A GENETIC ALGORITHM FOR POWER DISTRIBUTION SYSTEM PLANNING

A thesis submitted for the degree of Doctor of Philosophy

by

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Abstract

The planning of distribution systems consists in determining the optimum site and size of new substations and feeders in order to satisfy the future power demand with minimum investment and operational costs and an acceptable level of reliability. This problem is a combinatorial, non-linear and constrained optimization problem. Several solution methods based on genetic algorithms have been reported in the literature; however, some of these methods have been reported with applications to small systems while others have long solution time. In addition, the vast majority of the developed methods handle planning problems simplifying them as single-objective problems but, there are some planning aspects that can not be combined into a single scalar objective; therefore, they require to be treated separately. The cause of these shortcomings is the poor representation of the potential solutions and their genetic operators

This thesis presents the design of a genetic algorithm using a direct representation technique and specialized genetic operators for power distribution system expansion planning problems. These operators effectively preserve and exploit critical configurations that contribute to the optimization of the objective function. The constraints of the problems are efficiently handle with new strategies.

The genetic algorithm was tested on several theoretical and real large-scale power distribution systems. Problems of network reconfiguration for loss reduction were also included in order to show the potential of the algorithm to resolve operational problems. Both single-objective and multi-objective formulations were considered in the tests. The results were compared with results from other heuristic methods such as ant colony system algorithms, evolutionary programming, differential evolution and other genetic algorithms reported in the literature. From these comparisons it was concluded that the proposed genetic algorithm is suitable to resolve problems of large-scale power distribution system planning. Moreover, the algorithm proved to be effective, efficient and robust with better performance than other previous methods.

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Nomenclature

Symbols

3ϕ	Three-phase
Ω	ohm
λ	failure rate
r	failure duration
$\succ, \prec$	Pareto dominance relation
σ_i^k	Distance from point i to the k-th nearest point

Abbreviations

ACS	Ant colony system search
Amps	Amperes
EA	Evolutionary algorithm
ENS	Energy not supplied
f	failures
GA	Genetic algorithm
Km	Kilometers
kW	Kilowatts
kVA	Kilovoltamperes
kV	Kilovolts
Max/Min	Maximize / Minimize
Mill	Millions
MW	Megawatts
MVA	Megavoltamperes
MO-MST	Multiobjective minimum spanning tree
Pc	Recombination rate
Pm	Mutation rate
P.U.	Per unit
SPEA2	Strength pareto evolutionary algorithm 2

Declaration

The work described in this thesis has not been previously submitted for a degree in this or any other university, and unless otherwise referenced it is the author's own work.

Statement of Copyright

Chapter 1 Introduction

1.1 Background of the research

A power distribution system consists of substations (energy delivery nodes), feeders (electrical conductors connecting nodes and carrying power) and customers (power demand nodes). System planners must ensure that there is adequate substation capacity, feeder capacity and acceptable level of reliability to satisfy the power demand forecasts within the planning horizon. Planning these systems involves various tasks; some of these are:

- To find the site of substations and feeders
- To allocate substations and feeders capacities (substations and feeders sizes)
- To allocate the power load

These tasks have to be done simultaneously optimising various objectives such as economical cost and reliability of the systems.

The cost of a distribution system is divided into two parts: capital cost and operating cost. The capital cost (fixed cost) does not vary as a function of the system load while the operating cost (variable cost) varies as a non-linear function of the system load due to electrical losses.

An acceptable distribution system plan not only provides low cost but also must satisfy three main technical constraints: voltage drop limit, substation and feeder capacity limit and radial configuration.

Several power distribution system expansion planning methods have been developed and they differ in their degree of accuracy, complexity and applications. These methods can be divided into two groups:

a) Mathematical programming methods
b) Heuristic methods

1.1.1 Mathematical programming methods

There are various papers in which these methods are analysed in detail [Gonen and Ramirez-Rosado, 1986; Kathor and Leung, 1997; Willis and Northcote-Green, 1985; Vaziri et al., 2001; Quintana et al., 1993]. From these papers, the following conclusions are made:

- The planning horizon has been treated in two ways. Some methods define the duration of the plan as a single-stage (or single-period) whereas others consider the planning horizon as multi-stage (or multi-period). Single-stage refers to the assumption that the future power demand is constant throughout the planning horizon and the full expansion plan of the system is determined considering the planning horizon into only one period. On the other hand, multi-stage refers to the determination of the system expansion in successive plans over several stages. Multi-stage model is more challenging to formulate due to the interdependency between periods and limits the size of the system than can be studied but, the solution offers a more useful result. However, the vast majority of the developed planning methods have addressed the problem by single-period models.

- The mathematical representation of many requirements and restricting conditions specified by the system configuration is a very difficult task.

- Some methods divide the system into two subsystems: feeder subsystem and substation subsystem. The methods solve the subsystems separately; however, these subsystems interact heavily with one another and generally the true least-cost plan can be found only considering the substations and feeders together.

- Various approximations are necessary in the planning methods. For instance, some methods do not include feeder fixed costs in the total cost formulation because fixed costs produce discontinuity in the feeder cost function at the origin. Other methods either linearise or ignore the non-linear cost function of substations and feeders because the non-linear modelling results in longer solution times, smaller problem capacity, and greater likelihood of computational problems. Linearising or ignoring the fixed or variable costs affect the overall solution validity (the methods might provide very sub-optimal solutions).

- Various planing methods do not include voltage drop constraints into the planning problem formulation because they use approximate models of the feeders or calculate power flows based on economic considerations neglecting electrical parameters of the system.

- There are planning methods that can not consider the radiality constraint. Due to the network nature of distribution systems, the radial configuration constraint is particularly more difficult because usually the optimisation algorithm identifies that loop or network configurations will reduce cost since feeding the demand nodes with power from two or more different sources produces less energy losses.

- Some methods are able to solve large distribution systems, but sacrificing accuracy in modelling the physical system. Others have better accuracy but they can be applied to small systems only.

- The methods consider only one objective to optimise. This objective represents the economical costs of the system related to equipment, installation, energy losses and operation and maintenance. However, there are other important planning aspects that should be considered in the optimisation process but they can not be expressed in terms of economical costs such as reliability, environmental and social aspects.

The advantages of mathematical programming methods are that their convergence may be guaranteed; the stopping criteria can be well defined and they are based on mathematical analysis.

1.1.2 Heuristic methods

Since the early 1990's, planning methods based on modern heuristic search techniques have been developed. These heuristic techniques simulate physical phenomena, creature's evolution, and creature's behaviour. Many applications of these heuristics techniques to the distribution system optimisation have been tried in the last 10 years.

A fundamental idea of heuristic search is that of neighbourhood search [Rayword-Smith, 1996]. In the context of distribution system planning problem, let assume that a possible solution is specified by x, where the set of all feasible solutions is denoted by X, and the cost of solution x is denoted by $c(x)$ (objective function). Each solution x has an associated set of neighbours, $N(x) \subset X$, called the neighbourhood of x. Each solution $x' \in N(x)$ can be reached directly from x by an operation called "move", and x is said to move to x' when such an operation is performed. The choice criteria for selecting moves, and termination criteria for ending the search, are given by some external set of instructions. By specifying these instructions in different ways, the method can easily be altered to yield a variety of procedures.

Simulated annealing

The simulated annealing search technique works by searching the set of all possible solutions. Specifically, if a move from one solution x to another neighbouring solution results in a reduction Δc in the value of the objective function, the new solution is accepted (for minimisation problems). If the change increases the objective function value, the solution is accepted with a probability given by the Boltzmann factor:

$$e^{(-\Delta c/T)} < R$$

where T is a control parameter and $R \in [0,1]$ is a uniform random number. This process is repeated sufficient times for the current value of T; and then, T is slowly reduced and the entire process repeated until $T = 0$. The parameter T is initially high to allow many "bad" solutions to be accepted; therefore, reducing the T value, the probability of accepting bad solution is reduced.

This search technique is based on the manner in which liquids metal crystallise in the process of annealing.

In [Jonnavithula and Billinton, 1996], it is proposed a planning method based on simulated annealing. This method locates the feeder route minimising an objective function that sum the interruption, energy losses, investment and maintenance costs of the feeders. The investment and energy losses costs of substations are not considered. The capacity and voltage drop limit constraints for the feeders are considered but, the radiality constraint and the capacity of substations are not mentioned. The method is reported with an application to a system with 28 demand nodes and two substations. The computational effort of the method is not reported.

Another planning method based on simulated annealing is reported in [Parada et al., 2004]. This method is proposed to find the site and size of feeders and substations minimising the investment and energy losses costs of feeders and substations; however, the method was applied to theoretical distribution networks without considering substation costs and it is not mentioned the different feeder sizes used.

The major drawback of the simulated annealing technique is that repeatedly annealing with schedule is very slow.

Tabu search

Tabu search is based on neighbourhood search in a deterministic way. This technique not only keeps in memory local information (like the current value of the objective function) but also some information related to the exploration process. This systematic use of memory is an essential feature of tabu search. Specifically, a history record H is kept of moves which have been made in the recent past of the search, and which are "tabu" or forbidden for a certain number of iterations. This helps to avoid revisiting certain solutions in $N(x)$.

In [Ramirez-Rosado et al., 1999], a planning method based on the tabu search technique is proposed. This method considers an explicit modelling of the uncertainties of the future demands and the expansion costs in distribution systems by using possibility distributions in the context of the fuzzy set theory. The planning problem is formulated to find the site and size of substations and feeders considering fixed and variables costs. The method basically works as follows: an initial solution is achieved by a shortest path algorithm; then, topological changes in the system solution are allowed (movements); for instance, eliminating one feeder and including another (to keep the radial configuration); changing the size of a feeder or substation; and eliminating or including one substation. This method might be inefficient due to the huge number of possible moves in large distribution systems. In addition, for this search technique, distribution systems are complex in the sense that when small changes occur in the solution, big changes might result in the solution's desirability; for instance, including or eliminating a substation might result in big changes in the cost of the system or it might produce the violation of constraints in many parts of the system.

This method was applied to a large-scale system but, it is not mentioned which are the costs considered in the problem and what is the mechanism to select the moves or changes in the system.

Ant colony system search

Ant colony system search (ACS) is a heuristic technique based on the behaviour of real ant colonies. These insects have the abilities to find the shortest paths to their food sources without visual help, using a chemical substance (called pheromone) that is deposited as they walk. The ants use the 'pheromone trial' to communicate information among individuals regarding paths and to decide where to go.

Initially, a group of ants explore the surface without a predetermined direction. When the site of food is found, they carry food from the site to the nest secreting the pheromone. As all of the ants travel approximately at the same speed, the shortest paths will contain higher levels of pheromone because a higher number of ants choose the shortest paths (to go to the food site and to return from it). The differences among the pheromone deposits in the routes influence the decision of new ants, which will decide to choose the paths with higher levels of pheromone (the shortest paths).

A distribution system planning method based on ACS is proposed in [Gomez et al., 2004]. In this method, the fixed and variable costs are formulated to find the optimal site and size of substations and feeders. The voltage drop, capacity and radiality constraints are considered. The method works as follows: initially, a random level of pheromone is deposited in each feeder of an initial system. Then, a number of ants does independent explorations through the different feeders guided by an heuristic guide function and by the amount of pheromone deposited in the feeders, until all of the nodes of the network are connected, completing an expedition of that ant colony. After a predetermined number of sets of expeditions are performed, the fitness value of each ant is calculated. The objective function of the problem is used to evaluate the performance of the ants and the fitness values are used to update the pheromone intensity of each feeder. Afterwards, the process is repeated.

The disadvantage of this method is that it has seven parameters to be tuned for the proper performance of the method. It is stated, in the reference, that the method is robust despite this number of parameters, but it is not reported how robust it is and how is the interaction between the parameters. The algorithm was applied to a real large-scale distribution system which consists of one substation, 201 nodes and 227 feeders. The method found an approximate optimal solution in 1 hour, 33 minutes using a personal computer with an 800-MHz processor. It is not explained how the capacity and voltage drop constraints are handled.

Genetic algorithms

Genetic algorithms have been widely used to resolve optimisation problems. A genetic algorithm consists of a population of data structures. Each data structure represents a possible solution (encoded solution or chromosome) for the problem being optimised. These solutions are classified by an evaluation function, giving better value (fitness) to better solutions. The population evolves over several generations to a population including the optimal solution (or sub-optimal solutions) to the problem. This evolution is generated by two genetic operators: crossover, which creates new solutions (offspring) by combining parts from two solutions (parents), and mutation, which make changes in a single solution. A new population is created from the old population and the offspring population. This process is carried out in each iteration (generation) of the algorithm. The process is stopped by a predetermined stopping rule.

Recently, power distribution network planning methods have been developed using genetic algorithms [Miranda et al., 1994; Yeh et al., 1996; Ramirez-Rosado and Bernal-Agustin, 1998; Lin et al., 1998; Carvalho et al., 2000]. In these references, some methods are presented with applications to small systems; meanwhile others have a high solution time. The main causes of these limitations are the use of standard solution encoding techniques and genetic operators. The following problems have been observed [Carvalho et al., 2001; Gottlieb et al., 2001]:

- Low heritability. A significant number of offspring generated by the crossover operator hardly have substructures of their parents.
- Topological unfeasibility. Many offspring do not represent a topologically valid solution.

In [Miranda et al., 1994], a genetic algorithm is proposed to resolve the problem of optimal multistage distribution network planning. The authors proposed a binary codification to represent the topology of the network: each node is represented in a chromosome by a number of bits needed to encode the number of feeders connected to it - for example, if a node has four possible feeders to be connected to it, only two bits are needed (two bits can represent four different numbers). However, using this coding with conventional genetic operators can introduce illegal solutions. For example, for a node that has three possible feeders, using two bits for this node can introduce a number that would represent a non-existent connection. Another problem in this case is that one bit cannot represent the three possible connections (one bit can represent only two possibilities).

The method proposed in [Ramirez-Rosado and Bernal-Agustin, 1998] uses an *m*-vector for each chromosome; where *m* is the number of candidate feeders in the problem. The i-th element of the vector contains either a zero, if the feeder is not added, or a number that represents the size or type of the feeder to be added. In this method, the conventional genetic operators are used. These operators introduce many illegal solutions (solutions with non-radial configurations) since these operators are applied without avoiding the introduction of feeders that form loops.

The references [Yeh et al., 1996; Lin et al., 1998; Carvalho et al., 2000] present planning methods that use binary encoding. Binary encoding have severe drawbacks due to the existence of Hamming cliffs: pairs of encoding having a large Hamming distance while belonging to points of minimal distance in phenotype space (space of decoded solutions).

In [Carvalho et al., 2001] it is reported a planning method based on genetic algorithms in which, distributions systems are treated as spanning trees (graphs connecting all nodes with no loops). A spanning tree is encoded into two (n-1)-arrays, where n is the number of nodes. One array lists the nodes of the graph and the other array lists each node's predecessor in the path from the node to a previously selected root node in the represented spanning tree: If node i is the predecessor of node j, then node j is adjacent to node i and nearer the root. Thus, each candidate solution is represented by two arrays requiring space proportional to two times the number of nodes. With this encoding technique, a special recombination operator is proposed to produce only legal solutions (radial configurations). Basically, this special recombination operator consist of three steps: 1) Two nodes are selected randomly, 2) A path P_1 between these two nodes is found in parent T_1 and a path P_2 is found in parent T_2, 3) If possible, submit the P_1 to T_2, and submit P_2 to T_1. If not possible, return to step 1. This operator produces legal offspring with substructures of their parental solutions. However, it is possible that two parents (not necessarily with similar structure) have a pair of nodes with the same path between them. In this case, the recombination operator produces offspring that are identical to their parents, i.e. the recombination operation is not executed.

1.1.3 Multi-objective optimisation planning methods

The vast majority of the developed planning methods consider only one objective function to optimise. The objective function of these methods represents the economical costs of the system such as, investment, energy losses and interruption costs. However, there are other planning aspects that should be considered in the planning methods but they can not be expressed in terms of costs. For instance, environmental and social impact can be very important in some cases and they can not be expressed as economical costs. Reliability of the system is another planning aspect that have been expressed in terms of costs and considered in some planning methods but, it is required information about the economical impact of power interruptions on customers and suppliers. This information might be difficult to obtain in some cases. Therefore, some planning aspects to be considered need to be formulated as separate objective functions.

In multi-objective problems, the goal is to find a set of Pareto-optimal solutions (or approximate Pareto-optimal solutions). A Pareto-optimal solution is such that no other feasible solution presents the same or a better performance with respect to all objectives, with at least one being strictly better.

There are few multi-objective methods that have been proposed to resolve the problem of power distribution systems expansion planning with more than one objective function separately formulated. In [Kagan and Adams, 1993], a planning method is proposed to optimise three objective functions: economical cost, energy not supplied and total length of overhead lines. This method generates a set of Pareto-optimal solutions using the ε-constrained technique. This technique transforms two objectives into constraints, by specifying bounds to them (ε), and the remaining objective, which can be chosen arbitrarily, is the objective function to optimise. In other words, the multi-objective problem is transformed into a single-objective optimisation problem, which is resolved by classical single-objective algorithms. The bounds ε are the parameters that have to be varied in order to find multiple solutions.

Another planning method that uses the ε-constrained technique is reported in [Ponce-de-Leao and Matos, 1999]. This method resolves the single-objective problems using a simulated annealing algorithm.

The disadvantage of the ε-constrained technique is that the solution of the resulting single-objective problem largely depends on the chosen bounds ε. Some values of ε might cause that the single-objective problem has no feasible solution. Thus, no solution would be found. In addition, several optimisation runs are required to obtain a set of Pareto-optimal solutions.

In reference [Kagan and Adams, 1992], it is reported a planning method that uses the weighting technique to obtain non-dominated solutions. This technique consists in assigning weights to the different objective functions and combining them into a single-objective function. The Pareto-optimal solutions are identified by changing the weights parametrically with several optimisation runs.

One difficulty with this technique is that it is difficult to find a uniformly distributed set of Pareto-optimal solutions. In addition, many weight values can lead to the same solution and, in case of non-convex objective space, certain solutions can not be found.

In [Ramirez-Rosado and Bernal-Agustin, 2001], a multi-objective optimisation method based on genetic algorithms is presented. This method is able to find a set of approximate Pareto-optimal solutions in one single simulation run due to its population approach (genetic algorithms use a population of solutions in each iteration; therefore, the outcome is also a population of solutions). The method is formulated to find the site and size of substations and feeders optimising two objective functions: economical cost and energy non-supplied. The drawback of this method is that the genetic algorithm have to be run several times in order to obtain solutions closer to the optimal ones. Moreover, the method uses genetic operators that generate many illegal solutions and its encoding technique has low heritability, making the algorithm inefficient and ineffective.

1.2 Aims of the research

The main aims of this research were as follows:

1. To develop a method to find the site and size of substations and feeders, optimising economical costs and reliability of power distribution systems.

2. The method should be able to handle large-scale systems without sacrificing accuracy in modelling the physical system. The fixed costs and the non-linearity of the variable costs must be considered.

3. The technical constraint must be taken into account in the planning model. The method should be able to handle these constraints efficiently.

4. The planning method should be able to deal with single-objective and multi-objective planning problems. In multi-objective problems, the method should be able to optimise the objective functions separately.

5. The planning method should be more effective and efficient than previous developed methods reported in the literature.

1.3 Contributions of the research

The main contributions of this research to knowledge are as follows:

1. In this research, a new efficient and effective method for power distribution system expansion planning was developed. The method finds the site and size of substations and feeders optimising several planning aspects expressed in one or more objective functions. The fixed and non-linear variable costs are modelled in the planning method. The energy non-supplied index is formulated as a separate

objective function. The voltage drop limit, capacity limit and radiality constraints are considered. The method is based on the single-period planning model and it assumes that the candidates for facilities to be installed are known beforehand. The power demand forecast is the peak load. The magnitudes of voltage, current and power flow in the systems are calculated using a power flow program.

2. A genetic algorithm was developed to be used as the optimisation algorithm of the planning method. This genetic algorithm is based on an algorithm developed for the degree-constrained spanning tree problem. An encoding technique and special genetic operators were developed to overcome the problems of low heritability and topological unfeasibility. In addition, this encoding technique and genetic operators are able to enforce the radiality constraint.

3. The method uses an advanced strategy to generate a distributed set of approximate Pareto-optimal solutions for multi-objective planning problems. This strategy applies new concepts and ideas reported in the fields of multi-objective optimisation and evolutionary algorithms. With this strategy, the planning method is able to find approximate Pareto-optimal solutions in complex solutions spaces such as non-convex spaces.

4. In this research, it is proposed a strategy to handle the technical constraints for multi-objective problems of planning distribution systems. This strategy has the advantage that it does not require parameters to be tune.

1.4 Thesis layout

The thesis is organised as following:

Chapter 2. In this chapter, the planning problem of power distribution systems is explained. The problem is formulated as an optimisation problem with two objectives to optimise and four constraints to meet. The objectives are the economical cost and the energy non-supplied index of distribution systems. The constraints are the capacity limits of lines and substations; the maximum and minimum allowed voltage levels at the nodes of the system and the radial configuration.

Chapter 3. This chapter begins with a description of single-objective and multi-objective optimisation problems. The optimality conditions for any solution to be optimal in the presence of multiple objectives are also discussed. Thereafter, a description of genetic algorithms for single-objective optimisation and evolutionary algorithms for multi-objective optimisation is given.

Chapter 4. This chapter presents a new genetic algorithm developed in this research for optimal large-scale power distribution network expansion planning. The algorithm finds the optimum (or near optimum) location and size of substations and feeders to minimise a cost function of the network, which represent capital (fixed costs) and operational costs (non-linear variable costs). The algorithm was tested on three networks and the results were compared with the results from other genetic algorithms.

Chapter 5. This chapter presents a multi-objective evolutionary algorithm developed in this research for optimal large-scale power distribution network expansion planning. The algorithm is able to find a set of Pareto-optimal solutions (or a set of non-dominated solutions close to the Pareto-optimal front) to problems with more than one objective function to optimise. In this research, for power distribution network planning problems, two objectives were considered: economical cost function and energy non-supplied function. The algorithm was tested on three problems and some of the results were compared with results from another method.

Chapter 6. In this chapter, it is reported the application of the developed genetic algorithm to the problem of network reconfiguration for loss reduction. The goal of this chapter is to show the potential of the encoding technique and the special genetic operators, originally developed for the problem of distribution system expansion planning, to resolve operational problems.

Chapter 7. In this chapter, conclusions and suggestions for the future work are given.

Chapter 2 Power Distribution System Expansion Planning

2.1 Introduction

In this chapter, the planning problem of power distribution system is explained. The problem is formulated as an optimisation problem with two objectives to optimise and four constraints to meet. The objectives are the economical cost and the energy non-supplied index of distribution systems. The constraints are the capacity limits of lines and substations; the maximum and minimum allowed voltage levels at the nodes of the system and the radial configuration.

2.2 Power distribution systems

In general, the definition of an electric power system includes generation, transmission, and distribution systems. The distribution system typically starts with distribution substations, which are fed by one or more transmission lines, and each distribution substation serves one or more primary feeders. The function of a distribution system is to deliver electrical energy from the distribution substations to the customers.

The distribution system is an important part in the electric power system. This system is the closest one to the ultimate customer and its failures affect customer service more directly than failures on the generating and transmission systems. In addition, distribution systems have high investment cost. Therefore, distribution systems have particular characteristics that make them to play an important role in power systems.

There are three main goals in the function of a distribution system [Willis, 1997]: 1) to deliver power to the customers at their place of consumption, 2) to achieve the first goal at the lowest cost possible and 3) to provide acceptable reliability levels. The distribution system must deliver power to the customer by mean of electrical feeders with sufficient capacity to satisfy the customer's power demands. The feeders must be reliable in order to deliver all the power demanded during all of the time that this is demanded.

The planning of power distribution systems is a laborious task for the planners since a distribution system is composed of thousands of components which are spread throughout the service territory. The main components are: distribution lines, transformers, protective equipment, voltage regulation equipment, switches and capacitors. Each component must be selected and installed to function well in conjunction with the rest of the components in the system. Therefore, a distribution system is highly interconnected and integrated: the various decisions about equipment selection, siting, sizing, and line routing throughout the distribution system are interrelated. Therefore, the planning of distribution systems is a challenging problem to the network planners.

2.3 Power distribution system expansion planning

Power distribution system expansion planning is essential to assure that the growing power demand can be satisfied by distribution system additions, which must be technically adequate and economical; therefore, the goal of power distribution system planning is to define the expansion of the system to meet the future power demand with an acceptable level of reliability at minimum cost.

In the planning process of distribution systems, there are two types of planning according to the planning horizon [Willis, 1997]: short-range planning and long-range planning. The purpose of short-range planning is to make certain that the system can continue to supply power to the customers by distribution system additions that are to be made in the near future. On the other hand, the purpose of the long-range planning is to assure all short-range decisions have lasting value and contribute to a minimum-cost system; in other words, the purpose is to make certain that those decisions that are made in the short-range planning have a low present worth cost and fit the long-term needs.

These two types of planning are applied to two levels of distribution system planning [Willis, 1997]. One level is the planning that involves determining in detail the routing of a feeder and identifying its equipment to near engineering precision; for example, detailed equipment specification, pole locations, route maps, etc. This level of planning is done in the short-range planning period which can be from 1 to 5 years. The other level of planning involves the overall planning of distribution feeders in conjunction with the substations. This level of planning is done in the long-range planning period from 5 to 30 years and it includes the selection of sites and capacities of substations and feeders, optimising costs and considering constraints.

2.4 Planning process

The planning process of distribution systems consists of three steps: identifying alternatives; evaluating them against criteria and desired attributes; and selecting the best alternative [Willis, 1997]. In distribution system planning, there are thousands of alternatives involved since there is a high number of options to select equipment, sites, configurations, etc. The planner must consider the range of possibilities for resolving the planning problem in order to identify the best plan and make proper decisions. To identify and select the best plan, the alternatives must be evaluated against criteria and attributes that represent the utility's requirement, standards and constraints. This is a key step and it should be done carefully evaluating the alternatives completely.

The criteria and attributes are defined according to the problem. Criteria are the requirements and constraints the plan must meet, whereas the attributes are the qualities that are to be minimised (or maximised).

In order to use the same expressions in this thesis, the criteria and attributes are expressed as constraints and objectives to optimise, respectively. The typical objectives and constraints in distribution system planning are as follows [Willis, 1997; Lakervi and Holmes, 1995]:

Economical cost. One objective to minimise in any power system planning is cost. The economical cost of a power distribution system can be divided into two types: capital cost and operating cost. The capital cost includes the equipment, construction, installation and other costs associated with putting the equipment into operation. The operating cost corresponds to labour and equipment for operation, maintenance and electrical losses costs. The capital cost is considered as a one-time cost (once the equipment has been installed, the money has been spent) and the operating cost as a periodic cost. Moreover, the capital cost is seen as fixed cost whereas the operating cost is seen as variable cost. A fixed cost does not vary as a function of any variable element of the system, such as the loading of substations and lines. By contrast, a variable cost varies as a function of the loading of these components.

Reliability. Utilities have the mission to maintain the highest level of service reliability to customers and have the obligation to improve service reliability consistently by planning operations. However, the costs of building and maintaining the distribution system increase as greater reliability is desired. Therefore, reliability must be considered as another objective to optimise along with the economical cost objective. To consider reliability as an objective, a reliability assessment model is required to quantify reliability characteristics based on system topology and component reliability data.

Voltage drop constraint. Voltage drop constraint defines limits within which service is required. The constraint is established by electric utilities in order to define the level of service they provide and to maintain the system within these limits to assure its operation as expected. Considering the voltage drop constraint in the planning of distribution systems is important since the knowledge of the voltages at different locations can help to identify the strong and weak parts of a network

Capacity constraint. The capacity rating of an electrical equipment indicates how many KW o KVA it can handle. The equipment manufacturers give the capacity rating to maintain the equipment operating to that rating and to assure operation and lifetime as expected. In the planning of power distribution systems, the capacity constraint are used to determine the need to reinforce, add circuits and build new additions that must be built.

Radial configuration constraint. Most of power distribution systems are operated radially. The radial configuration is characterized by having only one path between each customer and a substation. Its predominance is due to two advantages: it is much less costly than other configurations and it is much simpler in planning and operating.

The planning process can take a very long time and can be extremely difficult and expensive if it is done manually. The number of alternatives is so large and they must be evaluated on the basis of the attributes and criteria. Therefore, the network planner requires computational optimisation methods to carry out the planning process with less effort and time, and to find the best alternatives with better justifications.

Optimisation methods involve a mathematical formulation of the planning problem and optimisation algorithms. To correctly formulate the problem, a precise problem definition is necessary. In the next section, the problem definition of distribution system expansion planning and its mathematical formulation, used in this research, is stated.

2.5 Problem statement

2.5.1 Problem definition

The problem of distribution system expansion planning is to find the location and capacity of substations and feeders to supply power to a given set of future power demand nodes at minimum cost with acceptable levels of reliability and meeting technical constraints. More specifically, the planning problem involves the selection of the number, location and size of substations and feeders such that the capital cost (fixed cost) and the cost of energy losses (variable cost) is minimum; maintaining the radiality of the network and at the same time not violating the capacity and voltage drop constraints in any part of the network.

This problem involves several substations and feeders at the same time; therefore, long-range planning period is applied.

Some planning analysis determines the full expansion requirements in one period (single-period planning model) whereas other analysis determines the requirements in successive expansion plans over several periods (multi-period planning model). Multi-period model is more challenging to formulate due to the interdependency between periods but the solution offers a more useful result. However, the vast majority of the developed planning methods have addressed the problem by a single-period model [Willis, 1997].

In this research, the single-period model was considered and the following assumptions were made for the formulation of the problem:

- Only peak load is considered
- Candidates for facilities, which are to be installed, are known beforehand

There are many optimisation methods that have been proposed to resolve the distribution system planning problem but, many of these methods have serious limitations due to the characteristics of the planning problem and the distribution systems. These characteristics are:

System size. Real distribution systems consist of thousands of nodes and lines. Some optimisation methods are not able to resolve large systems. Others can resolve them but with high computation time. In addition, there are methods that are fast and capable to resolve large systems but, they use approximations such as linearization.

Combinatorial problem. The problem of planning distribution system is categorised as combinatorial problem. Combinatorial problems are characterised by a finite number of feasible solutions. For practical distribution systems, this number of feasible solutions can be extremely high. In addition, when the number of components of the system increases, the number of solutions increases exponentially.

Non-linear costs. The costs to be minimised do not have a linear relationship. Despite of this fact, linear optimisation methods are often applied because they are fast and they can resolve large systems. However, this simplification of the problem might produce very sub-optimal solutions.

Technical constraints. An acceptable distribution system plan not only provides low costs but also satisfies a list of technical constraints. Among these constraints, the most important are: voltage drop, capacity limit and radial configuration. Some optimisation methods can not handle these constraints. Others use approximations.

Multiple objectives. Traditional power distribution system planning consider one objective to optimise -which is related to the economical cost. However, there are other attributes equally important that can not be translated into economical costs such as aesthetic, social and environmental impact.

2.5.2 Mathematical formulation

The total cost of an electrical line consists of three parts: initial installation cost, annual operation and maintenance cost and annual electrical losses cost. Initial installation cost is one-time cost and it is incurred whenever the line is built (generally, the initial cost is allocated in the beginning of the planning period). The annual operation and maintenance cost can be considered as constant. Electrical losses in the line contribute to the variable part of the line cost since electrical losses are function of the load and they occur whenever the system is in operation (which generally means 8760 hours per year). Therefore, the present value of the total cost of a three-phase line is determine by:

$$Cost_{cond} = FC + (Coeff)(PW)(3I^2R) \tag{2.1}$$

Where:

$$Coeff = (8760)(Cost\ of\ energy)(Loss\ factor) \tag{2.2}$$

$$PW = \frac{(1+d)^p - 1}{d(1+d)^p} \tag{2.3}$$

R = Resistance of the line

I = Current through the line

FC represents the present worth of the fixed cost of the line (which includes the installation and maintenance and operation costs); PW is the present worth factor; p is the planning period (in years) and d represents the discount rate. From equation (2.1), present worth cost of lines with different conductor sizes and load level can be calculated (Figure 2.1) [Mandal and Pahwa, 2002]. Figure 2.1 shows that the total cost of lines is non-linear. The cost of substations is determined by similar equations.

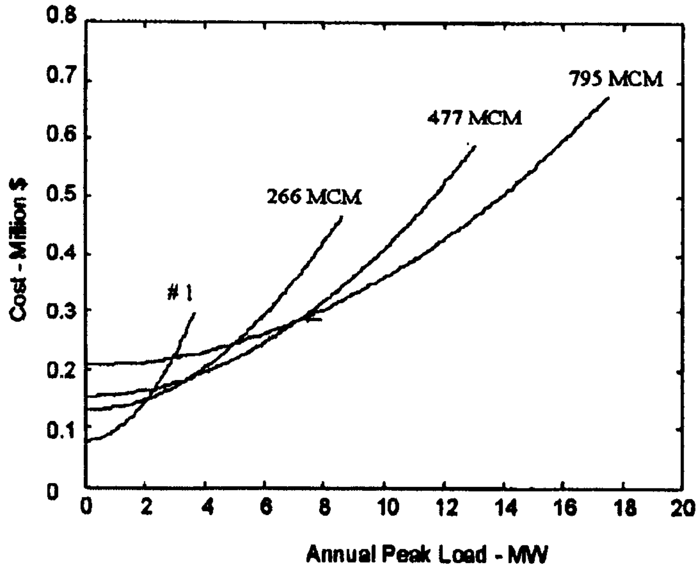


Figure 2. 1 Economic characteristics of a set of conductors

Therefore, the mathematical formulation of the planning problem is expressed as follows:

$$\text{Minimize} \quad F\cos t = \sum(\textit{fixed} \cos ts) + (\text{var}\,\textit{iable} \cos ts) \tag{2.4}$$

$$= \sum_{l\in Nl}\sum_{s\in Ns}\left\{(FC_l)_s(X_l)_s + (Coeff)(PW)(3I_l^2R_l)_s\right\} +$$

$$\sum_{l\in Nl}\sum_{c\in Nc}\left\{(FC_l)_c(X_l)_c + (Coeff)(PW)(3I_l^2R_l)_c\right\}$$

Subject to the conditions:

$$V_{min} \le V_j \le V_{max}$$

$$I_l \le Imax_l$$

$$T_l \le Tmax_l$$

$$\Sigma l = n\text{-}1$$

Where:

$Fcost$ = Objective function. Total cost

$(FC_t)_s$ = Fixed cost of substation t to be built with size s

$(X_t)_s$ = 1 if substation t with size s is built. Otherwise, it is equal to 0.

