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Asset Liability Management Using Stochastic Programming

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> "A banker is a fellow who lends you his umbrella when the sun is shining and wants it back the minute it begins to rain."
>
> Mark Twain

This chapter sets out to explain an important financial planning model called asset liability management (ALM); in particular, it discusses why in practice, optimum planning models are used. The ability to build an integrated approach that combines liability models with that of asset allocation decisions has proved to be desirable and more efficient in that it can lead to better ALM decisions. The role of uncertainty and quantification of risk in these planning models is considered.

ALM: AN INTRODUCTION

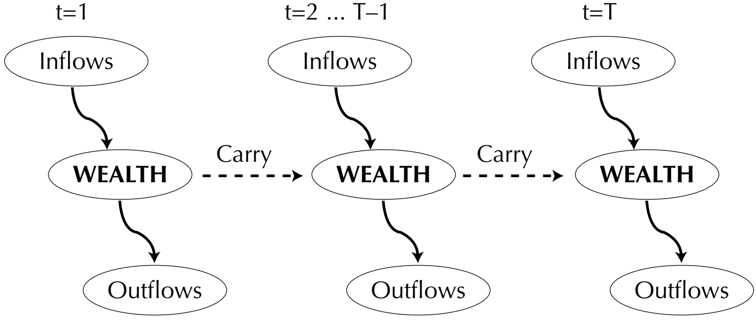
Many financial systems in a corporate as well as an individual context are underpinned by a cashflow-balancing (also called matching) activity. At an individual level, a young professional may set up savings after the birth of a child as he or she goes through the schools systems. The savings are typically assets suitably invested in bonds and shares and future payment for school fees are liabilities. At a corporate level, many institutions take contributions from the working employees of a corporation and invest these contributions by acquiring assets. These assets are, however, pledged to meet the pension payments of the individuals at future dates of their retirement. These pension payments are again the liabilities for the financial institution. A basic aspect of financial planning encompasses such matching activities of cashflows and is given the generic label of asset and liability management – ALM. From a mathematical perspective these models can be set up in an equational form involving non-negative variables that represent in- and outflow of funds and carry over retained assets and funds from one planning period to the next.

So what more can be expected from ALM than the established techniques? To answer this, it is necessary to ascertain the pitfalls and difficulties encountered when making investment decisions. It is important to understand the risks that are borne by an investor when investing in a particular security or portfolio of securities. Generally, the higher the risks undertaken, the higher the possible returns on that investment. But there are other constraints that cannot be ignored, such as the nature of uncertainty in the decision process, taxes and transactions costs. There may also be legal guidelines and other policy requirements such as institution-specific rules on asset mix.

Returning to the fundamental aspect that any company has both assets and liabilities, it is clear that in the course of business the company will benefit from cash inflows and also have to meet liabilities. When asset streams are greater than liability streams there is a surplus, and vice-versa; when liability streams are greater than asset streams, there is a deficit (see Figure 1). A company will always try to make sure that there is always a surplus but, in situations where there is a deficit, corrective measures can be taken to protect the company financially in the short-term. In the long term, however, a company continuing to accumulate shortfalls is likely to be in a serious financial position and may be on the verge of insolvency. Avoiding this financial quagmire requires advanced and meticulous financial planning, and for large organisations ALM is invaluable. Indeed at time of going to press, Bethlehem Steel, the third largest US steel maker has filed for Chapter 11 bankruptcy protection, its burgeoning pension liabilities being cited as one of the major reasons for the decision.

Asset allocation decision making is a crucial part of a company's risk

Figure 1 Illustration of an ALM stochastic programming model



management system. Currently, the idea of asset-class investing is becoming more complex than the usual preconceived notion of just investing in equities, fixed-income products or cash products. This is mainly due to the fact that hundreds of separate and distinct asset classes can be identified, and still more are flooding the markets. In addition, these asset classes have different risk and return combinations and their correlations to the other products vary. A landmark study by Brinson, Hood and Beebower (1986), demonstrates how powerful asset allocation is. The investment results of 91 very large pension funds were examined to determine how and why their results differed. They reasoned that only four elements could contribute to investment results:

1. investment policy;
2. individual security selection;
3. market timing; and
4. costs.

