The predictability of excess returns on UK bonds: a non-linear approach

Ilias Lekkos*
Monetary Instruments and Markets Division
Bank of England

and

Costas Milas
Department of Economics and Finance
Brunel University, UK

Abstract

This paper provides an empirical description of the behaviour of excess returns on UK government discount bonds in terms of risk factors such as the forward premium, the slope of the term structure, dividend yields and excess stock returns. We identify the existence of a time-varying term structure of expected excess returns. Further, the dynamics of the expected returns are characterised by regime-switching behaviour where the transition from one regime to the other is controlled by the slope of the term structure of interest rates. The first regime, which is characterised by flat or downward sloping term structures, occurs during periods of economic recession. The second regime, which is characterised by upward sloping term structures, occurs during periods of economic expansion. The main risk factors explaining expected returns are the slope of the term structure in the recessionary regime and the excess stock returns in the expansionary regime.

Keywords: Interest rates; Excess returns; Smooth transition; Regime-Switching models.

JEL classification: C51, C52, E43.

*This paper represents work undertaken by the author before joining the Bank of England, while he was at Lancaster University. The views expressed here are those of the authors and do not represent those of the Bank of England.
Tel: +44 20 7601 5678, *e-mail*: i.lekkos@bankofengland.co.uk

1. Introduction

The development of capital asset pricing models such as the CAPM and the APT has motivated a great amount of research on documenting the existence and dynamics of predictable excess returns in different financial markets. The ability of dividend yields, slope of the term structure, short-term interest rates and default spreads to predict future excess returns in equity markets has been examined in Keim and Stambaugh (1986), Fama and French (1989), Campbell and Shiller (1988), Campbell (1991) and Bekaert and Hodrick (1992). The significance of the forward premium for forecasting excess returns in foreign exchange markets has been studied by Cumby (1988), Bekaert and Hodrick (1992) and Lewis (1995).

The extension of these asset pricing models to bond returns is not always straightforward. This led to the development of a number of alternative theories on the behaviour of excess bond returns; theories that are not always consistent with each other. According to different versions of the expectations hypothesis of interest rates, excess one-period returns can be zero, non-zero but constant or even time-varying. [1] Alternatively, some authors (Backus, Gregory and Zin (1989), and Boudoukh (1993)) try to explain the predictable variation of excess bond returns within a general equilibrium framework. While it is true that equilibrium models can provide valuable insights to the behaviour of the term structure of interest rates, the existing attempts to explain time-varying risk premia as functions of the underlying process for inflation and consumption have been met with limited success.

Following a different line of research, Fama (1976, 1984) presents evidence that the forward interest rates can predict the variation of expected holding period returns as well as expected changes in spot interest rates. In subsequent research, Fama and French (1989, 1993) examine the predictability of excess returns on stock and bond portfolios as well as the existence of common risk factors driving both bond and stock returns. Their main finding is that there exist at least three stock market and two bond market factors driving excess returns. Stock and bond excess returns exhibit a certain degree of covariation, which can be attributed mainly to the impact of bond market risk factors on stock returns. Elton, Gruber and Blake (1995) develop relative pricing models to explain the structure of excess returns of mutual fund bond indices. They find that while stock and bond indices are important in explaining the time series of returns, unexpected changes in GNP growth and unexpected changes in inflation improve the ability of asset pricing models to explain expected returns. Later work by Elton, Gruber and Mei (1996) examines asset pricing models where both the factor risk premia and the factor loadings are allowed to vary with respect to the underlying term structure. They conclude that a two-factor model of this kind is adequate in describing the variation in expected bond returns. Finally, Ilmanen (1995) addresses the issue of predictability of stock returns within an international framework. He documents the ability of a small number of global instruments to predict excess returns on long maturity bond indices in six countries.

In this paper we provide a detailed characterisation of the entire term structure of expected bond returns of the UK government discount bonds with maturity from 2 to 10 years. The main question we address throughout the paper is whether we can identify a time-varying component in expected returns across maturities. We analyse the predictability of annual UK government bond returns in excess of the 1-year UK T-Bill. We examine annual rather than monthly excess returns for two reasons. The first reason is rather practical and is dictated by data availability. Estimation of monthly excess returns requires the existence of a highly developed and liquid T-

[1] See Shiller (1979) for an attempt to express all different expectations hypotheses into a CAPM-type framework.

Bill market. While this might not be an issue in the US, where the T-Bills are some of the most liquid instruments available, this is not the case for the UK. The UK T-Bills market is highly illiquid and short-term interest rates are regularly distorted due to institutional and regulatory constraints. The second reason is that, with the exception of Fama and Bliss (1987) and Fama and French (1989), most empirical research on excess returns has focussed on monthly returns. As a consequence, the short-term dynamics of the estimated system are employed to impute the excess returns behaviour over the long run. The drawback of this procedure is that conclusions about the long-term behaviour of excess returns are valid only when the estimated model captures precisely the dynamics of the underlying process. For these reasons we choose to address directly the issue of the term structure of UK excess returns over a holding period of one year.

The first issue related to the documentation of the predictability of excess returns is the selection of the risk factors or forecasting variables to be employed in the analysis. Based on theory and previous empirical research, we investigate the ability of different factors, such as the forward premium, the slope of the term structure, dividend yields and excess stock returns to predict bond returns across maturities. The wide selection of risk factors allows us to identify their relative significance in accounting for excess returns over the maturity spectrum. This framework allows us not only to investigate whether excess returns increase monotonically with maturity or fluctuate according to the stage of the business cycle of the economy, but also examine the link between the risk premia demanded by fixed income investors and the performance of the stock market.

The second important issue we tackle is that of the dynamics of expected returns. Elton, Gruber and Mei (1996) model both the risk factors and the bond excess returns sensitivity to these factors as quadratic functions of the underlying term structure. In our research, we allow for a more complicated non-linear relationship between the explanatory variables and excess returns. In particular, we characterise the behaviour of excess bond returns using the Smooth Transition Autoregressive (thereafter STAR) methodology. STAR models were originally introduced by Teräsvirta and Anderson (1992) in order to examine non-linearities over the business cycle. The statistical properties of the STAR models are discussed in Granger and Teräsvirta (1993) and Teräsvirta (1994). The STAR methodology has been applied mostly to macroeconomic time-series. Our paper is one of the first attempts to extend the application of the STAR models to finance research. [2] The STAR model can be interpreted as a regime-switching model, where the transition from one regime to the other occurs in a smooth way. The transmission mechanism between regimes is a function of the underlying explanatory variables, such as the slope of the term structure or the excess stock returns. The increased complexity of our methodology allows us to study the relationship between the underlying forecasting variables and excess returns in more detail compared to previous work. For instance, when the transition mechanism is controlled by the slope of the term structure, we can differentiate between the impact of the term structure on excess returns during periods of economic expansion (when the term structure is upward sloping), and its impact on excess returns during recession periods (when the term structure is downward sloping). Furthermore, we can identify threshold levels for the underlying variables that mark the transition from one regime to the other, as well as the speed at which this transition takes place.

The outline of the paper is as follows. Section 2 discusses the selection of variables that may affect the variability of excess bond returns. Section 3 introduces the theoretical aspects of non-linear models in the context of the STAR methodology. Section 4 describes the data and section

[2] Other applications of STAR models in finance include the term structure of interest rates (Anderson (1997) and van Dijk and Franses (2000)) and the relationship between spot and futures prices of the FTSE100 index (Taylor, van Dijk, Franses and Lucas (2000)).

5 estimates STAR models for excess returns on UK bonds. Section 6 presents a discussion of our findings and section 7 provides some concluding remarks.

2. Selection of risk proxies

Asset pricing theory suggests that excess returns offer compensation to investors for systematic (i.e. non diversifiable) sources of risk. Unfortunately, theory provides little guidance on what these systematic risk factors might be, especially in the case of fixed income instruments. In order to address this issue, we examine the ability of a set of economic variables, that are likely to contain information for expected returns, to act as proxies for the risk factors. [3] Most of the variables we employ have been shown to explain some part of excess return variation. In this paper we examine their relative significance as explanatory variables in the presence of alternative risk factors.

Our first proxy for a risk factor is the forward premium, i.e. the difference between the *i*-maturity one-year forward rate and the one-period spot interest rate, denoted by $fspr\_i_t$ where i = 2, 5, 7 and 10 years. The use of the forward premium as a risk factor is motivated by Fama (1976, 1984). Given that the forward rates can be decomposed to an expected holding period return and the riskless one period interest rate, the forward premium should capture the variation of expected excess returns. By definition, the forward premium is maturity specific in the sense that the forward rate employed should correspond to the maturity of the bond whose excess return

[3] Given that the aim of our study is to identify the predictability of excess returns, we focus on factors that require a significant premium over the one-period riskless rate of return, i.e. factors that are "priced". Constantinides (1980) and Chan, Karceski and Lakonishok (1998) show that these factors do not always coincide with factors that can account for substantial return comovements.

is under consideration. In that respect, the forward premium expresses the ability of a specific section of the yield curve to account for excess return predictability. Fama (1976, 1984) provides evidence that the forward premium captures the variation in the expected holding period return of the *i*-maturity bond. In addition, Stambaugh (1988) shows that the use of forward risk premium as conditioning variable for expected excess returns can be justified within the framework of an equilibrium interest rate model like the one by Cox, Ingersoll and Ross (1985). Both Fama (1984) and Stambaugh (1988) test the ability of the forward premium to account for excess returns on US T-Bills with maturity up to one year. In our study, we test the ability of the forward premium to predict annual excess returns on bonds with up to 10 years time to maturity.