I_t = Current through substation t

R_t = Resistance of the transformer in substation t

$(FC_l)_c$ = Fixed cost of feeder l to be built with size c

$(X_l)_c$ = 1 if feeder l with size c is built. Otherwise, it is equal to 0.

I_l = Current through feeder l

R_l = Resistance of feeder l

$Coeff$ = Cost factor

PW = Present worth factor

Nt = Number of proposed substations

Ns = Number of proposed sizes for substations

Nl = Number of proposed feeders

Nc = Number of proposed sizes for feeders

Vj = Voltage in node j

$Vmin$ = Voltage drop limit

$Imax_l$ = Current capacity limit of feeder l

$Tmax_t$ = Power capacity limit of substation t

Σl = Number of selected feeders

n = Number of nodes

A distribution system must be planned to serve power within loading and voltage limits and to provide acceptable level of reliability; therefore, a mathematical formulation is needed to quantify the level of reliability of the system. In this research, the energy non-supplied index was formulated [Billinton and Allan, 1996; Chen et al., 1995].

Usually, a power distribution system is either radial or operated radially; therefore, it consists of a set of series components, including lines, cables, busbars, etc. A customer connected to any load point of such system requires all components between himself and the supply point to be supplied with power. Radial power distribution systems are simple to evaluate reliability indices.

Suppose a radial distribution system with m components in series with failure rates λ and failure durations r. The system failure rate λ_m will be:

$$\lambda_m = \sum_{i=1}^{m} \lambda_i \qquad (2.5)$$

and the system failure duration:

$$r_m = \frac{\sum_{i=1}^{m} \lambda_i r_i}{\sum_{j=1}^{m} \lambda_j} \qquad (2.6)$$

The overall system interruption time U_m will be:

$$U_m = \lambda_m r_m = \sum_{i=1}^{m} \lambda_i r_i \qquad (2.7)$$

In the event of equipment failure, the downstream customers experience power interruption, which lasts for a period equal to the equipment failure duration. The yearly interruption time of a customer is counted by the summation of the failure time of its upstream equipment components.

Therefore, the energy not supplied (ENS) to the *i-th* customer due to the failures of the upstream elements is:

$$ENS_i = U_{m(i)} L_i = \sum_{j \in m(i)} \lambda_j r_j L_i \qquad (2.8)$$

where j is the *j-th* electric element on the feeder path between the customer and the supply point; λ_j and r_j are the failure rate and duration of the *j-th* element respectively; L_i is the peak load of the *i-th* customer; and *m(i)* is the set of upstream elements from the customer to the feeding point.

The total energy not supplied to all of the n customers on the system is:

$$ENS = \sum_{i=1}^{n} ENS_i = \sum_{i=1}^{n} \sum_{j \in m(i)} \lambda_j r_j L_i \tag{2.9}$$

The equation (2.9) can be formulated in terms of power flow. Let consider the system shown in figure 2.2. From equation (2.9), the ENS of the system will be:

$$ENS = \lambda_a r_a L_1 + [\lambda_a r_a + \lambda_b r_b] L_2 + [\lambda_a r_a + \lambda_b r_b + \lambda_c r_c] L_3 \qquad (2.10a)$$
$$= [L_1 + L_2 + L_3] \lambda_a r_a + [L_2 + L_3] \lambda_b r_b + L_3 \lambda_c r_c \qquad (2.10b)$$
$$= PF_a \lambda_a r_a + PF_b \lambda_b r_b + PF_c \lambda_c r_c \qquad (2.10c)$$

where PF_a, PF_b, and PF_c are the power flows on the line sections a, b, and c; respectively.

Thus, the energy not supplied index can be formulated from equation (2.10) as follows:

$$ENS = \sum_{i=1}^{N_L} PF_i \lambda_i r_i \tag{2.11}$$

where PF_i is the power flow on section line i and N_L is the number of section lines in the system.

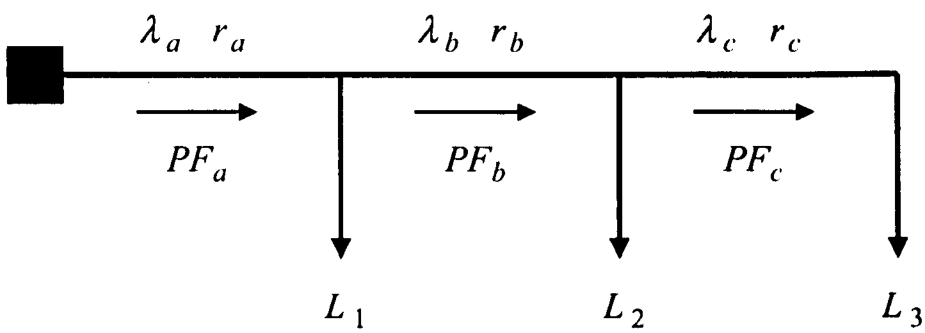


Figure 2. 2 A distribution system with three section lines, three power demand nodes and one substation

The above formulation is based on the assumption that the system has breaker on each line section for the perfect isolation of faults. The component failures are independent events and only one fault is assumed to occur on a feeder and no multi-faults are considered.

2.6 Conclusions

The problem of power distribution system expansion planning is a combinatorial, non-linear and constrained optimisation problem. Traditionally, this problem has been formulated as single-objective optimisation problem; however, in the real world, this problem has more than one objective to optimise.

In this chapter, the problem was formulated as an optimisation problem with two objectives functions and four constraints to meet. The formulation includes the non-linear variable costs and the fixed costs of feeders and substation for different sizes.

In practice, the number of alternative solutions to this problem is so large and they must be evaluated on the basis of the objectives and constraints in order to find the optimal (or sub-optimal) solution (or solutions); therefore, the system planner requires computational optimisation methods.

Classical optimisation method can not be used to solve this problem since its mathematical formulation involves a huge number of variables in real large-scale distribution systems; therefore, a method based on a modern heuristic technique was designed in this research.

Chapter 3 Evolutionary Algorithms for Optimisation Problems

3.1 Introduction

This chapter begins with a description of single-objective and multi-objective optimization problems. The optimality conditions for any solution to be optimal in the presence of multiple objectives are also discussed. Thereafter, a description of genetic algorithms for single-objective optimization and evolutionary algorithms for multi-objective optimization is given.

3.2 Single-objective optimisation problems

When an optimisation problem involves only one objective function, it is called single-objective optimisation problem. In general, a single-objective optimisation problem is represented mathematically as follows:

$$Min\ (or\ Max)\ f(\boldsymbol{x}) \tag{3.1}$$

subject to

$$\begin{aligned} &\boldsymbol{x}_l \le \boldsymbol{x} \le \boldsymbol{x}_u \\ &\boldsymbol{g}(\boldsymbol{x}) \le 0 \\ &\boldsymbol{h}(\boldsymbol{x}) = 0 \end{aligned}$$

where

$f(x)$ is the objective function

x	is a vector of independent variables
x_l	is a vector of lower limits
x_u	is a vector of upper limits
$g(x)$	is a vector of inequality constraints
$h(x)$	is a vector of equality constraints

The problem above must be solved for the values of the variables x that satisfy the constraints and meanwhile minimize (or maximize) the function $f(x)$. A vector that satisfies all the constraint is called a feasible solution to the problem. The collection of such solutions forms the feasible region.

The goal of a single-objective optimisation problem is to find a feasible point x^* such that $f(x) \geq f(x^*)$ (or $f(x) \leq f(x^*)$ for maximization) for each feasible point. Therefore, single-objective optimisation is the procedure of finding and comparing feasible solutions until no better solution can be found (the solutions are qualified as good or bad solution according to the value of the objective function).

3.3 Multi-objective optimisation problems

When an optimisation problem involves more than one objective function, the task of finding one or more optimum solutions is known as multi-objective optimisation. In general, a multi-objective optimisation problem is represented mathematically as follows:

$$Min \setminus Max\ f_m(x) \tag{3.2}$$

subject to

$$\begin{aligned} x_l &\leq x \leq x_u \\ g(x) &\leq 0 \\ h(x) &= 0 \end{aligned}$$

where $f_m(x)$ is a vector of objective functions and the constraints are the same defined in equation (3.1).

Each objective function, in the objective function vector, can be either minimized or maximized but; without loss of generality, it is assumed here that each of the m components of the objective vector is to be minimized.

Generally, in multi-objective optimisation, the objective functions represent incommensurable and competing objectives; therefore, there does not necessarily exist a single solution that is the best with respect to all objectives. To illustrate this, lets consider the figure 3.1 that shows the solutions to an optimisation problem with two objective functions, f_1 and f_2, to be minimized. Lets take the solution 7 as a reference for comparison. Comparing the solutions with solution 7, we can see that solutions 3 and 4 are better solutions than solution 7 since they have smaller values of the objective functions. On the other hand, solutions 8 and 9 are worse solutions. Solutions 1, 2, 5, 6 and 10 are indifferent or incomparable solutions because they are neither better nor worse than the solution 7 in both objective functions. Therefore, in multi-objective optimisation, it is rarely the case that there is a single solution that simultaneous optimizes all objective functions.

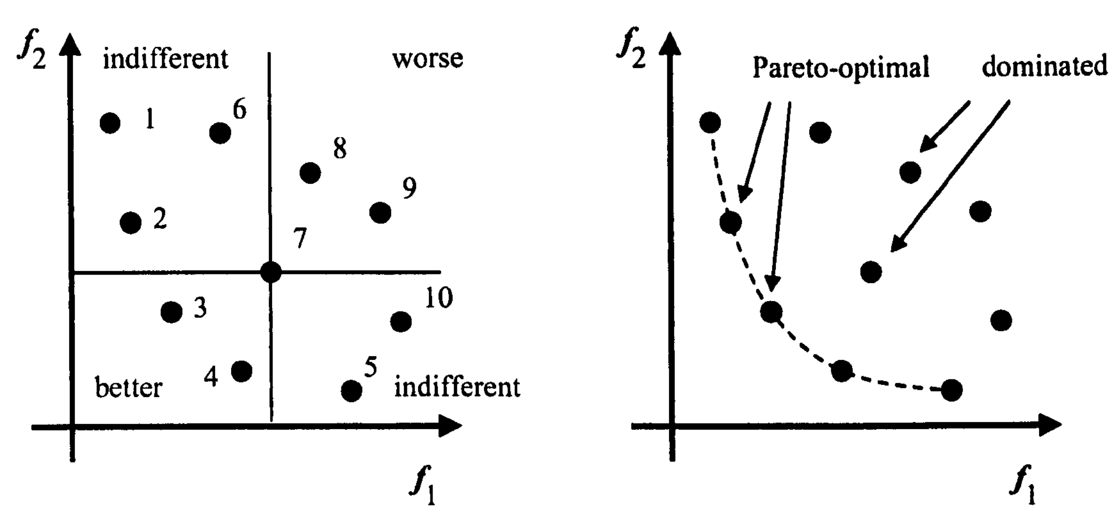


Figure 3. 1 Illustration of the concept of Pareto optimality

To resolve a multi-objective optimisation problem, a different notion of optimum is required. The most common notion of optimality in multi-objective optimisation is the one called Pareto-optimality [Pareto, 1896]. With this notion of optimality, the aim of multi-objective optimization is not to find an optimum solution but to find a set of optimal trade-off optimal solutions. For such solutions, no improvement is possible in any objective function without sacrificing at least one of the other objective functions.

3.3.1 Pareto-optimality

Pareto-optimality involves the following concepts [Deb, 2001]: domination, non-dominated set, Pareto-optimal set and Pareto front.

Domination. A solution $\boldsymbol{x}_1$ is said to dominate another solution $\boldsymbol{x}_2$ if both the following conditions are true:

1. The solution $\boldsymbol{x}_1$ is no worse than $\boldsymbol{x}_2$ in all objectives
2. The solution $\boldsymbol{x}_1$ is strictly better than $\boldsymbol{x}_2$ in at least one objective

If any of the above conditions is violated, the solution $\boldsymbol{x}_1$ does not dominate the solution $\boldsymbol{x}_2$.

If solution $\boldsymbol{x}_1$ dominate $\boldsymbol{x}_2$, it is said that solution $\boldsymbol{x}_2$ is dominated by solution $\boldsymbol{x}_1$. In addition, the concept of domination can be expressed mathematically as follows: If solution $\boldsymbol{x}_1$ dominates $\boldsymbol{x}_2$, the mathematical expression is $\boldsymbol{x}_1 \succ \boldsymbol{x}_2$.

Applying this concept to figure 3.1, it can be said that, for example, solutions 3 and 4 dominate solution 7; and solutions 8 and 9 are dominated by solution 7.

Non-dominated set. For a set of solutions *P*, the set of non-dominated solutions *P'* are those that are not dominated by any member of the set *P*.

In figure 3.1, if the ten solutions form the set *P*, the solutions 1, 2, 3, 4 and 5 form the non-dominated set.

Pareto-optimal set. The non-dominated set of the entire feasible search space S is the Pareto-optimal set.

If the solutions in figure 3.1 form the entire feasible search space, the non-dominated solutions 1, 2, 3, 4 and 5 form the Pareto-optimal set.

Pareto-optimal front. The Pareto-optimal front is the curve formed by joining the Pareto-optimal solutions.

Thus, in multi-objective optimisation, the problem is to find the Pareto-optimal set.

3.4 Genetic algorithms for optimisation

The best known algorithms in the class of evolutionary algorithms are genetic algorithms (GAs). GAs are powerful and broadly applicable stochastic search and optimisation techniques. They have been applied to optimisation problems of different areas such as science, commerce and engineering. The reasons for their success are their ease of use, global perspective and the ability to solve problems that are difficult to be solved by conventional methods.

In general, a genetic algorithm (GA) has five components [Michalewicz, 1996]:

1. A genetic representation of solutions to the problem
2. A method to create an initial population of solutions
3. An evaluation function rating solutions in terms of their fitness
4. Genetic operators
5. Values for the parameters of the genetic algorithm

Genetic algorithms are based on the principle of natural genetics and natural selection.

A GA generates an initial population of individuals (chromosomes). Each individual represents a possible solution for the problem being optimized. These solutions are classified by an evaluation function, giving better value (fitness) to better solutions. The population evolves over several generations to a population with the optimal solution (or near-optimal solution) to the problem. This evolution is generated by selection, recombination and mutation operations. In selection operation, individuals are selected from the current population according to their fitness for recombination operation (i.e. the more fit solutions have more probability of being selected). The recombination operation creates new solutions (offspring solutions) by combining parts from two individuals (parent solutions); and mutation operation makes changes in a single individual. Afterwards, the new solutions are evaluated and a new population is created from the old population and the offspring solutions selecting the better individuals. This process is carried out in each iteration (generation) of the algorithm. The process is stopped by a predetermined stopping rule. Figure 3.1 shows a general structure of genetic algorithms [Gen and Cheng, 2000].

```
Genetic Algorithm
Begin
      t = 0
      Generate an initial population P(t)
      Evaluate P(t)
      While (not termination condition) do
      begin
        Select parents Q(t)
        Recombine Q(t) to yield offspring population R(t)
        Modify some offspring
        Evaluate R(t)
        Select P(t+1) from P(t) and R(t)
        t = t + 1
      end
End
```

Figure 3. 2 A general structure of genetic algorithms

3.4.1 Representation of solutions

The first step in the design of a GA is to develop an encoding technique to represent each potential solution. The encoded solutions are called chromosomes.

Representing solutions of the problem is a critical point in the design of genetic algorithms. The encoding can be the factor that determines the success or failure of a GA. Many encoding techniques have been developed and they are classified as follows:

a) Binary encoding
b) Real number encoding
c) Integer or literal permutation encoding
d) General data structure encoding

An encoding technique is developed with the aim of meeting the nature of a given problem to represent all feasible solutions; and the mapping between chromosomes and solutions must be 1 to 1.

3.4.2 Genetic operators

In general, a search technique possesses one of these two abilities: random search and local search. Random search helps to explore the entire solution space. Local search helps to exploit the best solution. Hillclimbing is an example of a search technique that exploits the best solution for possible improvement but neglecting exploration of the search space. On the other hand, random search methods explore the search space ignoring the exploitations of promising solutions in the space.

Genetic algorithms are the search methods that posses these two search abilities. In GAs, accumulated information is exploited by the selection mechanism, while new regions of the search space are explored by means of genetic operators: recombination and mutation. Recombination operator is used to perform a random search to try to explore the area beyond a local optimum; and mutation operator is used to perform a local search to try to find an improved solution.

The recombination operator recombines features from two parent solutions to produce offspring. If this operation combines the best features from the parents, the offspring will have better fitness. And mutation operator produces random changes in various chromosomes.

3.4.3 Selection mechanism

The principle behind genetic algorithms is Darwin's theory of evolution by natural selection: natural selection occurs when some organisms leave more offspring than others, depending on their physical characteristics; if these characteristics are then inherited by their offspring, over time the population will become better and better adapted to the environment.

In genetic algorithms, the selection mechanism is applied to select individuals for recombination operation.

Selection provides the driving force in the search. But there are two important issues in the evolution process of the genetic search: population diversity and selection pressure. Selection pressure is the degree to which the best individuals are favoured. The higher the selection pressure, the more the best individuals are favoured. Population diversity and selection pressure are strongly related: an increase in selection pressure decreases the diversity of the population, and vice versa. Therefore, if the selection pressure is too low, the evolutionary progress will be slower than necessary making the search inefficient. If the selection pressure is too high, it is probable that the GA will terminate prematurely with a bad solution. Hence, a selection mechanism should preserve a balance between population diversity and selection pressure.

Many selections methods have been proposed [Gen and Cheng, 2000]. The common methods are as follows:

- Roulette wheel selection
- $(\mu+\lambda)$-selection
- Tournament selection

- Steady-state reproduction
- Ranking and scaling
- Sharing

3.4.4 Fitness function

Genetic algorithms work on two types of spaces: coding space (genotype space) and solution space (phenotype space). Genetic operators work on genotype space, and evaluation and selection work on phenotype space. When individuals in the genotype spaces are decoded and evaluated by the objective function, a fitness value is given to each individual in the phenotype space. This fitness value is assigned by means of a fitness function.

The objective function provides a measure of performance to a particular individual, according to its features as a coding solution, independently of any other individuals. The fitness function transforms that measure of performance into a value in the phenotype space. This value is defined with respect to other members of the current population. In most cases, however, the fitness is made equal to the objective function value [Deb, 2001].

As optimisation tools, GAs face the task of dealing with problem constraints therefore, some individuals can correspond to infeasible solutions (decoded solutions that lie outside the feasible region of a given problem). Another problem that GAs have to deal with is illegality. Illegality refers to the phenomenon that a chromosome does not represent a solution to a given problem. This phenomenon is originated from the nature of the encoding technique.

To handle constraint and infeasible solutions, many techniques have been developed. The common techniques are: rejecting technique, in which infeasible chromosome are discarded; repairing procedure, which uses a converter to transform an infeasible chromosome into a feasible one; and penalty approach, which transforms the constrained problem into an unconstrained problem by penalizing infeasible solutions with a penalty term.

In the penalty approach, there are two ways to define the fitness function with a penalty term. One way is the addition form in which the penalty term is added to the objective function. The second way is the multiplication form in which the objective function is multiplied by the penalty term.

The optimum solution in constrained optimisation problems occurs at the boundary of feasible regions; and frequently, an infeasible solution close to the optimum solution contains more information about it than a feasible solution far from the optimum. Therefore, the rejecting and repairing techniques often lead to poor solutions. On the other hand, penalty approach allows the genetic search to approach the optimum from both sides of the feasible and infeasible regions.

For illegal chromosomes, the penalty technique is not applicable because an illegal chromosome can not be decoded to a solution. Repairing techniques are usually adopted to convert an illegal chromosome to a legal one.

3.5 Evolutionary algorithms for multi-objective optimisation problems

Multi-objective optimisation involves two tasks: search and multi-criteria decision-making. In real-world optimisation problems, the space to be searched can be too large and complex to find the solutions by mathematical programming methods, local or gradient search. In addition, the multiple objectives to optimize are generally conflicting therefore, it can be difficult, for a decision-maker, to make tradeoffs in order to rank (order) the solutions and then, be able to make decisions. Evolutionary algorithms are ideal candidates to carry on both search and multi-criteria decision making task.

The integration of search and multi-criteria decision making is a key issue in evolutionary algorithms. Making some decisions before search can affect the fitness landscape of the search space whereas search before decision-making can eliminate the vast number of dominated solutions and focus decision making on a few alternative solutions.

Several approaches have been developed to resolve multi-objective optimisation problems. They can be classified according to how both search and multi-criteria decision making are integrated:

1) Making multi-criteria decisions before search
2) Search before making multi-criteria decisions
3) Interacting search with multi-criteria decision making

In this research, the focus was on the search process before making multi-criteria decisions using evolutionary algorithms (EAs).

In this context, the difference between single-objective optimization and multi-objective optimization is that in the former, there is only one goal, which is to find an optimum solution, whereas in the latter, there are two goals:

1. To find a set of solutions as close as possible to the Pareto-optimal front
2. To find a set of solutions as diverse as possible

Since both goals are equally important, a multi-objective optimisation method must pursue both goals.

There are three main issues in the design of multi-objective optimisation methods based on evolutionary algorithms: fitness assignment and selection, diversity preservation and elitism.

3.5.1 Fitness assignment and selection

Generally, in evolutionary algorithms for single-objective optimisation problems, the objective function and the fitness function are so closely related that the objective function is often identical to the fitness function. By contrast, in multi-objective optimisation, the fitness assignment and selection are more elaborate process since all objective functions must be equally considered.

Several fitness assignment and selection methods have been developed for multi-objective evolutionary algorithms and they can be categorized as follows: criterion-based, aggregation-based and Pareto-based methods.

Criterion-based methods

In these methods, a number of sub-populations are generated at each generation according to each objective function in turn. For example, in [Schaffer, 1985] it is reported a method that, for a problem with k objective functions and a population size of M, generates k sub-populations of size M/k using one of each of the k objectives. These sub-populations are shuffled to obtain a new population on which the genetic operators are applied.

In [Kursawe, 1991], a similar method is proposed. In this method, binary tournaments are conducted and one objective function decides each tournament. Each objective function is given a probability (either by random, fixed by the user or allowed to evolve with the population) that determines weather the objective will be considered for a tournament.

Aggregation-based methods

In these methods, the different objective functions are combined into a single parameterized function. The parameters of this function are varied during the optimisation run such that a set of non-dominated solutions can be found. In this case, the standard selection mechanisms can be used without modifications. An example of this type of method is reported in [Hajela and Lin, 1992]. This method multiply each objective function f_i by a weight w_i. The weighted function values are added together to calculate the fitness of a solution. Each individual in the population is assigned a different weight vector. The parameters of a weight vector are encoded on the chromosomes of its associated individual. After the fitness for each solution in the population is calculated, the roulette wheel selection is applied to create the mating pool.

In [Ishibuchi and Murata, 1996] a similar method to the above is suggested, except that a random normalized weight vector is assigned to each solution in the population.

Pareto-based methods

In [Goldberg, 1989] it is suggested the idea of using the non-dominated sorting concept in genetic algorithms. Under this concept, the solutions in a population are arranged according to Pareto dominance relationships: the current non-dominated solutions in a population are given rank one and then removed from the population. The newly non-dominated solutions in the reduced population are assigned rank two, and removed. This process continues until the population is ranked (with the non-dominated sorting concept, several non-dominated fronts are identified in the population). This non-dominated sorting method was later implemented in the algorithm reported in [Srinivas and Deb, 1994].

In the algorithm reported in [Fonseca and Fleming, 1993], the population is ranked according to the degree of domination; that is, the rank of a certain solution corresponds to the number of solutions in the current population by which it is dominated.

With these methods of ranking, versions of the roulette wheel selection are applied on the population.

3.5.2 Diversity preservation

Most multi-objective evolutionary algorithms use an additional mechanism to maintain diversity along the current approximate Pareto-optimal solutions. For example, the algorithm proposed in [Srinivas and Deb, 1994] uses the fitness sharing mechanism. This mechanism is based on the idea that individuals in a particular niche (sub-population of solutions) have to share the available resources (fitness). Therefore, the fitness value of a certain individual is the more degraded if the more individuals are located in its neighborhood.

The algorithm reported in [Knowles and Corne, 1999], divide the search space into a number of grids. The solutions in each box of the grid define the neighborhoods and the density around an individual is estimated by the number of solutions in the same box of the grid.

Another diversity-preserving mechanism is the nearest neighbor technique that was introduced in [Zitzler et al., 2001]. This technique takes the distance of a given solution to its *k*-th nearest neighbor into account in order to estimate the density in its neighborhood.

With diversity-preserving mechanisms, the chance of a solution of being selected is decreased the greater the density of individuals in its neighborhood.

3.5.3 Elitism

Elitism is the retention of some individuals in the population from one generation to the next, so that they can be considered for selection and reproduction for more than once throughout the evolutionary process. In evolutionary multi-objective optimisation algorithms, elitism plays an important role since good solutions may be lost during the optimisation process due to random effects.

One common elitist approach uses the idea of maintaining a secondary population (or external archive) to which promising solutions in the population are copied at each generation. For example, the algorithm reported in [Zitzler and Thiele, 1999] stores all non-dominated solutions separately from the general population. Individuals are chosen from the population and the non-dominated set as recombination partners, while the later get higher selection probabilities.

Another algorithm that uses an external archive is reported in [Knowles and Corne, 1999]. This algorithm is a multi-objective (1+1) evolution strategy in which the elitism is guaranteed by making the offspring and the parent solution to compete for a place in the elite population.

Not all elitist-multi-objective evolutionary algorithms explicitly incorporate an external archive. For example, in [Deb et al., 2002] the algorithm carries out a non-dominated sorting of a combined parent population and offspring population. Thereafter, the new population is filled by solutions of different non-dominated fronts, one at a time and starting from the front with the best non-dominated solutions. Each front is accepted until the population is filled.

3.6 Conclusions

Evolutionary algorithms (EAs) are very different from most of traditional optimization methods. The fundamental differences are:

- Evolutionary algorithms work with a coding of decision variables instead of the variable themselves; therefore, they can be applied to problems with a large number of variables.

- EAs work with a discrete search space; therefore, they are ideal to be applied to combinatorial problems.

- EAs work with a population of solutions instead of a single solution.

- EAs can give a set of solutions simultaneously, in one single simulation run; therefore, they can be used to find Pareto-optimal solutions for multi-objective problems.

- EAs do not impose any restriction to the objective function to be optimized; therefore, they can handle discrete, non-convex and non-linear functions.

Therefore, evolutionary algorithms are ideal algorithms for solving the distribution system planning problem formulated either as a single-objective or as a multi-objective optimization problem.

Chapter 4 A Proposed Genetic Algorithm for Power Distribution Network Expansion Planning

4.1 Introduction

This chapter presents a new genetic algorithm developed in this project for optimal large-scale power distribution network expansion planning. The algorithm finds the optimum (or near optimum) location and size of substations and feeders to minimise a cost function of the network, which represent capital (fixed costs) and operational costs (non-linear variable costs).

The algorithm was tested on three networks and the results were compared with the results from other genetic algorithms.

4.2 The proposed GA for power distribution network planning

4.2.1 Fundamental qualities that genetic algorithms should have for power distribution network planning

The qualities that any GA should have are [Raidl and Julstrom, 2002, 2003]:

- Space. Chromosomes should not require extravagant amounts of memory.
- Time. The time complexities of evaluating, recombining and mutating chromosomes should be small.
- Legality. All chromosomes should represent legal solutions.
- Coverage. The encoding should be able to represent all feasible and legal solutions.

- Bias. In general, representations of all solutions should be equally likely.
- Locality. A coding has high locality if mutating a genotype changes the corresponding phenotype slightly.
- Heritability. Offspring of recombination operation should represent solutions that combine substructures of their parental solutions.
- Constraints. Decoding of chromosomes and the recombination and mutation operator should be able to enforce problem-specific constraints.

From these qualities, we propose specific characteristics that a GA for power distribution network planning should have:

- All chromosomes should represent radial networks connecting all the nodes
- The result of decoding chromosomes and applying recombination and mutation operators should be radial networks
- Offspring of recombination operators should have not only components of their parental solutions but also the way these components are connected, that is, the configuration

The last point is important because the cost of a power distribution network is a function of capital and electrical losses costs, which are function of the network's configuration.

4.2.2 The proposed genetic algorithm

Methods based on genetic algorithms have been proposed to resolve the problem of power distribution expansion planning [Miranda et al., 1994; Yeh et al., 1996; Ramirez-Rosado and Bernal-Agustin, 1998; Lin et al., 1998; Carvalho et al., 2000]. However, some of these methods have been reported with applications to small networks and others have long solution time. The main problem of these methods is their poor representation of the potential solutions and their genetic operators.

To overcome such difficulties, a new genetic algorithm was developed in this project using an effective representation of the potential solutions and non-standard genetic operators.

The proposed genetic algorithm is based on a genetic algorithm developed for the degree-constrained minimum spanning tree problem [Raidl, 2000]. The algorithm uses a direct encoding technique and special recombination and mutation operators.

The degree-constrained minimum spanning tree problem belongs to the category called minimum spanning tree problem, which has a history in combinatorial optimisation. It was first formulated by Boruvka in 1926 [Graham and Hell, 1985]. Since then, the minimum spanning tree formulation has been widely applied to many combinatorial optimisations problems, such as telecommunication network design, transportation and distribution problems.

Consider a connected graph $G = (V, E)$, where $V = \{v_1, v_2, v_3, \ldots, v_n\}$ is a finite set of vertices or nodes and $E = \{e_1, e_2, e_3, \ldots, e_m\}$ is a finite set of edges or lines representing connections between these nodes. Each line has a positive real number denoted by $W = \{w_1, w_2, w_3, \ldots, w_m\}$, representing distance or cost. The minimum spanning tree problem is to seek a least-weight spanning tree (sub-graph connecting all nodes on the graph G with no loops).

A power distribution network when in operation is radial and connected, i.e., the network is a spanning tree. Thus, the formulation of power distribution network planning problem can be based on minimum spanning tree formulation.

In the following sections, the components of the proposed genetic algorithm are described.

4.2.2.1 Encoding technique

The encoding technique to represent each possible solution for the problem of planning power distributions networks consist in representing the solutions directly as sets of their lines. For example, figure 4.1 shows a power distribution network with 37 demand nodes, 3 candidate substations and 53-numbered candidate lines. A potential solution for this network is encoded as the set of numbers that represent the lines that form the solution (Figure 4.2).

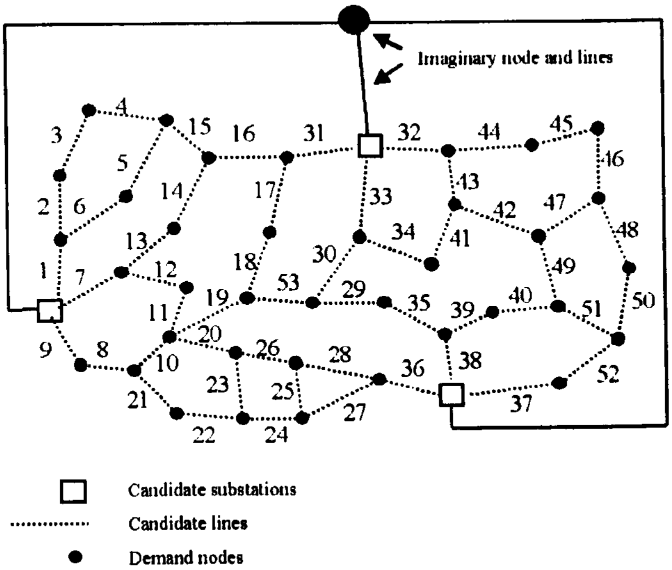
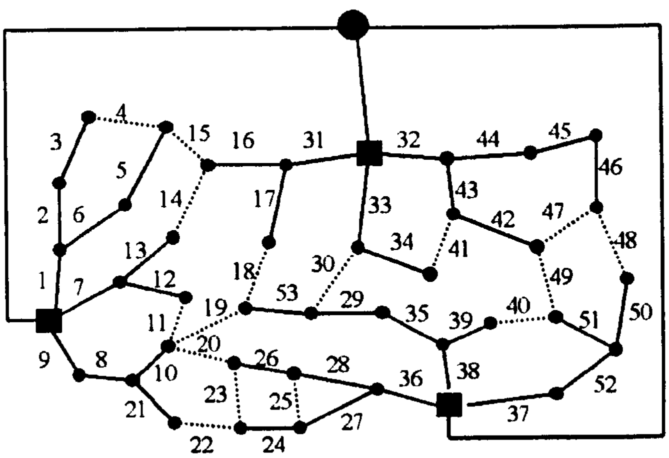


Figure 4. 1 A power distribution network for planning



Encoded solution =

{9,8,21,10,1,2,6,5,3,7,12,13,31,32,17,16,33,34,44,45,46,43,42,37,52,50,51, 38,39,36,27,28,24,26,35,29,53}

Figure 4. 2 An encoded solution for the network in figure 4.1

The imaginary node and lines are used to manipulate problems with more than one substation and to represent the networks as spanning trees. Therefore, each solution can be encoded with an array containing the lines of the solutions. The size of this array is equal to the number of nodes minus 1.

