ON ASSET PRICING AND THE EQUITY PREMIUM PUZZLE

A thesis submitted for the degree of Doctor of Philosophy

By

Claudius Pythias Bart-Williams

Department of Economics & Finance, Brunel University.

January 2000

CONTENTS

CHAPTER THREE (3) - CONTENTS

ACKNOWLEDGEMENTS

Doing work of this kind is never possible without the support of others; support which is acknowledged here.

Firstly, I must thank my family for all the love and support especially during the dissertation period; it has been a long 5 years. To Glenna, Nadim and Naya, thank you for putting up with me during the highs and lows, the good and the bad, the happy and the unhappy periods. I could not have done it without you and you know that. I now have no excuse for not helping around the house though I will think of something else. You will never know how much your patience and understanding helped. To Nadim and Naya; when are you ever going to sleep early and stop attempting to destroy my papers.

Mummy, before you ask about the research again, I should tell you that I have finished. You will have to find something else to ask, write or worry about; but that should not be difficult. Thanks for all you have done for me. As you know, not everything can be said but you know what I mean. I can trace the completion of this research directly to the early days. Remember this, "Claudius, have you done your homework yet". Adrian, I suppose you no longer have to listen to me waffle on about the research. If I manage to repay 1% of the help you gave me, I would have done well. Thanks brother. To my family, thank you for putting up with me over the last 5 years.

A huge thank you to Chris Martin and David Barr, my supervisors. I am well aware that I was "a pain" most of the time; all those emails, thousands I expect, to which you always responded offering invaluable advice and encouragement. I am also very grateful for the direction you gave me in terms of reading materials and computer programmes. I know that my work was made significantly easier by that help. I do however have an admission to make.

David, I tried to think of the process as being "just a Ph.d."; I fear that I failed. To David & Chris, thank you very much. My sincere thanks also to Jo Blume who never once expressed exasperation, which she must have felt at times, despite the many phone calls and requests. Jo, thank you very much. A great deal of thanks to other former and current members of staff at Brunel University, Tony Antoniou, Ian Garrett and Len Skerrat for all their helpful advice and encouragement. A big thank you also to other Ph.d students at Brunel University who provided invaluable insights into the process through the research seminars and general discussions; it was most invigorating. My sincere appreciation to Barclays De Zoete Wedd for the excellent data provided.

A great deal of thanks to colleagues at Thames Valley University, especially Chris Dawson and Pat Lord for keeping a watchful eye over me. Thanks also, to Thames Valley University for all the help provided, both financial and otherwise.

I cannot complete this acknowledgement without saying thank you to my Parents in Law, Oluniyi, Donald, Sadia, Brima, Madonna, Lena and many others, too numerous to mention, for all your support and encouragement. Just asking about progress was a spur in itself.

To everybody, thank you. It was all very much appreciated.

ABSTRACT

Brunel University, Uxbridge. Department of Economics & Finance

Claudius P. Bart-Williams

Asset Pricing & The Equity Premium Puzzle

January 2000

Doctor of Philosophy

Presented here are consumption and production related asset pricing models which seek to explain stock market behaviour through the stock premium over risk-free bonds and to do so using parameter values consistent with theory. Our results show that there are models capable of explaining stock market behaviour.

For the consumption-based model, we avoid many of the suggestions to artificially boost the predicted stock premium such as modelling consumption as leverage claims; instead we use the notion of surplus consumption. We find that with surplus consumption, there are models including the much-maligned power utility model, capable of yielding theory consistent estimates for the discount rate, risk-free rate as well as the coefficient of relative risk aversion, γ. Since real business cycle theory assumes a risk aversion coefficient of 1, we conclude that our model which gives a value close to but not equal to 1, provides an indication of the impact of market imperfections.

For production, we present many of the existing models which seek to explain stock market behaviour using production data which we find to be generally incapable of explaining stock market behaviour. We conclude by presenting a profit based formulation which uses deviations of actual from expected profits and dividends via stock price reaction parameters to successfully explain stock market behaviour. We also conclude that the use of a profit based formulation allows for a link to investment, output and pricing decisions and hence link consumption and production.

CHAPTER ONE (1)

Summary

Chapter 1 introduces the main subject area of the dissertation, which concerns the link between asset returns and the real economy. Essentially, consumption and production based asset pricing models are used to explain features of asset returns; particularly the mean stock return and implicitly the stock or equity premium.

Section 1.2 presents a review of the literature on consumption-based asset pricing models, which sought to explain the mean stock return starting with the original article by Mehra and Prescott (1985). To a large extent, these models fail to do so completely as some aspects and assumptions of the models are found to be inconsistent with theory. In other words, even models that are able to explain the mean stock premium are only able to do so with parameter estimates for risk aversion or the discount rate that are not consistent with theory. One of the more promising aspects of the literature introduced the concept of habit consumption and implicitly therefore, surplus consumption. The results are presented in chapters 3 and 4.

Section 1.3 presents production based asset pricing models outlined by Fama (1990), Cochrane (1991) and Basu and Vinod (1994) which seek to relate stock returns to production data. Cochrane (1991) and Basu and Vinod(1994) use production functions to do this. The results indicate a relationship between production and stock returns though not all the parameter values are consistent with theory. The results form the basis of our work on production based asset pricing in chapters 5 and 6.

Finally, there is an exposition of the Generalised Method of Moments (GMM) method of estimation (section 1.4) used quite extensively in our calibrations and estimations as well as of the key findings from the literature (1.5). Sections 1.6 and 1.7 detail the research questions and the dissertation map.

CHAPTER ONE

1.1. Stock Markets and the Real Economy

People, including professional economists, often wonder about the relationship between financial markets and the real economy not least because financial markets have often been seen to enjoy sustained periods of upward revisions in prices even though the general economy might be experiencing difficulties and vice versa. Indeed, financial markets have often been described as "casinos". Whilst, this dissertation cannot inform to that extent of saying everybody can become wealthy by dealing in the financial markets, there is some cause to say that there exists a link between financial markets and the real economy and that this link is an important one. One of the more meaningful ways of linking the real economy and financial markets is by linking consumer and producer behaviour and stock markets. Ultimately, all wealth exists because it has the capacity to make possible future consumption; for wealth on its own has no value. Wealth, however, is most visibly represented by stock market behaviour and which is synonymous with issues of price and value. Consumers save some of their current income to make for future consumption and the savings finds its way, directly or otherwise, into stock markets to at least maintain the value of wealth in the presence of ills such as inflation. Consequently, changes to consumer behaviour must in some way be reflected in stock market behaviour.

1.1.1 Stock Markets, Consumer and Producer Behaviour

Any model which claims to link stock market behaviour and consumption patterns should be able to use consumer information to give insights into stock market behaviour i.e. how does consumer behaviour impact on stock prices and ultimately stock returns? An answer to this question is sought in the context of a dynamic economy where innovations in economic variables are ever present such that there is a need to disentangle the effects of the many variables simultaneously at work in the economy.

If consumer behaviour can be used to explain stock market behaviour, then it follows that producer behaviour must itself be linked to stock markets. In a logical world, production choices must in some way, and to some extent, be related to consumption choices and therefore stock market behaviour. The link between production and stock markets could also be a more direct one, where the price of equity and its returns are based on the profit expectations from the producers who in turn seek financing from the stock market.

Any consumption or production model which seeks to explain stock market behaviour should not only be able to do so but should do so whilst also being able to yield other parameter values consistent with a priori expectations as well as known parameter values. Recent efforts to explain stock market behaviour have tended to concentrate on explaining the observed stock (risky) and bill (risk-less) returns and implicitly, the stock or equity premium. Herein follows a similar approach.

So far, we have assumed a direct link between stock markets and real economy via consumption and production. However, most households do not own stocks and even if they do, they do not do so directly. They tend to own stocks via financial intermediaries or agents whose particular functions are still the subject of much debate. Before looking at the consumption and production-based asset pricing models, attention turns to the role of such intermediaries.

1.1.2. The Role of Financial Intermediation

Santomero (1984) outlined 3 broad explanations for the existence of the firm to include asset transformation, bank liabilities and the two-sided nature of the financial firm.

Within the asset transformation explanation, two distinct views emerge i.e. asset diversification and asset evaluation. In asset diversification, intermediaries come into their own because individual investors are faced with a constrained investment opportunity set which implies sub-optimal portfolio choices. The intermediary is able to pool the resources of such individual investors and earn an economic profit for itself given that it is the intermediary who ensures optimal portfolio choices. Another aspect of this is transaction cost minimisation which is only so when resources are pooled.

The asset evaluation argument is based on the notion that the intermediary exists to assess the signals emanating from firms and decide on financial asset transactions because the individual participant is either unable or unwilling to assess the signals largely due to imperfect information about the value of a firm's underlying activities. In this context, it is perhaps not unreasonable to argue that the asset diversification and evaluation motives can be complementary. Assessing the signals is costly but these costs can be minimised by an intermediary acting on behalf of many individual investors. Also, asset transformation is only possible after asset evaluation and so perhaps the separation between asset transformation and asset evaluation is an unnecessary one.

The second reason for the existence of financial intermediaries focuses on the role of demand deposit liability, made possible by financial intermediaries, as a medium of exchange. This notion rests on the assumption that the deposit, as a monetary unit, has the ability to minimise transactions costs that arise as income is converted into an optimal consumption bundle. Another argument goes further to view money holdings as "part of the household's attempt to maximise a utility function in the first and second moments of consumption". For the intermediary, profit potential exists because of the supposed ability to attract deposits which are then invested at a positive spread but whose size is dependent upon the nature of competition.

The "two-sided nature of the financial firm" argument rests on the idea that there are levels of uncertainty relating to deposits and loans and therefore culminating in different returns. The financial intermediary is able to transform deposits into loans. The framework assumes a maximising firm in a financial market with uncertain rates of return but which can be extended to account for a positive spread across markets.

The existence of a positive spread for financial intermediaries raises questions about its source. Santomero (1984) offered explanations based around a "deviation from perfect market assumptions" such as non trivial transactions costs, information asymmetry or the existence of monopoly tendencies. Santomero (1984) presented an intermediary model of the general form,

$$\max E\left[V\left(\tilde{W}_{t+\tau}\right)\right] \tag{1.1.a}$$

subject to

$$W_{t+\tau} = W_t\left(1+\tilde{\Pi}_{t+1}\right)\left(1+\tilde{\Pi}_{t+2}\right)\cdot\cdot\cdot\left(1+\tilde{\Pi}_{t+\tau}\right) \tag{1.1.b}$$

$$\tilde{\Pi}_{t+k} = \frac{\Sigma_i \tilde{r}_{A_i} A_i - \Sigma_j \tilde{r}_{D_j} D_j - C\left(A_i, D_j\right)}{W_{t+k-1}} = \frac{\tilde{\pi}_{t+k}}{W_{t+k-1}} \tag{1.1.c}$$

where

$V(.) \equiv$ the objective function, where $\partial V/\partial W_{t+\tau} > 0$ and $\partial^2 V/\partial W_{t+\tau}^2 \leq 0$
$\left(\tilde{W}_{t+\tau}\right) \equiv$ the value of terminal wealth at the horizon time τ
$\tilde{\Pi}_{t+k} \equiv$ the stochastic profit per unit of capital during period $t+k$, where $0 \leq k \leq \tau$
$\tilde{r}_{A_i} \equiv$ the stochastic return from asset i
$A_i \equiv$ the asset category i, where $1 \leq i \leq n$
$\tilde{r}_{D_j} \equiv$ the stochastic cost for deposit j
$D_j \equiv$ the deposit category j, where $1 \leq j \leq m$
$C() \equiv$ the operations cost function, where $\partial C/\partial A_i \geq 0\ \forall i$ and $\partial C/\partial D_j \geq 0\ \forall j$.

Equation (1.1.*a*) allows for 2 types of behaviour i.e. maximisation of terminal wealth and also risk-aversion. The clear implication of the first derivative is that more wealth is preferred to less but the degree of marginal utility depends on the second derivative. The form of the second derivative either implies that the firm is a value maximising or a risk averse investor. A mean-variance efficient portfolio i.e. risk aversion, implies wealth concavity whilst if a mean-variance efficient portfolio is not a criterion, then profit maximisation is assumed. The case for the profit maximising intermediary rests on the assumption that individual investors have available to them, the same investment opportunity set as is available to the intermediary such that individual investors can duplicate a perfectly hedged portfolio. In these circumstances, the intermediary can only exist to maximise profit. However, arguments have also been advanced for the utility concavity associated with the risk averse intermediary such as insufficient owner diversification. Another argument is that investors have a linear utility function such that, according to Santomero (1984) they seek to "establish reward schedules for management that lead to risk-averse behaviour". Stiglitz (1972) introduced the concept of bankruptcy costs where the expected value maximiser behaves as if variance is negative due to the probability of default with the associated bankruptcy costs.

Equation (1.1.*b*) assumes interdependence between periods which makes the maximisation into a single period analysis whilst (1.1.*c*) defines profit per unit of capital invested by the owners or their agents.

The notion that intermediaries are profit maximisers implies that investors can perfectly duplicate any efficient intermediary portfolio. This would be particularly true in a perfect capital market. However it is conceivable that capital markets are not perfect. One such imperfection concerns the existence of credit rationing. Capital rationing is said to exist when there is excess demand for credit at the "going interest rate". Credit rationing is an attempt to explain the presence of financial market behaviour which departs from the standard neo-classical theory that lack of credit is not a problem because demand will always equal supply at the equilibrium price. Credit rationing models fall into 3 broad categories.

Firstly, credit rationing is said to exist owing to the relationship between the intermediary and the customer. Essentially, banks, as financial intermediaries between the providers and users of finance, have preferential customers with whom it prefers to trade during periods of credit tightening. Secondly, credit rationing prevails because the cost structures of banks make them refuse to grant credit at the going rate. Such costs may relate to monitoring costs which tend to be higher for smaller customers. Finally, credit rationing exists because of information asymmetry or adverse selection where the bank fails to categorise its potential customers as carrying varying degrees of risk.

It is in this context that an attempt is made to look at consumption and production asset pricing models.

1.2 Consumption Capital Asset Pricing

In 1985, using a simple power utility model, Mehra and Prescott outlined what they perceived to be a discrepancy between the observed equity returns and that predicted by the Consumption Capital Asset Pricing Model (CCAPM). Using USA annual data, 1889 to 1978, they concluded that the equity (stock) premium of equities over bonds as observed from the data is much higher than predicted by the CCAPM. In their mind, this disparity raised fundamental questions about the efficiency and relevance of the model. This inability of the CCAPM to explain the observed equity premium, even after the event, is the basis of their much discussed "equity premium puzzle".

Implicit in the Mehra and Prescott model is the assumption of a pure exchange economy with an elasticity of substitution between period t and $t+1$ with a discount rate between 0 and 1 and a concave utility function. They also assume that the equilibrium growth rate on consumption and asset returns is stationary. They further argue that the underlying principles of the CCAPM are present in many economic theories to do with long run economic growth and real business cycle in the rational expectations framework. Such a framework includes the notion that the parameter for risk should be close to 1, if it is indeed the case that individuals have time separable utility between consumption and leisure in time periods, t and $t+1$.

The formulation assumes a single representative household whose preferences over random consumption paths are represented by

$$\mathrm{E}_0\left\{\sum_{t=0}^{\infty} \delta^t U(C_t)\right\}, \quad 0 < \delta < 1, \tag{1.2.a}$$

where C_t is per capita consumption, δ is the subjective discount rate which would be between 0 and 1 owing to the assumption of a positive rate of time preference, $\mathrm{E}_0\{\ .\}$ is the expectation operator conditional on information available at time zero, which denotes the present time and $U: R_{+} \rightarrow R$ denotes the increasing concave utility function. The choice problem can be represented by

$$\frac{\delta\, U'(C_{j,t+1})}{U'(C_{jt})} > 0, \text{ and where } j \text{ is the representative investor.}$$

The utility function is further defined with a constant coefficient of relative risk aversion such that

$$U(C,\gamma) = \frac{C^{1-\gamma} - 1}{1-\gamma}, \quad 0 < \gamma < \infty, \tag{1.2.b}$$

where γ measures the curvature of the utility function. When γ equals 1, the utility function is logarithmic. This is the limit of the utility function in (1.2.*b*) as γ approaches 1.

The formulation further assumes that there is one productive firm producing the perishable consumption good and also that there is a competitively traded equity share in that producer. Since there is only one productive good, then the return on the market will be the same as the return on the equity share.

The output of the firm is constrained to be less than or equal to y_t, which is also the firm's dividend payment in period *t*. The growth rate of y_t is subject to the Markov process,

$$y_{t+1} = x_{t+1} y_t \tag{1.2.c}$$

where $x_{t+1} \in \{g_1, \ldots\ldots\ldots g_n\}$ is the growth rate, and

$$\Pr\{x_{t+1} = g_j; x_t = g_i\} = \phi_{ij}. \tag{1.2.d}$$

To price the security, they related ex-dividend and ex-interest payments at time t to time t consumption good such that[1]

$$P_t = \mathrm{E}_t\left\{\sum_{s=t+1}^{\infty} \delta^{s-t} \frac{U'(y_s)d_s}{U'(y_t)}\right\} \qquad (1.2.e)$$

where P_t is the price of any security with process $\{d_s\}$ on payments and y_s is the equilibrium consumption process. The price of the security is essentially a discounted utility (from consumption) in time period $t+1$ relative to time t.

Since $U'(C)=C^{-\gamma}$,

$$P_t^e = P^e(x_t, y_t) = \mathrm{E}\left\{\sum_{s=t+1}^{\infty} \delta^{s-t} \frac{y_t^\gamma}{y_s^\gamma} y_s \middle| x_t, y_t\right\} \qquad (1.2.f)$$

where P_t^e is the price of equity share and x_t and y_t are thought to capture the historical consumption process including consumption shocks. For the equity return, the capital letter denotes the expected return. Since $y_s = y_t \,.\, x_{t+1} \ldots x_s$, the price of the equity share is homogeneous of degree one in y_t. Furthermore, owing to the fact that the equilibrium values of the economies under consideration are time invariant functions of the state (x_t, y_t), then the subscript t can be ignored by redefining the state to be the pair (C, i), if $y_t = C$ and $x_t = g_i$. Hence P_t^e can be rewritten in terms of p^e where

$$p^e(C,i) = \delta \sum_{j=1}^{n} \phi_{ij}(\lambda_j C)^{-\gamma}\left[p^e(g_j C, j) + C g\right] C^\gamma, \qquad (1.2.g)$$

The period return is given as

$$r_{ij}^e = \frac{p^e(g_j C, j) + g_j C - p^e(C,i)}{p^e(C,i)}. \qquad (1.2.h)$$

[1] They assumed the existence of a Debreu (1954) competitive equilibrium.

Mehra and Prescott presented formulations for the expected equity return if the current state is *i*, such that

$$R_i^e = \sum_{j=1}^{n} \phi_{ij} r_{ij}^e, \qquad (1.2.i)$$

where capital letters denote expected return and ϕ is the first-order serial correlation of per capita real consumption growth. The risk-less return is represented by

$$R_i^f = 1/p_i^f - 1. \qquad (1.2.j)$$

For parameter estimates, Mehra and Prescott sought to match the observed values for the average growth rate of per capita consumption, μ, the standard deviation of the growth of per capita consumption, σ, and the first order serial correlation of the per capita consumption growth rate, ϕ. The observed values for the US economy were 0.018, 0.036 and -0.14 respectively, whereas that found by Mehra and Prescott were 0.018, 0.036 and 0.43. Given these parameters, they then search for values for γ, the curvature of the utility function and δ, the discount factor. The parameter γ measures the willingness of consumers to substitute intertemporally between successive time periods, a concept to be found in many areas of economics. Implicitly in this formulation, this parameter is assumed to be constant, justification for which is based on the work of Arrow (1971) who had concluded that only a curvature function value of 1 is consistent with theory. Mehra and Prescott also pointed to many of the earlier studies which indicated that curvature function values of around 2 and to their own formulation which restricts the curvature value to between 1 and 10. For the discount rate, they assumed a value between 0 and 1, which is consistent with a positive rate of time preference.[2]

---

[2] Consider the case of an investor who might wish to forgo $10 consumption now. With a negative rate of time preference (greater than one), such an investor is only going to be able to enjoy future consumption of $9.61 i.e. 10 divided by 1.04. With a positive rate of time preference, say 0.95, such an investor would enjoy future consumption level of $10.53 which is clearly preferable.

Table 1.1

Panel A

The Simple Return (%) on Equity and Bonds 1889-1978

	Real Equity Return	Real Risk-less Return	Real Equity Premium
Observed	6.98	0.80	6.18
As per the CCAPM	4.05	3.70	0.35

Source: Mehra and Prescott (1985)

As already mentioned, the results of the Mehra and Prescott model are not at all encouraging for the consumption-based asset pricing model as the model predicted an equity premium of under 0.50% compared to the observed value of over 6%. This equity premium is that predicted with the maximum plausible value for the risk aversion coefficient of 10. Furthermore, the CCAPM predictions for the risk-less rate is much higher than indicated by the historical data by a factor of about 5. The failure of the Mehra and Prescott model to explain the observed equity premium raises quite substantive questions not least because the principles underlying the CCAPM are part of economic theory i.e. long run economic growth, such that the failure of the CCAPM to explain the observed equity premium calls into question substantial areas of economic theory.

Abel (1990, 1991) proposed an adjustment to the Mehra and Prescott model which recognised that the ultimate reason for holding wealth was future consumption and as a result, the equity premium of equities over bonds should depend on the variability of consumption and its relationship with equity returns. Essentially, the idea is that since investors tend to be risk-averse, i.e. requiring a higher rate of return for holding riskier assets, then riskiness could be measured by the CCAPM. The CCAPM uses the relationship between the asset's returns and the marginal rate of substitution of the investor i.e. the value placed on additional funds by the investor. When the investor's overall level of wealth is low, then the investor lowers consumption and places a high value on having additional funds. The opposite is also true for a wealthy investor who is likely to care much less about additional funds for the future given that his consumption is already high. The risk element appears when an investor who places a high value on additional funds, and therefore has a low consumption level, holds an asset with low returns. For the high wealth individual who places a low value on additional funds, high asset returns in such circumstances can be considered risky.

Abel (1991) concluded that stock returns will on average be higher than bill returns though the extent of this will depend on 2 factors:

- the coefficient of relative risk aversion, γ, which gives an indication of the change in expected stock (equity) returns, rise or fall, of a change, fall or rise, in consumption and

- the covariance of consumption growth with stock (equity) and bond returns. This measures fluctuations in stock (equity) and bond returns and the relationship between the fluctuations in stock (equity) returns and consumption growth.

The data used by Mehra & Prescott provided much of the early attention in efforts to explain the failure of the CCAPM. Given that the principles underlying the CCAPM are part of economic theory, the failure of the CCAPM to explain the observed equity premium called into question substantial areas of economic theory. One argument used to explain the failure of the CCAPM was that the data for the period in question was incomplete in part for the sample period such that estimates and proxy data tended to be used. Siegel (1991) re-examined the data used by Mehra and Prescott and concluded that a different method of calculating the average rates of return was called for and indeed the values reported by him are somewhat different to that reported by Mehra and Prescott. Siegel (1991) used different measures for the stock price index, inflation and for part of the series, a different short-term interest rate. This data showed much more variability than that of Mehra and Prescott but despite this, Siegel found an equity premium for the period covered by Mehra and Prescott (1889-1978) similar to that identified by Mehra and Prescott although this premium was reduced from just under 7% to 4.6% when the sample period was 1802-1990 as can be seen from table 1.1 (Panel B).

Table 1.1

Panel B

The Simple Return (%) on Equity and Bonds 1802-1990

Period	Real Equity Return	Real Risk-less Rate of Return	Real Equity Premium
1802 - 1888	7.52	5.62	1.90
1889 - 1978	7.87	0.91	6.96
1979 - 1990	9.44	2.73	6.71
1802 - 1990	7.81	3.19	4.62

Source: Siegel (1991)

Even at 4.6%, this smaller equity premium was still greater than that predicted by the CCAPM of Mehra and Prescott. Furthermore, this reduction in the equity premium was largely due to an increase in the average real return on bills to above 3% from under 1%. Though equity returns showed greater variability than bill returns, it does not follow from this that average equity returns were significantly different from average bill returns. One way of investigating average returns is to look at moving average returns over time as done by Abel (1991) who calculated 30-year moving average rates of return and found that calculating a 30-year moving average yielded similar average rates of return.

Another method employed by Cecchetti, Lam & Mark (1991) involved testing the reliability of the historical average rates of return by attempting to estimate the closeness of the historical average rates of return to the underlying rates of return expected by investors making portfolio decisions. They find an equity premium of 6.03%, which is much in line with other studies.

Breeden, Gibbons & Litzenberger (1989) also investigated the possible consumption data deficiencies which included 1) the reporting of expenditures rather than consumption, 2) the reporting of an integral of consumption rates rather than consumption at a point in time, 3) infrequent reporting of consumption data relative to stock returns and 4) reporting aggregate consumption with sampling error since only a subset of the total population of consumption is measured. They concluded that the reporting of an integral of consumption rather than at a point in time amounted to a "summation bias". Breeden, Gibbons & Litzenberger (1989) viewed the "summation bias" as part of the reason for the failure of the CCAPM. They focussed on a version of the CCAPM whose performance is compared to one based on a market portfolio and even though the performance of the model based on the market portfolio is not too dissimilar to the CCAPM, argue that the CCAPM should be amended to take account of the fact that consumption tends to be measured discretely where consumption data is an integral of spot consumption rather than as a measure at a point in time. According to Breeden et al, this "summation bias" tended to underestimate the covariance of consumption and asset returns such that the reported coefficient of relative risk aversion tended to be only about ¾ of their true values.

Overall though, the data on consumption shows very little evidence of being responsible for the failure of the CCAPM to explain the observed equity premium. The equity premium puzzle therefore remains.

Other attempts to re-specify the model have included allowing for large and sudden fluctuations in consumption, relaxing the assumption of a single representative investor given that only a minority of households own stocks, relaxing the assumption that the elasticity of intertemporal substitution is the reciprocal of the coefficient of relative risk aversion. These efforts have all met with limited amounts of success in that they have only been able to reduce the unexplained equity premium and not to fully explain the observed equity premium.

An assumption fundamental to much of the early work in this area is the notion that there exists a relationship between the coefficient of relative risk aversion, γ, and the elasticity of intertemporal substitution, Ω, in that Ω is the reciprocal of γ. The basic idea is that the higher the coefficient of relative risk aversion, the lower the elasticity of intertemporal substitution because with higher risk, investors are less willing to substitute intertemporally. Consequently, much discussion revolved around the prospective values for the coefficient of relative risk aversion and implicitly therefore, the elasticity of intertemporal substitution.

As already mentioned, Mehra and Prescott considered that values for the coefficient of relative risk aversion, γ, must be between 0 and 10.[3] They further considered that larger values for γ were not justifiable not least because it called into question aspects of rational expectations theory. In their analysis, they pointed out that for the CCAPM to explain the observed equity premium of 6.18% required a risk coefficient of around 30, which they thought was implausible. In such circumstances, the determination of the coefficient of relative risk aversion becomes very important; especially since the coefficient of relative risk aversion is determined by consumption.

[3] Values for the coefficient of relative risk aversion, γ, of greater than 10 are thought to be implausible for should a risk averse investor have a 50-50 chance of either increasing wealth or reducing wealth, then there must be a limit to what that investor will pay to reduce risk. A calibration can be done using the formula $y = 1 - \left[(½)(1-x)^{1-\gamma} + (½)(1+x)^{1-\gamma}\right]^{1/(1-\gamma)}$, where x is the fraction of wealth that could be gained or lost with a 50-50 chance, γ is the coefficient of relative risk aversion, y is the fraction of your wealth that the investor could be willing to pay to avoid risk.

Efforts to explain the observed equity premium have also involved allowing for large and sudden changes in consumption which Mehra and Prescott did not. Allowing for large and sudden changes to consumption did explain the observed equity premium but the magnitudes of change necessary have never been reflected in the historical data according to Reitz (1988). Kandel and Stambaugh (1991) undertook a similar exercise and concluded that allowing for large and sudden swings in consumption resulted in an improvement in the predictability performance of the model but did not in itself solve the equity premium.

Mehra and Prescott, in their model, assumed that fluctuations in dividends were matched by fluctuations in consumption. Relaxing this assumption by using historical data on dividends to measure dividend variability and consumption data to measure consumption, has tended to improve the performance of the CCAPM.

Cecchetti, Lam and Mark (1991) presented a model which differed from earlier models in two ways. The authors separated consumption from dividends as well as developed a framework for measuring the ability of the model to match the data. They found that the CCAPM predicted equity premium is increased by almost 50% thus narrowing the gap between the predicted and the historically observed equity premium. This development though is not so significant so as to fully explain the historically observed rate in the context of a coefficient of relative risk aversion of less than or equal to 10. Cecchetti, Lam & Mark (1993) presented a revision of the earlier paper in which they attempted to match the moments of the data as well as present a re-calibration of a consumption-based model to generate both first and second moments of returns that match the observed data.

There follows a brief summary of that calibration. The utility function is defined as

$$U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}, \tag{1.2.k}$$

where $0 < \gamma < \infty$ is the coefficient of relative risk aversion. Based on the generalised Lucas economy in which consumption and dividends are not equal, the first order conditions satisfies

$$P_t^e = \delta \, \mathrm{E}_t \left\{ \frac{U'(C_{t+1})}{U'(C_t)} \left[P_{t+1}^e + D_{t+1} \right] \right\} \tag{1.2.l}$$

and

$$P_t^f = \delta \, \mathrm{E}_t \frac{U'(C_{t+1})}{U'(C_t)}, \tag{1.2.m}$$

where

P_t^e = real price of the traded asset, or equity
P_t^f = real price of the risk - free asset
C_t = per capita real consumption
D_t = dividend from owning one unit of equity
U' = marginal utility of the representative agent
δ = subjective discount rate, $0 < \delta < 1$ and
E_t = mathematical expectation conditional on information at time t.

Substituting (1.2.*k*) into (1.2.*l*) and (1.2.*m*),

$$P_t^e C_t^{-\gamma} = \delta \, \mathrm{E}_t C_{t+1}^{-\gamma} \left(P_{t+1}^e + D_{t+1} \right), \tag{1.2.n}$$

and

$$P_t^f = \mathrm{E}_t \, \delta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma}. \tag{1.2.o}$$

Importantly, the subjective discount rate is allowed to exceed 1, a feature justified by Kocherlatoka (1990). As indicated earlier, a subjective discount rate exceeding 1 implies that investors are not willing to substitute intertemporally. Intertemporal substitution however, is at the heart of many of the models in this area. Cecchetti et al assumed that consumption and dividends are governed by a bivariate version of Hamilton's (1989) Markov-switching model where the bivariate random walk with two-state Markov drift is

$$\begin{pmatrix} c_t \\ d_t \end{pmatrix} = \begin{pmatrix} c_{t-1} \\ d_{t-1} \end{pmatrix} + \begin{pmatrix} \alpha_0^c \\ \alpha_0^d \end{pmatrix} + \begin{pmatrix} \alpha_1^c \\ \alpha_1^d \end{pmatrix} M_t + \begin{pmatrix} \varepsilon_t^c \\ \varepsilon_t^d \end{pmatrix} \qquad (1.2.p)$$

where $c_t \equiv \ln C_t$, $d_t \equiv \ln D_t$ and M_t is a Markov random variable that takes on values of 0 and 1 with transition probabilities.

Furthermore, $\begin{pmatrix} \varepsilon_t^c \\ \varepsilon_t^d \end{pmatrix}$ is assumed to be an i. i. d normal with mean zero and co - variance matrix $\Sigma = \begin{pmatrix} \sigma_c^2 & \sigma_{cd} \\ \sigma_{cd} & \sigma_d^2 \end{pmatrix}$.

The formulations for the implied rates of return for equity and risk-free assets from time *t* to *t+1* are

$$R_t^e = R^e\left(M_{t+1}, M_t, \varepsilon_{t+1}^d\right) = \frac{P_{t+1}^e + D_{t+1}}{P_t^e} - 1 \qquad (1.2.q)$$

and

$$R_t^f = R^f\left(M_t\right) = \frac{1}{P_t^f} - 1. \qquad (1.2.r)$$

From this point, an equation for the equity premium is computed so that the covariance matrix of equity premium and the risk-free rate can be computed. Cecchetti et al test three models, Markov-Switching, Random Walk and Mehra and Prescott and concluded that the results of the Markov-Switching model dominate those of the other two models.

Table 1.1 (Panel C) reports results for the Markov-Switching model as well as the Random Walk and Mehra-Prescott models. The equity premium implied by these models is only able to match the historically observed value when the discount factor exceeds 1. A discount factor greater than 1, however, implies a negative rate of time preference. The estimates for the coefficient of relative risk aversion remain well in excess of the value of 10, the maximum value thought plausible by Mehra and Prescott.

Table 1.1

Panel C

First and second moments of asset returns : model values using Bivariate consumption-dividends processes (standard errors in parentheses)

β	γ	Risk-free rate	Equity premium	covariance	#2	#5[4]
			Markov-switching model			
0.999	9.9	11.65	2.42	-0.02	5.02	42.71
		(1.53)	(13.11)			
1.098	9.9	1.58	2.19	-0.02	6.33	32.32
		(1.39)	(11.90)			
1.056	15.0	5.02	3.52	-0.03	1.17	29.68
		(1.39)	(11.90)			
1.039	20.0	1.21	4.12	-0.02	0.11	26.72
		(3.68)	(11.31)			
0.825	29.0	0.80	1.28	0.02	0.02	24.96
		(6.20)	(10.62)			
			Random walk model			
0.999	9.9	12.11	2.25	0.00	5.65	-
		(0.00)	(13.95)			
1.098	9.9	2.00	2.05	0.00	6.56	-
		(0.00)	(12.69)			
1.056	15.0	6.64	3.26	0.00	2.46	-
		(0.00)	(13.41)			
1.039	20.0	5.23	4.31	0.00	1.07	-
		(0.00)	(13.36)			
0.825	29.0	15.16	6.89	0.00	0.43	-
		(0.00)	(14.89)			
			Mehra - Prescott model			
0.999	9.9	12.32	5.46	0.01	4.06	21.39
		(2.20)	(15.51)			
1.098	9.9	2.20	5.01	0.01	0.61	3.78
		(2.00)	(14.23)			
1.056	15.0	7.53	8.19	0.01	0.60	11.50
		(3.02)	(16.18)			
1.039	20.0	7.74	10.94	0.04	0.59	18.17
		(3.77)	(17.33)			
0.825	29.0	25.86	16.93	0.06	1.30	46.79
		(5.53)	(21.37)			

Source: Cecchetti, Lam and Mark (1993)

[4] #(2) tests for two means and #(5) tests for all five moments. The 5 percent critical value for $\chi^2_{(2)}$ is 5.99, for the $\chi^2_{(5)}$, it is 11.07. For the Random Walk Model, both the correlation and #(5) cannot be computed since R_t^f is nonstochastic.

So far, the discussions implicitly assume that investors have the same characteristics. This assumption makes for a simpler model but a simpler model is perhaps not quite capable of explaining the equity premium. Not least of these assumptions is that which implies that all households own stock when in fact that is not the case. A number of studies have shown that only a minority of households own stocks, a fact which calls into question the use of aggregate consumption data. One way around this is to take account also of stocks owned, but not directly held, by households such that stocks held by institutional investors would be considered. The rationale for this would be that institutional investors act only in the interests of their investors and are therefore unlikely to take actions, which run counter to the interests of investors. Since a large number of households hold pensions and other investment funds, we might then be able to use aggregate consumption data. Mankiw & Zeldes (1991), however, attempted a disaggregation of the consumption data to focus on the consumption of stockholding households. They find that the covariance of stock returns and consumption triples reflecting the fact that when compared to non-stockholders, the consumption of stockholders tend to be much more volatile such that the coefficient of relative risk aversion required to match the observed equity premium is significantly reduced. In the context of households that have significant liquid wealth, it might well be the case that they prefer to hold assets which provide insurance. Furthermore, an intriguing question on the nature of consumption by these stockholding households remains. It may well be the case that stockholding households use their stock returns for discretionary consumption rather than subsistence consumption.

The power utility model of the CCAPM implicitly makes two assumptions. Firstly, that consumption in any year affects only that year and secondly, that the utility function has a constant coefficient of relative risk aversion. The first assumption is feasible to the extent that the relevant consumption data used in these analyses is based on non-durable goods and services. The second assumption means that the share of wealth held in a portfolio is not dependent on the level of wealth. Clearly, an investor with little or no wealth is less likely to put a large share of wealth into risky assets as compared to an investor with substantial wealth bearing in mind that beyond a certain level of wealth, such a wealthy investor places less value on additional funds. All efforts discussed so far has focussed on trying to enrich the CCAPM to yield an equity premium that is much closer to the historically observed rate in the context of a coefficient of relative risk aversion equal to or less than 10. To a large extent, these have failed. In chapter 3, we return to the power utility model.

1.2.1 Volatility Bounds

Given that the stochastic discount factor features in asset pricing models, Hansen and Jagannathan (1992) have developed a lower bound for the volatility of the stochastic discount factors that, according to Campbell, Lo and Mackinlay (1997), "could be consistent with a given set of asset return data". This should be able to provide insights into the behaviour of the stochastic discount factor.

They re-write the familiar unconditional equation in a vector form as

$$E[Z_t M_t] = \iota \qquad (1.2.1a)$$

where ι is a vector of ones, Z_t is the vector of gross asset returns at time t and M_t is the stochastic discount factor. Hansen and Jagannathan assumed that the vector Z_t has a non-singular variance-covariance matrix Σ. This implies that no asset is unconditionally risk-less even though there could still exist an unconditional zero-beta (risk-less) asset with mean return equal to the unconditional mean of the stochastic discount factor. Implicitly, they assumed that a zero-beta asset does not exist such that the unconditional mean of the unconditional stochastic discount factor m, remains unknown. For the possible vector of values for m, Hansen and Jagannathan form a proxy discount factor $M_t^*(m)$ as a linear combination of asset returns where

$$M_t^*(m) = m + (Z_t - E[Z_t])' \beta_m \,. \qquad (1.2.1b)$$

Equation (1.2.1*b*) says that the proxy stochastic discount factor is determined by the mean value and the product of the derivative of the beta and the difference between actual and expected gross returns in time *t*.

Equation (1.2.1*b*) can now be written as E[Z_t, $M_t^*(m)$] which can in itself be expanded to

$$\begin{aligned}
& m\mathrm{E}[Z_t] + Cov(Z_t, M_t^*(m)) \\
= \quad & m\mathrm{E}[Z_t] + \mathrm{E}\left[(Z_t - \mathrm{E}[Z_t])(M_t^*(m) - m)\right] \\
= \quad & m\mathrm{E}[Z_t] + \mathrm{E}\left[(Z_t - \mathrm{E}[Z_t])(Z_t - \mathrm{E}[Z_t])' \beta_m\right] \\
= \quad & m\,\mathrm{E}[Z_t] + \mathrm{E}[\Sigma \beta_m] = \iota \qquad (1.2.1c)
\end{aligned}$$

where Σ is the previously mentioned unconditional variance-covariance matrix of asset returns. Consequently,

$$\beta_m = \Sigma^{-1}(\iota - m\mathrm{E}[Z_t]) \qquad (1.2.1d)$$

and the variance of the implied stochastic discount factor is

$$\mathrm{Var}(M_t^*(m)) = (\iota - m\mathrm{E}[Z_t])' \Sigma^{-1} (\iota - m\mathrm{E}[Z_t]). \qquad (1.2.1e)$$

The right hand side of (1.2.1*e*) is interpreted as the lower bound on the volatility of any stochastic discount factor with mean *m*.

These results can be placed into the benchmark portfolio framework. Despite the previously outlined assumption that a risk-less asset does not exist, if one were to exist, it would take the form $1/m$. So in any assessment of the risky return, the term $1/m$ has to be included. The benchmark portfolio return would then be

$$Z_{bt}(m) \equiv \frac{M_t^*(m)}{\mathrm{E}\left[M_t^*(m)^2\right]}. \qquad (1.2.1f)$$

Campbell, Lo and Mackinlay (1997) explicitly set out the characteristics of the term, $Z_{bt}(m)$:

a) That it is mean-variance efficient i.e. no other portfolio has a smaller variance and the same mean.
b) Any stochastic discount factor $M_t(m)$ has a greater correlation with Z_{bt} than with any other portfolio such that Z_{bt} is the "maximum correlation portfolio".
c) All asset returns conform to a beta pricing relation with the benchmark portfolio reflected in the form

$$\mathrm{E}\left[Z_{it} - (1/m)\right] = \beta_{ib}\left(\mathrm{E}\left[Z_{bt}\right] - (1/m)\right), \qquad (1.2.1g)$$

where $\beta_{ib} \equiv \mathrm{Cov}(R_{it}, R_{bt})/\mathrm{Var}(R_{bt})$ and which when substituted into (1.2.1g), yields a conventional asset pricing model.

d) The ratio of standard deviation to mean for the benchmark portfolio, $\sigma(Z_{bt})/\mathrm{E}[Z_{bt}]$ equals $(1/m - \mathrm{E}[Z_{bt}]/\sigma(Z_{bt})$. This is the Sharpe Ratio.

e) The ratio of standard deviation to mean for the benchmark portfolio is a lower bound on the same ratio for the stochastic discount factor i.e. $\sigma(Z_{bt})/\mathrm{E}[Z_{bt}] \leq \sigma(M_t(m))/\mathrm{E}[M_t(m)]$.

Even though an unconditional risk-less rate is assumed not to exist so far, Hansen and Jagannathan were able to re-specify the problem to account for the existence of an unconditional risk-less asset in the presence of a non-negative value for M_t. This formulation also presents us with an opportunity to look at the equity premium as outlined above. Mehra and Prescott indicated a mean excess stock return of 6.18% and a standard deviation of about 18%. The slope is therefore 0.0618/0.18 = 0.34, the Sharpe Ratio, indicating that the standard deviation of the stochastic discount factor must be at least 34% if it has a mean of 1. In results to be discussed, the stochastic discount factor approaches 1 but the standard deviation is significantly less than 34%. The Sharpe ratio will be further discussed later.

1.2.2 Separating Risk Aversion and Intertemporal Substitution

Mehra and Prescott (1985) used a representative agent economy with a constant elasticity of substitution and time additive expected utility preferences. This is quite a restrictive model not least in that the time additive aspect of it demands that the elasticity of intertemporal substitution is the inverse of the constant coefficient of relative risk aversion. This restriction necessarily involves imposing behavioural restrictions on preferences which have no justification in the theory. Kandel and Stambaugh (1991) have suggested that it was unnecessary to work within such a restrictive framework and go on to show that values for the coefficient of risk aversion greater than 10 are possible. This analysis together with that of Weil (1989) and Kocherlatoka (1990) have also attempted to explain the observed equity premium by using less restrictive models.

Weil (1989) studied a parametric class of Kreps-Porteus non-expected utility preferences in the Epstein and Zin (1987a,b) mode. The formulation, which used notation similar to that used by Mehra and Prescott starts with the technology. Weil assumed one perishable good (a fruit) which is "produced by nonreproducible identical trees whose number is normalised, without loss of generality, to be equal to the size of the constant population".

The number of fruits falling from the tree in time period t is denoted by y_t i.e. the dividend for holding a tree, whose growth rate is assumed to be normally distributed such that $g_{t+1} \equiv y_{t+1} / y_t$ and with change probabilities given by

$$\alpha_{ij} = \text{Prob}\{g_{t+1} = g_j | g_t = g_i\}, \qquad (1.2.2.1a)$$

where $i, j = 1,2 \ldots, I < \infty$, $g_j > 0$, and $\Sigma_{j=1}^{I} = 1, \forall i$.

For consumers, the one-period budget constraint facing a representative household is given as

$$c_t + p_t n_{t+1} = (p_t + y_t) n_t, \quad t \geq 0 \qquad (1.2.2.1b)$$

where p_t, n_t, and c_t are respectively, the fruit price of a tree at time period t, the number of (shares of) trees held at the beginning of period t and consumption at time period t of the representative household. Also, n_0 is assumed to be ≥ 0. If the rate of return on a tree, $R_{t+1} \equiv [p_{t+1} + y_{t+1}]/p_t$ and the beginning-of-period wealth is denoted by $w_t \equiv (p_t + y_t) n_t$, then the budget constraint can be more simply presented as

$$w_{t+1} = R_{t+1}(w_t - c_t). \qquad (1.2.2.1c)$$

With the von Neumann-Morgenstern utility function, the representative household is thought indifferent to the timing of the resolution of uncertainty on intertemporal choices. However, in order to separate risk aversion and intertemporal substitution, Weil views households as being anything but indifferent such that their preferences can be recursively represented by

$$V_t \equiv U[C_t, \text{E}_t V_{t+1}], \qquad (1.2.2.1d)$$

where E_t is the expectation conditional on information available in time period t. The utility function $U[.,.]$ could then be

$$U[C,V] = \frac{\left\{(1-\delta)C^{1-\gamma} + \delta\left[1+(1-\delta)(1-\rho)V\right]^{(1-\gamma)/(1-\rho)}\right\}^{(1-\rho)/(1-\gamma)} - 1}{(1-\delta)(1-\rho)}. \quad (1.2.2.1e)$$

This parameterisation of Kreps-Porteus preferences disentangles attitudes toward intertemporal substitution and risk aversion. The parameter, $\gamma > 0$, represents risk aversion, $\rho > 0$ represents the inverse of the elasticity of intertemporal substitution whilst $\delta \in (0,1)$ is the subjective discount rate. The von Neumann-Morgenstern time additive utility function then emerges as a special case of the Kreps-Porteus formulation in which γ equals ρ to give a coefficient of relative risk aversion which is the inverse of the elasticity of intertemporal substitution.

Weil goes on to outline a derivation similar to the von Neumann-Morgenstern function where

$$E_t\left\{\frac{U_{2t}U_{it+1}}{U_{it}} R_{t+1}\right\} = 1 \quad (1.2.2.1f)$$

or equivalently,

$$E_t\left\{\frac{U_{2t}U_{it+1}}{U_{it}} R_{kt+1}\right\} = 1, \quad (1.2.2.1g)$$

which would apply to any asset with a rate of return R_{kt+1} willingly held by the representative household. To complete the formulation of the optimal consumption process, Weil unveils

$$E_t\left\{\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right]^{(1-\rho)/(1-\gamma)} \left[R_{t+1}\right]^{(1-\rho)/(1-\gamma)-1} R_{kt+1}\right\} = 1, \quad (1.2.2.1h)$$

an equation which holds for the rate of return on trees ($R_{kt} = R_t$).

Finally, Weil was able to derive the following forms for the risky asset return, the risk-free rate and the resulting equity premium.

The expected rate of return on a tree in today's state i is

$$ER^i = \sum_{j=1}^{I} \alpha_{ij} g_j \frac{w_j + 1}{w_i}, \qquad (1.2.2.1h)$$

whilst the risk-free rate, RF^i is

$$RF^i = \frac{1}{\delta^{(1-\rho)(1-\gamma)} \left\{ \sum_{j=1}^{I} \alpha_{ij} g_j^{-\rho} \left(\frac{w_j + 1}{w_i} \right)^{(\gamma-\rho)/(1-\gamma)} \right\}}. \qquad (1.2.2.1i)$$

Consequently, the proportional equity premium follows

$$RP^i = \frac{\left\{ \sum_{j=1}^{I} \alpha_{ij} g_j \left(w_j + 1 \right) \right\} \left\{ \sum_{j=1}^{I} \alpha_{ij} g_j^{-\rho} \left(w_j + 1 \right)^{(\gamma-\rho)/(1-\gamma)} \right\}}{\sum_{j=1}^{I} \alpha_{ij} g_j^{1-\rho} \left(w_j + 1 \right)^{(1-\rho)/(1-\gamma)}}. \qquad (1.2.2j)$$

A feature in the equations (1.2.2.1*h*-*j*) is wealth, w. The implied wealth formulation is

$$w_i = \delta \left\{ \sum_{j=1}^{I} \phi_{ij} g_j^{1-\gamma} \left(w_j + 1 \right)^{(1-\gamma)/(1-\rho)} \right\}^{(1-\rho)/(1-\gamma)} \quad \text{for } i = 1, \ldots, I. \qquad (1.2.2.1k)$$

From (1.2.2.1*k*), Weil was also able to present a numerical solution for asset returns using the two-state Markov process identified by Mehra and Prescott where

$$g_1 = 1.054, \quad g_2 = 0.984, \tag{1.2.21l}$$

with transition probabilities

$$\alpha_{11} = \alpha_{22} = 0.43, \qquad \alpha_{12} = \alpha_{21} = 0.57. \tag{1.2.21m}$$

Given the three parameters, δ, γ and ρ which parameterise consumers' attitudes towards impatience, risk and intertemporal substitution, respectively, the probability, α, can be used to compute the long run average risk-free rate,

$$RF = \alpha RF^1 + (1-\alpha)RF^2, \tag{1.2.2.1n}$$

and the long run average equity premium,

$$RP = \alpha RP^1 + (1-\alpha)RP^2. \tag{1.2.2.1o}$$

Table 1.1, Panel D and E report the average long-run risk premia and risk-free rate for selected values for the elasticity of intertemporal substitution of between 1/45 and infinity.

Table 1.1

Panel D

Net Risk Premium (RP) and net risk-free rate (RF) - ($\delta = 0.95$)
(in percentages)

EIS		CRRA						
		0	0.5	1	5	10	20	45
∞	RP	0.00	0.05	0.10	0.48	0.94	1.77	3.01
	RF	5.25	5.24	5.21	5.01	4.78	4.40	4.09
2	RP	0.01	0.06	0.11	0.51	1.01	1.89	3.14
	RF	6.20	6.16	6.12	5.79	5.40	4.73	3.93
1	RP	0.01	0.07	0.12	0.55	1.08	2.00	3.27
	RF	7.14	7.08	7.03	6.56	6.02	5.06	3.76
0.2	RP	0.10	0.18	0.26	0.88	1.64	2.91	4.34
	RF	15.02	14.81	14.61	13.02	11.11	7.75	2.45
0.1	RP	0.24	0.35	0.45	1.31	2.33	4.04	5.72
	RF	25.73	25.32	24.96	21.68	17.87	11.23	0.85
0.05	RP	0.56	0.72	0.87	2.12	3.66	6.25	8.66
	RF	50.51	49.55	48.61	41.26	32.80	18.65	-2.23
1/45	RP	1.13	1.36	1.60	3.58	6.22	11.22	17.11
	RF	138.9	135.6	132.3	107.4	80.69	40.39	-9.22

Source: Weil (1989)

Table 1.1

Panel E

Net Risk Premium (RP) and net risk-free rate (RF) - ($\delta = 0.98$)
(in percentages)

EIS		CRRA						
		0	0.5	1	5	10	20	45
∞	RP	0.00	0.05	0.09	0.47	0.93	1.76	3.00
	RF	2.04	2.02	1.99	1.80	1.58	1.21	0.91
2	RP	0.01	0.06	0.11	0.51	1.00	1.88	3.13
	RF	2.95	2.91	2.87	2.55	2.17	1.53	0.75
1	RP	0.01	0.07	0.12	0.55	1.07	2.00	3.27
	RF	3.86	3.81	3.75	3.31	2.77	1.85	0.59
0.2	RP	0.10	0.18	0.26	0.89	1.65	2.93	4.37
	RF	11.49	11.29	11.10	9.56	7.72	4.45	-0.68
0.1	RP	0.25	0.36	0.47	1.33	2.37	4.10	5.79
	RF	21.87	21.47	21.08	17.95	14.26	7.83	- 2.24
0.05	RP	0.59	0.75	0.89	2.18	3.73	6.37	8.82
	RF	45.89	44.95	43.98	36.92	28.72	15.01	-5.22
1/45	RP	1.19	1.42	1.66	3.70	6.40	11.50	17.53
	RF	131.5	128.9	125.01	101.0	75.11	36.06	-12.00

Source: Weil (1989)

The results are similar to those of Mehra and Prescott in that the representative agent, complete markets model is overwhelmingly rejected as the model cannot replicate the risk-free rate and the risk premium simultaneously. To match the risk premium, a coefficient of relative risk aversion of around 20 is needed but the associated elasticity of intertemporal substitution of around 0.05 is consistent with a very large and implausible risk-free rate of around 15%. In consequence, the model is unable to explain the historically observed asset returns. Weil did point out that an assumption of a negative rate of time preference makes it possible to fit both the risk-free rate and the premium simultaneously. Weil (1989) concluded that the Kreps-Porteus preferences have not been able to explain the equity premium using reasonable parameters for risk aversion, the subjective discount rate and the elasticity of intertemporal substitution. In fact, Weil suggested the existence of another puzzle; the risk-free rate puzzle. This is discussed further in chapter 3.

Another attempt to separate risk aversion and intertemporal substitution was presented by Kocherlatoka (1990) who followed a similar path to Weil and arrived at the same conclusion that breaking the link between risk aversion and intertemporal substitution did not increase the ability of the models to match the observed equity premium.

Epstein & Zin (1991) also attempted a separation between risk aversion and intertemporal substitution. They considered utility derived from a single good by a representative agent. In period t, current consumption, C_t, is said to be deterministic whilst future consumption remains uncertain. Two assumptions which underlie the specification include that the representative agent forms a *certainty equivalent* of random future utility using his risk preferences and also that this *certainty equivalence* is combined with deterministic current consumption via an aggregator function to obtain the current-period lifetime utility. Lifetime utility will therefore take the form

$$U_t = W\left(C_t, \psi\left[\tilde{U}_{t+1} | I_t\right]\right), \tag{1.2.2.2a}$$

where W is the aggregator function, ψ is the certainty equivalent function reflecting the degree of risk aversion and I_t is the information available to the agent in the planning period.[5] The functional form for the aggregator function follows

$$W(C,Z) = \left[(1-\beta)C^\rho + \beta Z^\rho\right]^{1/\rho}, \quad 0 \neq \rho < 1, \tag{1.2.2.2b}$$

$$W(C,Z) = (1-\beta)\mathbf{log}(C) + \beta\mathbf{log}(Z), \quad \rho = 0, \tag{1.2.2.2c}$$

where $C, Z \geq 0$ and $\beta = 1/(1+\delta)$, $\delta > 0$.

According to Epstein and Zin, (1.2.2.2*c*) shows that "when future consumption ... is deterministic, the aggregator function results in a constant elasticity of substitution utility function with elasticity of substitution $\Omega = 1/(1-\rho)$ and rate of time preference, δ. Thus the parameter ρ is interpreted as reflecting substitution". Epstein and Zin then introduced a recursive structure for intertemporal utility where

$$U_t = \left[(1-\beta)C_t^\rho + \beta\left(\mathrm{E}_t\tilde{U}_{t+1}^\gamma\right)^{\rho/\gamma}\right]^{1/\rho}, \tag{1.2.2.2d}$$

where E_t is the conditional expectation operator given I_t and γ, the mean of ψ.

If $\gamma = \rho$, then (1.2.2.2*d*) becomes

$$U_t = \left[(1-\beta)\mathrm{E}_t\sum_{j=0}^{\infty}\beta^j\tilde{C}_{t+j}^\gamma\right]^{1/\gamma}. \tag{1.2.2.2e}$$

5 This is a recursive structure, introduced by Koopmans (1960) for deterministic models and is also consistent with Kreps-Porteus preferences.

This is the familiar expected utility specification where there does exist an indifference about the manner of uncertainty resolution.

The Euler equation for optimal consumption decisions is derived as

$$E_t\left[\beta\left(\frac{\tilde{C}_{t+1}}{C_t}\right)^{\rho-1}\tilde{M}_t\right]^{\alpha} = 1 \qquad (1.2.2.2f)$$

for all t and where $\alpha = \gamma / \rho$.

Putting the Euler equation in the previously discussed asset pricing framework of Hansen and Jagannathan, the intertemporal marginal rate of substitution, (IMRS) equals

$$\left[\beta\left(\frac{\tilde{C}_{t+1}}{C_t}\right)^{\rho-1}\right]^{\alpha}\left(\frac{1}{\tilde{M}_t}\right)^{1-\alpha}. \qquad (1.2.2.2g)$$

The attached weights to the IMRS are determined by α such that when α equals 1, consumption growth is seen as sufficient for discounting asset payoffs as per the intertemporal (or consumption) capital asset pricing model. When α equals 0, the market return is adequate for discounting individual asset payoffs as in the simple (or static) capital asset pricing model. For all other values of α, both the consumption growth and market return together determine the IMRS.

Epstein and Zin proceeded to discuss a number of data variations for the model, the results of which are too numerous to fully repeat here. They concluded that some broad patterns emerge that appear to hold over different time periods, consumption measures, asset returns and instrument sets. Specifically though, many of the results are sensitive to the consumption measure as well as the choice of instrumental variables. The elasticity of substitution is typically small i.e. less than one, and risk preferences are not much different from their logarithmic form. One of the concerns though is the negative rate of time preference, which is consistent with a discount rate greater than one. We have already discussed the implications of a negative rate of time preference, which undermines the theory that there exists intertemporal substitution.

Attannasio and Weber (1989), using UK data, have also attempted to separate the risk aversion and intertemporal substitution parameters. They looked at two formulations of the model where risk aversion and intertemporal substitution are not related. Firstly, they looked at the Selden (1978) model, herein represented by Epstein and Zin whose approach of intertemporal choices is based on the Ordinal Certainty Equivalence preferences and another of the Kreps-Porteus class, herein represented by Weil (1989). For asset returns, they used data on shares and building society deposits whilst for consumption, they used average consumption data from a single large cohort of married couples sampled in the Family Expenditure Survey. This approach avoided the problems associated with whether the economy is dominated by identical agents with infinite lives.

They assumed the presence of a representative agent who consumes a single, homogeneous good and has an investment opportunity set such that he maximises

$$\sum_{j=0}^{T} \delta^j U(C^*_{t+j}) \qquad (1.2.2.3a)$$

subject to :

$$V(C^*_{t+j}) = \mathrm{E}_t[V(C_{t+j})] \qquad j = 0,1,\ldots\ldots,T \qquad (1.2.2.3b)$$

$$C_{t+j} \leq Y_{t+j} + \sum_{t=1}^{N} R^i_{t+j} A^i_{t+j-1} - \sum_{t=1}^{N} A^i_{t+j}$$

$$A_T \equiv \sum_{t=1}^{N} A^i_T \geq 0,$$

where δ is the subject discount rate, C is consumption, Y is labour income, T is the length of the lifetime and A^i and R^i represents the holdings and the rate of return on asset i. Equation (1.2.2.3*a*) defines life-cycle utility as a function of the "certain equivalent" of consumption, C^*, rather than the actual consumption. Hence,

$$C^*_{t+j} = V^{-1}[\mathrm{E}_t V(C_{t+j})] = G[\mathrm{E}_t V(C_{t+j})], \qquad (1.2.2.3c)$$

where $G \equiv V^{-1}$. Substituting (1.2.2.3*c*) into (1.2.2.3*a*) yields

$$\mathrm{Max} \sum_{j=0}^{T} \delta^j U\{G[\mathrm{E}_t V(C_{t+j})]\}. \qquad (1.2.2.3d)$$

According to the authors, the essential point of the Ordinal Certainty Equivalence approach is the distinction between risk aversion and intertemporal substitution where the former is represented by the curvature of *V*, i.e. of the function that converts future consumption into certainty equivalence consumption and the latter by the curvature of *U*.

The traditional expected utility maximisation approach is consistent with $V = U$ where the first order condition for the optimisation problem (1.2.2.3*d*) is

$$\delta^{j+1} U'G'\mathrm{E}_t[V'(C_{t+j+1})R^i_{t+j+1}] = \delta^j U'G'\mathrm{E}_t[V'(C_{t+j})]$$

$$j = 0,1,2,\ldots.,T-t; i = 1,2,\ldots.,N. \qquad (1.2.2.3e)$$

The authors assume that V and U are isoelastic and take the form :

$$V(C) = C^{1-\gamma}/(1-\gamma), \quad U(C^*) = C^{*1-\rho}/(1-\rho),$$

where γ is the coefficient of relative risk aversion and $1/\rho$ is the elasticity of intertemporal substitution. Using the equality

$$C_t^* = C_t,$$

the authors derive

$$\delta \mathrm{E}_t\left[(C_{t+1}/C_t)^{-\gamma} R_{t+1}^i\right] \mathrm{E}_t\left[(C_{t+1}/C_t)^{1-\gamma}\right]^{(\rho-\gamma)/(1-\gamma)}. \qquad (1.2.2.3f)$$

The Ordinal Certainty Equivalence approach becomes the traditional expected utility model when $\rho = \gamma$. With 2 different assets, the ratio from (1.2.2.3*f*) becomes

$$\mathrm{E}_t\left[(C_{t+1}/C_t)^{-\gamma}\left(R_{t+1}^i - R_{t+1}^K\right)\right] = 0, \qquad (1.2.2.3g)$$

which can be estimated by NL2SLS or Generalised Method of Moments (GMM). With (1.2.2.3*g*), only the coefficient of relative risk aversion remains in the formulation.

Since the aim is also to estimate ρ, the authors make a number of assumptions including that the consumption growth rate and the real interest rate on all assets are normally distributed and conditional on information available in time t. Eventually, they derive the form (after taking logarithms),

$$r^i_{t+1} = h_{0i} + \rho x_{t+1} + v_{i,t+1}, \qquad (1.2.2.3h)$$

where

$$v_{t+1} = r^i_{t+1} - r^i - \rho(x_{t+1} - \mu_x)$$

and

$$x_{t+1} = \mathbf{log}(C_{t+1}/C_t), \mu_x = \mathrm{E}_t(x_{t+1}), r^i_{t+1} = \mathbf{log}(R^i_{t+1}), r^i = \mathrm{E}_t(r^i_{t-1}).$$

Note also that

$$Var(v_{i,t+1}) \equiv \sigma^2_{vi} = \rho^2\sigma^2_x - 2\rho\sigma_{xr^i} \text{ and}$$
$$h_{0i} = -\mathbf{log}\,\delta - \tfrac{1}{2}\{[\gamma - \rho(1-\gamma)]\sigma^2_x + \sigma^2_{r^i} - 2\gamma\sigma_{xr^i}\}$$

where $x_{t+1} = \log(C_{t+1}/C_t)$, $\mu_x = E_t(x_{t+1})$, $r^i_{t+1} = \log R^i_{t+1}$ and $r^i = E_t(r^i_{t+1})$.

According to Attanasio and Weber, "only by considering equation ... for more than one asset together with a reduced form for consumption growth, can we separately identify the various mean, variance and covariance terms, and the parameters of interest...". The unrestricted reduced form for consumption growth is presented as

$$x_{t+1} = a_0 + a(L)x(t) + b_1(L)r^1_t + b_2(L)r^2_t + \varepsilon_{t+1}. \qquad (1.2.2.3i)$$

Finally, they derived an explicit form for the coefficient of relative risk aversion from 'deep' error variances and their estimated equivalents:

$$\gamma = \rho + \tfrac{1}{2}(\sigma^2_{v_1} - \sigma^2_{v_2})/(\sigma_{xv_1} - \sigma_{xv_2}) + (h_{01} - h_{02})/(\sigma_{xv_1} - \sigma_{xv_2}), \qquad (1.2.2.3j)$$

where

$$\sigma_{xv_i} = Cov(v_{i,t+1}, \varepsilon_{t+1}) = -\rho\sigma^2_x + \sigma_{r^ix}, i = 1,2,..$$

Once the coefficient of relative risk aversion is determined, log δ can be determined using the intercept terms.

For the generalisation of the OCE approach, Attanasio and Weber used the reduced form mentioned above to reinterpret the previously discussed Epstein and Zin formulation in light of Kreps-Porteus analysis that allows for a distinction between intertemporal substitution and risk aversion whilst retaining the property of time-consistent choices. In these circumstances, the equivalent to (1.2.2.3*h*) is

$$r^{i}_{t+1} = h^{*}_{0i} + \rho x_{t+1} + v_{i,t+1} \qquad (1.2.2.3k)$$

where

$$h^{*}_{0i} = -\log\delta - \tfrac{1}{2}\left\{\left[(\rho-\gamma)/(1-\rho)\right]\sigma^{2}_{M} - \sigma^{2}_{r^{i}} - \rho^{2}\left[(1-\gamma)\right]^{2}\sigma^{2}_{x}\right\}$$
$$+\rho\left[(1-\gamma)/(1-\rho)\right]\sigma_{x^{i}} - \left[(\rho-\gamma)/(1-\rho)\right]\sigma_{M^{i}}. \qquad (1.2.2.3l)$$

The analogous form to (1.2.2.3*j*) for the coefficient of relative risk aversion is

$$\gamma = 1-(1-\rho)\left\{\begin{bmatrix}\tfrac{1}{2}\left(\sigma^{2}_{r^{1}} - \sigma^{2}_{r^{2}}\right) + \\ \left(h^{*}_{01} - h^{*}_{02}\right) - \left(\sigma_{m^{1}} - \sigma^{2}_{M}\right)\end{bmatrix} \Big/ \begin{bmatrix}\rho\left(\sigma_{x^{1}} - \sigma_{x^{2}}\right) - \\ \left(\sigma_{M^{1}} - \sigma_{M^{2}}\right)\end{bmatrix}\right\}. \qquad (1.2.2.3m)$$

The authors used data on consumption growth (from the Family Expenditure Survey) as well as building society deposit rates and stock market shares adjusting for inflation and taxes. In estimating (1.2.2.3*i*), consumption growth is estimated as a function of its own first to fourth lags, asset returns as a function of first, second and fourth lags, seasonal dummies and a constant. Table 1.1, Panels F and G reports the results.

Table 1.1

Panel F

Parameter estimates : Expected utility approach

Dependent variable: Regressors :	x	r^1 (shares)	x	r^2 (Building Society)
Intercept	0.018 (0.013)	- 0.022 (0.013)	0.025 (0.007)	- 0.022 (0.013)
s1	- 0.141 (0.026)	- 0.087 (0.043)	- 0.137 (0.023)	- 0.086 (0.043)
s2	- 0.067 (0.027)	- 0.109 (0.051)	- 0.031 (0.023)	- 0.109 (0.050)
s3	0.045 (0.022)	- 0.008 (0.038)	- 0.030 (0.038)	- 0.008 (0.038)
d74	0.001 (0.027)	- 0.273 (0.056)	0.011 (0.027)	- 0.273 (0.056)
d75	0.012 (0.038)	0.576 (0.081)	0.022 (0.039)	- 0.576 (0.081)
x(-1)	- 0.595 (0.131)		- 0.441 (0.123)	
x(-2)	- 0.292 (0.151)		- 0.261 (0.131)	
x(-3)	- 0.029 (0.148)		- 0.103 (0.125)	
x(-4)	- 0.388 (0.130)		- 0.192 (0.113)	
$r^1(-1)$	- 0.046 (0.037)		- 0.005 (0.030)	
$r^1(-2)$	- 0.002 (0.038)		- 0.033 (0.032)	
$r^1(-4)$	- 0.049 (0.038)		- 0.012 (0.031)	
$r^2(-1)$	0.109 (0.270)		- 0.352 (0.233)	
$r^2(-2)$	0.877 (0.323)		- 0.736 (0.287)	
$r^2(-4)$	0.250 (0.298)		- 0.177 (0.249)	
Parameters :				
ρ	1.459 (0.511)		0.380 (0.116)	
δ	0.976 (0.014)		1.001 (0.002)	

LR tests of over-identifying restrictions	6.500 (9)	12.341 (9)
GMM estimates of γ (equation 1.2.2.3g)	5.102 (4.07)	
Corresponding δ (OCE)	0.9806	0.9737

Note : Asymptotic standard errors in parentheses. The variables *s1, s2* and *s3* are zero sum quarterly seasonal dummies (leaving out the fourth quarter); *d74* takes value 1 in 1974:Q4 and 1975:Q1, 0 otherwise *d75* takes value 1 in 1975:Q2 and 0 otherwise.

Source: Attanasio & Weber (1989)

Table 1.1

Panel G

Parameter estimates. Ordinal Certainty Equivalence approach

Dependent variable:	x	r^1	r^2
Regressors :		(shares)	(Building Society)
Intercept	0.021 (0.007)	0.012 (0.013)	- 0.001 (0.002)
s1	- 0.132 (0.020)	- 0.068 (0.031)	- 0.039 (0.009)
s2	- 0.006 (0.020)	- 0.011 (0.031)	- 0.029 (0.009)
s3	- 0.028 (0.027)	- 0.002 (0.030)	- 0.002 (0.007)
d74	0.011 (0.027)	- 0.265 (0.057)	- 0.025 (0.009)
d75	0.009 (0.040)	0.541 (0.081)	- 0.044 (0.013)
x(-1)	- 0.255 (0.102)		
x(-2)	- 0.169 (0.106)		
x(-3)	- 0.107 (0.099)		
x(-4)	- 0.200 (0.099)		
$r^1(-1)$	- 0.031 (0.025)		
$r^1(-2)$	0.034 (0.026)		
$r^1(-4)$	0.008 (0.024)		
$r^2(-1)$	0.347 (0.198)		
$r^2(-2)$	0.427 (0.231)		
$r^2(-4)$	0.292 (0.207)		
Parameters :			
ρ	0.514 (0.183)		
δ	0.993 (0.011)		
γ	29.943 (33.603)		

LR tests of over-identifying restrictions		27.723 (19)
LM tests for serial correlation	- x :	11.221 (12)
	r^1:	19.415 (12)
	r^2:	16.638 (12)
Normality tests on RF equations	- x :	0.2044 (2)
	r^1:	0.7040 (2)
	r^2:	5.3140 (2)
3SLS estimate of ρl (time aggregation) :		0.445 (0.19)
GMM estimate of γ (equation 1.2.2.3*g*)		5.102 (4.07)

Source: Attanasio & Weber (1989)

According to the authors, the OCE framework "produces more convincing estimates..." as evidenced by Panel G of table 1.1 with estimates for ρ of around 0.514 (implying an elasticity of intertemporal substitution of close to 2) which is consistent with the data and a highly plausible value for δ of 0.993. The value for risk aversion is measured at 29.9 which the authors think is justified given the high value of the risk premium though the standard error is large enough not to accept the point estimate. They further concluded that separating the risk aversion and elasticity of substitution parameters could perhaps yield "fruitful" results despite the fact that the risk aversion parameter estimates might be "imprecise".

