Employees Provident Fund (EPF) Malaysia: Generic Models for Asset and Liability Management Under Uncertainty

A thesis submitted for the degree of Doctor of Philosophy

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Abstract

We describe Employees Provident Funds (EPF) Malaysia. We explain about Defined Contribution and Defined Benefit Pension Funds and examine their similarities and differences. We also briefly discuss and compare EPF schemes in four Commonwealth countries. A family of Stochastic Programming Models is developed for the Employees Provident Fund Malaysia. This is a family of ex-ante decision models whose main aim is to manage, that is, balance assets and liabilities. The decision models comprise Expected Value Linear Programming, Two Stage Stochastic Programming with recourse, Chance Constrained Programming and Integrated Chance Constraints Programming. For the last three decision models we use scenario generators which capture the uncertainties of asset returns, salary contributions and lump sum liabilities payments. These scenario generation models for Assets and liabilities were developed and calibrated using historical data. The resulting decisions are evaluated with in-sample analysis using typical risk adjusted performance measures. Out- of- sample testing is also carried out with a larger set of generated scenarios. The benefits of two stage stochastic programming over deterministic approaches on asset allocation as well as the amount of borrowing needed for each pre-specified growth dividend are demonstrated. The contributions of this thesis are i) an insightful overview of EPF ii) construction of scenarios for assets returns and liabilities with different values of growth dividend, that combine the Markov population model with the salary growth model and retirement payments iii) construction and analysis of generic ex-ante decision models taking into consideration uncertain asset returns and uncertain liabilities iv) testing and performance evaluation of these decisions in an ex-post setting.

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List of Abbreviations

ALM	Asset Liability Management
BPAM	Bond Pricing Agency Malaysia
BVAR	Bayesian Vector Autoregressive
CCP	Chance Constrained Programming
CD	Certificate of Deposits
CPF	Central Provident Fund, Singapore
CSaR	Conditional Surplus at Risk
CVaR	Conditional Value at Risk
DB	Defined Benefit
DC	Defined Contribution
EEV	Expectation of Expected Value
EPF	Employees Provident Fund
EQT	Equity
ETW	Expected Terminal Wealth
EVLP	Expected Value Linear Programming
EVPI	Expected Value of Perfect Information
FM	Fixed Mix
FMA	Financial Market Authority Austria
GARCH	Generalized Autoregressive Conditional Heteroscedasticity
GDP	Gross Domestic Product
GLC	Government Linked Company
HN	Here and Now
ICCP	Integrated Chance Constraints Programming
LDI	Liability driven investment
LP	Linear Programming
MA	Moving Average
MGS	Malaysian Government Securities
MGS10	Malaysian Government Securities-long term
MITB	Malaysian Investment Issues and Malaysian Islamic Treasury Bills
MMI	Money Market Instrument

MSP	Multistage Stochastic Programming
MTB	Malaysian Treasury Bill
PDS	Private Debt Securities
PF	Provident Fund
PROP	Property
RM	Ringgit Malaysia
SOCSO	Social Security Organization
SP	Stochastic Programming
TSP	Two Stage Stochastic Programming
VaR	Value at Risk
VAR	Vector Autoregressive
VECM	Vector Error Correction Model
VEqCM	Vector Equilibrium Correction Model
VSS	Value of Stochastic Solution
WS	Wait and See

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Chapter 1

Introduction and Problem Context

1.1 Introduction

Demographic changes affect the social and economic performance of the world. Current trends of demographic like decreasing fertility rates, decreasing mortality rates and increasing life expectancies are causing an "aging" population, where the proportion of elderly people in the total population is increasing (Giang, 2004). In the year 2000, less than 1 in 10 people were over 60 years old, but, by the year 2050, the approximation is, 1 in every 5 people will be over 60 years old (United Nations, 2000). Take the example of countries such as Japan, which is one of the accelerating aging nations in the world; the ratio of the population under the age of 20 to population over the age of 65 years, in the year 1950, was 9.3 however based on a forecast done, this ratio will decrease to 0.59 people under 20 for every person older than 65 (United Nations, 2000).

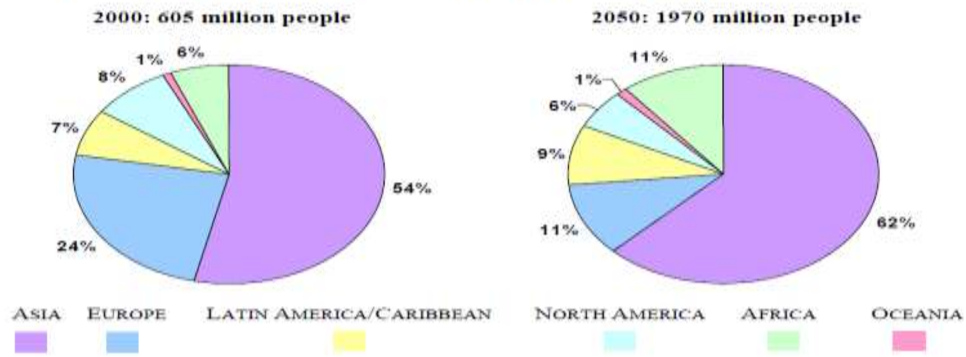
In Asia, the process of the aging population occurs faster compared to the Western countries. This issue has been highlighted by several sources, like Creighton et al., (2005), Westley and Mason (2002) and the World Bank (2007). Figure 1.1 shows the geographic distribution of population aged 60 and above in the year 2000 and the estimated distribution for the year 2050. Table 1.1 presents the projected growth of Asia's elderly population (people age 65 and above) from year 2000 to 2050.This data is taken from the article by Westley and Mason (2002); the estimation of people age 65 and above in Asia increases to reach 314% by the year 2050**.**

Table 1.1: Projected Growth of Elderly Population in Asia

	Population from the age 65 and above (1,000s)			Percent increment (2000-2050)
	2000	2025	2050	
Asia	206,822	456,303	857,040	314

Source: Westley and Mason (2002)

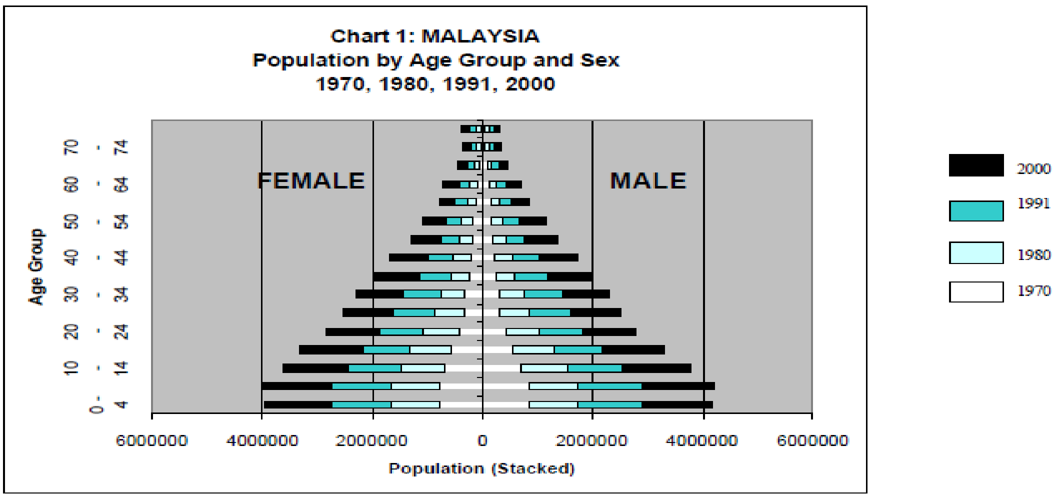
Figure 1.1: Geographic (Estimated) Distribution of Population Aged 60 and Above



Source: Creighton et al. (2005)

Malaysia is not spared from this phenomenon. Just like any other developing countries there is a demographic change in Malaysia resulting from improved health, rising life expectancy, decreasing mortality rate as well as fertility. Figure 1.2 shows the Malaysian population for both male and female compared to the age group. This figure shows the result of the census done from year 1970 to the year 2000. The population of people that are 15 years and below (younger age groups) decreased while the proportion of elderly is increasing. In the year 1970 the median age was recorded at 17.4. The median age increased to 21.9 in the year 1991 and the projection for the year 2020 is 27.1 (Mat and Omar, 2002). We can see that the median age rises by an average of 1.7 per decade in 30 years' time span (from 1991 to year 2020).

Figure 1.2: Malaysia Population by Age Group and Sex



Source : Mat and Omar (2002)

The elderly population doubled from the year 1980 to the year 2000, and the value of the elderly population is expected to increase to 3.4 million by the year 2020 (Ong, 2001). The old age dependency ratio that can be described as the number of the elderly population over the number of working age population is anticipated to rise to 15.7 in year 2020 from 10.5 in the year 1970. At present, Malaysia's population has not entered the aging population level even with the fast descent in the proportion of the young population age 0 to 14 years and a decline in the proportion of the older population age 65 years and above as compared to those in employment age population (15-55) years. Malaysia will only reach the matured or aging population level by the year 2020. The forecast shows that 9.5% of its population will be in the age group 60 and above (Ong, 2001). Although the rate of Malaysia's aging population increment is not as worrisome as other Asian countries like China, Japan and Singapore, the increase in the elderly population would make it difficult for the government to avoid the trouble they are going to face and the need to overcome the problems associated with social and economic changes due to population aging.

Caring for the aging population requires massive public expenditures to support pensions in both the state (public) pensions as well as private pensions and health care. Most of the pension plans introduced were based on the idea that the government or employers through contributions to a pension fund are able to provide the pension benefits when they fall due. Most pension schemes did not consider the impact of longevity risk. This may prove to be an unforeseen disaster to government budgets and long term fiscal sustainability of governments. Longevity, that is the increase in life expectancy, leads to a higher number of retirees withdrawing pension benefits from the pension funds to which only working persons are contributing. This causes deficits, especially in Defined Benefits (DB) pension plans. The DB Plan, also known as the final salary scheme, is a retirement account that defines the amount of pension income based on the length of employment and the final salary at retirement; the life annuity is paid monthly, from the retirement time until the time when the retiree dies.

In many countries, pension plans are shifting from DB towards the defined contribution (DC) schemes due to the challenges that arise from implementing DB pension funds. Another name for the DC pension plan is "money purchase" scheme. DC is a pension plan where members and employers contribute a fixed percentage of salary determined by contractual agreement on a monthly basis. The benefits (the amount of income received on retirement) vary depending on the accumulated contributions, the returns generated from investments in various assets as well as the price of an annuity at retirement. Some DC schemes guarantee an annual minimum return or dividends. Although a DC scheme could eliminate the issue of sustainability, DC scheme does not provide benefit payments throughout the lives of retirees. In some DC plans, the benefits are paid to participants as a lump sum at retirement, causing the pensioners a high risk of outliving their retirement saving.

Some of the most serious challenges that governments around the world need to address with respect to pension funds are (a) the best way to meet the needs of the elderly, and (b) ensure that the pension strategies which are implemented do not burden the younger generation and weaken the economic growth of a country (Rozinka and Tapia, 2007). In order to address the above challenges, it is essential to understand and compare pension systems or practices of other countries' retirement schemes.

1.2 Pension Funds in Asia and Employees Provident Fund

Britain introduced the pension concept in its former colonies in Asia. There were two different types: one was a DB pension scheme for civil servants and the other was a provident fund (PF) for the private sector employees (Lindeman, 2002). Employees Provident Fund (EPF), the first mandatory national provident fund, was founded in October 1951 and was firstly introduced and implemented in Malaysia (Thillainathan, 2000). PFs are still the main source of retirement income for private sector employees in Malaysia, Singapore, India and Sri Lanka at the present time. A comparative study has been carried out by the author which examines and contrasts various fund management policies. This study is given in Appendix D.

This research concentrates on the EPF of Malaysia which is a DC pension fund. EPF is mandatory for the formal private sector employees. Self-employed as well as employees in the informal sector can choose to become a member or not as it is not compulsory for them. The total number of active members in the year 2010 amounted to 6.04 million (EPF annual report 2010). Total accumulated assets including inactive members as at 2010 were recorded in Ringgit Malaysia (RM) 440.5 billion (£88.1 billion). On the 31st of December 2002, the EPF was the 20th largest pension fund in the world and ranked as the eighth largest pension fund in Asia (Ibrahim, 2004).

The main objective of EPF Malaysia is to provide for post-retirement securities through monthly compulsory saving for participants. The EPF was governed by the Employees Provident Fund Act, 1951, that was later substituted by the Employees Provident Fund Act, 1991 (Laws of Malaysia Act 452). EPF Board has representation from the Government, employers, employees and professionals. The investment panel is separated from the Board and report directly to the Ministry of Finance. Investment panel members are The CEO of EPF, Chairman, one representative from the Ministry of Finance, one representative from Malaysia Central Bank (Bank Negara Malaysia) and three Malaysian citizens that are experts in the field of finance and investment. The detail explanations of the EPF Malaysia scheme are explained in depth as follows.

i) Contributions and Accounts

EPF Malaysia holds and manages a large amount of assets that accumulate from the compulsory monthly contributions collected from participants and their employers. As of the end of 2010, the mandated contribution rate is within the range between 8% (minimum) to 11% of each member's monthly salary, while employers are obligated to contribute another 12% of an employee's salary to top up the members' savings.

EPF members' savings consist of two accounts. The first account, Account I, contains 70% of the members' monthly contribution, while the second account, Account II, stores 30%. Account I is for retirement; withdrawals from this account are restricted to a member that reaches the retirement age (55 years old), is deformed, leaves the country or passes away. Pre-retirement withdrawals from savings from Account II are permitted for active participants subject to the country's laws in respect of EPF.

ii) Pre-retirement Withdrawals

Over the years, other benefits beside retirement were added for participants. Active participants are allowed to withdraw for purposes like home ownership, children's education and health care. These withdrawals are allowed so that participants can balance their income to consumptions, especially during critical times in their lives; in some ways it forces individual to pursue financial planning (Dempster and Medova, 2011). These early withdrawal schemes are 'Life Cycle' benefits - a life span budget constraint dilemma in which individuals need to make the decision to withdraw from EPF when the need arises from being aware of their future salary, pension age and life expectancy (Adams and Prazmowski, 2003).

Most people rely on their monthly salaries as the main source of income; however, income varies and grows with age and experience. There are times when an individual needs to protect the household from unexpected emergencies, especially during the initial phase of life when consumptions are higher than incomes. Unlike income, expenditures decline with age. For employees with uninterrupted career, their expenditures will start to decline when their children finish school and leave home (Adams and Prazmowski, 2003). Based on this idea EPFs allow members to withdraw from their own account but subject to eligibility, as well as terms and regulations.

The detailed explanations on the pre- retirement withdrawals allowed by EPF Malaysia are as follows:

(a) **Housing Withdrawals**

Members can use the housing withdrawals to finance the purchase of a house, build houses or reduce their outstanding housing loans for their first house. For all the three purposes it can either be used individually or by sharing with a spouse or close family members. If the purpose of the withdrawal is for a second house, it can only be done after the first house purchased using EPF has been sold or disposal of ownership of property has taken place.

(b) Medical Expenses for Health Care Withdrawals

Members may utilize the medical expenses for health care withdrawals to remunerate the expense of members' medical treatment or to purchase medical aid equipment for family members such as parents, spouses or children who may have critical illnesses. However, the entitlement to withdraw from the medical expense savings is not valid if employers fully borne the employees' medical expenses.

(c) Education Withdrawals

EPF members can extract their savings to pay for their education fees or their children's education at diploma level and above at any authorised Institution of Higher Learning in Malaysia or abroad.

(d) Pensionable Employees Withdrawals

Public sector employees who choose to receive government pension scheme instead of EPF, may withdraw their contributions and dividends accrued from the contributions after returning the government share (contributions to EPF) to the Retirement Fund (Incorporated). Government servants who choose an early pension from the public sector are given the opportunity to withdraw all their share of the savings during their tenure of service. We will not include the pensionable Employees Withdrawals in our model.

(e) Investment Withdrawals

Starting from February 2008, EPF allows members to invest the excess savings amount in Account I. The Basic Saving amount is a specific amount of saving in Account I that is predetermined by members' age levels. The purpose of the basic saving amount in Account 1 is to enable participants to have a minimum saving amounting to RM 120 000 at retirement. There is an increase of investment withdrawals due to the higher number of members eligible. However, this investment withdrawal will not be considered in this work as it is not compulsory and there are a limited data, as it was only introduced in the year 2008.

(f) **Withdrawals for Saving Exceeding RM 1 million**

Another type of pre-retirement withdrawal that is available is the withdrawals for saving exceeding RM 1.05 million. Starting from year 2008, members who are eligible can withdraw the excess amount of savings and invest as well as manage it on their own. The RM 1.05 million does not include the savings that are invested by external fund managers (investment withdrawals) or the insurance companies for annuity. Eligible members can withdraw not less than RM 50 000 once in every three months.

iii) Retirement Withdrawals

On reaching retirement age, the balances from both accounts are merged and can be withdrawn. Members are to decide on one or a combination of the three alternatives of withdrawal options listed below.

a) **Lump sum Option**

The lump sum is a single payment of total accumulated wealth during employment received at retirement. This is the most popular choice for EPF members, especially those who need cash to pay for medical expenditure, other social obligations as well as, those who do not expect to live to an advanced age after retirement. Not many individuals are able to manage the lump sum especially with the higher rate of longevity.

b) **Phased Withdrawals Option**

Phased withdrawals can be a fixed amount at a fixed time (monthly withdrawal, annual withdrawals) or irregular withdrawals whereby the pensioner draws down a part of the accumulated wealth (and continued investment earnings). The remaining amount of the retirement capital in the retiree’s account at his/her death belongs to the beneficiaries. If a pensioner lives long after retirement, there is a likelihood of the retirement payments becoming inadequate in the later years. Under other type of withdrawals, there is also the risk of the wealth being completely exhausted before death. This option is recommended for participants that are unable to choose an annuity due to the accumulated balances are insufficient.

c) **Annuity**

An annuity offers a fixed payment/income, given to the retirees for life or for a fixed period. There are life annuities that offer additional guarantees, such as continued payment to the surviving spouse, or lump sum to the next of kin in the event of death. There are many options of the annuity product. However for EPF members, the monthly annuity option is only available for contributors that have at least RM 12,000 (£ 2400) in his/her balances; the monthly amount payable must not be less than RM200, and payments are made for at least 60 months (www.kwsp.gov.my). If the contributors die prior to retirement, the legal beneficiaries would receive the entire sum accumulated in both accounts. Full withdrawal is also permitted on account of permanent impairment and permanent emigration from the country.

Even though annuities protect against longevity risk, there are disadvantages as highlighted by Walliser (2000). Firstly, there is no cash available in the event that pensioners need to pay for large amounts during emergencies like paying for medical bills. Secondly, the retirees are unable to benefit from the higher rate of return as the amount of the annuity is fixed. Thirdly, they are unable to protect from inflation risk as the amount is not indexed. Individual annuities are not well developed especially in developing countries (Gokhale et al., 1996).

iv) Investments

The EPF can only invest in assets as stated in the EPF Act 1991 (www.epf.gov.my). EPF invests in five instruments; MGS, loans and bonds, equities, properties and money market instruments. In this research we assumed that EPF only invest locally. Although this had been the practice of EPF since it was first introduced in the year 1951, in November 2005, the Malaysian Government gave the approval for EPF to invest RM1.9 billion (£0.38 billion) - which is less than one per cent of its assets abroad.

The objective of investments is to ensure a return as per guaranteed (The EPF Act states that the EPF’s yearly dividend rate must not fall below 2.5%). EPF has to ensure that the fund is capable to meet liabilities (withdrawals) by members as and when they require to do so (Ibrahim, 2004). As the EPF’s main role is to protect the savings and retirement funds of Malaysians, investments are conservative and income-based. High risk leads to major challenges especially in the event of slow economic growth. Investment is also subject to quantitative asset restrictions (upper and lower bounds). In this research, we include the upper bound as well as the lower bound to determine the permissible minimum and the maximum amount of investment for each asset class.

Table 1.2 : EPF Malaysia’s Investment Portfolio

Assets	Variation (%)
Money Market Instruments	5 to 25
Malaysian Government Securities	15 to 35
Equities	10 to 25
Loans and Bonds	15 to 35
Property	0 to 10
International Investments	0 to 20

Source: Ghaffar (2005)

Table 1.2 shows the EPF Malaysia’s investment portfolio guidelines based on Ghaffar’s (2005) article. We took the current EPF’s asset allocation (year 2010) as the guideline in determining the upper and lower bound of each asset class.

Table 1.3 shows the asset allocation of EPF Malaysia in the year 2010 as well as the percentage of the total asset group. As we are not going to consider International Investments we exclude that data. The main difference between table 1.2 and 1.3 are firstly the percentage for Loans and Bonds which exceeded the upper bound suggested by Ghaffar (2005) and the percentage of investment in Property was too small as compared to the lower bound of the strategic asset allocation suggested by Ghaffar.

Table 1.3: EPF Malaysia's Asset Allocation for The Year 2010

Assets	Asset Allocation (RM)	Percentage (%)
Money Market Instruments	23 987 350 000	5.4
Malaysian Government Securities	118 517 140 000	26.9
Equities	153 531 270 000	34.9
Loans and Bonds	142 613 960 000	32.4
Property	1 867 480 000	0.4

Section 26 of the EPF Act 1951, stated that the EPF was required to invest 70 per cent of its funds in MGS (Ibrahim, 2004). However, the percentage declined in 1991 where only 41 per cent of the total assets were invested in MGS. With continued strong economic growth and the Malaysian Government commenced on privatization programs, there were decline in the issuance of MGS. In 1991, there were amendments to the EPF Act. Based on the present regulation, EPF needs to invest not less than 70 per cent of its total holdings in MGS but with the decline in new issues of MGS, the Ministry of Finance waived the requirement and 70% should include other instruments that are equivalent to MGS. The regulations for investment of EPF assets specify that (a) at least 70% must be invested in a low-risk fixed income instruments (MGS, loans and bonds and money market instruments) and (b) the amount invested in domestic equity must not exceed 25% (Act 452, EPF Act 1991).

The percentage of equities in Table 1.3 exceeded 25% as required in the EPF Act. However, in our model we consider the upper bound for equity to be 25%. The high percentage of investment in equities in the year 2010 is due to EPF taking advantage of the good performance and high returns on investment with the consent of the Ministry of Finance. For the year 2010, a total of RM10.94 billion (10.59%) was earned from equities.

EPF updates investments and pays dividends to participants on a yearly basis. Below are listed the descriptions of the assets considered in the research.

a) **Money Market Instruments (MMI)**

EPF, government, statutory agencies, banks and major corporations invest in MMI in order to raise funds. The money market is a place for trading of medium and short-term instruments. MMI is the most active market for securities as measured by daily trading volumes. Due to the highly liquid nature and short maturities, investors consider MMI as a safe place to invest. However, risk for example risk of default on securities sometimes happens. Money market securities include negotiable certificates of deposit (CDs), bankers' acceptances, Treasury bills, commercial paper, municipal notes, federal funds and repurchase agreements as listed and explained in www.bnm.gov.my.

b) **Equity**

EPF invests in long-term shares quoted on the Malaysian stock exchange (Bursa Malaysia). The allocation for equities was very small until the year 1993 where 10% was allocated for equities. The proposition was increased gradually and reached 20 per cent of the total fund in the year 2004. The rise in the equity allocation was mainly due to the demand for an avenue of investment after the decline in MGS issuance and the growth of EPF. The returns from equities have been high, same goes to the level of risk due to market uncertainty. So far EPF has been able to cope with the ups and downs of its equity investments.

c) **Malaysia Government Securities (MGS)**

Bank Negara Malaysia issues and manages MGS on behalf of the Government of Malaysia. The issuance of MGS is by auction and subscription. MGS are marketable debt instruments to escalate funds from Malaysia capital market to finance long term government development projects. MGSs are held to maturity as they are risk free. MGS were first issued by the Treasury in 1959, with the purpose to meet the investment needs of EPF, and other financial companies in Malaysia such as insurance companies and banks. By the late 1970s and early 1980s the reasons for MGSs issuance included financing public sector development programs. In the 1990s, MGS were used to fund the Government's shortcoming budget and payment of some of the Government's foreign debts. However during the years of fiscal surplus (1993-1997) MGS were issued to accommodate the market need for MGS. The different types of MGS available, taken from bnm.gov.my are tabulated in Table 1.4.

Table 1.4 : Different Types of MGS available in Malaysia

1.	Malaysian Government Securities (MGS)	Long term interest bearing bonds to raise funds from the domestic capital market for development expenditure.
2.	Malaysian Treasury Bills (MTB)	Short term securities for working capital
3.	Government Investment Issues and Malaysian Islamic Treasury Bills (MITB)	Long term and short term non-interest bearing Government securities which are issued based on Islamic principles.