This encoding technique requires less amounts of memory than the other techniques used in previous planning methods.

4.2.2.2 Initial population

An initial solution is created by selecting lines randomly and inserting them one by one in the array checking that a line does not introduce a loop. If a line introduces a loop, another line is selected. The procedure terminates when a complete spanning tree has been built, i.e. when the number of lines inserted in the array or set is one less that the number of nodes. This procedure will always produce solutions with radial configuration. To test efficiently whether a line introduces a loop, a union-find algorithm is used [Sedgewick, 1992; Weiss, 1993].

4.2.2.3 Recombination operation

Recombination operation consists of two steps: In the first step, a set of lines contained in both parents is selected to initialise the offspring. In the second step, lines are randomly and successively selected from the rest of the lines contained either in parent 1 or parent 2 (but not in both) to be included in the offspring (only lines that do not introduce loops are included).

Figure 4.3 shows an example of a recombination operation for the network of figure 4.1. Figures 4.3 a) and b) are the parent solutions P_1 and P_2, respectively. Figure 4.3 c) is the offspring initialised with lines contained in both parents. In this phase, the offspring has components disconnected. In the second step, the disconnected components are connected with lines contained either in P_1 or P_2; as it is shown in figure 4.3 d).

This recombination operation only produces legal offspring solutions with substructures of their parents resolving the problems of low heritability and topological unfeasibility.

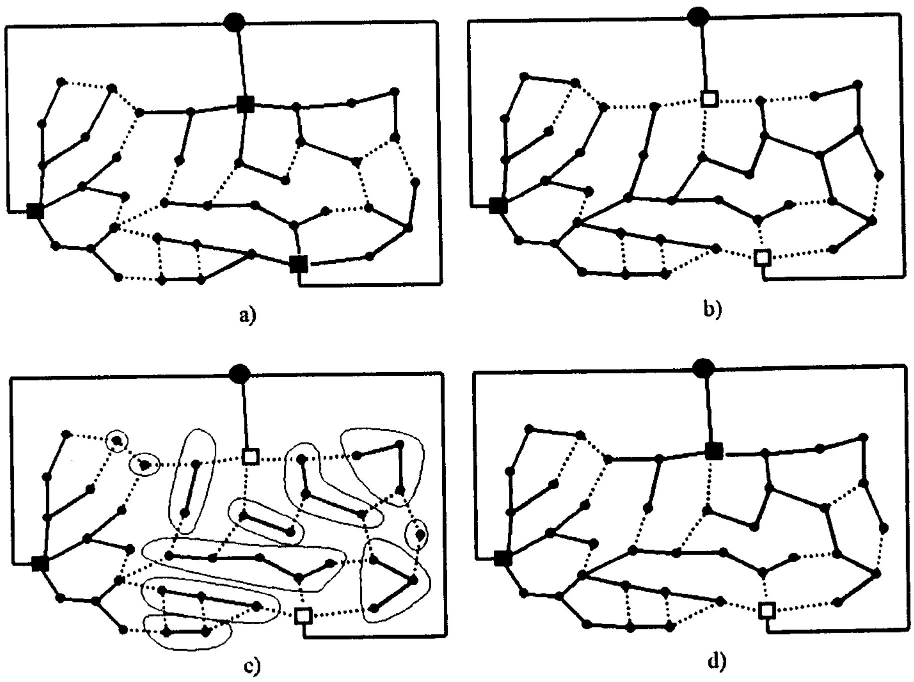


Figure 4. 3 Recombination operation. Figures a) and b) are the parent solutions; figure c) is the offspring initialized with lines contained in both parents; and figure d) is the resultant offspring solution

4.2.2.4 Mutation operation

Mutation operation is described as follows (see Figure 4.4): First, a candidate line currently not in the offspring is randomly chosen and inserted in the offspring, e.g. in figure 4.4 a) the line 30 (darker line) is inserted. A cycle will be formed with this action so, in a second step, a random choice among the lines in the cycle is made (excluding the new line inserted and the imaginary lines) and the chosen one is removed from the offspring. In Figure 4.4 b), the line 41 is removed.

To find the lines that conform a cycle, an adjacency list representation of the tree is created temporarily and a dept-first search algorithm is applied [Sedgewick, 1992; Weiss, 1993].

This mutation operation provides high locality making small changes in a solution: exchanging one line for another one.

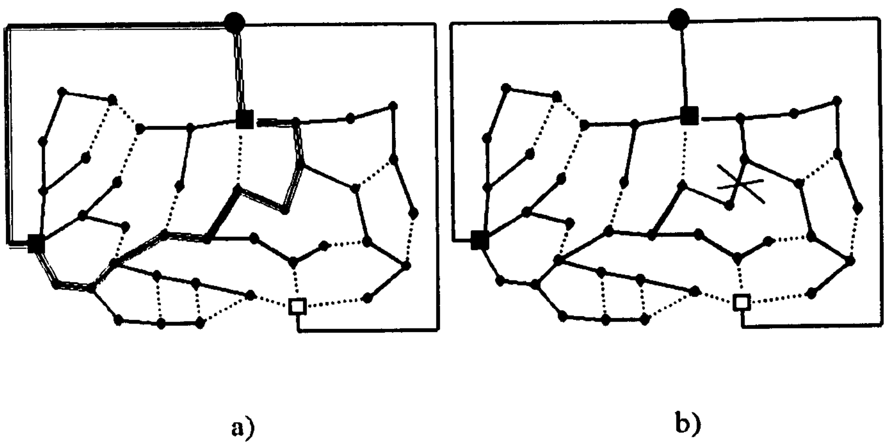


Figure 4. 4 Mutation operation for the offspring in figure 4.3. In figure a) the line 30 (darker line) is inserted and in figure b) the line 41 is removed

4.2.2.5 Fitness function

The problem of distribution network planning is to determine a network with the minimum cost satisfying the restrictions of radiality, voltage drop and capacity. The radiality constraint is satisfied with the GA manipulating only spanning trees. The other constraints are verified after computing power flow. The solutions that do not satisfy the voltage drop and capacity constraints are penalised by a penalty function. Thus, the fitness function is formulated as follows:

$$Fitness\ function = Objective\ function + Penalty\ function =$$
$$= \frac{\sum[(fixed\ costs) + (variable\ costs)]}{M} + Penalty\ function \quad (4.1)$$

The penalty function is defined as follows:

$$Penalty\ function = \alpha * s(tree) + \beta * l(tree) + \gamma * v(tree) \quad (4.2)$$

where

$$s(tree) = \frac{Number\ of\ substations\ above\ capacity\ limit}{Number\ of\ substations\ in\ the\ solution}$$

$$l(tree) = \frac{Number\ of\ lines\ above\ capacity\ limit}{Number\ of\ lines\ in\ the\ solution}$$

$$v(tree) = \frac{Number\ of\ node\_voltages\ above/below\ limit}{Number\ of\ nodes}$$

The penalty function is equal to zero if the solution is feasible; otherwise it is major than zero. The penalty pressure is incremented as the number of elements that violate the constraints increase.

The coefficients α, β and γ are adjusted according to the degree of violation of constraints. In this research, the genetic algorithm found the solutions with these coefficient taking values between 0.1 and 1.0.

The constant *M* is used to make the values of the fitness function be less than one. This constant can be of the order of the maximum cost of the network.

4.2.2.6 Selection mechanism

Selection mechanism utilised in this genetic algorithm is tournament selection. This selection scheme consists in selecting a *k* number of individuals randomly from the population and picking out the best chromosome from this set for recombination. This procedure is repeated until the mating pool is completed. The number of chromosome in the set is called tournament size. A common tournament size is 2 (this is called binary tournament).

Adjusting the tournament size, the selection pressure can be high or low. Bigger tournaments have the effect of increasing the selection pressure, since below average individuals do not have high probability of being selected.

4.2.2.7 Recombination and mutation rate

Recombination rate indicates the number of individuals that are selected from the population for recombination. For example, for a population size equal to 20 and a recombination rate equal to 0.25 (25%), the number of individuals selected for recombination is 6 (0.25*20). Mutation rate indicates the number of offspring selected for mutation.

4.3 Case studies

The proposed genetic algorithm was tested on three networks:

Case 1. In this case, the network is a complete graph (every node has a direct connection with the rest of the nodes) with 10 nodes and 45 lines. This network has 100 million possible solutions (spanning trees). The purpose of this case study is to evaluate the efficiency and effectiveness of the algorithm and compare the results with other genetic algorithms reported in reference [Carvalho et al., 2001].

Case 2. In this case, the network is a power distribution system with eight demand nodes, two candidate substations and fourteen candidate feeders. This problem was presented in references [Lin et al., 1998], [Ponnavaikko et al., 1987] and [Goswami, 1997]. The main objective of this case is to validate the proposed GA, applying it on a small problem where the optimum solution is recognised.

Case 3. The algorithm was tested on a real large-scale network presented in [Ramirez-Rosado and Bernal-Agustin, 1998]. The network has 201 nodes with 1 substation, 43 existing feeders and 184 proposed new feeders with 2 possible conductor sizes. The aim of this case is to demonstrate a practical application of the algorithm.

The procedure used in the proposed GA to build the new population in each generation consists in selecting individuals from the old and offspring populations (the better solutions are selected). With this procedure, the algorithm stops when the difference between the population fitness average and the best solution fitness is less than a predetermined tolerance value.

The tests were done using a PC compatible 1 GHz Pentium 4 with 128 Mb of RAM, Windows ME and a Visual C++ compiler.

4.3.1 Case 1

In this case, a target-spanning tree is defined, naming the lines that form this tree "correct lines". The fitness of any solution is the number of correct lines that it has. Thus, the target tree is the fittest solution.

In reference [Carvalho, 2001], it is reported the results of three different crossover operators tested on this network. One crossover operator is called canonical recombination, which is the conventional one-point crossover. The second one is called tree-generation recombination (TG), which comprises two steps: 1) a pool of lines from two parents is built and then 2) two spanning trees are generated selecting randomly lines from the pool. And the third crossover operator is the one proposed by the authors of this reference. This operator consists in selecting randomly two nodes in two parent solutions and then the paths found between these nodes in each parent solution are exchanged to produce two offspring solutions.

In order to compare the proposed genetic algorithm with the ones presented in reference [Carvalho, 2001], the following parameter values were used: Pc (recombination rate): 0.8, and Pm (mutation rate): 0.0. Five different population sizes were used and 30 different initial populations.

Table 4.1 shows the results of the proposed GA and the results of the other three genetic algorithms. The results are the percentage of correct lines (E) of the best solution found in 20 generations and the standard deviation (σ), for each population size.

Table 4. 1 Comparative table of the proposed GA and other three algorithms in [Carvalho, 2001]

Percentage of correct lines found in 20 generations								
	Proposed GA		Proposed approach in [Carvalho, 2001]		Canonical approach		TG approach	
Population size	E	σ	E	σ	E	σ	E	σ
40	84.07	10.99	85.24	6.77	48.90	13.10	64.54	9.09
50	91.48	7.95	92.57	6.34	49.29	13.88	67.43	11.93
60	97.04	4.91	92.78	6.52	50.26	15.82	68.45	12.91
70	96.66	5.09	94.91	5.73	51.78	12.18	69.25	12.22
80	97.04	4.91	96.82	5.15	56.73	11.89	69.65	13.11

The table shows that the proposed GA and the approach proposed in [Carvalho, 2001] are able to converge in less iteration than the canonical and TG approaches and; the proposed GA found more correct lines with smaller standard deviation than the approach proposed in [Carvalho, 2001], using population sizes bigger than 50.

The main causes of the poor performance of the canonical and TG approaches are:

- Low heritability. A significant number of offspring generated by the crossover operators hardly have substructures of their parents
- Topological unfeasibility. Many offspring do not represent a topologically

valid solution

Figure 4.5 illustrates the variation of the number of correct lines in the population using the proposed GA with a population size of 60. The number of correct lines increases with the generations, indicating that the recombination operator of the proposed GA has high degree of heritability.

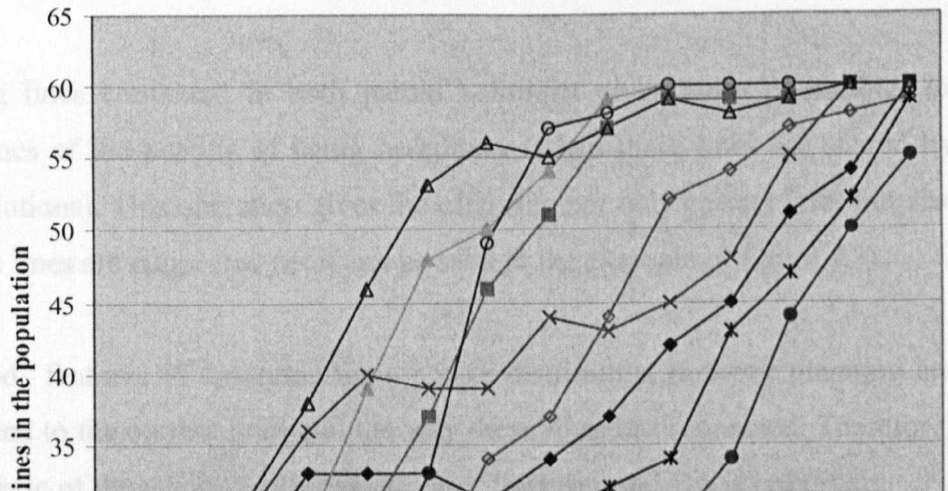


Figure 4. 5 Variation of the number of correct lines for the network in case 1

Comparing the results from the proposed GA and the TG approach it is confirmed the importance of one of the characteristics that a GA for distribution network planning should have, which is mentioned in section 4.2.1: Offspring of recombination operators should have not only components of their parental solutions but also the way these components are connected.

The difference between the proposed GA and the TG approach is that in the former, lines appearing in both parent solutions are favoured and automatically appear in the offspring; whereas in the latter, either all parental lines are treated equally.

Favouring lines contained in both parent solutions gives more probability to the correct lines of the parents of being hereditary (when these lines are shared by the parent solutions). This operation gives the offspring not only correct lines but also the way these lines are connected (as it can be seen in the example of figure 4.3).

The "good" features of solutions for a power distribution network planning are the ones related to the correct lines and the way these lines are connected. Therefore, this characteristic of the recombination operator of the proposed GA is important.

With these results, the proposed GA proved to be an efficient and effective algorithm in this case.

4.3.2 Case 2

In this case, the network to be planned is shown in figure 4.6 and the general data of the problem are given in appendix A. The problem is to select the optimum feeder routings and the locations of substations to supply the power demand nodes.

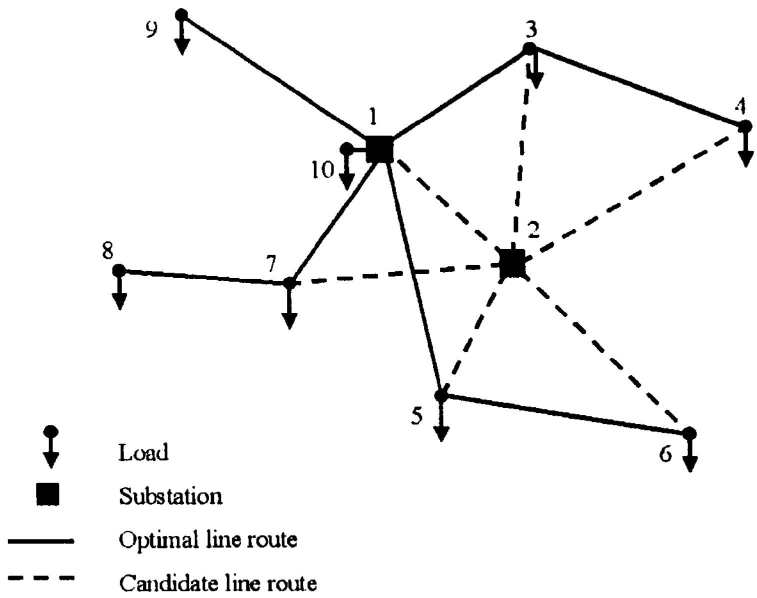


Figure 4. 6 Power distribution network for case 2

This is a small problem where the optimum solution is recognised. The problem was resolved in references [Lin et al., 1998], [Ponnavaikko et al., 1987] and [Goswami, 1997] using genetic algorithms, a simplex method and a branch exchange technique, respectively. Figure 4.6 shows the optimum configuration.

The total cost of the optimum solution is 12.62 (Million Rs). This cost consists of the following costs (Million Rs): total energy losses cost = 5.44; total feeder cost = 3.68; total feeder bay cost = 0.4 and substation cost = 3.1.

The proposed GA was tested on this problem with different combination of genetic operators and population sizes. For each operator combination, ten different initial populations were used for ten program runs. Figure 4.7 presents the average fitness of the solutions obtained over the 10 runs. Figure 4.8 shows the average generations.

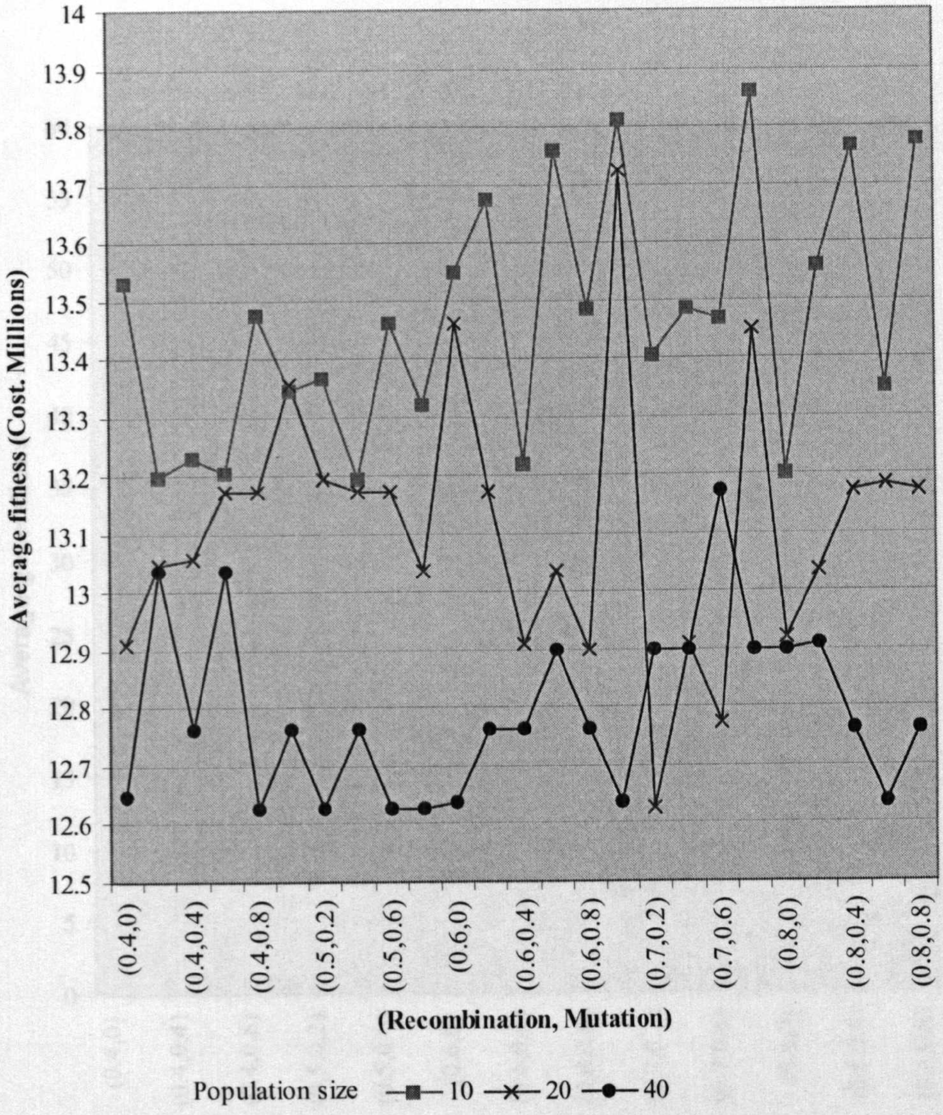


Figure 4. 7 Average fitness of solutions found over 10 program runs for case 2. The horizontal axis represents the combination of genetic operators (recombination rate increases by increments of 0.1 and mutation rate by 0.2).

The proposed GA gave the optimum solution over the ten program runs with population sizes of 20 and 40. With the population size of 20, the solution was obtained in 17 generations (average) with a recombination rate of 0.7 and a mutation rate equal to 0.2.

The proposed GA was able to obtain the solution in a single run, whereas the GA proposed in reference [Lin et al., 1998] obtained the same result by dividing the problem into three sub-problems and applying the algorithm to each of them.

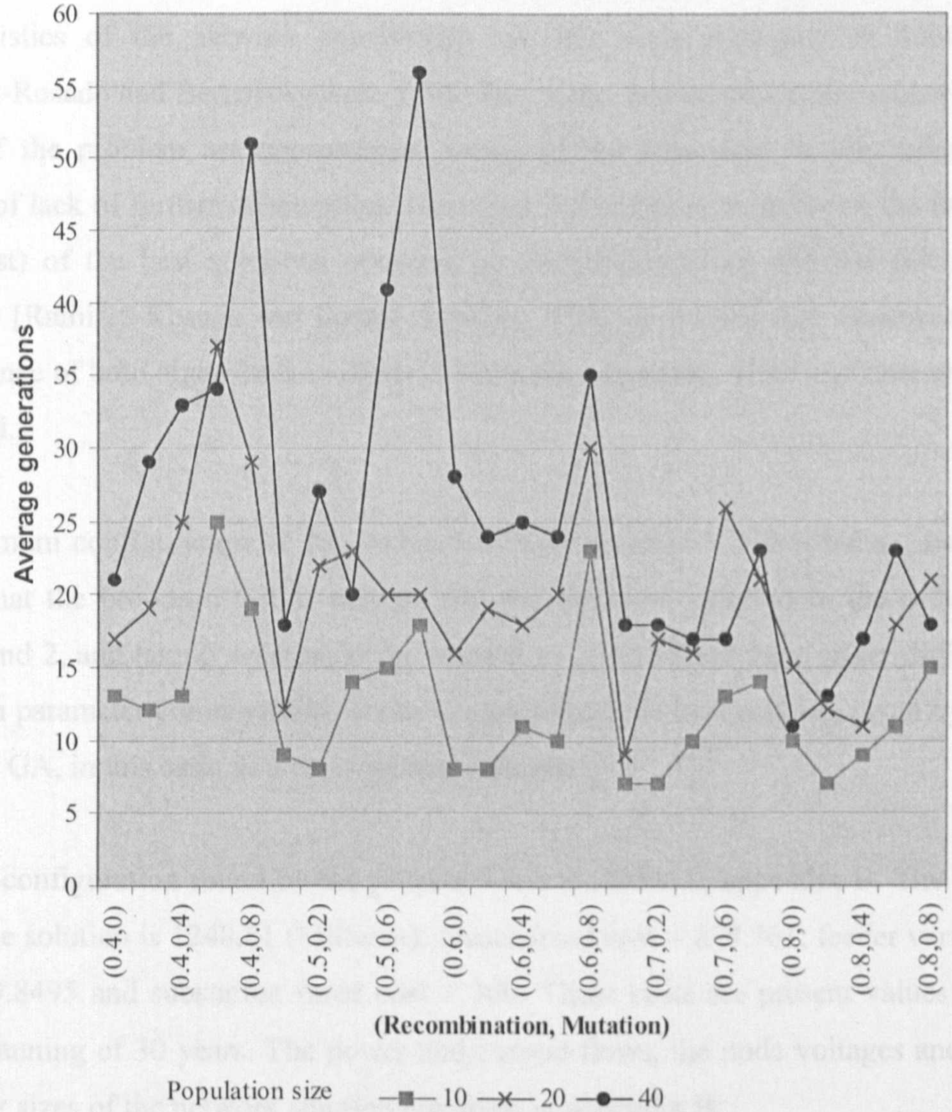


Figure 4. 8 Average generation of generations in which the algorithm stops over ten program runs for case 2. The horizontal axis represents the combination of genetic operators (recombination rate increases by increments of 0.1 and mutation rate by 0.2).

4.3.3 Case 3

The proposed genetic algorithm was tested on a real network reported in reference [Ramirez-Rosado and Bernal-Agustin, 1998]. Figure 4.9 shows this network where the continuous lines represent the existing feeders and the proposed routes are represented by dashed lines.

The general data of the problem are given in appendix A. The data related to electrical characteristics of the network components are the same presented in reference [Ramirez-Rosado and Bernal-Agustin, 1998] but, some data related to the economical aspect of the problem are approximate values of the data used in this reference because of lack of further information. Therefore, the comparison between the fitness (total cost) of the best solutions obtained by the proposed GA and the GA from reference [Ramirez-Rosado and Bernal-Agustin, 1998] is not possible; however, the performance of both algorithms in terms of iteration, population sizes and time can be compared.

The optimum configuration of this problem is not recognised. Nevertheless, having proved that the proposed GA is able to find the optimum solution in the previous cases 1 and 2, and taking account of the amount of experiments done under different algorithm parameter combinations, it can be considered the best solution given by the proposed GA, in this case, as a near-optimal solution.

The best configuration found by the proposed GA is shown in appendix B. The total cost of the solution is 1248.61 (Millions): feeder fixed cost = 878.761; feeder variable cost = 69.8495 and substation fixed cost = 300. These costs are present values in a period planning of 30 years. The power and current flows, the node voltages and the conductor sizes of the network solution are given in appendix B.

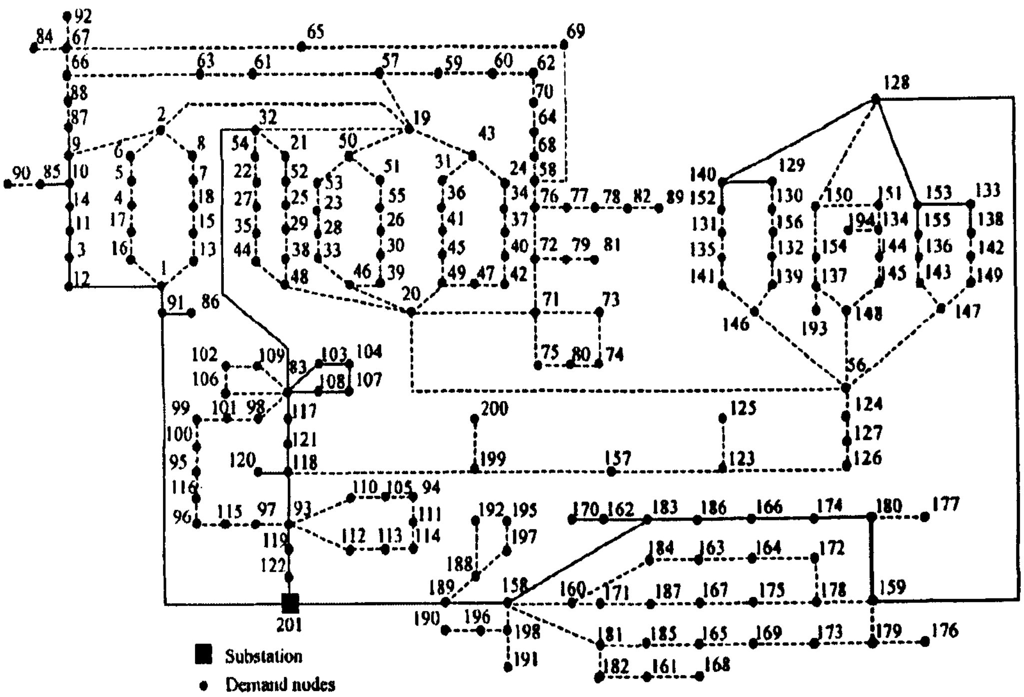


Figure 4. 9 Power distribution network for case 3

The proposed GA was tested on this problem with different combination of genetic operators and population sizes. For each operator combination, ten different initial populations were used for ten program runs. Figure 4.10 presents the average fitness of the solutions obtained over the 10 runs. Figures 4.11 and 4.12 show the average generation and average time, respectively.

In figure 4.10, it can be noted that the algorithm performance tends to follow a set pattern. For each population sizes and recombination rate, the algorithm gave the worst average fitness with a mutation rate equal to 0.0, whereas with a mutation rate of 0.8, the algorithm gave the best average fitness. And the quality of the solutions increases along with the rise in mutation rate and population size.

Increasing the mutation rate intensifies the local search activity helping the algorithm to find better solutions. With bigger population sizes, the diversity of solutions rises; therefore, the algorithm is more able to find better solutions.

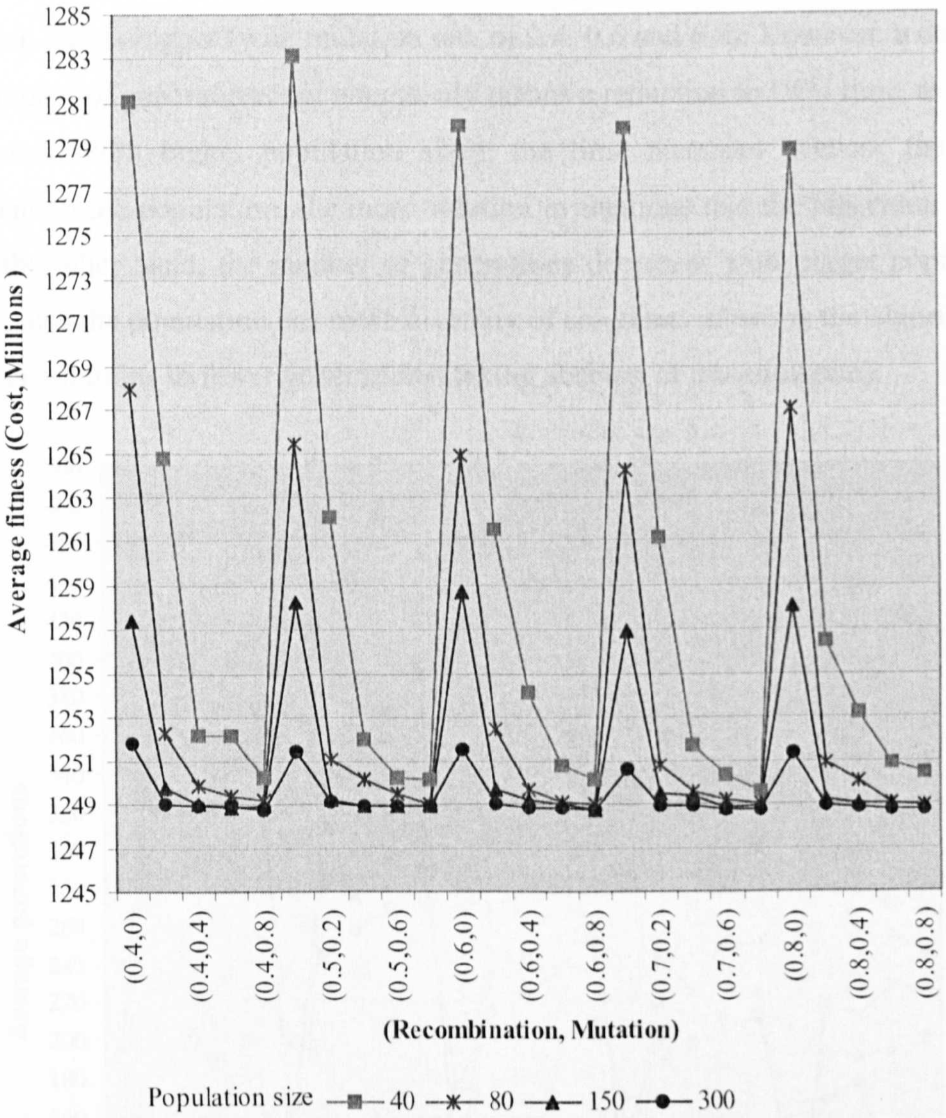


Figure 4. 10 Average fitness of solutions found over 10 program runs for case 3.

Apparently, from this figure, the recombination rate does not have any significant influence on the algorithm performance; however, looking at figure 4.11, it can be noted that the average generation decrease along with the increase of recombination rate.

Increasing the recombination rate intensifies the activity of exploring the entire solution space, helping the algorithm to find more solutions (covering more solution space); thus, the algorithm close in on the optimum (or near-optimum) solution in fewer number of generations.

In figure 4.11, it can be observed that the average generation decreases when the population size is bigger (with mutation rate of 0.4, 0.6 and 0.8). However, a decrease in the number of generations not necessarily means a reduction in CPU time, as figure 4.12 shows. With bigger population sizes, the time increases because the more individuals in the population, the more solution evaluations that the algorithm has to do. On the other hand, the number of generations decreases with bigger population sizes because the population has more diversity of solutions, allowing the algorithm to find better solutions in fewer generations (taking account of mutation rate).