1.2.3 Habit and Reference Consumption

Much of the earlier research, though encouraging, has been unable to come up with models capable of explaining the observed equity puzzle as well as simultaneously producing other parameter estimates consistent with the theory. One of the recent and more promising approaches by Constantinides (1990), Abel (1990), Pemberton (1993), Campbell and Cochrane (1995) and Campbell, Lo and Mackinlay (1997) focused on the notion of habit (reference) consumption formation by the investor. Part of the representative agent's consumption is the result of habit, variously defined, and the remainder, surplus consumption, determines asset returns.

a. The Constantinides (1990) Approach

The Constantinides (1990) framework involves an economy in which the time separability associated with von Neumann-Morgenstern preferences is relaxed in favour of one which "allows for adjacent complementarity in consumption, a property known as habit persistence". Habit persistence is introduced in a production economy where there exists one production good, which is also the consumption good and which is consumed or invested in two technologies. The technologies have constant returns to scale and rates of return over the period t, $t + dt$ of rdt and $\mu dt + \sigma dw(t)$ respectively, where r, μ and σ are constants and $w(t)$ is a standard Brownian motion in R. The representative consumer has capital $w(t)$ at time period t denominated in units of consumption good, investing a fraction $\alpha(t)$, where $0 \le \alpha(t) \le 1$ of the capital, in the risky technology and the remainder, $1 - \alpha(t)$, in a risk-less technology. The consumer consumes $c(t)dt$ in same period t, $t + dt$. The increase in capital over the period t, $t + dt$ is therefore

$$dW(t) = \{[(\mu - r)\alpha(t) + r]W(t) - c(t)\}dt + \sigma\alpha(t)W(t)dw(t). \qquad (1.2.3.a1)$$

Given a consumption and investment policy,

$$\{c(t), \alpha(t), t \ge 0\}$$

the expected utility of consumption is

$$E_0 \int_0^\infty e^{-\psi t} \kappa^{-1} [c(t) - x(t)]^\kappa dt \qquad (1.2.3.a2)$$

where

$$x(t) \equiv e^{-at} x_0 + b \int_o^t e^{a(s-t)} c(s) ds. \qquad (1.2.3.a3)$$

From (1.2.3.a2) and (1.2.3.a3), the habit level of consumption $x(t)$ is an exponentially weighted sum of past consumption. This is habit persistence.

Constantinides imposed conditions on consumer choice for consumption and investment policy to meet the following 4 properties : (i) consumption and investment decisions in time period t are based solely on information available at that time, (ii) consumption is non-negative and does not fall below the level of the habit, $(c(t) \geq x(t))$, (iii) investment in both technologies is non negative; i.e. $0 \leq \alpha(t) \leq 1$ for all t and (iv) capital is nonnegative, i.e. $W(t) \geq 0$ for all t. The optimal admission policy for consumption and investment and an associated derived utility of capital is defined by

$$V(W_0, x_0) \equiv \max_{\left\{ \substack{admissible \\ \alpha(s), c(s), s \geq 0} \right\}} \mathrm{E}_0 \int_0^\infty e^{-\rho s} \gamma^{-1} [c(s) - x(s)]^\gamma ds, \qquad (1.2.3.a4)$$

where $W(0) = W_0$ and $x(0) = x_0$.

Constantinides imposed restrictions on this model to derive formulations for optimal admissible consumption and investment policy denoted by

$$c^*(t) = x(t) + j\left[W(t) - \frac{x(t)}{r + a - b}\right] \qquad (1.2.3.a5)$$

and

$$\alpha^*(t) = m\left[1 - \frac{x(t) / W(t)}{r + a - b}\right], \qquad (1.2.3.a6)$$

where

$$j \equiv \left[\frac{r+a-b}{(r+a)(1-\kappa)}\right]\left[\psi - \kappa r - \frac{\kappa(\mu - r)^2}{2(1-\kappa)\sigma^2}\right] > 0. \quad (1.2.3.a7)$$

The derived utility of capital is

$$V(W(t), x(t)) = \frac{(r+a-b)j^{\kappa-1}}{(r+a)\kappa}\left[W(t) - \frac{x(t)}{r+a-b}\right]^{\kappa}. \quad (1.2.3.a8)$$

The capital is

$$W(t) = \frac{x(t)}{r+a-b} + \left(W_0 - \frac{x_0}{r+a-b}\right) \times \mathbf{exp}\left[\left(n - \frac{m^2\sigma^2}{2}\right)t + m\sigma w(t)\right] \quad (1.2.3.a9)$$

and the consumption growth rate is

$$\frac{dc(t)}{c(t)} = \left[n + b - \frac{(n+a)x(t)}{c(t)}\right]dt + \left[1 - \frac{x(t)}{c(t)}\right]m\sigma dw(t) \qquad (1.2.3.a10)$$

where

$$n \equiv \frac{r - \psi}{1-\kappa} + \frac{(\mu - r)^2(2-\kappa)}{2(1-\kappa)^2\sigma^2}. \qquad (1.2.3.a11)$$

Constantinides then proceeded to state the conditions under which the ratio $z(t) \equiv x(t)/c(t)$ has a stationary distribution which is then used to calculate the mean and variance of consumption growth which are given as

$$\frac{E(dc/c)}{dt} = n + b - (n+a)\int_0^1 zp_z(z)dz \qquad (1.2.3.a12)$$

and

$$\frac{\mathrm{var}(dc/c)}{dt} = m^2\sigma^2\int_0^1 (1-z)^2 p_z(z)dz \qquad (1.2.3.a13)$$

respectively.

Constantinides defined the relative risk aversion parameter, RRA (γ), as

$$\gamma = RRA = \frac{-WV_{ww}}{V_w} = \frac{1-\kappa}{1-\{x(t)/[W(t)(r+a-b)]\}} \qquad (1.2.3.a14)$$

which generalises the Giovanni and Weil (1988) and Kreps-Porteus preferences. This formulation defines risk aversion in terms of atemporal choices that changes the level of wealth but not current or future consumption at some future date. Equation (1.2.3.*a*14) also defines risk aversion in terms of the state variable $x(t)$ such that a sudden drop in wealth leaves $x(t)$ unchanged in the short run but increases the risk aversion coefficient. Constantinides, however argues that this fall in wealth is a temporary phenomenon as the risk aversion coefficient has a stationary distribution.

The elasticity of substitution in consumption (Ω) is presented here as the derivative of the expected consumption growth rate with respect to r, with $z(t)$, $\mu - r$ and σ^2 held constant to yield

$$\Omega = \left.\frac{d[\mathrm{E}(dc/c)]/dt}{dr}\right|_{z(t),\mu-r,\sigma^2} = \frac{1-z(t)}{1-\kappa}. \qquad (1.2.3.a15)$$

Since this formulation for the elasticity of substitution is based on consumption and that for risk aversion is based on wealth, it does imply that elasticity is not necessarily the inverse of risk aversion though there will exist a special case where it will be.

In the production context, Constantinides introduced a firm with capital $K(t)$ at time t that has access to the production technologies. The firm invests $\varphi_1 K(t)$ in the risky technology and the remaining capital, $(1-\varphi_1)K(t)$, in the risk-less technology, where φ_1 is a constant, $0 < \varphi_1 \leq 1$. The firm is financed with equity of value $P(t)$ and risk-less debt of value $B(t)$. The equity to total capital formulation is then $P(t)/[P(t)+B(t)]= \varphi_2$ with $0 < \varphi_2 \leq 1$. Since the firm has free access to the constant-returns-to-scale technology, the value of the firm equals its capital, i.e. $P(t)+B(t)=K(t)$. The rate of return on the risk-less asset is fulfilled by $dB/B = rdt$.

For the rate of return on equity,

$$dP(t) + B(t)rdt = \varphi_1 K(t)\left[\mu dt + \sigma dw(t) + (1-\varphi_1)K(t)rdt\right], \quad (1.2.3.a16)$$

which can be simplified to

$$\frac{dP(t)}{P(t)} = \left(\frac{\varphi_1}{\varphi_2}\right)\left[(\mu - r)dt + \sigma dw(t) + rdt\right]. \quad (1.2.3.a17)$$

Constantinides used the S&P Composite Stock Price Index to reflect the equity of the firm and takes the values indicated by Mehra and Prescott for the mean, range, standard deviation etc to estimate the model. The form φ_1 / φ_2 is set to 1, a value, according to Constantinides, consistent with any amount of leverage. Using observed data, Constantinides set

$$\frac{E(dP/P)}{dt} = \left(\frac{\varphi_1}{\varphi_2}\right)(\mu - r) = 0.06 \text{ per year.} \quad (1.2.3.a18)$$

and

$$\frac{\text{var}(dP/P)}{dt} = \left(\frac{\varphi_1}{\varphi_2}\right)^2 \sigma^2 = (0.165)^2 \text{ per year.} \quad (1.2.3.a19)$$

Finally, Constantinides reported values for pairs of a, b for which the mean and variance of consumption growth rate match that found in the observed data as well as the mean RRA and other parameters of interest.

Table 1.2

Mean and variance of the consumption growth rate generated by the model with habit persistence

Parameters a, per year	.1	.2	.3	.4	.5	.6
Parameter *b*	.093	.173	.250	.328	.405	.492
Mode *(z*)* of the state variable z	.86	.82	.81	.80	.79	.81
Mean annual growth rate in consumption:						
Unconditional mean	.018	.019	.018	.018	.018	.018
At z = z*	.011	.013	.014	.014	.014	.014
Standard deviation of the annual growth rate in consumption:						
Unconditional mean	.036	.036	.036	.036	.036	.036
At z = z*	.023	.029	.032	.033	.034	.032
RRA Coefficient:						
Unconditional mean	8.67	4.37	3.47	3.09	2.88	2.81
At z = z*	7.03	4.09	3.36	3.03	2.84	2.78
Elasticity of substitution *(s)* at z = z*	.06	.08	.09	.09	.09	.09
Ω. RRA at z=z*	.42	.33	.30	.27	.26	.25

Source: Constantinides (1990)

Constantinides argued that the equity premium is solved because the model is able to generate the mean and variance of consumption growth rate with a mean RRA coefficient as low as 2.81. The implied value for habit is about 0.8 and the elasticity of substitution is well below 1 at 0.09. This has clear implications for the relationship between elasticity of substitution and risk aversion. The last row shows figures for the product of the elasticity of substitution and the risk aversion parameter to be 0.25, much less than the implied value of 1 if the elasticity of substitution is the inverse of the risk aversion parameter.

Constantindes (1990) does have the capacity to resolve the equity premium puzzle though aspects of the model remain to be resolved; not least of which is the interpretation of habit, which is allowed to take on values to yield the required rates of return. As mentioned by Pemberton (1993), the Constantinides model has the implication that the model, with an habit/consumption ratio of 0.8 and its specification, to generate an infinite marginal utility of future consumption, which would run counter to theory. Also, as with much of the work in this area, the subjective discount rate is assumed to imply a positive rate of time preference. This could have been endogenously modelled.

b. <u>The Abel (1990) Approach</u>

Another variation of the habit formation model was presented by Abel (1990) who also used past consumption to determine habit. The basic idea is that the investor is concerned about consumption insofar as it relates to a predetermined benchmark based on past consumption or "Catching up with the Joneses" effect. With habit formation, investors are loath to hold risky assets to the extent that they could only be persuaded to do so by being offered a sizeable premium. The formulation is re-presented here and can be shown in a discrete time utility function written as

$$U \equiv \sum_{j=0}^{\infty} \delta^{j} u\left(C_{t+j}, X_{t+j}\right) \qquad (1.2.3.b1)$$

where X_{t+j} is a preference parameter which could be written as

$$X_t \equiv \left[C_{t-1}^{D} V_{t-1}^{1-D}\right]^{\psi} \qquad \psi \geq 0 \; and \; D \geq 0 \qquad (1.2.3.b2)$$

where C_{t-1} is the consumer's own consumption in period *t-1* and V_{t-1} is the aggregate per capita consumption in period *t-1*.

From (1.2.3.*b*2), if $\psi = 0$, then $X_t \equiv 1$ and the utility function is time-separable. If however $\psi > 0$ and D=0, then X_t depends on the lagged aggregate per capita consumption. This is the "catching up with the Joneses" model. Finally, when $\psi > 0$ and $D=1$, then X_t would depend on the consumer's own past consumption. This is the habit formation model according to Abel (1990).

One essential aspect of this formulation considers the effects on utility of changes in an individual's consumption at time period t whilst holding aggregate consumption unchanged. Such a prospect can best be viewed by substituting (1.2.3.*b*2) into (1.2.3.*b*1) and differentiating with respect to C_t such that

$$\frac{\partial U_t}{\partial C_t} = u_c(C_t, X_t) + \delta\, u_X(C_{t+1}, X_{t+1})\gamma\, D X_{t+1} / C_t. \qquad (1.2.3.b3)$$

Assuming an isoelastic form for the utility function, then

$$u(C_t, X_t) = \frac{(C_t/X_t)^{1-\gamma}}{1-\gamma} \qquad (1.2.3.b4)$$

where $\gamma > 0$ and is the coefficient of relative risk aversion. According to Abel, (1.2.3.*b*4) implies that "...utility depends on the level of consumption relative to some endogenous time-varying benchmark". If the assumption in (1.2.3.*b*4) of an isoelastic utility function is maintained, then (1.2.3.*b*3) can be represented as

$$\frac{\partial U_t}{\partial C_t} = \left[1 - \delta\psi D(C_{t+1}/C_t)^{1-\gamma}(X_t/X_{t+1})^{1-\gamma}\right].(C_t/X_t)^{1-\gamma}(1/C_t). \qquad (1.2.3.b5)$$

To determine equilibrium conditions, Abel (1990) introduced the concept of the perishable per capita consumption good produced from the capital stock and denoted by Y_t. If the assumption is that all output is consumed in the period in which it is produced, then owing to the fact that all consumers are identical, then $C_t = V_t = Y_t$ for all periods. Abel further defined a gross growth rate of output, g_{t+1}, defined as being equivalent to Y_{t+1} Y_t. As the assumption has already been made that $C_t = V_t = Y_t$, then $C_{t+1}/C_t = V_{t+1}/V_t = g_{t+1}$. In such circumstances, (1.2.3.*b*2) implies that $X_{t+1}/X_t = g\psi$.

This then allows a re-presentation of (1.2.3.*b*5) as

$$\frac{\partial U_t}{\partial C_t} = \left[1 - \delta\psi D g_{t+1}^{1-\gamma}\ g_t^{-\psi(1-\gamma)}\ X_t^{\gamma-1} C_t^{-\gamma}\right]. \qquad (1.2.3.b6)$$

To determine asset prices, Abel considered that if asset prices are in equilibrium, then a consumer who buys an asset in period t and then sells that asset in period $t+1$ will have no effect on the expected discounted utility. A consumer who reduces period t consumption, C_t, by 1 unit and then purchases an asset with gross return R_{t+1}, and then sells the asset in period $t+1$ increases consumption C_{t+1} by R_{t+1} units. The equilibrium return R_{t+1} must satisfy

$$E_t\left\{-\left(\frac{\partial U_t}{\partial C_t}\right) + \delta R_{t+1}\left(\frac{\partial U_{t+1}}{\partial C_{t+1}}\right)\right\} = 0. \qquad (1.2.3.b7)$$

Dividing both sides by $E_t\{\partial U_t\ \partial C_t\}$, (1.2.3.*b*7) can be rewritten as

$$E_t\left\{\delta R_{t+1} \frac{\left(\frac{\partial U_{t+1}}{\partial C_{t+1}}\right)}{\left(\frac{\partial U_t}{\partial C_t}\right)}\right\} = 1. \qquad (1.2.3.b8)$$

Equation (1.2.3.*b*8) represents the very familiar result that the conditional expectation of the product of the intertemporal rate of substitution and the gross rate of return equals 1. Looking at (1.2.3.*b*8), a formulation can be found for $(\partial U_{t+1}/\partial C_{t+1}) / E_t(\partial U_t\ \partial C_t)$ by using (1.2.3.*b*6) to yield

$$E_t\left\{\frac{\left(\frac{\partial U_{t+1}}{\partial C_{t+1}}\right)}{\left(\frac{\partial U_t}{\partial C_t}\right)}\right\} = \left[\frac{1 - \delta\psi D\, g_{t+2}^{1-\gamma} g_{t+1}^{-\psi(1-\gamma)}}{1 - \delta\psi D\, g_{t+1}^{1-\gamma} g_t^{-\psi(1-\gamma)}}\right] g_t^{\psi(\gamma-1)}\ g_{t-1}^{-\gamma}. \qquad (1.2.3.b9)$$

If P_t^s is the ex-dividend price of a share of stock in period t, then this can be regarded as a claim to a unit of risky capital. The rate of return on the stock is therefore $R^s_{t+1} \equiv \left(P^s_{t+1} + Y_{t+1}\right)/P^s_t$. It therefore follows that $P^s_t/Y_t \equiv W_t$ will be the dividend-price ratio. Consequently, $P^s_t = Y_t W_t$ and $P^s_{t+1} = Y_{t+1}W_{t+1}$ such that

$$R^s_{t+1} = \left(1 + W_{t+1}\right)g_{t+1}/W_t\,. \qquad (1.2.3.b10)$$

Substituting (1.2.3.*b*10) into (1.2.3.*b*8), results in

$$W_t = \delta\, E_t\left\{(1 - W_{t+1})g_{t+1} \cdot \left(\frac{\frac{\partial U_{t+1}}{\partial C_{t+1}}}{E_t\left\{\frac{\partial U_t}{\partial C_t}\right\}}\right)\right\}. \qquad (1.2.3.b11)$$

A one-period short-term risk-less asset (treasury bills) purchased in time period t at a price b_t. In period $t+1$, the short-term risk-less asset will then be worth 1 unit of consumption good such that the gross rate of return on the short term risk-less asset is $R^b_{t+1} = 1/b_t$. Substituting $1/b_t$ into (1.2.3.*b*8) results in

$$b_t = \delta\, E_t\left\{\frac{\left(\frac{\partial U_{t+1}}{\partial C_{t+1}}\right)}{E_t\left\{\frac{\partial U_{t+1}}{\partial C_{t+1}}\right\}}\right\}. \qquad (1.2.3.b12)$$

Equivalently, a longer term risk-less asset (consols) which can be purchased at an ex-coupon price P^c_t in period t and which pays one unit of consumption good in each period will take the form

$$P^c_t = \delta\, E_t\left\{\left(1 + P^c_{t+1}\right)\frac{\left(\frac{\partial U_{t+1}}{\partial C_{t+1}}\right)}{E_t\left\{\frac{\partial U_{t+1}}{\partial C_{t+1}}\right\}}\right\}. \qquad (1.2.3.b13)$$

If the assumption is that consumption growth is i.i.d over time, then it is possible to derive solutions for stock prices, bills and consols. The price-dividend ratio, W_t, can be written as

$$W_t = \frac{A\, g_t^{\Theta}}{J_t} \qquad (1.2.3.b14)$$

where
$$\Theta \equiv \psi(\gamma - 1)$$
$$A \equiv \delta E\{g^{1-\gamma}\}\left[1 - \delta\psi D E\{g^{(1-\gamma)(1-\psi)}\}\right] / \left[1 - \delta E\{g^{(1-\gamma)(1-\psi)}\}\right]$$
$$J_t = E_t\{H_{t+1}\} \equiv 1 - \delta\psi D E\{g^{1-\gamma}\} g_t^{\Theta} \text{ and}$$
$$H_{t+1} \equiv 1 - \delta\psi D g_{t+1}^{1-\gamma}\, g_t^{-\psi(1-\gamma)}.$$

The price of a one-period risk-less asset is

$$b_t = \frac{q\delta g_t^{\Theta}}{J_t}, \qquad (1.2.3.b15)$$

where
$$q = E\{g^{-\gamma}\} - \delta\psi D E\{g^{1-\gamma}\} E\{g^{\Theta-\gamma}\}$$

and the price of a consol is

$$P_t^C = \frac{Q g_t^{\Theta}}{J_t}, \qquad (1.2.3.b16)$$

where
$$Q \equiv \delta q / \left[1 - \delta E\{g^{\Theta-\gamma}\}\right].$$

With a distribution for g, the moments of g can be calculated and with it three asset prices. For time-separable preferences ($\psi = 0$) and for "catching up with the Joneses" ($\psi > 0$; $D = 0$), solutions are possible for the unconditional expected returns $E\{R^s\}$, $E\{R^B\}$ and $E\{R^c\}$.

Given the distribution for g, Abel derived closed-form solutions in terms of preference parameters and the moments of g for the unconditional expected asset returns:

$$E\{R^s\} = E\{g^{-\Theta}\}.\left[E\{g\} + A\,E\{g^{1+\Theta}\}\right]/A \qquad (1.2.3.b17)$$

$$E\{R^B\} = E\{g^{-\Theta}\}/\delta q \qquad (1.2.3.b18)$$

$$E\{R^c\} = E\{g^{-\Theta}\}\left[1 + QE\{g^{\Theta}\}\right]/Q. \qquad (1.2.3.b19)$$

Table 1.3 presents results from numerical solutions for time-separable preferences, relative consumption and habit formation. The first set of figures show the results calculated under an i.i.d process and those in brackets, under a lognormal distribution for g.

Table 1.3

Unconditional expected returns

γ	Stocks	Risk-less Asset	Consols
A. Time -separable preferences ($\psi = 0$)			
0.5	1.93	1.87	1.87
	(1.93)	(1.87)	(1.87)
1.0	2.83	2.70	2.70
	(2.83)	(2.70)	(2.70)
6.0	10.34	9.52	9.52
	(10.33)	(9.51)	(9.51)
10.0	14.22	12.85	12.85
	(14.13)	(12.72)	(12.72)
B. "Catching up with the Joneses" ($\psi = 1$; $D = 0$)			
0.5	2.80	2.76	2.73
	(2.80)	(2.76)	(2.73)
1.0	2.83	2.70	2.70
	(2.83)	(2.70)	(2.70)
6.0	6.70	2.07	5.84
	(6.72)	(2.06)	(5.86)
10.0	14.73	1.59	13.16
	(14.95)	(1.55)	(13.32)

C. "Habit formation ($\psi = 1; D = 1$)			
0.86	33.56	4.53	35.25
0.94	6.83	3.48	7.44
1.00	2.83	2.70	2.70
1.06	8.43	1.93	7.40
1.14	38.28	0.93	35.16

Source: Abel (1990)

Panel A reports results for time-separable preferences which indicates that at no stage is the model able to yield an equity premium close to that observed in the data. The closest the model comes to matching the observed data is in panel B (catching up with the Joneses), where the equity premium is 4.63 per cent in the presence of a risk-less rate of 2.07 per cent. The results are even less encouraging for the habit consumption model (Panel C) where the stock and consols returns are greater than 35 percent when γ equals 1.14. Here also, the results are very sensitive to the choice of γ as a value of 1.06 will generate an equity premium of 6.5 percent. Overall, the results are not encouraging for this form of the consumption model.

Even though the sensitivity of the model can be considered a problem, a larger one looms regarding values for the dividend-price ratio in relation to both the "catching up with the Joneses" and habit formation models. In the habit formation model for example, substituting the chosen parameter values results in a dividend-price ratio of over 100 which is about 5 times the size of the observed value. A further problem with the Abel model relates to the assumption that habit $X_t = C_{t-1}$. One implication of this model is that consumption can fall below habit which is inconsistent with accepted theory. Again, an endogenously determined discount rate provides an opportunity to explicitly model the discount rate which is prone to values in excess of 1 when endogenously determined.

c. <u>The Pemberton (1993) Approach</u>

Pemberton (1993) also presented a habit based model in the Kahnemann and Taversky (1979) framework i.e. prospect theory, whose central theme is choice under uncertainty. Two important features of Prospect Theory are firstly, that of diminishing sensitivity where the utility function is usually concave from below for gains relative to the reference point and convex from below for losses. The second feature is loss aversion where losses and gains of the same order have different magnitudes, so that the utility function is steeper in the negative region than the positive one.

In an intertemporal framework and unlike Constantinides (1990), the utility function in the general form is $u = u(C, X)$ where X is the reference level of consumption. This can be written in the more expansive form,

$$u(t) = \theta \frac{X(t)^{1-\gamma}}{1-\gamma} + (1-\theta)\frac{(C(t) - X(t))^{1-\gamma}}{1-\gamma}, \qquad (1.2.3.c1)$$

where *for* $C(t) \geq X(t), 0 \quad 0 < \gamma < 1$ or

$$u(t) = \theta \frac{X(t)^{1-\gamma}}{1-\gamma} - (1-\theta)\lambda \frac{(X(t) - C(t))^{1-\gamma}}{1-\gamma}, \qquad (1.2.3.c2)$$

where $C(t) < X(t), \lambda > 1, 0 < \gamma < 1$

where γ is the curvature parameter and λ is a sensitivity function. The first term of (1.2.3.*c*1) and (1.2.3.*c*2) is the constant elasticity function of the reference rather than the actual level of consumption with the parameter θ, which weights the influence on overall utility. The second term on the right hand side indicates deviations from the reference consumption level and their form ensures diminishing sensitivity whilst $\lambda > 1$ ensures loss aversion. The sensitivity function measures the impact of consumption changes on changes to utility and ultimately on the equity premium. Higher values for the sensitivity function implies the same for the equity premium.

That the reference consumption is implicitly the starting level from which gambles are taken means that the reference level does not automatically translate into an intertemporal framework. Pemberton presented a budget constraint of the form

$$C(t) = d(t)\,e(t-1) + p(t)^{e}\left[e(t-1) - e(t)\right] + f(t-1) - p(t)^{f} f(t), \quad (1.2.3.c3)$$

where *e(t)* and *f(t)* is the amount of equity and of one-period bills respectively, which the consumer holds in period *t* and to be held over to period *t+1* and *d(t)* is the dividend payment in period *t*. Though both equity and bills carry consumption capabilities, bills provide a guaranteed return whereas equity returns can be quite volatile i.e. equity has associated with it, surprises; pleasant or otherwise. According to Pemberton (1993), "my basic hypothesis about the reference consumption level is that it incorporates previously expected or planned equity outcomes, but not the current realisation of the stochastic element of the equity return". The term [*C(t)-X(t)*] reflects the "surprise" element in the equity returns. The reference consumption level is then defined by

$$X(t) = e(t-1)E_{t-1}\left[d(t) + p(t)^{e}\right] - E_{t-1}\left[p(t)^{e} e(t)\right] + f(t-1) - p(t)^{f} f(t). \quad (1.2.3.c3)$$

The first two terms on the right hand side reflect the contribution to consumption in period *t* of the expected dividend received and equity price and on planned equity retentions, sales and disposals. Combining the budget constraint and the reference consumption level,

$$C(t) - X(t) = e(t-1)\left[d(t) - E_{t-1}d(t)\right] + \left\{e(t-1)\left[p(t)^{e} - E_{t-1}p(t)^{e}\right] - \left[e(t)p(t)^{e} - E_{t-1}p(t)^{e}e(t)\right]\right\}. \quad (1.2.3.c5)$$

Firstly, since bill returns are predictable, they do not feature in (1.2.3.c5). The first term on the right hand side reflects the stochastic element of dividend payments whilst the second, in braces, reflect the unanticipated capital gains or changes in equity holdings relative to planned holdings. Pemberton concluded that the realised surprise in equity returns has an effect on utility via consumption directly.

Pemberton proceeded to maximise the utility function as per Mehra and Prescott subject to the same conditions but also to (1.2.3.c1) - (1.2.3.c5) to derive the following first order conditions:

$$\frac{\partial H(t)}{\partial f(t)} = -\theta p(t)^{f} X(t)^{-\gamma} + \delta\theta E_t X(t+1)^{-\gamma} = 0, \qquad (1.2.3.c6)$$

$$\frac{\partial H(t)}{\partial e(t)} = -(1-\theta)p(t)^{e}\lambda(t)(C(t)-X(t))^{-\gamma} + \delta\theta E_t\left(d(t+1)+p(t+1)^{e}\right)X(t+1)^{-\gamma}$$
$$+\delta(1-\theta)E_t\lambda(t+1)\left[\left|d(t+1)+p(t+1)^{e}-E_t\left(d(t+1)+p(t+1)^{e}\right)\right|\right]$$
$$\left(\left|C(t+1)-X(t+1)\right|\right)^{-\gamma} = 0. \qquad (1.2.3.c7)$$

To solve, k(T) is assumed equal to p(T)^{e}/C(T) as well as $C(t) = d(t)$, $e(t)=1$ and $f(t)=0$ for all t such that substituting (1.2.3.c3 - 1.2.3.c5) into (1.2.3.c1 - 1.2.3.c2) yields, $X(t) = [E(g)/g(t)]C(t)]$ and $E_t X(t+1) = E_t C(t+1)$.[6] Equation (1.2.3.c7) can now be rewritten as

$$(1-\alpha)k(t)\lambda(t)g(t)^{\gamma}\Delta^{-\gamma} = \delta\alpha E_t(1+k(t+1))g(t+1)(E(g))^{-\gamma}$$
$$+\delta(1-\alpha)E_t\lambda(t+1)\left[\left|(1+k(t+1))g(t+1)-E_t((1+k(t+1))g(t+1))\right|\right]\Delta^{-\gamma}, \qquad (1.2.3.c8)$$

where $\Delta = g_1 - \mathrm{E}(g) = \mathrm{E}(g) - g_2$.

[6] Such that k measures the price-dividend ratio.

According to Pemberton, "the expected value of the right hand side of (1.2.3.c8) is independent of the realisation of growth $g(t)$ in period t. On the left hand side, $\lambda(t) = 1$ or λ as $g(t) = g_1$ or g_2. Since the left hand side of (1.2.3.c8) must equal the right hand side for either realisation of growth, it follows that the price-dividend ratio $k(t)$ must take one of two values k_1 and k_2 as $g(t)$ equals g_1 or g_2".

Solutions for the price-dividend ratios, k_1 and k_2 are given by [7]

$$k_1 = \frac{\delta\theta(Eg)(Eg^{-\gamma}) + 0.5\delta(1-\theta)(1-\lambda)\Delta^{1-\gamma}}{(1-\theta)g_1^{\gamma} - 0.5\delta\theta(g_1 + Ag_2)(Eg)^{-\gamma} - 0.25\delta(1-\theta)(1+\lambda)(g_1 + Ag_2)\Delta^{-\gamma}}, \quad (1.2.3.c9)$$

$$k_2 = Ak_1, \qquad A = (1/\lambda)(g_1/g_2)^{\gamma}. \quad (1.2.3.c10)$$

Along the equilibrium path, (1.2.3.c6) can be rearranged such that

$$p_i^f = \delta g_i^{\gamma}, \qquad i = 1,2. \quad (1.2.3.c11)$$

From Mehra and Prescott equations, Pemberton derived the expected return on equity conditional on the realisation of growth rate g_i, i=1, 2 in any given period denoted by

$$0.5[g_i(1+k_1) + g_2(1+k_2)] / k_i - 1$$

whilst the unconditional expected equity return is

$$R^e = 0.25[g_1(1+k_1) + g_2(1+k_2)](k_1 + k_2)/k_1k_2 - 1. \quad (1.2.3.c12)$$

---

[7] The value of 0.5 follows from the assumption of an i.i.d process.

The rate of return on bills i.e. the risk-less return, conditional on realised growth rate is

$$\left[1 / p_i(t)^f\right] - 1 = \left(1 / \delta g_i^\gamma\right) - 1, \quad i = 1,2, \qquad (1.2.3.c13)$$

whilst the unconditional expected risk-free rate of return is

$$R^f = 0.5\left(g_1^\gamma + g_2^\gamma\right) / \delta g_1^\gamma g_2^\gamma - 1. \qquad (1.2.3.c14)$$

Table 1.4 shows the Pemberton results for the equity premium for which the following assumptions were made. Firstly, that $\gamma = 0.1$, $\delta = 0.98$, $g_1 = 1.054$, $g_2 = 0.982$ and growth is i.i.d. Secondly, the risk-less return, r^f equals 0.0187 which is based on the previously mentioned work of Siegel (1992) and finally, the dividend-price ratio is assumed to be 20 which is consistent with observed data.

Table 1.4

The equity premium

λ	R^e	$R^e - R^f$	θ
1.04	0.0694	0.0507	0.5631
1.15	0.0745	0.0558	0.5611
1.20	0.0803	0.0616	0.5598
1.25	0.0855	0.0688	0.5582
1.50	0.1222	0.1035	0.5470
1.75	0.1623	0.1436	0.5306
2.00	0.2156	0.1969	0.5086

Source: Pemberton (1993)

With a value for λ of 1.04, the equity return almost exactly matches the observed data though not the equity premium owing to the very high risk-less rate assumed. A value for λ of 1.20 is able to generate the historically observed equity premium though for a value of 1.25 for λ, the equity return estimates become unrealistically high. According to Pemberton, this amounts to a solution to the equity premium puzzle.

However, the model raises a number of issues that would benefit from further enquiry. Firstly, the process for the discount rate is not modelled endogenously; historically observed estimates are used. This is particularly important in the context of a value for γ of 0.1 because a value of this magnitude tends to be associated with a discount rate greater than 1 which amounts to a negative time preference. Similarly, values for the price-dividend ratio could be endogenously determined. Secondly, the choice of parameter values is quite crucial given that the model is very sensitive to such choices. The values for the risk-less return could have ranged from 0.008 to 0.0187 whilst that for γ could have taken any value up 2. Explicit modelling of these choices would avoid much uncertainty. Pemberton does not propose a process for assessing risk jointly between γ and λ, the loss aversion parameter, even though the suggestion is that they jointly account for risk attitudes.

d. <u>The Cooley and Ogaki (1991,1996) Approach</u>

Cooley and Ogaki (1996) "re-examines whether the time series properties of aggregate consumption, real wages and asset returns are consistent with a simple neoclassical representative agent economy" which has often been rejected "because the marginal rate of substitution between consumption and leisure does not equal the real wage implied by the first-order conditions of the model". Cooley and Ogaki (1996) also attempted an estimation of the long-run elasticity of intertemporal substitution for non-durable consumption which is used in the asset pricing formulation implied by the economy to derive parameter values for the discount rate and time-non-separability. The model is tested in the Hansen and Singleton (1992) Generalised Method of Moments framework to be discussed later in this chapter.

The authors introduced an economy with a population of N households who maximise utility with a function of the form

$$U = E_0\left[\sum_{t=}^{\infty} \delta^t u(t)\right] \qquad (1.2.3.d1)$$

where E_t denotes expectation conditioned on information in time t. Before proceeding to discuss time non-separability, Cooley and Ogaki presented a simple intra-period utility function which is assumed to be time and state separable in non-durable consumption, durable consumption and leisure:

$$U(t) = \frac{1}{1-\gamma}\left(C_t^{1-\gamma} - 1\right) \quad + v\left(l(t)\right) \qquad (1.2.3.d2)$$

where $v(.)$ is a "continuously differentiable" concave function, $C(t)$ is non-durable consumption, γ is the curvature parameter and $l(t)$ is leisure. The authors assume that real wages do not contain an insurance component such that the usual first-order condition for a household that equates the real wage with the marginal rate of substitution between consumption and leisure is

$$W(t) = \frac{v'\left(l(t)\right)}{C(t)^{-\gamma}}, \qquad (1.2.3.d3)$$

where $W(t)$ is the real wage rate. Assuming that leisure is a strictly stationary process, they concluded that the log of the real wage rate and the log of consumption are cointegrated with a cointegrating vector $(1, -\gamma)'$ which the authors used to identify the curvature parameter γ via cointegrating regressions. They concluded that this parameter could be different from one.

For time-non-separability, the intra-period utility function is written as

$$u(t) = \frac{1}{1-\gamma}\left(F_t^{1-\gamma} - 1\right) + v\left(l(t)\right), l(t-1), \ldots, l(t-k) \qquad (1.2.3.d4)$$

where F_t is the service flow from consumption purchases which is related to purchases of consumption by

$$F(t) = C(t) + \theta\, C(t-1). \qquad (1.2.3.d5)$$

According to (1.2.3.d5), when θ is negative then we have habit formation. This is the Cooley and Ogaki habit formation model. When θ is positive, this implies local substitutability or durability. Cooley and Ogaki also assumed that the long run elasticity of intertemporal substitution is the inverse of the curvature parameter such that $1/\gamma$.

The first-order condition for a household that equates the real wage rate to the marginal rate of substitution between leisure and consumption is given by

$$W(t) = \frac{\partial U/\partial l(t)}{\partial U/\partial C(t)} = \frac{E_t\left[\sum_{\tau=0}^{k} \delta^{\tau} \partial u(t+\tau)/\partial l(t)\right]}{E_t\left[\partial u(t)/\partial C(t) + \partial u(t+1)/\partial C(t)\right]}$$

$$= \frac{E_t\left[\sum_{\tau=0}^{k} \delta^{\tau} \partial v(t+\tau)/\partial l(t)\right]}{E_t\left[F(t)^{-\gamma} + \delta\lambda F(t+1)^{-\gamma}\right]}. \qquad (1.2.3.d6)$$

If ln $(C(t))$ is difference stationary in equilibrium, then

$$F(t-\tau)/C(t) = C(t+\tau)/C(t) + \lambda C(t+\tau+1)/C(t). \qquad (1.2.3.d7)$$

Combining (1.2.3d6) and (1.2.3d7) leads us to the conclusion that

$$W(t)C(t)^{-\gamma} = \frac{E_t\left[\sum_{\tau=0}^{k} \delta^{\tau} \partial v(t+\tau)/\partial l(t)\right]}{E_t\left[\{F(t)/C(t)\}^{-\gamma} + \delta\lambda\{F(t+1)/C(t)\}^{-\gamma}\right]} \qquad (1.2.3.d8)$$

is stationary.

Cooley and Ogaki proceeded to show that the formulation can also hold for aggregated households such that

$$\frac{E_t\left[\delta\left\{F_a(t+1)^{-\gamma}+\delta\theta F_a(t+2)^{-\gamma}\right\}R(t+1)\right]}{E_t\left[\left\{F_a(t)^{-\gamma}+\delta\theta F_a(t+1)^{-\gamma}\right\}\right]}=1 \qquad (1.2.3.d9)$$

where a denotes the aggregated data.

Using the standard asset pricing formulation with aggregate service flows, labour income, y_t, is

$$y(t)=\frac{E_t\left[\delta\left\{F_a(t+1)^{-\gamma}+\delta\theta F_a(t+2)^{-\gamma}\right\}W(t+1)[1-l(t+1)]\right]}{E_t\left[F_a(t)^{-\gamma}+\delta\theta F_a(t+1)^{-\gamma}\right]}. \qquad (1.2.3.d10)$$

To estimate the consumption curvature parameter and to test the model, Cooley and Ogaki combined both the cointegration approach of Ogaki and Park (1989) and Hansen and Singleton's (1992) GMM approach. Assuming that γ is the reciprocal of the long run intertemporal elasticity of substitution, the econometric model used for the GMM procedure is based on (1.2.3.d9) which implies that

$$E_t\left(\varepsilon_g^0(t)\right)=0,$$

where

$$\begin{aligned}\varepsilon_g^0 = {} & \delta\left[\left(C_a(t+1)+\theta C_a(t)\right)^{-\gamma}+\theta\delta\left(C_a(t+2)+\theta C_a(t+1)\right)^{-\gamma}\right]R(t+1)\\ & -\left[\left(C_a(t)+\theta C_a(t-1)\right)^{-\gamma}+\theta\delta\left(C_a(t+1)+\theta C_a(t)\right)^{-\gamma}\right] \qquad (1.2.3.d11)\end{aligned}$$

and where C_a indicates aggregate non-durable consumption.

Table 1.5 (Panel A) reports the cointegrating regression results from Cooley and Ogaki (1996). In column 1, the regressand is indicated as *w* (*ln(W)*) or *c* (*ln(C)*) with measures of consumption and the implicit deflator used to yield the real wage rate are indicated as NDS, ND and Food. Columns 2 and 3 details the estimations for the curvature parameter and its inverse, the elasticity of intertemporal substitution respectively with standard errors in parentheses. In Column 4 is the χ^2 test statistic for the deterministic cointegration restriction whilst columns 5-7 outlines χ^2 test statistics for stochastic cointegration. For columns 4-7, asymptotic *P*-values are in parentheses.

Table 1.5

Panel A : Canonical cointegrating regression results

Regressand (1)	γ (2)	$1/\gamma$ (3)	*H*(0,1) (4)	*H*(1,2) (5)	*H*(1,3) (6)	*H*(1,4) (7)
w, NDS	1.103	0.907	5.644	0.400	7.249	10.805
	(0.075)	(0.094)	(0.018)	(0.527)	(0.027)	(0.013)
c, NDS	1.916	0.522	0.033	1.944	3.039	4.085
	(0.214)	(0.112)	(0.856)	(0.163)	(0.219)	(0.252)
w, ND	2.361	0.424	0.744	2.060	7.243	8.584
	(0.098)	(0.019)	(0.388)	(0.151)	(0.027)	(0.035)
c, ND	2.370	0.422	1.486	0.005	0.032	0.062
	(0.142)	(0.025)	(0.223)	(0.942)	(0.984)	(0.996)
w, Food	2.939	0.340	1.297	1.774	1.808	3.260
	(0.226)	(0.027)	(0.255)	(0.183)	(0.405)	(0.353)
c, Food	2.870	0.348	0.228	0.089	1.446	3.798
	(0.221)	(0.028)	(0.633)	(0.765)	(0.485)	(0.284)

Source : Cooley and Ogaki (1996)

The estimates for the curvature parameter are positive and therefore consistent with theory, with the values remaining well within the maximum value of 10 suggested by Mehra and Prescott (1985). When the consumption measure is defined in terms of non durables, the point estimate is around 1, the value thought plausible by the neo classical Arrow-Debreu model.

Table 1.5 (panel B) presents results using conventional instrumental variables of consumption and asset returns where a constant, the real consumption growth rate lagged one period, real gross stock return lagged one period and real gross treasury bills lagged one period are used as instruments whilst panel C uses financial instrumental variables such as dividend yield, yield spread, value weighted stock returns and a constant for the treasury bill rate. Column 1 reports the measure of non durable consumption whilst columns 2-4 reports parameter estimates with standard errors in parentheses. Column 5 reports Hansen's *J*-test with asymptotic *P*-values in parentheses. Column 6 is the degree of freedom of Hansen's *J* in column 5. Column 7 reports the likelihood ratio type test with one degree of freedom for the restriction imposed with *P*-values in parentheses.

Table 1.5, Panel B : *Specification test results based on the asset pricing equation*

Regressand (1)	γ (2)	δ (3)	θ (4)	J_T (5)	DF (6)	C_T (7)
(1) Using consumption and returns as instruments						
NDS (R)	1.103	0.995	-0.522	9.573	4	...
		(0.003)	(0.120)	(0.048)		
NDS (U)	6.729	1.020	-0.181	6.039	3	3.534
	(2.948)	(0.013)	(0.148)	(0.110)		(0.060)
ND (R)	2.361	0.999	-0.143	7.150	4	...
		(0.002)	(0.089)	(0.128)		
ND (U)	3.092	1.001	-0.101	6.859	3	0.291
	(1.346)	(0.004)	(0.103)	(0.077)		(0.590)
Food (R)	2.939	0.589	-0.958	10.577	4	...
		(0.161)	(0.008)	(0.032)		
Food (U)	2.642	0.607	-0.960	10.545	3	0.032
	(2.988)	(0.360)	(0.017)	(0.014)		(0.858)

Source::Cooley and Ogaki (1996)

Table 1.5, Panel C : *Specification test results based on the asset pricing equation*

Regressand (1)	γ (2)	δ (3)	θ (4)	J_T (5)	DF (6)	C_T (7)
(2) Using financial instruments						
NDS (R)	1.103	0.992	-0.773	6.472	4	...
		(0.007)	(0.116)	(0.039)		
NDS (U)	7.129	1.011	-0.475	5.811	3	0.661
	(6.793)	(0.023)	(0.282)	(0.016)		(0.416)
ND (R)	2.361	0.996	-0.284	6.471	4	...
		(0.005)	(0.331)	(0.039)		
ND (U)	4.280	1.000	1.000	6.416	3	0.055
	(3.130)	(0.025)	(267.3)	(0.011)		(0.814)
Food (R)	2.939	0.820	-0.893	3.256	4	...
		(0.100)	(0.047)	(0.196)		
Food (U)	0.447	0.844	-0.971	2.802	3	0.454
	(2.075)	(0.151)	(0.040)	(0.094)		(0.501)

Source:Cooley and Ogaki (1996)

The statistically negative value for θ implies habit formation whilst the specification test, C_T test, does not reject the model at the 5% level. Cooley and Ogaki further concluded that the curvature value γ is the reciprocal of the long run elasticity of intertemporal substitution (IES). Finally, asset pricing equations for stock and nominal risk-free returns "... are satisfied when the second component of the cointegrating vector is used as the reciprocal of the long run IES".

Later on in chapters 3 and 4, Campbell and Cochrane (1995) and Campbell, Lo and Mackinlay (1997) reject the implication of the Cooley and Ogaki (1991, 1996) model that γ is the reciprocal of the long run elasticity of intertemporal substitution (IES). Furthermore, a clear implication of the habit specification is that consumption is allowed to fall below habit, but which does not accord with theory. For some of the results, the implied discount rate exceeds 1, which implies the existence of a negative rate of time preference. Even so, Cooley and Ogaki establish a role for habit consumption in linking asset returns and real economy.

e. The Campbell & Cochrane (1995) Approach[8]

Another model based on habit and the implicitly, the surplus consumption, is the Campbell and Cochrane (1995) formulation. This model is discussed in more detail in chapter 3 but a summary is presented here.

Essentially, they assumed a utility function of the form

$$E\sum_{t=0}^{\infty}\delta^t \frac{(C_t - X_t)^{1-\gamma} - 1}{1-\gamma}, \qquad (1.2.3.e1)$$

[8] This formulation is similar to that presented in Campbell, Lo and Mackinlay (1997).

where X is habit or reference consumption and γ is the curvature parameter. Campbell and Cochrane then defined a path for habit from which equations for the discount rate, δ, as well as for the risk-less and risky assets are derived. In chapter 3, there is a full discussion of the Cochrane and Campbell (1995) model together with results of our own replications of the model.

In section 1.5, there follows a summary of the conclusions from the consumption-based asset pricing models reviewed as well as the possible directions for our own research.

1.2.4. Imperfect Markets and Transactions Costs

Much of the work reviewed so far, including the original outline of the consumption-based asset-pricing problem by Mehra and Prescott (1985), implies a neo-classical approach where markets are broadly perfect. This means that capital markets clear and that markets are generally found in equilibrium. Here, attention turns to efforts which incorporate the notion of imperfect markets in attempting to solve the equity premium puzzle.

Aiyagari and Gertler (1991) explored the impact of incorporating a motive for holding liquid assets into an equilibrium asset pricing framework to help explain features of post-war US data i.e. the low real risk-free rate, the large spread between liquid asset returns and stock returns and the greater transaction velocity of the liquid assets relative to stock. They introduced a demand for liquid assets via uninsured individual risk and differential costs of securities trading. In this formulation, the economy moves out of the frictionless framework. More specifically, the model assumed an incomplete securities market as well as transactions costs where individuals face "idiosyncratic shocks to personal income". Since markets for claims on personal income are assumed not to exist, individuals "must self-insure i.e. buy and sell assets to smooth consumption". The formulation further assumed the existence of two securities; stocks and treasury bills, where stocks are assumed to be costly to trade and bills are freely exchanged. Finally, bills are assumed to be held directly by the household or repackaged by an intermediary who issues them to depositors unlike stocks. Implicitly then, bills are assumed to "have an edge over stocks as a vehicle for self-insurance".

The economy considered is a stationary, infinite horizon, pure exchange economy of type previously considered of the form

$$E_0\left\{\sum_{i=0}^{\infty}\delta^t U\left(C_t^i\right)\right\}, \quad 0 < \delta < 1, \tag{1.2.4a}$$

where time is discrete as represented by time t, C_t^i is consumption by individual i at time t, δ is the subjective discount factor and $E_0\{\ .\ \}$ is the mathematical expectation operator conditioned on information at time zero.

The individual is assumed to have two sources of the perishable consumption good; capital and labour. There exist $\overline{m}$ capital machines which costlessly produces output in each period and whose proceeds are distributed as dividends to shareholders who own the machines. Implicitly, there are $\overline{m}$ equity claims which are tradable and perfectly divisible and where one claim entitles the owner to $1/\overline{m}$ percent of the total output from all the machines in each period. The output per machine, d, is assumed to be constant over time. With regard to the labour source of perishable consumption good, each individual i receives an endowment of the consumption good, y_t^i, which obeys a stationary Markov chain. While individual labour incomes are assumed to be independent and therefore highly variable, per capita labour income is assumed to be smooth. Furthermore, while a market for capital income claims exists, none exists for labour income. Aiyagari and Gertler (1991) therefore interpreted variations in y_t^i as reflecting uninsured individual risk.

Aiyagari and Gertler then introduced a government sector to the economy which consumes g units per capita in each period and which is financed with a per capita lump sum tax, τ, and by the issuance of treasury bills. The government budget constraint can therefore be written as

$$g + \bar{b}_t = \tau + \bar{b}_{t+1}/(1 + r_t), \qquad (1.2.4b)$$

where $\bar{b}_t$ is the per capita quantity of treasury bills at the beginning of period t in terms of market value and r_t is the risk – less interest rate from t to $t+1$

In each period, the individual decides the level of consumption and amounts of stock and treasury bills to acquire. Trading in stocks is then assumed to carry a cost proportional to the value of the trade where α_b is the per unit buying cost and α_s is the per unit selling cost. The individual i's budget constraint is given by

$$\begin{aligned} &C_t^i + p_t\left(k_{t+1}^i - k_t^i\right) + b_{t+1}^i/(1 + r_t) \\ &= y_t^i + k_t^i d + b_t^i - \tau - \max\left\{\alpha_b p_t\left(k_{t+1}^i - k_t^i\right), \alpha_s p_t\left(k_t^i - k_{t+1}^i\right)\right\} \end{aligned} \qquad (1.2.4c)$$

where p_t is the period t price of equity.

In this formulation, short sales and borrowing are not allowed such that the following restrictions apply:

$$h_t^i \geq 0 \qquad (1.2.4d)$$

$$b_t^i \geq 0. \qquad (1.2.4e)$$

This assumption, which is later relaxed, is intended to isolate the individual's decision whether to buy and sell stocks. This requires that there exist no other source of funds to the individual. Aiyagari and Gertler then defined $F(k, b, y)$, the steady state, as the "joint cross-section distribution" of stock holdings at the beginning of t, bond holdings at the beginning of t, and labour income realisation at time t. In other words,

$$F(k,b,y) = \text{fraction of people at the beginning of } t \text{ for whom: } (k_t, b_t, y_t) \leq (k, b, y). \qquad (1.2.4f)$$

Individual labour incomes, which follow a Markov process can be written as

$$Y(y', y) = \text{prob}\left[y_{t+1} \leq y' \middle| y_t = y\right]. \qquad (1.2.4g)$$

Given the absence of aggregate uncertainty, the steady state consists of a constant over time stock price, p, a constant return on bonds, r, a constant per capita quantity of bonds, b, and a cumulative distribution function F(k, b, y) which is considered consistent with individual optimisation, the government budget constraint (1.2.4b) and market clearing at each date. Aiyagari and Gertler then used the Bellman equation for dynamic programming to describe a typical individual's dynamic optimisation. To look for the stationary equilibrium in which the interest rate, r, the stock price, p, and the cross-section distribution of asset holdings and income F(.) are all constant over time, they assumed that government expenditures and per capita bonds and taxes are constant over time. Consequently, the government budget constraint can be written as

$$g + r\bar{b}/(1+r) = \tau. \qquad (1.2.4h)$$

Aiyagari and Gertler also argued that "another advantage of fixing dividends is that we can isolate the impact of the frictions we have introduced. Since there is no dividend risk, any spread between the returns on stock and bonds is due only to the transactions costs operating in conjunction with the uninsured income risk".

Aiyagari and Gertler specified the computational procedure, which firstly involves specifying values for asset returns and taxes and then finding values for asset stocks and government purchases which support the aforementioned asset returns in equilibrium. They also choose values for the preference and technology parameter as well as for r_s, r and τ. According to Aiyagari and Gertler, "how successful the model is in explaining a particular configuration of asset returns then depends on how well the computed asset/income ratio and relative transaction velocities match with observed data". They assumed a Markov process for income and that agents can only buy and sell in discrete units, where they further assume that a unit equals 10 per cent of quarterly income. Furthermore, they also assumed that there are upper bounds to the quantity of stocks and bonds that can be held.

Consequently, they defined transaction velocities of the form

$$TVS = \frac{1}{2} \mathrm{E}\{ |\sigma_s - k\} \Big/ \bar{k} , \tag{1.2.4i}$$

$$TVB = \frac{1}{2} \mathrm{E}\{ |\sigma_b - b|\} \Big/ \bar{b} , \tag{1.2.4j}$$

where σ_k and σ_b are given by decision rules and

$$k'^{*} = \sigma_k(z') \tag{1.2.4k}$$

$$b'^{*} = \sigma_b(z') \tag{1.2.4l}$$

For parameters, Aiyagari and Gertler chose the values in the following way:

For preferences

$$\delta = 0.96(\text{annual}) \text{ and} \tag{1.2.4m}$$

$$U(C) = -(C^{-1} - 1). \tag{1.2.4n}$$

For the income process, Aiyagari and Gertler assumed a three-state Markov chain where the states are unemployment (u), low employment (l) and high employment (h). The low and high employment states are treated symmetrically such that the probability matrix is of the form

$$\pi^y = \begin{matrix}(u)\\(l)\\(h)\end{matrix}\begin{bmatrix} \pi_u & (1-\pi_u)/2 & (1-\pi_u)/2 \\ 1-\pi_e & \pi_e/2 & \pi_e/2 \\ 1-\pi_e & \pi_e/2 & \pi_e/2 \end{bmatrix}. \qquad (1.24o)$$

The terms π_u and π_e are determined from the following forms:

$$\theta_u = (1-\pi_e)/[(1-\pi_e)+(1-\pi_u)], \qquad (1.24p)$$

$$D_u = 1/(1-\pi_u), \qquad (1.24q)$$

where θ_u is the fraction of people in the unemployment state in a stationary equilibrium and D_u is the duration of unemployment. Assuming values for θ_u and D_u of 0.05 and 1.5 quarters to match the actual numbers, then

$$\pi^y = \begin{bmatrix} 0.3400 & 0.3300 & 0.3300 \\ 0.0350 & 0.4825 & 0.4825 \\ 0.0350 & 0.4825 & 0.4825 \end{bmatrix}. \qquad (1.24r)$$

Aiyagari and Gertler then determined income levels given the states where θ_e is the fraction of people in the employment state, l, which is also assumed to equal the fraction of people in employment state, h. Consequently,

$$\theta_e = (1-\theta_u)/2. \qquad (1.24s)$$

They further assumed that income while employed fluctuates by up to 30 per cent relative to y_e and also that income in the unemployed state is 30 per cent of average income. They concluded from this that

$$y^* = \theta_u y_u + \theta_e(y_l + y_h) = 1 \qquad (1.24t)$$

$$y_u = 0.3y_e, \quad y_l = 0.7y_e, \quad y_h = 1.3y_e \qquad (1.24u)$$

Since y^* is normalised to unity, the above formulations yield[9]

$$y_u = 0.3100, \quad y_l = 0.7254, \quad y_h = 1.3470. \tag{1.2.4v}$$

For transaction costs, buying and selling costs are assumed to be equal and experimenting with different values for the transactions costs parameter yields

$$\alpha_b = \alpha_s = \alpha \in \{0.02, 0.035, 0.05\}. \tag{1.2.4w}$$

For asset returns and asset/income ratios, the chosen values for asset returns and taxes are

$$r = 0, \tag{1.2.4x.1}$$

$$r_s = d/p = 0.03 \text{ (annual)} \tag{1.2.4x.2}$$

$$\tau = 0. \tag{1.2.4x.3}$$

They calculated a figure for the average annual (1949 - 1978) real Treasury Bill return of zero though they set the real stock return at 3 percent with a standard deviation of 7.03 per cent even though the historically observed real stock return is about 7.7 percent. They set the real stock return at 3 percent so as to avoid the claim that transaction costs are the sole explanation for the observed return differential. Finally, taxes are set to zero, which allows for some simplification, since at $r = 0$, the implied value of g is also zero, regardless of bonds.

The authors have already indicated the importance of asset/income ratios in assessing the performance of the model. They suggested that

$$\bar{k}/y = 0.65, \tag{1.2.4y.1}$$

$$\bar{b}/y = 0.35. \tag{1.2.4y.2}$$

[9]Aiyagari and Gertler argued that the representation of the income process is based on observed data and other studies in the field.

Aiyagari and Gertler reported results as follows:

Table 1.6
Panel A
Asset returns, transactions costs and uninsured individual risk

$R_s = 0.03$				
α	=	0.02	0.035	0.05
k/y	=	0.69	0.65	0.60
b/y	=	0.07	0.10	0.12
TVS	=	0.08	0.07	0.06
TVB	=	1.44	1.15	0.99
(all annual figures)				

Source: Aiyagari and Gertler (1991)

Where the stock return is assumed to equal 3 percent, the stock to income ratio matches the observed value quite well though the liquid assets to income ratio does not. The liquid assets transaction velocity is found to be about 16 times that of stocks.

The results in table 1.6 give an indication of the sensitivity of small changes to the spread i.e. where r_s is 2.6 percent (Panel B) and 3.4 percent (Panel C).

Table 1.6
Panel B
Asset returns, transactions costs and uninsured individual risk

$r_s = 0.026$				
α	=	0.02	0.035	0.05
k/y	=	0.60	0.54	0.49
b/y	=	0.07	0.11	0.14
TVS	=	0.09	0.07	0.06
TVB	=	1.40	1.15	0.91
(all annual figures)				

Source: Aiyagari and Gertler (1991)

Table 1.6
Panel C
Asset returns, transactions costs and uninsured individual risk

$r_s = 0.034$				
α	=	0.02	0.035	0.05
k/y	=	0.83	0.79	0.49
b/y	=	0.04	0.08	0.14
TVS	=	0.10	0.07	0.06
TVB	=	2.00	1.37	0.91
(all annual figures)				

Source: Aiyagari and Gertler (1991)

Even with a real stock return of 2.6 percent, Aiyagari and Gertler argued that "the average quantity of liquid assets is too low".

Table 1.6 (panel D) reports results for a model which incorporates the notion of costly borrowing. This is captured in the model by allowing negative values for bonds. Since there are transactions costs associated with borrowing, this is assumed to be a fixed percentage of the amount borrowed. The percentage borrowing costs is chosen to be 0.02 which implies an annual spread between the consumer loan rate and the risk-free rate of 8 percent which is consistent with historical data. They also imposed a credit limit on consumer loans of 40 percent of quarterly income.

Table 1.6
Panel D
Asset returns, transactions costs and uninsured individual risk - Impact of borrowing

$r_s = 0.03$				
α	=	0.02	0.035	0.05
k/y	=	0.61	0.61	0.56
b/y	=	0.05	0.04	0.07
TVS	=	0.08	0.06	0.06

TVB	=	*	*	*
LA/y	=	0.06	0.06	0.09
TVLA	=	1.48	1.32	1.10
(all annual figures)				

Source: Aiyagari and Gertler (1991)

LA and *TVLA* are the quantities of liquid assets and their transaction velocity respectively. The stock/income ratio becomes less sensitive to the transactions costs. Individuals have the option of borrowing to smooth out consumption thereby implying less need for a distress sale of stocks.

Table 1.6 (Panel E) reports results for a model that assumes that there are fixed transactions costs for trading stocks in addition to the constant marginal cost, α. The assumption made with regard to the borrowing level is maintained.

Table 1.6
Panel E
Asset returns, transactions costs and uninsured individual risk - transaction costs

$R_s = 0.03$				
α	=	0.02	0.035	0.05
k/y	=	0.58	0.53	0.49
b/y	=	0.07	0.09	0.12
TVS	=	0.07	0.05	0.05
TVB	=	*	*	*
LA/y	=	0.09	0.11	0.13
TVLA	=	1.42	1.16	0.99
(all annual figures)				

Source: Aiyagari and Gertler (1991)

The introduction of fixed transaction costs reduces the stock to income ratio as well as the transaction velocities of stocks and treasury bills though the relative transaction velocity of liquid assets to stocks remains largely unaffected. Most importantly, the ratio of liquid assets to income still falls short of observed values.

The main aim is to assess whether allowing an explicit demand for liquidity motive could help to resolve the risk-free and equity (stock) premium puzzles. They argued that the model performed satisfactorily on some grounds such as explaining the relative transaction velocities of stocks and liquid assets and the ratio of stocks to income but that it predicts too low a value for the ratio of liquid assets to income. Given the levels of stock ownership amongst households, where liquid assets are held by households who tend not to hold stocks and a small group of households who own a very high proportion of stocks, they concluded that allowing for heterogeneity of households could resolve the risk-free rate puzzle i.e. allowing for stockholding and non-stockholding households.

The Aiyagari and Gertler model determines the discount rate, $\delta = 0.96$. This is a potential weakness as many models that have endogenously dealt with the discount rate have reported values of over 1, which is consistent with a negative rate of time preference. They also set a "modest" stock return target of 3% in contrast to the observed rate of just under 8% but which does not provide as challenging a target as might otherwise have been the case. Studies that have tended to report a equity premium of the order of 3% have tended to do so in the context of a real risk-free rate of almost the same magnitude but which is inconsistent with the observed data.

Heaton and Lucas (1992) presented another attempt to investigate models with imperfect markets and transactions costs where they argue that "incomplete financial markets, coupled with undiversifiable idiosyncratic shocks, have the potential to explain a number of asset pricing puzzles. With trading costs or binding borrowing constraints, the risk-free rate falls and the risk premium rises relative to the complete markets case and the term structure exhibits a forward premium". Heaton and Lucas proposed a three-period, two person model where they are able to look at different trading frictions, borrowing constraints, short sales constraints, quadratic costs and proportional costs which they believed had the potential to solve the equity premium and low risk-free rate puzzles.

The economy has two agents distinguished by their income realisations which, like Aiyagari and Gertler, is derived from labour income, y^i, and aggregate stock dividend, d. Initially, each agent owns half the stock and holds no debt. Risk-free bonds are in zero net supply at time 0. The stock (equity) price is p^s and the bond price is p^b. Labour income is assumed to be uninsurable which implies that individual consumption volatility may be higher than aggregate consumption volatility as in Aiyagari and Gertler (1991).

Heaton and Lucas presented two variations of the basic model; firstly assuming asset markets close prior to the resolution of uncertainty and secondly, asset markets close after the resolution of uncertainty.

Model 1 : <u>Asset markets close prior to the resolution of uncertainty</u>

At time period 0, each agent i chooses consumption, c^i_0, stock (equity) purchases, e^i, and bond purchases, b^i, to maximise:

$$U\left(C_o^i\right) + \delta\, \mathrm{E}U\left(C_1^i\right), \qquad (1.2.4aa)$$

subject to the budget constraints

$$C_o^i = Y_o^i + \left(d_0/2\right) - b^i p^b - e^i p^s,$$

$$C_1^i = Y_1^i + \left(d_1/2\right) + b^i + e^i d_1. \qquad (1.2.4ab)$$

Market clearing requires

$$C_t^i + C_t^2 = Y_t^a + d_t, \qquad t = 0,1$$

$$e^1 + e^2 = 0,$$

$$b^1 + b^2 = 0. \tag{1.2.4ac}$$

Asset pricing satisfies

$$p^b = \delta \frac{\mathrm{E}[U'(C_1)]}{U'(C_0)}, \tag{1.2.4ad}$$

$$p^e = \delta \frac{\mathrm{E}[U'(C_1)\delta]}{U'(C_0)}. \tag{1.2.4ae}$$

Assuming that $U'''(C) > 0$, and that at time 0, all agents have equal labour and dividend income: $Y_0 + d_0/2$. At time 1, aggregate output is high with probability q and low probability $(1 - q)$. Aggregate dividend is $d_1 \in [0, d_H]$, and aggregate labour income is $Y_1^a \in [Y_L, Y_H]$. If realised output is high, each agent receives $Y_H/2$ whilst if realised output is low, then one agent receives $\varpi\, Y_L$, while the other receives $(1 - \varpi)\, Y_L$, with $\varpi \geq ½$.[10] According to the authors, this differential treatment can be viewed as the increased probability of becoming unemployed in a recession. Ex ante, each agent faces the same distribution of labour income and therefore cannot write a contingent claim on the realisation of this income. Owing to the notion that agents have the same wealth and information at time 0, then no trade occurs in each period. In which case, income can be substituted into (1.2.4*ad*) and (1.2.4*ae*) to find the price for stocks and bonds. Since the bond price is implied to increase in ϖ, then the equity premium is

[10] That $\varpi = ½$ is a common feature of standard representative agent models according to the authors.

$$E(d_1)/p^e - 1/p^b. \qquad (1.2.4af)$$

Heaton and Lucas then pointed to the assumption that $U'(0) = \infty$, for the equity premium becomes large as the share of income, λ, received by the employed agent approaches 1.[11] The authors referred to the work of Mankiw (1986) and Weil (1992) who suggested that when the stock (equity) premium is written as a ratio of the stock and bond (risk-less) returns, rather than their difference, the proportional premium increases with the level of idiosyncratic risk. As well as increasing the required return on stocks relative to bonds, idiosyncratic risk also has the effect of reducing required returns on stocks and bonds as there is an increased precautionary demand for assets. According to Heaton and Lucas "this analysis suggests that a model with incomplete markets has the potential to resolve the equity premium and risk-free rate puzzles".

Model 2 : Markets close after the resolution of uncertainty

In this model, agents can use the asset markets to partially offset income shocks which necessarily involves an extension of the above model to three periods. At time 0, agents are assumed to trade in one- and two-period risk-free bonds as well as stock. At time period 1, old two-period bonds can be resold, new one-period bonds can be issued and stock can be traded. The stock pays a dividend at time periods 1 and 2.

[11] When the stock premium is written as a ratio of the stock and bond returns rather than their difference, then the equity premium has been shown to become larger for more general income and dividend processes as the level of idiosyncratic risk rises.

As with the previous model, agents are assumed to be identical in time 0, but owing to employment shocks, potentially receive different shares of aggregate income in periods 1 and 2. Agents are assumed to choose asset holdings and consumption to maximise

$$U'(C_0^i) + \delta \mathrm{E}U(C_1^i) + \delta^2 \mathrm{E}U(C_2^i), \qquad (1.2.4ag)$$

subject to budget constraints

$$C_0^i = Y_0^i + (d_0/2) - b_0^{i1} p_0^1 - b_0^{i2} p_0^2 - e_0^i p_0^e,$$

$$C_1^i = Y_1^i + \left(\tfrac{1}{2} + e_0^i\right) d_1 + b_0^{i1} - p_1^i b_1^{i1} - e_1^i p_1^e,$$

$$C_2^i = Y_2^i + \left(\tfrac{1}{2} + e_0^i + e_1^i\right) d_2 + b_1^{i1} + b_0^{i2}, \qquad (1.2.4ah)$$

where b_t^{in} and e_t^i are the net purchases of the n-period bond and of stock respectively at time t by agent i, whilst p_t^n and p_t^e are the prices of the n-period bond and stock (equity) respectively at time t.

Agents are again assumed to be identical at time 0 such that, in equilibrium, no assets are traded: $b_0^{i1} = b_0^{i2} = e_0^i = 0$. When output is low, one agent is assumed to have a good idiosyncratic shock whilst the other receives a bad shock: $Y_1^1 = Y_H$ and $Y_1^2 = Y_L$, where $Y_H > Y_L$. Equilibrium portfolio and asset prices are then found by solving first order conditions for i =1, 2. Heaton and Lucas concluded that "whether asset markets will be used to smooth consumption depends critically on the persistence of the idiosyncratic shocks. The ability to self-insure diminishes as the shocks become more persistent, because more persistent shocks have a larger impact on permanent income and hence desired consumption".

For transitory shocks, the equity premium depends on the residual variability of consumption after trading has taken place. To calibrate the model, they assumed that δ equals 0.95 and $\gamma \in (1, 3, 5, 7)$, where γ is the constant coefficient of relative risk aversion. Each agent is also assumed to receive an equal share of dividend and labour income in time 0, with dividends amounting to 12.5% of total income. Dividend and labour income are assumed to grow at 2% each period and the probability associated with high or low aggregate dividends are the same, with $\beta_H = 1.3\delta_L$. In time period 1, high aggregate dividends result in equal shares for the agents unlike during periods of low aggregate dividends, when the first agent receives income equal to 1.62 the income of the second agent. In time 2, $Y_H = 1.10Y_L$. Heaton and Lucas reported results which showed a lower consumption variability relative to income which they claim accounted for the equity premium being lower than in the no trade case.

There are also permanent idiosyncratic shocks modelled by constraining labour income at time 2 to equal the realisation of labour income at time 1. As with Constantinides and Duffie (1991), they found a marked decrease in trading volumes and an increase in the equity premium. They concluded that where agents have no access to asset markets, or if idiosyncratic income shocks are permanent, then the standard specifications can produce a high equity premium and a low risk-free rate.

Next, they attempted to assess the impact of "moderate frictions" when income shocks are transitory in a framework similar to Aiyagari and Gertler; consequently, we proceed directly to review the results.

Table 1.7

Panel A

Asset returns, trading volume and consumption variability, with costly trading in stocks and frictionless trading in bonds

γ	1	1	1	5	5	5
τ	0.	1.	2.	0.	1.	2.
$E(r^e)$	0.076	0.076	0.076	0.163	0.162	0.161
$E(r^e - r^b)$	0.003	0.003	0.003	0.017	0.016	0.015
% stock trade cost	0.00%	0.02%	0.03%	0.00%	0.08%	0.16%
$E(p^e e)$	0.006	0.000	0.000	0.006	0.001	0.001
$E(p^b b)$	0.093	0.099	0.099	0.086	0.091	0.091
$\sigma(C_1)/E(C_1)$	0.088	0.088	0.088	0.093	0.093	0.093

Source: Heaton and Lucas (1992)

Panel A of table 1.7 shows that the imposition of a trading cost has little impact on asset prices and consumption with the cost parameter, τ^e, exogenously determined. In other words, agents prefer to smooth out portfolio composition rather than pay transactions costs. The estimates for stock returns and the stock (equity) premium are however not consistent with historical data.