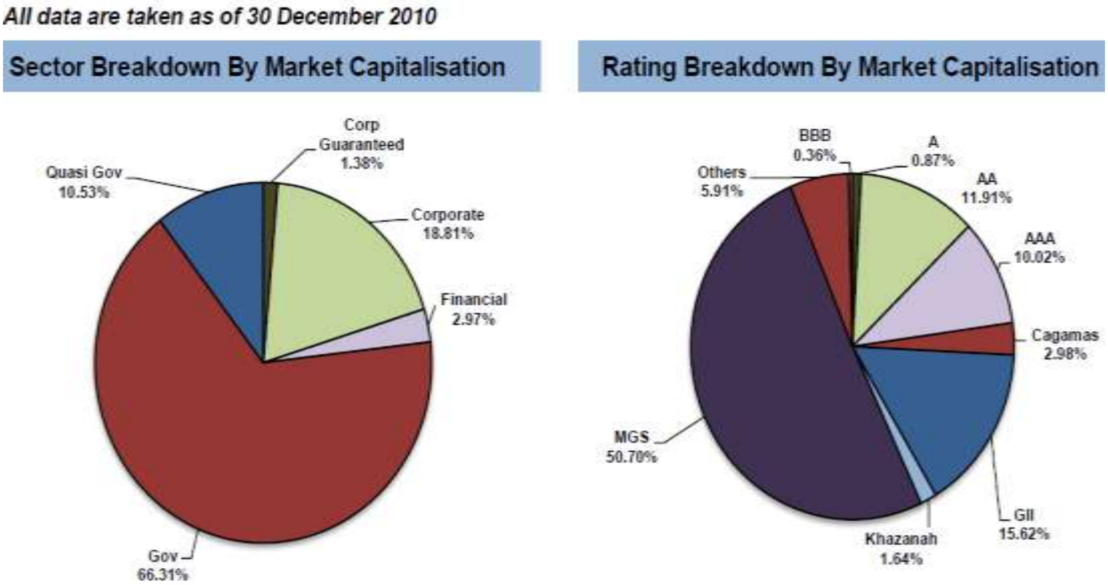
Source : bnm.gov.my

d) **Loans and Bonds**

Loans and Bond are mainly long term investments guaranteed by Government or Government linked entities. The loan portfolio can either be in the form of direct loans or in private debt securities (PDS). Bonds and PDS are low risk assets as they are assured by the Government and banks. From Figure 1.3, it can be seen that within this asset class 67.24 percent was allocated in the public sector whilst 32.76 percent was invested in the corporate sector. EPF has also increased its investment in Islamic bonds. The main issuers of Government debt are the Government of Malaysia, the central bank (Bank Negara Malaysia), and government linked institutions such as Khazanah, Danamodal and Danaharta. Private debt securities and asset-backed securities are issued by the National Mortgage Corporation (Cagamas Berhad), financial institutions and non-financial corporations (bnm.gov.my).

The EPF is one of the major investor in the Malaysian bond market. In order to maintain a low risk profile, EPF continue to invest the majority of its loans and bonds in high grade companies with credit ratings of AAA or AA. Islamic bonds are just like conventional bonds, Islamic Bonds always have a fixed term maturity. Islamic bonds are structured based on Islamic law (Syariah) whereby the issuance is not an exchange of paper for money consideration with the imposition of interest, but it is based on an exchange of an asset that allows investors to earn profits from the transactions (www.bondinfo.bnm.gov.my). Syariah law is also applied in terms of the approval of assets as well as the contract of exchange. The data for Islamic bonds (EPF) are only available from year 2007, therefore, we do not include Islamic Bonds in our research due to lack of data.

Figure 1.3: Malaysian Bonds and Loan Sector Breakdown and Rating Breakdown by Market Capitalisation



Source: http://www.bpam.com.my/

e) **Property**

The EPF's property investment is mainly on business complexes and purchase of tenancies by acquiring buildings that offer favourable yields. Property investment was at 0.4 per cent of EPF's assets, as at the end of year 2010. Although this is a small percentage, in Ringgit terms it is worth about RM1.87 billion (£ 0.374 billion).

Table 1.5: Summary of EPF Malaysia Scheme

	EPF (Malaysia)
Contributions	Employers' share – 12% of employees' salaries Employees share -8-11% of employees' salaries as at 2008 (Total – 20 to 23% of employees' salaries)
Accounts	Account I – 70% of total contribution Account II – 30% of total contribution
Investment decision by employees	Members are allowed to use their Account II EPF savings in their own investments.
Withdrawals	Account I – at retirement. Accountant II – can be withdrawn for the purpose of financing of housing (monthly housing loan payment), Health and Education.
Medical Expenses for Health Care	If the need arises -withdraw from Account II during pre-retirement. However post- retirement health benefit is not covered.
Annuity	Annuity only eligible if one has the minimum amount RM 12 000. Minimum annuity period (payment) is 60 months.
Asset Class	Money Market Instruments, Malaysian Government Securities, Equity, Loans and Bonds and properties.
Income tax	Both contributions and returns are tax free.
Insurance	NA

1.3 EPF Problem Formulation

Due to the growth of DC pension plans around the world, a large part of the world population depends on DC fund during retirement. In DC pension schemes, the uncertainty in the level of retirement income is high. With a fixed age of retirement, it is difficult to predict the amount of pension income that a participant will receive under a DC pension plan. Governments or pension providers promise a minimum pension income to protect from the uncertainty in retirement income. In the event that this amount is not met, Governments or pension providers need to take necessary action to meet the liabilities. In order for a DC pension plan to serve as a primary pillar of retirement savings, it would need to provide an adequate level of salary replacement during retirement.

In the financial industry, mutual funds and pension plan managers generally wish to control their long-term risk/return profile of their assets and liabilities; they wish to obtain acceptable returns while keeping only the risks to which they want to be exposed. The main source of risk for mutual funds and pension plans arises from long-term fluctuations in market prices and rates. In addition, pension plans and insurance companies have long-term liabilities in the form of annuities or insurance claim payments. As these institutions seek to match those liabilities with their assets, they become exposed to long-term market risks, making long-term risk management imperative for running their operations.

The main challenge faced by EPF is to provide members with a sufficient balance to last during retirement. Accumulated wealth in participants' accounts at retirement determines the retirement income especially if EPF is the only source of income. The high cost of living associated with longer life expectancy and the effect of inflation will increase the risk of outliving the savings. Unlike the situation in Malaysia, the pensioners in developed countries are financially well protected in their old age. Inadequacy arises for Malaysian citizens, particularly EPF participants, because of many factors. One of them is the current system that allows pre-retirement withdrawals for benefits such as mortgage, education and health. Another factor is the low investment returns. The ratio of EPF assets to Gross Domestic Product (GDP) (the total value of final goods and services produced within a country in a given period) for Malaysia is high due to high monthly mandated contributions from members. However, domestic financial markets are quite limited, and the capital markets in Asia including Malaysia are underdeveloped (Thillainathan, 2004). In most countries, EPF only invests locally and the investment is strictly regulated and weighted heavily towards public sector products and developments that further contribute to lower rates of return. Upon retirement, most provident funds pay out a lump-sum benefit, leaving the pensioner at risk of outliving his/her income. Based on a survey done in Malaysia, most retirees, especially the lower income group, spent the total lump sum withdrawn at retirement within 5 to 10 years and 60% of respondents depended on their children to survive during their retirements (Ibrahim, 2004).

Based on the above challenges Pension Fund providers, especially EPF, need to make dynamic decisions with multiple goals in order to satisfy stakeholders in term of regulations, investments, risk, etc. and at the same time ensure that the decisions implemented would not be detrimental to the economy of the country and the personal welfare of participants. One important aspect of the pension fund is the management of the investment portfolio. Long term sufficient investment returns are needed to cope with liabilities and at the same time maintain its working capital adequately.

1.4 Asset and Liability Management (ALM)

ALM is a mathematical tool that is used to address the integrated management of assets as well as liabilities of pension funds, insurance products, bank loan bookkeeping and Hedge funds (Ziemba and Mulvey (1998), Zenios and Ziemba (2007), Mitra and Schwaiger (2011)). ALM models also have been created for University Endowment Funds (Merton , 1991) and wealthy individuals (Ziemba (2003), Medova (2008)).

Most of the existing ALM models applied in Pension funds are designed for DB funds. The liabilities (benefits to participants) of DB plans are guaranteed prior to knowing the uncertain outcomes of future long investment horizons. The employers are responsible to fulfil the agreement even if the assets are short of the promised benefits. Among ALM research applied to DC plans are the Asset-liability management for Czech pension funds using stochastic programming formulated by Dupacová and Polívka (2009), DC pension fund benchmarking with fixed-mix portfolio optimization (Dempster et al., 2007) and InnoALM model by Geyer et al. (2008). However, besides DC pension fund, the InnoALM model is also applicable to DB pension funds.

1.5 Methodology

In this research, we formulate an Asset-Liability Management (ALM) model using deterministic or expected value (LP) as well as Stochastic Programming (SP) as a tool in the decision making process applied to EPF Malaysia. Schwaiger (2009) in her thesis implemented decision models using Linear Programming (LP), Two Stage Stochastic Programming (TSP), Chance Constrained Programming (CCP) and Integrated Chance Constrained Programming(ICCP) for a DB Pension Fund where the objective function is to minimize the deviation of the present value matching of the assets and liabilities over time. We are going to extend her research by applying decision models to EPF Malaysia, a DC pension scheme with different objective function, and include the asset classes that are important to EPF Malaysia.

Our decision models integrate interdisciplinary work that combines operations research, finance, mathematics, and Econometrics. The ALM models presented and formulated in this research are tailor made to EPF Malaysia. In many respects the models resemble those presented in the literature, but we include the unique features stemming from the Malaysian economic situation and statutory restrictions for EPF. Among the unique features are the pre-retirement withdrawals for housing, health and education (one of the uncertainties that no mathematical model has captured), minimum dividend (promised), different choices of income drawdown offered by the scheme. Another unique feature of EPF is the rule that excess amount of Basic Savings in Account I can be invested into the external fund managers approved by the Ministry of Finance. We are not going to model all the unique features due to the lack of data as some regulations have only recently been implemented. Refer to section 1.6 for the qualitative summary of the EPF scheme that we take into consideration in our ALM model.

We address interrelated areas in ALM including scenario generation for assets returns and liabilities, the population model and pension cash flows. Mathematical scenarios model of future change in pension fund assets and liabilities is generated independently. We generate a “fan” scenario tree with 2000 scenarios for each time period. We captured the uncertainties of the returns for each asset class. Following this, we implemented a Markov model to quantify the future population and their states (active participants, retired, etc.). We compute the future salary growth for active members. By combining the Markov population model with the salary model we gain the amount of annual contributions to EPF by active members. We calculate the lump sum liabilities as well as the pre-retirement withdrawals and use these uncertainties data as the input to the stochastic models. The average values of these scenarios are used as the input to LP model.

1.6 Summary of EPF characteristics and the ALM framework

We present a summary of the EPF characteristics of which we are going to use in the decision models.

The contributions for active participants are fixed at 23% of participants' salaries.

The pre-retirement liabilities considered are housing withdrawals, education withdrawals and health withdrawals.

EPF only invests locally and the funds are invested into 5 types of assets: the money market instruments, Malaysian Government Securities (long term), Malaysian Government Securities (short term), property and equity.

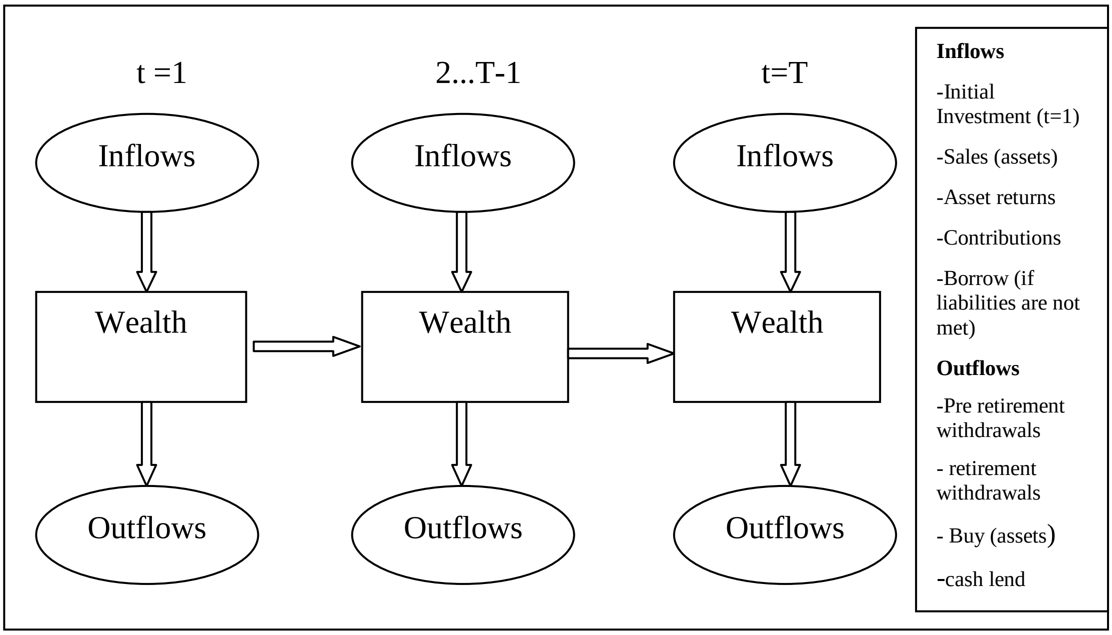
There is a restriction (upper and lower limit) for investment in each of the five asset classes mentioned in no 3 above.

Lump sum withdrawal of the accumulated wealth at age 55 (retirement age) is compulsory for all participants.

A lump sum equivalent to a member's accumulated wealth will be paid to next of kin if death occurs.

Figure 1.4 shows the ALM framework for EPF. The inflows (assets) include the contributions from both employers and employees, assets sold, asset returns, the initial investment in the first time period as well as the amount borrowed in the event the liabilities are not met. The outflows (liabilities) include retirement withdrawals, pre-retirement withdrawals, cash lent and assets bought.

Figure 1.4: ALM framework for EPF



With the objective function of maximising wealth, we report the optimal investment strategy as well as the impact of different percentage of dividends (growth dividend) towards the amount of borrowing (due to underfunding) needed by EPF.

1.7 Guided Tour

This thesis consists of seven chapters. In Chapter 1, we have discussed the current demographic trends and the impact of the population aging towards pension funds. We have introduced the available pension funds in Asia as well as in Malaysia and described the DC pension system of EPF Malaysia. We have also explained about ALM, the problem formulation, the methodology, and scope of our research.

Chapter 2 gives a detailed background of the solution methods that include linear programming and family of stochastic programming.

In Chapter 3 we describe the process of scenario generation for the uncertain parameters of our models that include assets return, liabilities (lump sum, pre-retirement), the population model, the salary growth for each age group and contributions (cash flow) from participants.

In Chapter 4 we present deterministic ALM model (Linear Programming) for EPF Malaysia, obtained by superseding the uncertain parameters with their expected values. The objective function is to maximise the terminal wealth. We consider a fixed mix portfolio as a benchmark for performance evaluation.

In Chapter 5 we formulate Two Stage Stochastic Programming (TSP) models for EPF Malaysia, in which we consider scenarios for the uncertain parameters obtained as described in chapter 3. One model is a TSP with recourse in which we maximize the expected terminal wealth. The feasibility is insured by considering borrowing and lending of cash variables for each time period and scenarios. The other models are the Chance Constrained Programming (CCP) and Integrated Chance Constraints Programming (ICCP) with the objective function to maximize the expected terminal wealth.

In Chapter 6 we apply decision evaluation techniques to test the results of the ALM decision models presented in Chapters 4 and 5. We used in sample analysis and out of sample testing. The data used for in sample analysis is as described in Chapter 3. A larger data set is generated for the out of sample analysis. For each time period we calculate the standard deviation, Sharpe ratio, Sortino Ratio, Solvency ratio and Funding Ratio. These computations are widely understood as risk-adjusted performance measurements in the industry by fund managers.

In Chapter 7 we present our findings in a summary form and outline the contributions of our research and the future research directions.

Chapter 2

Solution Methods

In this chapter we introduce the formulations of linear programming (LP), stochastic programming models (SP), that include the two-stage stochastic programming (TSP), chance-constrained programming (CCP) and integrated chance constraint programming (ICCP). These solution methods are used to solve our ALM decision models.

2.1 Linear Programming

LP is an established mathematical program. The objective functions along with the constraints are all linear.

The representation of LP model following the notation used by Dantzig (1955) anf Valente et al. (2009), has the form:

$$\begin{aligned} Z \quad &= \quad \min cx \\ \textit{subject to} \quad & Ax = b \\ & x \geq 0 \\ \textit{where} \quad & A \in R^{m \times n}; c, x \in R^{n}; b \in R^{m} \end{aligned} \tag{2.1}$$

The equal sign in the above equation can be substituted with other restrictions such as $\leq$ · (less or equal), $\geq$¸ (greater or equal) and ˜ (non-binding constraint). Integer linear programs are when some or all decision variables in the LP are restricted to integer values. The decision variable can be bounded between lower and upper bounds such that $l \leq x \leq u$.

2.2 Stochastic Programming

SP models are frequently used in the field of optimisation and operations research for various applications and industries settings. When SP models applied to ex-ante decision making problems, a decision must be made prior to the data which determine the model parameters evolve over time are revealed. For example, investment decisions on asset allocation and planning problems must be solved before future realisation of assets performance can be observed. Same goes to the power generation planning, where the decision makers must plan the capacity of the utility demand in the face of uncertainty. The goal of SP models is to find the best decision that is attainable for all the appropriate data instances and maximizes the expectation of particular function of the decision variables and the random variables. Uncertainties are indexed over the scenario dimension, which connects the data vector to the scenario tree structure. Event tree as well as the scenario generation method is explained in Chapter 3.

In SP formulation, (Ω,F,P) symbolises the probability space, $\omega\in\Omega$ represents a realisation of the uncertainties, $p(\omega)$, denotes the probability and F is the sigma space.

Equation (2.2) represents the realisations for a given ω.

$$(A,b,c)_\omega = \xi_\omega \quad or \quad \xi(\omega) \tag{2.2}$$

The canonical formulation of SP can be represented as :

$min \;\; f_0(x,\xi)$

Subject to

$$f_i(x,\xi) \le 0, \quad i=1,\ldots,I \tag{2.3}$$

There are three problems that can be associated with stochastic decision model: **expected value, wait and see** and **here and now**. The terms wait and see and here and now were introduced by Madansky (in Dantzig 1955). We attain knowledge pertaining to the distribution of the objective function value for possible realisations of the random parameters from the three associated decision models.

i) The expected value problem

When random parameters are replaced with the expected values, we obtain the Expected Value model which is a deterministic model instead of SP.

We define the corresponding constraint sets as :

$$C^{\omega} = \{x \mid Ax = b, x \geq 0\} \quad \textit{for} \quad (A,b,c)_{\omega} \;\; \textit{or} \;\; \xi(\omega) \tag{2.4}$$

We derive the expected value problem by substituting the uncertainties with their average values.

$$\bar{\xi}(\omega) = E[\xi(\omega)] = \sum_{\omega \in \Omega} p(\omega)\xi(\omega) \tag{2.5}$$

Hence,

$$\begin{aligned} Z_{ev} \quad &= \quad \min \bar{c}x \\ \textit{subject to} \quad &\bar{A}x = \bar{b} \\ &x \geq 0 \end{aligned} \tag{2.6}$$

x^*_{ev} symbolises the optimal solution of the expected value problem, for all possible scenarios $\omega \in \Omega$. After the objective function values for all scenarios are determined, the expectation of the expected value solutions Z_{eev} are calculated using Equation (2.7).

$$Z_{eev} = E\left[cx^*_{ev}\right] \tag{2.7}$$

ii) The wait-and-see problems

In the wait and see problem, it is assumed that the decision makers are able to wait until the uncertainty is revealed before deciding on the optimal decision. Decision makers make a decision at the first stage prior to knowing the outcome of the random parameters (in the second stage). In the second stage, the decision is made after the uncertainties in the second stage are resolved. This step is repeated for stage three and so forth (for a multiple stage SP mode). Here and now approach relies upon perfect information about the future. We solve this problem for each $\omega \in \Omega$ (realisation of the uncertain parameters). Thus the probability distribution of the optimal objective functions Z^{ω}:

$$\begin{aligned} & Z^{\omega} = \min cx \\ & \textit{subject to} \quad x \in C^{\omega}. \end{aligned} \tag{2.8}$$

The expected value of the wait and see solution is defined as:

$$Z_{ws} = E\left[Z^{\omega}\right] = \sum_{\omega \in \Omega} Z^{\omega} p(\omega) \tag{2.9}$$

iii) The here-and-now decision problem

The here-and-now decision problem is when decision makers need to make decisions at the present time before the uncertainties are resolved. This is the problem that decision makers normally need to solve in real life.

$$\begin{aligned} & Z_{hn} = \min E_{\omega}[cx] \\ & \textit{where} \quad x \in C \end{aligned} \tag{2.10}$$

$$\textit{and} \qquad C = \bigcap_{\omega \in \Omega} C^{\omega} \tag{2.11}$$

x has to be feasible for all scenarios (≡ realisations) $\omega \in \Omega$. The optimal solution $x^* \in C$ hedges against all possible future $\omega \in \Omega$.

iv) The inter relationship and bounds

The three solutions, Z_{ws}, Z_{hn}, Z_{eev} is connected by the following relationship (the relationship is true when considering minimisation problem, for maximisation problem, the relationship is the opposite of Equation (2.12)).

$$Z_{ws} \le Z_{hn} \le Z_{eev} \tag{2.12}$$

The difference between Z_{hn} and Z_{ws} is known as the expected value of perfect information (EVPI) or the cost of having the perfect information about the future uncertainties. Thus,

$$\text{EVPI} = Z_{hn} - Z_{ws} \tag{2.13}$$

Large EVPI means that it is costly when the information about the future uncertainties are incomplete.

The value of the stochastic solution (VSS), is the value of the difference between the solution of the stochastic optimisation problem and the expected solution of the expected value problem.

$$\text{VSS} = Z_{eev} - Z_{hn} \tag{2.14}$$

2.3 Two Stage Stochastic Programming

TSP is the most frequently applied and studied SP model.

TSP model with recourse can be represented as:

$$\begin{aligned} Z = \ & \min/\max && cx + E_{\omega}Q(x,\omega) \\ & \textit{subject to} && Ax = b \\ & && x \geq 0, \end{aligned} \tag{2.15}$$

and

$$\begin{aligned} Q(x,\omega) = \ & \min \ f(\omega)y \\ \textit{subject to} \ \ & D(\omega)y = d(\omega) + B(\omega)x \\ & y \geq 0. \end{aligned} \tag{2.16}$$

Matrix *A* and vector *b* are deterministic. *Q(x,ω)* denotes the recourse function that is defined by Equation (2.16). D(ω) is the recourse matrix or technology matrix, the right hand side d(ω). B(ω) represents the inter stage linking matrix and and f(ω) is the objective function coefficients. The recourse action y(ω) for a given realisation ω is obtained by solving Equation (2.16).

SP problems with recourse are dynamic LP models that include future uncertainties for some parameters. Two-stage linear programs with recourse are a substantial class of models that include stochasticity within an optimisation model. In formulating the stochastic model, firstly the decision maker identifies the "Here-and-Now" decision variables (for activities that can't be postponed) together with the deterministic parameters and then identifies the remaining decisions which are specified by recourse variables and related random parameters. The optimal first stage decision x must be feasible for all scenarios $\omega \in \Omega$. The second stage decision compensates and adapts to different scenarios ω after the outcomes ω are observed. The solution of this model provides the optimum solution of all scenarios $\omega \in \Omega$.

The recourse matrix D^{ω}, is called fixed recourse if it is fixed for all scenarios. Simple recourse is when D takes the form D= [I,-I] and the related variables extended to $y=[y^{+},y^{-}]$ the corresponding constraints become goal programming constraints.

2.4 Multi stage stochastic programming (MSP)

We provide a brief introduction on MSP in this section. The two stage SP problem can be extended to MSP. A multi stage recourse problem (Optirisk System, 2008) can be represented as:

.

$$
\begin{array}{llllll}
\min\limits_{x_1} & \left\{ c_1x_1 + E_{\xi_2}\left[\min\limits_{x_2} c_2x_2 + E_{\xi_3|\xi_2}\left[\min\limits_{x_3} c_3x_3 + \ldots + E_{\xi_T|\xi_{T-1}|\ldots|\xi_2} \min\limits_{x_T} c_Tx_T \right]\right]\right\} & & & & \\
st & A_{11}x_1 & & & & = b_1 \\
 & A_{21}x_1 + & A_{22}x_2 & & & = b_2 \\
 & A_{31}x_1 + & A_{32}x_1 + & A_{33}x_3 & & = b_3 \\
 & \vdots & & \ddots & & \vdots \\
 & A_{T1}x_1 + & A_{T2}x_1 + & A_{T3}x_3 + & \ldots & + A_{TT}x_T = b_T \\
 & \ell_t \le x_t \le u_t;\ t = 1,\ldots,T & & & &
\end{array}
\tag{2.17}
$$

In MSP a sequence of optimisation problems correspond to different stages: at time t=1 the decision maker has to make a decision where the outcome depends on the knowledge of the future realisations of the multidimensional stochastic data process. At stage t, $t \geq 1$, for each realization of the history ξ_t, a recourse problem is considered in which decisions are allowed to be a function of observed realization (x_{t-1},ξ_t) only.