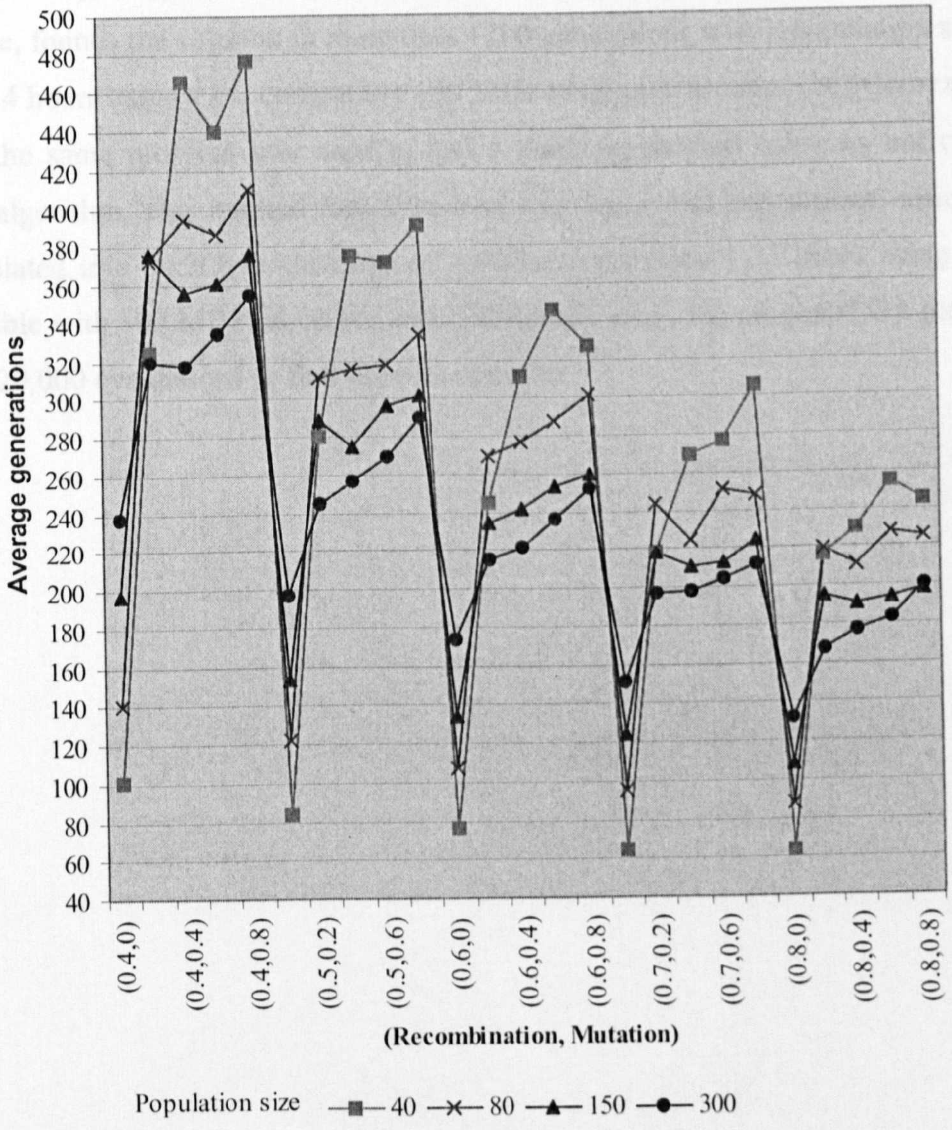


Figure 4. 11 Average generation of generations in which the algorithm stops over 10 program runs for case 3.

Another characteristic to observe in figure 4.12 is that the increase of recombination rate hardly increases the time with each population sizes (despite of the rise in the number of recombination operations). This is due to the reduction in the number of generations when the recombination rate grows.

The proposed GA is able to obtain good solutions with small population sizes. Figure 4.13 shows the performance of the algorithm with a population size of 80 and recombination and mutation rate of 0.8. The algorithm obtained the best solution in 224 generations (2 minutes 30 seconds). In reference [Ramirez-Rosado and Bernal-Agustin, 1998], this problem was used to test another GA. This GA, according to this reference, founds the solution in more than 1200 generations with a population size of 150 (2.14 hours using a PC compatible 150 MHz Pentium). In reference [Gomez et al., 2004], the same problem was used to test a planning method using an ant colony system algorithm. This method found the best solution in 562 expeditions, which can be translated into 84,300 evaluations of solutions (requiring 1.33 hours using a PC compatible with 800 MHz processor and 128 RAM). Here, the proposed GA required around 20,000 evaluations to find the best solution.

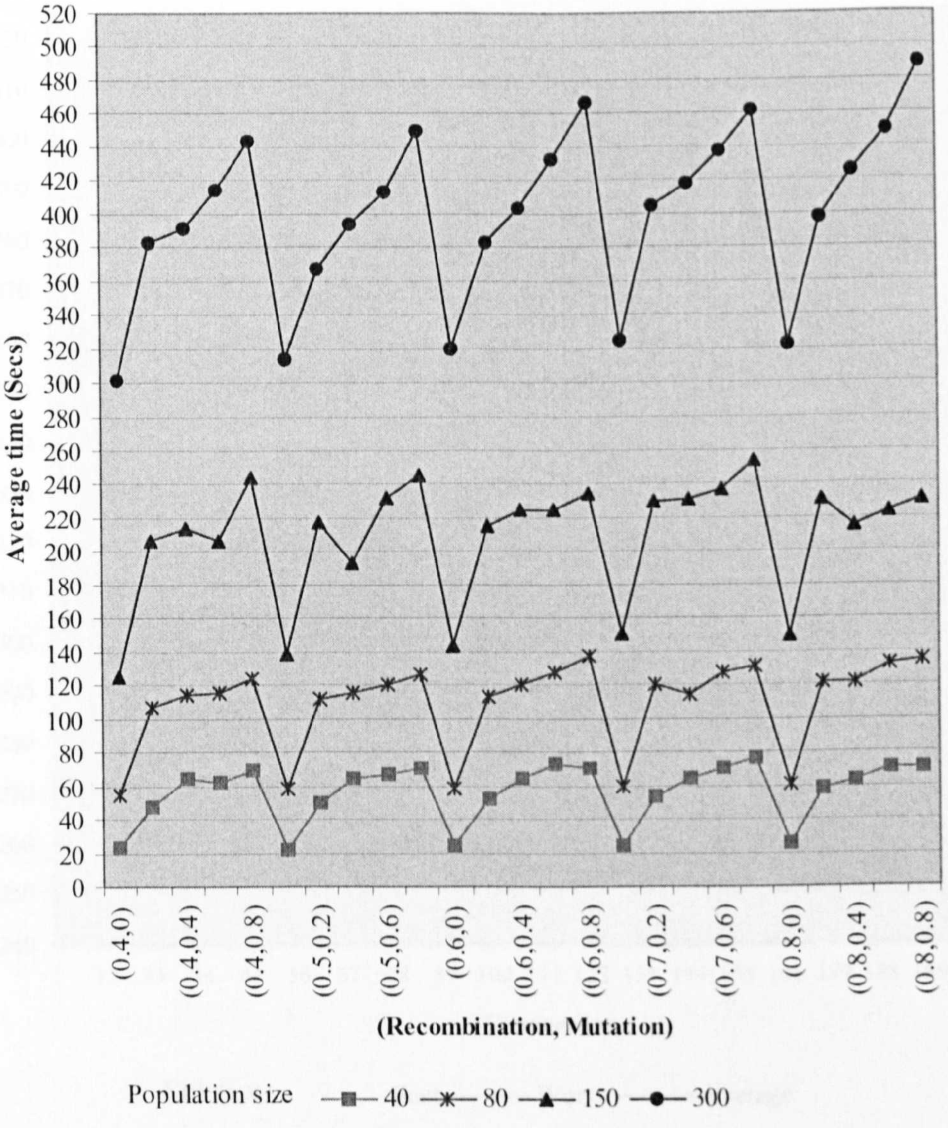


Figure 4. 12 Average time of times in which the algorithm stops over 10 program runs for case 3.

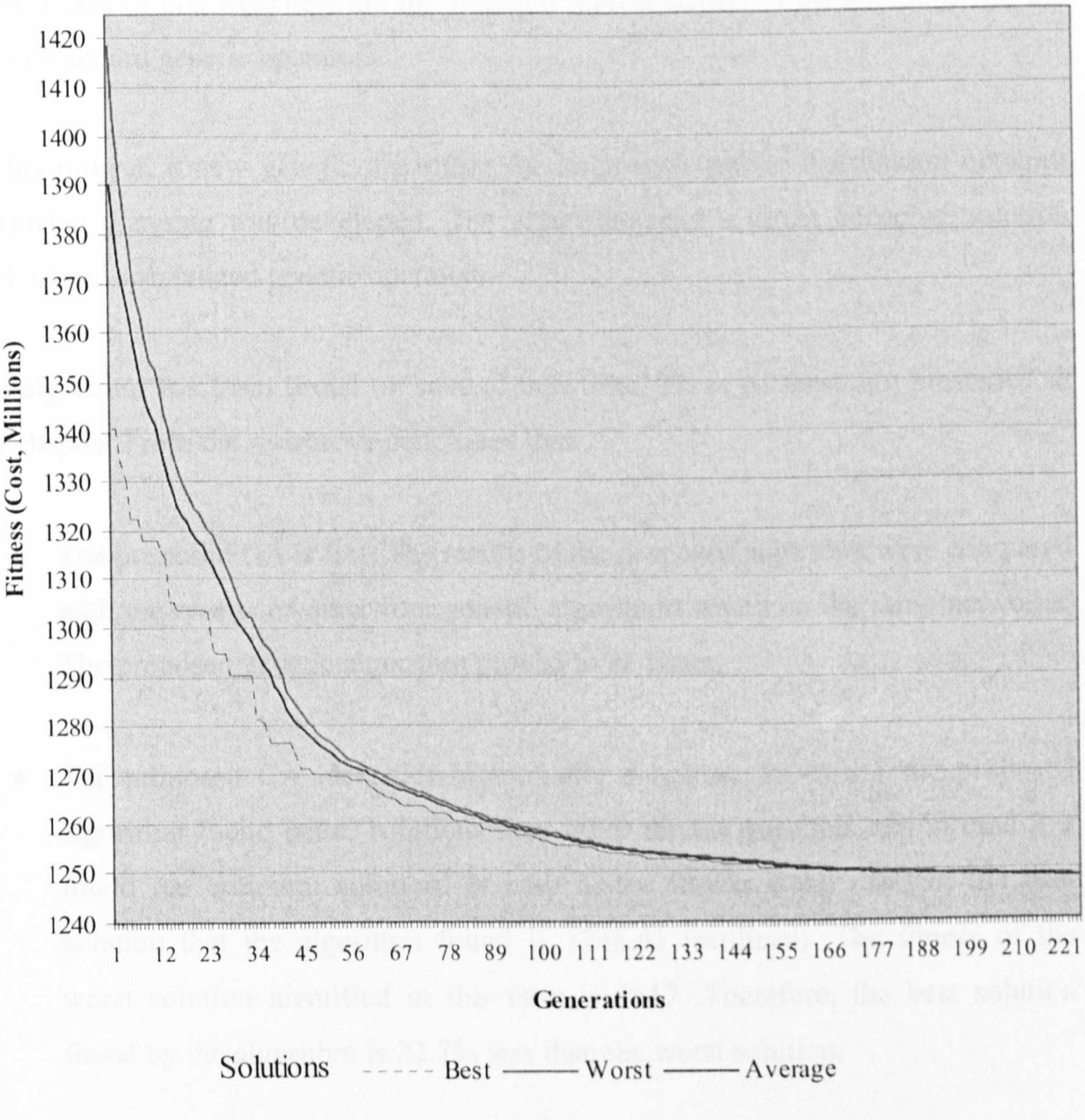


Figure 4. 13 Evolution of the best, worst and average solution from the proposed GA tested on case 3.

4.4 Conclusions

Previous power distribution network expansion planning methods based on genetic algorithms have the disadvantage of limited problem size and high solution time. The main causes of this weakness are the standard representation of the solutions and the use of standard genetic operators.

In this project, a new genetic algorithm for large-scale power distribution network expansion planning was developed. The algorithm uses a direct encoding solution method and specialized genetic operators.

The algorithm has been tested on several networks. Three of these are presented in this chapter. From the results we concluded that:

- The proposed GA is fast. The results of the proposed algorithm were compared with the results of other four genetic algorithms tested on the same networks. The proposed genetic algorithm proved to be faster.

- The proposed GA identifies high-quality solutions. In case 1 the proposed algorithm found better solutions than other three algorithms and in case 2 it found the optimum solution. In case 3, the fitness (total cost) of the best solution that the algorithm found is 1248.61 (millions). The fitness of the worst solution identified in this case is 1617. Therefore, the best solution found by the algorithm is 22.7% less than the worst solution.

- The proposed GA is robust. The algorithm has convergence stability. In case 1, the proposed GA found better solutions with smaller standard deviation. In case 3, the algorithm is able to find goods solutions with small population sizes and the population size does not influence significantly on the quality of solutions (with high mutation rate). Furthermore, the recombination operator does not affect considerably the solution time.

- The offspring generated by the recombination operator consist mostly of substructures of their parents, representing topological valid solutions.

- The proposed genetic algorithm is suitable to resolve the problem of large-scale power distribution networks expansion planning.

Chapter 5 A Proposed Multi-objective Evolutionary Algorithm for Power Distribution Network Expansion Planning

5.1 Introduction

This chapter presents a multi-objective evolutionary algorithm developed in this research for optimal large-scale power distribution network expansion planning. The algorithm is able to find a set of Pareto-optimal solutions (or a set of non-dominated solutions close to the Pareto-optimal front) to problems with more than one objective function to optimize. In this project, for power distribution network planning problems, two objectives were considered: economical cost function and energy non-supplied function.

The algorithm was tested on three problems and some of the results were compared with results from another method.

5.2 The proposed multi-objective evolutionary algorithm for power distribution network expansion planning

A multi-objective evolutionary algorithm is proposed to resolve the problem of power distribution network expansion planning. The algorithm is based on Strength Pareto Evolutionary Algorithm 2 (SPEA2) [Zitzler et al., 2001]. SPEA2 is a relatively recent technique for finding or approximating the Pareto-optimal set for multi-objectives optimisation problems. It has shown very good performance in comparison to other multi-objective evolutionary algorithms and it has been a point of reference in various investigations [Zitzler et al., 2001; Deb, 2001]. Thus, in this research, an algorithm was developed based on SPEA2 for power distribution network expansion planning.

5.2.1 Strength Pareto Evolutionary Algorithm 2

SPEA2 uses a regular population and an archive (external set). The overall algorithm is as follows [Zitzler et al., 2001]:

Step 1: Initialisation: Generate an initial population P_0 and create the empty archive A_0=0. Set t=0.
Step 2: Fitness assignment: Calculate fitness values of individuals in P_t and A_t.
Step 3: Environmental selection: Copy all non-dominated individuals in P_t and A_t to A_{t+1}. If size of A_{t+1} exceeds the archive size $\overline{N}$ then reduce A_{t+1} by means of a truncation operator; otherwise if size of A_{t+1} is less than $\overline{N}$ then fill A_{t+1} with dominated individuals in P_t and A_t.
Step 4: Termination: If $t \geq T$ (where T is the maximum number of generations) or another stopping criterion is satisfied then set $\bar{A}$ (non-dominated set) to the set of the non-dominated individuals in A_{t+1}. Stop.
Step 5: Mating selection: Perform binary tournament selection with replacement on A_{t+1} in order to fill the mating pool.
Step 6: Variation: Apply recombination and mutation operators to the mating pool and set P_{t+1} to the resulting population. Increment generation counter ($t = t + 1$) and go to Step 2.

5.2.1.1 Fitness assignment

The fitness assignment is a two-stage procedure. First, each individual i in the archive A_t and the population P_t is assigned a strength value $S(i)$, representing the number of solutions it dominates:

$$S(i) = |\{j \mid j \in P_t + A_t \wedge i \succ j\}| \qquad (5.1)$$

where $|\cdot|$ denotes the cardinality of a set, + stands for multiset union, $\wedge$ represents 'and' and the symbol $\succ$ corresponds to the Pareto dominance relation.

Second, the raw fitness of an individual i is determined by the strengths of its dominators in both archive and population:

$$R(i) = \sum_{j \in P_t + A_t,\, j \succ i} S(j) \tag{5.2}$$

Figure 5.1 illustrates the result of this assignment procedure for a minimization problem with two objective functions f_1 and f_2. $R(i)=0$ corresponds to a non-dominated individual, while a high $R(i)$ value means that i is dominated by many individuals which in turn dominate many individuals.

Additional density information is incorporated to discriminate between individuals having identical raw fitness values. The density information technique proposed in [Zitzler et al., 2001] is an adaptation of the *k-th* nearest neighbour method [Silverman, 1986] where the density at any point is a decreasing function of the distance to the *k-th* nearest data point. The adaptation consists in taking the inverse of the distance to the *k-th* nearest neighbour as the density estimate. In other words, for each individual i the distances (in the objective space) to all individuals j in archive and population are calculated and stored in a list. After sorting the list in increasing order, the *k-th* element gives the distance sought, denoted as σ_i^k. Afterwards, the density $D(i)$ corresponding to i is defined by

$$D(i) = \frac{1}{\sigma_i^k + 2} \tag{5.3}$$

Two is added in the denominator to ensure that the density value is $0 < D(i) < 1$. Finally, the fitness of an individual i is defined by:

$$F(i) = R(i) + D(i) \tag{5.4}$$

As a common setting, k is equal to the square root of the sample size. In this case, $k =$

$[N+\overline{N}]^{1/2}$; where N is the population size.

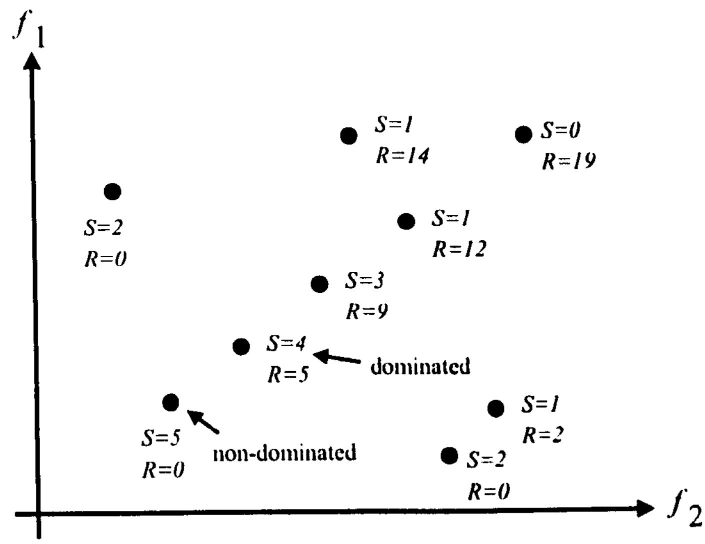


Figure 5. 1 Fitness assignment procedure in SPEA2 algorithm for a minimization problem with two objectives functions f_1 and f_2

5.2.1.2 Environmental selection

In the environmental selection, the first step is to copy all non-dominated individuals, i.e., those which have fitness values lower than one, from the archive and population to the archive of the next generation:

$$A_{t+1} = \{i \mid i \in P_t + A_t \wedge F(i) < 1\} \tag{5.5}$$

In this step, there can be three scenarios:

- The non-dominated set fits exactly into the archive ($|A_{t+1}| = \overline{N}$).
- The non-dominated set is smaller than the archive size ($|A_{t+1}| < \overline{N}$).
- The non-dominated set exceeds the archive size ($|A_{t+1}| > \overline{N}$).

In the first case, the environmental selection is completed. In the second case, the best $\overline{N} - |A_{t+1}|$ dominated individuals in the previous archive and population are copied to the new archive. This is implemented by sorting in increasing order the multiset P_t+A_t according to the fitness values and copy the first $\overline{N}-|A_{t+1}|$ individuals i with $F(i)\geq 1$ from the resulting ordered list to A_{t+1}. Finally, in the third case, an archive truncation procedure is invoked which iteratively removes individuals from A_{t+1} until $|A_{t+1}|=\overline{N}$. The truncation procedure is as follows:

At each iteration, an individual i is chosen for removal if:

- $\sigma_i^k=\sigma_j^k$ for every value of k in the range $0<k<|A_{t+1}|$ and $j \in A_{t+1}$ or
- $\sigma_i^q=\sigma_j^q$ and $\sigma_i^k<\sigma_j^k$ for every value of q in the range $0<q<k$ and any value of k in the range $0<k<|A_{t+1}|$.

In other words, the individual that has the minimum distance to another individual is chosen at each stage. If there are several individuals with minimum distance, the tie is broken by considering the second smallest distances and so forth.

5.2.1.3 Variation

In this step, a recombination and mutation techniques are applied to the mating pool. These techniques can be any proper ones to resolve the problem by evolutionary algorithms.

5.2.2 The proposed multi-objective evolutionary algorithm

The proposed multi-objective evolutionary algorithm for power distribution network expansion planning is based on SPEA2. In the following sections, the main components of the algorithm are described.

5.2.2.1 Encoding and genetic operators

The proposed multi-objective algorithm uses the encoding technique and genetic operators presented in chapter 4 but, the recombination and mutation rate used here are different from those used in chapter 4. In this case, recombination operation is controlled by a recombination probability parameter (P_{rc}): the probability parameter is set to a real number in the range [0.0-1.0) then, a real number is obtained by a random number generator. If the obtained number is smaller than P_{rc}, the recombination is executed; otherwise one of the parents is randomly chosen and copied to the offspring population. Mutation operation is applied to every individual in the offspring population: exactly one line in an individual is changed.

The size of the mating pool is equal to the population size and, in each iteration the old population is replaced with the offspring population.

5.2.2.2 Fitness function

The fitness function is defined by the strategy formulated in SPEA2 algorithm. The fitness of each individual takes into account the number of individuals it dominates and also the number of individuals it is dominated by; with a nearest neighbour density estimation added to the fitness.

In this research, the application of a different domination definition from the conventional one is suggested in order to handle the constraints of power network planning problems. This new concept of domination is called *constrain-domination* [Deb, 2001]:

A solution x_i is said to *constrain-dominate* a solution x_j if any of the following conditions is true:

- Solution x_i is feasible and solution x_j is not.
- Solution x_i and x_j are both infeasible but, solution x_i has a smaller constraint violation.
- Solution x_i and x_j are both feasible and solution x_i dominates solution x_j according to the usual domination definition (chapter 3).

From this definition, the Pareto-optimal set is the set of non-constrain-dominated individuals, which are those that are not constrain-dominated by any individual in the entire feasible search space.

The constrain-domination concept was introduced in reference [Deb, 2001] and it was tested on a number of test problems. The results were satisfactory compared with results from other constrain handling methods.

5.2.2.3 Selection mechanism

The binary tournament selection is the selection mechanism used in the phase where solutions are selected for recombination (step 6 of SPEA2). This mechanism selects two solutions randomly and picks out the solution with better fitness value. Using the concept of constrain-domination in the fitness function formulation, one of the following scenarios is created each time this selection mechanism is applied:

- If both solutions are feasible, the solution closer to the Pareto-optimal front is chosen
- If both solutions are infeasible, the solution with the smaller constraint violation is chosen
- If one solution is feasible and the other is not, the feasible one is chosen
- If both solutions are feasible and close to the Pareto-optimal front, the solution with the smaller density of individuals in its neighborhood is chosen.

Using the definition of constrain-domination with the binary tournament selection defines a strategy to handle the constraints for multi-objective problems [Deb, 2001]. This strategy does not require any penalty parameter and the infeasible solutions are not ignored, taking account of the constraint violations.

5.3 Case studies

The proposed multi-objective evolutionary algorithm was tested on three different problems:

Case 1. In this case, three complete graphs were considered to test the algorithm. Each graph has a vector of weights (non-negative real numbers) for each line and the problem was to find the set of Pareto-optimal spanning trees for each graph. This problem is known as multi-objective minimum spanning tree problem. The purpose of this case is to evaluate the effectiveness of the algorithm and compare the results with results from a greedy algorithm.

Case 2. In this case, the problem is to find a set of Pareto-optimal solutions of a complete graph which simulates a power distribution network. The main objective of this case is to validate the proposed algorithm applying it on a problem that is more similar to the problem of power distribution network planning. The Pareto-optimal solutions are recognized in this case.

Case 3. The algorithm was tested on a real network presented in [Ramirez-Rosado and Bernal-Agustin, 2001]. In this case, the problem was to find a set of Pareto-optimal solutions (or a set of non-dominated solutions close to the Pareto-optimal front) to a problem of power distribution network planning considering two objectives to optimise. The aim of this case is to demonstrate the practical application of the algorithm.

The parameter values of the algorithm used to resolve the problems of each case were: population size of 200; external archive size of 50; recombination probability of 0.8 and the maximum number of generations was 500. The tests were done using a PC compatible 1 GHz Pentium with 128 Mb of RAM, WindowsME and a Visual C++ compiler.

5.3.1 Case 1

The proposed algorithm was tested on three complete graphs (named as graphs A, B and C). Each graph has 10 nodes and 45 lines (every node has a direct connection with the rest of the nodes). Each line has a vector of weights ($w^1_{i,j}$, $w^2_{i,j}$) which are non-negative real numbers. The weights were randomly generated in the ranges [10.0-100.0] and [10.0-50.0] for $w^1_{i,j}$ and $w^2_{i,j}$, respectively. The problem was to find the set of Pareto-optimal spanning trees for each graph considering two objectives to minimize: one objective is the sum of weights w^1 and the other one is the sum of weights w^2. This problem is known as multi-objective minimum spanning tree problem (MO-MST).

In reference [Knowles and Corne, 2001], it was stated that a Pareto-optimal solution to MO-MST problems can be found by using any algorithm to the single-objective minimum spanning tree problem (such as Prim's and Kruskal's algorithms), by substituting the weight vector at each line with a weighted sum. In [Knowles and Corne, 2001], researchers adapted the Prim's algorithm to MO-MST problems. This modified Prim's algorithm is able to find a subset of Pareto-optimal solutions by finding a solution for different weighted-sum single-objective forms of the given problem. In this project, the Kruskal's algorithm was adapted employing the same strategy used in the Prim's algorithm adaptation, and it was applied on the three complete graphs of this case in order to identify the Pareto-optimal spanning trees.

Like the modified Prim's algorithm, the Kruskal's algorithm can not usually find the complete set of Pareto-optimal solutions but, it is able to find the solutions on the convex hull of the Pareto-optimal front (these solutions are called supported efficient solutions).

Figures 5.2-5.4 show the solutions for each graph found by the proposed evolutionary algorithm and the Kruskal's algorithm. In these figures, the horizontal and vertical axes represent the total sum of the weights w^1 and w^2, respectively. The proposed algorithm was able to find the set of supported efficient solutions which were identified by the Kruskal's algorithm. The other solutions can not be certainly considered as Pareto-optimal solutions; however most of these solutions were found by both algorithms.

The proposed algorithm found the solutions with five different initial populations for five simulation runs.

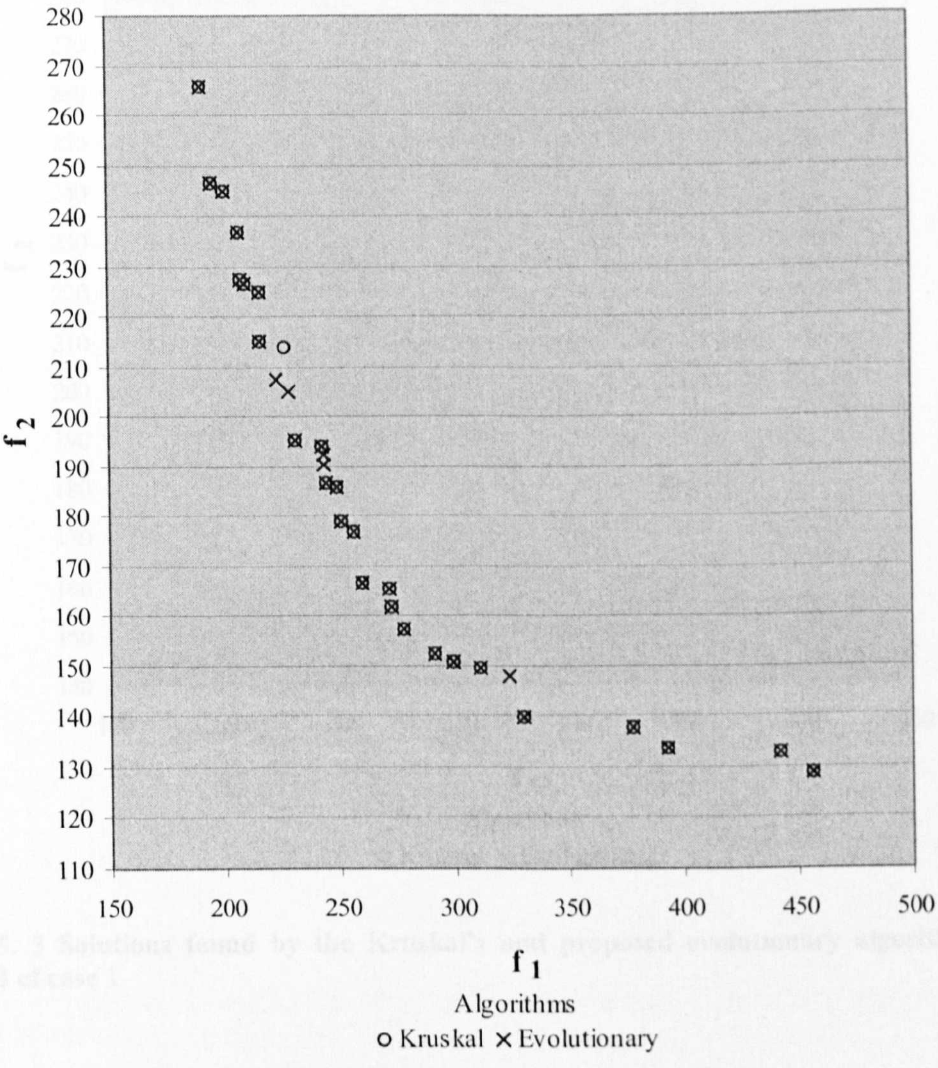


Figure 5. 2 Solutions found by the Kruskal's and proposed evolutionary algorithms for Graph A of case 1

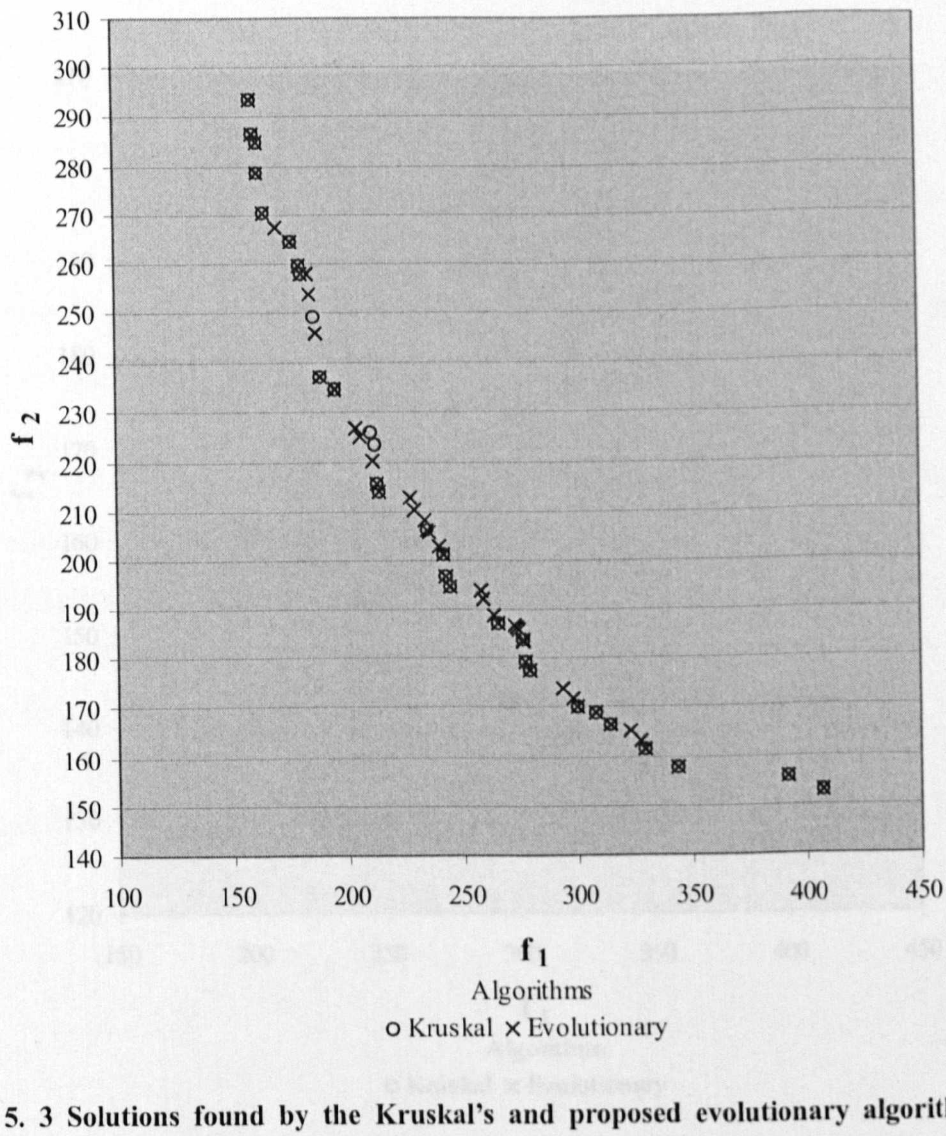


Figure 5. 3 Solutions found by the Kruskal's and proposed evolutionary algorithms for Graph B of case 1

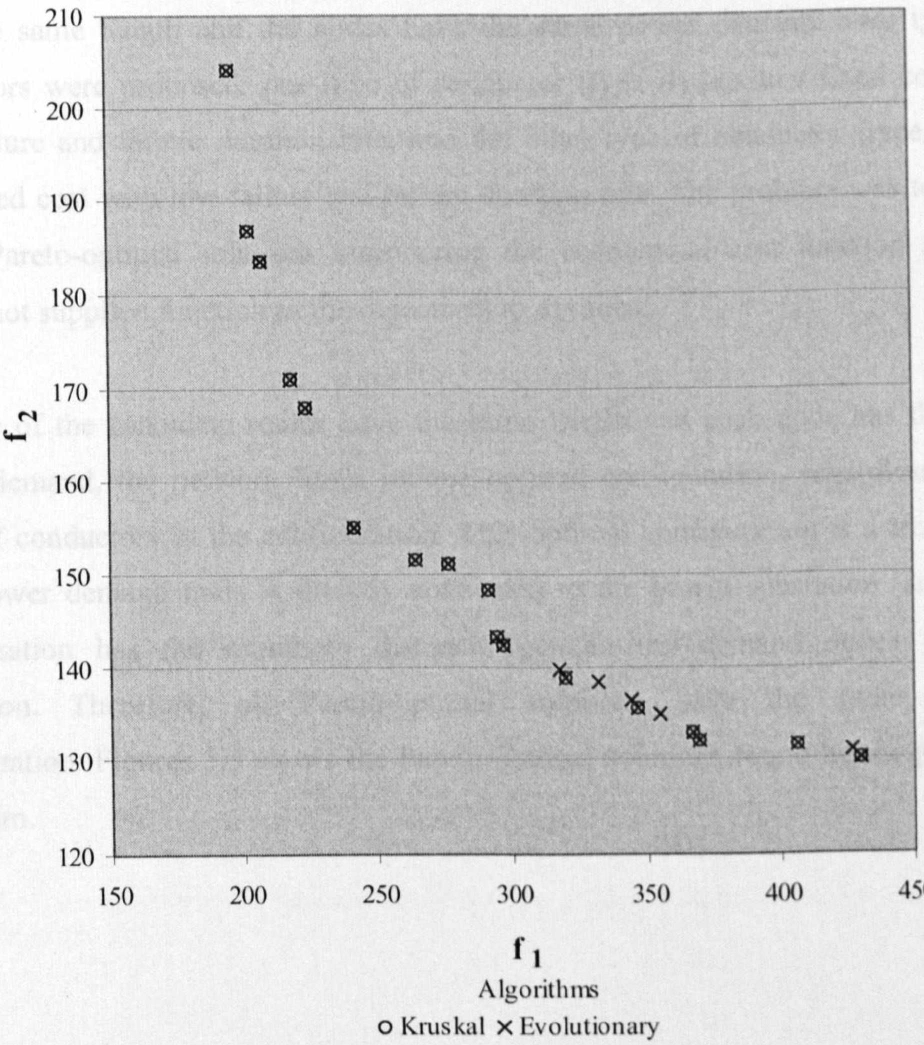


Figure 5. 4 Solutions found by the Kruskal's and proposed evolutionary algorithms for Graph C of case 1

5.3.2 Case 2

In this case, the proposed algorithm was tested on a theoretical power distribution network. The network has 9 power demand nodes to be connected, 45 candidate routes and one power substation. The network forms a complete graph: every node has a candidate route directly connecting the rest of the nodes. The candidate routes have the same length and the nodes have the same power demand. Two types of conductors were proposed: one type of conductor (type A) has low fixed cost with high failure and failure duration rate; and the other type of conductor (type B) has high fixed cost with low failure and failure duration rate. The problem was to find a set of Pareto-optimal solutions considering the economical cost function and the energy not supplied function as the objectives to optimise.