By using a regression analysis, they attributed the contribution (or lack of it) to each of the four elements. Their conclusions were quite astonishing. They concluded that the biggest single factor explaining performance was simply the investment policy (asset allocation) decision that determined how much a fund should hold in stocks, bonds or cash. On the whole, attempts at market timing amounted to a reduction in returns, and individual stock selection on average resulted in a reduction to the funds' returns. There was a wider variation in individual stock selection impact than in market timing, and a few managers were able to affect performance during the time period in a positive manner.

From this, the importance of distinguishing between *strategic* and *tactical* asset allocation decisions can be seen. Broadly, it could be said that tactical asset allocation (TAA) begins where strategic asset allocation (SAA) ends. SAA decisions are based upon long-term expected returns and estimations of risk, which are formed from a variety of factors, among which are past returns and volatilities, forecasts of long-term economic growth, and perhaps, assessments of political risk. But these allocations are formed infrequently, leaving the asset decision to drift in the intermediate term. On the other hand, TAA is designed to reposition the risk and return profile of the long-term strategic asset allocation in response to intermediate-term variations. There will be a reduction in those asset classes where risk has risen to abnormal levels, while exposure is increased in those asset classes likely to provide a more favourable return. This is done not by attempting to maintain a constant profile, but rather by evaluating the near-term relative risk and return characteristics of each of the underlying asset classes, and optimally shifting exposure away from asset classes showing uncharacteristic near-term weakness, and in the direction of those exhibiting much more promise in terms of returns.

Even the newspapers have run headlines on the debate of asset allocation. The *LA Times* (September 4, 1997) cited that "Academic studies have demonstrated that asset allocation among stocks, bonds and cash is the key to a portfolio's performance over time – much more important than the individual securities you select". Given this statement and the illustration of the study of the determinants of portfolio performance above, the idea of asset allocation and its importance starts to strike home. It is in this light that ALM has to come into play to make sure that the asset allocation decisions are optimal and try to smooth the cashflows of financial institutions.

AN OPTIMISATION APPROACH

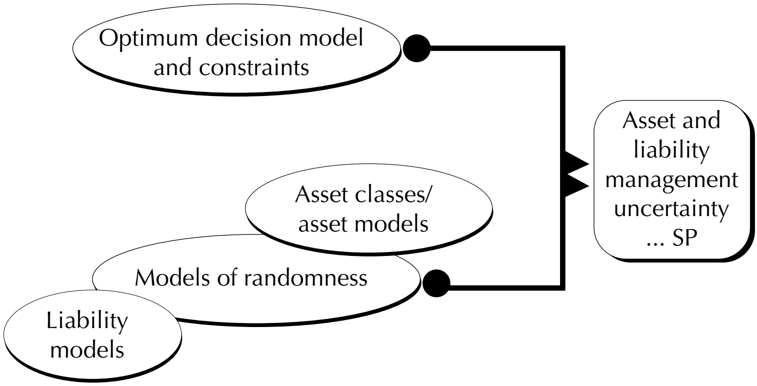
Mathematical programming (MP) is the generic name for the optimisation models that are used in planning. MP is characterised by the use of an objective function that must be optimised and a set of linear or non-linear equations or inequalities (called constraints) that must be satisfied. The objective function is introduced to obtain a desirable, or in some sense the best, solution. This is because in general there are many (often infinitely many) different ways in which the constraints can be satisfied. However, the MP models turn this into a question of making the best decision as opposed to any feasible decision. ALM as described above represents the requirements and the constraints of the cashflow matching, which can be achieved in (infinitely) many different ways. Through use of objectives and goals we, therefore, formulate optimisation models that lead to the best ALM 'matching' decisions.