2.5 Chance constrained programming (CCP)

CCP is a qualitative yet effective way of specifying constraints; the CCP formulation requires that constraints are satisfied with a given (high) probability, in contrast to LP and SP where constraints have to be satisfied exactly that is 'almost surely' (with probability =1). CCP was first introduced by Charnes and Cooper (1960). CCP is related to Value at Risk (VaR) measure (Klein Haneveld and Van der Vlerk (2006), Schwaiger (2009)).

In CCP, β indicates the probability of satisfying the constraint. C. Dert (1995) extended CCP for ALM approach, and includes binary variables to count the number of times a certain violation event happens. In a scenario-based SP problem, CCP allows some scenarios to be violated. The sum of probabilities of violated scenarios is bounded by β.

Consider a constraint $h(x,\xi) \geq 0$ where x is the decision variable and ξ is the uncertain parameter. In CCP, we relax this condition, requiring instead that the constraint is satisfied with a high probability.

$$P(h(x,\xi) \geq 0) \geq \beta \tag{2.18}$$

In ALM formulation the constraint is used to model and restrict the probability of underfunding. We require that the asset value is higher by a certain proportion with the liabilities $A_{t+1} \geq \gamma L_{t+1}$ with a high reliability level. L_t is the liability at time t and A_t is the asset at time t. The CCP formulation can be presented as:

$$P\{A_{t+1} - \gamma L_{t+1} \geq 0\} \geq \beta_t \qquad t = 1,\ldots,T-1 \tag{2.19}$$

Where β_t $(0 \leq \beta \leq 1)$ is the reliability level or the probability of satisfying a constraint that applies for all time period *t = 1...T* and scenario *s=1...S*. γ is level of meeting the liabilities in order to satisfy the constraints and is specified by users. CCP is modelled by considering additional binary variables that count the number of times when the constraint is violated.

$$M\delta_{t+1}^{s} \geq \gamma L_{t+1}^{s} - A_{t+1}^{s} \qquad t = 1,\ldots,T-1 \tag{2.20}$$

$$\pi_s \sum_{s=1}^{S} \delta_{t+1}^{s} \leq 1 - \beta_{t+1,} \qquad t = 1,\ldots,T-1 \tag{2.21}$$

$$\delta_{t+1}^{s} \in \{0,1\}, \qquad t = 1,\ldots,T-1 \tag{2.22}$$

Where

M	Big number
δ_{t+1}^{s}	Binary decision variable where $\delta_{t+1}^{s} \in \{0,1\}$,

γ	Level of meeting liabilities
A_{t+1}^{s}	Asset value at time t and scenario s
L_{t+1}^{s}	Liability value at time t and scenario s
β_t	Reliability level at time t

2.6 Integrated chance constraints Programming (ICCP)

ICCP was introduced by Klein Haneveld (1986). The idea behind ICCP is that not only the probability of underfunding is included but the amount of underfunding is also considered by introducing a new decision variable that measures the shortfall. The maximum acceptable level of deficit is pre-specified by the user. Conditional surplus at risk (CSaR) constraints which is a variant of conditional value at risk (CVar) are closely related to ICCP (Klein Haneveld and Van der Vlerk (2006) and Schwaiger (2009)).

We define individual ICCP based on the notation by Haneveld and van der Vlerk (2006) and Schwaiger (2009) as:

$$\mathbb{E}_{\omega}[\eta_i(x,\omega)^{-}] \leq \kappa_i, \quad \kappa_i \geq 0 \tag{2.23}$$

Joint CCP are defined as:

$$\mathbb{E}_{\omega}[max_{i \in I}\eta_i(x,\omega)^{-}] \leq \kappa \quad , \quad \kappa \quad \geq 0 \tag{2.24}$$

κ_i and κ is a user defined maximum allowed expected shortfall.

ICCP is an alternative and perhaps an enhanced formulation of CCP and is more appropriate especially for ALM where the quantitative value of shortfall, that is the amount of underfunding, is important. The constraints of ICCP model (for ALM) are simple linear restrictions and do not need binary variables. ICCP model for ALM can represented as:

$$H_t^s - \gamma L_t^s + shortage_t^s \geq 0 \tag{2.25}$$

$$\pi_s \sum_{s=1}^{S} shortage_t^s \leq \lambda \pi_s \sum_{s=1}^{S} L_t^s \qquad (2.26)$$

In ICCP, we quantify at each time period and for each scenario the shortfall of asset value (H_t^s) with respect to liability times γ (weight of liabilities in relation to the asset value). “Shortage” variables, measure the amount by which an asset value is less than the corresponding liability at each time period as shown in Equations (2.25). In Equation (2.26), we include another constraint to ensure the expected value of shortages to be equal or less than a pre-specified λ (small percentage) of liabilities.

2.7 Concluding Remarks

In this chapter we have presented the solution models that are going to be used in Chapter 4 and 5 to solve EPF ALM problems.

Later in this thesis The LP formulation is expanded to TSP to CCP (but due to the computational challenges associated with mixed integer programs the CCP results will not be included) and to ICCP.

Chapter 3

Scenario Models for Cash Inflows and Outflows

3.1 Modelling Paradigm for uncertainties

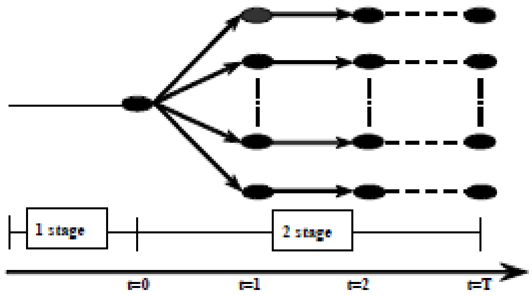
Uncertainties impinge on both assets and liabilities in our ALM models. Data on the real values of the stochastic parameters are disclosed in stages, and the decisions at every stage depends on the observations at that particular time, not on the future realizations. These uncertainties are captured using discrete models of randomness known as scenario generation. A finite number of scenarios is arranged and presented in a tree sequence known as a scenario tree (see Figure 3.1 and 3.2). A scenario tree does not forecast future values of random variables but generates a set of realistic possible scenarios (Vázsonyi, 2006).

The scenario generation model is separated into two parts; firstly the future realisation of pension fund asset return values and secondly is the future realisation of liabilities. The scenarios generated in this chapter are used as input into the optimisation models (both LP and SP models). For the LP model, the expected values from the scenarios generated are used as input. Various financial scenario generators are widely used by actuarial executives, financial planners and insurance agents and executives.

The scenarios for a TSP are organised in the form of a fan which is called a scenario tree as shown in Figure 3.1 also see Sutiene and Pranevisius (2007). At the initial planning horizon, a single node represents the information known or the deterministic data of the present states of the world. The sole node extends linearly to form branches of nodes that represent the uncertainties. From each of these nodes emerge flat scenarios. Node at each time period (from t = 1... T) is where an optimum decision is taken.

In TSP, the stages are fixed to only two however, the time periods for recourse actions are normally more than two depending on the planning horizon that are necessary or pertinent to the decision makers.

Figure 3.1: Two Stage Scenario Tree



The discrete probability associated with a scenario ω is denoted by p^{ω} or $p(\omega)$ and $p(\omega) \geq 0$. The event parameter takes the range of $\omega \in \Omega$, and $\Omega = \{1,...,S\}$ where S is the number of scenarios. The random parameter vector realisation is represented by $\xi(\omega)$.

$$\sum_{\omega \in \Omega} p(\omega) = 1 \text{ and } \quad \Xi = \bigcup_{\omega \in \Omega} \xi(\omega) \tag{3.1}$$

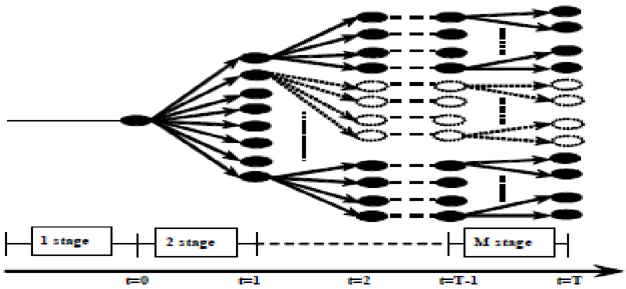
Where Ξ is the set of random parameter vectors.

In MSP Ω denotes the set of scenarios and $\Omega := \{\xi_t(1),...,\xi_t(S)\}$ where $t \in (1..T)$. The sequences of realisation are also called scenarios at stage t and can be presented as $\xi_t(\omega) = \{\xi_1(\omega),...,\xi_t(\omega)\} \quad \forall t > 1.$ The arc probabilities or the conditional probabilities are represented as $P(\xi_t(\omega) | \xi_{t-1}(\omega) | ... | \xi_2(\omega))$. The path probabilities of scenarios ω, p^{ω} are obtained by multiplication of related conditional probabilities.

$p^{\omega} \geq 0$ and $\sum p^{\omega} = 1$.

The decision at stage $t = 1,...,T$ depends on the realisation of random variables from the preceding stages. Multi stage scenarios tree structure is shown in Figure 3.2. Similar to the two stage scenario tree, a single node is associated at t=0 that later emerged to form branches of multiple of successor nodes. The branches from a node indicate the realisation of stochastic parameters. The successor nodes show the future conditions of the world, hence different realisations of the uncertainties that are relative on the data (state) attainable at a predecessor node. Additional decisions are taken based on the new revealed information therefore the branching continues at each stage until t=T.

Figure 3.2: Multistage Scenario Tree



Scenario generation for stochastic programming has been the subject of extensive research. There are vast literatures on different methods of scenario generation techniques. The goal of each method is to gain an adequate representation of uncertainty. Among the well- known methods used to generate scenarios in pension fund applications is the Wilkie Model with Cascade structure (Wilkie, 1984). The Wilkie model is a multivariate economy model that is composed of connected models of various economic series (price inflation, wage inflation, share yields, dividends, long term as well as short term interest rate) over time. Mulvey built the ALM model for Pacific Mutual (Mulvey, 1989) and also for Towers Perrin Tillinghast (Mulvey et al., 2000) where he applied the non- linear cascade Wilkie Model in generating the scenarios.

Alternatively, Econometric time series scenario generation models such as Moving Average (MA), vector auto-regressive (VAR) model (Cariño et al.,1994),Vector Error Correction Model (VECM) (Bosh-Princep et al., 2002), Bayesian vector autoregressive model (BVAR) (Collomb, 2004) and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) (Kulikova and Taylor, 2010) are among the choices in constructing the scenarios. Koivu et al. (2003) developed a multistage stochastic model for a Finnish pension insurance, where the stochastic parameters are captured using Vector Equilibrium Correction model. The uncertainties in the parameters are formulated using seven economic factors that include interest rate, bond yield, stock price index, dividend yield, property price index, rental yield and wage index (Koivu et al., 2003). The scenario generation also includes experts' views.

Statistical Approaches such as moment matching, moment fitting, quantile regression and forecasting are among other methods that are used in generating financial scenarios. Hoyland and Wallace (2001) developed the moment matching scenario generation technique where the distance between the statistical properties of the generated scenarios with the specified properties the first four moments (mean, standard deviation, kurtosis and skewness) and correlation of asset returns are minimised. Gulpinar et al. (2004) have proposed a novel hybrid approach for scenario generation. They report an empirical study with historical data which establishes the added value of the hybrid (simulation/optimization) approach. The most common techniques for scenario generations are explained and tabulated in Di Domenica et al. (2009) as well as Mitra (2006).

A forecast of future obligatory payments, in other word, liabilities, is needed in any ALM research. The future liabilities are usually computed by applying actuarial models of a company's workforce. The liabilities of pension funds and life insurance companies are not straightforward and consist of individual contracts. A model describing the total liability value needs to consider multiple sources of uncertainty. The liabilities scenarios are driven by current population both active as well as non-active and future numbers of participants, career advancement, salary, mortality and the benefits offered by the pension scheme. We make assumptions in respect of these and rely on actuarial calculations in order to generate the scenarios for liabilities. However, if the liabilities are influenced by the impact of uncertain financial variables such as interest rates or inflation then the liabilities scenarios can be generated with the appropriate models that correspond to the change of financial variables (Kyriakis, 2001).

As most of the ALM studies are based on DB pension fund rather than DC pension fund, the building up rights (accumulation phase) and earned rights (retirement benefits) that are based on the last drawn salary is considered. Another assumption

made in most research pertaining to the DB pension fund is that there are no new entrants into the current system which is a closed system (Schwaiger, 2009).
The uncertainties which we model in this chapter are:

i. Assets future returns, these are captured using Vector Autoregressive (VAR) scenario generation method.

ii. A population model to quantify the future population of EPF participants.

iii. Salary growth to define the future salary of the active participants based on age group.

iv. Contributions made by the participants. Contributions are calculated by combining the salary of active participants, the population currently holding the status as active participants in EPF as well as the monthly contribution rate.

v. Lump sum liabilities (lump sum payments for active as well as inactive participants at retirement, and lump sum payments to next of kin upon death).

vi. Pre-retirement withdrawals for active participants.

The rest of the chapter is organised in the following way. In section 3.2 we describe the assets returns scenarios generation method. In section 3.3 we explain about the population model. Section 3.4 covers the salaries model and contributions cash flows. In section 3.5 we explain about the formulation of liabilities scenarios before presenting the cash inflows and outflows in section 3.6 and conclusions in section 3.7.

3.2 Assets return model

i) Vector Autoregression

In a DB pension problem, the return on the invested assets should consider the technical interest rate, on the other hand, in a DC pension model, the return should consider the guaranteed minimal return (Frauendorfer et al., 2007). Most Economic scenario generation model methods require the modelling of multiple financial and economic series as well as the dependencies between them. These include various forms of non-stationary and time varying means and volatilities. In our study, we generate the asset return scenarios using a Vector Autoregressive Model (VAR).

VAR model was introduced by Sims in the year 1980. For his works in the development of VAR method, Sims was awarded the Nobel Prize in the area of Economic Sciences in the year 2011. VAR can be defined as a set of relationships between previous lagged values of the variables involved and the present value of each variable in the model (Collomb, 2004) that are jointly used to forecast future values. The scenario generation using VAR in the area of ALM was firstly introduced by Dert (1998). Carino (1998) explained the scenario generation of the Russel Yasuda Kasai model using VAR method. Besides these two ALM models**,** VAR model was used to generate scenario trees in ALM Model by Campbell and Viceira (2002), and Kouwenberg (2001). ORTEC-finance also uses VAR scenario generation to solve ALM problems (Steehouwer, 2006).

The strength of VAR models is in the ease of use and flexibility. Almost any covariance stationary time series process can be described with a VAR model. However, the weaknesses of VAR models are that their estimates may be imprecise when samples are small when compared to the number of parameters to be estimated. According to Mulvey (2000), VAR is a linear model, that is straightforward to calibrate, however can produce an unreliable results over planning periods. In order to avoid these problems, researchers use Vector Error Correction Model (VECM) instead of VAR.

VECM means time series are modelled as a function of its own lags and also the lags of its cointegrated pair and an error-correction component corrects the deviations from the equilibrium relationship in the previous period (Schmidt, 2008). Prior to deciding whether to apply the VAR or VECM model for our case study, we ran stationary test using Eviews to check whether the data of each asset class was stationary. Then we ran the Johansen Cointegration Test for all the non-stationary data. The results showed that our non- stationary data are not cointegrated, therefore we applied VAR model instead of VECM model.

There are a few companies that specialize in consulting and developing the scenario generators such as Barrie and Hibbert, and furthermore in our research we are considering TSP, rather than MSP. Therefore, the scenario generation for assets returns are based on VAR which is a simple but effective method of building a scenario tree.

ii) Source of Data

We have used the historical data from year 1990-2009 to represent the returns of the five asset classes (property, equities, money market instrument, Malaysian Government Securities (bond) for short and long period). The historical data is used to estimate and calibrate the model. Figure 3.3 shows the time series plot of the data. Table 3.1 shows the assets and their proxy that is used in our research.

Figure 3.3 : Time Series Plots of The Historical Data

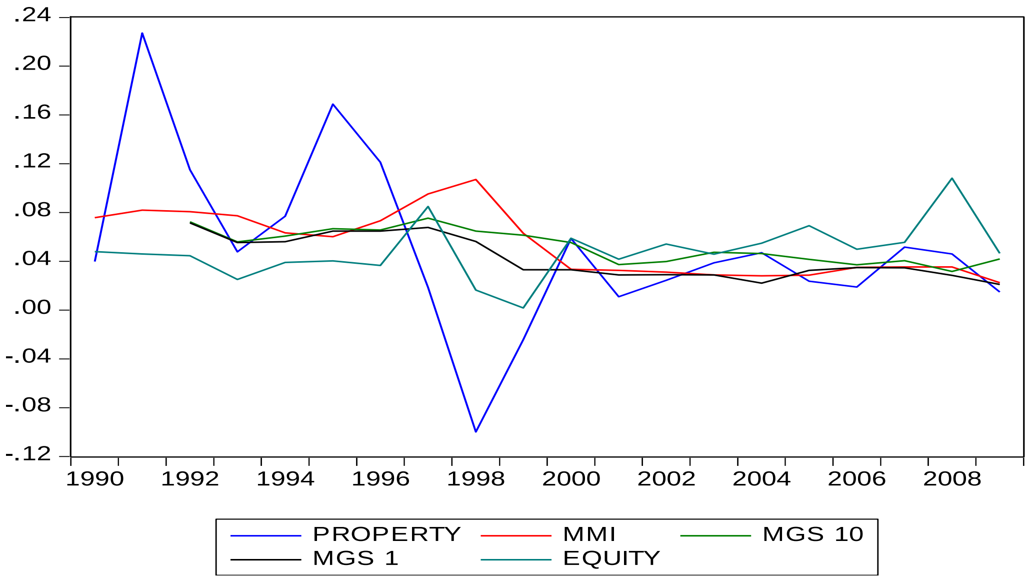


Table 3.1: Assets and Their Proxy

Asset	Proxy
MMI	Interest rate of interbank money market (weighted average 3 months).
MGS 1	Indicative yield bond (short term – 1 year remaining to maturity).
Equity	Earning yields– calculated from Net P/E ratio (composite index).
MGS 10	Indicative yield bond (long term – 10 year remaining to maturity).
Property	Annual percent change of property price indicators

EPF updates investments and pays dividends to participants on a yearly basis, therefore, monthly returns are less relevant; the returns data chosen are the annual data. The descriptive statistics of the annual and monthly data for the historical returns time series is tabulated in Table 3.2. The data for this study were obtained from various sources such as Bank Negara Malaysia (Central Bank of Malaysia), Bursa Malaysia and Bond Pricing Agency Malaysia (BPAM). The software package used to analyse the data is Eviews version 7.

Although EPF and most corporate organisations in Malaysia invest in commercial property to gain rental income rather than residential housing, there are limited reliable statistics on commercial properties in Malaysia. The returns taken from the residential statistics are used as a proxy. Seow (2007) mentioned that there is a high correlation between the residential house prices and office prices in Malaysia and used the residential data in his ALM model.

Table 3.2 : Descriptive Statistics of Annual and Monthly Returns

		MMI	**MGS 10**	**Equity**	**MGS 1**	**Property**
Annual	Mean	0.053452	0.053810	0.050055	0.043459	0.044760
	Standard deviation	0.027844	0.014269	0.025218	0.018046	0.060640
	Skewness	0.730477	0.185391	0.624347	0.474670	0.171815
	Kurtosis	2.257343	1.705832	4.094710	1.622768	4.009545
Monthly		**MMI**	**MGS 10**	**Equity**	**MGS 1**	**Property**
	Mean	0.041702	0.051511	0.054242	0.039547	0.031389
	Standard deviation	0.022403	0.013502	0.022805	0.017658	0.014914
	Skewness	1.590074	0.599879	-0.159653	1.363320	-0.029598
	Kurtosis	4.415202	2.082367	4.329960	3.926086	2.086703

iii) Design of Computational Experiment

Equation (3.2) represents the VAR model in the matrix form:

$$\begin{bmatrix} y_{1,t} \\ y_{2,t} \\ \vdots \\ y_{k,t} \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_k \end{bmatrix} + \begin{bmatrix} A^1_{1,1} & A^1_{1,2} & \cdots & A^1_{1,k} \\ A^1_{2,1} & A^1_{2,2} & \cdots & A^1_{2,k} \\ \vdots & \vdots & \ddots & \vdots \\ A^1_{k,1} & A^1_{k,2} & \cdots & A^1_{k,k} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \\ \vdots \\ y_{k,t-1} \end{bmatrix} + \cdots + \begin{bmatrix} A^p_{1,1} & A^p_{1,2} & \cdots & A^p_{1,k} \\ A^p_{2,1} & A^p_{2,2} & \cdots & A^p_{2,k} \\ \vdots & \vdots & \ddots & \vdots \\ A^p_{k,1} & A^p_{k,2} & \cdots & A^p_{k,k} \end{bmatrix} \begin{bmatrix} y_{1,t-p} \\ y_{2,t-p} \\ \vdots \\ y_{k,t-p} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \vdots \\ \varepsilon_{k,t} \end{bmatrix} \quad (3.2)$$

Another form of VAR is :

$$y_t = \mu + A_1 y_{t-1} + ... + A_p y_{t-p} + \varepsilon_t \qquad t = 1,...,T \qquad \varepsilon_t \sim \quad IIDN(0,\Sigma) \qquad (3.3)$$

$$y_{it} = \ln(1 + r_{it}) \qquad i = 1,..., I, t = 1,...,T \qquad (3.4)$$

r_{it} is the return of asset i at time period t. The historical returns of each asset (r_{it}) are transformed to ln (1+ return) to avoid heteroscedasticity problem.

Where:

p	Number of lag
ε_t	Vector of error distribution/ error term/residual
μ	Vector of Intercept
A_i	k x k Autoregressive Coefficient matrix k=(1,2,…p)
y_t	Vector of time net returns at time t
$\sum$	k x k covariance matrix

From the literature, it is common to apply first order VAR model (lag one) for asset returns for example in the research done by Boender et al (2005) and Dert (1998), however we follow the method of Kouwenberg's 2001 research and do not use lagged variables to model the returns to avoid unstable and spurious predictability. Besides the number of lags, decision makers also need to decide whether it is necessary to include other macroeconomic quantities beside returns in the VAR model. Boender et al. (2005) included currency rates of different countries and interest rates in the VAR model. Kouwenberg, 2001 included the returns of assets and salary in the scenario tree generation. We are going to include only the asset returns in our VAR model. It is not necessary to include salary in our VAR model as we model the salary growth separately.

Table 3.3 shows the VAR estimation of μ (intercept) for lag 0. Table 3.4 shows VAR equation used for each asset class.

Table 3.3:Vector Autoregression Estimates

	MMI	MGS_1	EQUITY	MGS_10	PROPERTY
μ	0.051746	0.042400	0.048572	0.052326	0.042197

Table 3.4: VAR Model Equations (Lag 0)

$\ln(1+property_t)$	$=0.042197+\varepsilon_{1t}$
$\ln(1+MMI_t)$	$=0.051746+\varepsilon_{2t}$
$\ln(1+MGS10_t)$	$=0.052326+\varepsilon_{3t}$
$\ln(1+MGS1_t)$	$=0.042400+\varepsilon_{4t}$
$\ln(1+equity_t)$	$=0.048572+\varepsilon_{5t}$

iv) Cholesky Decomposition

The purpose of the Cholesky decomposition is to unfold the normal equations in linear least squares problems. Cholesky decomposition of a symmetric positive definite matrix A, is a factorization into a unique upper triangular such that the lower triangular matrix M whose transpose M^T is the upper triangular.

$$M..M^T = A \tag{3.5}$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{31} & a_{32} & \cdots & a_{3n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & \cdots & 0 \\ l_{21} & l_{22} & \cdots & 0 \\ l_{31} & l_{32} & \vdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ l_{n1} & l_{n2} & \cdots & l_{nn} \end{bmatrix} \begin{bmatrix} l_{11} & l_{21} & \cdots & l_{n1} \\ 0 & l_{22} & \cdots & l_{n2} \\ 0 & 0 & \vdots & l_{n3} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & l_{nn} \end{bmatrix} \tag{3.6}$$

The Covariance matrix is a symmetric positive-definite and have the properties which is the same with components M^T

$$\Sigma^T{}_{ij} = \Sigma_{ji} \tag{3.7}$$

The value of

$$M_{ii} = \left(a_{ii} - \sum_{k=1}^{i-1} M^2{}_{ik} \right)^{1/2} \tag{3.8}$$

and

$$M_{ji} = \frac{1}{M_{ii}}\left(a_{ij} - \sum_{k=1}^{i-1} M_{ik} M_{jk}\right) \quad j = i+1, \ \ i = 2,...,I \tag{3.9}$$

$$l_{ij} = 0 \qquad \textit{for} \ \ i > j. \tag{3.10}$$

We apply Cholesky factorization of the variance covariance matrix of the residual (historical data), that are characterized by being $N(0,\Sigma)$ to be decomposed into residuals $N(0,1)$. The Cholesky decomposition is used to preserve the covariance structure of asset returns. Different values of the random numbers from a standard normal distribution are generated via a Monte Carlo simulation using Excel. The matrixes of standard normal variables are multiplied to the transpose of the Cholesky matrix.