The slope of the term structure, $slope_t$, is also employed as a risk factor. The slope of the term structure is significant in two respects. It is easy to show (see e.g. Campbell and Shiller (1987), Campbell and Ammer (1993) and Evans and Lewis (1994)) that the slope of the term structure reflects expectations about future real interest and inflation rates. Hence, it is not surprising that a number of studies (see e.g. Estrella and Hardouvelis (1991), and Chen (1991)) have shown that the slope of the term structure is a suitable indicator of real economic activity. [4] Furthermore, the slope contains information about expected excess returns of long bonds over rolling over one-period bonds. In that respect, the slope of the term structure will also capture information about the variation of markets' expectations of expected risk premia across maturities.

In addition to the two term structure factors, we also test the significance of two stock market

[4] Estrella and Hardouvelis (1991) show that a steep term structure of interest rates precedes periods of economic expansion and that a downward sloping term structure is an indicator of economic recession. Estrella and Mishkin (1997) extend these results to the major European countries. For evidence on the significance of the slope for predicting real economic activity in the UK, see also Davis, Henry and Pesaran (1994).

related factors, that is, the excess stock returns of the FTSE All Share index, $xretft_t$, and the dividend yield, $dyall_t$, of the FTSE All Share portfolio. We introduce these two factors in order to analyse the dynamic links between stock and bond returns across the term structure of interest rates. Fama and French (1989, 1993) provide clear evidence that stock and bond markets are far from being segmented. If markets are integrated then variation in factors like stock market returns and dividend yields should also contribute towards accounting for the variation of bond returns.

The next section of the paper discusses non-linear models in the context of the STAR methodology that will be empirically tested on the behaviour of excess bond returns.

3. Specification of STAR models

We write a k-dimensional Vector STAR model as:

$$y_t = \left(m_1 + \sum_{j=1}^{p} \Phi_{1,j} y_{t-j} \right)(1 - G(s_t)) + \left(m_2 + \sum_{j=1}^{p} \Phi_{2,j} y_{t-j} \right) G(s_t) + e_t, \tag{1}$$

where y_t is a (k x 1) time series vector, $\Phi_{1,j}$ and $\Phi_{2,j}$, j = 1,…, p, are (k x k) matrices, m_1 and m_2 are $(k \times 1)$ vectors, and $e_t \sim iid\,(0, \Sigma)$. $G(s_t)$ is the transition function, assumed to be continuous and bounded between zero and one. The STAR model can be considered as a regime-switching model which allows for two regimes, $G(s_t) = 0$ and $G(s_t) = 1$, respectively, where the transition from one to the other regime occurs in a smooth way. The regime that occurs at time t is determined by the transition variable s_t and the corresponding value of $G(s_t)$. Different functional forms of $G(s_t)$ allow for different types of regime-switching behaviour. In particular, asymmetric adjustment to positive and negative deviations of s_t relative to a parameter c, can be obtained by setting $G(s_t)$ equal to the 'logistic' function:

$$G(s_t; g, c) = \{1 + \exp[-g(s_t - c)/s(s_t)]\}^{-1}, g > 0, \tag{2a}$$

where $s(s_t)$ is the sample standard deviation of s_t. The parameter c is the threshold between the two regimes, in the sense that $G(s_t)$ changes monotonically from 0 to 1 as s_t increases, and takes the value of $G(s_t) = 0.5$ at $s_t = c$. The parameter g determines the smoothness of the change in the value of the logistic function and thus the speed of the transition from one regime to the other. When $g \to 0$, the 'logistic' function equals a constant (i.e. 0.5), and when $g \to +\infty$, the transition from $G(s_t) = 0$ to $G(s_t) = 1$ is almost instantaneous at $s_t = c$.

Another type of regime-switching behaviour describing asymmetric adjustment to small and large absolute values of s_t is obtained by setting $G(s_t)$ equal to the 'exponential' function:

$$G(s_t; g, c) = 1 - \exp\{-g(s_t - c)^2 / s^2(s_t)\}, g > 0.$$

A possible drawback of the 'exponential' function is that the model becomes linear if either $g \to 0$ or $g \to +\infty$. This can be avoided by setting $G(s_t)$ equal to the 'quadratic logistic' function:

$$G(s_t; g, c_1, c_2) = \{1 + \exp[-g(s_t - c_1)(s_t - c_2)/s^2(s_t)]\}^{-1}, g > 0, \tag{2b}$$

as proposed by Jansen and Teräsvirta (1996). In this case, if $g \to 0$, the model becomes linear, whereas if $g \to +\infty$, $G(s_t)$ is equal to 1 for $s_t < c_1$ and $s_t > c_2$, and equal to 0 in between.

The estimation of STAR models consists of three steps:

Step 1: Specification of a base VAR model in linear form.

Step 2: Identification of possible candidates for the transition variable s_t. For each one of them, test for linearity against the STAR model (1) using the linear model specified in *Step* 1 as the null hypothesis. The null hypothesis of linearity takes the form: $H_0 : m_1 = m_2$, and $\Phi_{1,j} = \Phi_{2,j}$, for $j = 1,\ldots, p$, in model (1). By taking a first-order Taylor approximation of $G(s_t)$ around $g = 0$, the test can be done within the reparameterised model:

$$y_t = \mathrm{M}_0 + \sum_{j=1}^{p} \mathrm{B}_{0,j} y_{t-j} + \sum_{j=1}^{p} \mathrm{B}_{1,j} y_{t-j} s_t + \sum_{j=1}^{p} \mathrm{B}_{2,j} y_{t-j} s_t^{\ 2} + \sum_{j=1}^{p} \mathrm{B}_{3,j} y_{t-j} s_t^{\ 3} + e_t, \qquad (3)$$

where e_t are the original errors ε_t plus the error arising form the Taylor approximation. Here, the null hypothesis of linearity is $H_0 : B_{1,j} = B_{2,j} = B_{3,j} = 0$, for $j = 1,\ldots, p$. This is a standard Lagrange Multiplier (LM) type test.

Step 3: Selection of the regime-switching mechanism. If the p-value associated with the LM test rejects linearity in *Step* 2, proceed by selecting the appropriate form of the transition function $G(s_t)$, that is, select between the 'logistic' function (2a) and the 'quadratic logistic' function (2b). This is done by running a sequence of LM tests nested within the non-linear model (3) of *Step* 2, namely:

$$\begin{aligned} &\mathrm{H}_{03} : \mathrm{B}_{3,j} = 0, \\ &\mathrm{H}_{02} : \mathrm{B}_{2,j} = 0 \,|\, \mathrm{B}_{3,j} = 0, \qquad (4) \\ &\mathrm{H}_{01} : \mathrm{B}_{1,j} = 0 \,|\, \mathrm{B}_{3,j} = \mathrm{B}_{2,j} = 0. \end{aligned}$$

In this case, the decision rule is to select the 'quadratic logistic' function (2b) if the p-value associated with the H_{02} hypothesis is the smallest one, otherwise select the 'logistic' function (2a). Having done that, proceed by estimating the non-linear STAR model (1), with the transition function $G(s_t)$ specified based on the sequence of tests in (4).

4. The data

We estimate our models using monthly observations from January 1976 to June 2000. The annual returns on discount bonds with maturity from 2 to 10 years are estimated as the change in log-price of a bond at month t and $t-12$. The prices of the discount bonds are estimated using the term structure of zero-coupon interest rates implied by the UK government bonds. This methodology, developed by Anderson and Sleath (1999), estimates the term structure of zero-coupon interest rates by fitting a set of cubic splines to the observed bond prices while penalising curvature in the forward curve. The roughness penalty is constant from day to day but depends on maturity. The excess returns are calculated by subtracting the one-year T-Bill yield from the estimated bond returns.

The zero-coupon term structure is also employed to estimate the one-year forward rates at various maturities. The difference between the one-year forward rates 2, 5, 7 and 10 years in the future and the one-year T-bill yield is our estimate of the forward premium for the corresponding maturities. The slope of the term structure is the difference between the 10-year and 1-year zero-coupon rates. Finally, the excess stock returns are estimated as the change in log-FTSE All Share index between month t and $t-12$ minus the one-year T-Bill rate, while the dividend yield is calculated as the ratio of the dividends of FTSE portfolio for the year ending on month t over the value of the FTSE portfolio ending on month t.

