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Worked**

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# PERSISTENCE AND CYCLES IN US HOURS WORKED

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## **Abstract**

This paper analyses monthly hours worked in the US over the sample period 1939m1 – 2011m10 using a cyclical long memory model; this is based on Gegenbauer processes and characterised by autocorrelations decaying to zero cyclically and at a hyperbolic rate along with a spectral density that is unbounded at a non-zero frequency. The reason for choosing this specification is that the periodogram of the hours worked series has a peak at a frequency away from zero. The empirical results confirm that this model works extremely well for hours worked, and it is then employed to analyse their relationship with technology shocks. It is found that hours worked increase on impact in response to a technology shock (though the effect dies away rapidly), consistently with Real Business Cycle (RBC) models.

**Keywords:** hours worked, fractional integration, cycles, technology shocks

**JEL classification:** C32, E24

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## 1. Introduction

This paper proposes a modelling approach for US hours worked, specifically average weekly hours in manufacturing. This is an important variable since it can be seen as an indicator of the state of the economy. Authors such as Glosser and Golden (1997) argue that firms tend to respond to business cycle conditions by decreasing or increasing hours worked, before hiring or laying off workers.

Although the relationship between business cycles and hours worked and their response to technology shocks has been extensively investigated, this is still a controversial issue. Gali (1999), Francis and Ramey (2005) and Gali and Rabanal (2004 - GR) found that, contrary to the implications of Real Business Cycle (RBC) models, they decline in response to a technology shock. These results were challenged, among others, by Christiano, Eichenbaum and Vigfusson (2003 - CEV) who presented evidence that instead hours worked increase following a technology shock.<sup>1</sup> Both types of studies use similar empirical (VAR) frameworks, the crucial difference between them being in the treatment of the hours worked variable. In particular, the former authors model it as a nonstationary I(1) variable whilst the latter assume that it is a stationary I(0) process. More recently, Gil-Alana and Moreno (2009) allow the order of integration of hours worked to be fractional, i.e. I(d), and find that the value of d depends on the specific series examined, although in general it lies in the interval between 0 and 1. They also find that per capita hours fall on impact in response to a technology shock.

All three approaches taken in the studies mentioned above implicitly assume a high degree of persistence in hours worked that should result in a large peak in the periodogram (or in any other estimate of the spectral density function) at the zero frequency. The model used in the present study is instead based on Gegenbauer processes and is characterised by autocorrelations decaying to zero cyclically and at a hyperbolic rate along with a spectral density that is unbounded at a non-zero frequency. The reason for choosing this specification is that the periodogram of the hours worked series is found not to exhibit a peak at the zero

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<sup>1</sup> For further evidence, see Gambetti, 2005 and Pesavento and Rossi, 2005.

frequency, as implied by the previous models, but instead at a frequency away from zero, which can be captured by Gegenbauer processes as explained in the following section. Our results confirm that this model works extremely well for hours worked, and it is then employed to analyse their relationship with technology shocks, finding a positive (though rapidly dying away) effect of such shocks, as suggested by Real Business Cycle (RBC) models.

The outline of the paper is as follows. Section 2 briefly describes the different types of long range dependence or long memory models used here. Section 3 presents the data. Section 4 discusses the empirical results and their implications for the debate on the relationship between hours worked and technology shocks, while Section 5 contains some concluding remarks.

## 2. A cyclical I(d) model

For the purposes of the present study, we define an I(0) process  $\{x_t, t = 0, \pm 1, \dots\}$  as a covariance stationary process with spectral density function,  $f(\lambda)$ , that is positive and finite at any frequency. Alternatively, it can be defined in the time domain as a process such that the infinite sum of the autocovariances is finite. This includes a wide range of model specifications such as the white noise case, the stationary autoregression (AR), moving average (MA), and stationary ARMA models.

In general, the I(0) condition is a pre-requisite for statistical inference in time series analysis. However, a series might be nonstationary, i.e. the mean, the variance or the autocovariances may change over time. For this case specifications with stochastic trends have usually been adopted, under the assumption that the first differenced process is stationary I(0), and thus valid statistical inference can be drawn after differencing once. More specifically,  $x_t$  is said to be I(1) if:

$$(1 - L)x_t = u_t, \quad t = 1, 2, \dots, \quad (1)$$

where L is the lag operator ( $Lx_t = x_{t-1}$ ) and  $u_t$  is I(0) as defined above. If  $u_t$  is ARMA(p, q), then  $x_t$  is said to be an ARIMA(p, 1, q) process.

The above model has been extended in recent years to the fractional case, since the differencing parameter required to render a series stationary  $I(0)$  is not necessarily an integer (usually 1) but might also have a fractional value. In this context,  $x_t$  is said to be  $I(d)$  if:

$$(1 - L)^d x_t = u_t, \quad t = 1, 2, \dots, \quad (2)$$

with  $x_t = 0, t \leq 0^2$ , and  $u_t$  is again  $I(0)$ . Note that the polynomial on the left-hand-side of equation (2) can be expanded, for all real  $d$ , as

$$(1 - L)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j = 1 - dL + \frac{d(d-1)}{2} L^2 - \dots$$

Thus, if  $d$  in (2) is an integer value,  $x_t$  will be a function of a finite number of past observations, while, if  $d$  is not an integer,  $x_t$  depends upon values of the time series in the distant past, and the higher the value of  $d$  is, the higher the level of dependence is between the observations.

If  $d > 0$  in (2)  $x_t$  displays long range dependence (LRD) or long memory. There are two definitions of LRD, one in the time domain and the other in the frequency domain. The former states that given a covariance stationary process  $\{x_t, t = 0, \pm 1, \dots\}$ , with autocovariance function  $E[(x_t - E x_t)(x_{t-j} - E x_t)] = \gamma_j$ ,  $x_t$  displays LRD if

$$\lim_{T \rightarrow \infty} \sum_{j=-T}^T |\gamma_j|$$

is infinite. A frequency domain definition may be as follows. Suppose that  $x_t$  has an absolutely continuous spectral distribution, and therefore a spectral density function, denoted by  $f(\lambda)$ , and defined as

$$f(\lambda) = \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \gamma_j \cos \lambda j, \quad -\pi < \lambda \leq \pi.$$

Then,  $x_t$  displays LRD if the spectral density function has a pole at some frequency  $\lambda$  in the interval  $[0, \pi]$ , i.e.,

$$f(\lambda) \rightarrow \infty, \quad \text{as } \lambda \rightarrow \lambda^*, \quad \lambda^* \in [0, \pi], \quad (3)$$

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<sup>2</sup> This condition is required for the Type II definition of fractional integration. For an alternative definition (Type I) see Marinucci and Robinson (1999).

