Approximate Factor Structures, Macroeconomic And Financial Factors, Unique And Stable Return Generating Processes And Market Anomalies: An Empirical Investigation Of The Robustness Of The Arbitrage Pricing Theory

by

Richard Priestley

Department of Economics
Brunel University

Thesis submitted for the degree of PhD in Economics

December 1994

For Liza, with love.

ABSTRACT

This thesis presents an empirical investigation into the Arbitrage Pricing Theory (APT). At the onset of the thesis it is recognised that tests of the APT are conditional on a number of preconditions and assumptions. The first line of investigation examines the effect of the assumed nature of the form of the return generating process of stocks. It is found that stocks follow an approximate factor structure and tests of the APT are sensitive to the specified form of the return generating process. We provide an efficient estimation methodology for the case when stocks follow an approximate factor structure. The second issue we raise is that of the appropriate factors, the role of the market portfolio and the performance of the APT against the Capital Asset Pricing Model (CAPM). The conclusions that we draw are that the APT is robust to a number of specified alternatives and furthermore, the APT outperforms the CAPM in comparative tests. In addition, within the APT specification there is a role for the market portfolio. Through a comparison of the results in chapters 2 and 3 it is evident that the APT is not robust to the specification of unexpected components. We evaluate the validity of extant techniques in this respect and find that they are unlikely to be representative of agents actual unexpected components. Consequently we put forth an alternative methodology based upon estimating expectations from a learning scheme. This technique is valid in respect to our prior assumptions. Having addressed these preconditions and assumptions that arise in tests of the APT a thorough investigation into the empirical content of the APT is then undertaken. Concentrating on the issues that the return generating process must be unique and that the estimated risk premia should be stable overtime the results indicate that the APT does have empirical content. Finally, armed with the empirically valid APT we proceed to analyse the issue of seasonalities in stock returns. The results confirm previous findings that there are seasonal patterns in the UK stock market, however, unlike previous findings we show that these seasonal patterns are part of the risk return structure and can be explained by the yearly business cycle. Furthermore, the APT retains empirical content when these seasonal patterns are removed from the data. The overall finding of this thesis is that the APT does have empirical content and provides a good description of the return generating process of UK stocks.

ACKNOWLEDGEMENTS

Many thanks to Liza for her support, encouragement, sacrifices, and total understanding throughout the writing of this thesis. Thanks to my parents and family for their generous support and encouragement without which this thesis would not have been written. A special thanks to my father for all his help over the last twenty five years.

To Professor Tony Antoniou, my co-supervisor, thanks for all the help, encouragements, discussions, guidance and opportunities, without which this thesis would not have been possible. Thanks, not least of all, for being a good friend and colleague.

To Ian Garrett, my supervisor, for very helpful discussions, supervision and providing time and patience to help improve this thesis. In addition, I would like to thank Ian for enlightening me on the relationship between writing a thesis and flying economy class during the night to Athens, for being a valuable colleague and friend and finally for making me look at least a little fashionable ! To Andrew Clare for very useful discussions and comments and being a good mate and also making me realise that the APT can not solve all the problems in the world.

I would also like to thank all other friends and colleagues at Brunel for their help over the last three years especially Andy Foster, Richard Batty, Mike Kagwa and Len Skerrett and staff at Manchester University Computer Centre.

Thanks to my flatmates who I have known while writing this thesis. Thanks to friends at home especially James Todd who contributed to this thesis a long time ago now but nonetheless it was very important. A special thanks to Richard Holmes for being a good friend, for support and encouragement and the countless pints of Guinness.

CONTENTS

List Of Tables

LIST OF FIGURES

INTRODUCTION

Asset pricing models have been at the forefront of research in financial economics for over three decades. The aim of any asset pricing model is to provide a measure of the relationship between risk and return. Our understanding of the relationship between risk and return is important given the effect agents' investment and consumption decisions can have on the overall efficient allocation of resources. In particular, if economic agents' measurement of the relationship between risk and return is inaccurate then there may well be a missallocation of resources and consequently a sub-optimal level of investment which may ultimately lead to a lower level of wealth in the economy. As a result, it is not surprising that asset pricing models have attracted a great deal of interest in the academic literature. The first asset pricing model to formally consider the relationship between risk and return is the Capital Asset Pricing Model (CAPM) developed by Sharpe (1964), Lintner (1965), and Mossin (1966). Extending the results of Markovitz (1959), the CAPM relates the return on a risky asset to its covariability with the market. The main property of the CAPM is that in equilibrium all securities will lie on the security market line such that there is a linear relationship between risk and return, where risk is measured by the covariability of a risky asset with a market wide measure of risk called beta.

Although appealing through its simplicity and intuitive nature the CAPM relies on some very strong assumptions. However, extensions of the CAPM have shown that the model is fairly robust to violations of these assumption. In addition, empirical evidence in the 1970's (see, for example, Fama and MacBeth (1973) and Black, Jensen, and Scholes (1972)) found strong support for the empirical version of the model. This led to the acceptance of the model both by academics and practioneers alike. However, the critique of Roll (1977) questioned the usefulness of the CAPM in any empirical context. The main focus of Roll's Critique is the assertion that:

> *"(a) No correct and unambiguous test has appeared in the literature, and (b) there is practically no possibility that such a test can be accomplished in the future."*

The crux of Roll's arguments arise from the unobservability of the true market portfolio. Since valid inferences from tests of the CAPM require that the true market portfolio is identified, then the implication from the unobservability of the market portfolio is that the CAPM is untestable. Consequently, tests and applications of the CAPM in an empirical framework must be treated with a degree of caution. Furthermore, recent empirical tests of the CAPM have found that alternative measures of risk other than beta are useful in describing the return generating process of stocks. For example, researchers found that firm size is an important variable in the risk return relationship (see Reinganum (1981)) as are variables such as dividend yield and price earnings ratios (Litzenberger and Ramaswamy (1979) and Basu (1977)). More recent evidence from Fama and French (1992) suggests that the inclusion of measures of firm size and book to market values in the risk return relationship can render beta insignificant in explaining the relationship between risk and return.

The aim of Ross (1976) was to derive a testable alternative asset pricing model under relatively weaker assumptions than those required for the CAPM. Principally, Ross (1976) relies on the absence of arbitrage in financial markets to arrive at an asset pricing model for risky assets. In its most basic form the APT states that the return on any risky asset depends on its relationship between k systematic risk factors. Once these k factors have been identified then they can be used to price any risky asset. While the theoretical development of the APT relies upon weaker assumptions than the CAPM recent empirical results regarding the APT have cast doubt on its empirical content. For example, it is apparent from the extant literature that the uniqueness condition of the k-factor APT may well not hold. This evidence emanates initially from the tests of the APT that employ factor analytic techniques to extract the k systematic risk factors. Results from Roll and Ross (1980), Drymes, Friend, and Gultekin (1984) and Drymes, Friend, Gultekin, and Gultekin (1986) indicate that the number of factors is not well determined and is conditional on the number of assets employed in empirical tests and the number of observations associated with each asset. Furthermore, empirical work which has specified observed variables as candidates for systematic risk factors has failed to identify a consistent set of risk factors (see, for example Chen, Roll, and Ross (1986), Burmeister and McElroy (1988), McElroy and Burmeister (1988) and Shanken and Weistein (1990) for US evidence and

Beenstock and Chan (1988), Poon and Taylor (1991) and Clare and Thomas (1994) regarding UK evidence). These findings imply that the return generating process is not unique. Therefore, the empirical version of the APT seems to suffer from an identification problem, similar in spirit to that suggested by Roll (1977) for the CAPM. While such findings do not question the validity of the APT as a theoretical model, they do suggest that the methodologies adopted to test its properties and implications are not necessarily correct.

The objective of this thesis is to reassess the empirical validity of the Arbitrage Pricing Theory by firstly developing a testing methodology which enables us to resolve some very important issues, ignored in empirical tests of the APT, and secondly investigating the empirical content of the APT. As shown in the thesis, failure to follow this testing procedure is likely to lead to invalid inferences regarding the risk return relationship and hence the rejection of the empirical version of the APT. Specifically, the thesis explicitly recognises that any test of the APT is a joint hypothesis between the properties of the APT and first, the structure of the return generating process, that is, whether returns follow a strict or an approximate factor structure, second, the factors chosen to approximate the return generating process and third, the way agents are assumed to form their expectations and as such the construction of the unanticipated components of the chosen factors that enter a candidates for systematic risk.

Although addressing the above stated issues is necessary in order to arrive at an empirical APT that is robust it is not sufficient since there are a number of other issues which require attention. The first issue stems from the predictions of the theory, namely that the return generating process is unique. This amounts to the systematic risk factors identified in a empirical study being able to price any other set of assets. For example, if we partioned a set of returns into two subsets and estimated the systematic risk factors from the first sample then it should follow automatically that these factors should be the only ones found to be significant in the second sample with the same estimated sign and size. If this is the case then we consider the APT to have a unique return generating process. Failure to find a unique return generating process questions the empirical validity of the APT. Evidently this issue is of paramount importance in any empirical test of the APT but has been ignored to date and has led Fama (1991) to suggest that:

> *'The flexibility of the Chen, Roll and Ross approach can be a trap. Since multifactor models offer at best vague predictions about the variables that are important in returns and expected returns, there is the danger that measured relations between returns and economic factors may be spurious, the result of special features of a particular sample (factor dredging). Thus the Chen, Roll and Ross tests, and future extensions, warrant extended robustness checks. For example, although the returns and economic factors used by Chen, Roll and Ross are*

available for earlier and later periods, to my knowledge we have no evidence on how the factors perform outside their sample.' (Fama (1991, p. 1595)).

In addition to this issue regarding the uniqueness of the return generating process, the estimated risk premia must be stable over time. In order to assess this, analysis of the behaviour of the risk premium over time is required. Failure to find stable risk premia would suggest that a time-varying approach to the estimation of the APT would be required. Finally, if the APT is robust to the above issues it is then and only then useful to proceed to examine issues such as market anomalies since tests of the Efficient Markets Hypothesis are a joint hypothesis of efficiency and the asset pricing model. Evidence of inefficiency may well be a result of a misspecified asset pricing model.

Chapter 1 of the thesis identifies the necessary issues which must be addressed in an empirical analysis of the APT. The contribution of the thesis to the literature lies in the originality of the following chapters. In Chapter 2 we consider the hypothesis regarding the form of the return generating process. We show that the assumed form of the return generating process can have severe effects on the inferences from tests of the APT. We also confirm earlier findings that tests of whether the return generating process follows a strict factor model or an approximate factor model are inconclusive. We present new evidence that indicates that the idiosyncratic return variance-covariance matrix characterises an approximate factor model. Give these results we extend the work of McElroy, Burmeister, and Wall (1985) to estimate both a strict and approximate factor model of stock returns. Empirical analysis reveals that markedly different results are achieved depending on whether or not a strict or an approximate factor model is specified. Overall the results of this chapter reveal that in empirical tests we should be using an approximate factor model.

In chapter 3 we consider the issue of the relative performance of the APT and the CAPM and the performance of the models as competing asset pricing models. This is of interest for two reasons. First, if agents are to make optimal decisions regarding their investment and consumption then the most appropriate model of risk and return must be employed. Second, this chapter considers the role of the market portfolio in tests of asset pricing models. If the CAPM dominates the APT in empirical tests then there appears little reason to extend tests of the APT. In short we find that the APT outperforms the CAPM and that the market portfolio does have a role to play in the APT specification. The results of the tests of the APT in chapters 2 and 3 reveal that the APT is not robust to the generation of unexpected components in the macroeconomic and financial factors. This issue forms the basis of chapter 4. We consider two techniques employed in extant tests of the APT for forming unexpected components of the factors. We find that both of these are deficient given the assumptions made regarding the way agents form their expectations of the macroeconomic and financial variables. This leads us to consider an alternative framework from which to generate expectations of the variables based upon specifying the expectations generating process as a learning mechanism.

Having considered the issues regarding the joint hypotheses that underlie tests of the APT we examine the empirical content of the APT in chapter 5 in more detail. This involves an analysis of the uniqueness issue. Using two samples of assets we test whether the same factors drive security returns in both samples. This is an imperative issue if the APT is to have any empirical content. Failure of the APT on this count would surely render the APT useless in empirical tests since it would have to be reestimated every time a new sample of assets was considered. Regarding the estimates of the APT's parameters, we also performs checks on the residuals from the APT estimation in order to assess whether they are serially uncorrelated and homoscedastic. Robustness to these assumptions is important for standard inferences with respect to the estimated parameters and their standard errors. In addition to this we also consider the stability of the estimated risk premia from the APT. Instability in these would question the usefulness of the APT in any practical context. We find that the APT does have a unique return generating process, that the residuals are robust to the assumptions regarding serial correlation and homoscedasticity and that the estimated risk premia are stable.

In chapter 6 of this thesis we consider the issue of seasonality in stock returns. Having found an APT model that is robust we are in a position where we can tackle the issues of apparent observed market anomalies and consequently make inferences regarding them that are free from problems encountered in tests that have used asset pricing models that are not robust. Our findings confirm that there are seasonal patterns in the UK stock market. These seasonal patterns are part of the risk return structure and furthermore they can be explained by the estimated risk factors from the APT. Additionally, we find that the APT retains empirical content when these seasonal patterns are removed from the data. The final chapter of the thesis provides a summary of the findings, discusses some possible policy implications and proposes future directions for research in this area.

CHAPTER 1

THE ARBITRAGE PRICING THEORY : A REVIEW

1.1 INTRODUCTION

We begin this thesis with a review of the extant literature regarding the Arbitrage Pricing Theory (APT). The purpose of this review is to assess the contribution researchers have made to date in identifying an asset pricing model that has empirical content. In section 1.2 we review the theoretical developments of the APT. This involves a discussion of Ross's (1976) seminal paper which developed the classic strict factor model version of the APT. Extensions of Ross's (1976) original model are discussed with particular reference to the structure of the return generating process. This section also involves a discussion regarding the testability of the model. In section 1.3 we discuss the testable implications of the APT. Section 1.4 reviews the empirical methodology and results of tests of the APT based upon techniques that extract statistical factors as candidates for systematic risk factors. The methodology and empirical results of tests of the APT which use observed factors are discussed in section 1.5. In section 1.6 we discuss the role of generating the unexpected components which enter as factors into the APT specification. Section 1.7 discusses the presence of seasonal patterns in stock returns and the role of asset pricing within this framework. A summary of the APT literature is discussed in section 1.8.

1.2 The Arbitrage Pricing Theory

The aim of Ross (1976) is to derive an asset pricing model under relatively weaker assumptions than the CAPM. Principally, Ross relies on the absence of arbitrage profits in financial markets to arrive at an asset pricing model for risky assets. In deriving the fundamental pricing relationship of risky assets Ross begins with the simple principle that in the absence of any arbitrage opportunities then the rates of return on two riskless assets, ω and ω' must be equal:

$$\omega = \omega' \tag{1.1}$$

If $\omega \neq \omega'$ then this would imply that agents could, without using any wealth or undertaking any risk, sell the overpriced asset and buy the under priced asset making riskless profits, that is, it would imply the existence of arbitrage profits. This process of arbitrage would continue until the rates of return on the two riskless assets were equal. It is this principle that Ross (1976) extends to the case of risky assets. Ross (1976) starts by assuming that agents have homogeneous beliefs regarding the return generating process such that asset returns are generated by a k factor model:

$$r_i = E[r_i] + b_{i1}f_1 + \cdots + b_{ik}f_k + u_i \tag{1.2}$$

where r_i is the random return on asset i; $E[r_i]$ is the expected return on asset i; b_{ik} is the sensitivity of asset i to the kth common systematic risk factor, f_k; u_i is a firm specific component representing unsystematic risk; and the following assumptions are made: $E[f_k] = E[u_i] = E[f_k u_i] = 0$, and $E[u_i u_i'] = \Omega$, where Ω is a diagonal matrix. In addition, it is assumed that capital markets are perfect and frictionless and agents are von Neumann-Morgensten utility maximizers. The next step is to define an arbitrage portfolio. An arbitrage portfolio is one which uses no wealth and imposes no risk and consequently must have a zero return. Taking **w** as change in wealth when forming the arbitrage portfolio, the additional return from the arbitrage portfolio is defined as:

$$r_{ap} = wE[r_{ap}] + wb_{i1}f_1 + \cdots + wb_{ik}f_k + wu_{ap} \qquad (1.3)$$

where r_{ap} is the return on the arbitrage portfolio. There are three conditions necessary to derive the riskless arbitrage portfolio: first, as assumed above, $w = 0$, that is, the change in wealth is zero; second Ross assumes that the portfolio is chosen such that the number of assets is large enough to invoke the law of large numbers, consequently as the number of assets approaches infinity $wu_{ap} \rightarrow 0$. This provides the condition that there is no unsystematic risk in the portfolio; and third the portfolio is chosen such that systematic risk is eliminated, that is, $wb = 0$. This has a mathematical interpretation: $wb=0$ means that the investment weights are orthogonal to the sensitivities vector. The assumption that $w=0$ implies that the vector of investment weights is orthogonal to a vector of ones. Using some simple linear algebra it can be shown that, if a vector is orthogonal to M-1 vectors then it must also be orthogonal to the Mth vector and thus the Mth vector can be expressed as a linear combination of the M-1 vectors. Therefore, for the APT where we have defined the orthogonal vectors as a vector of ones and a vector of sensitivities we can write expected returns as a constant times a vector of ones plus a constant times a vector of sensitivities given as

$$E(r_i) = \lambda_0 + \lambda_1 b_{i1} + \cdots + \lambda_k b_{ik} \tag{1.4}$$

If there exists a riskless asset whose rate of return is r_f then it follows that λ_0 is its rate of return. For a risky asset, λ_k is the price of risk associated with the kth systematic risk factor, f_k. It follows that for this risky asset the expected return is the sum of the risk free rate and a risk premium for bearing systematic risk. The prices of risk are constant, such that for two risky assets with the same sensitivity to the common risk factors, the expected returns must be identical. For two assets which have different levels of risk, the sensitivities of the assets to the common risk factor will be different while the price of risk will remain constant, thus giving different expected returns.

These results are derived without the assumption that the market is in equilibrium, or that the market portfolio is mean-variance efficient. All that is required is the absence of arbitrage opportunities, a much less restrictive assumption. To recap, the basic results of Ross's APT is that in competitive markets with the absence of arbitrage profits the return on all assets is a linear combination of k constant prices of risk associated with k systematic risk factors. When two stocks have identical risk characteristics they have identical b_{ik} coefficients and hence, given that λ_k is constant, they will have identical expected returns. For two stocks with different risk characteristics the expected returns will be different, this difference being caused by the differences in the b_{ik} coefficients.

The basic framework and extension of Ross's APT has been subject to much theoretical and empirical debate. The theoretical debate has been aimed at refining Ross's original formulation. For example, Ross (1976) assumes that agents are utility maximizers who have monotonically increasing utility functions. Hubermann (1982) derives Ross's result in a preference free framework making use only of a specific definition of no-arbitrage. However, like Ross, this model relies on an infinite number of assets, an assumption which has been the major source of debate in theoretical developments of the APT. By invoking the assumption of an infinite number of assets in the economy all idiosyncratic risk is diversified such that idiosyncratic returns are uncorrelated. This form of the APT has been termed a strict factor model. This form of the APT allows the covariance matrix of asset returns, $\mathbf{\Sigma}$, to be split into two parts. The first is the covariance matrix of assets' systematic factor risk, $\mathbf{\Psi}$, and the second is the covariance matrix of assets' idiosyncratic risk, $\mathbf{\Omega}$,

$$\mathbf{\Sigma} = \mathbf{\Psi} + \mathbf{\Omega} \tag{1.5}$$

The insight offered by Ross relates directly to this proposition that the covariance matrix of asset returns can be viewed as the sum of these two matrices. The condition of the existence of an infinite number of assets results in the diversifiability of all idiosyncratic risk. Consequently, there is no correlation amongst the idiosyncratic return for each asset and hence Ω is a diagonal matrix. As long as there is a bound on individual asset variance then as the portfolio gets larger, in terms of the number of assets, idiosyncratic risk should not matter[1]. Thus, the only important risk to be considered is the systematic non-diversifiable risk measured by Ψ. This version of the APT has formed the basis of empirical tests of the APT using factor analysis and measured macroeconomic factors.

The diagonality of Ω is sufficient, but not necessary in order to derive the APT. Chamberlain and Rothschild (1983) show that the diagonality restriction on Ω can be weakened to allow for some correlation amongst the idiosyncratic returns of individual assets. Given this possibility, as long as the first k eigenvalues of Σ are unbounded as the number of assets increases and the k+1st eigenvalue is bounded then the APT will hold. This specification of the APT has been called the approximate factor model and has been given

[1] Ross notes that as the number of assets increases then it is likely that wealth will also increase which may have the consequence of making some investors more risk averse. If the degree of risk aversion is increasing when the number of assets increases then this may cancel out the reduction in idiosyncratic variance as the number of assets increases and noise might persist. Ross shows that the necessary conditions required to avoid such situations can be achieved by considering agents whose coefficient of relative risk aversion is bounded.

empirical intuition by Connor and Korajczyk (1993) who point out that when concerned with empirical tests of the APT on a finite set of assets,

> *"It seems possible that a few firms in the same industry might have industry specific components to their returns which are not pervasive sources of uncertainty for the whole economy. For example, awarding a defense contract to one aerospace firm might affect the stock price of several firms in the industry. Assuming a strict factor structure would force us to treat this industry-specific uncertainty as a pervasive factor" (Connor and Korajczyk (1993, p.1264)).*

Consequently, rather than relying on the necessary conditions for a strict factor model to hold, it would seem more attractive to consider, in empirical tests, an approximate factor model.

A further area of interest which has concerned economists is the assumption that a portfolio can be formed so that it embodies zero residual risk since if residual risk can not be diversified away then the APT is not testable. The main advantage of the APT, as opposed to the CAPM, is that it is perceived to be testable since it is not subject to Roll's Critique (1977). In particular, there is no specific equilibrium role for the market portfolio in Ross' APT. However, Shanken (1982) questions whether the APT is in fact testable. Shanken (1982) notes that in actual empirical tests of the APT a finite number of assets is used and thus the linear algebra used by Ross (1976) is only an approximation. In this case tests of the APT rely on an assumption that the sum of squared pricing errors is bounded, that is,

$$\sum_{i=1}^{\infty}\left[E(R_i) - \lambda_0 - \lambda_i b_i\right]^2 < \infty \tag{1.6}$$

where b_i is the ith assets sensitivity to the systematic risk factors, r_i is the ith asset's return and λ_i is the price of risk associated with the ith systematic risk factor. For this relationship to hold deviations from the APT pricing relationship must be small. However this is not an empirically testable proposition since, as Shanken points out,

> *"A test of the APT must, of course, be implemented with a finite set of data. Since any finite sum of squared deviations is clearly finite, (4)* (equation (1.6) above) *is not an empirically testable condition. We should like to know, therefore, whether any empirically testable bound is implied by the theory"*

Shanken (1982) states that the empirical proposition tested by researchers has been equations (1.2) and (1.4), that is, the expected return vector is equal to a linear combination of a unit vector and the factor loadings. However, given the quote above this formulation is not a literal consequence of the theory and as such, rejection of the formulation can not be seen automatically as rejection of the theory. Using a two asset example Shanken shows that if the two assets have identical expected returns the factor structure for the two assets can still be very different, thus refuting the uniqueness property of the return generating process that underpins the APT.

However, the major drawback here is that it is not possible to measure, or test the bound and hence this has led to claims that the APT is in principle untestable. One way of avoiding this problem is to derive the APT in an equilibrium framework. Connor (1984) shows that in a competitive equilibrium framework, with an infinite number of assets, as long as investors assign a zero risk premium to unsystematic risk then proof of the APT follows. Grinblatt and Titman (1983) refine this model for the situation where there is a finite number of assets and show that the amount by which asset prices deviate from the pricing equation is small. An explicit bound on the pricing error is derived and applied by Dybvig (1983) and is shown, even in the worst case scenario, to be very small.

Shanken (1985) also discusses the equilibrium APT model of Connor (1984). Connor shows that if the market portfolio diversifies away unsystematic risk completely then the APT will hold and there is a special role for the market portfolio. In this instance there will exist a unique factor model. Unfortunately, as pointed out by Shanken, any model which relies on the identification of the entire market portfolio suffers from the Roll critique.

In response to the criticisms that the APT is untestable Dybvig and Ross (1985) claim that the approximation in the pricing relationship should not lead to falsification of the theory given that using a sub-set of assets can lead to tests of the APT in which the approximation error can be measured. Dybvig (1983) provides a measure for the approximate pricing error for an individual asset based on the following:

$$R_i \sigma_i^2 \alpha_i \tag{1.7}$$

where R_i is a measure of relative risk aversion, σ_i^2 is the unsystematic variance of asset i and α_i is a measure of the assets value in proportion to total market value. While α_i is unobservable (since we would require the value of the entire market portfolio to calculate α_i) Dybvig shows that it can be approximated by taking α_i relative to the value of a stock market index such as the S&P 500 (which will provide an estimate which is greater than the true α_i since the true market portfolio will always be much larger than a proxy such as the S&P 500). Also, estimates for relative risk aversion and unsystematic risk are used by Dybvig which are likely to overstate their true values. The resultant estimate of the bound for the pricing error is very small. Moreover, given that the assumptions used to derive the three estimates in equation (1.7) are extreme the actual true bound will be even smaller (as well as the measure of α_i being larger than the true α_i, Dybvig chooses a large value for σ_i^2). Dybvig shows that in an extreme "worst case" scenario the bound will be equal to four hundredths of one percent per year, small enough to be ignored.

Ross and Dybvig proceed to criticise the example used by Shanken (1982) that the factor structure is degenerate. They argue Shanken's conclusions are a result of the two asset example, that is, by construction of *arbitrary* portfolios the results will always obtain. A counter example using 1000 assets is developed by Dybvig and Ross (1985) and shown to have a unique factor structure. Dybvig and Ross agree that when assets are allowed to be transformed such as in Shanken (1982) then the transformation will destroy the unique factor structure. However, they argue that the APT will only be expected to make sense when transformations that exemplify the pricing error are ruled out. Dybvig and Ross claim that in any sensible application of the theory the factor structure will be unique. Only in transformations which include portfolios formed in rather inconceivable proportions will the factor structure be degenerate - a scenario which is unlikely to be observed.

This point notwithstanding, Dybvig and Ross (1985) argue that the APT is more likely to be rejected on spurious grounds than accepted. For the APT to make sense the factor structure must be present in a large number of assets. Absence of this condition would lead researchers to conclude that a true factor was idiosyncratic risk. If the APT makes sense in this context then it will be testable on subsets of assets. Dybvig and Ross (1985) give the following example : assume that there is a factor for land value that effects only a single asset and a researcher tests the APT using a group of stocks which are not effected by the land value factor, and one other asset, a real estate index which obviously will be effected by the land value factor. In a test of the APT the land value factor variance in the real estate index will appear as idiosyncratic variance and not conform to the APT when in fact it does. Consequently the model will be biased towards rejection of the theory. This case shows that the APT can lead to spurious rejection of the theory when a factor is omitted, but not spurious acceptance.

The issues regarding the testability of the APT have cast doubt on the usefulness of the APT as an empirical asset pricing model. However, recent results from Shanken have provided some empirically based tests which allow for the analysis of whether or not the approximation can be treated as an exact relationship. Shanken (1992) provides sufficient conditions for the pricing relationship (1.4) to hold (section 1.2 proposition 1 of Shanken (1992)). If a proxy for idiosyncratic risk is not a source of priced risk then equation (1.4) holds exactly. This allows for a testable hypothesis since in empirical work it is always possible to specify a proxy for idiosyncratic risk (residual variance proxied by the squares residuals) which can be entered as a factor into the APT specification and tested whether it is priced or not.

This debate surrounding the testability of the APT must not detract from two basic points. First, the innovative developments based on arbitrage offered by Ross (1976) and refined and expanded by others which have provided fresh insights into the possible determinants of asset returns. Second, we know that asset prices do react to changes in economic variables and therefore it is interesting to investigate the sources of systematic risk which affect every asset return. If we can identify a unique return generating process, then we can improve our understanding of the relationship between asset prices and the economy.

1.3 Testable Implications Of The APT

Before discussing the possible techniques aimed at identifying the number, and sources of risk premia in asset markets this section outlines some testable implications of the APT. The most important hypothesis of the APT is that return generating process is unique, that is, the number of factors identified and their associated risk premia are identical for each stock, or group of stocks. Moreover, for the APT to have any empirical content it must be possible to use the estimated factors and their risk premia to price any other asset outside the estimation sample of assets. Failure to confirm this hypothesis would imply that a fundamental result of the APT is violated. A second important issue is the performance of the APT with respect to some alternative hypothesis, either a specified hypothesis such as a firms own variance (a measure of idiosyncratic risk) should not be priced, and the APT should be robust to an unspecified alternative. For example, the residuals from a test of the APT should have a mean of zero. Rejection of this hypothesis for the assets under consideration would imply that while the APT can price some stocks, there is a proportion of other stocks which it fails to account for.

A further testable implication of the APT is its ability to explain average returns. Although the APT may be robust to specified and unspecified alternatives, and there may be a unique factor model, if it fails to explain average returns then its practical usefulness is limited. However, examining the absolute level of the APT's ability to explain average returns could be rather fruitless. For example, assume we have a robust APT model that can explain, say, 30% of average cross sectional returns. What does 30% imply, good performance or poor performance ? We would know that if the APT could explain 90% of average cross sectional returns this would be deemed good performance. However, the most important issue is its performance relative to an alternative asset pricing model. An important point to note is that however elegant a theory may be, at the end of the day its usefulness is judged on its practical applications and policy implications. To this end tests of the APT must, at some stage, involve a comparative analysis vis-a-vis an alternative such as the CAPM.

Empirical tests of the APT are usually undertaken without any regard for the specification of the econometric model. While the estimated parameters and their standard errors depend on the validity of the assumptions regarding the residuals, these are never analysed. However, if valid inferences are to made regarding the parameters and their statistical significance it is important that the residuals are robust to serial correlation and heteroscedasticity. Although this does not constitute a test of the APT it is an important issue regarding the practical usefulness of the model in terms of policy implications and portfolio management.

A further test of the APT is its robustness to seasonalities in stock returns. One of the empirical weaknesses of the CAPM is its inability to explain seasonal patterns in stock returns. This has led researchers to conclude that seasonal patterns in stock returns are anomalies to the efficient markets hypothesis. However, such claims involve a joint hypothesis that, first the asset pricing model is correct, and second seasonal patterns exist. Evidence indicating that the seasonalities exist in stock returns outside of the risk-return structure may be a result of an asset pricing model that does not correctly model the risk-return structure. Thus, a test of the APT could involve analysing whether the factors can account for the seasonal patterns in stock returns.

An important caveat is in order regarding the robustness of the APT to alternative hypotheses and tests. If the estimated APT model is not robust to alternative hypotheses and tests this does not imply that the theory is invalid. Rather, as we will see in the next sections, it is likely to be more symptomatic of problems in the estimation methodology employed.

1.4 Identifying Systematic Risk Premia : Statistical Techniques

Initial tests of the APT employed factor analysis as a technique for extracting the relevant factors (see, for example, Roll and Ross (1980)). Factor analysis is a statistical technique which aims to find partial correlations amongst a set of variables (in the case of the APT stocks) and factor these out such that when the factors are held constant the partial correlations vanish. In terms of tests of the APT the procedure is two fold. Firstly, factor analysis is run on equation (1.2), where $E[R_{it}]$ is replaced by a constant, for each individual stock or portfolio in the sample. The f_k's and b_{ik}'s are simultaneously estimated such that a prespecified proportion of the covariance of the residuals are minimized. For example, the researcher may prespecify that when the probability of identifying one more factor falls below 5% then the procedure stops and the number of factors identified at this stage represents the number of systematic risk factors that drive returns. This concludes the first stage. The second step involves performing a cross-sectional regression of the form given in (1.4). This step obtains estimates of the prices of risks, λ_k's, and their statistical significance. The use of factor analysis to extract factors from an APT framework automatically imposes a strict factor model on the return generating process of stocks, since Ω is forced to be a diagonal matrix through the estimation of the factors.

One of the main implication of the APT is that the estimated prices of risk should be the same across individual assets and, or, groups of assets. However, this is a test which has been largely ignored. In the first instance, tests of this hypothesis using traditional factor analytical techniques are not possible because it is not possible to know whether the first factor extracted is the same in one group of securities as another group of securities. A second testable hypothesis which is possible from a factor analytical framework is that the estimated constants in the empirical specification of the APT are equivalent. It is well known that the constant in factor analysis tests are unaffected by the choice of rotation of the factors and thus across samples/groups of securities they should be equal. If returns are expressed as excess returns then the intercept should be equal to zero. Alternatively, when returns are in 'raw' format then the intercept should be a measure of the zero beta or risk free rate. A joint test across securities/groups should enable some inference about the validity of one of the APT's predictions. This test has a further implication, namely that of mispricing in the APT (see Connor and Korajczyk (1988)). Assuming that there is no measurement error in the estimates of the factor and their loadings then the difference between the intercept and its theoretical level (either zero or equal to the risk free rate) is a measure of mispricing. This can lead not only to inferences regarding the performance of the APT model per se but can also be a criteria for judging the performance of competing models. Furthermore, augmenting the empirical APT with instruments such as measures of a stocks own variance allows a test of the APT against a specified alternative.

The first major test of the APT was the study of Roll and Ross (1980) who used daily returns on 1260 NYSE and AMEX stocks over the period July 3rd, 1962 to December 31, 1972. The stocks were grouped alphabetically into 42 portfolios of thirty stocks. Using factor analysis to extract the number of factors and their factor loadings and then using the cross-sectional Fama-MacBeth procedure to estimate the sign, size, and statistical significance of the risk premia they find three to four systematic risk factors are priced. Roll and Ross carried out a number of tests to analyse the robustness of the APT. First, they augmented the return generating process by including a measure of firms own variance along with the extracted factors. If the measure of own variance is significant then the APT fails an acid test that idiosyncratic risk is not priced. After correcting for positive skewness that creates dependence amongst sample means and sample standard deviations, they found that own variance is a priced factor in 21% of the groups. A second test that Roll and Ross perform is to examine whether the estimated risk free rate λ_0 is significantly different from zero. If the APT is the correct model, in each portfolio the estimate of λ_0 should be identical. Using an Hotelling T^2 statistic they could not reject the hypothesis that the risk free rate is equal across portfolios.

Even though Roll and Ross (1980) stress that their investigation is only preliminary it is worth noting that there are a number of associated problems. In the first instance, as noted above, there is no guarantee that the factors found significant in one sample of securities will be the same as those from another sample. This does not facilitate analysis of the major thrust of Ross's (1976) work that k factors generate returns. Furthermore, the use of factor analysis imposes an Errors in Variables (EIV) problem since in the cross-sectional stage of the analysis estimates of the b_{ik}'s are employed instead of their true values (see Shanken (1992) for a discussion of this problem and the effects on estimated prices of risk). Nevertheless, these problems associated with the use of factor analysis did not discourage researchers from concentrating on the estimation and analysis of mulitfactor models.

Following this seminal paper by Roll and Ross (1980) a plethora of tests of the APT using factor analysis have appeared in the literature. Chen (1983) uses returns on daily data over the period 1963 to 1984 to form four sub-periods for analysis and in each of these periods uses a sub-sample to estimate the factor sensitivities and form factor mimicking portfolios. The rest of the stocks in each sub period are used to estimate their factor sensitivities from the factor mimicking portfolios. Using a five factor APT Chen finds that the vector of risk premia obtained from a cross-sectional regression are significantly different from zero. Estimating a CAPM, Chen performs a number of tests for the competing models. The residuals from the CAPM, if the APT model is correct, should be explained by the APT factors. Using an OLS regression of the residuals from the CAPM on the APT factors Chen confirms that these residuals are explained by the APT factors. A second test that Chen performed is to take the fitted values from the APT and the CAPM and regress actual returns on these. If the APT is the correct model then the coefficient on the APT fitted values should be equal to one, and zero for the CAPM. Although Chen often rejects the hypothesis that the coefficient on the APT's fitted values is equal to one, he finds that the actual estimates of the coefficient on the APT's fitted values are very close to one.[2]

[2] There is a problem with this test proposed by Chen, namely it is constructed in favour of acceptance of the APT. This stems from the use of factor analysis, or any other statistical procedure that extracts the factors. If

Chen (1983) also finds that when stocks are ordered into portfolios according to the size of the own variance, and forced to have the same factor sensitivities there is no discernable difference in the expected returns. A similar procedure was undertaken to test for a firm size effect. In only one of the four sub-periods did firm size matter. However, this result contradicts results from Reiganum (1981), who, using the same estimation procedure, concludes that the APT can not explain the size anomaly. It appears that the multifactor model of stock returns, while identifying more than one factor as significant in the return generating process, also outperforms its predecessor the CAPM, both in term of the tests employed by Chen (1983), and also in terms of explaining some empirical inconsistencies which remain with the CAPM.

In an attempt to test the APT's most important prediction, that of the estimated prices of risk being the same in each sample, Brown and Weistein (1983) using the data set of Roll and Ross (1980) use a factor analysis technique which they term a 'bilinear paradigm' which tests whether two equations (portfolios) can be constrained to have the same risk premium. Their results for a three, five, and seven factor APT reject the null hypothesis of the risk premium being the same. Even after correcting for low power in the test they still reject the null hypothesis 50% of the time.

---

the first factor extracted from the APT is correlated with the market portfolio it is very likely that the correlation will be very high. Consequently the APT will outperform the CAPM in such a test as long as the APT has more than one significant factor.

In a series of papers Dhrymes, Friend, and Gultekin (1984) (DFG), Dhrymes, Friend, Gultekin, and Gultekin (1986) (DFGG) and Roll and Ross (1984) debate the usefulness of using factor analysis in empirical tests of the APT. In a critical review of the results from Roll and Ross (1980), DFG (1984) point out a number of major problems in using factor analysis. First they note there is no way of determining whether any one particular factor is priced or not since a t-test on an individual risk premium is meaningless. The only test possible on the estimated risk premia is a F-test on their joint significance. However, this requires that we know, *a priori*, the true number of factors in the return generating process. Second DFG state that factor analysing small groups of say thirty securities is not the same as factor analysing a large group of securities. Consequently, tests of the APT such as Roll and Ross (1980) and subsequent work employing small portfolios of securities are not sufficient to have the conditions required for the APT to hold. Thirdly, DFG find that as the number of securities in a portfolio is increased the number of factor extracted also increases. This violates the APT prediction that a small set of factors will be able to price all other subsets of assets.

In a reply to this Roll and Ross (1984) share the view with DFG (1984) that the joint test of significance is the only way test the empirical APT when factor analysis is employed, something which they claim they were well aware of in their original 1980 paper. Furthermore, Roll and Ross (1984) state that there is one test that is not invariant to the rotational patterns of factor analysis and this is the test of the intercept term in the empirical APT. The intercept should be equal to the risk free rate of return or alternatively equal to zero depending on the form of the returns in the portfolios. Thus, one way to test the APT is to test this hypothesis. In response to the claim by DFG that factor analysing a sub-group of securities is not the same as factor analysing the whole sample of securities Roll and Ross (1984) claim this technique is consistent and while obviously it would be more desirable to factor analyse the whole sample it is unfortunately impossible due to computational issues. With regard to the point that as the number of securities in a portfolio is increased the number of factors extracted increases Roll and Ross (1984) agree that this is a natural consequence of factor analysis and these extra extracted factors are industry or firm specific factors and simply will not be priced and thus are unimportant. Roll and Ross (1984) claim that the acid test of the APT is not the number of factors extracted but rather the performance of the model in terms of explaining returns. The problem with this argument is two fold. First, how do we know which of the factors are priced and which are not ? and second, although it is important that the APT does a good job in explaining returns, the number of factors extracted can not be relegated to second place when one considers a test of the APT. This is because the APT says that the number of factors in the return generating process is unique. Only when we know that the factors are systematic risk factors can we do such a test for uniqueness. The weakness of factor analysis is that it is not possible to answer this question. In a further paper DFGG (1986) found that in addition to the number of factors increasing as the number of securities in the portfolio increases, the number of factors extracted also increases when the number of observations in the sample increases.

In an attempt to perform a more robust test of the APT Cho (1984) uses inter-battery factor analysis which allows restrictions on the extracted factors to be imposed on the factor analysis estimation. In particular one of the main deficiencies of factor analysis is that it is not possible to determine whether the first factor in one group of assets is the same as the first factor in another group of asset. However inter-battery factor analysis allows a constraint that this condition is met in two different groups. Cho (1984) finds that there are five to six inter group common factors.

While all the studies considered so far have employed US data, in the UK two studies which use factor analysis are Beenstock and Chan (1986) and Abeysekera and Mahajan (1987). Beenstock and Chen find that twenty factors are present in the return generating process of UK stocks. They find that the model is linear, the APT is robust to the firm size effect and they reject the CAPM in favour of the APT. The only negative result for the APT was existence of priced own variance terms. However, like many before them the large number of factors extracted is indicative of the inherent problems of factor analysis identified by DFG. Ameysekera and Mahajan (1987) reject the APT as a description of stock returns in the UK. The authors find no evidence to suggest that there is a unique factor model in the UK. They emphasise that these findings may be a result of the large number of firms with relatively small market capitalizations. This would seem to indicate that there is a small firm effect that is biasing the results.

All the above studies have concentrated on testing the APT using factor analysis in a cross-sectional framework. Lehmann and Modest (1988) perform a time series test which involves estimating factor mimicking portfolios using factor analysis and then regressing returns on these factor mimicking portfolios and testing whether the constant in equation (1.2) is significantly different from zero, where $\mathbf{BF}_t$ is replaced by the factor mimicking portfolio. Portfolios were formed on the basis of dividend yield, own variance, and firm size and then the procedure outlined above was employed. They found that while the APT provides an adequate description of the relationship between risk and return for the portfolios formed on the basis of dividend yield and own variance, it was unable to explain the firm size effect.

The inherent problems of factor analysis are discussed by Raveh (1985) who quotes Harris (1962) and Kaiser and Hunka (1973) who show that the number of factors extracted will increase as the number of variables (securities) is increased. This result is a general feature of factor analysis. Moreover, this result, coupled with the finding of DFGG (1986) that the number of factors increases when the number of observations increases, has led Raveh to suggest that:

> *" essentially it means that.....there is no stable estimation for the loading factors, as well as their number. Hence there are*

no specific structures within each group of series and therefore a small number of factors m is insufficient evidence to support the APT model."

It is evident that the use of factor analysis in tests of the APT is limited. As a consequence the development of alternative statistical techniques to extract the factors has ensued.