Panel A of Table 1 reports the correlation between future excess returns and current bond market and stock market risk factors. The correlation between excess returns and bond market factors follows a U-shaped pattern as it starts from a specific level for $xret\_2_t$, then decreases for $xret\_5_t$, starts to increase again for $xret\_7_t$ and becomes even stronger for $xret\_10_t$. The dividend yield is positively correlated with excess returns across maturities and excess stock returns are negatively correlated with subsequent excess bond returns. Panel B of Table 1 reports the autocorrelation structure of the risk factors. All variables are highly autocorrelated but the autocorrelation declines rather rapidly in a few lags and drops close to zero after 8 lags (with the exception of the dividend yield). All variables in our analysis appear to be stationary over the sample period based on augmented Dickey-Fuller tests (results are available by the authors on request).

5. Estimation of STAR models

The starting point of our analysis is a linear VAR model for excess returns on UK bonds with maturity of 2, 5, 7 and 10 years. Following the discussion in *Step* 1 of Section 3, we specify four different VAR models, each one of them consisting of k = 5 endogenous variables:

$$y_t = [xret\_i_t, slope_t, fspr\_i_t, dyall_t, xretft_t]', \qquad (5)$$

where $xret\_i_t$, i = 2, 5, 7, 10, is the i-th maturity one-year excess return over the one year T-Bill rate, and all other variables have been defined in Section 2.

Taking into account that a high-order VAR may cause over-fitting and add considerably to the difficulties associated with getting converging estimates for the non-linear models, we restrict our analysis to first order VAR models (i.e. we set p = 1 in the linear VAR models (5) above). We have also tried second order VAR models but all second lags turned out to be insignificant. One issue related to the estimation of all linear and non-linear models in this paper is that of a priori autocorrelation due to the use of overlapping monthly observations of annual returns. To deal with this issue, all models reported in our paper have been estimated using the Generalised Method of Moments (GMM; see Hansen, 1982), which is robust to heteroskedasticity and autocorrelation of unknown form. All insignificant regressors are dropped based on the χ^2-version of the Wald test. A notable feature of this VAR is that the dividend yield ($dyall_{t-1}$) is not statistically significant in any of the $xret\_i_t$ (i = 2, 5, 7, 10) models. It only becomes significant when the FTSE excess returns ($xreft_{t-1}$) are excluded from the models. Given that all explanatory power of the dividend yield is absorbed by the excess stock returns, we drop the dividend yield as an explanatory variable in the subsequent analysis. In addition, $fspr\_5_{t-1}$ and $fspr\_10_{t-1}$ are also dropped as they turn out to be insignificant in the corresponding linear models. [5]

Having estimated the base linear models, we move on to *Step* 2 of our methodology which involves testing for the existence of non-linear dynamics in the $xret\_2_t$, $xret\_5_t$, $xret\_7_t$ and $xret\_10_t$ models. Given the lack of previous work that could guide our selection of transition variables suitable for controlling the non-linear dynamics, we test the performance of all three remaining variables, that is, $slope_{t-1}$, $fspr\_i_{t-1}$, (i = 2 and 7) and $xreft_{t-1}$, as possible transition candidates s_t. According to the results reported in Table 2, the null hypothesis of linearity, (i.e. H_0) is strongly rejected for all expected returns and all three transition variables. The sequence of tests (i.e. H_{03}, H_{02}, and H_{01}, respectively) favours the 'logistic' model (2a) as the appropriate transition function in all cases. The selection of this particular functional form indicates a

[5] To save space, the linear VAR estimates are not reported here but are available on request. In providing consistent parameter estimates, the GMM estimator makes use of a weighting matrix. We have used (i) the 'Bartlett' kernel to weight the autocovariances in computing the weighting matrix, and (ii) the fixed 'Newey-West' bandwidth to determine how the weights given by the kernel change with the lags of the autocovariances in the computation of the weighting matrix. The number of lags used to correct for autocorrelation depends on the autocorrelation structure we report in Table 1.

specific type of non-linear dynamics that allows for asymmetric adjustments of expected excess returns to positive and negative deviations of the transition variable from its threshold level. It is notable that in the case where more than one transition variable candidates exist, Teräsvirta (1994) suggests that the null hypothesis of linearity (i.e. H_0) can be used to select the appropriate transition variable in the STAR model. In particular, s_t is selected as the one for which the *p*-value of the test is the smallest one. The results in Table 2 suggest that the slope of the term structure is the appropriate transition variable in all cases but one. [6]

Next, we move to *Step* 3 of our analysis which involves the estimation of the non-linear models. Before doing that, it is worth mentioning that Granger and Teräsvirta (1993) and Teräsvirta (1994) stress particular problems like slow convergence or overestimation associated with estimates of the g parameter. For this reason, we follow their suggestion in scaling the 'logistic' function (2a) by dividing it by the standard deviation of the transition variable $\sigma(s_t)$, so that g becomes a scale-free parameter. Based on this scaling, we use $g = 1$ as a starting value and the sample mean of s_t as a starting value for the parameter c. The estimates of the first order parsimonious linear VAR equations for $xret\_i_t$, (i = 2, 5, 7, and 10) are used as starting values for the parameters in the STAR model (1).

Tables 3, 4, 5, and 6 report the GMM estimates of the non-linear models for the excess returns, $xret\_2_t$, $xret\_5_t$, $xret\_7_t$ and $xret\_10_t$, respectively, using $slope_{t-1}$ as the transition variable. For all estimated models, the error variance ratio of the non-linear relative to the linear models (i.e. s^2_{NL}/s^2_L) is less than one, indicating that the non-linear models have a better fit. In particular, the

[6] From Table 2 we can see that in the case of the *xret*_5 model, the smallest *p*-value associated with the H_0 hypothesis refers to $s_t = xretft_{t-1}$. However, our attempt to estimate a non-linear model for *xret*_5 using $xretft_{t-1}$ as the appropriate transition variable produced unsatisfactory estimates. For this reason, we decided to use $s_t = slope_{t-1}$ for this model as well. Estimates of the non-linear $xret\_i_t$ (i = 2, 5, 7, 10) models with $s_t = xretft_{t-1}$ and $s_t = fspr\_i_{t-1}$ (i = 2 and 7), respectively, are available by the authors on request.

s^2_{NL}/s^2_L ratio shows a reduction in the residual variance of the non-linear compared to the linear models ranging from about 4.24 percent for the $xret\_5_t$ model (see Table 4) to about 22.7 percent for the $xret\_7_t$ model (see Table 5). These results indicate that predictable excess returns are not only time-varying but that they also exhibit a regime-switching behaviour, which is successfully defined by the evolution of the slope of the term structure of interest rates.

The main parameters of interest in the STAR models are the estimates of the threshold level, c and the speed of adjustment, g. The c estimates reported in Tables 3 to 6 are statistically significant for all models. The estimates of the threshold parameter c range from 0.003 (or 0.3 percent) for both the $xret\_5_t$ (see Table 4) and $xret\_10_t$ models (see Table 6) to 0.011 (or 1.1 percent) for both the $xret\_2_t$ (see Table 3) and $xret\_7_t$ models (see Table 5). These estimates of the threshold parameter, c, indicate the existence of two regimes for predictable excess returns, one of which is characterised by a downward sloping term structure and an alternative one which is characterised by an upward sloping term structure. The economic implications of these results will be discussed in the following section. The estimates of the g parameter are rather similar for all models indicating that the transition from $G(s_t;\, g,\, c) = 0$ to $G(s_t;\, g,\, c) = 1$ at the estimated thresholds c is uniform across the term structure of expected returns. Notice, however, the rather high standard error of the g estimates for the fitted models. Teräsvirta (1994) and van Dijk, Teräsvirta and Franses (2000) point out that this should not be interpreted as evidence of weak non-linearity. Accurate estimation of g might be difficult, as it requires many observations in the immediate neighborhood of the threshold c. Further, large changes in g have only a small effect on the shape of the transition function implying that high accuracy in estimating g is not necessary.

6. Interpretation of the results

Our research identified the existence of a time-varying term structure of expected excess returns in the UK government bond markets. Moreover the expected returns exhibit a regime-switching behaviour according to the variation of $slope_{t-1}$. This result confirms the paramount importance of the slope of the term structure of interest rates as a factor affecting the evolution of excess bond returns across maturities. The slope of the term structure dominated both maturity specific changes in interest rates and excess stock returns as a proxy for the transition variable that defines the switching from one regime to the other. Furthermore, the regimes we identify have a straightforward economic interpretation. The first regime (i.e. $G(slope_{t-1}; g, c) = 0$), which is defined by a flat or downward sloping term structure, is identified with periods of recession or more generally with periods of subdued economic activity. Conversely, the second regime (i.e. $G(slope_{t-1}; g, c) = 1$), which is defined by an upward sloping term structure, is identified with periods of economic expansion. The relationship between the occurrence of a regime and the slope of the term structure is depicted in Figure 1, which plots the values of the transition function against $slope_{t-1}$. As discussed above, values of zero and one for the transition function are related to the occurrence of the first and second regime, respectively.

