Multithreading for High Performance Finance Risk Analysis

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by

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Declaration

The work described in this thesis has not been previously submitted for a degree in this or any other universities and unless otherwise referenced it is the author's own work.

Statement of Copyright

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Abstract

With the increasing of risks in the financial market, the models of risk management are developing quickly. The standard of the accuracy and effect of the models is improved continuously.

This thesis investigates Value at Risk (VaR) which is an important method for measuring the market risk. It reviews the three methods which can be used to quantify VaR. These methods are parameter method, historical data processing method and Monte Carlo simulation method.

Monte Carlo simulation has been widely employed for finance risk analysis. One challenge in Monte Carlo simulation is its computation complexity. For this purpose, this thesis researches into multithreading technique for high performance.

Keywords: Finance Risk Analysis, Value at Risk, Monte Carlo, Multithreading

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Chapter 1 Introduction

1.1 Background

Economic risk analysis of the project investment is the basis of project plan-selecting and scientific decision-making, which is based on the analysis of the calculation of the cost of inputs required and the expected benefits from the economic point of view, to assess the economic rationality of investment projects. The economic benefits of the investment projects are often affected by many random factors in the economic risk analysis should be considered.

Generally, the risk refers to the uncertainty or volatility of future results, such as the volatility or uncertainty of income, assets, or the value of the debt. Uncertainty is an objective facet in real life. It reflects a variety of possible outcomes of a specific event. The root cause of a variety of risks, including financial risks, uncertainty. The greater the uncertainty, the greater the risk will be. Due to the impact of economic globalization and financial integration, the development of modern financial theory and information technology and financial innovation and other factors, the global financial markets, the rapid development of financial market showing unprecedented volatility, industrial and commercial enterprises, financial institutions face with the increasingly serious risk.

In the early 1970s, the field of international finance unprecedented shocked. In 1973, the Bretton Woods system, which had been the international monetary and financial order for ten years, totally collapsed. Then the fixed exchange rate system was replaced by a floating exchange rate system. In the late 1970s, the major western countries had relaxed or even abolished the exchange controls and interest rate controls, leading to the boom of financial liberalization in the entire world. In this boom, financial risks were increasing, especially foreign currency risk and interest rate risk.

In recent years, the worldwide financial markets are perilous and storm occurs repeatedly. French Lyonnais credit bank engaged in real estate and other speculative industries which brought huge losses. British Barings bank collapsed because of the wanton operations of a trader. The devaluation of the Thai baht triggered the Asian financial crisis. The U.S. subprime mortgage crisis caused the financial crisis sweeping the world. All of these phenomena show the importance of strengthening the control and management of asset risk. Furthermore, the foundation and core of risk control and management is quantitative risk analysis and assessment.

But in the current situation, people are not helpless, there are a lot of evidences shows that financial crisis and economic risk can be predicted and controlled. The method is risk management and control. The basic and core of risk management and control is quantitative analysis. If you can finish the quantitative e analysis work correctly, you will get a big benefit. It is easier said than done. With the growing scale, more and more complex financial markets and the changing theory of financial, the quantitative analysis is more difficult than before. So, what the most important is how to finish quantities analysis work. There are a lot of risk measurement method which have different mathematical models. The risk predictor will select the appropriate mathematical models based on the current state. And then, he can help the manager make the best decision.

It does not have a long history. For the management and control, people create a very important value. Basel Agreement, signed in 1988, formulated the core adequacy ratio of the bank cannot be less than 8%. After updated and improved in several years, the senior managers can understand roughly the risk of a economic activity. But just understanding roughly is not enough, because it is difficult to distinguish the threshold.

Different risk measurement methods use different mathematical models, thereby affecting the choice of investment strategy. In 1988, for the management and control of financial risks, Basel Agreement formulated minimum core capital adequacy ratio of the bank should not be less than 8%. The agreement has been repeatedly modified and improved over time. In the early 1990s, people began to emphasize the risk report in the practice of risk managements, therefore the senior managers can timely and accurately understand the risks faced by the various investments based on risk-adjusted. The most influential risk report is a new tool for market risk measure proposed in July 1993 by the Group of Thirty – Value at Risk (VaR) [1]. VaR reflects in an asset or portfolio holding period, the maximum loss potential under a given confidence level. The measurement method is a major innovation, and it is also the effective complement to the Basel Agreement. As a comprehensive quantifying method of complex portfolio risk, VaR caused widespread concern of the financial sector. Initially, the report of the risk is just for derivative financial risk management. Subsequently, due to its simple definition and intuitive risk description, it has been widely used in the financial regulatory authorities and financial institution and many financial institutions have developed internal model system to support risk reporting. In recent years, VaR has emerged into a major approach for financial risk measurement.

In general, there are two major types of approaches to measure the VaR. The first one is using local methods to calculate VaR. These methods measure the risk by valuing the portfolio once at the initial position. Then it use local derivatives to forecast possible movements. Another type of VaR is full-valuation methods. That means these methods measure all the factors of a period of data to re-price portfolio. Historical Simulation approach and Monte Carlo approach is the main technique of these methods.

1.2 Motivations

This thesis focuses on Monte Carlo simulation for computing VaR. In fact, Monte Carlo Simulation employs the numerical methods, statistic and possibility theory. It generates a large number of random numbers for trails. Monte Carlo simulation is a competitive and effective approach to measure financial risk when enough historical data of portfolio is given. However, Monte Carlo Simulation normally involves a large amount of computation. For this purpose, this thesis researches into multithreading technique to speed up the computation process.

1.3 Major Contributions

The major contributions of the thesis are:

- It gives a comprehensive review on VaR approaches to the problem of finance risk management.
- It employs credibility theory in Monte Carlo Simulation.
- It researches into multithreading technique for high performance, to speed up Monte Carlo Simulation in computation.
- Both experimental and simulation results are presented and analysed.

1.4 Thesis Structure

The thesis is organized as follows:

Chapter 2 defines Value at Risk (VaR) and described its methods which are mainly parameter method, historical data processing method and Monte Carlo simulation method.

Chapter 3 introduces credibility theory and discusses how it can be used to deal with

VaR.

Chapter 4 researches into multithreading for speeded up performance in computation.

Chapter 5 evaluates the Monte Carlo simulation using both experimental and simulation tests. The performance evaluation results are presented and analysed.

Chapter 6 concludes the report and points out some future work

Chapter 2 Literature Review

2.1 Introduction of VaR

Financial activity becomes more and more complicated, and the harm of risk of finance becomes more and more serious. Especially, after 1870s, with the crash of the Bretton Woods system, the fixed exchange rate system based on dollars is replaced by a floating exchange rate system. Exchange rate varies in a large area frequently. Existent risk measurements, such as nominal amount, sensitivity method and fluctuation method, cannot satisfy the requirements of complicatedly financial market. In order to meet the daily requirement of "4.15 Report" required by Weatherstone, the president of JP Morgan Company, his risk managers introduce the Value at Risk (VaR) method that is quickly and widely used in the field of risk evaluation and management and becomes the mainstream of risk measure in financial market.

The Basel Committee on Banking Supervision, Securities and Exchange Commission and International Swaps and Derivatives Association consider VaR as a basic parameter for evaluation of internal risk capital, internal risk control, and risk report. In 1996, Basel Committee on Banking Supervision announced recommended VaR as an internal model to evaluate risk. In 1995, Securities and Exchange Commission recommended that companies should report their financial information via three methods including VaR. In 1995, International Swaps and Derivatives Association also pointed out that VaR is very important for risk measurement.

2.1.1 Definition of VaR

VaR is the abbreviation of Value at Risk. It means the maximum expected loss within a given time interval in a given market condition and the confidence level. As a measurement of using of probability theory and mathematical statistics method for evaluating the risk, this method is easy to understand and operate.

VaR can be defined as $\Pr ob\Delta t(\Delta P > VaR) = 1 - c$, where ΔP means the portfolio loss within Δt; c means The confidence level of risk measurement; VaR means the risk value under current confidence level.

For every given function of risk in a market, we can compute the value of VaR according to c, which can be used to intuitively express the level of risk. If the confidence level c=95% is given, we can solve （1-c）=5%, and further calculate VaR. For example, assuming that T-notes are 1.7 million dollars in a month, the maximum loss of portfolio will be $1.7 million in this month under a normal situation. This evaluation of the risk will give great help for advanced managers and stockholders for their decisions.

The greatest advantage of VaR is that it is comprehensive. It can integrate all risk factors into a value that presents the risk level. Therefore, it is easy to understand and utilise to help ones make decisions.

2.1.2 Foundation of VaR

The measurement of VaR is based on two parameters, holding period and confident level. Different models of VaR can be transformed into each other according to these two parameters.

Holding period is the time range for calculating VaR. It is decided by the finical company. Its usual value range is one day to one month. VaR will increase with the holding period. Generally, the shorter of the holding period, the better VaR is.

Parameter c, the measurement accuracy of VaR, is decided by the requirement of different financial company. Generally, c should be set to a low confidence level when it is used as an indicator of effectiveness of VaR; it should be set to a high confidence level to stratify the requirement of internal capital risk controlling and external monitoring.

2.2 Classical Methods of VaR

The classic VaR methods describe the possible maximum loss under the normal fluctuations in a market. Their essentials are to find out the distribution of the profits or losses of portfolios. The classical model of VaR includes three modules. The first modular is mapping. It expresses each position return of a portfolio as a function of factors of a market. The second one is fluctuation model of factors of a market. It is used to predict fluctuation of market factors. The third one is the evaluating model. It is used to measure values and distributions of portfolio according to the second one. The last two modules are the key components of VaR model. According to their differences, VaR model can be classified into different categories as shown in Table 2.1.

Table 2-1: Categories of VaR models [1].

Evaluation model / Fluctuation model	Analysis method	Simulation method
Historical data simulation	----	Evaluating portfolio by historical data
Monte Carlo simulation	----	Determining stochastic process by statistical parameters
Scenario analyses	Using single tool to analyse sensitivity	Limited scenarios
Risk Metrics model	Standard mapping with Covariance Matrix	Covariance Matrix Monte Carlo Method
GARCH model	Standard mapping with Covariance Matrix	Covariance Matrix Monte Carlo Method
Latent fluctuation	Standard mapping with Covariance Matrix	Covariance Matrix Monte Carlo Method
Random fluctuation	Standard mapping with Covariance Matrix	Covariance Matrix Monte Carlo Method

The VaR methods based on fluctuations can be classified into two categories, the analysis method (also known as local analysis method), and the simulation methods (also known as global simulation method).

The analysis method calculates profit and loss of portfolio mainly according to sensitivity of value of financial tools w.r.t market factors. It can be defined:

$\Delta V = f(s, \Delta r)$,

where, ΔV denotes the fluctuation range, s denotes sensitivity, Δr denotes the fluctuation of market factors.

The simplest method can be defined as $\Delta V = s * \Delta r$.

The sensitivity will close to practical situations just as the market fluctuation is small. For this reason, it is called local model. It simplifies VaR model using distribution and sensitivity. It is inefficient for non-linear problems in the practical financial market and will generate evaluating errors and risk.

The simulation method is based on the different simulating scenarios to evaluate profit and loss of portfolio. It is valid when the market factors fluctuate in a large range. Therefore, it is called a global simulation method. It can deal with non-linear problems well. Many experiments validate this conclusion.

Next, the thesis introduces classical simulation methods including historical data simulation method, parameter analysis method and Monte Carlo simulation method。

2.2.1 Historical Data Simulation Method

2.2.1.1 Principle

The simulation method based on historical data is the simplest VaR method. It uses historical data to determine the distribution functions of profit and loss of portfolio and uses quartile to estimates VaR under certain confidence level. In another words, it utilizes empirical distribution of returns ordered in time series $\{y_{t-j}\}_{j=1}^{t-1}$ to calculate

VaRt(α) where α denotes a confidence level.