Because of the candidate routes have the same length and each node has the same power demand, the network has a unique optimal configuration, regardless of the types of conductors in the configuration. This optimal configuration is a tree where each power demand node is directly connected to the power substation -since this configuration has the minimum distance between the demand nodes and the substation. Therefore, all Pareto-optimal solutions have the same optimal configuration. Figures 5.5 shows the Pareto-optimal solutions found by the proposed algorithm.

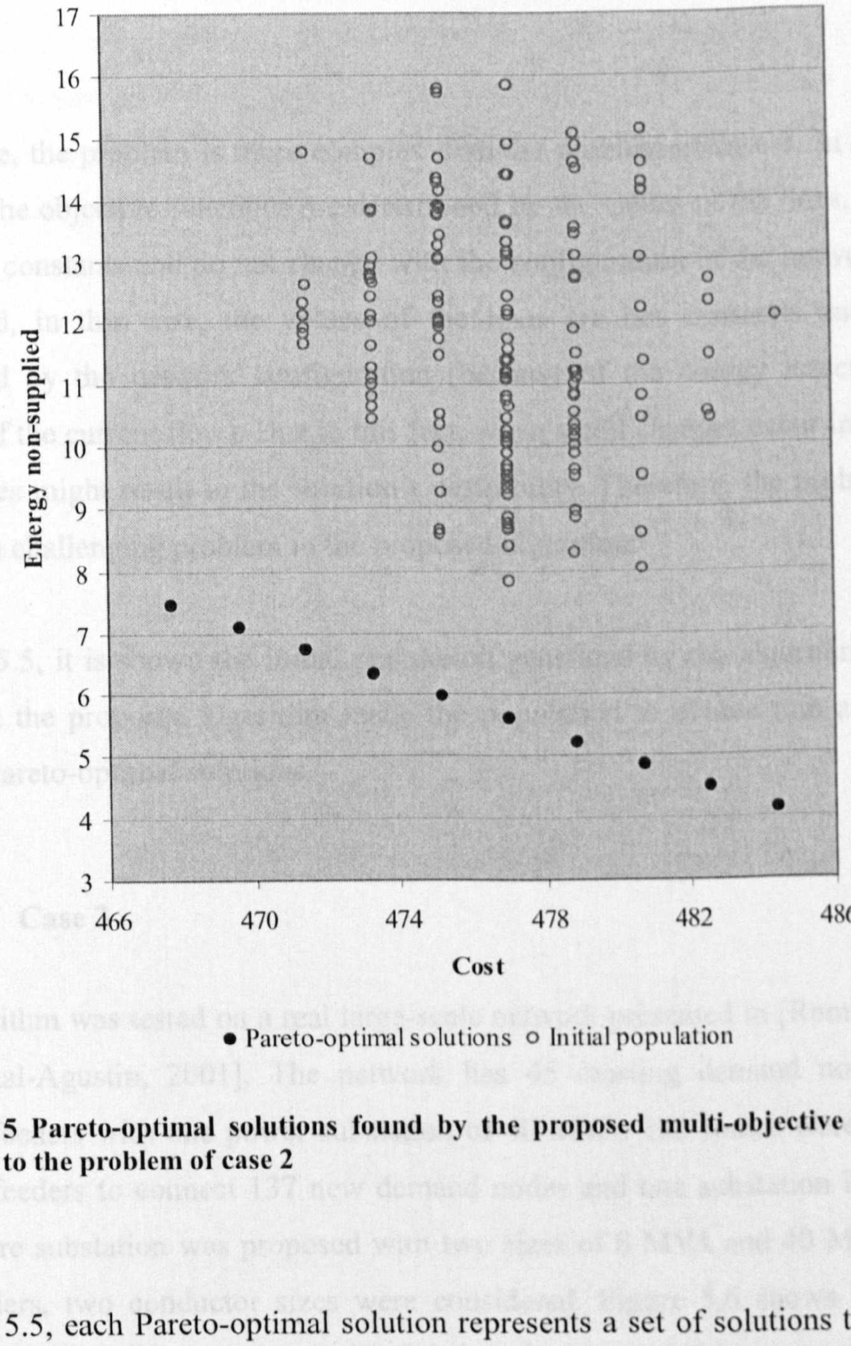


Figure 5. 5 Pareto-optimal solutions found by the proposed multi-objective evolutionary algorithm to the problem of case 2

In figure 5.5, each Pareto-optimal solution represents a set of solutions that have the optimal configuration and the same number of conductors of each type; except the solutions depicted by both extreme points. The left-extreme point is the solution with the optimal configuration and conductors of type *A*. The right-extreme point is the solution with the optimal configuration and conductors of type *B*. The rest of the solutions have different combinations of conductors; for instance, the second point, after the left-extreme point, represents the optimal solutions with eight conductors of type *A* and one conductor of type *B*; the next point represents the solutions with seven conductors of type A and two conductors of type B; the next point represents the solutions with six conductors of type A and three conductors of type B; and so on and so forth.

In this case, the problem is more complex than the problem of case 1. In case 1, the values of the objective functions are determined by the values of the lines. These line values are constants and do not change with the configuration of the network. On the other hand, in this case, the values of the lines are not constants and they are determined by the network configuration (because of the energy losses cost is a function of the current flow). Due to this fact, when small changes occur in a solution, big changes might result in the solution's desirability. Therefore, the problem of this case was a challenging problem to the proposed algorithm.

In figure 5.5, it is shown the initial population generated by the algorithm. It can be noted that the proposed algorithm made the population to evolve into a population with the Pareto-optimal solutions.

5.3.3 Case 3

The algorithm was tested on a real large-scale network presented in [Ramirez-Rosado and Bernal-Agustin, 2001]. The network has 45 existing demand nodes and 44 existing feeders with one power substation of 40 MVA. 163 routes were considered for new feeders to connect 137 new demand nodes and one substation in node 182. This future substation was proposed with two sizes of 8 MVA and 40 MVA. For the new feeders, two conductor sizes were considered. Figure 5.6 shows the network where the darker lines represent the existing feeders and the proposed routes are represented by thin lines.

The proposed conductors and substation sizes have different fixed cost. The substation size of 40 MVA costs 300 millions (unit of money), whereas the other substation size costs 136 millions. Similarly, the conductor with the bigger size costs more but, it has less failure and failure duration rate than the other conductor.

The general data of the problem are given in appendix A. The data related to electrical characteristics of the network components are the same presented in reference [Ramirez-Rosado and Bernal-Agustin, 2001] but, some data related to the economical aspect of the problem are approximate values of the data used in this reference because of lack of further information.

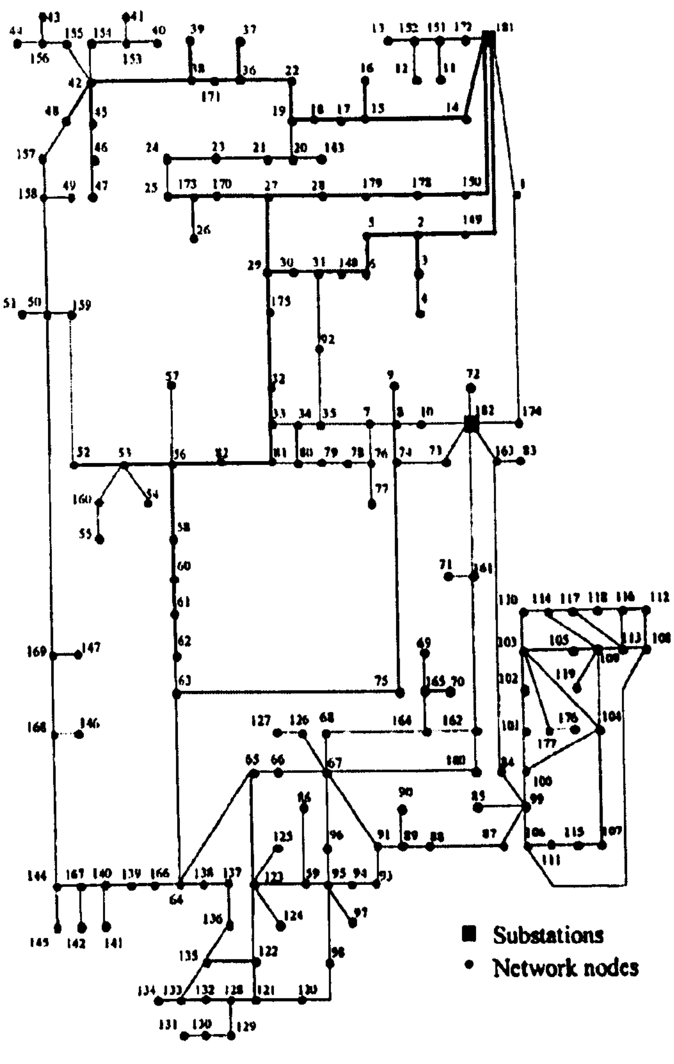


Figure 5. 6 Power distribution network for case 3

In this case, the problem was to find a set of Pareto-optimal solutions (or a set of approximate Pareto-optimal solutions) considering two objective functions to optimise: the economical cost function and the energy non-supplied function.

The set of Pareto-optimal solutions is not recognised in this problem. Nevertheless, having proved that the proposed algorithm is able to find the Pareto-optimal solutions in the previous cases 1 and 2, and taking account of the amount of experiments done under different algorithm parameter combinations, it can be considered the set of non-dominated solutions given by the proposed algorithm, in this case, as a set of non-dominated solutions close to the Pareto-optimal front.

Figure 5.7 shows the set of approximate Pareto-optimal solutions found by the proposed algorithm. Because of two substation sizes are proposed with different costs for the new substation, the Pareto front is divided into two fronts. The left front contains solutions with the new substation size of 8 MVA; and the other front contains solutions with the new substation size of 40 MVA.

Solutions with the substation size of 40 MVA provide more reliability (in terms of energy non supplied) than solutions with the substation size of 8 MVA because a bigger substation size can supply more power; therefore, the total power demand is more equally shared between the existing and new substation and it experiences less interruptions rate. However, in this case, the substation size of 40 MVA is more expensive.

Similarly, in each front, there are solutions with better reliability than others but they have higher costs. This is because the energy non-supplied is a function of the configuration and the types of conductors in the network. In this case, the cheaper conductor has the higher failure rate.

The solutions shown in figure 5.7 produce conflicting scenarios between both objective functions. If both objectives are equally important, none of these solutions is the best with respect to both objectives. However, this set of solutions can help the network planner to evaluate the solutions considering other criteria. The planner can assesses the advantages and disadvantages of each of these solutions based on other criteria which are still important; and compare them to make a choice.

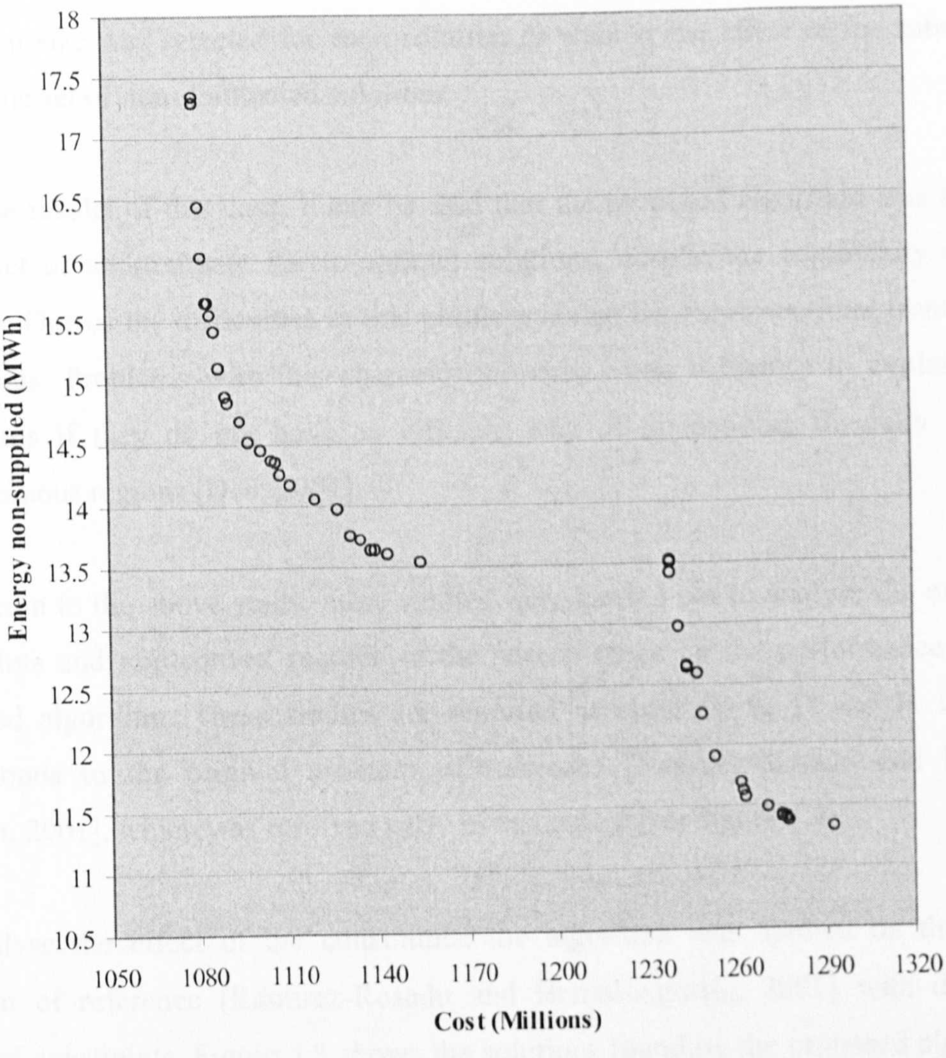


Figure 5. 7 Approximate Pareto-optimal solutions found by the proposed multi-objective evolutionary algorithm to the problem of case 3

This is the difference between the single-objective optimization and the multi-objective optimization formulation. In the first one, only one solution is obtained whereas in the second one, a number of solutions is acquired with more information related to the problem and its solutions. Thus, in the multi-objective optimization analysis, the goal is to find a set of trade-off solutions

Figure 5.7 is different from the figure that depicts the solutions for the same problem reported in reference [Ramirez-Rosado and Bernal-Agustin, 2001]. In this reference, the Pareto front is not divided into two fronts and it is not clear if the two proposed substation size have different fixed cost or not. Also, it is not mentioned which substation size was selected for each solution or what is the effect of the substation size on the set of non-dominated solutions.

From the results of this case, it can be said that the proposed algorithm was able to find a set of approximate Pareto-optimal solutions, despite the complexity of this problem. One of the difficulties in this problem is that the Pareto-optimal front is not continuous. Problems with this characteristic may cause difficulty to evolutionary algorithms if they do not have an efficient way of maintaining diversity among discontinuous regions [Deb, 2001].

In addition to the above study, more studies were carried out to analyse the effect of constraints and non-convex regions of the search space on the performance of the proposed algorithm. These studies are reported as cases B, C, D and E. Case A corresponds to the original problem of reference [Ramirez-Rosado and Bernal-Agustin, 2001], which was resolved early in this case 3 (see figure 5.7).

To analyse the effect of the constraints, the algorithm was applied on the same problem of reference [Ramirez-Rosado and Bernal-Agustin, 2001] with different levels of constraints. Figure 5.8 shows the solutions found by the proposed algorithm to the problem with the permissible level of voltage drop changed from the original 3.0 percent to 1.0 percent (case B). In figure 5.9, it is shown the solutions for the problem with the capacity limit of lines reduced by 50 percent (case C). Finally, figure 5.10 shows solutions to the problem with the proposed substation size of 40 MVA changed for a substation size of 9 MVA (the cost does not change) (case D).

Figures 5.8 and 5.9 show that the proposed algorithm was able to converge to one of the Pareto fronts previously found for the original problem. This Pareto front corresponds to the solutions with the new substation size of 40 MVA, which satisfy the new constraints. The other Pareto front of the original problem now lies on the infeasible region.

These solutions of the cases B and C were expected since, as it was mentioned early, solutions with the bigger substation size have the total power demand more shared between the existing and the new substation; therefore, the lines carry less amount of current and the voltage drop is lower.

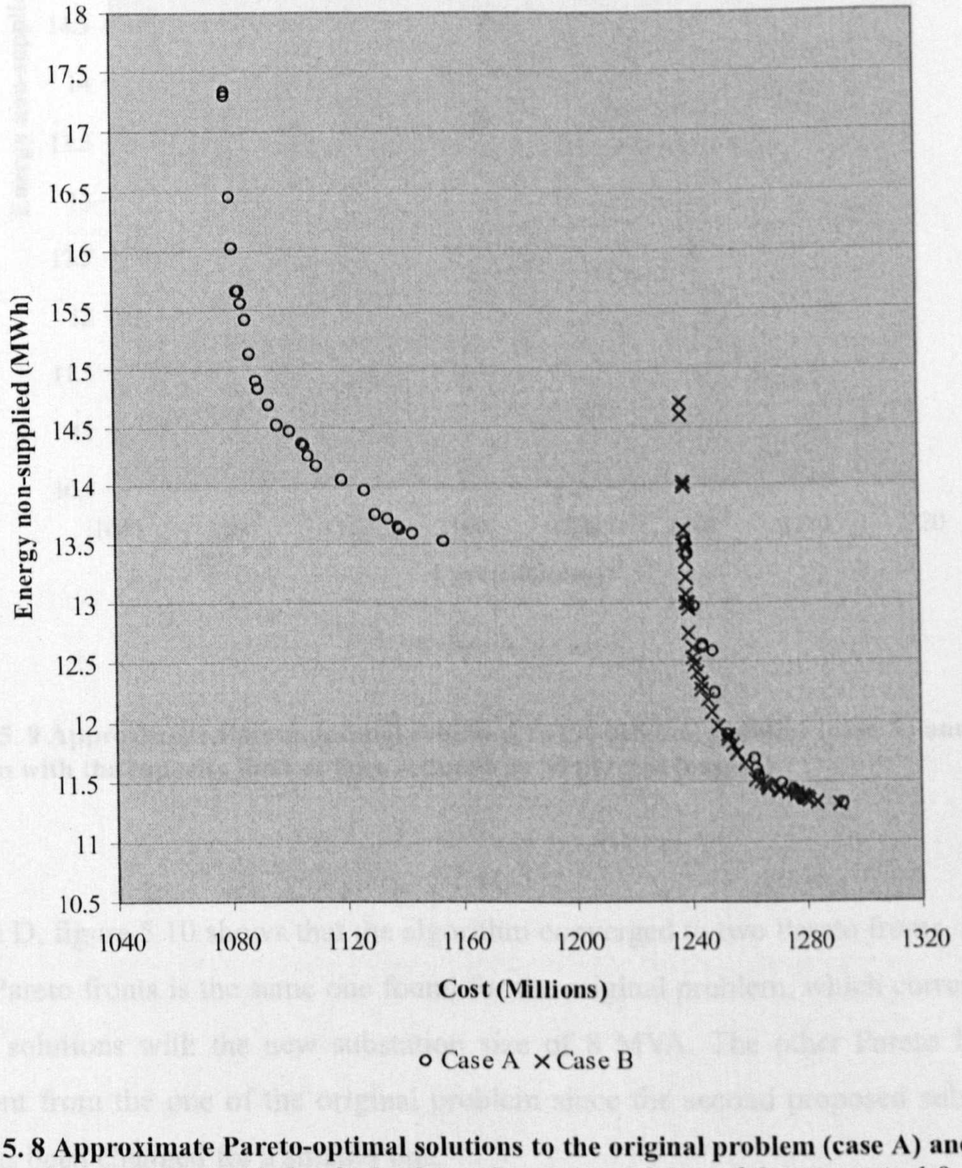


Figure 5. 8 Approximate Pareto-optimal solutions to the original problem (case A) and to the problem with the permissible level of voltage drop changed from 3.0 percent to 1.0 percent (case B)

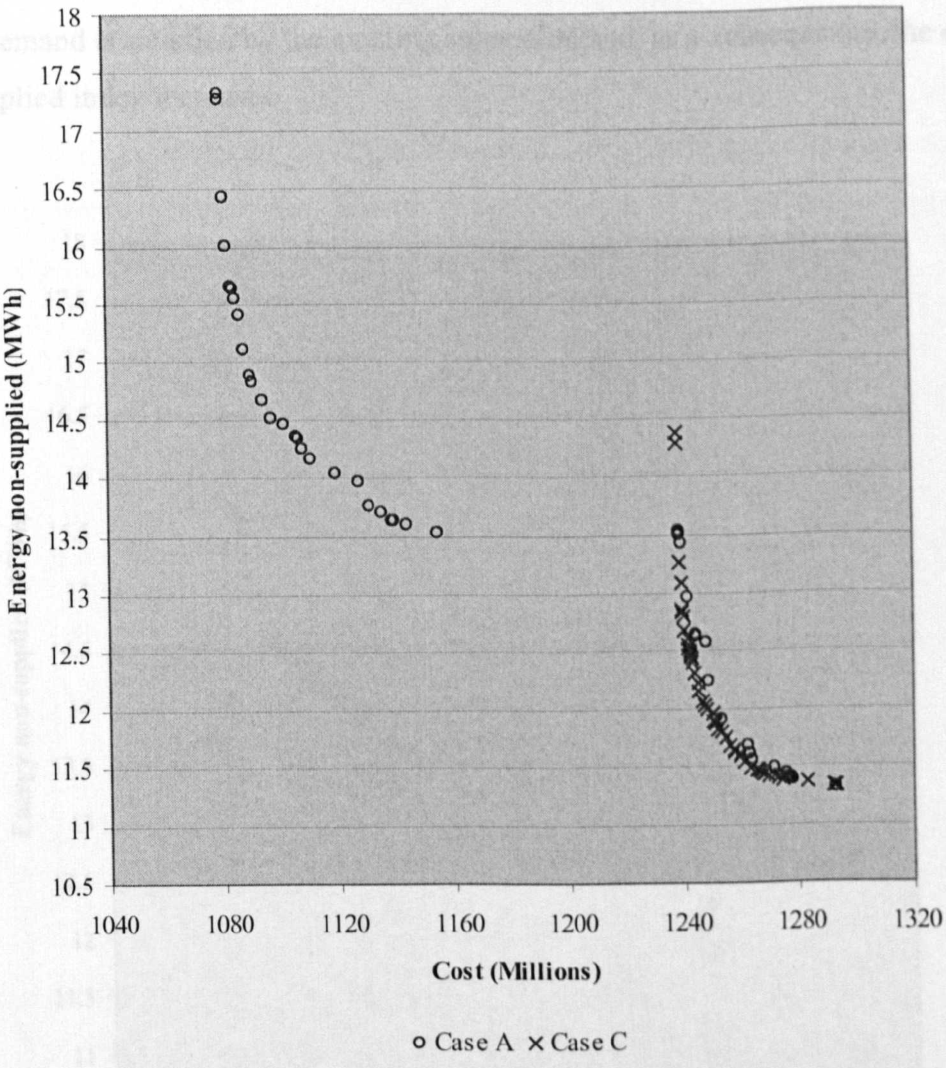


Figure 5. 9 Approximate Pareto-optimal solutions to the original problem (case A) and to the problem with the capacity limit of lines reduced by 50 percent (case C)

In case D, figure 5.10 shows that the algorithm converged to two Pareto fronts. One of these Pareto fronts is the same one found for the original problem, which corresponds to the solutions with the new substation size of 8 MVA. The other Pareto front is different from the one of the original problem since the second proposed substation size has been changed for a smaller one.

Similarly, the solutions of case D were expected. Because one of the proposed substation size was not changed, the algorithm converged to the corresponding Pareto front. The other Pareto front is different from the one of the original problem because the substation size of 9 MVA has less capacity to supply energy; therefore, more power demand is satisfied by the existing substation and, as a consequence, the energy non-supplied index increases.

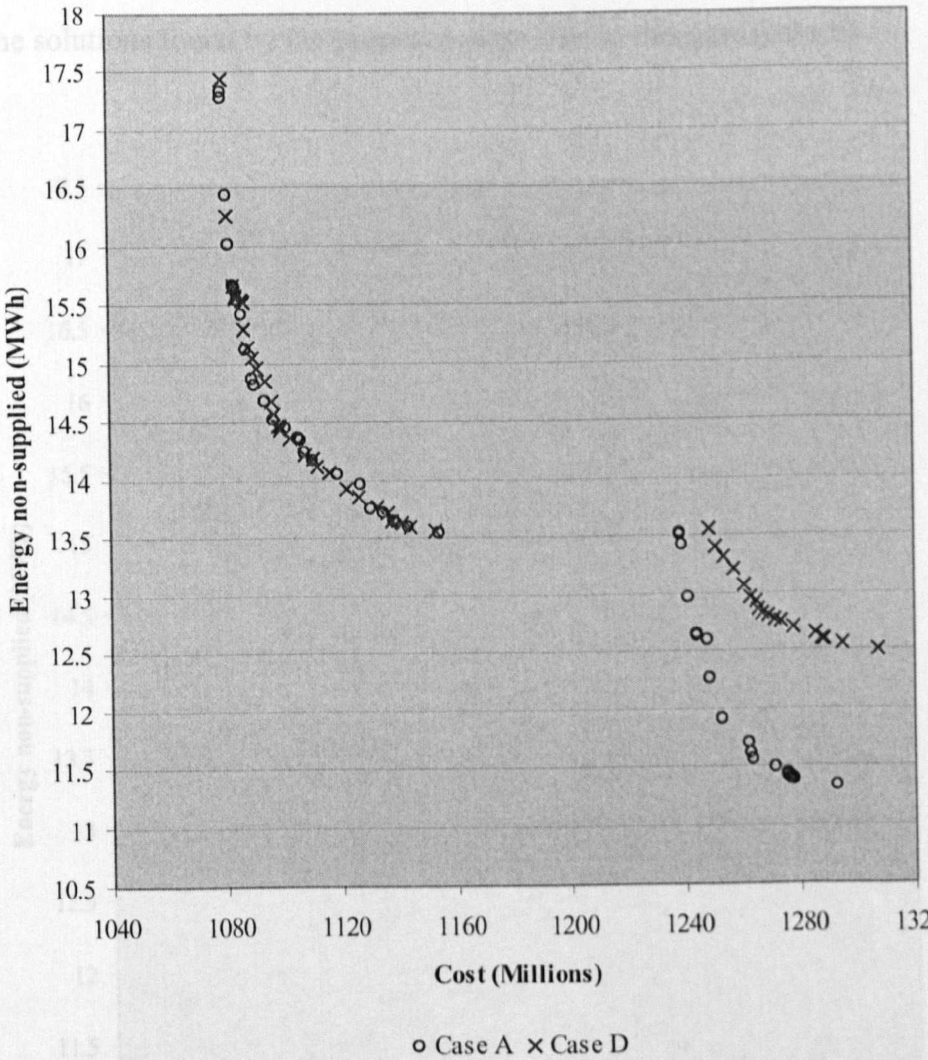


Figure 5. 10 Approximate Pareto-optimal solutions to the original problem (case A) and to the problem with the new proposed substation size of 40 MVA changed for a substation size of 9 MVA (case D)

The constraints can cause complications for some multi-objective evolutionary algorithms to converge to the Pareto-optimal front and to maintain a diverse set of Pareto-optimal solutions [Deb, 2001]. In this case, it can be said that the proposed algorithm was success in tackling these difficulties.

To analyse the effect of non-convex regions of the search space on the performance of the proposed algorithm, the algorithm was applied on the same original problem (reference [Ramirez-Rosado and Bernal-Agustin, 2001]) with the cost of the new substation size of 40 MVA reduced from 300 millions to 200 Millions. Figure 5.11 shows the solutions found by the proposed algorithm to this case (case E).

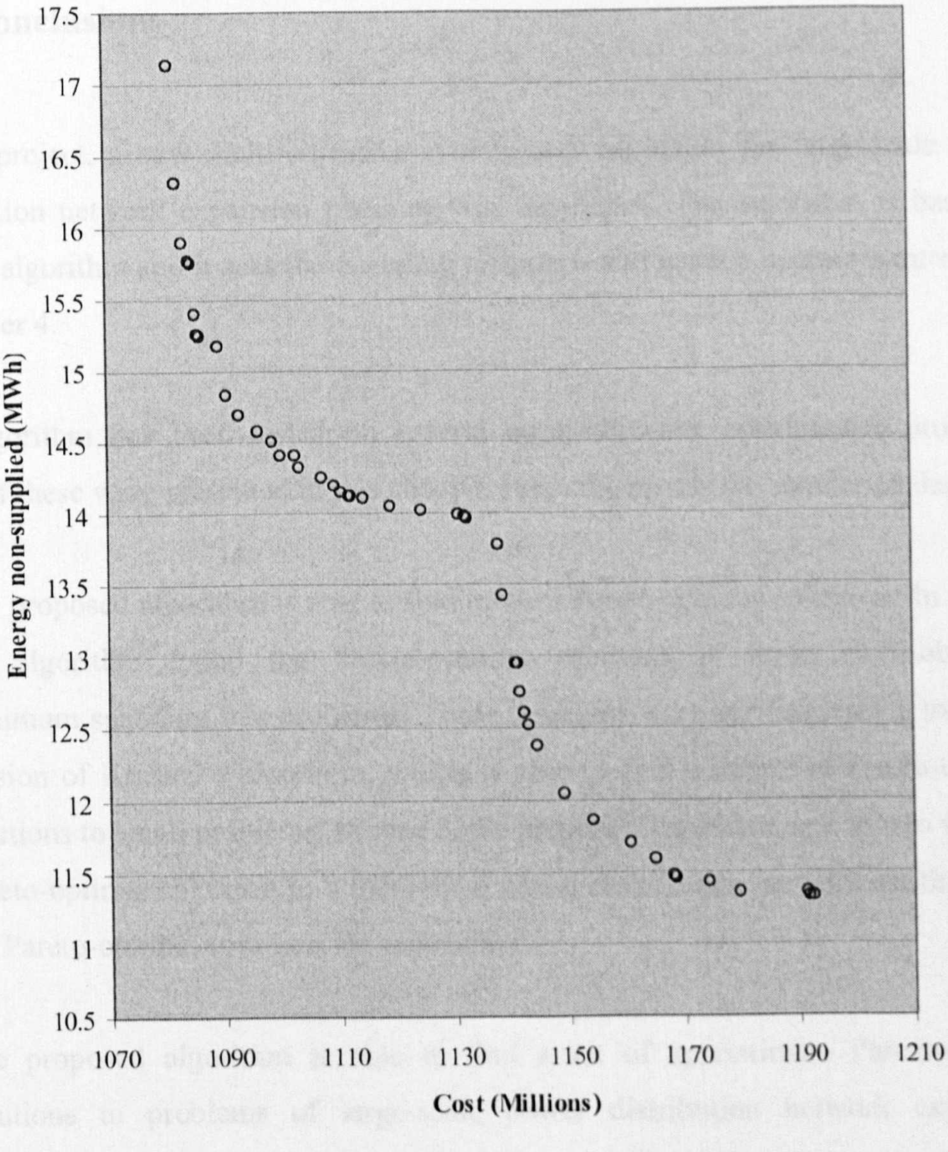


Figure 5. 11 Approximate Pareto-optimal solutions to the problem with the cost of the new proposed substation size of 40 MVA reduced from 300 millions to 200 millions (case E)

In this case E, there is a non-convex region in the Pareto front. The presence of several alternatives to build feeders and substations with different economical and electrical characteristics can produce this type of scenarios. Many multi-objectives optimisation methods that can handle problems with convex search space face difficulties in solving problems with non-convex search space. In this case, the proposed algorithm was able to find solutions in the non-convex region.

