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**The PPP Hypothesis Revisited: Evidence  
Using a Multivariate Long-memory Model**

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**THE PPP HYPOTHESIS REVISITED:  
EVIDENCE USING A MULTIVARIATE LONG-MEMORY MODEL**

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**Abstract**

This paper examines the PPP hypothesis analysing the behaviour of the real exchange rates vis-à-vis the US dollar for four major currencies (namely, the Canadian dollar, the euro, the Japanese yen and the British pound). An innovative approach based on fractional integration in a multivariate context is applied to annual data from 1970 to 2011. Long memory is found to characterise the Canadian dollar, the British pound and the euro, but in all four cases the results are consistent with the relative version of PPP.

**Keywords:** PPP, long memory, multivariate fractional integration

**JEL Classification:** C22, F31

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## **1. Introduction**

The absolute version of PPP postulates that the price levels in two different countries should converge when measured in the same currency, so as to equalise the purchasing power of the currencies. This implies that the real exchange rate should converge to 1. A less restrictive version of PPP is the relative PPP hypothesis, which implies that the real exchange rate in the long run should revert to a constant which may be different from 1. The rationale for this version of PPP is the existence of trade barriers, transport costs, and differences in price indices, which can result in some degree of divergence between price levels in different countries.

The empirical validity of the PPP hypothesis can therefore be examined by testing for mean reversion in the real exchange rate. In this paper we propose a new method based on a multivariate fractionally integrated model that has several advantages over standard tests (see Froot and Rogoff, 1995, for their well-known limitations). In particular, it is more flexible since it allows for fractional degrees of differentiation and not only for the standard  $I(0)$  and  $I(1)$  cases. Moreover, its multivariate nature enables one to take into account possible cross-sectional dependence that might affect the degree of integration of the individual series (see Caporale and Cerrato, 2006, for the issues arising in the context of panel approaches in the presence of cross-sectional dependence). Finally, it is a procedure that allows to test the null of PPP in individual series as opposed to the joint null of PPP in all series considered. The paper is organised as follows: Section 2 describes the methodology; Section 3 presents the empirical results, while Section 4 offers some concluding remarks.

## 2. Methodology

A covariance stationary process  $\{x_t, t = 0, \pm 1, \dots\}$  is defined as I(0) if its spectral density function is positive and bounded at all frequencies. This includes the class of stationary and invertible ARMA processes, which are characterised by an impulse response function decaying exponentially to zero. On the other hand, the unit root or I(1) class of models require first differences to render the series I(0) stationary and in this case shocks have permanent effects. In between, the I(d,  $0 < d < 1$ ) class of models are mean-reverting but display long-memory behaviour. This implies that the impulse responses decay hyperbolically to zero.

A process  $\{x_t, t = 0, \pm 1, \dots\}$  is said to be I(d) if it can be represented as

$$(1 - L)^d x_t = u_t, \quad t = 1, 2, \dots \quad (1)$$

where  $x_t = 0$ , for  $t \leq 0$  and I(0)  $u_t$ . Note that the polynomial  $(1-L)^d$  in (1) can be expressed in terms of its Binomial expansion, such that, for all real d:

$$(1 - L)^d = \sum_{j=0}^{\infty} \psi_j L^j = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j = 1 - d L + \frac{d(d-1)}{2} L^2 - \dots \quad (2)$$

and thus:

$$(1 - L)^d x_t = x_t - d x_{t-1} + \frac{d(d-1)}{2} x_{t-2} - \dots$$

In this context, d plays a crucial role as an indicator of the degree of dependence of the series: the higher the value of d, the higher the degree of correlation between the observations will be. Processes with  $d > 0$  in (1) display the property of “*long memory*”, and are characterised by autocorrelations decaying hyperbolically and a spectral density function unbounded at the origin. These processes were first analysed in the 1960s by Granger (1966) and Adelman (1965) who pointed out that for most aggregate economic

time series the spectral density increases dramatically as the frequency approaches zero. However, differencing the data frequently leads to over-differencing at the zero frequency. Robinson (1978) and Granger (1980) then showed that the aggregation of heterogeneous processes could be a source of fractional integration. Since then, fractional processes have been widely employed to describe the dynamics of many economic and financial time series (see, e.g. Diebold and Rudebusch, 1989; Sowell, 1992a; Baillie, 1996; Gil-Alana and Robinson, 1997; etc.) using different univariate procedures. However, univariate methods do not take into account the potential cross-dependence of the series.