(see McLeod and Hipel, 1978). Most of the empirical literature has focused on the case when the singularity or pole in the spectrum occurs at the zero frequency ( $\lambda^* = 0$ ). In fact, the I(d) model, defined as in (2), is characterised by a spectral density function which is unbounded at the origin. However, there might be cases when the singularity or pole in the spectrum occurs at other frequencies, for instance the spectrum might have a single pole at a frequency other than zero. Then the process still displays the property of LRD but the autocorrelations have a cyclical structure with slow decay. This is the case of the Gegenbauer processes defined as:

$$(1 - 2\cos w_r L + L^2)^d x_t = u_t, \quad t = 1, 2, \dots, \quad (4)$$

where  $w_r$  and  $d$  are real values, and  $u_t$  is I(0). For practical purposes we define  $w_r = 2\pi r/T$ , with  $r = T/c$ , and thus  $c$  indicates the number of time periods per cycle, while  $r$  stands for the frequency with a pole or singularity in the spectrum of  $x_t$  ( $\lambda^*$ ). Note that if  $r = 0$  (or  $c = 1$ ), the fractional polynomial in equation (4) becomes  $(1 - L)^{2d}$ , which is the polynomial associated to the common case of fractional integration at the long run or zero frequency.

Gray et al. (1989, 1994) showed that the polynomial in (4) can be expressed in terms of the Gegenbauer polynomial, such that, denoting  $\mu = \cos w_r$ , for all  $d \neq 0$ ,

$$(1 - 2\mu L + L^2)^{-d} = \sum_{j=0}^{\infty} C_{j,d}(\mu) L^j,$$

where  $C_{j,d}(\mu)$  are orthogonal Gegenbauer polynomial coefficients defined recursively as:

$$C_{0,d}(\mu) = 1, \quad C_{1,d}(\mu) = 2\mu d,$$

$$C_{j,d}(\mu) = 2\mu \left( \frac{d-1}{j} + 1 \right) C_{j-1,d}(\mu) - \left( 2 \frac{d-1}{j} + 1 \right) C_{j-2,d}(\mu), \quad j = 2, 3, \dots,$$

(see Magnus et al., 1966, Rainville, 1960, etc. for further details on Gegenbauer polynomials).

Gray et al. (1989) showed that  $x_t$  in (4) is (covariance) stationary if  $d < 0.5$  for  $|\mu = \cos w_r| < 1$  and if  $d < 0.25$  for  $|\mu| = 1$ .

The type of process described in (4) was introduced by Andel (1986) and subsequently analysed by Gray, Zhang and Woodward (1989, 1994), Chung (1996a,b), Gil-Alana (2001) and Dalla and Hidalgo (2005) among others.<sup>3</sup>

### 3. The dataset

The series examined here is the average number of hours worked per week by production workers in US manufacturing industries, monthly, over the sample period 1939m1 – 2011m10; the source is the Current Employment Statistics (CES) monthly survey of the US Bureau of Labor Statistics.

We analyse both seasonally adjusted and unadjusted data (HWSA11 and HWNSA11 respectively) for the whole sample period and also for a shorter sample ending in 2007m4 (HWSA07 and HWNSA07) in order to establish whether the 2007/8 crisis had an impact on hours worked.

**[Insert Figure 1 about here]**

Figure 1 displays the time series plots of both the seasonally adjusted and unadjusted series ending in 2011, which move very closely over time. It also shows the correlograms, which exhibit a clearly cyclical pattern. The periodograms, also displayed in the same figure, have the highest peak at frequency 7, as opposed to the zero frequency, which suggests that the I(d) and I(1) specifications estimated by other authors are not appropriate, and also that cycles have a length of approximately  $T/7 = 124.85$  months, i.e. around ten and a half years. The periodograms of the series ending in 2007 (not reported) have the highest value at frequency 6, namely  $T/6 = 136.66$  month ( $\approx 11.3$  years / cycle). These values imply longer cycles than those normally observed, typically with a periodicity between 6 and 10 years.<sup>4</sup>

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<sup>3</sup> LRD also admits processes with multiple poles or singularities in the spectrum (k-factor Gegenbauer processes - see Giraitis and Leipus, 1995; Woodward et al., 1998; etc.) but these are beyond the scope of the present study.

<sup>4</sup> Burn and Mitchell (1946), Romer (1986, 1994), Stock and Watson (1998), Diebold and Rudebusch (1992), Canova (1998), Baxter and King (1999), King and Rebelo (1999) among others showed that the average length of the cycle is approximately six years.

#### 4. Empirical results

As a first step we estimate the order of integration of the series using a standard I(d) model, i.e. assuming that the peak of the spectrum occurs at the long run or zero frequency. In other words, we consider a model such as (2) where  $x_t$  can be the errors in a regression model of the form:

$$y_t = \beta^T z_t + x_t, \quad t = 1, 2, \dots, \quad (5)$$

where  $y_t$  is the observed time series (hours worked),  $\beta$  is a  $(k \times 1)$  vector of unknown coefficients, and  $z_t$  is a set of weakly exogenous variables or deterministic terms that might include an intercept (i.e.,  $z_t = 1$ ), an intercept with a linear time trend ( $z_t = (1, t)^T$ ), or any other type of deterministic processes.

We estimate the fractional differencing parameter  $d$  using the Whittle function in the frequency domain (Dahlhaus, 1989), and also employ a testing procedure developed by Robinson (1994), which has been shown to be the most efficient one in the context of fractional integration. This method, based on the Lagrange Multiplier (LM) principle, tests the null hypothesis  $H_0: d = d_0$  in (2) and (5) for any real value  $d_0$  and has several advantages over other approaches. First, it allows to test for any real value of  $d_0$ , therefore encompassing both the stationary ( $d < 0.5$ ) and nonstationary ( $d \geq 0.5$ ) hypotheses. Moreover, the limiting distribution is  $N(0, 1)$  and this standard behaviour holds independently of the regressors used in the regression model (5) and the type of model for the I(0) disturbances  $u_t$  in (2). Finally, it is the most efficient method in the Pitman sense against local departures from the null (see Robinson, 1994).<sup>5</sup>

**[Insert Table 1 about here]**

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<sup>5</sup> Wald tests of fractional integration based on a similar approach to Robinson's (1994) have been proposed by Lobato and Velasco (2007). Their method, however, requires an efficient estimate of  $d$  and therefore Robinson's (1994) approach seems more attractive, at least computationally.



Table 1 displays the (Whittle) estimates of  $d$  (and the 95% confidence bands corresponding to the non-rejection values of  $d$  using Robinson's (1994) method) in the model given by equations (2) and (5) with  $z_t$  in (5) equal to  $(1, t)^T$ ,  $t \geq 1$ , 0 otherwise, i.e.,

$$y_t = \beta_0 + \beta_1 t + x_t, \quad (1 - L)^d x_t = u_t, \quad t = 1, 2, \dots, \quad (6)$$

assuming that  $u_t$  in (6) is white noise, AR(1), AR(2), seasonal AR(1) and finally adopting the exponential model of Bloomfield (1973) respectively. The latter is a non-parametric approach to modelling the  $I(0)$  disturbances that approximates ARMA structures with a small number of parameters and has been widely employed in the context of fractional integration (see Gil-Alana, 2004). In all cases we consider the three standard approaches of no regressors ( $\beta_0 = \beta_1 = 0$  a priori in (6)), an intercept ( $\beta_0$  unknown and  $\beta_1 = 0$  a priori) and an intercept with a linear trend ( $\beta_0$  and  $\beta_1$  unknown).