5.4 Conclusions

In this project, a new multi-objective evolutionary algorithm for large-scale power distribution network expansion planning was developed. The algorithm is based on SPEA2 algorithm and it uses the encoding technique and genetic operators introduced in chapter 4.

The algorithm has been tested on several multi-objective optimisation problems. Some of these were presented in this chapter. From the results we concluded that:

- The proposed algorithm is able to find a set of Pareto-optimal solutions. In case 1, the algorithm found the Pareto-optimal solutions to three multi-objective minimum spanning tree problems. These solutions were verified with a modified version of Kruskal's algorithm, which is able to find a subset of Pareto-optimal solutions to small problems. In case 2, the proposed algorithm was able to find the Pareto-optimal solutions to a theoretical power distribution network. In this case, the Pareto-optimal solutions are recognized.

- The proposed algorithm is able to find a set of approximate Pareto-optimal solutions to problems of large-scale power distribution network expansion planning considering the economical cost function and the energy non-supplied function as objectives to optimise. In case 3, the algorithm was tested on a real power distribution network.

- The encoding technique and genetic operators proposed in chapter 4 are effective in resolving multi-objective problems of power distribution network expansion planning.

- The proposed algorithm is able to find a relatively diverse set of solutions to multi-objective optimisation problems. The proposed algorithm uses a strategy to maintain the diversity of the Pareto set by incorporating density information into the selection process.

- The proposed algorithm is able to find a set of approximate Pareto-optimal solutions to problems with constraints. In case 3, the proposed algorithm was tested on problems in which the feasible search space was modified by changing the constraints. The algorithm found approximate Pareto-optimal solutions to these problems.

- The proposed algorithm is able to find approximate Pareto-optimal solutions in non-convex regions. In chapter 3, the proposed algorithm was tested on a power distribution network planning problem which has a non-convex search space.

- The proposed algorithm is able to find a set of approximate Pareto-optimal solutions in a single simulation run.

- The proposed algorithm is suitable to resolve the multi-objective problem of large-scale power distribution networks expansion planning.

Chapter 6 A Genetic Algorithm for Network Reconfiguration in Distribution Systems

6.1 Introduction

In this chapter, the application of the developed genetic algorithm to the problem of network reconfiguration for loss reduction is reported. The goal of this chapter is to show the potential of the encoding technique and the special genetic operators, originally developed for the problem of distribution system expansion planning, to resolve operational problems.

The algorithm was tested on two problems and some of the results were compared with results from another method.

6.2 Network reconfiguration in distribution systems

Distribution systems are planned and designed to reduce not only the investment cost but also the operating cost of the systems. As it was stated in the previous chapters, the planning of distribution systems consists in determining the optimum site and size of new substations and feeders in order to satisfy the future power demand with minimum investment and operational costs and an acceptable level of reliability. Thus, distribution systems are planned and designed to be economical, efficient and reliable systems under a determined configuration and normal operating conditions.

However, when unexpected forced outages of transformers or lines, maintenance or overloads happen, system operators need to reconfigure the system by controlling the on/off status of switches in order to improve the efficiency, reliability and operational cost of the system for the new operating conditions.

Two types of switches are used in primary distribution feeders. They are normally closed switches (sectionalizing switches) or normally open switches (tie switches). Both types are designed for both protection and reconfiguration.

According to the operating conditions, the networks are reconfigured for three purposes:

- Reconfiguration for loss reduction
- Reconfiguration for load balancing
- Reconfiguration for service restoration

The load balance or the reduction of power loss is considered as the goal for the network reconfiguration to eliminate the substation or feeder overloads and to improve the efficiency of the system under the normal operating condition. For the service restoration to supply the power to the unfaulted zone, the goal is to minimize the number of interrupted customers on the emergency conditions.

In this chapter, the focus is on the application of the genetic algorithm developed in this research to the network reconfiguration problem for loss reduction. However, the results developed provide an insight into the potential that the genetic algorithm developed for planning problems possesses to resolve operational problems in distribution systems.

6.3 Network reconfiguration for loss reduction

Each feeder in a distribution system has a different mixture of commercial, residential and industrial loads. The daily load variations of these loads are dissimilar. Therefore, the peak loads on substation transformers, on individual feeders or on feeder sections occur at different times. For example, a certain feeder or feeder section can be overloaded whereas others are lightly loaded.

Efficient operation of distribution systems can be achieved by altering the open/closed status of switches to transfer load from heavily loaded feeders (or substation transforms) to relatively less heavily loaded feeders (or substation transformers). Reducing the level of loads on the feeders or substations, the power losses are reduced and, in addition, the voltage profile along the feeders is improved.

In this context, the network reconfiguration problem is to identify a configuration with the minimum power losses while all system constraints are satisfied. This problem belongs to a combinatorial optimization problem since the problem is to determine open/closed status of all switches in large-scale distribution systems. Studies and methods have been developed and reported in the literature. Some of the recent publications are describe in this section.

In reference [Su and Lee, 2003], a method based on the mixed-integer hybrid differential evolution is proposed. The method has the common components of evolutionary algorithms (selection, crossover and mutation) and, in addition, two operations are included: acceleration and migration. The acceleration operation is applied if the best fitness is not further improved by the crossover and mutation operations. The migration operation is to improve the exploration of the search space in order to allow the method to use a smaller population.

In [Jeon et al., 2002], a method based on a simulated annealing algorithm is proposed. In this reference, a perturbation mechanism is introduced to generate network configurations. This mechanism take into account the topology of the system and the temperature parameter. A penalty factor, which is a function of the temperature parameter, is used to deal with unfeasible configurations. The polynomial-time cooling schedule is applied. Some parameters of the cooling schedule are determined based on the statistics calculated during the search.

Another method for network reconfiguration is proposed in [Hsiao and Chien, 2001]. In this reference, the problem is formulated as a multi-objective problem. The objectives to optimize are: power loss, load balancing and deviation of bus voltage. The method models these objectives by using fuzzy sets in order to evaluate the objectives by a common measure. An evolutionary programming algorithm is applied to find the optimal solution.

A network reconfiguration method for systems with dispersed generations is proposed in [Choi and Kim, 2000]. This method applies a genetic algorithm which uses binary strings to encode the solutions and the standard crossover operator (single/multiple crossover); therefore, this genetic algorithm generates a lot of unfeasible solutions.

In reference [Song et al., 1997] is reported a method that applied an evolutionary programming algorithm with a mutation fuzzy controller. This controller adaptively adjusts the mutation rate during the evolutionary process in order to speed up the search.

In this chapter, it is reported the application of the developed genetic algorithm in the problem of network reconfiguration for loss reduction. The goal of this chapter is to show the potential of the encoding technique and the special genetic operators, originally proposed for the problem of expansion planning, to resolve operational problems.

6.4 Problem formulation

Network reconfiguration for loss minimization can be formulated as follows:

$$Min \quad P_{Loss} = \sum_{l \in Nl} 3 I_l^2 R_l \tag{6.1}$$

Where

P_{Loss} = Total power loss of the system

I_l = Current through feeder section l

R_l = Resistance of section feeder l

N_l = Number of feeder sections in the system

Subject to

a) Radial network constraint
b) Isolation constraint: all nodes must be connected
c) Power source limit constraint
d) Voltage constraint
e) Current constraint: current magnitude of each branch (feeder, feeder section, laterals and switches) must not exceed their permissible capacity.

The site of the switches is known beforehand and the power demand at the nodes are peak loads modeled as constant values.

6.5 A network reconfiguration method based on the genetic algorithm with the direct encoding solution technique and special genetic operators

The genetic algorithm developed in this research for power distribution system expansion planning problems is applied in this operational problem. The adaptation of the algorithm to this problem is described in the following sections.

Encoding solutions

The direct encoding technique is used in the same way for the planning problem. Here, the feeder sections that do not have switches are considered as fixed components; i.e., these feeder sections will be in every solution since they do not have switches to isolate them. On the other hand, the feeder sections with switches are the components that will be affected by the genetic operators. Therefore, the feeder sections that do not appear in a solution indicate that the switches in these sections are open while the sections appearing in the solution indicate that their switches are closed.

Recombination operation

The recombination operator is applied in the same way as it is proposed for the planning problem. Since the fixed components (sections with no switches) appear in every solution, the offspring generated by the recombination operator will contain these components. The feeder sections with switches are the variable components that can be altered by the recombination operator.

Mutation operation

The mutation operator proposed for the planning problem is applied with a small difference. Here, the candidate feeder sections to be included or removed in this operation are the sections with switches. The sections with no switches do not participate in this process.

6.6 Case studies

The developed method was tested on two systems presented in two case studies:

Case 1. In this case, the network is a power distribution system with three feeders, 13 normally closed switches, three normally open switches and 13 power demand nodes. This problem was resolved in [Su and Lee, 2003] and [Song et al., 1997].

Case 2. The algorithm was tested on a real large-scale network presented in [Su and Lee, 2003]. The network has 94 nodes with 11 feeders, 83 normally closed switches and 13 normally open switches.

The tests were done using a PC compatible 1 GHz Pentium 4 with 128 Mb of RAM, Windows ME and a Visual C++ compiler.

6.6.1 Case 1

In this case, the network is shown in figure 6.1 where the dashed lines represent the tie switches, the continuous lines are sectionalizing switches and the dots represent the demand nodes. The general data of the problem are given in appendix A. The algorithm was applied with the following parameter values: Population size = 10, Pc (recombination rate) = 0.8, Pm (mutation rate) = 0.8. Table 6.2 shows the optimal configuration found by the algorithm, which is the same solution found by the method proposed in [Su and Lee, 2003]. Table 6.1 summarizes the results obtained by both methods. The proposed GA found the optimum solution in 12 generations.

This network was resolved by the evolutionary programming algorithm proposed in [Song et al., 1997]. This method found the same configuration in two generations but, using a big population size (100 individuals).

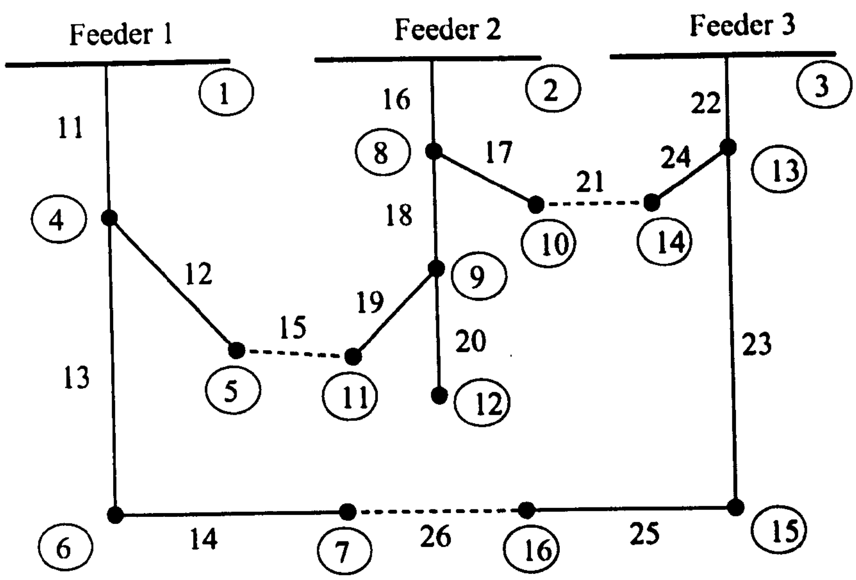


Figure 6. 1 Three-feeder distribution system for case 1

Table 6. 1 Numerical results of case 1 (Base three-phase complex power S_{base} = 100 MVA; base voltage V_{base} = 10 kV_{L-L})

Main items	Original configuration	Proposed method in [Su and Lee, 2003]	Proposed GA
Tie switches	15, 21, 26	19, 17, 26	19, 17, 26
Power loss (kW)	511.4	466.1	466.092
Voltage (P.U.)	V_{max} = 1.0 (Nodes 1, 2, 3)	V_{max} = 1.0 (Nodes 1, 2, 3)	V_{max} = 1.0 (Nodes 1, 2, 3)
	V_{min} = 0.9693 (Node 12)	V_{min} = 0.9716 (Node 12)	V_{min} = 0.971577 (Node 12)
Loss reduction		8.86 %	8.8596 %

Table 6. 2 Optimal configuration of case 1

Line Ni-Nj	Current (Amps)	Power flow ($MVA_{3\phi}$) Ni-Nj	Power flow ($MVA_{3\phi}$) Nj-Ni	Voltage (kV_{L-L}) Ni	Voltage (kV_{L-L}) Nj
8 - 9	564.655	9.59824	9.51976	9.81404	9.73379
13 - 15	181.064	3.11197	3.10373	9.92297	9.89671
4 - 5	210.488	3.61186	3.60161	9.90703	9.8789
1 - 4	548.948	9.50806	9.41966	10	9.90703
4 - 6	210.326	3.60909	3.59206	9.90703	9.86027
13 - 14	117.171	2.01384	2.01063	9.92297	9.90718
3 - 13	358.927	6.2168	6.16891	10	9.92297
9 - 12	285.822	4.8188	4.80988	9.73379	9.71577
2 - 8	818.205	14.1717	13.9082	10	9.81404
6 - 7	112.602	1.92307	1.92093	9.86027	9.84931
5 - 11	45.6467	0.781049	0.781017	9.8789	9.87849
15 - 16	131.166	2.2484	2.24721	9.89671	9.89144
10 - 14	78.463	1.34536	1.3464	9.8995	9.90718

With this case, the proposed genetic algorithm was validated. The proposed genetic algorithm found the same optimal configuration found in [Su and Lee, 2003] with almost the same power losses and the minimum voltage magnitude in the solution.

6.6.2 Case 2

The proposed genetic algorithm was tested on a real network reported in [Su and Lee, 2003]. Figure 6.2 shows this network where the continuous lines represent the sectionalizing switches (normally close switches) and the tie switches (normally open switches) are represented by dashed lines. The system is 3-phase and 11.4 kV. The general data of the problem are given in appendix A.

According to [Su and Lee, 2003], the system is a practical network of the Taiwan Power Company. The conductors used are: ACSR 477 KCM, for overhead lines, and copper conductor 500 KCM, for underground lines. The capacity of these conductors are: 670 and 840 amperes, respectively. The voltage magnitude limit is ± 5 % (V_{max} = 11.97 kV, V_{min} = 10.83 kV). Table B.4-1 (appendix B) shows the power flows, current flows and voltage magnitudes of the original configuration of the system. In this configuration, there are 10 nodes with voltages below the limit and the power loss is 531.786 kW.

The proposed genetic algorithm was applied with the following parameter values: Population size = 80; Pc = 0.8; Pm = 0.8. The configuration obtained by the proposed algorithm satisfies the voltage constraint and it has less power losses. In table B.4-2 (appendix B) it is shown the configuration solution for this case with voltage magnitudes, current and power flows. Table 6.3 summarizes the results obtained by the proposed algorithm and the method proposed in [Su and Lee, 2003].

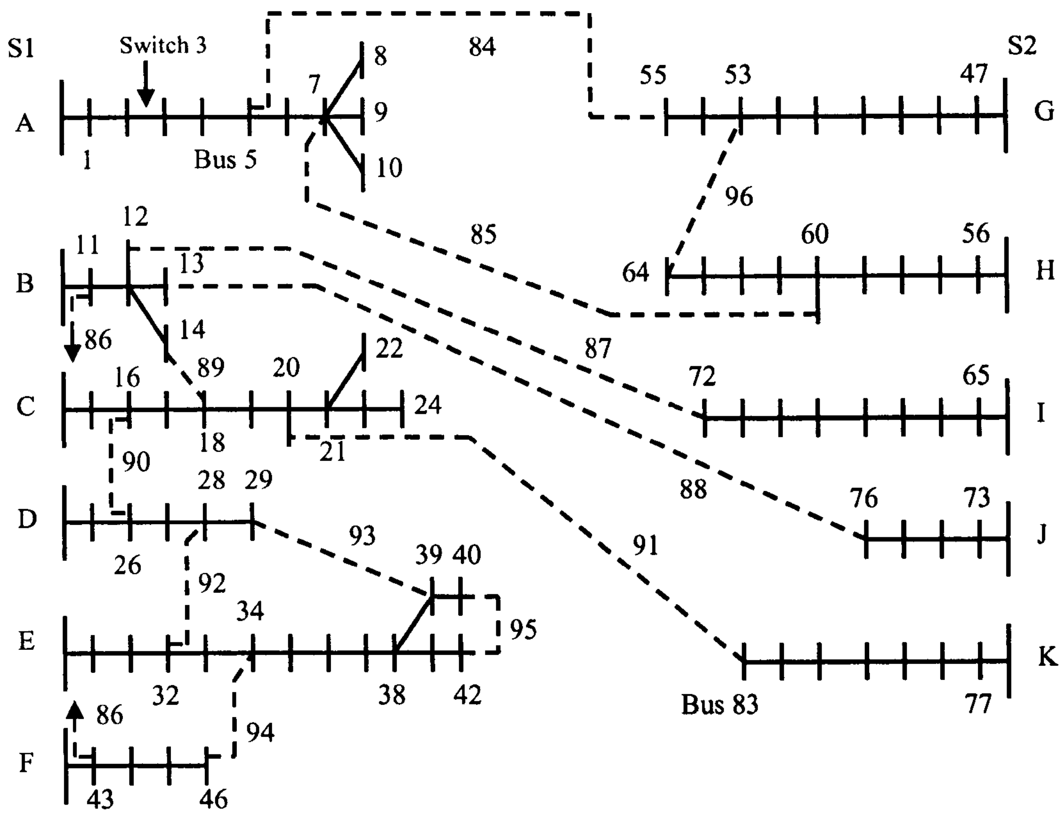


Figure 6. 2 Distribution system for case 2

Table 6. 3 Numerical results of case 2

Main items		**Original configuration**	**Method in [Su and Lee, 2003]**	**The proposed GA**
Tie switches		84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96	55, 7, 86, 72, 13, 89, 90, 83, 92, 39, 34, <u>41</u>, 62	55, 7, 86, 72, 13, 89, 90, 83, 92, 39, 34, <u>42</u>, 62
Power loss (kW)		531.786	469.88	469.795
Bus voltages (kV)	Substation	11.400	11.400	11.400
	V_4	10.7731	11.019	11.0191
	V_5	10.6435	10.951	10.9514
	V_6	10.6209	10.942	10.9415
	V_7	10.6028	10.912	10.9122
	V_8	10.5957	10.905	10.9053
	V_9	10.5856	10.895	10.8954
	V_{10}	10.5945	10.904	10.9041
	V_{71}	10.8178	10.866	10.8664
	V_{72}	10.8166	11.195	11.1947
	V_{83}	10.8049	10.958	10.9585
System V_{min} (kV)		10.5856	10.866	10.8664
Loss reduction		-	11.68 %	11.65 %
CPU time (seconds)		-	36.15	16.0
Iterations (generations)		-	-	61

In table 6.3, it is shown the voltage magnitudes below the limit in the original configuration. After network reconfiguration, these voltages satisfy the voltage constraint and the power losses are reduced by 11.65 %. In [Nara and Song, 2002] it is stated that up to 13% of the total power generation is wasted in the form of line loss at the distribution level; therefore, this reduction is significant.

The proposed genetic algorithm found a different solution from the one found by the proposed method in [Su and Lee, 2003]. The difference between both solutions is given by the tie switches 41 and 42; as it is shown in the table 6.3. However, the power losses and voltage magnitudes are almost the same between both configurations. The method in [Su and Lee, 2003] found the solution in 36.15 seconds using MATLAB and a Pentium II-266-MHz computer.

In figure 6.3 it is shown the evolution process of the proposed genetic algorithm for this case.

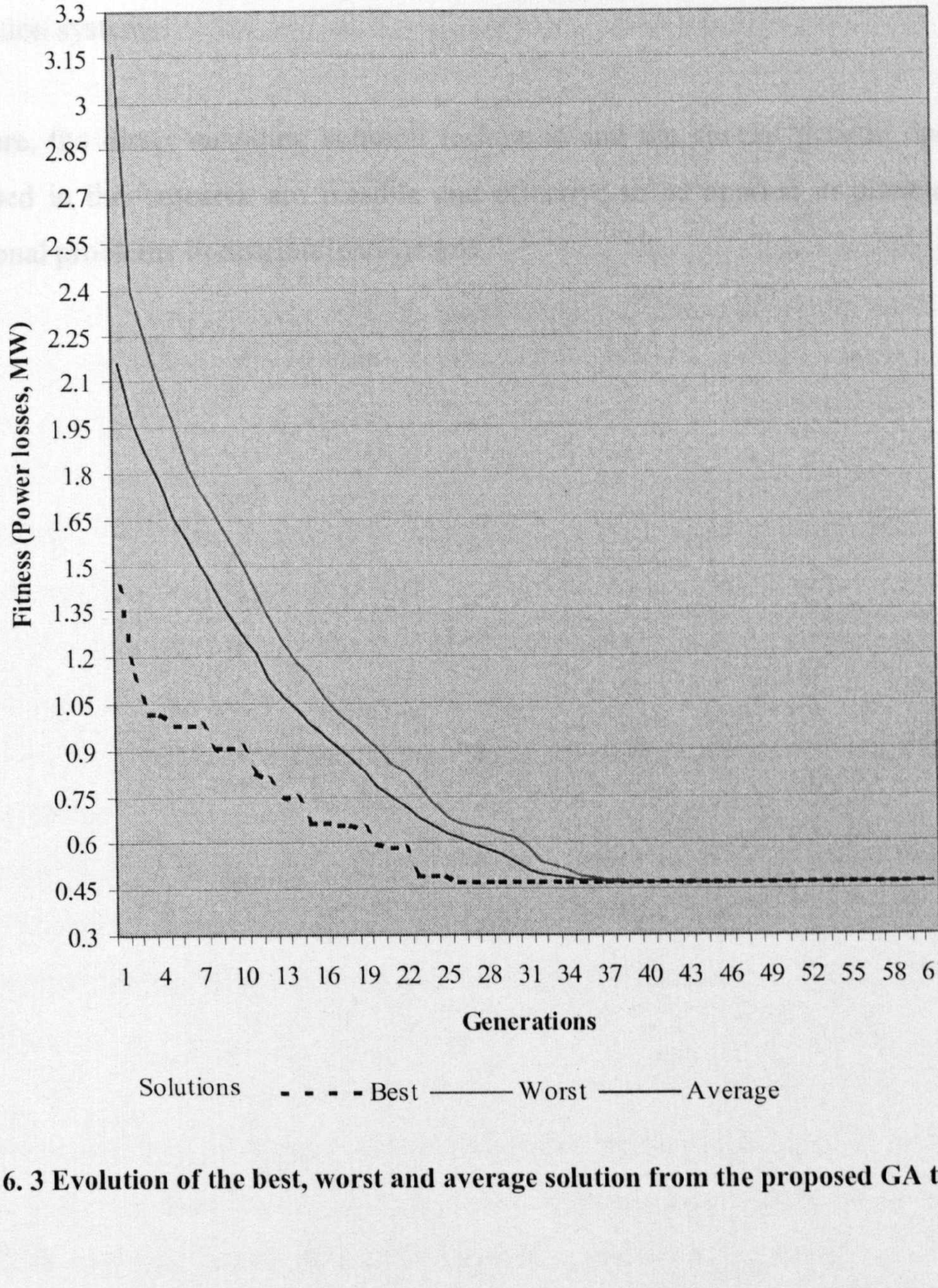


Figure 6. 3 Evolution of the best, worst and average solution from the proposed GA tested on case 2

6.7 Conclusions

In this chapter, it is reported the results obtained from the application of the developed genetic algorithm, originally proposed for distribution system expansion planning, to the problem of network reconfiguration for loss reduction. These results demonstrated the potential of this genetic algorithm in operational problems. These results also demonstrated that this algorithm can be applied on other types of problems such as optimal switching device placement and optimal open-loop configuration in distribution systems.

Therefore, the direct encoding solution technique and the special genetic operators developed in this research are feasible and effective to be applied in planning and operational problems in distribution systems.

Chapter 7 Conclusions and Future Work

7.1 Conclusions

A new method for power distribution system expansion planning was developed in this research. The method finds the site and size of substations and feeders optimizing various planning aspects expressed as objectives functions and considering the main constraints of the planning problem.

Various experiments were done to evaluate the efficiency, effectiveness, validity and practicability of the developed method. From these experiments, it can be concluded that the algorithm is effective, efficient and robust. Moreover, the algorithm proved to have better performance than other previous methods.

Previous planning methods based on genetic algorithms use encoding technique that have low locality. Low locality occurs when a small change in the representation generated by the mutation operator can lead to big changes in the represented network. This situation cause a problem called low heritability. Low heritability occurs when most of the individuals created by the recombination operators are not similar to their parents; therefore, the good properties of parent solutions are not passed to the offspring solutions. In this case, GAs work more like a random search than a guided search. Thus, the interaction between the coding technique for the potential solutions and the genetic operators play a key role in the performance of evolutionary algorithms.

In this research, it is proposed that the distribution system topologies be represented directly as sets of their feeders, and special recombination and mutation operators be used. This encoding technique and the genetic operators overcome the problems of low locality and heritability. In addition, the radiality constraint is enforced in each genetic operation.

Traditionally, the planning problem has been formulated to minimize the economical costs of the system being treated. However, the real-world problems are, in general, multi-objective. That is, the problems involve multiple objectives to be optimized, with the objectives often conflicting. Therefore, describing the solutions to a real-world problem in terms of a single objective function is impoverished in descriptive ability.

A distribution system involves other aspects such as reliability, environmental and social impact. If the solutions to a planning problem are described only in terms of economical costs, it might be difficult to qualify the solutions. If instead the solutions are described in terms of other aspects, there would be more information available to help the planner to compare and select options.

Evolutionary algorithms are ideal candidates to be applied on problems considering more than one objective since EAs work with a population of solutions; therefore, they are able to give a set of solutions as a result of the optimization process. However, this property of EAs has been little exploited. In this research, only one method based on EAs for multi-objective planning problem was found in the literature.

In this research, it is proposed to apply a strategy to find Pareto-optimal solutions (or approximate Pareto-optimal solutions) to the planning problem with two objectives to optimize. This strategy applies new concepts and ideas developed in the fields of multi-objective optimization and evolutionary algorithms. The proposed encoding technique and genetic operators used with this strategy proved to be effective in multi-objective problems.

Therefore, in this research, it was developed a planning method based on evolutionary algorithms that is able to resolve single-objective and multi-objective distribution system expansion planning problems.

7.2 Future work

Uncertainty. Power distribution system planning is subject to uncertainty due to various factors such as demand growth, inflation rates, interest rate, environmental regulation and public opinion. In the light of recent progress in the field of evolutionary multi-objective optimization, efficient techniques to deal with uncertainty can be developed.

Multi-stage planning. There have been efforts to develop planning methods to solve the problem in several stages rather than in a single stage. These methods involve a large number of variables; therefore, they have been applied on simplified systems. The outcome of evolutionary algorithms is a set of solutions. This characteristic could be exploited to formulate efficient and effective multi-stage planning methods. For example, instead of finding and treating a single solution in each planning stage, a set of solutions for each stage can be treated to have more information and options to make decisions.

Distributed generation. Historically, growth in electric load demand has been served by adding new large central station generating units, building transmission lines and extending traditional distribution systems. An alternative approach under consideration by utilities is to satisfy demand by investing in distributed generation (DG). DG can relieve capacity constraints on the generation, transmission and distribution systems and obviate the need to build new facilities. One way to evaluate a DG option is by determining the reduction in variable costs in the system and the value of deferring capacity investments. The application of multi-objective evolutionary algorithms in this case could prove to be fruitful.

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Papers related to the thesis

F. Rivas-Dávalos and M. R. Irving, "An efficient genetic algorithm for optimal large-scale power distribution network planning," in *Proc. Power Tech Conference, IEEE Bologna 2003*, vol. 3, pp. 797 - 801.

F. Rivas-Dávalos and M. R. Irving, "An approach based on the strength pareto evolutionary algorithm 2 for distribution system planning", in *Lecture Notes in Computer Science Volume 3410 / 2005: Evolutionary Multi-Criterion Optimization: Third International Conference, EMO 2005, Guanajuato, Mexico, March 9-11, 2005. Proceedings.* Editors: Carlos A. Coello Coello, Arturo Hernández Aguirre, Eckart Zitzler . ISBN: 3-540-24983-4. Springer-Verlag GmbH.

F. Rivas-Dávalos and M. R. Irving, "An efficient implementation of a genetic algorithm for distribution network planning", to be revised and resubmitted to IEEE Transactions on Power Systems.

F. Rivas-Dávalos and M. R. Irving, "An evolutionary algorithm for multi-objective optimisation of power distribution network planning", to be submitted to IEEE Transactions on Power Systems for review.

Appendix A Test systems used in the case studies

Nomenclature

Nr = Number

Nd = Node

Load = $MVA_{3\phi}$

Nd-i = Node (From)

Nd-j = Node (To)

Lgth = Length (km)

T = Type

St = Status

Each feeder is described by two nodes it connects; type of conductor and length. Status "0" describes the candidate feeders, while status "1" describes the existing feeders. The symbol "*" is for the imaginary feeders used to manipulate the systems with more than one substation and to represent the networks as spanning trees. Node "0" is the root node (imaginary node).

