Multifactor Consumption based Asset Pricing Model of the UK Stock Market: The US Stock Market as a Wealth Reference.*

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Abstract

Here a multifactor model of UK stock returns is developed, replacing the conventional consumption habit reference by a relation that depends on US wealth. Two step Instrumental Variables and Generalized Method of Moments estimators are applied to reduce the impact of weak instruments. The standard errors are corrected for the generated regressor problem and the model is found to explain UK excess returns by UK consumption growth and expected US excess returns. Hence, controlling for nominal effects by subtracting a risk free rate and conditioning on real US excess returns provides an appealing explanation of the equity premium puzzle.

Keywords: Consumption-CAPM, Excess Returns, Generated Regressor, GMM, Habits, Wealth Reference

JEL Classification: C52, E44, G12

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1 Introduction

Traditional Consumption-based Capital Asset Pricing Models (C-CAPM) have a record of poor performance in describing the relationship between returns and consumption growth. One important reason for the failure of C-CAPM is that consumption itself is not a good state-dependent variable. There is now a substantial body of literature that has documented alternative models that attempt to find better indicators, in particular, Campbell (2001) and Campbell and Cochrane (2000). In principle, these models try to augment consumption via the inclusion of habit formation to allow for time varying risk premium such that the consumption growth is sufficiently volatile to effectively explain the co-variation between the real economy and financial markets. Furthermore, asset prices are often normalized by aggregate inflation, even though inflation series prior to 1980 are often viewed as non-stationary or at best exhibit long memory.

This article revisits the explanation of stock performance driven by a consumption habit reference and find that a more appropriate comparator for asset pricing is an external wealth reference. This gives rise to a multi-factor C-CAPM model that is then applied to the UK and compared with extended C-CAPMs based on internal and external consumption habits. Recent experience and empirical work suggests via globalization that stock prices are inter-related, while the dynamics of consumption behaviour is more complicated than the simple habit explanation would have it. In particular, the US market can be regarded as the good proxy for the "world" market. This would suggest that it makes both theoretical and practical sense to draw together, a consumption based and external wealth based explanation of UK asset prices. Given that world interest rates are also highly inter-dependent and a monetary environment driven by the need to control inflation, the rate of return is normalized by a measure of the risk free rate. As a result, for the period considered, the excess return can be viewed as a real rate of return and has statistical properties directly comparable with similar data for the US. The analysis is complimented by a further study applied to quarterly data that permits investigation of recent events.

The rest of this paper is organized as follows. Section 2, contains a review of C-CAPM with consumption habit and the derivation of a generalized two-factor C-CAPM driven by wealth reference. Section 3 considers the methodology used to estimate such models with UK data, section 4 and 5 report the data descriptive statistics and results. Section 6 sets out the conclusions.

2 Consumption based Asset Pricing Models

The conventional C-CAPM theory introduced by Lucas (1978) and Breeden (1979) has been tested extensively on data for both the US and a wide range of other countries. However, the results associated with this research have been largely negative.[1] The failure of the C-CAPM has lead to a range of alternative models intended to solve the problem. For example, Gregoriou and Ioannidis

[1] See Gregoriou, Hunter and Wu (2009) for example.

(2007) have suggested the problem lies in market microstructure effects driven by transaction costs, while Smoluk and Vander-Linden (2004) extend the C-CAPM to take account of a US consumption reference

The various solutions to these empirical puzzles, have attempted to maintain a Constant Rates of Risk Aversion (CRRA) by incorporating habits or referencing current behaviour on past consumption. The notion that rational consumers wish to maintain their consumption position relative to some reference was first considered by Dusenberry (1949). In the context of asset pricing models these ideas were re-visited in a choice theoretic framework by Abel (1990), who has claimed that consumers are creatures of habit, and want to maintain their relative living standards, as measured by their capacity to continue to purchase a basket of consumption goods. Dusenberry describes this as a ratchet effect where by the utility of a current basket is viewed as being relative to the previous basket enjoyed by the household. Or in the aggregate, consumption today is seen relative to consumption in the past. Abel (1990) calls this behaviour an "external habit" or "catching up with the Joneses". In comparison, individual behaviour relative to current per capita consumption is called "internal habit" or "keeping up with the Joneses".

The notion of consumption habits developed by Abel (1990) has received some degree of support. However, the appropriate reference level to be used for comparison by the representative agent is still not easy to determine. More specifically, Campbell (2001) argues that the ratio of consumption relative to average per capita consumption used for habit utility in Abel (1990) can only explain constant risk aversion by an agent, because they prefer a habit function that includes the difference in consumption levels. To this purpose, Campbell and Cochrane (1999) develop a consumption-based model derived from a habit-formation economy, where the consumption-surplus ratio is defined as the extent to which the current level of consumption exceeds habit based consumption. It is this form of consumption reference that can give rise to cyclical variation in expected returns and volatility. Campbell and Cochrane (2000) use this ratio extensively to examine different forms of the CAPM and conclude that the poor performance of the C-CAPM is due to the low unconditional correlation between consumption growth and other state variables such as the price–dividend ratio.

The above results suggest that a state-dependent (conditional, reference level) C-CAPM is likely to perform better than the standard (state-independent or unconditional) C-CAPM. Campbell and Cochrane (1999) suggested that a good state-dependent variable that derives from the external habit-preference model is the log surplus consumption ratio, which is further proved by Li (2001) to perform almost as well as the finite-horizon, linear habit version of the model derived by Campbell and Cochrane (1999). Li (2001) analyses this type of model for the US, while Li and Zhong (2005) provide similar evidence for other national stock markets. Furthermore, Jacobs and Wang (2004) produce similar findings to Campbell and Cochrane (2000) when they add as an extra factor to the C-CAPM, cross-sectional consumption variation to capture the possibility of idiosyncratic risk.