2.2.1.2 Method and Process

This method uses historical data as a training set to train the value of factors of a market under a market model. It includes four steps.

- Mapping. It first identifies basic market factors, collects historical data varying from every day's data in 3-5 years, and uses market factors to calculate mark to market value.
- According to the variations of market factors in the past *M* periods of time series, the method calculates the profit and loss of $(N+1)^{th}$ period. If the calculated value is similar to the practice value of the $(N+1)^{th}$ period, it will be used to calculate the multiple possible situations.
- According to above simulated results and a price formula of Securities, it calculates multiple mark-to-market values of portfolios and produces their distributions.
- According to the profit distributions, it calculates the value of VaR under a confident level.

2.2.1.3 Discussion and Improvement

The simulation method based on historical data does not need any parameter. It can present the return distribution without limiting the sample data or parameters into a specific distribution. Therefore, it is very useful for these distributions that are different from normal one. However, it has some difficulties in practice. If the sample data is small, VaR will become inexact. If the sample data is large, it will destroy independent and identical distribution. Therefore, VaR will become inexact also. In order to answer this question, Butler and Schachter introduce kernel estimation into the model [2], so that VaR can be calculated based on continuous differentiability, and improve its accurate.

2.2.2 Parameter Analysis Method

2.2.2.1 Principle

The parameter analysis method is the most commonly used method. It uses the relation between market factors and value functions of portfolios, and the distribution of market factors to simplify the calculation of VaR.

2.2.2.2 Classifications and Comparison

According to the difference of portfolio value functions, parameter analysis methods can be divided into delta-model and gamma-model. The first-order approximation function of the value function of portfolio is adopted in the methods of delta model. Different models have different distributions of market factors. For example, delta-normal distribution model assumes that the distribution of market factors follows multivariate normal distribution; Delta-GARCH model uses GARCH model to describe market factors. The second-order approximation function of the value function of portfolio is adopted in the methods of Gamma model. The gamma-normal distribution model assumes that the distribution of market factors follows a multivariate normal distribution; Delta-GARCH model uses GARCH model to describe market factors. The methods of Delta model use a linear method to calculate VaR. It simplifies the calculation of VaR, but it cannot predict the non-linear risk.

2.2.2.3 Evaluation and Simulation

The parameter analysis methods are easy to get VaR since it only uses the first or second order Taylor expansion to estimate portfolio value. Since the market factors do not follow a normal distribution in practical situations, these methods cannot reach a high accuracy.

In 2000, Wang C. and Li G. introduced "Artificial neural networks" [3] method into this model in the proceeding of the 7th APFA. This method was widely used since its simplicity, parallel computing, global optimization, self-adaption and strong robustness. It also overcomes the incorrect assumption of normal distribution and improves its accuracy.

2.2.3 Monte Carlo Simulation Method

2.2.3.1 Principle

The Monte Carlo simulation method (so-called random simulation method) is the most promising method. Its idea is to model a stochastic process to make its parameters equal to the desired answer of a given question. Next step is to calculate the statistical characteristics of the parameters within the stochastic process. Finally, it gives the approximate values of the questions.

2.2.3.2 Method and Process

The Monte Carlo simulation method mainly includes three steps as shown as fellows:

- Scenario generation. It first chooses the distribution of market factors to simulate the future situations.
- Portfolio estimation. It utilizes the scenarios and pricing formulas to estimate portfolio values and their variations.
- Estimating VaR. it calculates VaR according to the distribution of variations of portfolio values under a confidence level.

2.2.3.3 Discussion and Improvement

The advance of Monte Carlos simulation method is that it can simulate all kinds of situations, such as non-linear questions, large fluctuations and fat tail problems. Its disadvantages lie on its heavy calculation load. It also relies on the specific stochastic process and the selected historical data. Therefore, it has some modelling risk. The cluster effect in the generation of pseudo-random number would decrease its efficiency.

Many improvements have been made in recent years, including antithetic variants [4], control variants [5]， moment matching methods [6], stratified sampling [7]， importance sampling [8], conditional Monte Carlo simulation [9]. These methods try to reduce the coefficients of errors. The Quasi-Monte Carlo method [10] uses the pre-selected sequence to replace the random sequence to overcome the cluster effect. The scenario Monte Carlo [11] method first finds out the mainly scenarios and then uses multinomial distribution to disperse the multivariate normal distribution of market factors, and finally generate limited mainly scenarios to improve its efficiency.

The Markov Monte Carlo simulation method [12] introduces Markov process into the model and makes it possible for dynamic simulation. It overcomes the statics of classic methods.

These three VaR methods are very different with each other and have their own characteristics, which have been shown in Table 2.2.

Table 2-2: The Comparison of VaR methods.

	Simulation Method Based on Historical Data	Parameter Analysis Method	Monte Carlo Simulation Method
Data collection	Difficult	Easy	Easy
Implementation	Moderate	Easy	Difficult
Processing speed	High	High	Low in complicated situations
Explanation to stakeholders	Easy	Moderate	Difficult
Non-linear	Unsuitable	Unsuitable excepting specific cases	Unsuitable excepting specific cases
Inspection for other assumption	N/A	Enabled for standard deviation and correlation coefficient	Suitable for all cases

2.3 Extreme VaR Method

The classical VaR methods can accurately measure VaR of portfolio in normal situations of financial market. But in many extreme situations, the fluctuations of market do not follow the normal distribution. They have the characteristics of fat tail which is valid for classical VaR methods. Therefore, the pressure test and extreme value theory are introduced in this sub-section.

2.3.1 Pressure Test

The pressure test [13] is a method to measure the loss of portfolio in extreme situations of financial market. It includes scenario analysis and system pressure test.

2.3.1.1 Scenario Analysis

The scenario analysis is used to analyse the affections of some special scenarios w.r.t the profit and loss of portfolio, and gives loss of portfolio in some scenarios. Although it cannot give probability of loss, it could complement with classical VaR methods.

Scenario analysis includes two steps, scenario generation and scenario evaluation.

Step 1: Scenario generation.

It aims to generate extreme scenarios in financial markets. It includes three methods.

- Historical data simulation method.。It uses historical data of extreme scenarios as a sample to generate the future ones that include single extreme scenarios and secular bear market one.
- Classically extreme scenario simulation method. It simulates their variations of market factors, such as interest rates, exchange rates, stock prices or commodity prices, to generate extreme scenarios.
- Special event assumption method. It assumes there will be a special event, such as natural disaster or political event and analyse what effect will bring by these

events to financial markets.

Step 2: Scenario evaluation.

It is used to analyse effect and consequent results after the generated extreme scenarios. It includes two kinds of methods.

- Sensitivity based scenario evaluation. It uses the sensitivity of positions w.r.t market factors to analyse the effect of extreme variations of market factors w.r.t positions. This method is valid for financial assets with simple structure, but it cannot accurately measure the complicated financial assets.
- Global scenario evaluation. It uses pricing formula to evaluate the portfolio after extreme fluctuations of market factors, subtracts the original value, and gets profit and loss. This method is suitable for complicated situations.

2.3.1.2 System Pressure Test

The idea of system pressure test is to integrate extreme fluctuations of all kinds of assets to generate series extreme scenarios, and evaluate their effect to generate a set of test results. It is more complicated and more comprehensive than scenario analysis, and thus needs more computations.

The system pressure test refers to two key questions, i.e. decision of type of risk and the selection of fluctuation of price.

- Selection of type of risk: Before selection of type risk, it should identify what risk the assets faces to, and then test pressure against different risks, such as Delta risk, Gamma risk and Vega risk.

- Selection of risk: Its process is to analyse every market factor in a specific period, finding out their maximum fluctuation and the recent maximum fluctuation, according to requirements and subjective experiences selecting them which needs to adjust with time.

The classical system pressure test methods include factors-driven analysis method [14], optimization of maximum loss [15] and worst case analysis method [16].

When applying this method, it should be noticed the small probability events that maybe generate big loss. System pressure test should be taken as a routine of risk management rather than a temporal operation. Its results should reflect positive maximum of risk of an organization.

2.3.2 Extreme Value Theory

The extreme value theory is another method of measurement of VaR in an extreme situation. It uses a statistical method to give VaR and its probability. With the help of abundant of historical data, this method is more accurate than pressure test.

It mainly includes two models, block-maxima model and peaks over threshold model. The former divides a period of historical data into many blocks and takes the maxima as extreme events. The latter takes the events that exceed the present threshold as extreme events. The former is usually used to seasons' data, while the latter focuses on the data rather than time. It is more effective for original data. Therefore, it is more popular nowadays.

2.3.2.1 Block-maxima Model

The block-maxima model is to find the distribution of maximums of blocks. Given an inter-independent variable sequence X_1 ,X_2, ……, they follow an unknown distribution of F. M=max(X_1,…,X_n) denotes the maximum of X_1,…,X_n. Assuming that we can find a sequence of real numbers a_n>0, b_n such that its normal maximums sequence（M_n-b_n）/a_n is convergent with the distribution. Namely, for a non-degenerate distribution function H(x), when $N \to \infty$, H(x) is called a maximum distribution. If $P\{(Mn - bn)/an \le x\} = Fn(an * x + bn) \xrightarrow{d} H(x)$, it is called that F lies in the maximum domain of attraction of H(x), denoted $F \in MDA(H)$

Distribution of maximums of blocks can be presented by a distribution with a parameter that is called generalized extreme value distribution. Its function is defined in Eq.(2-1).

$$H_\xi(x) = \begin{cases} \exp\left(1 - (1+\xi x)^{-1/\xi}\right) \text{......}(\xi \ne 0) \\ \exp\left(-e^x\right) \text{......................}(\xi = 0) \end{cases} \tag{2-1}$$

where x meets the condition of $1 + \xi x \succ 0$, where ξ is a parameter of shape. When $\xi \prec 0, \quad \xi = 0, \quad \xi \succ 0$, it respectively follows these distributions shown as follows.

Frechet distribution：
$$\varphi_{\xi}(x)=\begin{cases}0\,, & if\ x\leq (\\ e^{-x^{-\xi}}\,, & if\ x>0\end{cases} \qquad \xi>0 \tag{2-2}$$

Weibull distribution：
$$\psi_{\xi}(x)=\begin{cases}e^{-(-x)^{-\xi}}\,, & if\ x\leq 0\\ 0, & if\ x>0\end{cases} \qquad \xi\prec 0 \tag{2-3}$$

Gumbel distribution：
$$\Lambda_{\xi}(x)=e^{-e^{-\xi}}\,,\quad \mathrm{x}\in R\,, \qquad \xi=0 \tag{2-4}$$

Fisher-Tippett theorem [17] describes the extreme behaviours of maximums of normal samples.

Theorem 1.3.1(Fisher-Tippett) $F\in MDA(H)\Rightarrow H$ is a form of ξ.

From this theorem, we know that if a normal maximal sequence is convergent, its extreme distribution is a generalized extreme value distribution $H_{\xi,\mu,\sigma}(x)$ with specific values of their parameters.

If $\xi=0$，H_0 is Gumbel distribution. Normal distribution, exponential distribution, gamma distribution and logarithmic normal distribution belong to this domain of attraction. They are thick tail distributions.

if $\xi\prec 0$, H_{ξ} is Weibull distribution. Uniform distribution and beta distribution belong to this domain of attraction. Their tail is short, thus they are rarely applied in finical analysis.

If $\xi>0$, H_{ξ} is Frechet distribution. Pareto distribution, cauchy distribution, T distribution, Log-Gamma distribution and a variety of mixture distribution belong to this domain of attraction. Their tails are fat, hence they are suitable for simulating the fat tail in practice.

