran and Sims (1993)). Rogers (1997) shows how a long-range dependent process inevitably leads to arbitrage opportunities with derivative assets, but also shows how a modified process can retain long memory whilst eliminating arbitrage opportunities.

Although it is well established that R/S analysis can indeed detect long-range dependence in time series, it is suggested by Lo (1991) that it suffers from the drawback of being very sensitive to the presence of short-range dependence, which Lo and MacKinlay (1988, 1990) show is a common feature of stock returns data. Hence, any empirical investigation of long-term dependence must take into account the possible presence of higher frequency autocorrelations. To achieve this, Lo (1991) developed a *modified* R/S *statistic* which is designed to be robust with respect to short-range dependence structures in a time series. Using the new statistic, Lo reached different conclusions to Greene and Fielitz (1977) and others, indicating that there was often no long-range dependence once the effects of short-range dependence were accounted for.

Most recently, Willinger, Taqqu and Teverovsky (1999) have countered Lo's claims by identifying a number of problems associated with Lo's method and its use in practice. Most importantly, they have shown that Lo's modified R/S analysis has a strong preference for not rejecting the null hypothesis of no long-range dependence, irrespective of whether long memory is present or not. They concluded that Lo's method is not, on its own, an adequate test for long-range dependence, and that one should augment Lo's method with a diverse portfolio of graphical and statistical methods, including classical R/S analysis, and a variety of other methods, as detailed in Beran (1994), for instance.

1.2 The Behaviour of the Rate of Interest

The statistical behaviour of both nominal and real interest rates has been the subject of a number of econometric studies. These have produced a great deal of contradictory evidence. For example, Mishkin (1992) found stochastic trends in the nominal interest rate, and Rose (1988) concluded that real "ex-post" interest rates are non-stationary and that nominal rates have a unit root.

Evidence contradicting the above findings is presented by Garcia and Perron (1996) who conclude that the real rate of interest "ex-post" is stationary, albeit subject to a series of structural breaks in both its first two moments. Recently Phillips (1998), using non-parametric methods, finds that the ex-post real rate of interest has long memory and that it is not stationary (at the limit). A more recent survey of the evidence for European real interest rates can be found in Venetis and Siriopoulos (1999).

So there appears to be an absence of consensus regarding the empirical evidence on the time evolution of both the nominal and the real rate of interest. This may be due to the binary characterisation of the series as either I(1) or I(0). In the empirical literature most economic and financial time series are thus categorised. But recently there is mounting evidence that such a distinction may not be the appropriate one. There is a wide class of fractionally integrated processes I(d) ($0 < d < 1$), that provides a better description of the evolution of such time series data.

From conventional economic theory, under mild assumptions regarding the agent's utility function, the real rate of interest can be written as

$$r_t = a + b\mathbf{E}\delta(\log c_{t+1}) + \frac{1}{2}\sigma_t^2\delta(\log c_{t+1}), \tag{1}$$

where $\delta(\log c_{t+1})$ denotes the rate of growth of per capita consumption, a and b are parameters from the utility function and σ_t is the temporal volatility of the consumption growth rate.

Given that it is widely accepted that the rate of growth of consumption cannot be I(1), the finding that the ex-post real rate of interest is I(1) is somewhat perplexing. Moreover, the nominal rate of interest under the Fisher hypothesis is given as

$$R_t = r_t + \mathbf{E}\delta(\log p_{t+1}) + \mathrm{cov}(r_t, \delta(\log p_{t+1})), \tag{2}$$

where $\delta(\log p_{t+1})$ denotes the rate of inflation. On the basis of this specification and in the light of the findings of Hassler and Wolters (1995), the description of R_t as an I(1) process seems difficult to justify on a priori grounds.

The statistical behaviour of both the real and nominal rate of interest is of great importance for the testing of hypotheses regarding international capital market integration, nominal convergence and interest parity conditions.

In the face of the controversies detailed above, in this paper we compare a variety of methods for testing for long-range dependence across a number of time series of interest rates. Specifically, we use the following techniques:

- classical R/S analysis
- Lo's modified statistic
- variance-time plots
- maximum likelihood estimation of the fractional differencing parameter d (Granger (1980), Granger and Joyeux (1980), Hosking (1981), Whittle (1951)).

We apply each technique to daily interbank offer rate data from four different countries, to test for long-range dependence, or fractional integration. We benchmark our results on a short-range dependent series generated by a normal i.i.d. process. We also "shuffle" the interest rate data with a view to destroying, in turn, the short- and long-range dependence in the series, and repeat the tests. In this way we hope to shed light on the efficacy of the various methods that one can use to test for long memory. Interest rates are chosen as appropriate time series because they represent a suitable measure of "returns", and they are also a novel departure from the usual analysis of stock index returns.

Our results indicate that, whilst all the series examined show long-range dependence, inferring this requires the use of a variety of methods in conjunction with each other. In particular, the modified R/S statistic of Lo (1991) can sometimes lead to incorrect conclusions if it is not augmented with other, more traditional methods of analysis. This is in agreement with the findings in Willinger, Taqqu and Teverovsky (1999).

The rest of the paper is as follows. Section 2 outlines the methodology behind each of the tests for long-range dependence, and the various shuffling experiments carried out with the time series. Section 3 describes the data. Section 4 details our empirical results and Section 5 concludes.

2 Statistical Inference Methods for Long Memory

Given a covariance stationary time series $(X_i, i \geq 1)$, with mean μ, variance σ^2, and autocorrelation function $r(k), k \geq 0$, then suppose that $r(k)$ satisfies an equation of the form

$$r(k) \sim k^{-\beta} L_1(k), \text{ as } k \to \infty, \tag{3}$$

where $0 < \beta < 1$ and L_1 is a function which is slowly varying at infinity.[1] A process satisfying (3) is said to exhibit *long-range dependence*, and we see that its main characteristic is a slow (hyperbolic) decay of the autocovariance function as the lag k is increased. A consequence of (3) is $\sum_k r(k) = \infty$, which does indeed encapsulate our intuitive notion of long-range dependence, namely that while high-lag correlations might be individually small, their cumulative effect is not. This is to be contrasted with the case of short-range dependent processes, which are characterized by exponentially decaying autocorrelations, i.e. $r(k) \sim \rho^k$ as $k \to \infty$ for some $\rho \in (0,1)$, resulting in a summable autocorrelation function, $0 < \sum_k r(k) < \infty$.

If we work in a frequency domain as opposed to a lag ("wavelength") domain, then long-range dependence manifests itself in a spectral density that displays power-law behaviour near the origin. The equivalent statement to (3) is that there is long-range dependence if the spectral density function $f(\lambda) \equiv \sum_k r(k)\exp(ik\lambda)$ obeys

$$f(\lambda) \sim \lambda^{-\gamma} L_2(\lambda), \text{ as } \lambda \to 0, \tag{4}$$

where $0 < \gamma < 1$ and L_2 is slowly varying at the origin. So from the point of view of spectral analysis the nonsummability of autocorrelations in a long-range dependent model implies that $f(0) = \infty$; that is, the spectral density tends to infinity as the frequency λ approaches zero. On the other hand, short-range dependence is characterized by a spectral density function which is positive and finite for $\lambda = 0$.

From a statistical point of view, the most salient feature of long-range dependent processes is that the variance of the arithmetic mean decreases more slowly than the reciprocal of the sample size; that is, it behaves like $n^{-\beta}$ for some $\beta \in (0,1)$, instead of like n^{-1} for processes with short-range dependence. If we define, for each $m \geq 1$, a new covariance stationary process by averaging the new time series over non-overlapping blocks of size m, i.e. $X_k^{(m)} = m^{-1}(X_{km-m+1} + \cdots + X_{km})$, $k \geq 1$, $m \geq 1$, then it is shown in Cox (1984) that a specification of the autocorrelation function satisfying (3) (or, equivalently, of the spectral density function satisfying (4)) is the same as a specification of the sequence $(\mathrm{var}(X^{(m)}) : m \geq 1)$ with the property

$$\mathrm{var}(X^{(m)}) \sim am^{-\beta}, \text{ as } m \to \infty, \tag{5}$$

where a is a finite positive constant independent of m, and $\beta \in (0,1)$. In fact, the parameter β is the same as in (3) and is related to the parameter γ in (4) by $\beta = 1 - \gamma$. On the other hand, for covariance stationary processes that exhibit short-range dependence, it is easy to see that the sequence $(\mathrm{var}(X^{(m)}) : m \geq 1)$ satisfies

$$\mathrm{var}(X^{(m)}) \sim bm^{-1}, \text{ as } m \to \infty, \tag{6}$$

where b is a finite positive constant independent of m.

[1] By "slowly varying" we mean that $\lim_{t\to\infty} L_1(tx)/L_1(t) = 1$ for all $x > 0$. Examples of such functions are $L_1(t) = \text{const}$ and $L_1(t) = \log(t)$.