Thus far, the external consumption reference addressed by the types of state-variables considered above is better able to capture time-varying returns, since they eliminate the effect of the representative agent's habit preferences in the model. However, they all neglect the possible inter-relatedness between major world stock markets. It is well-known that the world's major stock markets are at least partially integrated in the globalized economy and there is increasing evidence of common real dynamics (Engsted and Tanggaard, 2004). In particular, since the introduction of cointegration, there has been a vast literature on international co-movement of financial markets and co-movement of economic fundamentals over long sample periods (Engsted and Lund, 1997). The work on market co-movement is largely explored using data for the UK and the US and this has highlighted that the UK market is strongly affected by the US.

Excepting, for the impact of large shocks, the notion that stock prices are inter-related does not seem obvious and once one moves away from effectively functioning highly capitalized markets the evidence in support of cointegration seems thin. However, if the observation of time-varying expected returns from developed countries' equity markets is consistent, with a world, consumption based asset pricing model with habits, then it is also more likely that the utility function of consumers/investors in these countries is also time-varying where the time variation may depend on the performance of the "world" market. Hence, agent decisions on consumption and savings may in turn depend on the world and based on the level of integration of the UK in global capital markets this type of explanation would appear particularly pertinent Here, two factors are considered that might influence a UK based C-CAPM: one is excess returns on the US stock market and the other is the aggregate habit preferences of UK consumers. The former affects the movement of the returns of economic agents in their domestic market and thus influences their decisions about consumption allocations. The latter implies that consumers/investors have to strive to close any gap in living standards in an attempt to maintain their own consumption levels for the preceding period.[2]

However, it may not be appropriate to treat US excess returns as exogenous to UK agent behaviour. Firstly, there are unobserved traits such as shocks that might affect both UK and US series, hence the error sequences will not be independent. Otherwise, one might view US consumption growth as a more appropriate proxy for this variable (see Li and Zhong, 2005). However, Gregoriou, Hunter and Wu (2009) suggest that although the US stock market is affected by the real domestic economy, this effect is dominated by the reverse impact of stock market windfalls on US consumption growth. Hence, US excess returns might be consumption based, but the relationship is interdependent and as a result US consumption growth is not an appropriate proxy for US wealth. Also given the timing differences in the opening of the two markets, it would appear more pertinent to explain UK excess returns by the expectation of US returns or some sort of long-term average. In what follows the C-CAPM is considered with both habit and external wealth references.

[2] Smoluk & Vander-Linden (2004) test for an international version of C-CAPM with local consumption catching up with that of American, but the poor performance of this model suggests that consumption across countries is not correlated.

2.1 Consumption-CAPM

The conventional C-CAPM theory relates asset prices to the economic agent's consumption and portfolio decisions over time.[3] The asset pricing model follows from maximizing agent utility over time:

$$E_t\left[\sum_{k=0}^{\infty}\beta^k U(C_{t+k})\right]$$

subject to the intertemporal budget constraint

$$W_{t+1}=(W_t-C_t)\sum_{i=0}^{N}x_i R_{it+1}$$

The solution to the problem gives rise to a first-order Euler equation:

$$1=E_t\left[M_{t+1}\left(1+r_{t+1}^e\right)\right] \tag{1}$$

where r_{t+1}^e is the excess returns on risky assets over risk-free rates, and

$$M_{t+1}=\beta\left[\frac{U^{'}\left(C_{t+1}\right)}{U^{'}\left(C_t\right)}\right]$$

is a Stochastic Discount Factor (SDF) or the Intertemporal Marginal Rate of Substitution (IMRS). It is common to make this problem operational for a single representative agent by selecting a specific utility function. A common specification in the C-CAPM literature is the power utility function,

$$U\left(C_t\right)=\frac{C_t^{1-\gamma}-1}{1-\gamma}$$

One implication of this choice is that γ, defines a rate of Constant Relative Risk Aversion (CRRA). Unfortunately, the C-CAPM model with a power utility function does not seem to satisfy the data (Campbell & Cochrane 2000).

2.2 The Utility Function with Consumption Habit Revisited

Dusenberry (1949) first suggested a reason for the observed inertia in consumption data based on a ratchet in aggregate consumption. This can be derived as a feature of optimal dynamic consumption and investment policy with extreme habit formation that prevents consumption from falling over time. This concept is what has entered the utility literature that then drives the habit based C-CAPM.

[3] An alternative proposition that gives rise to this type of analysis derives from the arbitrage pricing theory. However, for the models developed here this implies that both US excess returns and the growth in gross real consumption define pure innovations. The model devised by Gregoriou, Hunter and Wu (2009) and our further investigations would suggest that this is not the case.

Thus far the appropriate reference has been seen as past consumption, rather than some form of external wealth. However, there is evidence that consumption is driven by stock market wealth.[4] Thus far the literature on globalization and contagion has not paid specific attention to the underlying choice problem that might give rise to home asset pricing decisions driven by external current or future values of external asset prices. Here it is suggested that this arises via an external wealth reference. This is because of the need for investors to gauge their investment performance. In fact, many investors do not participate in markets directly, they do this via fund managers, who are usually required to hedge risk and perform on average better than the market. It is increasingly the case that UK assets are traded on the US stock markets and that fund managers diversify risk by holding assets from other markets. Therefore, they use a variety of benchmark indices to gauge the performance of their funds.