Theorem 2.3.2. A necessary and sufficient condition of deciding whether a distribution is a Frechet distribution is for $\xi > 0$,

$F \in MDA(H) \Leftrightarrow$ There exists a slowly changing function *L*(*x*) such that

$1 - F(x) = x^{-1/\xi} L(x)$

The Fisher-Tippett theorem proves that generalized extreme value distribution can be used for fitting the maximum of samples. However, when the sample data is insufficient, Peak Over Threshold (POT) is introduced.

2.3.2.2 Peak over Threshold

The idea of peak over threshold (POT) is to model all observed values that over a threshold.

The function of Generalized Pareto Distribution (GPD) is defined in Eq. (2-5).

$$G_{\xi,\beta}(x) = \begin{cases} 1-(1+\xi x/\beta)^{-1/\xi} ....(\xi \neq 0) \\ 1-\exp(-x/\xi)........(\xi = 0) \end{cases} \quad (2\text{-}5)$$

where, $\beta \succ 0$. If $\xi \geq 0$, x≥0; if $\xi \prec 0$, $0 \leq x \leq -\beta/\xi$. When $\xi > 0$, GPD is fat tail distribution. Its all order moments are no less than $1/\xi$ are infinite. Namely, if k>=$1/\xi$, E(X^k) is infinite.

2.3.2.3 Pickands-Balkama-de Haan Theorem

Assume that u denotes a big enough threshold, N_u denotes the number of the samples, $Y_1,\ldots Y_{Nu}$ denote excess values respectively, $Y_i=X_i$-u.X_0 denotes the right endpoint of a distribution F. Then the distribution functions of excess values can be defined in Eq. (2-6).

$$F_u(y) = P\{X - u \le y | X > u\} \qquad (y \in [0, x_0 - u]) \tag{2-6}$$

If F belongs to the maximum domain of attraction of H_ξ, the generalized Pareto distribution is the limit distribution of exceed values.

Theorem 2.3.3 Theorem of Pickands-Balkama-de Haan For every $\xi \in R, F \in MDA(H_\xi)$, if and only if for a positive measurement function $\beta(u)$ satisfies $\lim_{u \to x_0} \sup_{0 \le y \le x_{0-u}} \left| F_u(y) - G_{\xi,\beta(u)}(y) \right| = 0$

This theorem shows that for a large enough threshold u, the distribution of exceed values can be simulated by generalized Pareto distribution.

The extreme value theory (EVT) does not model the whole distribution, but it only focuses on its tail. The methods of simulating tails can be divided into two classes, semi-parametric method based on Hill-type estimator [18] and Hall bootmap [19] and full-parametric method based on generalized Pareto distribution [20].

The main advantage of EVT is that it can explicitly describe the quantile of the tail of a distribution. Its limitation is that it only can be used to describe the tail distribution and also waste many historical data. Besides, some assumptions of EVT are not conform to practical situations and need to be verified further.

2.4 Summary

Risk measure is the key for financial risk management. It is important to accurately calculate the size of deterministic and probabilistic, financial participants. Regulators can set the trading positions based on the measurement results cap, adjusting the portfolio asset structure and composition, the uncertainty in the pre-defined control that can withstand a level of inside. Therefore, from transnational syndicate to national regulators, financial risk measurement methods are committed to the research and application; international regulators are trying to build up a more internationally harmonized risk measurement and management standards. Although so far there is not a completely scientific method is widely accepted, the VaR system is being improved and more and more regulatory agencies and financial institutions use VaR as a risk management tool.

Chapter 3 Credibility Theory

3.1 Framework of Credibility Theory

The credibility model is emerging in 1990s. It is the most important method of experience rating in actuarial science of non-life insurance. In the insurance industry, premiums are determined from top to down. First, on the top level, enough premiums are collected to cover all insurances and balance payments. Then on the low level, premiums are apportioned among all policyholders. In this process, we need to exactly analyse policyholders' risk model. According to the traits of non-life insurances, companies generally uses credibility model to adjust loss experiences to adapt the changing of risk.

3.1.1 Definition

The credibility model is a model using loss data of policies portfolios (called experience data, denoted PM_e) and loss data of similar portfolios which are selected subjectively (called prior information data, denoted PM_0) to evaluate posteriori premiums with a credible factor Z, $Z \in [0,1]$. Its function is defined as follow.

$\mu(\Theta) = (1-Z) * PM0 + Z * PMe$.

3.1.2 Categories and Comparison

Credibility models can be classified into limited fluctuation credibility models and greatest accuracy credibility models according to the differences of experience data used in deciding the process of premium rates. The former decides the premium rates according to personal policyholders' historical data. It needs the personal data which are stable without big fluctuations. The latter uses the minimum mean square error formula to calculate the credibility premiums. It is the most accurate one. Therefore, it is widely used in insurance industry nowadays.

3.1.3 Framework of Limited Fluctuation Credibility Theory

The limited fluctuation credibility theory emerges from the beginning of last century. In 1914, Mowbray introduced the conception of full credibility [21]. Its main idea is to determine premiums according to experiential data. In 1918, Whitney proposed the conception of partial credibility [22]. Its idea is to make a balance between experiential data of individual claims and ones of claims of combination portfolios. These two methods both depend on the stability of experiential data of claims. They aims to solve a large enough z such the relative errors of $\hat{\mu}(\Theta)$ and $\mu(\Theta)$ do not exceed a threshold, i.e., $P_r\left(\left|\frac{\hat{\mu}-\mu}{\hat{\mu}}\right|<k\right)>\alpha$, where k and α is the given small positive numbers.

The item, $Z*(PMe-EPMe)$, denotes the total loss of random fluctuations. According to above two formulas, we can solve the distribution quantiles of PM_e and EPM_e and then solve out the value of Z.

3.1.4 The Framework of Highest Accuracy Credibility Theory

In 1945 [23] and 1950 [24], Bailey A.L. proposed the greatest accuracy credibility model. In 1967 [25] and 1969 [26], Hans Buhlmann proposed the idea that the Bayesian estimation should be limited in the range of combination of observed values. This method is also called the European credibility method.

3.1.4.1 Principle

The greatest accuracy credibility model has three key parameters - *m* denotes collective insurance premium which indicates the average level of risk; S^2 denotes homogeneous variance of risk which indicates the average value of variance among similar risks; and *a* denotes heterogeneous variance of risk which indicates the average value of variance among different risks.

Assuming that X denotes loss risk, its distribution function F(x; θ) denotes the conditional distribution $F_{x|\Theta}(x|\theta)$ with condition that Θ equals to θ. Assuming the distribution function of Θ is U(θ), the distribution function of X is $F_x(x) = \int F_{x|\Theta}(x|\theta)dU(\theta)$.

We have that $\mathrm{Var(Y)=E[Var(Y|\Theta)]+Var(E[Y|\Theta])=S^2+a}$,

where, $$\begin{cases} \mu(\Theta) = E[Y|\Theta] \text{...............} m = E[Y] = E[\mu(\Theta)] \\ \sigma^2(\Theta) = Var(Y|\Theta) = E[(Y-\mu(\Theta))^2|\Theta] \\ S^2 = E[\sigma^2(\Theta)] \text{..............} a = Var[\mu(\Theta)] \end{cases}.$$

According to net premium principle, $\mu(\Theta)$ in above formula is the risk premium of Y. There are two methods which can be used to evaluate it. They are the exact credibility estimate method and optimal linear and non-homogeneous one.

After obtaining a sample order X, denoted as X_1, X2,…,X_n, we can use minimum mean square error estimate X. In the exact credibility model, $\mu(\Theta)$ belongs to a posteriori Bayes estimation. It can be denoted $\hat{\mu}(\Theta) = E\left[\mu(\Theta) \middle| X_1, X_2, ..., X_n\right]$.

If the value of $\mu(\Theta)$ is limited in the estimated values of linear inhomogeneous of X_1,X_2,…,X_n, i.e., $c_0+c_1*X_1+c_2*X_2+…+c_n*X_n$, we can obtain the following formula based on the minimum mean square error estimate.

$$\begin{cases} \hat{\mu}(\Theta) = z * \overline{X} + (1 - z) * m \\ z = \dfrac{n}{n + s^2 / a} \end{cases}$$

Where, $\overline{X}$ denotes average value of samples, m denotes collective insurance premiums, z denotes factor of reliability.

We can use above two formula to determine the optimal linear non-homogeneous estimate value of $\mu(\Theta)$.

3.1.4.2 Classical Models

The greatest accuracy credibility models can be divided into two classes. One is the reliability model of least squares proposed by Buhlmann, and the other is the Bayesian reliability model.

- **Buhlmann Reliability Model**

In non-life actuarial science, Buhlmann established Buhlmann's Approach to Credibility, so called reliability model of least squares. It assumes that the distribution of variable x is decided by external variable Θ that has its own distribution called priori distribution. We can obtain some information about posterior distribution under a condition of $X=(x_1,x_2,......,x_n)$. $\mu(\theta)$ denotes the average value of posterior distribution, i.e., $\mu(\theta)$=E(θ|x). $\mu(\theta)$ can be presented as the linear combination of x_i (i=1,2,…..,n), i.e., $\mu(\theta)=\ \alpha_0+\alpha_1 x_1+......+\alpha_n x_n$.

Under the requirement of minimal square errors, we estimate α_j via least squares regression with n+1 normal equation.

If i=0， unbiasedness equations is defined.

E(x_{n+1})=$\tilde{\alpha}_0+\sum_{j=1}^{n}\tilde{\alpha}_j$ E(x_j);

If i=1,……,n， we can obtain other normal equations, get the conclusion that

Cov(x_i， x_{n+1})=$\sum_{j=1}^{n}\tilde{\alpha}_j\operatorname{cov}(x_i,x_j)$， hence the reliability of premium is $\tilde{\alpha}_0+\sum_{i=1}^{n}\tilde{\alpha}_i x_i$.

- **Bayesian Reliability Model**

The Bayesian method is proposed by Bayes in 1763. It has been widely used in many different areas. In the insurance field, the Bayesian method is mainly used to adjust insurance premium and claim.

The idea of Bayesian method is that given some priori information, we can solve its posteriori distribution and know about its mathematical characteristics with priori information.

The Bayesian reliability model takes unknown parameters of premium estimate as random variables denoted as θ. If θ is known, the joint probability density distribution of samples $X_1, X_2, \ldots, X_n$ can be used as a conditional density denoted as $P(X_1, X_2, \ldots, X_n/\theta)$. Then a priori distribution $\Pi(\theta)$ can be determined empirically. According to Bayesian formula, we can solve the density distribution of a posterior $h(\theta / X_1, X_2 ... X_N)$, denoted as $h(\theta/x)$, then we can obtain $h(\theta|x) = \dfrac{\Pi(\theta)P(x|\theta)}{\int \Pi(\theta)P(x|\theta)d\theta}$.

Based on the solved posterior distribution, we can estimate θ and posteriori premiums.

3.2 Credibility VaR Model

One of the difficulties in the financial market risk management is how to determine the loss distribution function. Risk units continue to enter and exit and the investment environment changes constantly, making financial loss distribution based on historical data fitting become increasingly untrusted. As mentioned earlier, the credibility method of the actuarial mathematics of non-life insurance actually adopts the weighted method combining the rate calculated by past experience (prior information) and the rate set by the risk change in a certain reasonable criteria (which needs experience and precise calculation).

Therefore, this idea can be fully applied to the VaR estimates. If we appropriately adjust the balance between VaR calculated by the distribution of the of historical data and the values of recent loss, which should be able to make VaR values more suitable for the real conditions and improve the overall fitting accuracy of the mode.

This report attempts to introduce parameter distribution method, historical simulation method and the extreme value theory into three forms of credibility theory (i.e. limited fluctuation credibility, Buhlmann credibility, Bayesian credibility) respectively and derive the credibility VaR model series.