ALM MODELS: OPTIMUM HEDGED DECISIONS UNDER UNCERTAINTY

In all real-world planning problems in general, and in financial planning problems in particular, time and uncertainty play key roles. Thus, optimum plans cannot be made in a deterministic way since the asset prices and the liabilities are not known with certainty in the future. Under these circumstances, the concept of optimum plans is extended to optimum hedged plans. In order to achieve this, optimum allocation models are brought together with models of randomness which include the possible future asset prices and future liabilities (as depicted in Figure 2). This combined paradigm of models is often known as "stochastic optimisation" (or stochastic programming, SP) models. Using such SP models, it is possible to compute hedged decisions. This may not be the best approach for any one realisation of the future but it is robust in respect of different realisations of the future. It is easily seen that a good description of uncertainty may significantly improve on ALM decisions.

Klaassen (1997) provides some insight into ALM techniques. Klaassen points out that a well-known problem that arises by using stochastic programming models in practice is that, only a limited amount of uncertainty

Figure 2 Optimisation models under uncertainty



can be included because of the numerical optimisation methods that are used. While the description should be representative of the true uncertainty, it also should be the case that uncertainty that does not affect optimal decisions can be left out.

Klaassen suggests that to get a good description of the uncertainty in future asset prices and returns, they have to be free of arbitrage opportunities and consistent with market prices. Yet, when stochastic programming models for portfolio investment problems are formulated, these properties are generally set aside. He shows that a violation of these properties may lead to optimal portfolios in stochastic programming models that are severely biased towards spurious profit opportunities.

Future uncertainties are often captured using an event tree, which is a simple but effective model of randomness. Figure 3 shows an event tree structure showing the possible future scenarios.

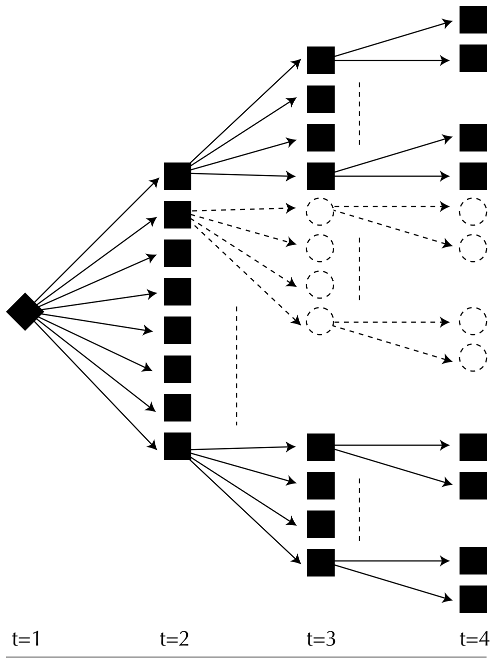
In this event tree structure, there are 64 scenarios; there are eight possible outcomes in the second stage, four conditional outcomes in the third stage and two conditional outcomes in the last stage, giving a total of $8 \times 4 \times 2 = 64$ scenarios.

RANDOM BEHAVIOUR OF ASSETS AND LIABILITIES: SCENARIO GENERATION

Asset and liability management models are in some sense related to the money and capital markets behaviour since the assets and the liability components are linked to the 'market'. The financial markets are both dynamic and volatile. Hence, assets arising in such a market are also (i) dynamic, that is, intertemporal in behaviour, or in other words, constantly changing with time, and, (ii) follow randomly fluctuations in their values.

From the modeller's view, the stochastic programming paradigm pro-

Figure 3 Event tree structure



vides a robust and realistic platform to adequately represent the uncertainty of random parameters in an optimisation problem. Scenario generation techniques capture this uncertainty arising in real world (markets). The basic random parameters of a financial-planning model consist of company goals and liabilities and asset classes in general and stocks, bonds and cash in particular. The quality of the optimal decisions (hedging against uncertainty) as generated by an asset allocation model depends on how well one is able to model randomness. Hence, the first component of stochastic programming, in the form of an optimum decision model and constraints, is essential for the realistic representation of the problem at hand. The second component, models of randomness, influences the quality of the decisions generated.