The future returns of our asset classes are constructed by solving the estimated equations in Table 3.4 combined with the residual values obtained from Monte Carlo simulation. For each node a different random vector of error distribution/residual is used to generate the scenarios for the asset returns. In this research we generate 2000 scenarios. All scenarios are equiprobable therefore the probability for each scenario is equivalent to $\frac{1}{2000}$ or 0.0005.

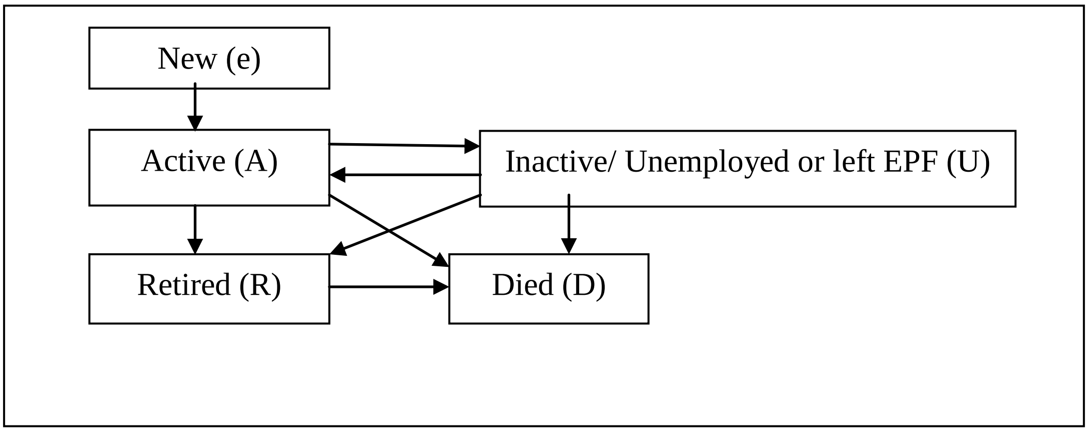
3.3 Population Model

The inflow and outflow of the members as well as the status of the members are simulated using a Markov probability model for population; a model which estimates the new entrants. The Markov model is a well-established approach to pension, life insurance and annuity calculation. Janssen (1966) connected and applied Markov processes to actuarial science for the first time. Mettler (2005) provides extensive explanation about the pension fund cash flows for a DB pension plan. He then elaborates with numeric examples of closed and open system pension schemes. In a closed system there is no staff turnover whereas in an open system new entrants join as active participants and others may leave the system.

i) Markov Model

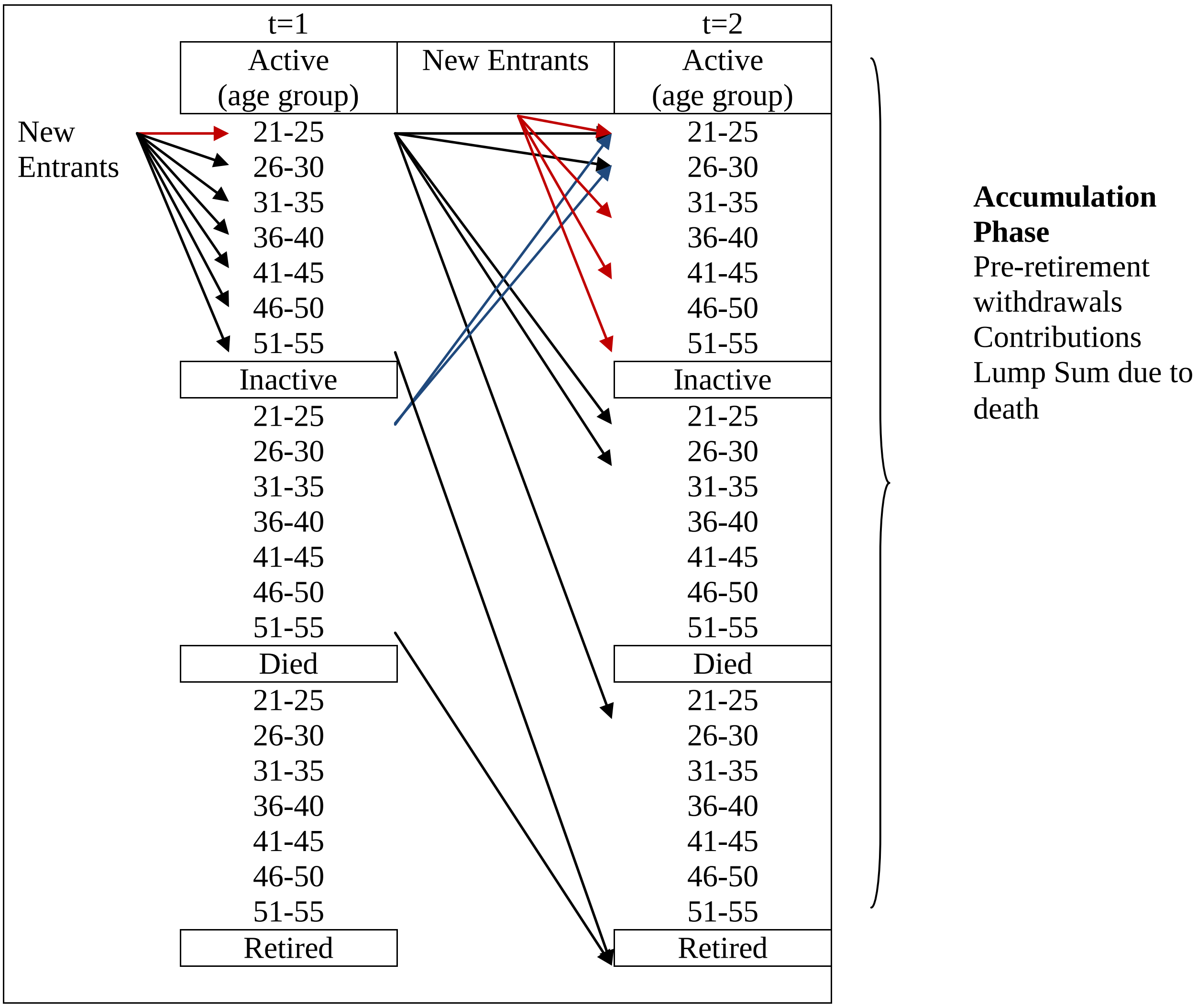
The states of the Markov Model are set out in Table 3.5 which shows the ten possible states that a member of the EPF scheme may occupy. By aggregating all the age groups and taking into consideration the new entrants we have an open system Markov population model as shown in Figure 3.4 (a). An expanded, that is, detailed view of the model is shown in Figure 3.4 (b). In this model (see Figure 3.4 (a) and 3.4 (b)) each arrow denotes a possible transition from one state to another. Only new employees can enter the pension scheme from outside. If participants who left the system return, becoming active participants again, then they are treated as new entrants. The Dead as well as the retired are absorbing states.

Figure 3.4 (a) : Aggregated View Markov Population Model for EPF



We assume that the age of entry into the plan is 21. All members retire at the age of 55. The inactive members are those that leave the EPF scheme before retirement either voluntarily or due to disablement. We assume a uniform distribution within the age group hence there is a 20% change of participants from preceding age groups to the next age groups annually.

Figure 3.4 (b) :Expanded Markovian Transition Model



We wish to estimate $p_{ij}(t)$ which is the probability of plan members belonging to state i moving to state j at time t, for time period, t=1…45. We compute a set of seven (observed) historical relative frequencies of transitions as found in pair wise historical data (2003-2004) until (2009-2010) supplied in EPF Malaysia annual reports (see Appendix B). The observed transitions (relative frequencies) for year 2003-2004 is set out in Table 3.6 and is given as an example. The first column of the transition probabilities shows the initial states and the top row of the table shows the destination states. Table 3.6 shows the 2003 and 2004 data in which the column ‘new entrants’ is derived (computed) information; this is explained below.

Table 3.5 : States of Markov Model

State	Status	Remarks
i = 1	**Active (age group)** 21-25	
i = 2	26-30	
i = 3	31-35	
i = 4	36-40	
i = 5	41-45	
i = 6	46 -50	
i = 7	51 -55	
i = 8	Inactive	Persons who leave EPF scheme.
i = 9	Retired	Persons (both active and inactive) who have retired.
i = 10	Dead	Persons in Active and Inactive states who have died. We do not consider persons who died after retirement in our calculation. We only consider the number of persons for whom pension payments have to be made.

Table 3.6: 2003 and 2004 Population Data

2003 data	Active Population $A_i(t)$	Survival ratio n_i	Active population year 2004 $(A_i(t) x\, n_i)$	Inactive $(A_i(t) x\, u(t))$	2004 data	New entrants
21 - 25	1529280	0.99073	1212083	45878	1574992	317031
26 - 30	960693	0.9886	759792	28821	988480	199866
31 - 35	739103	0.98493	582371	22173	751907	147362
36 - 40	588899	0.97867	461070	17667	607306	128569
41 - 45	474619	0.96927	368027	14239	491414	109148
46 - 50	348414	0.95103	265081	10452	364802	89268
51 - 55	203086	0.92198	149793	6093	216686	60800

Table 3.7: Relative Frequencies of Transition for Year 2003 to 2004

States	21 - 25	26 - 30	31 - 35	36 - 40	41 - 45	46 - 50	51 - 55	Inactive	Retired	Dead
21 - 25	0.7768	0.1924	0	0	0	0	0	0.0291	0	0.0016
26 - 30	0	0.7769	0.1920	0	0	0	0	0.0291	0	0.0020
31 - 35	0	0	0.7771	0.1913	0	0	0	0.0291	0	0.0024
36 - 40	0	0	0	0.8615	0.1182	0	0	0.0181	0	0.0022
41 - 45	0	0	0	0	0.7775	0.1884	0	0.0292	0	0.0049
46 - 50	0	0	0	0	0	0.7785	0.1851	0.0292	0	0.0072
51 - 55	0	0	0	0	0	0	0.7722	0.0290	0.1865	0.0123
Inactive	0	0	0	0	0	0	0	0.9793	0.0167	0.0040
Retired	0	0	0	0	0	0	0	0	1.0000	0.0000
Dead	0	0	0	0	0	0	0	0	0	1.0000

We derive (compute) the new entrants for each age group at time t, $e_i(t)$ in the following way. The value of $e_i(t)$ for each age group includes the survival ratio n_i for each age group i, (n_i value is taken from the Malaysian Abridged life table). In our notation n_i stands for a fraction of people, in age group (*i=1,…,7*) that survive. There is no distinction between female and male participants survival ratio, however, we take the readings of the male survival ratio data in our model, as the data of both sexes is not available. Equation (3.11) represents the number of new entrants by age group:

$$e_i(t) = AP_i(t) - (0.8 \times AP_i(t-1) \times n_i) - UP_i(t) \tag{3.11}$$

Where

AP_i	Active population
UP_i	Inactive population
e_i	New entrants
n_i	A fraction of the people in age group (i=1,…,7) that survive
i	age group (i=1,…,7)

We compute a set of seven columns of new entrants by age group and append these to the table of seven pair wise historical data (2003-2004) until (2009-2010).

ii) Generating Population Scenarios.

We use the bootstrap method (sampling with replacement), in which the precalculated seven transition probabilities as well as the number of new entrants by age group are used to simulate the population of EPF members throughout the 45 years planning horizon. The historical EPF population data in the year 2010 is the initial input of our Markov population model. The transition probabilities are multiplied with the previous year data and the new entrants are added to gain the number of population in the different states for the next time period. The simulation of the population model is repeated 20 times to generate 20 different population scenarios which are to be used in the stochastic decision models.

3.4 Salaries Model and Contributions

Salary is an important factor for determining future contributions. Based on the data in the year 2010 from the EPF's annual report, we determine the plan members' average salaries that correspond to their respective age groups. We assume 2% of annual growth in the total average salary for each age group. The salary of the plan members in employment for age group i, at time t is given as:

$$F_i\,(t) = F_i\,(t\text{-}1) \times 1.02 \tag{3.12}$$

Where

F_i Average Salary

i) Contributions

Contribution rate, number of active participants and the total amount of annual salaries for active participants determine the total amount of annual contributions. In the case of EPF the contribution rate is 23%. The contributions by the EPF members are computed by taking the simulation results of the population model and combining (that is, in this case multiplying) by the results of the salary model.

$$V_s(t) = \sum_{i=1}^{I} AP_{i,s}(t) \times F_i(t) \times k \tag{3.13}$$

k	Contribution rate – fixed at 23%
$F_i(t)$	Annual salary of active participants, age group i at time t.
$AP_{i,s}(t)$	Number of Active population age group i, at time t population i, scenario s.
$V_s(t)$	Total contributions at time t scenario s.

ii) Cash flow Implications

The future cash flows of the EPF are driven by the evolution of the population of participants as well as by the wages and also the historical value of average lump sums amount paid to members. The EPF scheme in each year provides the guarantee of a minimum dividend. The computational results are parametrically analysed for these growth dividends. Thus it is implied that there is a growth in each scheme member's fund value. In this research we assume a range of growth dividends which are 2.5%, 3.0%, 4.0%, 5.0% and 6.0%. This is used in calculating the lump sum payments to the active and inactive retirees and shown in Equations (3.15) and (3.16).

3.5 Liabilities

Liabilities which lead to cash outflows are divided into three categories. The first category is the pre-retirement withdrawals during the accumulation phase; the second category is the lump sum at retirement that is assumed to be compulsory for all members (although in the real case members are given three choices a) lump sum b) annuity payments and c) scheduled payment). The third category comprises liabilities due to death of active participants before reaching the age of 55.

i Pre- retirement withdrawals – 30% of total wealth (Account II)

Annual pre-retirement data (combination of health, mortgage and education withdrawals) are calculated based on the population of the active participants, mortality and wages. Based on historical data the annual average percentage of pre-retirement withdrawals are equivalent to 12% of the total annual contributions, therefore we assumed that the future pre-retirement has the same pattern.

$$z_s(t) = 0.12 \times V_s(t) \tag{3.14}$$

$z_s(t)$	Pre-retirement withdrawals at time t scenario s
$V_s(t)$	Contributions at time t scenario s

ii Lump sum Payments to different scheme cohorts (active and inactive)

Pension entitlements of the current participants are frozen until the retirement of the participants within EPF. This is known as Account 1. The retirement benefit is in the form of a lump sum at retirement.

The accumulated wealth of an employee at any time, it includes the contributions as well as the growth dividends that have accumulated over the years. In our optimisation model we consider different values of growth dividends, therefore different values of accumulated wealths are calculated for different values of dividend.

Our analysis of the historical data (EPF reports (Malaysia) 2005 to year 2010) shows that most members opt for lump sum withdrawals. We therefore construct our model in which retirement liability is in the form of a lump sum to the retired members (active as well as inactive).

In this study we have pre-calculated (using the data from the start year 2010) the average lump sum paid to participants at retirement for active as well as inactive members. The lump sum paid at retirement for the **active** and **inactive** participants are calculated using equations (3.15) and (3.16):

$$ARS\ \ (t-1)\times(1+G)\times ARP_s(t) \tag{3.15}$$

ARS Precalculated average lump sum paid at retirement for active participants.

G Growth dividend.

ARPs Number of people change status from being active to retired.

$$URS\ \ (t-1)\times(1+G)\times URP_s(t) \tag{3.16}$$

URS Precalculated average lump sum paid at retirement for inactive participants .

G Growth dividend .

URP_s Number of people change status from beaing inactive to retire.

In all our models we consider five different growth dividends that are 2.5%, 3.0%, 4%, 5% and 6%.

iii Lump Sum Payment on Death

Lump sum is equivalent to the average lump sum upon death from the previous time period (based on historical data from the start year 2010) times number of death participants at time t population scenarios and (1+ growth dividend).

$$DS\ \ (t-1)\times(1+G)\times DP_s(t) \tag{3.17}$$

DS Precalculated average lump sum paid due to death.

G Growth dividend .

DP_s Number of people died.

3.6 Combining Cash Inflow and Outflow Scenarios

In our scenario generation models, the population scenarios and asset return scenarios are entirely independent since they do not rely on any common factor. All the same since contributions depend on population scenarios, liabilities also depend on population scenarios. We can summarise the following relationship of cash inflows and cash outflows to population scenarios and asset returns scenarios.

Cash inflows: Depend on population scenarios and asset return scenarios.
Cash outflows: Depend on population scenarios.
Set of population scenarios: 20.
Set of assets returns scenarios: 100.

We make a set product of the 20 population scenarios (these affect contributions and lump sum liabilities) with the set of 100 asset return scenarios. These lead to the combined set of 2000 scenarios.

Figure 3.5 : Integrated View of Scenario Models

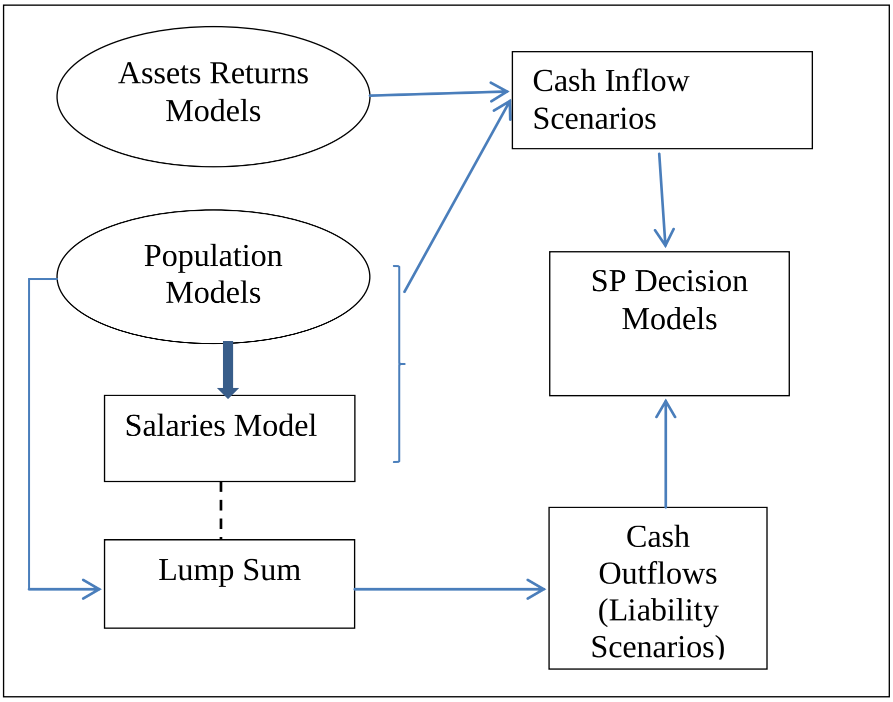


Figure 3.5 shows the integrated view of the scenario models. Dotted lines between salaries model and lump sum show that salary is considered in the calculation of outflows, however in this research the average amount of a lump sum paid at retirement and upon death (from the historical data) is used to calculate the cash outflows.

3.7 Concluding Remarks

In this chapter we have presented the models that are used to generate scenarios for the asset returns and liabilities of EPF Malaysia. The generated asset returns as well as liabilities are then used to formulate the stochastic programming models which are introduced in Chapter 5. For the out-of-sample simulation study in Chapter 6, again the same scenario generation techniques are used for a new and larger set of scenarios.

Chapter 4

Deterministic Model for ALM

4.1 Introduction to deterministic model

Asset liability management strategies have been applied to pension funds since the late 1980s (Drijver, 2005). The earliest ALM models introduced in the literature were formulated as a deterministic linear programming optimization model (Drijver, 2005). For a detailed explanation on ALM refer to the Asset and Liability Management Handbook (Mitra and Schwaiger Eds., 2011). From a methodological perspective, a deterministic model can be looked upon as an expected value problem, and in this chapter we introduce a deterministic formulation of the EPF scheme as an ALM problem.

4.2 Linear Programming ALM Model Components

Our model is formulated to capture the EPF plan for Malaysia where both employees and employers contribute to the scheme. The scheme invests the contributions and at the same time fulfils the obligations of both pre-retirement withdrawals as well as retirement withdrawals. Our formulation is used to determine the investment policy of the EPF, with the objective of maximising the terminal wealth of the fund. The constraints of the model ensure that the liabilities of the pre-retirement withdrawals and pensions are met. As a benchmark for performance, we use a fixed mix model.

Data Source

At the initial time period the data are taken from the EPF 2010 annual report.

The data used for the asset returns, cash flows, liabilities, pre-retirement liabilities are the average value of the scenarios generated in the previous chapter.

Model Components

The LP model is defined using two index sets:

i	Asset classes	$i= 1,\dots,I$ $(I=5)$
t	Time period	$t=1,\dots,T$ $(T=45)$

The time horizon of the EPF plan is taken as 45 years and the planning is considered to be annual. The base year is considered to be t=1, representing the year 2011. Index i denotes the asset class. Five assets classes for EPF have been taken into consideration; thus i=1...5 (see Table 3.1 in Chapter 3).

The parameters are defined as:

V_t	Aggregated contribution at time *t*
$r_{i,t}$	Aggregated return of asset i at time *t*
L_t	Aggregated liability of payment at time *t*
x_{i0}	Initial holding of asset *i* (RM)
α	Transaction cost expressed as a fraction of asset value; this is fixed at 2%
r^l	Lending rate equivalent to return of MMI at time t minus 0.005
r^b	Borrowing rate equivalent to return of MMI at time t plus 0.005
l_i	Lower bound of asset class *i* as a fraction of total asset portfolio
u_i	Upper bound of asset class *i* as a fraction of total asset portfolio

Decision variables:

W_t	Total wealth of the fund at time *t*
$x_{i,t} \geq 0$	Amount of asset *i* held at time *t*
$B_{i,t} \geq 0$	Amount of asset *i* bought at time *t*
$S_{i,t} \geq 0$	Amount of asset *i* sold at time *t*
$O_t \geq 0$	Amount of cash borrowed at time period *t*
$Q_t \geq 0$	Amount of cash lent at time period *t*
$H_t \geq 0$	Asset value at time period *t*

Objective Function

The terminal wealth is maximized.

$$\textit{Maximize } W_T \tag{4.1}$$

Wealth constraint

$$W_t = \sum_{i=1}^{I} x_{i,t} + Q_t(1 + r^l) - O_t(1 + r^b) \qquad \text{t=1,…,T} \tag{4.2}$$

Wealth in time t is equal to the total assets held and the amount of cash lent in the preceding time together with the lending rate received minus the amount of cash borrowed at time t-1 including the borrowing rate.

At any time period, EPF can invest by lending money or borrowing cash in the event that the liabilities cannot be met. At the next time period the amount lent is reinvested with return $(1 + r^l)$, while the borrowed money with return $(1 + r^b)$ needs to be paid back.

Asset holding constraints

$$x_{i,1}^s = x_{i0} + B_{i,1} - S_{i,1} \qquad t=1,\ i=1,\dots,I \tag{4.3}$$

$$x_{i,t} = x_{i,t-1}(1 + r_{it}) + B_{i,t} - S_{i,t} \qquad t=2,\dots,T,\ i=1,\dots,I \tag{4.4}$$

Asset holdings constraints rebalance the holding of assets over time. The amount of assets held is equal to the holding for each asset from the preceding time period and its returns plus the amount bought and minus the amount sold. During time period one the assets held are equivalent to the initial holdings (real data from year 2010) of assets with the addition of the amount of assets bought minus the amount of assets sold.

Asset value

$$H_t = \sum_{i=1}^{I} x_{i,t} \qquad t=1,\ldots,T \qquad (4.5)$$

Equation (4.5) shows the sum of assets held at time period t.

Cash balance constraints

$$\sum_{i=1}^{I}(1+\alpha)B_{i,1} + Q_1 + L_1 = V_1 + \sum_{i=1}^{I}(1-\alpha)S_{i,1} + O_1 \qquad t=1 \qquad (4.6)$$

$$\sum_{i=1}^{I}(1+\alpha)B_{i,t} + Q_t - \left(1+r^l\right)Q_{t-1} + L_t = V_t + \sum_{i=1}^{I}(1-\alpha)S_{i,t} + O_t - \left(1+r^b\right)O_{t-1}$$

t=2≤t≤T-1 (4.7)

$$\sum_{i=1}^{I}(1+\alpha)B_{i,t} - \left(1+r^l\right)Q_{t-1} + L_t = V_t + \sum_{i=1}^{I}(1-\alpha)S_{i,t} - \left(1+r^b\right)O_{t-1}$$

t=T (4.8)

These constraints capture the cash inflows and cash outflows that include selling off assets, contributions from participants, buying of assets, the amount borrowed and lent as well as meeting the liabilities of the EPF scheme. The amount invested in the purchase of assets at time t, the liability payments and the amount lent is equal to the cash borrowed, plus any cash generated from sales and the contributions from participants. The transaction cost (in a simplified form) for buying and selling as well as the rates of return for borrowing and lending are also included in this equation. The constraints are expressed in three part equation. Equation (4.6) shows the initial period. For time period t>1 the repayment of the amount borrowed from previous time period and the borrowing rate as well as the amount lent from previous time together with the lending rate is included. At the final time period no borrowing and lending are allowed.