Table 1.7

Panel B

Asset returns, trading volume and consumption variability, with costly trading in stocks and bonds $\tau^e = 0.2$, asymmetric transactions costs in the bond market

γ	1	1	1	5	5	5
τ^b	0.050	0.200	0.400	0.050	0.200	0.400
$E(r^e)$	0.076	0.077	0.077	0.164	0.171	0.175
$E(r^e - r^b)$	0.005	0.010	0.013	0.020	0.032	0.041
%bond trade cost	0.41%	1.28%	1.79%	0.38%	1.48%	2.48%
%stock trade cost	0.11%	0.38%	0.73%	0.10%	0.15%	0.37%
$E(p^e e)$	0.006	0.022	0.041	0.006	0.008	0.021
$E(p^b b)$	0.091	0.072	0.050	0.086	0.083	0.069
$\sigma(C_1)/E(C_1)$	0.088	0.092	0.094	0.093	0.094	0.095

Source: Heaton and Lucas (1992)

Table 1.7

Panel C

Asset returns, trading volume and consumption variability, with costly trading in stocks and bonds. τ^{s} = 0.2, symmetric transactions costs in the bond market

γ	1	1	1	5	5	5
τ	0.050	0.200	0.400	0.050	0.200	0.400
$E(r^e)$	0.076	0.077	0.074	0.167	0.165	0.167
$E(r^e - r^b)$	0.003	0.003	0.003	0.020	0.016	0.017
% bond trade cost	0.35%	0.78%	1.11%	0.38%	0.87%	1.28%
% stock trade cost	0.30%	0.84%	1.02%	0.10%	0.73%	0.95%
$E(p^e e)$	0.017	0.044	0.057	0.006	0.041	0.053
$E(p^b b)$	0.079	0.047	0.031	0.085	0.049	0.036
$\sigma(C_1)/E(C_1)$	0.090	0.094	0.096	0.094	0.095	0.096

Source: Heaton and Lucas (1992)

Table 1.7, Panel B, reports results for a range of costs under the assumption that only the borrower incurs a cost in the bond market whereas Panel C repeats the test of the same model as in Panel B except that borrowers and lenders are assumed to pay transactions costs in the bond market. When the borrower pays, Heaton and Lucas reported higher interest rates to the borrower, as one would expect, and a consequently lower equity premium. They also suggested that there are direct and indirect effects associated with transactions costs. For the direct impact, transactions costs mean that lenders require higher rates of return whilst borrowers respond by reducing their demand for funds causing the equilibrium borrowing rate to fall. Indirectly, higher costs reduce trading volumes, increases volatility and hence the equity premium.

Table 1.7

Panel D

Asset returns, trading volume and consumption variability, with costly trading in stocks and binding borrowing constraints in the bond market

γ	1	1	1	5	5	5
B	0.080	0.050	0.000	0.080	0.050	0.000
τ^e	0.050	0.500	0.500	0.500	0.500	0.500
$E(r^e)$	0.076	0.077	0.079	0.163	0.167	0.174
$E(r^e - r^b)$	0.008	0.019	0.035	0.026	0.041	0.065
% stock trade cost	0.51%	1.46%	3.08%	0.81%	1.90%	3.74%
$E(p^e e)$	0.012	0.033	0.069	0.018	0.043	0.084
$E(p^b b)$	0.083	0.053	0.000	0.073	0.046	0.000
$\sigma(C_1)/E(C_1)$	0.091	0.098	0.112	0.094	0.096	0.110

Source: Heaton and Lucas (1992)

Table 1.7, Panel D, reports results for the model that imposes a borrowing or short sales constraint by limiting the number of bonds but allowing costly trading in stocks. The model predicts a large stock premium as well as a small risk-free rate.

Table 1.7

Panel E : Asset returns, trading volume and consumption variability, with proportional costs in the stock market and a short sales constraint in the bond market

γ	1	1	1	5	5	5
B	0.050	0.050	0.050	0.050	0.050	0.050
τ^e	0.005	0.010	0.025	0.005	0.010	0.020
$E(r^e)$	0.081	0.086	0.103	0.168	0.175	0.190
$E(r^e - r^b)$	0.008	0.014	0.030	0.024	0.030	0.040
$E(p^e e)$	0.047	0.047	0.047	0.046	0.046	0.046
$E(p^b b)$	0.051	0.051	0.051	0.045	0.045	0.045
$\sigma(C_1)/E(C_1)$	0.088	0.088	0.089	0.094	0.094	0.094

Source: Heaton and Lucas (1992)

Table 1.7, Panel E, reports results for a model that introduces proportional costs in the stock market of between 0.5% and 2.5% as well as a borrowing constraint of approximately 5% of income. Compared to Panel D, more stocks are traded and consumption volatility is lower. The authors concluded that the introduction of trading costs would seem to be responsible for the predicted equity premia, which are not too dissimilar in Panels D and E.

According to the Heaton and Lucas, these results show the capacity of incomplete markets theory to explain the equity premium and risk-free rate puzzles because undiversifiable idiosyncratic risk affects the precautionary demand for assets and individual's attitude towards aggregate uncertainty. With the introduction of cost parameters and as their overall size increases, the risk-free rate is seen to fall and the equity premium tends to rise.

Although the model makes claims about the asset pricing puzzles, the model like many attempts before it, does not model the discount rate, δ, but rather choose a value of 0.95. This may be consistent with our expectations but as will be discussed later, many of these models are consistent with a negative rate of time preference i.e. a discount rate greater than 1. Also, it is difficult to accept that the value for the risk aversion coefficient is about 5 times that predicted by the real business cycle theory even after taking into account "reasonable" market frictions. If a coefficient of risk aversion around 1 is assumed, then there is a general failure for the predicted stock premium to match that implied by the observed data. Finally, the assumed borrowing constraint for the specification that yields the most encouraging results is only 5% of income. Instinctively, this looks unrealistically restrictive. Furthermore as Heaton and Lucas pointed out "small changes to the borrowing limit.... cause large changes in implied prices" to the extent that a model which assumes binding short sales in the stock market and costly trading in the bond market yields a negative equity premium. One solution to this problem could well be to endogenously deal with the borrowing constraints and or find an accurate empirical measure. Nevertheless, the model does provide some insights into the impact of market frictions on asset markets and ultimately on returns.

In section 1.5, there will be a discussion of the issues arising out of the consumption-based asset pricing literature and implicitly on the direction of our own research. Before that however, there is a review of the production-based asset pricing literature.

1.3 Production Capital Asset Pricing

Though the literature in this area of asset pricing remains not as comprehensive as for consumption, Fama (1990), Cochrane (1991), Basu and Vinod (1994) and Peng and Shawky (1997) have provided some insights into the relationship between production and stock returns.

1.3.1 Stocks Returns and Production Growth

Fama (1981) provided evidence that real activity explains larger fractions of stock return variations for longer horizon returns. Fama (1990) provided evidence "that information about the production of a given period is spread across many previous periods and so affects the stock returns of many previous periods". Short horizon returns are then viewed as having information about production growth rates for many future periods but "adjacent returns have additional information about the same production growth rates". As a result, Fama (1990) viewed regressions of long horizon growth rates on future production growth rates and vice versa as giving "a better picture of the cumulative information about production in returns". It follows from this that in regressions of the production growth rate from period t to $t+1$, $P(t, t+1)$ on lags of stock returns, $R(t - j, t - j - 1)$, more than one lag of the stock return should have an explanatory power. Fama presented results of such a regression using monthly data where

$$\begin{aligned}
P(t,t+1) \quad = \quad & \underset{(2.25)}{0.001} + \underset{(0.75)}{0.009}\,R(t-1,t) + \underset{(2.38)}{0.027}\,R(t-2,t-1) \\
& + \underset{(2.35)}{0.028}\,R(t-3,t-2) + \underset{(3.51)}{0.042}\,R(t-4,t-3) \\
& + \underset{(2.76)}{0.033}\,R(t-5,t-4) + \underset{(3.14)}{0.038}\,R(t-6,t-5) \\
& + \underset{(1.69)}{0.020}\,R(t-7,t-6) + \underset{(1.58)}{0.019}\,R(t-8,t-7) \\
& + \underset{(2.13)}{0.025}\,R(t-9,t-8) + \underset{(2.38)}{0.028}\,R(t-10,t-9) \\
& + \underset{(0.96)}{0.011}\,R(t-11,t-10) + \underset{(1.14)}{0.013}\,R(t-12,t-11) \\
& + e(t,t+1), \qquad (1.3.1a)
\end{aligned}$$

where the figures in parentheses are the t-statistics for the slopes and the R^2 is 0.14 (Table 1.8, Panel A). From (1.3.1a), up to 10 lags of the stock return can be used to forecast one month production growth i.e. information about production is spread across preceding periods. The accompanying table 1.8 (Panel B) also reports similar parameter values for the monthly and quarterly data. Fama pointed out that the slope on past monthly returns is a decaying one such that "implied constraints on the monthly slopes imposed in using quarterly returns have little effect on explanatory power".

<u>*Table 1.8*</u>

<u>*Panel A*</u>

Regression of Monthly, Quarterly and Annual Production Growth Rates on Contemporaneous and One-Year of Lags of Quarterly Real Returns on the Value-Weighted NYSE Portfolio : 1953-1987

$$
\begin{aligned}
P(t-T,t) &= a + b_1 R(t-3,t) + b_2 R(t-6,t-3) + b_3 R(t-9,t-6) \\
&\quad + b_4 R(t-12,t-9) + b_5 R(t-15,t-12) + b_6 R(t-18,t-15) \\
&\quad + b_7 R(t-21,,t-18) + b_8 R(t-24,t-21) + e(t-T,t)
\end{aligned}
$$

where $P(t-T,t)$ is monthly $(T=1)$, quarterly$(T=3)$, or annual $(T=12)$.

	Monthly *P(t-1,t)*		Quarterly *P(t-3, t)*		Annual *P(t-12, t)*	
	b	*t(b)*	*b*	*t(b)*	*b*	*t(b)*
Constant	0.00	2.27	0.00	1.94	0.02	2.29
R (t - 3, t)	0.01	2.24	0.00	0.04	- 0.09	-2.15
R (t - 6, t - 3)	0.03	4.52	0.10	3.96	0.05	1.36
R (t - 9, t - 6)	0.03	3.90	0.10	4.82	0.16	3.88
R (t - 12, t - 9)	0.02	3.92	0.06	3.11	0.26	6.52
R (t - 15, t - 12)			0.04	1.93	0.29	6.03
R (t - 18, t - 15)					0.20	6.26
R (t - 21, t - 18)					0.09	2.47
R (t - 24, t - 21)					0.02	0.50
R^2	0.14		0.30		0.44	
s(e)	0.01		0.02		0.05	
Observations	420		140		137	

Source: Fama (1990)

The relationship of most interest involves using production growth data to explain stock returns and Fama reported results to that effect in Table 1.8 (Panel B).

Table 1.8

Panel B

Regression of Monthly, Quarterly and Annual Continuously Compounded Real Returns on the Value-Weighted NYSE Portfolio on Contemporaneous and One-Year leads of Quarterly Production Growth: 1953-1987

$$R(t,t+T) = a + b_1P(t,t+3) + b_2P(t+3,t+6) + b_3P(t+6,t+9)$$
$$+ b_4P(t+9,t+12) + b_5P(t+12,t+15) + b_6P(t+15,t+18)$$
$$+ b_7P(t+18,t+21) + b_8P(t+21,t+24) + e(t,t+T)$$

where $R(t,t+T)$ is monthly $(T=1)$, quarterly$(T=3)$, or annual $(T=12)$.

	Monthly *R(t, t+1)*		Quarterly *P (t, t+3)*		Annual *P (t, t+12)*	
	b	*t(b)*	*b*	*t(b)*	*b*	*t(b)*
Constant	-0.00	-0.30	-0.00	-0.18	0.00	0.08
P (t, t + 3)	0.05	0.45	-0.46	-1.46	-0.96	-1.85
P (t + 3, t + 6)	0.29	4.52	1.10	3.09	0.35	0.92
P (t + 6, t + 9)	0.16	4.52	0.87	3.22	1.23	4.24
P (t + 9, t + 12)	0.18	4.52	0.37	1.50	2.11	7.02
P (t + 12, t + 15)			0.09	0.31	2.47	3.88
P (t + 15, t + 18)					1.18	2.14
P (t + 18, t + 21)					0.60	2.59
P (t + 21, t + 24)					0.39	0.78
R^2	0.06		0.20		0.43	
S(e)	0.04		0.08		0.13	
Observations	420		140		137	

Source: Fama (1990)

Fama (1990) pointed to the symmetry between the regressions in Table 1.8, (Panel A and B), where there is evidence that leads of quarterly production of up to three or four quarters can be used to explain monthly, quarterly and annual stock returns. The R^2 increases with horizon culminating in a value of 0.43 for annual stock returns. This is perhaps not surprising given that evidence has already been reported that regressions of shorter-horizon stock returns on production growth rates understate the explanatory powers because information on production is spread over a longer horizon. This evidence on production could be thought of as consistent with a view that production plans are based on the long-term view of productive possibilities.

1.3.2 Linking Stock and Investment Returns

Cochrane (1991) also attempted to link the stock market and the real economy via a production-based asset pricing model. Cochrane attempted to relate asset prices to implied contingent claims prices such that the firm sets production to meet sales, investment, output, stocks and labour inputs which can be represented by

$$max\ \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t Q_t (C_t - W_t L_t) \qquad (1.3.2a)$$

given initial capital stock, K_0, and where δ, is the discount factor, Q_t is scaled prices where Q_t equals P_t / δ_t. P_t, is the nominal contingent claims prices, A_t is sales, W_t, wages and L_t, the labour inputs. The constraints are

Production	$Y_t = f(K_t, \mathrm{L}_t, \vartheta_t)$	(1.3.2*b*)
Resources	$Y_t = A_t + I_t$	(1.3.2*c*)
Capital Accumulation	$K_{t+1} = g(K_t, I_t)$	(1.3.2*d*)

where Y_t is the output level, and K_t is the capital stock and ϑ_t represents uncertainty. The capital accumulation function is assumed to allow for adjustment costs to investment which according to Cochrane (1991), has much the same impact were the adjustment cost to be subtracted from output.

The first order conditions relating production to asset returns is

$$\delta E_t(q_{t+1} R_{i,t+1}) = 1, \qquad (1.3.2e)$$

where q_{t+1} is $p_{t+1}/\rho\pi\,(\vartheta_{t+1} \mid \vartheta_t)$, $p_{t-1} = P_{t-1}/P_t$ and where also π and δ (< 1) are the representative consumers' subjective probabilities and discount factor respectively. $R^I_{i,t+1}$ is the investment return from period t to $t+1$ such that

$$R_{i,t+1} = \left(f_{k,t+1} + \frac{g_{k,t+1}}{g_{I,t+1}} \right) g_{I,t}. \tag{1.3.2f}$$

where $f_{K,t+1} = \partial f(K_{t+1}, L_{t+1}, \vartheta_{t+1})/\partial K_{t+1}$, $g_{K,t+1} = \partial g(K_{t+1}, I_{t+1})/\partial K_{t+1}$ and also $g_{I,t+1} = \partial g(K_{t+1}, I_{t+1})/\partial I_{t+1}$. The form shown in (1.3.2*f*) can be interpreted as saying that the firm should adjust investment until the investment return equals the mimicking portfolio return. This implies that investment should be adjusted such that there should exist no arbitrage opportunities between the investment return and any mimicking portfolio.

The functional form herein outlined is as follows:

Production $$Y_t = mpk_t\,, K_t + mpl_t\,, L_t \tag{1.3.2g}$$

Resources $$Y_t = A_t + I_t \tag{1.3.2h}$$

Capital Accumulation $$K_{t+1} = (1 - \rho)\left[K_t + \left(1 - \frac{\alpha}{2}\left(\frac{I_t}{K_t} \right)^2 \right) I_t \right] \tag{1.3.2i}$$

where mpk_t and mpl_t are the marginal products of capital and labour, ρ is the depreciation rate (of the capital stock) and α is the adjustment cost parameter. Equation (1.3.2*i*) defines capital accumulation in period $t+1$ as a combination of the capital stock in period t minus the adjustment costs all further adjusted by the rate of depreciation. The adjustment cost parameter is accelerating in that as the investment-capital ratio increases, larger fractions of the investment are lost.

Consequently, the one-period investment return can be written, as in (1.3.2i), such that

$$R^I_{i,t+1} - R^I_{i,t} = (1-\rho)\left(mpk_t + \frac{1+\alpha(I_{t+1}/K_{t+1})^3}{1-(3/2)\alpha(I_{t+1}/K_{t+1})^2}\right)\left(1-\frac{3}{2}\alpha\left(\frac{I_t}{K_t}\right)^2\right). \quad (1.3.2j)$$

Equation (*1.3.2j*) says that when the investment-capital ratio is high in period *t*, the investment return is low with one reason being the progressively higher adjustment cost. It is the expectation that investment tends to be high in periods of low investment returns. In fact, it is the current low returns that induce the higher investment-capital ratio. However at time period *t+1*, the investment capital ratio may be high when the investment return is also high. In this time period *t+1*, the firm reduces investment to the originally planned levels. Consequently, a period of lower investment is likely to coincide with a period of high adjustment costs as the firm is likely to be able to sell larger quantities of the consumption good for every unit by which it reduces the capital stock. Cochrane (1991) found that the investment return has approximately the same sensitivity, measured by their partial derivatives, to investment-capital ratios at time periods *t* and *t+1* though with opposite signs. Implicitly, the investment return could be viewed as approximately proportional to changes in the investment-capital ratio or more easily to investment growth.[12] For a given firm, in equilibrium, the mimicking portfolio return is the return for owning a unit of capital which can be identified with the stock return.

---

12 The investment return is approximately proportional to investment growth because the capital stock changes remain less than investment. Implicitly, the capital stock is a smoother series.

A firm can transform a marginal unit of the consumption good in time period t into $g_{I,t}$ units of capital in time period $t+1$ via the capital accumulation equation (1.3.2*d*). Therefore, the price at time period t of a claim to a unit of time period $t+1$ capital is

$$P_t^{k_{t+1}} = \frac{1}{g_{I,t}(I_t, K_t)} = \frac{1}{(1-\rho)\left(1-(3/2)\alpha(I_t/K_t)^2\right)}. \qquad (1.3.2k)$$

At this stage, the market return is that achieved by buying a unit of capital at time t at a cost of $P_t^{k_{t+1}}$ and holding for a period until period $t+1$ when the buyer gets the marginal product of that capital, $f_{k,t+1}$. That extra unit of capital at time period $t+1$ depreciates and becomes $g_{k,t+1}$ units of capital at time period $t+2$. It therefore follows that in period $t+1$, this may be sold for $P_{t+1}^{k_{t+2}}$

The return from buying and holding it for a period is

$$\frac{f_{k,t+1} + g_{k,t+1} P_{t+1}^{k_{t+2}}}{P_t^{k_{t+1}}}. \qquad (1.3.2l)$$

From equation (1.3.2*k*), $P_t^{k_{t+1}} = 1/g_{I,t}$ and $P_{t+1}^{k_{t+2}} = 1/g_{I,t+1}$ such that substituting into (1.3.2*l*) leads us to the investment equation (1.3.2*f*). In equilibrium therefore, the investment return is equal to the mimicking portfolio return and in the case of a single firm, this would be tantamount to saying that the investment return is the same as the return on a share of the firm.

Given the conclusions to the previous section, the relationship between stock and investment returns can be summarised by

$$R_{t-1\to t} = R^I(I_{t-1}/K_{t-1}, I_t/K_t) \qquad (1.3.2m)$$

which forms the basis of the empirical work that follows.

In constructing the data, the first aim was to derive an investment-capital ratio using the capital accumulation rule of (1.3.2*i*) and which yielded

$$i_{t+1} = \frac{I_{t+1}}{I_t} \frac{i_t}{(1-\rho)\left(1+i_t-(\alpha/2)i_t^3\right)} \qquad (1.3.2n)$$

where $i_t = I_t/K_t$, ρ is the depreciation of the capital stock and α is the adjustment cost factor. From the investment-capital ratio as per (1.3.2*n*), the capital stock series is derived given investment, which is exogenously determined. The investment return is determined using (1.3.2*j*) and it is this that is viewed by Cochrane (1991) as being strongly correlated with stock returns.[13]

The relationship between investment returns and the investment-capital ratio is governed by three parameters; the adjustment cost parameter (α), depreciation (ρ), and the marginal productivity of capital, (*mpk*). Looking at (1.3.2*j*), the depreciation parameter and the marginal productivity of capital when taken together, results in an increase or decrease in the investment return by a constant. The adjustment cost parameter controls the sensitivity of the investment return to changes in the investment-capital ratio at time periods t and $t+1$. Implicitly then, the parameter controls the mean and standard deviation of the investment return. Cochrane (1991) however argued that this adjustment cost parameter has very little effect on the relative sensitivity of the investment return to the investment-capital ratio in other time periods such that the correlation of the investment return with the investment-capital ratio and other variables remain unaffected.

[13] Cochrane (1991) adjusted the stock return data to take account of the fact that stock returns are measured at a point in time compared to investment which was based on aggregated data.

Given that all the parameters α, ρ, and *mpk* control the mean and standard deviation of the investment returns but have a very limited effect on the correlation with other variables, the depreciation parameter is set to 0.10 and from where values for the other two parameters are found that equate the means of investment and stock returns as well as the standard deviation of the fitted values of a regression of the stock return on two leads and lags of the investment-capital ratio to that of a regression of the investment return on two lead and lags of the investment-capital ratio. The following parameters result:

Quarterly returns: $\rho = 0.10$, $\alpha = 13.04$, $mpk = 0.15$

Annual returns: $\rho = 0.10$, $\alpha = 13.22$, $mpk = 0.16$.

According to Cochrane (1991), "The reason for this choice of standard deviation is that the regression of the stock return on investment-capital ratios leaves a larger residual (lower R^2) than the corresponding investment return regression. This choice of the investment return standard deviation is designed to produce a series of about the same standard deviation as the investment return component of stock returns. Since most of the results are driven by the correlation of investment and stock returns, this scaling is not crucial to the results". Cochrane further pointed out that the investment return is quite insensitive to the arbitrary choice of ρ as long as the marginal product can be simultaneously adjusted to match the mean of the investment and stock returns. Table 1.9 reports means, standard deviations and autocorrelations of the investment-capital ratio, investment returns and the value-weighted returns.

Table 1.9

Means, Standard Deviations and Autocorrelations of Investment-Capital ratios, Investment Returns and Stock Returns

		Investment-Capital	Quarterly Investment Return	Quarterly Stock Return	Annual Investment Return	Annual Stock Return
Mean		0.137	1.70	1.70	7.33	7.33
Standard Deviation		0.009	3.42	7.24	9.37	15.53
Autocorrelations	1	0.90	0.45	0.11		
(by lag, in quarters)	2	0.71	0.10	0.04		
	3	0.49	-0.06	-0.04		
	4	0.28	-0.19	-0.03	-0.18	-0.07
	5	0.12	-0.23	-0.10		
	6	0.00	-0.13	-0.07		
	8	-0.17	-0.18	0.06	-0.20	-0.07
	12	-0.36	-0.19	0.04	-0.23	0.07

Source: Cochrane (1991)

Cochrane pointed to the highly autocorrelated nature of the investment-capital ratio which he suggested drives some of the regression results; some of which are re-presented here.

Table 1.10

Regressions of Real Stock Returns on Investment Returns, Investment Growth and GNP Growth
Panel A : Quarterly Returns

$$Stock\ Return(t-1 \to t) = a + b\ Right\ Hand\ Variable\ (t-1 \to t) + \varepsilon(t)$$

Right Hand Variable	*t-stat*	*%p value*	*Correlation of Stock, R.H.V*	*Standard Error of Correlation*
Investment Returns	3.163	0.186	0.241	0.069
Investment Growth	3.103	0.226	0.237	0.068
GNP Growth	3.914	0.013	0.294	0.074

Source: Cochrane (1991)

Panel B
Overlapping Annual Returns, with Corrected Standard Errors

$$Stock\ Return(t-4 \to t) = a + b\ Right\ Hand\ Variable\ (t-4 \to t) + \varepsilon(t)$$

Right Hand Variable	*t-stat*	*%p value*	*Correlation of Stock, R.H.V*	*Standard Error of Correlation*
Investment Returns	2.820	0.541	0.385	0.113
Investment Growth	3.060	0.259	0.360	0.103
GNP Growth	3.921	0.012	0.404	0.097

Source: Cochrane (1991)

Panel C

Regressions of Real Stock Returns on Investment Returns

(Annual Returns with No Overlap)

$$\textit{Stock Return}(t-4 \to t) = a + b\ \textit{Investment Return}\ (t-4 \to t) + \varepsilon(t)$$

Right Hand Variable	*t-stat*	*%p value*	*Correlation of Stock, inv. Return*	*Standard Error of Correlation*
(Annual Returns with No Overlap, First Quarter to First Quarter)				
First Quarter	2.885	0.634	0.449	0.128
Second Quarter	2.578	1.384	0.407	0.139
Third Quarter	1.851	7.173	0.306	0.141
Fourth Quarter	2.569	1.412	0.404	0.137

Source: Cochrane (1991)

Table 1.10 (Panels A, B and C) reports regression and correlation results used to assess the relationship between investment and stock returns. As well as pointing to the correlation coefficients that range from 0.241 for quarterly returns to about 0.45 for first quarter annual returns, Cochrane (1991) also pointed to the symmetry in the parameter estimates as evidence of the link between stock returns and investment returns.

Panel D

Forecasts of Stock Returns and Investment Returns

Single Regressions : Annual Returns

$$\textit{Return}(t-4 \text{ to } t) = a + b\, X\ (t-5) + \varepsilon(t)$$

Forecasting Variable	*Stock Return*		*Investment Return*		*Stock -Inv*
	b	*% p value*	*b*	*% p value*	*% p value*
Term	0.35	1.12	0.35	2.51	99.57
Corp	0.68	1.23	0.59	0.32	70.99
Return	0.12	50.97	0.24	0.66	48.86
d/p	5.02	0.28	0.80	48.47	0.02
I/k	-4.74	4.34	-7.40	0.00	25.35

Source: Cochrane (1991)

Panel E

Multiple Regressions : Annual Returns

$$Return(t - 4 \text{ to } t) = \alpha + b_1 \; Term\; (t - 5) + \ldots + b_5 \; I/K(t-5) + \varepsilon(t)$$

Forecasting Variable	*Stock Return* b	*Stock Return* % p value	*Investment Return* b	*Investment Return* % p value	*Stock - Inv* % p value
Term	0.26	18.17	0.23	2.36	92.19
Corp	0.41	12.91	0.06	51.05	18.64
Return	-0.30	8.02	-0.05	46.05	13.44
d/p	4.60	0.05	-0.57	39.89	0.00
I/k	-2.83	14.81	-7.10	0.00	3.94
R^2		0.22		0.52	0.18
Joint χ^2 *all variables*		0.01		0.00	0.00
Joint χ^2 *all but d/p*		1.29		0.00	7.09
Joint χ^2 *all but d/p, I/k*		1.01		5.42	30.68
Correlation of stock, investment return forecasts: 0.610, s.e.: 0.112					
	Regression without d/p				
R^2		0.11		0.51	0.03
Joint χ^2 *all variables*		4.03		0.00	58.61
Correlation of stock, investment return forecasts: 0.938, s.e.: 0.179					

Source: Cochrane (1991)

Panels D and E presents single and multiple regressions respectively of stock and investment returns and their difference on a set of forecasting variables.[14] From Panel D, one can see that with the exception of the lagged returns, the coefficients of stock returns on each of the forecasting variables are significant at conventional levels whilst the investment return delivers similar coefficient estimates to stock returns with the exception of the dividend-price ratio. Cochrane also regressed the difference between stock and investment returns "to test whether the coefficients are in fact equal to 1". With the exception of the dividend-price ratio, the restrictions can be rejected.

---

[14] The forecasting variables are as follows: term is the 10-year government bond return less treasury bill return. Corp is the corporate bond return less treasury bill return. Return is the real value weighted return. d/p is the dividend-price ratio and I/k is the investment-capital ratio.

Panel E presents the multiple regression results where all the forecasting variables of the model were used simultaneously. The variables are found to be individually insignificant as predictors of stock returns but jointly significant regardless of whether the dividend-price ratio is included in the regression. For the regression of the difference between the stock returns and investment returns on the forecasting variables, the forecasting variables are found to be jointly insignificant and with the exception of the dividend-price ratio and the investment-capital ratio, individually insignificant as well. Furthermore, the correlation between stock and investment return forecasts is about 0.61 with the dividend-price ratio included in the forecasting system and about 0.94 without it.

There are further regressions of returns on the investment-capital ratio and on GNP growth which are all interpreted in broadly the same manner. Based on these regressions of stock and investment returns on real economic variables, Cochrane concluded that there exists a link between stock and investment returns to the extent that "investment return equals stock return". In other words, stock return forecasts of economic activity are equal to investment return forecasts of economic activity. Cochrane further argued that the results are not sensitive to the adjustment technology and that overall, the production-based asset pricing model may prove more useful than its consumption counterpart for a number of reasons. Firstly, because it attempts to link asset returns with production variables such as output and investment whose movements are seen to more appropriately reflect economic fluctuations. Since firms are individually larger than consumers, then some of the reasons for the failure of the consumption-based model such as transaction and information costs might not apply to firms.

Another model used to link investment returns and stock returns was proposed by Restoy and Rockinger (1994). Like Cochrane (1991), they used production functions with adjustment costs parameters to show that "it is possible to establish, state by state, the equality between the return on investment and the market return of the financial claims issued by the firm". In addition to Cochrane (1991) however, they also included taxes in the formulation.

1.3.3. Production Asset Prices and Mean Reversion in Stock Prices (MRSP)

The significant negative autocorrelation in stock returns has often been viewed as representing mean reversion in stock prices. Porteba and Summers (1988) suggested the presence of "fads" that caused deviations from fundamental values in the short run. These "fads" could be the result of noise trading or speculative bubbles but according to Porteba and Summers (1988), these tend to disappear in the long run when fundamental values return to determine stock prices. Basu and Vinod (1994) suggested that this "fad hypothesis" is "not consistent with a full blown rational equilibrium asset pricing model". Basu and Vinod (1994) proposed a production-based asset pricing model to explore "the relationship between technological returns to scale and time series behaviour of equilibrium asset returns". The technological returns to scale is the preferred variable because the returns to scale of the technology is an important determinant of business cycles. Relating technological returns to scale to stock returns makes it possible to test whether growth in the economy is consistent with increasing or diminishing returns if one follows the underlying assumption that financial assets represent claims on the capital stock.

Basu and Vinod (1994) proposed a model for aggregate wealth, W_t, with Cobb-Douglas technology, of the form

$$W_{t+1} = (W_t - C_t)^{\alpha} \varepsilon_{t+1} \quad . \tag{1.3.3a}$$

where

C_t equals aggregate consumption,

α determines the returns to scale and

ε_{t+1} is a serially uncorrelated strictly positive productivity shock.

When $0 < \alpha < 1$, then the technology is subject to diminishing returns to scale whilst when $\alpha \geq 1$, then returns to scale are constant or increasing. The authors presented the consumer's intertemporal problem using the familiar maximisation

$$\max \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t U(C_t), \tag{1.3.3b}$$

subject to (1.3.3*a*). The price of each share to wealth is given by the also familiar form

$$P_t U'(C_t) = \delta \; E_t\left[U'(C_{t+1}).(P_{t+1} + D_{t+1})\right] \tag{1.3.3c}$$

where D_{t+1} is the dividend per share received at time $t+1$ and $U'(C_t)$ is the marginal utility of consumption. Basu and Vinod (1994) then assumed that there is only one share traded in the economy such that equilibrium consumption C_t equals D_t for all time. The equilibrium stock return is then given by

$$R_t = (P_t + D_t)/P_{t-1}. \tag{1.3.3d}$$

Given the isoelastic utility function,

$$U(C_t) = \left(C_t^{1-\gamma} - 1\right)/(1-\gamma), \tag{13.3e}$$

the authors outlined closed form solutions for two propositions i.e. when $\gamma = 1$ and when $\gamma \neq 1$.

For the first proposition when $\gamma = 1$ (also implying $U(C_t) = \log C_t$), Basu and Vinod (1994) presented forms for the equilibrium stock price ($\log P_t$) and stock returns ($\log R_t$) given by

$$\log P_t = B + \alpha \log P_{t-1} + \log \varepsilon_t \tag{1.3.3f}$$

where

$$B = (1-\alpha)\log\left[\frac{\delta(1-\alpha\delta)}{(1-\delta)}\right] + \alpha \log(\alpha\delta). \tag{1.3.3g}$$

$$\log P_t = \log \delta^* + \log P_{t-1} + \log \varepsilon_t \tag{1.3.3h}$$

For proposition 2, when $\gamma \neq 1$ and $\alpha = 1$, Basu and Vinod (1994) presented the equilibrium stock price ($\log P_t$) and stock returns ($\log R_t$) given by

$$\log P_t = \log \delta^* + \log P_{t-1} + \log \varepsilon_t \tag{1.3.3i}$$

where

$$\delta^* = \delta^{1/\gamma}\left\{E\left(\varepsilon_{t+1}^{1-\gamma}\right)\right\}^{1/\gamma} \tag{1.3.3j}$$

and returns are a stationary serially uncorrelated process where

$$R_t = \varepsilon_t. \tag{1.3.3k}$$

These two propositions can now be used to analyse the returns to scale technology as well as risk aversion in investigating the time series properties of stock returns. When $\gamma = 1$, Basu and Vinod (1994) presented the autocorrelation function for the log of stock returns represented by

$$\rho_1 = (\alpha - 1)/2 \quad \text{and} \tag{1.3.3l}$$

$$\rho_j = \alpha\rho_{j-1}, \text{ for } j > 1. \tag{1.3.3m}$$

Basu and Vinod used (1.3.3*f*) and (1.3.3*h*) to analyse 3 special cases.

(a) Increasing returns to scale where $\alpha > 1$ implying no MRSP. From (1.3.3*f*) and (1.3.3*h*), stock price and returns are not stationary and therefore non mean reverting.

(b) Constant returns where $\alpha = 1$ implying no MRSP. In this case, serial correlation in stock returns equals zero indicating the absence of mean reversion. Stock returns are stationary whilst stock price is also said to follow a geometric random walk with drift, hence no MRSP. A random walk model with drift is $I(1)$ and its first difference is $I(0)$.

(c) Diminishing returns where $0 < \alpha < 1$ implying the presence of MRSP as the stock price is purely trend stationary whilst stock returns exhibit negative autocorrelation

The main proposition however is that a positive risk aversion parameter which motivates smoothing as a necessary but insufficient condition for MRSP. To demonstrate the necessity of the positive risk aversion, the authors assumed an economy with zero risk aversion such that (1.3.3*c*) becomes

$$P_t = \delta E_t(P_{t+1} + D_{t+1}) \tag{1.3.3g}$$

which can be presented in terms of the expected stock returns where

$$E_t[(P_{t+1} + D_{t+1})]/P_t = 1/\delta \tag{1.3.3h}$$

and which is independent of the conditioning set. According to the authors, this implies zero serial correlation in stock returns such that there is no MRSP. In conclusion, risk aversion is necessary for the existence of MRSP. In the earlier propositions, the authors demonstrated that a diminishing return technology was a necessary condition for the presence of MRSP. To understand the relationship between diminishing returns and capital investment, the authors recalled the Cochrane (1991) result that stock returns (R_t) and investment returns (R_t^I) are equal in equilibrium and which is represented by

$$R_t^I = R_t = (1/\delta).(W_t/W_{t-1}). \qquad (1.3.3k)$$

A favourable realisation of ε_t which, given (1.3.3*a*), implies a higher value for W_t, leads to higher consumption and capital accumulation in time *t*. In the presence of diminishing returns, larger capital stock, K_{t+1}, would reduce the investment return R^I between period *t* and *t+1*. Consequently, the stock returns are also lower. An increase in stock returns between period *t-1* and *t* leads to a decrease in stock returns between period *t* and *t+1* and this is caused by diminishing returns.

Basu and Vinod proceeded to "examine the time series properties of the actual financial aggregates to understand the possible linkages between the technology and the stock market". For this, the authors performed unit root tests for log P_t and log R_t. Table 1.11 (panels A and B) reports the results for unit root tests for log P_t and log R_t.

Table 1.11

Panel A: Tests for unit root of log P_t (Real Stock Price)

Type	μ	τ	coeff	stat	1%	5%	Decision
ADF	0	0	0.9952	- 0.8998	- 3.446	- 2.842	11
PZA	0	0	0.9952	- 1.2650	-20.390	-13.870	11
PZT	0	0	0.9952	- 0.9033	- 3.446	- 2.842	11
ADF	0	1	0.9957	- 0.8015	- 3.446	- 2.842	11
PZA	0	1	0.9952	- 1.2650	-20.390	- 2.842	11
PZT	0	1	0.9952	- 0.9033	- 3.446	- 2.842	11
ADF	0	2	0.9962	- 0.7104	- 3.446	- 2.842	11
PZA	0	2	0.9952	- 1.2060	-20.390	-13.870	11
PZT	0	2	0.9952	- 0.8881	- 3.446	- 2.842	11
ADF	0	9	0.9984	- 0.2956	- 3.446	- 2.842	11
PZA	0	9	0.9952	- 1.2240	-20.390	-13.870	11
PZT	0	9	0.9952	- 0.8927	- 3.446	- 2.842	11
ADF	0	10	0.9988	- 0.2237	- 3.446	- 2.842	11
PZA	0	10	0.9952	- 1.2120	-20.390	-13.870	11
PZT	0	10	0.9952	- 0.8897	- 3.446	-2.842	11
ADF	1	0	0.9529	- 2.5310	- 3.991	-3.415	11
PZA	1	0	0.9529	-12.4700	-28.990	-21.650	11
PZT	1	0	0.9529	- 2.5450	- 3.991	- 3.415	11
ADF	1	1	0.9549	- 2.3900	- 3.991	- 3.415	11
PZA	1	1	0.9529	-12.4700	-28.990	-21.650	11
PZT	1	1	0.9529	- 2.5450	- 3.991	- 3.415	11
ADF	1	2	0.9546	- 2.3730	- 3.991	- 3.415	11
PZA	1	2	0.9529	-12.0400	-28.990	-21.650	11
PZT	1	2	0.9952	- 2.5020	- 3.991	- 3.415	11
ADF	1	9	0.9585	- 2.0340	- 3.991	- 3.415	11
PZA	1	9	0.9529	- 13.4100	-28.990	-21.650	11
PZT	1	9	0.9529	- 2.6535	- 3.991	- 3.415	11
ADF	1	10	0.9540	- 2.2460	- 3.991	- 3.415	11
PZA	1	10	0.9529	- 13.4300	-28.990	-13.870	11
PZT	1	10	0.9529	- 2.6380	- 3.991	- 3.145	11

Notes : ADF = Augmented Dickey Fuller test, PZA = Phillips' Z_α test and PZT = Phillips' Z_t test. The PZA and PZT tests use the Parzen Window with a large truncation parameter to compute the Newey-West type estimator for the variance of the partial sum of the residuals. Column π refers to the order of the polynomial where μ = 0 gives a test for nonstationarity around a constant term while μ = 1 refers to a test for nonstationarity around a linear trend. Column τ shows the number of lags included in the right hand side. "Coeff" is the estimate of the unit root and "stat" is the corresponding statistic using the ADF, PZA and PZT methods. Critical values are given at the 1% and 5% levels and when the decision = 11, the test fails to reject the null hypothesis of no unit root at both levels of significance i.e. 1 reflects non rejection of the null hypothesis of no unit root.

Source: Basu and Vinod (1994)

Since panel A of table 1.11 indicates non rejection of the null hypothesis of no unit root, this is taken as confirmation of the presence of a unit root although the authors reported rejection of the null hypothesis for $\Delta \log P_t$ which is therefore considered stationary. Panel B reports results for unit root tests of the real stock return.

Table 1.11

Panel B: Tests for unit root of log R_t (Real Stock Return)

Type	μ	τ	coeff	stat	1%	5%	Decision
ADF	0	0	- 0.05394	- 17.150	- 3.446	- 2.842	00
PZA	0	0	- 0.05394	-279.300	- 20.390	-13.870	00
PZT	0	0	- 0.05394	- 17.210	- 3.446	- 2.842	00
ADF	0	1	- 0.06039	- 11.810	- 3.446	- 2.842	00
PZA	0	1	- 0.05394	-279.300	- 20.390	-13.870	00
PZT	0	1	- 0.05394	- 17.210	- 3.446	- 2.842	00
ADF	0	2	0.10630	- 8.117	- 3.446	- 2.842	00
PZA	0	2	- 0.05394	-279.500	- 20.390	-13.870	00
PZT	0	2	- 0.05394	- 17.210	- 3.446	- 2.842	00
ADF	0	9	- 0.18650	- 5.467	- 3.446	- 2.842	00
PZA	0	9	- 0.05394	-285.500	- 20.390	-13.870	00
PZT	0	9	- 0.05394	- 17.190	- 3.446	- 2.842	00
ADF	0	10	- 0.25200	- 5.443	- 3.446	- 2.842	00
PZA	0	10	- 0.05394	-283.100	- 20.390	-13.870	00
PZT	0	10	- 0.05394	- 17.200	- 3.446	- 2.842	00
ADF	1	0	- 0.05403	- 17.120	- 3.991	- 3.415	00
PZA	1	0	- 0.05403	-279.300	- 28.990	-21.650	00
PZT	1	0	- 0.05403	- 17.210	- 3.991	- 3.415	00
ADF	1	1	- 0.06056	- 11.790	- 3.991	- 3.415	00
PZA	1	1	- 0.05403	-279.300	- 28.990	-21.650	00
PZT	1	1	- 0.05403	- 17.210	- 3.991	- 3.415	00
ADF	1	2	0.10620	- 8.101	- 3.991	- 3.415	00
PZA	1	2	- 0.05403	-279.600	- 28.990	-21.650	00
PZT	1	2	- 0.05403	- 17.210	- 3.991	- 3.415	00
ADF	1	3	- 0.03865	- 8.485	- 3.991	- 3.415	00
PZA	1	3	- 0.05403	-279.600	- 28.990	-21.650	00
PZT	1	3	- 0.05403	- 17.210	- 3.991	- 3.415	00
ADF	1	9	- 0.18640	- 5.456	- 3.991	- 3.415	00
PZA	1	9	- 0.05403	-285.500	- 28.990	-21.650	00
PZT	1	9	- 0.05403	- 17.190	- 3.991	- 3.415	00
ADF	1	10	- 0.25230	- 5.435	- 3.991	- 3.415	00
PZA	1	10	- 0.05403	-283.100	- 28.990	-21.650	00
PZT	1	10	- 0.05403	- 17.200	- 3.991	- 3.145	00

Notes : As detailed in Panel A.

Source: Basu and Vinod (1994)

Like $\Delta \log P_t$, the log R_t series is purely stationary. Since stock prices are $I(1)$ and returns are $I(0)$, then the two series are not cointegrated. According to Basu and Vinod, "these results indicate that the time series properties of real financial aggregates conform more closely to the second scenario (b), where the economy-wide technology displays constant returns to scale". Hence the results provide a reasonable basis to reject the hypothesis that the economy displays increasing returns to scale. Since constant returns to scale are very much a feature of endogenous growth theory, the implication is that financial aggregates support the notion of endogenous growth in the economy.

1.3.4. Productivity Shocks and Production based Asset Pricing

Peng and Shawky (1997) also looked at a production based asset pricing model which used an extension of Tobin's Q theory that "describes how exogenous shocks to productivity levels and the real wage are transmitted to equilibrium asset prices". In other words, the authors argued that exogenous economic shocks are causes of the time-varying behaviour of asset returns. The authors further linked expected asset returns with investment growth and other variables of the production state.

1.4. Generalised Method of Moments (GMM)

Some of the analysis and discussion in this and following chapters mentions the Generalised Method of Moments (GMM) method of estimation. This section is an outline of the GMM framework as per Hansen and Singleton (1992).

1.4.1. GMM : A Basic Framework

For the basic framework, consider the unconditional moment restrictions

$$E\big(f(X_t, \beta_0)\big) = 0 \qquad (1.4.a)$$

where { X_t: t = 1,2,...} is a collection of random vectors of X_t which are strictly stationary, β_0 a p-dimensional vector of the parameters to be estimated, and $f(X_t, \beta)$ a q-dimensional vector of functions. Assuming that the sample mean of $f(X_t, \beta)$ converges to its population mean, then

$$\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} f(X_t, \beta) = E\big(f(X_t, \beta)\big) \qquad (1.4.b)$$

with probability 1. Essentially, GMM estimation involves minimising a quadratic form of the sample means where

$$J_T(\beta) = \left\{\frac{1}{T}\sum_{t=1}^{T} f(X_t, \beta)\right\}' W_T \left\{\frac{1}{T}\sum_{t=1}^{T} f(X_t, \beta)\right\} \qquad (1.4.c)$$

with respect to β; where W_T is the weighting or distance matrix which satisfies the condition

$$\lim_{T\to\infty} W_T = W_0, \qquad (1.4.d)$$

with probability one for a positive definite weighting matrix W_0. The matrices W_T and W_0 are both viewed as the distance or weighting matrix whilst the estimator β_T is the solution to the minimisation problem in (1.4.b). Under general regularity conditions, the GMM estimator, β_T, is regarded as a consistent estimator for arbitrary distance matrices. Ogaki (1993) showed that linear regressions and non-linear instrumental variable estimations were special cases of GMM and that the GMM framework could be made applicable to cross sectional as well as panel data and also to habit formation and multiple goods models.

For the distribution of GMM estimators, Hansen and Singleton assumed that the central limit theorem applied to the disturbance of GMM, $u_t = f(X_t, \beta)$ so that

$$\left(1/\sqrt{T}\right)\Sigma_{t-1}^{T} u_t$$

has an asymptotic normal distribution with mean zero and covariance matrix ς in large samples. This is perceived as one of the main advantages of the GMM i.e. there is no need for a strong distributional assumption for u_t to be normally distributed. If u_t is serially uncorrelated, then

$$\varsigma = E\left(u_t u_t'\right). \tag{1.4.e}$$

If however, u_t is first order serially correlated, then

$$\varsigma = \lim_{l \to \infty} \sum_{-l}^{l} E\left(u_t u_{t-l}'\right). \tag{1.4.f}$$

The term for the covariance matrix is often referred to as the long run covariance matrix of u_t.

Hansen and Singleton define

$$\Gamma = \left(\partial f(X_t, \beta) / \partial \beta'\right) \tag{1.4.g}$$

as the expectation of the $q \times p$ matrix of the derivatives of $f(X_t, \beta)$ with respect to β where Γ is assumed to have full rank. Under regularity conditions,

$$\sqrt{T}(\beta_T - \beta_0)$$

has an approximately normal distribution with mean zero and the covariance matrix

$$Cov(W_0) = \left(\Gamma' W_0 \Gamma\right)^{-1}\left(\Gamma' W_0\, \varsigma\, W_0 \Gamma\right)\left(\Gamma' W_0 \Gamma\right)'^{-1} \qquad (1.4.h)$$

in large samples. When the number of moment conditions (q) equals the number of parameters to be estimated, then the system is just identified; in which case the GMM estimator does not depend on the choice of the distance matrix. When $q > p$, there are over-identifying restrictions and different GMM estimators result from different distance matrices. Asymptotically, the most efficient GMM estimator is given when the covariance matrix is minimised at $W_0 = \varsigma^{-1}$. With this distance matrix,

$$\sqrt{T}(\beta_T - \beta_0)$$

has approximately a normal distribution with mean zero and the covariance matrix

$$Cov\left(\varsigma^{-1}\right) = \left(\Gamma' \varsigma^{-1} \Gamma\right)^{-1} \qquad (1.4.i)$$

in large samples.

If ς_T is a consistent estimator of ς, then $W_T = \varsigma_T^{-1}$ can be used to obtain β_T. The resulting estimator is the optimal or efficient GMM estimator.

Hansen and Singleton also suggested that where $q > p$, a chi-square statistic can be computed and used to test the over-identifying restrictions of the model. Hansen (1992) has shown that T times the minimised value of the objective function $T\, J_T\, (\beta_T)$ has an asymptotic chi-square distribution with $q - p$ degrees of freedom if $W_0 = \varsigma^{-1}$. This is Hansen's J Test.

Appendix 2 details an exposition of the GMM framework in a two stage instrumental variable non linear estimation framework.

1.4.2. <u>GMM: The Application</u>

Hansen and Singleton (1992) proposed a representative consumer model to "explicitly characterise the restrictions on the joint distribution of asset returns and consumption implied by a class of general equilibrium asset pricing models and to obtain maximum likelihood estimates of the parameters describing preferences and the stochastic consumption process". Hansen and Singleton (1992) assumed that the joint distribution of consumption and returns is lognormal and which amounts to an imposition of restrictions on the production technology but which remains unspecified. In other words, the representation is a general one, which is intended to "accommodate a rich temporal covariance structure which might emerge when the investment environment faced by firms is more complicated than the environments in the models of Lucas (1978) and Brock (1982)". Such complications arise in the context of costly adjustment in altering capital stocks and gestation lags in producing new capital.

The utility function to be maximised by a representative consumer is the now familiar

$$E_o\left[\sum_{t=0}^{\infty}\delta^t U(C_t)\right], \qquad 0 < \delta < 1, \qquad (1.4.1a)$$

where δ is a discount rate, C_t is the aggregate real per capita consumption, $U(C)$ is the period utility function, $E_t(.)$ is conditioned on information, I_t, available in period t. I_t is assumed to include current and past information on current and past values of real consumption and asset returns.

Furthermore,

$$U(C_t) = C_t^{\gamma}/\gamma, \qquad (1.4.1b)$$

where γ, the constant coefficient of risk aversion, is less than 1. As is now familiar, consumers attempt to substitute intertemporally via assets i.e. risk-less as well as risky assets. Hansen & Singleton therefore defined w_t as holdings of N assets at time t, q_t as a vector of asset prices of the N assets net of any dividends and q^*_t denotes a vector of the values of the aforementioned dividends during time t. The feasible consumption and investment plan $\{c_t, w_t\}$ must satisfy the budget constraint of the form

$$C_t + q_t \cdot w_{t+1} \leq \left(q_t + q_t^*\right) \cdot w_t + y_t, \qquad (1.4.1c)$$

where y_t is the level of (real) labour income at time t. Consequently, the first-order conditions can be written as

$$E_t\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{\alpha} r_{it+1}\right] = 1; \quad i = 1,\ldots,N, \qquad (1.4.1d)$$

where $\alpha \equiv \gamma$-1.

In addition to assuming lognormality, $x_t \equiv C_t / C_{t-1}$ and $u_{it} \equiv x_t^{\alpha} r_{it}$, $i = 1,\ldots,n$, then (1.4.1d) can be written as

$$E_{t+1}(u_{it}) = 1/\delta, \quad i = 1,\ldots.n. \qquad (1.4.1e)$$

Let $X_t \equiv \log x_t, R_{it} \equiv \log r_{it}, Y_t = (X_t, R_{1t},\ldots,R_{nt})', U_{it} = \log u_{it} (i = 1,\ldots,n)$, and ψ_{t+1} denotes the information set $\{Y_{t-s} : s \geq 1\}$ with $\{Y_t\}$ assumed to be a stationary Gaussian process.

This assumption implies that the distribution of u_{it} conditional on ψ_{t-1} is normal with a constant variance σ_i^2 and a mean μ_{it-1} that is a linear function of past observations on Y_t. Consequently, Hansen and Singleton derived the formulation

$$E(u_{it}|\psi_{t-1}) = \exp\left[\mu_{it-1} + (\sigma_1^2/2)\right] \qquad (1.4.1f)$$

Given that $\psi_{t-1} \subset I_{t-1}$ and taking expectations of both sides of (1.4.1*e*) conditional on ψ_{t-1} to obtain

$$E(u_{it}|\psi_{t-1}) = 1/\delta. \qquad (1.4.1g)$$

Hansen and Singleton then equated the right hand sides of (1.4.1*f*) and (1.4.1*g*) and solve for μ_{it-1} which yields μ_{it-1} = $-\log\delta - (\sigma_i^2/2)$. They further defined the serially uncorrelated error term as

$$V_{it} \equiv U_{it} - \mu_{it-1} = \alpha X_t + R_{it} + \log\delta + (\sigma_i^2/2), i = 1,\ldots,n. \qquad (1.4.1h)$$

Consequently,

$$E(V_{it}|\psi_{t-1}) = \quad 0 \quad \text{and}$$
$$E(R_{it}|\psi_{t-1}) = -\alpha E(X_t|\psi_{t-1}) - \log\delta - (\sigma_i^2/2), i = 1,\ldots,n. \qquad (1.4.1i)$$

From (1.4.1*h*) and (1.4.1*i*), $\alpha = 0$ amounts to risk neutrality whereas $\alpha \neq 0$ assumes risk aversion.

Hansen and Singleton (1992) attempted to reach conclusions on asset predictability by deriving an expression for the coefficient of determination, R_i^2 from the regression of R_{it} onto ψ_{t-1} which given (1.4.1*i*) yields the form

$$R_i^2 = \frac{\mathrm{var}\left[E(R_{it}|\psi_{t-1})\right]}{\mathrm{var}(R_{it}|\psi_{t-1}) + \mathrm{var}\left[E(R_{it}|\psi_{t-1})\right]} \tag{1.4.1j}$$

where var is the variance operator. The variances of the predictable components of log r_{it} and log (C_t/C_{t+1}) are related by the expression

$$\mathrm{var}\left[E(R_{it}|\psi_{t-1})\right] = \alpha^2 \,\mathrm{var}\left[E(X_t|\psi_{t-1})\right]. \tag{1.4.1k}$$

Equation (1.4.1k) can therefore be written as

$$R_i^2 = \frac{\alpha^2 \,\mathrm{var}\left[E(X_t|\psi_{t-1})\right]}{\mathrm{var}(R_{it}|\psi_{t-1}) + \alpha^2 \,\mathrm{var}\left[E(X_t|\psi_{t-1})\right]}. \tag{1.4.1l}$$

From (1.4.1l), it follows that for agents to be risk averse, then $\alpha \neq 0$ which is a condition for asset returns to be forecastable.

For the estimation, Hansen and Singleton assumed that

$$E(X_t|\psi_{t-1}) = a(L)' Y_{t-1} + \mu_x \tag{1.4.1m}$$

where $a(L)$ is an $n + 1$ dimensional vector of finite order polynomials in the lag operator L. They derived the logarithm of the conditional likelihood function up to a constant term,

$$L(Y_1, \ldots, Y_T; \xi) = -(T/2)\,\mathbf{log}|\Sigma| - (\tfrac{1}{2})\sum_{t=1}^{T}\left[A_0 Y_t - A_1(L)Y_{t-1} - \mu\right]' \Sigma^{-1}$$
$$x \quad \left[A_0 Y_t - A_1(L)Y_{t-1} - \mu\right]. \tag{1.4.1n}$$

The maximum likelihood (ML) estimate of ξ is obtained by maximising (1.4.1n), where they use various definitions of consumption. They concluded that parameter estimates for risk aversion ranged between zero and two and that the test statistics "provided little evidence against the models using the value-weighted return on stocks listed on the New York exchange". However, for models using individual Dow Jones Shares and Treasury Bills, the estimated returns were "essentially zero".

1.5. Summary of Research

The existing literature on asset pricing, consumption particularly, is significant and substantial and has come a long way since the Mehra & Prescott paper in 1985. Many of the more recent models have focused on modelling consumption as leveraged claims to capital stock. Epstein and Zin (1989, 1991), Attanasio and Weber (1989) and Aiyagari and Gertler (1991) are typical of this approach. The introduction of habit consumption has provided additional insights into the behaviour of consumers with more consistent and recognisable parameter estimates. In this area, the works of Abel (1990), Constantinides (1990), Pemberton (1993), Campbell & Cochrane (1995) and Campbell, Lo and Mackinlay (1997) have been seminal. These models essentially view overall consumption as being driven by a persistent consumption habit.

Even so, there remain outstanding issues in asset pricing which require further attention. Fundamentally, there still remains to be found a model that resolves the equity premium and is consistent with theory in terms of endogenously generating a subjective discount rate of close to but less than 1, which is consistent with the observed risk-less rate whilst also avoiding infinite marginal utility. To that end, we now mention some of the particular problems associated with some of the models herein discussed.

- The Discount Rate - much of the literature has either exogenously determined the discount rate to be close to 1 rather than model this endogenously. Abel (1990), Aiyagari and Gertler (1991), Pemberton (1993) are typical of this approach. A discount rate close to 1 is important because only a value of that order is consistent with discounting theory. In other words, this is consistent with a positive rate of time preference which is necessary for the researcher to assume the existence of intertemporal consumption choices. Epstein & Zin (1992) endogenously modelled the discount rate but the implied values of more than 1 is only consistent with a negative rate of time preference. Hence, there still remains to be found a model where the discount rate is endogenously modelled and close to but less than 1.

- Exogenous Modelling - many of the parameter estimates are exogenously determined. For example, Pemberton (1993) did exogenously determine the sensitivity parameter, λ, rather than determine a value within the model. Many of the parameters in the literature mentioned above have been determined in much the same way.

- Artificial Mechanisms - Many of the models include artificial devices to boost the predicted equity premium such as modelling stocks as a leveraged claim to an unusually volatile part of the capital stock, as measured by industrial capital or the introduction of transactions costs, non heterogeneous consumers or monetary frictions (Heaton & Lucas (1991)).

- Consumption Habit - Abel (1990) modelled the consumption habit as the lag of previous period consumption. This however presents a problem in that if consumption in period $t+1$ falls relative to period t, then the habit consumption for period $t+1$ would be higher than the actual consumption in period $t+1$. This is considered inconsistent with theory. Hence, any model put forward as explaining the equity premium would have to define habit such that it is always below actual consumption. We also acknowledge problems with this formulation, which will be discussed in chapter 3. Additionally, models of this kind have tended to be associated with wild fluctuations in the risk-free rate but which is inconsistent with historically observed data.

- Asset Returns - additionally for many of these models, descriptive statistics for asset returns would help determine the validity of the asset return estimates. Some studies also present values for risk aversion but which do not explicitly model the associated stock and risk-free returns given the point value for risk aversion.

For the production-based asset pricing model, Cochrane (1991) presented a model which attempted to relate actual stock returns to predicted investment returns derived from investment data via a production function. This model used the correlation coefficient between the actual stock and predicted investment returns as evidence of their equality. The initial outline of the model followed a framework similar to that used for consumption. Hence, it provides a basis to link consumption and production asset pricing. Despite this, the formulation did not set out an explicit form for the coefficient of relative risk aversion. This is also true of the Fama (1990), Basu and Vinod (1994) and Peng and Shawky (1997) models. Essentially, these models seek to link the economy and asset markets in a less direct way compared to the consumption models.

1.6. Research Questions

- Can the notion of habit consumption be incorporated into the power utility model to produce estimates for parameters such as risk aversion consistent with known values to help in resolving the equity premium and if so, does the power utility model provide any insights into complete models that fully resolve the equity premium?

The author looks to give direction to the reasons for the failure or otherwise of the simple power utility model to explain the observed equity premium. This is particularly important as the simple capital asset pricing model remains a widely used technique for assessing required rates of return in financial economics.

- Are Consumption and Production Capital Asset Pricing Models capable of individually explaining the equity premium of stocks over bonds?

This is predicated on the premise that consumption and production models should be individually and independently capable of explaining the equity premium. However, it must be the case that producer choices are exercised in the context of consumer choices within a framework of constraints.

- Is it possible to link Consumption and Production Capital Asset Pricing Models to explain the equity premium of stocks over bonds?

The focus is on models of consumer and producer behaviour that do not need to employ some of the more recent techniques used to boost the equity premium on the basis that there should exist consumption and production models to explain the equity premium of equities over the risk-less rate in a framework consistent with theory. Such models will inform about the behaviour of producers relative to consumers. The results from the models analysed will have implications for significant areas of financial economics such as efficient capital allocation and the time value of money. It is also important that the parameter values obtained are consistent with known parameter values on the basis that this dissertation is primarily intended to link asset markets and the real economy.

1.7. Dissertation Map

So far in chapter 1, the discussion has been centred around consumption and production-based asset pricing models where much of the literature has been reviewed and the key features highlighted. This chapter is intended to provide a basis from which to present models and associated results that seek to explain the historically observed data.

Chapter 2 discusses the data used in our estimations as well as providing insights into the data that broadly follow previous efforts in this area. The data is annual UK-based data (1919-1991) providing an opportunity to look for a model that is historically consistent given that much of the work in this area uses post-war data. Normality tests are performed from which the characteristics of the data are found to be consistent with expectations and other studies.

Chapter 3 focuses entirely on the consumption-based asset pricing model and the replication of many of the relevant models. There is a basic outline of the power utility model and its log linear version as well as tests of its validity. As with many of the previous attempts, the model is rejected. A relationship between the coefficient of risk aversion and the elasticity of intertemporal substitution arises because if consumers are extremely risk averse, then they would be less willing to substitute intertemporally. Consequently, one of the earlier assumptions is that the elasticity of intertemporal substitution is the inverse of the coefficient of relative risk aversion. Estimations of both parameters are therefore attempted. On the basis that risk aversion and elasticity of intertemporal substitution estimates may be biased, the results of an instrumental variables estimation of the risk aversion parameter are also presented which tends to reject the notion that the elasticity of intertemporal substitution is the inverse of the risk aversion parameter. Overall, the results are not very encouraging for the power utility model. Chapter 3 also introduces the Generalised Method of Moments which makes no distributional assumptions about the data and which is used to estimate the parameter for risk aversion. The Epstein and Zin (1989, 1991) framework involves relaxing the link between the coefficient of relative risk aversion and the elasticity of intertemporal substitution. The results are less than encouraging with known parameter values not always consistent with predicted values. Chapter 3 also discusses and tests the habit consumption concept; ratio models as per Abel (1990) and difference models as per Campbell and Cochrane (1995) and Campbell, Lo and Mackinlay (1997). The difference models and its implications are discussed in some detail and there is also a discussion of the results presented by Campbell and Cochrane (1995) incorporating habit and the surplus consumption concept to show that there are models which go some way to simultaneously explain the observed equity premium and other parameters. The Instrumental Variable (IV) and Generalised Method of Moment (GMM) estimates of the risk aversion parameter are also presented as in Campbell, Lo and Mackinlay (1997).

In chapter 4, there are models presented aimed at improving the performance of the consumption-based models i.e. the power utility models as well as the Campbell and Cochrane model, in explaining the observed equity premium and accompanying parameters. The notions of habit and surplus consumption are incorporated into the power utility model and related to the components of real stock returns i.e. stock dividend and stock price. The results are very encouraging in that the resulting parameter estimates are more consistent with the observed data as well as expectations. In other words, the power utility model does work. Looking at the components of asset returns separately allows the notion of surplus consumption to explain the equity premium. This volatility of surplus consumption is captured and incorporated into amended versions of the Campbell and Cochrane (1995) and Campbell, Lo and Mackinlay (1997) models to explain the equity premium but also the risk-less return and the subjective discount rate. These results are also consistent with the Sharpe Ratio as outlined in Hansen and Jagannathan (1991). This is done using a model incorporating annual data where the standard deviation is 3 times the size of that found in the quarterly data. Implicitly, the predicted mean and standard deviation match that observed in the data. In other words, these models and the accompanying results provide much encouragement.

In chapter 5 can be found the results for production models such as that presented by Fama (1990), Cochrane (1991) and Basu and Vinod (1994). This produces some very interesting results sometimes consistent with a priori expectations, sometimes not. The results are at best only mildly encouraging for these versions of the production-based model. They, however give insights into appropriate models for explaining the equity premium.

In chapter 6, we present a profit-based model where stock price returns, and implicitly stock returns, are determined by deviations of actual from expected profits and dividends. The model also introduces stock price reaction parameters that are assumed to be equal to or greater than 1 and which transmits the effects of these deviations to stock prices. The predicted stock returns are found to quite closely match the observed stock returns.

Chapter 7 commences with a review of the results so far followed by explanations for the relatively low correlation between predicted and actual stock returns. These explanations centre around the notion of market distortions including transactions costs, information asymmetry and general market failure. As a simple test of the notion of market distortion, results for moving average returns, as per Abel (1989), are presented which show correlation coefficients between observed and predicted stock returns to be much higher than previously reported. The chapter is concluded with some discussion of the potential areas for future research, particularly that which further directly links asset markets and the real economy.

CHAPTER 2

Summary

The chapter presents, discusses and analyses the data to be used in our analysis. We start with a justification for the use of annual data 1919-1991 followed by the original variables. An historical perspective is also presented to put the original data into some context.

From these original variables, additional variables are derived which will form the basis of much of the estimation that follows. The key variables derived are the real stock return, real risk-less returns and the real equity premium of stocks over the risk-less rate. Other variables include per capita consumption growth, investment and profit.

The derived data for asset returns are compared to other related UK and USA based studies to establish the credibility of the data to be used in our analysis. A further historical perspective is provided by dividing the sample period into sub periods to better relate historical events to significant time periods. Much the same thing is done for log consumption and production growth.

Finally, we also present coefficients for skewness and kurtosis as well as Jarque-Bera and Anderson-Darling statistics.

CHAPTER 2

2.1 The Data

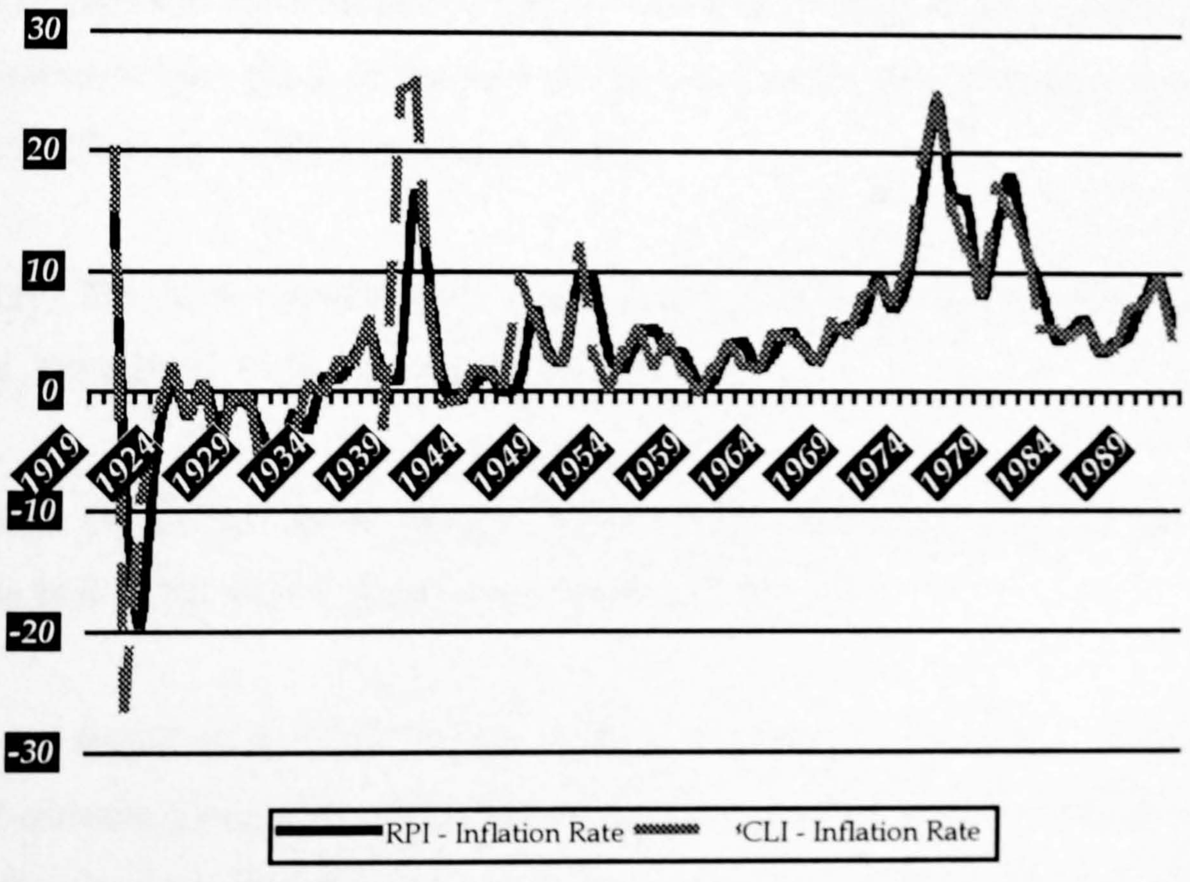
Our estimations are based on annual data series, 1919-1991. Given that much effort has been directed at using consumption and production data to explain the historically observed equity premium, it is important that the observed data to be explained is not perceived to be an easy target. As indicated by Campbell and Cochrane (1995), "annual data ...... have been the mainstay of the equity premium literature" though they used both pre and post-war data. Campbell, Lo and Mackinlay (1997) also used annual data, 1889 to 1994, in their analysis of the equity premium. Furthermore, since much of the effort here develops from Campbell, Lo and Mackinlay (1997), we follow their lead in using annual data. Also like Campbell and Cochrane (1995) and Campbell, Lo and Mackinlay (1997), the log form of the data is used extensively.

2.2. Original Data Series

- Barclays De Zoete Wedd, Cost of Living Index 1919-1991, Base year = 1918 (clix). The Cost of Living Index is a measure of retail prices derived from the General Retail Price Index (RPI). The General RPI is adjusted to reflect changes in consumption patterns though the series is not adjusted retrospectively. The correlation between the Cost of Living Index and the General Retail Price Index is 0.99 though the implied inflation rates have a correlation of 0.83. The relationship between the cost of living implied inflation rate and that implied by the retail price index as can be seen in Figure 2.1.

Figure 2.1

Cost of living and Retail Price Inflation Rates (%) 1919-1991



The BZW Equity Index is constructed to give an accurate measurement of the "performance of a representative portfolio of equities". The index was first calculated in 1956 and included the shares of the 30 largest companies as per the FT Index though this changed in 1962, when the index was then based on the FT Actuaries All-Share Index due to its "broader coverage" and which gave a more accurate picture of market movements. BZW argued that once the large quoted companies are included the index accurately reflects the general behaviour of the equity market.[14] Appendix 3 gives a listing of the constituents of the BZW Equity Index up to 1962.

- Barclays De Zoete Wedd Equity Price Index adjusted by the Cost of Living Index, 1919-1991, Base year = 1918 (epiax1).

The index is based on an arithmetic calculation with the index weighted by the number of shares in issue for each company at the beginning of the year though the calculations take place at the end of the year. Stock prices changes were used to calculate the index as outlined above.

- Barclays De Zoete Wedd Equity Price Index adjusted by the Retail Price Index, 1919-1991, Base year = 1918 (epiax2).

- Barclays De Zoete Wedd Equity Income Index adjusted by the Cost of Living Index, 1919-1991, Base year = 1918 (eiiax1).

The index is based on the BZW Equity Fund with the actual dividend received in the 12 months prior to the date of the index. The total value of dividends paid on the shares in that Fund formed the basis of the income index. Unlike many series before it, this series is based on actual receipts.

---

14 The original index excluded financial, mining and oil companies whose activities were primarily overseas.

- Barclays De Zoete Wedd Equity Income Index adjusted by the Retail Price Index, 1919-1991, Base year = 1918 (eiiax2).

The index is similar to that mentioned above except that the inflation adjustment is done using the retail price index. This is intended to test the robustness of the equity return calculations to specifications of the inflation series.

- Barclays De Zoete Wedd Equity Income Yield (%) 1919-1991, Base year =1918 (eiiyx).

The equity income yield is based on the previously outlined equity income and price indices.