The general representation of a fractionally integrated time series process (ARFIMA) is given by

$$\Phi(L)(1-L)^d X_i = \Theta(L)u_i, \tag{7}$$

where $\Phi(L)$ and $\Theta(L)$ are polynomials in the lag operator L and their roots lie outside the unit circle. The forcing variable is u_i and it is taken to be a white noise disturbance term with variance σ_u^2. If the fractional differencing parameter d lies within the range $-\frac{1}{2} < d < \frac{1}{2}$, then X_i is covariance stationary. For values of d in the range $0 < d < \frac{1}{2}$, X_i exhibits long-memory because, as was shown by Brockwell and Davies (1991), the sequence of autocovariances is not absolutely summable due to the very low rate of decay. Therefore at any moment in time the series X_i is path dependent.[2]

Historically, the importance of long-range dependence lies in the fact that it gives an explanation and interpretation of a long known empirical law known as *Hurst's Law* or the *Hurst effect*. Given a time series $(X_i, i \geq 1)$, with partial sum

$$Y(n) = \sum_{i=1}^{n} X_i, \qquad n \geq 1, \tag{8}$$

and sample variance

$$S^2(n) = n^{-1}\sum_{i=1}^{n}(X_i - n^{-1}Y(n))^2, \qquad n \geq 1, \tag{9}$$

the *rescaled adjusted range statistic* or *R/S-statistic* is defined by

$$\frac{R(n)}{S(n)} = \frac{1}{S(n)}\left[\max_{0\leq t\leq n}\left(Y(t) - \frac{t}{n}Y(n)\right) - \min_{0\leq t\leq n}\left(Y(t) - \frac{t}{n}Y(n)\right)\right], n \geq 1. \tag{10}$$

If we are considering a subset of the time series that starts at some time $t_0 > 1$ then the R/S statistic for this starting time is given by

$$\begin{aligned}\frac{R(t_0,n)}{S(t_0,n)} &= \frac{1}{S(t_0,n)}\left[\max_{0\leq t\leq n}\left(Y(t_0,t) - \frac{t}{n}Y(t_0,n)\right)\right. \\ &- \left.\min_{0\leq t\leq n}\left(Y(t_0,t) - \frac{t}{n}Y(t_0,n)\right)\right], n \geq 1, \end{aligned} \tag{11}$$

[2] If $d > 1/2$, then the series is non-stationary because its variance is not defined (Davidson and de Jong (1999)). However there is a misconception that the series is still "mean-reverting". This is not strictly true because if the variance does not exist then from Lyapunov's Central Limit Theorem, $\lim_{N\to\infty}\mathbf{P}\{\frac{1}{\sigma_N}\sum_{i=1}^{N}|X_i - \mu| \leq z\} = \Psi(z) := \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z}\exp(-z^2/2)dz$, provided that, amongst others, the following condition holds: $\mathbf{E}|X_i^{2+d}| < \infty$ for $d > 0$. For $d > \frac{1}{2}$, if the condition is not satisfied then the existence of the mean, as required by the theorem, is not obvious. Probably the most one can say is that there is some kind of 'median reversion'. That is the series returns to a value whereby half of the observations lie above it and half below it.

where $Y(t_0, n)$ is the partial sum staring at t_0:

$$Y(t_0, n) = \sum_{i=t_0+1}^{t_0+n} X_i, \qquad n \geq 1, \tag{12}$$

and $S(t_0, n)$ is the sample variance

$$S^2(t_0, n) = n^{-1} \sum_{i=t_0+1}^{t_0+n} (X_i - n^{-1}Y(t_0, n))^2, \qquad n \geq 1. \tag{13}$$

Hurst (1951) found that many naturally occurring processes with long-range dependence appear to be well described by the relation

$$\mathbf{E}[R(n)/S(n)] \sim c_1 n^H, \text{ as } n \to \infty, \tag{14}$$

with typical values of the *Hurst parameter* H in the interval (0.5,1.0), and c_1 a finite positive constant that does not depend on n. If, however, the observations X_i come from a short-range dependent model, then it is well known (Feller (1951), Mandelbrot and Van Ness (1968), Annis and Lloyd (1976)) that

$$\mathbf{E}[R(n)/S(n)] \sim c_2 n^{0.5}, \text{ as } n \to \infty, \tag{15}$$

where c_2 is independent of n, and finite and positive. The discrepancy between the relations (14) and (15) is referred to as the *Hurst effect.*

There is no universally agreed method that serves as an ultimate test of whether there is long-range dependence in a given time series. The main problems encountered are the lack of an infinite data set, and the obscuring effects of any short-range dependence that may be present. However, a variety of methods have been developed to the point where, if used in conjunction with each other, can give a reasonably conclusive view. We review the ones used in this study below. They are (i) *the variance-time function method* related to the slowly decaying variance property (5), (ii) *classical R/S-analysis*, (iii) *Lo's modified R/S-statistic*, (iv) Whittle's (1951) frequency domain maximum likelihood estimation technique. We also outline the shuffling procedures suggested in Willinger, Taqqu and Teverovsky (1999), that we used to destroy either short- or long-range dependence.

2.1 Variance-Time Plots

From (5) we see that long-range dependence gives rise to variances of the aggregated processes $X^{(m)}, m \geq 1$, that decrease linearly (for large m) in log-log plots against m with slopes arbitrarily flatter than -1. However, none of the short-range dependent models popular in finance can produce a power-law for the variances of the form (5). Whilst a long-range dependent process can be approximated for some transient period of time by a short-range dependent model with a large number of parameters, the variance of $X^{(m)}$ will eventually decrease linearly in log-log plots against m with a slope equal to -1 (see (6)). *Variance-time plots* are obtained by plotting $\log(\mathrm{var}(X^{(m)}))$ against $\log(m)$ ("time") and by fitting a simple least-squares line through the resulting points in the plane, ignoring points with small values of m. Typical variance-time plots are shown in Figure 2, for UK interest rates and for a normal i.i.d. process. Values of the estimate $\hat{\beta}$ of the asymptotic slope between -1 and 0 indicate long-range dependence, and an estimate $\hat{H}$ of the Hurst parameter is given by $\hat{H} = 1 - \hat{\beta}/2$. This method of testing for long-range dependence is slightly heuristic in nature, resulting in a point estimate of the Hurst parameter, and it will not be very reliable for empirical records with small sample sizes. However, for reasonably large records, variance-time plots are a useful and rather accurate tool (especially if used in conjunction with other tests) in helping to determine the presence or otherwise of long memory.

2.2 Classical R/S-Analysis

Classical R/S-analysis aims to infer from an empirical record the value of the Hurst parameter in (14) for the (supposed) long-range dependent process that generated the data. The procedure is somewhat heuristic (as are variance-time plots) and works as follows.

Given a sample of N observations $(X_k : k = 1, \cdots, N)$, we divide the sample into K non-overlapping blocks, each of size N/K. Then for each *lag* $n < N$ we compute the rescaled adjusted range $R(t_i, n)/S(t_i, n)$ for each of the new "starting points" $t_1 = 1, t_2 = 1 + N/K, \ldots, t_i = 1 + (i-1)N/K, i = 1, 2, \ldots, K$, which satisfy $t_i + n \leq N$. Thus, for a given i, all the data points prior to t_i are ignored. Furthermore, for a given lag n, we obtain many samples of $R(t_i, n)/S(t_i, n)$, as many as K for small values of n ($n < N/K$), and as few as one when n is close to the sample size N (when $n \geq N - N/K$). The values of $R(t_i, n)/S(t_i, n)$ corresponding to neighbouring values of t_i and n are strongly interdependent. That is, for a given $n > N/K$, the various estimates of the R/S statistic involve overlapping observations, and so do the estimates for different n but for a fixed starting point t_i. Next, we take logarithmically spaced values of n, starting with about $n = 10$, and plot $\log(R(t_i, n)/S(t_i, n))$ versus $\log(n)$, for all starting points t_i. This procedure results in a *rescaled adjusted range plot* (or *pox plot* or *pox diagram*) of R/S. An R/S plot for UK interest rates is shown in Figure 3, and Figure 4 shows the corresponding plot for a normal i.i.d. process.

When the parameter H in (14) is well defined, a typical pox plot starts with a transient zone representing the nature of short-range dependence in the sample, but eventually settles down and fluctuates in a straight "avenue" of a certain slope. Then an estimate $\hat{H}$ of the Hurst parameter H is given by the avenue's asymptotic slope (found by a least squares fit) which lies in the interval (0.5,1.0). To be effective, classical R/S-analysis, like variance-time plots, requires a large sample size. Its most useful feature is its robustness under changes in the marginal distribution of the data, whilst its weakness lies mainly in its sensitivity to the presence of explicit short-range dependence structures.

The lack of an underlying distribution theory for the statistic is also a barrier to its establishment as a rigorous statistical inference method.