Figure 2 plots the estimated transition function for each maturity against time in order to illustrate the succession of the regimes over the sample period. Periods from 1976 to 1980, 1982 to 1986 and 1994 to mid 1998 are classified into the expansionary regime. On the other hand, the estimated transition function for each maturity against time suggests that periods from 1980 to 1982, 1986 to 1988, mid 1989 to 1993 and mid 1998 to 2000 are classified into the recessionary regime.[7] In addition, we superimpose in Figure 2 the corresponding expected bond returns estimated as the fitted values from our non-linear models. The graphical evidence reveals a complex relationship between expected bond returns and the two regimes. For the period from 1977:01 to 1986:10, high expected returns coincide with the recessionary regime and low expected returns with the expansionary regime. This relationship seems to be reversed in the second half of the sample period. From 1986:11 to 2000:6, high expected returns correspond to the expansionary regime and low returns to the recessionary regime.

Up to this point our analysis has focussed on the non-linear features of the model and the use of the risk factors as potential transition variables. The fact the we have rejected the forward premia and excess stock returns as transition variables does not imply that they are not significant for explaining expected excess returns. On the contrary, our models allow for the behaviour and significance of the risk factors to vary across regimes. By comparing the coefficients for $slope_{t-1}$, $fspr\_i_{t-1}$, and $xretft_{t-1}$ in the two regimes (coefficients $f_{1,j}$ and $f_{2,j}$, j = 1,…,3 in Tables 3 to 6) we see that $fspr\_i_{t-1}$ is significant for the $xret\_2_t$ and $xret\_7_t$ models but only in the recessionary regime (see Tables 3 and 5, respectively). In these cases where $fspr\_i_{t-1}$ is significant, its effect on expected returns is opposite to that of $slope_{t-1}$.

The significance of $slope_{t-1}$ as a risk factor accounting for the variability of expected bond returns also varies across regimes. While $slope_{t-1}$ has no explanatory power in the expansionary regime, it is highly significant in the recessionary one. Further, its effect on expected returns

---

[7] One caveat has to be made with respect to the last period between mid 1998 and 2000. This period is characterised by a downward sloping term structure. Nevertheless, this is not due to the economy being in a trough of the business cycle. Instead, the inverted term structure is a result of regulatory changes in the UK that force UK pension funds to invest heavily in long-maturity government bonds, pushing bond prices higher and yields lower. This set of regulations is known as Minimum Funding Requirement (MFR). The impact of the MFR is compounded by the low levels of issuance of new debt due to improving government finances.

varies with bond maturity. In particular, $slope_{t-1}$ has a negative effect on $xret\_2_t$ (see Table 3) and a positive effect on $xret\_5_t$, $xret\_7_t$ and $xret\_10_t$ (see Tables 4 to 6, respectively), with the strongest relationship being with $xret\_7_t$. The positive relationship between expected returns and the slope of the term structure is counterintuitive. The expectations hypothesis advocates a negative relationship between expected returns and the slope of the term structure. This is based on the argument that in a situation where long rates are lower than short rates, the holder of the long bond has a yield disadvantage compared to the holder of a short bond. Long yields have to decrease in order to create capital gains that will offset the yield disadvantage of the long bond holder. The positive relationship we estimate between medium and long bond returns and the slope of the term structure means that the yield disadvantage is amplified rather than offset by term structure movements. This paradoxical result was first documented in the US bond market by Macaulay (1938) and more extensively studied by Campbell and Shiller (1991) and Shiller, Campbell and Schoenholtz (1983). [8]

Our results allow us to re-examine the relationship between the stock and bond markets. According to the results reported in Tables 3 to 6 (see coefficients $f_{1,3}$ and $f_{2,3}$ in the Tables), there exists a negative relationship between stock returns ($xretft_{t-1}$) and bond returns for all maturities ($xret\_i_t$, i = 2, 5, 7, and 10). This negative relationship is statistically significant only in the expansionary regime. The only exception is $xret\_2_t$ where $xretft_{t-1}$ is significant in both regimes. The negative relationship reported in our paper seems to contradict previous research by Fama and Schwert (1977), who report a positive relationship between stock and bond markets.

8 In fact, Campbell and Shiller (1991) and Shiller, Campbell and Schoenholtz (1983) identify the other side of the same paradox i.e. when the term structure is upward sloping the long rates do not rise enough to create capital losses that will eliminate the yield advantage of the long bond. One possible explanation for the different way that this paradox emerges in the US and the UK, is the fact that the UK term structure is more often downward sloping than the US term structure. Based on data reported in Ang and Bekaert (1998), between 1972:01 and 1996:08 the US economy was in recession for 50 month and in expansion for 247 months. The mean of the slope during the two regimes was 0.55% and 1.38%, respectively. This compares with 128 months in recession and 169 months in expansion for the UK and corresponding means for the UK slope of –0.49% and 0.80%.

Nevertheless, our results might not be as controversial as they seem at first. Campbell and Ammer (1993) report a positive effect of expected inflation on stock prices. [9] This can be justified on the grounds that stock dividends are relatively stable in real, not nominal terms. They also report a negative impact of expected inflation on bond prices. The combined effect of future inflation on the stock and bond market can create a negative relationship between the two markets. These results are corroborated by the results of Titman and Warga (1989) who report a positive relationship between stock returns and future inflation and a negative relationship between stock and bond returns. An alternative or possibly complementary argument in support of our finding is related to the linkage between the two markets through portfolio flows. Strong stock market performance has a negative impact on bond returns when funds are diverted from the bond to the stock market. On the other hand, during periods of stock market underperformance, investors pull out of the capital markets and invest heavily on bonds driving interest rates lower and bond prices and returns higher.

7. Conclusions

In this paper we model the behaviour of expected excess returns on UK government discount bonds as a function of a number of risk factors; namely the forward premium, the slope of the term structure, the excess stock returns of the FTSE All Share index and the dividend yield of the FTSE All Share portfolio. Using a non-linear framework we are able to identify a time-varying component of expected excess returns across maturities. The dynamics of the expected returns we identify exhibit a regime switching behaviour which is controlled by the slope of the term structure of interest rates. The first regime, which is characterised by flat or downward sloping

[9] As Campbell and Ammer (1993) point out, the positive effect between expected inflation and stock returns is consistent with a weak negative relationship between contemporaneous stock returns and inflation reported by Fama and Schwert (1977).

term structures, occurs during periods of economic recession. The second regime, which is characterised by upward sloping term structures, occurs during periods of economic expansion. The relationship between the economic regimes and expected bond returns is not uniform throughout the period we study. In the first half of the sample (from 1977:01 to 1986:06), bond holders require high returns during periods of recession and low returns during periods of expansion. This relationship is reversed in the second half of the sample.

The ability of the risk factors to account for the variability of expected returns varies across regimes. The slope of the term structure of interest rates is the main factor driving expected returns in the recessionary regime. The stylised fact, according to which the relationship between expected returns and slope moves against the direction predicted by the expectations hypothesis is also present in our results. The main, and in most cases only factor driving expected returns in the expansionary regime is excess stock returns. The negative relationship we estimate between the stock and the bond market maybe related to the differential impact of expected inflation on the stock and bond returns. Alternatively, portfolio flows from one market to the other may be responsible for this effect. Our methodology does not allow us to analyse the links between stock and bond markets to a greater extend. Nevertheless, we consider this to be a very important issue and we intend to examine it further in future research.

References

Anderson, H.M., 1997. Transaction costs and nonlinear adjustment towards equilibrium in the US Treasury Bill market. Oxford Bulletin of Economics and Statistics, 59, 465-484.

Anderson, N., and J. Sleath, 1999. New estimates of the UK real and nominal yield curves. Bank of England Quarterly Bulletin, 39, 384-392.

Ang, A., and G. Bekaert, 1998. Regime switches in interest rates. National Bureau of Economic Research, Working paper 6508.

Backus, D.K., A.W. Gregory, and S.E. Zin, 1989. Risk premium in the term structure: Evidence from artificial economies. Journal of Monetary Economics, 24, 371-399.

Bekaert, G., and R.J. Hodrick, 1992. Characterising predictable components in excess returns on equity and foreign exchange markets. Journal of Finance, 47, 467-509.

Boudoukh, J., 1993. An equilibrium model of nominal bond prices with inflation-output correlation and stochastic volatility. Journal of Money, Credit and Banking, 25, 636-665.

Campbell, J.Y., 1991. A variance decomposition for stock returns. Economic Journal, 101, 157-179.

Campbell, J.Y., and J. Ammer, 1993. What moves the stock and bond markets? A variance decomposition for long-term asset returns. Journal of Finance, 48, 3-37.

Campbell, J.Y., and R. Shiller, 1987. Cointegration and tests of present value models. Journal of Political Economy, 95, 1062-1088.

Campbell, J.Y., and R. Shiller, 1988. The dividend—price ratio and expectations of future dividends and discount factors. Review of Financial Studies, 1, 195-228.

Campbell, J., and R. Shiller, 1991. Yield spreads and interest rates movements: A bird's eye view. Review of Economic Studies, 58, 495-514.

Chan, L.K.C., J. Karceski, and J. Lakonishok, 1998. The risk and return from factors. Journal of Financial and Quantitative Analysis, 33, 159-188.

Chen, N., 1991. Financial investment opportunities and the macroeconomy. Journal of Finance, 46, 529-554.

