BIOMECHANICS OF THE HUMAN SPINE

THESIS PRESENTED FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

by

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In memory of my mother.

ABSTRACT.

The spinal column as a static structure is analysed in an attempt to quantify the mechanics of the system, of particular interest has been the derivation of forces, in operation in the muscles, required to maintain the equilibrium of the spine in various positions.

Three approaches to the solution of the structural problem have been used, namely:

(a) Establishing the equations of equilibrium for the thoracic and lumbar vertebrae, involving body weight, external dead load, muscle force and the intervertebral reactions. These equations are solved using the Linear Programming technique which minimizes the total force in the system. The solution gives numeric values for the muscle forces and intervertebral reactions.

(b) An iteration technique, which derives the material properties of a structure from displacement and applied load data, is used to analyse simple element structures involving bars and beams.

(c) Using both the Linear Programming technique and a structural analysis of the spine involving

bar and beam finite elements to form a complete static model of the spine. The Linear Programming as in (a) is used in an initial upright position. The structural analysis is used to calculate the vertebral forces required to deform the spine to a deflected position. Combining the two studies gives values for the intervertebral reactions in the deformed position, these, the body weight and the dead load are input into a modified set of equations of equilibrium which are solved by Linear Programming.

The method (a) has been used to give results for forward flexion, lateral flexion and a scoliotic curve with several orthopaedic supports. The approach (c) has been used for forward flexion alone.

Acknowledgements

Firstly, I wish to extend my deepest thanks to Mr. A. L. Yettram for his guidance, support and long-suffering in supervising this work. Also I would like to thank Professor G. Jackson for allowing the research to be carried out in his Department.

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Nomenclature

The symbols used in this thesis are, in general, specified locally. However, for ease of reference a list of the more major terms will be given here.

A	Cross-sectional area
E	Young's Modulus
F	Applied force
G	Shear Modulus
I	Second moment of area, about an axis in the plane of the cross-section
J	Second moment of area, about an axis perpendicular to the plane of the cross-section. (polar moment)
L	Length of element
M	Applied moment
{F}	Vector of applied force
{D}	Vector of displacement and rotation
[A]	Diagonal matrix of the inverse of elemental strain
[DS]	Matrix relating strain to displacement
[P]	Compliance or flexibility matrix
[ST]	Stiffness matrix
[R]	Correlation matrix between the movement of one node to that of another
[RT]	Transpose of [R]
X,Y,Z	Right-handed Cartesian co-ordinate system
σ	Stress
ε	Strain

Contents

CHAPTER 1.

INTRODUCTION

1.1. The Human Spine.

The vertebral column is probably one of the most sophisticated structures ever conceived. In general use it is able to meet the complete requirement demanded of it.

It must be able to:

(a) Carry the weight of the human frame, but in itself be light and occupy a minimum of space.

(b) Be flexible and allow bending and rotation in all directions, even under heavy load.

(c) Be hollow to allow delicate nerves and blood vessels to pass through it and emerge from its sides without being damaged during movement.

(d) Support and protect the internal organs of the trunk.

(e) Function for a lifetime, from birth until old age, and also develop in step with the rest of the body.

The vertebral column also contributes to an overall aesthetically pleasing body form.

The spine forms an integral unit with the body as a structure and also in the function of individual elements, e.g. the spine supports the ribcage allowing breathing, the ribcage acts as a stiffening box to the column and provides an anchorage for muscles which control movement.

The configuration of the spine can be seen in Fig. 1. The areas labelled 1, 2, 3, and 4 relate to the distinct regions of the vertebral column.

The regions are as follows:-

(1) The sacral curvature, fixed by fusion of the vertebrae.
(2) The lumbar curvature, concave posteriorly (lordosis).
(3) The thoracic region, convex posteriorly (kyphosis).
(4) The cervical region, concave posteriorly.

These curves, which develop during the first year of life, enable the person to stand upright requiring only a small amount of muscle effort. The