2.3 Lo's Modified R/S-Statistic

Lo (1991), motivated by the R/S-statistic's main shortcoming (its sensitivity to short-range dependence) devised a *modified R/S-statistic*. He uses only *one* lag, $n = N$, the length of the time series. Also, instead of using the sample standard deviation S to normalize R, he uses a weighted sum of autocovariances, $S_q(N)$, defined by

$$S_q^2(N) \equiv \frac{1}{N}\sum_{j=1}^{N}(X_j - \bar{X_N})^2 + \frac{2}{N}\sum_{j=1}^{q}\omega_j(q)\left(\sum_{i=j+1}^{N}(X_i - \bar{X_N})(X_{i-j} - \bar{X_N})\right), \tag{16}$$

where $\bar{X_N}$ is the sample mean of the time series, and the weights $\omega_j(q)$ are defined by

$$\omega_j(q) = 1 - \frac{j}{q+1},\ q < N. \tag{17}$$

Lo then defines the modified R/S-statistic, $V_q(N)$, by

$$V_q(N) = \frac{1}{\sqrt{N}}\frac{R(N)}{S_q(N)}, \tag{18}$$

with $R(N)$ given by the numerator in (10) with $n = N$.

So Lo's modification is to replace the denominator S (sample standard deviation) in the R/S-statistic by a consistent estimator of the square root of the partial sum's variance. Note that in the case of short-range dependence, this variance is not simply the sum of the variances of the individual terms but also includes autocovariances up to some *truncation lag*, q, which has to be chosen with some consideration of the data at hand. This choice of q is crucial in determining the effectiveness or otherwise of the procedure, as we shall see in the sequel.

Lo (1991) shows that

$$\lim_{N\to\infty} \mathbf{P}\{V_q(N) \in [0.809, 1.862]\} = 0.95, \tag{19}$$

so that the interval [0.809,1.862] is used as the 95% confidence interval for testing the null hypothesis that there is no long-range dependence in the time series.

2.4 Maximum Likelihood Estimation

An excellent survey of maximum likelihood methods can be found in Baillie (1996). The estimation of the model's differencing parameter, d (related to the Hurst exponent H by $H = d + \frac{1}{2}$), for the stationary model is usually undertaken in the frequency domain. A widely used technique relies on the algorithm proposed by Geweke and Porter-Hudak (1983) (GPH). Further refinements were proposed by Robinson (1995) who suggested more robust procedures by adopting semi-parametric methods.

The joint estimation of the parameters in the ARFIMA (p, d, q) model can be undertaken using maximum likelihood methods. This can be done in both the time and the frequency domains. Whittle's (1951) method consists of approximating the exact likelihood in the frequency domain. Sowell (1992) derives the exact MLE under the assumption of unconditional normality of the forcing process. The likelihood function assumes the usual form:

$$L = -\frac{N}{2}\log(2\pi) - \frac{1}{2}\log|\Omega| - \frac{1}{2}X'\Omega^{-1}X \tag{20}$$

where $\Omega_{i,j} = \gamma_{|i-j|}$ is the autocovariance matrix and $X = (X_i, 1 \leq i \leq N)$ is an N-dimensional vector of observations. This method is computationally cumbersome as it requires the inversion of a NxN matrix of nonlinear functions of the hypergeometric function at each iteration of the maximisation of the likelihood function. An appealing alternative whose performance is highly credible is an extension of the Whittle likelihood function which is based on the observation that by transforming the observations vector X into the frequency domain the autocovariance matrix Ω is diagonal and so the likelihood function can be approximated by

$$L = \sum_{i=1}^{N-1} \log(2\pi) f(\lambda_i) + \sum_{i=1}^{N-1} [I_N(\lambda_i)/f(\lambda_i)], \tag{21}$$

where $I_N(\lambda_i)$ is the periodogram evaluated at each frequency λ_i and $f(\lambda_i)$ denotes the spectral density at the relevant frequency.

2.5 Shuffling the Data

One of the difficulties one encounters in using classical R/S analysis to test for long memory is the presence of explicit short-range structure, which can sometimes lead to incorrect inferences being drawn. Indeed, this was one of Lo's (1991) main motivations for introducing the modified R/S statistic. More generally, it is difficult to know whether a time series contains both long- and short-range structure.

With the above in mind, we performed a number of experiments on our original time series, in which we shuffled the data in an attempt to destroy some of the autocorrelation structure. Then we repeated our statistical tests for long memory.

2.5.1 Destroying Short-Range Dependence

Suppose one suspects that a time series is long-range dependent, but it is also believed that the series contains some short-range dependent structure. In this case it is difficult to conclude that the results of any statistical test arise purely from the long-range dependence in the series. One possible way of removing the short-range structure and isolating the (perhaps hidden) long memory is to partition the time series into non-overlapping blocks of size s (e.g. $s = 10, 20$), and to then randomly "shuffle" the observations *within* each block, whilst leaving the position of each block unchanged. Intuitively, the effect of such a shuffling experiment is to destroy any particular structure of the autocorrelation function below lag s but to leave the longer lag structure essentially unchanged.

We performed such a shuffling experiment on each of our original time series, and carried out tests for long memory on the shuffled time series, to ascertain whether each test was indeed detecting long-range dependence, or whether our results were due to the presence of short-range structure.

2.5.2 Destroying Long-Range Dependence

To make our results yet more robust, we performed a second shuffling experiment as follows. Once again the original time series was split into non-overlapping blocks of size s, but now the blocks themselves were randomly shuffled, whilst leaving the observations within each block intact. Such a shuffle has the effect of eliminating any long memory in the data whilst maintaining the possible short-range dependence. We then tested the shuffled series for long memory.

3 The Data

The data consists of four time series of overnight interest rates, for the US, UK, France and Germany, staring in January 1981 and ending in December 1998. Each time series contains 4694 observations, and the interest rates are the "offer" rate.

We also generated by computer a series of 4000 independent standard normally distributed observations, and tested the resulting series for long memory. Since we are certain that this series does not display long-range dependence, it serves as a useful "control" in our statistical tests.

4 Empirical Results

For all four interest rates series the unit root hypothesis is tested by calculating the Phillips-Perron (PP) and the Augmented Dickey-Fuller (ADF) test statistics. Initially we perform the tests by excluding the time trend as there is no theoretical justification for its presence. To establish the robustness of our results to the chosen specification of the test equation we add the time trend.[3] The truncation lag for the PP statistics were set at 8, following the rule ($l = O(N^{1/4})$ where l is the number of lags and N is the sample size. The lags for the ADF statistic were selected so as to produce white noise residuals from the test equation.

[3]For the samples under consideration the data exhibited some mild trending. Our tests for long memory were carried out with the original time series and also with a trend-corrected series, with no change to our overall conclusions.

(a): Phillips-Perron Test		
Series/Test	With Trend	Without Trend
DM	-3.44	-3.12
FF	-4.90	-2.40
UK	-51.72	-43.70
US	-4.42	-3.97
Critical value 5%	-3.41	-2.86
Critical value 10%	-3.12	-2.56
(b): ADF Test		
Series/Test	With Trend	Without Trend
DM	-1.71	-1,59
FF	-4.02	-1.91
UK	-7.98	-6.01
US	-4.61	-4.01
Critical value 5%	-3.41	-2.86
Critical value 10%	-3.12	-2.56

Table 1: Unit root tests

The usual unit root tests (Phillip-Perron and the Dickey-Fuller) are consistent against $I(d)$ alternatives, although their power grows more slowly as d diverges from one than with the divergence of the first order autoregressive parameter from unity. Simulation studies have shown that the when the data are generated by a fractional process with $d = .75$ (non-stationary value) the rejection frequencies, although low, increase with the sample size (Hassler and Wolters (1995)).

The use of two testing procedures could in principle produce contradictory results regarding the same series, as one test may reject the null of non-stationarity whilst the other may fail to do so. This may be taken as evidence that the statistical behaviour of the series in question may not be adequately described by either an I(0) or an I(1) process but by an I(d).

Table 1 presents our results from the above mentioned testing procedures both with and without the time trend. Independently of the specification adopted nearly all the series are shown to be stationary at the 10% level of significance (the exceptions being the FF series using the PP statistic without trend, and the DM and FF series using the ADF test without trend and DM with trend).

As auxiliary evidence we present the autocorrelation functions for the rates of interest using 10% of the available sample of 4694. For all the series the autocorrelation function declines hyperbolically but at slower rate than would be the case for a stationary series. This is most evident in the case of the UK interest rates where the function seems to stabilise over a long number of lags to a constant value (approximately 0.35). The other series behave in a similar manner, but with the autocorrelation function remaining near unity longer that the one observed in the UK. Figure 1 shows the autocorrelation function for UK interest rates.