In order to extend the external consumption habit model to incorporate an external stock market wealth reference, we apply a simple Cobb–Douglas power type utility function:

$$U\left(C_t, X_t\right) = \beta \frac{C_t^{1-\gamma_1} X_t^{\gamma_2} - 1}{1-\gamma_1} \tag{2}$$

X_t is the level of the habit reference usually determined externally. If a consumption reference is considered as by Abel (1990), then $X_t = C_{t-1}$. However, instead of using past consumption as has occurred in the literature, an external wealth reference W_t is used here. Therefore:

$$U\left(C_t, W_{USt}\right) = \alpha\beta \frac{C_t^{1-\gamma_1} W_{USt}^{\gamma_2} - 1}{1-\gamma_1} \tag{3}$$

α is an implicit discount factor for the external wealth reference associated with the conventional consumers' optimization problem and the corresponding pricing kernel is:

$$M_{t+1} = \alpha\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma_1} \left(\frac{W_{USt+1}}{W_{USt}}\right)^{-\gamma_3} \tag{4}$$

Due to the dominant role of the US stock market in the global stock markets,[5] we choose a US stock index as a proxy of this wealth reference and test whether this is the factor that drives average non-US investor optimizing behaviour.

If we denote $r^e_{US,t}$as the excess return at the time t, the following equality can be satisfied:

$$\frac{W_{ust+1}}{W_{ust}} = 1 + r_{US,t}$$

Therefore:

$$M_{t+1} = \alpha\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma_1} \left(1 + \hat{r}^e_{US,t+1}\right)^{-\gamma_2} \tag{5}$$

[4] See Gregoriou, Hunter and Wu (2009) or Hall (1978) for example.
[5] This ratio is 44% from the IMF annual Report (2006).

In practice, we use $\hat{r}^e_{US,t+1}$, which is an estimate of the expectation of $r_{US,t+1}$ based on the information available at the time t. Specifically, $r^e_{US,t+1}$ can be proxied by expected returns on the S&P500 index. It follows for (5) to be consistent with agent rationality that $\alpha, \gamma_1, \gamma_2$ are all positive. Unfortunately, the subjective discount factor β in this model is not identified (Sargan, 1983). Following, Gregoriou and Ioannidis (2007) β is set to .99.

3 The Methodology

One advantage of a wealth reference over consumption habit formation is that the wealth effect as proxied by the US stock market index can capture transitory innovations as well as permanent shocks (Lettau and Ludvigson, 2004). Given the extreme volatility in stock prices, consumption-smoothing households may not want to vary their consumption to react to daily, monthly, or even yearly equity price movements. Thus an external wealth reference already captures an external consumption habit.[6]

As US excess returns are viewed as being endogenous we require some form of systems estimator. As the focus is on UK market behaviour and the feedback is viewed as being unidirectional we have restricted ourselves to Instrumental Variables (IV) and Generalized Method of Moments (GMM) estimators. It is reasonable to consider that UK information on consumption and returns might define reasonable forcing variables for UK consumption growth, but this is unlikely to be the case for US excess returns. In the light of the weak instrument problem (Stock, Wright and Yogo, 2002) a two step estimator is applied to help resolve this problem. Hence, future excess returns are estimated from the model of US excess returns developed by Gregoriou, Hunter and Wu (2009). This has the advantage that excess returns are explained by a well specified model that depends on the key relations driving the US economy. However, the two step approach, gives rise to inconsistency in the conventional estimate of the standard error that can be corrected either by the bootstrap or direct calculation of an appropriate asymptotic estimator of the equation variance.[7] Indeed, such a problem will always arise when the generated variable is correlated with the residuals ($E\left(\hat{r}^e_{US,t+1}\nu_{t+1}\right) \neq 0$),this may be the result of omitted explanatory variables or unobserved factors in the regression. This can also be caused by

[6] A similar three-factor model has also been tested but the coefficient on the consumption habit is not statistically significant. While, more recently Souza (2010) has considered a reference to past profits.

[7] The predicted variable is termed a "generated regressor" (see Pagan 1984) and with the exception of some very specific cases estimated standard errors are biased with their inclusion in estimated equations. One solution could be to bootstrap the problem, but bootstrap tests have to be considered carefully before applying them to two stage regression models since the impact of the residuals of the first step regression cannot be neglected. Greater caution is necessary in the GMM case, where inference can be biased, because the bootstrap estimates are based on an empirical distribution function that implements a moment condition that does not necessarily hold in the population of bootstrap samples. Moreover, even after some correction adjustment for the moment condition, bias in the augmented GMM bootstrap is reduced, but not eliminated.

the dynamic process generating the regressors, so when the residuals are neither serially correlated, nor heteroscedastic, and are normally distributed, then the bias may be small. Such generated variables are potentially useful, since good instruments are often difficult to obtain and this is particularly the case for return data. As is common in the case of IV regressions, the simple choices of different lagged values of returns may not be sufficient to describe the current behaviour of the variable. Moreover, the regressor selected here is consistent with the more fundamental view that rational expectations are model generated, where the model attempts to explain the complex inter-relations associated with the inter-action between financial and real sectors of the economy.