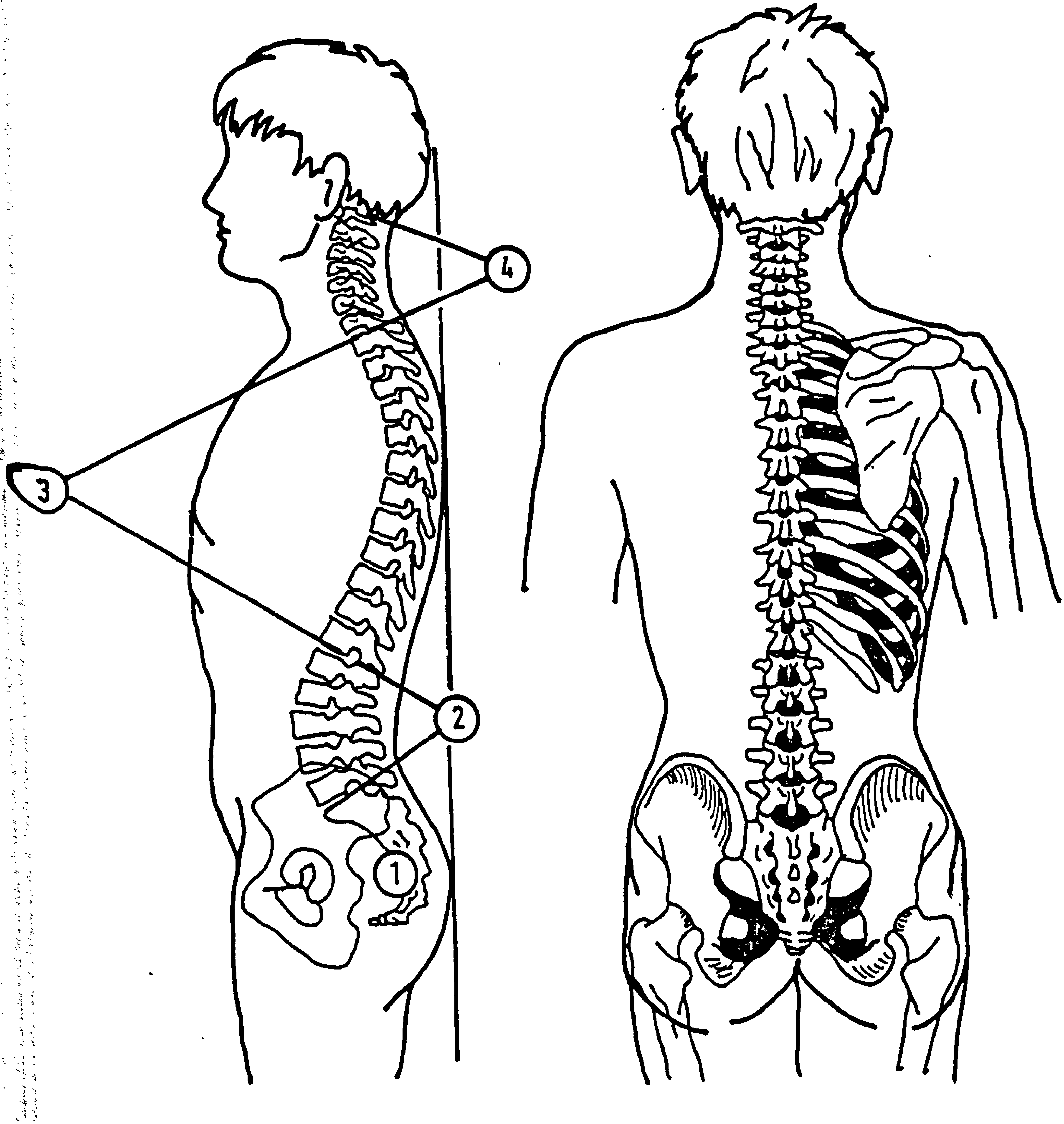


Fig.1 The Human Vertebral Column (From Kapandji[11])

centres of gravity of the various sections of the body are kept close to the centre line running through the whole structure. The curves are also able to absorb shock impulses which would otherwise be transmitted directly to the head and brain.

The vertebrae and the spinal canal will now be considered more closely. The movable section of the spine consists of twenty-four blocks of bone, five lumbar, twelve thoracic and seven cervical. They all have a common basic structure (excluding $C_1 - C_2$), although modifications occur in each region to suit their differing functions.

Each has a cylindrical structure in front called the vertebral body (1). See Fig. 2. The outer surface is of hard compact bone, but the inner core is of light honeycomb form. A cartilaginous plate is embedded in the superior and inferior surfaces of the body. A distinct rim is present on these surfaces and is formed from the epiphyseal plate which becomes fused to the body in the mid-teens. It has been suggested that the trabecular structure of the honeycomb medulla follows the lines of stress generated in the bone.[1] [2].