Short sales constraints

$$S_{i1} \le x_{i,0} \qquad t = 1, \;\; i = 1,..., I \tag{4.9}$$

$$S_{it} \le x_{i,t-1} \qquad t = 2,...,.T, \;\; i = 1,..., I \tag{4.10}$$

Following EPF rules, the fund only invests the money collected from the contributors. Therefore, the amount of assets sold must be less than the amount of assets held in the previous time period; this excludes the possibility of short selling.

Bound constraints

$$l_i \sum_{i=1}^{I} x_{it} \le x_{i,t} \le u_i \sum_{i=1}^{I} x_{i,t} \qquad t=1,...,T,\; i=1,...,I \tag{4.11}$$

Table 4.1: Upper Bound and Lower Bound for Each Asset Class

Asset Class	**Lower bound (%)**	**Upper bound (%)**
Money Market Instrument (MMI)	5	25
Malaysian Government Securities (MGS 1) short term	15	35
Equity	5	25
Malaysian Government Securities (MGS 10) long term	15	45
Property	0.1	5

The maximum and minimum bound of the portfolio weight of each asset held is included in the model. These constraints are based on the composition constraints enacted by EPF. The upper and lower bound for each asset class are displayed in Table 4.1.

4.3 Fixed Mix Strategy

Mulvey et al. (2003) compare fixed mix with buy and hold portfolios and the result showed that fixed mix shows superior efficient frontier of the expected return against return standard deviation. However when Fleten et al. (2002) compared the fixed mix model to four stage stochastic model, they found that dynamic stochastic programming solutions rule both the in sample as well as out of the sample of the fixed mix solutions. Table 4.2 presents the percentage of each asset class in the fixed mix asset allocation which is used for the purpose of comparison, that is, benchmarking.

The fixed mix strategy naturally satisfies the linear constraints Equation (4.1) to (4.11) of the LP formulation. So we simply use the objective functions to compute the terminal wealth. The assets weight or holdings are set at the fixed proportion at time t as shown in Equation (4.12).

$$
\begin{aligned}
&x_{i,t}^{f} = 0.24 \qquad t = 1,\dots,T \;,\; i = 1,\dots,4 \\
&x_{5,t}^{f} = 0.04 \qquad t = 1,\dots,T \;,\; i = 5
\end{aligned}
\tag{4.12}
$$

Table 4.2: Percentage of Assets for Fixed Mix Model

Asset Class	**Fixed mix (%)**
Money Market Instrument (MMI)	24
Malaysian Government Securities (MGS 1) short term	24
Equity	24
Malaysian Government Securities (MGS 10) long term	24
Property	4

The percentage of each asset for fixed mix strategy (see Table 4.2) is based on a value between the upper and lower limit restriction that we imposed on each asset.

4.4 Results

Figure 4.1:Growth Dividend VS Terminal Wealth (TW) for LP

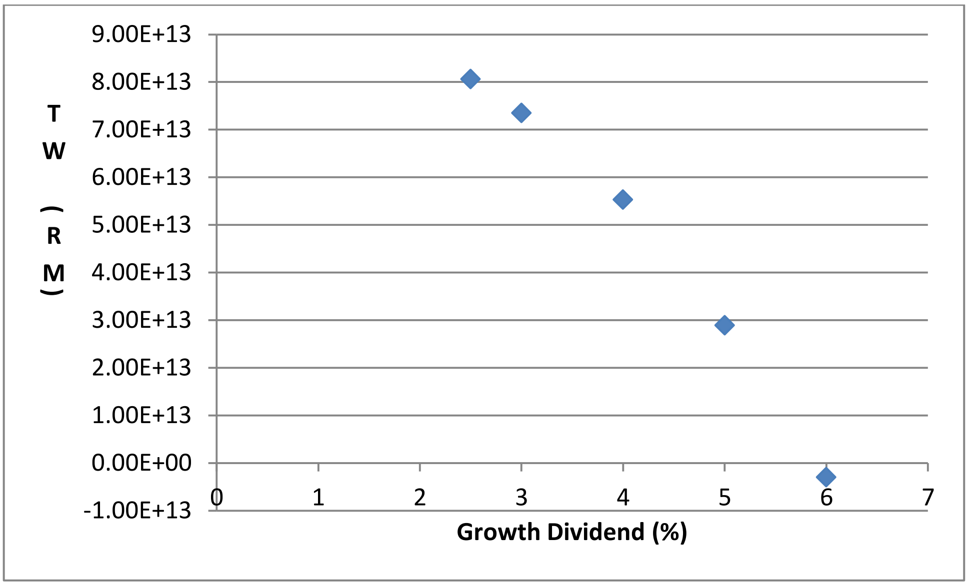


Table 4.3: Summary of LP Results

Dividend (%)	Terminal Wealth	Max Borrowing Required
2.5	8.06E+13	0
3	7.35E+13	0
4	5.53E+13	0
5	2.89E+13	0
6	-3.0E+12	5.39E+12

In figure 4.1 terminal wealth (t=45) against varying levels of growth dividend is plotted. With higher growth dividends the terminal wealth of the EPF is progressively reduced. Table 4.3 gives the summary of values of terminal wealth for an increasing level of the growth dividend (shown in column 1) as well as the maximum borrowing required using the LP decision model. We can see that no borrowing is necessary until 6% of growth dividends. At 6% growth dividend the necessary borrowing is RM 5.39E+12.

The amount of each asset holding at time period 1 and 45 for varying dividends in terms of monetary value and percentage allocation are displayed in table 4.4.

Table 4.4.:LP Asset Allocation for Each Asset Class at the Specified Time Period and Growth Dividend

time period	Dividend	Asset allocation (RM and percentage)				
		MMI	MGS	EQT	MGS10	PROP
Year 1	2.50%	6.84E+10	7.89E+10	9.81E+10	2.02E+11	1.87E+09
		15.2	17.6	21.8	45.0	0.4
Year 45		4.14E+12	1.45E+13	2.07E+12	1.86E+13	2.07E+12
		10.0	35.0	5.0	44.9	5.0
Year 1	3.00%	7.01E+10	7.87E+10	9.65E+10	2.02E+11	1.87E+09
		15.6	17.5	21.5	45.0	0.4
Year 45		3.77E+12	1.32E+13	1.88E+12	1.70E+13	1.88E+12
		10.0	35.0	5.0	45.1	5.0
1 Year	4.00%	7.38E+10	7.83E+10	9.31E+10	2.02E+11	1.87E+09
		16.43	17.44	20.73	45.0	0.42
Year 45		2.80E+12	9.80E+12	1.40E+12	1.26E+13	1.40E+12
		10.0	35.0	5.0	45.0	5.0
Year 1	5.00%	7.79E+10	7.78E+10	8.93E+10	2.02E+11	1.87E+09
		17.4	17.3	19.9	45.0	0.4
Year 45		1.43E+12	5.00E+12	7.14E+11	6.43E+12	7.14E+11
		10.0	35.0	5.0	45.0	5.0
Year 1	6.00%	8.23E+10	7.74E+10	8.51E+10	2.02E+11	1.87E+09
		18.34	17.25	18.97	45.02	0.42
Year 45		6.97E+07	4.18E+07	4.15E+07	1.25E+08	2.79E+05
		25.0	15.0	14.9	44.9	0.1

MMI - Money Market Instrument
MGS - Malaysian Government Securities - short term
EQT - Equity
MGS10 - Malaysian Government Securities -long term
PROP - Property

Comparison of LP and FM strategies

Figure 4.2 shows the comparison of the terminal wealth for the optimum LP and the fixed mix (FM) decisions. As expected, the terminal wealth of the LP model is higher than that obtained by the fixed mix strategy.When we compare the value of maximum borrowing required between LP and FM (see Table 4.5), borrowing starts at the same growth dividend level in the FM strategy which is at 6%. However the amount borrowed in the FM strategy is higher when compared to LP model. Therefore we conclude that LP based optimisation strategy is superior to the FM strategy.

Fixed mix Strategy

Figure 4.2 : Growth Dividend VS Terminal Wealth (TW) for LP and FM

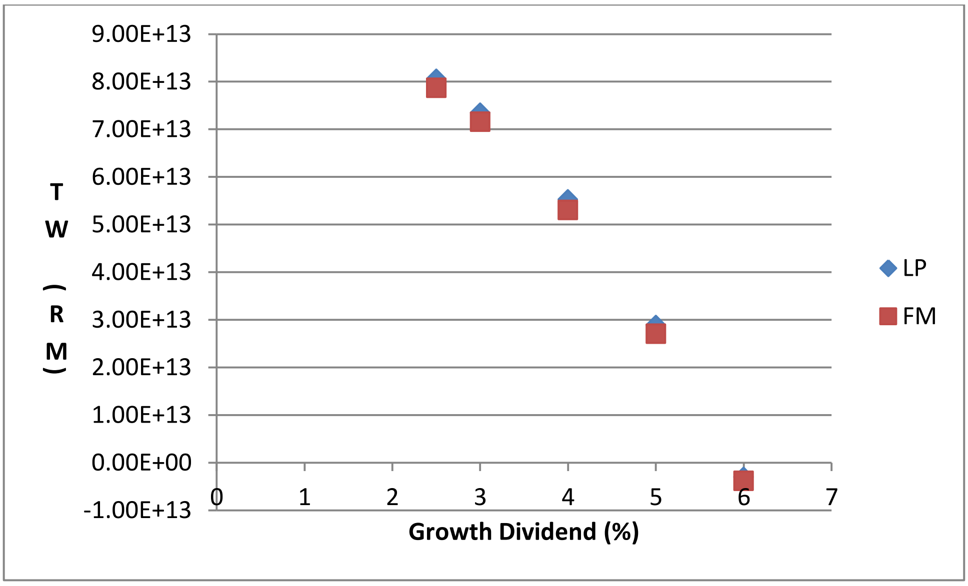


Table 4.5: Comparison of the Maximum Borrowing Required between FM and LP with Varying Growth Dividend.

Dividend (%)	Max Borrowing Required (LP)	Terminal Wealth (LP)	Maximum Borrowing Required (FM)	Terminal Wealth (FM)
2.5	0	8.06E+13	0	7.87E+13
3	0	7.35E+13	0	7.16E+13
4	0	5.53E+13	0	5.31E+13
5	0	2.89E+13	0	2.71E+13
6	5.39E+12	-3.0E+12	6.23E+12	-3.8E+12

4.5 Summary of the Expected value ALM Optimisation Model

Objective Functions

$$Maximize\ W_T \qquad (4.1)$$

Wealth constraint

$$W_t = \sum_{i=1}^{I} x_{i,t} + Q_t(1 + r^l) - O_t(1 + r^b) \qquad t=1,\ldots,T \qquad (4.2)$$

Asset holding constraints

$$x_{i,1}^{s} = x_{i0} + B_{i,1} - S_{i,1} \qquad t=1,\ i=1,\ldots,I \qquad (4.3)$$

$$x_{i,t} = x_{i,t-1}(1+r_{it}) + B_{i,t} - S_{i,t} \qquad t=2,\ldots,T,\ i=1,\ldots,I \qquad (4.4)$$

Asset value

$$H_t = \sum_{i=1}^{I} x_{i,t} \qquad t=1,\ldots,T \qquad (4.5)$$

Cash balance constraints

$$\textstyle\sum_{i=1}^{I}(1+\alpha)B_{i,1} + Q_1 + L_1 = V_1 + \sum_{i=1}^{I}(1-\alpha)S_{i,1} + O_1 \qquad \text{t=1} \qquad (4.6)$$

$$\sum_{i=1}^{I}(1+\alpha)B_{i,t} + Q_t - \left(1+r^{l}\right)Q_{t-1} + L_t = V_t + \sum_{i=1}^{I}(1-\alpha)S_{i,t} + O_t - \left(1+r^{b}\right)O_{t-1}$$

t=2≤t≤T-1 (4.7)

$$\sum_{i=1}^{I}(1+\alpha)B_{i,t} - (1+r^{l})Q_{t-1} + L_t = V_t + \sum_{i=1}^{I}(1-\alpha)S_{i,t} - (1+r^{b})O_{t-1}$$

t=T (4.8)

Short sales constraints

$$S_{i1} \le x_{i,0} \qquad t=1, i=1,\ldots,I \qquad (4.9)$$

$$S_{it} \le x_{i,t-1} \qquad t=2,\ldots,.T, i=1,\ldots,I \qquad (4.10)$$

Bound constraints

$$l_i \sum_{i=1}^{I} x_{it} \leq x_{i,t} \leq u_i \sum_{i=1}^{I} x_{i,t} \qquad t = 1,...,.T, i = 1,..., I \tag{4.11}$$

Fixed Mix Strategy

$$x_{i,t}^{f} = 0.24 \qquad t = 1,...,T \;, \; i = 1,...,4$$

$$x_{5,t}^{f} = 0.04 \qquad t = 1,...,T \;, \; i = 5 \tag{4.12}$$

4.6 Concluding Remarks

In this chapter we have considered a linear programming model and contrasted the optimal solutions against the fixed mix strategy. We have found that only at a 6% level of dividend does it become necessary to borrow cash in order to fulfil liabilities.

LP based strategy requires lower maximum borrowing than the fixed mix strategy.

LP based strategy leads to higher accumulated terminal wealth compared to the fixed mix strategy.

Chapter 5

The EPF Scheme Formulated as a Family of Stochastic Programming (SP) Models

5.1 A Review of Stochastic Programming Applied to ALM

The paradigm of SP models is a natural expansion of the LP approach to optimum decision models. In this chapter we broaden the LP formulation of the EPF problem and consider the uncertainties in the asset returns, contributions and the liabilities. In this section we first review the SP models in the domain of ALM. We then consider the data requirement of the scenario based SP models. In section 5.2 the ALM problem is formulated as a two stage SP model with recourse. We introduce the chance constrained programming (CCP) formulation of the ALM model in section 5.3. In section 5.4 we introduce the integrated chance constraints (ICCP) formulation of the ALM model. In section 5.5 we summarise the TSP, CCP and ICCP formulations. The conclusions are set out in section 5.6. We note that a further discussion of the in-sample testing and comparison of LP, SP and ICCP decisions is postponed until section 6.2.

Asset and Liability management modelling is one of the active topics of research in the SP field. According to Dupacova (1999), the benefits of stochastic programming is the ability to support the asset and liability management as well as the risk management decisions in diverse situations and reflect the objectives and constraints of the users. There have been abundant applications of the SP in ALM methodology to real life problems (case study). In the case of pension funds, among the models developed are multiple stage scenario-based models for analysing investment/funding policies of Dutch defined benefit pension plans by Dert (1995).

Geyer et al. (2008) present a multistage stochastic linear programming framework called the InnoALM model for Siemen AG Osterreich, the biggest privately owned industrial company in Austria which is a DC pension plan. InnoALM is used to evaluate the retirement fund asset allocation with the aim to maximize the expected current value of terminal wealth. In the InnoALM model, the authors restrict the maximum allocation for each asset class and at the same time made an assumption that the wealth grew at the rate of 7.5% in each period. Any occurrence of shortages are penalized by means of a piecewise linear convex risk measure. The authors recommended the pension fund to invest more in equity rather than bonds.

Dupacova and Polivka (2009) develop a model of Asset Liability Management for a defined contribution Czech Pension fund. Sadhak and Doss (2011) constructed an ALM model for New Pension System (NPS) India that is a DC pension fund. They use Monte Carlo simulation to project the forthcoming cash flows and liabilities: they present the result of investment decisions on the appropriate asset allocation to manage auto choice cycle funds. This means that the participants are not involved in deciding the asset allocations, however, the default model has an inbuilt option on how to invest for participants based on age-lifecycle portfolio. The auto choice cycle fund follows the investment regulation besides the need to earn maximum risk adjusted returns during accumulation phase. These results can be used as guidance to fund managers.

In the Netherlands, ALM is a risk management tool that is well accepted and frequently used; the most sophisticated ALM models for pension fund are the Dutch Pension Fund (Galo, 2009). The financial supervisory authority in Austria (FMA) developed a scenario analysis model in order to simulate the chain or the consequences of different investment returns on asset classes for Pensionskassen (pension fund) and see the effects toward members and beneficiaries and employers (Galo, 2009). Although ALM is accepted in some countries as mentioned above, there are still reservations against ALM, for example, in the United Kingdom. A major barrier to the adoption of an integrated approach to ALM is that it involves mathematical modeling techniques that are complex and hard to understand and interpret by non-qualitative fund managers (Galo, 2009).

In this research, we focus on three different classes of stochastic models that are:

Two Stage Stochastic programming (TSP),
Chance Constrained Programming (CCP),
Integrated Chance Constraints Programming (ICCP).

With the current progress in software and computer technologies, as well as improvement in solution algorithms, stochastic programming capabilities have been considerably extended; according to Laurent (2006) SP has become very applicable. Many solution algorithms have been reported for TSP problems (Zverovich et al., 2011).

An alternative way to model risk aversion is to use chance constraints. However CCP only considers the probability of a shortage rather than the magnitude of the shortfall of the funding ratio. Dert (1995) developed a multistage CCP formulation of an ALM model. This model includes binary variables to quantify the number of times underfunding events occur. Prékopa (1993) discusses methods to solve chance-constrained models by mean of gradient methods and introduces penalty functions. Ruszczynski (2002) formulates the CCP problems as a Mixed Integer Programming problem.

ICCP is an alternative and perhaps an enhanced formulation of CCP and is more appropriate, especially for ALM, where the quantitative value of the shortfall, that is the amount of underfunding, is important. Klein Haneveld et al. (2005) applied integrated chance constraints as short-term risk constraints in an ALM model for Dutch pension funds. Haneveld looked at the effect of ICCP on the first stage decision (contribution rate, remedial contribution and asset mix) with a range of predetermined values of maximal acceptable expected funding shortfall.

5.2 Two Stage Stochastic Programming ALM Model Components

Data Source

At the initial time period the data are taken from the EPF 2010 annual report.

The data used for the asset returns, cash flows, liabilities, pre-retirement liabilities are the value of the scenarios generated in the Chapter 3.

Model Components

In the two TSP model for EPF the objective function is a natural extension of the objective function of the expected value model as shown in Chapter 4. The objective function is maximizing the **expected** terminal wealth.

Objective Function

$$Maximize \sum_{s=1}^{S} W_T^s \times \pi_s \qquad \text{(Expected terminal wealth)} \qquad (5.1)$$

The model indices:

i	Assets	$i=1,\ldots,I,\ I=5$
t	Time period	$t=1,\ldots,T,\ T=45$
s	Scenarios	$s=1,\ldots,S,\ S=2000$

I and T are as defined in LP model. The additional index for TSP that differs from LP is the scenarios index s that represents the description of future uncertainties. Scenarios were produced using the model described in Chapter 3. For the prototype model described in this study we have used 2000 scenarios that is, S=2000.

The parameters:

V_t^s Contribution at time t under scenario s

π_s	Probability of scenario s occurring (fixed at 1/S)
$r_{i,t}^s$	Uncertain Return of asset *i* at time t under scenario *s*
rl_t^s	Lending rate at time *t* under scenario *s*
rb_t^s	Borrowing rate at time t under scenario *s*
L_t^s	Liabilities at time *t*, scenario *s*
x_{i0}	Initial holding of each asset at initial time period
α	Transaction cost expressed as a fraction of asset value fixed at 2%
l_i	Lower bound of asset class i as a fraction of total asset portfolio
u_i	Upper bound of asset class i as a fraction of total asset portfolio
G	Growth dividend

The parameters that are affected by uncertainty are L_t^s π_s , rl_t^s , rb_t^s and $r_{i,t}^s$.

Decision variables

W_t^s	Total wealth at time t scenario s (depending on the parameter value G, this can take positive and negative value)
$x_{i,t}^s \geq 0$	Amount of asset *i* held at time t scenario *s*
$B_{i,t}^s \geq 0$	Amount of asset *i* bought at time t scenario *s*
$S_{i,t}^s \geq 0$	Amount of asset *i* sold at time t scenario *s*
$H_t^s \geq 0$	Asset value
$O_t^s \geq 0$	Amount of cash borrowed at time t scenario *s*
$Q_t^s \geq 0$	Amount of cash lent at time t scenario *s*

Wealth Constraint

$$W_t^s = \sum_{i=1}^{I} x_{i,t}^s + Q_{t-1}^s(1 + rl_t^s) - O_{t-1}^s(1 + rb_t^s) \qquad t=1,\dots,T,\ s=1,\dots,S \qquad (5.2)$$

The total value of wealth at time t and scenario s is given by the total asset held in the time period and the amount of cash lent paid back including the lending rate deducting the amount borrowed and the borrowing rate (in the event that borrowing is necessary).

The lending rate at time t, scenario s is equivalent to MMI’s return at time t, scenario s minus 0.005 while the borrowing rate is equivalent to MMI’s return at time t, scenario s plus 0.005.

Asset holdings constraints

$$x_{i,1}^{s} = x_{i0} + B_{i,1}^{s} - S_{i,1}^{s} \qquad t = 1 \qquad (5.3)$$

$$x_{i,t}^{s} = x_{i,t-1}^{s}(1 + r_{i,t}^{s}) + B_{i,t}^{s} - S_{i,t}^{s} \qquad t \geq 2 \qquad (5.4)$$

Asset holdings constraints rebalance the holding of assets over time. The amount of assets held is equal to the holding for each asset from the preceding time period and its return, plus the amount bought and minus the amount sold. At time period one the asset held is equivalent to initial holding, plus the assets bought minus assets sold.

Asset value

$$H_t^s = \sum_{i=1}^{I} x_{it}^s \qquad t=1,\ldots,T,\ s=1,\ldots,S \qquad (5.5)$$

Asset value at time t is the total amount of assets held.

Cash balance constraints

$$\sum_{i=1}^{I}(1+\alpha)B_{i,1}^{s} + Q_1^s + L_1^s = V_1^s + \sum_{i=1}^{I}(1-\alpha)S_{i,1}^{s} + O_1^s \qquad t=1 \qquad (5.6)$$

$$\sum_{i=1}^{I}(1+\alpha)B_{i,t}^{s} + Q_t^s - (1+rl_t^s)Q_{t-1}^s + L_t^s = V_t^s + \sum_{i=1}^{I}(1-\alpha)S_{i,t}^{s} + O_t^s - (1+rb_t^s)O_{t-1}^s$$

$$t=2 \leq t \leq T\text{-}1 \qquad (5.7)$$

$$\sum_{i=1}^{I}(1+\alpha)B_{i,t}^{s}-(1+rl_{t}^{s})Q_{t-1}^{s}+L_{t}^{s}=V_{t}^{s}+\sum_{i=1}^{I}(1-\alpha)S_{i,t}^{s}-(1+rb_{t}^{s})O_{t-1}^{s} \quad t=T \qquad (5.8)$$

This set of constraints is interpreted as follows. The amount invested in the purchase of new assets plus the assets lent (reinvest spare cash) and all the liabilities is equal to the contribution income from participants plus any cash generated from assets sold including the amount of cash borrowed. The transaction costs are introduced in a simplified form, that is, (1+α) and (1- α) and α=2% for selling and buying of assets. Borrowing and lending are not allowed in the last time period.

Short sales constraints

$$S_{i,1}^{s} \le x_{i0} \qquad t=1, \; s=1,\ldots,S \qquad (5.9)$$

$$S_{i,t}^{s} \le x_{i,t-1}^{s} \qquad t \ge 2, \; s=1,\ldots,S \qquad (5.10)$$

We do not consider short sales in this problem. Amount of assets sold must be less than the amount of assets held in the last time period.

Bound constraints

$$l_i H_t^s \le x_{i,t}^s \le u_i H_t^s \qquad t=1\ldots T, \; i=1,\ldots,I, s=1,\ldots,S \qquad (5.11)$$

The maximum and minimum bounds of portfolio weights of assets held are included in the model as a constraint. The upper and lower bound for each asset class are the same as th LP model (see Table 4.1 in the previous chapter).

Non Anticipativity Constraints

$$x_{i,1}^{s} = x_{i,1}^{1} \qquad s=2,\ldots,S \qquad (5.12)$$

$$B_{i,1}^{s} = B_{i,1}^{1} \qquad s=2,\ldots,S \qquad (5.13)$$

$$S_{i,1}^{s} = S_{i,1}^{1} \qquad s = 2, \ldots, S \tag{5.14}$$

Decision at a given stage does not depend on the future realization of the random events but only the observed part of the scenario. Therefore, we include the non anticipativity or information constraints. The state variables take the same decisions if they share the same node in a scenario tree. The investment decisions at t=1 are the first stage decision variables, the rest is recourse variance.

Results

The stochastic Measures

Following the usual practice of investigating the two stage scenario based strategy, we consider the expectation of the expected value (EEV) solution, here and now (HN) and wait and see (WS) solutions for the growth dividend =2.5% and s =2000 scenarios. We calculate the expected value of perfect information (EVPI) and value of stochastic solution VSS. The results are set out in Table 5.1. The results show EVPI that is, the effect of not getting the perfect information leads to an expected loss of fund value. The advantage gained by including the randomness in the TSP model (VSS) equals to RM 2.000E+10. Table 5.2 shows the Z_{EEV} and Z_{HN} values for varying growth dividends.