- CSO Industrial Ordinary Share Price Index 1900-1991, 1980=100 (iospx).
- CSO Average 3-month tender rate on treasury bills (lruk3mx).
- CSO Consumer Expenditure at constant prices 1885-1992, Base year =1985 (cecpx).
- CSO Consumer Expenditure on Durable Goods at constant prices 1885-1992, 1985=100 (cedgx).
- CSO Population Trends 1885-1992 (popux).
- CSO Gross Domestic Product at Constant Factor Cost 1917-1991 (gdpcfc).
- CSO Gross Domestic Fixed Capital Formation 1917-1991 (gdfcf).
- CSO Gilt-Edge Return or Consols 1800-1992 (lgiltsx).
- The Open-Market Discount Rate on Prime Bank Bills 1800-1992 (lukshortx).
- CSO Current Account of Companies and Financial Institutions : Gross Trading Profits 1900-1991 (agtp).

Where appropriate, data for the period 1919-1991 is adjusted to ensure that all the series have the same base year.

2.2.1. Historical Perspective[15]

Figures 2.2 and 2.3 provide a basis on which to discuss the data and a context within which to discuss the economic and political environment of the time.

[15] Much of the historical information comes from the BZW Equity-Gilt Study 1992. Other sources include W.G. Hoffman, "British Industry 1900-1950" Oxford 1955, K.S. Lomax "Movements in the United Kingdom since 1900" in J.R.S.S (1959) and The Economist Annual.

Figure 2.2

Equity Income and Price Indices 1919-1991

(1919=100)

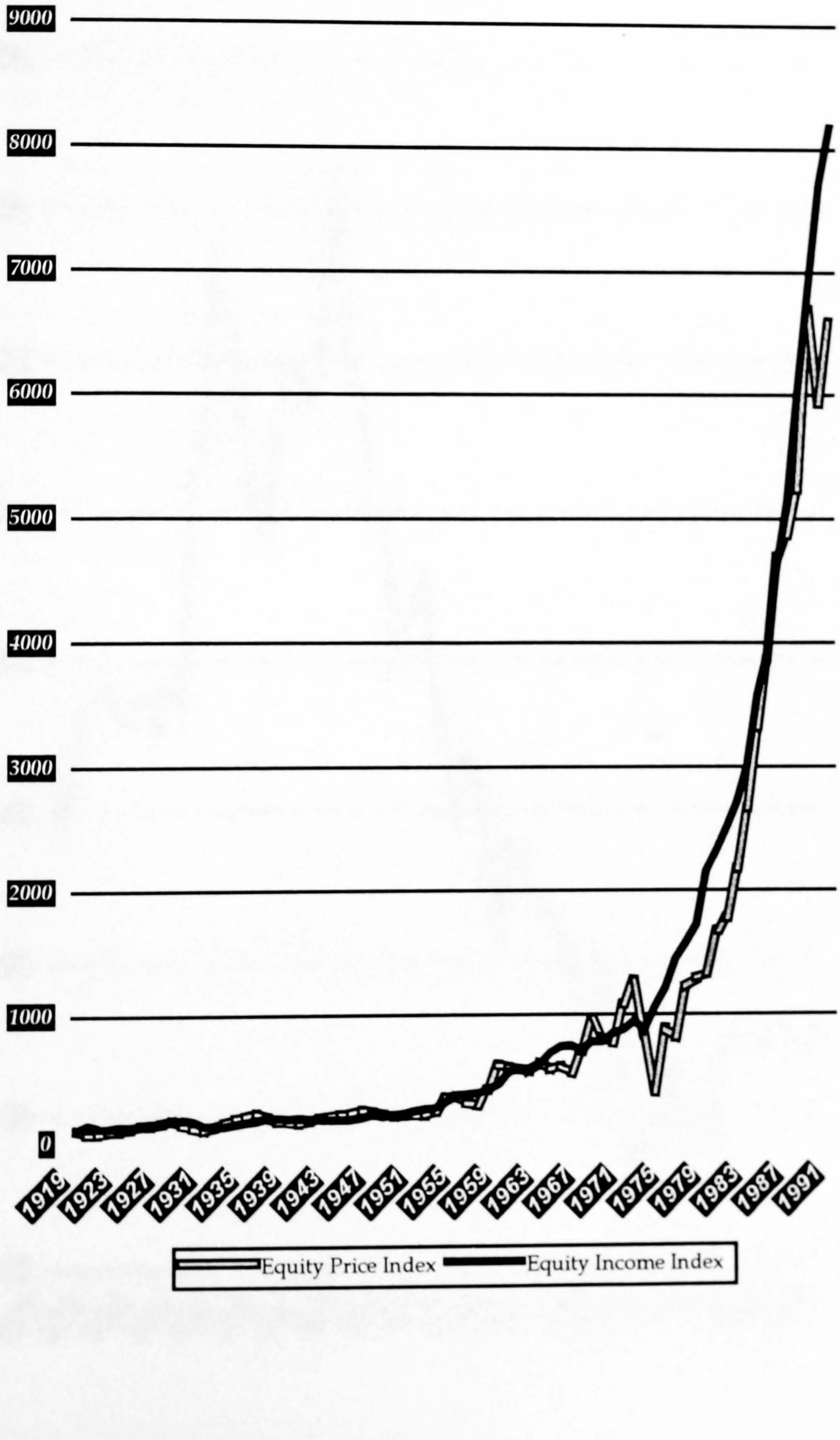
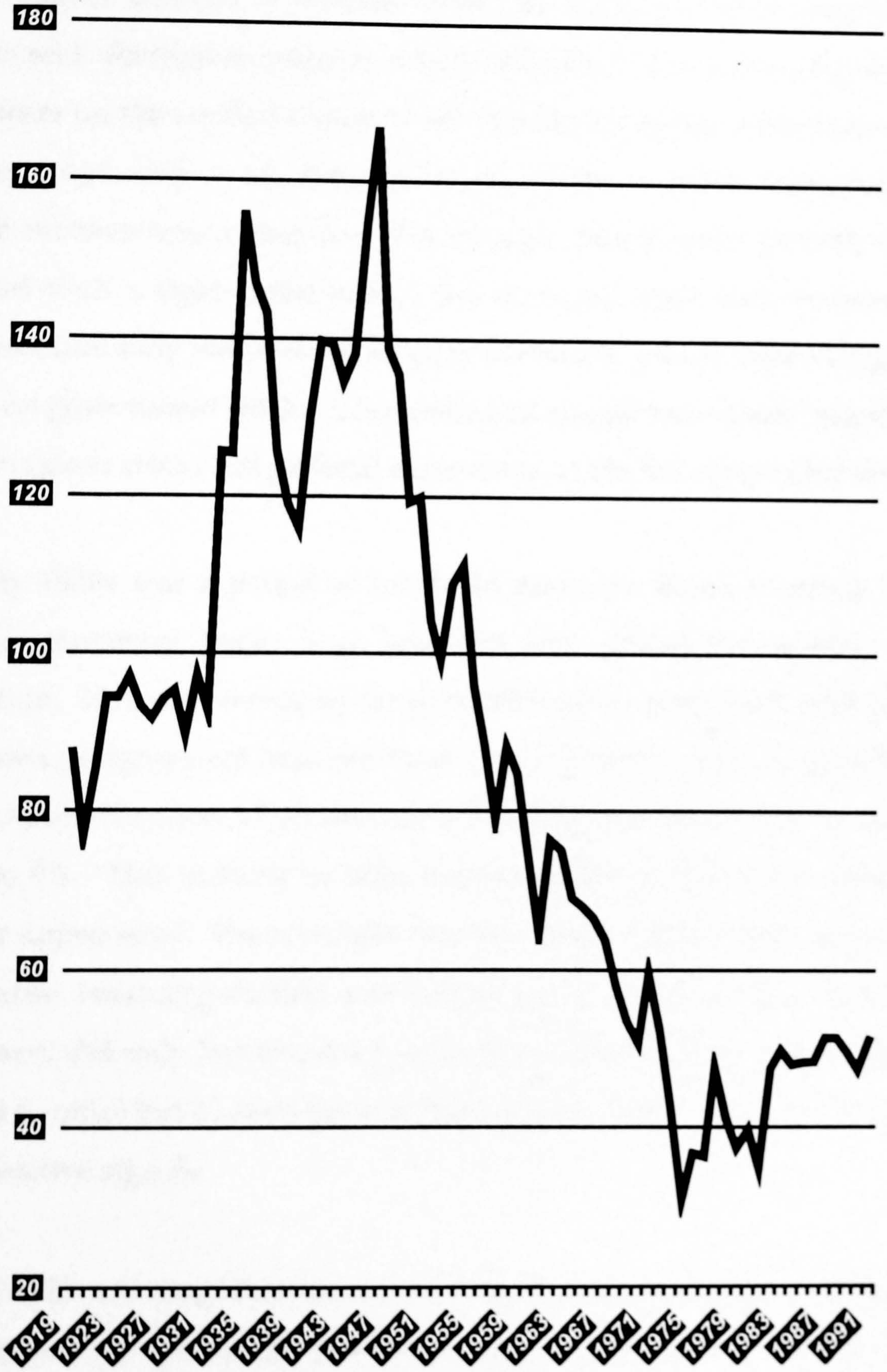


Figure 2.3

Gilt price Index 1919-1991

(1918=100)



After the great war of 1914-18, the UK government continued with a heavy expenditure programme to finance reconstruction. This expenditure was paid for in part by the issuing of more government stock, which depressed prices, but also higher taxation of income, which led many investors in government stocks to seek alternative assets in which to invest.[16] This focus of government expenditure on the civilian economy led to a strong equity performance as the economy responded to the lifting of many of the wartime restrictions. The postwar reconstruction that was not broadly based, soon petered out and combined with a tight credit policy, the economy went into recession. The recession eventually reduced inflationary pressures, which implied higher real returns on government stocks. This sentiment should have meant higher prices for government stocks but political uncertainty at the time prevented this.

The early 1920s was a period of sustained economic boom in equity and gilt prices as monetary policy was loosened and speculative interest gained momentum. However owing to the political worries associated with post-war reparations disagreement between France and Germany and the government's concern about the effect of the speculative bubble, interest rates were increased by 1% to 4%. This increase in rates depressed the domestic economy as the currency appreciated. The economic concerns at the time perhaps played a part in the nation returning the first ever Labour government in 1923. This Labour government did only last about a year before a Conservative government was returned to office but by then the economy and the equity market were already giving positive signals.

The year 1925 was a significant one primarily in terms of monetary policy as the UK returned to the Gold Standard and which was followed by the

[16] Many of these investors held government issued assets for the income.

strengthening of gilt-edged securities (gilts). In 1926, the indices of gilts and equities appeared to be flat but this masked swings within the year related to the General Strike. The market had discounted the effect of the strike but then this discount factor had been reversed when the strike was settled early. As one would expect, returning to the Gold Standard led to a reduction in inflation as implied by the tighter monetary policy discipline, which further implied an increase in real asset returns. Given this, the yield on gilts fell to the 3-3½% range prevalent before the Great War. The economy enjoyed the benefits of lower inflation until 1929 but the relatively poor performance in some sectors of the economy gave some cause for concern.

The great stock market crash of 1929 saw equities fall by up to 40% in real terms similar to that on Wall Street though the fall in the UK was spread throughout the year unlike on Wall Street where the crash started. Unsurprisingly perhaps, gilts did not perform as badly as equities given its status as an "asset of last resort". The early 1930s saw an upturn in economic fortunes despite continuing trade depression owing to the fact that Britain had a National Government, with a Conservative majority, committed to sound money. However, with the combined effects of a move away from the Gold Standard and cuts in interest rates, monetary policy was clearly not as tight as had previously been the case and this led to an expansion of credit. This period lasted until the late 1930s when worries about inflation emerged as a result of the re-armament programme associated with worries about the possibility of war. Gilt prices reflected the worry that the government would be expected to issue more gilts to finance the re-armament programme. Also, there were indications of a coming recession in the USA.

Eventually, the government decided to raise the funds to finance the re-armament programme by levying a tax on company profits. One cannot but note the similarity to the situation in 1997 when the government, expected to seek to raise revenue to meet electoral and other commitments, raised the necessary finance from companies i.e. the Windfall Tax. It appears that governments are more likely to seek to raise finance from companies to finance commitments when the economy has enjoyed a sustained period of growth. Unsurprisingly and unlike 1997 however, equity prices fell as a result of this levy in the late 1930s perhaps because the tax in 1997 was a one-off tax whilst that in the 1930s was not.

As war commenced in 1939, gilts performed reasonably well given that there were no alternative assets worth holding and retail prices were well under control through a combination of rationing and price controls. Equity prices suffered in anticipation of the war but by 1942 had begun to recover to reach 1937 levels. The Labour Party won a landslide in 1945 with plans for a national health service, education reform and nationalisation. These manifesto commitments did not find much favour with investors though equities and government stock prices rose for the first 2 years of the government owing to another round of "cheap money". The yield on government stocks fell to 2½% but with inflation remaining at 3%, this implied negative real returns to investors in gilts. In the post-war period, Britain had to meet the bills of war and the level of indebtedness required cash inflows to an extent that caused financial markets to worry as a balance of payments crisis developed. The equity market fell by something of the order of 20% though a recovery ensued when Sir Stafford Cripps became Chancellor of the Exchequer and the United States made a loan to the UK to ease fears of another balance of payments crisis. The UK economy showed signs of improvement in 1948 though the financial markets remained uncertain of economic prospects. This uncertain view of the world was justified in 1949, when another balance of payments crisis threatened and which saw the pound devalued from $4.03 to $2.80. By now, both equities and gilts were falling as investors became ever more disillusioned about UK economic prospects.

By 1950, the economic gloom had lifted, aided by the devaluation and which had improved the external financing position. The returning Labour government of 1950 pursued an expansionary fiscal policy with lower taxes and a general easing of restrictions. Though equities and gilts responded to the more favourable economic conditions, a Conservative government was returned in 1951. The fear of higher retail prices led to a fall in gilts prices therefore increasing the yield on gilts to around 4½%. There was a converse rise in equity prices as investors sought to protect themselves against inflationary tendencies in the economy. In 1952 however, the economy suffered yet another balance of payments crisis despite the deflationary policies of the Conservative government when interest rates were increased to 2½%, an excess profits tax introduced and import restrictions imposed. The balance of payment crisis led to a further retrenchment of the economy. Eventually, the austerity measures including a sound financial policy restored the economy to some health as the government sought to re-establish sterling as an international currency. Gilts prices also recovered, peaking in 1954 by which time the economy was beginning to overheat. Investors expected the government to introduce deflationary measures to slow down the economy. Equity prices also took an upward turn from mid 1952 and which turned out to be part of a long-term "bull" market. This period of Conservative government proved good for equities and gilts as the government enacted campaign promises headlined by the slogan "set the people free". Company profits and dividends grew significantly with the equity market given a further boost by the denationalisation programme. History however began to repeat itself as a period of economic expansion was about to be followed by the need to prevent the economy overheating as inflationary pressures began to appear. Gilts prices declined in the face of this economic situation with yields approaching 5%. The anticipation of another Conservative election victory acted to negate the impact of the actual economic circumstances in the expectation that the government would continue with "sound money" financial policy. The government did not disappoint in this regard with the introduction of restrictions on bank and hire purchase lending. However, inflationary pressures returned which in turn led to higher wage claims. For 1955 and 1956, political worries such as the nationalisation of the Suez Canal by President Nassar and the replacement by Harold Macmillan of Anthony Eden in preference to R.A. Butler dominated the economic picture. By 1957 however, the political worries had eased and together with bank rate reductions to boost consumption, equities and gilts prices were firmer. The rate reductions however served to increase cash outflows out of London with the consequence that rates were forced up to 7%. Consequently, equity and gilts prices came under further downward pressure. This negative sentiment was soon a thing of the past as the real economy continued to show signs of improvement with an improved balance of payments position and lower inflation with the prospect of further rate cuts and the removal of bank and hire purchase lending restrictions. Hence Harold Macmillan's "you have never had it so good". For 1958-9, prospects looked excellent and the financial markets responded accordingly with the yield on equities falling to under 4% below that on long gilts. The economy began to overheat with shortages of both labour and some imported items. Rates increased to 5% and the government introduced Special Deposit requirements on the banks to slow down the credit creation process. These measures together with a tightening of credit policy did not have a dramatic effect on equities. These restrictions stayed in place until 1961. There was a rise in equity prices led by banking and insurance shares due to proposed changes in the rules allowing trustees to acquire shares as part of their investment portfolio. By the time the rules were changed, the market had already risen too far for the trustees to actively participate in the market. The equity market therefore fell back towards previous levels. Also in 1961-2, gilts prices rose as inflationary pressures eased.

Political considerations came to the fore again in 1963, when De Gaulle vetoed Britain's application to join the EEC, Macmillan resigned to be replaced by Alec Douglas-Home and President Kennedy was assassinated. The financial markets coped reasonably well with these events preferring to focus on the expansionist approach of Chancellor Maudling, which was beginning to have a positive effect on the economy. In the later part of 1963 however, the perennial balance of payments problems arose and rates had to be increased to increase flows into the UK. The equity market held firm in the hope of another Conservative election victory. A Labour government resulted from the 1964 election campaign after which there was a sterling crisis caused by another balance of payments problem. These concerns eased somewhat as the incoming government introduced increases in taxation which led to a dramatic fall in equity values. This period did not last long as yet another sterling crisis threatened and measures to restrict credit led to further falls in equity values. Before equity values could fully recover in 1966, political worries again emerged e.g. Rhodesia, seaman's strike and the general election which increased the majority of the Labour government. The government introduced further restrictive measures in 1966 depressing equity values just as they were beginning to show signs of recovery. Gilts prices responded positively to these measures. Partly due to political pressures, the government began to ease policy in 1967 aided by the devaluation of sterling from $2.80 to $2.40 precipitated by the Arab-Israeli war in June which closed the Suez Canal[17] and by dock strikes in London and Liverpool. Eventually, the devaluation had to be negated and base rates were increased to 8% accompanied by the now familiar measures to restrict consumption in the economy. Political worries emerged in 1968; student riots in France and West Germany and the Russian invasion of Czechoslovakia but these had a limited impact on the equity and gilts markets. The deflationary policies from 1968 continued into 1969 and when combined

17 The Suez Canal was economically important to the UK because a significant part of its trade went through this route.

with labour disputes led to lower gilts prices with the yield up to 9.4% by June 1969. Political worries led to a fall in equities and this continued into 1969-70 as the prices and incomes policies were abandoned and the Labour government looked like winning the 1970 election. Unexpectedly for some, the Conservatives under Edward Heath won the election though the response of equity markets was somewhat muted as they waited to see the effects of government policy including the withdrawal of state support for "lame duck" companies such as Rolls Royce. Soon the equity market began to power ahead despite the higher inflation at the time of around 9% per annum but this was seen as a short term phenomenon. In 1971, the stock market was given added impetus by a wave of take-over bids. Gilts prices also remained buoyant with the real yields falling. The Conservative government began to ease monetary policy which led to an economic boom. Despite labour market problems, the equity market continued to show good gains though gilts prices came under downward pressure. By now, the government was "throwing money" at the various labour disputes though this only engendered further labour problems in other industries. Financial markets remained uncertain during this period and when combined with the oil price shock of 1973-4, equity and gilt prices came under further downward pressure. The oil price shock included restriction of supplies to the West that not only limited the productive capacity of British industry but also increased production costs significantly. The government responded with draconian measures such as the introduction of a 3-day working week, an increase in the minimum lending rate to 13% and an increase in special deposits by 2% to 6%. Furthermore, a deflationary budget was introduced as were restrictions on credit. Given the oil price crisis, the miners knew their strength and called a strike to which the Prime Minister responded with a call for a general election on the principle of "who rules, the trade unions or the Prime Minister"? The result was a Labour government with Liberal support and this led to a dramatic fall in equities. The fall was dramatic because the market had previously risen in anticipation of a Conservative victory, which would have been seen as a mandate to impose tough new restrictions on trade unions to solve the industrial disputes. In response to the perceived view of the market that the Labour government was incapable of dealing with the economic problems of the day, equities prices fell dramatically. Dennis Healey, the then Chancellor had promised to raise taxes on the wealthy with the slogan "....squeeze the rich until the pips squeak". By the end of 1974, equity values had halved and gilts values had fallen by almost 40%. Some institutions however, saw these lower prices as a buying opportunity given that yields were now very high with a strong possibility of capital gains since the market had to stop falling at some stage. In some sense, this is mean reversion in stock prices. Also at this time, the industrial grouping was now valued at only about 5 times earnings. Such views did have some merit in that in 1975, the market began to reverse the losses of 1974. In fact, equity prices doubled. Gilts prices, which had fallen by some way as well, recovered though unlike equities did not do so fully from earlier losses.

Despite the deflationary budget, the economy continued to show signs of recovery owing in part to the low short term interest rates at the time but which did not last as the sterling outflow caused concern. The minimum lending rate was increased and industrial output together with equity values started on a downward path. This downturn was short-lived and as indications emerged that union wage demands were moderating, optimism returned. Equities rose rapidly such that by January 1976, values were 40% higher than in July 1975.

In 1976, concerns emerged that the expansionary nature of government policy was unsustainable as the borrowing requirement was itself unsustainable. The Chancellor announced spending cuts of the order of £1bn which was not politically popular "on his own side". Worse still, investors thought this did not go far enough and refused to fund the government's borrowing requirement. The government then turned to the IMF for a loan of £3.9bn by which time equities and gilts prices had fallen by 28% and 15% respectively. This downward path of prices were soon reversed as sentiment changed; for it was increasingly the view that the conditions of the IMF loan meant that the government would have to pursue what equity and gilts investors perceived to be sensible economic policies. Despite the public expenditure cuts of 1977/8, equities were soon on an upward path as rates were reduced and the prices and incomes policies took effect. Gilts remained unmoved as it emerged that the prices and incomes policies which had been used to control inflation was merely a temporary squeeze and expectations for post-policy wage increases had been adjusted upwards to take account of the period of restraint. Reluctance by the government and employers to meet this "pent up" pay demand led to the "winter of discontent" as stoppages became the norm in many areas of economic life. As the discontent grew, gilts and equity prices, which initially came under some downward pressure, started to rise as expectations grew that the Conservatives would win the next election which they duly did in 1979. Given the monetarist policies proposed by Margaret Thatcher, it was clear that the short- term future was not entirely positive for equities as had been the case in 1971 when the Heath-led Conservative government had won the election. Hence the maxim that Labour governments are good for equities and the Conservatives for gilts. The monetarism of the new government led to higher interest rates and with it an appreciation of sterling with all the inherent consequences for the real economy. By this time, another oil price crisis had developed. In the expectation that the measures would deliver longer-term economic benefits, equity prices followed gilts prices in an upward direction in 1980. This continued until 1981 when fears of world economic disorder served to lower equity and gilts values but they recovered quickly after that. As inflation fell, nominal and real equity values rose and this signalled the start of another "bull" market. In 1982, equities continued on a mildly positive track though this became much more positive as it became clearer that industry was now in a far better position to compete given the huge restructuring process that had got underway following the new government in 1979. The government had also prevented trade union dissension from halting the restructuring process through successive legislative steps which had undermined the ability of the unions to prevent job losses thought necessary to improve competitiveness. With sterling falling, corporate UK was beginning to enjoy significant profitability growth. This formed the basis of the equity "bull" market in 1983/4 which continued its upward trend despite immediate term setbacks to do with Third World Debt and the miners strike. Sterling's weakness continued but now the government reacted with base rates of 14% as worries grew about the inflationary impact of the weaker currency. In 1985, the economic news continued to be positive as the Chancellor's budget statement painted an optimistic picture of the economy combined with tax reductions and an increasing number of take-over bids. There was another short-term setback for the equity market but this turned out to be a minor issue and the equity market continued to move ahead. The removal of exchange controls and other restrictions in 1979 had had the effect of further "internationalising" the UK market. Cecil Parkinson, the then Trade and Industry Secretary had secured the agreement, despite some resistance, of the London Stock Exchange to reform working practices and to remove restrictions on overseas members, the so-called "Big Bang". New firms, Japanese and American particularly, established a presence in London and this contributed to the generally optimistic view of the world. This period of "internationalisation" of the UK will perhaps come to be viewed as more significant in due course because it set the UK on a path which would be near impossible to reverse. The UK became much more susceptible to international economic circumstances to the extent that UK interest rates, for example, would never fully reflect domestic UK conditions. However, the expansionary sentiment was given a further boost by even lower oil prices which generated stronger hopes of non inflationary growth. Consequently, equity and gilts values continued their upward path as did the major markets of the world. This sentiment did not last long before doubts emerged which put a stop to the positive growth in equity and gilts values with nominal gilt yields returning to the 10-11% range. Worries about the Japanese and German economies contributed to the general sentiment of uncertainty which saw equities and gilts prices decline in 1986.

In 1987, with the renewed strength of the dollar and expected growth in UK corporate profits, equity and gilts prices continued to outperform especially when combined with the growing expectation that the Conservative government would be returned to office which they duly were. However the expected surge in Japanese buying of UK financial assets did not materialise but this was merely a temporary setback. Soon enough, the equity markets continued its upward path culminating in a near 50% rise in equities by mid July 1987, despite a 1% hike in base rates. Only in October 1987 was the market surge reversed. As equities had advanced, gilt price performance had been lacklustre such that the possibility of capital appreciation on gilts seemed possible. On the 19th of October, American share price fell on worries about a slowdown in economic expansion and by the persistently high negative trade and budgetary position. With equities reaching record levels, some investors switched to gilts hoping to benefit from capital appreciation but the programmed nature of the selling of equities led to a fall in world equity markets. Some of the losses were recovered by the year-end as fears of a world economic slump on the 1929 scale receded. The UK government concerned about this impact reduced interest rates and taxes further but this led to stronger inflationary pressures and a further worsening of the UK trade position which necessitated a rise in base rates to 15%. In hindsight, the government overestimated the negative impact of the 1987 stock market downturn and had perhaps reduced rates by a greater extent than was necessary. Part of the reason for the rise in base rates was due to the Chancellor's attempts to align sterling more closely with the German Mark in a quasi-ERM system. The Prime Minister did not entirely approve of this policy with the expression "you cannot buck the market". The Chancellor resigned and was replaced by John Major who was perceived to be less than fully committed to linking sterling with other European currencies. By this time, even though inflationary pressures had become stronger, the yield on gilts increased only mildly for 3 reasons according to BZW. Firstly, the government's anti-inflationary stance was still credible. Secondly, previous period budget surpluses had been used to redeem gilts thereby placing a "floor" on gilts prices and thirdly, the government had already put in place higher interest rates. This was therefore a period of negative real returns for gilts. Later on, gilts prices began to fall more substantially as further worries emerged about inflationary pressures in the economy and the prospect of a fourth conservative election victory looked distant. The period of sustained high interest rates led to the demise of major companies such as Collorol and British and Commonwealth. Eventually, share prices began to rise in view of the contention that prices had already been fully discounted to reflect the economic circumstances. In August, Iraq invaded Kuwait and the possibility of higher oil prices and the deflationary effects of war created much uncertainty. Eventually, the war was considered not to have a unmanageably detrimental effect on the economy and stock prices advanced accordingly. In 1991, the government decided to join the Exchange Rate Mechanism (ERM) of the European Monetary System (EMS) which led to a fall in long term bond yields but this was not good news for the equity markets as it implied a postponement of UK economic recovery.[18] In fact, the economy recovered from recession in the 3rd quarter of 1991. Furthermore, as the reunification of Germany proceeded, fears of higher German interest rates grew stronger due to the

[18] The ERM was a system of semi-fixed exchange rates where currencies were allowed to float within a band. For example, the Sterling band against the D-mark was 2.78 to 3.12 with a central rate of £1 to DM2.95. Postponement of UK economic recovery was implied by joining the ERM because one of the main ways of maintaining the bands was interest rates and since the stronger possibility was that sterling would fall below £1 to DM2.78, the prospects were then for higher interest rates with all the implied consequences for the real economy.

proposed conversion rate between the old East German Marks and West German D-Marks. The fears of higher German interest rates implied higher UK interest rates to prevent a damaging outflow of short-term cash.

There are general conclusions to be drawn from the data. Firstly, that there is a historical consistency in the general response of policymakers to particular events. For example, an economy growing too quickly soon leads to inflationary pressures, which usually demanded measures such as higher interest rates and restrictions on the availability and cost of credit. Higher interest however leads to an appreciation in the currency damaging export performance with opposite consequences for imports. This tighter monetary policy can also be accompanied by a tighter fiscal policy although this tends to be much more difficult to achieve not least because tax revenues accruing to the government tends to be more uncertain. Additionally, since 1945, the social security budget tended to increase during such a period making fiscal policy even less manageable. Once inflationary pressures have eased, interest rates can be reduced with the opposite effects. Though the policy response remains the same, its fundamental effectiveness depends on the structure of the national and international economy.

The historical perspective has also shown that real asset returns could be negative on 2 counts. Firstly that inflation might be unanticipated such that investors holding particular assets, usually government-issued, have inflation expectations which turn out to underestimate actual inflation. Secondly where interest rates are relatively low there is excess demand for assets thereby increasing the prices of these assets and reducing the implied yield. Such low interest rates might be the consequence of a deliberate government policy to achieve some desired outcome.

It is also clear that consumption and production choices adjust to economic circumstances. Consumption and production tend to fall during an economic downturn. However, when individuals and firms view a downturn as short term, the response would tend to be different compared to when the downturn is viewed as having a longer-term impact. There are clear indications that financial markets react negatively to uncertainty. Even when the causes of uncertainty have eased, the effects of that uncertainty will continue to be reflected in asset prices. Hence, it is possible that asset prices will temporarily at least reflect concerns rather than real economic variables. The 1987 Stock Market Crash provides a classic example of this.

Despite the fluctuating gilt yields, the long trend for gilts prices is negative implying ever higher yields. Secondly, the performance of equities post the late 1970s was dramatic generating wealth effects for the population as a whole with all the implicit implications for consumption and other asset values. Even though it is the case a only a relatively small part of the population own shares directly, many do through various investment and savings plans.

2.3. Derived Data[19]

From the original series, variables are constructed for use in our estimations and calibrations.

- To determine the equity (stock) return, the equity income yield and the change of the BZW equity price are taken together to give a measure of total equity return (eqrl) for 1919-1991. Both series are adjusted for inflation to give the real stock (reqrl). More precisely the formulation is

$$\frac{P_{t+1} - P_t}{P_t} + \frac{D_t}{P_t}$$

where,
P_{t+j} is the equity share price in period $t + j$ and
D_t is the dividend received at time t.

This formulation is exactly that used in the original article by Mehra and Prescott (1985).

- For the risk-free or risk-less rate, the average 3 month tender rate is used (lruk3mx). The average 3 month tender rate is thought most appropriate because other series related to treasury bills or gilts would be that derived from the secondary market where prices are subject to other forces. Furthermore, the average 3 month tender rate is adjusted by the cost of living index (clix) to give the real risk-less rate (rrfrl). The real risk-less rate is subtracted from the real stock return to yield the real equity premium (repl) of stocks over the risk-less rate. A retail price inflation adjustment is also calculated.

[19] The calculations are done using the following software, Gauss, Microfit and Minitab.

- The Industrial Ordinary Share Price Index (iospx) is adjusted by the cost of living (clix) index to obtain the real return on the Industrial Ordinary Share Prices (markp).

- For later calculations, a real equity income yield is derived by adjusting the equity income yield (eiiyx) by the cost of living index (clix). This is the dividend-price ratio (meiiyx).

- For consumption, the consumer expenditure on durable goods series at constant prices (cedgx) is subtracted from the total consumer expenditure at constant prices (cecpx). This is assumed to give a series on consumer expenditure on non-durable goods and services at constant prices (cepcx). This series is then divided by population to give per capita consumption on non durable goods and services. It is from this series that the per capita consumption growth series (rpccl) is derived. In choosing consumption on non-durables and services as the consumption variable, we follow the lead of Mehra and Prescott (1985), Campbell and Cochrane (1995) and Cooley & Ogaki (1996).

- The gross national product at constant factor cost (gnpcfc) is used to calculate economic growth (rgnpcfc) which is used in the discussion of existing production-based asset pricing models.

- The index of gross domestic fixed capital formation (gdfcf) is used to generate a series for investment growth (rgdfcf) which is also used in the discussion of production-based asset pricing models.

- Also used in the production-based asset pricing model is the term premia (term) which is difference between the long term gilts (giltsx) return or consols and the short term treasury bill return (ruk3mx).

- The corporate premium (corp) is derived using the short-term interest rate (mlukshort). There are limitations to this definition which are taken account of in the discussions in chapter 5 as the variable is used in single as well as multiple regressions.

2.3.1. <u>Comparative Data Analysis</u>

In this section is a comparative data summary for the UK (Table 2.1) as well as the USA (Table 2.2). Given the nature of the work, the comparative analysis focuses on asset returns and their standard deviation because as will be seen later, the mean and standard deviation form an integral part of the consumption and production models that are used to explain asset returns. Models that seek to explain asset returns should do so with values for the mean and standard deviation consistent with that observed in the historical data.

Table 2.1 shows representations of the derived data for the nominal and real returns on stocks, risk-free returns and the implied risk premium. Real asset returns are also calculated using the retail price inflation as well as that implied by the cost of living index. The eventual equity premium to be explained remains unchanged though there are only slight differences in the means and standard deviations. Table 2.1 also shows similar estimates for the range of variables with our estimates of asset returns slightly lower than that shown for BZW. The standard deviations remain broadly the same as well as providing clear evidence of the validity of the derived data. The standard deviation of the stock returns is just under 30%, a characteristic reflected in the equity premium. Both sets of data indicate that the post-war asset returns are quite similar for the full sample period with the real risk premium, the focus of much of our work, almost exactly the same. Hence using data from the pre-war period provides an historical perspective whilst maintaining the real equity premium which this research seeks to explain.

Table 2.1

Historical stock returns, risk free returns, inflation & the risk premia (%) 1919-1991

	Period	BZW Mean	BZW Standard Deviation (CLI adjusted)	Author (CLI Adjusted) Mean	Author (CLI Adjusted) S.D	Author (RPI Adjusted) Mean	Author (RPI Adjusted) S.D
Nominal Returns							
Equities	1919-91	14.43	26.05	13.76	24.57		
Treasury Bills	1919-91	-	-	5.29	7.11		
Inflation	1919-91	4.61	7.83	4.64	7.88	4.24	6.60
Equities	1946-91	16.33	29.52	15.89	28.21		
Treasury Bills	1946-91	7.18	4.54	7.11	4.25		
Inflation	1946-91	6.64	5.27	6.77	5.25	6.77	5.07
Real Returns							
Equities	1919-91	9.93	24.40	9.11	25.38	9.52	24.74
Treasury Bills	1919-91	-	-	0.64	7.41	1.04	5.65
Equities	1946-91	9.12	25.48	9.11	27.62	9.11	27.20
Treasury Bills	1946-91	0.62	4.30	0.33	4.74	0.34	4.42
Premia[20]							
Equity Risk Premium	1919-91	8.64	24.10	8.47	24.57	8.47	24.57
Equity Risk Premium	1946-91	8.61	26.97	8.78	28.22	8.78	28.22

Barclays De Zoete Wedd (1992)
Author's calculations using data on asset returns 1919-1991

The values for asset returns are also consistent with many of the UK based studies such those by Dimson and Brealey (1978) and Dimson & Marsh (1988) which have often indicated a real stock (equity) return of around 9% and risk premium of around 8%. In further analysis, the CLI adjusted data is used.

[20] The risk premium for the BZW figures do not exactly match the relevant differences between the real stock return and the treasury bill owing to calculation differences.

Table 2.2 gives another perspective on the data given that much of the work in this area is based on US data.

Table 2.2

Historical (%) Real Stock Returns, Real Risk Free Returns & the Real Risk Premia : USA & UK Data

	Period	Simple Returns		Log Returns	
		Mean	Standard Deviation	Mean	Standard Deviation
Author's (UK data)					
Equities	1919-91	9.11	25.38		
Treasury Bills	1919-91	0.64	7.41		
Inflation	1919-91	8.48	24.57		
Mehra & Prescott (1985) - (US data)					
Equities	1889-78	6.98	-		
Treasury Bills	1889-78	0.80	-		
Inflation	1889-78	6.18	-		
Author's Calculations					
Equity Return	1919-91			6.56	21.83
Risk-Free Return	1919-91			0.81	7.23
Risk Premium	1919-91			5.75	20.46
Campbell et al (1997) - US data					
Equity Return	1889-94			6.01	16.74
Risk-Free Return	1889-94			1.83	5.44
Risk Premium	1889-94			4.18	17.74

Barclays De Zoete Wedd (1992)
Campbell & Cochrane (1995)
Campbell, Lo and Mackinlay (1997)
Author's calculations using data on asset returns 1919-1991

Table 2.3 shows the mean and standard deviation for the real stock return, the risk-less asset return and the implied stock premium whilst figures 2.4 - 2.6 also provides a graphical illustration of the relationship between asset returns not least that between stock returns and the premium over the risk-less rate.

Table 2.3
Log real asset returns 1919-1991

	% real return on stocks (equity)		% real return on a relatively risk-less asset		% risk premium of equity over the risk-less return	
Time Periods	Mean (eqrl)	Standard Deviation	Mean (rfrl)	Standard Deviation	Mean (epl)	Standard Deviation
1919 - 1991	6.56	21.83	0.81	7.23	5.75	20.46
1919 - 1928	15.21	25.02	7.81	12.47	8.03	18.80
1929 - 1938	5.35	17.84	2.40	4.85	2.95	19.12
1939 - 1948	0.39	15.98	- 5.97	8.82	-6.36	8.74
1949 - 1958	7.45	17.39	- 1.08	3.95	8.53	16.67
1959 - 1968	9.97	18.00	1.97	1.04	8.00	17.52
1969 - 1978	-1.94	40.02	- 2.89	4.31	0.94	41.71
1979 - 1991	9.54	11.47	3.83	10.54	5.71	10.54

Author's calculations using data on asset returns 1919-1991

Even though the overall real risk-less rate is 0.81%, the 10-year average returns show some variability for the chosen sample periods. Given the high inflation and general economic uncertainty of the 1970s, it is unsurprising that the real risk-less rate is negative. A possible explanation is that levels of inflation were unanticipated such real returns on treasury bills were negative. The standard deviation for the real risk-less rate is, as expected, much less than that for the real stock return given that the real stock return is the source of risk. The variability associated with the real stock return is reflected in the stock premium with quite similar standard deviations for the two series. Predictably perhaps, the mean of the real risk premium is negative for the war period. The 1979-91 period shows the lowest variation in the real stock return and implicitly, the risk premium. This is consistent with a period of relative economic stability where low or zero inflation was the main objective of economic policy management. The standard deviation of the risk premium for the period is only about half the level of the overall data whilst that for the preceding period, 1969-1978 was the highest. The 1969-1978 period was characterised by significant economic uncertainty culminating in the government seeking assistance from the International Monetary Fund.

Figure 2.4 shows real stock returns to be consistently higher than treasury bill (risk-less) returns, as expected, with the risk-less rate following a relatively stable and less volatile path than stock returns.

Figure 2.4
<u>Real stock and treasury bill returns 1919-1991</u>

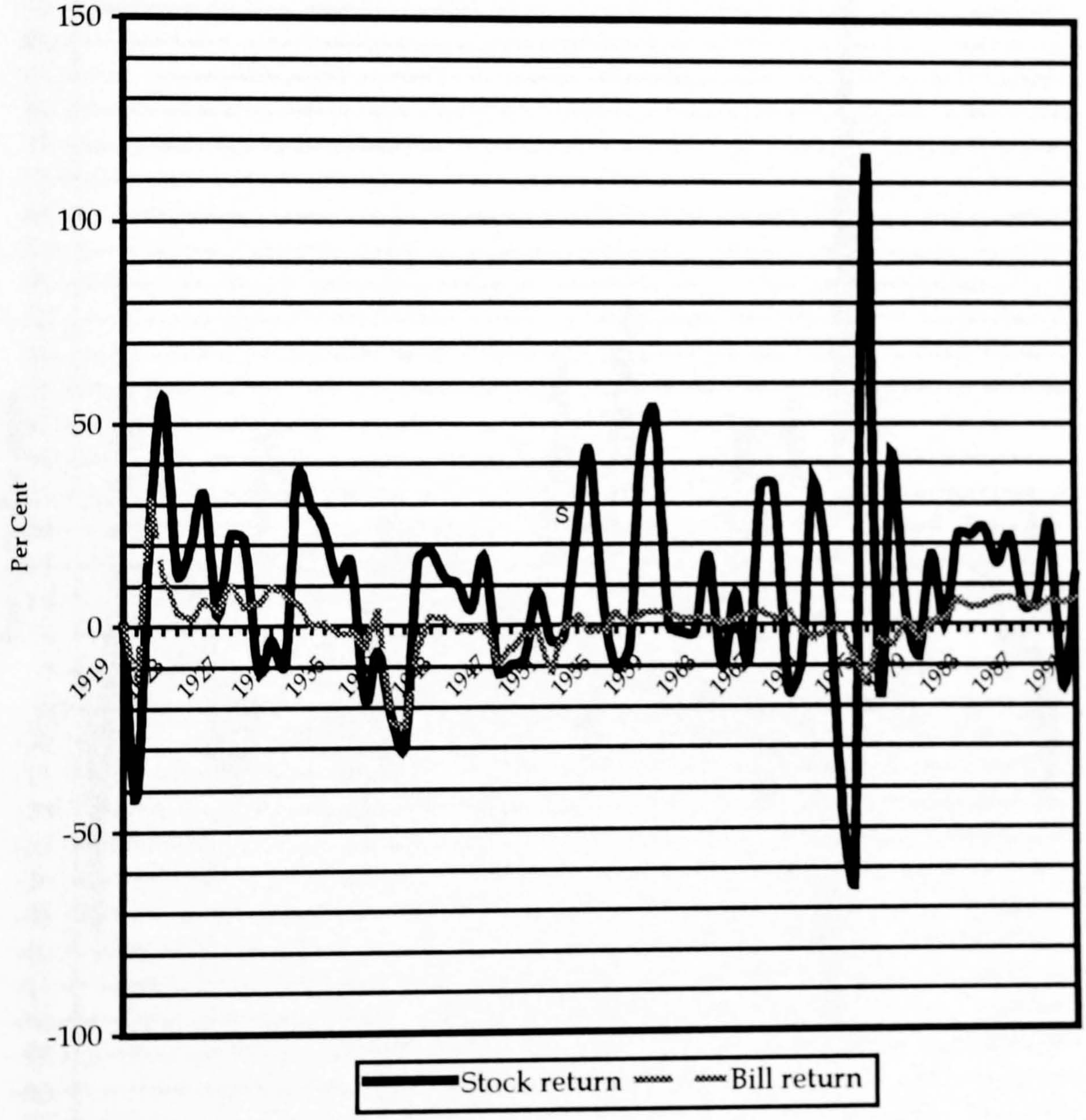


Figure 2.5 shows that volatility of the risk premium matches that of the stock return and that the premium is highly correlated with the real stock return.

Figure 2.5

Real annual asset returns - stocks and risk premia

1919-1991

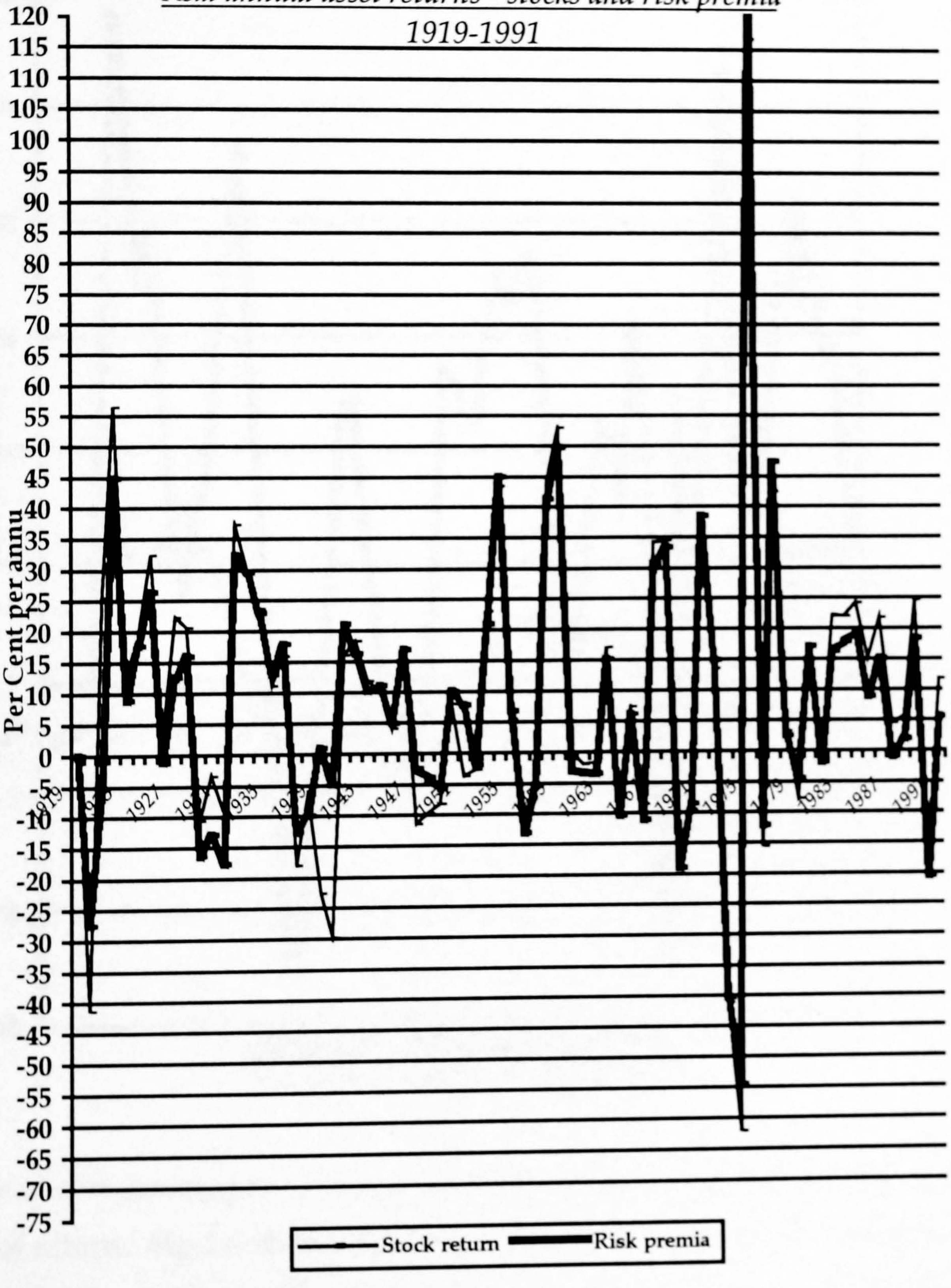
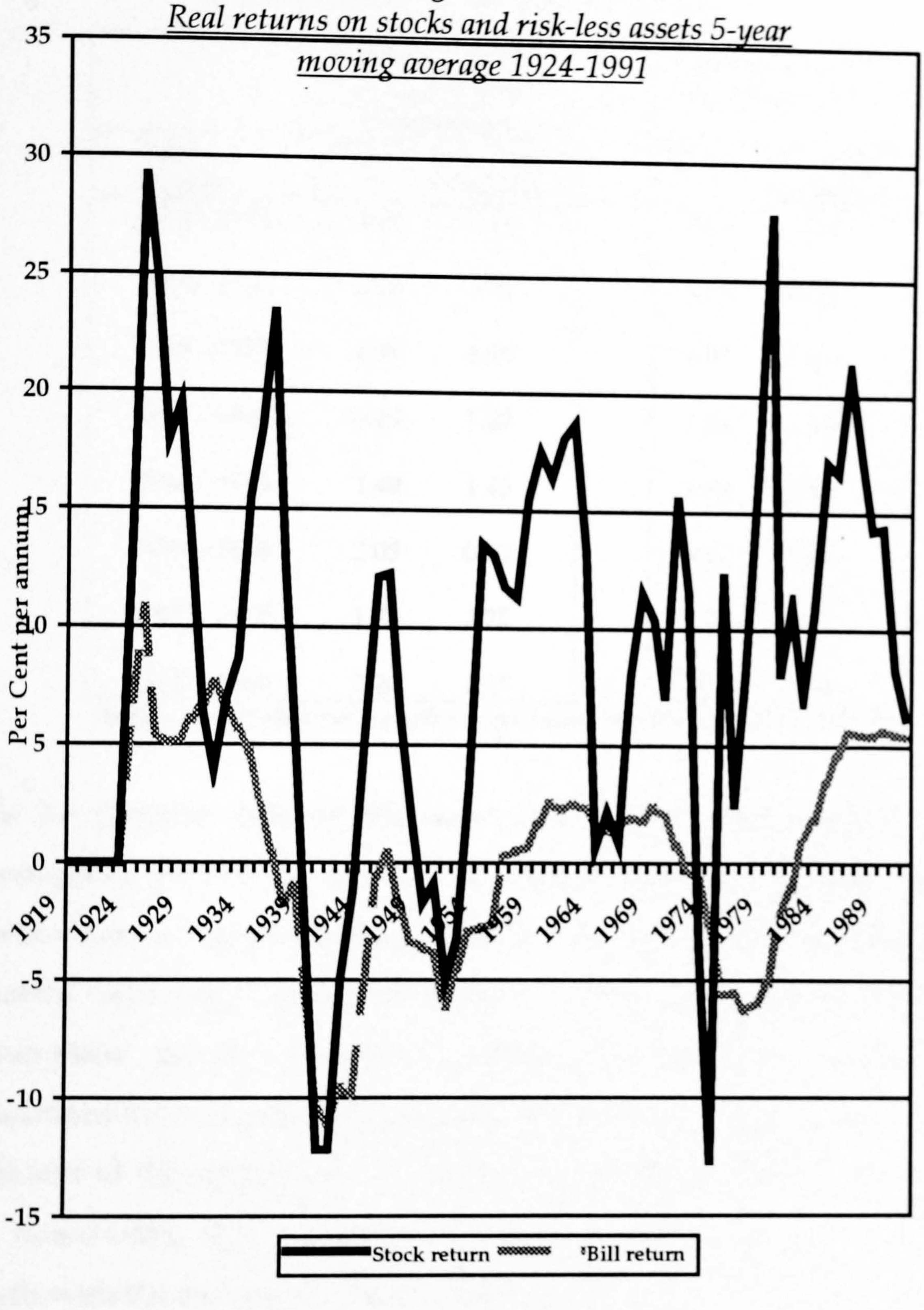


Figure 2.6

Real returns on stocks and risk-less assets 5-year moving average 1924-1991



One way of examining the average rates of return is to look at moving average rates of return. Fig 2.6 shows the 5-year moving average stock premium to be highest during the late 1950s and early 1960s. It can also be seen that the risk premium was on a declining trend between 1989 and 1991 as equity values reflected the economic uncertainty of the time.

Table 2.4
Log consumption & production growth

	% growth rate of per capita real consumption			% growth rate in real production	
Time Periods	Mean	Standard Deviation		Mean	Standard Deviation
1919 - 1991	1.47	3.14		2.46	5.65
1919 - 1928	0.87	3.10		4.27	13.16
1929 - 1938	1.20	1.25		2.83	5.81
1939 - 1948	0.40	7.07		1.58	2.88
1949 - 1958	1.49	1.43		2.99	2.92
1959 - 1968	2.05	0.82		3.57	1.05
1969 - 1978	1.72	2.25		1.75	3.63
1979 - 1991	2.26	2.25		0.87	3.54

Author's calculations using data on consumption and production 1919-1991

Table 2.4 presents data on the mean and standard deviation of per capita consumption growth as well as production growth. Overall, production growth shows a higher mean and standard deviation with the higher standard deviation indicating a more volatile series. This might seem to imply that the consumption growth process is different to that for production with implications for consumption and production asset pricing though a correlation coefficient of 0.4 might seem to suggest not entirely different processes. It is also noteworthy that production growth generally exceeded consumption growth with the exception of the 1979-91 period when the higher consumption growth reflected the aforementioned consumer led expansion with local firms perhaps unable to meet the demand generated by the growth in consumer expenditure. Consequently, we surmise that the demand must, to some extent at least, have been met by imported goods. Hence the significant growth in imports of the 1980s which led to a further worsening of the external trade position. The early 1980s was a period of contraction for production as industry was restructured; encouraged by the government of the day. During this period, much productive capacity was lost though the argument remains that this was necessary as much of the capacity was not very productive. Much of that lost capacity was export-orientated such that the restructuring led to a structural trade deficit.

2.4. Data Specification Tests

Table 2.5 presents test results for the derived variables to be used in our estimations and calibrations which seek to explain the observed equity premium. Since much of our work assumes a normal distribution, tests are computed to confirm the data distribution. For evidence of the distribution of the data, 5 tests are conducted: skewness, kurtosis as well as the Jarque-Bera, Kolmogorov-Smirnov and Anderson-Darling normality tests.

2.4.1. Skewness & Kurtosis

It is possible for two sets of data to have the same mean and standard deviation but be differently skewed. Skewness is concerned with the degree of lack of symmetry in the data. Since a normal distribution assumes there is data symmetry, the coefficient of skewness as measured by the Pearson Coefficient or the Third Moment should be close to zero.[21]

Two distributions could be similar to the extent that they have the same mean, standard deviation and skewness but could differ in terms of the shape of the distribution i.e. how they are "peaked". When the data series is fairly flat, then the distribution shape is mostly of a platykurtic type kind with a kurtosis value of less than 3. For a leptokurtic type, the value for kurtosis is greater than 3 and implies that the distribution shape is peaked.

Table 2.5 reports values for skewness and kurtosis for both the level and logarithmically transformed data.

---

[21] The test of skewness is based on the Pearson Coefficient which gives very similar results to that given by the Third Moment.

Table 2.5

Values for skewness & kurtosis: 1919-1991

Derived (Real) Variables	Skewness	Kurtosis
Stock Return (rreqrl)	- 0.01	3.55
Risk-less Return (rrfrl)	- 0.29	5.18
Risk-less Return (rrfrl)	- 0.49	4.01
Per Capita Consumption - Growth (rpccl)	- 0.16	9.14
Per Capita Consumption - Growth (rpccl)	0.07	2.20
Stock Premium (repl)	- 0.13	7.23
Stock Premium (repl)	- 0.09	0.51
Production growth (rprodl)	0.12	3.97
GNP growth (rgdpcfc)	- 0.52	3.41
Gross Domestic Fixed Capital - Growth (rgdfcfl)	- 0.15	37.66
Gross Domestic Fixed Capital - Growth (rgdfcfl)	- 0.31	1.76
Term Premium (rterml)	- 0.69	1.33
Corporate Premium (rcorpl)	0.53	2.37
Gross Trading Profits	0.40	5.08
Gross Trading Profits - Growth	- 0.13	2.30

Author's calculations using data for 1917-1991

Whilst not perfectly symmetrical, many of the skewness coefficients are negatively skewed. With kurtosis however, the distribution of some variables are extremely peaked. For these variables including gross domestic fixed capital formation (rgdfcfl) and real per capital consumption, a second computation is done which replaces the highest absolute value with the closest value with the same sign to check the possibility of a single outlier resulting in very high values for kurtosis. Although the values for skewness are generally close to 0, this does not necessarily indicate symmetry for as Kendall and Stuart

(1977) have shown, some asymmetric distributions have odd order moments that are zero. This would be particularly true of stock prices and therefore stock returns which is assumed to respond to ARCH effects. Hence, we report values for normality tests.

2.4.2. Jarque-Bera Test for normality

The Jarque-Bera (BJ) test first introduced in 1982 is a test of normality based on skewness and kurtosis and which takes the form

$$JB = \left[\frac{T}{6} sk^2 + \frac{T}{24}(k - 3)^2\right]$$

where
sk is the value for skewness,
k is the value for kurtosis and
T is the no. of observations.

and which is distributed as χ^2 (2). Table 2.6 (Panel A) gives the Jarque-Bera statistic for the derived variables.

Table 2.6

***Panel A**: Jarque-Bera test statistics for the data :1919-1991*[22]

Derived (Real) Variables	Jarque-Bera Statistic
Stock Return (rreqrl)	0.92
Risk-less Return (rrfrl)	15.34
Per Capita Consumption Growth (rpccl)	113.52
Per Capita Consumption Growth (rpccl)	1.97
Stock Premium (repl)	53.92
Stock Premium (repl)	18.69
Production growth (rprodl)	2.99
GNP growth (rgdpcfc)	24.30

[22] When the Jarque-Bera statistic exceeds the critical value, a second computation is done which replaces a single outlier with a dummy value given the sensitivity of the test to outliers.

Gross Domestic Fixed Capital - Growth (rgdfcf)	3604.98
Gross Domestic Fixed Capital - Growth (rgdfcf)	5.82
Term Premium (rterm1)	14.11
Corporate Premium (rcorp1)	6.57
Gross Trading Profits	35.93
Gross Trading Profits - Growth	1.64

Author's calculations using data for 1919-1991

Table 2.6(Panel B) reports Anderson-Darling (AD) test statistics and associated *p*-values for the data. The AD test is similar to the Komolgorov-Smirnov test in that they are both based on the cumulative distribution function.

Table 2.6

Panel B: Anderson-Darling (AD) normality tests,1919-1991[23]

Derived (Real) Variables	*Anderson-Darling (AD) test* *T-statistics*	*p-value*
Stock Return (rreqr1)	0.732	0.054
Risk-less Return (rrfr1)	2.758	0.000
Per Capita Consumption Growth (rpcc1)	2.453	0.000
Stock Premium (rep1)	1.354	0.002
Production growth (rprod1)	1.226	0.000
GNP growth (rgdpcfc)	2.493	0.000
Gross Domestic Fixed Capital - Growth (rgdfcf)	7.328	0.000
Term Premium (rterm1)	1.579	0.000
Corporate Premium (rcorp1)	2.029	0.000
Gross Trading Profits	0.936	0.017
Gross Trading Profits - Growth	0.344	0.479

Author's calculations using data for 1919-1991

[23] These calculations were done using Minitab

For the normality tests, only stock returns and the change in gross trading profits could be described as normally distributed though in the case of stock returns not at the 10% level. Significantly perhaps, for the stock premium, the null hypothesis is rejected. Many of the financial data series display ARCH effects reflected in fatter tails. Using the Jarque-Bera test, test statistics for per capita consumption growth and gross domestic fixed capital formation are particularly high. However, once the outliers are replaced, the test statistics fall well within the critical value in the test for normality. For per capita consumption growth, the outlier value was in 1940 at the beginning of the 2nd World War when consumption fell dramatically. For gross domestic fixed capital formation, the outlier values relate to 1946 when post-war reconstruction meant very significant increases in investment expenditure. For our estimations, these dummy variables are not used for they would have the effect of making the target equity premium to be explained slightly easier. Including the dummy variables does not in fact change the reported values in a significant way. Overall, inspection of the data reveals that many of the financial variables display ARCH features where there is volatility clustering and returns have 'fat' tails owing to the presence of outliers.

CHAPTER THREE (3)

Summary

In chapter 3, we present and test the original power utility model; the results of which are generally inconsistent with expectations or theory in that unreasonable values for risk aversion (greater than 10) and the discount rate are implied by the model. The power utility model essentially argues that the observed risk premium can be determined by the coefficient of relative risk aversion and its covariance with consumption. This model also implies that the elasticity of intertemporal substitution is the reciprocal of the coefficient of risk aversion although attempts to completely separate them has only met with limited success.

More successful has been the introduction of the notion of habit consumption though not all of the earlier models of Abel (1990) and Cooley and Ogaki (1996) could be regarded as successes given the parameter estimates reported in this study. More successful have been the work of Pemberton (1993), Campbell and Cochrane (1995) and Campbell, Lo and Mackinlay (1997). The models of Campbell et al in particular were able to report reasonable parameters for all the key parameters having endogenously modelled these key variables. It is in that light that the work of Campbell et al is viewed as having significantly advanced the work in consumption asset pricing. Our results demonstrate this success with one exception which is that although the predicted equity premium falls within the 95% confidence interval, the predicted equity premium is found to be less than half the observed values. The habit consumption model is also viewed as a generalisation of the power utility model such that the power utility model cannot be viewed as a complete failure. It is this and the attempt to more closely match the historically observed equity premium in the context of habit consumption that forms the basis of chapter 4.

CHAPTER THREE

3.1. The Consumption Capital Asset Pricing Model

The original Capital Asset Pricing Model (CAPM) is based on the simple proposition that risk averse investors, in contrast to risk neutral investors, are only willing to hold higher risk assets if offered higher average returns. Investors who prefer not to carry any risk at all will tend to hold government issued risk-less assets.

The basic Consumption Capital Asset Pricing Model is based on the premise that the reason for hoarding wealth is to provide for future consumption with such future consumption directly related to stock returns. Fluctuations in consumption are reflected in an investor's willingness to substitute consumption intertemporally, i.e. save and invest in period t for consumption in period $t+1$.

The basic framework which follows that by Campbell, Lo and Mackinlay (1997) now follows.

3.1.1. The Stochastic Discount Factor

The basic relationship under consideration here is

$$E_t\left[Z_{i,t+1}M_{t+1}\right]=1 \tag{3.1.1}$$

for all assets i in all time periods. We can further define $Z_{i,t+1} = 1+R_{i,t+1}$ as the gross return on asset i at time $t+1$ and M_{t+1} is a positive random variable known as the stochastic discount factor or marginal rate of intertemporal substitution. This is the basic relationship common in most of financial theory. Taking the unconditional expectations of (3.1.1) and lagging by one period yields

$$E[Z_{it}M_t] = 1. \tag{3.1.2}$$

The aim here is to derive a measure of asset returns and for this use is made of the unconditional form where

$$E[Z_{it}M_t] = E[Z_{it}]E[M_t] + Cov(Z_{it}M_t)$$

such that

$$E[Z_{it}] = \frac{1}{E[M_t]}(E[Z_{it}M_t] - Cov(Z_{it}M_t)). \tag{3.1.3}$$

Since

$$E[Z_{it}M_t] = 1$$

then

$$E[Z_{it}] = \frac{1}{E[M_t]}(1 - Cov(Z_{it}M_t)). \tag{3.1.4}$$

Equation (3.1.4) explicitly shows that expected asset returns is dependent on the inverse of the stochastic discount factor and the covariance of asset returns and the stochastic discount factor. Equation (3.1.4) shows that the lower the stochastic discount factor, the higher the expected asset return and also that the smaller the covariance of the expected asset return with the stochastic discount factor, the higher is the asset's expected return. Furthermore, should there exist a zero unconditional covariance, then a formulation can be derived for the risk-less asset. Where the covariance is zero, the asset is risk-less and can be represented by

$$E[Z_{ot}] = \frac{1}{E[M_t]}. \tag{3.1.5}$$

The derivations for expected risky asset returns have a non zero covariance of the asset return and consumption via the intertemporal substitution parameter (3.1.4) whilst that the risk-less asset is assumed to have a zero covariance (3.1.5).

Hence (3.1.5) can be combined with (3.1.4) to obtain an identity for the expected excess return or equity premium

$$E[Z_{it} - Z_{ot}] = \left[\frac{1}{E[M_t]}(1 - Cov(Z_{it}, M_t))\right] - \left[\frac{1}{E[M_t]}\right] \quad (3.1.6)$$

or more simply

$$E[Z_{it} - Z_{ot}] = \frac{1}{E[M_t]} Cov(Z_{it}, M_t). \quad (3.1.7)$$

This says that the excess asset return is a combination of the risk-less asset return and the covariance of the asset's return with the inverse of the stochastic discount factor. As expected, the inverse of the discount factor will affect asset returns regardless of the value of its covariance with the stochastic discount factor.

3.1.2 Consumption Capital Asset Pricing with Power Utility Model

The stochastic discount factor can be further explored via the consumption choices available to a representative individual *j*, who is assumed to have a time separable utility with a discount factor δ. This investor is faced with the consumption choice between times *t* and *t+1* such that should the investor wish to forgo consumption in time *t*, the cost associated with that loss of utility will be related to the benefits to be derived in time period *t+1* from having invested in time period *t* and realised the proceeds in time period *t+1*.

The representative investor's time separable power utility function can be written as

$$Max\sum_{j=0}^{\infty}\delta^{j}\frac{C_{t+j}^{1-\gamma}}{1-\gamma} \tag{3.1.8}$$

where γ is the coefficient of relative risk aversion. This function maximises consumption subject to a discount rate δ and a risk factor γ. The function exhibits several properties; firstly, it is scale-invariant with constant return distributions and a risk premia that does not change as economic variables change.

The power utility function can therefore be written as

$$U = \mathrm{E}_t\left[\mathrm{Z}_{i,t+1}\,\delta\left(\frac{C_{t+1}^{1-\gamma}}{C_t^{1-\gamma}}\right)\right]. \tag{3.1.9}$$

The first-order condition for (3.1.9) will therefore be

$$\mathrm{E}\left[\mathrm{Z}_{i,t+1}\,\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right] = 1, \tag{3.1.10}$$

as derived by Campbell, Lo and Mackinlay (1997) and earlier by Grossman and Shiller (1981).[15] An important aspect of being able to explain observed stock returns involves being able to estimate the coefficient of relative risk aversion, γ.

[15] This is the form present in much of the literature discussed in chapter 1.

This choice problem can be represented by the stochastic discount factor M_{t+1} which can be written as

$$M_{t+1} = \frac{\delta U'(C_{j,t-1})}{U'(C_{jt})} \qquad (3.1.11)$$

where M_{t+1} is assumed to be positive.[16]

Since

$$\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} = \frac{u'(c_{j,t+1})}{u'(c_{jt})}$$

then, substituting (3.1.11) into (3.1.1), leads to

$$E_t\left[Z_{i,t+1}\frac{\delta U'(C_{j,t+1})}{U'(C_{jt})}\right] = 1. \qquad (3.1.12)$$

Re-arranging (3.1.12), then becomes

$$\delta E_t\left[Z_{i,t+1}U'(C_{j,t+1})\right] = U'(C_{jt}). \qquad (3.1.13)$$

This is the ***Consumption Capital Asset Pricing Model (CCAPM)*** whose formulation can be viewed as an equilibrium condition where the left hand side of (3.1.13) is the marginal benefit from consuming the proceeds in time *t+1*, the benefits of an investment made in time *t*. The right hand side of (3.1.13) is the lost marginal utility from having consumed one unit less in time *t*. The model is based on the assumption that individuals can be aggregated to form a single representative investor. Grossman and Shiller (1982) have shown that a single

[16] This is the basic relationship in discounting models which attempt to place present values on future benefits.

individual representative model can be used in an aggregated framework. There is a discussion of the role of heterogeneous individuals in chapter 7.

In the CCAPM, risk exists when asset returns are low even though the marginal utility of investor *j* is high. A high marginal utility for the investor is consistent with current consumption being low. There exists risk because the asset fails to deliver wealth to the investor precisely when it is most needed and who will then demand a larger risk premium to hold the asset. Conversely, risk also exists when asset returns are high in the context of a low marginal utility to the investor because a low marginal utility is consistent with high current consumption in that the investor places a low value on additional funds.

3.2. The Lognormal Power Utility Model

Assuming lognormality, represented by lower case letters, and conditional homoskedasticity, then (3.1.2) can be re-presented as[17]

$$\log \mathrm{E}_t(\mathrm{Z}, \mathrm{M}) = \mathrm{E}_t \log(\mathrm{Z}, \mathrm{M}) + \tfrac{1}{2} Var_t(\log(\mathrm{Z}, \mathrm{M})). \qquad (3.2.1a)$$

From defining terms in (3.1.1), Z = (1+ R), the gross return on an asset, such that

$$\log Z = \log(1 + R) = r. \qquad (3.2.1b)$$

Also,.given that

$$M_{t+1} = \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \qquad (3.2.1c)$$

then the log form

$$m_{t+1} = -\gamma(c_{t+1} - c_t) = 0. \qquad (3.2.1d)$$

Given equations (3.2.1b to 3.2.1d), then equation (3.2.1a) can now be written as

$$\log \mathrm{E}_t(\mathrm{Z}, \mathrm{M}) = \mathrm{E}_t(r_{t+1} + m_{t+1}) + \frac{1}{2} Var(r_{t+1} + m_{t+1}). \qquad (3.2.1e)$$

It is now possible to derive a form for the return on a risky asset $r_{i,t-1}$ where

$$\mathrm{E}_t r_{i,t+1} + \log\delta + \mathrm{E}_t(-\gamma\Delta c_{t+1}) + \tfrac{1}{2}\mathrm{E}\left[r_{i,t+1} - \gamma\Delta c_{t+1} - \mathrm{E}(r_{i,t-1} - \gamma\Delta c_{t-1})^2\right] = 0. \qquad (3.2.1)$$

Lower-case letters are used to represent logs. Equation (3.2.1) was first derived by Hansen & Singleton (1983) and more recently by Campbell, Lo and Mackinlay (1997).

---

[17] A randomly distributed variable X when conditionally lognormal takes the notationally convenient form log E_tX=E_t Log X +½Var$_t$ Log X where Var$_t$ Log X ≡ E_t [(Log X - E_t Log X)^2]. In addition X could be conditionally homoskedastic in which case, Var$_t$ Log X = E [(Log X - E_t Log X)^2] = Var (Log X - E_t Log X).

Equation (3.2.1) can be further simplified to show that

$$\mathrm{E}_t r_{i,t+1} + \log\delta - \gamma \mathrm{E}_t \Delta c_{t+1} + \tfrac{1}{2}\left[\sigma_{ii} + \gamma^2 \sigma_{cc} + 2\gamma\sigma_{ic}\right] = 0, \qquad (3.2.2)$$

where σ_{ii} denotes the variance of asset returns, σ_{cc} is the variance of consumption growth and σ_{ic} is the covariance of asset returns and consumption growth.