Constandinides, G.M., 1980. Admissible uncertainty in the intertemporal asset pricing model. Journal of Financial Economics, 8, 71-86.

Cox, J., J. Ingersoll, and S. Ross, 1985. A theory of the term structure of interest rates. Econometrica, 53, 385-408.

Cumby, R.E., 1988. Is it risk? Explaining deviations from uncovered interest rate parity. Journal of Monetary Economics, 22, 279-300.

Davis, E.P., S. Henry and B. Pesaran, 1994. The role of financial spreads: Empirical analysis of spreads and economic activity. Manchester School of Economic and Social Studies, 62, 374-394.

Elton, E.J., M.J. Gruber, and C.R. Blake, 1995. Fundamental economic variables, expected returns and bond fund performance. Journal of Finance, 50, 1229-1256.

Elton, E.J., M.J. Gruber, and J. Mei, 1996. Return generating process and the determinants of term premiums. Journal of Banking and Finance, 20, 1251-1269.

Estrella, A., and G.A. Hardouvelis, 1991. The term structure as a predictor of real economic activity. Journal of Finance, 46, 555-576.

Estrella, A., and Mishkin, F.S., 1997. The predictive power of the term structure of interest rates in Europe and the United States: Implications for the European Central Bank. European Economic Review, 41, 1375-1401.

Evans, M.D.D., and K.K. Lewis, 1994. Do stationary premia explain it all? Evidence from the term structure. Journal of Monetary Economics, 33, 285-318.

Fama, E.F., 1976. Forward rates as predictors of future spot rates. Journal of Financial Economics, 3, 361-377.

Fama, E.F., 1984. The information in the term structure. Journal of Financial Economics, 13, 509-528.

Fama, E.F., and R. Bliss, 1987. The information in long-maturity rates. American Economic Review, 77, 680-692.

Fama, E.F., and K.R. French, 1989. Business conditions and expected returns on stocks and bonds. Journal of Financial Economics, 25, 23-49.

Fama, E.F., and K.R. French, 1993. Common risk factors in the returns on stocks and bonds. Journal of Financial Economics, 33, 3-56.

Fama, E.F., and G.W. Schwert, 1977. Asset returns and inflation. Journal of Financial Economics, 5, 115-146.

Granger, C.W.J., and T. Teräsvirta, 1993. Modelling nonlinear economic relationships. Oxford University Press, Oxford.

Hansen, L.P., 1982. Large sample properties of Generalised Method of Moments estimators. Econometrica, 50, 1029-1054.

Ilmanen, A., 1995. Time-varying expected returns in international bond markets. Journal of Finance, 50, 481-506.

Jansen, E.S., and T. Teräsvirta, 1996. Testing parameter constancy and super exogeneity in econometric equations. Oxford Bulletin of Economics and Statistics, 58, 735-768.

Keim, D.B., and R.F. Stambaugh, 1986. Predicting returns in the stock and bond markets. Journal of Financial Economics, 17, 357-390.

Lewis, K.K., 1995. Puzzles in international financial markets, in: Grossman G. and K. Rogoff, (Eds.), Handbook of International Economics, Vol. 3, North-Holland, Amsterdam, pp. 1913-1971.

Macaulay, F.R., 1938. Some theoretical problems suggested by the movements of interest rates, bond yields and stock prices in the United States since 1856. New York: National Bureau of Economic Research.

Shiller, R., 1979. The volatility of long-term interest rates and expectations models of the term structure. Journal of Political Economy, 87, 1190-1219.

Shiller, R., and A.E. Beltratti, 1992. Stock prices and bond yields. Can their comovements be explained in terms of present value models? Journal of Monetary Economics, 30, 25-46.

Shiller, R., J. Campbell and K. Schoenholtz, 1983. Forward rates and future policy: Interpreting the term structure of interest rates. Brookings Papers on Economic Activity, 1, 173-217.

Stambaugh, R.F., 1988. The information in forward rates: Implications for models of the term structure. Journal of Financial Economics, 21, 41-70.

Taylor, N.D., D. van Dijk, P.H. Franses, and A. Lucas, 2000. SETS, arbitrage activity, and stock price dynamics. Journal of Banking and Finance, 24, 1289-1306.

Teräsvirta, T., 1994. Specification, estimation, and evaluation of smooth transition autoregressive models. Journal of the American Statistical Association, 89, 208-218.

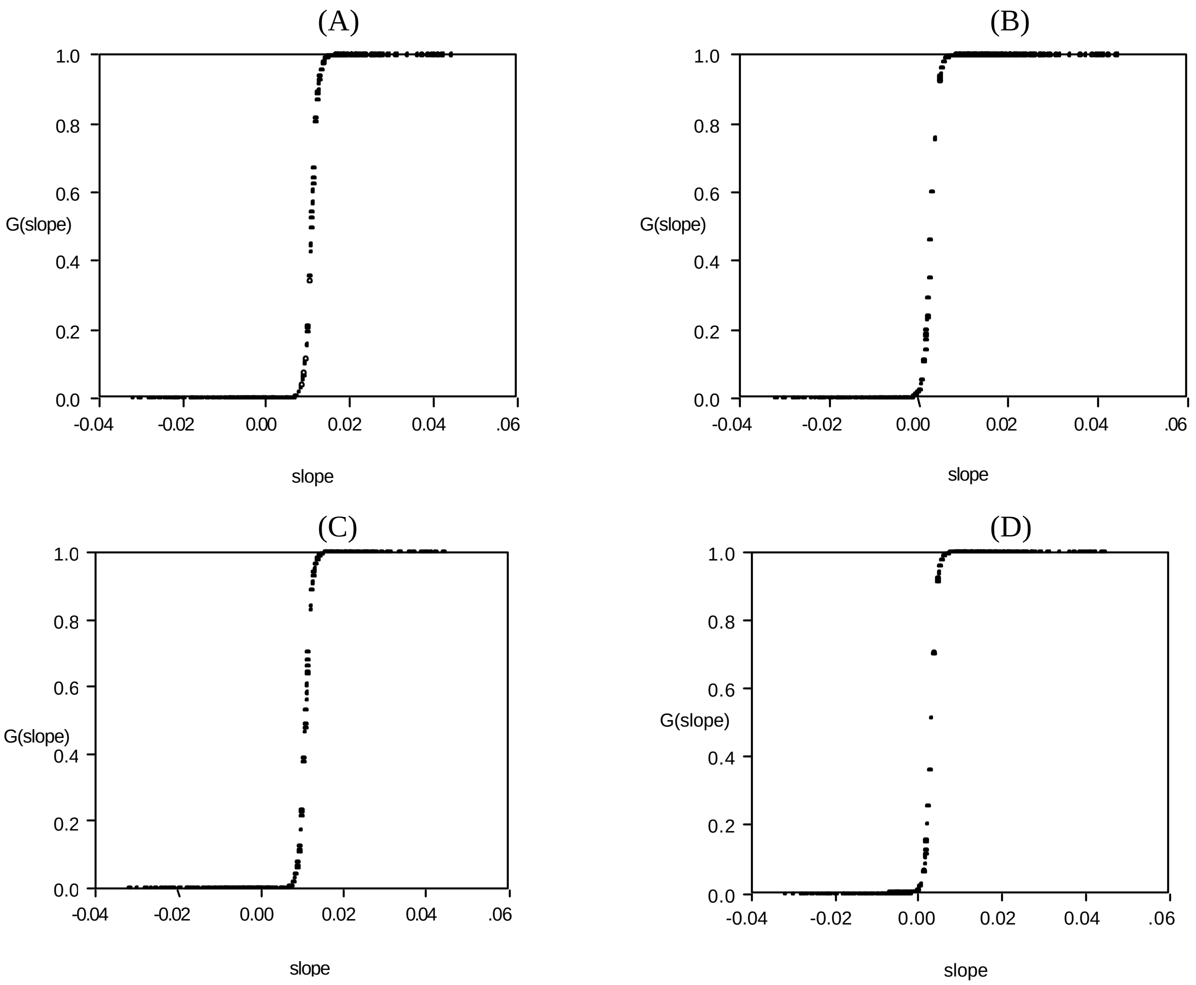
Teräsvirta, T., and H.M. Anderson, 1992. Characterizing nonlinearities in business cycles using smooth transition autoregressive models. Journal of Applied Econometrics, 7, S119-S136.

Titman, S., and A. Warga, 1989. Stock returns as predictors of interest rates and inflation. Journal of Financial and Quantitative Analysis, 24, 47-58.

van Dijk, D., and P.H. Franses, 2000. Nonlinear error-correction models for interest rates in the Netherlands, in: Barnett W.A., D.F. Hendry, S. Hylleberg, T. Teräsvirta, D. Tjøstheim and A. Würtz, (Eds.), Nonlinear econometric modelling in time series analysis Cambridge University Press, Cambridge, pp. 203-227.

van Dijk, D., T. Teräsvirta, and P.H. Franses, 2000. Smooth transition autoregressive models – a survey of recent developments. SSE/EFI Working paper series in Economics and Finance, No. 380, Stockholm School of Economics.