The use of principal components in tests of the APT arises for two main and complementary reasons. In the first instance principal components has distinct advantages over the use of factor analysis, and secondly principal components is a natural way to estimate the parameters from an approximate factor model. To see how the latter point arises we need to reexamine the work of Chamberlain and Rothschild (1983). Recall that the main result of Ross (1976) is that the covariance matrix of returns, Σ, can be broken down into two separate matrices:

$$\Sigma = \Psi + \Omega \tag{1.8}$$

where Ψ is the covariance of systematic risk factors, and Ω is the covariance matrix of idiosyncratic risk. Chamberlain and Rothschild (1983) show that Ω need not be diagonal and as the number of assets is increased then as long as the first k eigenvalues of Σ are unbounded and the k+1 eigenvalue is bounded then the first k eigenvalues are the systematic risk factors. This relationship corresponds to the definition of principal components and consequently it would seem natural to estimate the factors for the APT using this methodology. Returning to the former reason for the use of principal components, there are numerous advantages in extracting the factors this way rather than using factor analysis. First, it is possible to use t-tests on the extracted factors (eigenvectors) from principal components because they are unique, unlike factor analysis where they are only unique up to an orthogonal rotation. Second, the size of the eigenvalue represents its importance relative to other factors. Accordingly then, it is possible to compare components extracted from different groups of securities, that is, it is feasible to examine whether the first component in the first group of securities is the same as the first component in the second group of securities, and so on, a feat not possible with using factor analysis.

A further advantage of principal components, noted by Shukla and Trzcinka (1990), is that tests of the null hypothesis are more likely to be rejected because of measurement error than when using factor analysis. The reason for this is that factor analysis imposes fewer restrictions on the data than does principal components. Thus, *"a finding of no evidence against the eigenvectors-based model is stronger evidence in favour of the APT than the same finding with the factor based model."* (Shukla and Trzcinka (1990) p. 1542).

While the above final point is true it does, nevertheless, pose a problem.

Traditional methods of principal components do not estimate Ω but rather assume that all idiosyncratic risk is equal for each security and equal to zero (only then can the eigenvectors be used to measure systematic risk). In a finite economy this assumption may well be violated. In this case factor analysis will provide better estimates of the factor loadings since it simultaneously estimates the factor loadings matrix **B** and the variance of individual securities. Therefore the choice between factor analysis and principal components becomes one of whether we can assume that the economy is large enough, in terms of the number of assets, to make the assumption of equal residual risk.

A review of the performance of factor analysis and principal components is provided by Shulka and Trzcinka (1990). Unlike many previous studies using statistical methods to extract the factors/components weekly data rather than daily data was utilised. They note that the choice of weekly data arises through three considerations: first as observed by Roll and Ross (1980) there is skewness in daily data which can bias test results; the second consideration stems from Fama's (1976) finding that normality is not a good assumption to make regarding daily data; and finally Shanken (1987) shows that the covariance matrix estimated using daily data produced poor estimates of the factor structure. Their tests concentrate on two major themes, the size of the pricing error and the explanatory power of the model. Their comparisons take the following form: a one factor model, a one eigenvector model and a market model (β's were calculated using both an equally weighted and a value weighted market index) were estimated and comparisons were made regarding the above two criteria. In addition to this a five factor model and a five eigenvector model were compared. The pricing error was calculated by first estimating the factors, eigenvectors and the β's and then estimating the following cross-sectional regression:

$$R = \alpha + B\lambda + u \tag{1.9}$$

where **R** is a $N \times 1$ vector of average returns, **B** is a $N \times k$ matrix of systematic risk, λ is a $k \times 1$ vector of estimated risk premia, **u** is a $N \times 1$ vector of residuals, and α is a $N \times 1$ vector of intercepts. Since returns in Shulka and Trzcinka (1990) are not excess returns then the constant should be a measure of the risk free rate. Any deviation of the intercept from this is taken as the pricing error. In the first set of tests (1 factor/eigenvector/β) **B** is a $N \times 1$ vector and obviously the model with the best estimate of systematic risk will have the best fit and should therefore have the lowest pricing error. In the case of the five factor/eigenvector model then **B** is a $N \times 5$ matrix. For the case when **B** is $N \times 1$ they find that the 1 eigenvector model has the best fit with an adjusted R^2 of 34.17% and the lowest pricing error. With respect to the five factor/eigenvector model they find that these models do a better job than the single factor/eigenvalue/β models. Their results suggest that the five eigenvector model does at least as good a job as the five factor model. Interestingly they find that the 1-vector model has a smaller empirical pricing error then the 5-vector or 5-factor model.

The use of principal components has been pioneered by Connor and Korajczyk (1986,88,93) who adopted this technique to estimate the approximate factor model of Chamberlain and Rothschild (1983). The focus of attention in Connor and Korajczyk (1988) is to examine the pricing error in the APT and the CAPM. In order to facilitate this they estimate the following regression:

$$R_t = \alpha + BF_t + v_t \tag{1.10}$$

where: $\mathbf{R}_t$ is a vector of excess returns; $\mathbf{B}$ is a matrix of asset sensitivities to the vector of factors $\mathbf{F}_t$; and $\mathbf{v}_t$ is a vector of residuals. The mispricing in the APT is given by the level of the intercept in (1.10). They compare the size of the pricing error for the CAPM and the APT and show that the CAPM has a smaller pricing error. However, they point out that this finding may be due to the fact that the APT fits better and may reject more often because deviations are measured more accurately. This point is supported when the actual size of the intercepts are examined: the APT's intercept has a lower value.

In an extension to this work Connor and Korajczyk (1993) derive a test statistic for the number of factors extracted using principal components when Ω is no longer assumed to be diagonal. This test statistic is based on the result that when moving from the k factor model (assumed to be the correct model) to a k+1 factor model then there should be no reduction in idiosyncratic risk. The results of DGF (1984) show that as the number of assets increases then the number of factors increases. Connor and Korajczyk (1993) note that this is because there will be increased correlations amongst assets idiosyncratic returns. While these are not common factors, techniques which employ a diagonal Ω will pick these correlations up as common factors. Using monthly returns for the period January 1967 to December 1991 on all stocks with available data on the NYSE and AMEX stock exchanges they find evidence for between one and six factors, outside the month of January they find only one or two factors.

The developments of Connor and Korajczyk (1993) are important for testing the APT. It is important that tests can account for an approximate factor model since theoretically this model is more appealing in a finite economy. However, while the use of factor analysis and principal components can produce models with a good fit (in terms of R^2's) there is one obvious weakness with such models. It is not possible to place any economic interpretation or intuition on the factors extracted. It would be far more interesting if we could relate these extracted factors to the economy, whether this relationship is between business cycle movements, influences from other financial markets, or general macroeconomic movements. The identification of measured economic factors would allow for more practical applications of the results from tests of the APT. For example, fund managers could identify specific sources of risk and form portfolios which are hedges to such risk. Given the attractiveness of relating stock returns to measured economic state factors there is an increasingly expanding body of work in this area and we review this in the next section.

1.5 Identifying Systematic Risk Premia - Macroeconomic And Financial Factors

The identification of macroeconomic and financial variables as sources of risk premia in the stock market can only increase our understanding of price movements in financial markets. In this section we discuss the use of macroeconomic and financial variables as factors in the APT by reviewing the extant literature.

The work of Chen Roll and Ross (1986) (CRR) provides a watershed in tests of the APT by prespecifying a number of economic state variables as candidates for sources of risk premia in the US stock market. There is no general underlying theory which identifies certain macroeconomic or financial variables as factors which will command a risk premium in the stock market. However, CRR utilize the present discounted dividend formula as a rationale for identifying candidates for factors that may carry a risk premium:

$$P_{it} = \sum_{t=1}^{\infty} \frac{D_{it+1}}{(1 + \delta)^t} \tag{1.11}$$

where P_{it} is the price of stock i in time period t, D_{it+1} is the dividend paid in time t+1, and δ is the appropriate discount rate. From the present value model the rationale for choosing economic factors is that any variable which affects the discount rate or affects the future stream of dividends will affect the present stock price. Although the choice of economic state variables is still somewhat arbitrary in that the formula does not identify which variables are important, the present value formula provides a framework from which to prespecify likely candidates. CRR choose two groups of factors, those thought to influence expected future dividends, and those thought to influence the discount rate. With respect to the future dividends they postulate the following variables: the change in expected inflation (calculated as the expectation of inflation in time t+1 measured in time t minus expected inflation in time t measured in time t-1); unanticipated inflation (measured as actual inflation in time t minus the expectation of inflation in t generated at time t-1); and changes in real industrial production. For factors expected to influence the discount rate they postulate a measure of the term structure of interest rates and a measure of the risk premium (default risk). The term structure of interest rates is measured as the return on long government bonds in time t minus the return on short government bonds in time t-1, and the risk premium is measured as the return on a portfolio of low grade corporate bonds in time t minus the return on long government bonds in time t. The expectation of inflation was generated using the Fama-Gibbons (1984) methodology. Given that variables such as inflation and industrial production are announced monthly tests of the APT employing this methodology use monthly stock returns. In addition to those factors specified above CRR also specify a yearly industrial production factor.

If agents are rational and they observe that, say, industrial production carries a risk premium then they will form an expectation of next months change in industrial production and this expectation will be impounded in the current price of the stock. Consequently only unexpected changes (innovations) in industrial production can cause stock prices and hence returns to change. Thus, once the factors are prespecified, innovations in these factors must be formed. This issue, or rather the neglect of this issue in previous tests of the APT, will be addressed in the section 1.6 of this review.

In addition to these macroeconomic factors CRR also specify two stock market indices, one equally weighted and one value weighted. There are a number of reasons for this. CRR specify these indices in order to capture any factors omitted from the list above. Additional reasons for including market indices include the observation that if a market index is the only factor then, essentially, the model collapses to the CAPM. Also the market portfolio can include future expectations about the growth rate in the economy as a whole, something the researcher would find difficult to do (see Chan, Chen, and Hsieh (1985)).

Having specified the likely candidates CRR formed twenty equally weighted portfolios of stocks on the basis of market value over the period 1958 to 1984, the sample was split into three sub-periods as well as the whole sample period. The reason for forming portfolios is three fold: first it provides a good spread of risk and return which in turn improves the discriminatory power of the cross-sectional regression tests; second forming portfolio reduces the noise in individual stocks; and third, the use of the two-step procedure involves an errors in variables (EIV) problem since in the cross-section part of the regression estimates of the sensitivities are used rather than the actual true sensitivities. Black, Jensen, and Scholes (1972) and Fama and MacBeth (1973) show that forming a portfolio of stocks is likely to produce estimates of the sensitivities which are closer to the true sensitivities.

Following a variant of the Fama and MacBeth (1973) two-step estimation technique CRR, using an initial sample of sixty months of data, estimate time series regressions in order to estimate the factor sensitivities (b_{ij}'s). These sensitivities are then the independent variables in the next period, used to estimate the risk premia in a cross-sectional regression, one regression for each month in the following year. After the final month cross-sectional regression in the year, the sensitivities are updated to include the previous year (the year from which the first risk premia were estimated) in a new time series regression. This then allows another years risk premia to be estimated in a further cross-sectional regression. Once all years have been included in the sample a time series of risk premia is generated from which a t-test can be calculated to test the significance or otherwise of the estimated risk premia.

CRR found five factors were adequate to describe the return generating process, these being monthly shocks to industrial production, the term structure of interest rates, changes in the risk premium, unanticipated inflation, and changes in expected inflation. Interestingly, CRR found no role for the market indices. Similar results to these are reported in Chan, Chen, and Hsieh (1985) (CCH) who, adopting the same framework and factors as CRR, examine the firm size effect. They find that three factors carry significant risk premia: industrial production, unexpected inflation and the default risk. The residuals from their firm size portfolios are not significantly different across portfolios and consequently they claim that their APT model explains the firm size anomaly.

With regard to the UK, Clare and Thomas (1994) adopt the methodology of CRR and forming portfolios on the basis of firm size, over the period January 1978 to December 1990, find that two factors are important in the pricing of UK stocks: the retail price index and the comfort index.[3] To check the robustness of the APT to the ordering of stocks by different criteria they formed portfolios on the basis of security betas and repeated the estimation process. In this case the factors found significant were the two above plus the excess return on the market index.

Further tests of the APT in the UK employing this methodology are Poon and Taylor (1991) who form portfolios according to firm size over the period January 1965 to December 1984 and use the same factors specified by CRR but for the UK. They find that there is no relationship between any of these

---

[3] The comfort index is the ratio of consol to equity dividend yield and is used, according to Clare and Thomas (1994), by analysts as a means of evaluating the relative cheapness of equity.

factors and UK stock returns. Beenstock and Chan (1988) use a variant of the two-step procedure and find, forming portfolios over the period October 1977 to December 1983 according to a level of returns ranking, that four factors are priced in the UK stock market: an interest rate variable; sterling M3; and two inflation measures.

The results from these studies seems to provide support for the APT. However on further consideration there appears to be a subtle paradox materializing, namely that both in the UK and in the US different studies, independently, claim to find support for the APT, (except Poon and Taylor (1991) for the UK). However, using data over overlapping periods these studies find different factors to be significant and insignificant. This would seem to indicate that one of the conditions for the APT to hold, that of a unique return generating process, is violated. The non-uniqueness of the APT and non-robustness of the APT to alternative portfolio formation criteria is more likely to be symptomatic of the estimation techniques and the need to form portfolios within this framework than the lack of content in the theory. One of these particular issues is raised by Shanken and Weinstein (1990) and Shanken (1992) who reexamine the results of CRR and CCH using their factors and a similar time period. Shanken (1992) notes that the EIV problem inherent in the two step procedure needs to be corrected for, and the correction he derives is applied in Shanken and Weistein (1990). The result of this correction is to increase the standard errors of the prespecified factors and hence decrease the t-ratios. A further adjustment to the way portfolios are formed is applied. This involves forming portfolios at the beginning of the year of estimation rather than using the year end market value. This is to avoid inducing correlation between the estimation error in beta and the allocation of firms to portfolios.[4] The effect of these adjustments are such that only one factor, industrial production, is statistically significant over the whole sample period while none of the factors are significant in any of the three sub-periods. Furthermore, Shanken and Weistein (1990) find that the market portfolio is statistically significant. Thus, the implication from the studies above that measured economic state variables can render the market portfolio as insignificant are brought into question. This must, in some senses, question the ability of the APT to outperform the CAPM when using observed economic state variables.

An alternative framework for estimating the sensitivities and the risk premia for the APT is provided by McElroy, Burmeister, and Wall (1985). Extending the results of Gibbons (1982) they provide a time series econometric framework which allows the APT to be tested using individual security returns based on Non Linear Least Squares (NLLS) which simultaneously estimates the sensitivities and the prices of risk. This is an

---

[4] See also the so called data snooping bias (Lo and MacKinlay (1990)) in using portfolios that are formed using empirical criteria such as size rather than theoretical criteria such as dividend yield or own variance. The danger here is that grouping securities into portfolios using empirical criteria could lead to over rejection of the theory under investigation. For example, in considering Lehmann and Modest's (1988) test of the APT, Lo and MacKinlay (1990) show that if portfolios are ordered by size, the null hypothesis is rejected whereas adjustments for the fact that size is used as the criterion for grouping portfolios no longer leads to rejection.

important development since it automatically overcomes the two troublesome areas outlined in the last section. With the simultaneous estimation of the APT's parameters there is no need to form portfolios because the EIV problem is no longer relevant. Thus, within this non-linear framework more robust tests of the APT are possible.

As well as overcoming the problems with the two-step methodology, the NLLS technique has a further major advantage. Substitution of (1.4) into (1.2) yields:

$$R_{it} = \sum_{j=1}^{k} b_{ij} f_{jt} + \sum_{j=1}^{k} b_{ij} \lambda_j + \epsilon_{it} \tag{1.12}$$

It is evident that (1.12) contains non-linear restrictions that the price of risk (λ_k) is equal across all assets. The advantage of this is that, unlike previous techniques it allows the APT's principle conclusion that the price of risk is equal across every security to be tested. These non-linear across equation restrictions provide a system of single-security equations that are seemingly unrelated, and consequently can be estimated using a Non-linear Seemingly Unrelated Regressions (NLSUR) estimator. Using US data on seventy individual securities over the period January 1972 to December 1982 McElroy and Burmeister (1988) test the APT using a NLSUR framework. They prespecify four macroeconomic variables, a risk premia variable, the term structure of interest rates, unexpected inflation, and unexpected growth in real sales. They also form what they term a 'residual market factor' which is the result of regressing the market portfolio on the factors and using these residuals as a fifth factor. The basis for this is that these market residuals should capture any factor that is not included in the proposed list of measured variables. Four factors are significant at the 5% level while unexpected inflation is significant at the 10% level. The across equation restrictions that the prices of risk are equal are accepted, and when changing the residual market factor for the market index, they find that the four macroeconomic variables do add information to the return generating process over and above that contained in the market portfolio. This provides evidence that the APT performs better than the CAPM.

In an attempt to test the APT with observed and unobserved factors Burmeister and McElroy (1988) specify, along with three observed macroeconomic factors, three large portfolios to represent unobserved factors. An interesting feature of this work is the role of the market index as an unobserved factor. Burmeister and McElroy (1988) show that if the market portfolio enters as a factor into the APT specification then it should be treated as endogenous. Given this endogeniety a non-linear three stage least squares (NL3SLS) estimator is used[5]. As instruments Burmeister and McElroy choose returns on single assets that are not included in their estimation sample of seventy stocks. This choice of instruments is valid only when there is zero

---

[5] In addition, if we are interested in determining the risk premia associated with the equity market as a whole (proxied by the market portfolio) then we can specify an equation in the system for the market portfolio. As a result, the market portfolio becomes an endogenous variable on the right hand side of the remaining equations in the system.

correlation amongst the idiosyncratic elements of the individual stock returns, that is, when stocks follow a strict factor model. Burmeister and McElroy (1988) assume stocks follow a strict factor model and force the covariance matrix of idiosyncratic returns to be diagonal. Their results, which also include a test for a January effect, indicate that there are substantial differences to the significance of the estimated prices of risk under the assumption of endogeniety of the market portfolio. They find that their model of the APT can not explain the January effect, however they accept the APT in favour of the CAPM and the across equation restrictions the APT places on the linear factor model are accepted.

The developments of McElroy, Burmeister, and Wall (1985) offer a more attractive econometric framework from which to estimate the parameters of the APT. Furthermore, the results of Burmeister and McElroy (1988) provide an important insight on the role of the market portfolio in tests of the APT. Nevertheless there remains a number of unresolved issues. First there is an issue concerning the assumption that stocks follow a strict factor model which needs further clarification. If stock returns actually follow an approximate factor model then one can no longer use individual stock returns outside the sample as instruments. Moreover, if the results of tests of the APT are sensitive to the specification of a strict or an approximate factor model then this issue takes on increasing importance in evaluating the performance of the APT. In Chapter two of this thesis we address these two issues by testing whether stocks follow an approximate or strict factor model, and we show the dangers of estimating a strict factor model when returns actually follow an approximate factor model. These issues are important since to make inferences regarding the results and tests of the APT we must be at least approaching the issues from the correct angle. Secondly, while researchers have independently identified alternative sources of priced macroeconomic and financial factors, there is no supporting evidence that shows the factors are the same in another sample of assets over the same period. Given that the central conclusion of the APT, that there exists k factors which, once identified, should price all securities, has not been addressed it is apparent that any test of the empirical validity of the APT should incorporate this. An issue which is not a testable implication of the theory but rather can effect the results from tests of the APT when dealing with macroeconomic and financial factors is the issue of generating unexpected components in the factors. A necessary condition for the unexpected components is that they are mean zero and serially uncorrelated. However, how this condition is achieved may have implications for the number and estimated sign of statistically significant prices of risk. This concern is addressed in the next section.

1.6 Macroeconomic Variables, Expectations And The Apt

An issue which has received little concern from researchers in tests of the APT is the generation of innovations that enter as factors in the APT. As noted earlier, agents are assumed to form conditional expectations of the change in economic variables that carry a risk premia. As such then this expectation will be incorporated into the price of the stock before the announcement of the level of the variable. As a consequence of this only shocks in the variables will matter, that is, the innovation in the factor is the risk that agents hold.

In an attempt to capture this innovation researchers have used, in all but one case, either a 'rate of change' methodology or autoregressive techniques to eliminate any systematic dependencies that may still be present after first differencing. An alternative approach to generating innovations was adopted by Burmeister, Wall, and Hamilton (1986) who use Kalman filtering to extract an expectation of their inflation series to construct the innovation in inflation.

The problem with using the 'rate of change' methodology is clearly evident from CRR (1986, table 3 page 392) who provide the autocorrelation coefficients for their series of "innovations". The series are clearly autocorrelated in all cases when one examines the Box-Pierce statistics. Consequently they do not satisfy the requirement of the APT that they are innovations. In order to overcome this problem Beenstock and Chan (1988) and Clare and Thomas use autoregressive (AR) time series models to eliminate this systematic element in the first difference of the series. While this may satisfy the assumption that the unexpected components are innovations there is still an issue remaining. The APT assumes that agents are rational and if this is the case then any time variation in the series of factors will be noted by agents and subsequent adjustments regarding expectations will be made.

Unfortunately, AR models do not allow for this and given a change in time variation of one of the parameters in the AR models this will cause agents to make systematic forecast errors. Thus, it would seem more plausible to allow agents expectations formation process to be updated at each period to avoid these possible systematic forecast errors. One technique that allows for this is the Kalman filter. We employ this procedure in order to generate unexpected components.

1.7 Seasonalities And Stock Returns

The existence of seasonal patterns in stock returns has been the focus of much investigation. It is apparent that seasonal patterns do exist and consequently the interesting question that arises is whether or not they can be rationalised within an asset pricing framework. Failure of the asset pricing to explain these patterns has implications for the Efficient Markets Hypothesis (EMH). For example, Banz (1981) finds that a January dummy variable adds explanatory power to the CAPM. This, along with other results, has led to the term *'The January Anomaly'*. The rationalisation of such findings rely on such arguments as a tax effect or portfolio rebalancing. However, these explanations are somewhat clouded by the findings of some researchers (see, for example, Tinic and West (1984) and Gultekin and Gultekin (1987)) that the asset pricing model can account for the seasonality in the risk return structure but fails then to account for any other month in the year in the risk return structure.

The problem researchers face when analysing such issues is that they are faced with a joint hypothesis. On the one hand there is the hypothesis regarding market efficiency and on the other the assumption that the asset pricing model chosen is empirically valid. Claims of market inefficiency because a seasonal month can not be accounted for in the risk return relationship may well be due to an incorrectly specified asset pricing model. In essence, the testing of whether or not the APT can account for seasonal patterns in the risk return structure and, if they are present, analysing whether the APT has any empirical content once seasonal patterns are removed from the data, is another test of the robustness of the APT as well as an analysis of market efficiency. In addition, by specifying observed factors it may well be possible to examine why these seasonal patterns exist. These issues, as well as a more thorough review of the literature on seasonalities are considered in chapter 6.

1.8 SUMMARY

The contribution of this thesis is to reassess the empirical validity of the APT by providing a new framework for estimation of the APT. This new framework addresses a number of assumptions and preconditions that are made in tests of the APT and which have been ignored in empirical tests of the model. The above review has identified these preconditions and assumptions. The mixed results regarding the performance of the APT stem from two sources. First, failing to account for the possible effects assumptions and preconditions can have on empirical tests of the APT and second failure to actually consider the main thrust of the APT, that of a unique and stable return generating process. The first assumption that needs to be addressed is the issue of whether or not returns follow a strict or an approximate factor model. To date, all but one empirical test of the APT has employed an estimation technique that assumes that returns follow a strict factor model, whether the factors are observed or statistical in nature. Furthermore, attempts to identify whether stock returns do follow a strict or an approximate factor model are inconclusive. Consequently this issue needs to be addressed and the implications and results of estimating the APT under different assumptions regarding the nature of the return generating process also needs analysis.

The inherent problem of using factor analysis and the lack of economic intuition of this approach and the use of principal components in this respect suggests that the use of macroeconomic and financial factors may provide a more fruitful avenue for research. However, the usefulness of such an approach must be measured relative to a benchmark such as a competing asset pricing model. Establishing the link between asset returns and observed factors involves econometric considerations. The traditional two-step methodology suffers from a number of problems in this respect and consequently tests of the asset pricing model in terms of its predictions, its robustness to alternative hypotheses and competing models are all conditional on the estimator. As a result an alternative procedure which is free from such problems should be employed.

Associated issues with the use of macroeconomic and financial factors are the generation of unexpected components and the specification of expectation generating processes on the behalf of economic agents. As yet it is unknown how the effects of alternative methods of obtaining the factors that enter into the APT can effect the results of tests of the APT. The main issue that arises here is the plausibility of assumptions we make regarding the way agents form expectations and whether it is plausible to accept the assumption that agents will use simple AR models for expectations formation.

The most important issue regarding the empirical performance of the APT is the assumed uniqueness and stability of the return generating process. These issue have yet to be considered in an empirical test of the APT which involve macroeconomic and financial factors. Failure to test these hypotheses in ***any*** test of the APT surely renders that test of the APT unsubstantial and inconclusive regarding its empirical performance. Until results regarding these issues are provided little can be said on the usefulness and empirical validity of the APT.

The plethora of problems surrounding empirical tests of asset pricing models have a bearing on tests of market efficiency. For example, tests of weak form efficiency related to predictable seasonalities and tests of semi-strong efficiency based upon the announcement of publicly available information are all conditional on a joint hypothesis of market efficiency and the asset pricing model used in such tests being correctly specified. As we have seen from this review, the problems entailed in empirical estimation of asset pricing models could well effect the results from tests of market efficiency. It is apparent that such tests of market efficiency should only take place once an empirically valid asset pricing model has been identified.

In the following chapters we systematically address these issues in order to provide a framework from which tests of the APT can be performed free from the aforementioned problems.

CHAPTER 2

STRICT AND APPROXIMATE FACTOR MODELS OF STOCK RETURNS : IMPLICATIONS FOR EMPIRICAL TESTS OF ASSET PRICING MODELS

2.1 INTRODUCTION

Chapter 1 of this thesis identified a number of interesting issues when considering empirical tests of asset pricing models. Of particular importance, on both a theoretical and empirical level, is the notion of an approximate factor structure in the return generating process. In this chapter we address the issue of strict and approximate factor models of security returns from an empirical point of view. In particular, we examine the effects of tests of the APT that are performed from a strict factor model framework and tests of the APT that are performed from an approximate factor model framework. This thesis takes this issue as its starting point since if the structure of the idiosyncratic return variance-covariance matrix matters then all further tests will be conditional on the form that this matrix is assumed to take. Having assessed the possible effects the assumptions regarding the structure of the residual variance-covariance matrix can have on estimators for the APT when observed factors are used as systematic risk factors we precede to test whether a sample of UK stocks have an approximate or a strict factor structure. The remainder of the chapter then illustrates the possible invalid inferences that can be arrived at by making an incorrect assumption regarding the structure of the residual variance-covariance matrix.

To our knowledge the only paper which directly tests the APT from an approximate factor framework is Connor and Korajczyk (1993) who employ a modified test statistic for the number of factors in an approximate factor model estimated via principal components. As most economists would agree, measured macroeconomic factors provide a much richer mantle for interpretation than do unobserved factors extracted from factor analysis and/or principal components. Since all tests of the APT using observable factors have implicitly assumed a strict factor model of stock returns, it is interesting to evaluate this evidence in the light of the alternative structure we can give to factor models of security returns. The contribution of this chapter is to extend the results of Connor and Korajczyk (1993) such that, when and if appropriate, it is possible to estimate an approximate factor model version of the APT when the systematic risk factors are observable. This is an important area of research given the volume of published work concentrating on the relationship between stock returns and observed systematic risk factors. If the nature of the residual variance-covariance matrix matters then the results from these previous studies may well need to be reevaluated.

The framework that we adopt to address these issues is based upon econometric considerations. Principally, if stock returns do follow an approximate factor model then we require an estimator that allows for correlation across idiosyncratic returns. Following the ideas of Gibbons (1982), Burmeister, McElroy, and Wall (1985) provides a framework for estimating the APT. Their insight stems from the observation that by specifying the return generating process of stocks as a linear factor model the APT places across equation non-linear restrictions on this linear factor model. In turn, the APT's parameters can be estimated and the restrictions tested using a non-linear least squares estimator. Unlike the Fama and MacBeth (1973) technique, the principle results of the APT that the price of risk associated with each stock should be equal can be tested within this framework. In this chapter we show that this technique can also be extended to cope with the scenario that arises when stocks have an approximate or strict factor structure. Specifically, by placing restrictions on the residual variance-covariance matrix we can allow for the estimation of a strict or an approximate factor model. It should be noted that the existence of an approximate factor model implies further restrictions than just the identification of contemporaneous correlation of the idiosyncratic returns. Chamberlain and Rothschild (1983) show that the first eigenvalues of the return covariance matrix must be unbounded while the $k+1$th eigenvalue must be bounded, that is, it should be less than the largest eigenvalue from the covariance matrix of factor risk. In this chapter we address the issue of whether or not these restrictions hold and then use this information in the estimation of the APT using observed factors. To our knowledge this is the first time this issue has been addressed when specifying systematic risk factors that are observed. The rest of the chapter is organised as follows. In section 2.2 we illustrate the potential problems that arise when the APT is estimated under the assumption of a strict factor model when in fact returns follow an approximate factor structure. Section 2.3 discusses the role of the market portfolio in tests of the APT and its consequent effect on the assumed estimation technique. In Section 2.4 we test whether a sample of UK stock returns exhibit a strict or an approximate factor structure. In Section 2.5 we provide evidence which demonstrates the pitfalls of estimating a strict factor model when returns have an approximate factor structure. Section 2.6 concludes.

2.2 Strict And Approximate Factor Structures And Empirical Estimation

The essential feature of Ross's (1976) APT is that returns can be expressed as a linear function of a vector of expected returns, a matrix of k systematic risk factors plus a vector of idiosyncratic return, written as:

$$R_t = E[R_t] + BF_t + v_t \tag{2.1}$$

where $\mathbf{R}_t$ is a $N\times 1$ vector of asset returns, $E[\mathbf{R}_t]$ is a vector of excess returns, $\mathbf{B}$ is a $N\times k$ matrix of sensitivities of assets to systematic risk factors, $\mathbf{F}_t$ is a $k\times 1$ vector of systematic risk factors, $\mathbf{v}_t$ is a $N\times 1$ vector of idiosyncratic returns, and the following assumptions are made: $E[\mathbf{F}_t] = E[\mathbf{v}_t] = E[\mathbf{F}_t\mathbf{v}'_t] = 0$. A strict factor model arises with the assumption that the covariance matrix of idiosyncratic return, $\mathbf{\Omega}$, is a diagonal matrix, and thus the covariance matrix of asset returns, $\mathbf{\Sigma}$, can be written as:

$$\Sigma = \Psi + \Omega \tag{2.2}$$

where Ψ is the covariance matrix of systematic factor risk. As long as idiosyncratic risk can be diversified away completely this strict factor model will hold and empirical tests employing these assumptions will yield efficient estimates of **B**.

However, to guarantee that the assumption of zero idiosyncratic risk holds requires, in Ross's (1976) model, the existence of an infinite number of assets. While this condition is sufficient to guarantee zero idiosyncratic risk it is not necessary and has been weakened by Chamberlain (1983), and Chamberlain and Rothschild (1983) who show that the condition of zero idiosyncratic risk can be maintained at the expense of allowing for cross correlation of idiosyncratic risk, that is, Ω is no longer assumed to be diagonal. The problem that arises, and the one we address in this chapter, is, what are the effects of applying econometric estimators that assume Ω is diagonal if it is not the case ?

The aim of this section is to review the estimation technique employed in traditional tests of the APT using observed factors, and show that if the return generating process does follow an approximate factor model then the estimated parameters from traditional tests of the APT are not efficient. To motivate this we recap the Fama-MacBeth (1973) two-step estimation technique and show this leads to inefficient estimates of the parameters when returns have an approximate factor structure. Traditionally tests of the APT have employed a variant of the Fama-MacBeth two-step technique. This involves writing the return generating process as:

$$
\begin{aligned}
r_{1t} &= \alpha_{1t} + b_{11}f_{1t} + \cdots + b_{1k}f_{kt} + \epsilon_{1t} \\
r_{2t} &= \alpha_{2t} + b_{21}f_{1t} + \cdots + b_{2k}f_{kt} + \epsilon_{2t} \\
&\;\;. \\
&\;\;. \\
r_{it} &= \alpha_{it} + b_{i1}f_{1t} + \cdots + b_{ik}f_{kt} + \epsilon_{it}
\end{aligned}
\tag{2.3}
$$

where r_{it} is the excess holding period return on asset i, b_{ik} is the sensitivity of asset i to factor k, f_{kt} is the kth macroeconomic factor at time t, and ϵ_{it} is a stochastic error specific to firm i and is assumed not to be priced by the market. The two-step estimation technique takes the following form: in the first step an OLS time series regression is run equation by equation (equation (2.3)) to estimate the sensitivity of asset i to the macroeconomic factors over an initial period of n years. These b_{ij}'s and the constant are then used in a cross sectional regression to explain the actual return for each month in the next year (n+1). That is, we have one regression for each month with n $\times$ 12 observations for each b_{ij} and the α and this generates estimates of λ. These variables take the same value for each month of year n+1. The portfolio estimation is rolled forward one calender year at a time to give new beta estimates and hence new λ estimates. The time series of these λ's are then treated as a new variable and the following t-test is formed:

$$t_{\lambda_i} = \frac{\hat{\lambda}_i}{s(\hat{\lambda}_i)/\sqrt{N-1}} \tag{2.5}$$

where λ_i is the mean value of the ith estimated price of risk, $s(\lambda_i)$ is the estimated standard deviation of λ_i, and N is the number of observations. Thus the resultant t-tests give the statistical significance, or otherwise, of the estimated price of risk.

In the estimation procedure individual stock returns are replaced by portfolios of stocks. The major rationale for this is that it reduces the well known errors-in-variables (EIV) problem first recognised by Blume (1970). The problem arises because the expected risk-return relationship is in terms of the true b_{ij}'s but in empirical tests estimates of b_{ij} are employed. Blume (1970) shows that the b_{ij}'s of portfolios will be more precise estimates of the true b_{ij}'s than those for individual stocks if the errors in individual b_{ij}'s are not perfectly correlated.

However, the extent to which forming portfolios reduces the EIV problem is questionable. This issue is discussed by Shanken and Weinstein (1990) and Shanken (1992). In particular, they show that inferences about the means of the estimated λ_{jt}'s which are based on the sample variances of the estimates neglect the EIV problem. To tackle this issue Shanken derives a correction for the EIV problem and applies this to the CRR estimates. The results are highly significant. For example, with the correction applied to CRR's unanticipated inflation factor the relevant t statistic falls from the original -1.979 to -1.738. Of additional importance here is the results that the market portfolio, found to be insignificant in previous tests of the APT using observed factors which employ the two step technique, becomes statistically significant.

These problems aside, we now turn to the situation where stock returns are not generated by a strict factor model, but rather are generated by an approximate factor model, and examine the problem of using the two-step estimator in this case. Suppose we have a set of M regressions:

$$
\begin{aligned}
Y_{1t} &= \beta_{11}X_{1t} + \beta_{12}X_{2t} + \cdots + \beta_{1K_1}X_{1t,K_1} + \epsilon_{1t} \\
Y_{2t} &= \beta_{21}X_{2t} + \beta_{22}X_{2t} + \cdots + \beta_{2K_2}X_{2t,K_2} + \epsilon_{2t} \\
&\cdot \\
&\cdot \\
Y_{Mt} &= \beta_{M1}X_{Mt,1} + \beta_{M2}X_{Mt,2} + \cdots + \beta_{MK_M}X_{Mt,K_M} + \epsilon_{Mt}
\end{aligned}
\tag{2.6}
$$

where t = 1,2,...,T. Using matrix notation:

$$
\begin{aligned}
y_1 &= X_1\beta_1 + \epsilon_1 \\
y_2 &= X_2\beta_2 + \epsilon_2 \\
&\cdot \\
&\cdot \\
y_m &= X_m\beta_m + \epsilon_m
\end{aligned}
\tag{2.7}
$$

or

$$y_M = X_M\beta_M + \epsilon_M \tag{2.8}$$

where: $m = 1,2,\ldots,M$. y_M is a $(T \times 1)$ vector of values on the dependent variable, X_M is a $(T \times K_M)$ matrix of values on the explanatory variables, β_M is a $(K_M \times 1)$ vector of the regression coefficients, and ϵ_M is a $(T \times 1)$ vector of the disturbances. Imposing the following assumptions:

$$\begin{aligned} E(\epsilon_M\epsilon_M') &= \sigma_{MM}I_T \\ E(\epsilon_M\epsilon_p') &= 0 \end{aligned} \tag{2.9}$$

and assuming that $(X'_M X_M)/T$ is of full column rank we obtain the following variance-covariance matrix:

$$\Omega = \begin{bmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdots & \sigma_m^2 \end{bmatrix} \tag{2.10}$$

In essence this model obeys all the assumptions of classical linear regression and OLS applied to the system of M equations would produce estimated coefficients that are unbiased, consistent and efficient, where the estimate of β_M is given by:

$$\hat{\beta}_M = (X_M'X_M)^{-1}\, X_M'Y_M \tag{2.11}$$

This estimator forms the basis of the two step estimator in both the time series and cross sectional part of the process and given the assumptions in (2.9) above imposes a strict factor model on the return generating process.

To see how problems with this methodology arise when the return generating process follows an approximate structure we relax the assumption that Ω is diagonal. Classical linear regression assumes that all possible information is used in the estimation of the regression coefficients. However if there exists some other piece of information that is omitted from the regression, then the OLS estimates no longer have the desired properties. In particular, if it is assumed that the disturbances are in (2.8) are mutually correlated then the variance-covariance matrix is now given as:

$$\Omega = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1m} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2m} \\ \cdot & \cdot & \cdot & \cdot \\ \sigma_{m1} & \sigma_{m2} & \cdots & \sigma_m^2 \end{bmatrix} \qquad (2.12)$$

or more concisely:

$$E(\epsilon_m \epsilon_p') = \sigma_{mp} I_T \qquad (2.13)$$

and OLS applied to the system (2.8) will produce estimates that are no longer efficient. An efficient estimator of a system of equations, under certain conditions[6], which have mutually correlated errors is the Seemingly Unrelated Regression estimator of Zellner (1962). In this case, the estimator of β for the system of equations given in equation (2.8) is

$$\bar{\beta} = (X'\Omega^{-1}X)^{-1}(X'\Omega^{-1}y) \tag{2.14}$$

and the variance-covariance matrix of $\bar{\beta}$ is given as:

$$E(\bar{\beta}-\beta)(\bar{\beta}-\beta)' = (X'\Omega^{-1}X)^{-1} \tag{2.15}$$

An efficient estimator of $\bar{\beta}$ would be a generalized least squares estimator which takes account of the information embodied in the correlation of the disturbances. However Ω is unknown, and thus needs estimating. Zellner (1962) provides a method for estimating Ω and β, known as the two-stage Aitken estimator. In the first step OLS is performed and the residuals from this are used to form an estimate of Ω. The resulting estimate of β is:

$$\tilde{\beta} = (X'\hat{\Omega}^{-1}X)^{-1}(X'\hat{\Omega}^{-1}y) \tag{2.16}$$

which is asymptotically equivalent to a generalized least squares estimator and thus asymptotically efficient (it is also equivalent to a maximum likelihood

---

[6] For a system of equations SUR will only improve efficiency if there are different right-hand-side variables in each equation, or if there are restriction across equations on variables that are the same. Otherwise the system collapses to equation-by-equation OLS.

estimator (MLE)). The variance (standard error) of the estimated parameter is smaller than the resultant parameter estimate derived from OLS and thus gives the increase in efficiency.[7]

It is apparent, then, that if the error terms in a system of equations are mutually correlated, failure to take account of this can lead to coefficient estimates that are inefficient. Bringing these points back to the issue of stock returns, if we have an approximate factor model, ie, one where error terms are mutually correlated, then the traditional OLS two-step estimator will lead to inefficient estimates of **B**. To overcome this problem we can formulate the estimation of the asset pricing model with an approximate factor structure in a seemingly unrelated regression framework.

Such a framework has been developed, although from a different rationale, by McElroy, Burmeister, and Wall (1985) who set up a non-linear seemingly unrelated regression (NLSUR) framework in order to first impose the non-linear parametric across equation restrictions that the APT implies, namely that the price of risk must be the same for all securities, and second to simultaneously estimate **B** and λ, thus avoiding the EIV problem. To see how this NLSUR framework is formulated, specify the APT equations for a single asset as a linear factor model:

---

[7] For proof see Kmenta (1986 p.641-642)

$$r_{it} = \alpha_i + \sum_{j=1}^{k} b_{ij} f_{jt} + \epsilon_{it} \qquad (2.17)$$

where $E[r_{it}]$ is replaced by a constant. Note that $E[r_{it}]$ is defined as:

$$E[r_{it}] = \lambda_0 + \sum_{j=1}^{k} b_{ij} \lambda_j \qquad (2.18)$$

where λ_0 is the risk free asset, and λ_j is the price of risk on the jth factor, common across all assets. Now, substitute (2.18) into (2.17) to give,

$$R_{it} = \lambda_{0t} + \sum_{j=1}^{K} b_{ij} \lambda_j + \sum_{j=1}^{K} b_{ij} f_{jt} + \epsilon_{it} \qquad (2.19)$$

Equation (2.19) is a system of N nonlinear regressions over T time periods which entails T observations on N returns, from which K λ_j's and NK b_{ij}'s are to be estimated. It is clear that (2.19) places nonlinear restrictions on the linear factor model (2.16). Specifically, these are $\alpha_i = \sum_{j=1}^{k} b_{ij} \lambda_j$ Rewriting (2.19) in terms of excess returns for an individual asset we have,

$$R_i - \lambda_0 = \sum_{j=1}^{K} (\lambda_j \iota_T + f_j) b_{ij} + \varepsilon_i \qquad (2.20)$$

where $i=1,\ldots,N$ and N is the number of assets, $j=1,\ldots,K$, ι_T is a T vector of ones and $\mathbf{R}$, λ_0, $\mathbf{f}$ and ϵ are all Tx1 vectors, where T is the number of observations. Equation (2.20) can then be written as:

$$R_i - \lambda_0 = \left[(\lambda' \otimes \iota_T) + F\right]B_i + \varepsilon_i \tag{2.21}$$

where λ is a Kx1 vector of prices of risk, **F** is a TxK matrix of observations on the K factors, $\mathbf{B}_i$ is a Kx1 vector of sensitivities and $\otimes$ is the Kronecker product operator. Stacking the equations for i=1,...,N we have

$$R - \lambda_0 = \left\{I_N \otimes [(\lambda' \otimes \iota_T) + F]\right\}B + \varepsilon \tag{2.22}$$

where $\mathbf{I}_N$ is an identity matrix. As long as NT > NK+K, the estimators for the APT specification will exist and the NLSUR estimators, which provide joint estimates of λ and **B**, are those that solve the minimisation problem

$$\min_{\lambda, B} \ \varepsilon' (\hat{\Omega}^{-1} \otimes I_T) \varepsilon \tag{2.23}$$

where $\hat{\Omega}^{-1}$ is the residual variance-covariance matrix estimated from estimating equation (2.22) for all i=1,...,N, with $\lambda'\mathbf{b}_i$ replaced by a constant since λ cannot be identified at this stage. The three steps in estimating the parameters in equation (2.22) are as follows:

1. Estimate (2.22), equation by equation, using OLS. In this step λ is replaced by a constant and the residuals are estimated.
2. In the second step the residuals are used to estimate $\mathbf{\Omega}$.
3. In the third step the NLSUR estimator of λ and **B** is chosen to minimize equation (2.23).