Within ALM there are two basic types of scenario generation models:

1. models that are used to generate scenarios that correspond to the different financial variables and asset classes; and
2. models that are used to generate future paths of liabilities.

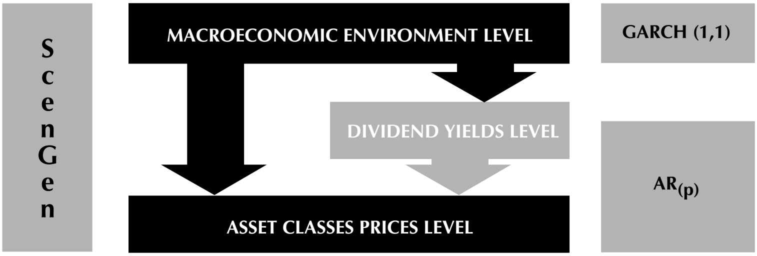
The first type involves stochastic processes or their discrete approximations. The second type is usually related to specific actuarial models for individual companies. Alternatively, if the liabilities are influenced by financial variables, such as interest rates or inflation, scenarios are generated by models that encapsulate the impact of changes in the financial variables to the corresponding liabilities.

The main challenge of generating scenarios in stochastic programming is not to accurately predict rates of return for different asset classes, but to construct scenarios for these asset classes that are consistent with economic and financial theory in general, and reflect the economic and financial environment observed in each country. Thus, academics and practitioners have developed several scenario generation systems that match these requirements. In general such systems, utilise both Gaussian and stochastic processes. Initially they are used to model the macroeconomic environment of the country in question, and then to predict changes in variables constituting the financial environment, which may be based on former results.

The "Wilkie model" (Wilkie, 1995) is an example of a non-linear scenario generation system that is based on a set of simultaneous finite difference equations. The approach makes extensive use of autoregressive and autoregressive with moving average equations and is based on actual data from the UK for the period 1923–1991. Similarly, Aon's TY model uses time-series analysis.[1] Inflation is the key variable and GARCH processes are used to estimate the coefficients for the system corresponding to UK data. Dert (1995) and Boender, van Aalst and Heemskerk (1998) make use of the VAR methodology to build integrated models that represent the Dutch economic environment. Tower Perrins' CAP:Link system utilises stochastic differential equations. The system can be applied in a global environment, with the US, Germany and Japan forming the three major economic powers. The economies of other countries are affected by, but do not themselves affect, the three major ones. Finally, the Falcon and Finish asset models follow a similar structure to the above representations.

A novel scenario generation system, developed by Kyriakis (2002) is ScenGen, which is conceptually comparable to the CAP:Link system of stochastic differential equations, but employs a different methodology. In

Figure 4 Levels and influences in the ScenGen



particular, it combines the GARCH(1,1) and AR(p) processes in such a way that it makes use of factors that influence the asset classes. The system is composed over three levels (macroeconomic, dividend yields and asset classes prices), and is used to generate scenarios for stocks, financial indices, industry sectors, Treasury bills, bonds and interest rate-linked liabilities. Figure 4 illustrates the three levels of ScenGen, their interdependencies and the methodologies employed in the different levels to generate the data parameters.

The macroeconomic level lies at the top of the hierarchy and constitutes the core of the system. The rationale behind this is the observation that, in complete and integrated financial markets, the actions of the whole market and the prices of securities reflect all the available information. Furthermore, financial markets relate, to a great extent, to the economic situation not only in the country under consideration but also to the economies of other countries. Therefore, explaining and replicating the economy of a country is the first step in obtaining valid information about the possible movement of the financial markets. However this information may not be sufficient to explain in full the movement of the financial market(s).