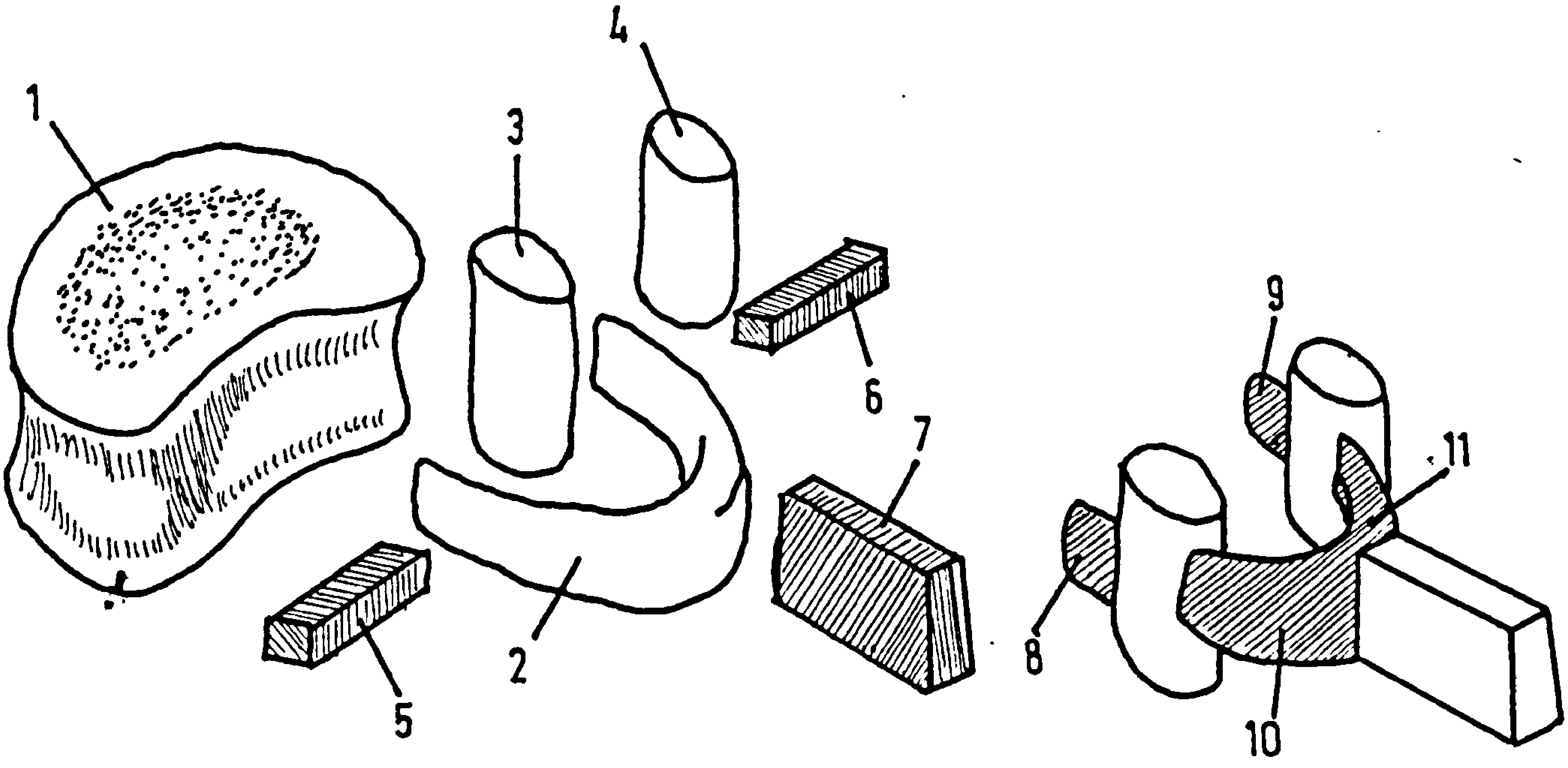


Fig. 2 Components Of A Vertebra (Kapandji [11])