The NLSUR estimators are, in the absence of normality, strongly consistent and asymptotically normal, and thus form the basis for classical asymptotic hypothesis testing. It is apparent that the NLSUR methodology has two useful contributions. Firstly it allows the direct testing of the APT's parametric across equation restrictions and provides estimates that are free from the EIV problem. Secondly, as developed here, it allows for the specification of an approximate factor structure, if returns follow such a process, by allowing $\hat{\Omega}$ to be non-diagonal. If it is the case that returns follow a strict factor structure then the NLSUR estimator can be modified so that $\hat{\Omega}$ is a diagonal matrix.

2.3 The Market Portfolio

In tests of the APT it is often typical to specify the market portfolio as one of the factors (see for example, Chen, Roll, and Ross (1986), Chan, Chen, and Hsieh (1985), Burmeister and McElroy (1988), Poon and Taylor (1989) and Clare and Thomas (1994)). There are a number of reasons for doing so. In the first instance the model will reduce to the CAPM if the market portfolio is the only priced factor. Second, the market portfolio can be used to proxy expectations of future growth rates as in Chan, Chen, and Hsieh (1985), and third the market portfolio can be used to proxy unobserved factors that may have been omitted from the prespecified macroeconomic factors as in Burmeister and McElroy (1988).

Results from the use of the two step procedure often find no role for the market portfolio when they specify observed macroeconomic factors.

However as Shanken and Weistein (1990) point out, and as discussed earlier, the EIV problem, and the use, or otherwise, of a correction for this, can affect the joint significance or otherwise of prespecified macroeconomic factors and the market portfolio. The issue of whether or not the insignificance of the market portfolio in tests of the APT is due to EIV can easily be resolved within the NLLS methodology since this eliminates the EIV problem.

There does, however, remain an issue raised by Burmeister and McElroy (1988) regarding whether the market portfolio should be included as an endogenous variable in tests of asset pricing models. To see why this matter arises, assume that the K factors can be partioned into those J that are observable and the remaining L which are unobserved:

$$B_K f_{Kt} = B_J f_{Jt} + B_L f_{Lt} \qquad (2.24)$$

Similarly, partition asset returns into the first N and the remaining M which gives

$$R_{Nt} = E(R_{Nt}) + B_{NJ} f_{Jt} + B_{NL} f_{Lt} + \epsilon_{Nt} \qquad (2.25)$$

$$R_{Mt} = E(R_{Mt}) + B_{MJ} f_{Jt} + B_{ML} f_{Lt} + \epsilon_{Mt} \qquad (2.26)$$

where $\mathbf{R}_{Nt}$ is Nx1, $\mathbf{R}_{Mt}$ is Mx1, $\mathbf{f}_{Jt}$ is Jx1, $\mathbf{f}_{Lt}$ is Lx1, ϵ_{Nt} is Nx1, ϵ_{Mt} is Mx1 and $\mathbf{B}_{NJ}$, $\mathbf{B}_{NL}$, $\mathbf{B}_{MJ}$ and $\mathbf{B}_{ML}$ are NxJ, NxL, MxJ and MxL respectively and have full column rank. Solving (2.26) for the unobserved factors and substituting into (2.25) yields

$$R_{Nt} = E(R^*_{Nt}) + B^*_1 R_{Mt} + B^*_2 f_{Jt} + u_{Nt} \tag{2.27}$$

where

$$E(R^*_{Nt}) = E(R_{Nt}) - B_{NL}B^{-1}_{ML}E(R_{Mt}) \tag{2.28a}$$

$$B^*_1 = B_{NL}B^{-1}_{ML} \tag{2.28b}$$

$$B^*_2 = B_{NJ} - B_{NL}B^{-1}_{ML}B_{MJ} \tag{2.28c}$$

$$u_{Nt} = \epsilon_{Nt} - B_{NL}B^{-1}_{ML}\epsilon_{Mt} \tag{2.28d}$$

Using the same line of reasoning for (2.18), we have

$$E(R_{Nt}) = \lambda_{0t}\iota_N + B_{NJ}\lambda_J + B_{NL}\lambda_L \tag{2.29}$$

$$E(R_{Mt}) = \lambda_{0t}\iota_M + B_{MJ}\lambda_J + B_{ML}\lambda_L \tag{2.30}$$

where λ_J is the price of risk for the J observed factors, λ_L is the price of risk for the L unobserved factors and ι_n is an n vector of ones. Solving (2.28a) for $E(\mathbf{R}_{Nt})$ gives

$$E(R_{Nt}) = E(R^*_{Nt}) + B^*_1 E(R_{Mt}) \tag{2.31}$$

Substituting (2.30) into (2.31), we have

$$E(R_{Nt}) = E(R^*_{Nt}) + B^*_1 (\lambda_{0t}\iota_M + B_{ML}\lambda_L + B_{MJ}\lambda_J) \tag{2.32}$$

Equating (2.32) and (2.29) and solving for $E(\mathbf{R}^*_{Nt})$ yields

$$E(R_{Nt}^{*}) = (\iota_N - B_1^{*}\iota_M)\lambda_{0t} + (B_{NJ} - B_1^{*}B_{MJ})\lambda_J \qquad (2.33)$$

Substituting (2.33) into (2.27) and rearranging yields

$$R_{Nt} - \lambda_{0t}\iota_N = B_1^{*}(R_{Mt} - \lambda_{0t}\iota_M) + B_2^{*}(\lambda_J + f_{Jt}) + u_{Nt} \qquad (2.34)$$

which, defining λ_{0t} as the risk-free rate of interest, gives us the APT. Moreover, comparing (2.34) with the general linear factor model given by

$$R_{Nt} - \lambda_{0t}\iota_N = A_0 + B_1^{*}(R_{Mt} - \lambda_{0t}\iota_M) + B_2^{*}f_{Jt} + u_{Nt} \qquad (2.35)$$

once again it is clear that the APT imposes testable, cross-equation nonlinear restrictions on the linear factor model, the restrictions being $\mathbf{A}_0 = \mathbf{B}_2^{*}\lambda_J$ which states that the price of risk for the jth factor must be the same for all assets. Note that in this formulation any unobserved factors are proxied by $\mathbf{R}_{Mt}$, the returns on those securities not included in $\mathbf{R}_{Nt}$. It is, in turn, feasible to proxy those securities not included in $\mathbf{R}_{Nt}$ by returns on a well diversified portfolio such as a market index. This insight offered by Burmeister and McElroy (1988) offers some flexibility in terms of the role of the market portfolio in the APT specification of the linear factor model since $\mathbf{B}_1^{*}$ can be set to zero if it is included as a priced factor or, if the market carries an insignificant risk premium, it can enter the APT unrestrictedly through $\mathbf{R}_{Mt}$ as a proxy for any unobserved factors that affect the pricing of security returns. However, specifying the model in this way automatically raises the issue of endogeniety of the market portfolio. The reason for this is that $\mathbf{R}_{Mt}$ represents returns on securities not included in the estimation sample. However, given that their inclusion explicitly in the estimation sample would make them endogenous, they may still be endogenous even when they are not included. If $\mathbf{R}_{Mt}$ is proxied by the market portfolio, then it is possible that the market portfolio may actually be an endogenous variable. If this is true, then estimation by NLSUR will generate biased and inconsistent estimates and a Nonlinear Three Stage Least Squares (NL3SLS) Estimator will be required instead.

To arrive at the NL3SLS estimators of the prices of risk and their sensitivities in the APT, assume that there are no unobserved factors but the market is priced and assume that there are N assets only. The NL3SLS estimators, which provide joint estimates of λ and $\mathbf{B}$, are those that solve the minimisation problem (Jorgenson and Laffont (1974)):

$$\min_{\lambda, B} \quad \epsilon' \left[\hat{\Omega}^{-1} \otimes \{Z(Z'Z)^{-1}Z'\}\right] \epsilon \qquad (2.36)$$

where $\hat{\Omega}^{-1}$ is the residual variance-covariance matrix estimated from estimating (2.34) for all i=1,...,N, with $\lambda'\mathbf{b}_i$ replaced by a constant since λ cannot be identified at this stage and $\mathbf{Z}$ is a matrix of instruments. The choice of instruments to use for the NL3SLS estimator depends crucially on the assumptions regarding the form of the covariance matrix of errors. Burmeister and McElroy (1988) choose a group of stock returns not in their sample to serve as instruments since they assume diagonality of the covariance matrix of errors. However, if the covariance matrix of errors is not diagonal then these stock returns will not serve as good instruments. Under this scenario the returns on alternative portfolios may be used as instruments as well as low polynomials of the variables (see Amemiya (1977) for a discussion on the choice of instruments in NL3SLS models).

2.4 Testing The Structure Of Stock Returns

Given the implications that the structure of stock returns can have on the empirical estimation of the APT it is of the upmost importance that before an empirical test is performed we determine whether returns have a strict or an approximate factor structure. However, as noted above the finding of contemporaneous correlation in the idiosyncratic return matrix is not enough to conclude that stock returns have an approximate factor model. The analysis of Chamberlain and Rothschild (1983) indicates that for an approximate k-factor structure the largest k eigenvalues of the return variance-covariance matrix grow unboundedly as the number of assets increases while the remaining eigenvalues are all bounded. Consequently, a natural test for the number of factors would be to examine the behaviour of the eigenvalues of the return variance-covariance matrix when the number of assets is increased. Results in the literature suggest, however, that this is not necessarily useful. Luedecke (1984) and Trzcinka (1986) both find that the first eigenvalue is by far the most important as the number of assets increases, while simultaneously all the remaining eigenvalues also increase. This suggests on the one hand that there is only one factor since only one eigenvalue dominates, but on the other hand all other eigenvalues increase and hence it is possible to argue that the number of factors is large. Simulations of a strict factor model in Brown (1989) show that while the first k population eigenvalues grow and the remaining k+1 eigenvalues are bounded as the number of assets increases, for a sample of the population the remaining k+1 eigenvalues also increase.

A Strict or an Approximate Factor Structure ? Empirical Results

A. Stock Price Data

The stock price data was collected from Datastream. Over the period December 1979 to December 1993 month end prices on 138 randomly selected stocks were chosen from the London Stock Exchange. Using the UK 3 month T-Bill as a proxy for the risk free rate of return, excess returns on the stock were calculated for the period January 1980 to December 1993. The first sample of sixty nine stocks, along with the excess return on the market index provides the first sample of 70 returns used in chapters 2, 3 and 4. In chapter 5 the second sample of stocks is used in order to validate the results of the first sample of stocks in terms of identifying common factors. In chapter 6, we use the first sample of stocks.

Total Returns and Price Changes

The data obtained from Datastream does not include dividends. Consequently, rather than using total returns this thesis uses price changes. While it is obviously preferable to use total returns the unavailability of dividends precludes this. The question that arises is the possible effects the omission of dividends may have on the empirical results in this thesis. First, the returns formed from price changes will be lower, however, given that month end returns are used then unless the dividend payment arises on the month end date then this addition return will be omitted from the monthly return series since all prices are adjusted for ex and cum dividend. Therefore, this problem is likely to be less serious than it may suggest. Second, it is unlikely that adding a constant twice a year to returns (if, by chance, they fall on the last day of the month) will alter the stochastic process of returns which is the important characteristic in terms of examining the correlations and covariation of returns with stochastic risk factors. Of course, we cannot be sure about these statements and it would be interesting to examine the impact, in future work, of using price changes as opposed to total returns.

Survivorship Bias

Throughout this thesis individual stock returns are used as opposed to portfolios of stocks. The rationale for this is two fold. In the first instance the reason for forming portfolios is the need to mitigate the Errors in Variables (EIV) problem encountered in the two-step estimation procedure. Since the econometric techniques used in the thesis provide simultaneous estimates of the parameters of the APT the EIV is not encountered. Although this does not preclude the formation of portfolios there exists a further reason why we avoid doing so. The results of Clare and Thomas (1994) reveals that the APT is not robust to the criteria on which portfolios are formed. Thus, given the central contribution of the thesis is to assess the empirical robustness of the APT, we avoid portfolio formation in order to provide evidence regarding the robustness of the APT free from the problems encountered in Clare and Thomas (1995).

Unfortunately, by choosing individual stocks, rather than portfolios, we restrict the sample to those stocks which have survived the estimation period (January 1980 to December 1993) and as a result this will bias the estimates of the risk premia downwards (assuming that the probability of failure is positively related to the level of risk). There is no feasible way of overcoming this problem with the use of individual stocks. However, the actual estimates of the risk premia (in terms of the size of the coefficients) was not the central aim of this thesis. Instead we concentrating on providing a testing methodology for the APT, free from inherent problems of extant tests, which allows for an examination of the central prediction of the APT - that of a common set of factors in the return generating process. This does not imply, in any manner, that the issue of survivorship bias should be ignored, but rather it is a secondary issue in terms of testing the APT and should be considered once a testing methodology has been established. Thus, we believe that this issue is worthy of consideration in future research where one may be interested in determining estimates of risk premia that will be applied to calculations should as the cost of capital or asset allocation in portfolio management.

B. Macroeconomic and Financial Factors

The data for the macroeconomic variables and the models used to generate the factors are listed below :

Unexpected Inflation	$\pi_t - E_{t-1}(\pi_t)$, where π is the change in the log of the United Kingdom Retail Price Index (All Items). The model used to generate the expectation is an AR(1) model.

Expected Inflation	$E_t(\pi_{t+1}) - E_{t-1}(\pi_t)$, where π and the model used to generate the expectations are as defined above.
Real Industrial Production	Log of United Kingdom industrial production deflated by the United Kingdom Producer Price Index; unobserved components model.
Real Retail Sales	Log of United Kingdom Retail Sales deflated by the RPI; AR(1) model.
Money Supply	Log of United Kingdom currency in circulation deflated by the RPI ; AR(2) model.
Commodity Prices	Log of the IMF All Commodity Price Index; unobserved components model.
Term Structure	$I_t - i_{t-1}$ where I is the yield on United Kingdom Government long term bonds and i is the yield on United Kingdom Government short term bonds; AR(1) model.
Default Risk	Defined as the difference between the FTA Debenture and Loan Stocks Redemption Yield and the yield on United Kingdom Government long term bonds; unobserved components model.
Exchange Rate	Log of the Sterling effective exchange rate; unobserved components model.
Market Portfolio	Returns on the FT All Share Index minus the one month Treasury Bill Rate.

Unless otherwise indicated, all data is as reported by the CSO and was obtained from Datastream.

Empirical Results

In order to determine whether we have an approximate factor structure for stock returns we analyse, in the first instance, the behaviour of the eigenvalues of the return variance-covariance matrix and the residual variance-covariance matrix for a random sample of returns on 69 UK stocks. To obtain the residual variance-covariance matrix we need to estimate the APT with the observed factors. The factors that we employ are those provided in table 2.1. We do not discuss the reasons for choosing this set of factors in this chapter since the results are simply used in order to provide an indication of the type of estimator we should be using in tests of the APT. However, it should be noted that their inclusion is justified in terms of the possible effects they can have on stock returns through their effect on the present value discounted dividends model. In order to provide unexpected components in the factors we follow the methodology of Chen, Roll, and Ross (1986), again this decision is entirely due to illustrative reasons and not because we believe that this is the most useful methodology. Table 2.2 reports the form of the unexpected components that enter into the APT specification. As a precursor we estimated equation (2.34) above for a sample of 69 stocks with an additional equation for the market portfolio. This provides an estimate of the idiosyncratic return variance-covariance matrix. Form this we extracted the eigenvalues. To facilitate the analysis of the Chamberlain and Rothschild (1983) restrictions for an approximate factor model we compare the eigenvalues from the return variance-covariance matrix, given in table 2.3, with the eigenvalues from the idiosyncratic variance-covariance matrix given in table 2.4. According to Chamberlain and Rothschild (1983), if we examine the return variance-covariance matrix then the number of systematic risk factors is equal to the number of eigenvalues that increase as the number of stocks increases, while the remaining eigenvalues should be bounded. Additionally, through examination of the idiosyncratic risk variance-covariance matrix the appropriate number of factors should be equal to the number of eigenvalues in the return variance-covariance matrix that are greater than the largest eigenvalue of the idiosyncratic variance-covariance matrix. Table 2.3 reveals that as the number of securities increases all the first ten eigenvalues increase which would seem to suggest that there are ten systematic risk factors in the return generating process. However, if we take table 2.3 in conjunction with table 2.4 the size of the first eigenvalue for the idiosyncratic risk is greater than the second eigenvalue of the return variance-covariance matrix. This implies that there is only one factor present in the return generating process.

These mixed results concur with the findings in the US of Luedecke (1984) and Trzcinka (1986) discussed above. Thus the problematic nature of this

Table 2.1

Economic State Variables

Factor	State Variable
f_1	Default Risk
f_2	Real Industrial Production
f_3	Exchange Rate
f_4	Real Sales
f_5	Money Supply
f_6	Unexpected Inflation
f_7	Change in Expected Inflation
f_8	Term Structure of Interest Rates
f_9	Commodity Prices
f_{10}	Excess Return on the Market Portfolio

Notes: Sample period 1980M1 - 1993m12. Source : Datastream

Table 2.2

Shocks In Economic State Variables

Shock	Form*
f_1 (Government bonds - Corporate Bonds)	L
f_2	FDL
f_3	FDL
f_4	FDL
f_5	FDL
f_6	FDL
f_7 (Inflation$_{t+1}$ - Inflation$_t$)	FDL
f_8 (Long Government Bonds$_t$ - Short Government bonds$_{t-1}$)	L
f_9	FDL
f_{10}	L

Notes: Sample period 1980M1 - 1993m12. Source : Datastream
*Key
L = levels
FDL = First Difference of Log of levels

Table 2.3

First 10 Eigenvalues Extracted from the Return Variance-Covariance Matrix
When the Number of Stocks, N, is Increased

N = 40	N = 50	N = 60
21.473546	27.247069	31.776337
9.8918278	10.025798	10.105154
4.8465819	5.5894319	5.6666193
3.9934365	4.2754269	4.5318344
2.9958588	3.6170483	3.9053788
2.6660652	3.1147494	3.3229392
2.0837117	2.6375473	2.9183134
1.9966277	2.5821143	2.6899743
1.8698918	2.2714048	2.4113293
1.7368022	2.1330497	2.2746254

Notes: Sample period 1980M1 - 1993m12. Source : Datastream

Table 2.4
First 10 Eigenvalues Extracted from the Residual Variance-Covariance Matrix
When the Number of Stocks, N, is Increased.

N = 40	N = 50	N= 60
10.265612	10.519034	10.592374
6.8499743	7.1275176	7.3282973
3.9191345	4.9906941	5.1080918
3.1307542	3.7678061	3.9408678
2.5151311	3.2667106	3.5100324
2.1478147	2.6322178	3.0294138
2.0594464	2.5088888	2.7296779
1.8489745	2.2772963	2.3321397
1.7765442	2.0508431	2.3002951
1.4717101	1.9803001	2.0790323

Notes: Sample period 1980M1 - 1993m12. Source : Datastream

Table 2.5
Correlations Amongst the Idiosyncratic Returns of the First 15 Companies

Comp. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1.00														
2	-0.20**	1.00													
3	0.07	-0.18**	1.00												
4	0.12	-0.04	0.05	1.00											
5	0.05	-0.17**	0.19	-0.03	1.00										
6	-0.01	-0.28***	-0.04	-0.14*	0.07	1.00									
7	-0.22***	-0.01	-0.15*	0.04	-0.14*	-0.07	1.00								
8	0.15**	-0.24***	0.11	0.13*	-0.01	-0.02	0.06	1.00							
9	0.15**	-0.12	-0.02	0.01	0.11	-0.04	-0.14**	0.09	1.00						
10	0.01	-0.07	0.03	0.07	0.00	-0.26***	0.16**	0.17**	-0.09	1.00					
11	0.11	-0.03	0.08	-0.04	0.13*	0.04	-0.21***	-0.01	0.06	-0.09	1.00				
12	0.13*	-0.08*	-0.13*	-0.03	0.00	0.01	0.01	0.15*	0.02	0.11	-0.08	1.00			
13	0.12	-0.06	0.01	0.14*	0.19***	0.05	-0.13*	-0.09	-0.05	0.32***	-0.34*	-0.14*	1.00		
14	0.03	0.07	-0.00	0.11	0.08	-0.01	-0.12*	0.05	-0.04	0.09	0.18**	-0.03	0.11	1.00	
15	0.07	-0.05	0.46***	0.04	0.09	-0.05	-0.29***	-0.01	0.13*	-0.15*	0.27***	-0.06	-0.09	-0.05	1.00

Notes
* denotes significant at 10%.
** denotes significant at 5%.
*** denotes significant at 1%.

Sample period 1980M1 - 1993m12. Source : Datastream

analysis does not facilitate the determination of the nature of the factor structure. However, if we consider the correlations of idiosyncratic returns then some indication of the relative importance of contemporaneously correlated residuals in the estimation of the APT can be attained. Table 2.5 above contains correlations amongst idiosyncratic returns for the first fifteen companies in the estimation sample to give some flavour of the interrelationship amongst the idiosyncratic returns for different companies. As is clear from the table, there are quite a number of correlations that are significant at the 10% level or less (33%), even just amongst the first fifteen companies. For the whole sample of companies (not reported here), approximately 30% of the correlations are significant at the 10% level or less which appears to confirm that an approximate factor structure is much more appropriate in describing observed security return behaviour.

2.5 Results From Tests Of The Apt When Observed Factors Employed

In this section we present evidence on the possible invalid inferences that may be obtained when it is assumed that returns follow a strict factor structure when they actually follow an approximate factor structure. We have shown that when the market portfolio enters as a factor into the APT then it should be treated as endogenous. Furthermore, we have shown that stock returns do follow an approximate factor structure. The implication of these two results is that we should be estimating the APT using NL3SLS where the variance-covariance matrix of errors is not constrained to be diagonal. The APT is estimated in two forms. First, we estimate the strict factor model by imposing a diagonal variance-covariance matrix of errors on the NL3SLS estimator. Second, we relax the restrictions on the variance-covariance matrix and thus estimate an approximate factor model. As instruments for the market portfolio we choose the fitted values from a regression of the market portfolio on the factors, the returns on the Standard and Poor 500 stock index, the returns on the New York Stock Exchange stock index, and squared values of these. For the exogenous variables we use their contemporaneous values and squared values. The results from this are presented in table 2.5 and 2.6 for the strict and approximate factor model respectively. By imposing a strict factor model on the return generating process we find that none of the factors are statistically significant at the 10% or less level. Only default risk appears to be any where near approaching a conventional significance level. However, the APT's across equation restrictions ($\mathbf{A}_0 = \mathbf{B}_2^* \lambda_J$) for the strict factor model hold: $\chi^2(60) = 0.38$.

These results contrast sharply for the same set of stock returns using an approximate factor model. There are six factors which have a statistically significant risk premia: the default risk, real industrial production, the change in expected inflation, commodity prices, and the market portfolio. The across equation restrictions for the approximate factor model also hold: $\chi^2(60) = 33.69$. This result is in direct conflict with the findings of Burmeister and McElroy (1988), who for their US sample of stock returns and factors report:

Table 2.6
NL3SLS Estimated Prices of Risk for APT
Strict Factor Model

Parameter	Estimate	T - Ratio
λ_1 (Default Risk)	-0.01074	-1.59
λ_2 (Real Industrial Prod)	0.00212	0.29
λ_3 (Exchange Rate)	0.00136	0.02
λ_4 (Real Retail sales)	-0.00061	-0.10
λ_5 (Money Supply)	0.00467	0.26
λ_6 (Unexpected Inflation)	-0.00567	-0.53
λ_7 (Expected Inflation)	-0.00304	-0.19
λ_8 (Term Structure of IR)	-0.03665	-0.12
λ_9 (Commodity Prices)	-0.00893	-0.19
λ_{10} (Market Portfolio)	0.00218	0.13

Notes:
* denotes significant at 10%
** denotes significant at 5%
*** denotes significant at 1%
Sample period 1980M1 - 1993m12. Source : Datastream

Table 2.7
NL3SLS Estimated Prices of Risk for APT
Approximate Factor Model

Parameter	Estimate	T - Ratio
λ_1 (Default Risk)	-0.01088	-17.37^{***}
λ_2 (Real Industrial Prod)	0.00292	4.80^{***}
λ_3 (Exchange Rate)	0.00323	0.69
λ_4 (Real Retail sales)	-0.00032	-0.73
λ_5 (Money Supply)	0.00199	1.61
λ_6 (Unexpected Inflation)	-0.00517	-6.58^{***}
λ_7 (Expected Inflation)	-0.00454	-3.05^{**}
λ_8 (Term Structure of IR)	-0.02318	-0.92
λ_9 (Commodity Prices)	-0.01365	-3.47^{***}
λ_{10} (Market Portfolio)	0.00181	2.02^{**}

Notes:
* denotes significant at 10%
** denotes significant at 5%
*** denotes significant at 1%
Sample period 1980M1 - 1993m12. Source : Datastream

"the similarity we found between...estimates probably indicates that the assumption of a diagonal covariance matrix for the ϵ_{ti}'s is empirically tolerable, at least with this set of factors."

The difference we find stems from the fact that, given restrictions across equations, the efficiency of the NL3SLS increases when there is information in the covariance matrix of errors such as significant covariance. This efficiency gain can be seen by analysis of the significant risk premia for the approximate factor model results with the corresponding risk premia for the strict factor model. This reveals that they have very similar estimates of the prices of risk. However, the standard errors from the strict factor model are always larger than the standard errors in the approximate factor model. When there is correlation amongst the error terms then the variance (standard error) of the approximate factor model parameter estimates will always be smaller than the variance (standard error) of the strict factor model parameter estimates.[8] Consequently, the t-ratios from the strict factor models will always under estimate the statistical significance of the estimated parameters.

These findings appear, at first sight, to be inconsistent with the results of those who have employed the OLS two-step procedure. For example, CRR (1986) and Clare and Thomas (1994) found four and three factors respectively. However, if we consider the results of Shanken and Weinstein

[8] For proof see Amemiya (1985)

(1990), who find that after a correction for EIV there is only one macroeconomic variable significant in the CRR estimates then the results we find here are consistent with the findings that once the EIV problem is removed a strict factor model estimate will tend to underestimate the true number of factors. These results are important in terms of determining the number of factor present in security returns and the implications this will have on any practical application.

2.6 Conclusion

An approximate factor structure, for this set of assets, seems to be a valid representation of the return generating process. In section 2.4 we found that there is statistically significant correlation between the errors in the APT. Given that we assume that the market portfolio should enter into tests of the asset pricing model as an endogenous factor we proceeded to examine the results from tests of the APT that assume the factor structure is, in the first instance strict, and secondly approximate. Results indicate that the use of a strict factor model in this situation will lead to estimates of standard errors that are large relative to the true variance. Thus, inferences from tests of asset pricing models which rely on a strict factor structure such as CRR (1986), CCH (1985) and Shanken and Weinstein (1990) in the US, and Poon and Taylor (1989) and Clare and Thomas (1994) in the UK, must be treated with caution. The aim of this chapter was to provide a framework from which to evaluate asset pricing models. The results indicate that the return generating process of UK stock returns follow an approximate factor structure, and we have shown that failure to account for this can lead to misleading inferences. These results provide a framework from which we can motivate tests of individual asset pricing models and test the relative performance of competing asset pricing models which are robust to the structure of the underlying return generating process.

CHAPTER 3

VALUATION MODELS AND ASSET PRICING:

THE APT AND THE CAPM

3.1 INTRODUCTION

The purpose of this chapter is to compare the performance of two competing asset pricing models, the Capital Asset Pricing Model (CAPM) and the Arbitrage Pricing Theory (APT) and to reassess the role of the market portfolio in tests of asset pricing models. This is of particular interest given the relationship between stock markets, valuation models, the cost of capital, and the ultimate investment and consumption decisions of economic agents. The choice of which asset pricing model to use in any practical context such as calculating the cost of capital and fair rates of return for regulated industries and evaluating portfolio performance, must ultimately be an empirical one. This chapter aims to address this issue within the framework outlined in chapter 2.

The issue of the appropriate asset pricing model and the specification of the model is interesting, not only for the above mentioned reasons, but also because of the recent mixed results on the performance and specification of asset pricing models in general. The results of Fama and French (1992) indicate that the role of beta may well be dead when variables such as book to market value and firm size are included as explanatory variables. Moreover, early results on tests of the APT as a contender to the CAPM (see, for example, Chen (1983)) suggest that the APT performs better in terms of describing actual returns. More recent results which specify observed factors in an APT specification also find that there is no role for the market portfolio, see, for example, Chen, Roll, and Ross (1986), Chan, Chen, and Hsieh (1985) for US evidence and Poon and Taylor (1989) with regard to UK evidence. It would appear, in the light of this evidence, that the role of beta is dead when alternative asset pricing models are considered. However, these results have been questioned on a number of grounds. Shanken and Weinstein (1990) show that once a correction for the Errors in Variables (EIV) problem, inherent in tests of the APT discussed above, is performed the ability of macroeconomic and financial factors specified in the above studies to render the market portfolio insignificant is seriously questioned. In the UK context Clare and Thomas (1994) provide results which show that the significance or otherwise of the market portfolio in an APT specification depends crucially on the criteria chosen to form portfolios of stocks. While in the US Kothari, Shanken, and Sloan (1994) show that it is data problems which generate the result in Fama and French (1992) that the market portfolio has no role to play.

What emerges form this analysis is clear evidence of the mixed results regarding tests of asset pricing models and in particular the role of the market portfolio in such tests. The contribution of this chapter is to reevaluate the performance of asset pricing models and the role of the market portfolio in such models. In particular, free from the inherent problems of the Fama and MacBeth (1973) methodology (that is, the EIV problem, the problem of forming portfolios, and the assumption of a strict factor model) we perform individual and comparative tests of the CAPM and the APT to determine their usefulness in terms of describing the return generating process of UK stock returns. This should provide clear indications of which asset pricing model performs best. This is extremely important given the relationship between asset pricing models and the investment and consumption decisions of agents outlined in the opening paragraph of this chapter. A final issue we address is the role of the market portfolio in tests of the APT. The results of this analysis will provide a contribution to help resolve the issue of which asset pricing model best describes the return generating process and what are the important variables in the asset pricing model. Furthermore, a secondary point is that, as far as the UK is concerned, these results will provide the first comparative tests of the APT and the CAPM and provide the first results which are robust to the EIV problem.

The individual asset pricing models can be estimated and numerous testable implications and restrictions can be analyzed. For example, both the CAPM's and the APT's price of risk is estimated under the hypothesis that expected returns are a linear function of the factor sensitivities and under this hypothesis inferences are made regarding these prices of risk. In order to analyse the robustness of the maintained hypothesis it is possible to undertake a number of test of this hypothesis. Such tests fall into one of two possible categories: either a test of linearity against an unspecified alternative, or a test of the hypothesis that a particular specified alternative is priced in competition to the factors.[9] However, a problem arises in the situation when the results of empirical tests indicate that for both models the maintained hypothesis is robust to the unspecified and specified alternatives. Namely, how do we decide which model is the best ? To overcome this problem there are a number of comparative tests of the competing models that we can undertake in order to determine if one model outperforms the other.

Within the debate regarding the performance of these two competing asset pricing models there is the interesting issue of the role of the market portfolio in tests of the APT - should the market portfolio be included as a factor in tests of the APT ? Given that we have a set of factors for the APT the question that arises is does the market portfolio improve the explanation of asset returns ? We also address this issue in this chapter. The rest of the chapter is organised as follows: Section 3.2 reviews the relationship between valuation models and asset pricing models in order to identify the importance of asset pricing models. Section 3.3 briefly discusses the role of the market portfolio in tests of the APT. The macroeconomic factors that we postulate as being candidates for economic state variables that may command a risk premium in the UK stock market are outlined in section 3.4. In section 3.5 we discuss the CAPM's testable implications and restrictions. Section 3.6 reviews the APT's testable implications and restrictions while the comparative

[9] For example, a test of the APT against the specific alternative that idiosyncratic variance is not priced tests the APT's prediction that this risk is not rewarded in financial markets. In addition to this it also tests whether the pricing relationship can be treated as an exact one (see Shanken (1992) section 1.2).

tests of the two models are discussed in Section 3.7. The empirical results of the tests of the individual asset pricing models are reported in Section 3.8 and the results of the comparative analysis of the two models, and the results regarding the role of the market portfolio are presented in Section 3.9. Section 3.10 concludes.

3.2 Valuation Models and Asset Pricing

The theory of valuation and the understanding of the behaviour of stock markets has interested economists for a great number of years. Fischer and Merton (1984) argue that the behaviour of stock markets is important for our understanding of business cycles, calculating appropriate discount rates and the overall efficient allocation of resources in the economy. The formal link between valuation theories and the behaviour of stock markets is provided by asset pricing models, which equate the price of a claim to a representative agents expectation of a future payoff, given their marginal rate of substitution. On the basis of this relationship agents form their utility maximising investment and consumption decisions.

With the advent of portfolio theory attention has focused on equilibrium asset pricing models such as the CAPM and no-arbitrage models such as the APT. The first model to formally explore the relationship between risk and return was the Sharpe (1964) and Lintner (1965) CAPM which relates expected returns on a single asset to the equilibrium return on the market portfolio and defines the risk of the asset as its covariability with the market portfolio. This market based valuation model has been extended to consumption based asset pricing, (see Breeden and Litzenberger (1978)), and into an intertemporal framework (see Breeden (1979)). The CAPM and its variants constitute an important contribution to the theory of valuation and the structure of equilibrium expected returns on risky assets.

The alternative asset pricing framework is based on no-arbitrage models such as the APT developed by Ross (1976)[10]. This class of multi-factor models aims to relate the return on an asset to systematic economy-wide factors, known as economic state variables. Like the CAPM, the APT has been extended theoretically where intertemporal and approximate pricing relationships have been derived. The intersection of asset pricing models and the empirical identification of economic state variables constitutes the important direction for research. A challenge to the theoretical developments in asset pricing models is to provide a particular set of economic state variables which are important in determining asset prices and returns, and to evaluate the practical usefulness of the competing asset pricing models.

The basic valuation equation has been derived formally by, amongst others, Rubenstein (1976), Lucas (1978), and Breeden and Litzenberger (1978) (see Constantinides (1988) for a review). Here we reproduce the basic framework of the valuation model (using the notation of Constatinides (1988)) and

[10] The APT can also be derived from an equilibrium framework. See, for example, Connor (1984).

highlight its relationship to asset pricing models. Initially there are a number of assumptions which are necessary in order to develop the model. There are n firms in the economy who pay a net dividend in time period t of $D^{t}(\phi^{t})$, where ϕ^{t} is the information set at time t. Agents are assumed to know the information set ϕ^{t} at time t, which is assumed to be a random process and agents have homogenous expectations. There is only one consumption good in the economy and the economy extends from period t=0,1,....,T, $T<\infty$. If an additional assumption that agents have standard von Neumann-Morgenstern utility functions is made then the analysis can be formalized in terms of a representative consumer who invests in all firms and consumes in time t an aggregate dividend $c^{t}=\Sigma D_{i}^{t}(\phi^{t})$, where D_{i}^{t} is the ith firms dividend in time t. Assuming no income from labour or other exogenous income is received, then the expected utility at t is given as:

$$E\left[\sum_{\tau=t}^{T} u^{\tau}(c^{\tau}) \mid \phi^{t}\right] \equiv E\left[\sum_{\tau=t}^{T} u^{\tau}\left[\sum_{i=1}^{n} D_{i}^{\tau}(\phi^{\tau})\right] \mid \phi^{t}\right] \qquad (3.1)$$

where $u^{\tau}(\cdot)$ is a monotone increasing and strictly concave utility function. The price of a stock in equilibrium represents its fundamental value and as such the representative consumer will be indifferent between buying or selling a proportion α of the stock and maximises the following objective function when $\alpha=0$:

$$\max_{\alpha} E\left[u^t\left(c^t - \alpha P_i^t\right) + \sum_{\tau=t+1}^{T} u^\tau\left(c^\tau + \alpha D_i^\tau\right) | \phi^t\right] \tag{3.2}$$

where P_i^t is the ex dividend price of stock i in time t. Constantinides (1988) points out that equation (3.2) is strictly concave in α and therefore provides conditions of optimality and further manipulation yields:

$$P_i^t = E\left[\sum_{\tau=t+1}^{T} \frac{u_c^\tau(c^\tau)}{u_c^t(c^t)} \cdot D_i^\tau \mid \phi^t\right] \tag{3.3}$$

where the subscript c is a derivative. This is the simple valuation model where the price of a stock is a function of the utility of future income streams and is one of the focal points of modern financial economics. Following Constantinides (1988) the model can be written in terms of returns between time periods t and t-1. Lagging (3.3) one period and using the law of iterated expectations:

$$P_i^{t-1} = E\left[\sum_{\tau=t}^{T} \frac{u_c^\tau(c^\tau)}{u_c^{t-1}(c^{t-1})} \cdot D_i^\tau \mid \phi^{t-1}\right] \tag{3.4}$$

$$= E\left[\frac{u_c^t(c^t)}{u_c^{t-1}(c^{t-1})}\cdot\left[D_i^t + \sum_{\tau=t+1}^{T}\frac{u_c^\tau(c^\tau)}{u^t(c_c^t)}\cdot D_i^\tau\right] \mid \phi^{t-1}\right] \tag{3.5}$$

$$=E\left[\frac{u_c^t(c^t)}{u_c^{t-1}(c^{t-1})}\cdot\left[D_i^t + E\left[\sum_{\tau=t+1}^{T}\frac{u_c^\tau(c^\tau)}{u_c^t(c^t)}\cdot D_i^\tau \mid \phi^t\right]\right] \mid \phi^{t-1}\right] \tag{3.6}$$

Substitution of (3.3) into (3.6) yields:

$$P_i^{t-1} = E\left[\frac{u_c^t(c^t)}{u_c^{t-1}(c^{t-1})}\cdot\left(D_i^t + P_i^t\right) \mid \phi^{t-1}\right] \tag{3.7}$$

Defining the rate of return as:

$$R_i^t \equiv \frac{P_i^t + D_i^t}{P_i^{t-1}} \tag{3.8}$$

and dividing both sides of (3.7) by P_i^{t-1} yields:

$$E\left[\frac{u_c^t(c^t)}{u_c^{t-1}(c^{t-1})}\cdot R_i^t \mid \phi^{t-1}\right] = 1 \tag{3.9}$$

Now, defining the riskfree rate of return as R_0^t, then the excess return, $r^t=R^t-R_0^t$, satisfies:

$$E\left[\frac{u_c^t(c^t)}{u_c^{t-1}(c^{t-1})}\cdot r_i^t \mid \phi^{t-1}\right] = 0 \qquad (3.10)$$

which can be rewritten simply as:

$$E\left[u_c^t(c^t)\cdot r_i^t \mid \phi^{t-1}\right] = 0 \qquad (3.11)$$

Equation (3.11) is the valuation model in terms of the excess return for stock i in time t. It is this equation that links with asset pricing models.

To arrive at the single period market based CAPM from equation (3.11) we take the result that at the end of the period the firm liquidates and pays a final dividend for consumption which will be equal to aggregate wealth W. Thus for period t=1 (3.11) is written as:

$$E\left[u_w^1(W^1)\cdot r_i^1 \mid \phi^0\right] = 0 \qquad (3.12)$$

Constatinides (1988) shows, via Stein's lemma, that by assuming the end-of-period dividends are multivariate normal, returns and wealth are also multivariate normal so that (3.12) becomes:

$$E\left[r_i^1 \mid \phi^0\right] = -\frac{E\left[u_{ww}^1(w^1) \mid \phi^0\right]}{E\left[u_w^1(W^1) \mid \phi^0\right]} \cdot cov(r_i^1, W^1 \mid \phi^0) \qquad (3.13)$$

From equations (3.12) and (3.13) a stocks return depends on the marginal utility of aggregate wealth. The CAPM takes the covariance of the stock return with the market portfolio to be the measure of expected returns on a stock and is written more traditionally as:

$$E(R_i) = R_f + [E(R_m) - R_f]\beta_i \qquad (3.14)$$

where $E(R_i)$ is the expected return on asset i, $E(R_m)$ is the expected return on the market portfolio, R_f is the return on the risk-free asset and β_i is a measure of the level of systematic risk of asset i relative to that of the market portfolio, that is, the covariance of asset i to the market portfolio. In this single factor model the investor only receives reward over and above the risk-free rate for bearing systematic risk relating to the stock market as a whole.

Recent interest has focused on multi-factor models stemming from the work of Ross (1976). Ross shows that the difference between actual returns and expected returns is given by a linear factor model:

$$R_i = E[R_i] + \sum_{j=1}^{k} b_{ij} f_j + \varepsilon_i \tag{3.15}$$

where: R_i is the excess return on asset i; E is the expectations operator; f_j is the jth factor common to all assets; b_{ij} is the sensitivity of asset i to factor j ; and ε_i is an error term assumed to be white noise where $E[\varepsilon_i] = E[f_j] = E[\varepsilon_i | f_j] = 0$, $E[\varepsilon_i \varepsilon_j] = \sigma_{ij}$ when i=j and = 0 when i≠j, and where the ε_i's are serially uncorrelated. Although Ross's APT can be derived immediately from the no-arbitrage restrictions framework, it can also be derived from the equilibrium valuation model. Constantinides (1988) shows that substitution of equation (3.15) into equation (3.11) yields:

$$E(r_i) = \sum_{j=1}^{k} \left[\frac{E[-u_c(c) \cdot f_j]}{E[u_c(c)]} \right] b_{ij} + \frac{E[-u_c(c) \cdot \varepsilon_i]}{E[u_c(c)]} \tag{3.16}$$

which in turn, assuming idiosyncratic risk is eliminated through diversification, simplifies to equation (3.17) where expected returns, in time period t are given by the following,

$$E_t[R_{it}] = \alpha_i + \sum_{j=1}^{k} b_{ij} \lambda_{jt} \tag{3.17}$$

where: λ_{jt} is taken as the price of risk for the jth factor, that is λ_{jt} represents the unit reward for undertaking a unit risk in the non-zero innovation in the jth factor. Assuming that only systematic risk is priced in the market, then, if all the f_{jt}'s are zero actual returns will equal expected returns. However when any of the f_{jt}'s are innovations they will be a source of systematic risk to which firms will be sensitive and although this sensitivity varies across firms, the price of risk for the jth factor will be the same in terms of estimated sign and size.