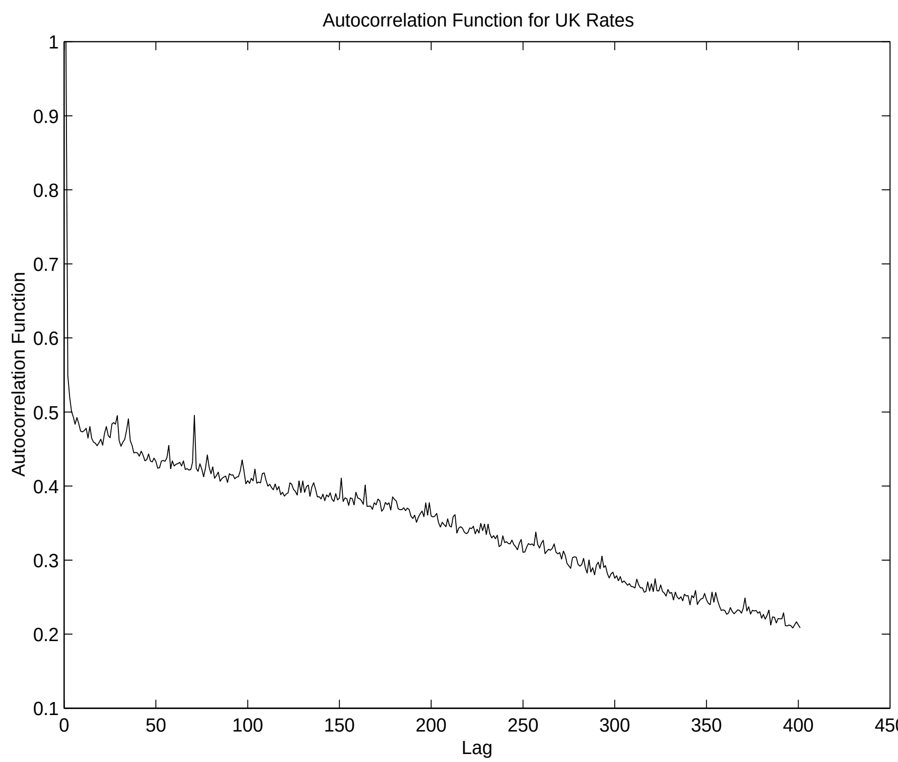

Figure 1: Autocorrelation function for UK interest rates

Figure 2 shows variance-time plots for UK interest rates and for a normal i.i.d. process. The graph for UK rates is relatively flat, indicating that the autocorrelation function is slowly decaying, which is consistent with the presence of long memory. The graph gives an estimate of the Hurst parameter of 0.96.

The variance-time plots for US, French and German interest rates (not presented here) are similar to those for UK rates, indicating the presence of long memory in each of the time series.

Figure 3 shows a classical rescaled range plot for UK interest rates. It is consistent with the results obtained from the variance-time plot in Figure 2, in that it gives an estimate of the Hurst parameter of 0.97, indicating that long memory is indeed present. This is to be contrasted with the R/S plot in Figure 4, for the normal i.i.d. process. In this case the estimate of the Hurst parameter is close to 0.5, as one would expect for a purely short-range dependent time

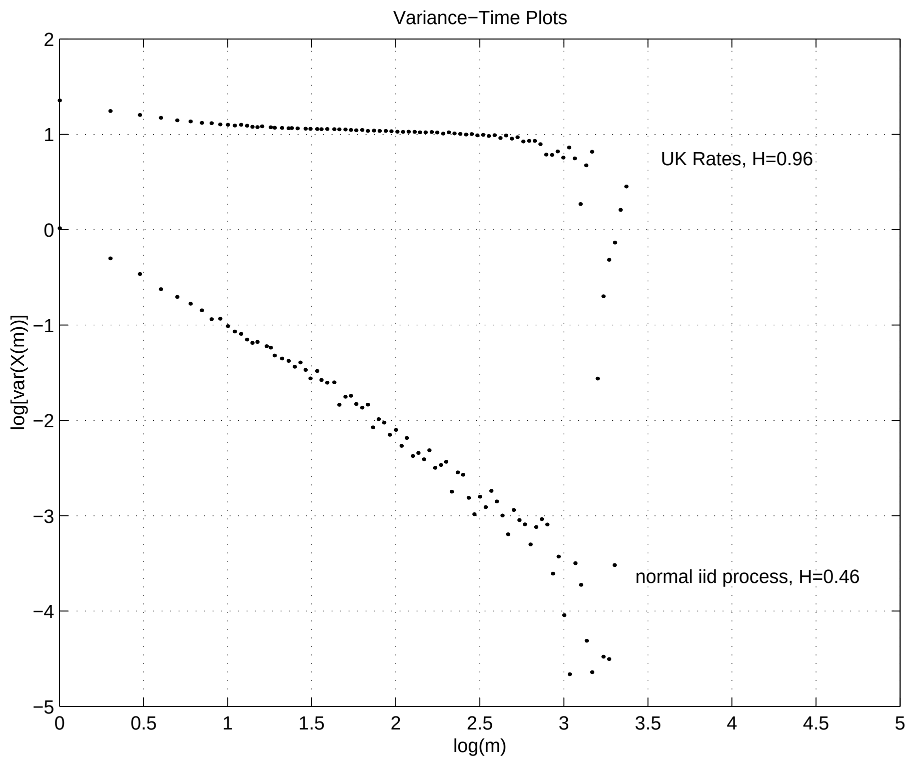

Figure 2: Variance-time plots for UK interest rates and for a normal i.i.d process

series.

Figures 5, 6 and 7 show R/S plots for US, French and German interest rates. Once again, they indicate the presence of long-range dependence, with estimates of the Hurst parameter broadly in line with those obtained from the variance-time plots.

Now we turn to the results obtained from Lo's modified R/S statistic. Figure 8 shows a plot of the modified R/S statistic $V_q(N)$, plotted for different values of the truncation lag q. The horizontal lines in the Figure indicate the interval $[0.809, 1.862]$. If the modified R/S statistic lies within this region, then according to Lo (1991), there is a 95% probability that the series is *not* long-range dependent. Figure 8 illustrates the problems one encounters with the new R/S statistic - it lies within the interval for no long-range dependence for some values of q, but not for others. If one uses a truncation lag below about $q = 250$, one would deduce that UK rates show long memory, but the opposite inference would be made for larger values of q.

Figure 9 shows a plot of the modified R/S statistic for a normally distributed

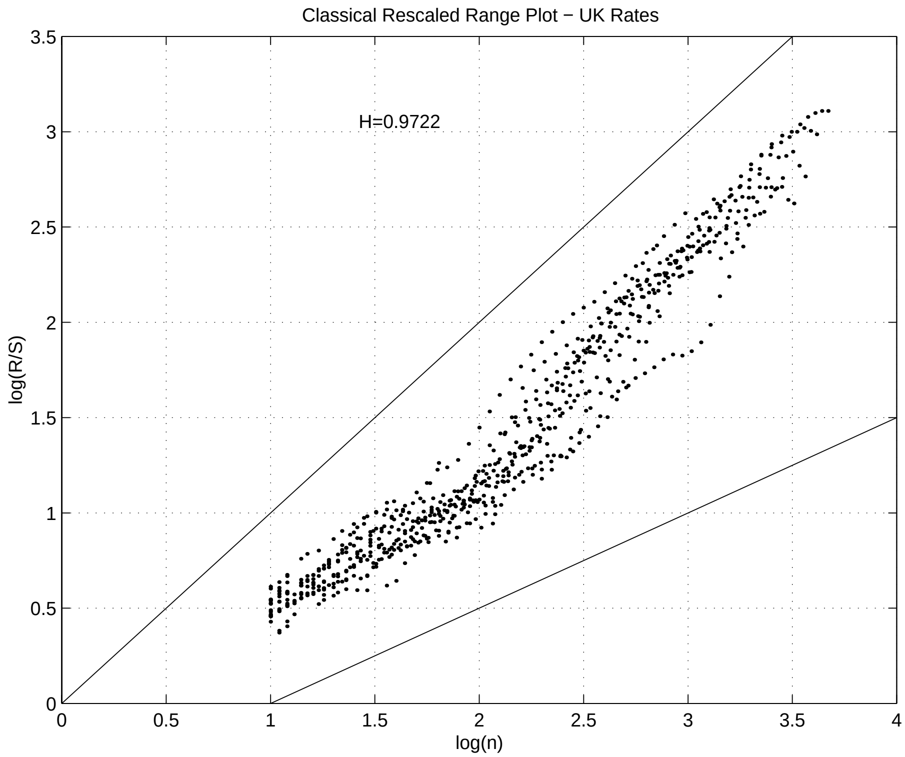

Figure 3: R/S plot for UK interest rates. The straight lines are of slope 0.5 and 1.0 and are included as visual aids.

i.i.d. process. For all values of the truncation lag q, the statistic lies within the 95% confidence interval for no long-range dependence. So we conclude that Lo's modified statistic is indeed able to correctly infer the absence of long memory.

Figures 10, 11 and 12 show plots of Lo's modified R/S statistic for US, French and German interest rates respectively. Once again we see that, depending on the choice of truncation lag q, one could infer both the presence and absence of long memory.

Using parametric methods based on fractionally differenced time series, we have estimated the fractional differencing parameter d, allowing for low orders of both autoregressive and moving average terms (≤ 1).[4] We found strong evidence that the series in question are characterised by long memory. In all four cases the parameter d was found to be significantly different from zero and from unity. These results are consistent with the bulk of evidence we presented using the non-parametric methods in the previous section.