Figure 1: Estimated transition functions against $s_{t\text{-}1}$ = $slope_{t\text{-}1}$:



Panels A, B, C and D plot the estimated transition functions for $xret\_2_t$, $xret\_5_t$, $xret\_7_t$ and $xret\_10_t$ against the transition variable $s_{t\text{-}1} = slope_{t\text{-}1}$ from the corresponding STAR models reported in Tables 3 to 6. The estimated transition functions are:

(A) $G(slope_{t\text{-}1}; g, c) = \{1 + \exp[-20.005(slope_{t\text{-}1} - 0.011)/\sigma(slope_{t\text{-}1})]\}^{-1}$ for the $xret\_2_t$ model
(B) $G(slope_{t\text{-}1}; g, c) = \{1 + \exp[-20.000(slope_{t\text{-}1} - 0.003)/\sigma(slope_{t\text{-}1})]\}^{-1}$ for the $xret\_5_t$ model
(C) $G(slope_{t\text{-}1}; g, c) = \{1 + \exp[-20.355(slope_{t\text{-}1} - 0.011)/\sigma(slope_{t\text{-}1})]\}^{-1}$ for the $xret\_7_t$ model
(D) $G(slope_{t\text{-}1}; g, c) = \{1 + \exp[-22.162(slope_{t\text{-}1} - 0.003)/\sigma(slope_{t\text{-}1})]\}^{-1}$ for the $xret\_10_t$ model

Figure 2: Expected Annual Returns Across Regimes

Figure 2A: Expected Annual Returns of a 2-year Bond Across Regimes

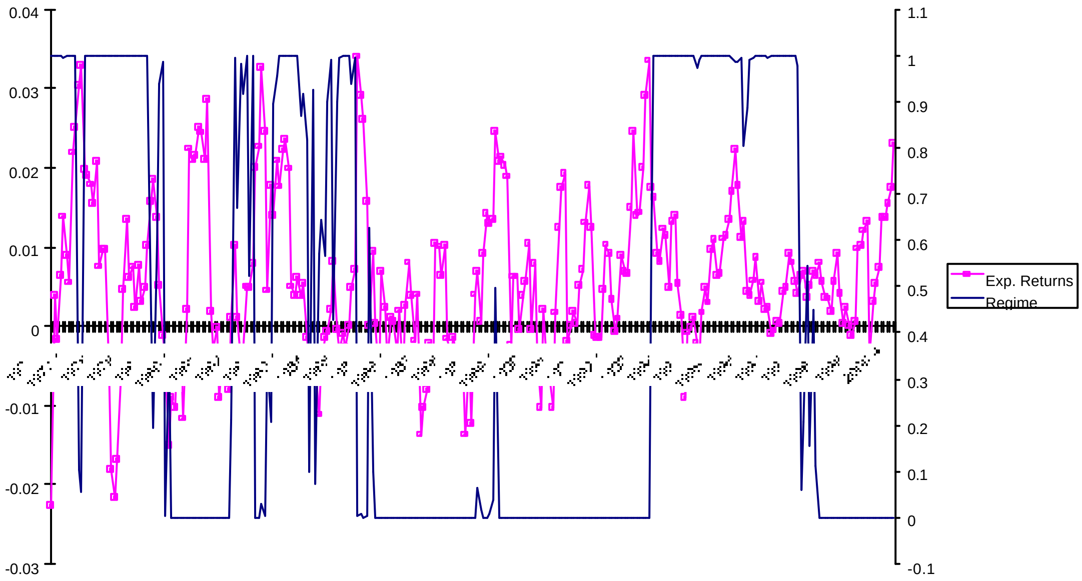
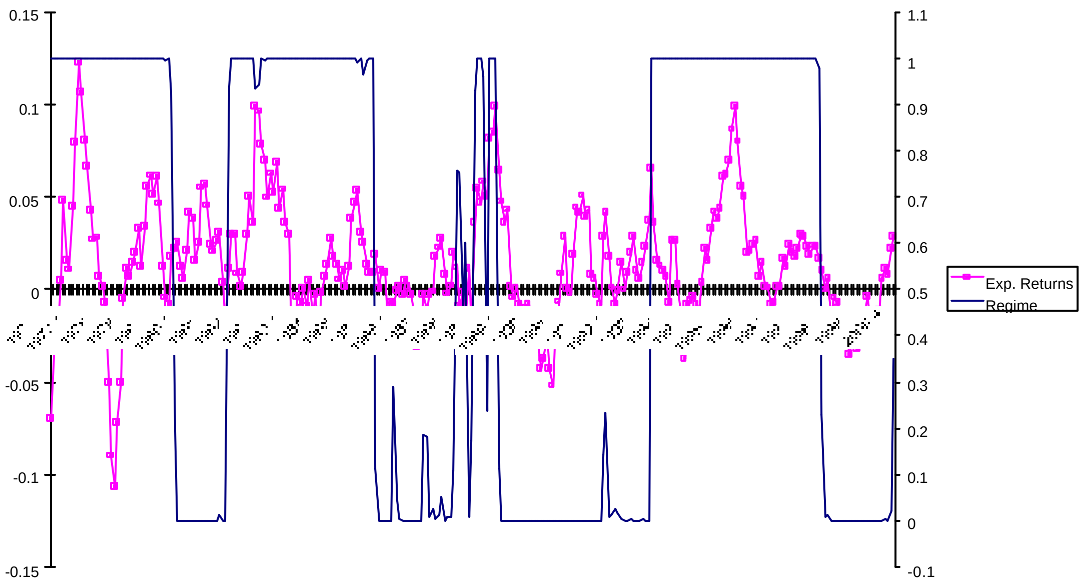


Figure 2B: Expected Annual Returns of a 5-year Bond Across Regimes



(continued)

Figure 2C: Expected Annual Returns of a 7-year Bond Across Regimes

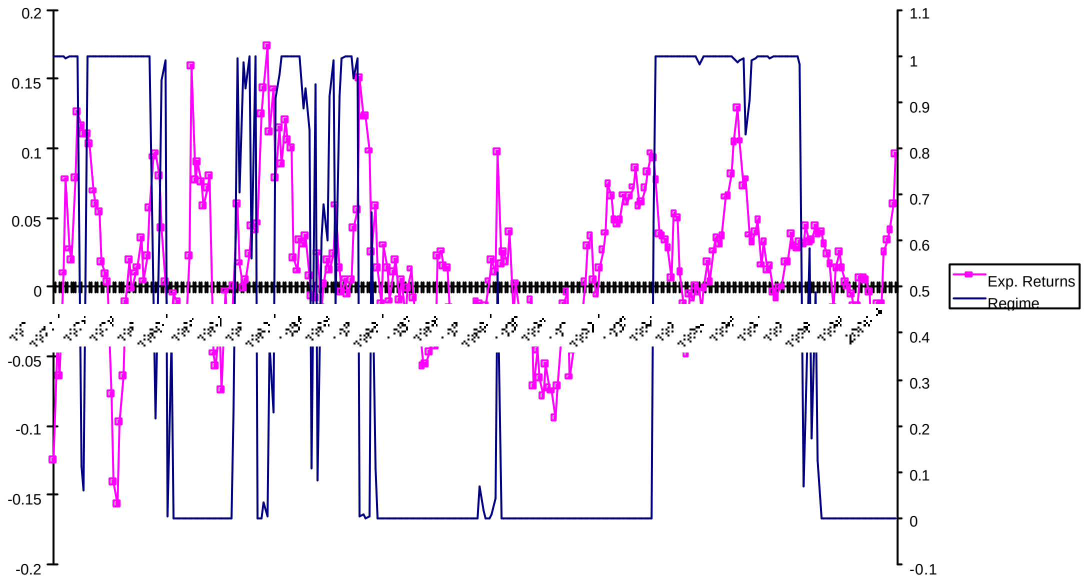
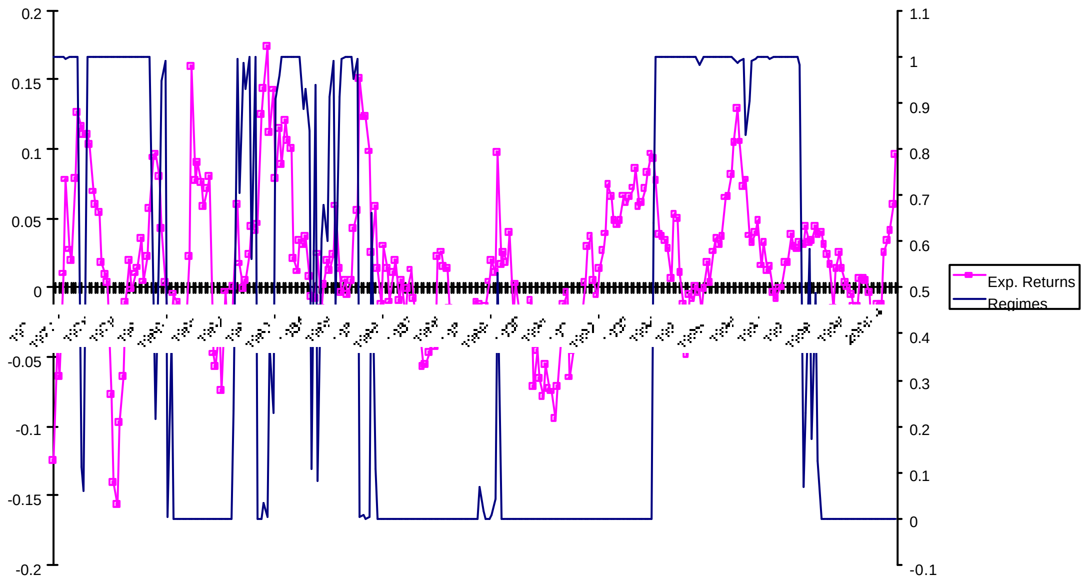


Figure 2D: Expected Annual Returns of a 10-year Bond Across Regimes



Panels A, B, C and D plot the expected annual excess returns for the 2-, 5-, 7- and 10-year UK government bonds (line with blocks, left hand side axis, returns in decimals). These are estimated as the fitted values of the STAR models reported in tables 3 to 6. The solid line (right hand side axis) plots the values of the transition function $G(slope_{t-1}; g, c)$ for each of the models. Extreme values of 0 and 1 of the transition function are associated with the two alternative regimes.