It is quite clear from the framework set out above that the role of asset pricing models is an extremely important one. Theoretically, the failure of the asset pricing model to adequately describe the return generating process can lead to investment and consumption decisions that may well be sub-optimal. One implication of this, with respect to real investment and consumption decisions, can feed through to investment policies of firms who employ asset pricing models in order to calculate the cost of capital and hence apply to net present value rules when undertaking investment appraisal. In turn, given that the firms investments generate future cash flows and hence future dividends these decisions can effect the actual consumption patterns of agents. Therefore, it is of paramount importance that we determine the appropriate asset pricing model to use in such circumstances.

3.3 The APT Versus The CAPM - The Role Of The Market Portfolio

As outlined in Chapter 1 the theoretical underpinnings of the market based CAPM rely on the identification of the entire market portfolio in empirical tests. However, the unobservability of the market portfolio is a damaging critique of the CAPM (see Roll (1977)). Moreover, the empirical evidence on the performance of the CAPM is rather disappointing with results indicating that the market portfolio is not the only factor that explains security returns. Banz (1981) and Reinganum (1981) document a firm size effect where small firms have returns in excess of those predicted by the CAPM. Litzenberger and Ramaswamy (1979) found that firms with high dividend yields also have returns in excess of those predicted by the CAPM. Calender anomalies such as the January effect have also been documented (Keim (1983, 1985). Rather more critical than these observed anomalies are the recent results of Fama and French (1992) who show that by including variables such as book to market value and firm size there is only a very weak positive relation between average returns and beta over the period 1941-90, while over the shorter period of 1963-90 there appears to be no relationship whatsoever. Such damaging evidence has led some commentators to put the final nail in the coffin of beta. However, Kothari, Shanken and Sloan (1994) present evidence which seriously questions these findings. Specifically, they show that data problems are what generate Fama and French's findings and consequently the market portfolio and hence beta has an important role to play in pricing risky assets. These findings bring to the forefront the role of the market portfolio in asset pricing models.

Following the apparent failure of the CAPM as a means of measuring the expected returns on risky assets as given by beta, attention has focused on the identification of pervasive factors that command a risk premium in the pricing of securities in the context of the Arbitrage Pricing Theory (APT). The factor model of Ross has been debated and tested widely and the first such tests used factor analysis and principal components to measure the number of pervasive factors present in the pricing of asset returns. The problem with this approach, especially when trying to determine the importance of the market portfolio, is that it is not possible to determine unambiguously whether one of the extracted factors is a proxy for the market portfolio. This problem has been overcome with the use of prespecifed economic state variables, where the market portfolio could be one of prespecifed pervasive factors. The pioneering work of CRR (1986) addressed this issue by postulating a number of macroeconomic variables as candidates for systematic risk factors. Empirical support for a number of priced macroeconomic factors in both the US and UK (see Chapter 1 for details) has led to strong support for a multi-factor model of returns.

Contrary to the extensive empirical support for the APT in the 1980's, more recent evidence by Shanken (1992), Shanken and Weinstein (1990), Fama and French (1992) and Kothari, Shanken and Sloan (1994) and Clare and Thomas (1994) has reopened the debate as to which asset pricing model best describes asset returns. Specifically, tests of the APT employing economic state variables identified a number of such factors as significant in explaining asset returns while finding at best a weak role for the market portfolio. However a number of problems have been identified regarding the methodologies employed by the above mentioned studies, with the implication that the role of the market portfolio in asset pricing models along with prespecified economic state variables needs to be reevaluated. Such an investigation should shed light on perhaps the more interesting question of the relative performance of the competing asset pricing models and ultimately the more appropriate model for valuation purposes.

All of the above mentioned approaches to estimating the APT, using measured factors, employ a variant of the Fama and MacBeth (1973) two step methodology. This methodology leads to the well known Errors in Variables Problem (EIV) and the consequent forming of portfolios in order to mitigate this effect. However, as Shanken (1992) demonstrated forming portfolios in itself does not overcome the EIV problem, but perhaps more damaging for this line of research, is the finding by Shanken and Weinstein (1990) that the significant macroeconomic factors found in the CRR study and the ability of such factors to render the market portfolio insignificant is very sensitive to the way in which portfolios are formed and whether or not a correction for EIV is made. It is clear that this issue needs readdressing and that the important question is whether the APT performs better than the CAPM and what is the role of the market portfolio in asset pricing models, if any ? Until we can overcome these problems inherent in previous tests of the APT it is not possible to compare the performance of the competing models, to assess the role of the market portfolio and ultimately evaluate the more appropriate model for valuation.

This chapter adopts an approach that will overcome the problems identified by

Shanken and Weinstein (1990) and it involves joint estimation of the parameters of the competing asset pricing models (as outlined in Chapter 2). The approach was developed Gibbons (1982) and McElroy, Burmeister, and Wall (1985) and uses non-linear least squares techniques which overcomes the EIV problem common to the two-step procedure and by implication avoids the need to form portfolios. Before any test of the APT can be performed it is first necessary to specify factors that may be contenders for economic state variables that command a risk premium in the UK stock market, we turn our attention to this issue in the next section.

3.4 Economic State Variables And Asset Pricing Models

The link between the valuation model and asset pricing models is provided by the empirical identification of the relevant economic state variables and hence the evaluation of the competing asset pricing models. However, it is first necessary to postulate candidates for these variables. In terms of empirical tests of the CAPM this is straight forward since we can proxy aggregate wealth and economic state variables with the returns on the market portfolio (although the empirical specification of the CAPM is straightforward there remains the problem of Roll's Critique (1977)). The APT, on the other hand, offers no guidance as to the relevant empirically observed state variables that influence asset returns and aggregate wealth. Asset prices are, however, known to react to news on the economy - a glance at the financial press reveals that unexpected changes in stock prices often follow unanticipated news on, say, industrial production or retail sales. Therefore, news regarding any variable that will effect the investment and consumption decisions and the opportunity set of economic agents would by implication have an impact through equation (3.3) on stock market returns. In considering the possible macroeconomic factors that may effect agents investment and consumption decisions it is common to express equation (3.3) as the present value of future dividends,

$$P_i = \sum_{t=1}^{\infty} \frac{E(D_{it})}{(1+\delta)^t} \tag{3.18}$$

where P_i is the price of stock i, E is the expectations operator, D_t is the dividend paid at the end of period t, and δ is the discount rate. It follows that any variable that will effect cashflows and/or the discount factor will have an impact on prices and hence returns. While the list of possible factors that could influence prices through equation (3.18) is extensive, utilising previous research and this model we postulate the factors presented in table 3.1 as candidates for economic state variables that command a risk premium in the UK stock market. These factors are expected to reflect both the real and financial sectors of the economy.

The rationale for choosing these factors is as follows. On the real side of the economy shocks to real industrial production will effect the business cycle and hence a firm's real cash flows and hence its prospective future dividend

Table 3.1

Candidates for Economic State Variables

Factor	Candidate Economic State Variable
f_1	Default Risk
f_2	Real Industrial Production
f_3	Exchange Rate
f_4	Real Sales
f_5	Money Supply
f_6	Unexpected Inflation
f_7	Change in Expected Inflation
f_8	Term Structure of Interest Rates
f_9	Commodity Prices

Notes: Sample period 1980M1 - 1993m12. Source : Datastream

payout. In addition a firm's cash flow could also be influenced by shocks to real retail sales. Shocks to commodity prices will affect a firm's input and output prices and thus cash flows and future dividends. On the financial side of the economy we consider interest rate factors, an exchange rate factor and inflation factors. To capture the interest rate factors we make use of default risk. This factor is measured as the difference between the yield on corporate bonds and the yield on long government bonds and its construction is hoped to capture the relative level of risk in the economy. As such, we would expect this to effect the discount rate in (3.18). A further interest rate variable that we would expect to influence the pricing operator is the term structure of interest rates, measured as the difference between the yield on long government bonds in time t and short government bonds in t-1. Given the open nature of the UK economy it may be expected that shocks to the exchange rate will have an impact on equation (3.18). A further indicator of the financial side of the economy to be considered in this chapter are shocks to the money supply which may have implications for future interest rates, inflation rates and hence future expectations regarding a firms cash flows. We also consider two inflation factors: the change in expected inflation and unexpected inflation. While the former factor could have an impact on both the numerator and denominator of (3.18) the latter will influence the investment opportunity set. Given the forward looking nature of equation (3.18) the empirical identification of economic state variables requires the generation of time series of unanticipated changes in the prespecified factors. Although this procedure is outlined in detail in Chapter 4, where we compare the effects of different assumptions regarding the way agents form their expectations on tests of the APT, we briefly outline the procedure here. Evidence from CRR (1986) indicates that the rate of change methodology does not produce unanticipated components that are innovations, defined as zero mean, serially uncorrelated, homoscedastic, white noise process. To counter this, an alternative technique for generating unanticipated components is to use autoregressive regressions in order to eliminate any systematic dependence amongst the residuals which are used as the unanticipated components (see, for example, Clare and Thomas (1994)). Therefore, all factors enter into a twelfth order autoregression in stationary form and the 'unrestricted' AR models are estimated and simplified by deleting insignificant lags. The final models should then be valid restricted, parsimonious representations of the unrestricted twelfth order AR regressions and the residuals from these regressions can be used as unanticipated factors that enter into the APT tests. Having outlined the candidate risk factors we now proceed to discuss the possible testable implications of the two asset pricing models.

3.5 The CAPM

The linear factor model (equation 3.15) has a number of testable implications in terms of both the CAPM and the APT. Consider the non-linear specification given in chapter 2, equation 2.19. Dealing first with the CAPM, assuming that the only relevant factor in the return generating process is the proxy for the market portfolio, then we can write equation (2.19) as:

$$R_t = B^m \lambda^m + B^m RM_t + \epsilon_t \tag{3.19}$$

where $\mathbf{R}_t$ is a N×1 excess return vector, $\mathbf{B}^m$ is a N×1 sensitivity vector, λ^m is the estimated market price of risk, $\mathbf{RM}_t$ is the excess return on the market portfolio and ϵ_t is an error vector assumed to be serially uncorrelated and homoscedastic, but can be correlated across assets.

The CAPM's Testable Implications

The testable implication of the CAPM is that the price of risk on the excess return on the market portfolio is statistically significant and equal across assets, that is, partioning the excess return vector, $\mathbf{R}_i$, into its specific components r_1, r_2....r_N and the corresponding sensitivity vector, **B**, into its specific components b_1, b_2....b_N, then:

$$R_{it} = B_i \lambda_i + B_i RM_t + \epsilon_{it} \tag{3.20}$$

implies that $\lambda_i = \lambda$ for all i where i = 1....N. This implication is tested by estimating (3.20) and estimating the unrestricted version:

$$R_{it} = \gamma + B_i F_t + u_{it} \tag{3.21}$$

where γ is a n×1 vector of intercept coefficients. The testable implication is that the following restriction holds:

$$\gamma = B\lambda \tag{3.22}$$

The likelihood functions from the two systems then form the basis of a likelihood ratio test for the null hypothesis that $\gamma = \mathbf{B}\lambda$.

First Testable Hypothesis

A test of the CAPM against an unspecified alternative is as follows: if the CAPM is the correct model then the residuals from the CAPM should have a mean of zero. Evidence contrary to this would indicate that part of the return generating process could not be explained by the market portfolio. Variants of this test have been used by Chan, Chen, and Hsieh (1985) and Shanken and Weistein (1990) in order to determine whether there is size effect in the sense that the empirical version of the CAPM fails to capture the risk-return relationship for small firms. In particular, if the CAPM fails to explain the small firm effect then we would expect that the residuals from the CAPM for small firms would be positive and significantly different from zero. To test this hypothesis we estimate the following equation:

$$\epsilon_{t,CAPM} = \alpha + \zeta_t \tag{3.23}$$

where $\epsilon_{t,CAPM}$ is the residual vector estimated from the CAPM, α is a constant vector and ζ_t is a vector of error terms. The t-ratio on the estimated coefficient then allows for a test whether the errors have a mean of zero.

Given a random sample of stocks, some of which will be small firms and some of which will be large firms, evidence of non-zero mean errors would lead to an examination of whether or not those stocks for which $\alpha=0$ are 'small'. The problem that we encounter with this test is that we know the residuals from the CAPM (and APT) are correlated across assets (see table 2.5). Therefore a test with more power would be to estimate the 'unrestricted' version given by equation (3.23) above and then impose an across equation restriction that $\alpha_1=\alpha_2=\ldots=\alpha_{69}$ and estimate this restricted model by ITNLSUR. First, if the restriction holds then it would appear that there is no firm size effect. Second, if the t-ratio on this restricted estimate of $\boldsymbol{\alpha}$ indicates that the residuals have a mean of zero we can infer that the asset pricing model is robust to an unspecified alternative. This test is also carried out.

Second Testable Hypothesis

This test compares the performance of the CAPM against a specific alternative. If the CAPM is the correct model then the differences in expected returns amongst stocks should only be due to different estimates of the sensitivity of a stock returns to the return on the market portfolio and no other factor. Thus augmenting equation (3.20) with a set of instruments to proxy an assets own variance should make no difference, that is, agents should not be rewarded for bearing unsystematic risk. To test this hypothesis we simply augment equation (3.20) with the squared residuals, defined as $\mathbf{v}$, from the CAPM regression,

$$R_t = B^m\lambda^m + B^m RM_t + b^v\lambda^v + b^v v + \epsilon_t \qquad (3.24)$$

where $\mathbf{B}^m$ is a N×1 matrix of asset sensitivities to the return on the market portfolio, $\boldsymbol{\lambda}^m$ is a 1×1 vector of the market price of risk, $\mathbf{b}^v$ is a N×1 vector of asset sensitivity to a proxy for own variance, and $\boldsymbol{\lambda}^v$ is the price of risk associated with the own variance factor. The hypothesis that own variance is unimportant implies that $\mathbf{b}^v\mathbf{v} = \mathbf{b}^v\boldsymbol{\lambda}^v = 0$. This can be tested by examining whether any of the individual assets have a significant price of risk.

3.6 The APT

Tests of the APT follow the approach taken to test the CAPM. In the first instance we estimate the restricted and unrestricted versions of the APT. This is simply estimation of equations (3.20) and (3.21), where for the APT $\mathbf{RM}_t$ is replaced by a N×k matrix $\mathbf{F}_t$ which is the time series of the observed factors given in table 3.1 (excluding the market portfolio), the vector $\mathbf{B}^m$ is replaced by a N×k matrix of estimated asset sensitivities to the factors, and $\boldsymbol{\lambda}^m$ is replaced by a k×1 vector of estimated risk premia. Accordingly we test the across equation pricing restrictions of equal price of risk for each factor given in equation (3.22).

The next step is to examine the statistical significance of the estimated risk premia. If a factor is not statistically significant we then delete this factor and estimate the reduced model. This reduced model places restrictions on the initial model. For example, assuming we start with the following model:

$$R_t = B\lambda + BF_t + v_t \tag{3.25}$$

where **B** and $\mathbf{F}_t$ are N×3 matrices, $\mathbf{R}_t$ is a N×1 vector and λ is a 3×1 vector. Assuming that the across equation restriction that $\gamma = \mathbf{B}\lambda$ holds, but the t-ratio indicates that $\lambda_3=0$, then we drop λ_3 and reestimate (3.25) where **B** and $\mathbf{F}_t$ are now N×2 matrices and λ is a 2×1 vector respectively. The restriction that $\lambda_3=0$ can be tested by comparing the likelihood functions of the two models in a likelihood ratio test. The next natural step, if these restrictions hold, is to test the across equation restrictions on the reduced model by estimating a new (reduced) unrestricted model.

As with the CAPM, a testable hypothesis of the APT model is that the errors from the APT regression should have zero means and the firm's own variance should not be a priced factor in the APT. Shanken (1992) shows that this tests is also a test of whether it is possible to treat the pricing relationship as an exact one. Under the null hypothesis that idiosyncratic risk is not priced then acceptance of the null also indicates that we can treat the pricing relationship as an exact one.

From this analysis we are able to make some inference about the performance of each model by comparing the results of tests for the two models. However the problem may arise that both models perform equally well in terms of the above tests. Moreover, while one model may not be robust to one of the tests while the other model is, the performance of the former may still be better in describing asset returns. Thus, in order to provide more rigorous and formal tests of the competing asset pricing models we also undertake some comparative tests.

3.7 Evaluating The Relative Performance Of The Models

First Comparison

To allow for the evaluation of the relative performance of the two competing models we carry out three tests. First is a test proposed by Chen (1983) and involves estimating

$$r_t = \beta + \alpha_1 \hat{r}_{t,APT} + \alpha_2 \hat{r}_{t,CAPM} + u_t \qquad (3.26)$$

where $\hat{r}_{t,APT}$ is a N×1 vector of fitted values from the APT model, $\hat{r}_{t,CAPM}$ is a N×1 vector of fitted values from the CAPM model. The null hypothesis that the APT is the correct model is that $\alpha_1 = 1$ and $\alpha_2 = 0$. This test, in the context of factor analysis (see Chen (1983), or the use of observed factors which include the market portfolio, is rather inconclusive. This inconclusiveness stems from the fact that in the Chen (1983) test the fitted values are obtained from both the CAPM regression, using a stock index as a proxy for the market portfolio, and the APT regression using factor analysis. It is highly likely that one of the factors extracted using factor analysis is highly correlated with the proxy for the market portfolio. Consequently the test proposed by Chen will be biased in favour of rejection of α_2=0 because the market portfolio will be included in both fitted values. In our first stage of testing the models we do not include the market portfolio as a factor in the APT and thus overcome this problem. We estimate equation (3.26) for the system as a whole and restricted $\alpha_1=1$ and $\alpha_2=0$. This restriction is easily tested against an unrestricted version.

Second Comparison

A further comparison between the two models is to calculate the average excess return and the average expected returns predicted from the CAPM and the APT in order to consider the extent to which the expected returns predicted by the APT and the CAPM capture the cross-sectional variation in average stock returns. In order to do this we estimate the following cross section regressions, one for the CAPM's predicted expected returns, and another for the APT's predicted expected returns:

$$r_i = \alpha_0 + \alpha_1 E[\hat{r}_i] + \nu \tag{3.29}$$

where $E[\hat{r}_i]$ is the expected return on asset i calculated from the APT and the CAPM. The preferred model will obviously explain most of the cross-sectional variation of asset returns. Thus, a comparison of the adjusted R^2's from the two regressions will indicate the performance of the two models in this respect.

Third Comparison

The final test we employ for comparison purposes is to test whether the residuals from the CAPM can be explained by the factors in the final APT specification. If the macroeconomic factors are important, over and above the contribution of the market portfolio, the CAPM's residuals should be explained by the macroeconomic factors. In analysing this we estimate the following regression:

$$\epsilon_{t\,CAPM} = B\lambda + BF_t + u_t \tag{3.30}$$

If the CAPM model (equation (3.20)) is the correct specification then the vector of residuals, ϵ_{tCAPM}, from (3.20) should not be explained by the factors, $\mathbf{F}_t$. Alternatively, if the APT is the correct specification then the CAPM residuals should be explained by the APT factors.

We now turn to the role of the market portfolio in tests of the APT. So far we have considered an APT specification using only measured macroeconomic factors. However, as discussed earlier, the role of the market portfolio in asset pricing models needs to be readdressed. Its inclusion as a factor can be motivated from a number of perspectives. Chan, Chen, and Hsieh (1985) use the market portfolio as a proxy for expected future growth in the economy. In addition to this, it is also possible to generalize the APT so that the market portfolio proxies for unobserved factors that have been left out of the list of candidates given in table 3.1 (see, for example, Chapter 2, and Burmeister and McElroy (1988)). Including the market portfolio as a factor in an APT specification also allows for another test of the CAPM - if the CAPM is the correct model then the measured factors should be insignificant and the market portfolio the only relevant factor. To ascertain whether the market portfolio has a role in the APT we estimate the full factor model with the inclusion of the market portfolio using NL3SLS.[11] The results from this analysis should shed some light on the issue of the relevance or otherwise of the market portfolio.

3.8 Empirical Results

In this section we present the results from the test of the CAPM and the APT outlined above. These tests of the individual asset pricing models are then followed by the comparative tests before finally evaluating the role of the market portfolio in an APT framework.

The CAPM

The NL3SLS estimates of the price of risk as predicted by the CAPM are presented in Table 3.2 along with the associated test of the restriction that the price of risk is equal across all securities. Panel A of table 3.2 indicates that the equity risk premium is 0.11% per month and statistically significant at the 5% level. This implies that investors in the stock market receive this return per unit of systematic risk they hold. Panel B of table 3.2 provides the results of the test of the pricing restrictions across all equations, indicating that the restriction of an equal price of risk across securities can not be rejected. At

[11] Following the results of chapter 2, we maintain the condition that the market portfolio enters the APT specification as an endogenous variable, and hence an instrumental variables estimator is required.

this stage the CAPM looks like a reasonable contender for the explanation of the behaviour of UK stock returns.

Now, given these apparent favourable results for the CAPM we consider other testable hypotheses outlined in section 3.5 in order to provide more robust evidence for, or against, the CAPM. In the first instance we test whether the residuals for the CAPM have a mean of zero. In order to do this we follow the procedure outlined in section 3.5 by initially regressing the time series of residuals on a constant. With the exception of two cases we can not reject the null hypothesis that the residuals have a mean of zero. However, as noted above, we augment this test to give it more power by restricting the system and testing whether the residuals have a mean of zero by estimating the system by ITNLSUR. These results, along with the test of the restriction it implies across the system are reported overleaf in table 3.3. The estimate of the constant from equation (3.31), panel A of table 3.3, restricting each equation to take on the same value produces a point estimate of -0.00012, which is insignificantly different from zero. Moreover, the restriction, in panel B of table 3.3, that the constants are equal to one another is accepted. Thus, there appears to be no evidence that any particular stocks, or group of stocks, such as small firms, have returns higher than those predicted by the CAPM. Finally, if the CAPM is correct then a firms own variance should not be a priced factor. However, considering table 3.4, for 23 firms out the total 69 the measure of 'own variance' is statistically significant.

Summarising these results, initially there is evidence to support the CAPM: the across equation pricing restrictions hold and the estimate of the market risk premium seems sensible. In addition, there is empirical support for the hypothesis that the residuals from the CAPM model have a mean of zero. As such then there appears to be no evidence of a firm size effect. However, there does appear to be evidence against the CAPM when confronted by a specific alternative. The results show that a stocks own variance is priced in one third of the firms. Such a finding questions one of the most important properties of the CAPM, namely in equilibrium investors will only be rewarded for bearing systematic risk.

The APT

The testing procedure for the APT follows directly the lines of that for the CAPM outlined in the previous section. We first estimate the unrestricted version of the APT, using the variables shown in table 3.1 (where the unexpected components of the factors have been derived from the autoregressive models) noting that at this stage the market portfolio is omitted and consequently the system of equations is estimated using ITNLSUR. Second, we test the restrictions of equal price of risk across all equations for the restricted version of the APT. Using likelihood ratio tests to determine whether first the across equation restrictions of equal price of risk on the factors are valid, see table 3.5, and second, whether sequential reduction of

Table 3.2
Estimate of Market Price of Risk and Test of Pricing Restrictions

Panel A
NL3SLS Estimate of Market Risk Premium

Parameter	Coefficient	T - Ratio
Excess Return on Market Portfolio	0.0011	2.10

Panel B
Test of the Across Equation Pricing Restrictions

Restriction	Calculated Value	Critical Value
H_o: $\gamma = \mathbf{B}\lambda$	67.34	$\lambda_2(68) = 88.179^{\dagger}$

Notes: Sample period 1980M1 - 1993m12. Source : Datastream

† Approximate 5% critical Value calculated as $\chi^2(v) = v\left[1 - \frac{2}{9v} + 1.64\sqrt{\frac{2}{9v}}\right]^3$ where v is the degrees of freedom

Table 3.3

A Test of Whether the Residuals from the Estimation of the CAPM are Jointly Equal to Zero

Panel A:

Restricted Estimate of α in Equation (26)

Parameter	Coefficient	T-Ratio
Constant	-0.00012	-0.25

Panel B :

Test of the Restriction that $\alpha_1 = \alpha_2 = ... = \alpha_{69}$

Null Hypothesis	Calculated Value	Critical Value
H_o: $\gamma_1 = \gamma_2 = ... = \gamma_{69}$	67.45	χ^2 (69) = 89.10†

Notes: Sample period 1980M1 - 1993m12. Source : Datastream

† Approximate 5% critical Value calculated as $\chi^2(v) = v\left[1 - \frac{2}{9v} + 1.64\sqrt{\frac{2}{9v}}\right]^3$ where v is the degrees of freedom

Table 3.4

Estimates of the Own Variance Factor in the CAPM (figures in parentheses are T-Ratios)

Firm No.	$\lambda_{o.v}$	Firm No.	$\lambda_{o.v}$	Firm No.	$\lambda_{o.v}$	Firm No.	$\lambda_{o.v}$
1	-0.0039 (-2.54)	18	-0.0145 (-0.33)	35	-0.0560 (-1.18)	52	-0.0108 (-2.24)
2	0.0247 (0.25)	19	-0.0003 (-0.15)	36	-0.0188 (-1.89)	53	-0.0066 (-2.02)
3	-0.0043 (-0.47)	20	-0.0081 (-1.28)	37	0.0761 (0.20)	54	-0.0201 (-4.36)
4	-0.0030 (-0.30)	21	-0.0006 (-0.16)	38	-0.0073 (-3.72)	55	-0.0157 (-2.76)
5	-0.0100 (-2.21)	22	-0.0005 (-0.36)	39	-0.0194 (-2.64)	56	-0.0146 (-2.44)
6	-0.0021 (-2.12)	23	0.0025 (0.47)	40	-0.0123 (-0.18)	57	-0.0041 (-0.09)
7	-0.0131 (-2.25)	24	-0.0033 (-1.42)	41	-0.0048 (-1.61)	58	-0.0015 (-2.28)
8	-0.0029 (-0.56)	25	0.0188 (0.11)	42	-0.0024 (-0.90)	59	-0.0055 (-2.91)
9	-0.0047 (-2.44)	26	-0.0016 (-0.57)	43	-0.0069 (-0.46)	60	-0.0041 (-0.23)

Table 3.5 cont

10	-0.0047 (-2.75)	27	-0.0036 (1.87)	44	-0.0176 (-2.28)	61	-0.0003 (-0.09)
11	0.0303 (0.17)	28	-0.0037 (-2.82)	45	0.0039 (0.83)	62	-0.0049 (-0.21)
12	-0.0012 (-0.51)	29	-0.0013 (-0.78)	46	-0.1399 (-1.36)	63	-0.0023 (-1.40)
13	-0.0048 (-1.35)	30	-0.0242 (-3.52)	47	-0.0042 (-1.84)	64	-0.0097 (-0.70)
14	-0.0028 (-2.47)	31	-0.0023 (-1.34)	48	-0.0064 (-0.91)	65	-0.0054 (-1.65)
15	-0.0038 (-0.26)	32	-0.0086 (-2.10)	49	-0.0141 (-2.02)	66	-0.0071 (-1.94)
16	0.00218 (0.03)	33	-0.0041 (-0.99)	50	-0.0289 (-0.77)	67	-0.0044 (-1.78)
17	-0.0087 (-1.28)	34	-0.0112 (-2.72)	51	-0.0141 (-3.08)	68	-0.0039 (-1.16)
						69	-0.0036 (-1.94)

Notes: Sample period 1980M1 - 1993m12. Source : Datastream

Table 3.5

Likelihood Ratio Tests of the APT's Across Equation Pricing Restrictions

APT Specification†	Calculated Value	Critical Value
All Factors (9)	59.85	χ^2 (60) = 79.082
First Reductions (7)	62.41	χ^2 (62)
Second Reductions (6)	62.51	χ^2 (63)
Third Reductions (5)	63.65	χ^2 (64)
Fourth Reductions (4)	67.04	χ^2 (65)

Notes:
† The numbers in parentheses indicate the number of factors at each stage
Sample period 1980M1 - 1993m12. Source : Datastream

insignificant factors are valid, see table 3.6, we arrive at the final APT specification the results for which are presented in table 3.7. In arriving at this final specification, the pricing restrictions across equations are accepted at each stage, and in addition the restrictions entailed in moving from one model to a reduced factor model are also accepted.

The factors found to command a risk premium in the UK equity market, according to the APT, are real industrial production, real sales, unexpected inflation, and commodity prices. Before making any inferences or interpretation of this final model some tests of the APT, following the lines of those for the CAPM, are presented. With respect to the assumption that the residuals from the APT regression have a mean of zero we carried out the same tests as those for the CAPM. In no cases, when the residuals were regressed on a constant , were any of the constants significantly different from zero. Table 3.8 reports the results from the second stage of this test. Panel A of table 3.8 shows that the restricted estimate of the constant is insignificantly different from zero and Panel B confirms that the constants can be restricted to be equal to one another. Although for both the CAPM and the APT this test indicates that the residuals do have a mean of zero, the restricted estimate of the constant, in the regression of the residuals on a constant, is positive for the APT and negative for the CAPM, which implies that the CAPM over predicts returns and the APT under predicts, although this is statistically insignificant. Additionally, the amount by which the CAPM over predicts returns is greater than the amount the APT under predicts returns.

Table 3.6

Likelihood Ratio Tests of Deletion of Insignificant Factors

Reduction	Calculated Value	Critical Value
First Reduction (9 to 7 factors)	140.64	χ^2 (140) = 168.52†
Second Reduction (7 to 6 factors)	70.12	χ^2 (70) = 90.53
Third Reduction (6 to 5 factors)	70.54	χ^2 (70)
Fourth Reduction (5 to 4 factors)	70.89	χ^2 (70)

Notes:
Sample period 1980M1 - 1993m12. Source : Datastream

† Approximate 5% critical Value calculated as $\chi^2(v) = v\left[1 - \frac{2}{9v} + 1.64\sqrt{\frac{2}{9v}}\right]^3$ where v is the degrees of freedom

Table 3.7

ITNLSUR Estimates of Factor Risk Premia for APT

Final Reduced Model

Factor Risk Premia	Coefficient Estimates	T - Ratio
λ_2 (Real Industrial Production)	0.00284	4.75^{***}
λ_4 (Real Retail Sales)	-0.00058	-1.76^{*}
λ_6 (Unexpected Inflation)	0.00138	2.39^{**}
λ_9 (Commodity Prices)	-0.00601	-2.10^{**}

Notes: Sample period 1980M1 - 1993m12. Source : Datastream
*** Indicates Significant at 1%
** Indicates Significant at 5%
* Indicates Significant at 10%

Table 3.8

A Test of Whether the Residuals from the Estimation of the APT are Jointly Equal to Zero
APT, Final Reduced Model

Panel A:

Restricted Estimate of α in Equation (26)

Parameter	Coefficient	T-Ratio
Constant	0.00003	0.07

Panel B :

Test of the Restriction that $\alpha_1 = \alpha_2 = \ldots = \alpha_{69}$

Null Hypothesis	Calculated Value	Critical Value
H_o: $\gamma_1=\gamma_2=\ldots=\gamma_{69}$	52.01	χ^2 (69) = 89.10

Notes: Sample period 1980M1 - 1993m12. Source : Datastream

† Approximate 5% critical Value calculated as $\chi^2(v) = v\left[1 - \frac{2}{9v} + 1.64\sqrt{\frac{2}{9v}}\right]^3$ where v is the degrees of freedom

Table 3.9

Estimates of the Own Variance Factor in the APT (T-Ratios in parentheses)

Firm No.	$\lambda_{o.v}$	Firm No.	$\lambda_{o.v}$	Firm No.	$\lambda_{o.v}$	Firm No.	$\lambda_{o.v}$
1	-0.0036 (-1.43)	18	-0.0544 (-0.56)	35	-0.0359 (-0.60)	52	-0.0109 (-0.86)
2	-0.0717 (-0.10)	19	0.00175 (0.55)	36	-0.0281 (-1.54)	53	-0.0033 (-0.59)
3	0.00021 (0.04)	20	-0.0065 (-0.55)	37	0.0118 (0.26)	54	-0.0124 (-1.35)
4	0.00290 (0.18)	21	0.00350 (0.70)	38	-0.0108 (-1.92)	55	-0.0189 (-1.76)
5	-0.0081 (-2.40)	22	0.00051 (0.20)	39	-0.0116 (-0.94)	56	-0.0176 (-1.69)
6	0.00100 (0.35)	23	0.00455 (0.33)	40	0.06574 (0.91)	57	-0.1233 (-0.18)
7	-0.0069 (-1.35)	24	-0.0039 (-0.82)	41	0.03426 (0.28)	58	-0.0012 (-0.52)
8	-0.0026 (-0.35)	25	-0.0091 (-1.20)	42	-0.0002 (-0.03)	59	-0.0093 (-2.02)
9	-0.0029 (-0.77)	26	-0.0175 (-1.36)	43	-0.0199 (-1.12)	60	-0.0023 (-0.08)

10	-0.0033 (-1.15)	27	0.00177 (0.40)	44	-0.0238 (-1.63)	61	0.00446 (0.47)
11	-0.00768 (-1.67)	28	-0.0052 (-2.82)	45	-0.0089 (-1.25)	62	-0.0093 (-0.74)
12	-0.0314 (-0.12)	29	-0.0001 (-0.05)	46	0.04283 (1.17)	63	-0.0008 (-0.34)
13	0.00042 (0.07)	30	-0.0316 (-3.81)	47	-0.0015 (-0.32)	64	-0.0185 (-0.96)
14	-0.0019 (-1.05)	31	0.00063 (0.17)	48	-0.0325 (-0.09)	65	-0.0237 (-1.50)
15	0.00501 (0.13)	32	-0.0104 (-2.28)	49	-0.0575 (-0.33)	66	0.01622 (0.25)
16	0.00686 (0.23)	33	-0.0041 (-0.71)	50	0.00094 (0.00)	67	-0.0085 (-1.63)
17	-0.0120 (-1.46)	34	-0.0105 (-2.22)	51	-0.0010 (-0.09)	68	0.00302 (0.40)
						69	-0.0041 (-1.22)

Notes: Sample period 1980M1 - 1993m12. Source : Datastream

The final test of the APT is whether or not a firms 'own variance' is priced. Table 3.9 above provides individual estimates and t-ratios for the 'own variance' proxied by the squared residuals from the final APT specification. In contrast to the CAPM, where 23 firms have a statistically significant 'own variance' factor, there are only six firms which have a statistically significant 'own variance' factor in the APT specification. A further implication of this is that we can treat the pricing relationship as an exact one given the conditions provided in Shanken (1992).

Summarizing the above evidence, for both the CAPM and the APT, it seems apparent that the APT is robust to all the tests, while the CAPM fails to account for a firms own variance. We are left with the conclusion, at this stage, that the APT is more robust in terms of the above tests. However, before we can unambiguously claim that the APT should be the preferred asset pricing model in a practical context we must determine the relative performance of the models against one another. The reason for this is that although the CAPM failed in one of the above tests while the APT was robust to all of them, the CAPM may prove to be a better description of UK asset returns and thus provide more accurate predictions. In order to establish whether there is any further support for the claims that the APT is a better asset pricing model the next section considers the relative performance of the two competing asset pricing models.

3.9 THE APT VERSUS THE CAPM

To compare the performance of the CAPM and the APT more directly we undertake the following tests. First we conduct a test in the spirit of Chen (1983) given in equation (3.26). To recap, if the APT is the correct model then α_1 in equation (3.26) should be equal to 1 and α_2 should be equal to zero. The results from these test are given in table 3.10 and we accept the restrictions that $\alpha_1=1$ and $\alpha_2=0$ at the 5% level. This provides formal evidence that the APT outperforms the CAPM in this context. The next test aimed at comparing the two models is to examine whether the factors in the APT model can explain the residuals from the CAPM. Results of this test, in table 3.11, show that the risk premia on the factors are all significant and the estimates are very similar to the original APT model. Thus, the unexplained part of returns in the CAPM can be explained by the APT where the estimates on the factors are the same sign as in the APT model and very close in magnitude. Again this is overwhelming evidence that the APT is outperforming the CAPM.

Finally, we consider the extent to which the expected returns predicted by the APT and the CAPM capture the cross-sectional variation in average security returns. We ran the cross sectional regression, given in equation (3.29), for both the CAPM and the APT and found that the APT predicted expected

Table 3.10
Likelihood Ratio Test of the Restrictions that : $\alpha_1 = 1, \ \alpha_2 = 0$

Restriction	Calculated Value	Critical Value
$\alpha_1 = 1, \alpha_2 = 0$	137.99	$\chi^2\,(138) = 166.32$

Notes: Sample period 1980M1 - 1993m12. Source : Datastream

† Approximate 5% critical Value calculated as $\chi^2(v) = v\left[1 - \frac{2}{9v} + 1.64\sqrt{\frac{2}{9v}}\right]^3$ where v is the degrees of freedom

Table 3.11

CAPM Residuals Regressed on APT Factors (ITNLSUR)

Risk Premia	Estimate	T - Ratio
λ_2	0.00226	4.52^{***}
λ_4	-0.00733	-2.36^{**}
λ_6	0.00097	1.90^{*}
λ_9	-0.00544	-2.09^{**}

Notes: Sample period 1980M1 - 1993m12. Source : Datastream
*** Indicates Significant at 1%
** Indicates Significant at 5%
* Indicates Significant at 10%

returns explains approximately sixty percent of average cross sectional returns, while the CAPM's predicted expected returns explains virtually none of the average cross sectional returns. This result is to be expected given the finding that the APT factors can explain the CAPM residuals. The reason for this apparently poor performance of the CAPM model becomes clear when we consider that the CAPM estimates a positive risk premium of 0.11% per month and generates β's which generally lie in the range 0.5 to 1.2. This, coupled with a positive yield on the risk-free asset means that the CAPM always predicts a positive expected total return which will always be greater than the risk-free rate.

Overall the APT, unlike the CAPM, is robust to all the individual tests we performed on each model. Moreover, in a battery of comparative tests the APT was seen to outperform the CAPM in its ability to explain a cross section of UK security returns. At this stage then, we are in a position to conclude that the APT is the more appropriate asset pricing model. However, the interesting question that remains is, can the addition of the market portfolio to the four macroeconomic factors improve the performance of the APT, that is does the market portfolio have a role to play in an APT specification ?

Within the APT framework the estimated price of risk on the market portfolio is very similar to the results reported for the CAPM in table 3.2 in terms of both the size of the estimated coefficient and the t-ratio. This result seems to shed light on the poor performance of the CAPM in terms of explaining the cross-section of actual returns, that is, along with the market portfolio there are four other factors which are important in explaining stock returns. To formally test whether or not the market portfolio is important in terms of explaining returns and its effect on the significance or otherwise of the other factors we undertake the following procedure: first we estimate the APT again, but using all the factors including the market portfolio; second, given we have identified the relevant factors in the first step we then test whether the macroeconomic factors add any information to the explanation of stock returns as compared to the market portfolio by comparing the likelihood functions of the APT model with the market portfolio with the likelihood function of the CAPM model. Basically, this second step is just a test for a zero restriction on the estimated matrix **B** and vector λ's on the macroeconomic factors, the null hypothesis being that the macroeconomic factors do not add anything to the information contained in the market portfolio.

Considering the first issue, the market portfolio was found to be significant throughout the deletion of insignificant macroeconomic factors, and in the final model the macroeconomic factors found significant for the APT in this context are the same as when the market portfolio was not included, that is, they correspond to those in table 3.7. It is quite clear that the role the market portfolio has to play in the APT is similar to the role it plays in the CAPM in that the estimate of the risk premia is similar, and the estimated sensitivities of assets to the market portfolio fall within the range of what would be termed

Table 3.12

NL3SLS Estimates of Risk Premia for APT Including Market Portfolio

Parameters	Estimates	T - Ratio
λ_2 (Real Industrial Production)	0.00228	4.43^{***}
λ_4 (Real Retail Sales)	-0.00078	-2.44^{**}
λ_6 (Unexpected Inflation)	0.00087	1.67^{*}
λ_9 (Commodity Prices)	-0.00621	-2.34^{**}
λ_{10} (Return on the Market Portfolio)	0.00160	2.19^{**}

Notes: Sample period 1980M1 - 1993m12. Source : Datastream
*** Indicates Significant at 1%
** Indicates Significant at 5%
* Indicates Significant at 10%

sensible β's. The across equation restrictions all hold and the reductions entailed from moving from one model to the reduced model are accepted, see tables 3.13 and 3.14 below.

As a further test for correct specification we estimated the APT model with only the market portfolio as a factor (trivially the CAPM estimated by NL3SLS), amounting to a test of whether or not the factors are important in explaining returns. The restrictions of zero coefficients on the other macroeconomic factors is easily rejected, the likelihood test statistic being 1132.54 which is distributed χ^2 (280) with an approximate 5% critical value of 323.189. Thus we know other factors as well as the market portfolio are important in explaining asset returns. These results regarding the significance of the market portfolio and its effect on other factors are in contrast to those found in Chen, Roll, and Ross (1986), Chan, Chen, and Hsieh (1985), Shanken and Weistein (1990) and Fama and French (1992).

3.10 Summary and Conclusion

In this chapter we used a framework for testing asset pricing models that was outlined in chapter 2. This framework is free from the problems identified with factor analysis and the Fama-MacBeth two step estimator and provides consistent and efficient estimators and tests of the across equation restrictions of equal price of risk in an approximate factor model framework. Real industrial production, unexpected inflation, the commodity price index, real sales, and the market portfolio were found to be important in explaining stock

Table 3.13

Likelihood Ratio Tests of APT Across Equation Restrictions When the Market Portfolio is Included

Model	Calculated Value	Critical Value
All Factors (10)	34.29	χ^2 (60) = 79.08
First Reductions (8)	36.62	χ^2 (62)
Second Reductions (7)	37.91	χ^2 (63)
Third Reductions (6)	38.90	χ^2 (64)
Fourth Reductions (5)	39.64	χ^2 (65)

Notes: Sample period 1980M1 - 1993m12. Source : Datastream

Table 3.14

Likelihood Ratio Tests of APT Reductions

Reduction	Statistic	Calculated Value
First Reduction (10 to 8 factors)	155.59	χ^2 (142) = 170.72†
Second Reduction (8 to 7 factors)	71.24	χ^2 (71) = 90.53‡
Third Reduction (7 to 6 factors)	83.26	χ^2 (71)
Fourth Reduction (6 to five factors)	81.43	χ^2 (71)

Notes: Sample period 1980M1 - 1993m12. Source : Datastream
‡ Critical Value Reported is χ^2 (70)

† Approximate 5% critical Value calculated as $\chi^2(\nu) = \nu\left[1 - \frac{2}{9\nu} + 1.64\sqrt{\frac{2}{9\nu}}\right]^3$ where v is the degrees of freedom

returns. Employing a battery of tests on both the CAPM and the APT we found that while the APT was robust to these tests, the CAPM was not in all cases. Consequently, we are left with the result that, as far as the UK stockmarket is concerned, the five factor APT, where one of the factors is the market portfolio, is the preferred model of stock returns.