The results suggest that there is no need to include a time trend, the intercept being sufficient to describe the deterministic part of the processes in all cases. When modelling the disturbances as a white noise, the estimates of  $d$  are in the interval  $(0.5, 1)$ , implying nonstationarity and mean reverting-behaviour. They are higher for the seasonally adjusted data and also slightly higher for the sample ending in 2011. When  $u_t$  is specified as an AR(1) process the same result holds for the unadjusted data; however, for the adjusted ones,  $d$  is slightly above 1 and the unit root null cannot be rejected at the 5% level. For the AR(2) model the estimated values of  $d$  are strictly above 1, unlike in the seasonal AR case where all the estimates are below 1. Finally, when employing the non-parametric approach of Bloomfield (1973)  $d$  is strictly above 1 for the seasonally adjusted data, and slightly below 1 (the unit root null not being rejected) in the case of the unadjusted data. Thus, the results change substantially depending on the specification of the error term.

We further investigate this issue by employing the parametric approach of Robinson (1994) described above assuming that the disturbances are white noise and autocorrelated in turn. In particular, we consider the set-up given by (6), testing  $H_0: d = d_0$ , for  $d_0$ -values from 0

to 2 with 0.001 increments in the case of white noise errors, and from -1.500 to 0.500 for AR(1) and AR(2)  $u_t$ . In other words, the tested (null) model is:

$$y_t = \beta_0 + \beta_1 t + x_t, \quad (1 - L)^{d_0} x_t = u_t, \quad t = 1, 2, \dots,$$

with I(0)  $u_t$ . Because the estimates of  $\beta_1$  were found to be statistically insignificant in all cases, we remove the time trend from the above equation. In general we should expect a monotonic decrease in the value of the test statistic with respect to the values of  $d_0$ . Such monotonicity is a consequence of the one-sided alternatives employed in this procedure. Thus, for example, we would expect that if  $H_0: d = d_0$  is rejected with  $d_0 = 0.250$  against the alternative  $H_a: d > 0.250$ , an even stronger rejection occurs when testing  $H_0$  with  $d_0 = 0.200$ .

**[Insert Figure 2 about here]**

We focus here on the series HSA11 using the model with an intercept. Similar results were obtained for the remaining three series. Figure 2(i) shows the values of the test statistic for the case of uncorrelated errors, and also displays the critical values (flat lines) of the testing procedure.<sup>6</sup> It can be seen that there is a monotonic decrease in the value of the test statistic with the values of  $d_0$  for which  $H_0$  cannot be rejected (also displayed in Table 1) ranging between 0.868 and 0.935. Thus, the unit root null hypothesis (i.e.,  $d = 1$ ) is marginally rejected in favour of mean reversion.

In the case of autocorrelated errors (see Figures 2(ii) and (iii)), monotonicity is not found in the values of the test statistic with respect to  $d$ : we obtain non-rejection values when  $d$  is close to 0 and 1 but rejection ones in between. This may be explained by the low power of this method if the roots of the AR polynomials are close to the unit circle. In fact, this is typical of all parametric procedures as a result of the competition between the fractional differencing parameter and the AR parameters in describing time dependence. When employing higher AR orders essentially the same results are obtained. However, this might also reflect model misspecification as argued in Gil-Alana and Robinson (1997) and in the present study as well:

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<sup>6</sup> Values between the two flat lines indicate non-rejections of the null hypothesis.

the classical I(d) model (even with positive integer degrees of differentiation) may not be a valid one to describe the behaviour of hours worked given the fact that the periodograms do not exhibit their highest values at the smallest (zero) frequency. Consequently, in what follows we consider the cyclical I(d) specification given by equation (4). In order to avoid distortions produced by the deterministic terms we use demeaned data (although the results are very similar to those obtained with the raw data), and apply again Robinson's (1994) method, which is very general since it allows to test stationary and nonstationary hypotheses, with one or more integer or fractional orders of integration of arbitrary order anywhere on the unit circle in the complex plane.

**[Insert Tables 2, 3 and 4 about here]**

Table 2 displays the estimated parameters of  $d$  and  $c$  ( $w_t = 2\pi/c$ ) in (4), assuming that the disturbances are white noise. The values of  $c$  are 137 (which correspond to 11.4 years per cycle) for the two series ending in 2007, and 125 and 124 (approximately a 10-year cycle) for the seasonally adjusted and unadjusted data ending in 2011. This is consistent with the shape of the periodograms, the highest values occurring at frequency 6 for HWSA07 and HWNSA07 and at frequency 7 for HWSA11 and HWNSA11 ( $T(=820)/6 = 136.66$ , and  $T(=874)/7 = 124.85$  respectively). Regarding the estimated values of  $d$ , these are above 0 but below 0.5, implying stationarity but mean-reverting behaviour. Very similar results are obtained under the assumption of autocorrelated errors (Tables 3 and 4): in these cases the values range between 0.4 and 0.5 for the seasonally adjusted data and between 0.3 and 0.4 for the unadjusted ones. Several diagnostic tests conducted on the residuals of the estimated models suggest that the AR(1) structure is sufficient to describe the short-run dynamics of the series.<sup>7</sup>

Next we investigate the relationship between technology shocks and hours worked. On the basis of the above evidence that supports the cyclically I(d) specification for hours worked,

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<sup>7</sup> Box-Pierce Q-statistics indicate that the models including AR(1) disturbances (see Table 3) are free of additional serial correlation.

and assuming that technology shocks are exogenous in this context, we consider the following model,

$$y_t = \alpha + \beta z_t + x_t, \quad (1 - 2\cos w_r L + L^2)^d x_t = u_t, \quad t = 1, 2, \dots,$$

(7)

where  $y_t$  is once more hours worked and  $z_t$  is the productivity series measured as output per hour.

In what follows we use two definitions of productivity. In particular, we employ the same variables as in Gali and Rabanal (GR, 2004) and Christiano et al. (CEV, 2003). Both GR and CEV work with quarterly data. However, while GR use data from the non-farm business sector, CEV employ data from all businesses, including farming activities. We perform our analysis with both variables, which were collected from the Federal Reserve Bank of St. Louis database (FRED). Non-farm business sector productivity is measured as output per hour of all persons (OPHNFB is the ID of the series). Non-farm business hours are computed as the ratio between the non-farm business sector hours of all persons (HOANBS) and the civilian non-institutional population over the age of 16 (CNP16OV). Total business productivity is measured as the output per hour of all persons (OPHPBS). Our dataset runs from the first quarter of 1948 to the fourth quarter of 2004. Using monthly seasonally unadjusted and adjusted hours worked we also construct quarterly series for the same sample period. Plots of the four series (productivity using the GR and CEV definitions respectively and seasonally adjusted/unadjusted hours worked, for 1948q1-2004q4) are displayed in Figure 3.