As previously mentioned, the risk-less rate, $r_{f,t+1}$, would be consistent with a zero covariance between asset returns and the intertemporal substitution of consumption implying that

$$r_{f,t+1} = -\log\delta - \tfrac{1}{2}\gamma^2\sigma_{cc} + \gamma\, \mathrm{E}_t \Delta c_{t+1}. \qquad (3.2.3)$$

Equation (3.2.3) is itself a linear function of consumption with a slope coefficient equal to the coefficient of relative risk aversion. Given (3.2.2) and (3.2.3), it is now possible to explicitly determine a formulation for the equity premium i.e.

$$\mathrm{E}_t\left[r_{i,t+1} - r_{f,t+1}\right] = -\frac{\sigma_{ii}}{2} + \gamma\,\sigma_{ic}. \qquad (3.2.4)$$

This says that the expected equity premium is a combination of half the variance of the asset return, and more relevantly, the coefficient of relative risk aversion multiplied by the covariance of asset returns and consumption growth. Campbell, Lo and Mackinlay (1997) pointed out that the first term on the right hand side is the Jensen Inequality adjustment "arising from the fact that we are describing expectations of log returns... the need for which can be eliminated by rewriting the equation in terms of the log of the expected ratio of gross returns" such that we focus only on the second term on the right hand side, $\gamma\,\sigma_{ic}$. Equation (3.2.4) shows that a higher equity premium is predicted by higher values of the coefficient of relative risk aversion as well as a higher covariance between asset returns and consumption growth. The inverse of the discount rate in (3.1.7) is equivalent to γ, the coefficient of relative risk aversion, found in (3.2.4). According to Campbell et al (1997), equation (3.2.3) can be reversed to express expected consumption growth as a linear function of the risk-less rate which is defined as the elasticity of intertemporal substitution. The elasticity of intertemporal substitution is denoted by Ω, which is the reciprocal of the coefficient of relative risk aversion, γ. The basic idea is that the higher the coefficient of relative risk aversion, the lower the elasticity of intertemporal substitution because with higher risk, investors are less willing to substitute intertemporally. Conversely, a lower coefficient of relative risk aversion implies a high degree of willingness to substitute intertemporally. This is an important derivation since there is much empirical focus on estimating the coefficient of relative risk aversion.

Mehra and Prescott considered that values for the coefficient of relative risk aversion must be between 0 and 10. They further considered that larger values for γ were not justifiable not least because it called into question whole aspects of rational expectations theory. In their analysis, they pointed out that for the Consumption Capital Asset Pricing Model to explain the observed equity premium of 6.18% will require a risk coefficient of around 30 which they thought was implausible.

3.2.1. <u>The Equity Premium & Risk-free Rate Puzzles</u>

For reasons of self-containment, Table 3.1 summarises, again, the Mehra and Prescott presentation of results of the Consumption Capital Asset Pricing Model which clearly fails to explain the observed premium of 6.18%. To a large extent, the reason for the failure has to do with the surprisingly high value for the risk-less rate. Consequently, the implied risk premium of 0.35% looks subjectively small given the higher levels of risk associated with equities.

Table 3.1

The mean simple return (%) on equity and bonds 1889-1978

	Real Equity Return	Risk-less Rate Return	Equity Premium
Observed	6.98	0.80	6.18
As per the CCAPM	4.05	3.70	0.35

Source: Mehra and Prescott 1985

Table 3.2

Moments of consumption growth and asset returns

Variable	Mean	Standard Deviation	Correlation With consumption growth	Covariance With consumption growth
Consumption growth	0.0172	0.0328	1.0000	0.0011
Stock return	0.0601	0.1674	0.4902	0.0027
Risk-less return	0.0183	0.0544	-0.1157	-0.0002
Equity premium	0.0418	0.1774	0.4979	0.0029

Source: Campbell, Lo and Mackinlay, 1997

Table 3.3

Moments of log consumption growth and asset returns

Variable	Mean	Standard Deviation	Correlation with consumption growth	Covariance with consumption growth
Consumption growth	0.0147	0.0314	1.0000	0.0009
Stock return	0.0657	0.2183	0.0485	0.0004
Risk-less return	0.0082	0.0723	0.0183	0.0001
Equity premium	0.0575	0.2046	0.0454	0.0003

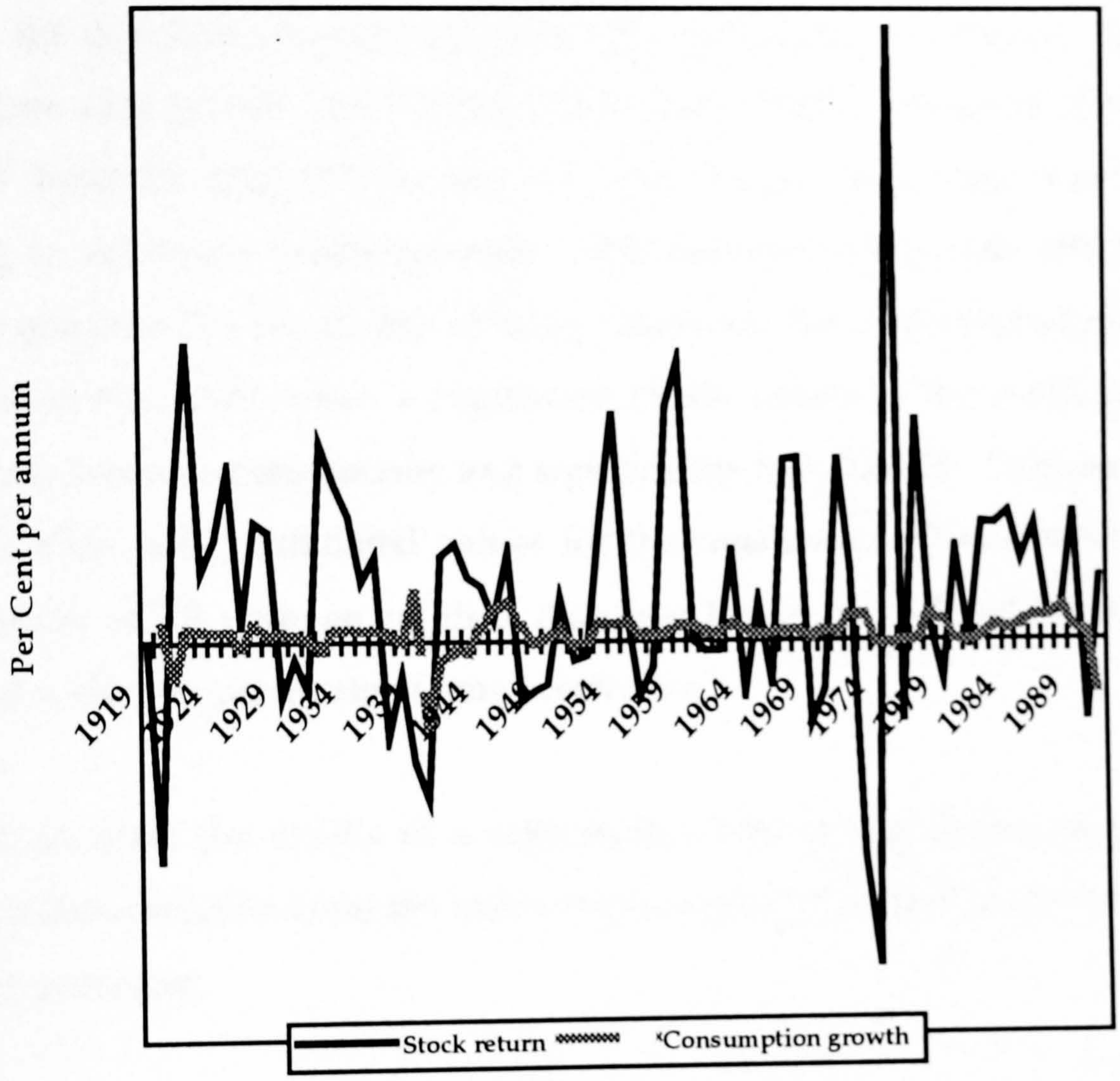
Source: Author's calculations using data on asset returns and consumption 1919-1991

Table 3.2 shows results from Campbell, Lo and Mackinlay (1997) whilst Table 3.3 shows results using the data described in chapter 2. The data in Table 3.3 shows the mean log equity premium to be higher than other studies have shown. Instinctively, there is an expectation of some correlation between consumption on the one hand and stock and stock premium returns on the other. This has tended to be the case with previous studies, which have shown correlation coefficients of around 0.5. This is much higher than the 0.05 revealed in our data and as indicated by figure 3.1.[18] This substantial difference could be the result of varying asset ownership patterns in the USA and UK. Wider and more direct ownership of stocks in the USA perhaps accounts for the much higher correlation between consumption growth and equity returns on one hand and the equity premium on the other. From Table 3.3, the log equity premium of 5.75% and a standard deviation of 20.5% correspond to a simple equity premium of just under 8% using the formula for the mean of the lognormal random variable. If the equity premium has a standard deviation of 21% then this implies that the stochastic discount factor must have a standard deviation of 21% if it has a mean of 1. From the data however, consumption (of non durable goods and services) is quite a smooth series with a standard deviation of only 3.1% such that the covariance of consumption and the excess stock return is less than 0.1% which is consistent with the relatively low correlation of consumption growth with the equity premium of around 5%. Substituting the moments into (3.2.4) shows that the coefficient of relative risk aversion required to fit the observed equity premium is 190. This is well above the value of 10 (by a factor of 19), the maximum thought plausible by Mehra and Prescott and clearly unrealistic. This is also significantly different from the Campbell, Lo and Mackinlay estimation that the required coefficient of relative risk aversion is 19 using US data. The key reason for this is the differing covariance of the equity premium with consumption growth with Campbell et al indicating a value of 0.003 and the author's calculations implying a value of 0.0003. The exactly explains why the required coefficient for risk aversion in our estimation is about 10 times the size of that indicated by Campbell et al using the power utility model.

[18] Campbell, Lo and Mackinlay (1997) report a correlation of 0.5.

Figure 3.1

Real risk premium & consumption growth 1919-1991



Equation (3.2.3) implies that the expectation of the unconditional risk-less interest rate is

$$\mathrm{E}r_{ft} = -\mathbf{log}\,\delta - \frac{1}{2}\gamma^2\sigma_{cc} + \gamma g \tag{3.2.5}$$

where g ($\Delta c_{t+1} = c_{t+1} - c_t$) is the growth rate of real per capita consumption, σ_{cc} is the variance of consumption growth. Substituting the moments into (3.2.3), implies a discount rate not consistent with theory. This is the result of the already mentioned low covariance between the asset return and consumption growth for if a covariance value of 0.003 is assumed, then the implied values for the coefficient of relative risk aversion (20) and discount rate (1.09) are similar to that reported by Campbell, Lo and Mackinlay (1997). In other words, the coefficient of relative risk aversion leads to an inconsistency in that it implies a value for the discount rate not consistent with expectations or theory. This is the risk-free rate puzzle first identified by Weil (1989). This risk-free rate puzzle is basically that if investors are risk averse, then they would be unwilling to substitute intertemporally. The risk-free rate puzzle effectively calls into question the possibility of large values for the coefficient of relative risk aversion especially when a regression yields values of the coefficient of relative risk aversion substantially and significantly less than 10. Campbell, Lo and Mackinlay (1997) estimated values for the constant coefficient of relative risk aversion of 19 with an implicit discount factor of 1.12, which is also consistent with a negative rate of time preference.

Table 3.4 presents the results of a regression of the equity premium on the product of the deviation from the mean of per capita consumption growth and the equity premium.

Table 3.4

Estimating the coefficient of relative risk aversion

Dependent Variable	*Explanatory Variable*	*Constant*	*CRRA* γ
Real stock return	*(i-mean(i))-(c-mean(c))*	0.067 (2.645) [0.010]	-7.421 (-1.342) [0.184]
$R^2 = 0.03$ F (1, 70) = 1.765 [0.188] SC $\chi^2(1)$ = 0.342 [0.559]	F (1, 70) = 1.765 (0.188) Durbin Watson = 1.777 JB Test $\chi^2(2)$ = 12.02 [0.002]	(T-statistics) [*p* - values]	

Source: Author's calculations using data on asset returns and consumption 1919-1991

Table 3.4 reveals a negative risk aversion coefficient which although not significant, is against theoretical expectations. The specification of the model is however undermined by Jarque-Bera (JB) test which indicates rejection of the null hypothesis and therefore undermining the theoretical underpinning of this form of the power utility model.

According to Campbell and Cochrane (1995), equation (3.2.2) can be written in the form of a regression:

$$r_{i,t+1} = \mu_i + \gamma_i \Delta c_{t+1} + u_{i,t+1} \qquad (3.2.6)$$

if the error term, $u_{i,t+1} \equiv r_{i,t+1} - \mathrm{E}_t\left[r_{i,t+1}\right] - \gamma\left(\Delta c_{t+1} - \mathrm{E}_t\left[\Delta c_{t-1}\right]\right)$. Since the coefficient of relative risk aversion is equivalent to the inverse of the stochastic discount factor, the regression can be reversed with the estimated parameter being the elasticity of intertemporal substitution.

Table 3.5 shows parameter estimates for regressions of consumption growth and asset returns on one another. The parameter estimates are positive as suggested by the theory with a risk aversion coefficient of 0.34 though not significantly different from zero.

Table 3.5

Regression of log asset returns and consumption

$$\text{Equation 1: consumption growth}_t = a + \gamma(\text{stock returns})_t$$

$$\text{Equation 2: stock returns}_t = a + \Omega(\text{consumption growth})_t$$

$$\text{Equation 3: consumption growth}_t = a + \gamma(\text{risk - less rate})_t$$

$$\text{Equation 4: risk - less rate}_t = a + \Omega(\text{consumption growth})_t$$

Equation	γ (t-stat) [p-value]	Ω (t-stat) [p-value]	F (1,70) [*p*-value]	DW	JB $\chi^2(1)$ [p-values]	SC $\chi^2(2)$ [p-values]	RESET $\chi^2(2)$ [p-values]
(1)		0.007 (0.277) [0.783]	0.165 [0.686]	1.85	497.5 [0.000]	0.186 [0.666]	0.318 [0.573]
(2)	0.337 (0.277) [0.783]		0.165 [0.686]	1.91	16.66 [0.000]	0.013 [0.998]	2.557 [0.110]
(3)		0.008 (0.045) [0.964]	0.023 [0.879]	1.86	517.8 [0.000]	0.170 [0.680]	13.900 [0.000]
(4)	0.042 (0.045) [0.964]		0.023 [0.879]	1.26	172.4 [0.000]	11.609 [0.001]	11.625 [0.001]

Source: Author's calculations using data on asset returns and consumption 1919-1991

The R^2 for the regression of consumption growth and stock returns is negligible at 0.018 whilst that for consumption growth and the risk-less rate is around 0.011. This suggests that consumption growth has a little or no capacity to forecast stock and risk-less returns. The results demonstrate that the link is a less direct one than is implied here. These simple regressions also present evidence against the notion that in the power utility model, the elasticity of intertemporal substitution is the reciprocal of the coefficient of risk aversion since the implied estimates, given risk aversion, are not consistent with those reported in table 3.5.

3.2.2. Instrumental Variables Estimation

Equation (3.2.2) implied that expected asset returns and expected consumption growth are perfectly correlated but that the standard deviation of asset returns is γ times as large as that of expected consumption growth. Since (3.2.6) is derived from (3.2.2), then there exists the possibility that the error term, $u_{i,j-1}$, will be correlated with realised consumption growth, Δc_{t+1}, such that OLS estimates as presented in Table 3.5 may be biased. Hence OLS is not an appropriate method of estimation. Hansen & Singleton (1983), Hall (1988) and Campbell et al (1997) have all proposed instrumental variables (IV) estimation as a way of solving this problem of correlated errors.

The Instrumental Variable (IV) estimation or the Generalised Method of Moments (GMM) methods can be used to solve this problem of correlated errors as these methods provide us with asymptotically more efficient and consistent estimates. With the IV estimation, the search is for variables to act as a proxy for Δc_{t+1} by being the regressors in a first stage regression with dependent variable Δc_{t+1}, from which fitted values for Δc_{t+1} can be computed. The variables chosen to act as regressors must have the characteristic of being highly correlated with Δc_{t+1} but not contemporaneously with the error term. In such circumstances, lagged variables can be used as instruments. A second stage regression can then be undertaken with $r_{i,t+1}$ as the dependent variable and the fitted values from the first stage regression as the independent variable for consumption growth.

Of relevance also in IV estimation is the question of over-identification i.e. are the choice of instruments used in the first stage regressions valid. If they are, the parameter is said to be identified. Alternatively, an over-identified equation is one in which the restrictions imposed by the instruments are over and beyond the minimum necessary to identify the equation. A commonly used test for over-identification is the R^2 and the associated significance level of a regression of the residuals from the second stage regression on the instruments. This is the standard test devised by Engel (1984).

Another relevant consideration at this stage concerns the difficulty in forecasting consumption growth. Campbell, Lo and Mackinlay (1997) pointed to established processes where the regression can be reversed as asymptotic theory can produce spurious results in the context of finite samples. So reversing (3.2.6), results in

$$\Delta c_{t+1} = \kappa_i + \Omega r_{i,t+1} + v_{i,t+1} \qquad (3.2.7)$$

where Ω is the reciprocal of γ.

Appendix 1 provides a self-contained exposition of the Instrumental Variables (IV) estimation.

Since the asset returns might be correlated with the error term, the choice of instruments for the estimation can be the lag of any variables including our initial variables. The instruments are therefore either 1 or 1 and 2 lags of the real risk-less return, real consumption of non-durable goods and services and alternately, the real stock return or the real log dividend-price ratio. Using the real stock return and the dividend-price ratio in place of one another is in part a consequence of the Heaton and Lucas (1992) formulation, which showed that the dividend-price ratio is equal to the stock return.

Tables 3.6, Panel A and B, reports results for (3.2.6) and (3.2.7). The results include R^2 statistics and the significance levels for the first stage regressions of the asset returns and consumption growth on the instruments. Column 1 shows the asset return being estimated in the regression and the number of lags of the instruments whilst Column 2 shows the R^2 and joint significance levels of the explanatory variables in regressions of asset returns and consumption growth on the instruments. Column 3 shows the parameter estimates and standard errors (in parentheses) for γ and Ω from the second stage regressions. Finally in column 4, are the R^2 statistics and the significance levels of a regression of the residuals on the instruments used as a test of the over-identifying restrictions of the model.

Table 3.6

Panel A: IV estimates of asset returns and consumption

$$Test(1) \quad (3.2.6) \quad r_{i,t+1} = \mu_i + \gamma \Delta c_{t+1} + u_{i,t+1}$$

$$Test(2) \quad (3.2.7) \quad \Delta c_{t+1} = \kappa_i + \Omega r_{i,t+1} + v_{i,t+1}$$

Return (Instruments)	First Stage Regressions r Δc	γ (s.e)	Ω (s.e)	Test(1) Test(2)
Risk-less Return (1)	R^2 0.147 0.236 Sig (0.014) (0.000)	1.551 (0.532)	0.488 (0.107)	R^2 0.040 0.070 Sig (0.485) (0.184)
Stock Return (1)	R^2 0.156 0.236 Sig (0.019) (0.000)	2.803 (1.652)	0.093 (0.044)	R^2 0.121 0.186 Sig (0.036) (0.003)
Risk-less Return (1 and 2)	R^2 0.451 0.307 Sig (0.000) (0.001)	1.337 (0.393)	0.242 (0.088)	R^2 0.357 0.230 Sig (0.001) (0.009)
Stock Return (1 and 2)	R^2 0.232 0.307 Sig (0.009) (0.001)	2.840 (1.496)	0.076 (0.035)	R^2 0.191 0.257 Sig (0.032) (0.004)

Source: Author's calculations using data on asset returns and consumption 1919-1991

Panel A: IV estimates of asset returns and consumption - diagnostics

Return (Instruments)	*Durbin Watson*	*Serial Correlation*	*Normality*	*Functional Form (FF)*
Risk-less Return (1)				
Equation (3.2.6)	*1.06*	*9.22 [0.00]*	*619.87 [0.00]*	*0.020 [0.89]*
Equation (3.2.7)	*1.30*	*3.07 [0.08]*	*463.93 [0.00]*	*9.816 [0.00]*
Stock Return (1)				
Equation (3.2.6)	*1.79*	*0.34 [0.56]*	*15.11 [0.00]*	*0.439 [0.51]*
Equation (3.2.7)	*1.70*	*1.14 [0.29]*	*43.72 [0.00]*	*2.827 [0.09]*
Risk-less Return (1 & 2)				
Equation (3.2.6)	*1.34*	*7.13 [0.01]*	*243.31 [0.00]*	*0.906 [0.34]*
Equation (3.2.7)	*1.92*	*0.03 [0.86]*	*155.35 [0.00]*	*9.718 [0.00]*
Stock Return (1 & 2)				
Equation (3.2.6)	*1.91*	*0.02 [0.95]*	*19.51 [0.00]*	*0.691 [0.41]*
Equation (3.2.7)	*1.84*	*0.43 [0.50]*	*119.20 [0.00]*	*2.835 [0.09]*

Source: Author's calculations using data on asset returns and consumption 1919-1991

Panel A of Table 3.6 reports results for a model which uses the log dividend-price ratio as the risky asset return instrument. In this formulation, there exist some evidence, with the R^2, that the real commercial paper rate, the real stock return and real consumption growth are all forecastable with consumption growth the most forecastable. As expected, the estimates for γ and Ω are positive, as implied by the theory and are significant at conventional levels. Even though the parameter estimates are positive, the over-identifying restrictions of the model are rejected when 1 and 2 lags of instruments are used in the estimation of the stock return. In fact, the over-identifying restrictions of the model are only not rejected when only 1 lag of the instruments are used in an estimation of the risk-less return. Looking at the values for γ and Ω in table 3.6 (Panel A), the model also tends to reject the notion that the elasticity of intertemporal substitution is the reciprocal of the coefficient of relative risk aversion. In other words, only in an estimation of the risk-less return using 1 lag of the instruments does the value for Ω come close to being the reciprocal of the coefficient of relative risk aversion. With the exception of normality tests, the diagnostics are much more supportive of the model with stock returns as opposed to the risk-less rate.

Table 3.6

Panel B: *IV estimates of asset returns and consumption*

Return (Instruments)	First Stage Regressions	r	Δc	γ (s.e)	Ω (s.e)		Test(1)	Test(2)
Risk-less Return	R^2	0.136	0.273	1.386	0.543	R^2	0.037	0.085
(1)	Sig	(0.021)	(0.000)	(0.497)	(0.130)	Sig	(0.469)	(0.117)
Stock Return	R^2	0.154	0.273	2.054	0.079	R^2	0.132	0.239
(1)	Sig	(0.011)	(0.000)	(1.551)	(0.046)	Sig	(0.024)	(0.002)
Risk-less Return	R^2	0.480	0.267	2.103	0.311	R^2	0.241	0.112
(1 and 2)	Sig	(0.000)	(0.003)	(0.376)	(0.081)	Sig	(0.006)	(0.261)
Stock Return	R^2	0.222	0.267	1.219	0.029	R^2	0.215	0.260
(1 and 2)	Sig	(0.012)	(0.003)	(1.638)	(0.036)	Sig	(0.015)	(0.003)

Source: Author's calculations using data on asset returns and consumption 1919-1991

Panel B: *IV estimates of asset returns and consumption - diagnostics*

Return (Instruments)	*Durbin Watson*	*Serial Correlation*	*Normality*	*Functional Form (FF)*
Risk-less Return (1)				
Equation (3.2.6)	1.03	9.20 [0.00]	625.49 [0.00]	0.001 [0.98]
Equation (3.2.7)	1.21	6.17 [0.01]	538.05 [0.00]	9.464 [0.00]
Stock Return (1)				
Equation (3.2.6)	1.82	0.45 [0.50]	19.34 [0.00]	2.271 [0.13]
Equation (3.2.7)	1.70	1.02 [0.31]	80.04 [0.00]	2.697 [0.10]
Risk-less Return (1 & 2)				
Equation (3.2.6)	1.64	1.44 [0.23]	217.19 [0.00]	0.290 [0.59]
Equation (3.2.7)	1.87	0.16 [0.69]	142.40 [0.00]	5.083 [0.02]
Stock Return (1 & 2)				
Equation (3.2.6)	1.91	0.03 [0.87]	23.86 [0.00]	2.074 [0.15]
Equation (3.2.7)	1.82	0.71 [0.40]	464.38 [0.00]	0.381 [0.54]

Source: Author's calculations using data on asset returns and consumption 1919-1991

This feature of the results in table 3.6, Panel A is repeated in table 3.6, Panel B, where the lag of the stock return is used as a risky return instrument. The results are very similar to when the log dividend-price ratio is used as the risky asset return instrument. The parameter estimates of 1.2 to 1.5 remain reasonable and similar to estimates of more recent studies. The R^2 remains high, indicating reasonable levels of variable forecastability.

The results provide support for the power utility model by not rejecting the possibility of a positive γ, which would explain the equity premium puzzle, and that of a low positive Ω, which would explain large fluctuations in consumption growth. The results also provide some evidence that consumption-based model explanations of the observed equity premium are perhaps more promising when the consumption horizon is relatively short i.e. no more than 1 lag. The idea that the elasticity of intertemporal substitution is the reciprocal of the risk aversion parameter is not wholly supported in the context of stock returns though some support exists in estimations of a relatively low risk asset return. In other words, it is possible that in estimations of returns for a relatively low risk asset, the elasticity of intertemporal substitution could well take a value close to the reciprocal of the constant risk aversion parameter.

These results are not inconsistent with a view that consumers use past consumption as a benchmark in decisions about present and future consumption. The only reservation relates to the precise values for the coefficient of relative risk aversion which would need to be close to 1 to be consistent with a time separable utility function between consumption and leisure.

Table 3.6

Panel C: IV estimates of asset returns and consumption

Return (Instruments)	First Stage Regressions r	Δc	γ (s.e.)	Ω (s.e.)	Test(1)	Test(2)
Risk-less Return	R^2 0.275	0.034	-1.984	-0.088	R^2 0.106	0.028
(1)	Sig (0.000)	(0.485)	(1.318)	(0.113)	Sig (0.004)	(0.234)
Stock Return	R^2 0.080	0.034	-6.365	-0.100	R^2 0.008	0.007
(1)	Sig(0.071)	(0.485)	(5.428)	(0.091)	Sig (0.673)	(0.705)
Risk-less Return	R^2 0.297	0.102	-0.953	-0.118	R^2 0.221	0.091
(1 and 2)	Sig (0.000)	(0.145)	(0.567)	(0.109)	Sig (0.000)	(0.096)
Stock Return	R^2 0.110	0.102	-0.235	-0.008	R^2 0.105	0.097
(1 and 2)	Sig (0.105)	(0.145)	(1.650)	(0.059)	Sig (0.056)	(0.075)

Source: Campbell, Lo and Mackinlay (1997)

Panel C of Table 3.6 reports results from Campbell, Lo and Mackinlay (1997) for a model which uses the log dividend-price ratio instead of the stock return as a return instrument. The real commercial paper rate is found to be forecastable though much less so is stock return and consumption growth. The IV estimates for γ and Ω are negative though not significantly from zero. Furthermore, the over-identifying restrictions in estimates of γ are overwhelmingly rejected whenever the risk-less return is used as the asset.

The Campbell, Lo and Mackinlay (1997) results are much less promising than those reported in Table 3.6, Panels A and B in terms of the forecastability of variables and the parameters signs. The over-identifying restrictions of the models, and the conclusions, are broadly the same with the exception of the estimations including the risk-less return. The overall conclusions therefore remain that the power utility model is incapable of explaining the equity premium.

3.2.3 Power Utility and the Generalised Method of Moments

So far, much of the focus has been on the loglinear model of the CCAPM. However, Hansen and Singleton's (1982) Generalised Method of Moments (GMM) makes it possible to test the power utility model without making assumptions about the distribution of the data. This is a relaxation of the linear assumption of the traditional power utility model.

Appendix 2 provides an exposition of the Generalised Method of Moments in an instrumental (two-stage) variables context. The model developed by Hansen, Heaton & Ogaki (1993) is used to estimate the risk aversion parameter. Table 3.7, Panel A, presents results for a single asset model where asset returns (column 1) are used in an estimation of the coefficient of relative risk aversion as in (3.2.6). The second and third columns show the estimated risk aversion parameter values and their associated standard errors together with chi-squared statistics and associated probability values in a test of the over-identifying restrictions. The results show that estimations using the risk-less return and dividend-price ratio as asset returns are able to deliver positive risk aversion estimates although when 1 lag is used, the over-identifying restrictions are not all rejected. The negative parameter found with the stock return is maintained when 1 and 2 lags are used though the over-identifying restrictions of the model are rejected only for the dividend-price ratio. These negative parameter estimates are inconsistent with the theory but consistent with the results of Campbell, Lo and Mackinlay (1997). The negative parameter found using GMM provides fairly strong evidence against the power utility model and its ability to explain the observed equity premium with a reasonable parameter for risk aversion.

Table 3.7
Panel A
GMM estimates of the coefficient of relative risk aversion in a single asset power utility model

Asset Returns (Instrument lags)	Initial Weighting Value for γ^{**}	After 2 iterations γ^{**} (s.e)	After 2 iterations Chi Square Test for OIR (p-value)	After 5 iterations γ^{**} (s.e)	After 5 iterations Chi Square Test for OIR (p-value)
Risk-less Return (1)	1.945	1.105 (0.792)	1.247 (0.264)	1.019 (0.580)	1.385 (0.239)
Dividend-Price Ratio (1)	2.723	1.723 (1.307)	1.082 (0.298)	1.107 (0.858)	3.777 (0.052)
Stock Return (1)	- 1.177	-7.689 (3.789)	1.393 (0.237)	-5.823 (4.019)	0.923 (0.337)
Risk-less Return (1 and 2)	2.993	2.059 (0.942)	5.544 (0.136)	2.495 (0.586)	7.555 (0.056)
Dividend-Price Ratio (1 and 2)	3.540	0.322 (1.618)	10.409 (0.015)	0.277 (0.617)	19.032 (0.000)
Stock Return (1 and 2)	- 1.562	-9.033 (2.986)	2.209 (0.530)	-7.753 (3.977)	1.325 (0.723)

Source: Author's calculations using data on asset returns and consumption 1919-1991

Table 3.7
Panel B
GMM estimates of the coefficient of relative risk aversion in a multiple asset power utility model

Asset Returns (Instrument lags)	Initial Weighting Value for γ^{**}	(After 2 iterations) γ^{**} (s.e.)	Chi Square Test for OIR (p-value)	(After 5 iterations) γ^{**} (s.e.)	Chi Square Test for OIR (p-value)
Dividend-Price ratio	-1.233	-0.993	38.065	-0.697	37.754
/Risk-less Return (1)		(0.693)	(0.000)	(0.658)	(0.000)
Stock Return	-2.689	-2.556	14.599	-0.689	14.438
/Risk-less Return (1)		(2.094)	(0.024)	(0.637)	(0.025)
Dividend-Price Ratio	-2.367	-2.250	43.667	-1.912	43.945
/Risk-less Return (1 & 2)		(0.547)	(0.000)	(0.501)	(0.000)
Stock Return /	-10.300	-5.112	22.239	-0.963	29.093
Risk-less Return (1 & 2)		(2.801)	(0.034)	(0.302)	(0.004)

Source: Author's calculations using data on asset returns and consumption 1919-1991

Table 3.7, Panel B, reports results for a multiple asset model whose results are directly comparable to the instrumental variable estimation results of table 3.6 and where combinations of assets (column 1) are used as instruments in estimating the parameters and thereby determine values for the risk aversion parameter. At the asymptotically adequate two-stage level, all the parameter estimates are negative though not significantly so when 1 lag of the instruments is used. The over-identifying restrictions of the model are rejected at the 5% level for all estimations.

These results are not at all encouraging for the power utility model especially given the negative parameter estimates though they do not rule out the possibility of a high coefficient of risk aversion which would explain the equity premium and a low elasticity of intertemporal substitution which would explain intertemporal substitution. These multiple asset results are consistent with those of Campbell, Lo and Mackinlay (1997). Note is also taken of the fact that the GMM results are much less promising than those of the IV estimation perhaps owing to the strength of the GMM estimation.

3.3. Separating Risk Aversion and Intertemporal Substitution

The simple power utility model previously outlined implied that the elasticity of intertemporal substitution, Ω, is the reciprocal of the coefficient of relative risk aversion, γ. Although a higher coefficient of relative risk aversion implies less willingness to substitute intertemporally, Epstein & Zin (1989, 1991) however, rejected the tight link between risk aversion and intertemporal substitution and propose an objective function recursively defined by

$$U_t = \left\{ (1-\beta) C_t^{\frac{1-\gamma}{\theta}} + \beta \left(\mathrm{E}_t U_{t+1}^{1-\gamma} \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \tag{3.3.1}$$

where $\theta \equiv (1-\gamma)/(1-1/\Omega)$. The objective function is subject to an intertemporal budget constraint which can be written as

$$W_{t+1} = Z_{m,t+1}(W_t - C_t) \tag{3.3.2}$$

where $Z_{m,t+1}$ is the return on the 'market' portfolio of all invested wealth, C_t is consumption and W_t denotes wealth.

Epstein and Zin then combined (3.3.1) and (3.3.2) to show that

$$\mathrm{E}_t \left[\left\{ \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\Omega}} \right\}^{\theta} \left\{ \frac{1}{Z_{m,t+1}} \right\}^{1-\theta} Z_{i,t+1} \right] = 1. \tag{3.3.3}$$

Maintaining the assumption of homoskedasticity and joint lognormality, (3.3.3) implies that

$$r_{f,t+1} = -\log \delta + \frac{\theta - 1}{2} \sigma_{mm} - \frac{\theta}{2\Omega^2} \sigma_{cc} + \frac{1}{\Omega} \mathrm{E}_t \Delta c_{t+1}, \tag{3.3.4}$$

$$E_t\left[r_{m,t+1} - r_{f,t+1}\right] = (1/2 - \theta)\sigma_{mm} + \frac{\theta}{\Omega}\sigma_{cc}. \qquad (3.3.5)$$

Consequently, the risk premium on assets other than that contained in the market portfolio can be represented by

$$E_t r_{i,t+1} - r_{f,t+1} = -\frac{\sigma_{ii}}{2} + \theta\frac{\sigma_{ic}}{\Omega} + (1-\theta)\sigma_{im} \qquad (3.3.6)$$

The derivation is quite similar to the traditional CAPM in that asset returns are combined with a 'market' portfolio. In fact, the traditional CAPM is derived if we assume that $\theta = 0$, whereas if $\theta = 1$, we have the power utility model.[19]

Equation (3.3.6) can be written in the more convenient form

$$E_t r_{i,t+1} - r_{f,t+1} = -\frac{\sigma_{ii}}{2} + \sigma_{im} + \theta\left(\frac{\sigma_{ic} + \Omega\sigma_{im}}{\Omega}\right). \qquad (3.3.7)$$

Overall these attempts to break the link between the intertemporal elasticity of substitution and the coefficient of relative risk aversion have met with limited success in that the explanatory powers of the models have not been able to yield coefficient estimates able to explain the equity premium.

[19] Weil (1989) used a model based on the KrepsPorteus non expected utility preferences which also relaxes the constraint familiar in time additive models that the intertemporal elasticity of substitution is the inverse of a constant coefficient of relative risk aversion. Weil's model assumes a constant elasticity of intertemporal substituiton as well as a constant but unrelated coefficient of relative risk aversion. According to Weil (1989), the time additive expected utility function as developed by Epstein & Zin (1987) and Weil (1988) is rejected by the data. Intuitively, this separation should work better than the time additive model but it does not. In fact Weil (1989) argued that a new puzzle is highlighted; the risk free rate puzzle. In other words, why is the risk free rate so low given the assumption that investors are averse to intertemporal substitution. The issue here is that a low risk free rate is only likely in equilibrium if investors have a negative rate of time preference. Kocherlatoka (1990), using the Generalised Method of Moments, finds no increase in the explanatory powers of the model when the link between the elasticity of intertemporal substitution and the coefficient of relative risk aversion is broken such that the equity premium puzzle as outlined by Mehra and Prescott remains unresolved. Some commentary on this literature is presented in chapter 1.

Table 3.8 (Panel A) reports results for equation (3.3.7) using observed data.

Table 3.8

Panel A: Moments of log consumption growth & asset returns

Variable	Mean	Standard Deviation	Covariance with Stock Return	Correlation with Stock Return
Consumption growth	0.014	0.031	0.001	0.051
Stock return	0.067	0.224	0.050	1.000
Market return	0.007	0.196	0.033	0.755
Stock premium	0.060	0.209	0.044	0.944

Author's calculations using data on asset returns and consumption 1919-1988

The market return is based on the Industrial Ordinary Share Price Index which reports a mean return of less than 1% (Table 3.8, Panel A) which is relatively low. However, the standard deviation matches that of the stock returns and the correlation with the market return is 0.76.

Table 3.8 (Panel B) reveals values for the coefficient of risk aversion for given values of the elasticity of intertemporal substitution. For the parameter estimates, (3.3.7) is explicitly rewritten to find a value for θ,

$$\theta = \frac{E_t rp_{i,t+1} + \frac{\sigma_{ii}}{2} - \sigma_{im}}{\frac{\sigma_{ic}}{\Omega} + \sigma_{im}} \qquad (3.3.8)$$

where $E_t rp_{i,t+1} = E_t r_{i,t+1} - r_{f,t+1}$.

The coefficient of relative risk aversion, γ, is rewritten as

$$1-\theta\left(1-\frac{1}{\Omega}\right). \qquad (3.3.9)$$

Using observed data, parameter estimates are then found as presented in table 3.8, panel B.

Table 3.8

Panel B : Coefficient estimates of the Epstein & Zin Model (Eq 3.3.7)

Value of Ω	θ	γ
0.05	1.329	26.24
0.10	1.460	14.14
0.20	1.535	7.14
0.30	1.562	4.64
0.40	1.576	3.36
0.50	1.585	2.59
1.00	1.602	1.00

Author's calculations using data on asset returns and consumption 1919-1988

The results are a success in that the model delivers parameter estimates for the elasticity of intertemporal substitution of less than 1 which is consistent with a positive coefficient of relative risk aversion of less than 10, the maximum value assumed plausible. The results of the model are less of a success given that a coefficient of risk aversion value of 1 is only possible with a value for the elasticity of intertemporal substitution of 1 as well. The results therefore raise the possibility of a relatively high parameter value for the coefficient of relative risk aversion. Furthermore, many of the more recent studies have tended to indicate a value for the elasticity of intertemporal substitution of around 0.2 which implies a value for the risk aversion coefficient of just over 7, which looks subjectively high. The model breaks the link between intertemporal substitution and risk aversion in that the elasticity of intertemporal substitution is never quite the reciprocal of the risk aversion coefficient.

3.4. Substituting Consumption out of the model

Given some of the problems so far encountered with the power utility model and its extensions, one way around this is to substitute consumption out of the model such that mean returns are related to the underlying state variables that determine consumption. Campbell (1993) and Campbell, Lo and Mackinlay (1997) suggested a model obtained by loglinearizing the intertemporal budget constraint (3.3.2) around the mean of log consumption-wealth ratio to obtain

$$\Delta w_{t+1} \approx r_{m,t+1} + k + \left(1 - \frac{1}{\rho}\right)(c_t - w_t), \qquad (3.4.1)$$

where $\rho \equiv 1 - \exp(c - w)$ and k is a constant. Campbell et al (1997) then proposed combining the left hand sides of (3.4.1) with the following "trivial equality" $\Delta w_{t+1} = \Delta c_{t+1} - \Delta(c_{t+1} - w_{t+1})$ to obtain a difference equation in the log consumption-wealth ratio, $c_t - w_t$ which can be solved forward by assuming that $\lim_{\to\alpha} \rho^j (c_{t+j} - w_{t+j}) = 0$ such that

$$c_t - w_t = \sum_{j=1}^{\infty} \rho^j (r_{m,t+j} - \Delta c_{t+j}) + \frac{\rho k}{1-\rho}. \qquad (3.4.2)$$

According to Campbell (1993), whilst equation (3.4.2) holds ex post, it can also hold ex ante if expectations are taken of equation (3.4.2) for as the left hand side remains unchanged and the right hand side becomes the expected discounted value such that

$$c_t - w_t = \mathrm{E}_t \sum_{j=1}^{\infty} \rho^j (r_{m,t+j} - \Delta c_{t+j}) + \frac{\rho k}{1-\rho}. \qquad (3.4.3)$$

Equation (3.4.3) shows clearly that where the consumption-wealth ratio is high, then the investor is either expecting high returns on wealth in the future or expects low consumption growth rates. Equation (3.4.3) can now be combined with (3.3.4) and (3.3.5) to obtain a consumption to wealth formulation of

$$c_t - w_t = (1-\Omega)E_t\sum_{j=1}^{\infty}\rho^j r_{m,t+j} + \frac{\rho(k-\mu_m)}{1-\rho}. \quad (3.4.4)$$

The log consumption-wealth ratio $(c_t - w_t)$ is therefore defined as (1-Ω) times the discounted value of the expected returns on wealth invested plus a constant term. When Ω is less than 1, the consumer is thought of as unwilling to substitute intertemporally such that the income effect dominates, whereas a value for Ω greater than 1 implies a greater willingness to substitute intertemporally i.e. a substitution effect.

According to Campbell, Lo and Mackinlay (1997), (3.4.4) implies

$$c_{t+1} - E_t c_{t+1} = r_{m,t+1} - E_t r_{m,t+1} + (1-\Omega)(E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j r_{m,t+1+j}. \quad (3.4.5)$$

Equation (3.4.5) suggests that an increase in expected future returns could raise or lower consumption although the precise effect of this depends on whether Ω is greater or less than 1. Campbell, Lo and Mackinlay (1997) also pointed out that when the market return is mean reverting, there is a negative correlation between current returns and revisions in expectations of future returns with the effect of reducing consumption variability if Ω is less than 1 but increasing it if Ω is greater than 1. From (3.4.5), the covariance of any asset with consumption growth can be rewritten in terms of the covariance of the market returns with changes in expectations about future returns such that

$$Cov_t(r_{i,t+1}, \Delta c_{t+1}) \equiv \sigma_{ic} = \sigma_{im} + (1-\Omega)\sigma_{ih} \quad (3.4.6)$$

where

$$\sigma_{ih} \equiv Cov_t\left(r_{i,t+1}, (E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j r_{m,t+1+j}\right). \quad (3.4.7)$$

σ_{ih} is covariance of asset i returns with upward revisions in expected future returns.[20]

Campbell, Lo and Mackinlay substituted (3.4.6) into (3.3.6) and used the definition of θ in place of γ and Ω to obtain

$$E_t r_{i,t+1} - r_{f,t+1} = -\frac{\sigma_{ii}}{2} + \gamma\sigma_{im} + (\gamma - 1)\sigma_{ih}. \tag{3.4.8}$$

Equation (3.4.8) can be written in the more convenient form

$$E_t r_{i,t+1} - r_{f,t+1} = -\frac{\sigma_{ii}}{2} - \sigma_{ih} + \gamma(\sigma_{im} + \sigma_{ih}). \tag{3.4.9}$$

The asset pricing model derived makes no reference to consumption data either in terms of its covariance with another variable or in terms of Ω, the elasticity of intertemporal substitution. The coefficient of relative risk aversion, γ, remains the only parameter in the formulation. The term $(\gamma - 1)\sigma_{ih}$ in (3.4.8) implies that when γ is less than 1, assets that perform well with good news about future market returns will have lower mean returns as opposed to when γ is greater than 1, in which case such assets would show higher mean returns.

Substituting observed data into (3.4.9), the covariance of the return on asset i, with good news about future returns on the market is 0.038 to yield a required estimate for the coefficient of relative risk aversion, γ, of 1.75. This estimate is not far away from the value of 1 thought consistent with a time separable utility function in a rational expectations framework.

[20] Upward revisions to expected future returns are referred to as good news about future returns. Barr and Pesaran (1997) discussed this 'good news' concept in the context of the bond market which even though only indirectly relevant here provides an exposition of the conceptual framework.

So far, the results have generally provided evidence against the power utility model that explicitly includes a consumption variable, where estimates of the risk aversion parameter exceeded that which could be considered plausible. The Generalised Method of Moments estimation, however, rejects the power utility at almost every opportunity. The results relating for (3.4.8) does provide support for a version of the power utility model but this formulation does not include consumption. Finally, separating the coefficient of relative risk aversion from the elasticity of intertemporal substitution would require values for the latter well in excess of the historically observed levels to obtain reasonable values for the risk aversion coefficient. Conversely, historically observed values for the coefficient of risk aversion are only able to deliver a value for the elasticity of intertemporal substitution well above historically observed and reasonable values.

3.5. Habit formation and "Catching up with the Joneses" : Ratio Models

Given the uncertainties of the power utility model in explaining the observed equity premium, efforts have more recently focused on variations of the basic model. One such variation is the incorporation of the notion of habit formation. The basic idea is that the investor is concerned about consumption insofar as it relates to a predetermined benchmark based on past consumption or "Catching up with the Joneses" effect.[21] With habit formation, investors are loath to hold risky assets to the extent that they could only be persuaded to do so by being offered a sizeable equity premium. Models incorporating habit formation are either ratio models (Abel (1990) and Cooley and Ogaki (1993)) or difference models (Campbell and Cochrane (1995). In this section, attention focuses on the ratio models whilst in section 3.6, the discussion will turn to the difference models of habit formation.

3.5.1. Habit Formation -Abel (1990)

We have already discussed this model in chapter 1 though for reasons of self containment, we present a brief summary here. The formulation can be shown in a discrete time utility function written as

$$U \equiv \sum_{j=0}^{\infty} \delta^j u\left(C_{t+j}, X_{t+j}\right) \tag{3.5.1}$$

where X_{t+j} is a preference parameter which could be written as

$$X_t \equiv \left[C_{t-1}^D V_{t-1}^{1-D}\right]^{\psi} \qquad \psi \geq 0 \text{ and } D \geq 0 \tag{3.5.2}$$

where C_{t-1} is the consumer's own consumption in period *t-1* and V_{t-1} is the

---

21 Studies to have focused on these areas include Constatinides(1990), Pemberton(1993), Campbell, Lo and MacKinlay (1997), Abel(1990) and Mehra and Prescott(1988). A review of these models is presented in chapter 1.

aggregate per capita consumption in period t-1.

From (3.5.2), if $\psi = 0$, then $X_t \equiv 1$ and the utility function is time-separable. If however $\psi > 0$ and D=0, then X_t depends on the lagged aggregate per capita consumption. This is the "catching up with the Joneses" model. Finally, when $\psi > 0$ and D=1, then X_t would depend on the consumer's own past consumption. This is the habit formation model according to Abel (1990).

With a distribution for g, the moments of g can be calculated and with it three asset prices. For time-separable preferences ($\psi = 0$) and for "catching up with the Joneses" ($\psi > 0; D = 0$), solutions are possible for the unconditional expected returns $E\{R^s\}$, $E\{R^B\}$ and $E\{R^c\}$. Given the distribution for g, Abel derived closed-form solutions in terms of preference parameters and the moments of g for the unconditional expected asset returns:

$$E\{R^s\} = E\{g^{-\Theta}\}.\left[E\{g\} + A\,E\{g^{1+\Theta}\}\right]/A \qquad (3.5.3)$$

$$E\{R^B\} = E\{g^{-\Theta}\}/\delta q \qquad (3.5.4)$$

$$E\{R^c\} = E\{g^{-\Theta}\}\left[1 + QE\{g^{\Theta}\}\right]/Q. \qquad (3.5.5)$$

Table 3.9 presents results from numerical solutions for time-separable preferences, relative consumption and habit formation. Panel A reports results for time-separable preferences which indicates that at no stage is the model able to yield an equity premium close to that observed in the data. The closest the model comes to matching the observed data is in panel B (Catching up with the Joneses), where the equity premium is 4.63 per cent in the presence of a risk-less rate of 2.07 per cent. The results are even less encouraging for the habit consumption model (Panel C) where the stock and consols returns are greater than 35 percent when γ equals 1.14. Here also, the results are very sensitive to the choice of γ as a value of 1.06 will generate an equity premium of 6.5 percent. Overall, the results are not encouraging for this form of the habit consumption model.

Table 3.9 - Unconditional expected returns

γ	Stocks	Risk-less Asset	Consols
A. Time -separable preferences ($\psi = 0$)			
0.5	1.93	1.87	1.87
	(1.93)	(1.87)	(1.87)
1.0	2.83	2.70	2.70
	(2.83)	(2.70)	(2.70)
6.0	10.34	9.52	9.52
	(10.33)	(9.51)	(9.51)
10.0	14.22	12.85	12.85
	(14.13)	(12.72)	(12.72)
B. "Catching up with the Joneses" ($\psi = 1; D = 0$)			
0.5	2.80	2.76	2.73
	(2.80)	(2.76)	(2.73)
1.0	2.83	2.70	2.70
	(2.83)	(2.70)	(2.70)
6.0	6.70	2.07	5.84
	(6.72)	(2.06)	(5.86)
10.0	14.73	1.59	13.16
	(14.95)	(1.55)	(13.32)
C. "Habit formation ($\psi = 1; D = 1$)			
0.86	33.56	4.53	35.25
0.94	6.83	3.48	7.44
1.00	2.83	2.70	2.70
1.06	8.43	1.93	7.40
1.14	38.28	0.93	35.16

Source: Abel (1990)

In the "Catching up with the Joneses" model, when D=0, Campbell, Lo and Mackinlay (1997) make the assumption of homoskedasticity and joint lognormality of asset returns and consumption growth, and thus show that the risk-less real interest rate would obey

$$r_{f+1} = -\log\delta - \gamma^2\sigma_{cc}/2 + \gamma E_t \Delta c_{t+1} - \psi(\gamma - 1)\Delta c_t. \quad (3.5.6)$$

Consequently, the risk premium would obey

$$E_t\left[r_{i,t+1} - r_{f,t+1}\right] = \frac{\sigma_{ii}}{2} + \gamma\sigma_{ic}. \quad (3.5.7)$$

The risk-less asset return equals the value derived under the original power utility model less $\psi(\gamma-1)\Delta c_t$. Equation (3.5.7) is exactly the same as obtained under the original power utility model. The "catching up with the Joneses" idea generally has the potential to solve the risk-less rate puzzle identified by Weil (1989) despite the questionable implication of a more variable real risk-less return. The model, however, does not have the potential to solve the equity premium puzzle itself.

3.5.2. Habit Formation: Cooley and Ogaki (1991, 1996)

The authors introduced an economy with a population of N households who maximise utility with a function of the form

$$U = E_0\left[\sum_{t=0}^{\infty} \delta^t u(t)\right] \quad (3.5.8)$$

where E_t denotes expectation conditioned on information in time t. Before proceeding to discuss time non-separability, Cooley and Ogaki presented a simple intra-period utility function which is assumed to be time and state separable in non-durable consumption, durable consumption and leisure:

$$U(t) = \frac{1}{1-\gamma}\left(C_t^{1-\gamma} - 1\right) \quad + v(l(t)) \quad (3.5.9)$$

where $v(.)$ is a "continuously differentiable" concave function, $C(t)$ is non-durable consumption, γ is the curvature parameter and $l(t)$ is leisure. The authors assumed that real wages do not contain an insurance component such that the usual first-order condition for a household that equates the real wage with the marginal rate of substitution between consumption and leisure is

$$W(t) = \frac{v'(l(t))}{C(t)^{-\gamma}}, \tag{3.5.10}$$

where $W(t)$ is the real wage rate. Assuming that leisure is a strictly stationary process, they concluded that the log of the real wage rate and the log of consumption are cointegrated with a cointegrating vector $(1, -\gamma)'$ which the authors used to identify the curvature parameter γ via cointegrating regressions. They concluded that this parameter could be different from one.

For time-non-separability, the intra-period utility function is written as

$$u(t) = \frac{1}{1-\gamma}\left(F_t^{1-\gamma} - 1\right) + v(l(t)), l(t-1), \ldots . l(t-k) \tag{3.5.11}$$

where F_t is the service flow from consumption purchases which is related to purchases of consumption by

$$F(t) = C(t) + \theta\, C(t-1). \tag{3.5.12}$$

According to (3.5.12), when θ is negative then we have habit formation. This is the Cooley and Ogaki habit formation model. When θ is positive, this implies local substitutability or durability.

Cooley and Ogaki (1991, 1996) derived first order conditions and using standard asset pricing formulation with aggregate service flows, labour income, y_t, determined to be

$$y(t) = \frac{E_t\left[\delta\left\{F_a(t+1)^{-\gamma} + \delta\theta F_a(t+2)^{-\gamma}\right\}W(t+1)\left[1 - l(t+1)\right]\right]}{E_t\left[F_a(t)^{-\gamma} + \delta\theta F_a(t+1)^{-\gamma}\right]}. \qquad (3.5.13)$$

To estimate the consumption curvature parameter and to test the model, Cooley and Ogaki combined both the cointegration approach of Ogaki and Park (1989) and Hansen and Singleton's (1992) GMM approach. Assuming that γ is the reciprocal of the long run intertemporal elasticity of substitution, the econometric model used for the GMM procedure is based on formulation for aggregated households which implies that

$$E_t\left(\varepsilon_g^0(t)\right) = 0,$$

where

$$\varepsilon_g^0 = \delta\left[\left(C_a(t+1) + \theta C_a(t)\right)^{-\gamma} + \theta\delta\left(C_a(t+2) + \theta C_a(t+1)\right)^{-\gamma}\right]R(t+1)$$
$$- \left[\left(C_a(t) + \theta C_a(t-1)\right)^{-\gamma} + \theta\delta\left(C_a(t+1) + \theta C_a(t)\right)^{-\gamma}\right] \qquad (3.5.14)$$

and where C_a indicates aggregate non-durable consumption.

Table 3.10 reports results for a model that uses the real stock return or the dividend-price ratio as the risky asset return. The model used is that developed by Cooley & Ogaki (1993) which yields parameters estimates for the coefficient of relative risk aversion, γ. The model uses the real per capita consumption growth together with a combination of the real stock return, the real dividend-price ratio and the real risk-less asset return. The table also presents χ^2 statistics in tests of the over-identifying restrictions of the model.

Table 3.10
GMM estimates of the coefficient of relative risk aversion under habit formation

Asset Return (Instrument lags)	After 2 iterations		After 5 iterations	
	γ (s.e)	Chi Square Test for OIR	γ (s.e)	Chi Square Test for OIR
Dividend-Price /	0.202	9.528	0.162	8.354
Risk- less Asset	(0.364)	(0.049)	(0.353)	(0.079)
Stock Return /	1.024	20.754	1.009	12.458
Risk-less Asset	(5.063)	(0.000)	(9.146)	(0.014)

Source: Author's calculations using data on asset returns and consumption 1919-1991

Using the real stock return as an instrument lag results in a positive parameter of around 1; the value thought to be consistent with real business cycle theory. However the over-identifying restrictions of the model are rejected and the parameters are never significantly different from zero. When the dividend-price ratio is used as an return instrument, the parameter estimates for risk aversion are much lower than is thought plausible with the parameters not significant at conventional levels though the over-identifying restrictions are accepted at the 5% level but not at the 10%.[22] These results do not appear to provide unqualified support for this version of the habit consumption model.

22 The model uses the Durbin method to estimate the distance matrices though using the QS Kernel estimator does not significantly change the results. With the GMM method, a QS Kernel estimator is a nonparametric estimator used to calculate the distance matrices by smoothing (weighted averaging) observational errors. However, in smoothing, a choice of scalar (to control bandwidth) and weights are necessary so as to avoid the weighted average being too smooth and therefore not exhibit the genuine nonlinearities.

3.6. Habit formation: Difference Models

This model follows the work of Campbell and Cochrane (1995) and Campbell, Lo and Mackinlay (1997).

The utility function of a representative agent is assumed to be

$$E\sum_{t=0}^{\infty} \delta^{t} \frac{(C_t - X_t)^{1-\gamma} - 1}{1-\gamma} \tag{3.6.1}$$

In this utility function, C_t represents consumption, X_t is the consumption habit, δ is the discount factor and γ is the utility curvature parameter. Consumption is assumed to follow a random walk and is lognormally distributed.

Campbell, et al argued that it is more convenient to work with the transformation of the habit/consumption ratio i.e. the surplus consumption ratio which can be represented by

$$S_t \equiv \frac{C_t - X_t}{C_t}. \tag{3.6.2}$$

Given that $s_t \equiv \log(S_t)$, the lognormal version would read as

$$s_t \equiv \ln\left(1 - \frac{X_t}{C_t}\right) \tag{3.6.3}$$

which assumes that $X < C$ and therefore positive, but less than the infinite marginal utility. This assumes that consumers are always able to adjust to new information. For reasons of consistency and comparability, we maintain this assumption in this chapter but relax this in our formulation presented in chapter 4 (section 4.4).

Another implication of (3.6.2) being assumed to always be positive is that there is large negative covariance between overall consumption, C, and habit consumption, X. Given (3.6.2), $\mathbf{var}(S_t) = \mathbf{var}(C_t) + \mathbf{var}(X_t) - 2\,\mathbf{cov}(C_t, X_t)$ and since $\mathbf{var}(C_t)$ and $\mathbf{var}(X_t)$ are thought to be small, then the implication is of a large negative value for $\mathbf{cov}(C_t, X_t)$. One possible explanation for this is that a rise in C_t relative to its mean increases expectation on the part of consumers of a fall in consumption, in a future period, C_{t+k}, where $k \geq 1$. At this point, consumers begin to adjust their habit consumption level, X in period t, downwards. This adjustment to habit is more likely if consumers believe that the original increase in C_t is transitory. A fall in C_t relative to its mean will lead to the opposite effect.

Using lower case letters to denote the logs of corresponding upper-case letters, the consumption model is therefore

$$\Delta c_{t+1} = g + v_{t+1} \sim i.i.d.\ \mathrm{N}(0, \sigma^2),$$

where consumption follows a random walk with drift, g, which is represented by the mean consumption growth and v_{t+1} represents innovations i.e. shocks to consumption. Furthermore, the surplus consumption S is assumed to be the result of an AR(1) process from per capita consumption driven by shocks to log consumption v_{t+1} such that

$$s_{t+1} = (1-\phi)n + \phi s_t + \lambda(s_t)v_{t+1}. \quad (3.6.4)$$

In (3.6.4), n is the steady state surplus consumption ratio and the function $\lambda(s_t)$ is the parameter that controls the sensitivity of s_{t+1} and therefore the log habit, x_{t+1}, to contemporaneous consumption shocks v_{t+1}. The parameter ϕ determines the effect of the surplus consumption ratio on future consumption patterns. The first term on the right hand side of (3.6.4) says that surplus consumption is related to past and current consumption. Implicitly, (3.6.4) describes current habit as a non-linear function of past and current consumption for which an approximation around the steady state $s = n$ and $c_{t+1} - c_t = g$ will read

$$x_{t+1} - c_{t+1} - h \approx \phi(x_t - c_t - h) - \frac{\lambda(n)}{N-1}(c_{t+1} - c_t - g) \qquad (3.6.5)$$

where h is the log of the steady state value of X/C. Campbell, Lo and Mackinlay (1997) imposed the condition that $\lambda(n) = N-1$ such that the c_{t+1} term disappears and habit, x_{t+1}, is therefore predetermined and not affected by c_{t+1} at the steady state. The formulation for habit is then

$$x_{t+1} \approx \phi x_t + (1-\phi)h + (1-\phi)c_t + g. \qquad (3.6.6)$$

Equation (3.6.4), however, provides a more convenient form with which to work.

The marginal utility of consumption is defined as

$$u'(C_t) = (C_t - X_t)^{-\gamma}. \qquad (3.6.7)$$

Given (3.6.2)[23], the marginal rate of intertemporal substitution is therefore

$$\mathrm{M}_{t+1} = \delta \frac{(S_{t+1})^{-\gamma}}{(S_t)^{-\gamma}} \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma}. \qquad (3.6.8)$$

---

[23] Given (3.6.2) and (3.6.3), then $\mathrm{M}_{t+1} = \delta G^{-\gamma} e^{\gamma[(\phi-1)(s_t-n)-(1-\lambda(s_t))]v_{t+1}}$

Assuming lognormality,

$$\mathrm{E}_t r_{i,t+1} + \ln(\delta) - \gamma g + \gamma((1-\phi)(s_t - n)) +$$

$$+ \tfrac{1}{2}\left[\sigma_i^2 - 2\gamma\sigma_{iv}^2 + \gamma^2\sigma_v^2\right](1+\lambda(s_t))^2 = 0. \qquad (3.6.9)$$

Consequently, the log risk-free rate is therefore

$$r_{f,t+1} = -\ln(\delta) + \gamma g - \gamma(1-\phi)(s_t - n) - \frac{\gamma^2\sigma_v^2}{2}\left[1+\lambda(s_t)\right]^2. \qquad (3.6.10)$$

In (3.6.9) and (3.6.10), (s_t -n) is reflective of mean aversion in marginal utility such that as s increases, the surplus consumption ratio, $S=(C-X)/C$, declines towards the consumption habit accompanied by an increase in the marginal utility of consumption. Furthermore, s is expected to revert back to its mean such that the expected decline in the marginal utility of consumption results in a higher risk-free rate. Campbell, Lo and Mackinlay (1997) further suggested that the last term in (3.6.10) is a precautionary savings term linear in $[1+\lambda(s_t)]^2$. As uncertainty increases, consumers are more inclined to save and therefore reduce the risk-free rate. Consequently, any increase in uncertainty associated with v_t will reduce the risk-free rate.

The risk-free rate shows very little variation, with a variance of 0.006, but for this variance to be replicated by the data will require the serial correlation parameter ϕ to be close to 1 and/or $\lambda(s)$ rise with s so that the last precautionary saving term in (3.6.11) offsets the intertemporal substitution term, (s – n), which is also the mean-reversion in marginal utility. The authors found that the serial correlation parameter is close to 1 as is the discount rate, which is consistent with theory. Tables 3.11-3.13 reports the results of Campbell and Cochrane

(1995) as well as those of the author in this regard.

Campbell & Cochrane (1995) further defined a mean-standard deviation slope as

$$Slope = \frac{E_t(R^e)}{\sigma(R^e)} = \frac{\sigma_t(M_{t+1})}{E_t(M_{t+1})} = \left(e^{\gamma^2\sigma^2(1+\lambda(s_t))^2} - 1\right)^{\frac{1}{2}}. \quad (3.6.11)$$

Its convenience arises from the fact that the risk premia can be studied without referring to any specific asset even though such work will only be possible if $\lambda(s)$ varies with s to be able to generate a time varying slope of the mean-standard deviation frontier of the kind encountered with volatility bounds.

The sensitivity function, λ, was chosen in the habit consumption framework to meet 3 conditions. Firstly, that the risk-free rate is constant and this would be achieved, if $\lambda(s)$ is of the form

$$\lambda(s_t) = \left[A - \frac{2}{\gamma\sigma^2}(1-\phi)(s_t - n)\right]^{\frac{1}{2}} - 1. \quad (3.6.12)$$

Secondly, habit is predetermined at the steady state $s = n$. If habit were to be fixed, then there will exist the possibility that consumption may fall below habit. Furthermore, were habit to change one-for one with consumption, then there will not exist the possibility of a time varying premia as it is this uncertainty which generates the stock premia. From (3.6.5), this condition implies $\lambda(n) = N\text{-}1$, where $n = \log N$. This can be viewed more directly by finding the derivative of log habit with respect to log consumption and then imposing the condition to determine A and obtain

$$\lambda(s_t) = \left[N^2 + \frac{2(1-\phi)(s_t - n)}{\gamma\sigma^2}{}_t\right]^{\frac{1}{2}} - 1. \quad (3.6.13)$$

The third condition ensures that habit is pre-determined near the steady state and moves positively with consumption everywhere by requiring

$$\frac{d}{ds}\left(\frac{dx}{dc}\right)_{\downarrow s=n} = 0.$$

Taking the derivative and setting to zero at $s = n$ obtains

$$\lambda'(n) = N.$$

Additionally, taking the derivative of λ from (3.6.13) implies that

$$N^2 = \frac{1-\phi}{\gamma\sigma^2}. \tag{3.6.14}$$

Substituting back into (3.6.14), the expression for λ can be written as

$$\lambda(s_t) = \left[\frac{1-\phi}{\gamma\sigma^2}\right]^{\frac{1}{2}}\left[1-2(s-n)\right]^{\frac{1}{2}} - 1. \tag{3.6.15}$$

Campbell and Cochrane further derived a form for the steady state surplus consumption ratio, given that habit is locally predetermined, $\lambda'(n) = N$. The steady state surplus consumption ratio, S, is therefore

$$S = \sigma\left(\frac{\gamma}{1-\phi}\right)^{\frac{1}{2}}. \tag{3.6.16}$$

Table 3.11 reports results from Campbell and Cochrane (panel A) and the author (Panel B) on log consumption growth and asset returns. The values reported in table 3.11, panels A and B are broadly similar and consistent with expectations despite the different time horizons of the data. From panel B, the log equity premium is just under 6% with a standard deviation of just over 20% to give a Sharpe Ratio of 0.28. This Sharpe Ratio is higher than reported by Campbell et al using US data but is consistent with the higher observed mean equity premium. The AR(1) in price-dividend ratio found in our data is 0.55 which is less than the 0.72 found in the Campbell and Cochrane results.

Table 3.11

***Panel A**: Log consumption growth & asset returns*[24]

	Mean	Standard Error	Standard Deviation
Ln stock return-ln risk-free rate (%)	3.69	1.59	17.39
Sharpe Ratio = mean / std deviation	0.21		
Ln consumption growth (%)	1.72	0.33	3.32
Stock price-dividend ratio	3.07	0.03	0.28
Stock Price/Dividend	22.3	0.56	6.06
AR(1) in price-dividend ratio	0.72	0.06	

Campbell and Cochrane (1995)

***Panel B**: Log consumption growth & asset returns*

	Mean	Standard Error	Standard Deviation
Ln stock return-ln risk-free rate (%)	5.75	2.74	20.46
Sharpe Ratio = mean / std deviation	0.28		
Ln consumption growth (%)	1.47	1.42	3.14
Stock price-dividend ratio	3.00	0.02	0.24
Stock Price/Dividend	20.6	0.05	4.60
AR(1) in price-dividend ratio	0.55	0.02	

Author's calculations using data on asset returns & consumption 1919-1991

[24] Campbell and Cochrane (1995) use annual data, 1889-1992. For the stock return, they use quarterly value-weighted NYSE (Centre for Research in Securities Prices), for the Treasury Bill (Risk-less) rate, they use the quarterly SBBI and the consumption data is derived by non-durables + services per capita (CITIBASE series GCNQ + GCSQ/GPOP).

The mean and standard deviation of log consumption growth, g and σ, are taken directly from the data to estimate 3.6.9 and 3.6.10. Furthermore, the serial correlation parameter, ϕ, is taken from the serial correlation of log price/dividend ratios. The discount rate is then determined given these parameters to yield the 0.8% real risk free rate found in the data. The curvature value, γ, remains the only value to be determined and this is done by matching the price of risk, found in the data, using the model.

Table 3.12
Panel A
Effect of curvature parameter on model predictions for mean and standard deviation of excess stock returns (%) and their ratio

γ	$E(r-r_f)$	$\sigma(r-r_f)$	$E(r-r_f)/\sigma(r-r_f)$
1.800	1.81	8.34	0.217
1.902	2.23	8.17	0.281
2.000	2.75	8.05	0.342

Author's calculations using data on asset returns & consumption 1919-1991

Table 3.12

Panel B

Assumptions & derived parameters

Assumptions

Constant interest rate (%)	0.815
Mean log consumption growth, g (%)	1.472
Std. Deviation of log consumption growth	3.139
Curvature	1.902
AR(1) p-d coefficient, ϕ	0.546

Derived parameters

Discount rate δ	0.896
Steady state surplus consumption ratio (C-X)/C	0.064
Maximum surplus consumption ratio	0.207
Sensitivity value at the steady state surplus cons	14.559
Correlation - actual and predicted returns	0.239

Author's calculations using data on asset returns & consumption 1919-1991

Table 3.12 presents results for the model which assumes a slow moving consumption habit as implied by (3.6.4). The model assumes that habit remains below consumption such that surplus consumption over habit is always positive. Table 3.12 shows a value for the curvature function of just under 2 as that which is able to deliver a Sharpe Ratio i.e. the price of risk, of 0.28. This parameter is positive but also well within the value of 10 considered to be the maximum plausible value by Mehra and Prescott. The mean and standard deviation of the risk premium, however, remain much lower than observed in the data, though the mean of the risk premium falls within the 95% confidence intervals, given the high standard errors. When viewed with the positive correlation (0.20) between the historically observed and predicted returns, the results are very encouraging. Furthermore, the model is able to deliver a value for the discount rate of 0.90 and which is consistent with a positive rate of time preference. These results are less encouraging given that the mean and standard deviation of the equity premium falls below that observed from the data. Table 3.13 reports results from Campbell and Cochrane (1995) for a model similar to that for which results are presented in table 3.12 but the data is quarterly post-war data.[25]

As can be seen from table 3.1.3, Campbell and Cochrane (1995) identified a

[25] The results based on the annual data series are unavailable to the author given that the paper is unpublished.

mean quarterly return of 1.6%, (annual equivalent 6.5% per year) or 7.4% per year with annual data, which is consistent with many previous studies. With the quarterly data, the price of risk is 0.21. They also pointed out that the 95% confidence interval for the mean return extends from 0.5% to 2.7% per quarter within which the observed mean quarterly rate falls.

Table 3.13
Panel A
Effect of curvature parameter on model predictions for mean and standard deviation of excess stock returns (%) and their ratio

γ	$E(r-r_f)$	$\sigma(r-r_f)$	$E(r-r_f)/\sigma(r-r_f)$
Post-war Data	1.61	7.74	0.21
1.000	1.00	7.19	0.14
2.000	1.11	5.83	0.19
2.372	1.14	5.51	0.21
3.000	1.20	4.83	0.25
4.000	1.25	4.17	0.30
5.000	1.28	3.74	0.34

Campbell and Cochrane (1995)

Table 3.13

Panel B

Assumptions & derived parameters

Assumptions

Constant interest rate (%)	0.250
Mean log consumption growth g (%)	0.444
Std. Deviation of log consumption growth (%)	0.555
Curvature	2.372
AR(1) p-d coefficient, ϕ	0.970

Derived parameters

Discount rate δ	0.973
Steady state surplus consumption ratio (C-X)/C	0.049
Maximum surplus consumption ratio	0.081

Campbell and Cochrane (1995)

The curvature value required to yield the price of risk of risk is 2.372 not far off the value found in our model. However, one feature of the Campbell and Cochrane results is that an assumed curvature value of 1 yields quite similar results to that achieved with a value for γ of 2.372.

Figure 3.1 shows a negative relationship between the sensitivity function and the surplus consumption ratio. As the surplus consumption ratio declines, the sensitivity values increase; this is the required behaviour for a constant risk free interest rate and a rising price of risk and implying a higher equity premium.