3.2.1 Limited Fluctuant Credibility Parameter Distribution Method

According to the basic principles of credibility theory, we make use of historical data by adopting moving average model to get the experience value of the financial asset VaR denoted by $V\tilde{a}R(\alpha)$, regarded as a priori information data, the recent period of observations n is denoted by $\hat{VaR}_t(\alpha)$.

Therefore, the credibility VaR value can be denoted by Eq. (3-1).

$$C\hat{VaR}_t(\alpha,\beta)=(1-\chi)V\tilde{a}R(\alpha)+\chi\hat{VaR}_t(\alpha) \tag{3-1}$$

Where the α value is the loss probability of the original VaR as a quantile, β is the probability of credibility.

Based on limited wave theory, we have:

$C\hat{VaR}_t(\alpha, \beta)=(1-\chi)\hat{VaR}(\alpha)+\chi E(\hat{VaR}_t(\alpha))+\chi[\hat{VaR}_t(\alpha)-E(\hat{VaR}_t(\alpha))]=(1-\chi)V\tilde{a}R(\alpha)+\chi . \mu_t+\chi \phi_t^{-1}(\alpha)\sigma_t$ (3-2)

Where the third entry of the right equation $\chi \phi_t^{-1}(\alpha)\sigma_t$ is expressed as the return value of random fluctuation, and has:

$$\hat{\mu}_t = \frac{1}{n}\sum_{i=1}^{n} y_{t-i} \tag{3-3}$$

$$\hat{\sigma^t}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_{t-i} - \hat{u}_t)^2 \tag{3-4}$$

The limited fluctuation credibility meaning of VaR is to seek a sufficiently large value χ making the $C\hat{VaR}$ and the relative error of CVaR does not exceed a certain limit probability, i.e.: $\mathbf{Pr}(|\frac{C\hat{VaR}-CVaR}{C\hat{VaR}}|< k) > \beta$ (k, β is a given positive number)

Therefore, $\mathbf{Pr}\ (\frac{|\chi(\phi_t^{-1}(\alpha)\sigma^t)|}{\mu_t}\rangle k) < 1-\beta \Rightarrow \Pr(\phi_t^{-1}(\alpha)\rangle \frac{k\mu_t}{\chi\sigma_t}) < 1-\beta$.

And because of the median $\phi_t^{-1}(\alpha) \sim U(0,1)$ (i.e. subject to [0,1] uniformly distribution), so $\frac{k\mu_t}{\chi\sigma_t} = u_{t-\beta} = 1-\beta$

That is: $\chi = \frac{k\mu_t}{(1-\beta)\sigma_t}$ (3-5)

Therefore, the limited fluctuation measurements credibility risk (CVaR) estimator of the parameter distribution law is:

$$\hat{CVaR}_t(\alpha,\beta)=\frac{(1-\beta)\sigma_t-k\mu_t}{(1-\beta)\sigma_t}\tilde{VaR}(\alpha)+\frac{k\mu_t}{(1-\beta)\sigma_t}\hat{VaR}_t(\alpha) \quad (3\text{-}6)$$

3.2.2 Buhlmann Credibility Historical Simulation

Buhlmann credibility method has the advantage of handling a large number of non-homogeneous risk. We can adopt the Buhlmann credibility model to the calculation of VaR historical simulation:

3.2.2.1 BCVaR（Buhlmann Credibility VaR）

The idea of the Buhlmann credibility method is to infer possible future losses based on the occurred loss, on the premise of some characteristic information (such as the origin of the moment, the central moments) of given variables distribution. For example, if a loss of large mean deviation has occurred recently, we can infer that the large loss will not occur in a short time, therefore the influence of the mean of recent results to calculate predicted values should be reduced, which means the weight χ should be reduced.

In the classical model of Buhlmann credibility, provided external parameters is given in the case where Θ = 0, the conditional distribution of Xi is assumed to be independent and identically distributed, Θ ’s probability density function (pdf) is π (θ). On this basis we can obtain E (xi) = μ, Var(x_i)=$\sigma^2=v+a,(i=1,....,n)$ Cov(x_i,x_j)= ρ, according to the canonical equation, $\tilde{\alpha}_0 = \frac{(1-\rho)\mu}{1-\rho+n\rho}=\frac{v\mu}{na+v}$

$\tilde{\alpha}_i=\frac{\rho}{1-\rho+n\rho}=\frac{a}{na+v}(i=1,2,....,n)$.

Resulting in getting reliability premiums:

$$\tilde{\alpha}_0 + \sum_{i=1}^{n} \tilde{\alpha}_i x_i = \chi \bar{x} + (1-\chi)\mu \tag{3-7}$$

Then: $\chi = \dfrac{n}{n + \dfrac{v}{a}} = \dfrac{n}{n+k}$ (3-8)

Therefore, we adopt the Buhlmann credibility model to the calculation of VaR historical simulation, which is assumed to be based on historical data. We know that VaR(α) experience mean is μ, experience variance is σ^2, $\mu = \frac{1}{n}\sum_{i=0}^{n} VaR_{t-i}$, and σ^2 can be derived from MA, ARCH, GARCH and other models. Meanwhile, m times return on investment value has been taken in the most recent wave cycle as a priori information. Then the above method has been used to obtain parameters the most recent $V\tilde{a}R_t(\alpha)$ which is $BC\hat{V}aRt(\alpha) = \chi V_a R(\alpha) + (1-\chi) V\tilde{a}R_t(\alpha)$.

$$\chi = \frac{m\rho}{1-\rho+m\rho} \tag{3-9}$$

3.2.2.2 BC CaVaR（Buhlmann CaVaR）

CaVaR stands for Conditional Autoregressive VaR model. In 2000, Engle et al. proposed VaR can be directly used for self-quantile regression model instead of using the entire distribution auto-regression (e.g. GARCH) method:

$$VaR_t = a_0 + a_1 VaR_{t-1} + f(x_t \mid \theta) \tag{3-10}$$

In the actual analysis with the two sets of models:

Symmetry CaVaR model: $VaR_t = a_0 + a_1 V_a R_{t-1} + a_2 |y_{t-1}|$

And asymmetric CaVaR Model:

$$VaRt = a0 + a1VaRt - 1 + a2 | yt - 1 | + a3 | yt - 1 | (yt - 1 < 0)$$

As a complement to historical simulation methods, this model actually has identified the VaR autoregressive relationship, the linear combination of historical VaRi $(\alpha\alpha(i = t - 1, t - 2, \ldots, 1)$ can be used to represent VaR (α).So,

$$VaR_t = \alpha_0 + \alpha_1 VaR_{t-1} + f(x_t | \theta) = a0 + a1(a0 + a1VaRt - 2 + f(xt - 1 |) + f(xt |)$$

$$= \ldots\ldots\ldots\ldots = (a_0 + a_1 a_0 + a_1^2 a_0 + \ldots\ldots + a_1^{t-1} a_0) + a_1^t V_\alpha R_1$$

$$+ f(x_t | \theta) + a_1 f(x_{t-1} | \theta) + a_1^2 f(x_{t-2} | \theta) + \ldots + a_1^{t-1} f(x_1 | \theta) \qquad (3\text{-}11)$$

If the first term from the regression equation is VaRm, there is

$$VaR_{m+t}(\alpha) = a_0 \frac{1 - a_1^t}{1 - a_1} + a_1^t VaRm(\alpha) + f(x_{m+t} | \theta) + \ldots a_1^{t-1} f(x_m | \theta) \qquad (3\text{-}12)$$

Therefore, let $M = -\frac{a_0}{a_1}$,

$$VaRm+t(\alpha) = a_1^t VaRm(\alpha) - \frac{a_1(1 - a^t)}{1 - a_1} M + a_1^{t-1} f(x_m | \theta) + \ldots + f(x_{m+t} | \theta)$$

VaRm+t(α) is a weighted average with the sum of coefficients of 1, and

$$VaRt(\alpha) = \alpha_0 VaR(\alpha) + \alpha_1 M + \alpha_2 f(x_{t-n} | \theta) + \ldots.\alpha_{n+2} f(x_t | \theta) \qquad (3\text{-}13)$$

Where $\alpha_0 = \alpha_1^t \qquad \alpha_1 = -\frac{\alpha_0}{\alpha_1} \qquad \alpha_2 = a_1^{t-1} \ldots.a_{n+2} = 1$

3.2.3 Bayesian Reliability Extreme Value VaR Model

Based on a similar principle, we can introduce Bayesian into VaR model, solving the future value of VaR estimation based on some of the known prior VaR conditions.

Note $X = (x1, x2, ..., xn)'$ are the variable values of first n periods, x_{n+1} is the value of the (n+1)th term, the conditional distribution of x_{n+1}

$$f_{X_{n+1}|X}(x_{n+1} \mid x) = \frac{f_{X_1, X_2, \cdots, X_n, X_{n+1}}(x_1, x_2, \cdots, x_n, x_{n+1})}{f_X(x)}$$
$$= \frac{\int f_{X_{n+1}|\Theta}(x_{n+1} \mid \theta) \cdot \pi_{\Theta|X}(\theta \mid x) \cdot f_X(x) d\theta}{f_X(x)} = \int f_{X_{n+1}|\Theta}(x_{n+1} \mid \theta) \cdot \pi_{\Theta|X}(\theta \mid x) d\theta \tag{3-14}$$

We assume that distribution of x is known under the premise of a given parameter Θ (Similarly, given parameter Θ the conditions set distribution of x_{n+1} is $f_{X_{n+1}|\Theta}(x_{n+1} \mid \theta)$), and the priori distribution (π (θ)) is known. From the sample x_1, x_2,...,x_n, we can obtain the conditional distribution $\pi_{\Theta|x}(\theta \mid x)$ given samples Θ.

Therefore,

$$F_{X_{n+1}|X}(x_{n+1} \mid x) = \iint f_{X_{n+1}|\Theta}(x_{n+1} \mid \theta) \cdot \pi_{\Theta|X}(\theta \mid x) d\theta dxn + 1$$
$$= \int F_{X_{n+1}|\Theta}(x_{n+1} \mid \theta) \cdot \pi_{\Theta|X}(\theta \mid x) d\theta \tag{3-15}$$

Since VaR of F (x) at a given confidence level of the p quantile, there is

BVaRp= F^{-1}(1-p) (3-16)

According to the Peak Over Threshold Gradually Theorem, we can obtain:

Fu(x-u)= $\frac{F(x)-F(u)}{1-F(u)}$ (3-17)

So, $\frac{F_{X_{n+1}|\Theta}(x_{n+1}\mid\theta)-F_{X_{n+1}|\Theta}(u)}{1-F_{X_{n+1}|\Theta}(u)}=G_{\xi\sigma}(y)$ (3-18)

And, $F_{X_{n+1}|\Theta}(x_{n+1}\mid\theta)=G_{\xi\sigma}(y)(1-F_{X_{n+1}|\Theta}(u))+F_{X_{n+1}|\Theta}(u)$

However, due to the current theory community has not yet found the distributed expression of $\pi_{\xi,\sigma|x}(\xi,\sigma\mid x)$, it is impossible to obtain the analytical type of $F_{x_{n+1}|x}(x_{n+1}\mid x)$, and thus we cannot get formula of BvaRp. Neither can be achieved by data. But this is a novel method, which should have several insights.

3.3 Evolution of Risk Measurement Methods

When looking back at history, we can see that the risk measure is advancing accompanied by financial practices and regulatory and the development of economic and financial theory.