**[Insert Figure 3 about here]**

The two hours worked series behave in a very similar way, being relatively stable across the sample period, whilst the two productivity series are increasing over time. Tables 5 and 6 display the estimates of  $d$  along with those of the intercept ( $\alpha$ ) and the slope ( $\beta$ ) in a model including hours worked ( $y_t$ ) and productivity ( $z_t$ ) assuming that the regression errors are

I(d) and the d-differenced process is white noise, AR and Bloomfield respectively. In other words, the model is now:

$$y_t = \alpha + \beta z_t + x_t, \quad (1 - L)^d x_t = u_t, \quad u_t \approx I(0), \quad t = 1, 2, \dots, \quad (7)$$

implying the existence of a pole or singularity in the spectrum at the zero frequency. Tables 5 and 6 present the results for seasonally unadjusted and adjusted hours worked respectively. Starting with the former (in Table 5) we notice that the estimates of d range between 0.405 (CEV with AR(1)) and 0.525 (CEV with white noise) and the two hypotheses of integer degrees of differentiation (i.e.,  $d = 0$  and  $d = 1$ ) are decisively rejected in all cases. In general the results are very similar for the two productivity series (CEV and GR). Also, the estimates of the slope coefficient  $\beta$  are all positive though not statistically different from zero.

**[Insert Tables 5 and 6 about here]**

Table 6 displays the results for the seasonally adjusted data. Here we observe large variability in the estimates of d depending on the specification of the I(0) disturbances. However, unlike in the previous case, the estimates of  $\beta$  are now all significantly positive implying that on impact hours worked increase in response to a technology shock, in line with the findings of CEV (2003). It is noteworthy that the results presented in these two tables are based on the case when the spectrum is unbounded at the origin, a feature whose presence is not supported by the empirical evidence presented below.

**[Insert Figure 4 about here]**

Figure 4 displays the periodograms for the two hours worked series employed for this part of the analysis, i.e., for the sample period 1948q1 – 2004q4. It can be seen that now the highest peak occurs at frequency 6, implying cycles of approximate length of  $T/6 \approx 38$  quarters or roughly 9.5 years. In what follows, we consider the model given by equation (4) again with white noise and correlated (AR(1), AR(2) and Bloomfield) errors. The results, for seasonally unadjusted and adjusted hours worked respectively, are reported in Tables 7 and 8.

## **[Insert Tables 7 and 8 about here]**

The estimated value of  $c$  (not reported) is 38 in all cases, consistently with the periodograms displayed in Figure 4, whilst the estimated values of  $d$  are in all cases in the interval  $(0, 1)$  but smaller than for the seasonally unadjusted data (in Table 7), implying long memory and mean reverting behaviour.<sup>8</sup> These figures also show that, when using the cyclical  $I(d)$  specifications, hours worked increase in response to a technology shock and this happens for both seasonally adjusted and unadjusted hours worked. Next we investigate which of the potential models for the disturbances is the most adequate for the two series examined. We perform various diagnostic tests on the residuals which suggest that the model with  $AR(1)$  disturbances is the most appropriate one for both series (CEV and GR) and both adjusted and unadjusted data. We then consider a 1-standard error technology shock and estimate the impulse responses of the selected model for each series. The results are displayed in Figure 5.

## **[Insert Figure 5 about here]**

It can be seen that on impact the effect is positive and statistically significant in all four cases, and more sizeable for the GR series. However, after two years it becomes negligible and it disappears in the long run in all cases. In other studies the cyclical pattern in hours worked is modelled as a simple  $AR(2)$  processes with complex roots (see, e.g., Bernardi et al., 2008) which produce autocorrelations (and impulse responses) decaying at an exponential rate. When using our approach the rate of decay is hyperbolic, i.e. much slower, and given the length of the cycles in this context, it does not produce in the short run a clear cyclical pattern in the figures.

## **5. Conclusions**

This paper analyses monthly hours worked in the US over the sample period 1939m1 – 2011m10 using a cyclical  $I(d)$  model based on Gegenbauer processes, which are characterised by a spectral density function unbounded at a non-zero frequency. The motivation for adopting

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<sup>8</sup> For the seasonally unadjusted data, the estimated values of  $d$  range between 0.123 and 0.272, whilst for the seasonally adjusted ones the variability is much higher, the values ranging between 0.068 and 0.705.

this type of framework is the observation that the periodogram of the hours worked series has a peak at a frequency away from zero. This is in contrast to the models normally found in the literature (e.g., Gali, 1999; Christiano, Eichenbaum and Vigfusson, 2003; Gil-Alana and Moreno, 2009) that, although differing in the degree of integration assumed for hours worked, are all based on hours worked being a highly persistent series with a peak at the zero frequency in the spectrum.

The evidence presented here suggests that such a specification is not empirically supported, our chosen framework being the most suitable one for capturing the cycle length in the case of hours worked, with the cycles having a periodicity of about ten years, and the order of integration of the series being positive though smaller than 0.5, implying stationary and mean-reverting behaviour.

When including productivity as a weakly exogenous variable further evidence is obtained supporting the Gegenbauer model, the order of integration again being in the interval (0, 0.5). Moreover, hours worked are found to increase on impact in response to a technology shock (although its effects disappear after two years). This result is consistent with the findings of Christiano, Eichenbaum and Vigfusson (2003), despite the use of a completely different methodology, and represents an important contribution towards settling the ongoing debate on the relationship between hours worked and technology: it shows that, when the shape of the periodogram is duly taken into account by specifying an appropriate statistical model, a positive rather than negative effect of technology shocks on hours worked is estimated, consistently with Real Business Cycle (RBC) models.