Considering the issues raised at the beginning of this chapter regarding the mixed results of tests of asset pricing models, the role of the market portfolio in such tests and the implications of asset pricing models for valuation the results of this chapter are of particular importance. We have shown that the APT, in which the market portfolio does have a role, is robust to a number of individual tests. Furthermore, this model outperforms the CAPM. These results are of use in general to those wishing to test asset pricing models since the framework from which these tests are performed is important in determining the correctly specified model.

Finally, there is a further interesting point to note. In this chapter we argued that the unanticipated components of the factors should be generated by autoregressive models since the use of the rate of change methodology does not necessarily lead to the result the these unanticipated components are innovations. Comparing the estimated prices of risk for the approximate factor model in chapter 2 and those in this chapter, it is evident that the way the unanticipated factors are generated effects the estimated sign, size and significance of the risk premia. Thus, the issue of how unexpected components are generated is a crucial one. In the next chapter we examine in detail the possible techniques of generating unexpected components and their effects on the estimates of the prices of risk.

CHAPTER 4

THE APT, MACROECONOMIC AND FINANCIAL VARIABLES AND EXPECTATIONS GENERATING PROCESSES

4.1 INTRODUCTION

Empirical tests of the APT using observed factors implicitly rely on the assumption that stock returns react to news regarding macroeconomic and financial variables. Fundamental to this is the notion that news is unanticipated. Consequently, this implies that agents form expectations of the factors that command a risk premium in asset markets. In tests of the APT it is therefore necessary to postulate an expectations formation process in order to generate the unanticipated components which enter into the APT specification. The APT says nothing about how agents form their expectations of observed factors. However, a condition required of the unanticipated components is that they should be zero mean, serially uncorrelated white noise processes. Accordingly, any expectations process we specify must, at least, provide unanticipated components that satisfy these properties. Failure to satisfy this requirement surely renders the test of the APT questionable.

The objective of this chapter is to address the issue of generating unexpected components for the observed factors and show that previous techniques employed in this area may well lead to results that are misleading. To date, two techniques have been employed extensively. These are simple 'rate of change' models and autoregressive models. The former technique simply uses the first difference of the factor as the unanticipated component and essentially assumes the factors follow a random walk where the expectation is then the current value. The latter technique is more general, assuming that agents use autoregressive models to form expectations. The unanticipated components are then given by the residuals from these models. In this chapter we highlight the weaknesses of these two techniques as mechanisms for generating unexpected components and suggest an alternative based upon expectations formation as a learning process. We find that the rate of change methodology fails to provide the basic criteria that the unexpected components are serially uncorrelated and the autoregressive methodology fails to provide an expectations generating process that avoids the possibility of agents making systematic forecast errors. This results in these two techniques being invalid as methods of generating unexpected components. The alternative methodology we specify for generating unexpected components, based upon the Kalman filter, does meet the basic requirements of providing unexpected components which are innovations and an expectations generating process which does avoid the possibility of agents making systematic forecast errors. Furthermore, these findings do matter in tests of the APT for, as we show, the estimated prices of risk are sensitive to the way the unexpected components are generated. Therefore, tests of the APT which use the rate of change methodology or autoregressive techniques to generate unexpected components will lead to invalid results in terms of which factors command a risk premium. The rest of the chapter is organised as follows. In Section 4.2 we present a technique for generating the unexpected components that enter into the APT based upon expectations generating processes as a learning scheme. Section 4.3 provides estimates of the unanticipated components for a number of observed factors using three techniques to generate the unanticipated components. In Section 4.4 we provide evidence that the results from tests of the APT are very sensitive to the way the unanticipated components have been generated. Section 4.5 concludes.

4.2 Generating Unanticipated Components For Factors In The APT

In tests of the APT the necessity to generate unanticipated components in the factors is readily seen if we consider the model of Ross (1976) which assumes the return generating process is a function of k systematic risk factors:

$$R_{it} = E[R_{it}] + b_{i1}f_{1t} + \cdots + b_{ik}f_{kt} + u_{it} \qquad (4.1)$$

where R_{it} is the return on asset i in time t; $E[\cdot]$ is the expectations operator; b_{ik} is the sensitivity of asset i to the kth factor; f_{kt} is the kth factor with $E[f_{kt}] = 0$ and u_{it} is an error term which represents idiosyncratic returns with $E[u_{it}] = 0$, $E[f_{kt}u_{it}] = 0$, and $E[u_{it}u_{jt}] = 0$ when $i \neq j$, or σ^2 when $i = j$. Subtracting expected returns from both sides of (4.1) gives the following expression where the right hand side must be unanticipated:

$$R_{it} - E[R_{it}] = b_{i1}f_{1t} + \cdots + b_{ik}f_{kt} + u_{it} \qquad (4.2)$$

We can define the ith factor, f_{it} as:

$$f_{it} = (X_{it} - E_{t-1}[X_{it}]) \tag{4.3}$$

where X_{it} is the actual value of the ith observed factor in time t and $E_{t-1}[X_{it}]$ is the expectation taken at time t-1. Clearly, defining the factors in this way raises two issues. First, the assumption that $E[f_{it}] = 0$ must be satisfied. This is not a problem since we can always ensure the f_{it} has zero mean by construction. Second, we must consider the expectations formation process regarding X_{it} since this may alter the form of f_{it}.

The APT and Expectations: Existing Methodologies

There are two major extant approaches to generating unexpected components in the economic state variables. These are the simple 'rate of change' methodology and the autoregressive methodology. In previous work (for example, see CRR (1986)) the 'rate of change' methodology has failed to provide unexpected components that are white noise . There appears to be a large systematic component in the time series of the differenced factors. For example, table 3 in CRR (1986) reports autocorrelation coefficients for the first twelve lags of their series of unexpected components and a Box-Pierce test for 24th order autocorrelation. In all cases their unexpected components reveal significant autocorrelation at the 5% level.

The use of AR models to form conditional expectations aims to eliminate any systematic dependence in the unexpected component. This form of expectation formation process assumes that the agent limits the information set to 'own' past values. While this technique may provide unexpected components that are innovations, and thus satisfy an important condition in expectation formation, it also places a restrictive assumption on agents expectation formation process. This restrictive assumption concerns the issue of stability of AR models over time. Implicitly AR models assume that the agent does not update the estimated parameters when there is a change in regime or when variables time-vary. However, if the estimated parameters of the AR model change over time then agents will make systematic forecast errors. For example, when dealing with macroeconomic variables such as interest rates, exchange rates, inflation, and the money supply agents will be aware that they are subject to government influences and this influence changes from time to time given current government policy.

A similar problem arises if adaptive expectations are considered. Although adaptive expectations are a more sophisticated form of expectations formation than simple AR models they also suffer from the problem that the coefficient of adaption is fixed, and once again given, say, a change in policy regime, agents would still make systematic forecast errors. These models assume some irrationality on behalf of the agent when the change takes place. To counter the criticisms of irrationality the development of rational expectations has ensued in the economic literature. These models offer an alternative that implicitly makes the assumption that agents act in a rational manner and by definition will not make systematic forecast errors.

The Rational Expectations Hypothesis

The APT assumes that agents are rational and therefore it would seem natural that we generate innovations in the factors by forming expectations that are rational. One obvious way forward here would be to employ the REH framework of expectations formation. Following the seminal work of Muth (1961) a growing body of literature on both the specification and implications and the estimation of rational expectations has developed. The assumptions underpinning the REH stem from two basic axioms. Firstly the assumption that agents know the true structure of the model generating the variable of interest, known as the available information assumption and secondly the assumption of information exploitation, that is, given this information then agents use it to optimize their decisions. These assumptions provide a relationship between the expectation and the outcome as follows:

$$X_{t+1} = E(X_{t+1}) + v_{it+1} \tag{4.4}$$

where the following conditions are assumed to hold:

$$\begin{aligned} &E\left(v_{it+1} \mid \Omega_t\right) = E\left(v_{it+1} \mid \Lambda_t\right) = 0 \\ &E\left(v_{it+1}^2 \mid \Omega_t\right) = \sigma_v^2 \\ &E\left(v_{it+1}\, v_{it+1-j} \mid \Omega_t\right) = 0 \end{aligned} \tag{4.5}$$

where j = 1,2,3 . . . ∞. forecast errors are unbiased, uncorrelated and have constant variances, that is, error in the expectation is a white noise process and an innovation conditional on the complete information set Ω_t, and is orthogonal to a subset of the complete information set ($\Lambda_t \in \Omega_t$). In order to assess how RE models work consider the following model which the agent believes to be the true model:

$$z_\tau = x_\tau' \beta + \varepsilon_\tau \tag{4.6}$$

where z_τ is a variable in time τ (τ=-T_0,...,t-1) related to a vector of predetermined variables $\mathbf{x}_\tau$, β is a vector of fixed coefficients, and ε_τ is a white noise error term. Assuming the agent wishes to form expectations of future values of z_t conditional on the predetermined value of $\mathbf{x}_t$ dated up to and including time period τ. The expectations process can be written as:

$$E_{t-1}(z_t) = x_t' \beta_{t-1} \tag{4.7}$$

where

$$b_{t-1} = \left(X_{t-1}' X_{t-1}\right)^{-1} X_{t-1}' z_{t-1} \tag{4.8}$$

is the OLS estimator of β. As new information appears in the subsequent periods it is used optimally to provide the future periods expectation, thus the estimator for β for time period t+n is:

$$b_{t+n-1} = \left(X_{t+n-1}' X_{t+n-1}\right)^{-1} X_{t+n-1}' z_{t+n-1} \tag{4.9}$$

This expectations process will satisfy the information exploitation assumption of Muth's REH since the OLS estimator is unbiased, consistent and efficient, that is, it provides the minimum variance estimator. There seems to be no great contention with this assumption and it is reasonable to assume that agents would use information optimally. However, there is a great deal of contention surrounding the issue of information availability, that is, the assumption that the agent knows the true model which generates the variable of interest. The plausabilty of this assumption has been drawn into question by, amongst others, Friedman (1979) who shows that given the expectation errors,

$$z_t - E_{t-1}(z_t) = x_t'(\beta - b_t) + \varepsilon_t \tag{4.10}$$

the error orthoganality condition of the REH will only be met if $\mathbf{b}_t = \beta$. This implies that agents know not only the true model but also the coefficients in the model. An example of the plausibility or otherwise of such a condition is outlined by Friedman (1979) using the Sargent and Wallace (1975) rational expectations model:

> *"to derive the conditional expectation $E_{t-1}(p_t)$* (prices), *people must solve out the five equation model using the model's own mathematical expectation of p_t* (price) *as the fifth equation; and to perform this solution people must know not only the specification of the other four equations but also the values of their coefficients"*

Typically such issues arise in models invoking the RE and have led researchers such as Friedman to examine alternative forms of expectation formation. In particular, Friedman (1979) alludes to a simple linear model with learning. The attractiveness of such an approach is its ability to capture short run dynamics as it allows agents to update their expectations as new information becomes available. These learning models are an extension of adaptive expectations models where instead of having a fixed adaptive parameter, the parameter is updated as new information is obtained. In addition, given an AR model with time varying parameters, these time-varying parameters can be estimated using a Kalman Filter, thus countering the arguments of irrationality of fixed parameter models. Such learning models have been termed 'weak' rational expectations (Hall (1990)) assuming agents expectations are on average correct and they do not make systematic forecast errors.

Expectations, Learning and the Kalman Filter

The Kalman Filter has been applied as a technique to produce expectations that satisfy the criteria for 'weak' rational expectations. Cuthbertson (1988) provides a link between Friedman's objections to the strong form of RE and his alternative of a learning process which is characterised by a simple linear model with time-varying parameters as a proxy for the more complex true model. This link is the Kalman filter. The Kalman filter can be used to estimate models of expectation formation which include variables that time-vary and it can also be used to estimate unobserved components models, where the unobserved component is the expectation. Both models assume learning by the agent. Following Friedman (1979), agents may use a simple linear model that updates the expectation as new information comes along on the variable of interest. The emergence of new information will cause the variable to change and this allows the expectation to be updated. Given that information flow is random then the changes in expectations will be governed by a random process. The Kalman filter allows the modelling of this random process and hence models the evolution of the expectations series.

To rationalise the use of the Kalman filter and provide an intuitive framework in the context of expectations we begin with the variable parameter adaptive expectations model described by Cuthbertson (1988). Suppose we have

$$Y_t = Y_t^* + u_t \tag{4.11}$$

where Y_t is some variable observed by the agent, Y_t^* is the expectation of Y_t, and u_t is the unanticipated factor (the difference between the expected and actual outcome of Y_t). Clearly Y_t^* is unobserved and this is what is estimated. Assuming we have a prior estimate of Y_0, Y_0^*, then we update this as new observations on Y become available. Assuming that expectations (Y^*) evolve as a random walk and that a fixed amount of u_t, h_t (assumed to vary over time but to be known by the agent) constitutes a permanent change in Y, and $(1-h_t)u_t$ constitutes a transitory change, then we can write the model in two parts. The first part is known as the measurement equation which gives actual Y_t equal to the expected value Y_t^* plus $(1-h_t)u_t$:

$$Y_t = Y_t^* + (1-h_t)u_t \tag{4.12}$$

The second part is known as the transition equation which represents the assumed evolution of Y_t^* through time:

$$Y_t^* = Y_{t-1}^* + h_t u_t \tag{4.13}$$

Substituting (4.13) into (4.12), multiplying by h_t and then substituting from (4.13) for $h_t u_t$ gives:

$$Y_t^* = Y_{t-1}^* + h_t(Y_t - Y_{t-1}^*) \tag{4.14}$$

Equation (4.14) is the updating equation for Y_t^*. In terms of expectation formation, assuming that h_t is known, and we have data on Y and a prior estimate of Y_0^* then (4.14) can be used to provide future expectations of Y_t^* The drawback of this type of model is the assumption that h_t is known by the agent. To make this model realistic the Kalman filter provides an estimate of h_t. Assuming that shocks to Y_t and Y_t^* are statistically independent and that the changes in Y_t^* are time-varying with parameter γ_{t-1} which evolves as a random walk (these are less restrictive assumptions than the REH and become intuitively appealing if we view information as arriving randomly) then the model can be written as:

$$Y_t = Y_t^* + u_t \tag{4.15}$$

$$\begin{aligned} Y_t^* &= Y_{t-1}^* + \gamma_{t-1} + \zeta_t \\ \gamma_t &= \gamma_{t-1} + \omega_t \end{aligned} \tag{4.16}$$

Equation (4.15) is the measurement equation, and (4.16) are the transition equations which determine the evolution of the unobserved component. These equations characterise the stochastic environment the agent faces where the expectations are assumed to be generated by a stochastic trend. The Kalman filter can also be used to estimate models which have time-varying parameters. Such models, in the context of expectations formation, are those where the parameters of the model are subject to change overtime. It is important when agents form their expectations that they take account of any time variation in the parameters arising out of, say, changes in government policy. In terms of the Kalman filter the time-varying parameters model is specified as:

$$Y_t = X_t\beta_{it} + \varepsilon_t \qquad i=1....m \tag{4.17}$$

$$\begin{aligned} \beta_{1t} &= \beta_{1t-1} + \omega_{1t} \\ . &= . \quad . \; . \\ . &= . \quad . \; . \\ \beta_{mt} &= \beta_{mt-1} + \omega_{mt} \end{aligned} \tag{4.18}$$

(4.17) is the measurement equation and (4.18) the transition equation. The parameters of the model vary over time with coefficient β_i, which evolves as a random walk.

To demonstrate the workings of the Kalman filter in estimating (4.15) and (4.16) we can set up the model in state space form by first defining the follow vectors and matrices:

$$\boldsymbol{x}' = [1, 0],\ \boldsymbol{\beta}_t = [X_t^*, \gamma_t]',\ \boldsymbol{T} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix},\ \boldsymbol{\eta} = [\zeta_t, \omega_t]' \tag{19}$$

The measurement and transition equations are:

$$\begin{aligned} x_t &= \boldsymbol{x}'\boldsymbol{\beta}_t + \varepsilon_t \\ \boldsymbol{\beta}_t &= \boldsymbol{T}\boldsymbol{\beta}_{t-1} + \eta_t \end{aligned} \tag{20}$$

where ε_t has variance $\sigma^2 h_t$ and η_t has variance $\sigma^2 \mathbf{Q}_t$. Defining $\hat{\boldsymbol{\beta}}_{t-1}$ as the best estimate of β_{t-1} and $\mathbf{P}_{t-1}$ as the covariance matrix of $\hat{\boldsymbol{\beta}}_{t-1}$ then we have the following prediction equations:

$$\hat{\boldsymbol{\beta}}_{t|t-1} = \boldsymbol{T}\hat{\boldsymbol{\beta}}_{t-1} \tag{23}$$

$$\boldsymbol{P}_{t|t-1} = \boldsymbol{T}\boldsymbol{P}_{t-1}\boldsymbol{T}' + \boldsymbol{Q}_t \tag{24}$$

As observations on X_t become available the prediction equations are updated and thereby updating the estimate of X_t^*. The updating equations are given by:

$$\hat{\boldsymbol{\beta}}_t = \hat{\boldsymbol{\beta}}_{t|t-1} + \boldsymbol{P}_{t|t-1}\boldsymbol{x}_t\,(X_t - \boldsymbol{x}_t'\,\hat{\boldsymbol{\beta}}_{t|t-1}) \,/\, \boldsymbol{x}_t'\,P_{t|t-1}\,\boldsymbol{x}_t + h_t \tag{25}$$

$$\boldsymbol{P}_t = \boldsymbol{P}_{t|t-1} - \boldsymbol{P}_{t|t-1}\,\boldsymbol{x}_t\boldsymbol{x}_t'\,\boldsymbol{P}_{t|t-1} \,/\, \boldsymbol{x}_t'\,\boldsymbol{P}_{t|t-1}\,\boldsymbol{x}_t + h_t \tag{26}$$

The prediction and updating equations define the Kalman filter. The likelihood function can then be expressed as a function of the one-step-ahead prediction errors and the model can be estimated by maximum likelihood. To recap briefly, the intuition for using the Kalman filter to extract expectations is that we can approximate the general model used to form the expectations using an unobserved components model as long as this model generates unanticipated components that are serially uncorrelated. However, if there is evidence of 1st order serial correlation we can augment the model by including time-varying lags of the dependent variable and we can repeat this process until we have white noise unanticipated components.

4.3 Empirical Models For Generating Unexpected Components

In this section the three approaches to generating unexpected components, as outlined above, are considered. The factors that we consider are those outlined in chapters 2 and 3. Although we have listed these factors before, they are presented again in table 4.1 for convenience.

Rate of Change Methodology

In order to generate the unexpected components from the 'rate of change' approach the economic state variables are simply first differenced and this first difference then enters as an unexpected component in the APT. However, it should be noted that when dealing with the term structure of interest rates and default risk, which are differences in two interest rates, these enter simply as levels and are not differenced (see, for example, Chen, Roll, and Ross (1986)).

The assumptions underlying the APT are that the unexpected components are mean zero innovations. In order to assess whether these unexpected components are mean zero we regress the first difference on a constant. The results are presented in table 4.2 and clearly show that for default risk, unexpected inflation, unexpected money supply and unexpected industrial production the mean value of the series is not equal to zero.

With regard to the condition that the series should be serially uncorrelated table 4.3 presents a χ^2 test for first order serial correlation in the factors in order to determine whether or not this technique provides unanticipated components that are innovations. In all but one case (real industrial production) the test indicates that the unexpected components from the rate of change methodology are autocorrelated, and consequently they fail to meet

Table 4.1

Economic State Variables

	Factors
f_1	Default Risk
f_2	Industrial Production
f_3	Exchange Rate
f_4	Retail Sales
f_5	Money Supply
f_6	Unexpected Inflation
f_7	Change in Expected Inflation
f_8	Term Structure of Interest Rates
f_9	Commodity Prices

Notes: Sample period 1980M1 - 1993m12. Source : Datastream

Table 4.2
Unexpected Component from Rate of Change Methodology
Regressed On a Constant

Factor	Coefficient	Standard Error	T-Ratio
Term Structure	-.015362	.0143351	-1.0715
Default Risk	.0116113	.3098E-3	37.4738**
Industrial Production	-.001024	.2501E-3	-4.0786**
Money supply	-.001557	.6038E-3	-2.5713*
Retail Sales	.1635E-3	.2268E-3	.72082
Exchange Rate	-.002366	.0024008	-.98582
Commodity Prices	-.001288	.0015976	-.80636
Expected Inflation	-.668E-4	.5344E-3	-.12510
Unexpected Inflation	.0051715	.4648E-3	11.1264**

Notes: Sample period 1980M1 - 1993m12. Source : Datastream

** denotes significant at the 1% level
* denoted significant at the 5% level

Table 4.3

Autocorrelation tests for rate of change factors

Model	χ^2 test for 1st Order Serial Correlation[a]
f_1	139.91
f_2	0.086
f_3	21.89
f_4	13.75
f_5	30.47
f_6	18.55
f_8	2.97
f_9	15.73

Notes: Sample period 1980M1 - 1993m12. Source : Datastream

[a] the test statistic is distributed χ^2 which has a critical value of 3.841 at the 5% level.

the requirements for being an innovation. Hence these unexpected components become redundant in terms of estimating the APT. While these unexpected components obviously fail one of the basic assumptions of the APT and as such should not be employed in tests of the APT they are used in section 4.4 to outline the potential problems that might arise when unexpected components do not satisfy the basic requirements of being white noise innovations.

Autoregressive Methodology

An alternative to the rate of change methodology is to specify a time series regression with the aim of removing any time dependence in the series by the use of lags of the dependent variable. In order to achieve this the stationary form of the variable enters a twelfth order autoregression of the following form:

$$\Delta X_t = \alpha_0 + \delta_i \Delta X_{t-i} + u_t \tag{4.25}$$

where X_t is the variable of interest. The models can then be simplified by omitting insignificant lags and the valid restricted, parsimonious version of the model is assumed to be the model employed by agents for expectations generation. The results from estimating the parsimonious versions of the models are reported in table 4.4, along with a χ^2 test for 1st order serial correlation and an F-test that the restricted models are valid. It should be noted that the term structure of interest rates follows a random walk. The

Table 4.4

Parsimonious versions of the autoregressive models

PANEL A : Real Industrial Production

$$\underset{(0.0003)}{\Delta RIP_t = -0.0004} + \underset{(0.0729)}{0.155\Delta RIP_{t-3}} + \underset{(0.0715)}{0.171\Delta RIP_{t-5}} + \underset{(0.0742)^{c}}{0.196\Delta RIP_{t-8}}$$

$$\chi^2_{s.c} = 0.846^{a} \qquad F(9,157) = 1.69^{b}$$

PANEL B : Commodity Prices

$$\Delta CP_t = \underset{(0.002)}{0.0003} + \underset{(0.069)}{0.3346\Delta CP_{t-1}}$$

$$\chi^2_{s.c} = 0.016 \qquad F(11,168) = 0.500$$

PANEL C : Retail Sales

$$\Delta RS = \underset{(0.0002)}{0.0003} + \underset{(0.0779)}{0.3273\Delta RS_{t-1}} - \underset{(0.0781)}{0.1302\Delta RS_{t-2}}$$

$$\chi^2_{s.c} = 0.543 \qquad F(10,152) = 0.850$$

PANEL D : Exchange Rate

$$\underset{(0.002)}{\Delta XR_t = -0.0016} + \underset{(0.078)}{0.4152\Delta XR_{t-1}} - \underset{(0.077)}{0.1381\Delta XR_{t-2}}$$

$$\chi^2_{s.c} = 2.544 \quad \xi_2(10,152) = 0.49$$

PANEL E : Inflation

$$\Delta I = \underset{(0.0001)}{0.0006} + \underset{(0.0592)}{0.2317\Delta I_{t-1}} + \underset{(0.0622)}{0.1594\Delta I_{t-6}} + \underset{(0.0617)}{0.4652\Delta I_{t-12}}$$

$$\chi^2_{s.c} = 5.994 \quad F(9,157) = 0.550$$

PANEL F : Default Risk

$$DR_t = \underset{(0.0003)}{0.0005} + \underset{(0.0694)}{0.8384DR_{t-1}} - \underset{(0.0912)}{0.2084DR_{t-2}} + \underset{(0.0692)}{0.3251DR_{t-3}}$$

$$\chi^2_{s.c} = 0.209 \quad F(9,157) = 0.970$$

PANEL G : Money Supply

$$\Delta MS_t = \underset{(0.0005)}{-0.0006} - \underset{(0.0588)}{0.3617\Delta MS_{t-1}} + \underset{(0.0572)}{0.1019\Delta MS_{t-5}} + \underset{(0.0634)}{0.1441\Delta MS_{t-8}} + \underset{(0.0706)}{0.2183\Delta MS_{t-9}} -$$

$$\underset{(0.0662)}{0.1507\Delta MS_{t-10}} + \underset{(0.0566)}{0.4905\Delta MS_{t-12}}$$

$$\chi^2_{s.c} = 0.827 \quad F(6,152) = 1.00$$

Notes: Sample period 1980M1 - 1993m12. Source : Datastream

[a] The test statistic is a χ_2 test for 1st order serial correlation which has a critical value of 3.841 at the 5% level.

[b] The test statistic is an F test that the restrictions involved from moving from the unrestricted model to the parsimonious model are valid.

[c] Standard errors are in parenthesis.

residuals from this model serve as the unanticipated components. All the residuals are serially uncorrelated at the 5% level of significance, except inflation which is serially uncorrelated at the 1% level. Thus, at this stage it would appear that the autoregressive methodology satisfies the requirement that the unexpected components are innovations. We now turn our attention to examining the feasibility of autoregressive models as a mechanism for expectations formation. As noted earlier, one requirement of autoregressive models, if they are to be employed as mechanisms for expectations generation, is that the estimated parameters are stable. In order to test this the parsimonious versions of the models were estimated using Recursive Least Squares (RLS). The coefficients on the recursive estimates, are reported in figures 4.1 through to 4.8 for selected lags. It is evident that with the exception of the default risk, which increases throughout the whole period, all other parameters exhibit instability. To analyse this formally we performed one-step ahead Chow tests which confirmed this instability. For example, with respect to commodity prices the critical value of the one-step ahead Chow test is exceeded on 23 occasions, for the exchange rate on 30 occasions, for the money supply on 15 occasions, for the term structure of interest rates on 29 occasions. The Chow tests reveal less instability for inflation, real retail sales, and real industrial production, where the critical value for the Chow test was exceeded on 9, 5, and 3 occasions respectively. Overall, it appears that the recursive coefficients are unstable, to varying degrees, throughout the period under investigation. Consequently, it would appear that if agents are assumed to use past information efficiently then they would take

Figure 4.1 Real Industrial Production : RLS Estimate of the Coefficient on ΔRIP_{t-5}

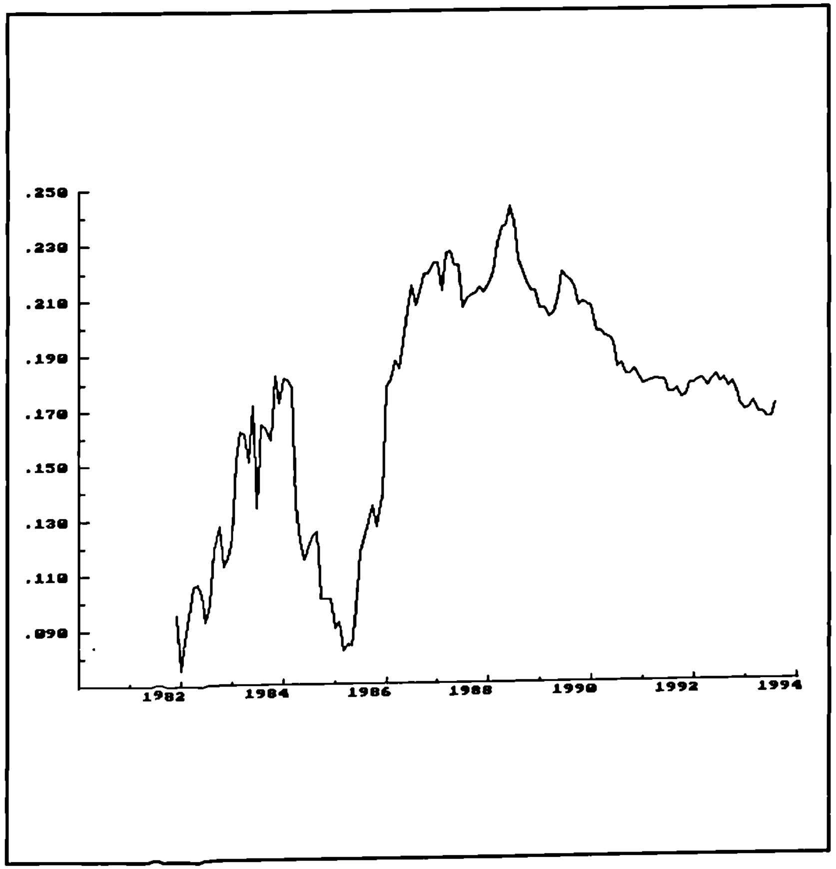


Figure 4.2 Commodity Prices : RLS Estimate of the Coefficient on ΔCP_{t-1}

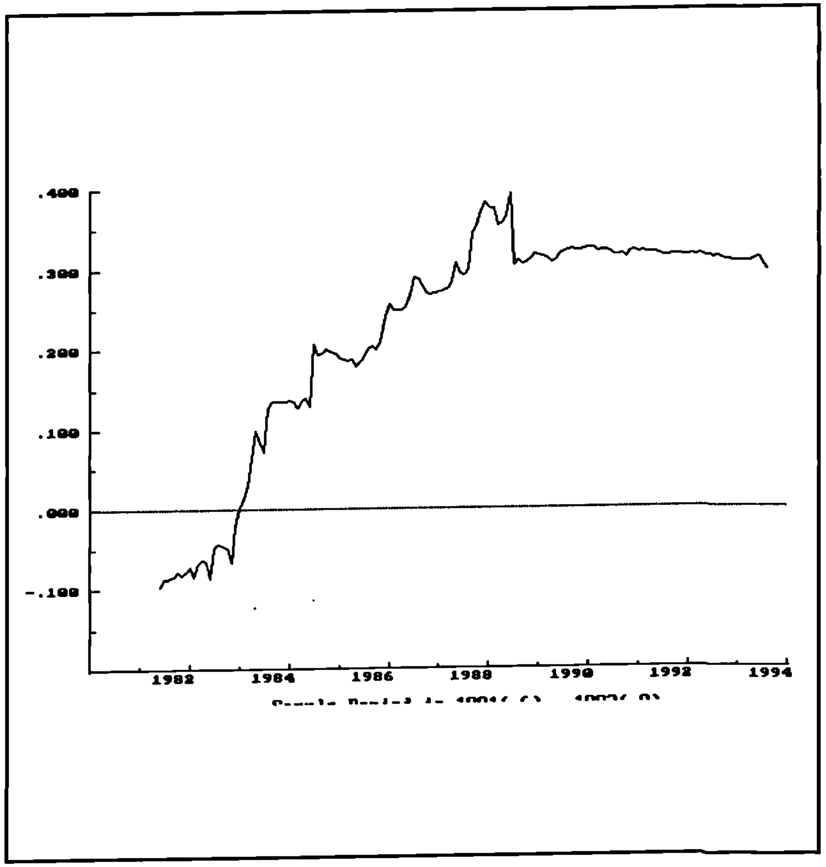


Figure 4.3 Retail Sales : RLS Estimate of the Coefficient on ΔRS_{t-2}

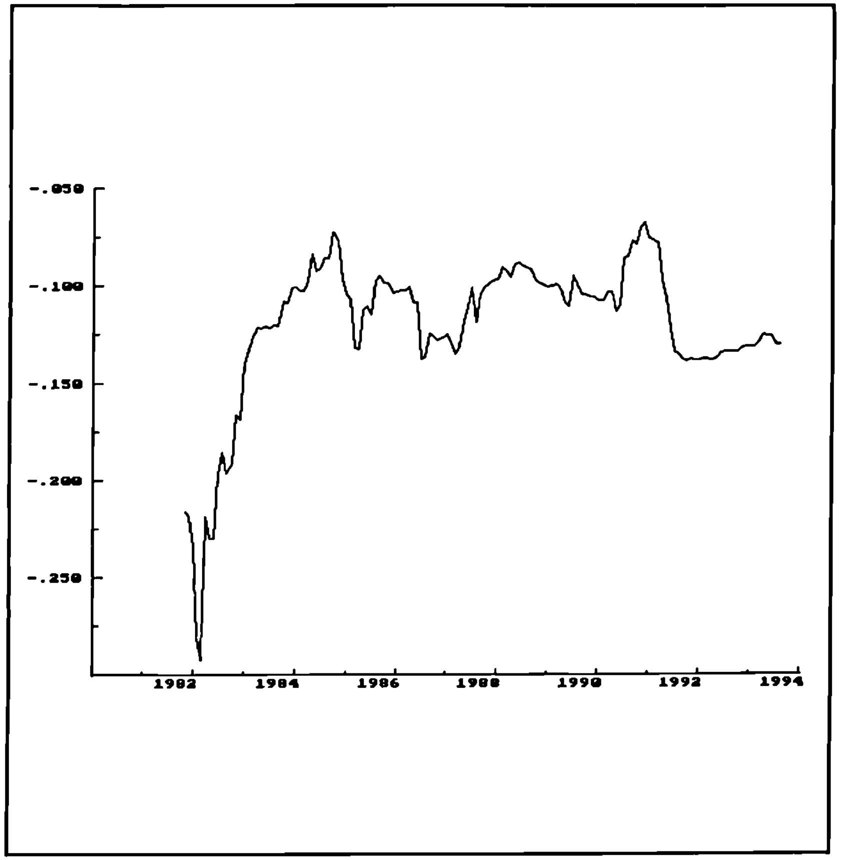


Figure 4.4 Exchange Rate : RLS Estimate of the Coefficient on ΔXR_{t-2}

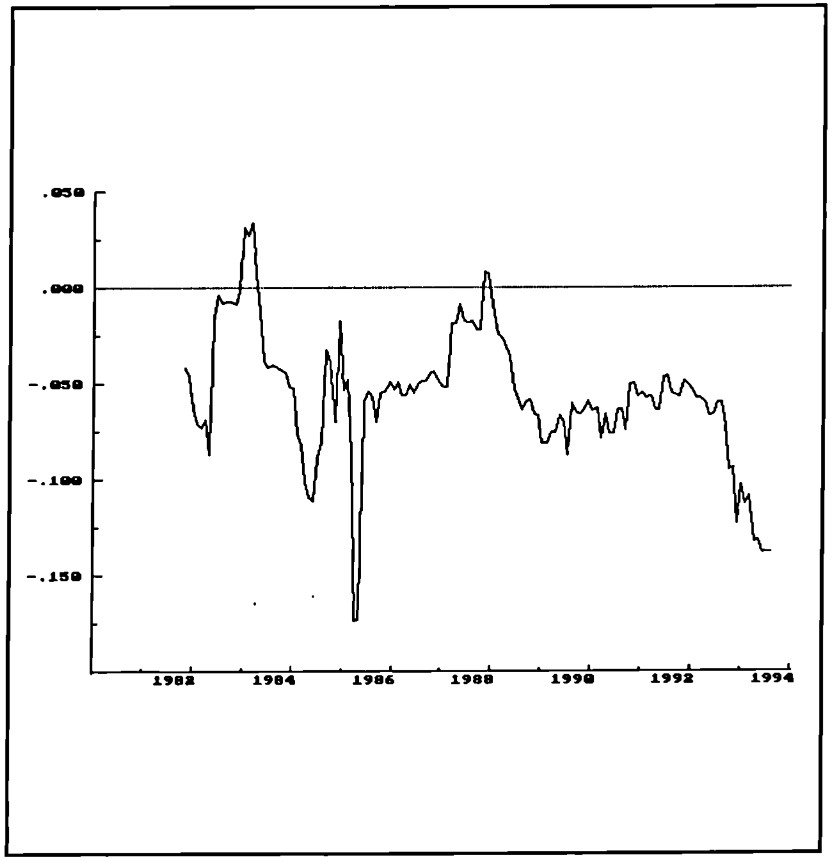


Figure 4.5 Inflation : RLS Estimate of the Coefficient on ΔINF_{t-1}

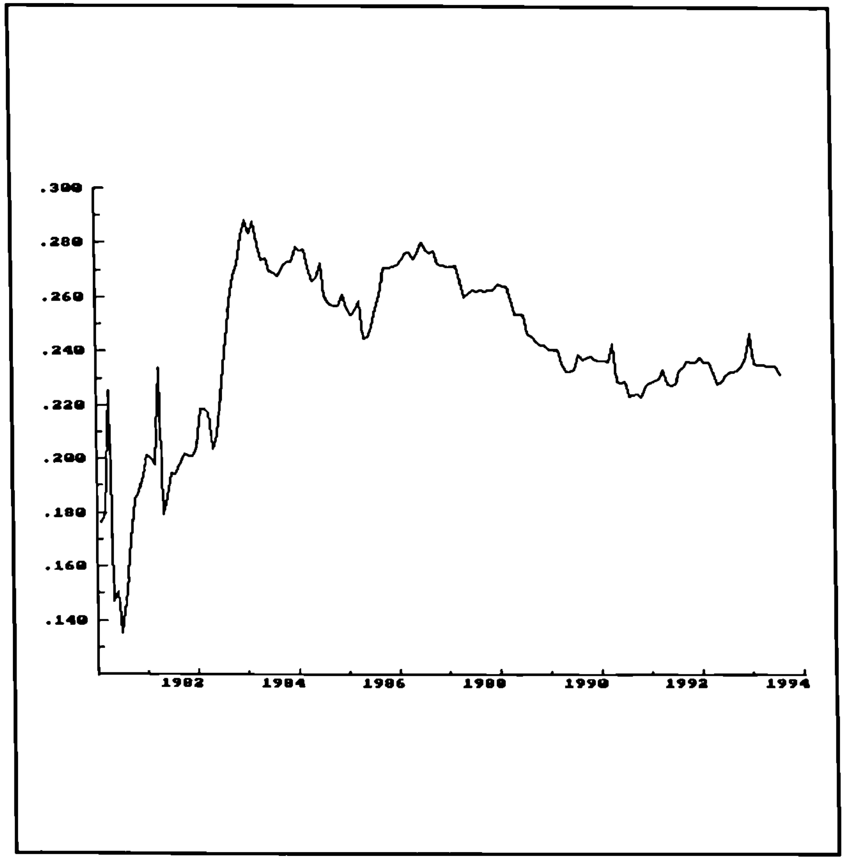


Figure 4.6 Default Risk : RLS Estimate of the Coefficient on DR_{t-1}

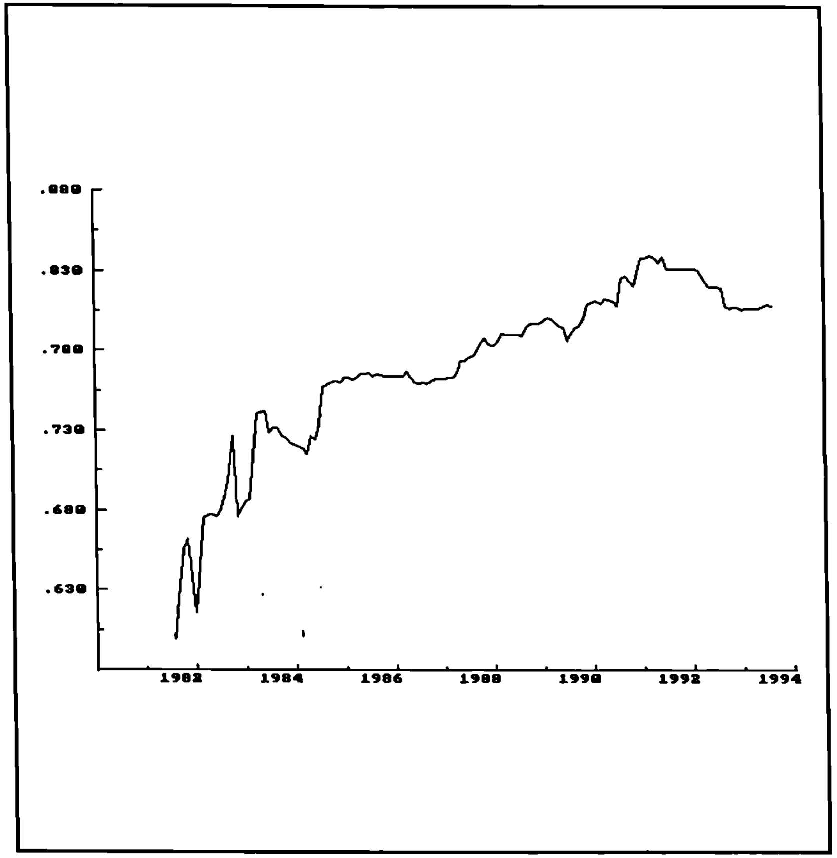


Figure 4.7 Term Structure of Interest Rates : RLS Estimate of the Coefficient on ΔTS_{t-1}

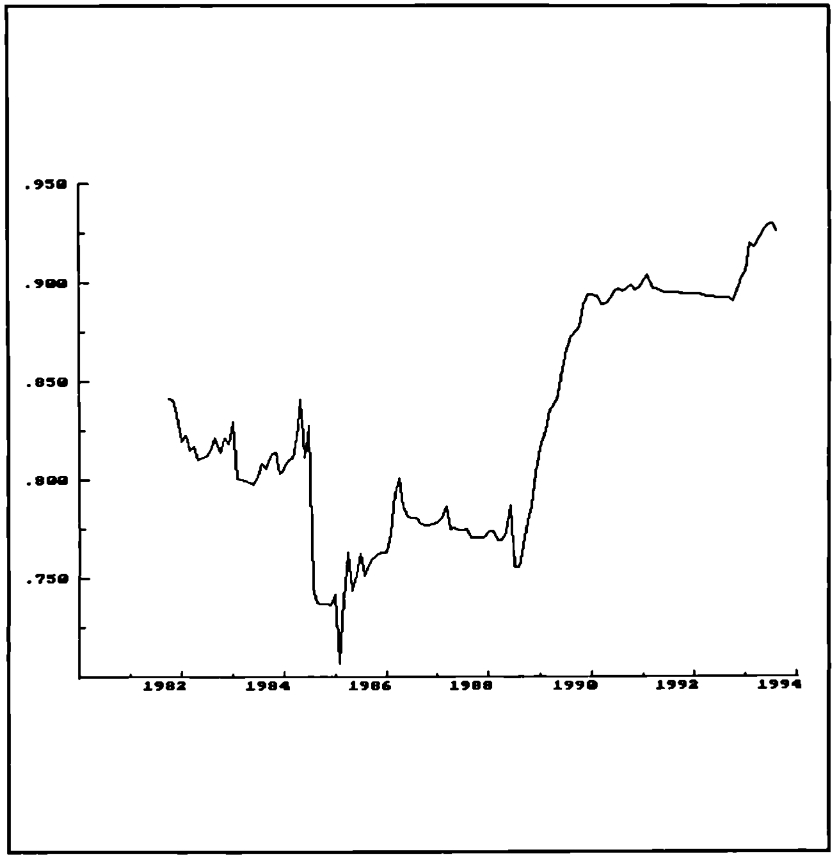
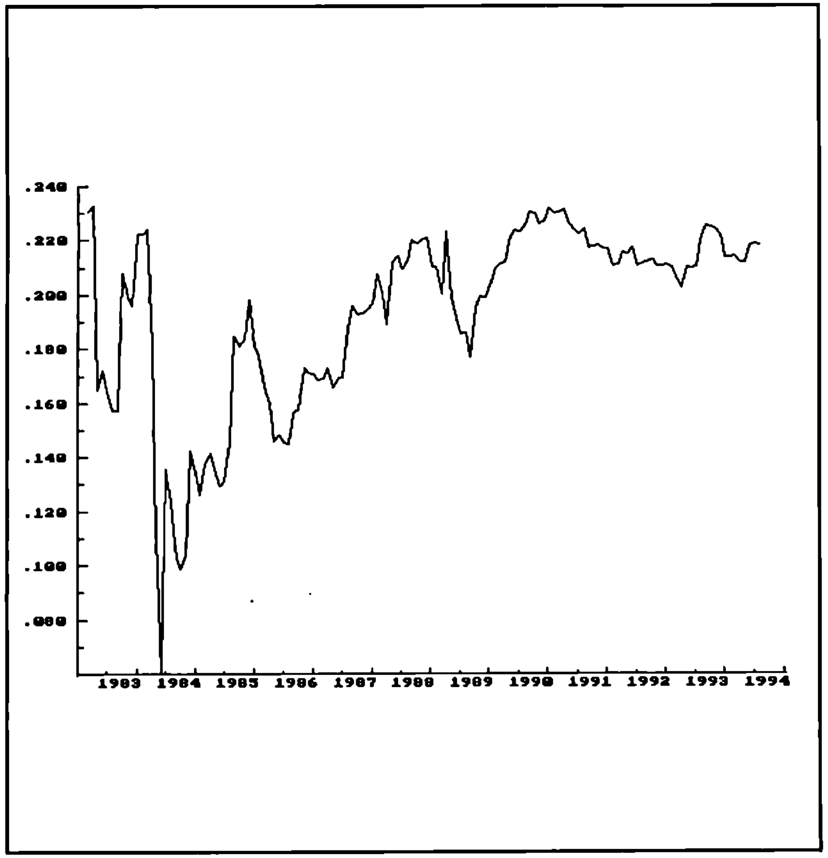


Figure 4.8 Money Supply : RLS Estimate of the Coefficient on ΔMS_{t-9}



account of these changes. The use of autoregressive models rules out this possibility and may result in agents making systematic forecast errors. It appears that the use of autoregressive models, although providing the requirement of serially uncorrelated residuals, fails to provide an expectations formation process that rules out agents making systematic forecast errors. In order to eliminate this prospect an alternative expectations formation scheme such as the Kalman filter outlined above is required to take account of these changes.