Table 1

Correlation structure of excess bond returns and risk factors

Panel A

	$slope_{t-1}$	$fspr\_i_{t-1}$	$dyall_{t-1}$	$xretft_{t-1}$
$xret\_2_t$	0.125	0.182	0.296	-0.300
$xret\_5_t$	0.063	0.055	0.254	-0.347
$xret\_7_t$	0.100	0.066	0.237	-0.346
$xret\_10_t$	0.176	0.146	0.214	-0.340

Panel B

	r_1	r_2	r_3	r_8
Bond risk factors				
$slope_t$	0.953	0.915	0.843	0.549
$fspr\_2_t$	0.942	0.898	0.802	0.495
$fspr\_5_t$	0.943	0.854	0.782	0.511
$fspr\_7_t$	0.945	0.853	0.812	0.505
$fspr\_10_t$	0.946	0.870	0.830	0.573
Stock market risk factors				
$dyall_t$	0.954	0.917	0.881	0.708
$xretft_t$	0.819	0.637	0.490	-0.282

Notes: *xret*_2, *xret*_5, *xret*_7 and *xret*_10 denote the one-year returns of the 2-, 5-, 7- and 10-year UK government bonds in excess of the 1-year T-Bill. *slope* is defined as the 10-year discount rate minus the 1-year T-Bill rate. *fspr*_2, *fspr*_5, *fspr*_7 and *fspr*_10 denote the forward rate spreads estimated as the *i*-th maturity forward rate minus the 1-year T-Bill rate. *xretft* is the return of the FTSE All Share index in excess of the 1-yeat T-Bill and *dyall* is the dividend yield on the FTSE All Share portfolio. The sample period is from 1976:1 to 2000:6. Panel A reports the correlations between risk factors and future excess returns. Panel B reports the autocorrelation structure of the risk factors. r_i is the *i*-th order autocorrelation coefficient.

Table 2

Linearity testing of excess bond returns

Panel A: *xret*_2 model

	H_0: $B_{1,j} = B_{2,j} = B_{3,j} = 0$	H_{03}: $B_{3,j} = 0$	H_{02}: $B_{2,j} = 0 \mid B_{3,j} = 0$	H_{01}: $B_{1,j} = 0 \mid B_{3,j} = B_{2,j} = 0$	Model, transition
$slope_{t\text{-}1}$	0.000052**	0.001081*	0.628205	0.004399	L, *slope*
$fspr\_2_{\,t\text{-}1}$	0.000116	0.003755*	0.315761	0.109004	L
$xretft_{t\text{-}1}$	0.001085	0.002542*	0.006856	0.223328	L

Panel B: *xret*_5 model

	H_0: $B_{1,j} = B_{2,j} = B_{3,j} = 0$	H_{03}: $B_{3,j} = 0$	H_{02}: $B_{2,j} = 0 \mid B_{3,j} = 0$	H_{01}: $B_{1,j} = 0 \mid B_{3,j} = B_{2,j} = 0$	
$slope_{t\text{-}1}$	0.005353	0.017876*	0.944807	0.028208	L
$xretft_{t\text{-}1}$	0.000206**	.815562	0.024413	0.000071*	L, *xretft*

Panel C: *xret*_7 model

	H_0: $B_{1,j} = B_{2,j} = B_{3,j} = 0$	H_{03}: $B_{3,j} = 0$	H_{02}: $B_{2,j} = 0 \mid B_{3,j} = 0$	H_{01}: $B_{1,j} = 0 \mid B_{3,j} = B_{2,j} = 0$	
$slope_{t\text{-}1}$	0.000001**	0.000029*	0.654914	0.000916	L, *slope*
$fspr\_7_{\,t\text{-}1}$	0.000061	0.000333*	0.686182	0.000348	L
$xretft_{t\text{-}1}$	0.015102	0.423954	0.133355	0.006698*	L

Panel D: *xret*_10 model

	H_0: $B_{1,j} = B_{2,j} = B_{3,j} = 0$	H_{03}: $B_{3,j} = 0$	H_{02}: $B_{2,j} = 0 \mid B_{3,j} = 0$	H_{01}: $B_{1,j} = 0 \mid B_{3,j} = B_{2,j} = 0$	
$slope_{t\text{-}1}$	0.000105**	0.001066*	0.111595	0.009710	L, *slope*
$xretft_{t\text{-}1}$	0.013591	0.981437	0.165436	0.001305*	L

Notes: The Table reports the *p*-values of the linearity tests developed in section 3. *xret*_2, *xret*_5, *xret*_7 and *xret*_10 denote the one-year returns of the 2-, 5-, 7- and 10-year UK government bonds in excess of the 1-year T-Bill. *slope* is defined as the 10-year discount rate minus the 1-year T-Bill rate. *fspr*_2, *fspr*_5, *fspr*_7 and *fspr*_10 denote the forward rate spreads estimated as the *i*-th maturity forward rate minus the 1-year T-Bill rate. *xretft* is the return of the FTSE All Share index in excess of the 1-yeat T-Bill and *dyall* is the dividend yield on the FTSE All Share portfolio. The first column reports the H_0 test for selecting the most suitable transition variable (numbers refer to *p*-values of the F-version of the LM test). A double asterisk indicates the lowest *p*-value for the H_0 test. The second, third and fourth columns report the *p*-values of the nested H_{03}, H_{02} and H_{01} tests for selecting between the 'logistic' model and the 'quadratic logistic' model for the transition function of the STAR models. An asterisk indicates the lowest *p*-value for the tests. Column 5 reports the transition variable and transition function selected by the testing procedure. L refers to the 'logistic' model.

Table 3

Estimated non-linear model for *xret*_2

The Table reports the GMM estimates of the following STAR model:

$$xret\_2_t = (m_1 + f_{1,1}\, fspr\_2_{t-1} + f_{1,2}\, slope_{t-1} + f_{1,3}\, xretft_{t-1})(1 - G(slope_{t-1}; g, c)) +$$
$$(m_2 + f_{2,1}\, fspr\_2_{t-1} + f_{2,2}\, slope_{t-1} + f_{2,3}\, xretft_{t-1})G(slope_{t-1}; g, c)$$

$$\text{where } G(slope_{t-1}; g, c) = \{1 + \exp[-g\,(slope_{t-1} - c)/\,\sigma(slope_{t-1})]\}^{-1},$$

is the logistic transition function , with $slope_{t-1}$ as the transition variable. Values of 0 and 1 of the transition function are associated with the two alternative regimes. The $xret\_2_t$ dynamics in the first regime, when $G(slope_{t-1}; g, c) = 0$, are: $xret\_2_t = m_1 + f_{1,1}\, fspr\_2_{t-1} + f_{1,2}\, slope_{t-1} + f_{1,3}\, xretft_{t-1}$. In the second regime, when $G(slope_{t-1}; g, c) = 1$, its dynamics are: $xret\_2_t = m_2 + f_{2,1}\, fspr\_2_{t-1} + f_{2,2}\, slope_{t-1} + f_{2,3}\, xretft_{t-1}$. For intermediate values of $G(slope_{t-1}; g, c)$, i.e. $0 < G(slope_{t-1}; g, c) < 1$, $xret\_2_t$ dynamics are a weighted average of the two equations. The speed of transition between the two regimes is determined by the parameter g . The threshold level of $slope_{t-1}$, that marks the half-way point between the two regimes, is determined by c.

$xret\_2_t =$	(0.009	$+2.576\ fspr\_2_{t-1}$	$-1.605\ slope_{t-1}$	$-0.054\ xretft_{t-1})\ (1 - G(slope_{t-1}; g, c))+$
	(0.003)	(0.604)	(0.550)	(0.017)
	(0.005	$-0.402\ fspr\_2_{t-1}$	$+0.462\ slope_{t-1}$	$-0.081\ xretft_{t-1})\ G(slope_{t-1}; g, c)$
	(0.005)	(0.322)	(0.281)	(0.015)

where

$G(slope_{t-1}; g, c) = \{1+$ $\exp[-20.005(slope_{t-1}$	$-0.011)\ /\ \sigma(slope_{t-1})]\}^{-1}$
(38.324)	(0.001)

$s_{NL} = 0.0180$, $s^2_{NL}/s^2_{L} = 0.8789$, adjusted $R^2 = 0.2275$

Notes: Standard errors are given in parentheses below the estimated coefficients. s^2_{NL}/s^2_{L} refers to the error variance ratio of the non-linear model relative to the linear one.