3.1 Extended C-CAPM Equations

Inserting (5) into (1) results in the following criterion:

$$E_t \left[\left(1 + r^e_{UK,t+1}\right) \alpha\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma_1} \left(1 + \hat{r}^e_{US,t+1}\right)^{-\gamma_2} \right] = 1. \tag{6a}$$

Equation (6a) is a nonlinear form of the generalized C-CAPM. If the error is viewed as being multiplicative or the joint distribution of consumption and returns log-normal, then taking logs of (6a) gives rise to the following model.[8]

$$r^e_{UK,t+1} = -\log\alpha - \log\beta + \gamma_1 cg_{UK,t+1} + \gamma_2 \hat{r}^e_{US,t+1} + \epsilon_{t+1}. \tag{6b}$$

It is a statistical convention for (6a) and (6b) to be evaluated using expectations based on information available at time $t-1$ instead of time t. Therefore:

$$r^e_{UK,t} = -\log\alpha - \log\beta + \gamma_1 cg_{UK,t} + \gamma_2 \hat{r}^e_{US,t} + \epsilon_t. \tag{7a}$$

$$E_{t-1} \left[\left(1 + r^e_{UK,t}\right) \alpha\beta \left(\frac{C_t}{C_{t-1}}\right)^{-\gamma_1} \left(1 + \hat{r}^e_{US,t}\right)^{-\gamma_2} \right] = 1. \tag{7b}$$

The differences between linear and nonlinear models only relates to nature of the econometric methodology and the linear approximation. As the expectation is conditional on information at time $t-1$, then the expected value of the dynamic equation explaining excess returns from Gregoriou, Hunter and Wu (2009) is adjusted for the influence of contemporaneous shocks such as the dummies that capture the effect of large outliers related to the stock market crash of 1987 and the Asian markets crisis.

3.2 Correcting the Equation and Coefficient Variance

Although the generated variable may be econometrically plausible, in the sense that the innovations are white noise, the standard errors of the coefficients are

[8] Similar log-linear Consumption-based CAPM have already been reported in the finance literature, and also been extended to time-varying models by Hodrick and Zhang (2001).

not correct. Consider the variance-covariance matrix of the parameters from the IV/GMM estimator in the two-stage regression:

$$\operatorname{var}\left(\tilde{\beta}\right) = \tilde{s}^2\left(X^{*'}P_uX^*\right)^{-1}$$

where $\tilde{s}^2 = \frac{\tilde{\epsilon}'\tilde{\epsilon}}{n}$is the sum of the squared residuals $\tilde{\epsilon}$. For linear models $\tilde{\epsilon} = Y - X^*\beta$ and for nonlinear ones, $\tilde{\epsilon}$ is calculated by some (possibly nonlinear) orthogonal function of the parameters and a set of instrumental variables Z. $X^*$are explanatory variables including any generated regressors. P_u is the orthogonal projection matrix of the instrument variable set. For IV estimation $P_u = Z\left(Z'Z\right)^{-1}Z'$,and for linear GMM, $P_u = ZWZ'$where W is a weighting matrix. To obtain the optimal GMM estimator, W is required to be the inverse of the variance-covariance matrix of the moment conditions $g\left(\hat{\beta}\right) = \frac{1}{n}E\left(Z'\tilde{\epsilon}\right)$, that is $W = S^{-1}$ and $S \equiv \frac{1}{n}E\left(Z'\tilde{\epsilon}\tilde{\epsilon}'Z\right)$. Consequently, the standard error of the i^{th} coefficient is

$$SE(\hat{\beta}_i) = \tilde{\epsilon}/\lambda_{ii}$$

where λ_{ii} is the i^{th} diagonal value of $\left(X^{*'}P_uX^*\right)^{-1}$. The conventional estimate of the residual variance is calculated as

$$\operatorname{var}\left(\hat{\beta}_{IV}\right) = \hat{s}^2\left(X'P_uX\right)^{-1}.$$

$\hat{s}^2 = \frac{\hat{\epsilon}'\hat{\epsilon}}{n}$, $\hat{\epsilon} = Y - f\left(X, \hat{\beta}\right)$ and X are explanatory variables including the actual values corresponding to the generated regressors. That is, to correct the bias in the standard errors, we need to calculate $SE(\hat{\beta}_{IV})_{BC}$ that are based on residuals computed using actual values of variables instead of the generated ones. Comparing the two formulae above, the standard errors are correct to a factor that relates to the differential in the squared residuals, when $\left(X^{*'}P_wX^*\right)^{-1}$and $\left(X'P_wX\right)^{-1}$asymptotically converge to the same limit. The latter requirement is satisfied when the instruments are stationary and residuals of the first step regression have the normal as their limiting distribution. Then the corrected standard errors are given by re-scaling using the factor $\sqrt{\tilde{\varepsilon}'\tilde{\varepsilon}/\hat{\varepsilon}'\hat{\varepsilon}}$:

$$SE\left(\hat{\beta}_{IV}\right)_{BC} = SE\left(\hat{\beta}_{IV}\right) \times \sqrt{\tilde{\varepsilon}'\tilde{\varepsilon}/\hat{\varepsilon}'\hat{\varepsilon}}. \tag{8}$$

Alternatively, the bias in the standard error can be removed by using the actual return as a regressor and the prediction as an instrument. This resolves the prospective inconsistency in the equation variance, but not in the moment matrix of the data. Further, there is a tension between the appropriate specification of the model and appropriateness of inference (Davidson and MacKinnon, 2004). Hence, what happens when the estimated parameters, associated with what may be considered as two alternate forms of the same model are materially different, because predicted excess returns feed into the mean equation the restricted impact of a range of external US macroeconomic variables. More specifically, Muellbauer (1983) tested the surprise rational expectations model of consumption and found that such restrictions relative to more general dynamic specification were not always accepted. Hence, the dynamic error correction model can provide a better or an equivalent explanation of UK aggregate consumption behaviour.