Consumer price index, long-run and short-run interest rates represent the macroeconomic environment, which translates into a closed-form economy, since an open economy requires the inclusion of the exchange rates dynamics. Despite this restrictive assumption, for the purpose of this study where we only concentrate on portfolios with domestic stocks, considering a closed economy does not negatively affect the quality of the generated scenarios. That is because extending the system to an open economy becomes crucial when considering internationally diversified portfolios. The three macroeconomic variables are only influenced by each other. Their outcome, however, affects the dividend yields on the second level and the variables on the third level.

The dividend yields level is second in the hierarchy. Thus, when moving from the macroeconomic to the microeconomic point of view, an investigation into the dividend policy of a company becomes essential. There are three reasons that make dividend policy interesting. First, dividend payout is one of the major financial decisions of a company. Further, announcements of dividends can influence the market sentiment positively, and vice versa. Finally, understanding the dividend policy helps decision makers in gaining a better insight of additional financial conditions and corporate policies of the company, such as asset price, capital structure, merger and acquisitions.

In the third level of the system, asset classes prices, there are two categories of variables:

1. those that are influenced directly only by interest rates; and
2. those that are influenced by both dividend yields and interest rates together.

The first set of variables consists of interest rate-linked liabilities, bonds, Treasury bills and cash, while the second consists of financial indexes, industry sectors and individual stocks.

Figure 5 shows how each of these stages inter-relate.

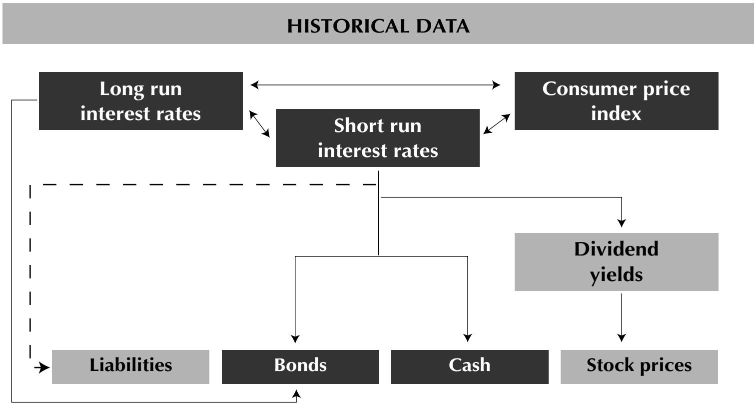
RISKS FACED BY FINANCIAL INSTITUTIONS AND RISK QUANTIFICATION

In the finance world, future unpredictability is termed volatility; the volatility of asset prices and uncertain liabilities clearly affects financial plans. In general, such uncertainties lead to possible financial loss or in other words financial risk. But that does not mean uncertainty equates to or is synonymous with risk. Depending upon the decision-maker or the fund manager's utility, there are many alternative measures of risk.

The choice of an appropriate risk measure that captures an individual's investment preferences has been, and continues to be, the subject of a long debate between academics and practitioners. This is not surprising, since without prior assumptions on the risk preferences of the individuals or the forms of the alternative distributions, it is likely that two individuals will consider risk from alternative perspectives.

In general, risk measures can be divided in two groups depending on the perception of risk. The first group contains the so-called "dispersion risk measures" that quantify risk in terms of the probability-weighted dispersion of results around a specific reference point, usually the expected value. Measures in this category penalise negative as well as positive deviations from a pre-specified target. Two of the most well-known and widely applied risk measures in this group are Markowitz's (1952, 1959) variance or standard deviation and the expected or mean absolute deviation of Adikson (1970) and Konno and Yamasaki (1991).

Figure 5 Relation between the stages in ScenGen



The second group comprises of measures that quantify risk according to results and probabilities below reference points, selected either subjectively or objectively. Such risk measures include expected value of loss from Domar and Musgrave (1944), Roy's (1952) safety first criterion, the semi-variance proposed by Markowitz (1959) and Fishburn's α–t criterion (1977). In 1993, a JP Morgan analyst further extended the concept of loss beyond a target, and introduced value-at-risk (JP Morgan, 1993) as a measure of loss to level beyond a given percentile of distribution.