Figure 3.1
Sensitivity Function as a function of the Surplus Consumption ratio

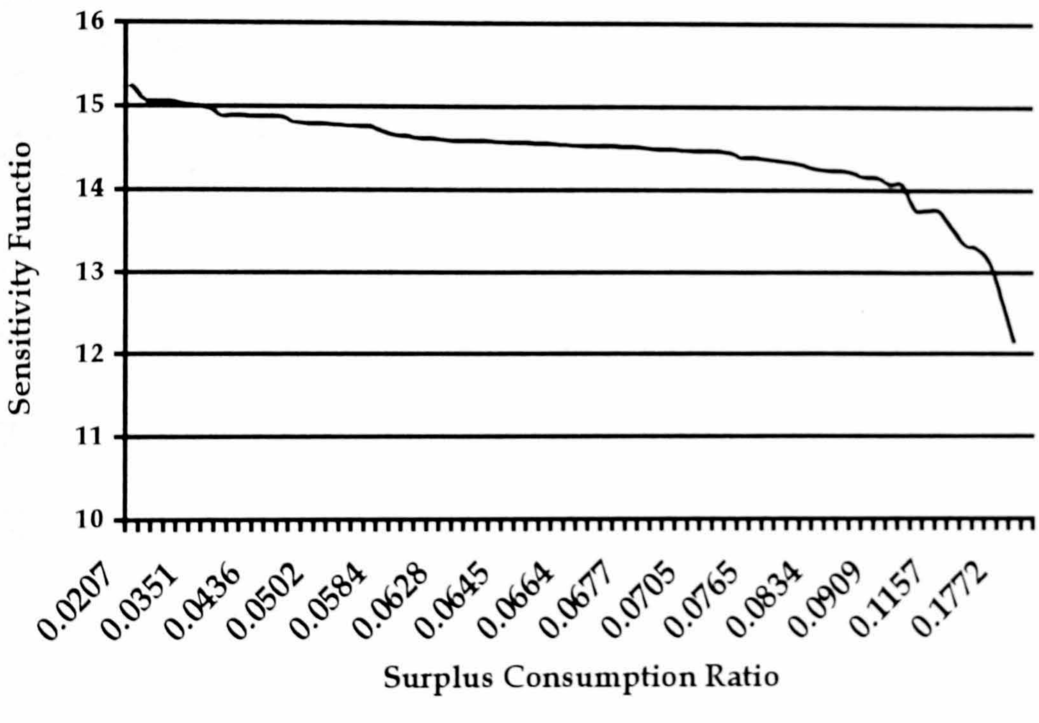
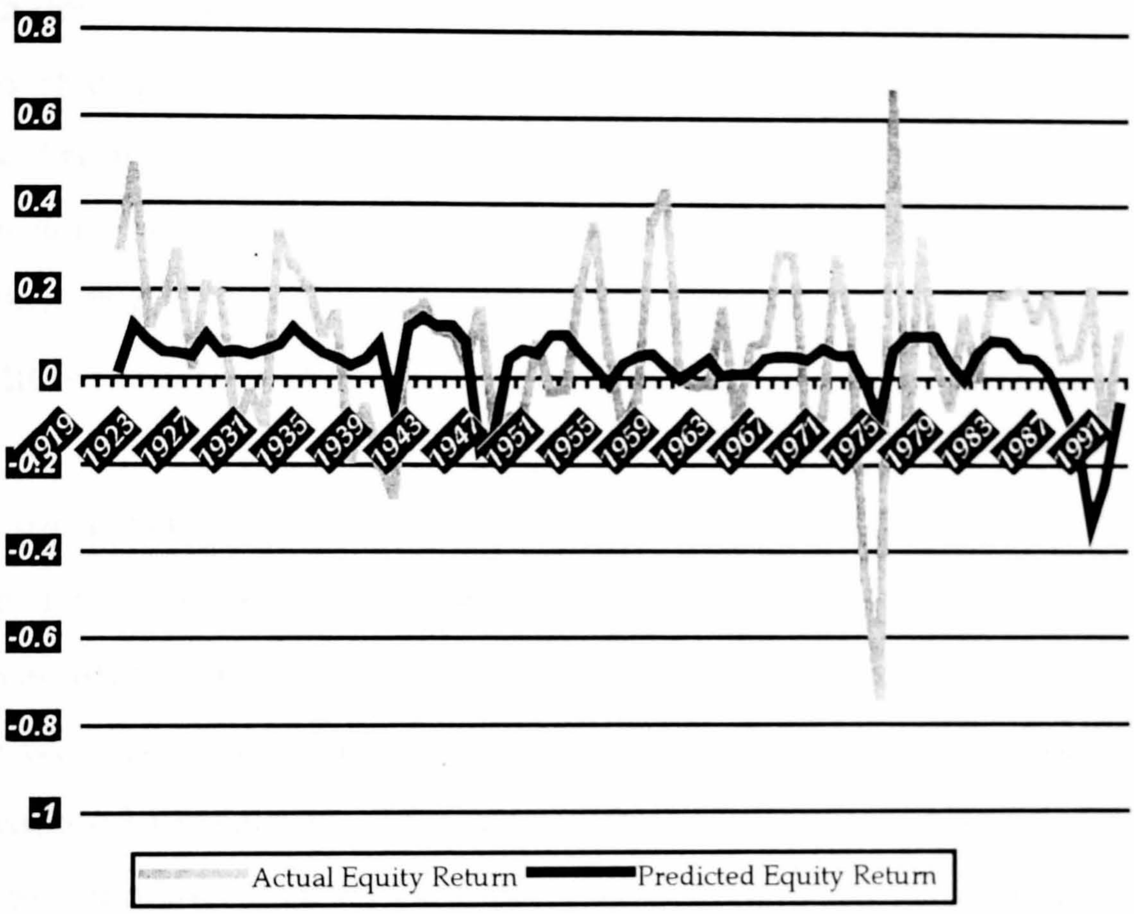


Figure 3 shows the relationship between actual and predicted stock returns.

Figure 3.2
Actual and predicted real stock returns 1919-1991



The diagram confirms the correlation coefficient of just under 0.25 with the standard deviation for the predicted stock returns much lower than that for the actual stock returns. The predicted stocks returns are unable to match the actual returns particularly during periods of significant positive or negative returns.

Conclusion

Overall, the power utility model has been unable to explain the observed equity premium with reasonable values for risk aversion and other parameters. However, the surplus consumption, and implicitly habit, formulation does not necessarily invalidate the power utility model; in fact, it could be complementary. The set up of the original power utility model does not allow for estimates of the risk premium for it is the historically observed risk premium that is used to generate the constant coefficient of relative risk aversion. From (3.6.8), if $S_{t+1}/S_t = 1$, then the formulation is the standard power utility model. Given the comparable risk aversion coefficient estimates, then it must follow that S_{t+1}/S_t must, on average, be close to 1. At the very least, the power utility model is a special case of the surplus consumption model.

The first presentation was the power utility model which despite its limitations provided a basis from which to proceed and develop more realistic models. The power utility model implied that a very high coefficient of relative risk aversion was required to help explain the observed equity premium but this was considered unrealistic. The Instrumental Variables (IV) and Generalised Method of Moments (GMM) estimation were also used to estimate the risk aversion parameter for there existed the possibility of biased estimates owing to serial correlation of the error term in an OLS regression of consumption growth and stock returns. The results of these estimations are not entirely encouraging for the power utility model because although the instrumental variables estimations yielded positive parameter estimates for risk aversion as expected, the over-identifying restrictions of the model tended to be rejected at conventional significance levels. With the GMM estimates, however, some of the parameter estimates turn out to be negative and given the power of the GMM test, explained in appendix 2, this is tantamount to strong evidence against the power utility model. Other tests[26] including relaxing the link between the coefficient of relative risk aversion and the elasticity of intertemporal substitution tended to give better results using ordinary least squares estimations than with GMM. Overall therefore, the power utility model was deemed to be incapable of providing explanations for the size of the observed equity premium. Finally, the model proposed by Campbell, Lo and Mackinlay (1997) was used and like them, the parameters were found to be as expected and provided many explanations for the equity premium phenomena. However, the results still fail to provide a basis for using consumption-based data to predict stock returns and the associated equity premium over short term government securities even after the event. More particularly, the mean and standard deviation remained well below that observed in the data. Given this, an attempt is made to discuss and re-present previously mentioned models (chapter 4) in an attempt to throw new light on some of the issues. The notion of habit formation has given further insights into consumer behaviour and its relationship to stock market returns. However, there remains an issue of refining the model to more closely match the historically observed equity premium whilst also maintaining reasonable parameter values for the discount rate, coefficient of relative risk aversion and the real risk-less rate. Essentially, the surplus consumption models needs to capture the volatility of surplus consumption.

[26] These tests are all based on the power utility model.

CHAPTER FOUR (4)

Summary

Chapter 4 attempts to relate habit and surplus consumption to stock income and stock price returns respectively on the basis that habit and stock income tends to follow a more predictable pattern as opposed to surplus consumption and stock prices which are seen to follow a more unpredictable pattern with similar standard deviations. The conclusion is that it is the surplus consumption and the stock price to a large extent that drives the observed equity premium. The relationship between consumption and stock returns is also discussed and tested in a life cycle-permanent income framework as per Hall (1978). These relationships are also tested within the context of the power utility and Generalised Method of Moments (GMM) framework and the results are, on the whole, supportive of this separation between stock income and stock prices.

In section 4.4 is a model which attempts to capture the volatility of surplus consumption in explaining the equity return. The model is a redefinition of the earlier surplus consumption model of Campbell, Lo and Mackinlay (1997) and which implies that habit consumption is a very slow moving habit. The model is deemed to be a success in that the predicted mean and standard deviation values for the model are close to that observed from the actual data (as opposed to the earlier model in chapter 3) and also that the implied coefficient of risk aversion is well within acceptable boundaries. In fact, the implied coefficient of relative risk aversion is close to 1; the value thought consistent with time separable utility in the Arrow-Debreu framework. The model also yields a value for the discount rate of less than 1 implying a positive rate of time preference. Much encouragement is therefore taken from this model.

CHAPTER FOUR

4.1. Consumption-based Asset Pricing Models

In chapter 3, consumption models were presented which attempted to estimate the risk aversion and elasticity of intertemporal substitution. The aim was to find values for risk aversion, in particular, which helped to explain the observed equity premium of stocks over bonds of just under 6%.

4.1.1. Surplus Consumption and the Power Utility Model

One of the problems with the power utility model is the implication of a constant excess stock return which makes it impossible to see how well the model fits the data. However, the power utility model is used here as a starting point and is anticipated to provide indications of the likely direction of a model which will yield a closer relationship between observed and predicted stock returns. Reflecting on the original power utility model in chapter 3, it is worth remembering that the one of the more noticeable features of the consumption data was its smoothness relative to stock returns and the stock (equity) premium.[36]

One implication of trying to relate stock returns to consumption growth is that consumption takes place in the context of asset returns which are uncertain.[37] The Permanent Income Hypothesis (PIH) or the Life-Cycle Hypothesis (LCH) indicates that individuals plan consumption on a longer term view than is implied by stock returns. Consumers therefore assess their longer term

---

36 The standard deviation of real consumption growth is 3.14% compared to almost 22% for the real stock return and just over 20% for the stock premium.

37 With stock returns, the larger element of uncertainty relates to the stock price element. Even though stock income does carry an element of uncertainty about future income, it is less uncertain that its associated price component.

consumption and set consumption accordingly. Consequently, consumption is generally expected to follow a steadier path, relative to stock returns, to the extent that current consumption should be a good predictor of future consumption. Were this to be related to stock returns, then the clear implication would be that stock returns are easily forecastable. The different standard deviations of stock returns and consumption growth provides some evidence of this. All formulations presented so far have attempted to use current consumption of varying descriptions to evaluate asset returns.

Hall (1978) commenced with a model of life-cycle consumption under uncertainty where the maximisation is represented by

$$E_t \sum_{\tau=0}^{T-t} (1+\delta)^{-\tau} u(C_{t+\tau}), \tag{4.1.1}$$

subject to

$$\sum_{\tau=0}^{T-t} (1+r)^{-\tau} (C_{t+\tau} - W_{t+\tau}) = A_t. \tag{4.1.2}$$

In (4.1.1) and (4.1.2),

E_t = mathematical expectation conditional on all information available in t;
δ = rate of subjective time preference;
r = real rate of interest ($r \geq \delta$), assumed constant over time;
T = length of economic life
$u(\,)$ = one-period utility function, strictly concave;
C_t = consumption;
W_t = earnings;
A_t = assets apart from human capital.

Earnings, W_t, is stochastic and the only source of uncertainty such that the consumer chooses consumption, C_t, to maximise lifetime utility with full knowledge of W_t. If the consumer does indeed maximise utility as stated above, then according to Hall

$$E_t u'(C_{t+1}) = \left[\frac{(1+\delta)}{(1+r)}\right] u'(C_t). \tag{4.1.3}$$

Hall then presented a series of corollaries which culminates in the "conclusion that the simple relationship $C_t = g\,C_{t-1} + \varepsilon_t$ where ε_t is unpredictable at time t-1 and is a close approximation to the stochastic behaviour of consumption under the life cycle-permanent income hypothesis". In this formulation, g is the rate of growth represented by the formulation [38]

$$g_t = \left(\frac{1+\delta}{1+r}\right)^{u'(C_t)/C_t u''(C_t)}. \tag{4.1.4}$$

Hall pointed to the disturbance term, ε_t, as summarising the "impact of all information that becomes available in period t about the consumer's lifetime well-being". This can be seen in relation to assets, human capital etc.

Assets, A_t, evolves according to

$$A_t = (1+r)(A_{t-1} - C_{t-1} + W_{t-1}). \quad (4.$$

Human capital, H_t, is defined as the current earnings plus the expected present value of future earnings represented by $H_t = \sum_{\tau=0}^{T=t}(1+r)^{-\tau} E_t W_{t+\tau}$ where $E_t W_t = W_t$ such that H_t evolves according to

[38] g will exceed one if u'' is negative.

$$H_t = (1+r)(H_{t-1} - W_{t-1}) + \sum_{\tau=0}^{T-1}(1+r)^{-\tau}(E_t W_{t-\tau} - E_{t-1} W_{t-\tau}), \quad (4.1.6)$$

The second term represents the present value of the set of changes in expectations of future earnings between periods *t-1* and *t*. This term can be represented by η_t such that $E_{t-1}\ \eta_t = 0$. The first term in (4.1.6) introduces an intertemporal dependence into H_t with the implied stochastic equation for total wealth represented by

$$A_t + H_t = (1+r)(A_{t-1} + H_{t-1} - C_{t-1}) + \eta_t. \quad (4.1.7)$$

According to Hall, the changes to total wealth then depends on the relationship between the new information about wealth, η_t, and by induced changes in consumption, ε_t. Under certainty equivalence, justified by quadratic utility or by the small size of ε_t, Hall proposed a form for ε_t where

$$\varepsilon_t = \left[\frac{1+g}{(1+r)} + \ldots\ldots + \frac{g^{T-t}}{(1+r)^{T-t}}\right]\eta_t = \alpha_t \eta_t. \quad (4.1.8)$$

According to Hall, this is the "modified annuity value of the increment in wealth" implying that the stochastic equation for total wealth is

$$A_t + H_t = (1+r)(1-\alpha_{t-1})(A_{t-1} + H_{t-1}) + \eta_t \quad (4.1.9)$$

and which is assumed to follow a random walk with trend.

In this formulation, consumers determine the appropriate level of consumption by converting data on current and future earnings as well as taking account of financial assets. According to Hall, there are predictable and unpredictable elements in earnings. Table 4.1 reports results from Hall (1978) of a regression of consumption on lagged consumption.

Table 4.1

Panel A

$$C_t^{-1/\sigma} = \gamma\, C_{t-1}^{-1/\sigma} + \varepsilon_t$$

Regression results for the basic model 1948-77

Equation	σ	Constant	γ	SE	R^2	D-W Statistic
1.1..........	0.2	-	0.983 (0.003)	0.000735	0.9964	2.06
1.2..........	1.0	-	0.996 (0.001)	0.00271	0.9985	1.83
1.3..........	- 1.0	- 0.014	1.011 (0.003)	0.0146	0.9988	1.70

Source: Hall (1978)

Table 4.1, Panel A, reports the results of a regression fitting the relation between current and lagged marginal utility as predicted by the hypothesis. Equations 1.1 and 1.2 are for the constant-elasticity utility function, with σ = 0.20 and 1.0 respectively. Equation 1.3 is for the quadratic function exactly, or for any utility function approximately which is simply a regression of consumption on its own lagged value and a constant. The results show a close fit of the regressions with parameter values significantly different from zero at conventional levels. Looking at the residuals, Hall argued that "the data contain no obvious refutation of the unpredictability of the residuals from the basic model...." though he proceeds to reports results for other tests in an assessment of the life cycle-permanent income hypothesis. These results come from tests of whether consumption can be predicted from its own past values and also whether disposable income and wealth can be used to predict consumption.

Hall reported the following regression results where current consumption is the dependent variable with lags of past consumption as the independent variables:

$$C_t = \underset{(8.3)}{8.2} + \underset{(0.092)}{1.130}C_{t-1} - \underset{(0.142)}{0.040}C_{t-2} + \underset{(0.142)}{0.030}C_{t-3} - \underset{0.093}{0.113}C_{t-4}; \quad (4.1.10)$$

$$R^2 = 0.9988 \qquad SE = 14.5 \qquad \text{D - W} = 1.96.$$

(standard errors in brackets)

As is consistent with expectation, consumption lagged by more than one period has no predictive power for current consumption. Essentially, consumers are expected to offset any cyclical patterns and "restore the non-cyclical behaviour of consumption predicted by the hypothesis". Joint significance tests of the coefficients for 2 and 3 lags of consumption are found to be insignificant.

Table 4.1

Panel B

Equations relating consumption to lagged consumption and past levels of real disposable income

	R^2	SE	D - W	F	F^*
Eq 1: $C_t = -\underset{(11)}{16} + \underset{(0.044)}{1.024}C_{t-1} - \underset{(0.032)}{0.010}Y_{t-1}$	0.9988	14.7	1.71	0.1	3.9
Eq 2: $C_t = -23 + \underset{(0.047)}{1.076}C_{t-1} + \underset{(0.043)}{0.049}Y_{t-1}$					
$- \underset{(0.052)}{0.051}Y_{t-2} - \underset{(0.051)}{0.023}Y_{t-3} - \underset{(0.037)}{0.024}y_{t-4}$	0.9989	14.4	2.02	2.0	2.4
Eq 3: $C_t = -\underset{(11)}{25} + \underset{(0.054)}{1.113}C_{t-1} +$					
$\sum_{i=1}^{12}\beta_i Y_{t-1} \quad \Sigma\beta_i = \underset{(0.040)}{0.077}$	0.9988	14.6	1.92	2.0	2.7

Source: Hall (1978)

The life cycle-permanent income hypothesis is refuted if lagged income is found to have substantial predictive powers. Furthermore, this could be taken as evidence that the consumers are very sensitive to current income or that consumers use past income as a basis for consumption choices. Table 4.1, Panel B, reports regression results along these lines.

Equation 1 shows that a single lagged level of disposable income has essentially no predictive power for current consumption with test statistics less than the critical values. Equation 2 uses a year-long distributed lag as independent variables and the results are quite encouraging for the life cycle-permanent income hypothesis in that longer lags yield negative parameter estimates. The long run marginal propensity to consume, measured by the sum of all the coefficients, is also negative though the joint F test would confirm non-rejection of the life cycle-permanent income hypothesis at the 5% level but not at the 10% level. Equation 3 uses a 12-quarter lag to see if a long distributed lag can be a useful predictor of current consumption. The results are again broadly supportive of the life cycle-permanent income hypothesis. Hall concluded that even though the results are broadly supportive, there is possibly a hint that recent levels of disposable income could be a predictor, of some significance, of current consumption.

Hall also proposed a test where a measure of wealth is used as an explanatory variable. The chosen measure of wealth is stock prices, denoted by P. The regression takes the form:

$$C_t = \underset{(8)}{-22} + \underset{(0.004)}{1.012}\,C_{t-1} + \underset{(0.051)}{0.223}\,P_{t-1} - \underset{(0.083)}{0.258}\,P_{t-2} + \underset{(0.083)}{0.167}\,P_{t-3} - \underset{(0.051)}{0.120}\,P_{t-4} \quad (4.1.11)$$

$$R^2 = 0.9990; \quad SE = 14.4; \quad \text{D - W} = 2.05.$$

The coefficients are not only individually significant (*t-tests*) but also jointly so (*F tests*) indicating a rejection of the life cycle-permanent income hypothesis. Hall, however, proposed an amendment to the hypothesis to explain this observation. The suggestion is that even though consumption does depend on permanent income, some part of that consumption does take time to adjust to a change in permanent income. Consequently, any variable correlated with permanent income in *t-1* can be used to predict the change in consumption since part of the change in consumption in period *t* is a lagged response to the previous change in permanent income. In other words, lagged changes in stock prices are found to have predictive power in current consumption changes but which is consistent with a modified life cycle-permanent income hypothesis which recognises a "brief lag between changes in permanent income and the corresponding changes in consumption".

Even though our data is not directly comparable to that used by Hall (1978), results for similar regressions are reported.

Equations (4.1.12) and (4.1.13) reports results for regressions that includes stock prices as a measure of wealth as one of the explanatory variables.

$$C_t = \underset{(29.45)}{-58.280} + \underset{(0.011)}{1.019}\, C_{t-1} + \underset{(0.089)}{0.190}\, P_{t-1} \qquad (4.1.12)$$

$$R^2 = 0.993 \qquad SE = 63.71 \qquad \text{D - W} = 1.60.$$

$$C_t = \underset{(27.08)}{-61.8} + \underset{(0.010)}{1.030}\, C_{t-1} + \underset{(0.127)}{0.424}\, P_{t-1} - \underset{(0.127)}{0.306}\, P_{t-2} \qquad (4.1.13)$$

$$R^2 = 0.994 \qquad SE = 57.96 \qquad \text{D - W} = 1.91.$$

As with Hall, there is evidence that stock prices (P) are capable of forecasting current consumption against the implications of the life cycle-permanent income hypothesis.

These results provide us with some insights into consumption which can prove useful in further discussions of the equity premium. Firstly, models which have gone some way to resolving the equity premium have tended to use only previous period (1 lag) consumption data on the right hand side. Secondly, as Hall acknowledged, there are predictable and unpredictable element of earnings which take account of financial assets accumulated from the past. In our framework, the predictable element of earnings is associated with stock dividends whilst the unpredictable element is associated with stock prices. In such circumstances, it is difficult to view overall consumption growth as being directly linked to stock returns. Perhaps, more realistic is the notion that surplus consumption, that level of consumption above the amount required to survive (habit), is more closely related to stock returns than is total consumption. More precisely, it is surplus consumption that generates the high standard deviation of around 20% associated with stock returns (via stock prices), and implicitly therefore, the equity premium. Intuitively, habit consumption and stock dividends would appear to be more predictable than their counterparts, surplus consumption and the stock price. Realistically, investors are able to quite accurately estimate the level of future dividends as well as their habitual consumption. This is not to say that habit consumption may not be subject to change; rather that any change in habit consumption is more predictable than is surplus consumption. In the Campbell and Cochrane context, habit is determined near or at the steady state. This is however not the case with surplus consumption and the stock price which exhibit far less predictability. This is the source of risk. Implicitly, the assumption here is that total consumption has an unpredictable element associated with surplus consumption. In the context of Hall, stock prices are related to surplus consumption whilst stock dividends are related to habit consumption.

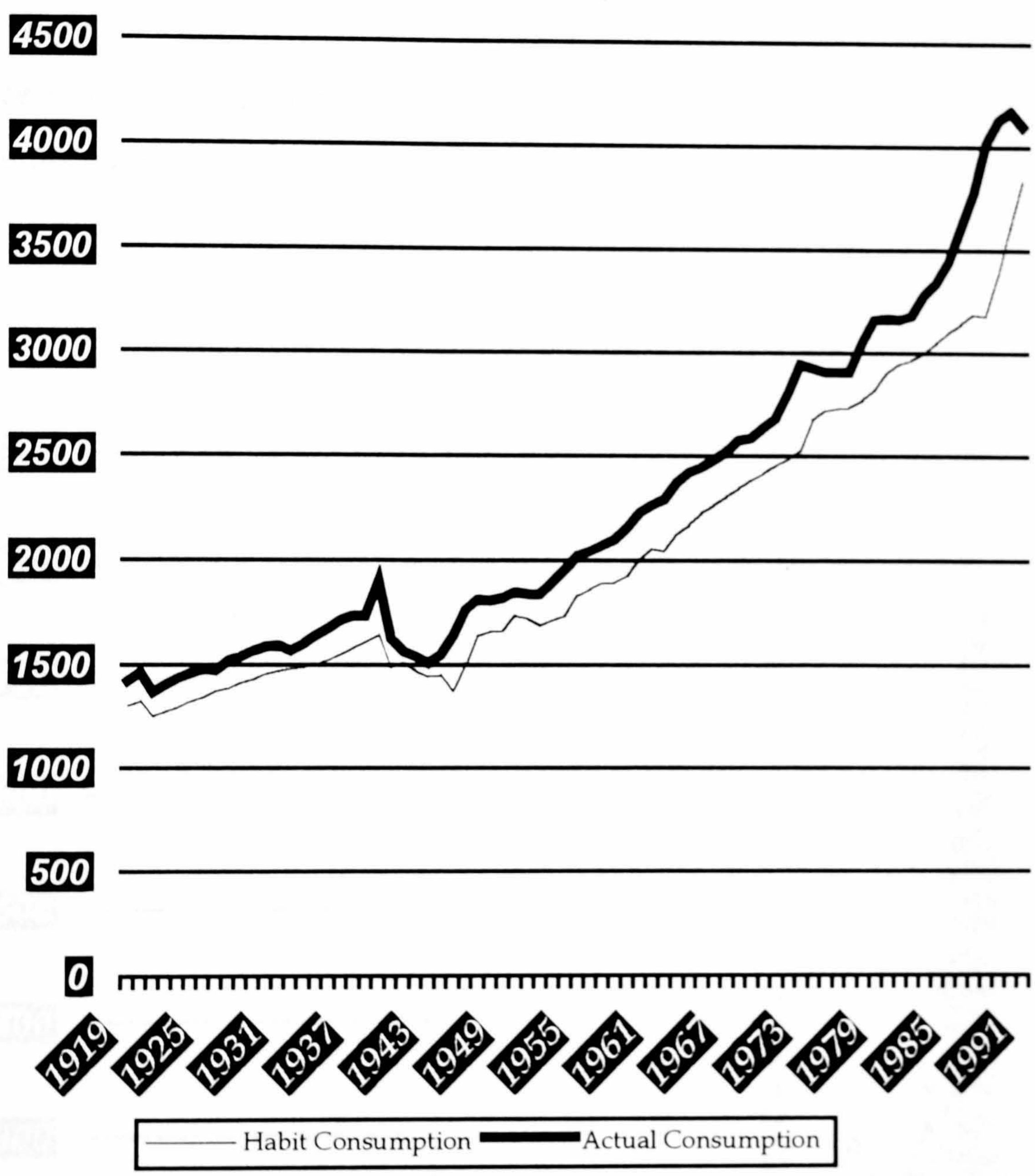
Given this conclusion, we suggest that Equation 3.2.4 can be re-presented as

$$E_t\left[r_{i,t+1} - r_{f,t+1}\right] = -\frac{\sigma_{ii}}{2} + \gamma\,\sigma_{is} \qquad (4.1.15)$$

where σ_{is} is the covariance of asset return and surplus consumption growth, γ remains the coefficient of relative risk aversion and the left hand term is the equity premium. We propose (4.1.15) on the basis of our earlier conclusion that it is surplus consumption that is more closely related to the equity premium. The surplus consumption is derived from the surplus consumption ratio estimated using the Campbell, Lo and Mackinlay (1997) model in chapter 3.[39] Figure 4.1 shows the relationship between actual and habit consumption of non-durable goods and services.

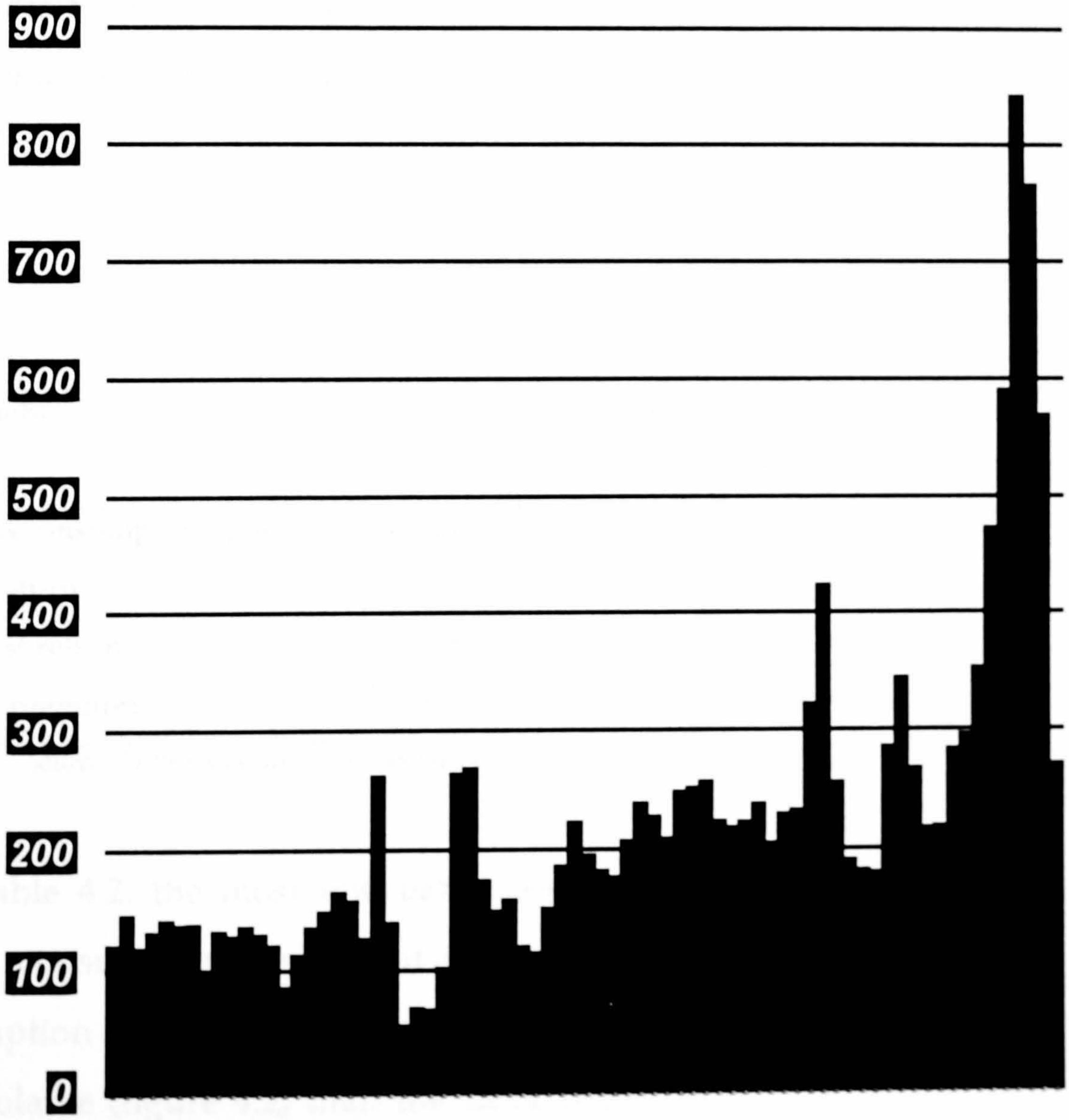
[39] Using this definition of the surplus consumption ratio will give an indication of the role of surplus consumption in this type of analysis. Later on, there will be a discussion of the precise form for determining surplus consumption.

Figure 4.1
Real per capita consumption 1919-1991 (£1985 prices)



Habit consumption is always below actual consumption as defined in the Campbell, Lo and Mackinlay (1997) model. Only during the 2nd world war did habit almost match actual consumption whilst the two series are furthest apart in the 1980s. Implicitly, the significant expansion of consumer expenditure in the mid to late 1980s was essentially an expansion of surplus consumption which responded to wealth effects generated in part at least by higher stock prices of the time. As the economy slowed down in the early 1990s and the stock price surge eased, surplus consumption is seen to fall substantially and back to levels comparable with the pre 1980s expansionary period.

Figure 4.2
Real per capita surplus consumption, 1919-1991
(£1985 prices)



In our view, the diagram also reflects the sustained nature of the upward revisions in stock prices. The implied values for surplus consumption are shown in figure 4.2. This data is used in a test of the power utility model in 2 ways: firstly, the ability of surplus consumption to explain overall stock returns and secondly, the ability to explain stock price returns.

Table 4.2 presents results for this approach using the power utility model and where the surplus consumption is that derived from the surplus consumption ratio calculated using the Campbell and Cochrane (1995) formulation in chapter 3. From figure 4.2, the surplus consumption is always positive with the highest values recorded during the 1980s. Since the results provide an indication of being able to provide insights into the equity premium or aspects of it, this provides a useful starting point. An alternative specification is presented later in the chapter.

Table 4.2

Moments of log surplus consumption growth and asset returns

Variable	Mean	Standard Deviation	Correlation with consumption growth	Covariance with consumption growth
Surplus consumption growth	0.0114	0.2977	1.0000	0.0886
Stock return	0.0657	0.2183	0.0934	0.0061
Risk-less return	0.0082	0.0723	0.0249	0.0005
Equity premium	0.0575	0.2046	0.0909	0.0056

Source: Author's calculations using data on asset returns and consumption 1919-1991

From table 4.2, the most noticeable feature is the standard deviation of the surplus consumption growth of just over 29% compared to that for overall consumption of just over 3%. Surplus consumption growth is shown to be even more volatile (figure 4.2) than the stock return and the equity premium. The correlation of surplus consumption growth and stock returns is about twice the level found between consumption growth and stock returns. Even so, the estimate remains well below that found for consumption growth by Campbell, Lo and Mackinlay (1997) of around 0.5. The coefficient of relative risk aversion required, as per (4.1.15), to explain the equity premium is just over 10 which though positive exceeds the maximum plausible value of 10. The implied values for the risk-free and discount rates are also well in excess of values thought reasonable. The mean of the surplus consumption growth is about a third of overall consumption growth which demonstrates the volatility of surplus consumption given that surplus consumption ratio is only about 0.10 on average. The results are more positive for the power utility model than the original model in chapter 3 where the coefficient of risk aversion required to explain the observed equity premium was significantly in excess of 10.

4.2. Surplus Consumption and the Stock Price Return

Building on the earlier formulations, attention now turns to the relationship between surplus consumption and the stock price return.

Table 4.3 presents diagnostic results.

Table 4.3

Moments of log surplus consumption growth and asset returns

Variable	Mean	Standard Deviation	Correlation with consumption growth	Covariance with consumption growth
Surplus consumption growth	0.0114	0.2977	1.0000	0.0887
Stock Price Return	0.0156	0.2394	0.1130	0.0081
Risk-less return	0.0082	0.0723	0.0249	0.0005
Equity Price premium	0.0074	0.2263	0.1116	0.0075

Source: Author's calculations using data on asset returns and consumption 1919-1991

The results in table 4.3 are similar to those in table 4.2 especially in relation to the standard deviations of the variables. The correlation of the variables with surplus consumption growth is similar to that reported in table 4.2. Given these results, it is not surprising that the coefficient of relative risk aversion required to explain the equity premium is about 0.99, a value which is positive and consistent with a priori expectations. Furthermore, substituting the moments into the surplus consumption equivalent of (3.2.3) reveals a discount rate of 0.96 which is indicative of a positive rate of time preference and which is consistent with the theory. The lower mean real stock price premium (0.74%) return relative to the overall equity premium (5.75%) is to a large extent responsible for the reasonable parameter values implied. When the mean stock price return (1.56%) is used, the implied values are 0.85 for the discount rate and 1.93 for the coefficient of relative risk aversion. These results are encouraging for the power utility model though they do not provide conclusive evidence in favour of the power utility model. However, the results provide support for the notion that surplus consumption forms an important and relevant element in explaining observed asset returns.

4.2.1. IV and GMM Estimates of Risk Aversion

Table 4.4 reports results for the now familiar instrumental variables estimation which uses as instruments either 1 or 1 and 2 lags of the real risk-less return, surplus consumption growth and either the real stock price return (Table 4.4, Panel A) or the log dividend-price ratio (Table 4.4, Panel B). Column 1 shows the return asset being estimated in the regression and in parentheses, the number of lags of the instruments whilst column 2 shows the R^2 and joint significance levels of the explanatory variables in regressions of asset returns and surplus consumption growth on the instruments. Column 3 reports parameter estimates for γ and the associated standard errors whilst column 4 reports the R^2 and joint significance levels of the explanatory variables in regressions of residuals from the instrumental variables (IV) estimation on the instruments. The second part reports associated diagnostic results.

The equation to be estimated is

$$r_{p,t+1} = \mu_p + \gamma \Delta s_{p,t+1} + u_{p,t+1}, \qquad (4.2.1)$$

where $r_{p,t+1}$ is the stock price return and Δs_{t+1} is the change in surplus consumption. The subscript p is used for convenience to denote that the return under consideration is a stock price return.

Table 4.4
Panel A
IV Estimates of Asset Returns and Consumption

Return (Instruments)	First Stage Regressions r_p	Δs	γ (s.e)	Test(1)
Risk-less Return (1)	R^2 0.124 Sig (0.027)	0.166 (0.007)	0.165 (0.071)	R^2 0.060 Sig (0.249)
Stock Price Return (1)	R^2 0.076 Sig (0.156)	0.166 (0.013)	0.225 (0.239)	R^2 0.064 Sig (0.225)
Risk-less Return (1 and 2)	R^2 0.442 Sig (0.000)	0.221 (0.013)	0.190 (0.045)	R^2 0.294 Sig (0.001)
Stock Price Return (1 and 2)	R^2 0.117 Sig (0.232)	0.221 (0.013)	0.212 (0.097)	R^2 0.102 Sig (0.322)

Source: Author's calculations using data on asset returns and consumption 1919-1991

***Panel A:** IV estimates of asset returns and consumption - diagnostics*

Return (Instruments)	*Durbin Watson*	*Serial Correlation*	*Normality*	*Functional Form (FF)*
Risk-less Return (1) Equation (4.2.1)	*1.32*	*3.55 [0.06]*	*189.46 [0.00]*	*4.840 [0.03]*
Stock Price Return (1) Equation (4.2.1)	*1.89*	*0.19 [0.66]*	*435.92 [0.00]*	*0.816 [0.37]*
Risk-less Return (1 & 2) Equation (4.2.1)	*1.67*	*0.92 [0.34]*	*419.47 [0.00]*	*11.22 [0.00]*
Stock Price Return (1 & 2) Equation (4.2.1)	*2.01*	*0.03 [0.87]*	*10.18 [0.00]*	*2.850 [0.09]*

Source: Author's calculations using data on asset returns and consumption 1919-1991

Table 4.4 (Panel A) uses the real stock price return as the risky asset instrument and the parameter estimates for risk aversion, γ, are positive but quite low though significant relative to previous estimates. This is unsurprising because asset returns and the surplus consumption growth are extremely volatile such that a much lower coefficient of risk aversion is required to match the two series. There is evidence that the surplus consumption and asset returns are forecastable given the R^2 values. Of added significance is the fact the over-identifying restrictions are only rejected when 1 and 2 lags of the instruments are used with the risk-less return in contrast to the original results in chapter 3 which tended to reject all the over-identifying restrictions. As with previous estimations, the diagnostic results indicate that the model including the stock price return is a better specified model relative to that for the risk-less rate.

Table 4.4 (Panel B) reports results similar to those in Panel A.

Table 4.4

Panel B: IV Estimates of Stock Asset Returns and Consumption

Return (Instrument lags)	First Stage Regressions	r_p	Δs	γ (s.e)	Test(1)	
Risk-less Return	R^2	0.138	0.127	0.196	R^2	0.064
(1)	Sig	(0.020)	(0.020)	(0.081)	Sig	(0.222)
Stock Price Return	R^2	0.083	0.127	0.489	R^2	0.039
(1)	Sig	(0.124)	(0.020)	(0.270)	Sig	(0.454)
Risk-less Return	R^2	0.413	0.221	0.130	R^2	0.349
(1 and 2)	Sig	(0.000)	(0.013)	(0.047)	Sig	(0.000)
Stock Price Return	R^2	0.159	0.221	0.436	R^2	0.091
(1 and 2)	Sig	(0.090)	(0.013)	(0.192)	Sig	(0.400)

Source: Author's calculations using data on asset returns and consumption 1919-1991

Panel B: IV estimates of asset returns and consumption - diagnostics

Return (Instruments)	*Durbin Watson*	*Serial Correlation*	*Normality*	*Functional Form (FF)*
Risk-less Return (1) Equation (4.2.1)	*1.15*	*7.92 [0.00]*	*155.75 [0.00]*	*3.030 [0.08]*
Stock Price Return (1) Equation (4.2.1)	*1.90*	*0.11 [0.74]*	*3459.6 [0.00]*	*1.319 [0.18]*
Risk-less Return (1 & 2) Equation (4.2.1)	*1.57*	*2.28 [0.13]*	*172.08 [0.00]*	*8.717 [0.00]*
Stock Price Return (1 & 2) Equation (4.2.1)	*1.91*	*0.07 [0.79]*	*2946.4 [0.00]*	*2.863 [0.09]*

Source: Author's calculations using data on asset returns and consumption 1919-1991

These results are similar to estimates reported in table 4.4, Panel A, where the over-identifying restrictions are only rejected when 1 and 2 lags of the instruments together with the risk-less asset return are used.

As in chapter 3, Generalised Method of Moments (GMM) estimates for risk aversion are also presented in a further test of the validity of the parameter estimates. As explained in chapter 3, GMM provides a rigorous method by which to further test the parameter estimates for risk aversion. In the original version of the model in chapter 3, the GMM estimates were broadly negative even though the OLS estimates had been positive. This was viewed as a rejection of the power utility model given the nature of the GMM test.

Table 4.5 reports results for the power utility model tested using GMM where consumption variable is surplus consumption and the risky asset return is the stock price return. In other words, table 4.5 is the GMM equivalent of table 4.4.

Table 4.5

GMM estimates of the coefficient of relative risk aversion in a multiple asset framework

Asset Return (Instrument lags)	Initial Weighting Value	(After 2 iterations) γ (s.e.)	Chi Square Test for OIR[40]	(After 5 iterations) γ (s.e.)	Chi Square Test for OIR
Risk-less Return / Stock Price Return (1)	0.378	0.804 (0.224)	3.838 (0.698)	0.879 (0.091)	6.816 (0.338)
Risk-less Return / Stock Price Return (1 and 2)	0.714	0.905 (0.074)	13.646 (0.324)	0.900 (0.033)	23.503 (0.024)

Source: Author's calculations using data on asset returns and consumption 1919-1991

The supportive evidence for the power utility model becomes much stronger with the multiple asset model where not only do the parameter estimates approach 1[41], the over-identifying restrictions of the model are not rejected by the data for the most part. Furthermore, the parameter estimates are not only significant at the 5% level but strongly so. This result is particularly significant in that the original specification of the model was rejected by both the IV and GMM estimations. Since the IV and GMM-based results do not now reject the model, these results are taken as evidence of support for the power utility model when consumption is defined in terms of its surplus value. These results of the IV and GMM estimations also indicate that only 1 lag of the consumption is capable of predicting asset returns as was found with the earlier regressions as per Hall (1978).

[40] Test for the over-identifying restrictions.

[41] The value of 1 is that which is consistent with neo classical theory, which indicates that only a value of 1 is consistent with time separable utility.

4.3. The Consumption Habit and Stock Income Return

One implication of the formulation discussed in section 4.1. is that the consumption habit is more closely related to stock income rather than overall stock returns; for stock income returns are more predictable and thus more closely related to subsistence consumption i.e. consumption habit. If this is indeed the case, then implicitly, this is a source of less risk. Consequently, our expectation is that compared to the parameter values found with surplus consumption and stock price return, the risk aversion parameter is expected to be much higher and or that the associated parameters are found to be unrealistic or both.

Data on habit consumption is taken from the results using the Campbell, Lo and Mackinlay (1997) formulation presented in chapter 3. Table 4.6 presents some results.

Table 4.6

Moments of log consumption habit growth and stock income returns

Variable	Mean	Standard Deviation	Correlation with consumption growth	Covariance
Habit consumption growth	0.0150	0.0269	1.0000	0.0007
Stock income return	0.0186	0.0833	0.2002	0.0004
Risk-less return	0.0081	0.0723	- 0.0026	0.0000
Stock Income premium	0.0105	0.0831	0.2029	0.0004

Source: Author's calculations using data on asset returns and consumption 1919-1991

As expected the standard deviation of log habit consumption growth is much lower than overall consumption as is that for the stock income and stock income premium over the risk-free rate. Of note also, is that the correlation between stock income growth and habit consumption growth is higher, at around 0.20, relative to overall consumption. In the context of the power utility model, it is unsurprising that a risk aversion coefficient of around 23 is required to explain the asset returns and the implied discount rate is 1.16 which is indicative of a negative rate of time preference and which implies that consumers may be unwilling to substitute intertemporally.

Table 4.7 presents instrumental variables estimations of the risk aversion parameter in line with previous efforts. The instruments used in this formulation are either 1 or 1 and 2 lags of the real risk-free return, the habit consumption growth and either the real stock income growth or the dividend-price ratio. Column 1 shows the return asset being estimated in the regression and the number of lags of the instruments whilst column 2 shows the R^2 and joint significance levels of the explanatory variables in regressions of asset returns and consumption growth on the instruments. Column 3 reports parameter estimates for γ whilst column 4 reports the R^2 and joint significance levels of the explanatory variables in regressions of the residuals from the instrumental variables (IV) estimation on the instruments. The second part reports diagnostic test results for the regressions as specified.

Table 4.7, Panel A, uses the stock income return as the risky asset return in the instrumental variables system. The equation to be estimated is

$$r_{d,t+1} = \mu_d + \gamma \Delta h_{t+1} + u_{d,t+1}, \qquad (4.3.1)$$

where $r_{d,t+1}$ is the stock income (dividend) return and Δh_{t+1} is the change in the consumption habit. The parameter, γ, continues to represent the constant coefficient of relative risk aversion.

The results are as expected, with the over-identifying restrictions of the model overwhelmingly rejected at the 5% level only when 1 and 2 lags of the instruments are used in an estimation of the risk-less return and at 10% for all regressions except for when 1 lag of the instruments are used in estimating stock income.

Table 4.7

***Panel A:** IV Estimates of Stock Income Returns and Habit Consumption*

Return (Instruments)	First Stage Regressions	r_d	Δh	γ (s.e.)	Test(1)
Risk-less Return (1)	R^2	0.250	0.141	2.897	R^2 0.102
	Sig	(0.000)	(0.018)	(0.790)	Sig (0.067)
Stock Income Return (1)	R^2	0.196	0.141	2.922	R^2 0.083
	Sig	(0.002)	(0.018)	(0.945)	Sig (0.124)
Risk-less Return (1 and 2)	R^2	0.428	0.176	2.226	R^2 0.310
	Sig	(0.000)	(0.050)	(0.592)	Sig (0.001)
Stock Income Return (1 and 2)	R^2	0.244	0.176	2.176	R^2 0.175
	Sig	(0.006)	(0.050)	(0.875)	Sig (0.051)

Source: Author's calculations using data on asset returns and consumption 1919-1991

Panel A:** IV estimates of asset returns and consumption - **diagnostics

Return (Instruments)	*Durbin Watson*	*Serial Correlation*	*Normality*	*Functional Form (FF)*
Risk-less Return (1) Equation (4.2.1)	*0.89*	*9.00 [0.00]*	*432.00 [0.00]*	*0.311 [0.58]*
Stock Income Return (1) Equation (4.2.1)	*1.39*	*5.34 [0.02]*	*18.19 [0.00]*	*0.379 [0.54]*
Risk-less Return (1 & 2) Equation (4.2.1)	*1.02*	*14.89 [0.00]*	*26.28 [0.00]*	*7.621 [0.00]*
Stock Income Return (1 & 2) Equation (4.2.1)	*1.42*	*8.21 [0.00]*	*34.59 [0.00]*	*2.167 [0.14]*

Source: Author's calculations using data on asset returns and consumption 1919-1991

Table 4.7, Panel B, uses the dividend-price in the instrumental variables where the only significant change concerns the fact that the coefficient estimates are not significant when the stock income is used in the estimation.

Table 4.7

***Panel B:** IV Estimates of Stock Income Returns and Habit Consumption*

Return (Instruments)	First Stage Regressions	r_d	Δh	γ (s.e)	Test(1)	
Risk-less Return (1)	R^2	0.157	0.119	2.397	R^2	0.068
	Sig	(0.010)	(0.038)	(0.893)	Sig	(0.195)
Stock Income Growth (1)	R^2	0.083	0.119	2.009	R^2	0.036
	Sig	(0.123)	(0.038)	(1.070)	Sig	(0.477)
Risk-less Return (1 and 2)	R^2	0.440	0.123	3.018	R^2	0.281
	Sig	(0.000)	(0.203)	(0.689)	Sig	(0.002)
Stock Income Growth (1 and 2)	R^2	0.179	0.123	1.596	R^2	0.152
	Sig	(0.047)	(0.203)	(1.078)	Sig	(0.097)

Source: Author's calculations using data on asset returns and consumption 1919-1991

Panel B:** IV estimates of asset returns and consumption - **diagnostics

Return (Instruments)	*Durbin Watson*	*Serial Correlation*	*Normality*	*Functional Form (FF)*
Risk-less Return (1) Equation (4.2.1)	*0.83*	*11.20 [0.00]*	*449.90 [0.00]*	*0.819 [0.37]*
Stock Income Return (1) Equation (4.2.1)	*1.34*	*5.59 [0.02]*	*28.63 [0.00]*	*1.095 [0.30]*
Risk-less Return (1 & 2) Equation (4.2.1)	*1.14*	*10.70 [0.00]*	*26.43 [0.00]*	*3.514 [0.00]*
Stock Income Ret (1 & 2) Equation (4.2.1)	*1.36*	*5.46 [0.002*	*39.70 [0.00]*	*2.083 [0.15]*

Source: Author's calculations using data on asset returns and consumption 1919-1991

Since we have argued against this model being able yield parameter estimates for risk aversion, we are unsurprised by the failure of diagnostic tests relating to serial correlation including Durbin Watson). The parameters are significant at all levels and, as expected, are significantly higher than found in section 4.2 with surplus consumption and stock prices. The results are significant insofar as higher values for risk aversion are implied relative to those found in the surplus consumption-stock price formulation as expected.

Table 4.8 reports GMM estimates for the same model whose results are presented in tables 4.7.

Table 4.8

GMM estimates of the coefficient of relative risk aversion in a multiple asset framework

Asset Return (Instrument lags)	Initial Weighting Value	(After 2 iterations) γ^* (s.e.)	Chi-Square Test for OIR	(After 5 iterations) γ^* (s.e.)	Chi-Square Test for OIR
Risk-less Return / Stock Income Return (1)	- 5.904	- 2.962 (2.386)	- 4.912 (0.555)	- 2.632 (0.971)	7.051 (0.317)
Risk-less Return / Stock Income Return (1 and 2)	- 3.938	- 4.556 (1.169)	15.125 (0.235)	- 5.392 (1.566)	14.503 (0.269)

Source: Author's calculations using data on asset returns and consumption 1919-1991

The results are not altogether surprising in that not only are the parameter estimates negative, they are significantly so for the most part with the over-identifying restrictions of the model not rejected at conventional levels. These results do provide support for the assertion that the source of risk is surplus consumption and the stock price return. Consequently, models that incorporate habit and stock income are not expected to yield reasonable or realistic parameter estimates for risk aversion.

Overall, the results do provide some evidence that the power utility model can indeed give indications of the likely parameter values for risk aversion via the stock price and surplus consumption. The power utility can only be seen in that context because the implications of the model of a constant risk premium does not make it possible to match the data. In fact, the Campbell, Lo and Mackinlay (1997) model can be seen to reflect features found in the power utility. Such features include the fact that the surplus consumption model with 1 lag of the explanatory variables is likely to give better results than a model with 1 and 2 lags. Furthermore, it is the stock price and the surplus consumption that drives the equity premium in that they are more volatile with higher standard deviations. Since our conclusion is that it is growth in surplus consumption that drives the equity premium, this notion can be incorporated into the Campbell, Lo and Mackinlay (1997) model. In other words, an adjustment to the surplus consumption model presented by Campbell, Lo and Mackinlay (1997) in chapter 3 is proposed.

4.4. A Slow Moving Habit and the Equity Premium

For reasons of self containment, the Campbell, Lo and Mackinlay (1997) AR(1) surplus consumption process, equation (3.6.4), is re-presented here as

$$s_{t+1} = (1-\phi)n + \phi s_t + \lambda(s_t)v_{t-1} \quad (4.4.1)$$

where s_t is surplus consumption, n is the steady state surplus consumption ratio and v_{t+1} reflects consumption shocks. This process was defined in the context of a utility function where

$$\mathrm{E}\sum_{t=0}^{\infty}\delta^t \frac{(C_t - X_t)^{1-\gamma} - 1}{1-\gamma}. \quad (4.4.2)$$

In this utility function, C_t is consumption, which is assumed to follow a random walk and lognormally distributed, X_t is the level of habit, δ is the discount factor and γ is the utility curvature parameter reflecting risk aversion.

The surplus consumption ratio, S_t, is

$$S_t \equiv \frac{C_t - X_t}{C_t}. \quad (4.4.3)$$

or the log normal version

$$s_t \equiv \ln\left(1 - \frac{X_t}{C_t}\right)$$

which assumes that $X < C$ and therefore positive but less than the infinite marginal utility for the most part. In previous models where consumption was allowed to fall below habit, this implied infinite or negative marginal utility. Hence the Campbell & Cochrane (1995) assumption that surplus consumption is always positive. However, in our formulation we allow for the temporary possibility that $X > C$ during periods of extreme shocks.

From chapter 3, the surplus consumption model predicts an equity premium of just over 2%. However, this prediction is in the presence of a curvature function value of 1.902, which is significantly above the value of 1 thought consistent with balanced utility between consumption and leisure. The reported standard deviations were also well below those found in the observed data. Consequently, the search is on for a model that yields a curvature value that is consistent with predictions for the equity premium that is closer to the historically observed values. From work in earlier parts of this chapter, the volatility of surplus consumption is seen to match that of the observed equity premium such that (4.4.1) is redefined.

Equation (4.4.1) is redefined to capture fluctuations in surplus consumption and which is represented by

$$s_{t+1} = (1 - \phi_{t+1})n + \phi_{t+1}(s_t) + \lambda(s_t)v_{t+1} \qquad (4.4.4)$$

where

$$\phi_{t+1} = \theta \frac{P_t}{P_{t-1}},$$

$$\Delta c_{t+1} = g + v_{t+1} \text{ and}$$

where P_{t-1} is stock prices in period t - 1,
g is the mean consumption growth rate,
v_{t+1} is the consumption shock,
θ is an AR(1) coefficient of stock prices and
λ is the sensitivity function to consumption shocks.

Equation (4.4.4) involves a simple modification of (4.4.1) and says that surplus consumption in period $t+1$, and implicitly the consumption habit, is determined by a combination of surplus consumption in the recent past, s_t, long term surplus consumption as represented by n, the steady state surplus consumption ratio, and by consumption shocks. From the earlier work in this chapter, the stock price was determined to be related to surplus consumption such that the persistence parameter, ϕ, is driven by stock prices (wealth effects). The choice of stock prices, which implies uncertainty and volatility, suggests that investors prefer to view stock price returns in period t before determining surplus consumption in period $t+1$. Such a view of stock prices is consistent with the Hall (1978) formulation discussed in the earlier part of this chapter. Stock prices were found to represent wealth effects which had a persistent effect on consumption. One very important implication of (4.4.4) is that surplus consumption could be negative during periods of extreme shocks such as when stock prices fall dramatically relative to the previous period. The reason for this is that consumers might not be able to fully and immediately respond to dramatic shocks hence X will be greater than C until consumers are able to fully adjust.

Given the marginal rate of intertemporal substitution,

$$M_{t+1} = \delta\left(\frac{S_{t+1}}{S_t}\right)^{-\gamma}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \tag{4.4.5}$$

and assuming lognormality, the stock (risky asset) return will obey

$$r_{i,t+1} = -\ln(\delta) + \gamma g - \gamma(1-\phi_{t+1})(s_t - n) - \frac{1}{2}\left[\sigma_i^2 - 2\gamma\sigma_{iv}^2 + \gamma^2\sigma_v^2\right]\left(1+\lambda(s_t)\right)^2 \tag{4.4.6}$$

where σ_i is the standard deviation of the stock premium, σ_v is the standard deviation for surplus consumption growth and σ_{iv} is the covariance of the stock premium and surplus consumption growth. Assuming that the risk-less return is determined around the steady state, n, its form will be

$$r_{f,t+1} = -\ln(\delta) + \gamma g - \gamma(1-\phi_{t+1})(s_t - n) - \frac{1}{2}\left[\gamma^2\sigma_v^2\right]\left(1+\lambda(s_t)\right)^2 \tag{4.4.7}$$

The sensitivity function λ is determined to meet 3 conditions in this long term surplus consumption, and implicitly habit, framework. Firstly, that the risk-free rate is constant and this would be achieved, where $\lambda(s)$ is of the form

$$\lambda = \left[A - \frac{2}{\gamma\sigma^2}(1-\phi)(s_t - n) \right]^{\frac{1}{2}} - 1 \tag{4.4.8}$$

and where A is a constant.

Secondly, habit is predetermined at the steady state $s = n$. If habit were to be fixed, then there will exist the possibility that consumption may fall below habit. Furthermore, were habit to change one-for one with consumption, then there will not exist the possibility of a time varying premia as it is this uncertainty which generates the stock premia. From (3.6.5), this condition implies that $\lambda(n) = N-1$ where $n = \log N$. This can be viewed more directly by finding the derivative of log habit with respect to log consumption and then imposing the condition to determine A and obtain[42]

$$\lambda = \left[N^2 + \frac{2(1-\phi)(s_t - n)}{\gamma\sigma^2} \right]^{\frac{1}{2}} - 1. \tag{4.4.9}$$

The third condition ensures that habit is pre-determined near the steady state and moves positively with consumption everywhere by requiring

$$\frac{d}{ds}\left(\frac{dx}{dc}\right)_{\downarrow s=n} = 0$$

[42] Campbell and Cochrane (1995).

Taking the derivative and setting to zero at $s = n$ obtains

$$\lambda'(n) = N$$

Taking the derivative of λ from (4.4.9), this condition implies

$$N^2 = \frac{1-\bar{\phi}}{\gamma\sigma^2}. \tag{4.4.10}$$

Substituting back into (4.4.10), the expression for λ can be written as

$$\lambda = \left[\frac{1-\phi}{\gamma\sigma^2}\right]^{\frac{1}{2}}\left[1-2(s-n)\right]^{\frac{1}{2}} - 1. \tag{4.4.11}$$

As with Campbell and Cochrane (1995), the steady state surplus consumption ratio is defined as

$$S = \sigma\left(\frac{\gamma}{1-\phi}\right)^{\frac{1}{2}}. \tag{4.4.12}$$

The parameters g and σ are taken from the data whilst the discount rate is determined within a framework of an 0.82% real risk-less rate as observed in the data. The curvature parameter, γ, is then determined by matching the Sharpe Ratio of 0.281 found in the data.

Table 4.9 shows curvature values together with the associated mean and standard deviation of excess stock returns for the model.

Table 4.9

Panel A:* *Effect of curvature parameter on model predictions for mean and standard deviation of excess stock returns (%) and their ratio

γ	$E(r-r_f)$	$\sigma(r-r_f)$	$E(r-r_f)/\sigma(r-r_f)$
1.00	3.03	23.33	0.130
1.21	4.97	17.63	0.281
1.50	7.36	14.29	0.517

Author's calculations using data on asset returns & consumption 1919-1991

Panel B: ***Assumptions & derived parameters***

Assumptions

Constant interest rate (%)	0.815
Mean log consumption growth g (%)	1.472
Std. Deviation - Log consumption growth	3.139
Curvature	1.205
Mean (persistence) coefficient, ϕ.	0.992

Derived parameters

Discount rate δ	0.838
Steady state surplus consumption ratio (C-X)/C	0.035
Maximum surplus consumption ratio	0.189
Sensitivity value at the steady state surplus consumption	27.880
Correlation - actual and predicted return	0.134

Author's calculations using data on asset returns & consumption 1919-1991

Table 4.9 reports results of an estimation of the parameters of the model which provides very positive results in that the parameter estimates are consistent with our expectations and theory. The predicted mean equity premium and standard deviation are not too dissimilar to that found in the observed data. The associated curvature value of around 1.21 is much closer to 1 compared to the value of 1.90 found in our first attempt using the Campbell, Lo and Mackinlay (1997) in chapter 3. The associated discount rate remains below 1, which is indicative of a positive rate of time preference. If a curvature value of 1 is assumed, the estimates for the equity premium, though lower, still falls within the 95% confidence interval limits and the standard deviation of the equity premium remains reasonable to the data.

Figure 4.3
Surplus consumption ratio 1919-1991

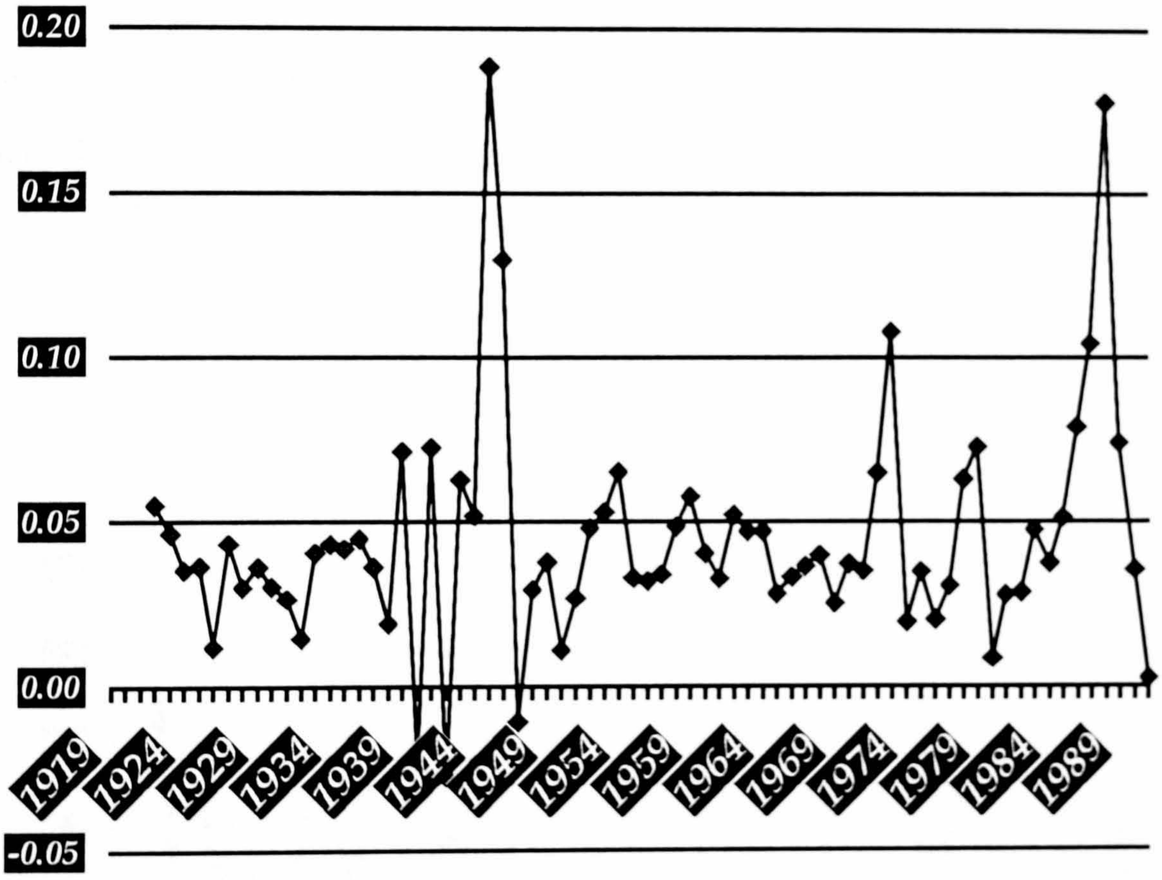
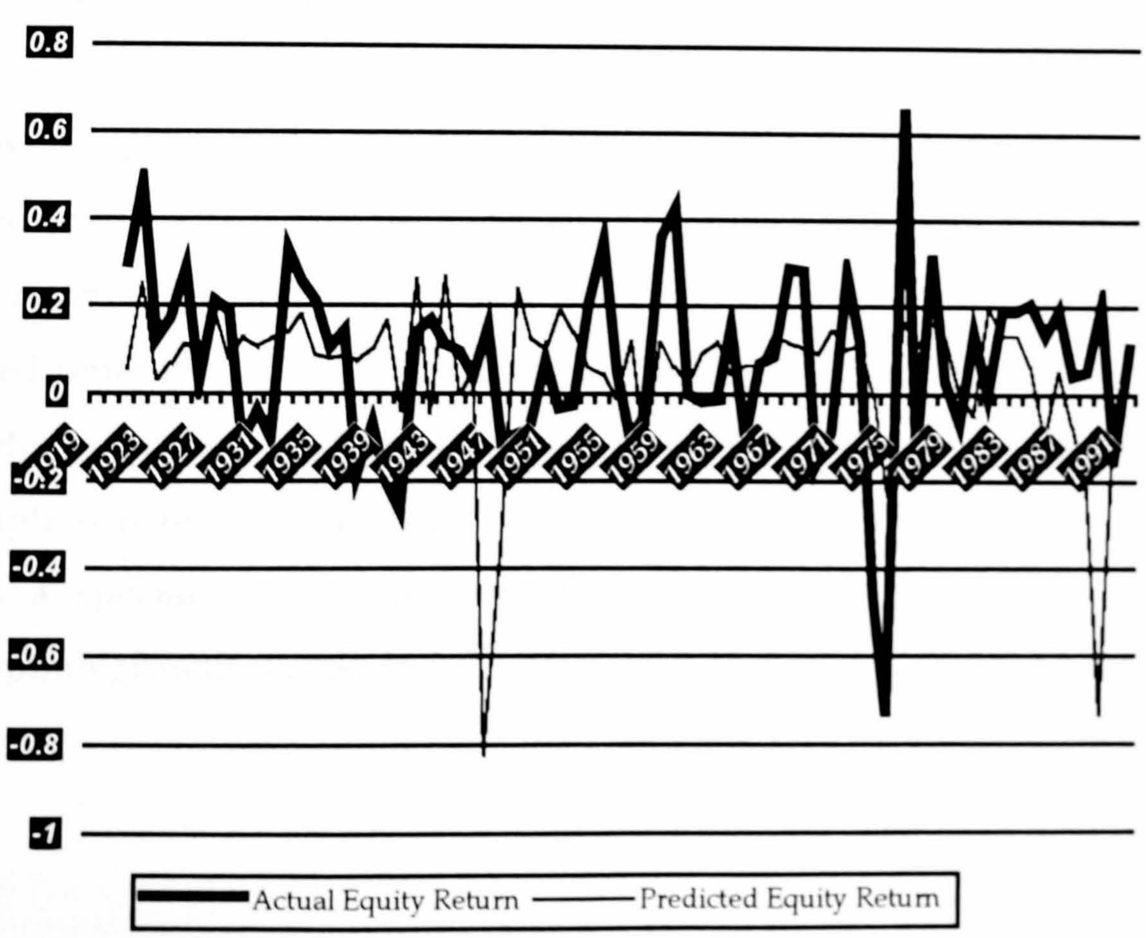


Figure 4.3 shows the surplus consumption ratio where values are mostly though not wholly positive as expected. Negative or very low positive values are found during periods of severe shocks such as during the 2nd world war and the late 1980 and early 1990s. The might be indicative of a very slow moving habit where negative consumption shocks of more than 3% tends to induce a slow moving habit. Intuitively, consumers might not be able to fully and

immediately adjust to a strong negative shock of this magnitude such that the effects of the shocks are further accounted for in subsequent periods. Hall (1978) recognised the presence of a slow moving habit, which was discussed earlier in this chapter, and arrived at much the same conclusion about the behaviour of consumption. From figure 4.1, it can be seen that periods of economic booms are associated with high surplus consumption whilst periods of economic difficulty are generally associated with relatively low surplus or, exceptionally, negative surplus consumption.

Figure 4.4 shows plots of the actual and predicted stock returns of the model that confirms the positive correlation coefficient of 0.13. This provides some evidence that observed stock returns are forecastable such it is possible to match the moments of the observed stock returns.

Figure 4.4
Actual and predicted real log stock returns 1919-1991



Conclusion

These results are encouraging insofar as the model simultaneously generates parameter values that explain the observed risk-less and risky asset returns in a framework consistent with a positive rate of time preference. The correlation between actual and predicted stock returns is lower than found in the calibration in chapter 3. Some thoughts on this are advanced in chapter 7.

That there exists a direct relationship between surplus consumption and stock returns is confirmed by the correlation coefficient of -0.96. The negative coefficient indicates that a decline in surplus consumption increases the risk premium by almost the same magnitude. The decline in surplus consumption implies less consumption as investors place a higher value on additional funds and hence the higher risk premium. Similarly an increase in surplus consumption has the effect of lowering the risk premium. Consequently, the very slow moving habit model is viewed as having advanced the theory linking consumer behaviour and stock returns.

The power utility model, whilst not formulated to match the moments of observed stock returns, has been shown to produce reasonable parameters for risk aversion and the discount rate when surplus and habit consumption are discussed separately. The power utility model also informs that estimations of asset returns do not yield positive enough results when more than 1 lag of consumption is used in the estimation. The power utility model can also be seen as a special case of the surplus consumption model when surplus consumption growth is close to 0 (Equation. 4.4.5).

CHAPTER FIVE (5)

Summary

Section 5.1 presents results from production-based models discussed in chapter 1 from Fama (1990), Cochrane (1991) and Basu and Vinod (1994). The results, using UK data, within the Fama specification are less encouraging than those reported by Fama on the link between stock returns and production growth. Fama had argued that regressions of stock returns and production growth and vice versa are symmetrical such that production growth can be used to predict stock returns.

Section 5.2 reports results from the Cochrane (1991) model which used production functions to derive an "investment return" which the author argued was highly correlated with actual stock returns. As with Cochrane, stock and investment returns were regressed on a set of explanatory variables and the results used as evidence to test the hypothesis that stock and investment returns were equal. According to Cochrane, the significance or otherwise of the parameter estimates should be the same for both sets of regressions. In our estimations, the results were much less encouraging than those reported by Cochrane in that the coefficients of the explanatory variables were sometimes significantly different in estimations of the stock and investment returns. The level of correlation between stock and investment returns were reasonable though remain lower than that reported by Cochrane.

Section 5.3 reports results of Augmented Dickey-Fuller tests, as per Basu and Vinod (1994), of asset returns as a test of the economy-wide technological returns to scale implied by financial assets. Our results indicate the possibility of a constant returns to scale technology in the economy.