Since the disintegration of the Bretton Woods system in the 70's, the dollar-based fixed exchange rate system is replaced by floating exchange rate system, derivative financial products as a means of avoiding risk is under a lot of development. Because the limited human rationality and incompleteness of information and other issues emerged in the financial activities, the financial risks have a greater diffusibility, concealment, unexpectedness. In the second year after the former Federal Republic of Germany Herstatt bank and the United States Franklin National Bank went bankrupt, in September 1975, the first Basel Accord was unveiled. Economists applied the regulation theory (The chasing theory, the public interest theory and the new regulation theory) to the banking sector and examine how to achieve effective banking supervision. While the mainstream view in the financial circle is "risk management irrelevance theory", which considered that in a frictionless market (no transaction costs, no taxes, and no information asymmetry), corporate financial policy (debt / equity ratio) neither increase nor reduce the value of the business, namely the famous M-M theory. Financial Risk Measurement methods have been developed from "nominal amount method" to "sensitivity method" and the "volatility method".

In 1982, the Mexican government announced that they were unable to pay foreign debt in time, unveiling the debt crisis in Latin American countries for several years. In early 1980s, Argentina's financial crisis almost destroyed its entire economic system, making GNP value losses up to 55%, which had led to the commercial banks overseas debt crisis of developed countries led by the U.S. Out of international concerns about credit risk, Basel, in 1988, has taken an important step by strengthening the financial risk management by stipulating the minimum capital adequacy ratio of commercial banks. Economists paid more attention to apply the positive transaction costs in institutional economics to bank ownership analysis. While the risk control "VaR (value at risk) approach" research has become a topic among scientists in major banking and financial research department.

In the 1990s, the market risk is significantly increasing. In early 1995 a trader from a branch of the British Barings Bank failed of profiteering for yen derivatives, which led to the old bank with 230 years of history collapsed overnight. The deposits of the Queen of England were gone; In July 1997, the outbreak of the financial crisis in Thailand quickly spread to Southeast Asia, resulting in more than ten countries currency devaluation and the global stock market swung. Under this circumstance, in April 1995 the Basel Committee proposed an expansion of market risk models to allow allowed banks to use their own internal risk measurement models to determine their capital requirements for the first time, and formally acknowledge the scientific system of VaR. Economists eagerly discussed the impact of financial system risk management on the economy effectiveness. Arrow-Debreu's "full market model" and Merton's "Dynamic full market models" had both proven that through effective risk management of economic behaviours in the financial system, we can achieve the effective allocation of risk–bear, which has benefits to improve the efficiency and overall economic welfare. While the financial, statistical academia start to focus on the extreme VaR approach, especially on extreme value theory in that market risks, and the small probability events of operational risk occur frequently and the serious consequences.

3.4 Summary

As can be seen from the above, the economic base determines the superstructure; No matter what kind of financial practice, it will need the corresponding theoretical and guidance. As the world economic integration and the surging wave of financial globalization, competition in the financial sector, especially among international banks is getting increasingly fierce. Financial innovation makes the banking business tend to be diverse and complex and the gap between the operating levels of managers can lead to bank profits gap growing. It is reasonably to predict, after the credit risk, market risk, and operational risk, the operational risk will be one of important risk types which worth paying attention to by banking participants, regulators.

Chapter 4 Multithreading

4.1 Model

We know that concurrency control has been used in the multi-database system to assure that multiple users can execute queries to the database system at the same time. This means that the database system can handle several requests at the same time. Depending upon this idea, we apply the Java concurrency feature to the client application side. At the client application side, multiple threads are created simultaneously, and each thread can execute the same query to the different part in the database. Compare with the traditional way that only one request is sent to the whole database, this method can reduce the Round Trip time that is measured for each request and response in the database application. It can be better understood by the Figure 4-1.

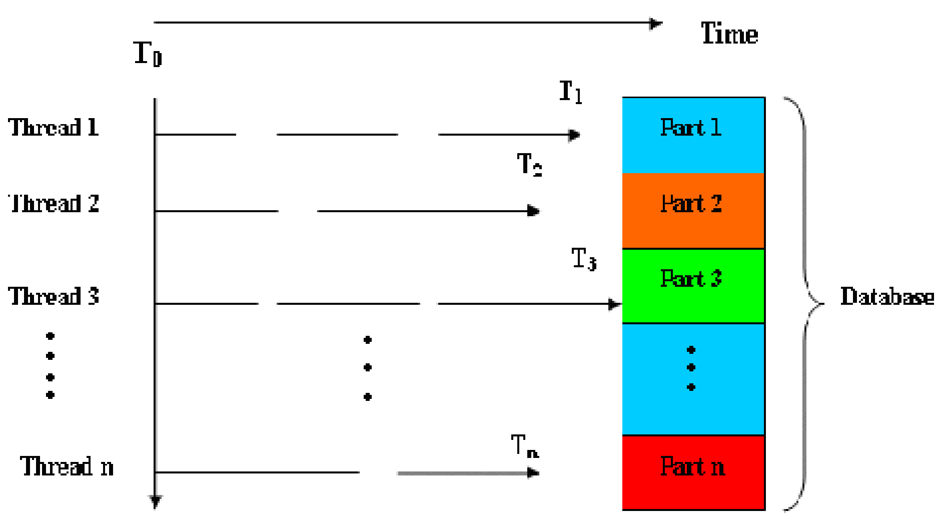

Figure 4-1: Thread process.

Figure 4-1 indicates that the database is divided into several parts, and each part is counterpart to a thread. All of the threads send the same query to their own database simultaneously at time $T_{0.}$ In the process of executing query, the time that is spent on requesting and retrieving the records from each part of the database, is different. After all threads finishing, the longest time is $T_{3,}$ which is the Round Trip time for requesting and retrieving all the records from the whole database. However, it is shorter than the time that spent on the whole database for only one thread. Hence, the application of multi-threads in the client side can improve the performance of database that it can reduce Round Trip time spent on database application.

4.2 Thread Architecture

This section describes the architecture of the model component for multi-threads application. A highlight of the architecture is that the application of multi-threads improves the performance of database system. The thread model architecture is shown in Figure 4-2.

From Figure 4-2, we know that this model is similar with the traditional database application architecture that contains three parts: Client Application, JDBC middleware and Database three parts. However, it has own highlight that multi-threads are adopted by this model.

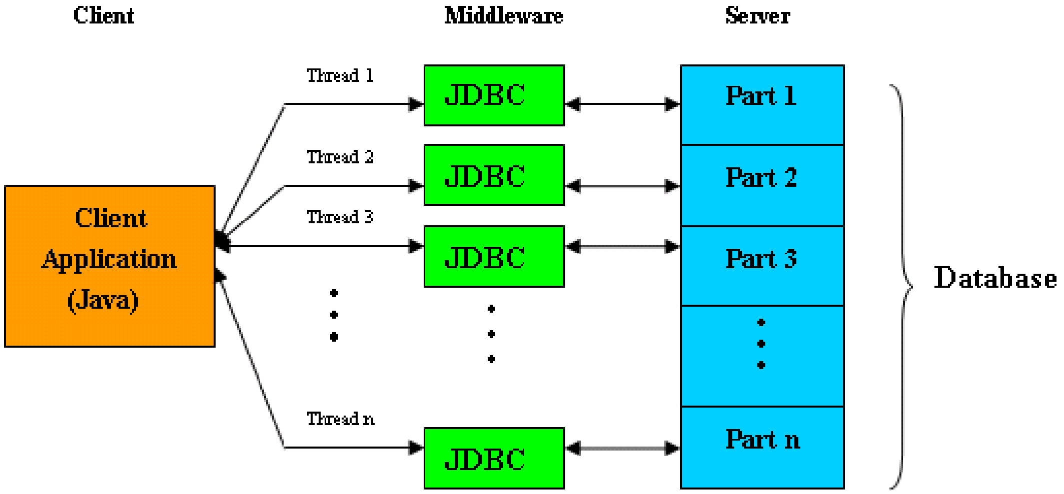

Figure 4-2: Thread architecture.

Client Application: this part is aim to create multi-threads at the same time, and each thread sends the same request to JDBC middleware through SQL statement. The application on client side is based on Java language which has the capability of performing concurrency actions at a time by using multi-threads.

JDBC middleware: As a standing interface for Java Database Connectivity, which provide a standard library for accessing relational databases. JDBC is used to create connection between client application and database server. In this case, JDBC sends SQL statement to database and then retrieve the records from the database.

Database: All the data is stored in the database in the form of two-dimensional data structures that defined by rows and columns. In this case, the database as a bookstore database contains millions of book information. The information of each book includes book ID, title, author and price. However, in this model, the book database is divided into several parts, and each part is counterpart to a thread.

4.3 Socket Design

Although thread model is one of choices to reduce the Round Trip time in the database application, it sometimes cannot perform perfectly when it applied to a good-sized database on a computer. This because of the restriction on computer hardware, the computer I/O interface has no capability to handle much of data at the same time. Much of data need to queue as a line to wait for handling by the computer. This problem may results in spending much time on retrieving records from bookstore database. To conquer this problem, we will introduce a new method in this section, which is the socket model that takes advantage of Java socket to build a distributed database system.

4.3.1 What is Socket?

Socket is used to establish a communication channel between client and server on the Internet or Intranet. The channel is based on the host IP address and the port that the server is listening on. Both client and server can send or receive data through the established channel between them. The process of establishing channel is shown in Figure 4-3.

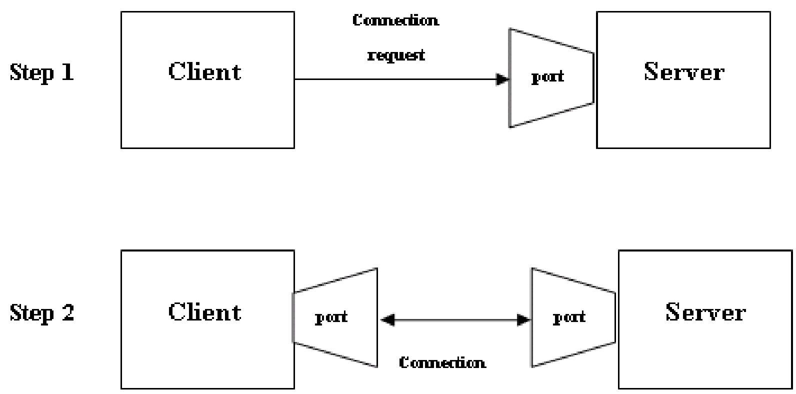

Figure 4-3: Socket establishment.

The Figure 4-3 shows that the client sends the connection request to the server port that the server is listening on. Then the server receives the request and checks the port's state. If the port is not occupied by the other tasks, the server sends a response to client that agrees to establish a connection. After the connection is established, the data can be transmitted through it. If the transmission of data is finished, the socket connection between client and server will be closed.

There are two kinds of sockets: TCP socket and UDP socket. TCP socket is connection-oriented, which provides a stable connection between client and server. In contrast with TCP socket, UDP socket is a connectionless socket, which can not guarantee the security of data transmission. However, it has a faster transmission speed than TCP socket.

4.3.2 Methodology

In the thread model, it is not capable of resolving the problem of computer hardware restriction for handling much of data. In this section, we will propose a new way that builds a distributed database system to conquer the problem by using Java socket.

Distributed database systems are distributed systems where all databases are modeled as a set of cooperating objects. The goal of most distributed database systems is to provide the ability to build a database on one host and transmit the records in the database to another host[16]. Based on this concept, we can design a distributed model for database application that one client interacts with multiple servers where the same database resides. In this case, the client can communicate with multiple distributed servers at the same time, so that it improves the performance of the distributed database system that reduce the Round Trip time in the application on the distributed database system.

To achieve the deployment of distributed database system with high speed interconnection, Java sockets are adopted as communication strategy. They are used to transmit data between a client and multiple servers in parallel to achieving efficient performance.