Regulation of risk is naturally important in the context of banks and financial institutions' planning and operations. The Bank of International Settlements (BIS) started their work in Basel in 1988 (Basel Accord) and since then introduced regulatory requirements that are frequently updated (a new Accord, Basel II, is expected to be implemented in 2006). These regulations are globally followed by financial institutions. The risks faced by financial institutions come from different sources of uncertainty. These are then classified accordingly. At the time of writing, the accepted areas of risk are credit, liquidity, systemic, political, operational and legal.

Of these, the first four are financial risks, broadly those risks for which part of their uncertainty relates to the returns of assets arising from unanticipated and unpredictable events. These events may initiate runs on banks or create a banking panic.

"Credit risk" is a risk that arises in the event that a counterparty defaults on its obligations. The losses can be very substantial for any firm. For example, defaults on mortgage payments or companies not honouring their bond repayments. "Liquidity risks" are defined as an event when it is difficult or expensive to make changes in the composition of one's portfolio. This usually takes place when there are crises in the global markets or following some unexpected political events.

"Political risks" are usually country-specific and relate to the political uncertainties and policies of a particular government. An example of the existence of political risks in 2003 could be that of Zimbabwe where recent events have created some instability and reduced investment in the economy. A situation where the financial sector has collapsed and where runs on banks are present and problems of liquidity and defaults surface – an 'apocalyptic' situation in a sense – can be defined as "systemic risk".

It is a widely accepted notion that financial institutions (and some banks) are in the business of managing risks. The better they manage these risks, the better they are placed in dealing with very rare but possibly commercially destructive events. Moreover, as the financial markets' adage states, "a company's reputation is only as good as its last transaction"; hence any let-up in controlling the different aspects of a business could severely dent its future expansion.

PAST, PRESENT AND FUTURE: A DECISION-MAKING PERSPECTIVE

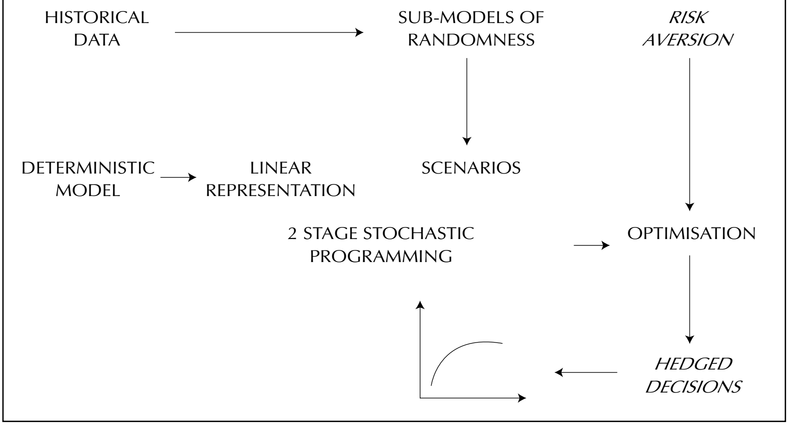
There are well-known models, such as the Markowitz mean variance model, which has been used to capture uncertainty and make hedged decisions. The finance industry has progressively embraced the portfolio optimisation models for asset allocation (see Mitra *et al.*, 2003 and OptiRisk, 2001). Unfortunately, the Markowitz mean variance model relies entirely on history and makes a single-period static decision. The real question that should be asked is: whether history should be taken into account to make future decisions? History does not always repeat. However, our models should be forward-looking with event trees of future scenarios. The flow of data and processing this into an analytic database (datamarts) and the use of models that support hedged optimum decisions are shown in Figure 6.