Table 5.1: Stochastic Measures for growth dividend 2.5%

Z_{EEV}	8.315E+13
Z_{HN}	8.317E+13
Z_{WS}	8.319E+13
EVPI	2.000E+10
VSS	2.000E+10

Table 5.2 : Z_{EEV} and Z_{HN} values for varying growth dividends

	Z_{EEV}	Z_{HN}
3%	7.587E+13	7.589E+13
4%	5.702E+13	5.705E+13
5%	3.049E+13	3.051E+13
6%	-1.6990E+12	-1.970E+12

Table 5.3 shows the asset allocation first stage decisions in time period one for varying levels of growth dividend. An examination of Table 5.3 reveals that the highest asset held is the long term Malaysian Government Securities as the upper bound for MGS 10 is high.

Table 5.3: TSP Model – Growth Dividend VS Expected Terminal Wealth (ETW) and 1st Stage Asset Allocation for Each Asset Class.

Dividend %	ETW (RM)	Asset Allocation				
		MMI	MGS	EQT	MGS10	PROP
2.5	8.32E+13	3.05E+10	1.19E+11	1.11E+11	1.62E+11	2.10E+10
		6.88	26.83	25.03	36.53	4.74
3	7.59E+13	3.11E+10	1.19E+11	1.11E+11	1.62E+11	2.10E+10
		7.00	26.80	24.99	36.48	4.73
4	5.70E+13	3.22E+10	1.19E+11	1.11E+11	1.62E+11	2.10E+10
		7.23	26.73	24.93	36.39	4.72
5	3.05E+13	3.34E+10	1.19E+11	1.11E+11	1.61E+11	2.11E+10
		7.50	26.71	24.92	36.14	4.74
6	-1.97E+12	3.72E+10	1.19E+11	1.12E+11	1.6E+11	1.97E+10
		8.31	26.57	25.01	35.72	4.40

Figure 5.1 shows the growth dividend VS the expected terminal wealth. The expected terminal wealth decreases progressively as higher growth dividends are given to participants. We note that compared with the LP result, the expected wealth of SP is higher than the LP.

Figure 5.1: TSP Model – Growth Dividend VS Expected Terminal Wealth (ETW)

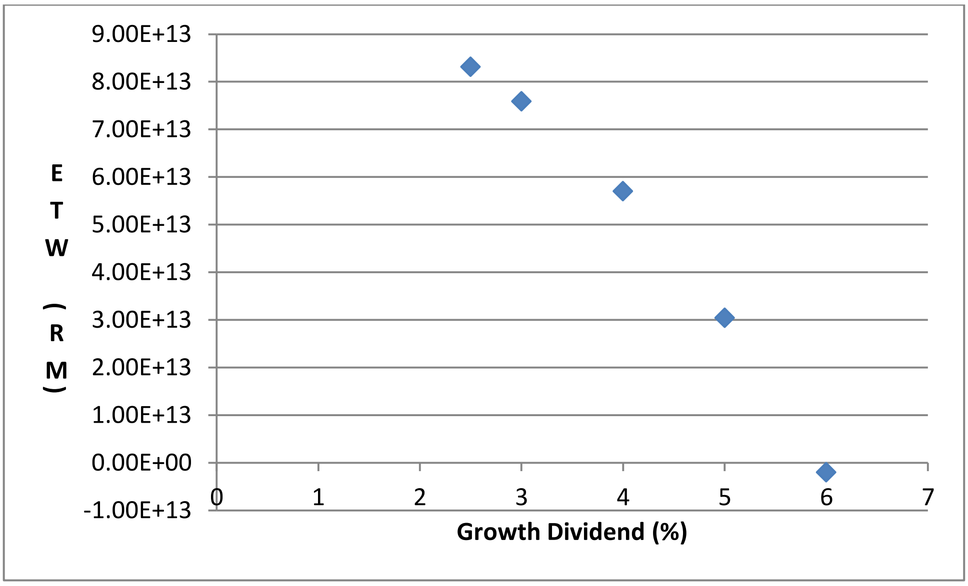


Table 5.4 shows the maximum expected borrowing needed for respective growth dividends. A higher amount of expected borrowing is required as higher growth dividend is paid to participants. At time t=1 the expected borrowing remains zero even when the growth dividend is set at 6%. At the level of div=5% the maximum expected borrowing is equal to RM5.34E+10.

Table 5.4:TSP Model-Maximum Expected borrowing and Expected Borrowing at t=1 VS Growth Dividend

Growth Dividend (%)	Maximum Expected Borrowing (RM)	Expected Borrowing at t=1 (RM)
2.5	0	0
3	0	0
4	0	0
5	5.34E+10	0
6	5.47E+12	0

5.3 Chance Constrained Programming ALM Model Componets

The CCP formulation can be presented as:

$$P\{A_{t+1} - \gamma L_{t+1} \geq 0\} \geq \beta_t \qquad t = 1,\dots,T-1 \tag{5.15}$$

We consider β_t as 0.95 (for all time period). γ or the level of meeting the liabilities, is equal to 1.10. CCP is modelled by considering additional binary variables that count the number of times when the constraint is violated.

$$M\delta_{t+1}^{s} \geq \gamma L_{t+1}^{s} - A_{t+1}^{s} \qquad t=1,...,T-1, s=1,...,S \tag{5.16}$$

$$\pi_s \sum_{s=1}^{S} \delta_{t+1}^{s} \leq 1-\beta_{t,} \qquad t=1,...,.T-1 \tag{5.17}$$

$$\delta_{t+1}^{s} \in \{0,1\}, \qquad t=1,....,T-1 \tag{5.18}$$

M	Big number
δ_{t+1}^{s}	Binary decision variable where $\delta_{t+1}^{s} \in \{0,1\}$,
γ	Level of meeting liabilities
A_{t+1}^{s}	Asset value at time t scenario s
L_{t+1}^{s}	Liability value at time t scenario s
β_t	Reliability level at time t

The binary variable in the equation (5.18), has value 1 if liabilities are greater than assets and 0 otherwise, at each time period and scenarios. Equation (5.17) restricts the number of underfunding scenarios to be less or equal to (1-β_t).

Refer to equations (5.1) to (5.14) for the objective function, Wealth Constraints, Asset holdings constraints, Asset value, Short sales constraints, Cash balance constraints, Bound constraints and Non Anticitivity constraints.

Refer to equations (5.16) to (5.18) for CCP equations.

Different reliability levels can be set for each time period. As most ALM models include long planning time, there is a high possibility that the results will differ from the forecasted scenarios. Usually the reliability is set higher at an earlier time period. The model can be re-run with the new input later on. Drijver (2005) introduces a short term chance constraint only for one year as the supervisor in the Netherlands only considers the short term financial position of pension funds. Schwaiger (2009) on the other hand includes the CCP up to t=3. We do not include the results of CCP model.

Our reasons are: (i) the CCP model takes a long time to solve even when we restrict the CCP constraints for only one time period. The introduction of binary decision variables makes the stochastic programming become a mixed integer stochastic programming model. (ii) The ICCP formulation provides additional information (on the extent of underfunding, not only on the probability).

5.4 Integrated Chance Constraints Programming ALM Model Components

The ICCP model has the following advantages compared to TSP and CCP. Firstly, the decision maker can reflect his/her risk aversion in ICCP model. The decision maker can restrict both the number of underfunding events and the amount of a possible deficit. Secondly, the ICCP model is computationally more tractable as it does not include any binary variables.

The constraints of ICCP model can be represented as:

$$H_t^s - \gamma L_t^s + shortage_t^s \geq 0 \qquad (5.19)$$

$$\pi_s \sum_{s=1}^{S} shortage_t^s \leq \lambda \pi_s \sum_{s=1}^{S} L_t^s \qquad (5.20)$$

In Equation (5.20), we ensure the expected value of shortages to be equal to or less than a pre-specified λ (small percentage), in our case 5% of liabilities, while maximising the terminal wealth.

γ	Level of meeting liabilities. In this research this is fixed at 1.10.
λ_t	Maximum expected shortfall. Assumed to be equal to 5%.
M	Large number (e.g. maximum value the investment portfolio is likely to reach)
G	Growth dividend

Non implementable stochastic decision variable

$shortage_t^s \geq 0$ Amount of underfunding at time t scenario s

Refer to equations (5.1) to (5.14) for the objective function, wealth constraint, asset holdings constraints, asset value, short sales constraints, cash balance constraints, bound constraints and non Anticitivity constraints.

For ICCP refer to Equations (5.19) and (5.20).

Results:

Table 5.5, Table 5.6 and Figure 5.2 show the result of ICCP with the value of λ=0.05 and γ =1.10.

Table 5.5: ICCP Model –Growth Dividend VS Expected Terminal Wealth (ETW) and 1st stage Asset Allocation for Each Asset Class

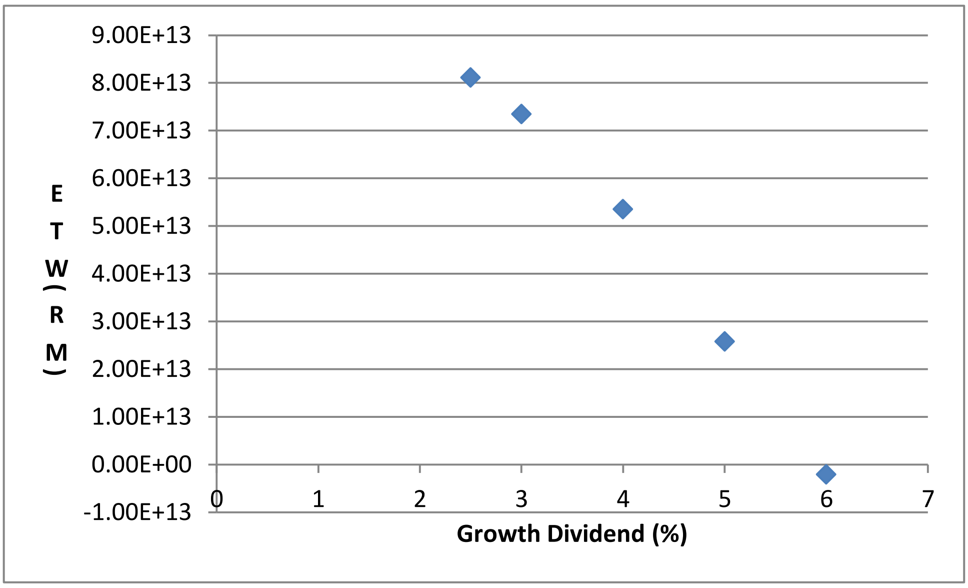
Growth Dividend %	ETW (RM)	Asset Allocation				
		MMI	MGS	EQT	MGS10	PROP
2.5	8.11E+13	3.17E+10	1.19E+11	1.11E+11	1.62E+11	2.11E+10
		7.1	26.8	25.0	36.4	4.7
3	7.35E+13	3.21E+10	1.19E+11	1.11E+11	1.62E+11	2.11E+10
		7.2	26.7	24.9	36.4	4.7
4	5.35E+13	3.30E+10	1.19E+11	1.11E+11	1.62E+11	2.11E+10
		7.4	26.7	24.9	36.3	4.7
5	2.58E+13	3.44E+10	1.19E+11	1.11E+11	1.60E+11	2.11E+10
		7.7	26.7	24.9	35.9	4.7
6	-2.10E+12	3.83E+10	1.19E+11	1.12E+11	1.60E+11	1.98E+10
		8.5	26.5	24.9	35.6	4.4

Similar to LP and TSP results, the highest asset held in the first time period for each promised dividend is the long term Malaysian Government Securities (MGS10).

Table 5.6:ICCP Model-Maximum Expected Borrowing and Expected Borrowing at t=1 VS Growth Dividend

Dividend (%)	Maximum Expected Borrowing (RM)	Expected Borrowing at t=1 (RM)
2.5	0	0
3	0	0
4	0	0
5	5.99E+11	0
6	1.41E+13	0

Figure 5.2: ICCP Model-Growth Dividend VS Expected Terminal Wealth (ETW)



The expected wealth decreases as higher growth dividend is given to participants. In ICCP we imposed additional constraints requiring a match between assets and liabilities times γ (weight of liabilities with respect to the asset value), causing the expected terminal wealth to be lower when compared to TSP. Since we imposed an additional restriction, requiring that the expected shortage is less than 5% of the total liabilities, the amount borrowed is higher in ICCP as compared to TSP and LP. However, in the first time period, even at 6% guaranteed dividend, the liabilities are not funded by borrowing.

The generated model statistics and the solution time for LP, TSP and CCP are display in Apendix A.2.

5.5 Summary of the Stochastic Programming ALM Optimisation Model

Two Stage Stochastic Programming (TSP)

Objective Function

$$Maximize \sum_{s=1}^{S} W_T^s \times \pi_s \tag{5.1}$$

Wealth Constraint

$$W_t^s = \sum_{i=1}^{I} x_{i,t}^s + Q_{t-1}^s(1 + rl_t^s) - O_{t-1}^s(1 + rb_t^s) \qquad t=1,\dots,T,\ s=1,\dots,S \qquad (5.2)$$

Asset holdings constraints

$$x_{i,1}^s = x_{i0} + B_{i,1}^s - S_{i,1}^s \qquad t = 1 \qquad (5.3)$$

$$x_{i,t}^s = x_{i,t-1}^s(1 + r_{i,t}^s) + B_{i,t}^s - S_{i,t}^s \qquad t \ge 2 \qquad (5.4)$$

Asset value

$$H_t^s = \sum_{i=1}^{I} x_{it}^s \qquad t=1,\dots,T,\ s=1,\dots,S \qquad (5.5)$$

Cash balance constraints

$$\sum_{i=1}^{I}(1+\alpha)B_{i,1}^s + Q_1^s + L_1^s = V_1^s + \sum_{i=1}^{I}(1-\alpha)S_{i,1}^s + O_1^s \qquad t=1 \qquad (5.6)$$

$$\sum_{i=1}^{I}(1+\alpha)B_{i,t}^s + Q_t^s - (1+rl_t^s)Q_{t-1}^s + L_t^s = V_t^s + \sum_{i=1}^{I}(1-\alpha)S_{i,t}^s + O_t^s - (1+rb_t^s)O_{t-1}^s$$

$$t=2 \le t \le T\text{-}1 \qquad (5.7)$$

$$\sum_{i=1}^{I}(1+\alpha)B_{i,t}^s - (1+rl_t^s)Q_{t-1}^s + L_t^s = V_t^s + \sum_{i=1}^{I}(1-\alpha)S_{i,t}^s - (1+rb_t^s)O_{t-1}^s$$

$$t=T \qquad (5.8)$$

Short sales constraints

$$S_{i,1}^s \le x_{i0} \qquad t = 1,\ s=1,\dots,S \qquad (5.9)$$

$$S_{i,t}^s \le x_{i,t-1}^s \qquad t \ge 2,\ s=1,\dots,S \qquad (5.10)$$

Bound constraints

$$l_i H_t^s \le x_{i,t}^s \le u_i H_t^s \qquad t=1...T\ , i=1,...,I, s=1,...,S \tag{5.11}$$

Non Anticipativity Constraints

$$x_{i,1}^s = x_{i,1}^1 \qquad s=2,\ ...,\ S \tag{5.12}$$

$$B_{i,1}^s = B_{i,1}^1 \qquad s=2\ ,\dots,\ S \tag{5.13}$$

$$S_{i,1}^s = S_{i,1}^1 \qquad s=2,\ \dots,\ S \tag{5.14}$$

The chance constrained Programming (CCP) are formulated as follows:

$$M\delta_{t+1}^s \ge \gamma L_{t+1}^s - H_{t+1}^s \qquad t=1,...,T-1 \tag{5.16}$$

$$\pi_s \sum_{s=1}^{S} \delta_{t+1}^s \le 1-\beta_{t+1}, \qquad t=1,...,T-1 \tag{5.17}$$

$$\delta_{t+1}^s \in \{0,1\}, \qquad t=1,...,T-1 \tag{5.18}$$

The Integrated Chance Constraints (ICCP) are formulated as follows:

$$H_t^s - \gamma L_t^s + shortage_t^s \ge 0 \tag{5.19}$$

$$\pi_s \sum_{s=1}^{S} shortage_t^s \le \lambda \pi_s \sum_{s=1}^{S} L_t^s \tag{5.20}$$

5.6 Concluding Remarks

In this chapter we formulated three different stochastic decision models. The two stage stochastic programming (TSP), chance constrained stochastic programming (CCP) (which restricts the number of underfunding events) and integrated chance constraint programming (ICCP) (which restricts the expected amount of underfunding).

Solutions of TSP and ICCP were compared in term of asset allocation as well as the expected terminal wealth and expected borrowing required.

A discussion of the in –sample testing and comparison of LP, SP and ICCP decisions is postponed until section 6.2.

Chapter 6

Benchmarking and Evaluation of Results

6.1 Introduction

In this chapter we examine and evaluate the results obtained for the decisions of Fixed Mix (FM), Expected value (LP) (deterministic approach), Two stage Stochastic Programming (TSP) and Integrated Chance Constraint Programming (ICCP) which are described in Chapter 4 and Chapter 5 respectively and compare to the out-of-sample model. A new set of larger scenarios are generated in the same method as described in Chapter 3 for the out-of-sample models. In this section we review the literature on benchmarking and performance evaluation. In section 6.2 we report the in- sample analysis for FM, LP, TSP and ICCP models and in section 6.3 we set out and report the out-of-sample analysis for FM, LP, and TSP. Our analyses are based on two sets of scenarios. We compute the common risk adjusted performance measures; these include both symmetric and asymmetric down-side risk measures, such as Sharpe Ratio, Sortino Ratio, Solvency Ratio and Funding Ratio for FM, LP and TSP. We report the results in section 6.5. We present our conclusions in section 6.6.

It is common practice to evaluate a model by applying backtesting methodology which uses historical data; this is in addition to the analysis done by in-sample testing as well as out-of sample evaluation. In the context of our investigation the historical data is limited, therefore we do not perform such backtests. Zenios et al. (1998) evaluated a multistage SP for fixed income portfolio management under uncertainty using historical backtesting and Monte Carlo simulation. The SP models are compared with portfolio immunization models and single period models. The results showed that the multi period SP models outperform the portfolios generated by single period models and portfolio immunization.

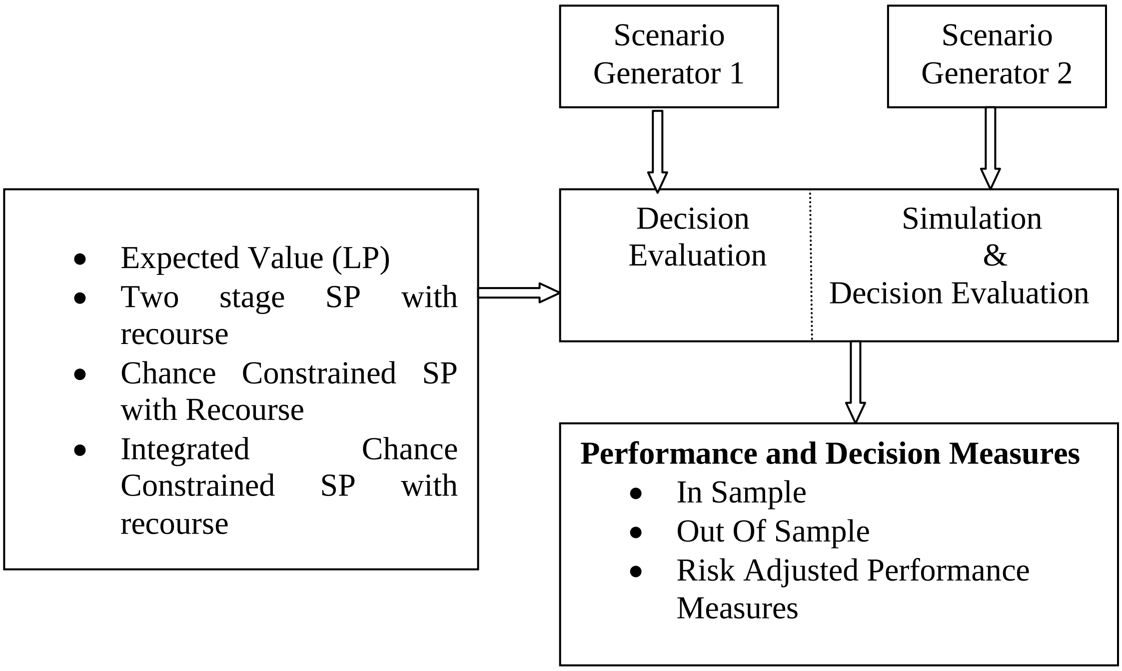
Mulvey et al. (2000) employed stress testing procedure in the Towers Perrin-Tillinghast ALM model. They identified poor performance scenarios using a systematic approach and set up the CAP:Link system (scenario generator). Four sets of 500 scenarios were generated to represent normal economy conditions, equity market crash, the equity bear market and disinflation were generated and compared. This approach enables the decision makers to prepare a contingency plan should any of the adverse scenarios materialise.

Kouwenberg (2001) conducted rolling horizon simulations and investigated the annual optimal decisions of the ALM model for a pension fund in Netherland for five years. The rolling horizon simulations are compared to a fixed mix model. The objective function is to minimize the average contribution rate and penalize shortfall in the funding ratio. Using a simulation study Kouwenberg demonstrates that the trade off between risk and costs are superior in the SP model and concludes that the SP model leads to better solutions than the FM strategy.

Fleten et al. (2002) evaluate and compare the performance of a multistage stochastic programming model with a fixed mix ALM insurance model by in sample and out of sample testing. Schwaiger (2009) proposes two stage and multistage simulation methodologies and six decision evaluation techniques for a defined benefit pension fund. In the two stage simulation, the first stage decisions are fixed and the recourse actions are examined. In the multistage simulation, Schwaiger implements a rolling forward setting, and compares the wealth distribution at each time period.

Figure 6.1 shows the overall decision framework of our research. Evaluation of the models is done by in sample testing as well as out of sample testing. We include the calculation of risk adjusted performance measures for the ALM models. Scenario Generator 1 provides scenarios which are used in our stochastic optimization model. Scenario generator 2 provides scenarios for the out of sample analysis.

Figure 6.1: Decision making framework



6.2 In- Sample Analysis

Data

Set of 2000 scenarios (described in chapter 3) are used in the in-sample analysis.

The growth dividend for in-sample analysis is equal to 5%.

Design of Computational Experiments

We fix the first stage (here and now) decision variables. These are the decisions taken in the first time period. We then optimise the recourse part (second stage) of the SP model and compute the terminal wealth values for all the scenarios. Thus the methodology for the in-sample analysis can be summarised as:

1. Fix the first stage decision (FM, LP, TSP or ICCP).
2. Solve the remaining model for all in-sample scenarios.
3. Compute the terminal wealth values and analyse the results.