The Kalman Filter

For each factor the unobserved components model given in equations (4.15) and (4.16) was estimated and tested for 1st order serial correlation. If 1st order serial correlation is evident then lags of the dependent variable are included and a time-varying AR model (equations (4.17) and (4.18)) is estimated to take account of the instability observed in the simple AR models discussed in the previous section, and again the residuals are tested for 1st order serial correlation. For commodity prices, inflation, default risk, and the exchange rate unobserved component models were estimated and the tests for 1st order serial correlation, reported in table 4.5, indicate that the unobserved components models adequately describe the evolution of the expectations series. However, for the money supply, real retail sales, real industrial production, and the term structure of interest rates unobserved components models resulted in evidence of 1st order serial correlation. Consequently time varying parameter models were estimated and the tests for 1st order serial correlation are provided in table 4.6. The results from tables 4.5 and 4.6 indicate that in the case of six models the null hypothesis of no 1st order serial correlation is accepted at the 5% level and the remaining two at the 1% level. Accordingly, the Kalman filter models seem to do an adequate job at generating expectations. In the case of the time-varying parameter models plots of the time-varying coefficients are reported in figures 4.9 through to 4.11. Examination of these plots indicate substantial variation over time.

The expectations generated from the Kalman filter overcome the restrictive nature of the autoregressive models regarding the stability of the parameters of the model and allow agents to use a learning process. This not only produces expectations which are more likely to reflect agents actual expectations, under the assumption that agents do not make systematic forecast errors, but it also meets the requirement that the unanticipated components are innovations. It appears that the only factors that should enter into the APT specification are those generated from the Kalman filter approach. However, as mentioned above, the unexpected components generated from the rate of change methodology and the autoregressive methodology will also be used in tests of the APT to highlight two issues. First, the possible invalid inference that can be made from results of the APT when the unexpected components fail to satisfy the basic requirement of being white noise innovations, as is the case with the rate of change unexpected components. Second, by estimating the APT using the unexpected components generated from the autoregressive framework we can illustrate the potentially misleading

Table 4.5

1st Order Serial Correlation Tests

Unobserved Components Models

Factor	Coefficient	Ljung-Box
Comm. Prices	-0.1251	2.643
Inflation	-0.1258	2.644
Exchange Rate	-0.0042	0.003
Default Risk	0.1038	1.862

Notes: Sample period 1980M1 - 1993m12. Source : Datastream

Table 4.6

1st Order Serial Correlation Tests

Time Varying Parameters Models

Factor	Coefficient	Ljung-Box
Money Supply	-0.1759	5.074
Retail Sales	-0.1845	5.585
Industrial Prod	-0.0684	0.768
Term Str. I.R.	0.0217	0.079

Notes: Sample period 1980M1 - 1993m12. Source : Datastream

Figure 4.9 - Time Varying Parameter Estimate of $DRIP_{t-1}$

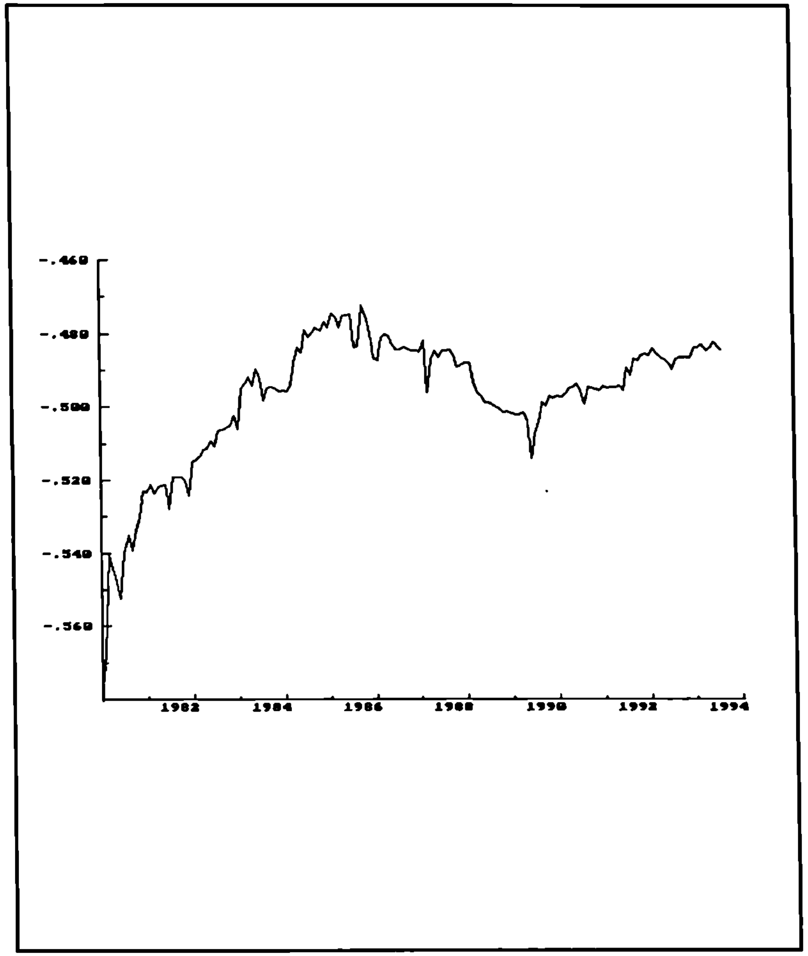


Figure 4.10 - Time Varying Parameter Estimate of $DRRS_{t-1}$

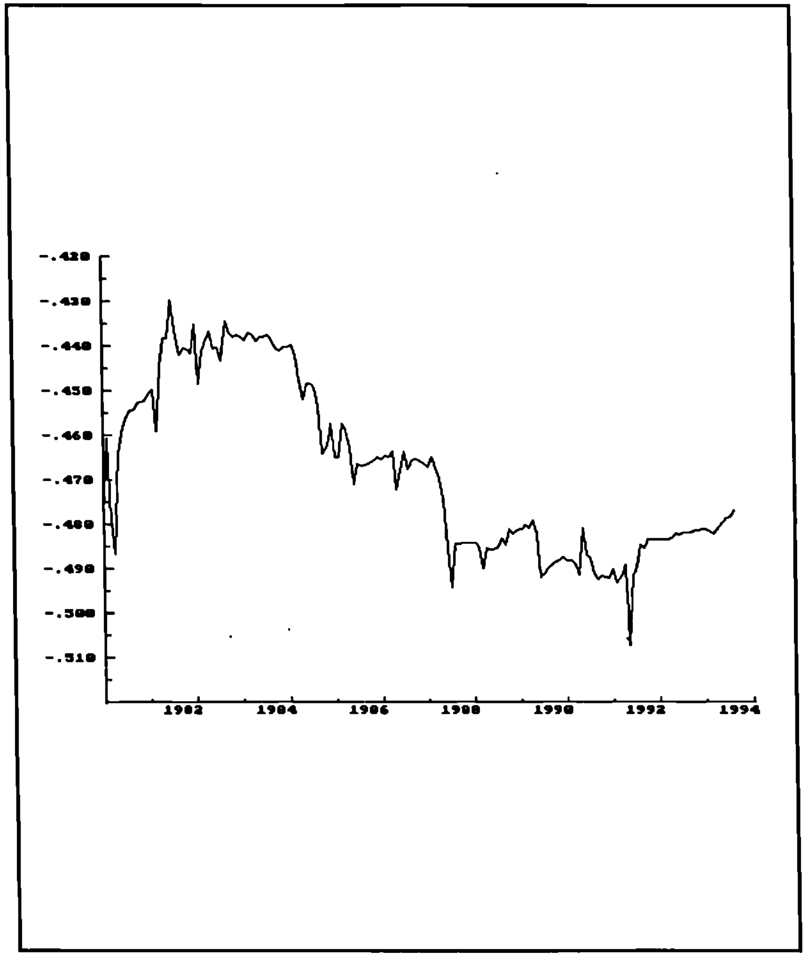


Figure 4.11 - Time Varying Parameter Estimate of DMS_{t-1}

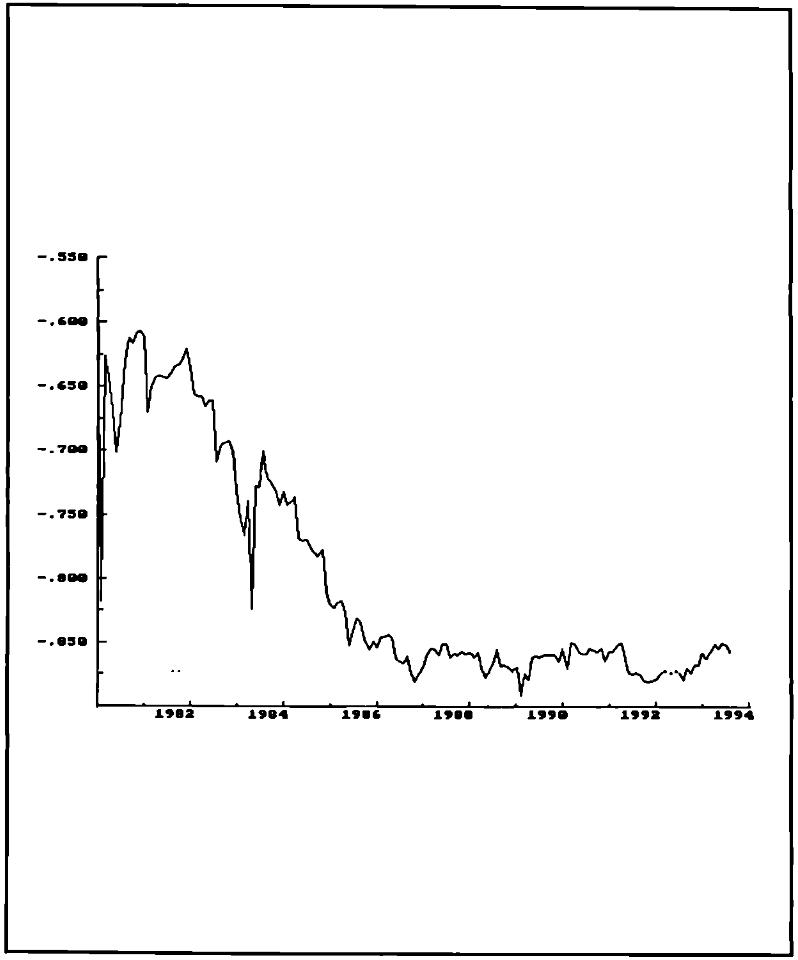
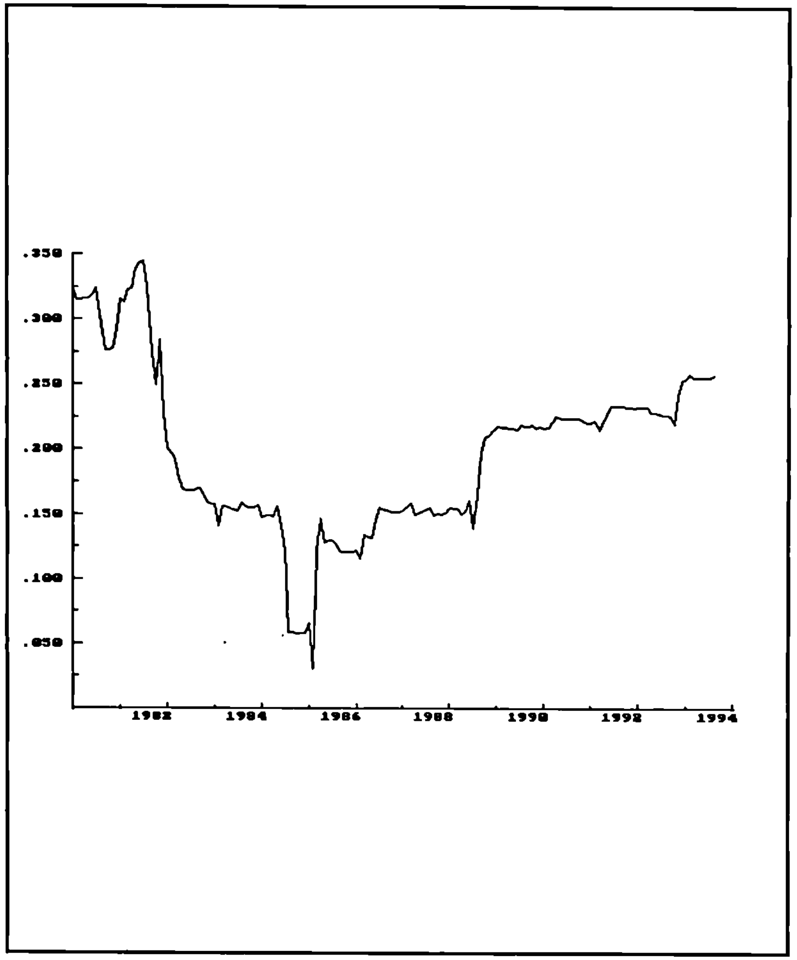


Figure 4.12 - Time Varying Parameter Estimate of DTS_{t-1}



inferences from results of tests of the APT which employ unexpected components that do satisfy the basic requirement of being white noise innovations but place restrictive assumptions on the way it is assumed that agents generate their expectations.

4.4 A Comparison Of Expectations Generated From The Kalman Filter And The Autoregressive Models

Although there is a large literature on the comparisons of alternative forecasts, (see *inter alia* Wallis (1989)), and the use of combining forecasts to provide *'better'* forecasts, in this section we provide some simple comparisons of the expectations derived from the AR models and the Kalman filter. We perform two basic comparisons on the premise that the best forecast (expectation) is the one that is closer to the actual value. The first is to simply examine the correlation of the two expectations with the actual outcome, and the second is to examine the error variances of the two models. In this case the best forecast is the one with the smallest error variance. Table 4.7 through to table 4.14 provides correlations amongst the expectations derived from the Kalman Filter, AR models and the actual outcome.

Analysis of these correlation matrices reveals that the expectations derived from the Kalman filter, in every case, have a higher correlation to the actual outcome than the AR models. For example, the correlation coefficient for the expectations from the Kalman filter for the exchange rate (table 4.14) with the actual is 0.7632, while the correlation coefficient for the expectation from the

AR model with the actual exchange rate is 0.3217. Similar results are recorded across all the tables. The correlation coefficient for the expectations from the Kalman filter with the actual outcomes for all factors ranges from 0.7287 to 0.9889, while the correlation coefficient between the expectations from the AR models and the actual outcomes range from 0.1517 to 0.9295.

Given these results there is a clear indication that the assumption of learning by agents provides expectations of economic variables that are much closer to the actual values than AR models. To emphasise this point further comparisons of the Kalman filter and AR models error variances are reported below in table 4.15. In all cases the Kalman filter models have a lower error variance than the AR models, indicating once again that the Kalman filter provides more accurate forecasts than the corresponding AR models. In addition to this considering the weakness of the AR models with respect to time variation in the estimated parameters it seems more appropriate to use the assumption that agents use a learning framework when forming their expectations. In the next section we turn our attention to the issue of how the different assumptions regarding expectations formation affect results from tests of the APT.

Table 4.7 : Correlation Matrix For Inflation*

	E(KF)	E(AR)	ACTUAL
E(KF)	1	0.7106	0.9336
E(AR)		1	0.7214
ACTUAL			1

Notes: (Tables 4.8-4.14) Sample period 1980M1 - 1993m12. Source : Datastream

Notes:

* E(KF) is the expectation of the factor derived from the Kalman filter
  E(AR) is the expectation of the factor derived from the AR model

Table 4.8 : Correlation Matrix For Commodity Prices

	E(KF)	E(AR)	ACTUAL
E(KF)	1	0.8058	0.7605
E(AR)		1	0.3086
ACTUAL			1

Table 4.9 : Correlation Matrix For Real Industrial Production

	E(KF)	E(AR)	ACTUAL
E(KF)	1	0.5377	0.7570
E(AR)		1	0.3743
ACTUAL			1

Table 4.10 : Correlation Matrix For The Term Structure Of Interest Rates

	E(KF)	E(AR)	ACTUAL
E(KF)	1	-0.0563	0.7287
E(AR)		1	0.1517
ACTUAL			1

Table 4.11 : Correlation Matrix For The Default Risk

	E(KF)	E(AR)	ACTUAL
E(KF)	1	0.9702	0.9889
E(AR)		1	0.9295
ACTUAL			1

Table 4.12 : Correlation Matrix For The Money Supply

	E(KF)	E(AR)	ACTUAL
E(KF)	1	0.7591	0.8617
E(AR)		1	0.7542
ACTUAL			1

Table 4.13 : Correlation Matrix For Real Retail Sales

	E(KF)	E(AR)	ACTUAL
E(KF)	1	0.0404	0.7676
E(AR)		1	0.3217
ACTUAL			1

Table 4.14 : Correlation Matrix For The Exchange Rate

	E(KF)	E(AR)	ACTUAL
E(KF)	1	0.7354	0.7632
E(AR)		1	0.3881
ACTUAL			1

Table 4.15

Comparisons of Error Variances from Kalman filter and AR Models

Factor	Kalman Filter	AR
Inflation	0.1744 E-4	0.2533 E-4
Commodity Prices	0.1875 E-3	0.3795 E-3
Real Industrial Prod	0.8222 E-5	0.9035 E-5
Term Structure of IR	0.3436 E-4	0.6819 E-4
Default Risk	0.1439 E-5	0.2924 E-5
Money Supply	0.1911 E-4	0.2674 E-4
Real Retail Sales	0.6740 E-5	0.7734 E-5
Exchange Rate	0.3828 E-3	0.8150 E-3

Notes: Sample period 1980M1 - 1993m12. Source : Datastream

4.5 ALTERNATIVE UNEXPECTED COMPONENTS AND RESULTS FROM TESTS OF THE APT

Rate of Change Innovations and the APT

The final specification of the APT using the rate of change innovations is given in table 4.16. Default risk, unexpected inflation, real industrial production and commodity prices are significant at the 1% level, expected inflation at the 5% level, and the money supply and the market portfolio at the 10% level. The across equation restrictions of the APT are accepted at each reduction (see table 4.17) and each reduction is accepted (see table 4.18)

Autoregressive Innovations and the APT

Table 4.19 provides the estimated prices of risk for the factors derived from the AR models at the final stage of reduction. Table 4.20 shows that the across equation pricing restrictions are accepted at each stage and table 4.21 indicates that the reductions of insignificant factors are accepted. The final reduced model estimates of the prices of risk show that when using innovations derived from an AR process there are four factors significant at the 5% level. These are real industrial production; real retail sales; commodity prices; and the return on the market portfolio. In addition there is one factor significant at the 10% level, unexpected inflation.

Table 4.16

NL3SLS Estimated Prices of Risk for APT (Final Model)
Rate of Change Factors

Parameter	Estimate	T - Ratio
λ_1 (Default Risk)	-0.01072	-16.71^{***}
λ_2 (Real Industrial Prod)	0.00280	4.57^{***}
λ_5 (Money Supply)	0.00198	1.65^{*}
λ_6 (Unexpected Inflation)	-0.00501	-6.65^{***}
λ_7 (Expected Inflation)	-0.00450	-3.21^{**}
λ_9 (Commodity Prices)	-0.01521	-3.68^{***}
λ_{10} (Market Portfolio)	0.00172	1.92^{*}

Notes: Sample period 1980M1 - 1993m12. Source : Datastream
* denotes significant at 10%
** denotes significant at 5%
*** denotes significant at 1%

Table 4.17

Likelihood Ratio Tests of APT Across Equation Restrictions: H_o:$\mathbf{A}=\mathbf{B}\lambda$
Rate of Change Factors

Model	Calculated Value	Critical Value*
All Factors	38.37	$\chi^2(60)$
First Reductions	39.69	$\chi^2(62)$
Second Reductions	31.26	$\chi^2(63)$

Notes: Sample period 1980M1 - 1993m12. Source : Datastream
* $\chi^2(60)$ = 79.08

Table 4.18

Likelihood Ratio Tests of APT Reductions: H_o:$\lambda_i=b_{ji}=0$
Rate of Change Factors

Reduction	Calculated Value	Critical Value
First Reduction ($\lambda_3=\lambda_4=0$)*	137.85	$\chi^2(142)$
Second Reduction ($\lambda_8=0$)	79.57	$\chi^2(71)$

Notes: Sample period 1980M1 - 1993m12. Source : Datastream
* the reduction implies $b_{j3}=b_{j4}=\lambda_3=\lambda_4=0$, thus for the system of seventy equations the number of degrees of freedom is 70×1 b_{j3}'s + 70×1 b_{j4}'s + 1 λ_3 + 1 λ_4 = 142.

Kalman Filter Innovations and the APT

The results from estimating the APT using the Kalman filter innovations are presented below in table 4.22. The tests of the across equation pricing restrictions and the restrictions implied by omitting insignificant factors are presented in tables 4.23 to 4.24. There are six factors which have a statistically significant risk premium when the Kalman Filter innovations are used. The across equation restrictions and the restrictions implied by omitting insignificant factors are accepted. These results illustrate the importance of generating unexpected components in the macroeconomic and financial factors. As illustrated above, the rate of change factors, in all but one case, do not satisfy the requirement of being white noises process. Whilst the autoregressive factors do satisfy this criterion we have shown that the estimated parameters from the models we assume agents use to form their expectations are unstable. As a result of this instability we can not rule out the possibility that agents will make systematic forecast errors when forming their expectations. This immediately rules out these two techniques as methods of generating the unexpected components in tests of the APT and hence renders the results of tests of the APT using these techniques as invalid. This has been illustrated by the findings in this section that the APT is very sensitive to the way the unexpected components are generated. On the other hand, we have shown that the Kalman filter provides estimates of the unexpected components that satisfy the criteria of being white noise processes and additionally can adequately deal with the problem of instability in estimated parameters, thus ruling out the possibility that agents may make

Table 4.19

NL3SLS Estimates of Risk Premia for APT (final model)
Autoregressive Factors

Parameters	Estimates	T - Ratio
λ_2 (Real Industrial Prod)	0.00228	4.43***
λ_4 (Real Retail Sales)	-0.00078	-2.44**
λ_6 (Unexpected Inflation)	0.00087	1.67*
λ_9 (Commodity Prices)	-0.00621	-2.34**
λ_{10} (Market Portfolio)	0.00160	2.19**

Notes: Sample period 1980M1 - 1993m12. Source : Datastream

* denotes significant at 10%
** denotes significant at 5%
*** denotes significant at 1%

Table 4.20

Likelihood Ratio Tests of APT Across Equation Restrictions: $H_0{:}A=B\lambda$
Autoregressive Factors

Model	Calculated Value	Critical Value*
All Factors	34.29	79.08 (60)
First Reductions	36.62	79.08 (62)
Second Reductions	37.91	79.08 (63)
Third Reductions	38.90	79.08 (64)
Fourth Reductions	39.64	79.08 (65)

Notes: Sample period 1980M1 - 1993m12. Source : Datastream
* $\chi^2(60) = 79.08$

Table 4.21

Likelihood Ratio Tests of APT Reductions: $H_0{:}\lambda_i=b_{ji}\ 0$
Autoregressive Factors

Reduction	Calculated Value	Critical Value
First Reduction ($\lambda_7=\lambda_8=0$)*	155.59	(142)
Second Reduction ($\lambda_3=0$)	71.24	(71)
Third Reduction ($\lambda_1=0$)	83.26	(71)
Fourth Reduction ($\lambda_5=0$)	81.43	(71)

Notes: Sample period 1980M1 - 1993m12. Source : Datastream
* the reduction implies $b_{j7}=b_{j8}=\lambda_7=\lambda_8=0$, thus for the system of seventy equations the number of degrees of freedom is 70×1 b_{j7}'s + 70×1 b_{j8}'s + 1 λ_7 + 1 λ_8 = 142.

Table 4.22

NL3SLS Estimated Prices of Risk for APT
Kalman Filter Factors

Parameter	Estimate	T - Ratio
λ_1 (Default Risk)	-0.00041	-2.78^{***}
λ_3 (Exchange Rate)	-0.01723	-3.89^{***}
λ_5 (Money Supply)	0.00178	2.28^{**}
λ_6 (Unexpected Inflation)	-0.00333	-3.91^{***}
λ_7 (Expected Inflation)	0.00125	2.33^{**}
λ_{10} (Market Portfolio)	0.00144	1.67^{*}

Notes: Sample period 1980M1 - 1993m12. Source : Datastream
* denotes significant at 10%
** denotes significant at 5%
*** denotes significant at 1%

Table 4.23

Likelihood Ratio Tests of APT Across Equation Restrictions: H_o:$A=B\lambda$
Kalman Filter Factors

Model	Calculated Value	Critical Value*
All Factors	32.68	χ^2 (60)
First Reductions	34.85	χ^2 (61)
Second Reductions	36.52	χ^2 (62)
Third Reductions	37.51	χ^2 (63)
Final Model	38.49	χ^2 (64)

Notes: Sample period 1980M1 - 1993m12. Source : Datastream
* $\chi^2(60)=79.08$

Table 4.24
Likelihood Ratio Tests of APT Reductions: H_o:$\lambda_i=b_{ji}=0$
Kalman Filter Factors

Reduction	Calculated Value	Critical Value**
First Reduction ($\lambda_4=0^*$)	70..57	$\chi^2(71)$
Second Reduction ($\lambda_{10}=0$)	77.03	$\chi^2(71)$
Third Reduction ($\lambda_8=0$)	85.18	$\chi^2(71)$
Fourth Reduction ($\lambda_2=0$)	64.75	$\chi^2(71)$

Notes: Sample period 1980M1 - 1993m12. Source : Datastream
* the reduction implies $b_{j9}=\lambda_9=0$, thus for the system of seventy equations the number of degrees of freedom is 70×1 b_{j9}'s + 1 λ_9 = 71.
** the critical value at 5% for $\chi^2(70)$ = 90.53.

systematic forecast errors when forming expectations. This technique seems more appropriate since it overcomes the problems with the two former techniques.

4.6 Conclusion

Three techniques for generating unexpected components of factors which enter into the APT were analysed. We found that the extant techniques, namely the rate of change methodology and the autoregressive methodology both have problems which render tests of the APT invalid. The rate of change methodology, in all but one case, failed to meet the requirement that the unanticipated components are innovations. With respect to the autoregressive components, although the unexpected components were innovations, the expectations generating process was found to be unstable. Results from previous tests of the APT, such as Chen, Roll, and Ross (1986) and Chan, Chen, and Hsieh (1985) in the US who use the rate of change methodology, and Clare and Thomas (1994) and Beenstock and Chan (1989) in the UK who employ autoregressive techniques, should be treated with caution. In this chapter we have shown that an alternative method of generating expectations based upon learning provides very different results in terms of results from tests of the APT to the two previously used methods. This has important implications for practioners and policy makers who wish to use these models as part of their decision making process. For example, fund managers may seek to hedge against certain types of risk. As a consequence of the method used to generate the unexpected component while they may believe they are hedging against a certain risk they may actually be increasing their exposure to this risk.

CHAPTER 5

THE EMPIRICAL VALIDITY OF THE APT : UNIQUENESS AND STABILITY

5.1 INTRODUCTION

One of the most important aspects of an asset pricing model is that it be robust whilst at the same time offering economic insight into the determinants of security returns. It is quite straightforward to place economic interpretation on the determinants of security returns, since one can simply prespecify macroeconomic variables as factors rather than using statistical factors such as those extracted from factor analysis and principal components. In chapter 3 of this thesis the APT was estimated from the framework of an approximate factor model using macroeconomic factors and consequently economic interpretation could be placed on security returns through the existence of statistically significant prices of risk. However, in chapter 4, it was also noted that the APT was not robust to the specification of the way agents are assumed to form their expectations of the future values of macroeconomic variables that enter into the APT as factors. To this end we employed a learning procedure to estimate expectations of macroeconomic variables in order to generate factors that are more likely to be representative of agents actual observations of the factors.

Considering the issues of economic interpretation of factors that drive security returns, the use of an approximate factor model, and the robustness of the APT to the specification of expectations is in itself not enough to generate a valid APT model for as Fama (1991) points out,

> *'The flexibility of the Chen, Roll and Ross approach can be a trap. Since multifactor models offer at best vague predictions about the variables that are important in returns and expected returns, there is the danger that measured relations between returns and economic factors may be spurious, the result of special features of a particular sample (factor dredging). Thus the Chen, Roll and Ross tests, and future extensions, warrant extended robustness checks. For example, although the returns and economic factors used by Chen, Roll and Ross are available for earlier and later periods, to my knowledge we have no evidence on how the factors perform outside their sample.' (Fama (1991, p. 1595)).*

Thus, even when we take into consideration the use of an approximate factor model, and we employ sophisticated estimation techniques such as NL3SLS and we perform a number of tests of the APT such as those carried out in chapter 3, and we form expectations of the macroeconomic variables from a learning perspective this may not be enough to guarantee that the APT has empirical content. Any evaluation of the empirical validity of the APT must also focus on its ability to price assets outside of the sample used for estimation. Specifically, this requires the return generating process is unique. This is similar to the point made by Connor and Korajczyk (1992) who argue that if the return vector **R** is partitioned in two subsets, $\mathbf{R}_1$ and $\mathbf{R}_2$, the two vectors λ_1 and λ_2 must be equal. This issue of uniqueness is often ignored in empirical tests of the APT, but is an important issue that must be addressed. A non-unique APT would have very limited use in any practical context since it would also have to be reestimated every time a new set of assets was under consideration. In this chapter we consider the issue of uniqueness by estimating the APT using two separate sets of securities returns, the first we will call the estimation sample, the second we will call the validation sample. Both sets of securities contain 69 randomly selected stocks from the UK equity market over the period January 1980 to August 1993.

The second issue we consider in this chapter is the ability of the APT to perform its task in explaining the cross sectional behaviour of asset returns, a point made by Roll and Ross (1984) who have argued, in the context of factor analysis,

> *'...the number of extracted factor is, at best, a secondary issue since the acid test of the APT is how well the factors explain pricing and, in particular, how well they fare against alternative hypotheses.'*

While this may be the case for factor analysis[12] it is certainly not the case when the factors are prespecified macroeconomic variables. Indeed the issue of the number of priced factors and their ability to explain the cross section of average returns must be at least equal in terms of importance. Therefore, when considering the empirical performance and validity of the APT we are interested in the issues of uniqueness of the return generating process and the performance of the model in terms of explaining observed return behaviour. A natural way to assess the APT's ability to explain observed return behaviour is to calculate the average excess security return and construct the expected excess return predicted by the APT which is given as:

$$E(\hat{r}_i) = \sum_{j=1}^{k} \hat{b}_{ij} \hat{\lambda}_j \qquad (5.1)$$

where $\hat{b}_{ij}$ is the sensitivity of asset i to the price of risk on factor j and $\hat{\lambda}_j$ is the estimated price of risk associated with the jth factor. Denoting the average return as $\hat{r}_i$ then the APT's ability to explain observed return behaviour can be assessed by employing the following cross-sectional regression:

---

[12] The arguments of Roll and Ross (1984) stem from the finding that as the number of securities included in a portfolio increases so does the number of factors extracted. This result is a natural consequence of factor analytical techniques (see Raveh (1985) for a discussion of this point) and the extra factors extracted are likely to be firm specific factors, unfortunately there is no way of knowing this.

$$\hat{r}_i = \alpha_0 + \alpha_1 E(\hat{r}_{i,APT}) \tag{5.2}$$

The analysis of the adjusted R^2 from this regression will give an indication of the performance of the APT with respect to its ability to explain observed security returns.

As a third issue, an important aspect of any econometric model is that it be robust to the underlying assumptions regarding the stochastic processes of the residuals. In particular, the assumptions underlying the APT are that the residuals are serially uncorrelated and homoscedastic and as such the estimated residuals from the econometric model must be robust to these assumptions. This is a further issue in empirical tests of the APT that is ignored. The effects of serial correlation of the residuals can effect the parameter estimates and hence the inferences that we make regarding the prices of risk and sensitivities of assets to shocks in the factors. Furthermore, evidence of serial correlation may be indicative of misspecification of the econometric model. Analysis of the results regarding the existence or otherwise of significant serial correlation can provide evidence regarding whether we need to consider alternative factors in the return generating process. A further possible misspecification of the residuals from the econometric model is heteroscedasticity. In the presence of heteroscedasticity the standard errors from the regression model will be effected and consequently the t-ratios will be biased. Given the implications of misspecification of the residuals from the APT estimation we carry out tests for serial correlation and heteroscedasticity.

A fourth issue worthy of consideration is the stability of the estimated risk premia from the APT. This issue has received recent attention in the literature by way of estimating asset pricing models using the ARCH family of econometric models. Following the developments of Merton (1973) researchers have attempted to model the relationship between asset returns and time varying variances and covariances. However, to date there has been no evidence that the risk premia associated with macroeconomic factors used in tests of the static APT actually do time vary. This issue is important because if the risk premia are time varying then the practical use of the APT is once again brought into question. For example, practical applications such as calculating the cost of capital for firms requires that the estimate can be used to discount future cash flows. However, if the risk premia change in the future then the discount rate calculate from ex post data may not be representative of the future discount rate given the change in risk premia. In order to evaluate the stability of the estimated risk premia the APT is recursively estimated over the period March 1990 to August 1993. This provides a time series of estimated risk premia which can then be checked for stability. The rest of this chapter is organised as follows: in section 5.2 we discuss the issues of uniqueness of the return generating process. Section 5.3 presents the results from estimating the model using the estimation sample and the validation sample in order to determine whether or not the APT has a unique factor structure. We also consider the performance of the APT in terms of its ability to explain the cross sectional variation in average returns. In section 5.4 we present tests for serial correlation and heteroscedasticity of the residuals from the final estimated version of the APT. In Section 5.5 the results from recursive estimates of the APT are reported in order to assess the stability or otherwise of the risk premia. Section 5.6 concludes.

5.2 Uniqueness Of The Return Generating Process

As mentioned above one of the implications of Ross's (1976) APT is that the return generating process is unique. However, with regard to the UK and the US this condition seems to be violated. For example, in the UK, while Beenstock and Chan (1986) find over twenty factors are present in the UK stock market using factor analysis, Abeysekera and Mahajan (1987) also using factor analysis, and covering the period used by Beenstock and Chan (1986) find that there are no priced factors in the UK stock market. Similar results emerge when considering the use of macroeconomic variables as factors. Beenstock and Chan (1988) find an interest rate factor, a measure of the money supply, and two inflation factors as statistically significant while Poon and Taylor (1991) find no relationship between security returns and the factors specified by Chen Roll and Ross (1986) in the US, for the UK. In addition, Clare and Thomas (1994) find that their results are not robust to within sample ordering of portfolios. For example, given the criteria for forming their portfolios they find a different number of factors are priced depending on the portfolio formation criteria chosen. A similar pattern emerges in the US with results from Chen, Roll, and Ross (1986) and Chan, Chen, and Hsieh (1985) being contradicted by Shanken and Weistein (1990).

It would be easy to argue that this observed non-uniqueness is evidence against the empirical validity of the APT as an asset pricing model. However, before such a claim can be made we must consider the problems entailed in estimating the APT. Failure to identify a unique return generating process can not in itself invalidate the APT theoretically. Rather, it may well be more symptomatic of the problems with the methodology used to test the model. All the above studies employ some variant of the Fama and MacBeth (1973) two-step estimator, and form portfolios of stocks in order to reduce the errors in variables problem. We have noted in chapter 1, and shown in chapter 2, that there are severe problems with this technique which can lead to problems in empirical tests of the APT. The techniques used in testing the APT in this thesis aim to overcome the problems of the Fama and MacBeth (1973) estimator, the forming of portfolios and the use of a strict factor model. Consequently, we are in a stronger position to go on and address the issue of whether the return generating process is unique, free from the problems faced by earlier studies.

5.3 Is The Return Generating Process Unique ?

To facilitate the assessment of whether or not the APT has a unique return generating process we briefly repeat the results of the test of the APT undertaken in Chapter 4. We found that, when expectations are formed according to a learning scheme, there are six priced factors in the UK equity

Table 5.1

NL3SLS Estimated Prices of Risk for the APT
Estimation Sample

Parameter	Estimate	T - Ratio
λ_1 (Default Risk)	-0.00041	-2.78***
λ_3 (Exchange Rate)	-0.01723	-3.89***
λ_5 (Money Supply)	0.00178	2.28**
λ_6 (Unexpected Inflation)	-0.00333	-3.91***
λ_7 (Expected Inflation)	0.00125	2.33**
λ_{10} (Market Portfolio)	0.00144	1.67*

Notes: Sample period 1980M1 - 1993m12. Source : Datastream
* denotes significant at 10%
** denotes significant at 5%
*** denotes significant at 1%

market represented by a sample of 69 stocks. We will call this sample the estimation sample. Table 5.1 above recaps these results, noting that the APT parametric across equation restrictions hold, and each sequential reduction holds. The relevant factors are default risk, the money supply, the return on the market, unexpected inflation, the change in expected inflation, and the exchange rate. This model seems an adequate description of the return generating process of stocks and with the exception of the exchange rate, which represents the open nature of the UK economy, the factors found significant are similar to those found relevant in the US.

The issue now is whether or not these factors are unique. In order to evaluate this we begin by specifying the six factors found to be priced in the first sample as potential sources of systematic risk for the validation sample. The results from estimating this model for the validation sample are reported in table 5.2. Before examining the price of risk, it is worth noting that the APT pricing restrictions can not be rejected. Comparing tables 5.1 and 5.2 it is evident that while there are six factors priced in the estimation sample, only five of these factors have significant price of risk in the validation sample, with expected inflation being very insignificant with a t statistic of only 0.208. Moreover, of those five factors that are significant only three of them have the same sign in both samples, these being the money supply, the market portfolio, and unexpected inflation. Default risk and the exchange rate have different signs in the two samples. A test of whether the five common factors' estimated price of risk in the validation sample can be restricted to be

Table 5.2

NL3SLS Estimated Prices of Risk for APT
Validation Sample

Parameter	Estimate	T - Ratio
λ_1 (Default Risk)	0.00035	3.67^{***}
λ_3 (Exchange Rate)	0.00748	3.04^{***}
λ_5 (Money Supply)	0.00073	1.68^{*}
λ_6 (Unexpected Inflation)	-0.00803	-2.31^{**}
λ_7 (Expected Inflation)	0.00009	0.21
λ_{10} (Market Portfolio)	0.00128	1.98^{**}

Notes: Sample period 1980M1 - 1993m12. Source : Datastream
* denotes significant at 10%
** denotes significant at 5%
*** denotes significant at 1%

the same as the estimation sample is easily rejected at conventional significance levels (χ^2 (5) = 22.39). At first sight it appears that while the APT pricing restrictions hold in both samples, the condition underlying the APT that the return generating process is unique is violated both in terms of the number of statistically significant factors and their respective signs. It may be tempting at this stage to claim that the APT is invalid as an empirically useful model. However, it is possible to restrict the three factors with the same sign on the price of risk in both samples to be the same as one another. The results from imposing the restrictions that the price of risk on unexpected inflation, the money supply, and the market portfolio are the same in the validation sample as they are in the estimation sample are reported in table 5.3. The likelihood ratio test of the restrictions is χ^2 (3) = 6.074 which is easily accepted at conventional levels. Thus we have three common factors in the return generating process. While at this stage the evidence against the non-uniqueness of the APT is still apparent given the non-common factors in each sample, it is possible to take this a step further when we consider the performance of the APT in explaining observed security returns.