MODEL-BASED APPLICATIONS OF ALM

The basic concepts of ALM models under uncertainty were developed by Kallberg, White and Ziemba (1982) and Kusy and Ziemba (1986). Afterwards, large-scale applications were developed, including the Russell-Yasuda Kasai model (see Carino *et al.*, 1994). This is an asset liability model for a Japanese insurance company, using multistage stochastic programming, which, according to the authors, has enabled Yasuda to make use of a state-of-the-art decision-making and risk-management tool that provides valuable insights into the complex choices and restrictions to which the business is exposed.

Consigli and Dempster (1998) present the computer-aided asset liability management (CALM) stochastic programming model for dynamic ALM, which has been designed to deal with uncertainty affecting both assets (in

Figure 6 From historical data to optimum ALM decisions



either the portfolio or the market) and liabilities (in the form of scenario dependent payments or borrowing costs). The discussion continues with the presentation of Zenios' (1995) paper on "ALM Under Uncertainty for Fixed Income Securities", makes it possible to capture the increasing complexities of the fixed-income markets. The paper also finds that the use of stochastic models is well justified given their superior performance over traditional immunisation techniques.

Finally, Dert (1995) presents a scenario-based optimisation model for analysing the investment policy and funding policy of pension funds, taking into account the development of the liabilities in conjunction with the economic environment. The ALM model presented can be used to compute ALM strategies that specify investment decisions and contribution levels to be set under a wide range of future circumstances. Dert (1995) finds that decisions reached were different when using dynamic ALM strategies compared to static policies. Also, the use of the ALM model resulted in strategies with lower funding costs; the probabilities of under-funding were substantially smaller and the magnitude of deficits, reflected by the costs of remedial contributions, has been reduced dramatically.

Given that there are various complex financial products being traded in the markets and over-the-counter, not only has it become much harder to assess the potential risks of some of these stand-alone products but also the problem of integrating these risks in the risk management system has arisen. Some research has been done on this topic; Holmer (1998) looks at integrated ALM, which he views as a new management perspective that is creeping its way into the more inventive financial intermediaries in reaction to problems inherited from the older functional management perspective. For the latter, an organisation has to be structured into different functional units (eg, marketing, asset management, etc), the decisions of which are synchronised by a corporate plan based on macroeconomic forecast. However, the lack of precision in predicting macroeconomic variables has forced the hands of some banks in looking for alternative management perspectives.

Hence, the new concept of integrated ALM. This perspective, as its name implies, is more focused on integrating the various units of the organisation in order to include all the functional activity related to a line of business. Decisions are taken with the help of computer models, also trying to ascertain the uncertainty of the future business environment, and to generate profitable strategies by structuring the assets and liabilities of the business line across a series of alternative future scenarios. By comparing these alternative ways, a crucial difference can be spotted, which is that decisions are made using profitability calculations based on a single-scenario planning forecast, while, for the integrated ALM, decisions are made using risk-adjusted or hedged profitability calculations based on multiplescenario possibilities. This is just a brief synopsis of how ALM can be used in the decision-making process.

CONCLUSION

In this chapter, important recent developments and the pragmatic approaches to the ALM models and systems have been presented, as has an explanation to show how ALM is becoming the linchpin of firms' financial management, especially under conditions of uncertainty that require risk management. An illustration of how ALM and SP are integrated using an optimisation and risk control paradigm have also been discussed.

Quantitative analysts in the finance industry have developed highly sophisticated asset pricing and simulation models. However, these descriptive methods do not address the central problem of making optimum risk decisions; such decisions can be only made within a stochastic programming framework. This novel approach integrates the notion of optimum hedged decisions and risk considerations within a unified modelling paradigm. Ziemba (2003) addresses this important topic.

SP has proved to be a powerful modelling approach to optimum decision-making under uncertainty. It has been shown to be more appropriate in many applications. Improvements in the new technologies and solution methods have made SP a viable optimisation tool, especially in the domain of asset and liability management.

1 See Yakoubov, Teeger, and Duval (1999) and http://www.aon.com.

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