Results

Figure 6.2: Histogram of Terminal Wealth Distribution. Strategy: FM, LP, TSP and ICCP

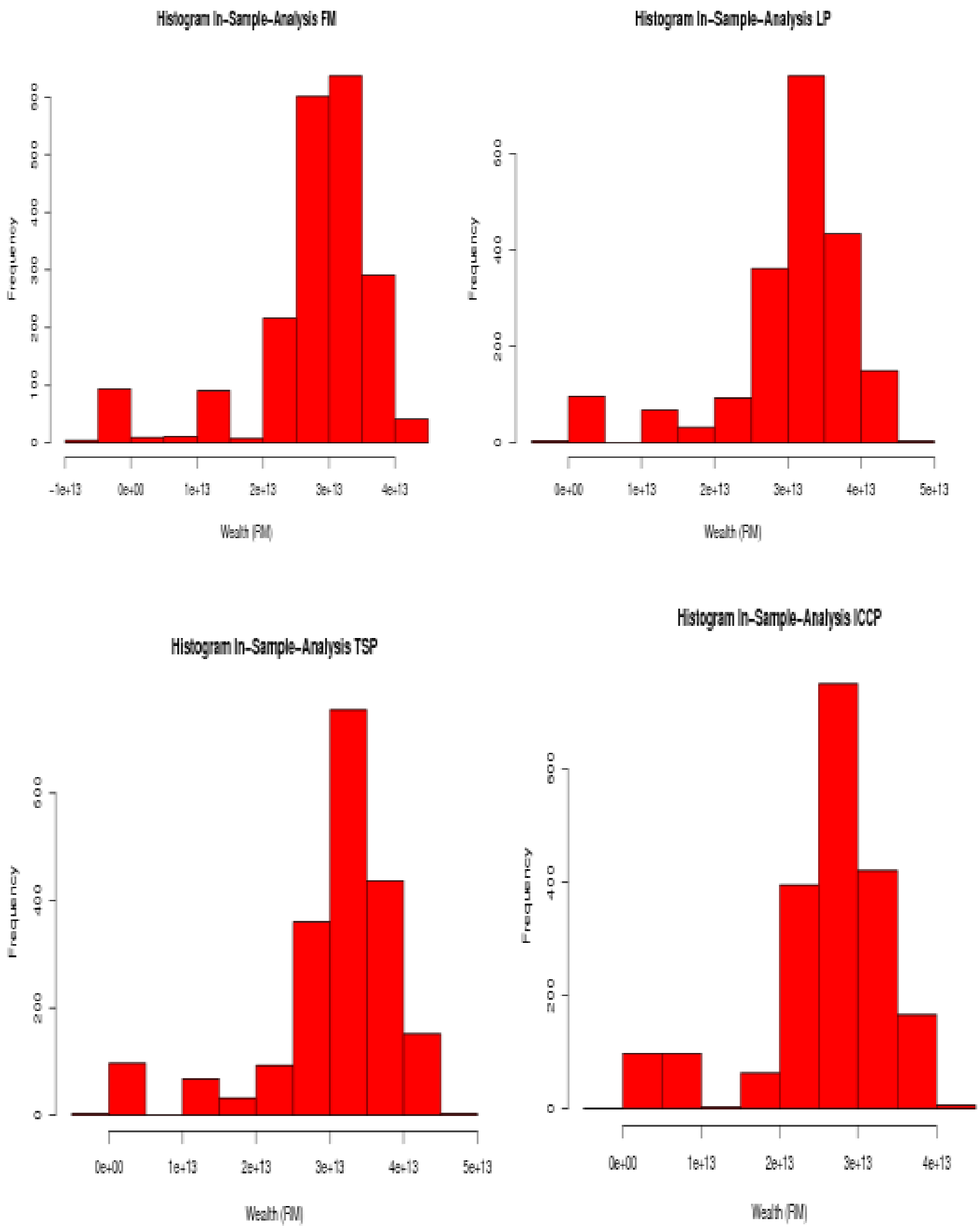


Table 6.1: Descriptive Statistics for In-sample Analysis

In-Sample Analysis of Terminal Wealth (RM)				
	FM	LP	TSP	ICCP
Mean	2.791E+13	3.0488E+13	3.051E+13	2.583E+13
Maximum	4.403E+13	4.625E+13	4.628E+13	4.122E+13
Minimum	-5.449E+12	-1.365E+11	-1.364E+11	-1.718E+11
Standard Deviation	9.399E+12	9.106E+12	9.109E+12	8.295E+12
Skewness	-1.811	-1.761	-1.761	-1.359
Kurtosis	3.468	3.379	3.381	1.891
Range	4.948E+13	4.638E+13	4.642E+13	4.139E+13

Figures 6.2 shows the histograms for the distribution of the terminal wealth for the FM, LP, TSP and ICCP strategies with the growth dividend fixed at 5%. The descriptive statistics of these results in a summary form are given in Table 6.1. All four distributions are skewed to the left. The expected terminal wealth values of the LP and TSP are seen to be comparable. The FM and ICCP have much lower expected terminal wealth. The ICCP is the least skewed. Finally, the minimum value, that is, the left tail terminal wealth distribution in the FM model is, an order of magnitude higher than that of LP, SP and ICCP. This shows that the FM strategy could result in much larger losses.

Many pension schemes which follow Liability Driven Investment (LDI) explicitly minimise the deviation between assets and liabilities (see Schwaiger, 2009). In our study we are however, concerned with asset allocation decisions and maximise expected terminal wealth, improving constraints on the expected shortfall of assets matching liabilities. Hence ICCP constraints reduce the terminal wealth. Since our objective is not to minimize deviation or to minimize borrowing in case of a shortfall in the present modelling framework, the evaluation of ICCP strategies is of limited value. The ICCP model (strategy) is therefore omitted from our out of sample analysis.

6.3 Out- of- sample Testing : Decision Evaluation

Data

Set of 4000 scenarios (out-of-sample) are generated and used to compute the out-of-sample analysis.

The growth dividend for the out of sample analysis is equal to 5%.

Design of Computational Experiments

Our approach to out-of-sample analysis is described below:

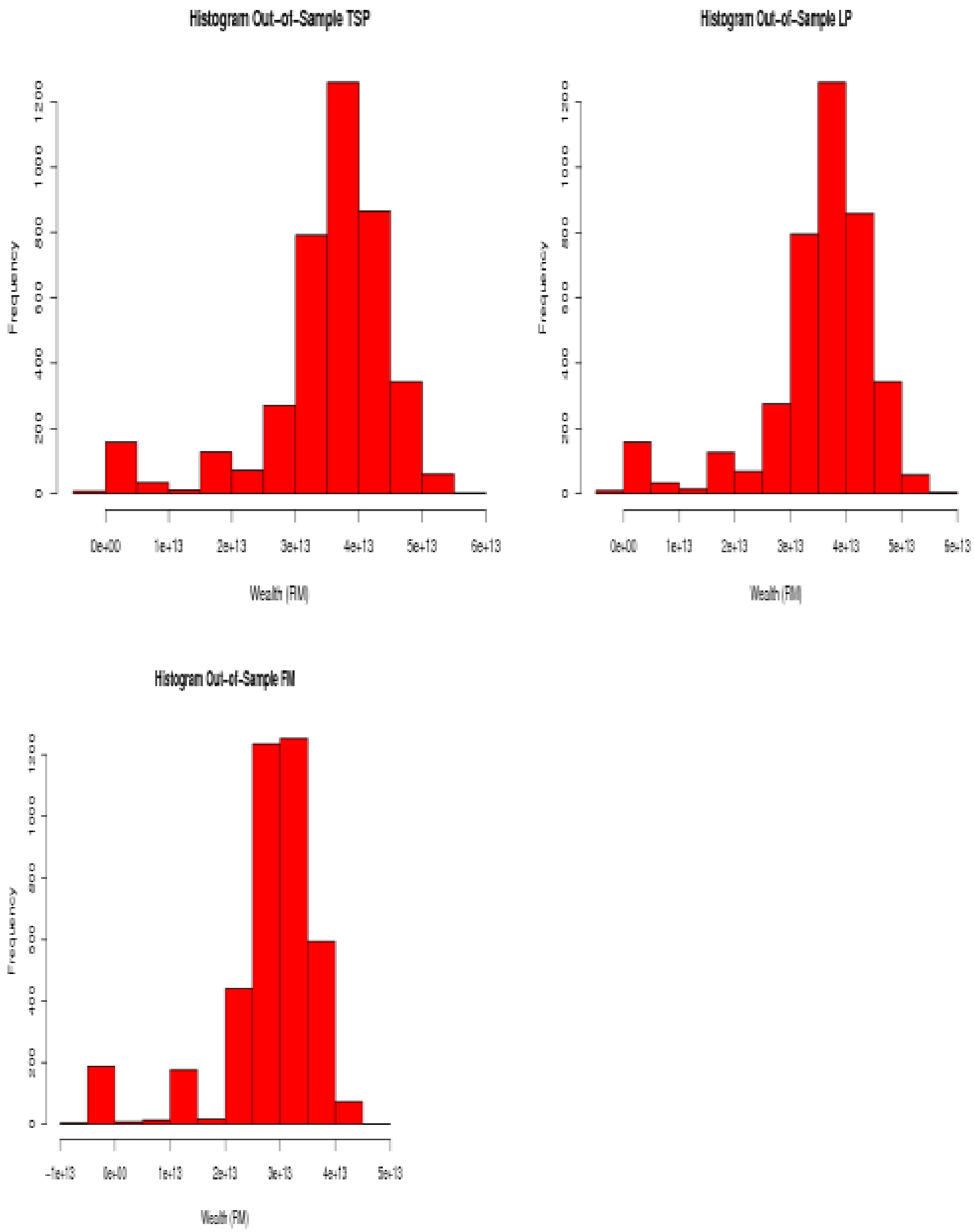
1. Fix the first stage decision (FM, LP, TSP).
2. Solve the remaining model for all out-of-sample scenarios.
3. Compute the terminal wealth and analyse the results.
4. Compute risk adjusted performance indices across the cross section of the time periods.

Results

Table 6.2: Descriptive Statistics for Out-of-Sample Analysis

Out-of-Sample Analysis of Terminal Wealth (RM)			
	FM	LP	TSP
Mean	2.798E+13	3.529E+13	3.533E+13
Maximum	4.535E+13	5.763E+13	5.737E+13
Minimum	-5.449E+13	-5.818E+11	-6.333E+11
Standard Deviation	9.223E+12	1.00E+13	1.00E+13
Skewness	-1.843	-1.642	-1.644
Kurtosis	3.752	3.201	3.219
Range	5.08E+13	5.821E+13	5.8E+13

Figure 6.3: Histogram of Terminal Wealth Distribution. Strategy: TSP, LP and FM



The descriptive statistics of these results in a summary form are given in Table 6.2. As in the in-sample analysis, the out-of-sample histograms for the distribution of the terminal wealth (Figure 6.3) shows that these values are skewed to the left. The mean values of LP and TSP in the out-of-sample analysis are higher than the mean values in the in-sample analysis. However the standard deviation and the range are higher indicating that the terminal wealth values are spread out over a larger range of values in the out of sample analysis. The lowest value, the left tail of the wealth distribution in the FM stratey, is two orders of magnitude higher than of LP and TSP.

6.4 Computation of Performance Indices in time buckets

Mean Variance analysis by Markowitz (1959) is the common conventional method used to quantify the trade off between risk (measured by the variance of return) and reward (modelled by the mean of the return). Various other risk adjusted performance measures (RAPM) are employed in finance for example the Treynor ratio, the Sharpe ratio and Jensen's alpha. RAPM is currently an active area for research and is very substantial to investors and mutual fund managers who need to make strategic asset allocation decisions and also evaluate such decisions. The common feature of the performance measures is that it measures a given fund’s returns in regard to risk; however, how risk is defined and measured in each RAPM is different. Usually, the achievement of the asset is measured based on the performance of any benchmark asset which can be a broad-based market index, a specialized index, or a customized index.

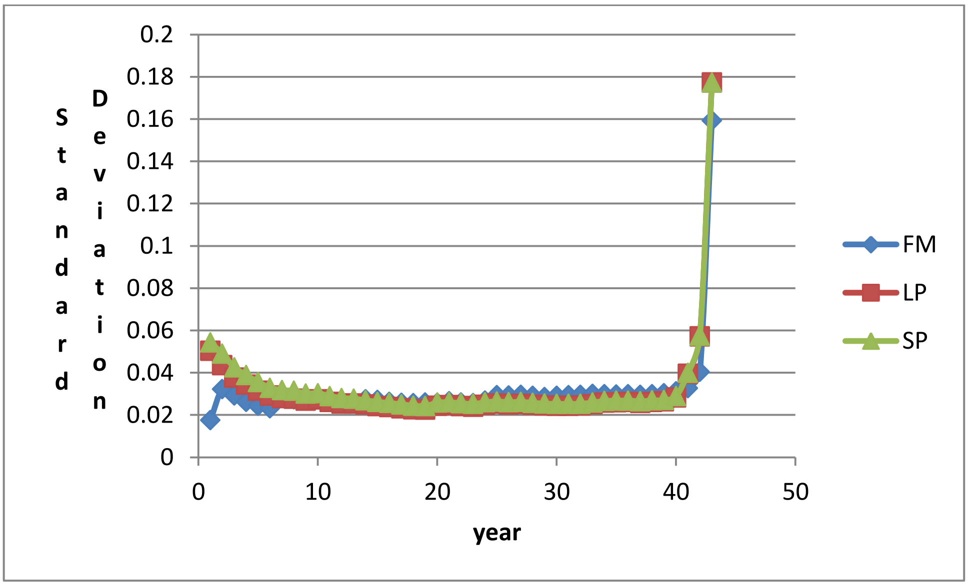
Schmid (2010) discusses the applications of risk adjusted performance measures in the financial industry and test the RAPMs using empirical data. Schmid combines the two important allocation problems which are to maximize investors’ expected utility and secondly, optimize the risk capital of a financial institution’s allocation for the different risky business activities.

We consider the out-of-sample scenarios and the wealth distribution for each year as described in section 6.1 and 6.3. In this section we consider the following performance measures i) standard deviation of returns, ii) Sharpe Ratio, iii) Sortino Ratio, iv) Solvency Ratio, and Funding Ratio. The decisions proposed by the different models are evaluated and compared.

i. Standard Deviation

Standard deviation is the basic measure of volatility therefore, it is also known as historical volatility. Standard deviation is also a symmetric measure of risk. The tracking error symbolises the standard deviation of excess return over the risk-free rate or some benchmark. Figure 6.4 shows the standard deviations for FM, LP and SP for fund wealth over the 45 years planning horizon.

Figure 6.4: Standard Deviation for fund wealth over the 45 years



ii. Sharpe Ratio

One of the popular risk adjusted performance measure is the Sharpe ratio that was introduced by Sharpe (1966). The Sharpe ratio shows the excess return one receives per unit of risk in investment.

The ex ante Sharpe ratio is given as :

$$S = \frac{\bar{d}}{\sigma_d} \qquad (6.1)$$

Where

$\bar{d}$ = the average of excess return expected value of the difference between return of the fund over the benchmark.

σ_d =is the standard deviation of the excess return of the fund over the benchmark.

$$\overline{d} = E[R_p - R_I] \tag{6.2}$$

R_p = return for the fund.

R_I =5% (considered as a benchmark)

We compute the ex-post Sharpe ratios using the out of sample scenarios. For the ex-post computation of Sharpe ratio at each time point t, we have followed the method recommended by Sharpe (1994). This is calculated as follows:

$$S_{ht} = \frac{\overline{D}_t}{\sigma_{Dt}} \tag{6.3}$$

and

$$D_t = R_{Pt} - R_{It} \tag{6.4}$$

$$\overline{D}_t = \frac{1}{t}\sum_{t'=1}^{T} D_{t'} \tag{6.5}$$

$$\sigma_{Dt} = \sqrt{\frac{\sum_{t'=1}^{t}(D_t' - \overline{D})^2}{t-1}} \tag{6.6}$$

R_{Pt} = the expected return on the fund in period t.

R_{It} = the return on the benchmark portfolio in period t. This is set to 5% for all t.

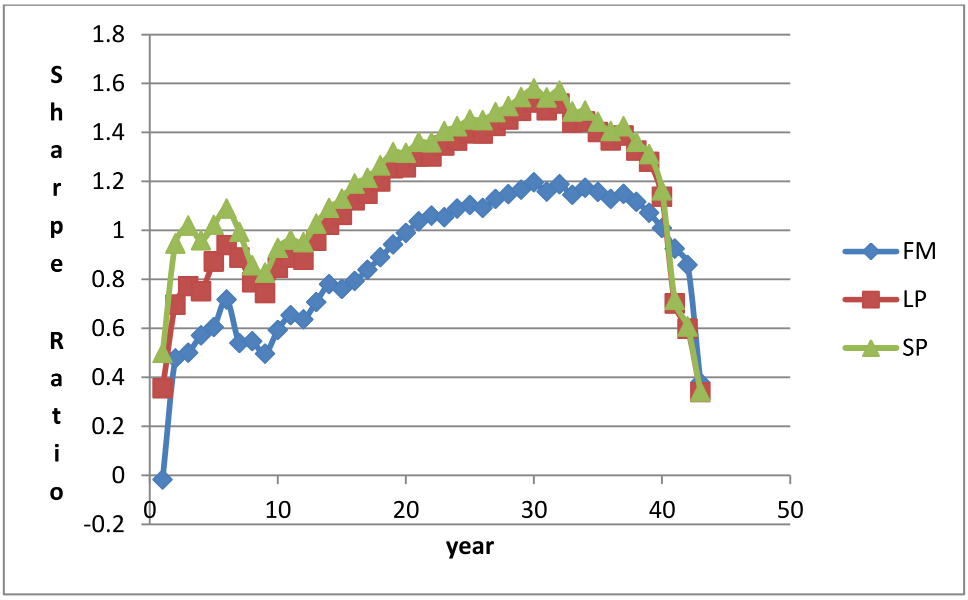
D_t = the expected excess return of the fund over the benchmark at time period t

$\overline{D}$ = the average value of D_t over the period from t'=1…t.

σ_{D_t} = standard deviation of excess return over the period t'=1…t

With the Sharpe ratio, a direct comparison of the risk-adjusted performance of any two funds, regardless of their volatilities and their correlations. The higher the Sharpe ratio value, the better the risk adjusted performance. Figure 6.5 shows the Sharpe Ratio of FM, LP and TSP strategies for fund wealth over the 45 years.

Figure 6.5: Sharpe Ratio for fund wealth over the 45 years



iii. Sortino Ratio

Sortino Ratio developed by Sortino and Van der Meer (1991) takes into consideration down side risk. In this respect it is an enhancement of the Sharpe ratio as it considers only those returns falling below a user-specified target, or pre-specified rate of return, unlike the Sharpe ratio that penalizes both upside as well as downside return deviations. Sortino ratio therefore treats risk more pragmatically, compared to the Sharpe ratio. Following Schwaiger (2009) we compute the ex-post Sortino ratios at each time period t=1…T using geometric mean return:

$$S_t = \frac{R_{Pt} - R_{It}}{\sigma_{dt}} \tag{6.7}$$

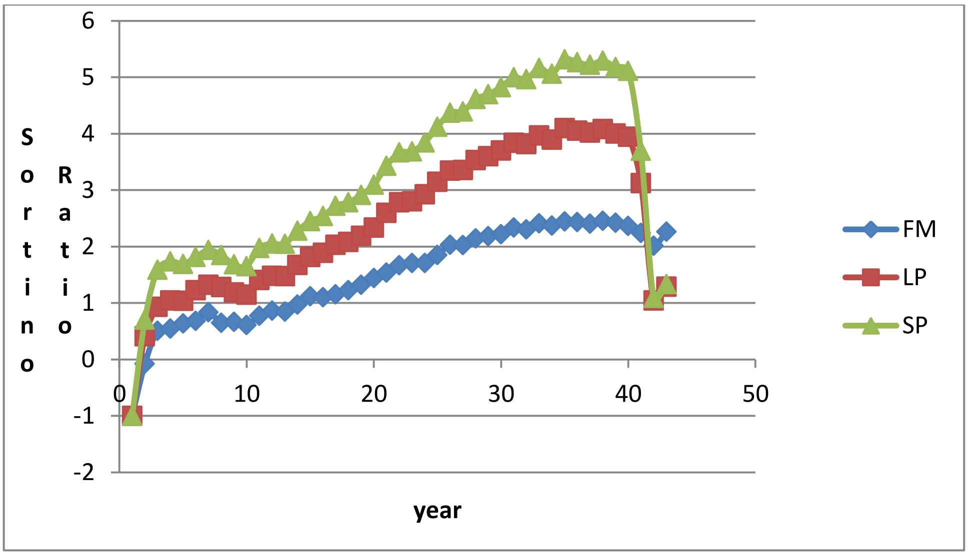
where

$$R_{Pt} = \left(\prod_{t'=1}^{t}(1 + R_{Pt'})\right)^{\frac{1}{t}} - 1 \tag{6.8}$$

$$\sigma_{dt} = \sqrt{\frac{1}{t}\sum_{t'=1}^{t}\left(\min\left(R_{Pt} - R_{It}, 0\right)\right)^2} \tag{6.9}$$

The term σ_d is also called the downside deviation or **target** semideviation. Only at the time period in which the portfolio's return falls below the benchmark are penalized. The benchmark or target return R_{It} is set to 5%.

Figure 6.6: Sortino Ratio for fund wealth over the 45 years



iii. Solvency Ratio and Funding Ratio

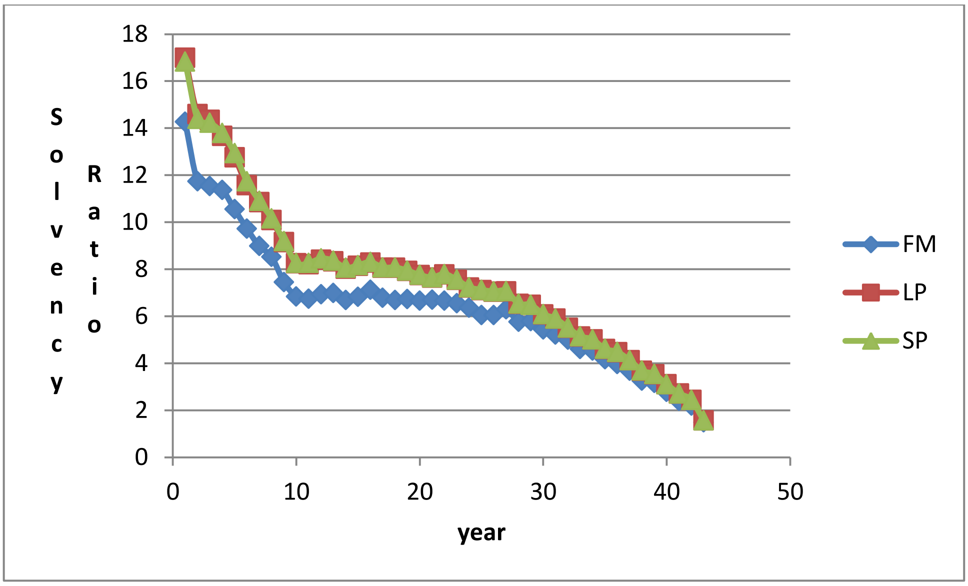
Solvency ratio is used to measure a fund's or an investor's ability to meet long term obligations and can be presented as:

Solvency Ratio =

(Asset net present value – Liability net present value)/(Liability net present value) (6.10)

Solvency Ratio shows how likely it is that a fund will be able to meet its debt obligations.

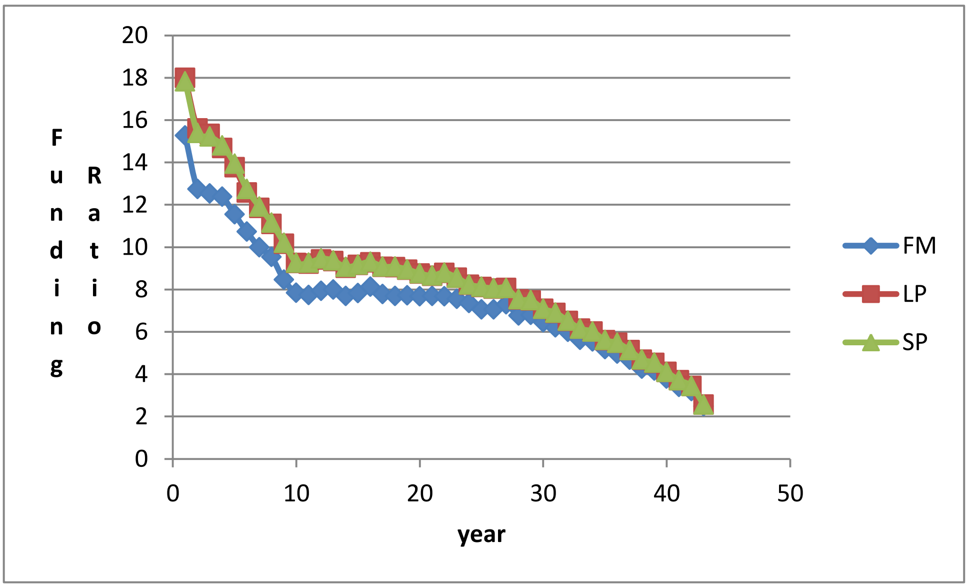
Figure 6.7: Solvency Ratio for fund wealth over the 45 years



Funding ratio is a ratio of a pension scheme's assets to its liabilities. A funding ratio indicates to what extent the pension scheme is able to cover all payments it is obligated to make.

Funding Ratio = Asset net present value/Liability net present value (6.11)

Figure 6.8: Funding Ratio for fund wealth over the 45 years



6.5 Discussion of Results

From Figure 6.4 which displays the standard deviation of the returns, we find that all three strategies have fairly comparable standard deviation. Given that the LP and TSP lead to superior expected wealth value the latter are desirable over the former.

We have reported the ex-post Sharpe ratio and ex-post Sortino ratio plotted over the planning horizon. The TSP strategy as shown in Figure 6.5 gives higher (desirable) Sharpe ratio compared to LP and FM. Between the TSP and LP strategies, we observe that in the first 8 years SP leads to higher values than the LP. After this period the difference is indistinguishable. In comparing the results with Sortino Ratio (Figure 6.6) it shows that TSP dominates the LP and FM.

We observe that towards the final periods year 43 of the time horizon it becomes necessary to borrow funds to meet the growing lump sum payment liabilities (calculated at the growth dividend level 5%). This coincides with a reduction of wealth value, thereby indicating the negative return of the EPF fund. This is reflected in the sudden drop in Sharpe and Sortino ratios.

Figure 6.7 and Figure 6.8 display the solvency ratio and the funding ratio for TSP, LP and FM. The Funding ratio and solvency ratio for TSP and LP are fairly comparable and higher than FM throughout the planning horizon. However in the last three time periods the difference between the solvency ratio and the funding ratio of TSP, LP and FM strategy is small.

The fact that the TSP is evaluated as a better strategy than LP and FM with respect of Sharpe Ratio and Sortino Ratio measure can be interpreted as a reason for choosing TSP over LP and FM. This is because the immediate future is more meaningful than a distant horizon of longer years as the economic conditions are likely to change and the model results are less dependable.