A.1 System data used for case 2 in chapter 4

Table A.1-1 General data of the system for case 2 in chapter 4

Load factor	0.40
Loss factor	0.21
Cost of energy	0.30 Rs/kWh
Discount rate	0.10
Power factor	0.80
System life	25 years
System voltage	33 kV

Table A.1-2 Power demand data of the system for case 2 in chapter 4

Number	Node	Load ($MVA_{3\phi}$)	Power factor
1	3	5	0.8
2	4	3	0.8
3	5	4	0.8
4	6	3	0.8
5	7	3	0.8
6	8	5	0.8
7	9	6	0.8
8	10	5	0.8

Table A.1-3 Substation data of the system for case 2 in chapter 4

Number	Feasible site node	Capacity ($MVA_{3\phi}$)	Fixed cost (Millions Rs.)	Feeder bay cost (Millions Rs.)
1	1	50	3.100	0.1
2	2	50	3.100	0.1

Table A.1-4 Conductor data of the system for case 2 in chapter 4

Type	Resistance (Ω/km)	Reactance (Ω/km)	Fixed cost (Millions/km)	Capacity (Amps.)
1	0.3432	0.1425	0.04	209

Table A.1-5 Feeder data of the system for case 2 in chapter 4

Number	Node (From)	Node (To)	Type	Status	Length (km)
0	0	1	*	*	*
1	0	2	*	*	*
2	1	3	1	0	12.0
3	1	5	1	0	13.0
4	1	7	1	0	10.0
5	1	10	1	0	0.0
6	2	10	1	0	10.0
7	2	3	1	0	10.0
8	2	5	1	0	8.0
9	2	6	1	0	16.0
10	2	4	1	0	16.0
11	2	7	1	0	14.0
12	7	8	1	0	12.0
13	10	9	1	0	16.0
14	3	4	1	0	15.0
15	5	6	1	0	14.0
16	1	9	1	0	16.0

A.2 System data of case 3 in chapter 4

Table A.2-1 General data of the system for case 3 in chapter 4

Load factor	0.55
Loss factor	0.3768
Power factor	0.80
Cost of energy	9.6 Pts/kWh
Discount rate	0.10
System life	30 years
System voltage	10 kV
Voltage drop limit	3.0%

Table A.2-2 Power demand data of the system for case 3 in chapter 4

Nr	Nd	Load	Nr	Nd	Load	Nr	Nd	Load	Nr	Nd	Load
1	2	0.135	44	56	0.013	87	102	0.079	130	152	0.067
2	5	0.127	45	57	0.086	88	103	0.086	131	153	0.086
3	6	0.057	46	58	0.055	89	104	0.216	132	154	0.216
4	7	0.092	47	60	0.067	90	105	0.135	133	155	0.062
5	9	0.135	48	61	0.079	91	106	0.061	134	156	0.135
6	10	0.135	49	62	0.135	92	107	0.042	135	158	0.003
7	11	0.086	50	63	0.086	93	108	0.024	136	159	0.001
8	12	0.055	51	64	0.106	94	110	0.086	137	160	0.058
9	13	0.075	52	65	0.134	95	111	0.135	138	161	0.216
10	14	0.079	53	66	0.086	96	112	0.042	139	162	0.011
11	17	0.086	54	67	0.114	97	113	0.135	140	163	0.142
12	18	0.033	55	68	0.058	98	114	0.03	141	164	0.018
13	19	0.013	56	69	0.063	99	115	0.071	142	165	0.041
14	20	0.001	57	70	0.19	100	120	0.088	143	166	0.171
15	21	0.216	58	71	0.134	101	121	0.274	144	168	0.023
16	22	0.086	59	72	0.043	102	123	0.055	145	169	0.087
17	23	0.086	60	73	0.132	103	124	0.055	146	170	0.023
18	24	0.086	61	74	0.054	104	125	0.055	147	171	0.03
19	25	0.135	62	76	0.055	105	126	0.055	148	172	0.114
20	26	0.116	63	77	0.049	106	127	0.055	149	173	0.135
21	28	0.086	64	78	0.09	107	128	0.086	150	174	0.08
22	29	0.27	65	79	0.147	108	129	0.135	151	175	0.055
23	30	0.086	66	80	0.216	109	130	0.069	152	176	0.135
24	31	0.086	67	81	0.023	110	131	0.117	153	177	0.216
25	32	0.05	68	82	0.135	111	132	0.135	154	179	0.086
26	33	0.134	69	83	0.003	112	133	0.086	155	180	0.091
27	34	0.216	70	84	0.058	113	134	0.086	156	181	0.059
28	35	0.135	71	85	0.114	114	136	0.094	157	182	0.135
29	36	0.086	72	86	0.023	115	137	0.009	158	183	0.071
30	39	0.107	73	87	0.023	116	138	0.135	159	184	0.096
31	40	0.135	74	88	0.023	117	139	0.039	160	185	0.091
32	41	0.078	75	89	0.005	118	140	0.061	161	187	0.186
33	42	0.064	76	90	0.023	119	141	0.17	162	190	0.146
34	44	0.135	77	92	0.023	120	142	0.351	163	191	0.023
35	45	0.135	78	93	0.02	121	143	0.117	164	192	0.023
36	46	0.135	79	94	0.135	122	144	0.022	165	193	0.023
37	47	0.096	80	95	0.129	123	145	0.105	166	194	0.023
38	48	0.086	81	96	0.008	124	146	0.042	167	195	0.023
39	50	0.135	82	97	0.071	125	147	0.092	168	197	0.023
40	51	0.135	83	98	0.054	126	148	0.056	169	200	0.058
41	52	0.135	84	99	0.018	127	149	0.086			
42	54	0.047	85	100	0.086	128	150	0.135			
43	55	0.135	86	101	0.132	129	151	0.113			

Table A.2-3 Substation data of the system for case 3 in chapter 4

Number	Node	Capacity ($MVA_{3\phi}$)	Fixed cost (Millions Pts.)
1	201	40	300.0

Table A.2-4 Conductor data of the system for case 3 in chapter 4

Type	Resistance (Ω/km)	Reactance (Ω/km)	Fixed cost (Millions/km)	Capacity (Amps.)
1	0.2570	0.0870	18.636	255
2	0.1020	0.0950	20.489	515
3	0.0510	0.00475	32.434	1030

Table A.2-5 Feeder data of the system for case 3 in chapter 4

Nr	Nd-i	Nd-j	T	St	Lgth	Nr	Nd-i	Nd-j	T	St	Lgth
0	0	201	*	*	*	113	158	183	2	1	0.62
1	65	67		0	0.18	114	127	126		0	0.74
2	69	65		0	0.175	115	124	127		0	0.06
3	58	69		0	0.147	116	199	118		0	0.21
4	67	84		0	0.08	117	199	200		0	0.02
5	67	92		0	0.23	118	157	199		0	0.06
6	59	57		0	0.03	119	123	157		0	0.28
7	64	70		0	0.195	120	198	191		0	0.04
8	68	64		0	0.085	121	198	196		0	0.19
9	58	68		0	0.058	122	158	198		0	0.74
10	62	60		0	0.3	123	196	190		0	0.03
11	70	62		0	0.062	124	158	189	3	1	1.003
12	60	59		0	0.12	125	158	160		0	0.23
13	57	61		0	0.354	126	160	171		0	0.24
14	63	66		0	0.123	127	171	187		0	0.25
15	66	67		0	0.236	128	187	167		0	0.25
16	61	63		0	0.165	129	175	178		0	0.18
17	57	19		0	1.62	130	167	175		0	0.31
18	201	91	2	1	1.064	131	160	184		0	0.36
19	91	1	2	1	0.933	132	184	163		0	0.25
20	91	86	1	1	0.044	133	166	174	2	1	0.31
21	1	16		0	0.24	134	174	180	2	1	0.29
22	5	6		0	0.11	135	159	179		0	0.12
23	6	2		0	0.142	136	179	176		0	0.15

Table A.2-5 (*Continued*)

Nr	Nd-i	Nd-j	T	St	Lgth	Nr	Nd-i	Nd-j	T	St	Lgth
24	17	4		0	0.335	137	168	161		0	0.21
25	4	5		0	0.135	138	161	182		0	0.23
26	16	17		0	0.32	139	182	181		0	0.09
27	1	13		0	0.045	140	159	180	2	1	0.25
28	7	8		0	0.26	141	180	177		0	0.16
29	8	2		0	0.04	142	170	162	1	1	0.22
30	15	18		0	0.43	143	162	183	1	1	0.26
31	18	7		0	0.06	144	159	128	1	1	0.78
32	13	15		0	0.27	145	128	140	1	1	0.11
33	1	12	1	1	0.11	146	130	156		0	0.1
34	3	11	1	1	0.23	147	129	130		0	0.15
35	11	14	1	1	0.36	148	140	129	1	1	0.16
36	14	10	1	1	0.455	149	156	132		0	0.31
37	12	3	1	1	0.155	150	128	150		0	0.09
38	10	85	1	1	0.12	151	144	145		0	0.09
39	85	90		0	0.26	152	145	148		0	0.15
40	9	10	1	1	0.16	153	151	134		0	0.21
41	2	9		0	0.295	154	134	144		0	0.39
42	9	87	1	1	0.375	155	134	194		0	0.025
43	87	88		0	0.22	156	150	151		0	0.15
44	88	66		0	0.295	157	201	189	3	1	0.603
45	2	19		0	1.67	158	189	188		0	0.2
46	116	96		0	0.11	159	188	197		0	0.3
47	97	93		0	0.11	160	197	195		0	0.1
48	115	97		0	0.17	161	188	192		0	0.064
49	56	146		0	0.43	162	148	137		0	0.35
50	152	140	1	1	0.25	163	137	154		0	0.29
51	146	141		0	0.29	164	137	193		0	0.035
52	141	135		0	0.22	165	154	150		0	0.19
53	131	152		0	0.24	166	125	123		0	0.08
54	135	131		0	0.41	167	153	133	1	1	0.27
55	56	148		0	0.19	168	147	56		0	0.4
56	56	124		0	0.25	169	183	186	2	1	0.12
57	126	123		0	0.08	170	186	166	2	1	0.29
58	19	50		0	0.24	171	75	71		0	0.15
59	50	51		0	0.19	172	74	80		0	0.23
60	51	55		0	0.32	173	80	75		0	0.145
61	55	26		0	0.26	174	73	74		0	0.29
62	26	30		0	0.28	175	71	72		0	0.27
63	30	39		0	0.29	176	72	79		0	0.2
64	39	46		0	0.25	177	72	76		0	0.21
65	50	53		0	0.15	178	76	77		0	0.14
66	53	23		0	0.2	179	78	82		0	0.66
67	23	28		0	0.26	180	81	79		0	0.2
68	28	33		0	0.315	181	77	78		0	0.06
69	33	46		0	0.475	182	82	89		0	0.331
70	46	20		0	0.23	183	76	58		0	1.06
71	19	43		0	0.04	184	122	201	3	1	0.073
72	43	24		0	0.35	185	119	93	3	1	1.079
73	24	34		0	0.35	186	119	122	3	1	1.253
74	34	37		0	0.22	187	113	114		0	0.13

Table A.2-5 (***Continued***)

Nr	Nd-i	Nd-j	T	St	Lgth	Nr	Nd-i	Nd-j	T	St	Lgth
75	37	40		0	0.285	188	114	111		0	0.11
76	40	42		0	0.32	189	93	112		0	0.09
77	42	47		0	0.22	190	112	113		0	0.21
78	47	49		0	0.2	191	105	94		0	0.23
79	19	32		0	0.18	192	94	111		0	0.09
80	83	32	2	1	0.115	193	93	110		0	0.18
81	27	35		0	0.25	194	110	105		0	0.18
82	35	44		0	0.36	195	118	120	1	1	0.035
83	48	44		0	0.12	196	93	118	2	1	0.29
84	54	22		0	0.21	197	117	83	2	1	0.11
85	22	27		0	0.19	198	118	121	2	1	0.14
86	32	54		0	0.12	199	121	117	2	1	0.55
87	20	49		0	0.1	200	103	104	1	1	0.17
88	31	43		0	0.7	201	83	103	2	1	0.2
89	36	31		0	0.29	202	108	107	1	1	0.17
90	41	36		0	0.185	203	83	108	2	1	0.13
91	45	41		0	0.07	204	109	83		0	0.12
92	45	49		0	0.1	205	102	109		0	0.21
93	20	48		0	0.37	206	106	102		0	0.15
94	38	29		0	0.32	207	83	106		0	0.18
95	48	38		0	0.24	208	96	115		0	0.12
96	52	21		0	0.11	209	83	98		0	0.36
97	29	25		0	0.15	210	100	95		0	0.29
98	25	52		0	0.275	211	99	100		0	0.16
99	21	32		0	0.12	212	101	99		0	0.15
100	20	56		0	0.59	213	98	101		0	0.14
101	20	71		0	0.53	214	95	116		0	0.215
102	71	73		0	0.06	215	132	139		0	0.29
103	163	164		0	0.09	216	139	146		0	0.42
104	164	172		0	0.38	217	128	153	1	1	0.12
105	172	178		0	0.31	218	136	143		0	0.26
106	178	159		0	0.19	219	143	147		0	0.26
107	158	181		0	0.28	220	153	155	1	1	0.11
108	181	185		0	0.25	221	155	136		0	0.23
109	185	165		0	0.16	222	142	149		0	0.27
110	165	169		0	0.16	223	149	147		0	0.06
111	169	173		0	0.1	224	133	138	1	1	0.41
112	173	179		0	0.27	225	138	142		0	0.17

A.3 System data of case 1 in chapter 5

Table A.3-1 Vector of weights for graph A of case 1 in chapter 5

Nr	w^1	w^2	Nr	w^1	w^2
0	0.0000	0.0000	23	17.331	42.8048
1	31.6778	24.9924	24	36.6631	38.747
2	47.3549	27.8523	25	14.0287	12.9911
3	98.9595	23.4323	26	51.6129	22.213
4	70.1867	23.615	27	88.5226	10.7425
5	57.971	14.0801	28	42.3963	25.052
6	24.7702	42.5451	29	25.0725	16.6975
7	92.1846	39.8229	30	73.0808	19.0767
8	15.6921	39.8862	31	92.4496	20.6944
9	30.3895	19.8877	32	48.3898	19.9456
10	74.2169	42.847	33	94.3424	47.9267
11	87.851	10.5506	34	49.3415	42.3923
12	81.4457	36.4435	35	75.702	46.515
13	34.9696	42.3834	36	88.4566	39.5314
14	24.7319	15.5488	37	72.4092	32.6584
15	18.9167	44.3563	38	39.4639	23.5686
16	38.8372	36.9854	39	64.0833	20.0846
17	69.9022	12.4811	40	36.0414	45.31
18	38.6389	27.9288	41	39.8044	12.8133
19	81.3701	23.3797	42	69.8949	38.8251
20	53.6563	42.8405	43	77.2821	27.221
21	65.2281	23.6438	44	21.8463	20.9746
22	45.8769	14.388	45	18.4494	29.9366

Table A.3-2 Vector of weights for graph B of case 1 in chapter 5

Nr	w^1	w^2	Nr	w^1	w^2
0	0.0000	0.0000	23	17.3314	34.806
1	31.6776	32.9922	24	72.6631	46.7469
2	65.3549	35.8537	25	68.0291	44.9918
3	44.9595	23.4323	26	51.6128	22.2112
4	16.1865	23.6162	27	16.5226	34.7425
5	21.971	38.0799	28	96.3964	17.0508
6	60.7701	42.5447	29	25.0725	16.6977
7	20.1846	39.823	30	37.0809	11.0776
8	15.6921	31.8862	31	74.4496	28.6943
9	12.3896	43.8891	32	66.3899	35.9456
10	38.2169	42.847	33	22.3423	31.9243
11	87.851	18.5533	34	85.3415	26.3923
12	27.4456	44.4433	35	75.702	30.5113
13	52.9695	34.3833	36	52.4566	15.5317
14	24.7319	23.5492	37	18.4093	32.6586
15	18.9167	28.3563	38	39.4639	39.5681
16	38.8376	36.9862	39	46.0833	28.0846
17	69.9022	36.481	40	54.0409	37.3085
18	56.6392	27.931	41	39.8044	20.8134
19	81.37	39.3791	42	69.8945	46.8211
20	53.6563	34.8405	43	41.2821	11.2217
21	47.2281	15.6442	44	39.8463	12.9746
22	99.8769	30.3881	45	90.4494	21.9358

Table A.3-3 Vector of weights for graph C of case 1 in chapter 5

Nr	w^1	w^2	Nr	w^1	w^2
0	0.0	0.0	23	17.3324	26.8091
1	31.677	40.9917	24	18.6631	14.7468
2	83.3549	43.8576	25	32.0303	36.9936
3	80.9595	23.4323	26	51.6127	22.206
4	52.1862	23.6193	27	34.5226	18.7425
5	75.971	22.0796	28	60.3968	49.0476
6	96.7698	42.5435	29	25.0725	16.6982
7	38.1846	39.8233	30	91.0813	43.0799
8	15.6921	23.8862	31	56.4495	36.694
9	84.3899	27.893	32	84.3901	11.9457
10	92.2169	42.847	33	40.342	15.9175
11	87.8511	26.5605	34	31.3415	10.3923
12	63.4456	12.4428	35	75.7022	14.5013
13	70.9693	26.3831	36	16.4566	31.5324
14	24.7319	31.5503	37	54.4098	32.6593
15	18.9167	12.3563	38	39.4638	15.5667
16	38.8388	36.9884	39	28.0833	36.0845
17	69.9022	20.4809	40	72.0394	29.3043
18	74.6402	27.9371	41	39.8044	28.8135
19	81.3699	15.3776	42	69.8933	14.81
20	53.6563	26.8405	43	95.2822	35.2235
21	29.2281	47.6453	44	57.8464	44.9746
22	63.8769	46.3882	45	72.4493	13.9338

A.4 System data of case 3 in chapter 5

Table A.4-1 General data of the system for case 3 in chapter 5

Load factor	0.55
Loss factor	0.3768
Power factor	0.80
Cost of energy	9.6 Pts/kWh
Discount rate	0.10
System life	30 years
System voltage	10 kV
Voltage drop limit	3.0%

Table A.4-2 Power demand data of the system for case 3 in chapter 5

Nr	Nd	Load	Nr	Nd	Load	Nr	Nd	Load	Nr	Nd	Load
1	1	0.224	36	39	0.032	71	77	0.011	106	116	0.037
2	2	0.224	37	40	0.01	72	78	0.14	107	117	0.047
3	3	0.081	38	41	0.005	73	79	0.224	108	118	0.058
4	4	0.09	39	43	0.02	74	80	0.353	109	119	0.048
5	5	0.128	40	44	0.05	75	81	0.091	110	120	0.14
6	6	0.131	41	45	0.036	76	82	0.14	111	121	0.119
7	7	0.353	42	46	0.068	77	83	0.024	112	122	0.224
8	8	0.179	43	47	0.09	78	84	0.014	113	123	0.14
9	9	0.14	44	48	0.14	79	85	0.079	114	124	0.027
10	10	0.066	45	49	0.025	80	86	0.112	115	125	0.112
11	11	0.027	46	50	0.015	81	87	0.125	116	126	0.09
12	12	0.027	47	51	0.02	82	88	0.14	117	127	0.172
13	13	0.005	48	52	0.14	83	90	0.038	118	128	0.025
14	14	0.14	49	53	0.006	84	91	0.092	119	129	0.034
15	15	0.062	50	54	0.029	85	92	0.066	120	130	0.14
16	16	0.069	51	55	0.02	86	93	0.14	121	131	0.014
17	18	0.035	52	56	0.14	87	94	0.161	122	132	0.062
18	19	0.176	53	57	0.09	88	95	0.224	123	133	0.133
19	20	0.14	54	59	0.14	89	96	0.056	124	134	0.14
20	21	0.224	55	60	0.046	90	97	0.046	125	135	0.448
21	22	0.045	56	62	0.031	91	98	0.224	126	136	0.09
22	23	0.09	57	63	0.033	92	99	0.085	127	137	0.197
23	24	0.14	58	64	0.09	93	100	0.179	128	138	0.168
24	25	0.224	59	65	0.224	94	101	0.53	129	139	0.01
25	26	0.02	60	66	0.353	95	102	0.075	130	140	0.032
26	27	0.224	61	67	0.196	96	103	0.015	131	141	0.015

Table A.4-2 (*Continued*)											
Nr	**Nd**	**Load**	**Nr**	**Nd**	**Load**	**Nr**	**Nd**	**Load**	**Nr**	**Nd**	**Load**
27	28	0.118	62	68	0.14	97	104	0.004	132	142	0.025
28	29	0.194	63	69	0.007	98	106	0.073	133	143	0.05
29	30	0.193	64	70	0.034	99	108	0.067	134	145	0.04
30	31	0.126	65	71	0.067	100	109	0.105	135	146	0.005
31	32	0.224	66	72	0.062	101	111	0.021	136	147	0.01
32	34	0.224	67	73	0.14	102	112	0.074	137	148	0.089
33	35	0.448	68	74	0.131	103	113	0.02	138	176	0.095
34	36	0.224	69	75	0.14	104	114	0.037	139	177	0.095
35	37	0.14	70	76	0.224	105	115	0.023			

Table A.4-3 Substation data of the system for case 3 in chapter 5

Number	Node	Capacity ($MVA_{3\phi}$)	Fixed cost (Millions Pts.)
1	181	40	300.0
2	182	8	136.0
3	182	40	300.0

Table A.4-4 Conductor data of the system for case 3 in chapter 5

Type	Resistance (Ω/km)	Reactance (Ω/km)	Fixed cost (Mill/km)	Capacity (Amps.)	Failure rate (f/km*yr)	Outage time (Hrs)
1	0.2570	0.0870	18.636	255	0.096	10.75
2	0.1020	0.0950	20.489	515	0.064	8.95

Table A.4-5 Feeder data of the system for case 3 in chapter 5

Nr	Nd-i	Nd-j	T	St	Lgth	Nr	Nd-i	Nd-j	T	St	Lgth
0	0	181	*	*	*	104	138	64		0	0.186
1	0	182	*	*	*	105	128	129		0	0.073
2	181	149	2	1	0.62	106	129	130		0	0.114
3	149	2	2	1	0.064	107	130	131		0	0.075
4	150	178	2	1	0.064	108	128	132		0	0.16
5	2	3	1	1	0.175	109	169	147		0	0.325
6	3	4		0	0.17	110	169	50		0	0.08

Table A.4-5 (*Continued*)

Nr	Nd-i	Nd-j	T	St	Lgth	Nr	Nd-i	Nd-j	T	St	Lgth
7	2	5	1	1	0.354	111	7	76		0	0.184
8	178	179	1	1	0.354	112	76	77		0	0.09
9	5	6	1	1	0.205	113	113	117		0	0.198
10	182	174		0	0.03	114	100	104		0	0.12
11	7	8		0	0.105	115	109	114		0	0.168
12	8	9		0	0.32	116	109	104		0	0.226
13	8	10		0	0.153	117	110	103		0	0.288
14	10	182		0	0.154	118	95	59		0	0.131
15	174	1		0	0.41	119	59	86		0	0.098
16	1	181		0	1.32	120	59	123		0	0.295
17	172	151		0	0.285	121	123	125		0	0.015
18	151	11		0	0.311	122	123	65		0	0.284
19	151	152		0	0.585	123	123	124		0	0.132
20	152	12		0	0.55	124	123	122		0	0.24
21	152	13		0	0.137	125	122	121		0	0.185
22	181	14	2	1	0.951	126	121	120		0	0.26
23	14	15	2	1	0.29	127	120	98		0	0.192
24	15	16	1	1	0.34	128	98	95		0	0.104
25	15	17	2	1	0.18	129	121	128		0	0.086
26	17	18	2	1	0.008	130	64	166		0	0.024
27	18	19	2	1	0.22	131	166	139		0	0.003
28	19	22	1	1	0.243	132	139	140		0	0.075
29	22	36	1	1	0.28	133	140	141		0	0.31
30	36	37	1	1	0.213	134	133	134		0	0.153
31	36	171	1	1	0.184	135	140	167		0	0.49
32	38	39	1	1	0.02	136	175	29	1	1	0.095
33	38	42	1	1	0.286	137	167	142		0	0.025
34	42	154		0	0.015	138	167	144		0	0.315
35	154	153		0	0.35	139	144	145		0	0.16
36	153	41		0	0.05	140	144	168		0	0.145
37	153	40		0	0.2	141	168	146		0	0.36
38	42	155		0	0.025	142	168	169		0	0.57
39	155	156		0	0.14	143	34	35		0	0.115
40	156	43		0	0.035	144	7	35		0	0.21
41	156	44		0	0.015	145	31	148	1	1	0.26
42	42	45	1	1	0.125	146	148	6	1	1	0.11
43	45	46		0	0.144	147	179	28	1	1	0.141
44	46	47		0	0.199	148	28	27	1	1	0.216
45	42	48	1	1	0.167	149	181	150	2	1	0.62
46	113	108		0	0.191	150	31	92		0	0.205
47	108	112		0	0.215	151	92	35		0	0.19
48	112	116		0	0.187	152	20	143		0	0.08
49	116	118		0	0.338	153	177	176		0	0.305
50	118	117		0	0.182	154	103	177		0	0.161
51	117	114		0	0.19	155	108	106		0	0.27
52	114	110		0	0.228	156	113	116		0	0.223
53	48	157		0	0.118	157	67	68		0	0.233
54	157	158		0	0.015	158	68	164		0	0.3
55	158	49		0	0.055	159	164	165		0	0.07
56	158	50		0	0.385	160	165	69		0	0.275
57	50	51		0	0.515	161	165	70		0	0.045

Table A.4-5 (*Continued*)

Nr	Nd-i	Nd-j	T	St	Lgth	Nr	Nd-i	Nd-j	T	St	Lgth
58	50	159		0	0.27	162	164	162		0	0.205
59	159	52		0	0.136	163	162	161		0	0.08
60	52	53	1	1	0.157	164	162	180		0	0.72
61	53	160		0	0.161	165	161	71		0	0.045
62	160	55		0	0.11	166	161	182		0	0.125
63	53	54		0	0.075	167	182	163		0	0.78
64	53	56	1	1	0.27	168	163	83		0	0.285
65	56	57		0	0.29	169	163	84		0	0.16
66	56	82	1	1	0.184	170	180	67		0	0.847
67	82	81	1	1	0.227	171	67	126		0	0.105
68	81	33	1	1	0.19	172	126	127		0	0.15
69	33	32	1	1	0.095	173	67	96		0	0.138
70	32	175	1	1	0.04	174	96	95		0	0.138
71	29	30	1	1	0.156	175	95	94		0	0.203
72	29	27	1	1	0.09	176	95	97		0	0.103
73	27	170	1	1	0.16	177	94	93		0	0.062
74	170	173	1	1	0.135	178	93	91		0	0.251
75	173	26		0	0.01	179	67	91		0	0.27
76	173	25	1	1	0.008	180	91	89		0	0.046
77	25	24		0	0.075	181	89	90		0	0.015
78	24	23		0	0.19	182	89	88		0	0.172
79	23	21		0	0.19	183	88	87		0	0.128
80	21	20		0	0.24	184	87	99		0	0.511
81	20	19		0	0.23	185	99	85		0	0.3
82	182	72		0	0.039	186	181	172		0	0.076
83	182	73		0	0.316	187	99	84		0	0.222
84	73	74		0	0.229	188	99	100		0	0.255
85	74	8		0	0.382	189	100	101		0	0.152
86	74	75		0	0.317	190	101	102		0	0.124
87	75	63		0	0.315	191	102	103		0	0.272
88	62	63		0	0.188	192	103	104		0	0.57
89	61	62	1	1	0.216	193	104	107		0	0.138
90	60	61	1	1	0.13	194	107	115		0	0.607
91	58	60	1	1	0.175	195	115	111		0	0.288
92	56	58	1	1	0.188	196	111	106		0	0.198
93	171	38	1	1	0.091	197	106	99		0	0.214
94	63	64		0	0.198	198	103	105		0	0.19
95	64	65		0	0.295	199	105	109		0	0.346
96	65	66		0	0.272	200	109	119		0	0.035
97	66	67		0	0.428	201	109	113		0	0.198
98	132	133		0	0.114	202	76	78		0	0.132
99	133	135		0	0.195	203	78	79		0	0.094
100	135	122		0	0.15	204	79	80		0	0.17
101	135	136		0	0.177	205	80	34		0	0.135
102	136	137		0	0.08	206	80	81		0	0.147
103	137	138		0	0.115	207	34	33		0	0.228

A.5 System data of case 1 in chapter 6.

Table A.5-1 System data for case 1 in chapter 6

Line Ni-Nj	Section resistance (P.U.)	Section reactance (P.U.)	End node real load (MW)	End node reactive load (MW)	End node fixed capacitor (MVAR)
1 - 4	0.075	0.1	2.0	1.6	
4 - 5	0.08	0.11	3.0	1.5	1.1
4 - 6	0.09	0.18	2.0	0.8	1.2
6 - 7	0.04	0.04	1.5	1.2	
2 - 8	0.11	0.11	4.0	2.7	
8 - 9	0.08	0.11	5.0	3.0	1.2
8 - 10	0.11	0.11	1.0	0.9	
9 - 11	0.11	0.11	0.6	0.1	0.6
9 - 12	0.08	0.11	4.5	2.0	3.7
3 - 13	0.11	0.11	1.0	0.9	
13 - 14	0.09	0.12	1.0	0.7	1.8
13 - 15	0.08	0.11	1.0	0.9	
15 - 16	0.04	0.04	2.1	1.0	1.8
5 - 11	0.04	0.04			
10 - 14	0.04	0.04			
7 - 16	0.12	0.12			

A.6 System data of case 2 in chapter 6

Table A.6-1 System data for case 2 in chapter 6

Nr	Nd-i	Nd-j	Section resistance (Ω)	Section reactance (Ω)	End node real load (MW)	End node reactive load (MVAR)
0	0	A	*	*	0	0
1	0	B	*	*	0	0
2	0	C	*	*	0	0
3	0	D	*	*	0	0
4	0	E	*	*	0	0
5	0	F	*	*	0	0
6	0	G	*	*	0	0
7	0	H	*	*	0	0
8	0	I	*	*	0	0
9	0	J	*	*	0	0
10	0	K	*	*	0	0
11	84	1	0.1944	0.6624	0	0
12	1	2	0.2096	0.4304	0.1	0.05
13	2	3	0.2358	0.4842	0.3	0.2
14	3	4	0.0917	0.1883	0.35	0.25
15	4	5	0.2096	0.4304	0.22	0.1

Table A.6-1 (*Continued*)

Nr	Nd-i	Nd-j	Section resistance (Ω)	Section reactance (Ω)	End node real load (MW)	End node reactive load (MVAR)
16	5	6	0.0393	0.0807	1.1	0.8
17	6	7	0.0405	0.138	0.4	0.32
18	7	8	0.1048	0.2152	0.3	0.2
19	7	9	0.2358	0.4842	0.3	0.23
20	7	10	0.1048	0.2152	0.3	0.26
21	85	11	0.0786	0.1614	0	0
22	11	12	0.3406	0.6944	1.2	0.8
23	12	13	0.0262	0.0538	0.8	0.6
24	12	14	0.0786	0.1614	0.7	0.5
25	86	15	0.1134	0.3864	0	0
26	15	16	0.0524	0.1076	0.3	0.15
27	16	17	0.0524	0.1076	0.5	0.35
28	17	18	0.1572	0.3228	0.7	0.4
29	18	19	0.0393	0.0807	1.2	1
30	19	20	0.1703	0.3497	0.3	0.3
31	20	21	0.2358	0.4842	0.4	0.35
32	21	22	0.1572	0.3228	0.05	0.02
33	21	23	0.1965	0.4035	0.05	0.02
34	23	24	0.131	0.269	0.05	0.01
35	87	25	0.0567	0.1932	0.05	0.03
36	25	26	0.1048	0.2152	0.1	0.06
37	26	27	0.2489	0.5111	0.1	0.07
38	27	28	0.0486	0.1656	1.8	1.3
39	28	29	0.131	0.269	0.2	0.12
40	88	30	0.1965	0.396	0	0
41	30	31	0.131	0.269	1.8	1.6
42	31	32	0.131	0.269	0.2	0.15
43	32	33	0.0262	0.0538	0.2	0.1
44	33	34	0.1703	0.3497	0.8	0.6
45	34	35	0.0524	0.1076	0.1	0.06
46	35	36	0.4978	1.0222	0.1	0.06
47	36	37	0.0393	0.0807	0.02	0.01
48	37	38	0.0393	0.0807	0.02	0.01
49	38	39	0.0786	0.1614	0.02	0.01
50	39	40	0.2096	0.4304	0.02	0.01
51	38	41	0.1965	0.4035	0.2	0.16
52	41	42	0.2096	0.4304	0.05	0.03
53	89	43	0.0486	0.1656	0	0
54	43	44	0.0393	0.0807	0.03	0.02
55	44	45	0.131	0.269	0.8	0.7
56	45	46	0.2358	0.4842	0.2	0.15
57	90	47	0.243	0.828	0	0
58	47	48	0.0655	0.1345	0	0
59	48	49	0.0655	0.1345	0	0
60	49	50	0.0393	0.0807	0.2	0.16

Table A.6-1 (*Continued*)						
Nr	**Nd-i**	**Nd-j**	**Section resistance (Ω)**	**Section reactance (Ω)**	**End node real load (MW)**	**End node reactive load (MVAR)**
61	50	51	0.0786	0.1614	0.8	0.6
62	51	52	0.0393	0.0807	0.5	0.3
63	52	53	0.0786	0.1614	0.5	0.35
64	53	54	0.0524	0.1076	0.5	0.3
65	54	55	0.131	0.269	0.2	0.08
66	91	56	0.2268	0.7728	0	0
67	56	57	0.5371	1.1029	0.03	0.02
68	57	58	0.0524	0.1076	0.6	0.42
69	58	59	0.0405	0.138	0	0
70	59	60	0.0393	0.0807	0.02	0.01
71	60	61	0.0262	0.0538	0.02	0.01
72	61	62	0.1048	0.2152	0.2	0.13
73	62	63	0.2358	0.4842	0.3	0.24
74	63	64	0.0243	0.0828	0.3	0.2
75	92	65	0.0486	0.1656	0	0
76	65	66	0.1703	0.3497	0.05	0.03
77	66	67	0.1215	0.414	0	0
78	67	68	0.2187	0.7452	0.4	0.36
79	68	69	0.0486	0.1656	0	0
80	69	70	0.0729	0.2484	0	0
81	70	71	0.0567	0.1932	2	1.5
82	71	72	0.0262	0.0538	0.2	0.15
83	93	73	0.324	1.104	0	0
84	73	74	0.0324	0.1104	0	0
85	74	75	0.0567	0.1932	1.2	0.95
86	75	76	0.0486	0.1656	0.3	0.18
87	94	77	0.2511	0.8556	0	0
88	77	78	0.1296	0.4416	0.4	0.36
89	78	79	0.0486	0.1656	2	1.3
90	79	80	0.131	0.269	0.2	0.14
91	80	81	0.131	0.269	0.5	0.36
92	81	82	0.0917	0.1883	0.1	0.03
93	82	83	0.3144	0.6456	0.4	0.36
94	5	55	0.131	0.269		
95	7	60	0.131	0.269		
96	11	43	0.131	0.269		
97	12	72	0.3406	0.6994		
98	13	76	0.4585	0.9415		
99	14	18	0.5371	1.0824		
100	16	26	0.0917	0.1883		
101	20	83	0.0786	0.1614		
102	28	32	0.0524	0.1076		
103	29	39	0.0786	0.1614		
104	34	46	0.0262	0.0538		
105	40	42	0.1965	0.4035		
106	53	64	0.0393	0.0807		

Appendix B Results of the case studies

Nomenclature

Gen = Number of generations

#Lines = Number of correct lines in the best solution

Ni = Node (From)

Nj = Node (To)

B.1 Results of case 1 in chapter 4

Table B.1-1 Results of the simulation runs of case 1 in chapter 4 (Population size = 40)

Run	Gen	# Lines	Run	Gen	# Lines	Run	Gen	# Lines
0	19	6	10	20	6	20	20	9
1	16	7	11	20	8	21	15	7
2	17	7	12	16	7	22	20	6
3	20	9	13	20	8	23	19	8
4	20	6	14	20	7	24	20	9
5	20	9	15	20	6	25	20	7
6	20	7	16	20	8	26	20	7
7	20	8	17	20	9	27	20	7
8	20	8	18	19	9	28	16	8
9	18	8	19	20	8	29	20	8
Population size = 40 Sum = 227 Average = 7.566				Percentage = 84.07 %s (standard deviation) = 10.99				

Table B.1-2 Results of the simulation runs of case 1 in chapter 4 (Population size = 50)

Run	Gen	#Lines	Run	Gen	#Lines	Run	Gen	#Lines
0	20	9	10	20	8	20	18	7
1	18	7	11	15	7	21	20	9
2	19	8	12	20	8	22	20	7
3	20	9	13	20	8	23	20	9
4	15	8	14	20	8	24	20	8
5	20	9	15	20	9	25	20	9
6	20	8	16	20	8	26	20	8
7	20	8	17	20	9	27	18	8
8	20	8	18	20	9	28	14	7
9	20	9	19	20	9	29	20	9
Population size = 50 Sum = 247 Average = 8.23				Percentage = 91.48 %s (standard deviation) = 7.95219				

Table B.1-3 Results of the simulation runs of case 1 in chapter 4 (Population size = 60)