The multivariate methodology employed in this paper addresses this issue. The fractionally integrated vector autoregressive model (FIVAR or VARFIMA) can be written as:

$$D L X_t = v_t \quad (3)$$

$$I - F_p L v_t = w_t \quad (4)$$

where  $X_t$  is a  $N \times 1$  vector of variables for  $t = 1, \dots, T$ ,  $L$  is the lag operator,  $I$  is an  $N \times N$  identity matrix and  $w_t$  is an  $N \times 1$  vector of i.i.d errors with 0 mean and  $N \times N$  variance-covariance matrix  $\Omega$ . The VAR(p) process in (4) is assumed to be stationary.  $D L$  is a diagonal  $N \times N$  matrix with fractional integration polynomials on the main diagonal given by (2).

To estimate the process given by (3) and (4) we use the approximate frequency domain maximum likelihood (Whittle) estimator proposed by Boes et al. (1989). A discussion of the multivariate version of this procedure can be found in Hosoya (1996).

### 3. Empirical Results

We use data on real exchange rates vis-à-vis the US dollar for four currencies, namely the Canadian dollar, the Japanese yen, the euro and the British pound, obtained from Datastream. The series are annual and the sample period goes from 1970 to 2011.

**[Insert Table 1 about here]**

Table 1 displays the parameter estimates for the model given by equations (3) and (4) under a VAR(1) specification for the short-memory polynomial in (4). This lag length was selected according to the Akaike information criterion. The four estimates of the fractional differencing parameters are in the interval (0, 1), which implies fractional integration. Their values are: 0.5534 for the Canadian dollar; 0.3625 for the Japanese yen; 0.2980 for the British pound and 0.6665 for the euro, implying stationary behaviour ( $d < 0.5$ ) for the Japanese yen and the British pound, and non-stationarity ( $d \geq 0.5$ ) one for the Canadian dollar and the euro. The standard errors, displayed below the estimates in Table 1, indicate that one cannot reject the null of short memory (i.e.  $d = 0$ ) in the case of the Japanese yen, whilst this hypothesis is rejected in favour of long memory (i.e.  $d > 0$ ) for the remaining three exchange rates. As for the unit root hypothesis ( $d = 1$ ), this is rejected in favour of mean reversion (i.e.  $d < 1$ ) for the four series examined, thus supporting the relative version of PPP. Concerning the short-run dynamics, the highest degree of persistence is found for the euro, with a coefficient close to 1 in the VAR representation of the series (0.8856).

### 4. Conclusions

This paper uses an innovative multivariate fractional integration approach to test for PPP by examining the behaviour of the real exchange rates vis-à-vis the US dollar of four major currencies (the Canadian dollar, the Japanese yen, the euro, and the British

pound). The method used is more flexible than standard tests and takes into account possible cross-dependence. Moreover, unlike other multivariate procedures that only allow to reject/not reject the PPP hypothesis for the whole sample of countries included in the analysis, it sheds light on the stochastic behavior of each individual exchange rate and its consistency (or lack of) with PPP in its different versions.

Using this more advanced method, we find evidence of long memory for the Canadian dollar, the British pound and the euro, but in all four cases the results are consistent with the relative version of PPP, in contrast to the results often obtained with standard testing procedures.

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**Table 1: Estimates of the parameters of the model given by equation (2)**

Currency	d	Matrix F				Cholesky representation			
		Canada	Japan	U.K.	Europe	Canada	Japan	U.K.	Europe
Canadian dollar	0.5534 (0.2104)	0.5972 (0.1969)	-0.3150 (0.1321)	0.0605 (0.1365)	0.3569 (0.1866)	0.3356 (0.0398)			
Japanese yen	0.3625 (0.3187)	-0.4980 (0.2416)	0.2942 (0.3313)	-0.2000 (0.2156)	0.4751 (0.2730)	0.1644 (0.1016)	0.6097 (0.0697)		
British pound	0.2980 (0.1005)	-0.3366 (0.2257)	-0.2235 (0.2519)	0.4800 (0.2488)	0.4318 (0.3159)	0.2130 (0.1035)	0.3454 (0.0906)	0.5165 (0.0571)	
Euro	0.6665 (0.1750)	-0.2423 (0.2179)	-0.2689 (0.2230)	-0.3167 (0.2046)	0.8856 (0.3339)	0.2219 (0.0978)	0.4374 (0.0804)	0.3304 (0.0527)	0.2437 (0.0273)

Standard errors in parentheses.