4.3.2 Socket Architecture

The design of socket model architecture is based on the principles of distributed system. The client connects with multiple servers in parallel, so that it can send requests to multiple server databases at the same time. The connection between client and multiple servers is built by Java socket technology. After the connection is established, data is transmitted between client and servers through it.

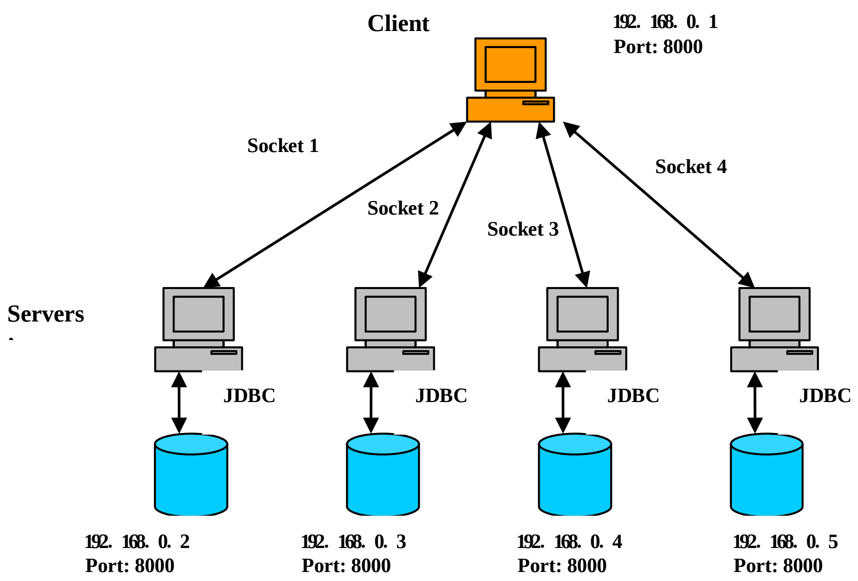

Figure 4-4: Sockets architecture.

Figure 4-4 shows the sockets model architecture, that a client and four servers consist of a distributed database system. The client established four connections with four server nodes where the same database resides. The connections are based on the host IP address and the port that the server is listening on.

In this case, we use UDP socket as the communication strategy. At first, the server ports are listened on to wait for receiving the packets that sent from client side. Then the client sends a packet with request to search book records from the bookstore database on each server simultaneously based on the server IP address and the port number. After each server receives the packet, they will execute the request and retrieve book records from the bookstore database on the server. At the same time, the results about book records will be sent to the client through the sockets connection.

Although the database on each server is the same bookstore database, it is divided into several parts based on the numbers of servers. By this way, we can assure that each server is responsible for different part of the bookstore database in the process of searching book records and the records can be gained simultaneously from the different part bookstore database by the users. As a result, the application of UDP socket can reduce the Round Trip time that the users spend on searching book records from the bookstore database and returning the results to users.

4.4 Finance Risk Analysis in MATLAB

Monte Carlo simulation method is used to calculate the VaR of the portfolio. Monte Carlo simulation method is a mathematical method. It is difficult to get precise and fast results of portfolio VaR due to the limitation of the computational ability. When a large scale of portfolios which have too much correlated factors to be taken account need to be simulated, it is hardly possible to work out the VaR accurately in time by Excel. That is why a powerful software is required in this simulation.

In fact, there are several good software which has enough computational ability to performance the large scale of Monte Carlo simulation. In this thesis, MATLAB Version 7.0 is chosen. MATLAB is a multifunctional mathematical tool which aims to solve a variety of tasks in different disciplines from medicine to financial modeling. There are a lot of advantages of MATLAB. Firstly, MATLAB is a professional mathematical software, therefore it can easily deal with large amounts of data. Secondly, MATLAB has plenty of practical algorithms and new algorithms can be also created by people to finish the specific tasks. Thirdly, MATLAB can easily work with Excel, such as reading data from Excel or putting data into Excel.

The calculation of VaR using Monte Carlo simulation is based on the concepts mentioned in Chapter 2 and Chapter 3.

The detailed process of the design of the simulation is expressed as follows:

1^{step}: Start the program and get the data from Excel file. Generate the matrix of the prices of portfolio.

2^{step}: Input the figures which are used to measure the VaR of portfolio, including number of shares, time, confidence level and number of simulations.

3^{step}: Call Monte Carlo function.

4^{step}: In the part of Monte Carlo function,

5^{step}: Calculate the mean value and standard deviation according the historical data. Then simulate the possible prices at the specified time according the Geometric Brownian motion.

6^{step}: Repeat step 5 n times, thereby getting a distribution of possible prices.

7^{step}: Calculate the standard deviation of the return of Monte Carlo simulation and get the VaR according the formula $\mathrm{VaR} = -\alpha\sigma W\sqrt{\Delta t}$.

4.5 Summary

This Chapter described both the thread model and sockets model. Although we adopted different technologies on the two models, the aim of the design was to reduce the Round Trip time in computation.

This Chapter also presented the design of the Monte Carlo simulation in MATLAB, and described the steps involved.

Chapter 5 Results and Analysis

To analyze the performance of our proposed models with threads and sockets, we designed and launched a serial of simulations. Notice that we can change the number of activated threads in the thread model and change the number of servers in the sockets model. The results and analysis of these simulations will be described in this chapter.

5.1 Case 1

In this case, we only need to change the number of threads in each test. The size of bookstore database is static, and which contains 300,000 items book records. The results are shown in Table 5-1.

Table 5-1: Round Trip time in Case 1.

Num of thread	**1 thread**	**2 threads**	**4 threads**	**6 threads**	**8 threads**	**10 threads**
Round Trip Time	63.625s	53.344s	55.469s	53.641s	53.531s	53.422s

The result can be explicitly represented in graph, which is shown in Figure 5-1.

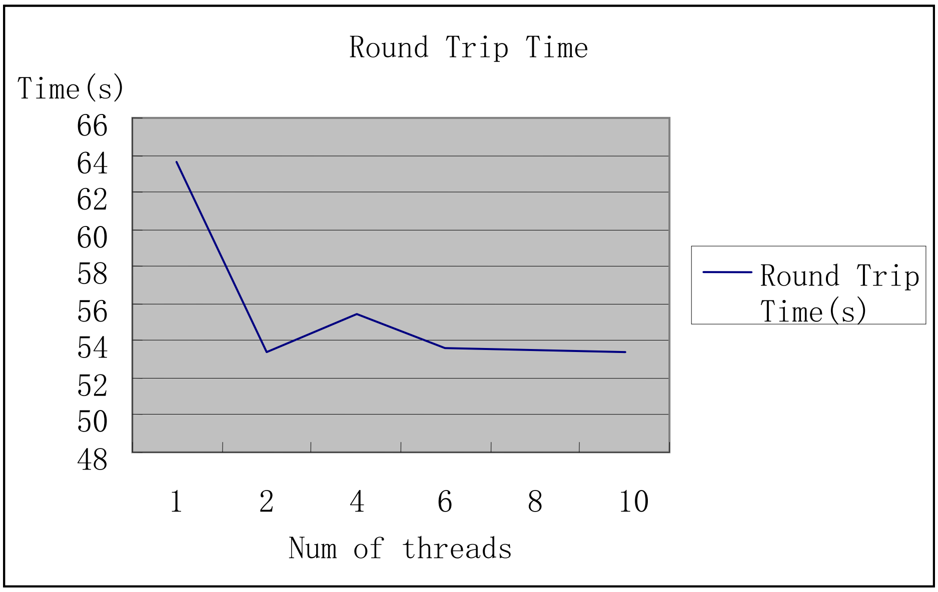

Figure 5-1: First result in thread model.

Figure 5-1 represents the results of Round Trip time with the number change of threads. When multiple threads are applied to this application, the Round Trip time is significant less than that a thread is adopted. This means that the application of multiple threads in this case actually improve the performance of the database system that the Round Trip time is reduced dramatically. However, there is no obvious change about Round Trip time when the number of thread increased from 2 to 10 discretely.

In theory, the Round Trip time should be reduced with the increment of thread number. Actually, there is no significant effect for Round Trip time when the number of thread is increased. The reason caused the problem may be the restriction of computer hardware. We know that multiple threads share memory and other resources of a single process. Shared resources such as caches and memory have to be enough large to assure computing hardware has the ability of allowing quick switching between different threads. Otherwise, the computing hardware restriction becomes the bottleneck of improving database system performance. Actually, in this case, because of the restriction of computing hardware, we can not gain the desired result that the Round Trip time is reduced with the increment of threads.

5.2 Case 2

In these simulations, the size of database is changed from 10,000 to 1,100,000 items book records discretely. We simulated the thread model using 1 thread and 2 threads respectively at each database size. The purpose of this design is to see the effect of database size on the multiple threads application. The result is shown in Figure 5-2.

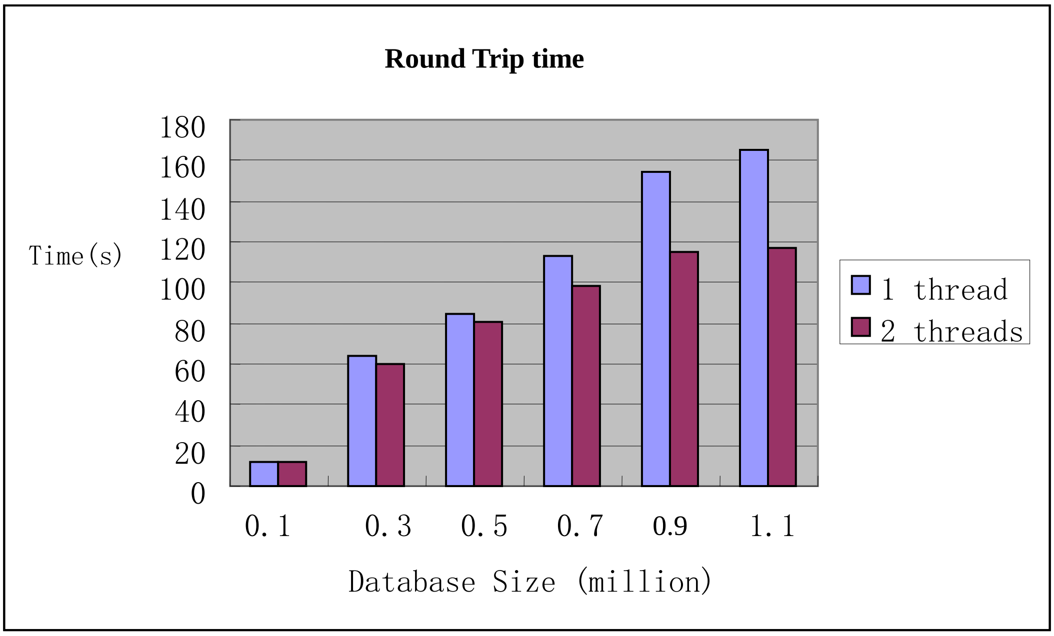

Figure 5-2: Second result in thread model.

Figure 5-2 shows the Round Trip time for no thread application and two threads application with the increment of database size. According to the chart, the Round Trip time in two threads application has a little decrease when the database size less than 700,000 records. But it is obvious from the chart that the Round Trip time reduces significantly with the increment of database size from 700,000 items record to 1,100,000 items record discretely. So we can conclude that the application of multiple threads in the database system is good for the large size of database.

5.3 The Socket Model

In this model, the function of sockets model is same as in the thread model that is to search book records from bookstore database. The main difference between the two kinds of model is that the sockets model is a distributed system which is consisted by a client and several servers in an Intranet. Moreover, each server contains the same bookstore database with 300,000 items book records. In this simulation, we need to add server node to see the performance of the model in each simulation.

In each test, the server sockets are ran firstly to listen on the query that sent from the user on the client. After the server sockets had been ran, we run the client socket to send a query that searches book records from the bookstore database on each server, and receive the results from each server database.