The expected returns and actual average returns were calculated for both samples under the following conditions: for the estimation sample all six factors, as estimated in table 5.1, were used to calculate expected returns as in equation (5.1). For the validation sample the five factors were used with unexpected inflation, the money supply and the market portfolio restricted to have the same price of risk as in the estimation sample (table 5.3). Equation

Table 5.3
NL3SLS Estimated Prices of Risk for APT *(Validation Sample, Significant Factors)* with the price of risk for unexpected inflation, money supply, and the market portfolio restricted to be the same as the estimates in the estimation sample.

Parameter	Estimate	T - Ratio
λ_1 (Default Risk)	0.00087	5.04^{***}
λ_3 (Exchange Rate)	0.03080	5.49^{***}
λ_5 (Money Supply)	0.000178	†
λ_6 (Unexpected Inflation)	-0.00333	†
λ_{10} (Market Portfolio)	0.00144	†

Notes: Sample period 1980M1 - 1993m12. Source : Datastream
* denotes significant at 10%
** denotes significant at 5%
*** denotes significant at 1%
† denotes a parameter constrained to take on a specific value.

(5.2) was estimated for both samples and the resultant adjusted R^2's are reported in table 5.4 The high adjusted R^2's are very encouraging with regard to the APT's ability to explain observed security returns. This compares favourably with the results from the US (for example, see Shulka and Trzcinka (1990) who report cross sectional adjusted R^2's of up to 42% when using principal components and factor analysis).

The interesting question that arises, given the APT's ability to adequately describe the observed movements of security returns, is how much of this movement is due to the factors that are outside the three common factors. In order to assess this we reestimated the APT including only the three factors that are common in both samples and restricted their price of risk to be equal to -0.00333, 0.00178, and 0.001445 respectively. At this stage the adjusted R^2's from the cross sectional regression of actual returns on excess returns, reported in table 5.5 , both fall to around 50%, still impressive, but the obvious reduction comes from the fact that the three factors are constrained to be the same while the other factors are removed. A comparison of the expected and average returns, with the three common factors constrained to be equal, are presented in figures 5.1 and 5.2. Inspection of the two plots reveals that there are two outliers in both samples at company numbers 28 and 62 for the estimation sample and company numbers 35 and 45 for the validation sample. In the first instance we placed a dummy variable, which takes the value of one for the company where the outliers are present and zeros elsewhere, in the cross sectional regression. The adjusted R^2's from this

Table 5.4

Adjusted R^2's from Cross Sectional Regression of Average Returns on Predicted excess Returns

Sample	Adjusted R^2
Estimation	75.1%
Validation	78.3%

Table 5.5

Adjusted R^2's from Cross Sectional Regression of Average Returns on Predicted excess Returns

(Three Common Factors Constrained to have the same Price of risk as the Estimation Sample)

Sample	Adjusted R^2
Estimation	50.5%
Validation	43.7%

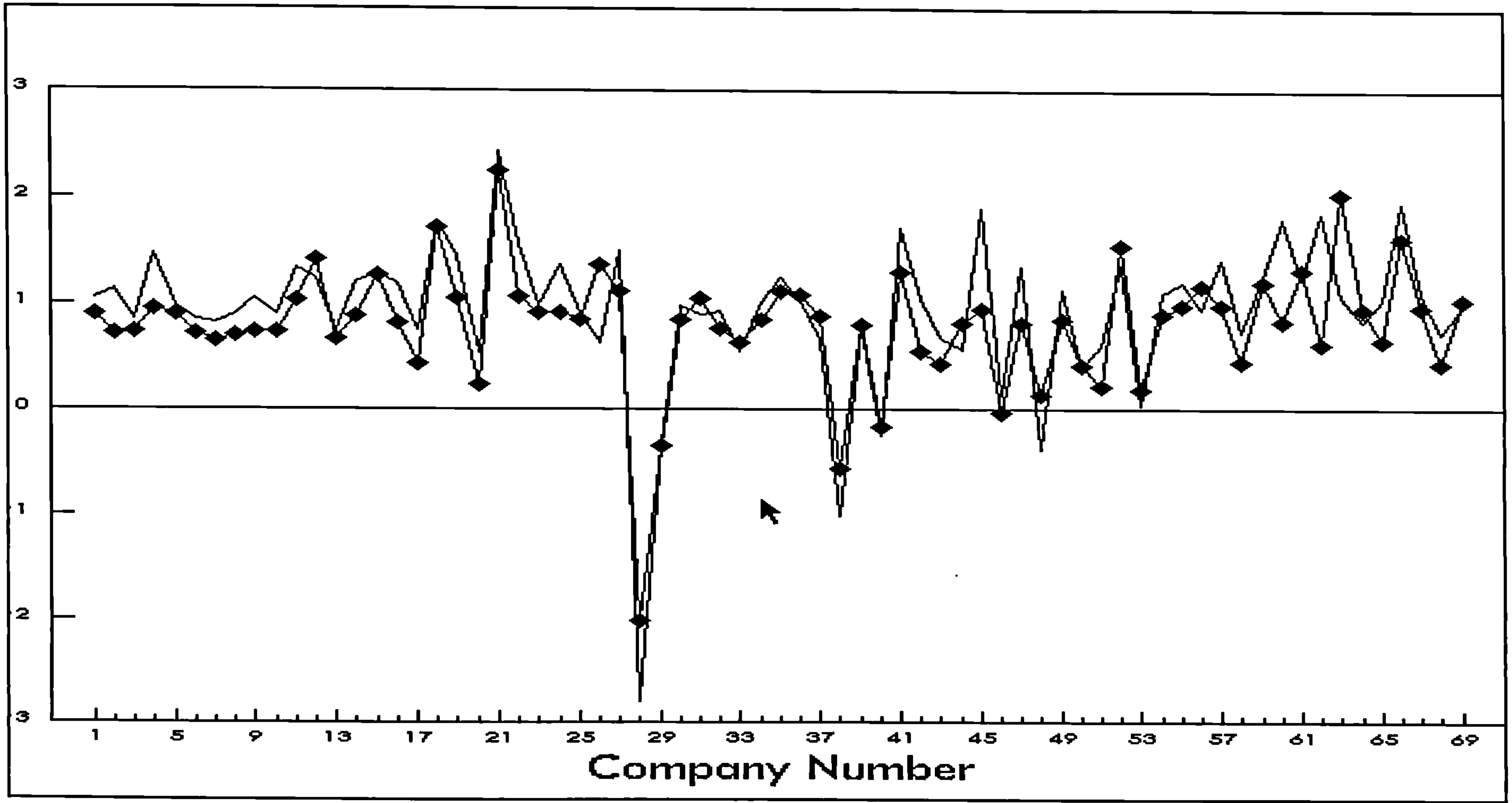


Figure 5.1 Actual and Expected Returns : Estimation Sample (3 Factors)

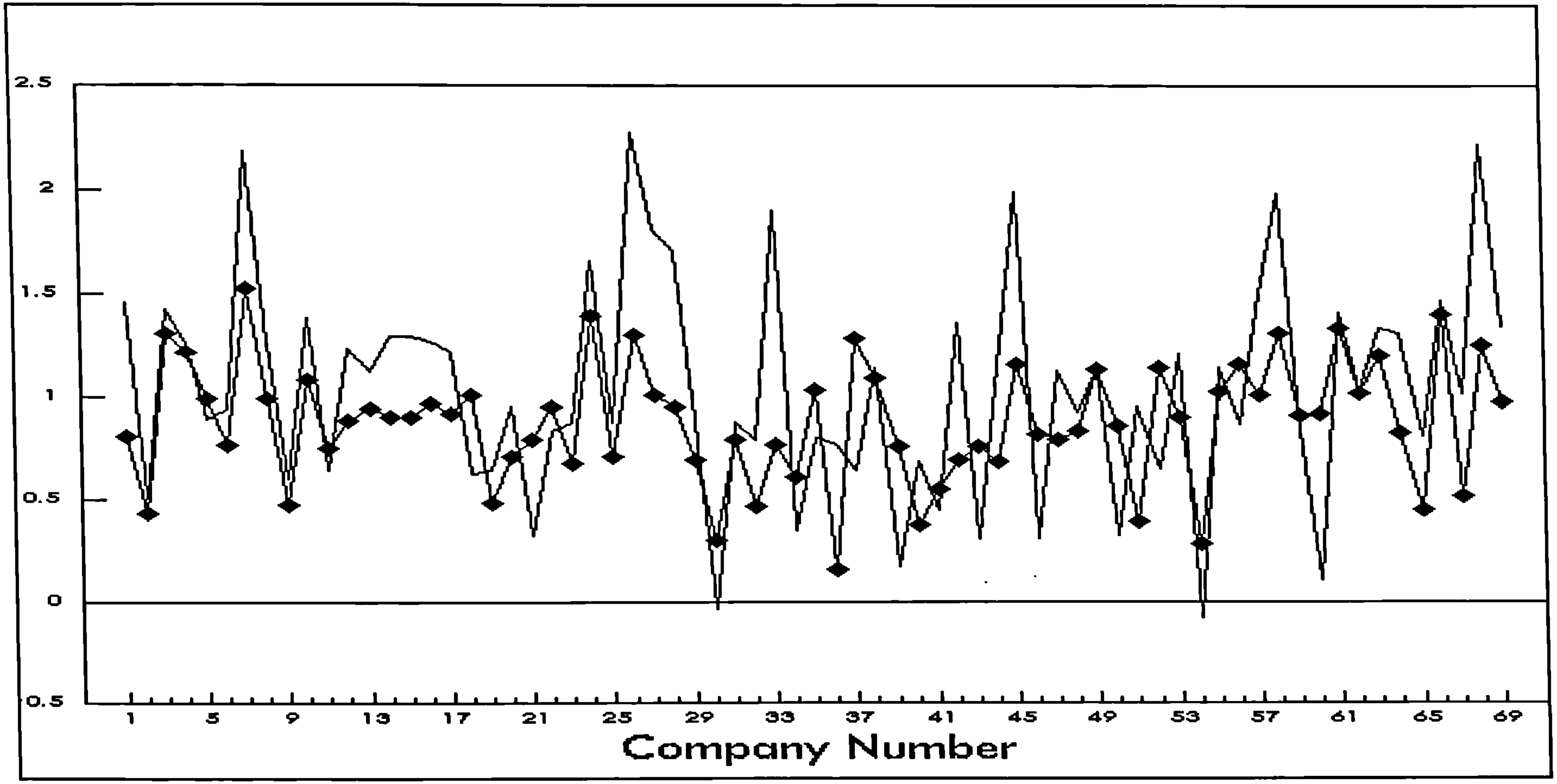


Figure 5.2 Actual and Expected Returns : Validation Sample (3 Factor where the coefficient estimates are constrained to take on the values of the estimation sample).

regression are reported in table 5.6. The removal of the outliers improves the adjusted R^2's back towards their original values with all six and five factors respectively. This indicates that the additional three factors in the estimation sample and the two factors in the validation sample add very little in terms of explanatory power. Indeed, taking the estimation sample, it is possible to drop the three additional factors from the model and reestimate with the three factors. The restrictions that this imposes on the six factor model are accepted with a likelihood ratio statistic of 223.24 which is distributed χ^2 (213) which has an approximate critical value of 247.936.[13] Thus we are left with only three factors in the estimation sample with the estimated price of risk are 0.0013 for the money supply, -0.0057 for unexpected inflation and 0.0015 for the market portfolio and the test of the APT's pricing restrictions are accepted (χ^2 (67) = 46.69). The restrictions that the validation samples price of risk are the same value as in the new three factor estimation sample model is also accepted at the 2.5% significance level (χ^2 (3) = 8.67). What emerges from this analysis is a three factor APT, where the three factors can be restricted to be the same in each sample of assets. This fulfils the requirement that the APT be unique. Although it was initially found that each sample required more than three factors, these additional factors add extremely little in terms of explaining the risk return relationship for the vast majority of the stocks in

---

[13] The approximate 5% critical value is calculated from,

$$\chi^2(\nu) = \nu \left[1 - \frac{2}{9\nu} + 1.64 \sqrt{\frac{2}{9\nu}} \right]^3$$

where v is the degrees of freedom.

Table 5.6

Adjusted R^2's from Cross Sectional Regression of Average Returns on Predicted excess Returns
(Three Common Factors Constrained to have the same Price of risk as the Estimation Sample, and a Dummy Variable for the two outliers)

Sample	Adjusted R^2
Estimation	73.3%
Validation	59.9%

each sample.

An important issue now is how well does the three factor model do in terms of explaining the observed security returns. To facilitate a comparison between the earlier specifications of the APT and the new three factor model we calculated the expected returns given the new estimates of the price of risk for the estimation sample. The adjusted R^2 for the cross sectional regression of actual excess returns on expected excess returns is 71.94% which compares favourably with the three factor model estimated earlier when the price of risk took on the values from the estimation with all six factors (50.5%). Moreover, it is very close to the three factor model with the dummy variable to account for the outliers (73.3%). This leads to the conclusion that reducing the model to only three factors and reestimating provides an excellent description of average observed returns. With respect to the validation sample when the three factors price of risk are restricted to take on the values as in the estimation sample leads to an adjusted R^2 of 81.19%. It appears that the three factor APT does an excellent job at describing average returns, both for the estimation sample and the validation sample.

To summarize thus far, starting with a six factor APT for the estimation sample, it was found that the imposing these factors on the validation sample leads to rejection of the same coefficient estimates for three of the six factors. However, further analysis reveals that these additional three factors in the estimation sample add little to the overall determination of the return generating process. On reestimating the model with the three common factors to obtain new parameter values we are left with a three factor APT that can explain up to three quarters of the movement in individual asset returns. Imposing these three new price of risk estimates on the validation sample (the restrictions were accepted) leads to an adjusted R^2 of 81.19%. In conclusion, on the validity of the APT in terms of the uniqueness of the return generating process, we find that the APT does have empirical content as shown by out of sample tests. In the next section of this chapter we turn to the issue of the robustness of the estimates to any underlying misspecification of the residuals.

5.4 Misspecification Tests Of The Residuals From The APT

At this stage of the analysis it appears that the empirical version of the APT provides an excellent description of the return generating process of UK stocks. However, one of the assumptions that we have made in interpreting the empirical results is that the residuals are serially uncorrelated and homoscedastic. In this section we outline the possible effects invalid assumptions regarding the nature of the residuals can have on the empirical results and proceed to tests whether or not the assumptions are valid. In the first instance we deal with the issue of the assumptions regarding the variances of the residuals. A standard assumption in the econometric model used to estimate the APT is that the variances of the residuals are constant:

$$Var(\epsilon_t) = \sigma^2 \qquad \textit{for all } t \tag{5.3}$$

This property of the regression residuals is known as homoscedasticity. In the case where this assumption is violated we have:

$$E(\epsilon_t^2) = \sigma_t^2 \tag{5.5}$$

which implies that the variance of the residuals is no longer constant and varies between each period. The important issue is the effect that heteroscedasticity will have on the estimates of the parameters of the model. It is well known that the presence of heteroscedasticity will render the estimates inefficient. Consequently, the standard errors will be imprecise and thus the hypotheses tested regarding the significance or otherwise of the estimated coefficients will be invalid. Thus, if we are to make valid inference regarding the significance of the estimated parameters we need to be aware of any misspecification with respect to the variances of the residuals. In order to test for the presence of heteroscedasticity the residuals from the estimated equations were extracted and squared to proxy for the variance terms. This new time series of observations, one series for each equation, is then tested for heteroscedasticity using a Box-Pierce test which is distributed Chi-Squared under the null hypothesis of homoscedastic residuals.

Turning now to the issue of serial correlation, the underlying assumption is that the residuals are not serially correlated over time,

$$E(\epsilon_t \epsilon_{t-i}) = 0 \tag{5.6}$$

In the case when this assumption is violated then residuals are no longer independent throughout time and the result is parameter estimates that are inefficient and estimated variances that are biased. Given these results regarding the presence of serial correlation it is necessary to test whether the residuals have the desired properties of no serial correlation. In order to do this we employ a Lagrange Multiplier test which involves regressing the estimated residuals on the fitted values and lagged residuals,

$$\hat{\epsilon}_{it} = \alpha_0 + \alpha_1 \hat{\epsilon}_{it-1} + \alpha_2 X_{it} + u_{it} \tag{5.7}$$

where $\hat{\epsilon}_{it}$ is the residual estimated from the APT for the ith equation, X_{it} are the fitted values from the APT estimation for the ith equation, and u_{it} is an error term. The null hypothesis of no serial correlation is H0: $\alpha_1 = 0$ which is accepted if the calculated value of TR^2 (where T is the number of observations) from (5.7) is less than the critical value for a χ^2 distribution with one degree of freedom. Having outlined the two tests for the presence of heteroscedasticity and serial correlation in the residuals from the APT estimation we report results from them in tables 5.7 and 5.8. Dealing first with the tests for serial correlation. It is apparent from table 5.7 that in 11 of the 70 equations there is evidence of serial correlation at the 5% or less level. However, if we consider the 1% level only there are just 6 cases of serial correlation. With regard to the tests for heteroscedasticity (table 5.8)

Table 5.7

Chi-Square Test[a] for Serial Correlation of Residual from the APT (NL3SLS Estimation)

Equation No.	$\chi^2(1)$	Equation No.	$\chi^2(1)$	Equation No.	$\chi^2(1)$	Equation No.	$\chi^2(1)$	Equation No.	$\chi^2(1)$
1.	5.346*	15.	2.592	29.	7.938**	43.	0.585	57.	0.303
2.	2.754	16.	2.106	30.	10.84**	44.	3.402	58.	0.817
3.	2.106	17.	1.964	31.	0.794	45.	11.34**	59.	3.248
4.	2.911	18.	2.102	32.	1.519	46.	1.944	60.	3.712
5.	3.454	19.	3.245	33.	2.025	47.	0.567	61.	0.437
6.	3.243	20.	1.992	34.	3.175	48.	3.241	62.	1.458
7.	1.458	21.	0.953	35.	0.278	49.	2.268	63.	1.442
8.	0.858	22.	3.156	36.	0.437	50.	1.116	64.	1.944
9.	1.766	23.	15.71**	37.	4.374*	51.	3.478	65.	2.674
10.	7.786**	24.	0.107	38.	2.592	52.	0.340	66.	1.702
11.	5.671*	25.	2.754	39.	8.132**	53.	4.861*	67.	1.893
12.	0.972	26.	2.431	40.	0.956	54.	2.089	68.	1.269
13.	6.486**	27.	1.928	41.	2.268	55.	3.402	69.	2.312
14.	1.246	28.	2.365	42.	2.642	56.	1.264	70.	1.453

Notes: Sample period 1980M1 - 1993m12. Source : Datastream
[a] The test is a Breusch-Godfery LM test for serial correlation distributed χ^2 (1).
* Indicates significant at 5%
** Indicates significant at 1%

Table 5.8

Chi-Square Test[a] for Heteroscedasticity of Residual from the APT (NL3SLS Estimation)

Equation No.	$\chi^2(1)$	Equation No.	$\chi^2(1)$	Equation No.	$\chi^2(1)$	Equation No.	$\chi^2(1)$	Equation No.	$\chi^2(1)$
1.	3.245	15.	21.72**	29.	1.626	43.	1.432	57.	2.971
2.	0.013	16.	1.825	30.	0.807	44.	1.751	58.	0.313
3.	2.258	17.	4.193*	31.	1.205	45.	0.713	59.	2.081
4.	0.837	18.	0.006	32.	19.22**	46.	1.528	60.	0.155
5.	0.295	19.	0.613	33.	2.131	47.	1.444	61.	2.710
6.	3.574	20.	0.674	34.	2.515	48.	6.703**	62.	0.661
7.	0.036	21.	0.037	35.	2.489	49.	0.672	63.	1.789
8.	2.258	22.	1.128	36.	21.39**	50.	7.202**	64.	1.871
9.	0.837	23.	7.516**	37.	0.165	51.	0.004	65.	2.211
10.	0.295	24.	1.264	38.	4.945*	52.	1.339	66.	2.696
11.	3.572	25.	1.218	39.	1.104	53.	0.290	67.	0.029
12.	0.036	26.	0.031	40.	0.224	54.	0.198	68.	0.237
13.	4.676*	27.	0.127	41.	0.553	55.	0.594	69.	0.020
14.	0.565	28.	1.042	42.	3.325	56.	0.048	70.	0.019

Notes: Sample period 1980M1 - 1993m12. Source : Datastream
[a] The test is a Box-Pierce test for heteroscedasticity distributed χ^2 (1).
* Indicates significant at 5%.
** Indicates significant at 1%.

there appears to be 9 equations which have residuals that are heteroscedastic at the 5% level. At the 1% level there are only 6. Overall the presence of misspecification of the residuals from the APT estimation does not appear to be of much significance. If we consider the 1% critical level then less than 10% of the estimated equations suffer from either serial correlation or heteroscedasticity. This is an encouraging result given the possible implications of misspecification. Having dealt with this issue we now turn to examining the stability of the estimated risk premia overtime.

5.5 Stability Of The Estimated Risk Premia

Since the seminal work of Merton (1973) there has been a growing interest in asset pricing models that allow factor price of risk and asset betas to vary over time. The family of ARCH models have been employed to estimate both CAPM versions (for example, see French, Stewart, and Stambaugh (1987) and Chou (1988) and APT versions of intertemporal asset pricing models (for example, see Engle, Ng, and Rothschild (1989) and Shanken (1990)). In this section we test whether the risk premia from the three factor APT estimated in the previous section, for the estimation sample, vary over time. In order to facilitate this we estimate the APT using the first 124 observations, that is January 1980 to February 1990, and then recursively update the estimates of the risk premia one observation (month) at a time over the period March 1990 to August 1993. For each month we use the equation for the market portfolio and calculate the equity market risk premia ($b_{mj}\lambda_j$ where b_{mj} is the sensitivity of the market portfolio to the jth factor) associated with each factor which provides a time series of fifty six observations of the estimated risk premia. Analysis of this time series can then determine whether the risk premia are stable.

Figure 5.5, 5.6, and 5.7 are the plots of the recursively estimated risk premia for unexpected money supply, unexpected inflation, and the market portfolio respectively. With respect to the market portfolio, which is simply the price of risk since in this case the sensitivity of the return on the market portfolio to itself is one, this risk premia seems quite stable over time. A similar pattern emerges for the risk premia associated with the unexpected money supply. The risk premia on unexpected inflation again appears to be fairly stable through the period under investigation. There appears to be a little more variation in the unexpected inflation risk premia.

5.6 CONCLUSION

This chapter has evaluated four important issues in the empirical implementation of the APT. First, for the APT to have any empirical content, and for it to have any practical use, there must exist a unique return generating process. Second, the APT must be able to perform adequately well in terms of being able to explain the average excess return movements. Third, for inferences to be valid then the assumptions regarding the residuals from the estimation of the APT must also be valid. Finally, the risk premia must be stable. Considering the first issue we found that the return generating process is unique: there are three factors present in the UK equity market that

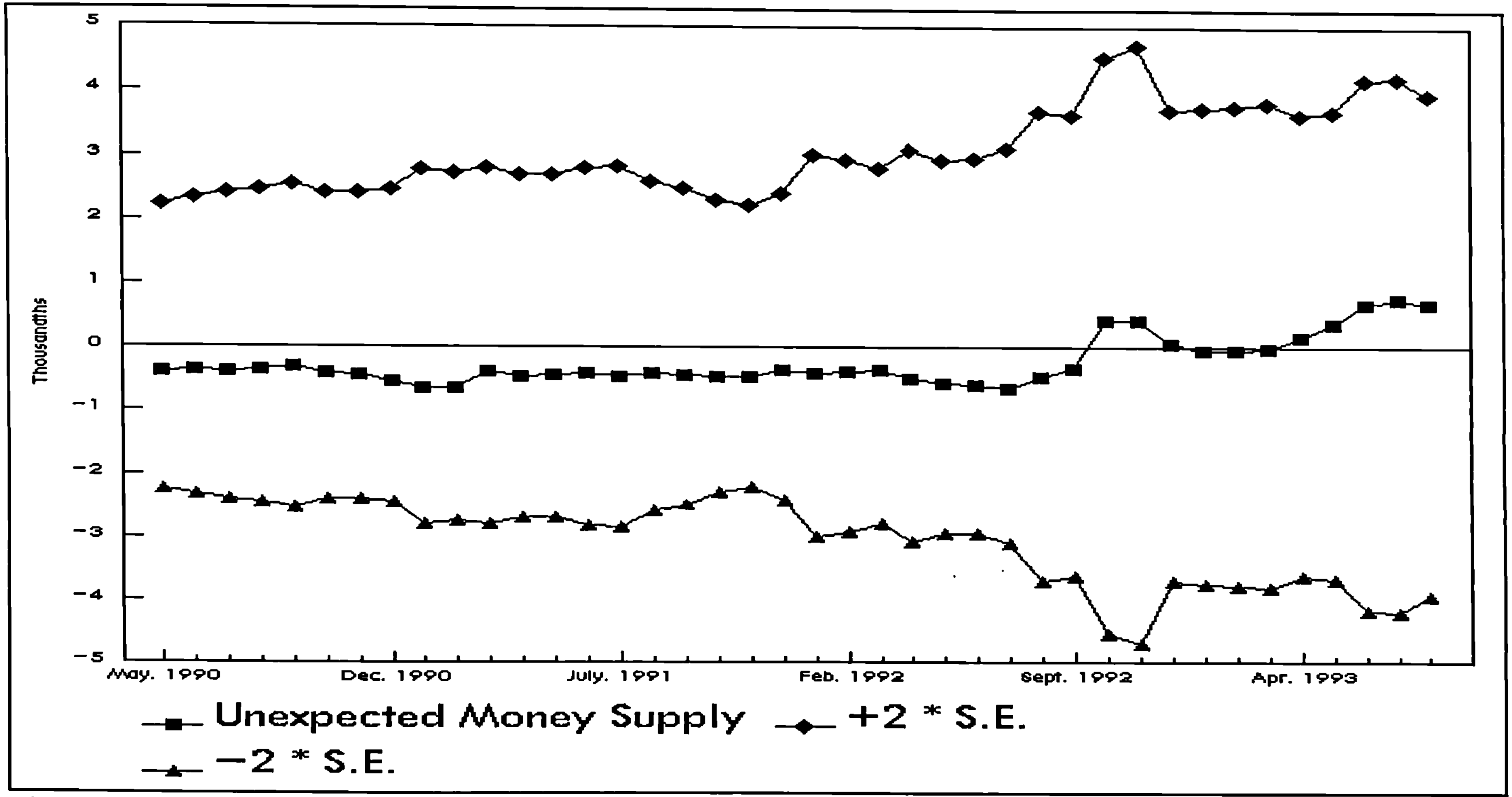


Figure 5.5 Recursively Estimated Market Risk Premium Associated with Unexpected Money Supply

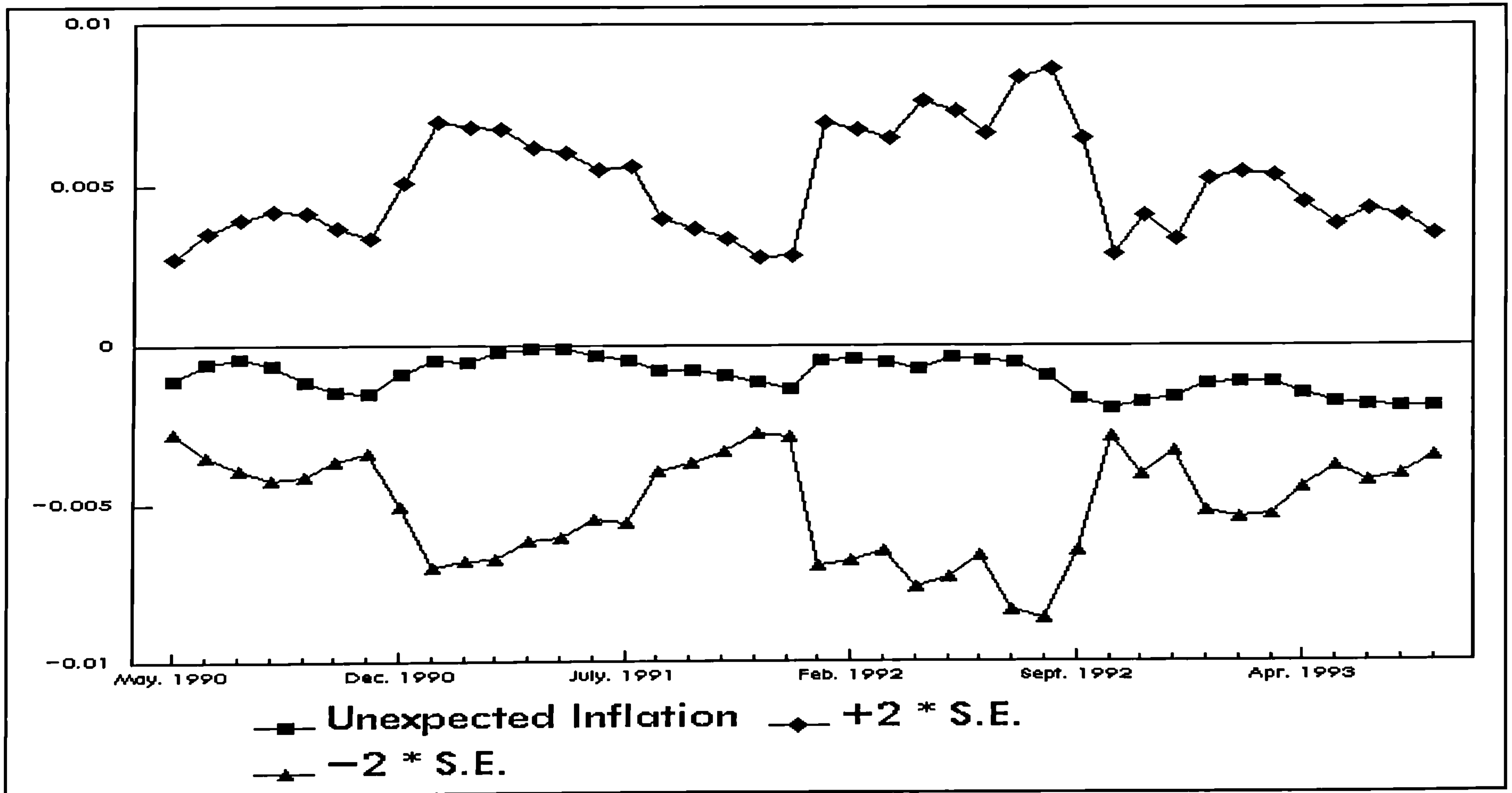


Figure 5.6 Recursively Estimated Risk Premium for the Market Associated with Unexpected Inflation

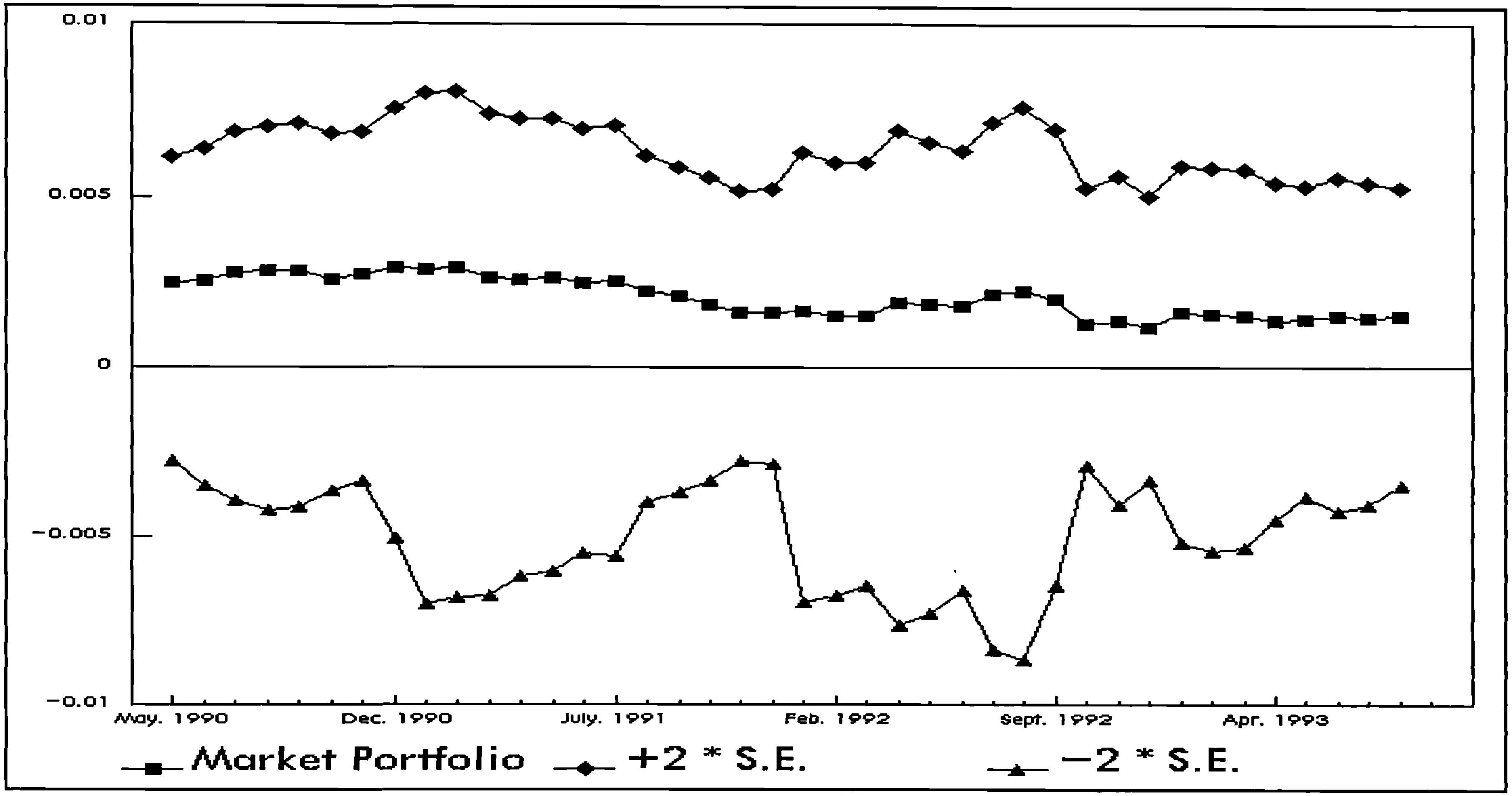


Figure 5.7 Recursively Estimated Risk Premium Associated with the Market Portfolio

carry statistically significant prices of risk, these being the excess return on the market portfolio, unexpected inflation, and unexpected money supply. These three factors explain over 70% of average excess returns in the estimation sample and over 70% in the validation sample. These are very encouraging results for the APT in the light of previous tests both in the US and the UK, where it appeared that the condition of uniqueness of the return generating process was violated. However, when tests of the APT are approached in a way that is free from the problem of the traditional tests based on strict factor models and the Fama and MacBeth (1973) two step procedure, then the results of tests of the APT are supportive. Furthermore, we have found that the residuals estimated from APT appears to be well specified on the whole. With respect to the final issue, this chapter found that the risk premia are stable over time. In conclusion, this chapter has provided extensive robust test of the empirical content of the APT which have to date been ignored in the literature. We have found that the APT provides an excellent description of the return generating process for UK stocks and is robust to underlying assumptions.

CHAPTER 6

RISK, RETURN AND SEASONALITIES : RESOLVING THE PUZZLE

6.1 INTRODUCTION

There is a growing body of empirical evidence, in both the US and Europe, demonstrating that security returns exhibit seasonal patterns. In the US, for example, Rozeff and Kinney (1977) find evidence that returns in January are significantly higher than returns in any other month. With regard to the UK, Clare, Psaradakis, and Thomas (1994) find evidence of statistically significant seasonal patterns in the months of January, April, September, and December. Attempts to explain observed seasonal patterns in stock returns are generally divided into one of two categories: either the tax loss selling hypothesis and the portfolio rebalancing hypothesis or the risk-return hypothesis. The actual category into which the observed seasonal pattern falls is driven by empirical findings. Empirical tests of seasonalities are usually conducted from an asset pricing model framework in the following way. First, a series of returns is taken and examined for the existence or otherwise of seasonal patterns. The positive identification of a seasonal pattern then leads to the second part of the analysis which involves testing whether the assumed risk return structure can account for the seasonality. The results of this step then categorise the explanation for them. If the risk return structure accounts for the seasonal pattern then the seasonality is said be a product of the risk in the equity market. However, in the circumstance when this is not the case the seasonality is rationalized in the context of the tax loss selling hypothesis or the portfolio rebalancing hypothesis. The tax-loss selling hypothesis arises as a result of investors realizing losses before the beginning of the new tax year. This causes downward pressure on stock prices in the month prior to the end of the tax year and a subsequent upward pressure on stock prices in the first month of the new tax year as selling pressure disappears and investors reenter the market. Thus, with respect to the US, where the new tax year begins in January, the evidence of statistically significant higher returns in January has been attributed to tax-loss selling (see, for example, Branch (1977)). The same pattern emerges in the UK where the new tax year begins in April. However, the evidence from the UK of an observed seasonal pattern in January can not be attributed to tax-loss selling unless this is via contagion from the US market. The repositioning of professionally managed funds at the turn of the year works in the following way: fund managers sell "loser" stocks at the end of the year to "window dress" their fund and then repurchase in the first month of the new tax year (see Haugen and Lakonishok (1988) and Ritter and Chopra (1989) who find evidence to support this hypothesis for the US). If this hypothesis is true then it will have the same effect on stock returns as the tax-loss selling hypothesis.

Generally it is the case that the tax loss selling hypothesis and the portfolio rebalancing hypothesis arise as a consequence of negative results regarding the ability of the asset pricing model to account for the observed seasonality. In terms of the implications of seasonal patterns in stock returns on economic theory it is not surprising that such explanations have appeared in the literature, since, if the asset pricing model can not account for the seasonal behaviour of returns, and there is no other rationale for its existence then ultimately it must be an anomaly to the Efficient Markets Hypothesis (EMH).

These issues are complicated by the findings that in some cases the asset pricing model can account for the seasonal pattern in stock returns and thus the seasonal patterns are a product of the risk return structure. However, in this context the asset pricing model fails to find any statistically significant relationship between risk and return in any other month (see for example Tinic and West (1984 and 1986) and Gultekin and Gultekin (1987)). Under this scenario the implications are severe for asset pricing models. In spite of this an important caveat must be noted. Tests of seasonal patterns in stock returns are undertaken under the joint hypothesis that first the market is efficient and second that the asset pricing model chosen to represent the risk return structure is empirically valid. It is quite possible that the failure of the asset pricing model to capture the seasonality, or the observation that the asset pricing model has no empirical content outside of the month where a seasonal pattern has been observed is due to the asset pricing model being empirically invalid. Given the result of chapter 3, that the CAPM provides an inadequate description of the risk return structure, it may not be surprising that tests for seasonalities employing the CAPM find that either the seasonality is outside of the risk return structure or that when it is in the risk return structure there is no statistically significant relationship between risk and return in any other month.

In this chapter we aim to reassess the evidence of seasonalities in the UK equity market using the framework outlined in the previous chapters. Using an approximate factor model, non-linear time series regressions, and the unique factor model found in chapter 5, we test whether the previously observed seasonalities in the UK equity market[14] are part of the risk return structure as successfully modelled in the previous chapters. The approach adopted is two fold. Initially, we test whether the seasonalities are in the risk return structure by augmenting the three factor APT with a dummy variable that takes the value of one for the month under consideration and zero elsewhere.[15] If the dummy variable is found to be statistically significant then we can infer that the seasonality is outside the risk-return structure since it can not be explained by the APT specification. Alternatively, if the dummy variable is found to be statistically insignificant we are faced with one of two possible outcomes: either the seasonality is in the risk-return structure or the observed seasonality is not present in this sample. To test this we omit the month under

---

[14] With respect to the UK equity market the following monthly, seasonal patterns have been observed in empirical tests: January (Clare, Psaradakis, and Thomas (1994), Demos, Sentana, and Shah (1993), and Corhay, Hawawini, and Michel (1987)); April (Clare, Psaradakis, and Thomas (1994), and Corhay, Hawawini, and Michel (1987)); September (Clare, Psaradakis, and Thomas (1994)); December (Clare, Psaradakis, and Thomas (1994)).

[15] Given the relatively short sample (14 January's) this test may not be very powerful. However, we employ further tests which examine the performance of the APT when seasonal patterns are removed from the data. For example, if the seasonal dummy variable appears to be insignificant, but this is due to the lack of power of the test, then by analysing the adjusted R^2 from a cross sectional regression of average returns on expected returns when seasonal months are omitted from the data and included in the data we would expect the adjusted R^2 to be significantly higher for the data with the seasonal month included than the adjusted R^2 for the data with the month excluded.

consideration from the data set and reestimate the three factor APT. If the seasonality is not in the risk-return structure then there should be no statistically significant difference between the estimated risk premia when all months are included and the estimated risk premia when the month is omitted. Such a proposition is easily tested within our framework.

On the other hand, if we find that the seasonality is in the risk return structure then the question that arises is whether or not the APT has any empirical content outside the month under question. In order to examine this we calculate the expected returns predicted by the APT when the month is omitted. If the APT does have empirical content outside of the month under consideration then there should still be a positive, statistically significant relationship between actual and expected returns. This is analyzed by regressing the actual returns with the month omitted on the expected returns predicted by the APT when the month is omitted. We can then compare the adjusted R^2's for the APT with no month omitted and the APT with the month omitted.

It may be possible that the above two hypotheses, that is, that the seasonality is either in the risk return structure or outside it, are not mutually exclusive. For example, we may find that the risk-return structure captures the seasonality, in the sense that the risk premia are different when the month is omitted from the data, and that the APT does have empirical content outside of these months. However, there is the possibility that when we omit the month from the data there is an improvement in the APT's explanation of actual returns. This would imply that not all of the seasonality is captured by the APT's risk-return structure. It then becomes necessary to examine by how much this matters. In order to assess this an analysis of the relative mispricing of the APT when no months are omitted and a month is omitted can indicate by how much there is an improvement, or dilapidation of the performance of the APT by omitting a month where the seasonality is present. For example, if the seasonality in a particular month is found to be part of the risk-return structure but the omission of the month improves the adjusted R^2 in the cross sectional regression of actual returns on expected returns, then analysis of the change in mispricing would indicate the relative improvement of the APT as a model which seeks to price risky assets. The rest of the chapter is organised as follows: in section 6.2 the extant literature is reviewed. In section 6.3 the methodology and tests used to examine the seasonalities in the UK equity market are described. In section 6.4 the results from the tests for seasonalities are reported and finally section 6.5 summarizes and concludes the chapter.