4 Data Description

The primary analysis relates to the seasonally adjusted aggregate consumption expenditure data $C^N_{UK,t}$ used in the study of the UK by Gregoriou and Ioannidis (2007), and for comparison, seasonally adjusted US personal consumption expenditure data $C^N_{US,t}$. The UK FTSE100 index and the 3-month UK government Treasury bill rate are used respectively as the risky asset returns $R_{UK,t}$ and risk-free rate of return $R^f_{UK,t}$. US excess returns are calculated from actual returns $R_{US,t}$ on SP500 index less the returns $R^f_{US,t}$ on 3-month US Treasury Bills. The expected values are measured using fitted values $\left(\hat{r}^e_{US,t}\right)$of US excess returns generated by the system of equations estimated by Gregoriou, Hunter and Wu (2009). Nominal consumption data C^N_t have been deflated by the CPI index π_t, and for this purpose, we set π_t over the period 1980:01 as the base value. Then real consumption is denoted C_t and continuously compounded consumption growth cg_t. The principle study is based on monthly series for the period 1980:01-1999:12, which is the extent of the monthly consumption data available from the Office for National Statistics (ONS), while for estimation we have used the sample 1983:01-1999:12. Table 1 reports the correlations between these variables.

It should be noted that based upon the Table 1, it would appear that volatility in UK excess returns would seem to be transmitted from the US stock market as is indicated by the strong correlation between the two markets. For instance, the correlation coefficient between UK excess returns and US excess returns is 0.75, and even that associated with our estimates of the conditional expectations of the mean of US excess returns is close to 0.50. Further, such volatility transmission can be readily detected through extreme observations such as those associated with the stock market crash in October 1987 and the Asian markets crisis. Consequently it would seem necessary to account for this co-movement of returns as an explanatory variable in any UK asset pricing model. This may be compared with the weak association of consumption growth between the two countries (0.130), which is suggestive of the possibility that a representative agent from the UK might be less likely to share common consumption habit behaviour with similar agents in the US and thus to base their consumption and asset pricing decisions by direct reference to US consumption.[9]

[9] This is evidence that the idea of a reference that implies catching up with the American

Table 1 Correlations of Excess Returns and Consumption Growth for both the UK and the US

	r^e_{UK}	r^e_{US}	$\hat{r}^e_{US}$	cg_{UK}
r^e_{UK}	1			
r^e_{US}	0.735	1		
$\hat{r}^e_{US}$	0.499	0.546	1	
cg_{UK}	0.044	0.054	0.016	1
cg_{US}	0.054	-0.074	0.064	0.130

There is an issue of stock market timing that is related to the expectations of returns on extreme observations, since these shocks are not predictable. To purge the equations of the influence of these extreme observations the expectations are calculated only using information available before time t-1.[10] Thus, revised fitted values of the observations associated with 1987 stock market crash and the Asian Crisis are obtained as 0.019257 and 0.026543, respectively.

5 Empirical Results

This section, considers the two-factor C-CAPM models (7a) and (7b), results based on a range of different instruments are considered and then the correction of the standard errors using (8). The analysis is supported by a study of similar models using quarterly data for the same period and a more recent sample.

Removing expectations in any model unveils an error in variables that can be resolved either via IV (Sargan, 1958) or GMM estimation (Hansen, 1982). The IV and GMM objective function can be estimated for both linear and non-linear models. However, underlying the early treatment of IV is the notion that the error process is driven by measurement error and that this relates ostensibly to well defined structures (Sargan, 1959), but more recently this distinction between errors driven by shock and measurement error has been diluted (Arrelano, 2002). If we consider linear IV estimators, then the key criterion is that the moment matrix of the data has full rank, the moment matrix has a limit and the cross moment matrix for the regressors and the instruments has a limit (Sargan, 1988). Solving, the IV problem depends on the nature of the

consumer as suggested in Smoluk and Vander-Linden (2004) is not supported by the data applied here.

[10]Nevertheless, shock dummies are necessary for the correct specification of a US asset pricing model, but since the shocks are unforeseeable they have to be excluded from any estimates of what may be seen as rational expectations.

instrument set used. More specifically the limit condition on the moment matrix means they ought to be stationary and appropriately dimensioned. Should serial correlation be an issue then this might preclude the use of certain types of lagged information. On the basis of selecting an optimal set of instruments (Sargan, 1959), an efficient estimator will yield consistent parameter estimates that are asymptotically normal and give rise to conventional inference on the parameters and with respect to the specification of the model (Sargan, 1988).

Sargan first described the IV problem in terms of a set of moment conditions that according to Arrelano (2002) might be best viewed as sufficient statistics for the underlying Data Generation Process. Although Sargan (1959) extended the IV estimator to consider non-linear forms, it is now more usual to estimate such models by GMM. Hansen (1982) extended this use of moment conditions in a non-linear context to develop an estimator deemed to be robust to pure error autocorrelation and heteroscedasticity. Although contemporary use of the IV and GMM method have removed the need to specify likelihood functions and systems of equations, this emphasis on consistent estimation is often bought at a cost. In the first instance this relates to efficiency, and Davidson and MacKinnon (2004) warn that it makes little sense to base inference on inefficient estimators as such inference is significantly more difficult. While from our earlier discussion of bootstrapping, such methods are not assured to improve inference. Secondly, when serial correlation follows from dynamic misspecification, then the expectation structure may no longer be identified and as result this may give rise to bias and in dynamic models inconsistency. This suggests when applying GMM it is still key that the model is appropriately specified.