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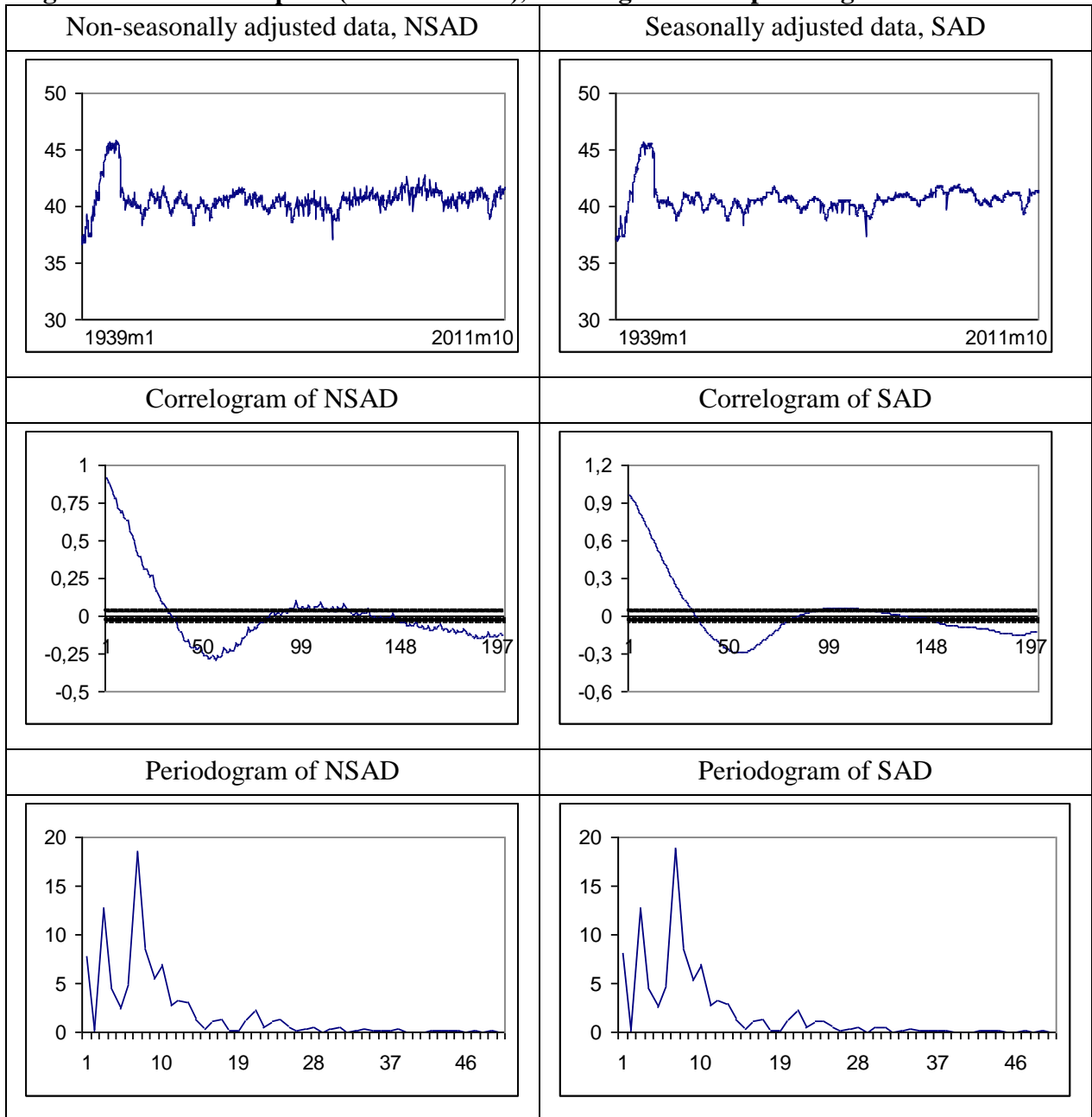
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**Figure 1: Time series plots (hours worked), correlograms and periodograms**



The thick lines in the correlograms represent the 95% confidence band for the null hypothesis of no autocorrelation. In the periodograms, the horizontal axis refers to the discrete Fourier frequencies  $\lambda_j = 2\pi j/T$ ,  $j = 1, \dots, T/2$ .

**Table 1: Estimates of d based on an I(d) model with the singularity at the 0-frequency**

Disturbances	Series	No regressors	An intercept	A linear trend
White noise	HWSA07	0.996 (0.956, 1.042)	<b>0.899</b> <b>(0.868, 0.935)</b>	0.900 (0.869, 0.935)
	HWNSA07	0.981 (0.942, 1.027)	<b>0.745</b> <b>(0.713, 0.781)</b>	0.748 (0.717, 0.783)
	HWSA11	0.998 (0.959, 1.045)	<b>0.903</b> <b>(0.873, 0.938)</b>	0.904 (0.874, 0.938)
	HWNSA11	0.984 (0.946, 1.028)	<b>0.748</b> <b>(0.717, 0.784)</b>	0.751 (0.721, 0.786)
AR (1)	HWSA07	1.351 (1.268, 1.446)	<b>1.038</b> <b>(0.988, 1.093)</b>	1.037 (0.989, 1.093)
	HWNSA07	1.285 (1.186, 1.389)	<b>0.868</b> <b>(0.822, 0.921)</b>	0.870 (0.824, 0.921)
	HWSA11	1.352 (1.271, 1.446)	<b>1.045</b> <b>(0.997, 1.100)</b>	1.045 (0.997, 1.099)
	HWNSA11	1.281 (1.180, 1.380)	<b>0.870</b> <b>(0.823, 0.922)</b>	0.871 (0.825, 0.922)
AR (2)	HWSA07	1.887 (1.724, 2.031)	<b>1.175</b> <b>(1.089, 1.268)</b>	1.173 (1.089, 1.266)
	HWNSA07	1.866 (1.710, 2.042)	<b>1.102</b> <b>(1.001, 1.226)</b>	1.101 (1.001, 1.224)
	HWSA11	1.886 (1.721, 2.055)	<b>1.185</b> <b>(1.099, 1.276)</b>	1.184 (1.099, 1.274)
	HWNSA11	1.861 (1.712, 2.074)	<b>1.103</b> <b>(1.003, 1.223)</b>	1.102 (1.003, 1.221)
Seasonal AR (1)	HWSA07	0.999 (0.959, 1.047)	<b>0.903</b> <b>(0.871, 0.938)</b>	0.903 (0.872, 0.939)
	HWNSA07	0.979 (0.938, 1.024)	<b>0.759</b> <b>(0.721, 0.802)</b>	0.760 (0.723, 0.803)
	HWSA11	1.002 (0.964, 1.047)	<b>0.907</b> <b>(0.876, 0.942)</b>	0.908 (0.877, 0.942)
	HWNSA11	0.981 (0.941, 1.027)	<b>0.766</b> <b>(0.729, 0.808)</b>	0.767 (0.730, 0.809)
Bloomfield (1)	HWSA07	1.008 (0.948, 1.091)	<b>1.108</b> <b>(1.034, 1.195)</b>	1.107 (1.034, 1.194)
	HWNSA07	0.999 (0.934, 1.082)	<b>0.938</b> <b>(0.872, 1.016)</b>	0.939 (0.873, 1.017)
	HWSA11	1.011 (0.946, 1.079)	<b>1.121</b> <b>(1.047, 1.202)</b>	1.114 (1.046, 1.200)
	HWNSA11	1.003 (0.943, 1.091)	<b>0.939</b> <b>(0.872, 1.012)</b>	0.939 (0.873, 1.014)

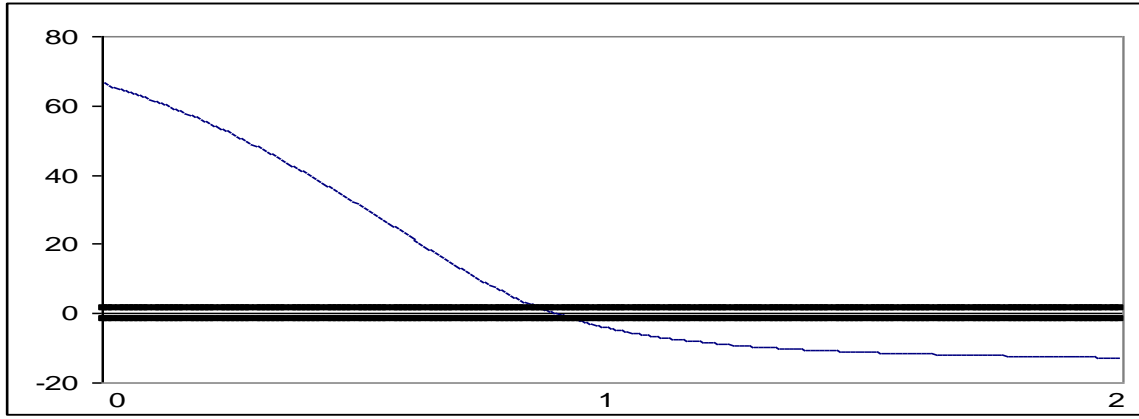
The estimates are the Whittle estimates of d and the values in parentheses are the 95% confidence band of the non-rejection values using Robinson's (1994) method. In bold, the significant cases with the deterministic terms.