6.6 Concluding Remarks

In this chapter we have described the simulation and evaluation methods used for the in- sample analysis as well as the out- of- sample analysis.

We reported the results of the in- sample analysis and out- of- sample analysis using descriptive statistics and histograms representing the distribution of terminal wealth for each of the three models (strategies) FM, LP and TSP.

We computed the risk adjusted performance indices across the cross section of the time periods. The RAPMs include the i) standard deviation of returns, ii) Sharpe Ratio, iii) Sortino Ratio, iv) Solvency Ratio and Funding Ratio. The decisions proposed by these three different models were evaluated and compared.

Chapter 7

Conclusions and Future Directions

7.1 Thesis Summary

In this thesis we have presented an ALM framework for the main pension scheme for private sector employees of Malaysia, the Employees Provident Fund (EPF). The EPF is a defined contribution scheme: upon retirement, participants are paid out of the lump sum payments depending on the contributions accumulated over the employment period. The EPF's liabilities are not just the pension payments. The liabilities also include early withdrawals; during the employment period, participants can withdraw up to 30% of money for healthcare, mortgages and education and the lump sum of the accumulated wealth is given to next of kin when members die. The participants receive a (guaranteed) growth dividend on their contribution, of at least 2.5% per annum, which is reinvested into the fund; this protects the participants against the uncertainties in the retirement income. In the event that the aggregated liabilities are not matched in a given year, the EPF needs to fulfil the liabilities by borrowing cash and repay the amount borrowed over a period of future years.

EPF is a defined contribution retirement scheme; thus, the increase in life expectancy brings the risk that participants may outlive their savings. We have considered a planning horizon of 45 years and investigate investment allocation strategies that maximise the terminal wealth of the EPF. We have constructed a family of decision models: LP, TSP, CCP, ICCP which encapsulates the ex-ante decision problem for EPF Malaysia. We include a fixed mix (FM) strategy for the purpose of comparison of results. The maximum and minimum bounds of portfolio weights of asset holdings are included in the model. These guidelines are based on the composition constraints enacted in the EPF Act. The funds are invested into 5 types of assets: the money market instruments, Malaysian Government Securities (long term), Malaysian Government Securities (short term), property and equity.

The formulation of the stochastic programming models includes three major sources of uncertainties, namely the returns of the assets, contributions and the liabilities. We have captured the uncertainties of the asset returns using a vector autoregressive (VAR) scenario generation model.

The uncertainty in the future contribution payments and liabilities is driven by the population. We implemented a Markov model and quantified the future evolution of populations and their states (active participants, retired, etc.). We used the bootstrap method (sampling with replacement), in which the pre-calculated transition probabilities as well as the number of new entrants by age group are used to simulate the population of EPF members throughout the 45 years planning horizon. The historical EPF population data in the year 2010 is the initial input of our Markov population model. We assumed 2% of annual growth in the total average salary for each age group. By combining the Markov population model with the salary model we quantified the contribution cash flows.

In this study we have pre-calculated (using the data from the start year 2010) the average lump sum paid to participants at retirement for active as well as inactive members and lump sum payment due to death. These pre-calculated data are combined with the growth dividend and population scenarios to generate lump sum payments throughout the planning horizon. We make a set product of two sets i) the set of population scenario (these affect contributions and lump sum liabilities) with ii) the set of asset returns scenario leading to the combined set of scenarios. The asset returns, contributions and liabilities in the form of discrete scenarios are used to instantiate the family of SP decision models. The liabilities data are calculated for different values of growth dividends considered in this research. We generate a "fan" shaped scenario tree with 2000 scenarios spanning all the time periods. The expected values of the uncertain parameters, at each time period, are used as inputs to the LP (deterministic) model.

We have compared the investment strategies obtained with each of these models by carrying in-sample analysis and out-of-sample testing. For the out of sample analysis, we have used the first stage decisions obtained with the in-sample scenarios. We then generated 4000 scenarios, solved the remaining model for each of the scenarios and analysed the distribution of the terminal wealth. We found that the solutions obtained with TSP give the best distribution of the terminal wealth as evaluated in and out-of-sample. The LP (deterministic) model leads to marginally worse solutions. Both TSP and LP clearly outperform the FM strategy. As somehow expected (due to additional constraints at each time period) from the ICCP model results, it can be said that the distribution of the terminal wealth are with weaker characteristics, e.g. a lower value of terminal wealth.

We calculated the typical risk adjusted performance measures that are commonly used in the industry for the decision models. The performance measures include the standard deviation, Sharpe ratio, Sortino Ratio, Solvency ratio and Funding Ratio.

7.2 Contributions

The contributions of our research are summarised as follow:

i. We provide an insightful overview of the Provident Funds, especially Employees Provident Fund (EPF) Malaysia. The EPF scheme provides flexibility and value to participants by allowing drawdowns during accumulation phase. We have also presented a comparative study of EPF in other Commonwealth countries (see Appendix F).

ii. We have developed an innovative population model of persons joining and leaving the EPF scheme. Our modelling is based on an open system of the population model unlike the normal practice of assuming a closed system. We introduce further innovation in applying this population model to describe the liability as well as contribution scenarios.

iii. We have developed and presented a generic optimum decision model under uncertainty for the EPF Malaysia. This family of models is used to investigate different aspects of asset allocation and liability matching decisions.

iv. We test and valuate the performance of our models in an ex-post setting. The decision evaluation of the rebalancing decisions are carried out via in-sample analysis and out-of-sample testing. We calculate risk adjusted performance measures that are variously used in the industry to evaluate decision models for asset allocation. Through our empirical study we have established that TSP provides a superior asset allocation strategy.

7.3 Further Research Directions

The work presented in this thesis could be extended in a number of ways, which are summarised below.

The proposed stochastic programming models (two-stage, chance constrained and integrated chance constraints) with recourse can be widened from two stages to a multistage stochastic programming setting.

Other assets that are important to EPF can be included in the model for example Islamic bonds or introduce strategies such as interest rate swaps and index futures which can be used to minimize risk. The data can also include overseas investment and their performance as EPF has started to invest outside of Malaysia eventhough in a small percentage of the total wealth.

This model can also be run with other alternative scenario generation methods. The uncertainties in the pre-retirement withdrawals –education, health and mortgage can also be modelled separately and then introduced in the SP decision model.

Lastly a comparative study of the four available provident funds (India, Sri Lanka, Singapore and Malaysia) could be undertaken by applying the proposed decision models to see the impact of the models for each country based on regulations and economic conditions.

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[124] Central Provident Fund (CPF) Singapore website at http://mycpf.cpf.gov.sg/

[125] Employees Provident Fund Organisation India website at http://epfindia.nic.in/epfbrief.htm

[126] Employees Provident Fund Malaysia website at www.kwsp.gov.my

[127] Employees Provident Fund Sri Lanka website at http://www.epf.lk/

[128] Life expectancy. (http://www.indexmundi.com/malaysia/life_expectancy_at_birth.html)

[129] Unemployment rate. http://www.indexmundi.com/malaysia/unemployment_rate.html

[130] USA Internal Revenue Service website (http://www.irs.gov/).

APPENDIX A :

A.1 The AMPL Models

```
# Generic Model
# 1 Scenario EVLP
# Multiple Scenario-TSP,ICCP

param tbuy;
param tsell;


#param NS;
param NSContributions;
param NSReturns;
param NA;
param NT;

#SETS
set assets := 1..NA;
set tp := 1..NT;

#SCENARIO
set contributionsscenarios := 1..NSContributions;
set returnsscenarios := 1..NSReturns;

set scen := 1.. (NSContributions*NSReturns);

#RANDOM PARAMETERS
param return{assets, tp, scen};
param liabilities {tp, scen};
param contributions {tp, scen};

param b{t in tp, s in scen} := return[1, t, s] - 0.005;
param c{t in tp, s in scen} := return[1, t, s] + 0.005;

#PROBABILITIES
# Assuming uniform probabilities
param ProbContributionScen{contributionsscenarios} :=
1/card(contributionsscenarios);
param ProbReturnScen{returnsscenarios} := 1/card(returnsscenarios);

# Calculated when reading data
param Prob{scen};

#PARAMETERS
param initialholdings{assets} ;
param lowbound {assets};
param upperbound {assets};


#VARIABLES
var amounthold{a in assets, t in tp, s in scen } >=0;
var amountbuy{ a in assets, t in tp, s in scen } >=0;
var amountsell{ a in assets, t in tp, s in scen } >=0;
var marketvalue{t in tp, s in scen};
```

```
var assetvalue{t in tp, s in scen}>=0;
var borrow{t in tp,s in scen}>=0;
var lend{t in tp,s in scen}>=0;
var expborrow {t in tp}>=0;

#OBJECTIVE
maximize wealth:sum{s in scen}Prob[s]*(marketvalue[NT,s]);

subject to

asetmarketvalue{s in scen}:
	marketvalue[1,s]=sum{a in assets}amounthold[a,1,s];

asetmarketvalue1{t in 2..NT,s in scen}:
	marketvalue[t,s]=sum{a in assets}amounthold[a,t,s]+(1+b[t,s])*lend[t-
1,s]-borrow[t-1,s]*(1+c[t,s]);

asetvalu1{t in tp,s in scen}:
    assetvalue[t,s]=sum {a in assets} amounthold[a,t,s];

assetholdings1{a in assets, s in scen}:
	amounthold[a,1,s]=initialholdings[a]+amountbuy[a,1,s]-
amountsell[a,1,s];

assetholdings2{a in assets,t in 2..NT, s in scen }:
	amounthold[a,t,s]=amounthold[a, t-
1,s]*(1+return[a,t,s])+amountbuy[a,t,s]-amountsell[a,t,s];

cashbalance{s in scen}:
	sum{a in assets} amountbuy [a,1,s]*tbuy+lend[1,s]+
liabilities[1,s]=sum {a in
assets}amountsell[a,1,s]*tsell+contributions[1,s]+borrow[1,s];

cashbalances{t in 2..44, s in scen}:
	sum{a in assets} amountbuy [a,t,s]*tbuy+ liabilities[t,s]+lend[t,s]-
(1+b[t,s])*lend[t-1,s]=sum {a in
assets}amountsell[a,t,s]*tsell+contributions[t,s]+borrow[t,s]-
(1+c[t,s])*borrow[t-1,s];

cashbalancess{s in scen}:
	sum{a in assets} amountbuy [a,45,s]*tbuy+ liabilities[45,s]-
(1+b[45,s])*lend[44,s]=sum {a in
assets}amountsell[a,45,s]*tsell+contributions[45,s]-
(1+c[45,s])*borrow[44,s];


shortsaleconstraint{ a in assets,  s in scen}:
	amountsell[a,1,s]<=initialholdings[a];

shortsaleconstraints{ a in assets, t in 2..NT,s in scen}:
	amountsell[a,t,s]<=amounthold[a,t-1,s];

boundconstraints{a in assets, t in tp, s in scen}:
	assetvalue[t,s]*upperbound[a]>=amounthold[a,t,s];

boundconstraints2{a in assets, t in tp, s in scen}:
	amounthold[a,t,s]>=assetvalue[t,s]*lowbound[a];

#Non anticipativity constraints
```

```
NA1{a in assets, s in scen}:
       amounthold[a, 1, 1] = amounthold[a,1,s];
NA2{a in assets, s in scen}:
       amountbuy[a, 1, 1] = amountbuy[a,1,s];

NA3{a in assets, s in scen}:
       amountsell[a, 1, 1] = amountsell[a,1,s];

# Definition of expectations
epbr{t in tp}:
expborrow [t]=sum{s in scen}borrow[t,s]*Prob[s];
```

Solve SP Model

```
# Execute (and change) this script to load the SP model
include ../base/basemodel.ampl;
include ../base/basedata.ampl;
include ../scripts/scriptparameters.ampl;
let ModelType := "SP";

################ INPUT PARAMETERS #########################
# Put the following to 0 to read from Excel, to 1 to read from TAB files
(generated through the script ExportToTabFiles)
let loadFromExcel := 0;
# Path to datafile to load. In case of XLS, omit the extension (i.e.
"../DataFiles/table200x20"), in case of tab files put
# the base file name
let FILENAME := "table200x20";
# Percentage for liabilities. i.e., for 2.5%, put 2.5, for 3% put 3 and so
on
# It tries to find it in the corresponding table in the excel file
# or in the corresponding bit file
let liabpercentage := 5;
# Number of contributions scenarios to read (and use in the SP)
let contributionScenariosToRead := 20;
# number of returns scenarios to read (and use in the SP)
let returnScenariosToRead := 100;

############### END OF INPUT PARAMETERS ####################



let NSContributions := contributionScenariosToRead;
let NSReturns := returnScenariosToRead;

# Load SP Model Data
include ..\scripts\DoSPLoadData.ampl;
# Solve
include ..\scripts\DoSolve.ampl;
# Save solution
include ..\scripts\savesolution.ampl;
```

Solve ICCP Model

```
Execute (and change) this script to load the ICCP model
include ../base/basemodel.ampl;
include ../base/baseICCConstraints.ampl;
include ../scripts/scriptparameters.ampl;
include ../base/basedata.ampl;

let ModelType := "ICCP";


################ INPUT PARAMETERS #########################
let gamma := 1.10;
let lamda := 0.05;
let displaySolveDetails := 0;
# Put the following to 0 to read from Excel, to 1 to read from TAB files
(generated through the script ExportToTabFiles)
let loadFromExcel := 0;
# Path to datafile to load. In case of XLS, omit the extension (i.e.
"../DataFiles/table200x20"), in case of tab files put
# the base file name
let FILENAME  := "table200x20";
# Percentage for liabilities. i.e., for 2.5%, put 2.5, for 3% put 3 and so
on
# It tries to find it in the corresponding table in the excel file
# or in the corresponding bit file
let liabpercentage := 5;
# Number of contributions scenarios to read (and use in the SP)
let contributionScenariosToRead := 20;
# number of returns scenarios to read (and use in the SP)
let returnScenariosToRead := 100;

############### END OF INPUT PARAMETERS ####################



let NSContributions := contributionScenariosToRead;
let NSReturns := returnScenariosToRead;

# Load SP Model Data
include ..\scripts\DoSPLoadData.ampl;
# Solve
include ..\scripts\DoSolve.ampl;
# Save solution
include ..\scripts\savesolution.ampl;
```

Solve LP Model

```
# Execute (and change) this script to load the LP model
include ../base/basemodel.ampl;
include ../base/basedata.ampl;
include ../scripts/scriptparameters.ampl;
let ModelType := "LP";


################ INPUT PARAMETERS #########################
# Put the following to 0 to read from Excel, to 1 to read from TAB files
(generated through the script ExportToTabFiles)
let loadFromExcel := 0;
# Path to datafile to load. In case of XLS, omit the extension (i.e.
"table200x20"), in case of tab files put
# the base file name.
# The system defaults to the DataFiles folder, so to get the
../DataFiles/table200x20.xls file just specify
# table200x20
let FILENAME  := "table200x20";
# Percentage for liabilities. i.e., for 2.5%, put 2.5
let liabpercentage := 5;
# Number of contributions scenarios to read
let contributionScenariosToRead := 20;
# number of returns scenarios to read
let returnScenariosToRead := 100;
############### END OF INPUT PARAMETERS ####################



# Load LP Model Data
include ..\scripts\DoLpLoadData.ampl;
# Solve
include ..\scripts\DoSolve.ampl;
# Save solution
include ..\scripts\savesolution.ampl;
```

A.2 Generated Model Statistics

Model Class	No. of Scenario/s	No. of Constraints	No of Variables
LP	1	1085 3584 non zeroes	899
TSP	2000	2080045 7098035 non zeroes	1708045
ICCP	2000	2170090 7368035 non zeros	1798045

The models were generated and as deterministic equivalent problems (as above). All models were written in AMPL and solved using CPLEX 12.4 on Intel Core i5 3.3GHz machine with 16Gb RAM.

Solution Time LP : 0.0312 seconds.

TSP : 197 seconds.

ICCP : 614 seconds.

Appendix B: Observed Relative Frequencies of Transitions

The method of computing the observed relative frequencies of transitions using the historical population data is explained in Chapter 3, Section 3.3. Historical data covering (2004 t - 2005) until (2009 -2010).

Observed Relative Frequencies of transitions (2004 - 2005)

States	21 to 25	26 – 30	31 - 35	36 - 40	41 - 45	46 - 50	51 - 55	Inactive	Retired	Dead
21 - 25	0.7655	0.1971	0	0	0	0	0	0.0358	0	0.0017
26 - 30	0	0.7633	0.1985	0	0	0	0	0.0361	0	0.0021
31 - 35	0	0	0.7640	0.1974	0	0	0	0.0361	0	0.0025
36 - 40	0	0	0	0.7644	0.1960	0	0	0.0360	0	0.0036
41 - 45	0	0	0	0	0.7597	0.1983	0	0.0368	0	0.0052
46 - 50	0	0	0	0	0	0.7570	0.1978	0.0374	0	0.0077
51 - 55	0	0	0	0	0	0	0.7483	0.0392	0.1986	0.0139
Inactive	0	0	0	0	0	0	0	0.9806	0.0165	0.0030
Retired	0	0	0	0	0	0	0	0	1.0000	0
Dead	0	0	0	0	0	0	0	0	0	1.0000

Observed Relative Frequencies of transitions (2005 - 2006)

States	21 to 25	26 - 30	31 - 35	36 - 40	41 - 45	46 - 50	51 - 55	Inactive	Retired	Dead
21 - 25	0.7793	0.1862	0	0	0	0	0	0.0329	0	0.0016
26 - 30	0	0.7669	0.1963	0	0	0	0	0.0348	0	0.0021
31 - 35	0	0	0.7670	0.1957	0	0	0	0.0348	0	0.0025
36 - 40	0	0	0	0.7650	0.1963	0	0	0.0351	0	0.0036
41 - 45	0	0	0	0	0.7644	0.1953	0	0.0353	0	0.0051
46 - 50	0	0	0	0	0	0.7575	0.1983	0.0365	0	0.0078
51 - 55	0	0	0	0	0	0	0.7490	0.0378	0.1994	0.0138
Inactive	0	0	0	0	0	0	0	0.9804	0.0171	0.0025
Retired	0	0	0	0	0	0	0	0	1.0000	0
Dead	0	0	0	0	0	0	0	0	0	1.0000

Observed Relative Frequencies of transitions Year (2006 - 2007)

States	21 to 25	26 - 30	31 - 35	36 - 40	41 - 45	46 - 50	51 - 55	Inactive	Retired	Dead
21 - 25	0.7764	0.1911	0	0	0	0	0	0.0309	0	0.0016
26 - 30	0	0.7667	0.1990	0	0	0	0	0.0322	0	0.0021
31 - 35	0	0	0.7683	0.1972	0	0	0	0.0320	0	0.0025
36 - 40	0	0	0	0.7675	0.1968	0	0	0.0322	0	0.0036
41 - 45	0	0	0	0	0.7664	0.1961	0	0.0324	0	0.0051
46 - 50	0	0	0	0	0	0.7620	0.1972	0.0332	0	0.0077
51 - 55	0	0	0	0	0	0	0.7433	0.0347	0.2081	0.0139
Inactive	0	0	0	0	0	0	0	0.9824	0.0166	0.0009
Retired	0	0	0	0	0	0	0	0	1.0000	0
Dead	0	0	0	0	0	0	0	0	0	1.0000

Observed Relative Frequencies of transitions Year (2007 - 2008)

States	21 to 25	26 - 30	31 - 35	36 - 40	41 - 45	46 - 50	51 - 55	Inactive	Retired	Dead
21 - 25	0.7649	0.2001	0	0	0	0	0	0.0333	0	0.0017
26 - 30	0	0.7644	0.2001	0	0	0	0	0.0334	0	0.0021
31 - 35	0	0	0.7664	0.1979	0	0	0	0.0332	0	0.0025
36 - 40	0	0	0	0.7641	0.1988	0	0	0.0335	0	0.0036
41 - 45	0	0	0	0	0.7640	0.1973	0	0.0336	0	0.0052
46 - 50	0	0	0	0	0	0.7564	0.2009	0.0349	0	0.0079
51 - 55	0	0	0	0	0	0	0.7504	0.0362	0.1994	0.0140
Inactive	0	0	0	0	0	0	0	0.9790	0.0186	0.0023
Retired	0	0	0	0	0	0	0	0	1.0000	0
Dead	0	0	0	0	0	0	0	0	0	1.0000

Observed Relative Frequencies of transitions Year (2008 - 2009)

States	21 to 25	26 - 30	31 - 35	36 - 40	41 - 45	46 - 50	51 - 55	Inactive	Retired	Dead
21 - 25	0.7751	0.1882	0	0	0	0	0	0.0351	0	0.0016
26 - 30	0	0.7649	0.1963	0	0	0	0	0.0367	0	0.0021
31 - 35	0	0	0.7653	0.1955	0	0	0	0.0367	0	0.0025
36 - 40	0	0	0	0.7656	0.1942	0	0	0.0367	0	0.0035
41 - 45	0	0	0	0	0.7610	0.1964	0	0.0375	0	0.0051
46 - 50	0	0	0	0	0	0.7578	0.1963	0.0382	0	0.0077
51 - 55	0	0	0	0	0	0	0.7490	0.0397	0.1977	0.0137
Inactive	0	0	0	0	0	0	0	0.9783	0.0190	0.0027
Retired	0	0	0	0	0	0	0	0	1.0000	0
Dead	0	0	0	0	0	0	0	0	0	1.0000

Observed Relative Frequencies of transitions   Year (2009 - 2010)

States	21 to 25	26 - 30	31 - 35	36 - 40	41 - 45	46 - 50	51 - 55	Inactive	Retired	Dead
21 - 25	0.7637	0.1994	0	0	0	0	0	0.0352	0	0.0017
26 - 30	0	0.7633	0.1993	0	0	0	0	0.0353	0	0.0021
31 - 35	0	0	0.7650	0.1974	0	0	0	0.0351	0	0.0025
36 - 40	0	0	0	0.7660	0.1955	0.0000	0.0000	0.0350	0	0.0036
41 - 45	0	0	0	0	0.7646	0.1951	0.0000	0.0352	0	0.0051
46 - 50	0	0	0	0	0	0.7568	0.1989	0.0366	0	0.0078
51 - 55	0	0	0	0	0	0	0.7512	0.0380	0.1968	0.0139
Inactive	0	0	0	0	0	0	0	0.9767	0.0201	0.0032
Retired	0	0	0	0	0	0	0	0	1	0
Dead	0	0	0	0	0	0	0	0	0	1

Appendix C: Multidimensional Tables Used in Scenario Generation

C.1 Population Data from 2010 to 2011

		population 2010			2011			
Active-Retired	Inactive-Retired	Active			Active	Inactive-Retired	Active-Retired	Active-Died
		1619693	21-25		1673472			
		1227437	26-30		1351145			
		887172	31-35		964427			
		706437	36-40		767789			
		584020	41-45		634673			
		478262	46-50		528901			
		323821	51-55		345967			
30689	89246					99000	37988	36993

C.2 Annual salary growth (2011-2019)

Age	Salary								
	2011	2012	2013	2014	2015	2016	2017	2018	2019
21-25	1000	1020	1040	1061	1082	1104	1126	1149	1172
26-30	1700	1734	1769	1804	1840	1877	1914	1953	1992
31-35	2200	2244	2289	2335	2381	2429	2478	2527	2578
36-40	2700	2754	2809	2865	2923	2981	3041	3101	3163
41-45	3200	3264	3329	3396	3464	3533	3604	3676	3749
46-50	3700	3774	3849	3926	4005	4085	4167	4250	4335
51-55	4200	4284	4370	4457	4546	4637	4730	4824	4921

C.3 Annual average lump sum due to death for different growth dividends (2011-2019)

Year	2011	2012	2013	2014	2015	2016	2017	2018	2019
2.5%	16511	16923	17347	17780	18225	18680	19147	19626	20117
3%	16591	17089	17602	18130	18674	19234	19811	20405	21017
4%	16752	17422	18119	18844	19598	20382	21197	22045	22927
5%	16913	17759	18647	19579	20558	21586	22666	23799	24989
6%	17074	18099	19185	20336	21556	22850	24220	25674	27214

C.4 Annual average lump sum due to retirement (active/inactive) for different growth dividends (2011-2019)

	2011	2012	2013	2014	2015	2016	2017	2018	2019
Active	143311	146894	150567	154331	158189	162144	166197	170352	174611
Non Active	23276	23858	24454	25065	25692	26334	26993	27667	28359
Active	144010.48	148331	152781	157364	162085	166948	171956	177115	182428
Non Active	23389	24091	24814	25558	26325	27115	27928	28766	29629
Active	145408	151225	157274	163565	170108	176912	183988	191348	199002
Non Active	23616	24561	25543	26565	27628	28733	29882	31077	32321
Active	146806	154147	161854	169947	178445	187367	196735	206572	216901
Non Active	23843	25036	26287	27602	28982	30431	31952	33550	35228
Active	148204	157097	166523	176514	187105	198332	210232	222845	236216
Non Active	24070	25515	27046	28668	30388	32212	34144	36193	38365