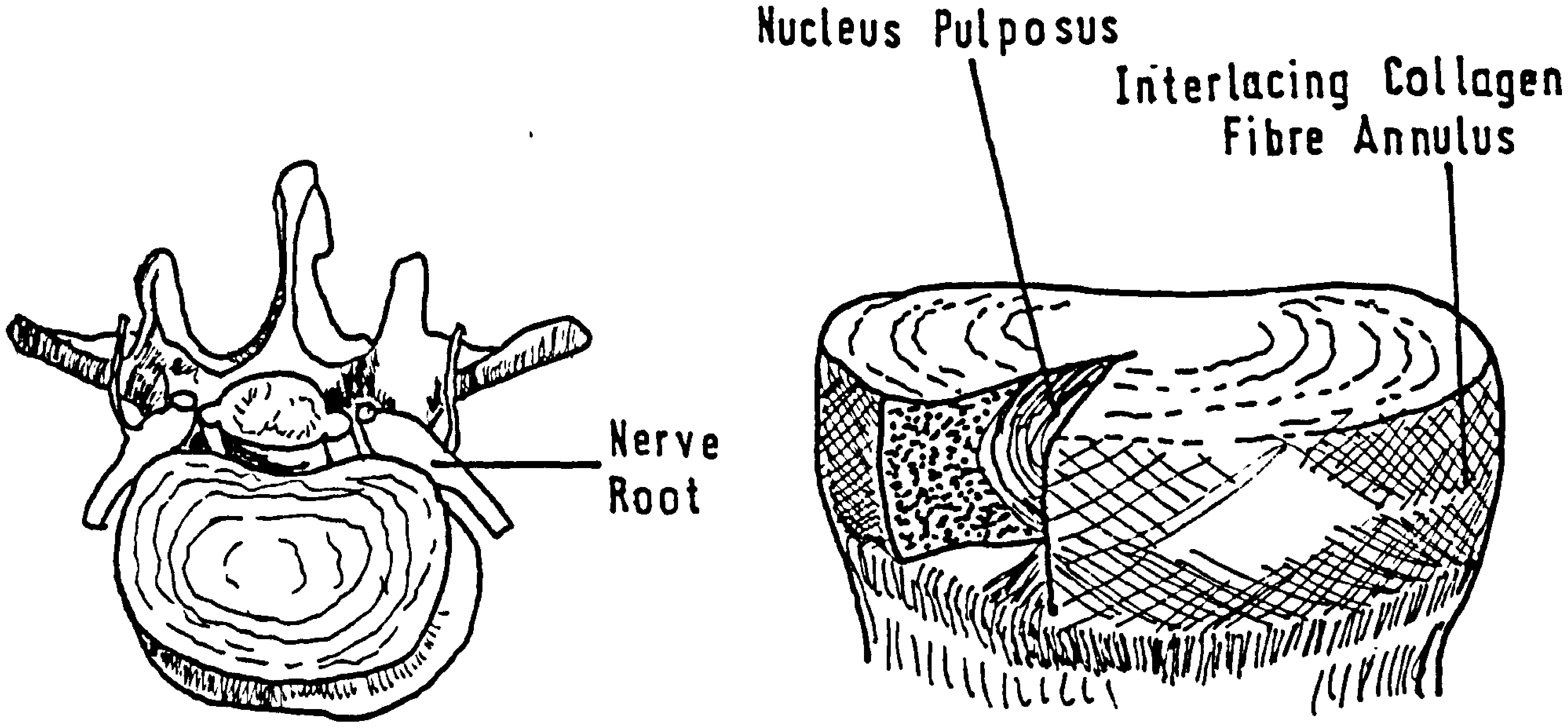


Fig. 3 A Lumber Vertebra An Intervertebral Disc

(From Jayson [3])

An arch (2) projects from the back of the vertebral body. This provides the vertebral foramen along which the spinal cord lies, the nerve roots pass out from the spinal cord through the intervertebral foramen, this is the space between the arches of adjacent vertebrae. The arch is split into three parts by the two articular processes (3 & 4), the anterior part called the pedicles (8 & 9), the posterior section the laminae (10). The spinous process is attached medially and rearward from the laminae (7). The transverse processes (5 & 6) protrude laterally from the arch near the articular processes. An assembled view of these parts is shown in Fig.. 3, and the intervertebral disc is also drawn.

Two vertebral bodies are joined by an intervertebral disc. The disc consists of two distinct regions, the central part, the nucleus pulposus, a gelatinous substance which contains up to 90% water in childhood but reduces to nearly 70% in old age, the outer part is the annulus fibrosus which is made up of interlacing bundles of collagen fibres. Collagen fibre has a tensile strength of 100-500 MPa similar to many metal wires, (Jayson) [3], it is arranged in layers in which the fibre orientation alternates between 30° and 150°

to the vertebral body ends. Thus a structure is formed which can withstand very high internal pressure but allows the spine to move easily in flexion and extension.

The adjacent vertebra are also in contact at the articular processes and form synovial joints there. These processes are orientated to transmit load and aid location of the vertebrae. Figures 4 and 5 show clearly the differences between lumbar and thoracic vertebrae respectively. The body is more substantial in the lower region, the articular facets are also heavily built. The facet orientation suggests a centre of horizontal plane rotation posterior to the articular processes, the intervertebral disc is subjected to shear in this mode of movement so the range of motion is limited. The centre of rotation for the thoracic vertebrae lies near to the centre of the disc and this permits easier turning, up to 3° per level, this is three times as great as that recorded for the lumbar vertebrae. The range of the thoraco-lumbar region in flexion and extension is 105° and 60° respectively. In lateral bending the movement amounts to 40° either side of the vertical.