HWSA07 and HWNSA07 stands respectively for hours worked, seasonally adjusted and unadjusted, ending in 2007m4.

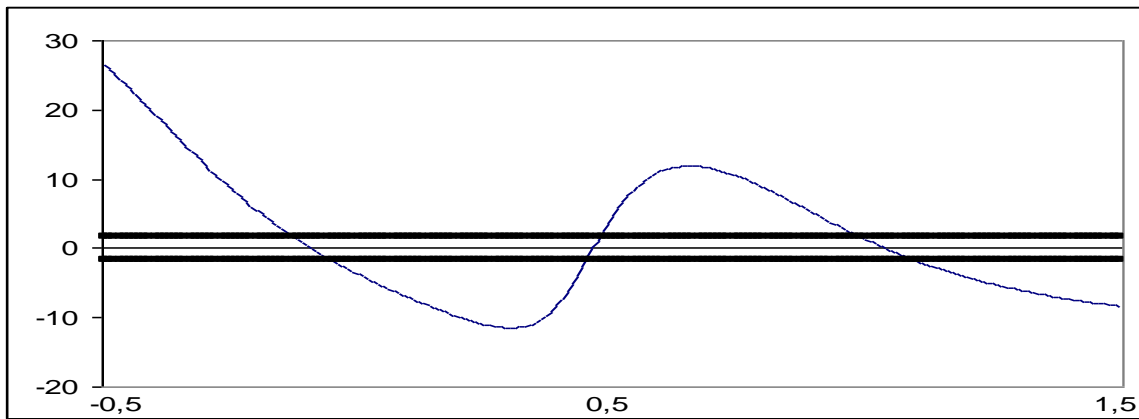
HWSA11 and HWNSA11 stands respectively for hours worked, seasonally adjusted and unadjusted, ending in 2011m10.

**Figure 2: Test statistic (Robinson, 1994) for different  $d_0$ -values**

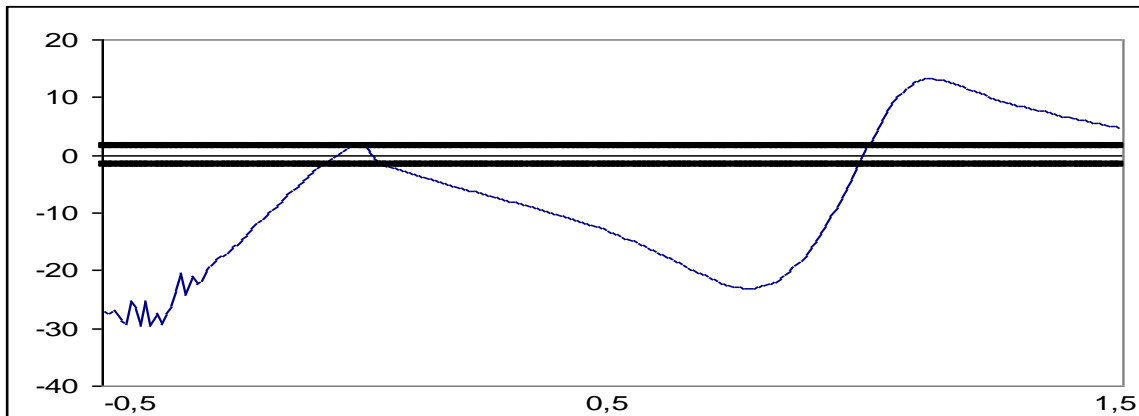
i) White noise disturbances



ii) AR(1) disturbances



iii) AR(2) disturbances



On the horizontal axis are the values of  $d$  under  $H_0$ , and on the vertical one the values of the test statistic; the bold lines refer to the 95% non-rejection bands.

**Table 2: Estimates of the parameters in the model given by equation (4) with white noise  $u_t$**

Series	$c$ (months)	$c/12$ (years)	$d$ (95% interval)
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HWSA07	137	11.416	0.419 (0.394, 0.450)
HWNSA07	137	11.416	0.343 (0.318, 0.373)
HWSA11	125	10.416	0.421 (0.397, 0.452)
HWNSA11	124	10.333	0.343 (0.319, 0.374)

The values in parentheses are the 95% confidence band of the non-rejection values of d using Robinson (1994).

**Table 3: Estimates of the parameters in the model given by equation (4) with AR(1)  $u_t$**

Series	c (months)	c/12 (years)	d (95% interval)	AR coef.
HWSA07	137	11.416	0.479 (0.433, 0.533)	-0.209
HWNSA07	136	11.333	0.390 (0.347, 0.441)	-0.175
HWSA11	125	10.416	0.482 (0.436, 0.535)	-0.212
HWNSA11	125	10.416	0.388 (0.345, 0.439)	-0.163

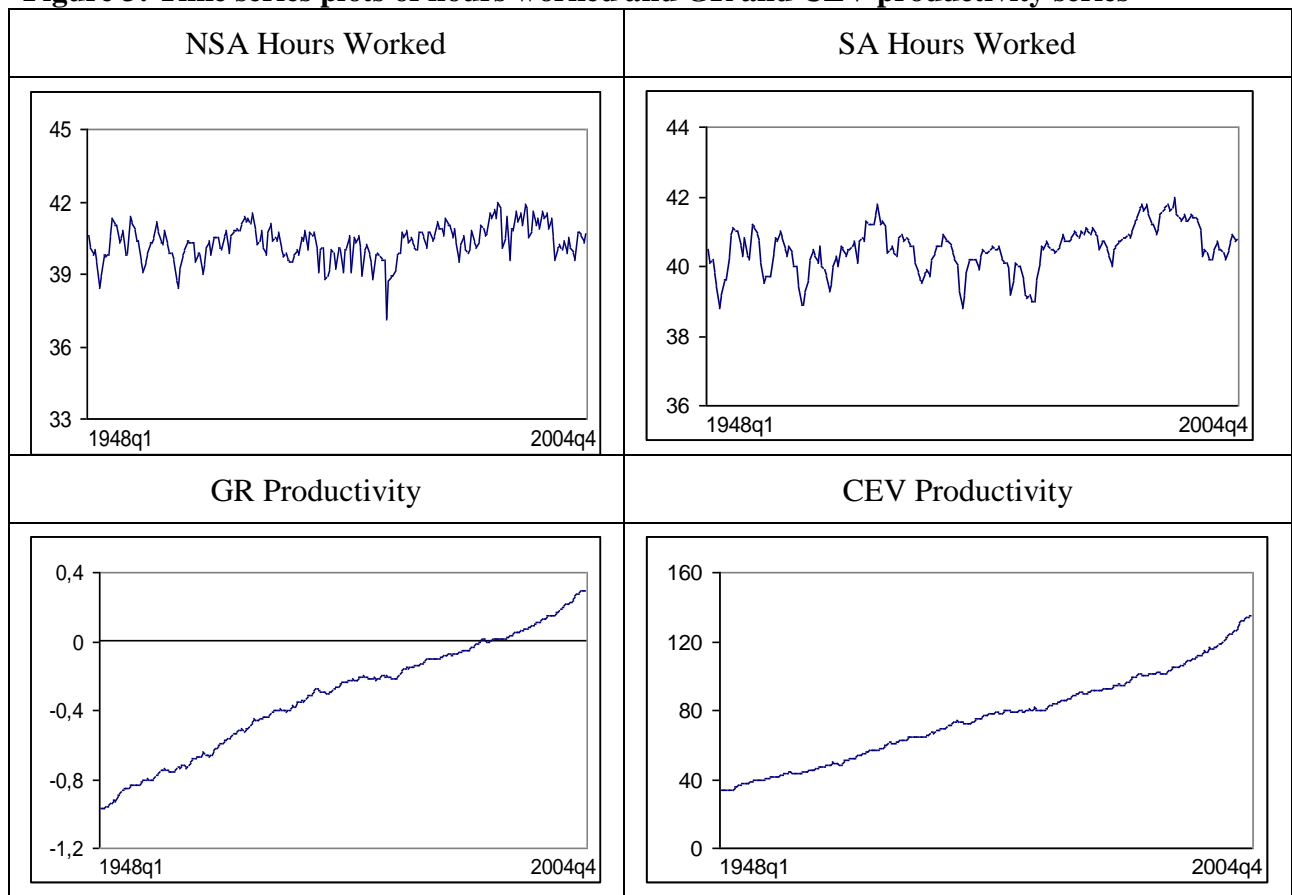
The values in parentheses are the 95% confidence band of the non-rejection values of d using Robinson (1994).