5.3.1 Results

After finishing a series of tests, Round Trip time can be retrieved, which is represented in Table 5-2.

Table 5-2: Round Trip time in sockets model.

Num of server node	**1**	**2**	**3**
Round Trip Time (s)	**59.437**	**44.703**	**39.219**

The results can also be represented in graph, which is shown in Figure 5-3.

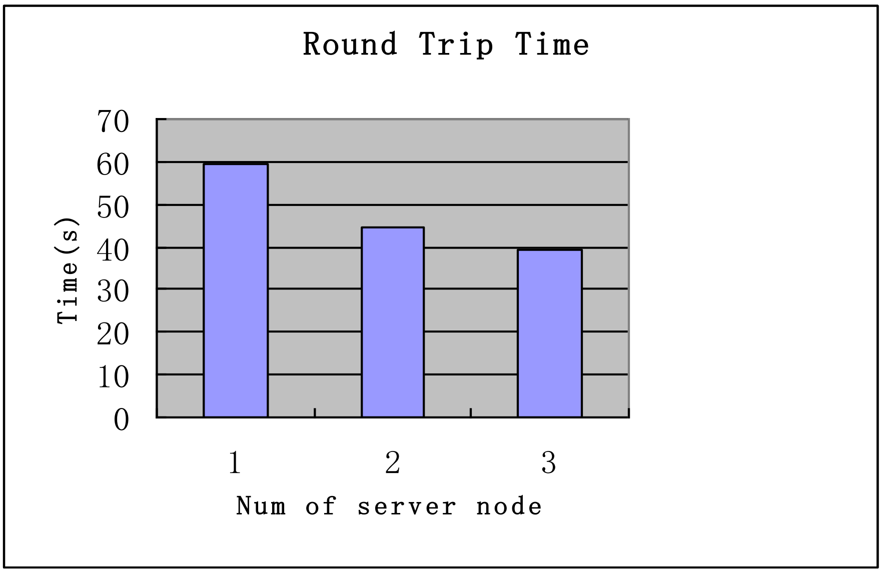

Figure 5-3: Round Trip time in sockets model.

From Figure 5-3, we know that the Round Trip time in socket model gradually decreased with the increment of server node. When the server node is added to three, the user spent the least time on searching book records from bookstore database. Hence, it proves that the application of sockets can improve the performance of searching function in database system.

In the sockets model, Java sockets as the communication strategy contribute to the distributed database system, and which guarantee data transmission is so fast between client and servers. Moreover, each server in the distributed system is responsible for different part of the bookstore database. This method can avoid the hardware restriction problem that a server has no capability to handle the whole database so fast. Hence, based on the two advantages of thread model, we gain the desired result that the Round Trip time is decreased with the increment of server node.

5.3.2 Model Comparison

The purpose of this section is to compare the performance of database system in the two different models. We consider the database size as the variable parameter which is increased from 150,000 to 700,000 items book records discretely. Then the Round Trip time is tested with the increment of database size in each mode. We should pay attention that there are two threads in threads model and three nodes in sockets model. The results of Round Trip time is shown in Figure 5-4.

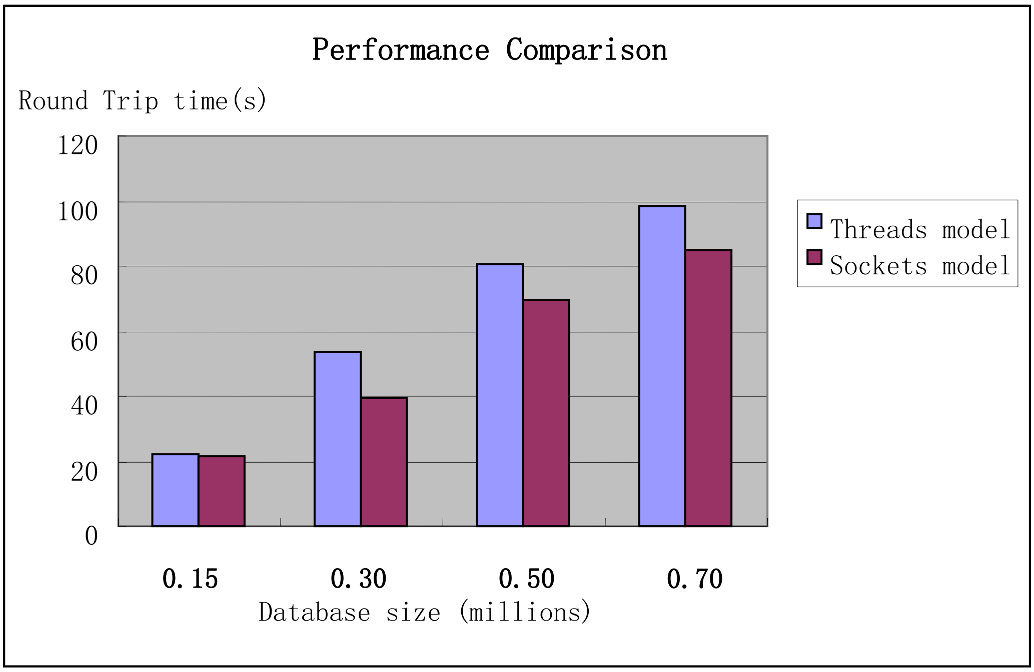

Figure 5-4: Results comparison.

From the above Figure 5-4, we know that the performance of database system in sockets model is almost the same as the threads model when the database size is less that 0.15 million items book records. However, with the increment of database size, the Round Trip times in sockets model is less than those in the threads model. This means that the performance of sockets model is better than the threads model's. This advantage of sockets model is suited to the users whose aim is to gain the best performance and regardless of hardware cost.

5.4 VaR Performance in MATLAB

5.4.1 Datasets

In this thesis, the historical data of 9 assets in 3 years is captured from yahoo finance for the Monte Carlo simulation. The 9 assets are respectively ARM, BARC, BAY, BGY, LOG, RTK, TSCO, VOD and WTD. The closing prices of these 9 assets in the London Stock Exchange (LSE) within 3 years are selected. The numbers of the shares of each asset are respectively 565, 686, 895, 743, 160, 250, 353, 689 and 754. Then the VaR of this portfolio will be estimated with different parameters. The parameters include confidence level (conlvl), number of simulations (numsim) and holding period (time).

5.4.2 VaR Performance

To make the comparison, all the results generated with different confidence levels and holding periods are shown in Table 5.3.

Table 5-3: Testing parameters.

Confidence level / Holding period	90%	95%	99%
20 days	521990	669970	947550
30 days	639700	821050	1161200
50 days	817590	1049400	1484100

From the results in the Table 5.3, under the same confidence level, when the holding period became longer, the VaR would be larger. It means the risk that the investors faced is larger. In the meantime, the selection of the holding period will influence the reliability of the VaR. Therefore, the choice of holding period is important for the portfolio risk. When choosing the holding period, two main factors will be considered. One is the distribution of the return of the portfolio, another one is the liquidity of the market and exchange frequency of the cash position.

To calculate VaR, the probability distribution of the return of the portfolio should be determined first. Generally, there are two methods to determine it. The first one is to assume the return following a type of distribution directly. Normal distribution is used most frequently. However, the actual distribution usually does not follow the normal distribution. But when the holding period is shorter, the calculation of VaR under assumption of normal distribution will be more effective and more reliable. So a short holding period should be chosen under the assumption of normal distribution of return. Another method to determine the probability distribution of the return is using the historical data to simulate the distribution. In this method, the availability and effectiveness should be considered. The longer the holding period is, the more the historical data will be. Then the problems and difficulties will be more. Thus the shorter holding period should be chosen.

Another factor in choosing the holding period is the liquidity of the market and exchange frequency of the cash position. When calculating the VaR, the cash position of the portfolio is usually assumed as invariant during the holding period. So the VaR is not reliable if the changes of cash position in the holding period are not taken account. When the market is liquid enough, the exchange will happen frequently. Then the investors can adjust the portfolio easier and the probability of the change of the cash position is higher. To ensure the reliable of VaR, the shorter holding period should be chosen. When the market is not liquid enough, the probability of the change of the cash position is lower. Then the longer holding period should be chosen. Investors often hold the cash position in plenty of different markets and the liquidity in different markets are diverse. Then investors should set the holding period according the proportion of the portfolio.

From the results presented in the Table 5.3, within the same holding period, when the confidence level became higher, the VaR would be larger. It means the risk that the investors faced is larger under a higher confidence level. To choose and set the confidence level, three factors should be taken account.

The first one is the availability and sufficiency of the historical data. In the practical operation, historical data is often used to calculate VaR. High confidence level means a larger VaR. To ensure the reliability and effectiveness of the calculation, more historical data samples are need. However, if the confidence level is too high, the probability that the loss exceeds the VaR is low. Therefore, in the historical data, the number of samples that the loss exceeds the VaR is low. To ensure the reliability, effectiveness and calculability of VaR, confidence level should be set as a proper level according the availability and sufficiency of the historical data.

The second factor is the conveniences of the comparison and analysis. Investors always compare the risk of different financial transactions using VaR. Because the comparison of VaR under different confidence level makes no sense, the selection and set of confidence level should follow the industry standard.

The last factor should be considered is the use of VaR. If VaR is only used to compare the market risk in a company or between some companies, the confidence level is not very important. The most important thing is the whether the confidence level can ensure the reliability and effectiveness of VaR. it depends on the availability and sufficiency of the historical data. If the financial institution wants to determine the economic capital demand using VaR, the confidence level is very important. It depends on the risk averse of the institution and the costing when the loss exceeds VaR. The higher the risk averse is, the larger costing of loss is. And the capital that is needed to make up the loss is larger.

5.4.3 Results in MATLAB

At the end of the simulation, two plots are generated by the program. The first one is the price paths of Monte Carlo simulation of 9 assets, which is Figure 5-4.

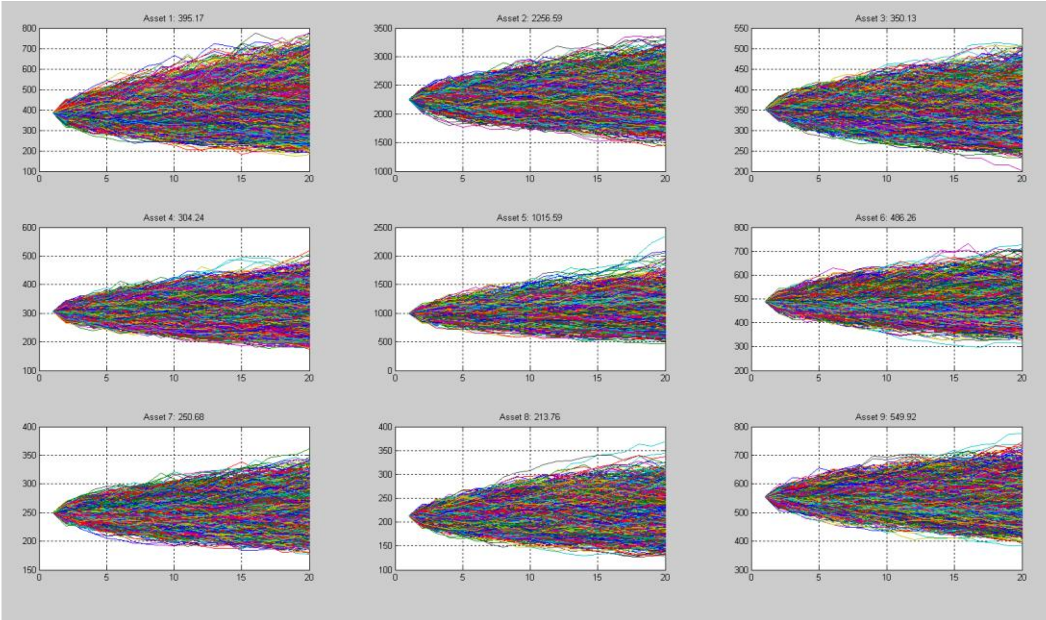

Figure 5-4: Yield paths of Monte Carlo simulation of 9 data assets.