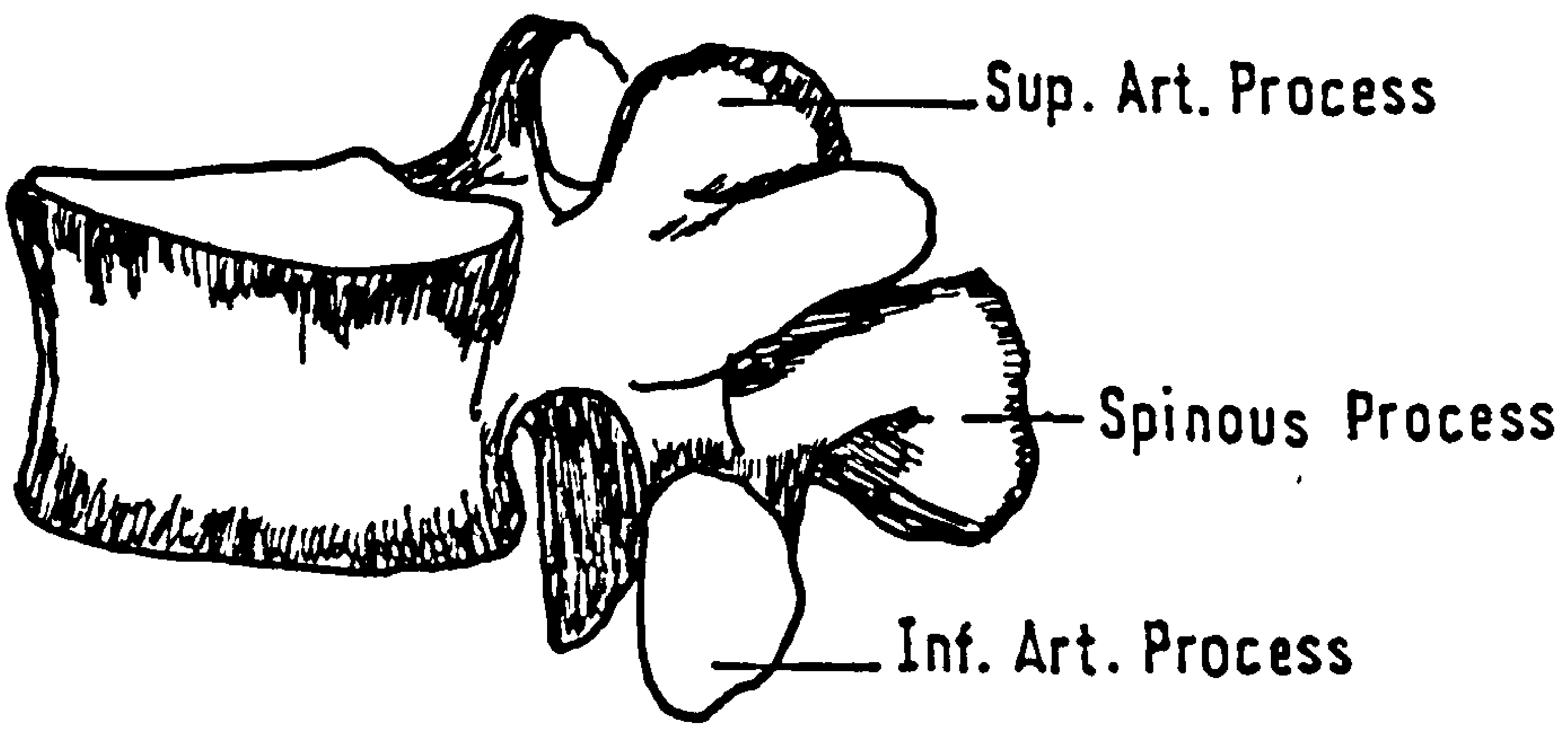
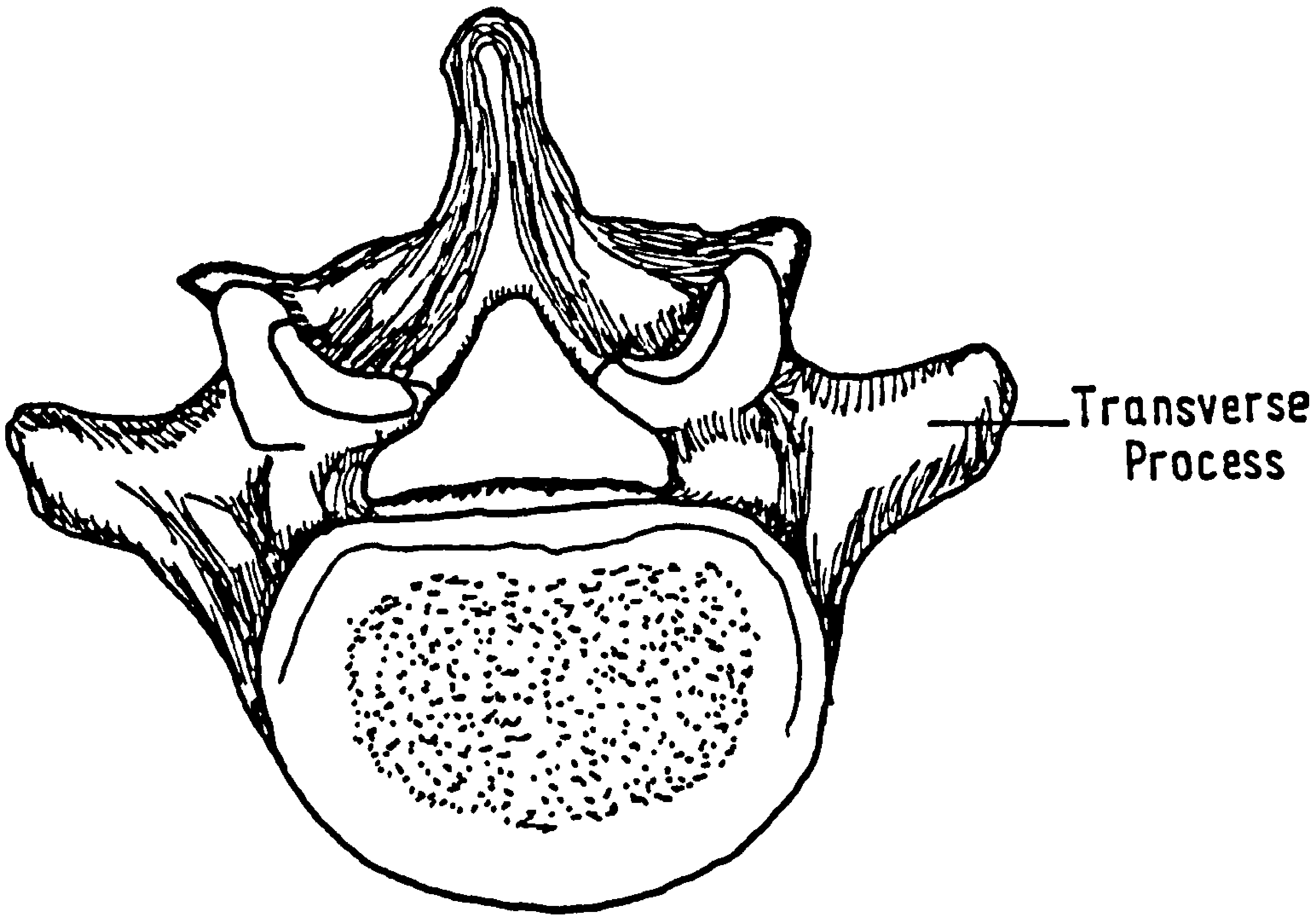