**Table 4: Estimates of the parameters in the model given by equation (4) with Bloomfield  $u_t$**

Series	c (months)	c/12 (years)	d (95% interval)	Bloomf. c.
HWSA07	137	11.416	0.419 (0.403, 0.437)	-0.093
HWNSA07	137	11.416	0.343 (0.326, 0.361)	-0.089
HWSA11	125	10.416	0.420 (0.404, 0.439)	-0.082
HWNSA11	125	10.416	0.343 (0.326, 0.361)	-0.077

The values in parentheses are the 95% confidence band of the non-rejection values of d using Robinson (1994).

**Figure 3: Time series plots of hours worked and GR and CEV productivity series**



NSA and SA stand respectively for non-seasonally adjusted and seasonally adjusted data.

GR stands for the productivity series used in Gali and Rabanal (2004)., whilst CEV is the productivity series used by Christiano et al. (2003).

**Table 5: Estimates of the relationship between hours worked and productivity using an I(d) specification for the error term (Seasonally unadjusted hours worked)**

		d (95% conf. interval)	$\alpha$ (t-value)	$\beta$ (t-value)	Short run par.
White noise	CEV	0.525 (0.436, 0.641)	39.85071 (71.7345)	0.00791 (0.91655)	---
	GR	0.523 (0.435, 0.637)	40.75346 (70.7800)	0.71560 (1.0770)	---
AR(1)	CEV	0.405 (0.181, 0.620)	39.81537 (94.9110)	0.00705 (1.25671)	0.163
	GR	0.408 (0.174, 0.617)	40.5726 (121.9811)	0.59901 (1.3651)	0.158
AR(2)	CEV	0.492 (0.299, 0.956)	39.8419 (97.9311)	0.00753 (1.19703)	0.079, -0.057
	GR	0.500 (0.298, 0.929)	40.70159 (101.982)	0.67994 (1.4450)	0.068, -0.062
Bloomf.	CEV	0.413 (0.272, 0.612)	39.81771 (94.2940)	0.00707 (1.24201)	0.159
	GR	0.412 (0.272, 0.607)	40.57617 (121.3546)	0.60178 (1.3681)	0.161

In parentheses, in the 3<sup>rd</sup> column, the 95% confidence bands for the non-rejection values of d; in the 4<sup>th</sup> and 5<sup>th</sup> columns, the corresponding t-values.

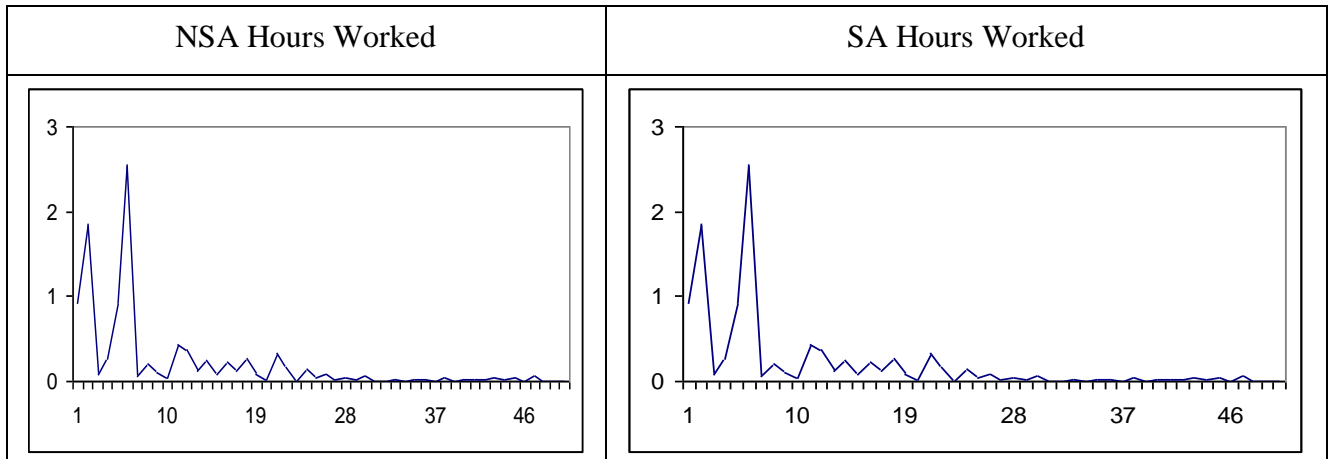
**Table 6: Estimates of the relationship between hours worked and productivity using an I(d) specification for the error term (Seasonally adjusted hours worked)**

		d (95% conf. interval)	$\alpha$ (t-value)	$\beta$ (t-value)	Short run par.
White noise	CEV	0.994 (0.866, 1.146)	38.51816 (43.908)	<b>0.05922</b> <b>(2.395)</b>	---
	GR	0.968 (0.849, 1.116)	45.67521 (26.384)	<b>5.36218</b> <b>(3.042)</b>	---
AR(1)	CEV	0.245 (0.091, 0.390)	39.80768 (193.225)	<b>0.00906</b> <b>(3.563)</b>	0.740
	GR	0.245 (0.092, 0.388)	40.73681 (342.447)	<b>0.73619</b> <b>(3.710)</b>	0.738
AR(2)	CEV	xxx	xxx	xxx	Xxx
	GR	xxx	xxx	xxx	Xxx
Bloomf.	CEV	0.555 (0.368, 0.852)	39.84810 (123.258)	<b>0.00984</b> <b>(1.856)</b>	0.519
	GR	0.550 (0.372, 0.859)	41.00297 (114.785)	<b>0.92400</b> <b>(2.296)</b>	0.519

In parentheses, in the 3<sup>rd</sup> column, the 95% confidence band for the non-rejection values of d; in the 4<sup>th</sup> and 5<sup>th</sup> columns, the corresponding t-values. In bold, significant coefficients for the slope term. xxx indicates that convergence is not achieved.