In Figure 5-4, the paths of yield rate change of each asset over 10000 times are presented in the corresponding sub graph. For each sub graph, there are 10000 curves and each curve stands for the changing path of the yield rate in a single simulation in the 20 days. In the title of each sub graph, it shows the asset number and the mean value of all the yield rates of this asset in all the simulations. The range of the yield rate change is depended on the volatility of the historical data of the asset, which standard deviation multiplied by the square root of delta t.

Another graph is the Q-Q (quantile-quantile) plot of the historical data of the 9 assets, which is shown in Figure 5-6.

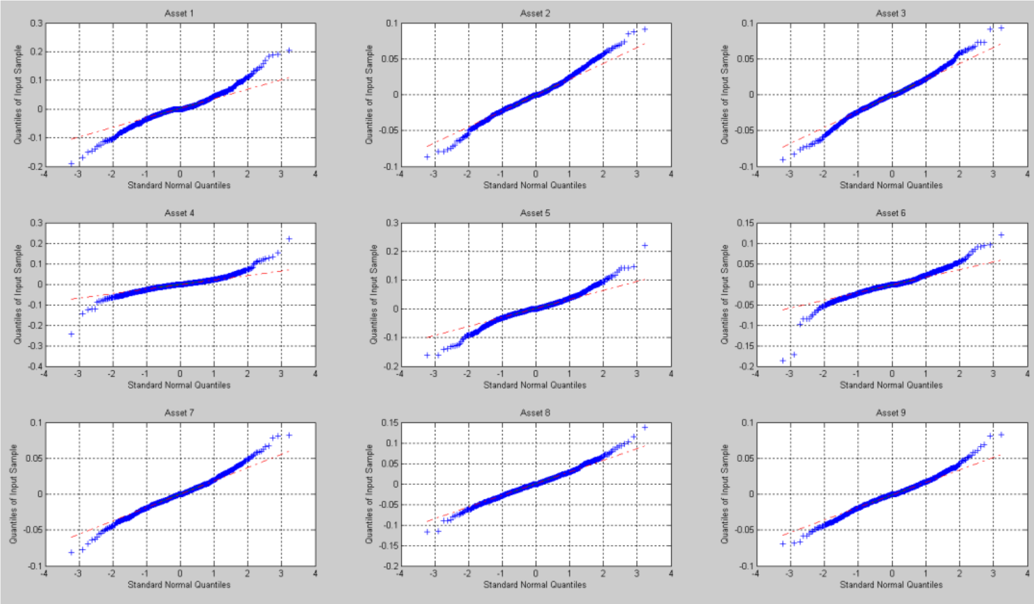

Figure 5-6: Q-Q plot of the historical data.

Q-Q plot is one of the most intuitional methods to check the normal distribution of yield rate of the samples. If the yield rate of samples follows the normal distribution, the splashes will level off to a straight line. From the plot, the distribution of the splashes presents a shape that similar to the letter s, but not a strict straight line. It means the distributions of yield rate of the samples are not strict normal distribution.

Generally, the Monte Carlo simulation is based on the assumption of the normal distribution of the yield rate and uses the standard deviation to measure the volatility. However, the realistic distribution of the yield rate is not following the normal distribution strictly. That will lead to the errors of the VaR. That is the limitation of Monte Carlo simulation.

5.5 Multithreading in MATLAB

The first chart is about accuracy. In the X axis, 1 is for 100 times, 2 is for 1000 times, 3 is for 10000 times, 4 is for 100000 times, In the Y axis, the number is the time of processing in seconds.

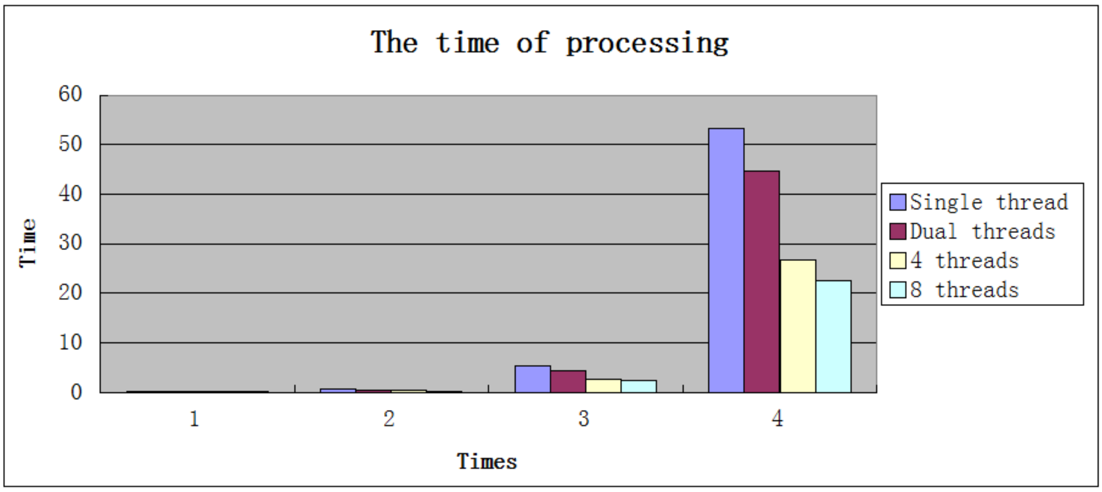

Figure 5-6: The time of processing.

In the single thread, the time of processing are 0.07s, 0.56s, 5.35s and 53.31s. In the Dual threads, the time of processing are 0.1s, 0.48s, 4.32s and 44.64s. In the 4 threads, the time of processing are 0.11s, 0.3s, 2.7s and 26.75s. In the 8 threads, the time of processing are 0.16s, 0.29s, 2.28s and 22.47s.

So we can find that in the first node, the time is unusual. Because the speed of single thread is more faster than the others. And in the other nodes, the time of processing can be expected. The more threads, the time more less. And we can find another problem is that the difference of 4 threads and 8 threads are very tiny. In the next paragraphs, I will explain the reasons.

First of all, I will explain why the time of processing in the first node is unusual. That is because there are two parts of the time in multi-threading. The first one is the time for work, and the other one is for creating the threads and distributing the tasks to the other labs. So, when the number of tasks is few, the time of creating the threads and distributing + the time of working in multi-threads > the time of working in single thread.

Secondly, I will explain why the time of processing in multi-threads are less than the time of processing in Single thread. In the mode of multi-threads, the computer will divide the tasks into several pieces so that the other thread can run it at the same time. In this method, it can save a lot of time. For example, if we have 100 tasks, each of them will cost 1 second, and the time of creating threads are 10 seconds. So the time of the single thread is 100 * 1=100 seconds. But when we use Multi-threading, such as dual threads, the time is (100 / 2) * 1 + 10 = 60 seconds. So, we can save about 40% of the original time.

After these explanations, the next part will be very important.Finally, I will explain why the difference between the time of 8 threads and the time of 4 threads is few. That is because the mechanism of Multi-threading. There is a very significant point. It is the time of threads switching. It is mean, even in Multi-threading works, the time is not a simply addition. The performance will declined after a threshold because of the too mach times of switching. And the speed of declining in Multi-threads is more faster than the speed of declining in single thread. So, I find the speed of processing is almost constant after 4 threads. And I believe the times maybe will increase after a large threads.

Figure 5-7 shows the accuracy of the multithreaded Monte Carlo simulation. In the X axis, 1 is for 100 times, 2 is for 1000 times, 3 is for 10000 times, 4 is for 100000 times, In the Y axis, the number is the VaR. Figure 4.2 shows the values of VaR in 4 testing scenarios, using 1 thread, 2 threads, 4 threads and 8 threads.

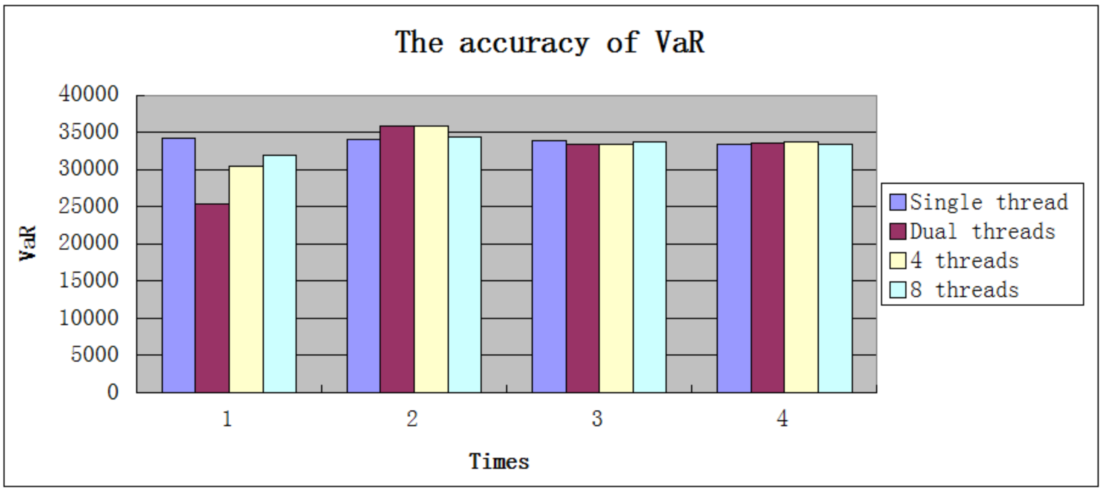

Figure 5-7: The accuracy of multithreadeded VaR.

In the single thread, the VaR are 34273, 33997, 33810 and 33412. In the Dual threads, the VaR are 25413, 35813, 33352 and 33541. In the 4 threads, the VaR are 30419, 35831, 33331 and 33694. In the 8 threads, the VaR are 31829, 34279, 33643 and 33345.

5.6 Summary

This Chapter implemented multithreading and presented the performance results of using multiple threads. As can be observed from the results, the Monte Carlo Simulation computation is speeded up with an increasing number of threads. This phenomenon is reflected from both the experimental results and the simulation results.

Chapter 6 Conclusion and Future Work

6.1 Conclusion

Traditional risk estimation methods are not appreciated by the investors and Monte Carlo simulation has been used in many investment institutions as a standard in the industry. Compare with the advantages of Monte Carlo simulation, its disadvantages are insignificant.

The function of the Monte Carlo simulation technique is very powerful, and the application is also flexible. It can be used for assumptions of different yield movements and simulation analysis of different yields distributions. Monte Carlo simulation techniques use computer simulations to generate a large number of scenarios and get a more reliable and more comprehensive conclusion comparing with the risk ratio analysis method when measuring the risk. In addition, Monte Carlo simulation method is a full-value estimate, which reflects the convexity of the non-linear assets and effectively solves the difficulties of the analytical methods in handling non-linear and non-normality problems.

In this thesis, three types of traditional risk estimation model were studied and compared. Monte Carlo simulation was employed empowered with multithreading techniques for speed up in computation. Both experimental and simulation results were analysed and discussed.

6.2 Future Work

Because the fluctuations of prices of the financial products are random and different assets have different types of fluctuations, it is difficult to be indicated by a single type of model. The choice of model will lead to a certain number of errors which needs to be addressed.

Multithreading was employed for speeding up the computation of Monte Carlo simulation. However, multithreading works for single computers. For a large scale Monte Carlo simulation, working on big data [27], scalable computing technologies such as MapReduce model would be considered [28].

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