Run	Gen	#Lines	Run	Gen	#Lines	Run	Gen	#Lines
0	20	8	10	18	8	20	18	9
1	17	9	11	20	9	21	20	9
2	20	9	12	20	9	22	17	8
3	19	8	13	19	9	23	20	9
4	20	9	14	19	9	24	20	9
5	20	9	15	18	9	25	20	9
6	20	9	16	20	9	26	18	9
7	19	8	17	17	9	27	20	9
8	18	8	18	19	8	28	18	9
9	17	9	19	20	9	29	16	8
Population size = 60 Sum = 262 Average = 8.73				Percentage = 97.037 %s (standard deviation) = 4.9135				

Table B.1-4 Results of the simulation runs of case 1 in chapter 4 (Population size = 70)

Run	Gen	#Lines	Run	Gen	#Lines	Run	Gen	#Lines
0	19	8	10	16	9	20	19	9
1	20	8	11	20	9	21	20	9
2	20	8	12	20	9	22	19	8
3	20	9	13	20	9	23	20	8
4	19	8	14	20	9	24	20	9
5	20	9	15	19	9	25	20	9
6	20	9	16	18	9	26	20	9
7	19	9	17	20	9	27	20	9
8	20	9	18	20	8	28	20	9
9	20	9	19	20	8	29	17	8
Population size = 70 Sum = 261 Average = 8.7				Percentage = 96.66 %s (standard deviation) = 5.09175				

Table B.1-5 Results of the simulation runs of case 1 in chapter 4 (Population size = 80)

Run	Gen	#Lines	Run	Gen	#Lines	Run	Gen	#Lines
0	20	9	10	20	8	20	20	9
1	20	9	11	20	9	21	20	9
2	20	8	12	18	9	22	20	9
3	20	9	13	20	9	23	20	8
4	20	8	14	19	9	24	20	9
5	20	9	15	20	8	25	20	9
6	20	9	16	20	9	26	20	9
7	20	9	17	20	9	27	20	8
8	20	9	18	20	9	28	20	9
9	20	8	19	20	9	29	20	8
Population size = 80 Sum = 262 Average = 8.733				Percentage = 97.037 %s (standard deviation) = 4.91				

B.2 Results of case 2 in chapter 4

Table B.2-1 Power flows and voltage magnitudes of the optimum solution of case 2, chapter 4

Line Ni-Nj	Capacity (Amps.)	Current (Amps.)	Power flow (MVA3ϕ) Ni-Nj	Power flow (MVA3ϕ) Nj-Ni	Voltage kV_{L-L} Ni	Voltage kV_{L-L} Nj
0 – 1	*	614.788	35.1398	35.1397	33	32.9999
0 – 2	*	0	0	0	33	33
5 – 6	209	55.004	3.04547	2.99969	31.9668	31.4863
7 – 8	209	91.9233	5.10898	4.99937	32.0884	31.4
1 – 7	209	145.899	8.33922	8.10889	32.9999	32.0884
10 – 9	209	108.535	6.20357	5.99976	32.9999	31.9157
1 – 3	209	145.634	8.32406	8.04874	32.9999	31.9084
3 – 4	209	55.1681	3.04897	2.99963	31.9084	31.392
1 – 5	209	127.245	7.273	7.04531	32.9999	31.9668
1 – 10	*	196.011	11.2035	11.2035	32.9999	32.9999

B.3 Results of case 3 in chapter 4

Table B.3-1 Power flows and voltage magnitudes of the best solution of case 3, chapter 4

Line Ni-Nj	Capacity (Amps)	Current (Amps)	Power flow (MVA3ϕ) Ni-Nj	Power flow (MVA3ϕ) Nj-Ni	Voltage (kV_{L-L}) Ni	Voltage (kV_{L-L}) Nj
0 - 201	*	894.556	15.4942	15.4942	10	10
201 - 91	515	155.294	2.68976	2.6791	10	9.96035
91 - 1	515	153.96	2.6561	2.64691	9.96035	9.92588
91 - 86	255	1.3332	0.023	0.023	9.96035	9.96032
1 - 12	255	149.598	2.57191	2.57	9.92588	9.91852
3 - 11	255	146.396	2.51243	2.50861	9.90838	9.89333
11 - 14	255	141.378	2.42261	2.41704	9.89333	9.87058
14 - 10	255	136.757	2.33804	2.33146	9.87058	9.84278
12 - 3	255	146.396	2.515	2.51243	9.91852	9.90838
10 - 85	255	8.03636	0.137006	0.137	9.84278	9.84235
9 - 10	255	120.802	2.05765	2.05945	9.83415	9.84278
9 - 87	255	81.7441	1.39237	1.39043	9.83415	9.82045
152 - 140	255	10.8534	0.184007	0.184029	9.78834	9.78955
83 - 32	515	266.757	4.54903	4.54562	9.84559	9.83822
158 - 183	515	243.146	4.19446	4.17922	9.95976	9.92358
158 - 189	1030	329.519	5.68448	5.69875	9.95976	9.98477
166 - 174	515	227.065	3.89364	3.887	9.90025	9.88335

Table B.3-1 (*Continued*)						
Line Ni-Nj	**Capacity (Amps)**	**Current (Amps)**	**Power flow (MVA3ϕ) Ni-Nj**	**Power flow (MVA3ϕ) Nj-Ni**	**Voltage (kV_{L-L}) Ni**	**Voltage (kV_{L-L}) Nj**
174 - 180	515	222.391	3.807	3.80104	9.88335	9.86788
159 - 180	515	204.428	3.48968	3.49402	9.85561	9.86788
170 - 162	255	1.33818	0.023	0.023	9.92322	9.92335
162 - 183	255	1.97817	0.034	0.034001	9.92335	9.92358
159 - 128	255	181.52	3.09863	3.07874	9.85561	9.79236
128 - 140	255	57.2487	0.970987	0.970708	9.79236	9.78955
140 - 129	255	42.7978	0.725679	0.725452	9.78955	9.78649
201 - 189	1030	333.51	5.77655	5.76776	10	9.98477
153 - 133	255	33.7691	0.572548	0.57231	9.78885	9.78478
183 - 186	515	237.037	4.07422	4.07142	9.92358	9.91675
186 - 166	515	237.037	4.07142	4.06464	9.91675	9.90025
122 - 201	1030	405.753	7.02627	7.02784	9.99776	10
119 - 93	1030	405.753	6.99921	6.97594	9.95926	9.92615
119 - 122	1030	405.753	6.99921	7.02627	9.95926	9.99776
118 - 120	255	5.13042	0.088001	0.088	9.90311	9.90303
93 - 118	515	330.851	5.68819	5.67499	9.92615	9.90311
117 - 83	515	296.726	5.06411	5.06008	9.85342	9.84559
118 - 121	515	312.717	5.36394	5.35825	9.90311	9.8926
121 - 117	515	296.726	5.08425	5.06411	9.8926	9.85342
103 - 104	255	12.6686	0.21602	0.215999	9.84474	9.84378
83 - 103	515	17.7121	0.302046	0.30202	9.84559	9.84474
108 - 107	255	2.46296	0.042001	0.042	9.84547	9.84528
83 - 108	515	3.87035	0.066001	0.066001	9.84559	9.84547
128 - 153	255	65.4553	1.11018	1.10978	9.79236	9.78885
153 - 155	255	26.6139	0.451233	0.451173	9.78885	9.78754
133 - 138	255	28.6948	0.486311	0.48605	9.78478	9.77953
150 - 151	255	13.0933	0.222024	0.222004	9.7902	9.78932
158 - 198	255	9.80053	0.169067	0.169012	9.95976	9.95652
136 - 143	255	17.4105	0.295081	0.29502	9.78519	9.78317
78 - 82	255	8.28003	0.140033	0.139998	9.76422	9.76178
82 - 89	255	0.295717	0.005	0.005	9.76178	9.76173
156 - 132	255	22.7949	0.386242	0.386118	9.78278	9.77963
7 - 8	255	7.34234	0.124999	0.12501	9.82906	9.82992
126 - 123	255	3.20739	0.055	0.055001	9.90032	9.90043
161 - 182	255	13.8656	0.239	0.239034	9.95176	9.95318
61 - 63	255	4.6535	0.078999	0.079002	9.80127	9.80161
139 - 146	255	12.5223	0.212071	0.21202	9.77771	9.77536
151 - 134	255	6.42888	0.109005	0.108999	9.78932	9.78872
123 - 157	255	9.62214	0.165001	0.165021	9.90043	9.90164
19 - 50	255	53.6638	0.913908	0.913374	9.83242	9.82667
20 - 71	255	64.0177	1.08515	1.08347	9.78652	9.77137
106 - 102	255	4.63303	0.079002	0.079	9.84493	9.84462
71 - 72	255	32.3415	0.547363	0.547145	9.77137	9.76747
159 - 179	255	12.9476	0.221022	0.221006	9.85561	9.85492
160 - 184	255	14.8491	0.256078	0.256017	9.95661	9.95422
39 - 46	255	6.31373	0.106999	0.107006	9.78435	9.78505

Table B.3-1 (*Continued*)

Line Ni-Nj	Capacity (Amps)	Current (Amps)	Power flow (MVA3ϕ) Ni-Nj	Power flow (MVA3ϕ) Nj-Ni	Voltage (kV_{L-L}) Ni	Voltage (kV_{L-L}) Nj
160 - 171	255	12.5281	0.216052	0.216022	9.95661	9.95526
41 - 36	255	10.1515	0.172018	0.172004	9.78325	9.78241
73 - 74	255	15.9591	0.270082	0.270025	9.77073	9.76866
114 - 111	255	15.713	0.270025	0.270004	9.92165	9.92087
36 - 31	255	5.07593	0.086005	0.085999	9.78241	9.78175
131 - 152	255	6.90151	0.116999	0.117007	9.7876	9.78834
158 - 160	255	30.7404	0.530298	0.53013	9.95976	9.95661
62 - 60	255	9.03362	0.15302	0.153001	9.7797	9.77849
70 - 62	255	17.0033	0.288032	0.288018	9.78017	9.7797
51 - 55	255	19.8122	0.337128	0.337031	9.82432	9.82149
158 - 181	255	45.6589	0.787654	0.787203	9.95976	9.95405
76 - 77	255	16.4988	0.279069	0.27904	9.76561	9.76458
171 - 187	255	10.7883	0.186022	0.186	9.95526	9.95406
184 - 163	255	9.28106	0.160017	0.16	9.95422	9.95318
20 - 48	515	118.489	2.00847	2.01063	9.78652	9.79704
137 - 154	255	19.9499	0.338107	0.338196	9.78484	9.78743
37 - 40	255	0	0	0	9.78088	9.78088
143 - 147	255	10.5059	0.178021	0.177999	9.78317	9.78195
125 - 123	255	3.20739	0.055	0.055001	9.90032	9.90043
27 - 35	515	139.453	2.372	2.36998	9.82034	9.81197
24 - 34	255	12.6898	0.216042	0.215998	9.82933	9.82735
179 - 176	255	7.90936	0.135007	0.134999	9.85492	9.85439
172 - 178	255	6.67938	0.113999	0.11401	9.85384	9.85477
50 - 51	255	27.7457	0.472241	0.472127	9.82667	9.82432
35 - 44	515	131.509	2.23498	2.23239	9.81197	9.8006
97 - 93	255	33.1251	0.569413	0.569506	9.92452	9.92615
189 - 188	255	3.99004	0.069004	0.069002	9.98477	9.98442
168 - 161	255	1.33436	0.023	0.023	9.95163	9.95176
17 - 4	255	5.0524	0.085999	0.086006	9.82737	9.82812
198 - 196	255	8.46682	0.146012	0.146002	9.95652	9.95581
113 - 114	255	17.4587	0.300056	0.300025	9.92266	9.92165
100 - 95	255	16.8881	0.290032	0.290096	9.91526	9.91745
28 - 33	255	7.87724	0.134014	0.133999	9.82236	9.82125
40 - 42	255	7.96875	0.134998	0.135014	9.78088	9.78202
60 - 59	255	5.0778	0.086002	0.086	9.77849	9.77822
185 - 165	255	15.261	0.263054	0.263025	9.95176	9.95067
154 - 150	255	32.6913	0.554194	0.554351	9.78743	9.7902
95 - 116	255	24.3979	0.419095	0.419194	9.91745	9.91979
175 - 178	255	3.22229	0.055	0.055001	9.85451	9.85477
116 - 96	255	24.3979	0.419194	0.419245	9.91979	9.92099
93 - 110	255	12.8562	0.221031	0.221008	9.92615	9.92512
169 - 173	255	7.83386	0.135005	0.135	9.94975	9.9494
178 - 159	255	9.90167	0.169011	0.169026	9.85477	9.85561
101 - 99	255	10.8323	0.186001	0.186014	9.91369	9.91441
81 - 79	255	1.35964	0.023	0.023	9.76645	9.76657
25 - 52	255	23.7853	0.405026	0.405147	9.83137	9.83429

Table B.3-1 (*Continued*)						
Line Ni-Nj	**Capacity (Amps)**	**Current (Amps)**	**Power flow (MVA3ϕ) Ni-Nj**	**Power flow (MVA3ϕ) Nj-Ni**	**Voltage (kV_{L-L}) Ni**	**Voltage (kV_{L-L}) Nj**
132 - 139	255	14.8251	0.25112	0.25107	9.77963	9.77771
88 - 66	255	79.0386	1.34333	1.3419	9.81256	9.80214
67 - 92	255	1.35565	0.023	0.023	9.79537	9.79523
188 - 192	255	1.32998	0.023	0.023	9.98442	9.98438
146 - 141	255	10.0417	0.17002	0.169998	9.77536	9.77406
72 - 76	255	19.7504	0.334132	0.334068	9.76747	9.76561
165 - 169	255	12.8822	0.222025	0.222005	9.95067	9.94975
112 - 113	255	25.3137	0.435159	0.435055	9.92503	9.92266
188 - 197	255	2.66006	0.046002	0.046	9.98442	9.98406
72 - 79	255	10.0494	0.170014	0.169998	9.76747	9.76657
145 - 148	255	7.49604	0.126999	0.127005	9.78152	9.78202
93 - 112	255	27.7569	0.477213	0.477159	9.92615	9.92503
4 - 5	255	5.0524	0.086006	0.086009	9.82812	9.82843
53 - 23	255	17.9864	0.306096	0.306046	9.82547	9.82386
66 - 67	255	64.254	1.09089	1.09014	9.80214	9.79537
163 - 164	255	1.04412	0.018	0.018	9.95318	9.95314
77 - 78	255	13.6016	0.23004	0.230032	9.76458	9.76422
64 - 70	255	28.2194	0.47815	0.47803	9.78263	9.78017
65 - 67	255	52.7606	0.894753	0.89514	9.79113	9.79537
148 - 137	255	18.0617	0.306019	0.306108	9.78202	9.78484
56 - 148	255	7.26053	0.123007	0.123015	9.7814	9.78202
149 - 147	255	5.0759	0.085999	0.086	9.78181	9.78195
124 - 127	255	3.24662	0.055	0.054999	9.78068	9.78059
43 - 24	255	17.7412	0.302127	0.302041	9.83211	9.82933
74 - 80	255	12.7676	0.216026	0.215997	9.76866	9.76735
2 - 9	255	31.1321	0.530059	0.53028	9.83005	9.83415
138 - 142	255	20.7249	0.351052	0.350995	9.77953	9.77795
115 - 97	255	28.9947	0.498302	0.498413	9.92232	9.92452
199 - 118	255	13.0039	0.223025	0.223053	9.90189	9.90311
110 - 105	255	7.85353	0.135008	0.135	9.92512	9.92448
58 - 68	255	37.8978	0.642291	0.642226	9.78492	9.78394
94 - 111	255	7.85663	0.135	0.135004	9.92056	9.92087
21 - 32	255	44.3896	0.75623	0.756413	9.83585	9.83822
129 - 130	255	34.8336	0.590454	0.590313	9.78649	9.78416
54 - 22	515	144.506	2.46136	2.45953	9.83398	9.8267
55 - 26	255	11.8764	0.202032	0.202004	9.82149	9.82011
19 - 32	255	72.1683	1.22904	1.22977	9.83242	9.83822
182 - 181	255	21.6965	0.374034	0.374067	9.95318	9.95405
68 - 64	255	34.4752	0.584227	0.584149	9.78394	9.78263
180 - 177	255	12.6388	0.216019	0.215999	9.86788	9.86697
83 - 106	255	8.21032	0.140011	0.140002	9.84559	9.84493
23 - 28	255	12.9322	0.220047	0.220013	9.82386	9.82236
18 - 7	255	1.93839	0.033	0.033	9.82901	9.82906
16 - 17	255	0	0	0	9.82737	9.82737
99 - 100	255	11.8805	0.204014	0.204032	9.91441	9.91526
98 - 101	255	3.14489	0.054	0.054001	9.91349	9.91369

Table B.3-1 (***Continued***)

Line Ni-Nj	Capacity (Amps)	Current (Amps)	Power flow (MVA3φ) Ni-Nj	Power flow (MVA3φ) Nj-Ni	Voltage (kV_{L-L}) Ni	Voltage (kV_{L-L}) Nj
69 - 65	255	44.8591	0.760482	0.760754	9.78762	9.79113
46 - 20	255	14.2791	0.242005	0.242041	9.78505	9.78652
32 - 54	515	147.265	2.50944	2.50836	9.83822	9.83398
5 - 6	255	12.5127	0.213008	0.213021	9.82843	9.82904
63 - 66	255	9.71916	0.165001	0.16501	9.80161	9.80214
47 - 49	255	17.4114	0.295036	0.295083	9.78317	9.78473
181 - 185	255	20.5404	0.354136	0.354054	9.95405	9.95176
42 - 47	255	11.7461	0.199013	0.199037	9.78202	9.78317
50 - 53	255	17.9864	0.306134	0.306096	9.82667	9.82547
87 - 88	255	80.3919	1.36743	1.36633	9.82045	9.81256
6 - 2	255	15.8608	0.270021	0.270048	9.82904	9.83005
144 - 145	255	1.29853	0.022	0.022	9.78147	9.78152
29 - 25	255	15.8574	0.269998	0.270027	9.8303	9.83137
155 - 136	255	22.9567	0.389173	0.38908	9.78754	9.78519
197 - 195	255	1.33003	0.023	0.023	9.98406	9.984
45 - 49	255	22.721	0.385028	0.385068	9.78371	9.78473
20 - 49	255	40.1324	0.680275	0.68015	9.78652	9.78473
59 - 57	255	5.0778	0.086	0.085999	9.77822	9.77815
22 - 27	515	139.453	2.37353	2.372	9.8267	9.82034
196 - 190	255	8.46682	0.146002	0.146	9.95581	9.95569
56 - 124	255	6.49321	0.110007	0.109999	9.7814	9.78068
85 - 90	255	1.34919	0.023	0.023	9.84235	9.8422
52 - 21	255	31.7108	0.540146	0.540231	9.83429	9.83585
157 - 199	255	9.62214	0.165021	0.165025	9.90164	9.90189
26 - 30	255	5.05646	0.086005	0.085999	9.82011	9.81948
96 - 115	255	24.8635	0.427245	0.427302	9.92099	9.92232
58 - 69	255	41.1429	0.69729	0.697482	9.78492	9.78762
45 - 41	255	14.7546	0.250029	0.250018	9.78371	9.78325
1 - 13	255	4.3625	0.075001	0.075	9.92588	9.92579
128 - 150	255	53.7458	0.911575	0.911374	9.79236	9.7902
130 - 156	255	30.7621	0.521314	0.521241	9.78416	9.78278
48 - 44	515	123.557	2.09663	2.09739	9.79704	9.8006
67 - 84	255	3.41859	0.058	0.057999	9.79537	9.79525
8 - 2	255	7.34234	0.12501	0.125012	9.82992	9.83005
71 - 73	255	23.7589	0.402107	0.402081	9.77137	9.77073
137 - 193	255	1.35709	0.023	0.023	9.78484	9.78482
134 - 194	255	1.35655	0.023	0.023	9.78872	9.7887
19 - 43	255	17.7412	0.302136	0.302127	9.83242	9.83211
199 - 200	255	3.38181	0.058	0.058	9.90189	9.90186
135 - 131	255	0	0	0	9.7876	9.7876
48 - 38	515	0	0	0	9.79704	9.79704
80 - 75	515	0	0	0	9.76735	9.76735
198 - 191	255	1.33371	0.023	0.023	9.95652	9.9565
167 - 175	255	0	0	0	9.85451	9.85451
109 - 83	255	0	0	0	9.84559	9.84559
13 - 15	255	0	0	0	9.92579	9.92579

B.4 Results of case 2 in chapter 6

Table B.4-1 Power flows and voltage magnitudes of the original system of case 2, chapter 6

Line Ni-Nj	Current (Amps)	Power flow (MVA3ϕ) Ni-Nj	Power flow (MVA3ϕ) Nj-Ni	Voltage (kVL-L) Ni	Voltage (kVL-L) Nj
A - 1	224.341	4.4297	4.34561	11.4	11.1836
1 - 2	224.335	4.3455	4.28124	11.1836	11.0182
2 - 3	218.56	4.17101	4.1025	11.0182	10.8372
3 - 4	199.372	3.74234	3.72018	10.8372	10.7731
4 - 5	176.328	3.29019	3.25063	10.7731	10.6435
5 - 6	163.52	3.01452	3.0081	10.6435	10.6209
6 - 7	89.6354	1.64892	1.64611	10.6209	10.6028
7 - 8	19.6326	0.360543	0.360304	10.6028	10.5957
7 - 9	20.6044	0.37839	0.377776	10.6028	10.5856
7 - 10	21.6214	0.397066	0.396757	10.6028	10.5945
B - 11	170.994	3.37635	3.36248	11.4	11.3532
11 - 12	170.994	3.36247	3.30282	11.3532	11.1518
12 - 13	51.7932	1.00041	0.999979	11.1518	11.147
12 - 14	44.5838	0.861154	0.860213	11.1518	11.1396
C - 15	229.071	4.5231	4.47227	11.4	11.2719
15 - 16	229.071	4.47226	4.45553	11.2719	11.2297
16 - 17	212.111	4.12565	4.11122	11.2297	11.1904
17 - 18	180.652	3.50147	3.47	11.1904	11.0899
18 - 19	139.173	2.67327	2.66851	11.0899	11.0701
19 - 20	57.7098	1.10653	1.10299	11.0701	11.0348
20 - 21	35.6774	0.681893	0.680077	11.0348	11.0054
21 - 22	2.82516	0.053853	0.053846	11.0054	11.0041
21 - 23	5.47814	0.104423	0.104396	11.0054	11.0025
23 - 24	2.67557	0.050988	0.050984	11.0025	11.0016
D - 25	142.022	2.80428	2.79467	11.4	11.3609
25 - 26	139.069	2.73656	2.72434	11.3609	11.3102
26 - 27	133.136	2.60811	2.58152	11.3102	11.1949
27 - 28	126.84	2.45944	2.4529	11.1949	11.1651
28 - 29	12.0662	0.233343	0.233233	11.1651	11.1599
E - 30	234.952	4.63922	4.57265	11.4	11.2364
30 - 31	234.951	4.57263	4.52785	11.2364	11.1264
31 - 32	110.369	2.12698	2.11742	11.1264	11.0764
32 - 33	97.347	1.86759	1.8661	11.0764	11.0676
33 - 34	85.8143	1.64503	1.63746	11.0676	11.0167
34 - 35	33.4438	0.638157	0.637811	11.0167	11.0107
35 - 36	27.337	0.521348	0.519139	11.0107	10.9641
36 - 37	21.2079	0.402745	0.402639	10.9641	10.9612
37 - 38	20.0434	0.380532	0.380437	10.9612	10.9585
38 - 39	2.35611	0.04472	0.044718	10.9585	10.9579
39 - 40	1.17809	0.02236	0.022358	10.9579	10.9571
38 - 41	16.5601	0.31432	0.31399	10.9585	10.947

Table B.4-1 (*Continued*)					
Line Ni-Nj	**Current (Amps)**	**Power flow (MVA3ϕ) Ni-Nj**	**Power flow (MVA3ϕ) Nj-Ni**	**Voltage (kVL-L) Ni**	**Voltage (kVL-L) Nj**
41 - 42	3.07556	0.058315	0.058303	10.947	10.9448
F - 43	68.6421	1.35536	1.35333	11.4	11.3829
43 - 44	68.6421	1.35333	1.35216	11.3829	11.3731
44 - 45	66.8241	1.31635	1.31268	11.3731	11.3414
45 - 46	12.7385	0.250233	0.25	11.3414	11.3308
G - 47	169.288	3.34266	3.28519	11.4	11.204
47 - 48	169.287	3.28516	3.27403	11.204	11.166
48 - 49	169.286	3.27402	3.26291	11.166	11.1281
49 - 50	169.286	3.2629	3.25623	11.1281	11.1054
50 - 51	156.024	3.00114	2.98988	11.1054	11.0637
51 - 52	104.006	1.99306	1.99061	11.0637	11.0501
52 - 53	73.5429	1.40756	1.40511	11.0501	11.0309
53 - 54	41.7159	0.797026	0.796517	11.0309	11.0238
54 - 55	11.2848	0.215469	0.215385	11.0238	11.0195
H - 56	93.6184	1.84853	1.83179	11.4	11.2968
56 - 57	93.6179	1.83178	1.80347	11.2968	11.1222
57 - 58	91.7463	1.76741	1.76477	11.1222	11.1055
58 - 59	53.6719	1.03239	1.03142	11.1055	11.095
59 - 60	53.6725	1.03143	1.03075	11.095	11.0877
60 - 61	52.5217	1.00865	1.00822	11.0877	11.0829
61 - 62	51.371	0.986128	0.984462	11.0829	11.0642
62 - 63	38.9402	0.74624	0.744072	11.0642	11.0321
63 - 64	18.8714	0.360596	0.360525	11.0321	11.0299
I - 65	178.005	3.51477	3.50121	11.4	11.356
65 - 66	178.005	3.50121	3.46766	11.356	11.2472
66 - 67	175.039	3.40988	3.37725	11.2472	11.1396
67 - 68	175.032	3.37713	3.31878	11.1396	10.9471
68 - 69	146.727	2.78207	2.77309	10.9471	10.9118
69 - 70	146.724	2.77305	2.75961	10.9118	10.8589
70 - 71	146.722	2.75957	2.74914	10.8589	10.8178
71 - 72	13.3406	0.249963	0.249935	10.8178	10.8166
J - 73	96.8007	1.91137	1.88527	11.4	11.2443
73 - 74	96.8003	1.88526	1.88266	11.2443	11.2288
74 - 75	96.8003	1.88266	1.87811	11.2288	11.2017
75 - 76	18.0381	0.349975	0.349851	11.2017	11.1978
K - 77	233.74	4.61529	4.49848	11.4	11.1115
77 - 78	233.728	4.49824	4.43863	11.1115	10.9642
78 - 79	205.591	3.90429	3.88729	10.9642	10.9165
79 - 80	79.5699	1.5045	1.49945	10.9165	10.8798
80 - 81	66.6238	1.25549	1.25194	10.8798	10.8491
81 - 82	33.8568	0.63621	0.635563	10.8491	10.8381
82 - 83	28.7489	0.539676	0.538025	10.8381	10.8049

Table B.4-2 Power flows and voltage magnitudes of the solution of case 2, chapter 6

Line Ni-Nj	Current (Amps)	Power flow (MVA3ϕ) Ni-Nj	Power flow (MVA3ϕ) Nj-Ni	Voltage (kVL-L) Ni	Voltage (kVL-L) Nj
51 - 52	145.7	2.76889	2.76396	10.972	10.9525
78 - 79	175.935	3.35979	3.34758	11.0255	10.9854
77 - 78	203.842	3.93743	3.89273	11.1521	11.0255
16 - 17	241.338	4.68436	4.66556	11.2063	11.1614
80 - 81	37.7407	0.716626	0.71553	10.9628	10.9461
7 - 10	21.0168	0.397226	0.396935	10.9122	10.9041
F - 43	148.171	2.92569	2.91641	11.4	11.3638
56 - 57	129.813	2.53017	2.4748	11.2531	11.0068
45 - 46	92.0295	1.7972	1.7851	11.2748	11.1989
E - 30	147.859	2.91953	2.89289	11.4	11.296
62 - 63	12.6323	0.238496	0.238717	10.9003	10.9104
47 - 48	211.636	4.08734	4.06973	11.1504	11.1024
1 - 2	142.482	2.78087	2.75567	11.2683	11.1662
7 - 9	20.0283	0.378543	0.377963	10.9122	10.8954
69 - 70	132.797	2.51896	2.50795	10.9514	10.9036
2 - 3	136.746	2.64474	2.61863	11.1662	11.056
20 - 21	35.8988	0.681888	0.680049	10.9666	10.937
49 - 50	211.633	4.0521	4.04157	11.0544	11.0257
57 - 58	127.922	2.43876	2.43353	11.0068	10.9832
17 - 18	209.816	4.05617	4.01347	11.1614	11.0439
58 - 59	89.4558	1.70176	1.69897	10.9832	10.9652
H - 56	129.814	2.56323	2.5302	11.4	11.2531
44 - 45	146.342	2.87515	2.85784	11.3431	11.2748
21 - 22	2.84264	0.05385	0.053843	10.937	10.9357
21 - 23	5.51201	0.104417	0.104389	10.937	10.9341
15 - 16	258.295	5.03489	5.01348	11.2542	11.2063
30 - 31	147.859	2.89289	2.87495	11.296	11.2259
7 - 8	19.0854	0.360722	0.360496	10.9122	10.9053
3 - 4	117.918	2.25808	2.25055	11.056	11.0191
74 - 75	150.316	2.89858	2.88761	11.1332	11.091
53 - 64	51.9772	0.983226	0.982587	10.9215	10.9144
52 - 53	115.006	2.1817	2.17552	10.9525	10.9215
12 - 72	12.8932	0.250343	0.249997	11.2102	11.1947
63 - 64	32.9209	0.62212	0.622344	10.9104	10.9144
G - 47	211.639	4.17888	4.08739	11.4	11.1504
79 - 80	50.5909	0.962609	0.960628	10.9854	10.9628
70 - 71	132.796	2.50792	2.49938	10.9036	10.8664
11 - 12	131.496	2.58829	2.5532	11.3642	11.2102
18 - 19	168.23	3.21799	3.21102	11.0439	11.0199
50 - 51	198.253	3.78603	3.7676	11.0257	10.972
7 - 60	87.1464	1.6471	1.65322	10.9122	10.9527
27 - 28	132.246	2.5625	2.55541	11.1872	11.1563
67 - 68	161.01	3.11239	3.06302	11.1604	10.9834

Table B.4-2 (*Continued*)					
Line Ni-Nj	**Current (Amps)**	**Power flow (MVA3ϕ) Ni-Nj**	**Power flow (MVA3ϕ) Nj-Ni**	**Voltage (kVL-L) Ni**	**Voltage (kVL-L) Nj**
K - 77	203.849	4.02508	3.93756	11.4	11.1521
23 - 24	2.6921	0.050984	0.05098	10.9341	10.9333
A - 1	142.483	2.81338	2.78089	11.4	11.2683
48 - 49	211.635	4.06971	4.05213	11.1024	11.0544
65 - 66	163.977	3.2263	3.19784	11.3595	11.2594
20 - 83	28.3499	0.5385	0.538099	10.9666	10.9585
43 - 44	148.171	2.91641	2.91109	11.3638	11.3431
66 - 67	161.015	3.14007	3.11248	11.2594	11.1604
40 - 42	3.02123	0.058318	0.058308	11.1445	11.1426
34 - 35	27.5715	0.534459	0.534222	11.1916	11.1867
34 - 46	79.1421	1.53413	1.53512	11.1916	11.1989
19 - 20	86.3996	1.64912	1.64114	11.0199	10.9666
36 - 37	15.5511	0.300314	0.300256	11.1494	11.1473
68 - 69	132.799	2.52634	2.51899	10.9834	10.9514
75 - 76	70.6921	1.35801	1.35598	11.091	11.0744
C - 15	258.296	5.10015	5.03491	11.4	11.2542
53 - 54	30.8345	0.583282	0.582996	10.9215	10.9161
37 - 38	14.4133	0.278287	0.278237	11.1473	11.1453
73 - 74	150.317	2.90487	2.89859	11.1573	11.1332
4 - 5	95.4004	1.82078	1.80959	11.0191	10.9514
60 - 61	1.17852	0.022357	0.022357	10.9527	10.9526
I - 65	163.977	3.23779	3.2263	11.4	11.3595
J - 73	150.319	2.9681	2.90492	11.4	11.1573
35 - 36	21.5666	0.417871	0.41648	11.1867	11.1494
D - 25	147.437	2.91121	2.90088	11.4	11.3596
31 - 32	24.2834	0.472164	0.471715	11.2259	11.2153
5 - 6	71.7617	1.3612	1.35998	10.9514	10.9415
13 - 76	52.5322	0.999921	1.00764	10.9896	11.0744
26 - 27	138.546	2.71332	2.68459	11.307	11.1872
B - 11	131.496	2.59644	2.58829	11.4	11.3642
59 - 60	89.4584	1.69902	1.69708	10.9652	10.9527
25 - 26	144.483	2.84275	2.82958	11.3596	11.307
12 - 14	44.3515	0.861155	0.860224	11.2102	11.198
81 - 82	5.50632	0.104395	0.104382	10.9461	10.9447
28 - 29	17.41	0.336417	0.336191	11.1563	11.1487
32 - 33	11.5119	0.223624	0.223605	11.2153	11.2143
38 - 41	13.2788	0.256335	0.256121	11.1453	11.1359
5 - 55	11.3587	0.215456	0.21537	10.9514	10.947
39 - 40	4.17713	0.080651	0.080631	11.1474	11.1445
29 - 39	5.33381	0.102997	0.102984	11.1487	11.1474