Here GMM is used to estimate the non-linear first-order condition associated with C-CAPM, as there is no direct requirement for the data to be stationary. The non-linear approach is applied as it does not impose the restriction of the linear form that forces the conditional covariance between returns and marginal rates of substitution to be constant through time.

The chosen instrument sets are different between models. In the case of the C-CAPM and the model extended to include habits, lags in UK gross excess returns ($\hat{r}^e_{UK,t}$) and gross consumption growth ($cg_{UK,t}$) are used as instruments. When the US wealth reference is introduced we also include lagged US excess returns ($r^e_{US,t}$). The same instrument sets are used for linear and non-linear models, though with both linear and non-linear GMM a Newey-West weighting matrix is used with fixed bandwidth and no pre-whitening. In undertaking the analysis instrument sets with 2, 4, 6 and 12 lags were considered when the simple C-CAPM was estimated.[11] The model estimate of US excess returns ($\hat{r}^e_{US}$) derived from Gregoriou, Hunter and Wu (2009) is included as a variable in the monthly models, while current values and lags are used as instruments.[12] To choose the most appropriate model, in economic terms we consider the sign and

[11] The use of a similar lag length for IV and GMM was first suggested by Hansen and Singleton (1982). For comparison purposes results using 2 and 12 lags are reported here, this range characterises the range of findings in between.

[12] Applying generated instruments will not lead to inconsistency of 2SLS estimates (Wooldridge, 2002, pp. 117), provided that they are not correlated with the residuals.

size of the coefficients and their significance to determine whether the models are coherent in theoretical terms. The econometric importance of the model relies on the correct coefficients, there significance and the model being well specified. To this end, a number of tests are applied to the residuals, namely, where appropriate, Ljung-Box tests of the autocorrelation structure in the residual and squared residual correlogram, Lagrange Multiplier (LM) tests for serial correlation and autoregressive Conditional Heteroscedasticity(ARCH), and the Jarque-Bera test for normality (for further details see Davidson and MacKinnon, 2004).

Correct inference relies on asymptotic normality and it should be addressed here as an important criterion for model selection. In particular, it can be used as a means of detecting omitted variables or unobserved variables in the regression. However, such a test has always been neglected in the C-CAPM

Table 2 IV and GMM estimates for the UK C-CAPM with habit preferences

	IV		GMM			
			Linear		Non-linear	
l	2	12	2	12	2	12
$\alpha \times .99$	0001 (.99)	$-.002$ (.52)	$-.004$ (.46)	$-.006$ (.00)	1.006 (.00)	1.004 (.00)
γ_1	3.35 (.41)	.967 (.21)	$-.027$ (.46)	.83 (.027)	$-.298$ (.93)	.82 (.27)
γ_2	-1.24 (.44)	$-.364$ (.51)	$-.55$ (.99)	$-.46$ (.19)	.65 (.95)	.457 (.19)
df	2	22	2	22	2	22
$\chi^2_{IV\,df}$	1.95 (.38)	16.78 (.76)	3.34 (.19)	17 (.76)	3.33 (.19)	17.27 (.76)
χ^2_{AR}	7.47 (.82)	10.85 (.54)	10.68 (.56)	11.8 (.46)	10.54 (.57)	11.78 (.46)
χ^2_{ARCH}	2.04 (1.0)	3.09 (1.0)	3.72 (.99)	3.7 (.99)	3.93 (.99)	3.77 (.99)
χ^2_N	275 (0)	570 (0)	578 (0)	594 (0)	565 (0)	573 (0)

† Notes: Standard errors are corrected for autocorrelation and heteroscedasticity in the IV estimation, and p-values are given in parentheses. Dynamic tests are carried out up to 12 lags for residuals, and relate both to the LM test and the Ljung-Box Q statistics for the IV estimator, and only the Box-Ljung Q statistics for the GMM estimation. The instruments are the constant and the lagged explanatory variables plus the UK excess returns up to the lag l and the test for the validity of over-identifying restrictions are given by Sargan's test for the IV estimator and Hansen's J-test for the GMM estimator. Number of observations used 204.

literature where there seems to have been an over emphasis on tests of instrument validity, such as the J-test (Hansen, 1982) and t-tests. The J-test has been demonstrated to be a weak model criterion since it is only considers whether the instruments can be accepted based on a set of over-identifying restrictions, but this is not then a direct test of the models specification. It should be noted that the forecasts that derive from the model generating predictions of US excess returns already take the major shocks into account and as a result when they are included in the UK model, they give rise to models that exhibit error behaviour that appears normal.

Consider the results in Table 2 that relate to the C-CAPM with habit preferences using 2 and 12 lagged instrument sets. These dominate the conventional C-CAPM without habits, but do not explain the non-normality and such results are not improved upon by the inclusion of an external habit reference.[13] The results in table 2 provide some evidence in favour of a linear version of C-CAPM models, but these results are not robust to the specification and are

Table 3 IV and GMM Estimates of the C-CAPM for the UK with US Wealth reference

	IV		GMM			
			Linear		Non-linear	
l	2	12	2	12	2	12
$\alpha \times .99$	003 (.41)	.0012 (.59)	$-.0004$ (.89)	.0098 (.00)	1.01 (.00)	1.02 (.00)
γ_1	1.55 (.14)	.68 (.22)	.987 (.022)	1.03 (.00)	.993 (.24)	1.21 (.00)
γ_2	.943 (.00)	.948 (.00)	$-.77$ (.00)	1.1 (.00)	.787 (.00)	1.116 (.00)
df	5	35	5	35	5	35
$\chi^2_{IV\,df}$	9.57 (.09)	29.78 (.72)	7.3 (.2)	30.5 (.68)	7.31 (.19)	30.6 (.63)
χ^2_{AR}	7.62 (.81)	9.52 (.66)	9.53 (.66)	8.84 (.716)	9.38 (.67)	8.83 (.72)
χ^2_{ARCH}	11.36 (.58)	10.83 (.54)	9.51 (.61)	10. (.53)	8.88 (.71)	10.87 (.54)
χ^2_N	.05 (.98)	.1 (.95)	.43 (.78)	.32 (.85)	1.34 (.54)	.76 (.68)