[4] Estimates of d for higher orders of AR and MA polynomials are available from the authors.

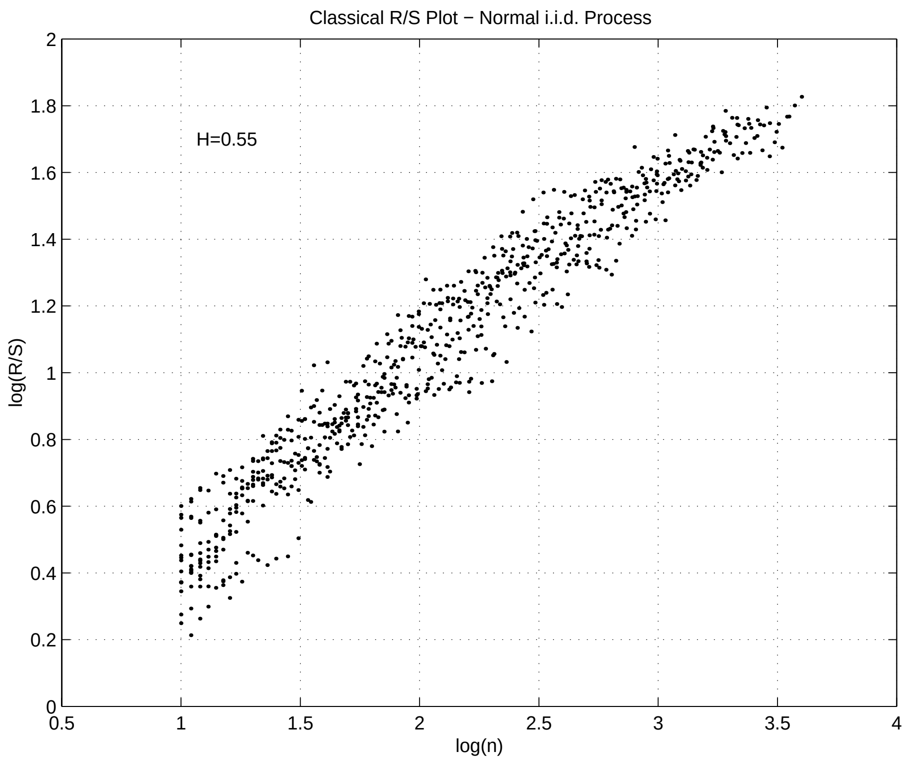

Figure 4: R/S plot for normal i.i.d. process

However the results from the parametric estimation using Whittle's likelihood cast doubt on the stationarity of the series. Unlike the results of the previous section we find that with the exception of the UK interest rates the other three series were found to be non-stationary (in the strict sense) as the estimated values of d exceed 0.5. Table 2 presents our results. The second column reports our estimates of the differencing parameter d when both the MA and AR orders have been set to zero. The entries in brackets are the standard errors. Subsequent columns report our estimates of d for differing orders of AR and MA polynomials.

The parametric estimation of d provides conclusive evidence of long memory. All the estimated coefficients are well below unity and significantly different from zero. Furthermore, the d estimates are comparatively impervious to the chosen order of the AR and/or MA polynomials.

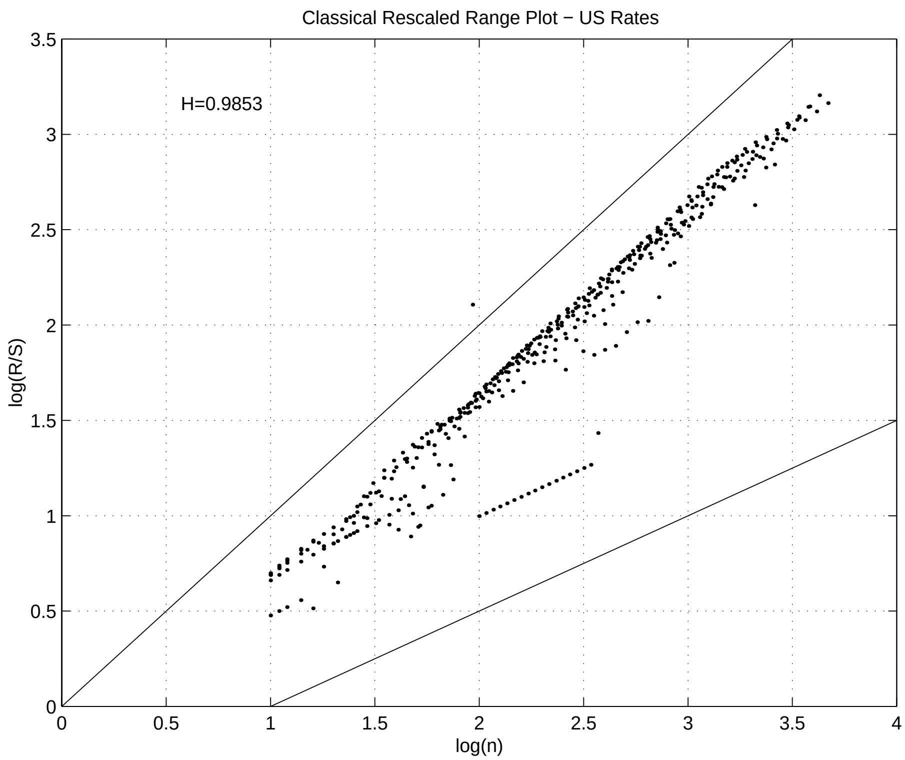

Figure 5: R/S plot for US interest rates

4.1 Results from Shuffled Data

As mentioned in Section 2.5 the data is shuffled to destroy, in turn, long and short-term memory, with a view to repeating the R/S analysis, taking on board the comments of Willinger, Taqqu and Teverovsky (1999) regarding the weaknesses of Lo's modified statistic.

In Figure 13 we show a classical R/S plot for UK rates, in which non-overlapping blocks of length $s = 10$ (i.e two trading weeks) have been randomly shuffled, leaving the observations within each block intact. We see that this shuffling experiment has indeed destroyed the long-range dependence in the time series, resulting in an estimate of the Hurst exponent which is in the region of 0.5. This is confirmed in Figure 14, which shows Lo's modified R/S statistic for the time series with long-range dependence destroyed.

In Figure 15 we show a classical R/S plot for UK rates, but with a random shuffling of observations *within* blocks of size $s = 10$, whilst the positions of the blocks was left unchanged. This shuffling procedure has destroyed the short-range structure in the time series, *but has left the long memory in the series*

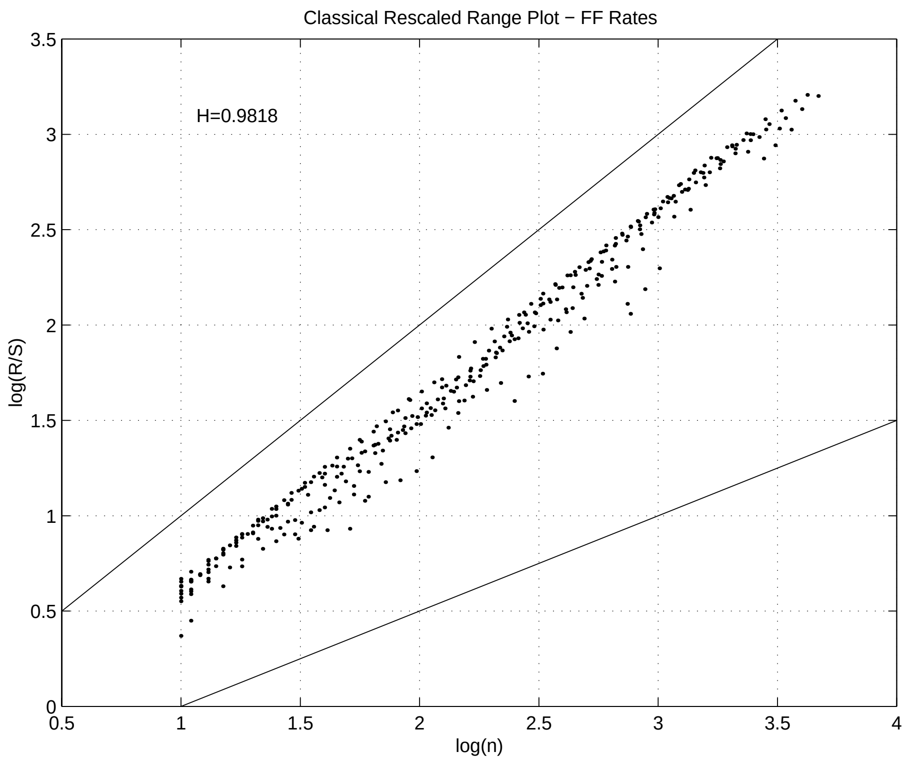

Figure 6: R/S plot for French Franc interest rates

intact, as Figure 15 confirms. Figure 16 shows Lo's modified statistic for this shuffled series, and wee see once again that, depending on the choice of the truncation lag q, the modified R/S statistic can suggest the absence of long memory, when it is highly likely that long-range dependence is still present. This highlights the care that must be taken in using the new test for long memory.