CHAPTER FIVE (5)

5.1. Stock Returns and Production Growth

As outlined in chapter 1, Fama (1990) attempted to establish a link between stock returns and the real economy via production growth. Fama suggested that long lags of both stock returns and leads of production growth were needed to explain the behaviour of the other. Tables 5.1 and 5.2 reports results for single and multiple regressions of stock returns and production growth on lags and leads respectively of each other with the interpretation in the context of Fama (1990).

Table 5.1

Single Regressions
Regressions of production growth on stock returns (Column 1) and stock returns on production growth (Column 2)

				DW	SC	JB	FF[43]
					[p-values]		
	Production Growth						
Stock return	b	$t(b)$	R^2				
t	0.103	0.578	0.005	2.351	5.197 [0.023]	93.169 [0.000]	13.514 [0.000]
$t-1$	0.488	3.090	0.125	1.868	0.054 [0.816]	18.062 [0.000]	16.025 [0.000]
$t-2$	- 0.128	-0.930	0.013	1.576	1.869 [0.172]	1.926 [0.382]	7.626 [0.006]
$t-3$	- 0.153	-1.175	0.020	2.351	1.356 [0.244]	1.887 [0.389]	0.000 [1.000]
	Stock Return						
Production Growth	b	$t(b)$	R^2				
t	0.046	0.578	0.005	0.324	41.206 [0.000]	47.136 [0.000]	1.327 [0.249]
$t+1$	0.256	3.090	0.125	0.406	40.386 [0.000]	16.589 [0.000]	3.617 [0.057]
$t+2$	- 0.100	-0.930	0.013	0.328	38.205 [0.000]	18.067 [0.000]	1.454 [0.228]
$t+3$	- 0.132	-1.175	0.020	0.318	37.504 [0.000]	20.097 [0.389]	0.619 [1.000]

Source: Author's calculations using data on asset returns & production 1919-1991

[43] Fama (1990) did not present diagnostics for the regressions.

In column 1, only 1 lag of the stock return displays a limited capacity to explain production growth with an R^2 of 0.125 but also with significant and positive parameter estimates. Parameter estimates beyond 1 lag of the stock return are negative though insignificant and are generally less able to explain production growth. Column 2 however reports results which show production growth and its leads (save for 1 lead of the production growth) to be individually incapable of predicting stock returns. On the basis of the diagnostic results, the results are only symmetrical on the basis that only 2 and 3 lags and leads respectively given any indication of being reasonably well specified. Overall, the results are less than encouraging especially when viewing the diagnostics.

Tables 5.2, Panel A, reports coefficient estimates, test-statistics and correlation coefficients (with the dependent variable) from a multiple regression of production growth on the lags of stock returns.

Table 5.2
Panel A: Multiple Regressions
Regressions of production growth on lags of continuously compounded stock returns

Dependent Variable: Production Growth			
Stock return	*b*	*b*	*b*
t	1.211	1.170	1.114
t(b)	0.194	1.890	1.866
correlation with dep.var	0.222	0.222	0.222
t - 1	-0.921	- 0.496	- 1.016
t(b)	-1.533	- 0.737	- 1.490
correlation with dep.var	0.171	0.171	0.171
t - 2		- 0.433	0.720
t(b)		- 1.375	1.284
correlation with dep.var		0.073	0.073
t - 3			- 0.680
t(b)			- 2.448
correlation with dep.var			- 0.142

R^2	0.082	0.108	0.184
Joint (P) F	0.060	0.058	0.010
Serial Correlation	0.042	0.061	0.330
	[0.838]	[0.806]	[0.561]
Jarque-Bera (JB) Test	21.278	0.168	0.330
	[0.000]	[0.979	[0.852]
Functional Form (FF)	25.786	0.842	2.326
	[0.000]	[0.359]	[0.127]

Source: Author's calculations using data on asset returns & production 1919-1991

Table 5.2 (Panel A) shows that none of the explanatory variables are significant in explaining production growth. In fact, not only are lags of the stock return found to have negative coefficient estimates, the evidence against the model becomes stronger with longer lags. As expected, the R^2 increases with the return horizon, up to 2 lags, though a value of under 0.20 is not very encouraging for the model. The diagnostic results are also more supportive of higher order lags.

Panel B : Multiple Regressions
Regressions of continuously compounded stock returns on leads of production growth

Dependent Variable: Stock Returns			
Production Growth	*b*	*b*	*b*
t	0.114	0.121	0.107
t(b)	1.435	1.515	1.311
correlation with dep.var	0.057	0.057	0.057
t + 1	0.290	0.287	0.283
t(b)	3.392	3.342	3.285
correlation with dep.var	0.353	0.353	0.353
t + 2		- 0.091	- 0.076
t(b)		- 0.905	- 0.730
correlation with dep.var		- 0.113	- 0.113
t + 3			- 0.089
t(b)			- 0.818
correlation with dep.var			- 0.142

R^2	0.151	0.162	0.170
Joint (P) F	0.004	0.009	0.016
DW	0.336	0.407	0.377
Serial Correlation (1)	42.665	39.242	40.321
	[0.000]	[0.000]	[0.000]
JB (2)	16.509	21.711	14.224
	[0.000]	[0.000]	[0.001]
FF (1)	0.0259	0.146	0.960
	[0.872]	[0.702]	[0.327]

Source: Author's calculations using data on asset returns & production 1919-1991

Table 5.2 (Panel B) reports that only 1 lead of production growth provides any indication of being able to explain stock returns. Increasing the horizon does not significantly increase the ability of the model to explain the stock returns.

The multiple regression results are not very encouraging for predicting production and stock returns especially when the time horizon extends beyond period 1 period. Clearly, it must be the case that in forecasts of production growth and stock returns, other variables such as investment need to be included in the system.

5.2. Intertemporal Production - based Asset Pricing

The Production Capital Pricing Model presented by Cochrane (1991) provides a complement to the previously discussed consumption model, which attempts to link the real economy and financial markets by using production growth to predict stock returns. The production-based asset-pricing model is analogous to the standard consumption model and attempts to use producers and production functions in place of consumers and utility functions. The production model attempts to link asset returns with the marginal rate of transformation.[44] This stochastic intertemporal marginal rate of transformation is derived from the producer's first order conditions. Essentially, the production-based model could be viewed as attempting to determine asset returns for given levels of investment and or its associated production variables. In that sense it is clearly analogous to the consumption model.

Like consumption, the production based model is merely a statement of the producer's first order conditions such that producers wish to reduce output of the consumption good in time period, *t*, so as to add to the capital stock and therefore make available resources for capital investment in period *t* so as to be able to increase output in time period *t+1*, whilst leaving the capital stock and output plans unchanged from period *t+2* onwards. The investment return i.e. the marginal rate of transformation, is thus determined by the rate at which the producer can transform period *t* consumption goods into period *t+1* consumption goods. However, this investment process is subject to risk on the basis that expected investment returns between periods *t* and *t+1* is likely to be subject to changes in productivity, labour demand and sales decisions. Hence, the investment return is not risk free. The next stage is to relate the investment return to the asset return. Assuming that firms have access to financial

[44] Much of this work is presented in Cochrane (1991). Cochrane (1991) refers to the stochastic intertemporal marginal rate of transformation as the investment return.

markets, then the firm can construct a portfolio of assets whose returns in time *t+1* are perfectly correlated with the investment return such that if the asset's returns have a price greater than 1, the firm can short sell the representative portfolio and invest the proceeds. The investment return can then be used to pay off the asset portfolio. Given the possibility that the results are sensitive to the assumed production function and choice of parameters, Cochrane (1991) showed that the investment growth path follows, quite closely, the investment return calculated with an adjusted cost production function. It should be the case that the relationship between asset returns and production is totally independent of that between asset returns and consumption. However, this does not entirely rule out a link between consumption and production for if these variables can provide insights into asset returns, there must exist a relationship between the two variables.

5.2.1. Intertemporal Marginal Rate of Transformation

Relating asset prices to implied contingent claims prices, the firm sets production to meet sales, investment, output, capital stocks and labour inputs production to meet sales, investment, output, stocks and labour inputs which can be represented by

$$\max E_0 \sum_{t=0}^{\infty} \delta^t Q_t (C_t - W_t L_t) \qquad (5.2.1)$$

given initial capital stock, K_0, and where δ, is the discount factor, Q_t is scaled prices where Q_t equals $P_t \cdot \delta_t$. P_t is the nominal contingent claims prices, A_t is sales, W_t, wages and L_t, the labour inputs. The constraints are

Production	$Y_t = f(K_t, L_t, S_t)$	(5.2.2*b*)
Resources	$Y_t = A_t + I_t$	(5.2.2*c*)
Capital Accumulation	$K_{t+1} = g(K_t, I_t)$	(5.2.2*d*)

where Y_t is the output level, and K_t is the capital stock and S_t represents uncertainty. The capital accumulation function is assumed to allow for adjustment costs to investment which according to Cochrane (1991), has much the same impact were the adjustment cost to be subtracted from output. This is the formulation presented in chapter 1.

From (1.3.2*j*), the one-period investment return is written as

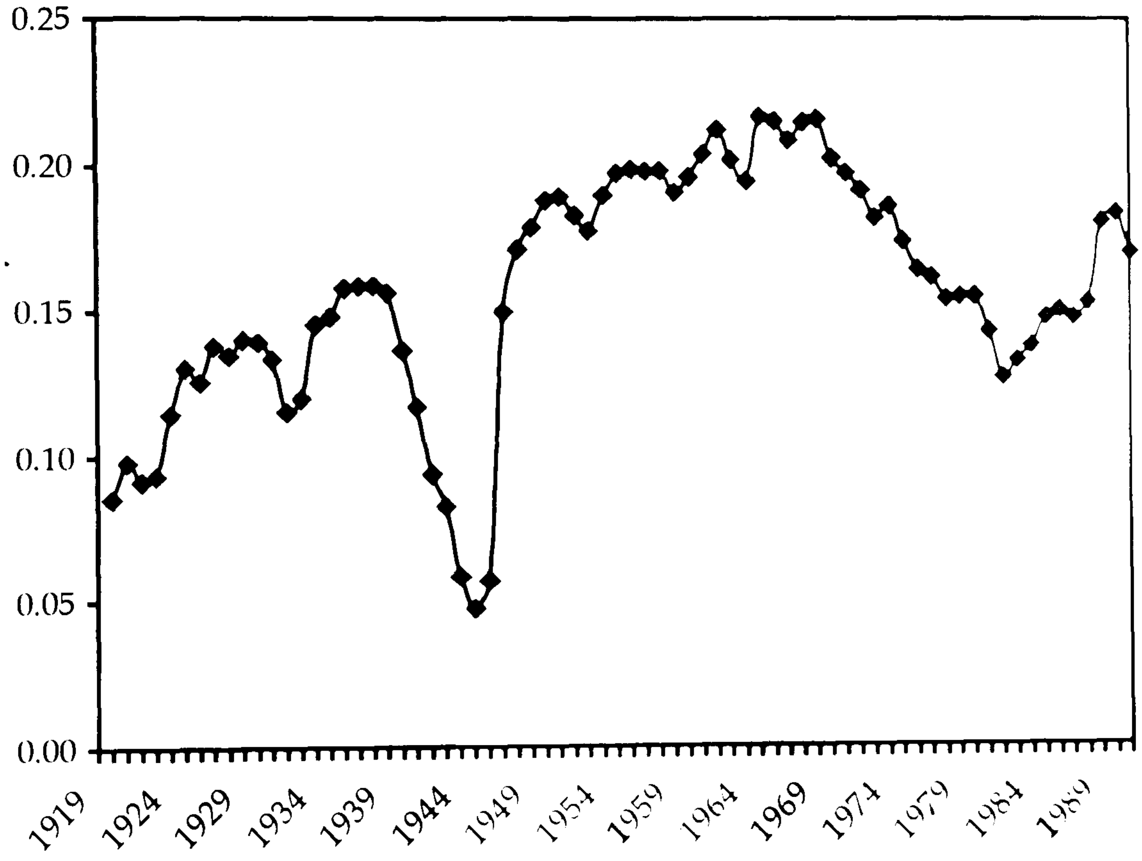
$$R_{i,t+1} - R_{i,t} = (1-\rho)\left(mpk_t + \frac{1+\alpha(I_{t+1}/k_{t+1})^3}{1-(3/2)\alpha(I_{t+1}/k_{t+1})^2}\right)\left(1-\frac{3}{2}\alpha\left(\frac{I_t}{k_t}\right)^2\right). \quad (5.2.3)$$

where the investment-capital ratio, $i_t \equiv I_t/k_t$ is

$$i_{t+1} = \frac{I_{t+1}}{I_t}\frac{i_t}{(1-\rho)(1+i_t-(\alpha/2)i_t^3)}. \quad (5.2.3)$$

I is investment, ρ is the depreciation of the capital stock, α is the adjustment cost parameter and *mpk* is the marginal cost of capital. Given that the parameters α, ρ, and *mpk* control the mean and standard deviation of the investment returns but have a very limited effect on the correlation with other variables, the depreciation parameter (0.11) and the other two parameters are set to match the mean of the stock return (9.41%) as well as the standard deviation (6.19%) determined from the fitted values derived from a regression of the investment return on two leads and lags of the investment-capital ratio. The standard deviation of 6.19% was determined using the fitted values derived in a regression of the stock return on two leads and lags of the investment-capital ratio. The other resulting parameters are 13.2 for the adjustment cost parameter α, which rises when the investment-capital ratio increases and larger fractions of the investment is lost, and 0.267 for the marginal product of capital, *mpk*.[45] Figure 5.1 shows the implied values for the investment-capital ratio whilst figure 5.2 shows the relationship between stock returns and the implied investment returns. This will be discussed in more detail in due course.

Figure 5.1
The Investment-Capital ratio 1919-1991

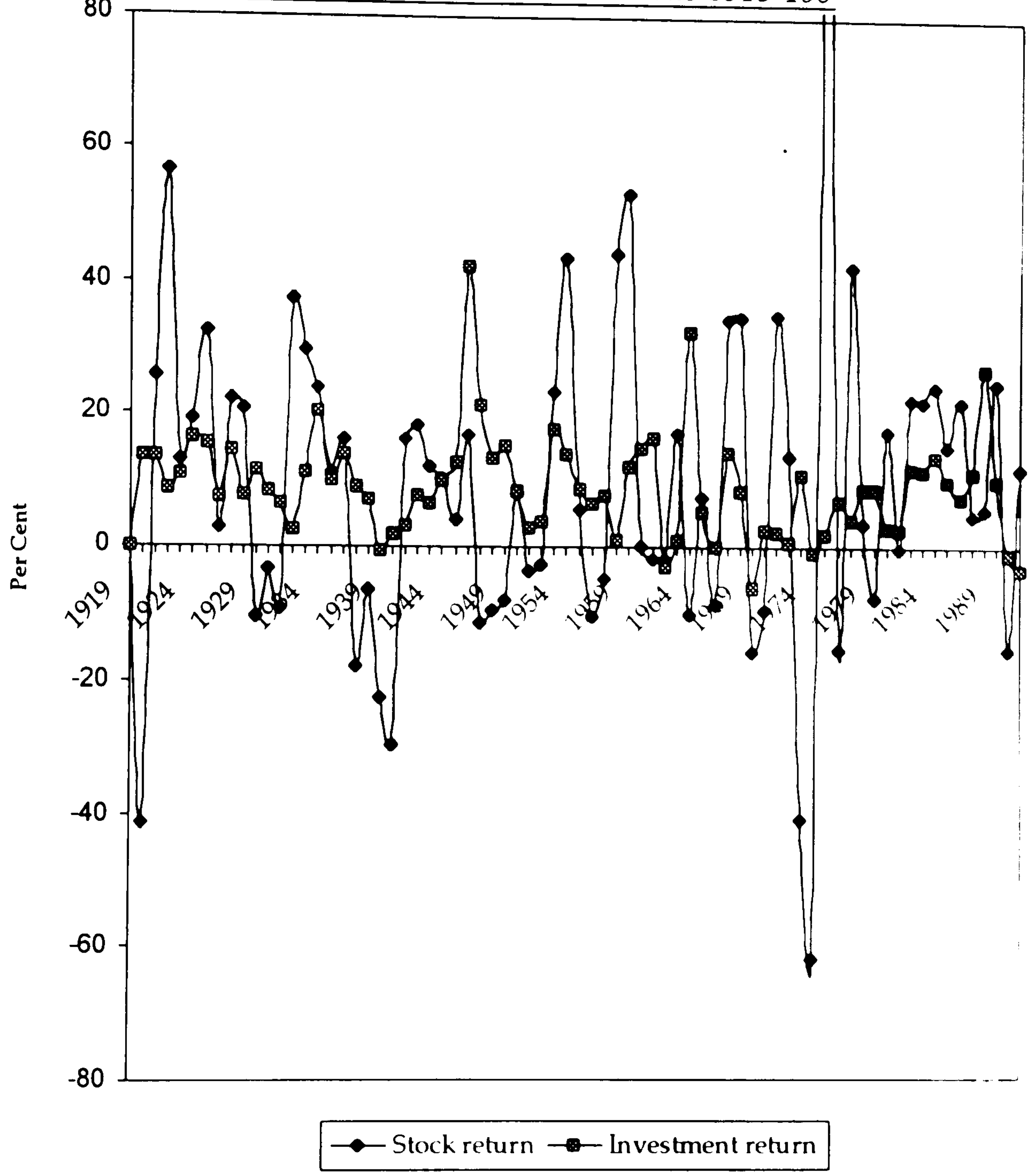


The investment-capital ratio is defined to be positive and from which the capital stock series can be calculated given the investment data series. The investment-capital ratio series show a dramatic fall during the war years as resources were directed towards the war effort. Periods of decline tend to be associated with economic downturns with the most recent declines associated with the downturns of the early 1980s and that of the late 1980s and early 1990s.

45 Unlike Cochrane, we use all 3 parameters in determining the mean and standard deviation Cochrane arbitrarily set the depreciation parameter to 0.10.

Figure 5.2

<u>*Real stock and investment returns 1919-199*</u>



The investment return series is clearly not as volatile (lower standard deviation) as stock returns though there is clearly some correlation between the 2 series.

Table 5.3 presents means, standard deviation and autocorrelations of investment returns, stock returns and the investment capital ratio as previously defined.

Table 5.3

Means, Standard deviations and autocorrelations of investment-capital ratio, investment returns & stock returns

	Investment Capital	Investment Return	Stock Return
Mean	0.153	9.11%	9.11%
Standard Deviations	0.043	7.89%	25.38%
Autocorrelations 1	0.925	0.258	-0.092
2	0.806	- 0.094	-0.115
3	0.699	0.108	-0.082

Source: Author's calculations using annual data 1919-1991

From table 5.3, the investment-capital ratio is a highly correlated series whilst the stock return displays a standard deviation almost 3 times the size of the investment return. The highly autocorrelated nature of the investment-capital ratio will inevitably drive some of the regression results that follow.

Table 5.4 reports results for regressions of stock returns on current, 1 lag and 1 lead of the explanatory variables in an attempt to assess the link between stock and investment returns.

Table 5.4

Panel A: Regression of real stock returns on investment returns, investment growth and GNP growth

$$\text{Stock Return}_{t-1\to t} = \alpha + \beta \, \text{Right Hand Variable}_{t-1\to t} + \varepsilon_t$$

Right Hand Variable	Test Statistics	%p value	Correlation of stock, R.H.V
Investment Returns	1.271	42.9	0.124
Investment Growth	-0.742	72.4	-0.006
GNP Growth	0.038	80.7	0.012

DW = 1.971 R^2 = 0.023 P(F) = 0.655

SC(1)=0.002[0.989], JB(2) = 12.942[0.000], FF(1)=1.117[0.291]

Source: Author's calculations using annual data 1919-1991

Panel B: Regression of real stock returns on investment returns, investment growth and GNP growth

$$\text{Stock Return}_{t-1\to t} = \alpha + \beta \, \text{Right Hand Variable}_{t-2\to t-1} + \varepsilon_t$$

Right Hand Variable	Test Statistics	%p value	Correlation of stock, R.H.V
Investment Returns	-0.142	88.4	-0.025
Investment Growth	0.127	88.9	-0.006
GNP Growth	-2.206	3.10	-0.261

DW = 2.032 R^2 = 0.068 P(F) = 0.188

SC(1)=0.025[0.873], JB(2)=10.069[0.007], FF(1)=1.271[0.260]

Source: Author's calculations using annual data 1919-1991

Panel C: Regression of real stock returns on investment returns, Investment growth and GNP growth

$$\text{Stock Return}_{t-1\to t} = \alpha + \beta \, \text{Right Hand Variable}_{t\to t+1} + \varepsilon_t$$

Right Hand Variable	Test Statistics	%p value	Correlation of stock, R.H.V
Investment Returns	1.248	21.0	0.211
Investment Growth	0.142	88.7	0.145
GNP Growth	1.941	5.6	0.251

DW = 2.055 R^2 = 0.097 P(F) = 0.076

SC(1)=0.051[0.821], JB(2) = 5.492[0.064], FF(1) = 5.474[0.019]

Source: Author's calculations using annual data 1919-1991

Table 5.4 reports positive correlation between stock returns on the one hand and investment returns and investment growth on the other when current and 1 lead of the explanatory variables are used. The correlation between stock and investment returns ranges from 0.12 (current) to 0.21 (with 1 lead). Unsurprisingly perhaps, there is a positive correlation between stock returns and 1 lead of GNP growth indicating that stock returns are a leading indicator of GNP growth. Only when leads of the explanatory variables are used in the estimation are the signs of the parameter estimates as expected.

Tables 5.5 reports the results of regressions of investment returns on explanatory variables in an attempt to further assess the statistical links between stock and investment returns.

Table 5.5

Panel A: Regression of investment returns on stock returns, investment growth and GNP growth

$$\text{Investment Return}_{t-1\to t} = \alpha + \beta\,\text{Right Hand Variable}_{t-1\to t} + \varepsilon_t$$

Right Hand Variable	Test Statistics	%p value	Correlation of stock, R.H.V
Stock Returns	1.271	20.8	0.124
Investment Growth	5.442	0.0	0.548
GNP Growth	0.417	67.8	0.080

DW = 1.410 R^2 = 0.319 P(F) = 0.000

SC(1)=6.277[0.012], JB(2) = 93.812[0.000], FF(1) = 19.407[0.000]

Source: Author's calculations using annual data 1919-1991

Panel B: Regression of investment returns on stock returns, investment growth and GNP growth

$$\text{Investment Return}_{t-1\to t} = \alpha + \beta\,\text{Right Hand Variable}_{t-2\to t-1} + \varepsilon_t$$

Right Hand Variable	Test Statistics	%p value	Correlation of stock, R.H.V
Stock Returns	1.873	6.5	0.211
Investment Growth	1.157	25.1	0.120
GNP Growth	-2.085	4.1	-0.230

DW = 1.622	R^2 = 0.116	P(F) = 0.039
SC(1) = 2.610[0.106],	JB(2) = 17.140[0.000],	FF(1) = 0.030[0.862]

Source: Author's calculations using annual data 1919-1991

Panel C: *Regression of investment returns on stock returns, investment growth and GNP growth*

$$\text{Investment Return}_{t-1\to t} = \alpha + \beta\,\text{Right Hand Variable}_{t\to t+1} + \varepsilon_t$$

Right Hand Variable	Test Statistics	%*p* value	Correlation of stock, R.H.V
Stock Returns	-0.472	62.3	-0.025
Investment Growth	2.727	0.8	0.294
GNP Growth	-1.349	18.2	-0.108
DW = 1.762	R^2 = 0.112	P(F) = 0.046	
SC(1) = 0.889[0.346],	JB(2) = 15.390[0.000],	FF(1) = 2.239[0.135]	

Source: Author's calculations using annual data 1919-1991

Table 5.5 shows positive correlation between investment returns and investment growth throughout with the correlation reaching 0.55 (Panel A) for the current period though the estimates are only significant for the current and 1 lead of investment growth. The results do not indicate a high level of symmetry between the regression estimates of stock and investment returns on the independent variables in contrast to the indications given by Cochrane (1991). The implication is that the investment return is not a good enough proxy for stock returns which might in part be due to the implied value of the marginal product of capital which is twice that suggested by Cochrane. We have already seen evidence that production is not the sole predictor of stock returns. It is conceivable that the relationship between production and stock returns is an indirect one in that production plans are transmitted to stock returns via profit.

Table 5.6 compares parameter estimates for stock and investment returns and their difference. The independent variables, similar to those used by Cochrane(1991), includes the term premium, the corporate premium, the lagged real stock return, the dividend-price ratio and the investment-capital ratio.[46] According to Cochrane (1991), these variables "... are just a few well known representative variables, picked in particular for their association with economic activity". Given that the investment-capital ratio is serially correlated, it is included as one of the forecasting variables.

Table 5.6

Forecasts of stock returns and investment returns
Panel A : Single Regressions

$$\text{Stock Return}_{t-1\to t} = \alpha + \beta \text{ Right Hand Variable}_{t-1\to t} + \varepsilon_t$$

Forecasting Variable	Stock Return β	Stock Return *%p value*	Investment Return β	Investment Return *%p value*	Stock-Inv *%p value*
Term	0.19	90.5	- 0.08	86.8	86.6
Corp	-11.54	17.3	- 5.97	1.5	51.2
Stock Return	- 0.01	90.5	0.06	7.7	52.4
d/p	- 8.92	0.0	- 1.26	6.1	0.1
I/k	- 0.94	24.4	- 0.05	85.1	26.7

Forecasting Variable	Stock Return *Durbin Watson*	Stock Return R^2	Investment Return *Durbin Watson*	Investment Return R^2	Stock-Inv *Durbin Watson*
Term	1.971	0.000	1.423	0.000	2.038
Corp	2.040	0.027	1.498	0.082	2.074
Stock Return	1.950	0.000	1.541	0.045	1.944
d/p	1.464	0.219	1.489	0.061	1.600
I/k	2.007	0.020	1.427	0.001	2.062

Source: Author's calculations using annual data 1919-1991

Panel A shows the single regression coefficients from regressions for the stock return (column 2), the investment return (column 3) and the difference return

[46] For the corporate premium, we were unable to find a suitable data series. After assessing many variations of the data, we used the short-term (3-month) market interest rates as a proxy for the corporate premium.

between stock and investment return (column 4). In forecasting stock returns, all the forecasting variables, except d/p, are found to be individually insignificant at conventional significance levels, whereas for the investment return coefficients, the corporate premium is found to be significant. Furthermore, using the difference between stock return and investment return as the dependent variable, the coefficients are not significant with the exception of the d/p ratio as found by Cochrane (1991). Generally then, the notion that single regression coefficients of investment and stock returns are equal cannot be rejected though the results are not all that encouraging for the hypothesis.

Panels B and C show multiple regression results, coefficients and probability values, for all variables (panel B) and all variables excluding the d/p ratio (panel C).

Table 5.6

Forecasts of stock returns and investment returns
Panel B : Multiple Regressions

$$\text{Stock Return}_t = \alpha + \beta_1 \text{ Term}_t + \ldots\ldots\ldots\ldots + \beta_5 \text{ I}/k_t + \varepsilon_t$$

Forecasting Variable	Stock Return β	Stock Return *%p value*	Investment Return β	Investment Return *%p value*	Stock - Inv *%p value*
Term	- 0.30	86.8	- 0.86	16.0	53.1
Corp	0.80	34.6	- 0.64	2.7	7.9
Stock Return	- 0.23	4.0	0.04	27.9	1.8
d/p	-11.64	0.0	- 0.46	53.4	0.0
I/k	- 1.47	4.6	0.06	79.2	4.1
R^2	*0.31*		*0.12*		*0.26*
Joint P (F)	*0.00*		*0.14*		*0.10*
Durbin Watson	*1.03*		*1.59*		*1.14*
SC(1)	*46.124* *[0.000]*		*2.065* *[0.151]*		*31.220* *[0.000]*
JB(2)	*3.459* *[0.177]*		*29.581* *[0.000]*		*3.392* *[0.183]*

FF(1)	*5.652* [0.017]	*1.906* [0.167]	*1.661* [0.197]
Correlation of Stock and investment return forecasts = 0.54			

Source: Author's calculations using annual data 1919-1991

Table 5.6

Forecasts of stock returns and investment returns
Panel C: Multiple Regressions

$$\text{Stock Return}_t = \alpha + \beta_1 \text{ Term}_t + \ldots\ldots\ldots + \beta_4 \text{ I}/k_t + \varepsilon_t$$

Forecasting Variable	Stock Return β	Stock Return *%p value*	Investment Return β	Investment Return *%p value*	Stock - Inv *%p value*
Term	1.02	56.8	0.50	33.2	77.1
Corp	-15.31	14.4	- 6.24	3.9	38.8
Stock Return	- 0.09	48.7	0.03	46.5	35.5
I/k	- 0.64	44.8	0.09	72.1	39.2
R^2	0.05		0.12		0.04
Joint P (F)	0.47		0.11		0.67
Durbin Watson	1.88		1.59		1.89
SC(1)	0.874 [0.350]		2.073 [0.150]		0.712 [0.399]
JB(2)	16.128 [0.000]		32.473 [0.000]		14.438 [0.001]
FF(1)	0.266 [0.606]		1.291 [0.256]		1.222 [0.269]
Correlation of Stock and investment return forecasts = 0.55					

Source: Author's calculations using annual data 1919-1991

In panel B, the forecasting variables are jointly significant in the prediction of stock returns with an *F(P)* value of 0 and R^2 value of 0.31. The dividend-price ratio and the lag of the stock return are found to be the only individually significant predictor of stock returns though the lag of the stock return is not significant in estimating stock returns when the dividend-price ratio is excluded from the forecasting system (panel C). With the investment return however, the dividend-price ratio is an insignificant predictor of the investment return as are the other variables together (panel B). When the difference between the stock return and the investment return is the dependent variable, all the forecasting variables are individually insignificant with the exception of the dividend-price ratio and the lagged stock return though when the dividend-price ratio is excluded from the system (panel C), the variables are all individually and jointly insignificant. This is consistent with the theory of equivalence between stock and investment return forecasts in that the forecasting variables are not significant predictors, either individually or jointly, of the difference return. This result is similar to those reported by Cochrane (1991). The multiple regressions as represented in Panel B provide an indication of the link between stock and investment return forecasts by their correlation of 0.55. This correlation, even though lower than that found in Cochrane (1991), confirms the view that the dividend-price ratio is indeed an individually significant predictor of the return difference. One possible explanation for the lower correlation is given by Cochrane (1991) who views the variables excluding the dividend-price ratio as having a "common business cycle" component that forecasts stock and investment returns equally. The dividend-price ratio, however, contains elements that can predict longer-term aspects of stock returns but not investment returns. Also of note is that specification of the model as represented by the Durbin Watson statistic is called in question in Panel B but not in Panel C where the dividend-price ratio is excluded from the system. We suggest that is in part due to the possible multicollinearity between stock returns and the dividend-price ratio. Heaton and Lucas (1992) showed that under certain conditions, stock returns and the dividend price ratio are equal. In chapter 3, the dividend-price ratio was a key feature in producing reasonable parameters for the consumption-based asset-pricing model. These results do not reject the Cochrane idea that stock and investment return forecasts are equal though they are not as encouraging as the Cochrane results not least because the correlation between stock and investment returns reported by Cochrane is much higher than is reported in our results.

Table 5.7 show results for a regression of stock, investment and stock-investment returns on the contemporaneous, lead and lag values of the investment-capital ratio.

Table 5.7

***Panel A:** Multiple regressions of stock returns on investment-capital ratios*

$$\text{Stock Return}_t = \alpha + \beta_1 \, 1/k_{t-1} + \ldots\ldots\ldots + \beta_3 \, 1/k_{t+1} + \varepsilon_t$$

Forecasting Variable	Stock Return β	Stock Return % p value	Stock Return β	Stock Return % p-value
I/k_{t-1}	-2.32	26.9	-1.35	55.6
I/k_t	1.31	53.9	-1.83	61.3
I/k_{t+1}			2.77	28.4
R^2	0.04		0.05	
Durbin Watson	2.00		2.06	
P(F)	0.32		0.33	
SC (1)	0.03 [0.87]		0.00 [0.96]	
JB (2)	18.72 [0.00]		15.92 [0.00]	
FF (1)	0.21 [0.65]		5.26 [0.02]	

Source: Author's calculations using annual data 1919-1991

***Panel B:** Multiple regressions of investment returns on investment-capital ratios*

$$\text{Investment Return}_t = \alpha + \beta_1 \, 1/k_{t-1} + \ldots\ldots\ldots + \beta_3 \, 1/k_{t+1} + \varepsilon_t$$

Forecasting Variable	Investment Return β	Investment Return % p-value	Investment Return β	Investment Return % p-value
I/k_{t-1}	-4.77	0.00	-4.50	0.00
I/k_t	4.05	0.00	4.49	0.00

I/k_{t+1}			-0.34	30.70
R^2	0.77		0.77	
Durbin Watson	1.92		1.92	
P(F)	0.00		0.00	
SC (1)	0.16 [0.69]		0.04 [0.85]	
JB (2)	44.71 [0.00]		49.79 [0.00]	
FF (1)	18.98 [0.00]		21.59 [0.00]	

Source: Author's calculations using annual data 1919-1991

Panel C: Multiple regressions of investment returns on investment-capital ratios

$$\text{Stock - Investment Return}_t = \alpha + \beta_1\, I / k_{t-1} + \ldots\ldots + \beta_3\, I/k_{t+1} + \varepsilon_t$$

Forecasting Variable	Stock-Inv Return β	% *p-value*	Stock-Inv Return β	% *p-value*
I/k_{t-1}	2.04	33.10	3.16	16.8
I/k_t	- 2.75	19.90	- 6.32	8.3
I/k_{t+1}			2.81	22.3
R^2	0.03		0.05	
Durbin Watson	2.04		2.11	
P(F)	0.37		0.33	
SC (1)	0.00 [0.91]		0.01 [0.91]	
JB (2)	13.50 [0.00]		11.39 [0.00]	
FF (1)	0.34 [0.00]		2.18 [0.14]	

Source: Author's calculations using annual data 1919-1991

Panel A details the results when the stock return is the dependent variable. The explanatory variables are not found to be significant predictors of stock returns regardless of the time horizon of the explanatory variables. Unlike stock returns, regression of investment returns (panel B), as with Cochrane (1991), show higher coefficients but more significantly, the estimates are statistically significant and with the R^2 approaching 0.8 gives every indication of the investment return being forecastable using the investment-capital ratio. The addition of 1 lead of the investment-capital ratio does not add to the explanatory potential of the investment-capital ratio. The results for the stock and investment returns are not at all encouraging for the notion that stock and investment returns are equal. The results are in contrast to those reported by Cochrane (1991) who found the stock and investment return parameter estimates to be insignificant. The parameter estimates of the stock-investment return difference are not significant at conventional levels as expected. The serial correlation associated with the investment-capital ratio should disappear in a multivariate regression such that the explanatory capacity of the previous period investment-capital ratio of the investment return cannot be so easily explained away. The theory of equality between investment and stock returns is furthered undermined by the rejection of the functional form of the regressions are functional form of the regressions.

Finally, table 5.8 presents forecasts of GNP growth from lagged stock and investment returns and their difference.

The results of the single (Panel A) and multiple (Panel B) regressions show that current and past stock and investment returns, and their difference, to be insignificant predictors of GNP growth, both individually and jointly.

Table 5.8

Panel A: Single Regressions - Forecasts of GNP growth by stock and investment returns

$$\text{GNP Growth}_{t-1\rightarrow t} = \alpha + \beta \text{ Return}_{t-x-1\rightarrow t-x} + \varepsilon_t$$

Return Date	Stock Return β	Stock Return % *p-value*	Investment Return β	Investment Return % *p-value*	Stock-Investment β	Stock-Investment % *p-value*
t-2	0.003	82.6	-0.047	40.5	0.007	65.6
t-1	0.017	31.4	-0.055	33.7	0.020	19.9
t	-0.011	50.1	0.067	23.7	-0.017	30.8
t+1	-0.025	13.2	-0.112	4.0	-0.015	37.4

Return Date	Stock Return *Durbin Watson*	Investment Return *Durbin Watson*	Stock-Investment *Durbin Watson*
t-2	1.507	1.499	1.506
t-1	1.487	1.486	1.478
t	1.509	1.374	1.481
t+1	1.519	1.514	1.509

Source: Author's calculations using annual data 1919-1991

Panel B: Multiple Regressions - Forecasts of GNP growth by stock and investment returns

$$\text{GNP Growth}_{t-1\rightarrow t} = \alpha + \beta_1 \text{ Return}_{t-4\rightarrow t-3} + \dots\dots\dots\dots \beta_3 \text{Return}_{t-2\rightarrow t-1} + \varepsilon$$

Return Date	Stock Return β	Stock Return % *p-value*	Investment Return β	Investment Return % *p-value*	Stock-Investment β	Stock-Investment % *p-value*
t-3	-0.01	31.8	0.04	53.1	-0.02	29.7
t-2	0.00	87.9	-0.05	43.5	-0.01	69.4
t-1	0.02	63.0	-0.05	41.9	0.01	44.6
R^2	0.02		0.02		0.01	
P(F)	0.64		0.73		0.82	
Durbin Watson	1.45		1.59		1.43	
SC (1)	4.76 [0.03]		4.28 [0.04]		5.01 [0.03]	
JB (2)	23.76 [0.00]		16.25 [0.00]		24.05 [0.00]	
FF (1)	0.41 [0.52]		0.01 [0.93]		0.24 [0.62]	

Source: Author's calculations using annual data 1919-1991

5.3. Relationship between Asset Returns and the Production Technology

In chapter 1.3 (section 3), results were presented for a model by Basu and Vinod (1994) which attempted to link the negative autocorrelation in stock returns with the real economy via the technological returns to scale. Given that the technological returns to scale is an important determinant of business cycles, relating technological returns to scale to stock returns makes it possible to test whether growth in the economy is consistent with increasing or diminishing returns if one follows the underlying assumption that financial assets represent claims on the capital stock. Basu and Vinod (1994) proposed that tests of the unit roots of asset returns could be taken as evidence of the link between financial assets and the technological returns to scale in the wider economy particularly given that economic growth could be argued to respond to a particular technology. The formulation was presented in 1.3.3 though for reasons of self containment, we re-present the formulation here.

Basu and Vinod (1994) proposed a model for aggregate wealth, W_t, with Cobb-Douglas technology, of the form

$$W_{t+1} = (W_t - C_t)^{\alpha} \varepsilon_{t+1} \tag{5.3.1}$$

where

C_t equals aggregate consumption,

α determines the returns to scale and

ε_{t+1} is a serially uncorrelated strictly positive productivity shock.

When $0 < \alpha < 1$, then the technology is subject to diminishing returns to scale whilst when $\alpha \geq 1$, then returns to scale are constant or increasing. The authors presented the consumer's intertemporal problem using the familiar maximisation

$$\max E_0 \sum_{t=0}^{\infty} \delta^t U(C_t) \tag{5.3.2}$$

subject to (5.3.1). The price of each share to wealth is given by the also familiar form

$$P_t U'(C_t) = \delta\, E_t\left[U'(C_{t+1}).(P_{t+1} + D_{t+1})\right] \tag{5.3.3}$$

where D_{t+1} is the dividend per share received at time $t+1$ and $U'(C_t)$ is the marginal utility of consumption. Basu and Vinod (1994) then assumed that there is only one share traded in the economy such that equilibrium consumption, C_t, equals D_t for all time. The equilibrium stock return is then given by

$$R_t = (P_t + D_t)/P_{t-1}. \tag{5.3.4}$$

Given the isoelastic utility function,

$$U(C_t) = \left(C_t^{1-\gamma} - 1\right)/(1-\gamma), \tag{5.3.5}$$

the authors outline closed form solutions for two propositions i.e. when $\gamma = 1$ and when $\gamma \neq 1$.

For the first proposition when $\gamma = 1$ (also implying $U(C_t)$=log C_t), Basu and Vinod (1994) presented forms for the equilibrium stock price (log P_t) and stock returns (log R_t) given by

$$\log P_t = B + \alpha \log P_{t-1} + \log \varepsilon_t \tag{5.3.6}$$

where

$$B = (1-\alpha)\log\left[\frac{\delta(1-\alpha\delta)}{(1-\delta)}\right] + \alpha\log(\alpha\delta). \quad (5.3.7)$$

$$\log R_t = -(1-\alpha)\log\delta + \alpha\log R_{t-1} + \log\varepsilon_t - \log\varepsilon_{t-1} \quad (5.3.8)$$

For proposition 2, when $\gamma \neq 1$ and $\alpha = 1$, Basu and Vinod (1994) presented the equilibrium stock price ($\log P_t$) and stock returns ($\log R_t$) given by

$$\log P_t = \log\delta^* + \log P_{t-1} + \log\varepsilon_t \quad (5.3.9)$$

where

$$\delta^* = \delta^{1/\gamma}\left\{E\left(\varepsilon_{t+1}^{1-\gamma}\right)\right\}^{1/\gamma} \quad (5.3.10)$$

and returns are a stationary serially uncorrelated process where

$$R_t = \varepsilon_t. \quad (5.3.11)$$

These two propositions can now be used to analyse the returns to scale technology as well as risk aversion in investigating the time series properties of stock returns. When $\gamma = 1$, Basu and Vinod (1994) presented the autocorrelation function for the log of stock returns represented by

$$\rho_1 = (\alpha - 1)/2 \quad \text{and} \quad (5.3.12)$$

$$\rho_j = \alpha\rho_{j-1}, \text{ for } j > 1. \quad (5.3.13)$$

Basu and Vinod used (5.3.6) and (5.3.8) to analyse 3 special cases.

(a) Increasing returns to scale where $\alpha > 1$ implying no MRSP. From (5.3.6) and (5.3.8), stock price and returns are not stationary and therefore non mean reverting.

(b) Constant returns where $\alpha = 1$ implying no MRSP. In this case, serial correlation in stock returns equals zero indicating the absence of mean reversion. Stock returns are stationary whilst stock price is also said to follow a geometric random walk with drift, hence no MRSP. A random walk model with drift is $I(1)$ and its first difference is $I(0)$.

(c) Diminishing returns where $0 < \alpha < 1$ implying the presence of MRSP as the stock price is purely trend stationary whilst stock returns exhibit negative autocorrelation

The main proposition however is that a positive risk aversion parameter motivates smoothing as a necessary but insufficient condition for MRSP. To demonstrate the necessity of the positive risk aversion, the authors assumed an economy with zero risk aversion such that (5.3.3) becomes

$$P_t = \delta E_t(P_{t+1} + D_{t+1}) \tag{5.3.14}$$

which can be presented in terms of the expected stock returns where

$$E_t[(P_{t+1} + D_{t+1})]/P_t = 1/\delta \tag{5.3.15}$$

and which is independent of the conditioning set. According to the authors, this implies zero serial correlation in stock returns such that there is no MRSP. In conclusion, risk aversion is necessary for the existence of MRSP. In the earlier propositions, the authors demonstrated that a diminishing return technology was a necessary condition for the presence of MRSP. To understand the relationship between diminishing returns and capital investment, the authors recalled the Cochrane (1991) result that stock returns (R_t) and investment returns (R_t^I) are equal in equilibrium and which is represented by

$$R_t^I = R_t = (1/\delta).(W_t/W_{t-1}). \tag{5.3.16}$$

A favourable realisation of ε_t which, given (5.3.1), implies a higher value for W_t, leads to higher consumption and capital accumulation in time t. In the presence of diminishing returns, larger capital stock, K_{t+1}, would reduce the investment return R^I between period t and $t+1$. Consequently, the stock returns are also lower. An increase in stock returns between period $t-1$ and t leads to a decrease in stock returns between period t and $t+1$ and this is caused by diminishing returns.

Basu and Vinod proceeded to "examine the time series properties of the actual financial aggregates to understand the possible linkages between the technology and the stock market". For this, the authors performed unit root tests for log P_t and log R_t. Table 5.9 (panels A and B) reports the results for unit root tests for log P_t and log R_t. Although, we interpret our results in the context of the Basu and Vinod formulation, our approach differs on two counts. Firstly, we include a trend with the ADF maintained regressions and we do so by inspection of the graphs (Figure 5.3) for the series as well as the autocorrelations of levels and first differences. Secondly, we start with the $I(2)$ tests on the basis that starting with an $I(1)$ may result in incorrect conclusions about whether a series contains a unit root or contains only one unit root. The procedure for this involves testing for unit roots for first differences in the first instance. This amounts to a test of two unit roots as one unit root has already been imposed by the taking of first differences. Only if the null hypothesis is rejected do we move to testing for one unit root i.e. testing the original level of the variable rather than the first difference.

PAGE MISSING IN ORIGINAL

Figure 5.3

***Panel C**: Autocorelation coefficients, 1920-1991*

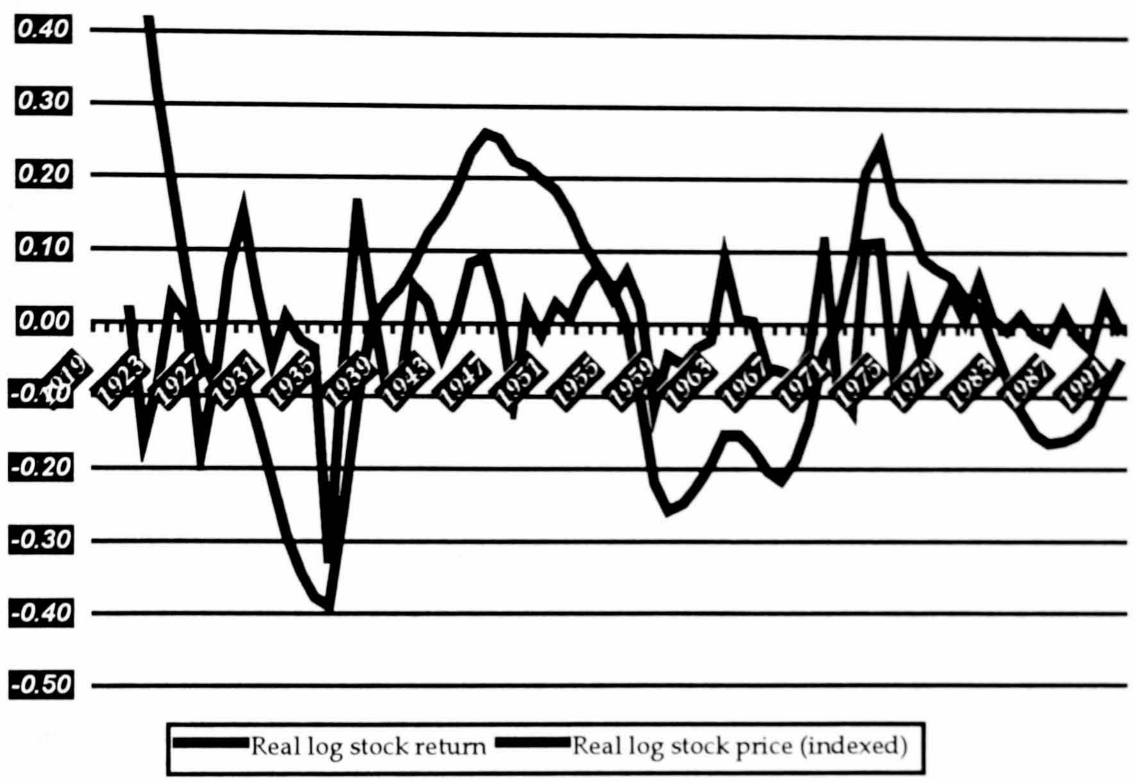


Figure 5.3

***Panel D**: Autocorelation of first differences 1920-1991*

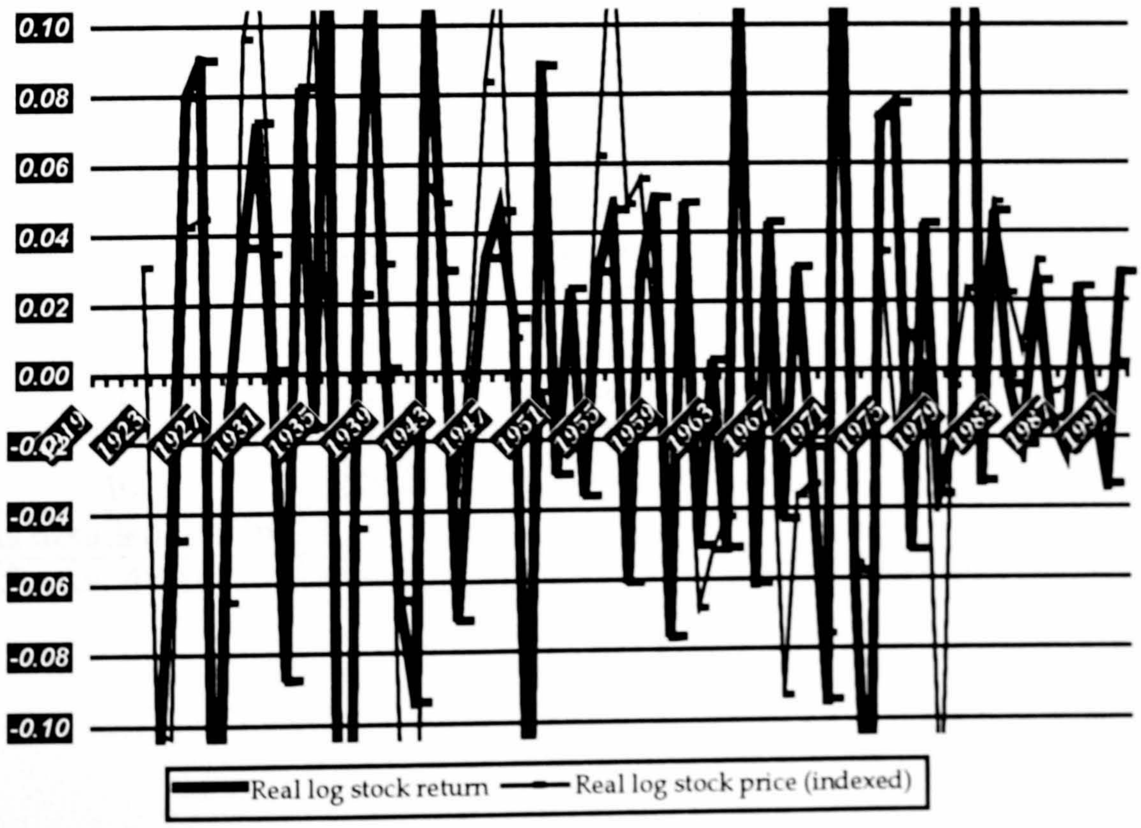


Table 5.9

Panel A: ADF tests for unit root of log ΔP_t (Real stock price)

π	τ	Stat	5%	Decision
1	1	-6.7322	-3.4739	0
1	2	-5.5993	-3.4749	0
1	3	-4.4744	-3.4759	0
1	4	-3.9362	-3.4769	0
1	5	-4.4079	-3.4779	0
1	6	-4.0425	-3.4790	0
1	7	-3.5210	-3.4801	0
1	8	-3.0778	-3.4812	1
1	9	-2.4920	-3.4836	1
1	10	-1.9533	-3.4849	1

Notes: ADF = Augmented Dickey Fuller test. Column π refers to the order of the polynomial where $\pi = 1$ refers to a test for nonstationarity around a trend. Column τ shows the number of lags included in the right hand side. Given that the data is annual data, 10 lags are used. "stat" is the corresponding statistic using the ADF method. Critical values are given at the 5% level and when the decision = 1, the test fails to reject the null hypothesis of no unit root at the chosen significance level. When the decision = 0, then the test rejects the null hypothesis implying that no more unit roots remain.

Source: Author's calculations using data on asset returns 1919-1991

Panel B: ADF tests for unit root of log P_t (Real stock price)

π	τ	Stat	5%	Decision
1	1	-3.8764	-3.4730	0
1	2	-3.4698	-3.4739	1
1	3	-3.0114	-3.4749	1
1	4	-3.2667	-3.4759	1
1	5	-3.2653	-3.4769	1
1	6	-2.5429	-3.4779	1
1	7	-2.5826	-3.4790	1
1	8	-2.5179	-3.4801	1
1	9	-2.5217	-3.4812	1
1	10	-2.9462	-3.4824	1

Notes: As detailed in Panel A.

Source: Author's calculations using data on asset returns 1919-1991

***Panel** C: Tests for unit root of log R_t (Real Stock Return)*

π	τ	stat	5%	Decision
1	1	-6.7117	-3.4739	0
1	2	-5.4988	-3.4749	0
1	3	-5.4988	-3.4759	0
1	4	-3.9043	-3.4769	0
1	5	-4.3612	-3.4779	0
1	6	-3.9343	-3.4790	0
1	7	-3.4877	-3.4801	0
1	8	-3.0596	-3.4812	1
1	9	-2.4913	-3.4824	1
1	10	-2.2424	-3.4836	1

Notes: As detailed in Panel A.

Source: Author's calculations using data on asset returns 1919-1991

When the first differences (Table 5.9, Panel A) are tested, $I(2)$ test, the results indicate a rejection of the null hypothesis indicating that there are no more unit roots and that the series is stationary (up to 7 lags) whilst Table 5.9, Panel B indicates non rejection of the null hypothesis (for 2-10 lags) implying the presence of a unit root. Hence log P_t is an $I(1)$ series.

As with Δlog P_t, the log R_t is purely stationary up to 7 lags. In the Basu and Vinod (1994) framework, since stock prices are $I(1)$ and stock returns are $I(0)$, the immediate implication is that stock prices and stock returns are not cointegrated. According to Basu and Vinod (1994), this is consistent with a constant returns to scale technology. Another implication of these tests is that the notion of an increasing returns to scale technology is rejected.

Conclusion

Overall, there is some evidence of a link between stock and investment returns though this does not necessarily argue that the link extends as far as to suggest that stock and investment returns are equal. Such evidence comes in the form of generally consistent parameter estimates though the signs are not always so. More positively, the correlation coefficient between stock and investment return forecasts reaches 0.55, which provides a link between stock markets and the real economy given the derivation of the investment return. However, the derivation of the investment return, and implicitly the investment-capital ratio, leave ample opportunity to further develop the models in this area. Cochrane (1991), for example, assumed that the depreciation parameter is constant with a value of 0.10. Furthermore, the implied standard deviation value for the proxy stock return i.e. the investment returns, is only about a third of the actual stock return and in the context of the consumption model discussed earlier, such a value would be incapable of explaining the observed equity premium. Given the nature of our work, an explicit form for risk would be desirable especially since the model acknowledges the presence of such risk. The Basu and Vinod (1994) presentation provided further evidence of the link between financial assets and the real economy but this is in the absence of a formal derivation of the link. The Basu and Vinod framework whilst not able to link moments of asset returns with the production economy provides a methodology for evaluating the technological form.

<u>CHAPTER 6</u>

Summary

Chapter 6 presents a profit-based model that uses a simple profit formulation to derive values for the real stock return which is then compared to the historically observed real stock return. Like the consumption-based model, this profit-based model yields estimates for the mean and standard deviation of the real stock return which are similar to that for the observed real stock return.

Section 6.1.2 derives predicted values for profit based on a standard profit model which is in turn based on the previous period and steady state profit level. We find very strong correlation between the actual and predicted profit levels.

In section 6.1.3, we link stock returns to profit via the profit formulation and a sensitivity function to yield estimates for the mean and standard deviation of stock returns. The model is able to capture the volatility of actual stock returns which is generated on the basis that actual profits and dividends may exceed or fall below expectations with share prices reacting accordingly.

In section 6.1.4, we define the sensitivity function by which the deviations of actual from expected profit is related to stock prices. This function is defined to reflect information asymmetry of stock prices.

Since the model yields estimates consistent with observed data, we conclude that the model represents something of a success.

CHAPTER SIX

6.1. Production-Based Asset Pricing Models

In chapter 5, effort was directed at linking stock returns to marginal rates of transformation via investment and GDP data through a production function. Generally, the overall results were consistent with Cochrane (1991) where a correlation coefficient between the predicted stock and investment returns of up to 0.59 was found. In this chapter, we seek to link production and stock returns via a profit formulation to yield values for the mean and standard deviation of stock returns and implicitly therefore, the risk premium.

6.1.1. The Profit Variable

The profit term used in this formulation has been so defined to enable an assessment of underlying profitability which provides a better insight into the profitability of the firm as opposed to "headline" profit figures. In our estimations, the profitability data used is based on "gross trading profits of companies and financial institutions" since this data avoids many of the distortions and measurement problems associated with net profits.[47] Net profits usually include non-recurring items which are not fundamental to the longer terms prospects for the firm. Hence, gross trading profits provide a more reasonable basis on which to assess fundamental profitability. Gross trading profits also better captures the historical persistence feature common in many profit based models.

[47] We considered using a series that did not include financial institutions but were unable to produce a reliable series especially for the pre 1945 period.

6.1.2. The Profit formulation

Firms are assumed to maximise profit:

$$\max E_{t+\kappa-1}(\pi_{t+\kappa}) = (1-\theta)\,\pi_{t+\kappa-1} + \theta(\hat{\pi}_{t+\kappa}) + \varepsilon_{t+\kappa} \qquad (6.1.1)$$

where

$\kappa \geq 0$,

θ is a profitability parameter where $\theta \leq 1$,

π is real profit level,

$\hat{\pi}$ is the steady state real profit level and

$\varepsilon_{t+\kappa}$ is an error term where,

$$\varepsilon_{t+\kappa} = \left[\pi_{t+\kappa-1} - \hat{\pi}_{t+\kappa-1}\right] \qquad (6.1.2)$$

Equation (6.1.1) says that profitability in period $t+\kappa$ depends on actual profit in the previous period and steady state profit in the current period. The steady state profit level is that which is consistent with the economic growth path. The relative importance of previous period profits and steady state profits is governed by the profit parameter θ. When $\theta = 1$, the steady state profit level determines current period profits whereas when $\theta = 0$, only previous period profits determine current period profits. Hence a value for θ closer to 0 implies that previous period profits is a more important determinant of current period profits. Equation (6.1.2) implies an effect on profits in period $t+\kappa$ of actual reported profits being different from steady state profit expectations in period $t+\kappa$-1. More precisely, profit out-turns which exceed expectations in $t+\kappa$-1 causes upward revisions to profits in $t+\kappa$ capturing the effects of structural shifts in the business outlook to do with issues such as technology. This feature is also part of our stock price formulation. Stock prices and therefore returns are assumed to respond to profitability out-turns that exceeds or falls below expectations. This will be discussed in more detail in section 6.1.3.

We attempted to estimate the profit parameter, θ, by re-writing equation (6.1.1) to make for a testable model. Equation (6.1.1) can be differenced to yield the stationary formulation (with parameter estimates)

$$\Delta\pi_t = \underset{(-0.7415)}{-0.0009} - \underset{(-4.8328)}{0.3509}\left(\pi_{t-1} - \hat{\pi}_{t-1}\right) + u_t \qquad (6.1.3)$$

$\theta = -0.3509$, 't' statistics in brackets, DW = 1.87, P(F) = 0.000, SC(1) = 0.222 [0.633], FF(1) = 1.629 [0.202], JB(2) = 0.690 [0.741]

The parameter estimates for θ as well as for the model are significant with no evidence of serial correlation. As indicated earlier, the steady state real profit level is assumed to form around the economic growth path.

Figure 6.1
The Real Profit Level
Actual and Predicted @ 1985 Prices, 1919-1991

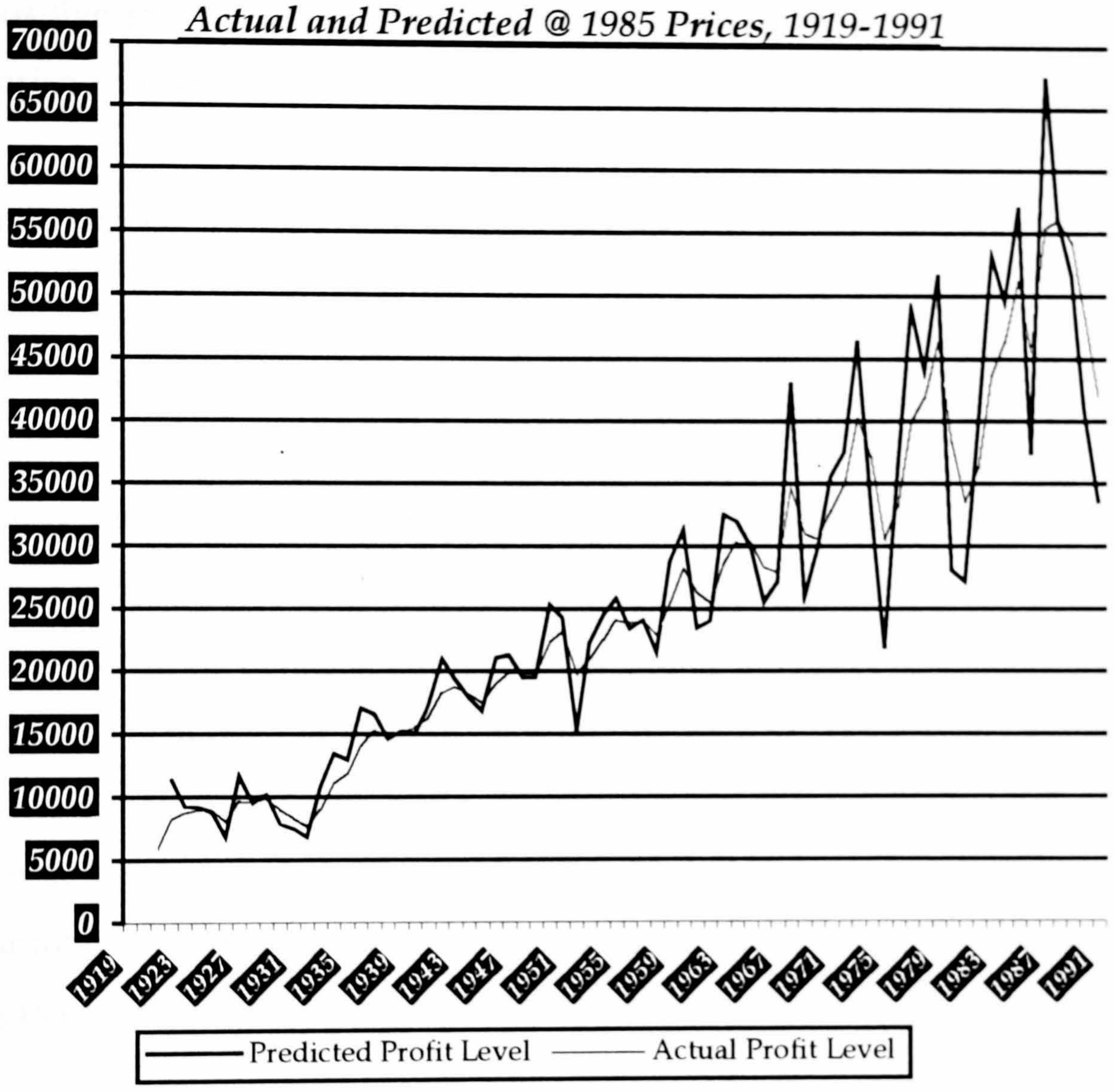


Figure 6.1 shows values for the actual and predicted values for the real profit level. The diagram shows a high degree of correlation between actual and predicted profits. The actual and predicted real profit levels also indicate a clear upward trend over the sample period.

6.1.3. Stock Returns and Profit

In this section, we seek to relate stock returns to our profit formulation on the basis that stock returns respond to profit and dividend announcements such that positive growth announcements that exceed expectations yield a higher stock price and consequently higher stock returns.

Relating stock returns to profit announcements,

$$r_{t+k} = \left[\frac{d_{t+k}}{P_{t+k}}\right] + \frac{P_{t+k} - P_{t+k-1}}{P_{t+k-1}} = \left[\frac{d_{t+k}}{P_{t+k}}\right] + \lambda_{t+k}\left[\frac{\pi_{t+k} - E_{t+k-1}(\pi_{t+k})}{E_{t+k-1}(\pi_{t+k})}\right] + \upsilon_{t+k} \quad (6.1.4)$$

where d_{t+K} is actual dividends and P_{t+K} is stock prices in period $t+k$ such that d_{t+K} $/ P_{t+K}$ is dividend or stock income return in period $t+k$ and υ_{t+k} is the error term. The second term on the right hand side represents the impact on stock prices of deviations of actual profit from its expected value with the term λ representing a reaction or sensitivity parameter. Since we have already established that the stock price is the unpredictable component of stock returns as well as the source of volatility in stock returns, we focus our effort on the relationship between stock prices on one hand and profits on the other.

The error term, υ_{t+K}, represents the deviations of actual dividends from expected dividends with expected dividends defined around its mean growth rate. Hence

$$\upsilon_{t+k} = \left[\frac{d_{t+k} - E_{t+k-1}(d_{t+k})}{E_{t+k-1}(d_{t+k})}\right]. \quad (6.1.5)$$

It can be seen from (6.1.4) that when profits in period $t+k$ exceeds expectations, stock prices, P_{t+k}, reacts positively. Falling stock prices are the result of profit announcements falling short of expectations, a not unknown occurrence.

Theoretical models and practical experience have indicated that expectations of market professionals do not always concur with the actual data. Implicitly therefore, a key feature is the process by which expectations are determined.

6.1.4. The Sensitivity function

In (6.1.4), the term λ represents a stock price (implicitly stock return) reaction parameter which measures the responsiveness of stock prices to profit announcements with either exceed or fall below market expectations. The notion that stock prices and implicitly stock returns overreact is reflected in the work of De Bondt and Thaler, (1985), Porteba and Summers, (1988), Campbell and Ammer, (1993), and Basu and Vinod, (1994). Porteba and Summers as well as Campbell and Cochrane, (1995) imply that the existence of mean reversion in stock prices, is in effect arguing that stock prices implicitly overreact.

Given that stock prices are assumed to follow a GARCH process, we define the sensitivity function, λ, with the following conditions. Firstly, that where actual profit falls below expectations, such bad news will have a greater negative impact on stock prices than an equivalent positive out-turn. Secondly, the function is defined to be positive at all times and thirdly, that the sensitivity function has the capacity to maintain the first condition in the context of general macro-economy. For example, the first condition should hold in the context of changing interest rates such that a 1% rise in interest rates will imply a higher sensitivity function value relative to a 1% fall in interest rates. The lack of an overreaction to profit announcements will be reflected in reaction parameter values of around 1, hence we define $\lambda_{t+k} = 1 + \lambda^{*}_{t-k}$.

In defining the sensitivity function, we present a stock valuation (V) formulation,

$$V_{t+k} = \lambda^{*}_{t-k}\left[\frac{\pi_{t+k}}{\varpi_{t+k}}\right] + d_{g,t+k} \qquad (6.1.6)$$

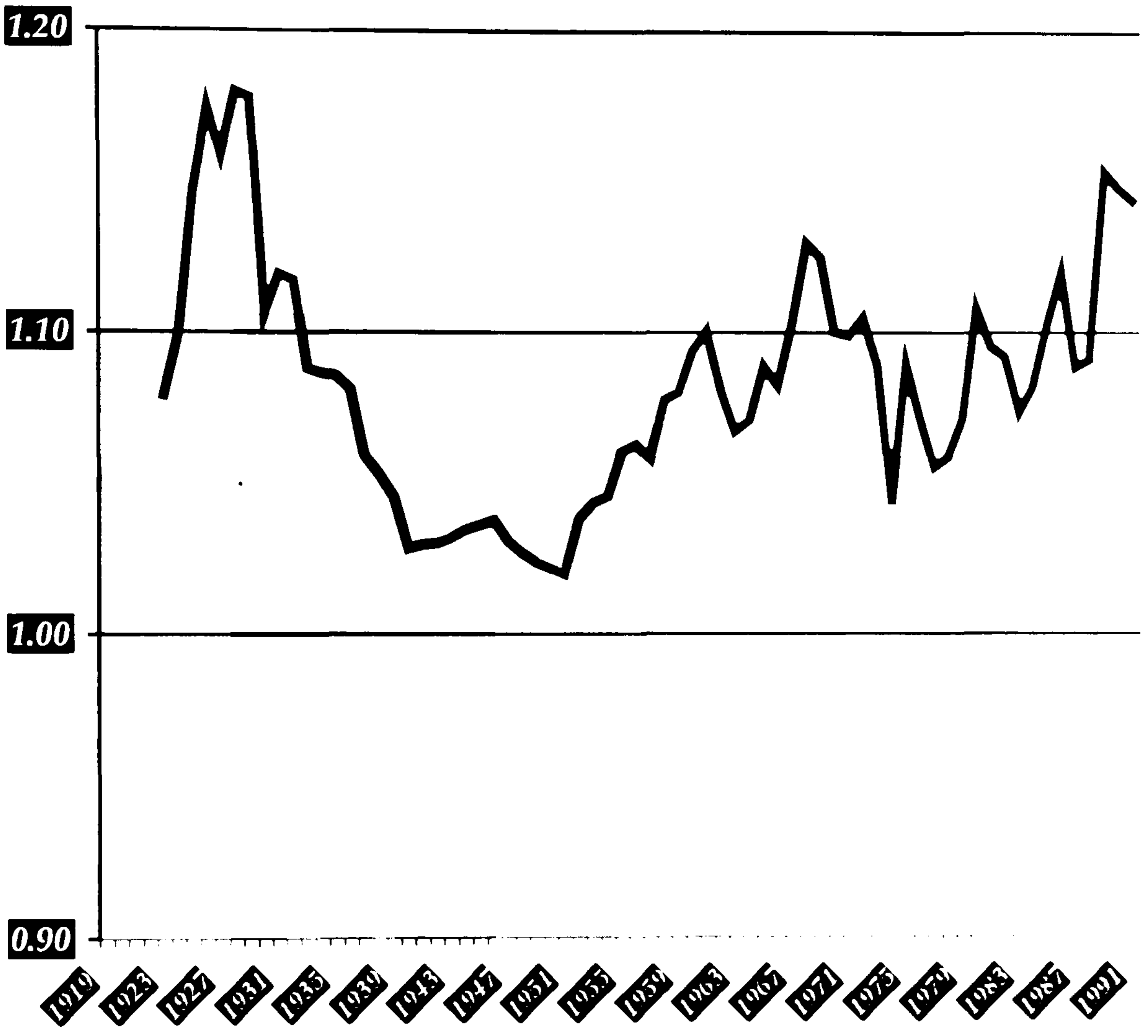
where λ^{*} is the period reaction parameter, ϖ_{t+k} is the cost of capital and $d_{g,t+k}$ is the growth rate of dividends. Equation (6.1.6) says that the value of a share reacts to discounted profits, π_{t+k}/ϖ_{t+k}, and the level of dividends in the period. Equation (6.1.6) could be rewritten as

$$\lambda^{*}_{t+k} = \left[\frac{V_{t+k} \cdot \varpi_{t+k}}{\pi_{t+k} + \varpi_{t+k} \cdot d_{g,t+k}} \right] \qquad (6.1.7)$$

from which we conclude that the conditions as set out have been met. In estimating sensitivity, we use base interest rates to represent ϖ and V is represented by the current period stock price (P_{t+k}) and stock income/ dividend return (d_{t+k} / P_{t+k}). Both V and π are represented by indexed data.