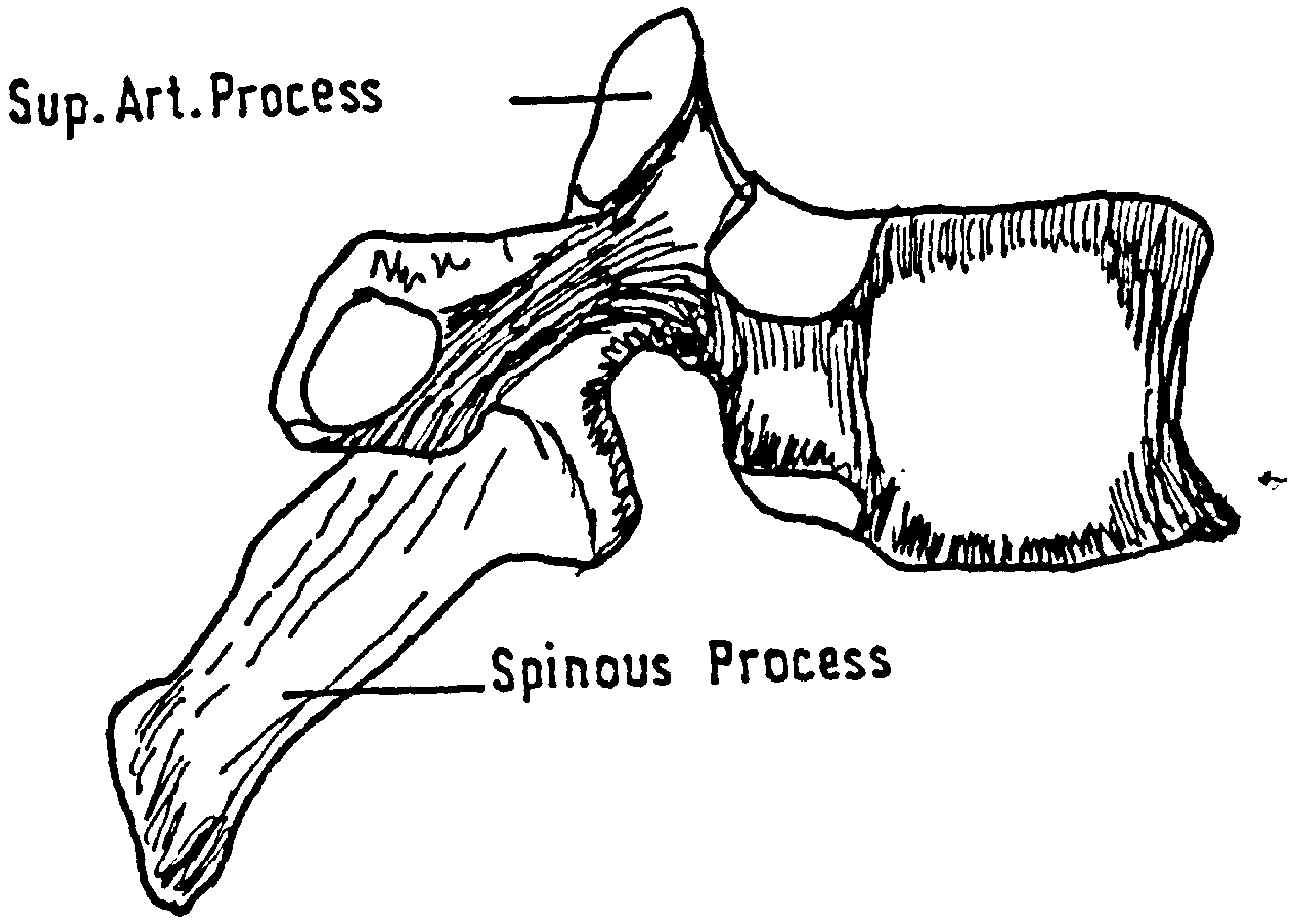