Similar shuffling experiments were performed on the US, French and German interest rate series, with very similar results.

5 Conclusions

In this paper we have analysed a number of methods for testing whether a time series displays long-range dependence, for daily interest rate series from four countries. Our goal was to illustrate that our results were not the outcome of a single testing methodology but that they are robust to the use of both parametric and non-parametric methods. We have found strong evidence of

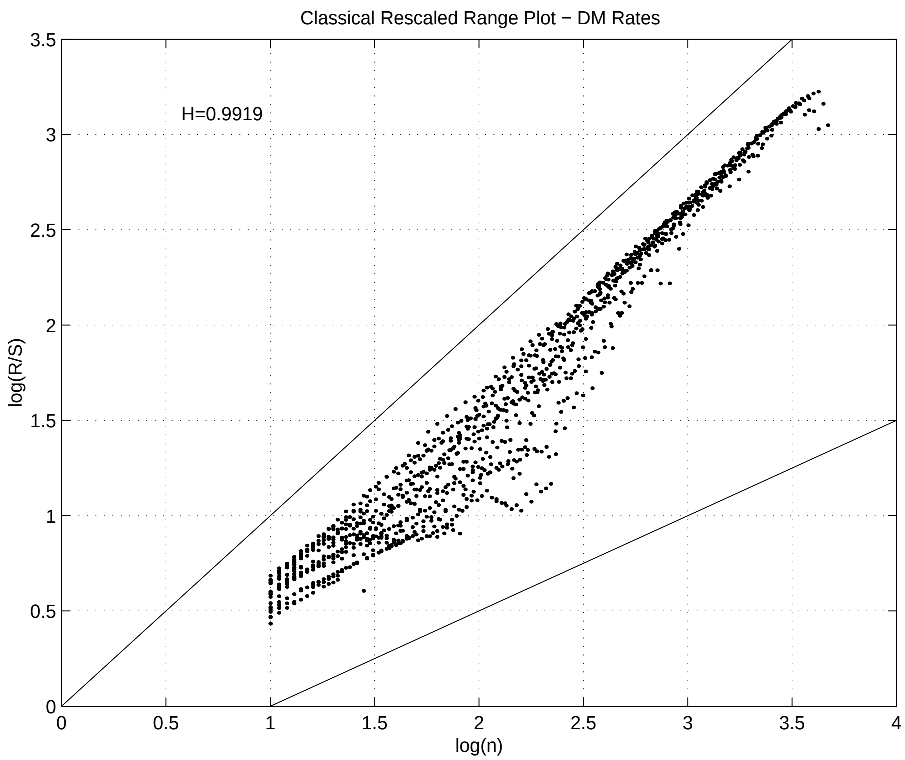

Figure 7: R/S plot for DM interest rates

long-range dependence in the nominal interest rates, and this is consistent with the findings regarding the rate of inflation reported by Hassler and Wolters (1995).

The results form Lo's modified R/S statistic are worthy of detailed comment. For time series which appear to display long memory (when tested by other methods), Lo's statistic lies, for a large range of values of the truncation lag q, within the confidence interval for no long-range dependence, as was also found by Willinger et al (1999). However, there are some (low) values of q for which the statistic lies outside the interval. For processes with no long-range dependence, the statistic always lies within the confidence interval for no long memory. Therefore, the use of Lo's statistic should be carried out across a range of values of the truncation lag, since it can sometimes lie within the 95% confidence interval for no long memory, even when other techniques strongly suggest that a series is indeed long-range dependent.

There are however remaining questions regarding the stationarity of the nominal rate. As we cannot reconcile on a-priori grounds the existence of non-

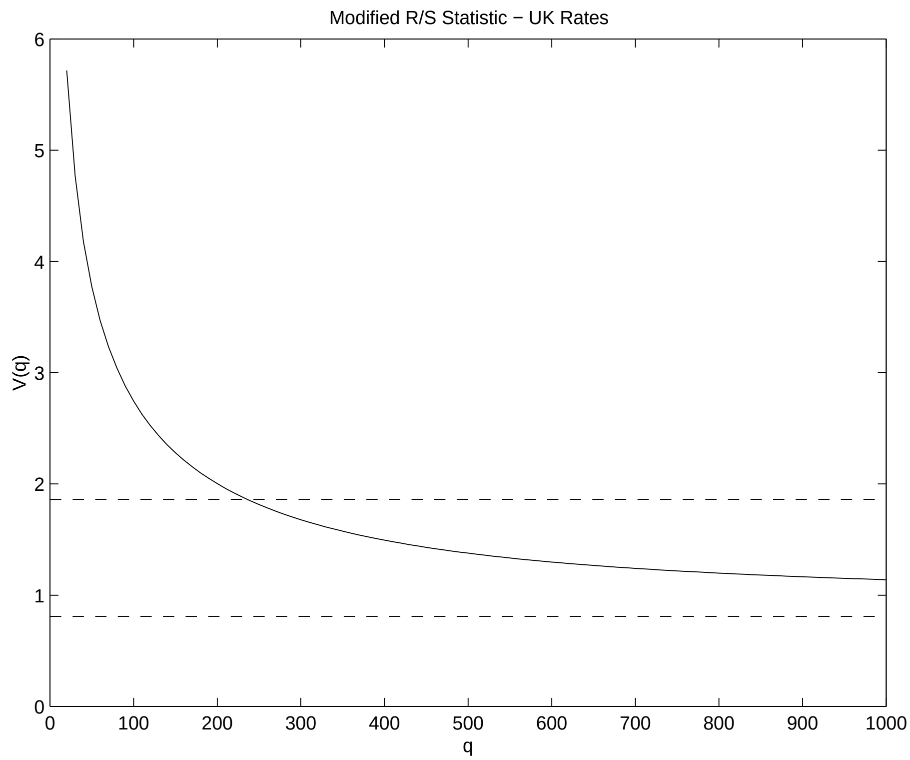

Figure 8: Lo's modified R/S statistic for UK interest rates

stationarity we conjecture that this finding may be consistent with a series of structural breaks that characterised the monetary policies of the countries in question. In a recent paper Granger and Hyung (1999) show that a series with breaks can mimic some of the properties of I(d) processes. From the countries we analysed, only the UK had the same government in power throughout the time of the sample with policy decisions taken by the Treasury and implemented by the Bank of England. The USA experienced at least two monetary regime changes and two governors of the Federal Reserve over the period. France entered a variety of exchange rate regimes during the early part of the sample, with the alignment of the French Franc to the Deutschemark. Finally, Germany experienced a major monetary shock with unification that placed severe fiscal pressures on the treasury and corrective actions by the Bundesbank. We are reasonably confident, from the UK results, that we have correctly identified long memory when it is present, but the nonstationarity may be due to structural breaks.

Our results indicate that most methods can correctly identify when a se-

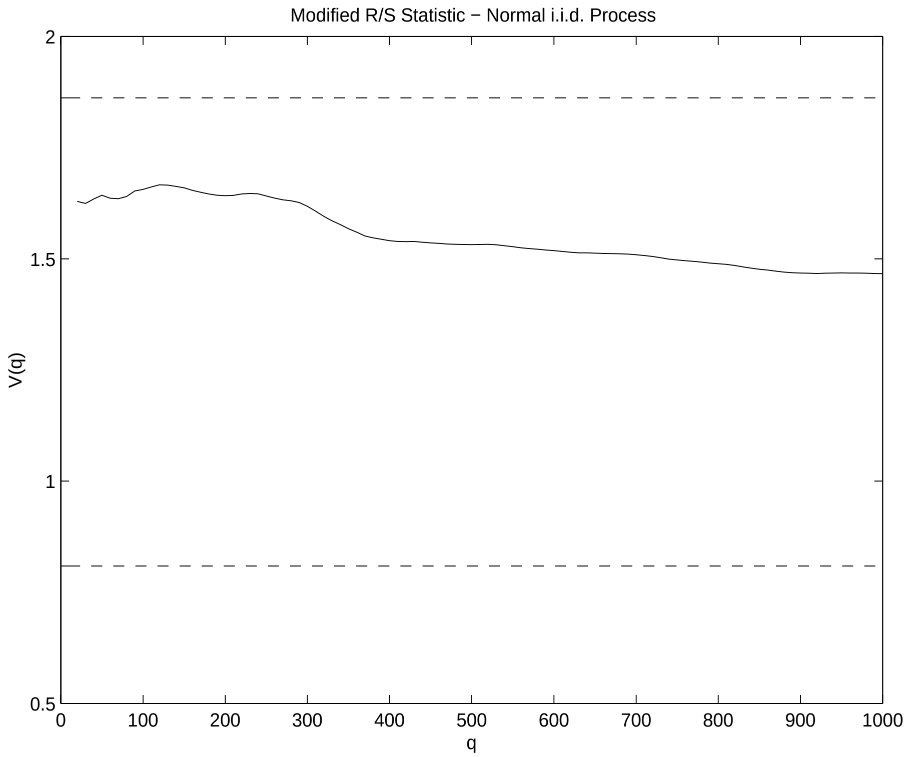

Figure 9: Lo's modified R/S statistic for normal i.i.d. process

ries does not exhibit long-range dependence, but when the series is suspected to display long memory, then the situation is less clear cut. In this case, one should not place too much reliance on one particular method. Instead, a variety of techniques should be used in conjunction with each other. These include variance-time plots, classical rescaled range plots, maximum likelihood estimation of fractionally differenced models, and Lo's modified R/S statistic. The use of Lo's statistic should be carried out across a range of values of the truncation lag, since it can sometimes lie within the 95% confidence interval for no long memory, even when other techniques strongly suggest that a series is indeed long-range dependent.