Figure 6.2

Sensitivity function values, 1919-1991



The sensitivity values indicate an upward trend since 1945 explaining, in part at least, the increased volatility associated with stock prices. The lowest levels of sensitivity were found in the immediate post-war period when the economy enjoyed substantial economic growth. Higher values were found for the early 1970s and the late 1980s; periods consistent with economic uncertainty.

In the model, expectations for (6.1.5) are assumed to be formed around its long-term growth rate and is therefore represented by

$$d_{t+k-1} = g_d + \Phi_1 d_{t+k-2} + \varepsilon_{dt} \tag{6.1.8}$$

where Φ_1 is a parameter estimate close to 1. Equation (6.1.8) can be rewritten such that

$$d_{t+k-1} - \Phi_1 d_{t+k-2} = g_d + \varepsilon_{dt} \tag{6.1.9}$$

where g_d is the mean real growth rate of dividends (2.2%).

Table 6.1 reports estimates for the mean and standard deviation of real stock returns.

Table 6.1

***Panel A**: Model Predictions for the log mean and standard deviation*

	Mean of Log Real Stock Returns	Standard Deviation of Log Real Stock Returns	Sharpe Ratio (Col 2/Col 3)
Actual	0.0575	0.2046	0.281
Predicted	0.0667	0.1737	0.384

Author's calculations using data on asset returns 1920-1991

***Panel B**: Assumptions and derived parameters*

Assumptions	
Mean real dividend growth rate, g_d	0.022

Derived Parameters	
Mean sensitivity value	1.0825
Maximum sensitivity value	1.1798
Minimum sensitivity value	1.0197
Profit parameter, θ,	- 0.3509

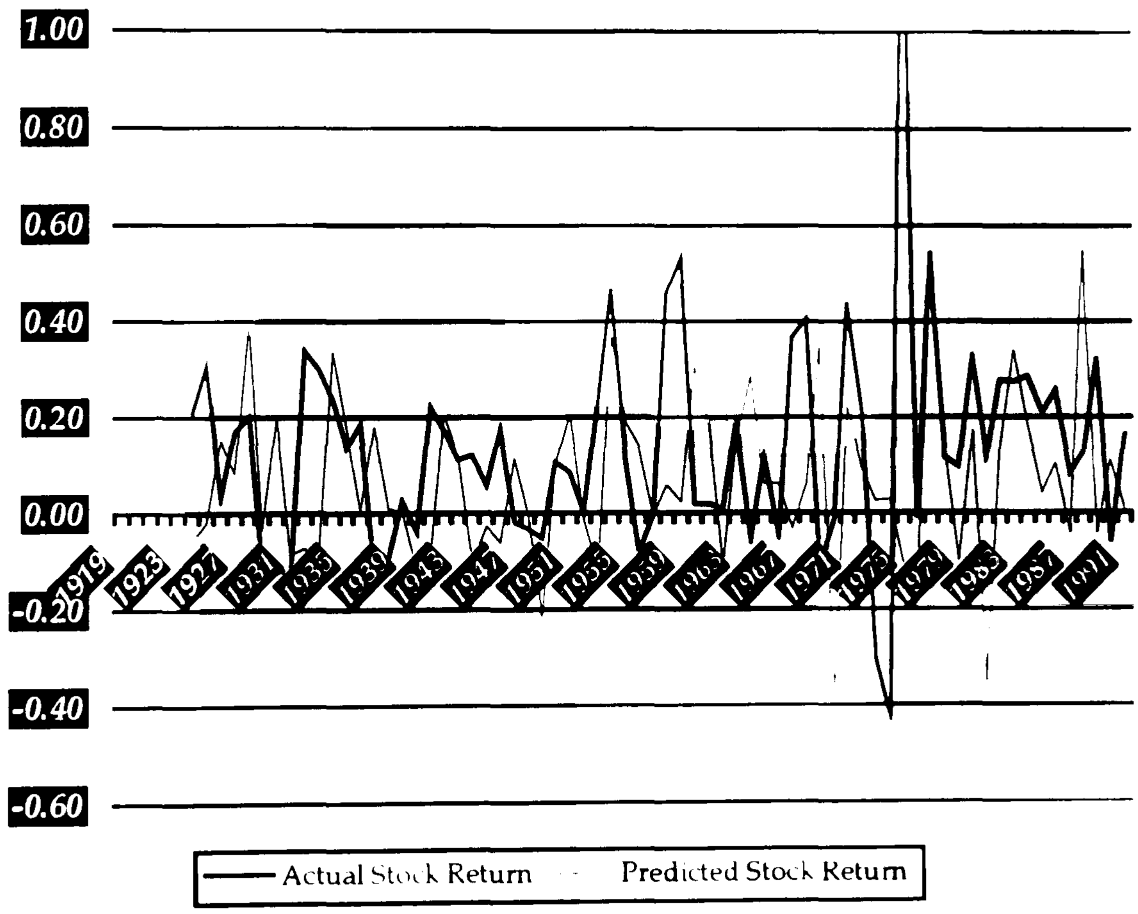
Author's calculations using data on asset returns 1920-1991

The results presented in Table 6.1 provides support for the profit formulation as well as for the model linking stock returns and profit in that the estimates for the mean and standard deviation of stock returns are realistic and reasonably close to the historically observed levels. The predicted mean stock premium is also comfortably within the limits of a 95% confidence interval. This we find to be most encouraging given the definitions of parameters of interest such as the sensitivity values which reflected our assumption that stock prices follow a

GARCH process implying asymmetric responses to information i.e. stock prices are more volatile in response to negative shocks as opposed to positive shocks of the same magnitude. The correlation between actual and predicted stock returns is positive (0.12) but which increases dramatically when we look at 2-year and 3-year averages in chapter 7. Figure 6.3 reports values for the predicted and actual stock returns.

Figure 6.3

Actual and Predicted Nominal Stock Returns, 1919-1991



Conclusion

Ordinarily, we would expect profit growth to be reflected in higher stock prices. However, we have seen many instances of negative stock price reaction to generally positive growth in reported profits. This is because stock prices would have already taken into account profit growth expectations of market participants such that should actual profit growth fall below expectations, stock prices are likely to respond in a negative manner. In other words, expectations are "already in the price" before profit announcements are made.

The model is considered something of a success in that the model is able to replicate historical data on stock returns by matching the mean and standard deviation. This is done using a model which links stock returns to deviations of actual from expected profitability performance via a reaction parameter. Furthermore, the formulation is able to account for much discussed features of stock returns i.e. mean reversion and over-reaction.

A reaction parameter value in excess of 1 implies that there exists the possibility of over-reaction to profit announcements which further implies that there will exist profit-making opportunities. This is consistent with the notion that there exists a "herd instinct" in financial markets such that market participants are likely to trade on the basis that other market participants are doing the same. When viewed with market participants who at least aim to match returns of the various indices, then there is the possibility of an over-reaction.

This profit-based model can also be linked to the consumption-based model in a full equilibrium framework. Furthermore, the model can also be linked to output, investment and pricing decisions. We suggest a methodology for this in chapter 7.

CHAPTER 7 (SEVEN)

Summary

The chapter starts with a summary of the results from the earlier chapters relating to both consumption (section 7.1) and production via profit (section 7.2) models. The summary traces a process of model developments which culminate in proposed models which yield realistic parameter values.

Section 7.3 discusses the relationship between the predicted and observed stock returns and concludes that the presence of market distortions account for the relatively low correlation between actual and predicted returns. The notion of market distortion focuses on information asymmetry, liquidity constraints, stock price manipulation, the agency problem and heterogeneous consumers. Correlation coefficients for 2-year and 3-year averages, moving and otherwise, are presented which show much stronger correlations. For the profit-based production model in chapter 6, the correlation coefficient approaches 0.62.

Section 7.4 considers how the consumption and profit-based production models could be integrated in a full equilibrium framework.

Section 7.5 considers the wider implications for macroeconomic modelling and forecasting and concludes that market distortions need to be an integral part of models which seek to explain financial, product or labour market behaviour. Finally in this section, are the conclusions which directly responds to the research questions posed in chapter 1.

Section 7.6 discusses further potential areas for future research.

CHAPTER SEVEN

7.1. Evidence from the consumption-based models

So far, our results for consumption and production-based models have been encouraging in linking stock markets, via stock returns, and the real economy. A summary of some of the results are reported in Table 7.1.

Table 7.1

Summary of results : consumption-based models

Model and method of estimation	Assumed Variables (logs)	Estimated Variables (logs) (or implied)
Power utility: a calibration using the covariance of the stock premium with consumption growth. Table 3.3.	Mean Stock Return =0.0657 Mean Stock Prem = 0.0575 -	CRRA = 190 - -
Regression of stock premium on the product of deviation from the mean of stock premium and consumption growth. Table 3.4.	- - - -	CRRA = -7.421 Std. Err = (5.585) - -
Regressions of log asset returns and consumption growth. Table 3.5.	Dep. Var : Stock Return Ind. Var : Cons Growth	CRRA = 0.377 Std. Err = (5.585)
	Dep. Var : Risk-Free Rate Ind. Var : Cons Growth	CRRA = 0.042 Std. Err = (0.275)
	Dep. Var : Cons Growth Ind. Var : Stock Return	EIS = 0.007 Std. Err = (0.017)
	Dep. Var : Cons Growth Ind. Var : Stock Return	EIS = 0.008 Std. Err = (0.052)
IV Estimations: Table 3.6 (Panel A) Using 1 lag of the instruments Table 3.6, Panel A	IV Estimations[48] Dep.Var : Risk-less Return Dep.Var: Stock Return	 CRRA: 1.55 (0.53) CRRA: 2.80 (1.65)

[48] Dependent variables are the fitted values for the variables. These IV results are those which use lags of the stock return as instruments. Results for regression which use the dividend-price instead (Table 3.6, panel B) are quite similar and therefore not presented here.

[49] GMM estimates up to 2 iterations.

	Dep.Var : Cons Growth Dep.Var: Cons Growth	EIS: 0.49 (0.11) EIS: 0.09 (0.04)
Using 1 and 2 lags of the instruments Table 3.6, Panel A	Dep.Var : Risk-less Return Dep.Var: Stock Return	CRRA: 1.34 (0.39) CRRA: 2.84 (1.50)
	Dep.Var : Cons Growth Dep.Var: Cons Growth	EIS: 0.24 (0.09) EIS: 0.08 (0.04)
GMM estimates in a multiple regression model similar to presentation in table 3.6. (Table 3.7, Panel B)	GMM[49] (1 lag of Instruments) D/P ratio & Risk-less Return Stock return / Risk-less Return	 CRRA: -0.99 (0.69) CRRA: -2.56 (2.10)
	GMM(1&2 lags of Instruments) D/P ratio & Risk-less Return Stock return / Risk-less Return	 CRRA: -2.25 (0.55) CRRA: -5.11 (2.80)
GMM estimates of the coefficient of risk aversion under HABIT FORMATION (Cooley and Ogaki (1991, 1996) - Table 3.10	GMM(1&2 lags of Instruments) D/P ratio & Risk-less Return Stock return / Risk-less Return	 CRRA: 0.20 (0.36) CRRA: 1.02 (5.06)
Parameter estimates of the Campbell et al surplus consumption model - Table 3.12	Calibration of the surplus consumption model as per Campbell et al. Mean Stock Return = 0.0657 Mean Stock Premium = 0.0575 Mean Consumption G = 0.0147	CRRA: 1.902 Stock Return: 2.2% Std. Dev: 8.17% Sharpe Ratio: 0.28 Discount rate: 90% Correlation (actual & predicted stock returns): 0.239
Power utility model : a calibration using the covariance of the stock premium with surplus consumption growth. Table 4.2.	Mean Stock Return = 0.0657 Mean Stock Premium = 0.0575 Mean Surplus Cons G = 0.0114	CRRA: approx 10
Power utility model: a calibration using the covariance of the stock price premium with surplus consumption growth. Table 4.3.	Mean Stock Price Ret = 0.0156 Mean Stock Price Prem = 0.007 Mean Surplus Cons G = 0.0114	CRRA: 0.99 Discount rate: 0.96
IV Estimations Using 1 lag of the instruments Table 4.4, Panel A	IV Estimations Dep.Var: Risk-less Return Dep.Var: Stock Price Return	 CRRA: 0.16 (0.07) CRRA: 0.23 (0.24)
Using 1 and 2 lags of the instruments Table 4.4, Panel A	Dep.Var : Risk-less Return Dep.Var: Stock Price Return	CRRA: 0.19 (0.05) CRRA: 0.21 (0.10)
GMM estimates in a multiple regression model similar to presentation in table 3.6. Using 1 lag of the instruments (Table 4.5)	GMM (1 lag of Instruments) Risk-less Return/Stock Price R	 CRRA: 0.81 (0.22)
Using 1 and 2 lags of the instruments	GMM(1&2 lags of Instruments) Risk-less Return/Stock Price R	 CRRA: 0.91 (0.07)

Power utility model : a calibration using the covariance of the stock income premium with habit consumption growth. Table 4.6.	Stock Income Return = 0.0186 Stock Income Premium = 0.011	CRRA: Around 23 Discount rate: 1.16
IV Estimations Using 1 lag of the instruments Table 4.7, Panel A	IV Estimations Dep.Var : Risk-less Return Dep.Var : Stock Income Return	 CRRA: 2.90 (0.79) CRRA: 2.92 (0.94)
Using 1 and 2 lags of the instruments Table 4.7, Panel A	Dep.Var : Risk-less Return Dep.Var : Stock Income Return	CRRA: 2.22 (0.59) CRRA: 2.18 (0.88)
GMM estimates in a multiple regression model. (Table 4.8)	GMM (1 lag of Instruments) Risk-less Return/Stock Income	 CRRA: -2.96 (2.39)
	GMM(1&2 lags of Instruments) Risk-less Return/Stock Income	 CRRA: -4.56 (1.17)
Parameter estimates of the "Very Slow Moving Habit" model by the author - Table 4.9	Calibration of the "Very Slow Moving Habit" model by the author - Table 4.9	CRRA : 1.21 Stock Return: 4.9% Std. Dev: 17.6% Sharpe Ratio: 0.28 Discount rate: 84% Correlation (actual & predicted stock returns) : 0.134

NOTES
CRRA : Coefficient of Relative Risk Aversion — *Std. Dev : Standard Deviation*
EIS: Elasticity of Intertemporal Substitution — *Std. Err : Standard Error*
Dep. Var : Dependent Variable

Author's calculations

7.1.1. The Basic Power Utility Model

Chapter 3 and 4 presents results for consumption-based models which attempted to derive values for known parameters whilst being able to match observed asset returns. The results for the consumption version of the basic power utility model were not very encouraging as found in Campbell, Lo and Mackinlay (1997). The coefficient of risk aversion required to explain the observed data was well in excess of the maximum value of 10 thought plausible by Mehra and Prescott (1985). Whilst it is not possible for the specification of the power utility to give anything more than parameters estimates for risk aversion, the hope remains that it could at least provide a basis from which to develop richer models. The failure of the power utility model was compounded by the Weil (1989) inspired risk-free rate puzzle which implies that the power utility model can also imply a negative rate of time preference.

7.1.2. IV and GMM Estimates of Risk Aversion

Given the possibility of biased estimates, IV and GMM estimates were generated which provided some encouragement. For consumption, the parameter estimates are positive, in contrast to the Campbell, Lo and Mackinlay (1997) results, and significant though the over-identifying restrictions of the model tended to be rejected. With 1 lag of the instruments, the risk aversion parameter was estimated at around 4. This value, though positive, is quite a way from the real business cycle-inspired value of 1. The results also presented evidence against the notion that the elasticity of intertemporal substitution is the inverse of the coefficient of relative risk aversion. Campbell and Cochrane (1995) also presented evidence against this notion. The GMM estimates are less encouraging, and given the power of GMM, provides strong evidence against the power utility model. The parameter estimates are negative and the over-identifying restrictions of the model are not rejected. This negative parameter estimate does indeed present a problem for the power utility model though it perhaps did not quite indicate its death.

7.1.3. The Habit and Surplus Consumption

The incorporation of habit into the consumption framework has been somewhat of a success in that they have been able to yield consistent and reasonable parameters for risk aversion in the presence of values for the discount and risk-less rates consistent with observed data. For consumption, results using the Campbell, Lo and Mackinlay (1997) and Campbell and Cochrane (1995) formulations of the problem, are very encouraging with a risk aversion parameter estimate of 1.90. This value was found in the presence of a risk-less return of 0.82% and a discount rate of 0.82, which is indicative of a positive rate of time preference. The predicted stock premium of 2.23% was lower than found in the data (5.75%) as was the associated standard deviation of (8.17%) compared to that found in the observed data of over 20%. In other words, even though the observed Sharpe Ratio of 0.281 was matched by the data, this was done with lower values for the mean and standard deviation of the risk premium.

7.1.4. Disentangling the components of Stock Return

The results were extremely encouraging for this link and formed the basis of a model presented by the author which redefined surplus consumption to explicitly capture the volatile growth in surplus consumption which according to the author drives the stock return and implicitly the premium. In chapter 4, price and income components were separated on the basis that it is the stock price that generates the variation in stock returns. Consequently, surplus consumption is much more closely related to the stock price whereas habit consumption is related to the stock income. Testing the power utility model in this context, the standard deviation of surplus consumption growth matches the stock price return and the risk aversion coefficient required to match the observed stock price return is around 1. On the basis that surplus consumption generates the risk aversion parameter, the expectation must be that the power utility model incorporating habit consumption cannot generate the observed stock premium with reasonable parameters. The value required to explain the observed return is unsurprisingly about 50 though the implied discount rate is about 0.71.

7.1.5. A Slow Moving Habit

Since surplus consumption notion works well in the power utility context, it is at this juncture that an amendment to the Campbell and Cochrane (1995) model is presented which attempts to capture the changes to the surplus consumption which generates the stock return and consequently the stock premium. The result is that the mean and standard deviation of the stock premium almost exactly match that observed in the data to yield the observed Sharpe Ratio of 0.28. The implied value for the risk aversion parameter is 1.21 much closer to the value of 1 consistent with real business cycle theory. By any measure, this model is regarded as something of a success.

Overall, the incorporation of habit, and implicitly surplus consumption, into the formulation has taken us closer to fully explaining the observed stock premium.

7.2 Evidence from Production-based Models

Attention also turned to production-based models which attempted to link production and stock returns via production functions, Tobin's Q theory, production technologies and profit functions but with varying degrees of success. Table 7.2 provides a summary of some of the key results.

Table 7.2

Summary of results: production-based models

Model and method of estimation	Assumed Variables (logs)	Estimated Variables (logs) (or implied)
OLS regressions (as per Fama 1990) : Table 5.2 (Panel A)	Multiple regression of production growth on current and 1 lag of stock return (continuously compounded).	R^2 : 0.125 Joint (*P*) *F* : 0.012
OLS regressions (as per Fama (1990) : Table 5.2 (Panel A)	Multiple regression of production growth on current, 1 and 2 lags of stock return (continuously compounded).	R^2 : 0.204 Joint (*P*) *F* : 0.002
OLS regressions (as per Fama (1990) : Table 5.2 (Panel A)	Multiple regression of production growth on current and 1, 2 and 3 lags of stock return (continuously compounded).	R^2 : 0.184 Joint (*P*) *F* : 0.010
OLS regressions (as per Fama (1990) : Table 5.2 (Panel B)	Multiple regression of stock return (continuously compounded) on current and 1 lead of production growth.	R^2 : 0.151 Joint (*P*) *F* : 0.004
OLS regressions (as per Fama (1990) : Table 5.2 (Panel B)	Multiple regression of stock return (continuously compounded) on current, 1 and 2 leads of production growth.	R^2 : 0.162 Joint (*P*) *F* : 0.009
OLS regressions (as per Fama (1990): Table 5.2 (Panel B)	Multiple regression of stock return (continuously compounded) on current, 1, 2 and 3 leads of production growth.	R^2 : 0.170 Joint (*P*) *F* : 0.016

Using the Capital Accumulation rule Cochrane (1991). Table 5.3. Further results from Tables 5.4 - 5.8.	Using the capital accumulation rule to derive the investment returns (IR) which Cochrane argues is equal to stock returns (SR).	IR SR Mean : 9.1% 9.1% Std.d : 7.9% 25.4% A(1) : 0.26 -0.09 A(2) : -0.09 -0.12
ADF Tests for unit roots as a test of the production technology (Basu and Vinod (1994) : Table 5.9 (Panel A).	ADF Tests of the log change of the real stock price	π τ stat 5% D 1 1 -6.7 -3.5 0 1 2 -5.6 -3.5 0 1 3 -4.5 -3.5 0 1 4 -3.9 -3.5 0 1 5 -4.4 -3.5 0 1 6 -4.0 -3.5 0 1 7 -3.5 -3.5 0 1 8 -3.1 -3.5 1 1 9 -2.5 -3.5 1
ADF Tests for unit roots as a test of the production technology (Basu and Vinod (1994) : Table 5.9 (Panel B).	ADF Tests of the log of the real stock price	1 1 -3.9 -2.9 0 1 2 -3.5 -2.9 1 1 3 -3.0 -2.9 1 1 4 -3.3 -2.9 1 1 5 -3.3 -3.5 1 1 6 -2.5 -3.5 1 1 7 -2.6 -3.5 1 1 8 -2.5 -3.5 1 1 9 -2.5 -3.5 1
ADF Tests for unit roots as a test of the production technology (Basu and Vinod (1994): Table 5.9 (Panel C).	ADF Tests of the log real stock return	1 1 -6.7 -2.9 0 1 2 -5.5 -2.9 0 1 3 -5.5 -2.9 0 1 4 -3.9 -2.9 0 1 5 -4.4 -3.5 0 1 6 -3.9 -3.5 0 1 7 -3.5 -3.5 0 1 8 -3.1 -3.5 1 1 9 -2.5 -3.5 1
Author model on the relationship between profit and stock returns: Table 6.1 (Panels A and B)	Calibration of the model with the profit parameter θ, stock price-profit reaction parameter value λ.	Nominal Returns Stock Return: 6.7% Std. Dev: 17.4% Sharpe Ratio: 0.38 θ = -0.3509, Correlation (actual & predicted stock returns): 0.12

NOTES (ADF Tests: Basu and Vinod (1994))

Notes: ADF = Augmented Dickey Fuller test. Column π refers to the order of the polynomial where π=0 gives a test for nonstationarity around a constant term while π = 1 refers to a test for nonstationarity around a linear trend. Column τ shows the number of lags included in the right hand side. Given that the data is annual data, 4 lags are used. "stat" is the corresponding statistic using the ADF method. Critical values are given at the 5% level and when the decision = 1, the test fails to reject the null hypothesis of no unit root at the chosen significance level. When the decision = 0, then the test rejects the null hypothesis.

Author's calculations

7.2.1 Production and Stock Returns

Fama (1991) suggested that regressions of stock returns on leads of production and of production on lags of stock returns could be used to draw inferences about the relationship between stock returns and production. Our results following this approach, was at best mildly encouraging for the notion that production growth could directly predict stock returns. One disadvantage of this approach was that other parameter values were not implicitly modelled.

7.2.2. Investment and Stock Returns

Results based on the Cochrane (1991) production model are somewhat encouraging in that the correlation between stock returns and investment returns takes values of up to 0.59 despite the fact that the parameter signs are not always consistent with the theory. This model derived an investment return from production data via a production function and suggested that the investment return closely mimicked stock returns. Again, this model did not explicitly model the discount rate and or other associated parameters of interest.

7.2.3. The Production Technology

Basu and Vinod (1994) attempted to link production and asset returns via the production technology. They derived a model which used asset returns to infer the production technology using unit root tests and concluded that the production technology was a constant returns to scale. According to the authors, since asset returns displayed a constant returns to scale technology which is consistent with real business cycle theory, then the clear implication is that financial assets, as claims to the capital stock, are inextricably linked to the economy. Our results broadly supported this conclusion using the same interpretative framework. We recall that the consumption and production models which delivered reasonable and realistic parameter estimates explicitly assumed mean reversion in surplus consumption and profit and implicitly therefore in asset returns.

7.2.4 The Profit Function

In chapter 6, a model was presented by the author which used deviations of actual from expected profits and dividends to generate values for the stock returns. The generated values compared well with the observed historical data in terms of the mean and standard deviation and the associated Sharpe Ratio.

One of the outstanding issues in our results so far has been the low positive correlation between actual and predicted stock returns even after the event. It is not unrealistic to expect that models which claim to be able to predict stock returns should also be able to generate values which are reasonably well correlated with the actual data. In our models, correlation levels remained relatively low and there now follows an exposition of some of the reasons for despite the fact that the models have been able to match the mean and standard deviation of observed stock returns.

7.3. Predictions of Stock Returns

Though reasonable, the relatively low correlation between predicted and actual stock returns provides some cause for concern. This is particularly so given that this is after the event. Models which claim to explain aspects of stock market behaviour must to some extent be able to predict stock returns as well as show a stronger correlation between actual and predicted returns. The explanations for this phenomenon are no doubt many and varied but they are often based around the notion of imperfect capital markets. Perhaps a more apt term might be market distortions. The basic point is that actual stock returns at a point in time might not always be reflective of predicted returns since actual returns are subject to short term pressures which do not necessarily exist in the longer term. This point was noted by Porteba and Summers (1988) and Fama and French (1988) who indicated the presence of short-term "fads".

7.3.1. Imperfect Capital Markets - The Intermediation / Agency Problem

Another aspect of the CCAPM which requires further consideration is the implicit assumption that agents, acting on behalf of individual investors, act as their clients would have done such that the intermediation role is a purely passive one. The rationale for agency is based on the premise that the agent is able to pool the risk of individual investors thereby reducing the collective risk. Furthermore, this approach is assumed to generate higher returns via lower transactions costs or the same return with lower risks. The assumption that agents act only in the interests of the investors is an interesting one given that the returns to the agent is not determined in the same way as that to their investors. The agent's remuneration typically includes a fixed fee element arrangement whereas the return to the investor is based purely on returns achieved. One could well argue that the agent has an incentive not to maximise returns lest they pursue an opportunity set that is so risky as to put at risk their own economic profit. The key question remains whether consumer choices do indeed feed through to the stock markets.

7.3.2. Imperfect Capital Markets - Stock price manipulation

Allen & Gorton (1992), using a Glosten and Milgrom (1985) model, discussed a scenario where an uninformed investor can manipulate a market by driving up share prices and then selling at a profit. Such an approach is legitimately part of an investor's trading strategy although it would seem to indicate a less than efficient market. Such a possibility is said to exist because of information asymmetry.

In a Glosten and Milgrom perfectly competitive market, a market specialist indicates a bid (buying) price B and an ask (selling) price A. In this market, all orders are market based and only one unit of stock can be traded at a point in time. After the specialist has indicated bid and ask prices, a trader indicates a willingness either to purchase or sell a stock at the specified bid and ask prices. After this trade, the specialist could well adjust the bid and ask prices on the basis of this demand. The stock value at some future date is therefore uncertain in that it may be high, V_H, or low, V_L. Allen and Gorton then defined the specialist's initial probability of a high value as d and that of an arriving trader being an insider as α. Also bear in mind that private informed traders are informed about the future value of the stock, V_H or V_L unlike the liquidity trader who has to buy or sell at prevailing prices for exogenous reasons. Allen and Gorton define the probability that the liquidity trader is a seller as β and assume that all agents are risk-neutral. At this stage, the specialist is aware of the potential for entry into the market by other specialists and so sets prices so as not to be surprised by the expected prices regardless of whether the next transaction is a buy or a sell. Allen and Gorton outlines a situation where the specialist's current expectations prior the stock value being V_H is d. The specialist then anticipates that if the next trader is a purchaser, "then the share should be sold at a price which takes into account the fact that the trader may be informed and therefore knows that the stock has a value V_H". The specialist is then able to update his or her expectations, using Bayes' rule to obtain

$$d_1^A = d_0[\alpha + (1-\alpha)(1-\beta)] / \{d_0[\alpha + (1-\alpha)(1-\beta)] + (1-d_0)(1-\alpha)(1-\beta)\}. \quad (7.3.2a)$$

Allen and Gorton then defined the specialist's ask price as

$$A_1 = d_1^A V_{11} + (1 - d_1^A)V_L. \quad (7.3.2b)$$

Similarly in the case of a seller, the specialist's knows that the seller may know the expected stock price is V_L such that

$$d_1^B = d_0(1-\alpha)\beta / \{d_0(1-\alpha)\beta + (1-d_0)[\alpha + (1-\alpha)\beta]\} \quad (7.3.2c)$$

and

$$B_1 = d_1^B V_H + (1 - d_1^B)V_L. \quad (7.3.2d)$$

Following the trade, sale or purchase, the specialist then changes expectations from d_0 to either (7.3.2*a*) or (7.3.2*c*) depending on whether the trade is a purchase or a sale. The authors went on to show that profitable manipulation is not possible when the model is symmetric. The suggested strategy is a { buy, buy, sell, sell } where the manipulator buys stock in period dates 1 and 2 that drives up the ask and bid prices but which is not driven by liquidity purchases. These positions ".... can then be unwound without moving the bid price down by very much because there are many liquidity sales". Allen and Gorton further discussed and illustrated a situation where asymmetry is derived from short sale constraints and ill-informed sellers. They concluded that it is possible to make profits from manipulation in such circumstances. Short sales are a well established trading strategy in asset markets. In such circumstances, it is clearly the case that the observed stock price, which forms part of the overall stock returns, will be inconsistent with that implied by consumption-based and profit-based models. In the longer term, such manipulation of stock prices should revert to a path consistent with economic fundamentals. That manipulation exists is contrary to the wishes of policymakers who have always sought ways to eliminate such stock manipulation. The reasons for this include possible loss of market confidence by participants, not least the less informed. Attempts to outlaw insider trading around the world is a step in the direction of eliminating such manipulation.

The presentations herein outlined provide us with reasons for the relatively low correlation between actual and observed returns and these are based around the notion that capital markets, the source of much of the observed data, are not always efficient to the extent that stock return estimates may be misleading or inconsistent with underlying state variables in the short term. The reasons for these distortions includes over-reacting to news, good or bad, the agency problem that sees a divergence between the interests of managers and shareholders and the much discussed "herd instinct" prevalent in the capital markets. The "herd instinct" sees a market specialist assessing their performance relative to other market specialists or to the general stock market indices. Where other market specialists react negatively to some shock, other participants are also likely to react in the same way on the basis that their performance should not deviate from that of other market specialists. Implicitly, this means that a market specialist prefers to make bad decisions as long as other specialists are making the same bad decisions rather than deviate from the general market sentiment and perform differently, either better or worse. This is the "there is safety in numbers" approach to investment. The investment behaviour so far described does not in any way indicate profit maximising behaviour. In fact, this behaviour contributes to the previously identified phenomenon of "overreaction" where market specialists overreact to news; good or bad. There is now substantial evidence to show that capital markets overreact. De Bondt and Thaler (1985) and Chopra et al (1992) provide some evidence of this. In the medium to long term, the market reverts to a path consistent with underlying state variables.

One way of testing out this hypothesis of capital market distortion is simply to calculate averages and moving averages of predicted asset returns on the basis that if there exists inefficiency in the capital markets, to the extent that stock prices may not fully reflect all available information, then such price inefficiencies would be remedied in the near term. This is more likely where the source of stock price inefficiency is either the result of stock price manipulation or an overreaction to shocks. Table 7.3 reports results for 2 and 3 year averages and moving averages from the models presented in earlier chapters. For the consumption model, Model 1 is the original surplus consumption model as per Campbell and Cochrane (1995) in chapter 3 whilst Model 2 is that which implies a slow moving habit in chapter 4. For the production model, Model 1 is the profit-based model presented in chapter 6.

Table 7.3

Panel A

Correlation between actual and predicted stock returns

	2-year Moving Averages		3-year Moving Averages	
	Consumption	Production (Profit-based)	Consumption	Production (Profit-based)
Model 1	0.269	0.520	0.255	0.585
Model 2	0.163	-	0.178	-

Source: Author's calculations using data on asset returns, consumption and profit 1919-1991

Table 7.3

Panel B

Correlation between actual and predicted stock returns

	2-year Averages		3-year Averages	
	Consumption	Production (Profit-based)	Consumption	Production (Profit-based)
Model 1	0.344	0.542	0.286	0.623
Model 2	0.281	-	0.190	-

Source: Author's calculations using data on asset returns, consumption and profit 1919-1991

Overall, the results show the correlation between actual and predicted stock returns to be higher than previously found. Model 1 for the profit-based model works shows 2-year moving average coefficients approaching 0.60 and the 3-year average of 0.62. This is marked increased in the correlation values reported in chapters 4, 5 and 6.

7.4. Linking production and consumption models

Given the formulations for the consumption and production models presented in chapters 4 and 6, it is now possible to combine the formulations in a full equilibrium framework. Whilst it is beyond our scope to fully evaluate this possibility here, we bring together the relevant equations from earlier chapters.

Equating equations (4.4.6) and (6.1.4),

$$
\begin{aligned}
r_{i,t+k} &= -\ln(\delta) + \gamma g - \gamma(1 - \phi_{t+k})(s_t - n) - \frac{1}{2}\left[\sigma_i^2 - 2\gamma\sigma_{iv}^2 + \gamma^2\sigma_v^2\right](1 + \lambda_c(s_t))^2 \\
&= \lambda_{\pi,t+k}\left[\pi_{t+k} - E_{t+k-1}(\pi_{t+k})/E_{t+k-1}(\pi_{t+k})\right] + \upsilon_{t+k} \qquad (7.4.1)
\end{aligned}
$$

where δ is the consumer's subjective discount rate, γ is the coefficient of relative risk aversion, g is the mean growth rate of consumption, s_t is log surplus consumption, n is the steady state surplus consumption ratio, ϕ is a serial correlation parameter, λ_c is the sensitivity function relating to consumption and σ represents the standard deviation where v is the consumption shock defined as a deviation from the mean growth rate of consumption. For the profit formulation, λ_π is the sensitivity of stock returns to deviation of actual from expected profit and υ is an error term.

To further evaluate (7.4.1), it is perhaps useful to further define profits in terms of its state variables where $\pi = f(y)$ i.e. profit is a function of output (y).

Profit can be directly related to output via

$$\pi_{t+k} = [p(y)]_{t+k} - [h_1(y)]_{t+k} - [h_2]_{t+k} + \varepsilon_{t+k} \qquad (7.4.2)$$

where p is the selling price of output, h_1 is the direct (variable) production cost of the output, h_2 is other operating (fixed) costs not related to the level of output. Furthermore, this formulation can be linked to investment decisions using a framework similar to that outlined by Cochrane (1991). According to Cochrane, stock returns equals investment returns (from 1.3.2j), r^I_{t+k}, which reads

$$r^I_{t+k} = (1-\rho)\left(mpk_t + \frac{1+\alpha(I_{t+k}/K_{t+k})^3}{1-(3/2)\alpha(I_{t+k}/K_{t+k})^2}\right)\left(1-\frac{3}{2}\alpha\left(\frac{I_t}{K_t}\right)^2\right) \qquad (7.4.3)$$

where I is investment, K is the capital stock, mpk is the marginal product of capital and ρ and α are capital adjustment cost and depreciation parameters respectively.

Whilst our results (Chapter 5) from testing the Cochrane (1991) focussed on whether stock and investment returns are equal, the derivation of the investment returns provides a potentially useful formulation in relating profit and therefore stock returns to output, investment and capital decisions and thereby fully link consumption to production.

7.5. Final Thoughts

With regard to our research questions, we can conclude that

- Firstly, the power utility model incorporating habit consumption can be made to yield parameters for risk aversion consistent with other known parameter values. This we find is only possible when the stock return is viewed in terms of its predictable element (stock income) and less predictable element (stock price). We conclude that the stock price is more closely related to surplus consumption and stock income to habit consumption. Implicitly, attempts to work with overall consumption and stock returns are not deemed to be a success.

- Secondly, we find that there are models capable of explaining the stock return and consequently the stock premium. These models, in chapters 4 and 6, are also capable of doing so in the context of known parameter values for risk aversion, the discount rate, price of risk and the risk-less return in the case of the consumption model and the profit and stock price reaction parameters for the production related model.

- Thirdly, we find it possible to link both the consumption and production asset pricing models via stock prices and stock returns. In the consumption-based model, we use stock prices as an indicator of wealth that helps determine consumption whilst in the production model, stock prices are derived using profit as well as profit and stock reaction parameters. Hence, stock prices are viewed as having a role in consumption and production models.

Much of the literature reviewed implied an Arrow-Debreu frictionless economy in the traditional neo-classical economy. In such an economy, asset pricing is efficient in the sense that all available information is reflected in period prices. However there now exists substantial evidence to show that markets are less than efficient at all times. Our results show that even when the model is able match the observed stock return and associated stock premium, the associated value for the risk aversion is well above the value of 1 thought consistent with real business cycle neo-classical theory. In our view, that the risk aversion coefficient value is not around 1 is caused by short term market irrationality which have already been discussed. There are not many other plausible explanations. In other words, market irrationality increases the risk aversion. Neo-classical models as represented by real business cycle theory often assume that economic fluctuations are caused by exogenously determined shocks and that perfect markets lead to innovation. Clearly, however, market failure exists. More recently, events in Asia demonstrate this. It is difficult to believe that one of the causes of the crisis i.e. the parlous financial state of Japanese financial institutions, is any worse today compared to say a year ago. However the sudden reaction of financial markets would seem to suggest that the perceived problem is sudden and unexpected. In a perfectly efficient market, market reaction should have been much more gradual thus avoiding a sudden shock.

Our model does carry implications for macroeconomic modelling and forecasting. In our discussions, use was made of representative models where individuals are aggregated into a single representative entity. The same is often done with firms but there are clearly limitations to this, and as Stiglitz (1991) points out, this is particularly so in the presence of liquidity constraints which tend to be non linear. The neo-classical preference for aggregation assumes that all market participants react in the same way and that everybody knows how everybody else will react. This is clearly subject to some doubt.

The speed of the adjustment process also presents us with another area in which the neo-classical approach could prove misleading for it fails to recognise that some variables adjust more quickly than others. As well as being able to understand the economic dynamics, accounting for adjustment speed can help in assessing the short run consequences of policy changes in policy. In the absence of adjustments costs and in the presence of perfect information, a variable is considered to be in equilibrium to the extent that the variable is at the level it should be given the above conditions. However, since information is not costless, it is possible for prices to reflect the views of the informed such that arbitrage is possible.

Neo-classical economics also argues that government actions have no real effect on the economy and recent evidence comes from Japan where the government have spent the last few years introducing measures to stimulate the economy to no avail. There is however, substantial evidence to the contrary which suggests that governments can indeed influence the economy though perhaps not in the intended manner.

In the context of the labour market for example, the key pointers which indicate an inability of neo-classical economics to explain events would focus on the unemployment - real wage debate. Much of the recent fall in unemployment in the UK and USA has been achieved in the presence of ever higher wage rates. A vertical supply curve does not seem to fully answer the question and neither does the notion that supply shifts are matched by demand shifts such that they cancel out one another. More recent theories on the labour market has focused on other explanations such as the insider-outsider theory discussed by Lindbeck and Snower (1986), where insiders have a relatively strong bargaining position which they use to keep their wages relatively high and prevent the hiring of new outsiders. Since the insiders are aware of the costs associated with hiring new workers, they are able to make calculated demands on the firm knowing that a rational firm will continue to employ the insiders. Efficiency wage theories suggest that equilibrium is consistent with high and varying levels of unemployment since a worker's productivity is affected by the wages they receive. In these circumstances, to reduce wages would lead to even greater falls in productivity. Hence, relatively high wages remain even in the presence of unemployment contrary to market-clearing expectations. More recently, a number of studies have tended to view employment as an investment decision especially in the context of skill shortages. Consequently, employment decisions are evaluated as such.

The neo-classical view of capital market also suggests that money should have no real effects such that by extension, the state of public finances should be irrelevant. The gyrations caused by the US budget deficit in the 1980s provides a basis to disagree. Heaton and Lucas (1992) did incorporate incomplete markets and trading costs in an explanation of asset returns in the context of a consumption-based model. They concluded that "with trading costs or binding borrowing constraints, the risk-free rate falls and the risk premium rises relative to the complete markets case, and the term structure exhibits a positive forward premium". In our context, such a model which incorporates market imperfections is able to show patterns more consistent with the observed patterns.

The product markets also provide a quandary not least given the failure to explain the 1973/4 oil price shock. Failure to fully explain output changes also further undermines the neo-classical case though this has been reflected in more recent work which has attempted to focus on incomplete markets and imperfect information.

This is not to say neo-classical theories are not capable to telling us anything about the macro economy, rather they are likely to tell us about how it should be rather than how it is. Furthermore, this is not to fully accept models which seek to incorporate market imperfections which are also liable to giving explanations not consistent with the evidence. For example, models which suggest that presence of trade union is a major cause of labour market imperfection have still the align this view with the very limited and declining coverage of trade unions in the economy. Also with the labour market, the implicit contract theory which argues that observed real wages have very little to do with current economic activity is still not an explanation of wage rigidity or the pattern and form of unemployment. Furthermore, that there exists market imperfections is not automatically a case for government intervention because although government actions can have an effect, the effect might not be as intended. Essentially, the explanations that link stock markets and the real economy via consumption should seek more realistic models for even though the existing models are able to report parameter estimates similar to the actual data, the possibility exists, as in our model to explain the stock premium, that inferences drawn will be wrong. Despite the fact that our model is able to produce a mean and standard deviation similar to that found in the data, the relatively low positive correlation gives an indication of this possibility. The clear implication from all that is said so far is that markets do not always clear and that being the case, macroeconomics has to take account of imperfection in the economy. These imperfections are present in all markets, labour, capital and product and can be caused by imperfect information, transaction costs and general rigidity in the respective markets. In other words, economists, of any description, should run away from seemingly easy solutions that imply that the world is a perfect place.

7.6. Potential areas for research

One of the uncertain elements in all that has been presented so far concerns the data. Much of the work reviewed is based around USA data whilst much of that done by the author is based on UK data. Perhaps, there needs to be a step back before any further forward steps to ascertain the differences and similarities in the data. The Instrumental Variables (IV) estimation in chapter 3 shows results using UK and USA data which are contradictory to the extent that the signs of the parameter estimates are different as are the levels of significance in the tests for over-identifying restrictions even though the mean and standard deviations might be similar. More particularly, the covariance and between asset returns and consumption growth found with USA data is about 10 times that found with UK data. This is a source of much thought especially since the mean stock returns are quite similar.

There also exists a need to explicitly model habit which is not only positive but incorporates stock dividends as part of the habit formation process in that they are predictable or certainly far more predictable than the stock price could ever be. The standard deviation of the stock income return is only about a third of that for the stock price. The clear implication of this is that the coefficient of risk aversion required to explain the stock income return is likely to be higher than with the stock price. Evidence has already been presented which suggests that models which separate stock income from the stock price are more likely to yield reasonable results. In fact models considered incapable of explaining the stock premium, i.e. power utility models, are suddenly able to much better explain it.

Finally, though not least concerns a formulation which explicitly deals with production and consumption in the same formulation to yield parameter values consistent with the observed data. It is something of a challenge to define the technology and then for the model to yield reasonable values for the relevant parameters but also to explain the observed equity premium and to achieve a reasonable level of positive correlation between the predicted and actual stock returns. Such a model whilst linking consumer and producer behaviour should also fulfil the condition of each aspect of the model by being able to individually explain the observed equity premium.

The sensitivity function used in the consumption and stock reaction parameters in the profit-based asset pricing model were assumed to be constant. It would be interesting to see whether a consumption-based formulation can be found which represents the economic state. Preliminary work in the profit formulation already reveals a much higher correlation between actual and predicted asset returns.

Finally, there remains a need to relate our profit function in chapter 6 to state variables, investment, capital stock and production via production functions.

APPENDICES

Appendix One (1)

Instrumental Variables Estimation

There now follows a discussion of IV(2SLS) estimation in the context of a GMM framework to be discussed later.[50]

Equation (3.2.5) can be written in the unconditional form for convenience to obtain

$$r_i = \mu_i + \gamma_i \Delta c_t + u_t \qquad (A.1.1)$$

where

- r_i is an $N \times 1$ vector with N observations of a dependent variable.
- Δc_t is an $N \times K_c$ matrix with N observations of the independent variable.
- γ_i is a $K_c \times 1$ parameter vector and
- $u(\gamma_i)$ is an $N \times 1$ vector with N observations of the error term.

If the error term, $u(\gamma_i)$, is assumed to be serially uncorrelated with the regressor and homoskedastic, with variance σ^2, then the variance of $u(\gamma_i)$ is $\sigma^2 I_N$ when I_N is an identity matrix. If however we assume that the error term, $u(\gamma_i)$, is serially correlated with the regressor Δc_{t+1}, then instruments have to be sought that can produce unbiased estimates for Δc_{t+1} which can be used as the regressor in the main equation.

The instruments chosen here are the lagged variables of the main variables, real consumption growth and real asset returns since lagged variables are uncorrelated with the error term $u(\gamma_i)$.

---

[50] The presentation follows that by Campbell, Lo and Mackinlay and no doubt benefits from it as well.

The instruments are represented here by a N x K_L matrix containing N observations and K_L number of instruments where the instruments are assumed to be serially uncorrelated with the error term such that

$$\mathrm{E}[L_t u(\gamma_t)] = 0. \qquad (A.1.2)$$

This is the orthogonality condition used by the IV regression to estimate the model. In this formulation, the residual of the regression will be defined by

$$u_t(\gamma) = r_i - \Delta c_t \gamma. \qquad (A.1.3)$$

This means that there will exist a K_L x 1 column vector of the cross product of the instrument vector with the residual represented by

$$f_t(\gamma) \equiv L_t' u_t(\gamma) \qquad (A.1.4)$$

since a perfect instrument will yield similar residuals to the original regressor. The expectations of this cross product is a vector of zeros at the true value such that

$$\mathrm{E} f_t(\gamma_t) = 0 \qquad (A.1.5)$$

but since we do not know the true value of f, the aim must be to chose instruments that come as close as possible to satisfying (A.1.5) using the sample mean of f denoted by $g_N(\gamma)$ such that we can define

$$g_N(\gamma) \equiv N^{-1} \sum_{t=1}^{N} L_t' u_t(\gamma) = N^{-1} L' u(\gamma). \qquad (A.1.6)$$

This equation says that the sum of the cross products of the error term divided by the number of observations yields g_N, the sample mean of f. The point is that in our instrumental variables estimation, $g_N(\gamma)$, the sample mean, is used since the true value is unknown. Essentially, the problem is a minimisation of the quadratic form

$$Q_N(\gamma) \equiv g_N(\gamma)' W_N \, g_N(\gamma), \qquad (A.1.7)$$

where $Q_N(\gamma)$ is a weighted sum of cross products of sample average products of different instruments with the residual, W_N is an K_N x K_N weighting matrix. Given (A.1.6) then

$$Q_N(\gamma) = \left[N^{-1} u(\gamma)' L \right] W_N \left[N^{-1} L' u(\gamma) \right]. \qquad (A.1.8)$$

Instrumental variables regression seeks a value of γ to minimise the quadratic form $Q_N(\gamma)$. For simplicity, Δc equals C.

The first order condition for the minimisation problem is

$$C'L\, W_N L'r = C'L\, W_N L'C(\tilde{\gamma}). \qquad (A.1.9)$$

If the number of instruments, L, equals the number of parameters to be estimated, then the minimisation gives

$$\tilde{\gamma} = (L'C)^{-1} L'r. \qquad (A.1.10)$$

Since the number of instruments equals the number of estimates, i.e. $K_L = K_u$, the orthogonality conditions are perfectly satisfied such that the estimates are independent of the weighting matrix. This is exactly the same as the OLS regression that would have been run in the absence of the instruments.

However in our analysis, the instruments exceed the number of estimates i.e $K_L > K_c$, such that the model is over-identified. In this case, the weighting matrix is used to give weights to the various instruments being used. The solution is therefore

$$\tilde{\gamma} = (C'LW_N L'C)^{-1} C'LW_N L'r. \qquad (A.1.11)$$

Substituting (*A*.1.11) into (*A*.1.1) and re-arranging, leads to

$$N^{-\frac{1}{2}}(\tilde{\gamma} - \gamma_i) = (N^{-1}C'W_N N^{-1}L'C)^{-1} N^{-1}C'LW_N N^{-\frac{1}{2}}L'u(\gamma_i). \qquad (A.1.12)$$

So far, the error term is assumed to be serially uncorrelated and homoskedastic such that the term in (*A*.1.12) converges to a normal distribution with mean zero and variance $\sigma^2 M_{LL}$ where M_{LL} is a non singular matrix. As with OLS, then

$$N^{\frac{1}{2}}(\tilde{\gamma} - \gamma_i) \xrightarrow{d} N(0, V) \qquad (A.1.13)$$

$$\text{with Variance}, V = \sigma^2 (M_{cL} W M_{LC})^{-1} M_{CL} W M_{LL} W M_{LC} (M_{CL} W M_{LC})^{'-1} \qquad (A.1.14)$$

To minimise the asymptotic variance matrix, *V*, a value for the weighting matrix is chosen such that $W^* = (\kappa M_{LL})$ where κ is any positive scalar. It can be seen from (*A*.1.12) that this weighting matrix W^* is proportional to the inverse of the asymptotic covariance matrix of $N^{-\frac{1}{2}}L'u$. The basic idea is to place more weight on precise orthogonality conditions and less on noisy ones.

The weighting matrix could therefore be

$$W_N^* = (\kappa_N N^{-1} L'L)^{-1} \qquad (A.1.15)$$

implying that as κ_N converges towards κ, the true value, then W_N will converge towards $W^* = (\kappa M_{LL})^{-1}$ as N, the number of observations, increases.

As a result,

$$\tilde{\gamma}^* = \left[C'L(L'L)^{-1}L'C\right]^{-1}C'L(L'L)^{-1}L'r. \qquad (A.1.16)$$

This is equivalent to

$$\tilde{\gamma}^* = \left(\hat{C}'\hat{C}\right)^{-1}\hat{C}'r \qquad (A.1.17)$$

where

$$\hat{C} = L(L'L)^{-1}L'C.$$

This is the two stage least squares method where the first regression is a regression of C on the instruments L and then a second regression of r on the fitted value of C from the first regression.

As a choice for the scalar κ, the variance of the error term u, σ^2, can be used such that the scalar κ_T is then any consistent estimate of σ^2. However this choice of estimate is problematic in that to get a consistent estimate for σ^2, an initial estimate of γ_i, is needed. However N times the minimised objective function is asymptotically χ^2 with $(T_L - T_c)$ degrees of freedom under the null hypothesis that (A.1.1) holds. With the estimated value of σ^2, then

$$W_T = \left(\hat{\sigma}^2 N^{-1}L'L\right)^{-1}. \qquad (A.1.18)$$

The objective function to be minimised is

$$\left[N^{-1}u(\tilde{\gamma}^*)'L\right]\left(\hat{\sigma}^2 N^{-1}L'L\right)^{-1}\left[N^{-1}L'u(\tilde{\gamma}^*)\right] \qquad (A.1.19)$$

Given that the instruments exceed the number of parameters being estimated, then a test of the over-identifying restrictions is needed and for which the test devised by Engle (1984) is used. In this standard test of over-identifying restrictions, N times the R^2 in a regression of the residuals from the second stage regression on the instruments is asymptotically $(T_L - T_c)$ degrees of freedom. Since the asset returns might be correlated with the error term, the choice of instruments for the estimation can be the lag of any variables including our initial variables.

Appendix Two (2)

Power Utility and the Generalised Method of Moments

So far, much of the focus has been on the loglinear model of the CCAPM. However, using Hansen's (1982) Generalised Method of Moments (GMM) makes it possible to test the power utility model without making assumptions about the distribution of the data. Hence the linear assumption of the traditional power utility model is abandoned.

For an exposition of the GMM, we revisit the model outlined in (*A*.1.1) where

$$r_{it} = \mu_i + \gamma_i \Delta c_t + u_{it}. \qquad (A.2.1)$$

Since, C is used to denote Δc_t, the residuals, *u*, can be defined as a vector $u(C_{t+1}, \gamma)$ where C_{t+1} now includes all relevant data such that no distinction is made between *r* and C and γ is a vector of K_γ coefficients.

This formulation is a more general case of the instrumental variables regression because firstly, (*A*.2.1) can be a column vector with K_u elements rather than a scalar; and secondly, it can be non linear as opposed to the linear specification of the instrumental variables estimation.

Given (3.1.13), *u* would be a vector with elements

$$\left(\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} Z_{i,t+1} - 1\right) \qquad (A.2.2)$$

for assets i=1.....N_u. As previously outlined, there is a 1 x K_L row vector of instruments L_t and that the model implies $u(C_{t+1}, \gamma)$ is orthogonal to the instruments such that

$$f_{t+1}(\gamma) \equiv L_t' \otimes u(C_{t+1,}\gamma) \qquad (A.2.3)$$

where γ_t is the true parameter vector and f_{t+1} is the cross product of each element of u with each instrument. Implicitly, the elements of f would be determined by

$$\left(\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} Z_{i,t+1} - 1\right)L_{jt} \qquad (A.2.4)$$

for all assets i=1....K_N and instruments j=1..K_c such that f is a column vector with $N_f = N_u\ N_L$ elements. In the context of the aforementioned instrumental variables model, the equivalent of (A.1.6) is therefore

$$\mathrm{E}f_{t+1}(y_t) = 0. \qquad (A.2.5)$$

As with the IV analysis, the equivalent for (A.1.6), the sample averages can then be defined as

$$g_N(\gamma) = N^{-1}\sum_{t=1}^{N} f_{t+1}(\gamma). \qquad (A.2.6)$$

The quadratic form to be minimised is therefore

$$Q_N(\gamma) = g_N(\gamma)'\ W_T\ g_N(\gamma). \qquad (A.2.7)$$

The problem is now a non linear one such that the minimisation must be done numerically where the first order condition is

$$G_N(\gamma^*)'W_T\ g_N(\gamma^*) = 0, \qquad (A.2.8)$$

where $G_N(\gamma)$ is a matrix of partial derivatives with its *(i,j)* elements determined by $\partial g_{Ni}(\gamma)/\partial\gamma_j$. The large sample properties dictate that the coefficient estimate

$$\gamma^* \quad \text{tends to} \quad N^{\frac{1}{2}}(\gamma^* - \gamma_i) \xrightarrow{d} N(0, V_G) \qquad (A.2.9)$$

where

$$V_G = (G_0' W G_0)^{-1} G_0' W S_W W G_0 (G_0' W G_0)^{-1}. \qquad (A.2.10)$$

G_0 is equivalent to $E\partial f(C_{t+1},\gamma_i)/\partial\gamma_i$ which is equivalent to M_{LX} in the instrumental variables model. S_W is the variance covariance matrix of the time average of $f_{t+1}(\gamma)$ and is equivalent to

$$\underset{n\to\infty}{Lim}\, \mathrm{E}\left[N^{-1}\left(\sum_{t=1}^{N} f_{t+1}(\gamma_i)\right)\left(\sum_{t=1}^{N} f_{t+1}(\gamma_i)'\right)\right]. \qquad (A.2.11)$$

which can be rewritten as

$$\lim_{N\to\infty} \mathrm{E}\left[f_{t+1}(\gamma)f_{t+1}(\gamma_i)' + \sum_{j=1}^{N-1}\left(\frac{N-J}{N}\right)\left(f_{t+1}(\gamma_i)f_{t+1-j}(\gamma_i) + f_{t+1}(\gamma_i)f_{t+1-j}(\gamma_i)'\right)\right]$$

$$(A.2.12)$$

This is the same as that derived by Campbell, Lo and Mackinlay (1997). The model $u(C_{t+1}, \gamma_i)$ can be further simplied if we assume that the process is white noise such that S_w is the variance of $f_{t+1}(\gamma_i)$. As with the IV model, there is an optimal weighting matrix $W^* = S_{w'}^{-1}$. As a consequence, the coefficient estimate of γ has asymptotic covariance matrix

$$V_G^{*} = \left(G_0' S_W^{-1} G_0\right)^{-1}. \qquad (A.2.13)$$

With this optimal weighting matrix, the objective function to be minimised is distributed χ^2 with $(K_f - K_\gamma)$ degrees of freedom, where K_f is the number of orthogonality conditions and K_γ is the number of parameters being estimated.

The estimation starts with an arbitrary weighting matrix and (A.2.7) is minimised to get an initial consistent estimate γ^*. Furtheron, we estimate (A.2.10) by continually replacing its elements by consistent estimates such that G_0 is replaced by the estimate of $G_N(\gamma^*)$, W by W_T and S_W by $S_{WT}(\gamma^*)$. Using these estimates, a new weighting matrix can then be constructed such that W_T^* $= S_{WT}(\gamma^*)^{-1}$. A second stage estimation can be carried out to yield a second stage estimate, γ^{**}. The asymptotic variance of this second stage estimation would be

$$V_{GN}^{*} = \left(G_N\left(\gamma^{**}\right)' S_{WT}\left(\gamma^{**}\right)^{-1} G_N\left(\gamma^{**}\right)\right)^{-1}, \qquad (A.2.14)$$

with the objective function distributed χ^2 with $(K_f - K_\gamma)$ degrees of freedom.

Appendix Three (3)

BZW Equity Index : Constituents

Constituents at December 1918	Constituents at December 1945	Constituents at December 1962
Associated Cement	Associated Portland Cement	Associated Portland Cement
Bass	Bass Ratcliff	Bass Mitchells and Butlers
Rover	Austin Motors	British Motor
J. and P. Coats	J. and P. Coats	Coats Patons
Powell Duffryn	Bolsover Colliery	Cory (William)
Courtaulds	Courtaulds	Courtlands
Distillers	Distillers	Distillers
Dunlop	Dunlop	Dunlop
Gramaphone Co. (EMI)	EMI	EMI
Fine Spinners	Fine Spinners	Fine Spinners & Doublers
General Electric	General Electric	General Electric
Guest Keen	Guest Keen	Guest Keen
Armstrong Whitworth	Hawker Siddeley	Hawker Siddeley
Harrods	Harrods	House of Fraser
Explosives Trades (ICI)	Imperial Chemical	ICI
Imperial Tobacco	Imperial Tobacco	Imperial Tobacco
Maypole	International Stores	International Stores
Dorman Long	Dorman Long	Leyland Motors
Neuchatel Asphalte	London Brick	London Brick
Mond Nickel	Murex	Murex
Bradford Dyers	Patons & Baldwins	P & O Steam Navigation
Rolls - Royce	Rolls - Royce	Rolls - Royce
Spillers	Pinchine Johnson	Swan Hunter
Wall Paper	Tate & Lyle	Tate & Lyle
Callender's Cables	Callender's Cables	Tube Investments
Bells United Asbestos	Turner and Newall	Turner and Newall
United Steel	United Steel	United Steel
Vickers	Vickers	Vickers
Newcastle Breweries	Watney Combe Reid	Watney Mann
Maple	Woolworth	Woolworth

BIBLIOGRAPHY

Abel, A. B., 1990, "Asset Prices under habit formation and Catching up With the Joneses," *American Economic Review*, 80, 38-42.

Abel, A. B., 1991, "The Equity Premium Puzzle," *Federal Reserve Bank of Philadelphia - Business Review* (September/October), 3-14.

Aiyagari, S. R., and M. Gertler, 1991, "Asset Returns with Transactions Costs and Uninsured Individual Risk," *Journal of Monetary Economics*, 27, 311-331.

Allen, F., and G. Gorton, 1992, "Stock price manipulation, market microstructure and asymmetric information," *European Economic Review*, 36, 624-30.

Attansio, O. P., and G. Weber, 1989, "Intertemporal substitution, risk aversion and the Euler equation for consumption," *Economic Journal*, 99, 59-73.

Barclays De Zoete Wedd, 1992, "The BZW Equity - Gilt Study 1992," Barclays De Zoete Wedd Research Limited (January).

Barr, D. G., and B. Pesaran, 1995 "An Assessment of the Relative Importance of Real Interest Rate, Inflation, and Term Premiums in determining the Prices of Real and Nominal U.K. Bonds," Bank of England, Working Paper Series No. 32.

Barr, D. G., and J. Campbell, 1995, "Inflation, Real Interest Rates, and the Bond Market: A study of UK Nominal and Index-Linked Government Bond Prices," Discussion Paper 1732, Harvard Institute of Economic Research, Harvard University.

Basu, P., and H. D. Vinod, 1994, "Mean Reversion in Stock Prices: Implications from a Production Based Asset Pricing Model," *Scandinavian Journal of Economics*, 96, No. 1, 51-65.

Blake, D., 1990, *Financial Market Analysis*, McGraw Hill London.

Blanchard, O. J., and S. Fischer, 1989, *Lectures on Macroeconomics*, MIT Press.

Breeden, D. T., M. R. Gibbons, and R. H. Litzenberger, 1989, "Empirical Tests of the Consumption Orientated CAPM," *Journal of Finance*, 44, No 2, 231-261.

Campbell, J. Y., 1993, "Intertemporal Asset Pricing without Consumption data," *American Economic Review*, 83, 487-512.

Campbell, J. Y., and J. Ammer, 1993, "What Moves the Stock and Bond Markets? A Variance Decomposition for Long-Term Asset Returns," *Journal of Finance*, 48, 3-37.

Campbell, J. Y., and J. H. Cochrane, 1995, "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behaviour," Unpublished paper, Harvard University and University of Chicago.

Campbell, J. Y., A. Lo, and A C. Mackinlay, 1997, "*The Econometrics of Financial Markets*," Princeton University Press.

Campbell, J. Y., and R. J. Shiller, 1991, "Yield Spreads and Interest Rate Movements: A Bird's Eye View," *Review of Economic Studies*, 58, 495-514.

Cecchetti, S., P. Lam, and N. Mark, 1993, "The Equity Premium and the Risk-Free Rate: Matching the Moments," *Journal of Monetary Economics*, 31, 21-45.

Chopra, N., J. Lakonishok and J. Ritter 1992, "Measuring abnormal performance: Do stocks overreact?," *Journal of Finance*, 47, 1579-1593.

Christiano, L. J. and M. Eichenbaum, 1995, "Liquidity Effects, Monetary Policy and the Business Cycle," *Journal of Money, Credit & Banking*, 27, No. 4 (Part 1), 1113-1136.

Clare, A. B., M. Maras, S. H. Thomas, 1995, "The Integration and efficiency of international bond markets," *Accounting and Business Research*, 25, 313-322.

Cochrane, J. H., 1991, "Production-Based Asset Pricing and the Link Between Stock Returns and Economic Fluctuations," *Journal of Finance*, 46, No.1, 209-237.

Cochrane, J. H., 1992, "A Cross Sectional Test of a Production-Based Asset Pricing Model," NBER Working Paper Series No 4025 (March).

Constantinides, G. M., 1990, "Habit formation: A resolution of the Equity Premium," *Journal of Political Economy*, 98, No. 3, 519-543.

Cooley, T.F., and M. Ogaki, 1991, "A Time Series Analysis of Real Wages, Consumption, and under Optimal Labor Contracting : A cointegration-Euler Approach," RCER Working Paper No. 285, University of Rochester.

Cooley, T.F., and M. Ogaki, 1996, "A Time Series Analysis of Real Wages, Consumption, and Asset Returns," *Journal of Applied Econometrics*, 11, 119-134.

Cuthbertson, K., S. Hall and M. Taylor, 1992, *Applied Econometric Techniques*, Philip Allan (London).

DeBondt, W., and R. Thaler, 1985, "Does the stock market overreact?," *Journal of Finance*, 40, 793-805.

Dimson, E., and R. Brealey, 1978, "The Risk Premium on UK Equities," *The Investment Analyst*, (December), .

Dimson, E., and P. Marsh, 1988, "Cases in Corporate Finance" John Wiley & Sons (Chichester).

Dimson, E., and P. Marsh, 1988, Instructor's Guide to Cases in Corporate Finance, London Business School.

Elston, J. A., 1993, "Are Capital Markets Perfect? Theory and Evidence from US Panel Data," Unpublished paper (February).

Engel, R., 1984, "Wald, Likelihood Ratio, and Lagrange Multiplier Tests in Econometrics," in Z. Griliches and M. Intriligator (eds.), *Handbook of Econometrics* 2, North-Holland, Amstrerdam, chap. 13.

Epstein, L., and S. Zin, 1989, "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica*, 57, 937-968.

Epstein, L., and S. Zin, 1991, "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: An Empirical Investigation," *Journal of Political Economy*, 99, 263-286.

Fama, E. F., 1990, "Stock Returns, Expected Returns and Real Activity," *Journal of Finance*, 45, No. 4, 1089-1109.

Fama, E. F., 1991, "Efficient Capital Markets: 11," *Journal of Finance*, 46, No. 5, 1575-1618.

Fama, E. F., and K. R. French, 1988, "Dividend Yields and Expected Stock Returns," *Journal of Financial Economics*, 22, 3-25.

Furest, T. S., 1995, "Monetary and Financial Interactions in the Business Cycle," *Journal of Money, Credit and Banking*, 27, No.4 (Part 2), 1321-1353.

Gorton, G., and R. Rosen, 1992, "Corporate Control, portfolio choice and the decline of banking," *NBER Working Paper Series* 4247 (December).

Griner, E. H., and L. A. Gordon, 1993, "Internal Cash Flow, Insider Ownership, and Capital Expenditures: A Test of the Pecking Order and Managerial Hypotheses," *Journal of Business Finance & Accounting*, 22, No. 2, 179-199.

Hall, R. E., 1978, "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence," *Journal of Political Economy*, 86, No. 6, 971-88.

Hansen, L. P., 1982, "Large Sample Properties of Generalised Method of Moments Estimators," *Econometrica*, 50, 1029-1054.

Hansen, L. P. and K. J. Singleton, 1982, "Generalised Instrumental Variables Estimations of Nonlinear Rational Expectations Models," *Econometrica*, 50, 1269-1286.

Hansen, L. P. and K. J. Singleton, 1983, "Stochastic Consumption, Risk Aversion, and the Temporal Behaviour of Asset Returns," *Journal of Political Economy*, 91, 249-268.

Hansen, L. P., K. J. Singleton and M. Ogaki, 1993, "Program for the generalized method of moments (GMM) procedure," Unpublished.

Heaton, J., and D. Lucas, 1992, "The Effects of Incomplete Insurance Markets and Trading Costs in a Consumption-Based Asset Pricing Model," *Journal of Economic Dynamics and Control*, 16, 601-620.

Intriligator, M. D., 1978, Econometric Models, Techniques and Applications, Prentice Hall Europe (London).

Kandel, S., and R. F. Stambaugh, 1990, "Expectations and volatility of consumption and Asset Returns," *Review of Financial Studies*, 3, 207-32.

Kendall, M., and A. Stuart, 1977, "The Advance Theory of Statistics" (4th edn), Charles Griffin, London.

Kennedy, P., 1992, *A Guide to Econometrics*, 3rd Edition. Basil Blackwell (Oxford)

Kocherlakota, N. R., 1990, "Disentangling the Coefficient of Relative Risk Aversion from the Elasticity of Intertemporal Substitution: An Irrelevance Result," *Journal of Finance*, 45, No. 1, 175-190.

Labadie, P., 1995, "Financial Intermediation and Monetary Policy in a General Equilibrium Banking Model," *Journal of Money, Credit and Banking*, 27, No. 4, (Part 2), 1290-1353.

Lakonishok, J., A. Shleifer and R. Vishny, 1994, "Contrarian Investment, Extrapolation and Risk," *Journal of Finance*, 49, No. 5, 1541-1578.

Lindbeck, A., and D. Snower, 1986, "Wage Setting, Unemployment and Insider-Outsider Relations," *American Economic Review*, 76, 235-239.

Mankiw, N. G., and S. P. Zeldes, 1991, "The Consumption of Stockholders and Non-stockholders," *Journal of Financial Economics*, 29, 97-112.

Mehra, R., and E. Prescott, 1985, "The Equity Premium, A puzzle," *Journal of Monetary Economics*, 15, 145-161.

Mehra, R., and E. Prescott, 1988, "The Equity Premium: A solution?" *Journal of Monetary Economics*, 22, 133-36.

Merton, R., 1990, *Continuous-Time Finance*, Blackwell Publishers, Cambridge MA.

Merton, R., 1973, "An Intertemporal Capital Asset Pricing Model" *Econometrica*, 41, No. 5, 867-887.

Mishkin, F., 1990, "What does the term structure tell us about future inflation?," *Journal of Monetary Economics*, 25, 77-95.

Ogaki, M., 1993, "Generalised Method of Moments: Econometric Applications," in G.S. Maddala, C.R. Rao and H.D Vinod (eds.) *Handbook of Statistics, 11: Econometrics*, Elsevier Science Publishers B.V.

Ogaki, M., 1993, "GMM: A User Guide," University of Rochester, Working Paper No. 348 (April), (with accompanying GMM programme by Hansen, Lars P., John Heaton and Masao Ogaki).

Patterson, K., 2000, "An Introduction to Applied Econometrics, a time series analysis," Macmillan.

Pemberton, J., 1993, "The Equity Premium Puzzle: Prospects for a Solution," Discussion paper, 2, No. 17 (December), Department of Economics, University of Reading, Berkshire.

Peng, Y., and H. Shawky, 1997, "Productivity Shocks and Capital Asset Pricing," *International Review of Economics and Finance*, 6, Issue 3, 303-317.

Porteba, J. M., and L. H. Summers, 1988, "Mean Reversion in Stock Prices, Evidence and Implications," *Journal of Financial Economics*, 22, 27-60.

Reitz, T., 1988, "The Equity Risk Premium: A Solution," *Journal of Monetary Economics*, 22, 117-31.

Restoy, F., and M. Rockinger, 1994, "On Stock Market Returns and Returns on Investment," *Journal of Finance*, 49, No.2, 543-556.

Santomero, A., 1984, "Modeling the Banking Firm," *Journal of Money, Credit & Banking*, 16, No. 4 (Part 2), 576-616.

Siegel, J., 1991, "The Real Rate of Interest From 1800-1990: A study of the US and the UK," Working Paper 9-91 (March), Rodney L. White Center for Financial Research, University of Pennsylvannia.

Silver, M., 1992, "Business Statistics" McGraw Hill (London).

Shiller, R., 1990 "The Term Structure of Interest Rates," in B.M. Friedman and F.H. Hahn, *Handbook of Monetary Economics*, 1, Chapter 13, 627-722.

Stiglitz, J., 1991, "Methodological Issues and the New Keynesian Economics," The New Keynesian Macroeconomics Chp3.

Weil, P., 1989, "The Equity Premium Puzzle and the Risk Free Rate Puzzle," *Journal of Monetary Economics*, 24, 401-421.