† Notes: The subjective discount factor is restricted to assume the value of $\beta = 0.99$. Standard errors are corrected for autocorrelation and heteroscedasticity in the IV estimation, and p-values are given in parenthesis. For the linear C-CAPM, dynamic tests are carried out up to 12 lags for residuals, and are reported by both the LM test and the Ljung-Box Q statistics for the IV estimation, and only the Box-Ljung Q statistics for the GMM estimation. The instruments are the constant and the lagged explanatory variables plus the UK excess returns up to the lag n=NLAG, and the test for the validity of over-identifying restrictions are given by Sargan's test for the IV estimation and Hansen's J-test for the GMM estimation.

[13] Other C-CAPM models were estimated for the UK in both linear and nonlinear forms, but these models either perform as poorly as the habit preference model or are worse than the models that include the US wealth reference.

very sensitive to the inclusion of the habit reference that appears not to be significant in any of the cases estimated. There are also quite considerable shifts in the coefficients for the non-linear GMM models. The errors though generally uncorrelated are not normal that will call into question any inference in such models. Certainly the error bands are likely to be greater than those ordinarily applied.

In Table 3 the model based on the US wealth reference is considered. Firstly, the results in Table 3, imply that in no case can the null of normality be rejected even at the 10% level, also the null of no serial correlation and ARCH behaviour in the residuals cannot be rejected. Further, the coefficients on the US return reference are all significant ranging from 0.943 to 0.948 for IV estimation, and from 0.77 to 1.067 for GMM estimation. However, estimates of the constant and risk aversion coefficient are only significant in the linear GMM case with 12 lags and the non-linear GMM case. According to the model selection criteria, and compared with the results of other specifications, it would seem that estimates based on models with 12-lagged instruments should be chosen as the best models. All three coefficients are statistically significant at 1% level (bias unadjusted), the null hypothesis for normality of the residuals cannot be rejected at 10% level and there is also no sign of either autocorrelation or ARCH. The coefficient on US excess returns, (1.096) is even bigger than that of risk aversion, (1.028) suggesting that the risk associated with investment comparisons made relative to the US stock market cannot be neglected. If agents are engaged in keeping up with the Joneses, they live in the US or more pertinently, a rational investor ought to determine their asset allocations based on the highest returns obtainable across a portfolio of assets that do not suffer from a home bias.

Thus far, we have examined all the specifications of the UK C-CAPM models, and for both economic and econometric purposes, the best models appear to be the non-linear model with US expected return preference with 12 lagged instruments. Generally, the nonlinear models perform better than corresponding linear ones, and the significance of the risk aversion parameter improves with the order of the lags.

It is not surprising that when compared with IV, the GMM estimator performs better, since the former is more sensitive to the quality of the instrument set and poor instruments can affect statistical inference. Furthermore, the GMM weights yield a minimum that is optimized as close to zero as is possible, while the weigh matrix defined by GMM is the covariance matrix of the sample moments that in the limit is the minimum variance estimator. As both linear and non-linear GMM estimators rely on a different instrument set to IV, they are invariably over-identified, and hence larger covariance terms with respect to the orthogonality conditions associated with the instruments have smaller weights in the objective function and this ought to yield GMM estimates that are less sensitive to the selection of instruments.

Furthermore, the optimal linear and nonlinear estimation by GMM yields similar coefficient estimates when the same instrument set and sample are used. For example, in the linear model, 0.987 and 1.02, compare with.993 and 1.021 for the nonlinear model. This would suggest a degree of consistency over the estimated parameters applied to the different methods. Though, deciding which model is preferred on the basis of p-values or t-statistics cannot be considered without correcting the standard errors. For this purpose, equation (8) is used and adjusted p-values for one tail t-tests that reflect the theoretical restriction that the signs are positive are reported in Table 4. Table 4 demonstrates that all coefficients are significant at the 1% level when a one-tailed test is applied, and the only two exceptions are the constant and the coefficient of risk aversion in the linear consumption model estimated by IV with 12 lagged

Table 4 IV and GMM Estimates for Optimal UK C-CAPM with Correction for Standard Errors and Market Timing

Estimates	Constant	γ_1	γ_1
Linear IV	$-.0104$ (.443)	.471 (.149)	.468 (.01)
Linear GMM	$-.012$ (.01)	2.862 (.00)	.613 (.00)
Non-linear GMM	1.012 (.00)	2.908 (.00)	.645 (.00)

† Note: One-tail p-values are given in parenthesis. The rescaling factors for the standard errors are 0.758, 0.787 and 0.786, respectively. *,**,***: statistical significance for one-tail test at 10%, 5% and 1%, respectively.

instruments. This suggests that on statistical grounds preference might be given to non-linear GMM over the linear IV methodology as the enhanced t-values ought to reflect the relative efficiency of the estimator, when the residuals are heteroscedastic. The similarity of the estimates in the linear and nonlinear GMM cases, suggests that the assumption of log-normality embedded in linear GMM is satisfied and that the conditional covariance between returns and IMRS is constant. If estimating C-CAPM by a non-linear estimator has the virtue of depicting the nonlinearity, then this is not obvious for the relation between returns and IMRS for the UK. However, the coefficient estimates are somewhat different when adjusted excess returns are used.