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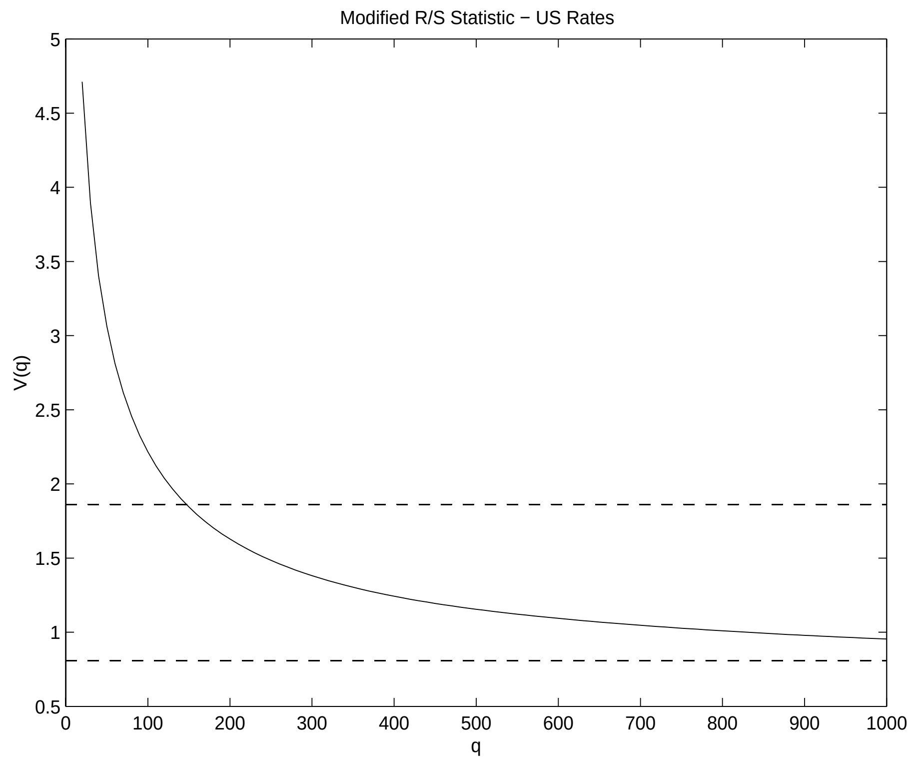

Figure 10: Lo's modified R/S statistic for US interest rates

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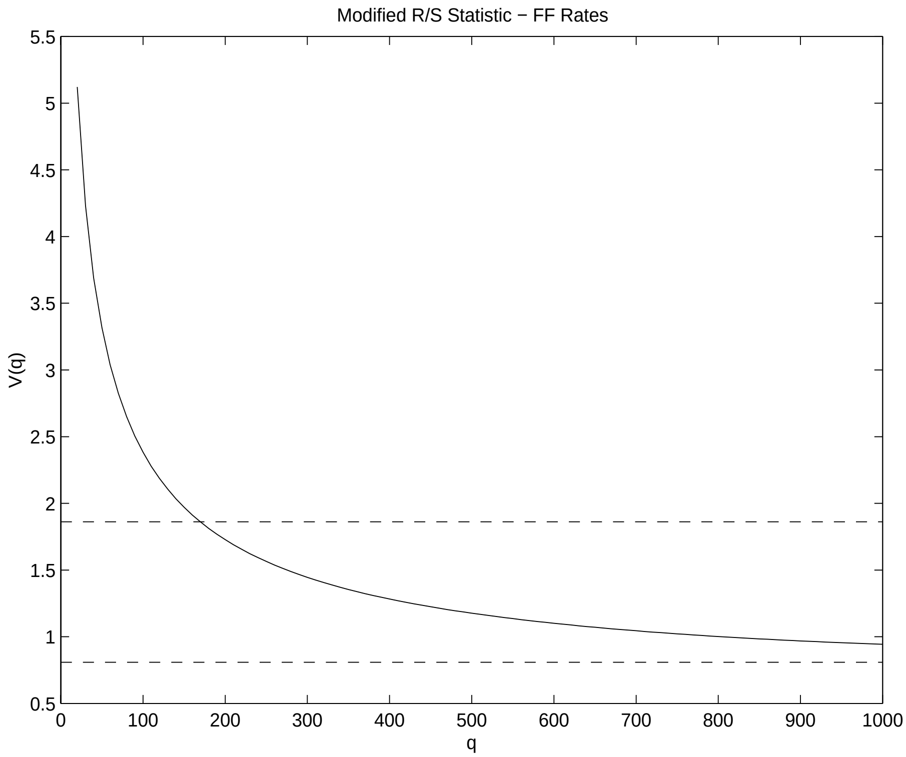

Figure 11: Lo's modified R/S statistic for FF interest rates

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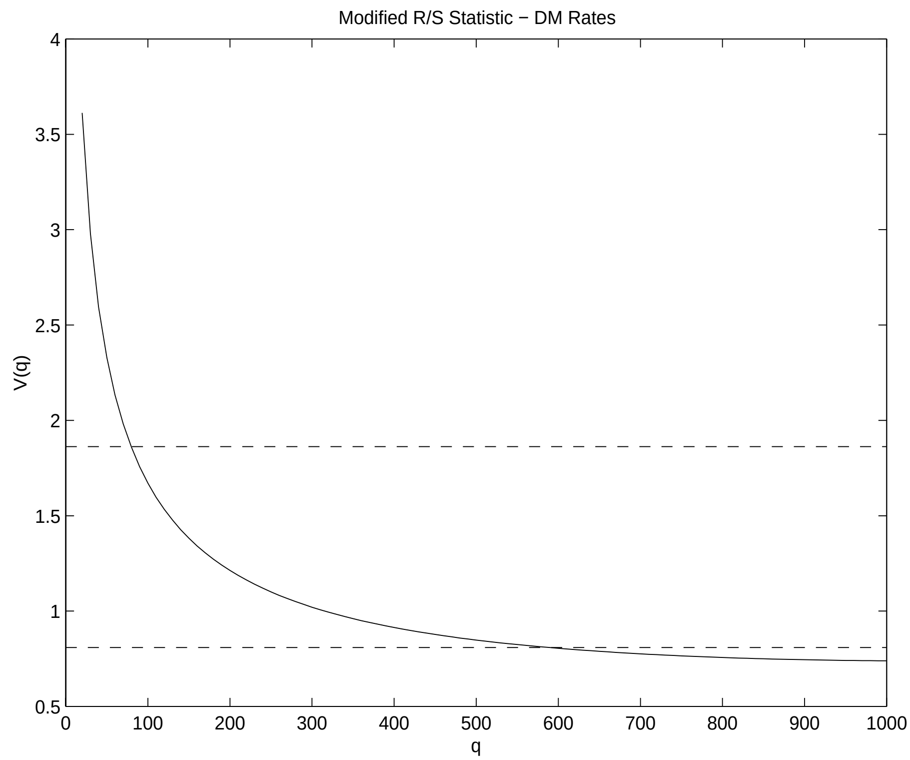

Figure 12: Lo's modified R/S statistic for DM interest rates

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Estimates of d				
Series	AR=0; MA=0	AR=1; MA=0	AR=0; MA=1	AR=1; MA=1
UK	0.252 (0.031)	0.307 (0.028)	0.335 (0.037)	0.506 (0.075)
US	0.891 (0.012)	0.688 (0.027)	0.729 (0.018)	0.689 (0.027)
DM	0.635 (0.023)	0.632 (0.031)	0.629 (0.041)	0.613 (0.029)
FF	0.901 (0.011)	0.860 (0.017)	0.843 (0.021)	0.851 (0.012)