6.2 THE EXTANT LITERATURE

The first tests of seasonalities in stock returns took the form of calculating mean returns in a specific month and testing their statistical significance. For example, Rozeff and Kinney (1976) find evidence of a statistically significant higher return in the month of January in the US stock market. Given these observed seasonalities researchers turned to trying to determine whether they could be explained by an asset pricing model. Early evidence from Banz

(1981) found that when the CAPM is augmented with a January dummy variable then beta is no longer the only significant factor in the risk-return structure. Further evidence in the US (Keim (1983) and Reinganum (1981)) related this observed seasonality to a firm size effect where the majority of higher returns in January are due to small firms. This would seem to provide strong evidence against the EMH and the CAPM.

Direct tests of whether observed seasonalities are present in the risk return structure have provided little in the way of explaining the existence of these patterns. Tinic and West (1984 and 1986) employ the Fama and MacBeth (1973) two step procedure to test whether a January seasonal is present in the estimated risk premia on the market portfolio. Their results suggest that this is the case. However, there appears to be no statistically significant relationship between risk and return in any other month. International evidence provided by Corhay, Hawawini, and Michel (1987) confirms the above findings for the US as well as suggesting similar results for the equity markets in the UK, France and Belgium. In the UK they found that April had a statistically significant positive return as well as January. To analyse whether these patterns are present in the risk return structure they formed twenty portfolios for each stock market, ranked by individual asset beta and using the methodology of Tinic and West (1984 and 1986), found the existence of persistent seasonalities in the risk premia in all four stock markets. The relationship between systematic risk and average returns is only significantly positive in January for the US and Belgium stock exchanges. In the UK this relationship holds for the month of April and no other month. For France, while the risk premia in January is positive and greater than in other months it is not significantly different from zero.

Given the recent empirical findings that the APT, when compared to the CAPM, provides a better description of risky asset returns (see, for example, Chen (1983) and Chapter 3) attention has focused on whether or not the APT can capture the seasonal patterns of stock returns and still provide a description of asset returns outside of this month. Gultekin and Gultekin (1987) examine this issue by estimating the APT using factor analysis to extract the systematic risk factors. However, using this specification of the risk return structure leads to the same conclusions as Tinic and West (1984 and 1986), namely that while the risk return structure captures the January seasonal in the US the APT has nothing to say regarding the other eleven months of the year. However, this contradicts the results of Burmeister and McElroy (1988) who estimate an APT model with three observed factors and three unobserved factors, and a January dummy. They find that a January dummy is significant, thus ruling out its existence in the risk-return structure they estimate.

More recently, using an approximate factor model framework and principal components to extract the factors, Connor and Korajczyk (1993) find that, for the US, in none January months there are two factors present in the risk-return structure whereas when January is included there are up to six factors present.

The problem of employing techniques which extract statistical factors as proxies for the systematic risk factors is that it is not possible to place any interpretation on the factors and as a consequence even if the factors can account for the seasonality and the model can explain return outside of the seasonal months we are no nearer to explaining why these patterns exist.

In two recent papers, Demos, Sentana, and Shah (1993) and Clare, Psaradakis, and Thomas (1994), present evidence of seasonalities in UK stock returns. Demos, Sentana, and Shah (1993) examine the risk-return structure of the UK equity market by estimating a dynamic version of the APT, under a strict factor model assumption, that explicitly allows for a different conditional factor structure in January. A two factor model, in which one factor represents the market index and the other a January factor, of asset returns with time-varying volatility is estimated using a generalized quadratic autoregressive conditional heteroscedasticity (GQARCH) formulation. They find that there is a positive, statistically significant relationship between risk and return in non-January months, thus contradicting the earlier results of Corhay, Hawawini, and Michel (1987). However, even under the two-factor, dynamic version of the model a January seasonal dummy variable is still significant in the mean of the equation. Thus, while their model can explain returns outside of January it can not explain the January seasonal, that is, the January seasonal is not part of the risk-return structure as they have modeled it. Consequently, under this specification of the risk-return structure, the January seasonality would be treated as an anomaly.

In addition to a January seasonal, Clare, Psaradakis, and Thomas (1994) find significant seasonal patterns in April, September, and December. In order to assess whether these seasonal patterns are a function of the risk-return structure they use a dynamic version of the CAPM. Specifically, a GARCH model for the risk-return structure is estimated including a dummy variable for each month. They find that these seasonalities are still present even when they use a dynamic model of the risk-return structure within a CAPM framework, once again suggesting that the seasonal patterns in the UK stock market are not part of the risk-return structure as they model it.

The problem here is that, as noted above, all the above results rely on the assumption that the risk-return structure is correctly specified. While we have shown in previous chapters that the CAPM is not representative of the risk-return structure, the use of an APT model does not automatically imply that the risk-return structure will be correctly specified either. For example, the dynamic APT model of Demos, Sentana, and Shah (1993) is likely to have omitted factors, and the GQARCH terms are likely to be picking up misspecification of the underlying model. The same can be said of the Clare, Psaradakis, and Thomas (1994) GARCH specification of the CAPM. As such then, to obtain reliable results from test of seasonalities it is first necessary to have the correct model of the risk return structure before any test, such as those for seasonalities, are undertaken. Using the unique factor model outlined in chapter 5, which is known to explain up to seventy percent of the cross-sectional variation of average returns, we re-evaluate the evidence on seasonalities in the UK stock market by testing for seasonalities in January, April, September, and December using the APT.

6.3 Testing For Seasonalities In The Risk Return Structure

Before considering whether the observed seasonalities in the United Kingdom equity market (see Clare, Psaradakis, and Thomas (1994), Corhay, Hawawini, and Michel (1987) and Demos, Sentana, and Shah (1993)) are a product of the risk-return structure, there are three points that require consideration. First, before any tests of seasonal patterns in the risk return structure can take place it is necessary that we have an asset pricing model which adequately describes the risk return structure when all months are considered. The second point is that this asset pricing model should perform equally well when all seasonal patterns are removed from the data. Third, if the above two points are satisfied it must be possible to relate individual seasonal months to characteristics of the asset pricing model. In addressing these three points we adopt the following methodology:

Estimating the APT

In the first instance we estimate the APT using the methodology outlined in previous chapters. Specifically the final, three factor APT specification of chapter 5 is employed using the estimation sample of stock returns. To assess the adequacy of the asset pricing model we consider two measures of performance. The first involves a cross sectional regression of the form:

$$\hat{r}_i = \alpha_0 + \alpha_1 E[\hat{r}_i] + u_i \tag{6.1}$$

where: $\hat{r}_i$ is the average excess return on asset i; $E[\hat{r}_i]$ is the expected excess return on asset i predicted by the APT; u_i is an error term; and α_0 is a constant. The adjusted R^2 from this regression provides a measure of the performance of the APT in terms of describing the cross section of asset returns. The second measure of performance is also derived form the cross sectional regression (6.1) where the constant can be interpreted as a general measure of mispricing in the APT model. Large values for the measure of mispricing would indicate that the empirical formulation of the APT may well be improved upon. For example, if the risk return structure does not capture the seasonal patterns in returns then a dummy variable for, say, January may improve the APT's ability to explain actual returns.

Testing for Seasonalities

Once we arrive at a benchmark APT model, the next step is to determine whether or not the previously observed seasonal patterns in the United Kingdom stock market are present in our data set. To facilitate this we augment the final version of the APT with dummy variables for the months of January, April, September and December. This enables us to determine whether or not these seasonal patterns are part of the risk return relationship. In particular, we estimate the following:

$$\boldsymbol{R}_t = \boldsymbol{B}^f \lambda^f + \boldsymbol{B}^f \boldsymbol{F}_t + \gamma^\delta \lambda^\delta + \gamma^\delta \delta_t + \epsilon_t \tag{6.2}$$

where: $\mathbf{R}_t$ is a N×1 vector of excess returns; λ^f is a K×1 vector of the prices of risk on the observed factors; $\mathbf{B}^f$ is a N×K matrix of sensitivities of the securities to the observed factors, $\mathbf{F}_t$ is a K×1 matrix of observed factors; γ^δ is a N×1 vector of sensitivities of the assets to the seasonal dummy; λ^δ is a scaler risk premium associated with the dummy variable; δ_t is a dummy variable where 1 appears for the month under investigation and zeros elsewhere. There are two advantages of this approach: first, if a seasonal dummy variable is significant, specifying a risk premium on this variable enables us to determine whether or not returns are higher, or lower, in this month than other months and second, by specifying a system of individual, randomly selected stocks, then assessing whether or not the prices of risk are equal over all stocks forms a test of the firm size effect and seasonalities[16].

To test the statistical significance of the seasonal dummy the following model is estimated:

$$R = B^f \lambda^f + B^f F + \epsilon \tag{6.3}$$

Equation (6.2) implies a restriction on equation (6.3), namely $\lambda^\delta \gamma^\delta = \gamma^\delta \delta = 0$, which can be tested with a likelihood ratio test. If we accept the null hypothesis that $\lambda^\delta \gamma^\delta = \gamma^\delta \delta = 0$, then the seasonality is not outside the specified risk-return structure and we do not need additional seasonal factors. This

[16] If we reject that the price of risk on the seasonal dummy variable is equal across all securities then it may well be the case that small firms have a higher risk than large firms in this month (see Clare, Psaradakis, and Thomas for a discussion of firm size and seasonalities in the UK).

leaves two possible explanations of seasonality: first the seasonality is already captured by the risk-return structure or second, seasonalities are not present in our data set. Evaluating these alternatives involves estimating equation (6.3) with the relevant month omitted from the data. This is then compared to the same model estimated with the price of risk coefficients restricted to be the same as those from the APT model with no months omitted. Testing the restriction that the coefficients are equal in both scenarios is a test of whether or not the seasonality is captured by the risk-return structure. Acceptance of the restrictions that the prices of risk are equal indicates that the risk premium is the same, irrespective of that months inclusion in the data, and hence there is no seasonality in this month. The results of this stage of the analysis will provide the evidence on which months have seasonal patterns in the United Kingdom stock market.

The Performance of the APT when Seasonal Patterns are Removed from the Data

Having determined which months have seasonal patterns in the United Kingdom, and whether or not they are part of the risk return structure, we turn to examining the performance of the APT when these months are omitted simultaneously. This step involves repeating the cross sectional regression (6.1) in order to provide the adjusted R^2 and level of mispricing for the APT when all seasonal months are removed from the data and comparing them to the corresponding results when all months are included.

Relating Seasonal Patterns in Stock Returns to Shocks in Macroeconomic Factors

The above three steps are important in addressing the issues of determining whether the seasonal patterns in the United Kingdom are part of the risk return structure, and determining the performance of the APT when the seasonal months are omitted as compared to when they are included. If we do have seasonal patterns in the risk return structure then it should be possible to relate these seasonal patterns to the estimated factors in the APT. We attempt to relate the seasonal patterns in the risk return structure by omitting the seasonal months from the data individually and reestimating the APT. By comparing the estimated risk premium from the APT with all months included, and the APT with the seasonal month omitted we can determine the cause of the seasonality in the risk return structure.

6.4 EMPIRICAL RESULTS

The APT

To recap briefly, the results of the previous chapter regarding the performance of the APT, in terms of the adjusted R^2 from the cross sectional regression of actual returns on expected returns, and the general measure of mispricing are provided in table 6.1. The model seems quite capable of explaining the cross section of average returns, the adjusted R^2 is 71.94% and the measure of monthly mispricing is 0.12%. We now turn to assessing whether or not there are seasonal patterns in the United Kingdom stock market.

Tests for Seasonalities

In this section we report results from testing for seasonal patterns in stock returns and examine whether in this sample these seasonal patterns are present in stock returns when we model the risk return relationship using the APT. Table 6.2 presents the results from testing the restrictions that the dummy variables, included individually as explanatory factors in the APT model, are zero. The likelihood ratio test resoundingly accepts the null hypothesis that the dummy variables, in each month, have no statistically significant effect on the returns of the system of stocks[17]. We now turn to examining whether the seasonal patterns are part of the risk return structure or, alternatively, the seasonal patterns observed in previous empirical work are not present in the sample period used in this study. Restricting the estimates of the risk premia,

---

[17] To assess the power of the test using the dummy variable we also omitted the seasonal month from the data and estimated the cross sectional regression (6.1). An adjusted R^2 from this regression greater than from the regression with all months data included would suggest that the model is not fully capable of picking up the seasonalities and the dummy variable does not capture this seasonality. The results from estimating (6.1) with each seasonal month omitted individually provide adjusted R^2's of around 80%. This indicates that the model with all months included can be improved upon by omitting months with seasonal patterns. However, the extent of this appears to be small and the following results regarding the existence of seasonal patterns in the risk return structure confirm this.

Table 6.1

The Performance of the APT When All months are Included in the Data: Cross Sectional Adjusted R^2 and Measures of Mispricing

Adjusted R^2	Monthly Mispricing	Annual Mispricing
71.94%	0.12%	1.45%

Table 6.2

Likelihood Ratio Tests for the Significance of the Seasonal Dummy Variables in the APT Specification

Restriction	Calculated Value	Critical Value*
January	9.676	χ^2 (70) ≈ 90.531
April	1.879	χ^2 (70) ≈ 90.531
September	1.232	χ^2 (70) ≈ 90.531
December	1.012	χ^2 (70) ≈ 90.531

Notes: Sample period 1980M1 - 1993m12. Source : Datastream

Table 6.3

Likelihood Ratio Tests for Seasonality in the
Risk Return Structure of the APT

Restriction	Calculated Value	Critical Value*
January	40.86	χ^2 (3) = 7.815
April	29.42	χ^2 (3) = 7.815
September	1.691	χ^2 (3) = 7.815
December	17.18	χ^2 (3) = 7.815

Notes: Sample period 1980M1 - 1993m12. Source : Datastream

when the relevant month is omitted from the data, to be the same as the risk premia when all months are included provides a test of whether or not the seasonality is part of the risk-return structure. The results from these tests are provided in table 6.3 above. With respect to September, we find that it is possible to restrict the estimated risk premia to be the same when September's data is omitted from the data as the risk premia from the model when all months are included. Consequently, we do not find a role for a September seasonal in this data set, either as part of, or separate from the risk-return structure. Turning to the other three months under investigation, the results in table 6.3 indicate that the risk return structure is different when these months are omitted from the data.

The Performance of the APT when Seasonal Patterns are Removed From the Data

The analysis so far seems to imply that the observed seasonalities in January, April and December are part of the risk-return relationship. An important consideration that needs to be addressed now is how well does the APT perform if we omit all three seasonal months from the data set. It is important in terms of asset pricing theory that our model can still account for the movements in risky assets when all the seasonal months are left out of the data set. Table 6.4 reports the estimated risk premium for the APT when all 3 seasonal months are omitted from the data. While it is important to note that all three factors remain statistically significant, the estimated sign on the money supply factor changes.

In terms of comparative performance with the APT model with all 12 months included we consider the results from the cross sectional regression of average excess returns on predicted expected excess returns. The adjusted R^2 from regressing actual average returns with the three months omitted on the expected returns with the three months omitted is 80.24%. It is quite clear that there still exists a strong positive relationship between the actual returns and the expected returns. This is a slight improvement over the same result when all data is included in the sample. To consider whether this matters or not we again look at the mispricing when these months are omitted. The estimate of α_0 is 0.14% per month, or 1.69% per anum when all three months are omitted from the data. There is an increase in the adjusted R^2 when the three seasonal months are omitted from the data, which reflects the greater accuracy of the 3 factors in measuring non-seasonal months returns. However, importantly the APT still has empirical content, both when seasonal months are included, and seasonal months are excluded. The interesting issue that remains is relating the seasonal patterns in stock returns to shocks in the macroeconomic factors.

Table 6.4

Regression of the System of Seventy Stock Returns on the Factors using NL3SLS When January, April and December are Omitted From the Data Simultaneously

Risk Premium	Estimate	T - Ratio
λ_5	-0.00128	-2.46^{**}
λ_6	-0.00126	-2.75^{***}
λ_{10}	0.04075	4.90^{***}

Notes: Sample period 1980M1 - 1993m12. Source : Datastream
* denotes significant at 10%
** denotes significant at 5%
*** denotes significant at 1%

Relating Seasonal Patterns in Stock Returns to Shocks in Macroeconomic Variables

While we have shown that the APT is quite capable of explaining seasonal patterns in stock returns, and moreover, the omission of the seasonal months from the data set does not effect the empirical performance of the APT in terms of being able to describe the cross section of average returns, it is interesting to try and relate the seasonal months to the estimated factors. To investigate this we present the results of the estimated risk premia, in table 6.5, when January is omitted from the data (panel A), when April is omitted from the data (panel B), and when December is omitted from the data (panel C). The results from table 6.5 indicate that the seasonal patterns in stock returns are related to shocks in the money supply and inflation. Considering December, this month is important regarding levels of output, sales and restocking for subsequent months in the year. For example, large sales in the Christmas period would imply higher output levels and restocking in following months. Thus, November inflation figures, which are announced in December provide important information regarding likely levels of sales and output activity over the Christmas period. The absence of December from the data leads to an increase in average returns to 0.133%, which implies that in the absence of December investors require a higher rate of return for the perceived risk of not having the information contained in the shocks to inflation. This is readily apparent when we consider the fact that when December is omitted the risk premium on unexpected inflation becomes positive. With respect to omitting January, average returns fall to 0.05543%

Table 6.5

Regression of the System of Seventy Stock Returns on the Factors using NL3SLS When January, April and December are Omitted From the Data Individually
(Panel A: January Omitted, Panel B: April Omitted, Panel C: December Omitted)

Risk Premium	Estimate	T - Ratio
	PANEL A	
λ_5	0.00073	1.02
λ_6	-0.00004	-0.08
λ_{10}	0.06732	4.58^{***}
	PANEL B	
λ_5	0.00076	0.80
λ_6	-0.00328	-1.86^{*}
λ_{10}	0.10521	2.89^{***}
	PANEL C	
λ_5	0.00242	3.17^{***}
λ_6	0.00213	2.35^{**}
λ_{10}	0.06112	4.22^{***}

Notes: Sample period 1980M1 - 1993m12. Source : Datastream
* denotes significant at 10%
** denotes significant at 5%
*** denotes significant at 1%

and the risk premium on unexpected money supply and inflation becomes insignificant. This implies that when January is included there is an increase in risk associated with the announcement of December's money supply and inflation figures. Once again both these factors measure the yearly business cycle, and in particular, the performance of the economy in the important Christmas period and the consequent effect for subsequent months production levels. Shocks to theses factors cause an increase in risk. Thus by omitting them from the data this risk is avoided and hence returns are smaller.

Finally, April is the month when the United Kingdom government announces it expenditure and income plans for the coming year and is also the end of the UK tax year. The possibility of changes in government policy regarding, for example, tax levels and changes in monetary and fiscal policy will have an effect on factors which measure the expected levels of these changes. In addition, the possibility of large tax payments by companies can effect the future cash flows of firms. Given that the money supply is sensitive to changes in tax levels and monetary policy, then a shock in April to the money supply figures reflects the risk associated with expectations of changes in government policy in April. Consequently, when April is omitted there is a small fall in average returns to 0.0969% and the risk premium on money supply becomes insignificant.

6.5 Summary And Conclusion

In this chapter we have attempted to relate the shocks to macroeconomic factors to seasonal behaviour in stock returns. It seems, then, that the seasonalities are part of the risk return structure and their behaviour can be linked to the importance of the prices of risk at different important points in the yearly business cycle. We have found that there are significant seasonal patterns in the months of January, April and December. Modelling the risk return structure using a three factor APT we find that these seasonal patterns are captured by the risk return relationship. Importantly, as far as asset pricing theory is concerned, we also find that the three factor APT still has empirical content when these months are omitted from the data individually and at the same time. Moreover, we present arguments that relate the seasonal months to shocks to factors that provide information regarding the future performance of the economy which is a direct result of the important economic activity around the Christmas period.

This is an important contribution, both to the EMH literature and the asset pricing literature, given previous results which indicate either the asset pricing model can not explain the seasonal patterns of stock returns, or alternatively when the asset pricing model does capture seasonal patterns in returns for the other months in the year the asset pricing model has no empirical content. The important message seems to be that the tests of observed regularities in financial markets are subject to the joint hypothesis which includes the asset pricing model being correctly specified. Our results would seem to indicate that when the asset pricing model is correctly specified then the apparent seasonal anomalies disappear and the asset pricing model still has empirical content outside of these months.

CHAPTER 7

CONCLUSION

7.1 SUMMARY OF FINDINGS

The importance of asset pricing models to our understanding of the nature of price movements in asset markets can not be overstated. They have implications for the theory of valuation, for the ultimate investment and consumption decisions of economic agents as well as implications for portfolio management. While the theoretical literature on asset pricing models has expanded and provided numerous insights into the determination of the returns on risky assets the empirical literature has lagged somewhat behind these developments. This thesis has sought to investigate the empirical content of the Arbitrage Pricing Theory (APT). As described in chapter 1, extant tests of the APT have led to generally mixed results regarding the empirical performance of the model. In particular, the results to date offer, at most, a vague idea of the relationship between the returns and systematic risk factors. Chapter 1 concluded that any test of the APT should involve a systematic testing methodology which in turn needs to consider a number of issues before any thorough investigation into the empirical content of the APT can ensue. The remainder of the thesis identified and analysed these issues and then proceeded to consider the empirical validity of the APT and then finally apply this model to an issue regarding the Efficient Markets Hypothesis.

Specifically, this thesis assessed the form of the factor structure which we showed had implications for the estimation of an asset pricing model. Having identified the appropriate form of the factor structure we then considered the issue of the appropriate factors and the role of the market portfolio and the CAPM in tests of the APT. The final issue we considered as a precursor to testing the empirical validity of the APT was the generation of unexpected components of the factors. This involved an analysis of the assumed form of the expectations generating process that agents use regarding the future values of variables that we specify as candidates for systematic risk factors. Having investigated these crucial issues we then turned to assessing the empirical validity of the APT. Identifying the issues of uniqueness and stability as the crucial concepts in testing the APT we carry out tests in order to determine whether the APT does have any empirical content. Finally, we consider an issue of market efficiency which involves the implementation of the empirical asset pricing model developed throughout earlier chapters.

The general finding of the thesis is that the issues identified in earlier chapters are crucial to the empirical results obtained from testing the APT. Using these results we find that the APT does have empirical content and that once the APT is correctly specified previously documented market anomalies disappear while at the same time the APT retains empirical content. Notably, in chapter 2 we identified that the issue of whether returns are assumed to follow a strict or an approximate factor model has implications for the form of estimation technique employed. The crucial issue with regard to testing the APT is whether we have an approximate or a strict factor model. The evidence in chapter 2 indicates that inferences are different under the different factor structures and consequently the assumed form of the factor structure does matter. The evidence we provide regarding the characteristics of the idiosyncratic return covariance matrix suggests that returns have an approximate factor structure. When the APT is estimated from an approximated factor model framework we find six significant risk factors, while from a strict factor model framework their is only one factor which is marginally significant. These results support the findings of Shanken and Weinstein (1990) who show that once the Errors in Variables problem is corrected for then the number of factors found significant in the Chen, Roll, and Ross (1986) methodology (a strict factor model) reduces dramatically.

Having established the form of the factor structure the next issue we consider, in chapter 3, is the relative performance of the CAPM and the APT. Overall we find that the APT is superior to the CAPM both in terms of tests on each individual model and on comparative tests. Thus, proceeding with our analysis of the APT from an approximate factor framework in chapter 4 we considered the issue of generating unexpected components in the variables that we postulate as candidates for systematic risk factors. Having established, through the results of chapters 2 and 3, that the APT is sensitive to the specification of the unexpected components we examined the validity of alternative methods of generating the unexpected components. We showed that the extant techniques used for this purpose are not robust to a number of assumptions we make regarding the form of the unexpected components and the expectations generating process. As an alternative we put forth the use of a learning scheme for expectations generation in order to generate the unexpected components. This produces expectations and unexpected components which satisfy our prior assumptions.

Having considered the above issue which establishes a framework within which we can test asset pricing models we proceed in chapter 5 to test the empirical validity of the APT. We found that the return generating process is unique and that, through the use of recursive estimation, the estimated risk premia are stable over time. Furthermore, in this chapter we showed that the residuals from the econometric model are robust to the assumptions of homoscedasticity and no serial correlation. Consequently we are able to make valid inferences regarding the parameter estimates and their statistical significance. We find that a three factor APT, where the factors are unexpected money supply, unexpected inflation and the return on the market portfolio, provides an excellent description of the return generating process of UK stocks.

Armed with this empirically robust three factor APT we then turned to examining the issue of observed seasonal patterns in stock returns. Tests of market anomalies rely on the joint hypothesis of the Efficient Markets Hypothesis and the validity of asset pricing model used to control for risk. Consequently rejection of the former may be due to misspecification of the latter. Our results of the analysis of previously documented seasonal patterns in the UK stock market confirm that there are seasonal patterns in the months of January, April, and December. These seasonal patterns are captured in the risk return relationship thus ruling out the existence of evidence against the Efficient Markets Hypothesis with respect to monthly seasonality. In addition to this we also find that the APT has empirical content when the seasonal patterns are removed from the data. These results reconcile the conflicting findings of those who find that the asset pricing model can not account for the seasonal patterns and those who find that they can account for them in the risk return structure but the asset pricing model retains no empirical content once the seasonal patterns are removed from the data.

7.2 Policy Implications And Directions For Future Research

The overall aim of this thesis has been to arrive at an empirically valid asset pricing model. The justification for this stems from the relationship asset pricing models have with decision making processes of agents. An empirically invalid asset pricing model can lead to policy decisions which may well have contrarian prescriptions. However, we have shown that the APT does have empirical content and we have addressed issues which have cast doubt on the empirical validity of the APT. Consequently, the APT should succeed the CAPM as the means of measuring the returns on risky assets. This has implications for a number of issues. For example, firms should use the APT required rate of return as the cost of capital in their net present value calculations. Furthermore, on a related issue, the calculation of fair rates of return and discount rates by regulators of utilities has received increased attention in recent years. Although the CAPM offers an easy and intuitive measure the expected return from a risky asset, regulators should not sacrifice accuracy for simplicity. The APT should come to the forefront in such work.

Further developments with respect to research should concentrate on increasing the size of the data set to allow for a larger cross section. This would improve the reliability of the cross sectional results presented in chapter 5. It would be interesting to compare the perforce of the APT with the models in the spirit of Fama and French (1988) who specify factors such as book-to-market value, firm size, dividend yield and price earnings ratios. In addition, including measures of the volatility of risk factors and extending this analysis to an intertemporal framework could provide an interesting avenue for future research. With regard to the results presented in chapter 6 it would be interesting to examine the reaction of stock prices to the announcements of inflation and money supply figures on a daily basis, in particular it should be possible to relate the monthly seasonal patterns to one day in each month.

On a more general level, having found an empirical valid model which is robust opens up a plethora of research interests. For example, market efficiency can be reexamined within an APT framework as well as examining issue such as the performance of managed funds.

Tow further issues require comment. The use of price changes as opposed to total returns and the issue of survivorship bias should be addressed in future research. This is especially important, having provided a testing methodology for the APT, if we wish to analyse issues such as the cost of capital and specific estimates of risk factors. It may be possible that the above two issues will effect the point estimates of the risk factors and consequently the calculated risk premia.

BIBLIOGRAPHY

Abeysekera, Sarath and Arvind Mahajan (1987). 'A test of the APT in pricing UK stocks.' *Journal of Business Finance and Accounting* 14, 377-391.

Amemiya, Takeshi (1977). 'The maximum likelihood and the Nonlinear Three-Stage Least Squares estimator in the general nonlinear simultaneous equation model.' *Econometrica* 45, 955-968.

Banz, R.W. (1981). 'The relationship between return and the market value of common stocks.' *Journal of Financial Economics* 10, 3-18.

Basu, S. (1977). 'Investment performance of common stocks in relation to their price-earnings ratios: a test of the efficient market hypothesis.'*Journal of Finance* 663-682.

Beenstock, Michael and K.F. Chan (1986). 'Testing the arbitrage pricing theory in the UK.' *Oxford Bulletin of Economics and Statistics* 48, 121-141.

Beenstock, Michael and K.F. Chan (1988). 'Economic forces in the London stock market.' *Oxford Bulletin of Economics and Statistics* 50, 27-39.

Black, Fisher, Micheal C. Jensen, and Myron Scholes (1972). 'The capital asset pricing model: some empirical tests'. In *Studies in the Theory of Capital Markets.* Ed. by Micheal C. Jensen. New York: Praeger.

Blume, M. (1970). 'Portfolio theory: a step towards its practical application' *Journal of Business* 152-173.

Branch, B. (1977). 'A tax loss trading rule.' *Journal of Business* 50, 198-207.

Douglas, T (1979). 'An Intertemporal Asset Pricing Model with StochasticConsumption and Investment Opportunities.' *Journal of Financial Economics* 7, 265-296.

Breedon, Douglas T and R.H. Litzenberger (1978). 'Prices of state contingent claims implicit in option prices.' *Journal of Business* 51, 621-651.

Brown, Stephen (1989), 'The number of factors in security returns.' *Journal of Finance* 44, 1247-1262.

Brown, Stephen and Mark I. Weinstein (1983). 'A new approach to testing asset pricing models: the bilinear paradigm'. *Journal of Financial Economics* 14, 711-743.

Burmeister, Edwin, Kent D. Wall, and James D. Hamilton (1986). 'Estimation of unobserved expected monthly inflation using Kalman filtering' *Journal of Business and Economic Statistics* 4, 147-160.

Burmeister, Edwin and Marjorie B. McElroy (1988). 'Joint estimation of factor sensitivities and risk premia for the Arbitrage Pricing Theory.' *Journal of Finance* 43, 721-735.

Chamberlain, Gary (1983), 'Funds, factors and diversification in arbitrage pricing models.' *Econometrica* 51, 1283-1300.

Chamberlain, Gary and Michael Rothschild (1983). 'Arbitrage and mean variance analysis on large asset markets.' *Econometrica* 51, 1281-1304.

Chan, K.C. (1986). 'Can tax loss selling explain the January seasonal in stock returns ?.' *Journal of Finance*, 1115-1128.

Chan, K.C, Nai-Fu Chen, and David A. Hsieh (1985). 'An exploratory investigation of the firm size effect.' *Journal of Financial Economics* 14, 451-471.

Chen, Nai-Fu (1983). 'Some empirical tests of the Arbitrage Pricing Theory.' *Journal of Finance* 38, 1393-1414.

Chen, Nai-Fu, Richard Roll and Stephen A. Ross (1986). 'Economic forces and the stock market.' *Journal of Business* 59, 383-403.

Cho, D. Chinhyung (1984). 'On testing the arbitrage pricing theory: inter-battery factor analysis.' *Journal of Finance* 39, 1485-1502.

Chou, Ray Y. (1988). 'Volatility persistence and stock valuation: some empirical evidence using GARCH.' *Journal of Applied Econometrics* 3, 279-294.

Clare, Andrew C. and Stephen H. Thomas (1994). 'Macroeconomic factors, the APT and the UK stockmarket.' *Journal of Business Finance and Accounting* 21, 309-330.

Clare, Andrew C., Zacharis Psaradakis, and Stephen H. Thomas (1994). 'An analysis of seasonality in the United Kingdom equity market.' Forthcoming *Economic Journal*, March 1995.

Connor, G. (1984). 'A Unified Beta Pricing Theory.' *Journal of Economic Theory* 34, 13-31.

Connor, Gregory and Robert A. Korajczyk (1986). 'Performance measurement with the Arbitrage Pricing Theory : a new framework for analysis.' *Journal of Financial Economics* 15, 373-394.

Connor, Gregory and Robert A. Korajczyk (1988). 'Risk and return in an equilibrium APT: application of a new test methodology.' *Journal of Financial Economics* 21, 255-289.

Connor, Gregory and Robert A. Korajczyk (1992). 'The Arbitrage Pricing Theory and multifactor models of asset returns.' *LSE Financial Markets Group Discussion Paper No. 149,* London School of Economics, forthcoming in Robert Jarrow, Vojislav Maksimovic and William Ziemba, eds.: *Finance Handbook.*

Connor, Gregory and Robert A. Korajczyk (1993) 'A test for the number of factors in an approximate factor model.' *Journal of Finance* 48, 1263-1291.

Constantinides, George M. (1989) 'Theory of Valuation : Overview and Recent Developments,' in *Theory of Valuation : Frontiers of Modern Financial Theory, Volume 1* (eds. G.M. Constantinides and S. Bhattacharya), New Jersey : Rowman and Littlefield.

Corhay, Albert, Gabriel Hawawini, and Pierre Michel (1987). 'Seasonality in the risk return relationship: some international evidence.' *Journal of Finance* 42, 49-68.

Cuthbertson, Keith (1988). 'Expectations, learning and the Kalman Filter.' *The Manchester School* 56, 223-245.

Demos, Antonis, Enrique Sentana, and Mushtaq Shah (1993). 'Risk and return in January: some United Kingdom evidence.' *LSE Financial Markets Group Discussion Paper* No. 162.

Dhrymes, Phoebus J., Irwin Friend and N. Bulent Gultekin (1984) 'A critical reexamination of the empirical evidence on the Arbitrage Pricing Theory.' *Journal of Finance* 39, 347-350.

Drhymes, Phoebus J., Irwin Friend, Mustafa N. Gultekin, and N. Bulent Gultekin (1985). 'New tests of the APT and their implications.' *Journal of Finance* 40, 659-674.

Dybvig, Philip H. (1983). 'An explicit bound on deviations from APT pricing in a finite economy.' *Journal of Financial Economics* 12, 483-496.

Dybvig, Philip H. and Stephen A. Ross (1985). 'Yes, the APT is testable.' *Journal of Finance* 40, 1173 1188.

Fama, Eugene F. (1976). *Foundations in Finance*. New York, Basic books.

Fama, Eugene F. (1991). 'Efficient capital markets : II.' *Journal of Finance* 46, 1575-1617.

Fama, Eugene F. and James D. MacBeth (1973). 'Risk, return and equilibrium: empirical tests.' *Journal of Political Economy* 71, 607-636.

Fama, Eugene F. and Micheal R. Gibbons (1984). 'A comparison of inflation forecasts'. *Journal of Monetary Economics* 14, 319-338.

Fama, Eugene F. and Kenneth R. French (1988). 'Dividend Yields and Expected Stock Returns.' *Journal of Financial Economics* 22, 3-25.

Fama, Eugene F. and Kenneth R. French (1992). 'The Cross-section of Expected Returns.' *Journal of Finance* 47, 427-465.

Ferson, W.E., and C.R. Harvey (1991). 'The variation of economic risk premiums.' *Journal of Political Economy* 99, 385-415.

Fischer S. and R. C. Merton (1985). 'Macroeconomics and finance: the role of the stock market.' in K. Brunner and A.H. Meltzer (eds.) *Essays on Macroeconomic Implications of Financial and Labor Markets and Labor Process.* Carnegie-Rochester Conference Series on Public Policy, Volume 21. Amsterdam: North-Holland

French, Kenneth R., G. William Stewart, and Robert F. Stambaugh (1987). 'Expected stock returns and volatility.' *Journal of Financial Economics* 19, 3-30.

Friedman, Benjamin M. (1979). 'Optimal expectations and the extreme information assumptions of 'rational expectations' macromodels.' *Journal of Monetary Economics* 5, 23-41.

Gibbons, Michael R. (1982). 'Multivariate tests of financial models : a new approach.' *Journal of Financial Economics* 10, 3-27.

Grinblatt, Mark and Sheridan Titman (1983). 'Factor pricing in a finite economy' *Journal of Financial Economics* 12, 497-507.

Gultekin, Mustafa N., and N. Bulent Gultekin (1987). 'Stock return anomalies and tests of the APT.' *Journal of Finance*, 42, 1213-1224.

Hall, Stephen G (1990). 'Modelling the sterling effective exchange rate using expectations and learning.' *Bank of England Discussion Papers, Technical Series* No.33

Harris, W. (1962). 'Some Rao-Guttman relationships'. *Psychometrika* 19 149-161

Harvey, Andrew (1987). 'Applications of the Kalman filter in econometrics.' in: T.F Bewely, ed., *Advances in econometrics: Fifth world congress*, Vol. 1 (Cambridge University Press).

Haugen, R.A. and J. Lakonishok (1988). *The incredible January effect: the stock markets unresolved mystery.* Dow Jones-Irwin, Homewood, Illinois.

Hubermann, Gur (1982). 'A simple approach to arbitrage pricing.' *Journal of Economic Theory* 28, 183-191.

Jorgenson, Dale W. and J. Laffont (1974). 'Efficient estimation of nonlinear simultaneous equations with additive disturbances.' *Annals of Economic and Social Measurement* 3, 615-640.

Kaiser, H.F. and S. Hunka (1973). 'Some empirical results with Guttman's stronger lower bound for the number of common factors.' *Educational and Psychological Measurement* 33, 99-102.

Keim, D.B. (1983). 'Size related anomalies and stock return seasonality: further empirical evidence.' *Journal of Financial Economics*, 12, 13-32.

Kemeta, Jan (1986). *Elements of Econometrics* 2nd Ed. New York: Macmillan Publishing Company.

Kothari S.P., Jay Shanken, and Richard G. Sloan (1994). 'Another look at the cross-section of expected returns ' *Forthcoming, Journal of Finance*

Lehmann, Bruce N. and David Modest (1988). 'The empirical foundations of the Arbitrage Pricing Theory.' *Journal of Financial Economics* 21, 213-254.

Lintner, John (1965). 'The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets.' *Review of Economics and Statistics* 47, 13-37.

Litzenberger, R. and K. Ramaswamy (1979). 'The effect of personal taxes and dividends and capital asset prices: theory and empirical evidence' *Journal of Financial Economics* 163-195.

Lo, Andrew W. and A. Craig MacKinlay (1990). 'Data-snooping biases in tests of financial asset pricing models.' *Review of Financial Studies* 3, 431-467.

Lucas, Robert E. (1976). 'Econometric policy evaluation : a critique.' *Carnegie-Rochester Conference Series on Public Policy* 1, 19-46.

Luedecke, Bernd P (1984). 'An empirical investigation into arbitrage and approximate k-factor structure on large asset markets.' *Doctoral Dissertation.* Department of Economics, University of Wisconsin.

Markowitz, H.M. (1959). *Portfolio Selection: Efficient diversification of Investments* (Coles Foundation Monograph 16). Yale University Press, New Haven.

McElroy, Marjorie B. and Edwin Burmeister (1988). 'Arbitrage Pricing Theory as a restricted nonlinear multivariate regression model : ITNLSUR estimates.' *Journal of Business and Economic Statistics* 6, 29-42.

McElroy, Marjorie B. and Edwin Burmeister and Kent D. Wall (1985). 'Two estimators for the APT model when factors are measured.' *Economic Letters* 19, 271-275.

Mei, Jianping (1993). 'A semiautoregression approach to the Arbitrage Pricing Theory.' *Journal of Finance* 48, 599-620.

Merton, Robert (1973). 'An intertemporal capital asset pricing model.' *Econometrica* 41, 867-887.

Mossin, J. (1966). 'Equilibrium in the capital asset market.' *Econometrica* 768-783.

Muth, John F. (1961). 'Rational expectations and the theory of price movements.' *Econometrica* 29, 315-355.

Ng, Victor, Robert F. Engle and Michael Rothschild (1992). 'A multi-dynamic-factor model for stock returns.' *Journal of Econometrics* 52, 245-266.

Poon, S. and Stephen J. Taylor (1991). 'Macroeconomic factors and the UK stock market.' *Journal of Business Finance and Accounting* 18, 619-636.

Raveh, Adi (1985). 'A note on factor analysis and arbitrage pricing theory' *Journal of Banking and Finance* 9, 317-321.

Reinganum, Marc (1981). 'The Arbitrage Pricing Theory: some empirical tests.' *Journal of Finance* 36, 313-322.

Ritter, J.R. and N. Chopra (1989). 'Portfolio rebalancing and the turn-of-the-year effect.' *Journal of Finance* 44, 149-166.

Roll, Richard (1977) 'A critique of asset pricing theory's tests-Part I: on past and present potential testability of the theory.' *Journal of Financial Economics* 4, 129-176.

Roll, Richard and Stephen A. Ross (1980). 'An empirical investigation of the Arbitrage Pricing Theory.' *Journal of Finance* 35, 1073-1103.

Roll, Richard and Stephen A. Ross (1984). 'A critical reexamination of the empirical evidence on the Arbitrage Pricing Theory : a reply.' *Journal of Finance* 39, 347-350.

Ross, Stephen A. (1976). 'The arbitrage theory of capital asset pricing.' *Journal of Economic Theory* 13, 341-360.

Rozef, M.S and Kinney, W.R. (1976) Capital market seasonality: the case of common stock returns.' *Journal of Financial Economics* 3, 379-402.

Rubenstein, M (1976). 'The valuation of uncertain income streams and the pricing of options.' *Bell Journal of Economics and Management Science* 7, 407-425.

Sargent, Thomas J. and Neil Wallace (1975). 'Rational expectations, the optimal monetary instrument and the optimal money supply rule.' *Journal of Political Economy* 83, 241-254.

Shanken, Jay (1982). 'The arbitrage pricing theory : is it testable?' *Journal of Finance* 37, 1129-1140.

Shanken, Jay (1985). 'Multi-beta CAPM or equilibrium APT: a reply' *Journal of Finance* 40, 1189-1196.

Shanken, Jay (1987). 'Nonsynchronous data and the covariance-factor structure of returns'. *Journal of Finance* 42, 221-231.

Shanken, Jay (1992), 'On the estimation of beta-pricing models.' *Review of Financial Studies* 5, 1-33.

Shanken, Jay and Mark Weinstein (1990). 'Macroeconomic variables and asset pricing: estimation and tests.' Working paper, University of Rochester.

Sharpe, William F. (1964). 'Capital Asset Prices : a Theory of Market Equilibrium Under Conditions of Risk.' *Journal of Finance* 19, 425-442.

Shukla, Ravi and Charles Trzcinka (1990). 'Sequential tests of the Arbitrage Pricing Theory: a comparison of principal components and maximum likelihood factors.' *Journal of Finance* 45, 1541-1564.

Tinic, S.M. and R.R West (1984) 'Risk and return: January vs. the rest of the year.' *Journal of Financial Economics* 13, 126-147.

Tinic, S.M. and R.R West (1986) 'Risk, return, and equilibrium: a revist.' *Journal of Political Economy* 94, 126-147.

Trzcinka, Charles (1986). 'On the number of factors in the arbitrage pricing model.' *Journal of Finance* 41, 347-368.

Wallis, Kenneth F (1989). 'Macroeconomic forecasting: a survey.' *Economic Journal* 99, 28-61.

Zellner, A. (1962). 'An efficient method of estimating seemingly unrelated regressions and tests of aggregation bias' *Journal of American Statistical Association* 57, 348-368.