In the next table we consider results associated with estimating (7a) and (7b) on quarterly data over two periods: (1) 1983q1-1999q4 and (2) 1983q1-2010q1. In this case the optimal instruments include four lags in consumption growth, UK excess returns and augmented predictions that arise from the US economy model of Gregoriou, Hunter and Wu (2009). Again the linear IV and GMM estimators are applied along with non-linear GMM.[14] All the series are aggregated to the quarterly frequency and where necessary augmented by quarterly observations of UK consumption growth. In the quarterly case, it is necessary to augment using quarterly observations for the UK. In the quarterly case, actual consumption growth and US excess returns are used as variables and the

[14] The weight matrix in the case of GMM is based on Newey-West weights using the same bandwidth as in the monthly case using the quadratic spectral kernel. All estimation is in Eviews 6.0 and 7.0.

Table 5 IV and GMM Estimates of Quarterly UK C-CAPM with US wealth reference

	IV		GMM			
			Linear		Non-linear	
sample	1	2	1	2	1	2
$\alpha \times .99$	$-.01$ (.08)	$-.01$ (.11)	$-.04$ (.00)	$-.016$ (.00)	.95 (.00)	.97 (.00)
γ_1	.28 (.75)	$-.01$ (.99)	1.42 (.01)	.65 (.04)	1.26 (.24)	.65 (.45)
γ_2	1.14 (.00)	.91 (.00)	1.32 (.00)	.84 (.00)	1.48 (.00)	.854 (.00)
df	10	10	10	10	11	10
$\chi^2_{IV\,df}$	7.17 (.71)	10.88 (.72)	9.82 (.46)	7.59 (.67)	8.67 (.56)	7.7 (.66)
χ^2_{AR}	11.54 (.32)	8.17 (.66)	7.94 (.63)	5.95 (.82)	8.07 (.62)	6.13 (.8)
χ^2_{ARCH}	7.29 (.69)	4.75 (.91)	9.0 (.53)	3.19. (.98)	10.06 (.44)	3.26 (.98)
χ^2_N	3.71 (.15)	4.51 (.11)	.38 (.83)	2.13 (.34)	.19 (.44)	2.83 (.3)

Notes: Standard errors are corrected for autocorrelation and heteroscedasticity in the IV estimation, and p-values are given in parentheses. Dynamic tests are carried out up to 10 quarterly lags for residuals, and relate both to the LM test for the IV estimator, and Ljung-Box Q statistics for the GMM estimation. The test for the validity of over-identifying restrictions are given by Sargan's test for the IV estimator and Hansen's J-test for the GMM estimator. *,**,***: Statistically significant at the 10% level, 5% and 1%, respectively. Number of observations used is 68 for sample (1) and 109 for sample (2).

predictions are then used as instruments. One reason for this is that the US models were not optimized on quarterly data, and the IV and GMM equations derived using the actual data give rise to residuals that also do not suffer from non-normality.

If the models in table 5 are considered, then with the exception of the IV models there is no sign of misspecification in terms of ARCH or error autocorrelation, the residuals are all normal and the instrument validity test cannot be rejected. In the IV case there is some sign of first order serial correlation and for this reason the first lag in the dependent variable was excluded as an instrument. The quarterly results are probably most easily considered relative to the results using US excess return predictions not adjusted for the inclusion of dummies. Based on the standard errors derived from the quarterly data models, the coefficient for the US wealth reference is within a standard error of the coefficient estimated using the monthly data, and the same applies for the risk aversion parameter. The estimates based on the model using corrected data are also not dissimilar when the standard errors from the quarterly data models are used. As the sample expands we still find that US excess returns and consumption growth have a significant effect on UK excess returns.

6 Conclusion

Many augmented models have been developed to improve the performance of the C-CAPM. In essence, they are trying to find or construct state-dependent variables for consumption that might help remove any excess smoothness or heterogeneity. However, all but a small number of these models neglect external factors, here the co-movement across markets that has arisen with globalization, and this suggests why returns seem to be less directly dependent on consumption growth.

It is shown here that C-CAPM for the UK can be fruitfully extended by replacing the consumption habit by a wealth reference that can be proxied by the US stock market. As a result, it is argued that a primary driver of UK agent behaviour is a US wealth reference. Thus, the Intertemporal Marginal Rate of Substitution depends both on domestic consumption and movements in the US market. Consequently, future US excess returns and the growth rate of UK consumption are key factors in explaining UK excess returns controlled for the risk free rate. The empirical results suggest that this two-factor model can well explain the equilibrium between UK returns and domestic consumption, since after correction of the standard errors, both linear and nonlinear models reveal the statistically significant and substantial effect of the US market on the UK. Further, this two-factor C-CAPM suggests that it is not external habit effects or comparison with external consumption, i.e. US consumption that has driven C-CAPM models for the UK.

The findings based on the quarterly sample for the period 1983q1-1999q4 do not contradict the proposition that a Consumption based asset pricing model augmented by US excess returns can explain UK excess returns. In the absence of monthly consumption data beyond 1999, it is necessary for an analysis of the more recent past to rely on quarterly data. The quarterly and monthly coefficients suggest something similar about the impact of US excess returns, though the analysis is less robust for consumption growth. To this end, the results for the extended period 1983q1-2010q1 are not dissimilar and the proposition that the two sets of results are the same cannot be rejected. A question still unanswered is the most appropriate way of handling expectations and to this end the impact of outliers.

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