Table 2: MLE results for fractional differencing parameter d

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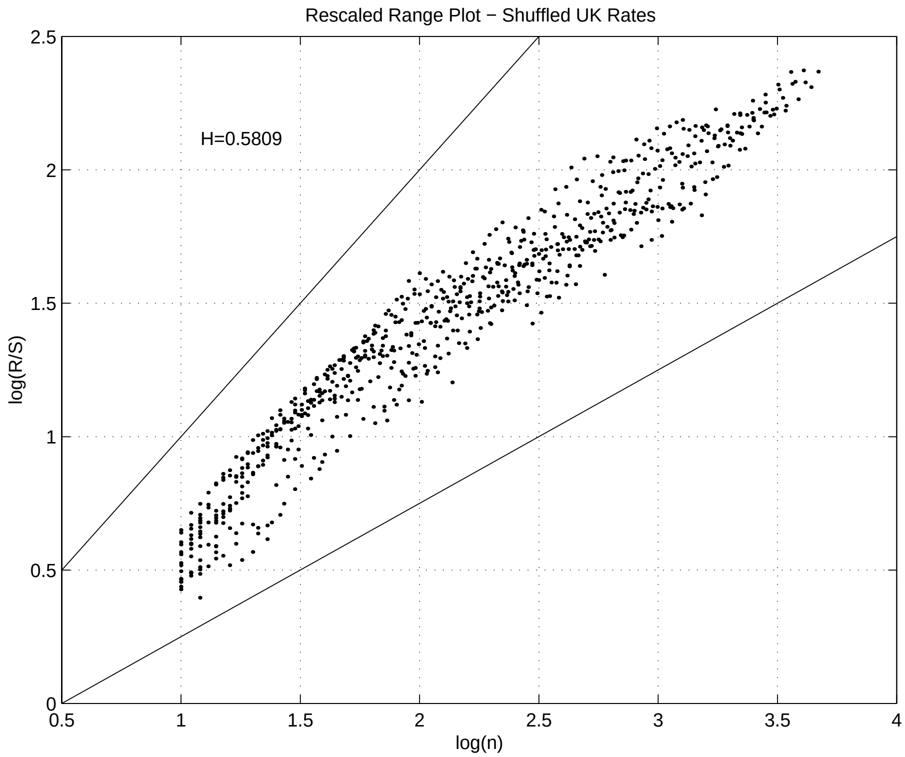

Figure 13: R/S statistic for shuffled UK interest rates, with long-range dependence destroyed

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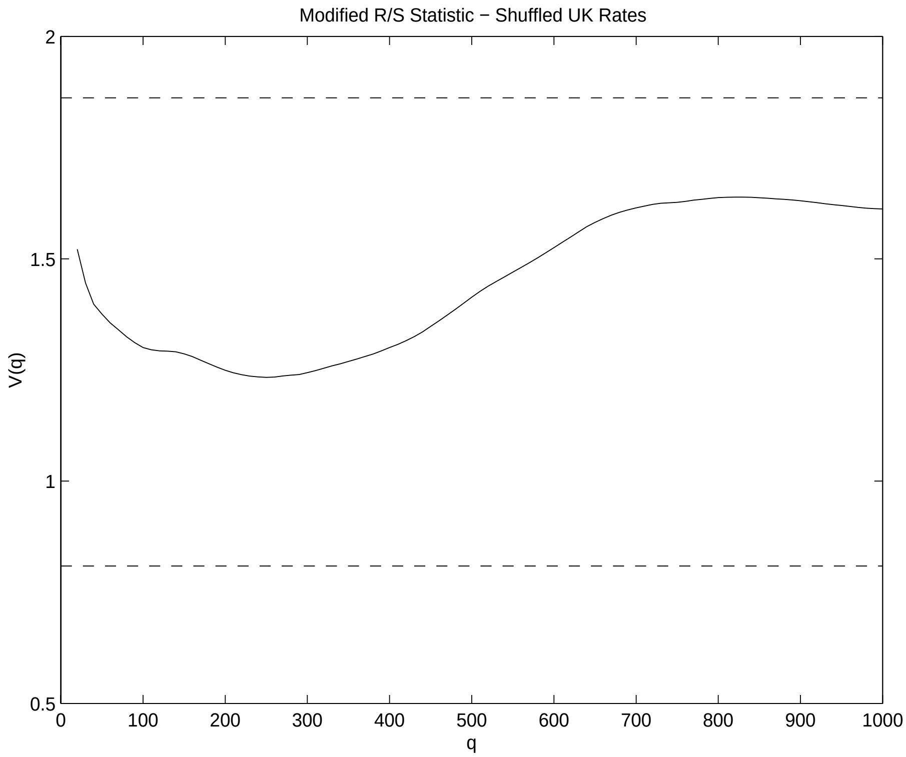

Figure 14: Lo's modified R/S statistic for shuffled UK interest rates, with long-range dependence destroyed

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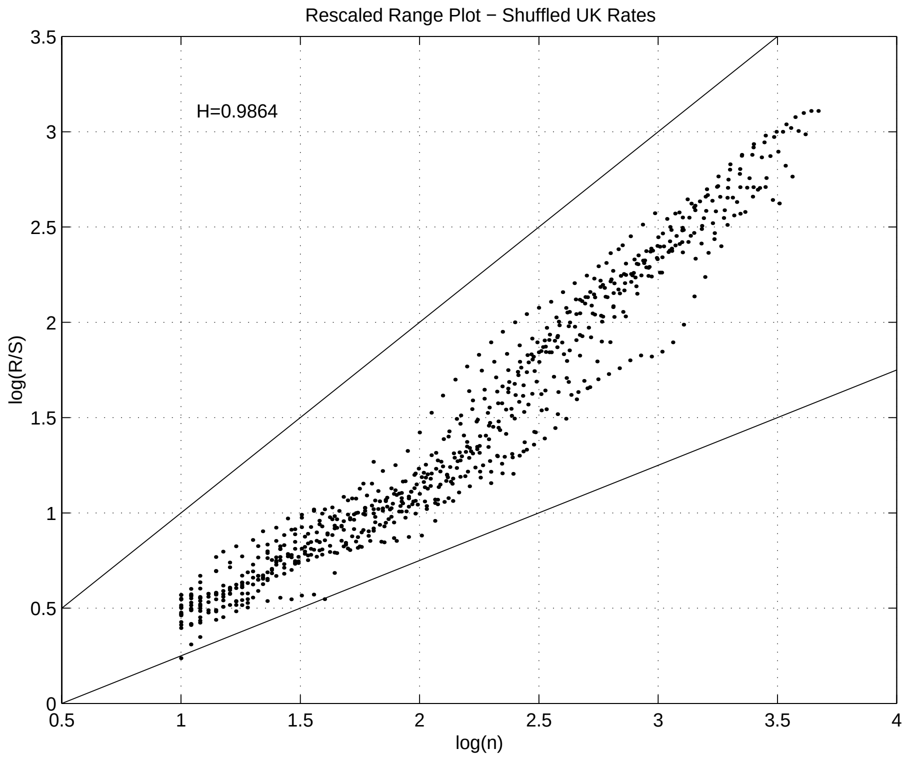

Figure 15: R/S statistic for shuffled UK interest rates, with short-range dependence destroyed

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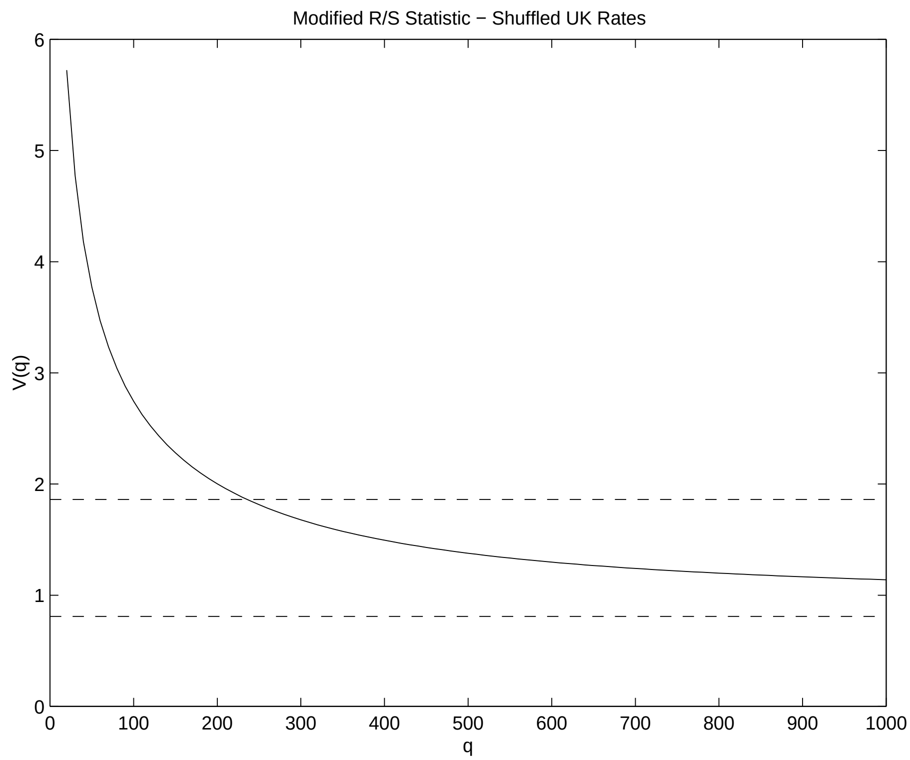

Figure 16: Lo's modified R/S statistic for shuffled UK interest rates, with short-range dependence destroyed