**Figure 4: Periodograms of NSA and SA Hours Worked series (1948q1 – 2004q4)**





On the horizontal axis the discrete Fourier frequencies  $\lambda_j = 2\pi j/T$ ,  $j = 1, \dots, T/2$ .

**Table 7: Estimates of the relationship between hours worked and productivity using a cyclical I(d) specification for the error term (Seasonally unadjusted hours worked)**

		d (95% conf. interval)	$\alpha$ (t-value)	$\beta$ (t-value)	Short run par.
White noise	CEV	0.238 (0.170, 0.330)	39.57827 (87.851)	<b>0.00900</b> <b>(1.937)</b>	xxx
	GR	0.239 (0.172, 0.332)	40.49730 (211.408)	<b>0.77002</b> <b>(1.918)</b>	xxx
AR(1)	CEV	0.188 (0.110, 0.306)	39.65396 (117.780)	<b>0.00812</b> <b>(1.966)</b>	0.179
	GR	0.187 (0.108, 0.309)	40.47605 (282.42)	<b>0.66808</b> <b>(2.021)</b>	0.180
AR(2)	CEV	0.272 (0.148, 0.460)	30.50404 (71.635)	<b>0.00989</b> <b>(1.974)</b>	0.015, -0.093
	GR	0.270 (0.146, 0.459)	40.51716 (177.390)	<b>0.85784</b> <b>(1.689)</b>	0.018, -0.091
Bloomf.	CEV	0.123 (0.001, 0.281)	39.70797 (165.988)	<b>0.00754</b> <b>(2.556)</b>	0.307
	GR	0.124 (0.002, 0.282)	40.46488 (388.985)	<b>0.59925</b> <b>(2.529)</b>	0.310

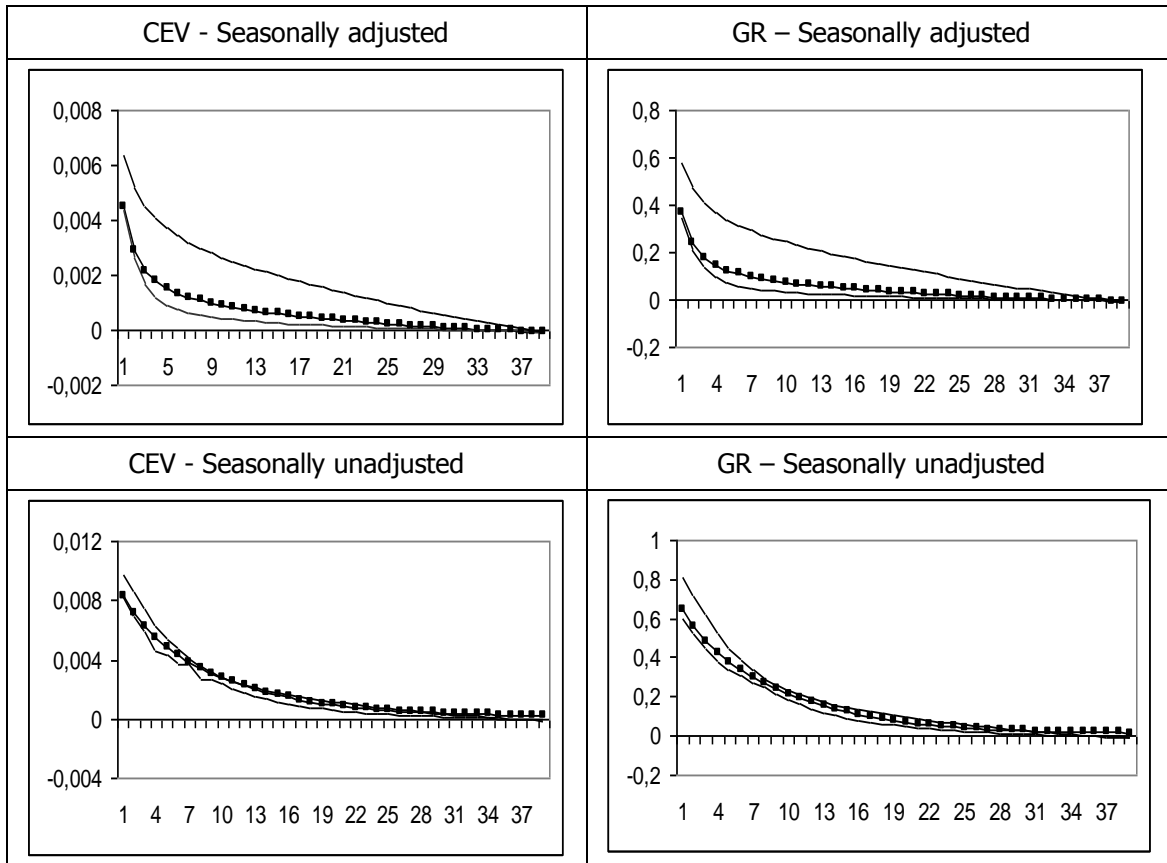
In bold, significant coefficients for the slope term.

**Table 8: Estimates of the relationship between hours worked and productivity using a cyclical I(d) specification for the error term (Seasonally adjusted hours worked)**

		d (95% conf. interval)	$\alpha$ (t-value)	$\beta$ (t-value)	Short run par.
White noise	CEV	0.489 (0.394, 0.610)	38.52378 (34.374)	<b>0.02463</b> <b>(1.875)</b>	xxx
	GR	0.486 (0.394, 0.604)	41.10432 (95.498)	<b>2.43490</b> <b>(2.284)</b>	xxx
AR(1)	CEV	0.071 (-0.054, 0.184)	39.76926 (266.937)	<b>0.00942</b> <b>(5.111)</b>	0.799
	GR	0.068 (-0.043, 0.182)	40.71395 (63.2416)	<b>0.74526</b> <b>(5.133)</b>	0.803
AR(2)	CEV	0.705 (0.513, 0.874)	36.74870 (18.240)	<b>0.05298</b> <b>(2.192)</b>	-0.202, - 0.241
	GR	0.705 (0.531, 0.854)	42.07008 (31.068)	<b>3.76092</b> <b>(2.217)</b>	-0.215, - 0.238
Bloomf.	CEV	0.204 (0.052, 0.457)	39.72078 (160.421)	<b>0.00982</b> <b>(3.237)</b>	0.659
	GR	0.215 (0.059, 0.454)	40.72335 (368.549)	<b>0.84427</b> <b>(3.288)</b>	0.629

In bold, significant coefficients for the slope term.

**Figure 5: Impulse responses of a 1-standard error technology shocks to hours worked**



The dotted line represents the response to a 1-standard error technology shock. The thin lines are the 95% confidence intervals.