Table 4

Estimated non-linear model for *xret*_5

The Table reports the GMM estimates of the following STAR model:

$$xret\_5_t = (m_1 + f_{1,1} xret\_5_{t-1} + f_{1,2} slope_{t-1} + f_{1,3} xretft_{t-1})(1 - G(slope_{t-1}; g, c)) +$$
$$(m_2 + f_{2,1} xret\_5_{t-1} + f_{2,2} slope_{t-1} + f_{2,3} xretft_{t-1}) G(slope_{t-1}; g, c)$$

where $G(slope_{t-1}; g, c) = \{1 + \exp[-g\,(slope_{t-1} - c)/\,\sigma(slope_{t-1})]\}^{-1}$,

is the logistic transition function , with $slope_{t-1}$ as the transition variable. Values of 0 and 1 of the transition function are associated with the two alternative regimes. The $xret\_5_t$ dynamics in the first regime, when $G(slope_{t-1}; g, c) = 0$, are: $xret\_5_t = m_1 + f_{1,1} xret\_5_{t-1} + f_{1,2} slope_{t-1} + f_{1,3} xretft_{t-1}$. In the second regime, when $G(slope_{t-1}; g, c) = 1$, its dynamics are: $xret\_5_t = m_2 + f_{2,1} xret\_5_{t-1} + f_{2,2} slope_{t-1} + f_{2,3} xretft_{t-1}$. For intermediate values of $G(slope_{t-1}; g, c)$, i.e. $0 < G(slope_{t-1}; g, c) < 1$, $xret\_5_t$ dynamics are a weighted average of the two equations. The speed of transition between the two regimes is determined by the parameter g . The threshold level of $slope_{t-1}$, that marks the half-way point between the two regimes, is determined by c.

$xret\_5_t =$	(0.024	$-0.269\ xret\_5_{t-1}$	$+2.264\ slope_{t-1}$	$-0.095\ xretft_{t-1})\ (1 - G(slope_{t-1}; g, c))+$
	(0.015)	(0.179)	(0.873)	(0.064)
	(0.043	$-0.147\ xret\_5_{t-1}$	$-0.413\ slope_{t-1}$	$-0.226\ xretft_{t-1})\ G(slope_{t-1}; g, c)$
	(0.016)	(0.082)	(0.714)	(0.058)

where

$G(slope_{t-1}; g, c) = \{1+$	$\exp[-20.000(slope_{t-1}$	$-0.003)\,/\,\sigma(slope_{t-1})]\}^{-1}$
	(12.328)	(0.001)

$s_{NL} = 0.0594$, $s^2_{NL}/s^2_L = 0.9576$, adjusted $R^2 = 0.2115$

Notes: Standard errors are given in parentheses below the estimated coefficients. s^2_{NL}/s^2_L refers to the error variance ratio of the non-linear model relative to the linear one.

Table 5

Estimated non-linear model for *xret_7*

The Table reports the GMM estimates of the following STAR model:

$$xret\_7_t = (m_1 + f_{1,1} fspr\_7_{t-1} + f_{1,2} slope_{t-1} + f_{1,3} xretft_{t-1})(1 - G(slope_{t-1}; g, c)) +$$
$$(m_2 + f_{2,1} fspr\_7_{t-1} + f_{2,2} slope_{t-1} + f_{2,3} xretft_{t-1}) G(slope_{t-1}; g, c)$$

$$\text{where } G(slope_{t\text{-}1}; g, c) = \{1 + \exp[-g\,(slope_{t\text{-}1} - c)/\,\sigma(slope_{t\text{-}1})]\}^{-1},$$

is the logistic transition function , with $slope_{t\text{-}1}$ as the transition variable. Values of 0 and 1 of the transition function are associated with the two alternative regimes. The $xret\_7_t$ dynamics in the first regime, when $G(slope_{t\text{-}1}; g, c) = 0$, are: $xret\_7_t = m_1 + f_{1,1} fspr\_7_{t-1} + f_{1,2} slope_{t-1} + f_{1,3} xretft_{t-1}$. In the second regime, when $G(slope_{t\text{-}1}; g, c) = 1$, its dynamics are: $xret\_7_t = m_2 + f_{2,1} fspr\_7_{t-1} + f_{2,2} slope_{t-1} + f_{2,3} xretft_{t-1}$. For intermediate values of $G(slope_{t\text{-}1}; g, c)$, i.e. $0 < G(slope_{t\text{-}1}; g, c) < 1$, $xret\_7_t$ dynamics are a weighted average of the two equations. The speed of transition between the two regimes is determined by the parameter g . The threshold level of $slope_{t\text{-}1}$, that marks the half-way point between the two regimes, is determined by c.

$xret\_7_t =$	(0.031	$-16.962\ fspr\_7_{t\text{-}1}$	$+23.593\ slope_{t\text{-}1}$	$-0.089\ xretft_{t\text{-}1}$)	$(1 - G(slope_{t\text{-}1}; g, c))+$
	(0.013)	(3.487)	(4.262)	(0.077)	
	(0.068	$+1.617\ fspr\_7_{t\text{-}1}$	$-3.191\ slope_{t\text{-}1}$	$-0.403\ xretft_{t\text{-}1}$)	$G(slope_{t\text{-}1}; g, c)$
	(0.031)	(1.236)	(2.194)	(0.077)	

where

$G(slope_{t\text{-}1}; g, c) = \{1+$	$\exp[-20.355(slope_{t\text{-}1}$	$-0.011) / \sigma(slope_{t\text{-}1})]\}^{-1}$	
	(12.008)	(0.001)	

$s_{NL} = 0.0728$, $s^2_{NL}/s^2_L = 0.7730$, adjusted $R^2 = 0.3412$

Notes: Standard errors are given in parentheses below the estimated coefficients. s^2_{NL}/s^2_L refers to the error variance ratio of the non-linear model relative to the linear one.

Table 6

Estimated non-linear model for *xret*_10

The Table reports the GMM estimates of the following STAR model:

$$xret\_10_t = (m_1 + f_{1,1} xret\_10_{t-1} + f_{1,2} slope_{t-1} + f_{1,3} xretft_{t-1})(1 - G(slope_{t-1}; g, c)) +$$
$$(m_2 + f_{2,1} xret\_10_{t-1} + f_{2,2} slope_{t-1} + f_{2,3} xretft_{t-1}) G(slope_{t-1}; g, c)$$

where $G(slope_{t-1}; g, c) = \{1 + \exp[-g\,(slope_{t-1} - c)/\,\sigma(slope_{t-1})]\}^{-1}$,

is the logistic transition function , with $slope_{t-1}$ as the transition variable. Values of 0 and 1 of the transition function are associated with the two alternative regimes. The $xret\_10_t$ dynamics in the first regime, when $G(slope_{t-1}; g, c) = 0$, are: $xret\_10_t = m_1 + f_{1,1} xret\_10_{t-1} + f_{1,2} slope_{t-1} + f_{1,3} xretft_{t-1}$. In the second regime, when $G(slope_{t-1}; g, c) = 1$, its dynamics are: $xret\_10_t = m_2 + f_{2,1} xret\_10_{t-1} + f_{2,2} slope_{t-1} + f_{2,3} xretft_{t-1}$. For intermediate values of $G(slope_{t-1}; g, c)$, i.e. $0 < G(slope_{t-1}; g, c) < 1$, $xret\_10_t$ dynamics are a weighted average of the two equations. The speed of transition between the two regimes is determined by the parameter g . The threshold level of $slope_{t-1}$, that marks the half-way point between the two regimes, is determined by c.

$xret\_10_t =$	(0.050	+0.015 $xret\_10_{t-1}$	+5.254 $slope_{t-1}$	–0.175 $xretft_{t-1}$) $(1 - G(slope_{t-1}; g, c))+$
	(0.030)	(0.222)	(2.328)	(0.116)
	(0.065	–0.169 $xret\_10_{t-1}$	+0.832 $slope_{t-1}$	–0.367 $xretft_{t-1}$) $G(slope_{t-1}; g, c)$
	(0.039)	(0.126)	(1.758)	(0.142)

where

$G(slope_{t-1}; g, c) = \{1+$	$\exp[-22.162(slope_{t-1}$	$-0.003)\,/\,\sigma(slope_{t-1})]\}^{-1}$
	(12.972)	(0.001)

$s_{NL} = 0.1040$, $s^2_{NL}/s^2_L = 0.9255$, adjusted $R^2 = 0.2360$

Notes: Standard errors are given in parentheses below the estimated coefficients. s^2_{NL}/s^2_L refers to the error variance ratio of the non-linear model relative to the linear one.