Fig. 4 A Lumbar Vertebra





·Fig.5 A Thoracic Vertebra

The ligaments join vertebrae together and are listed below. Their typical locations are shown in Fig. 6.

Anterior Longitudinal.	A strong band running down the front of the column. Attaches to the disc and lip of the vertebral body.
Posterior Longitudinal.	A strong band running down the rear side of the vertebral body. Attaches as above, but is "waisted" between the pedicles.
Ligamentum Flavum.	Well-developed and important. A broad band joining the laminae.
Interspinous Ligament.	Thin and membranous, joins the vertebrae along the length of the spinous processes.

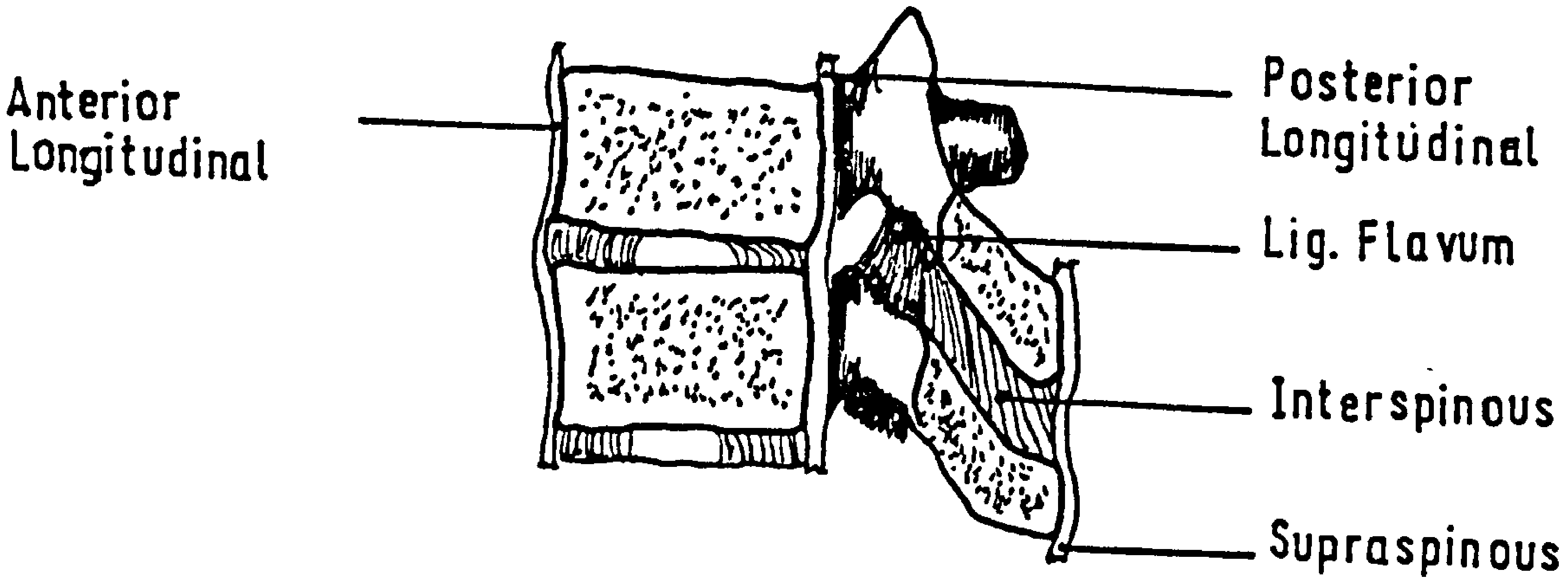


Fig. 6 The Major Ligaments

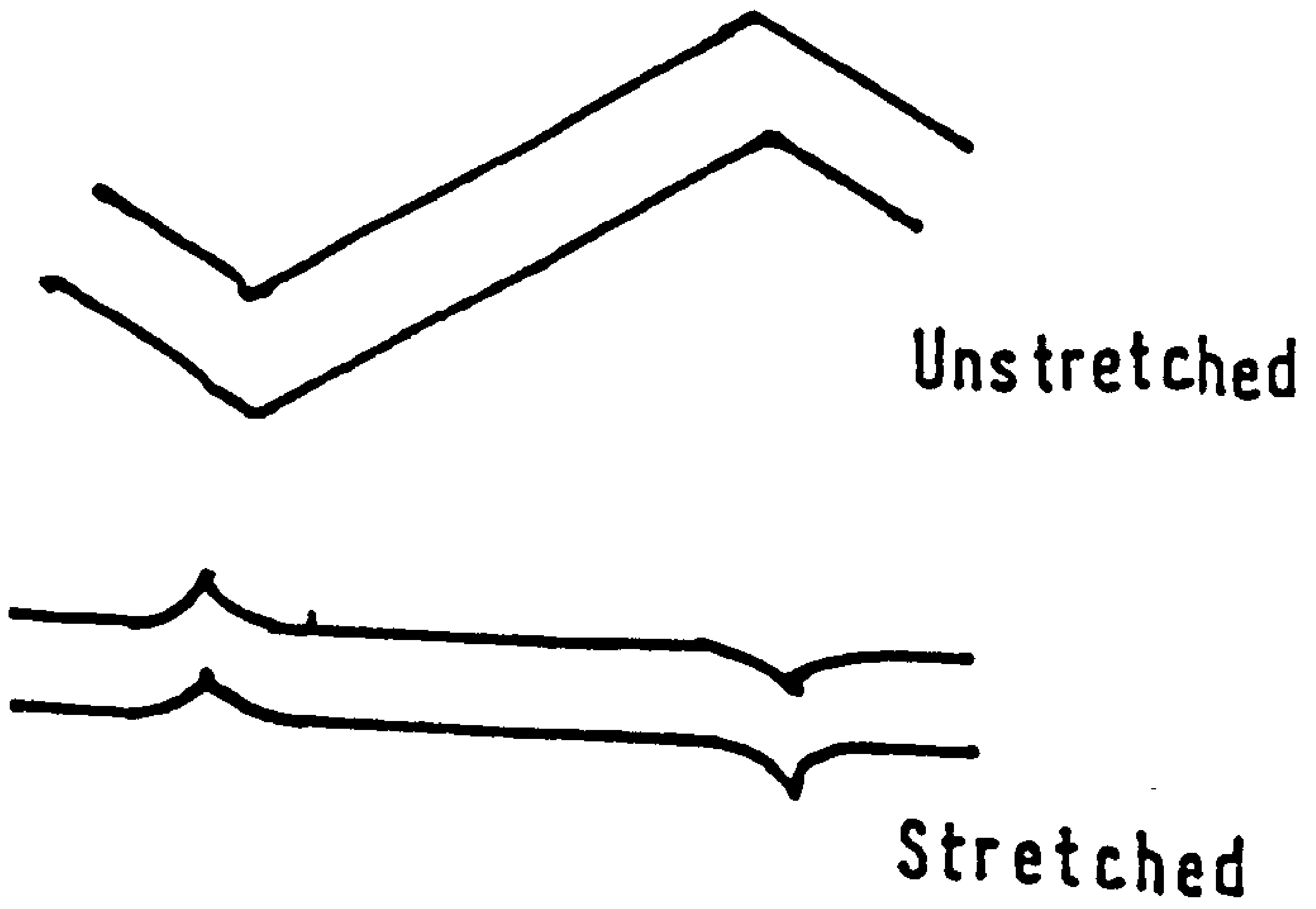


Fig. 7 Extension Of Collagen Fibre

Supraspinous Ligament.	A strong fibrous cord joining the apices of the spinous processes. Sometimes does not directly connect S_1, L_5, and L_4.
Intertransverse Ligament.	A thin band joining the transverse processes. Often part of the muscles that insert in that region.
Capsular Ligament.	A loose and relatively small ligament across the articular facet joints.

The function of the ligaments is to stabilise the spine (Steindler) [4], and to prevent excessive motion of one level relative to the next. There is a divergence of opinion as to the exact operation of the ligaments, whether they are slack or in tension with the spine in its upright position. Most researchers agree that the vertebral column is pre-stressed by the ligamentum flavum as suggested by Nachemson and Evans [5]. However, Farfan [6] maintains that the posterior ligaments i.e. inter- and supraspinous ligaments are slack and only come into play towards the end of possible motion.

Three-dimensional microscopy shows that the fibres lie in a zig-zag manner (Fig. 7), on elongation the fibres straighten out but only in the latter stages of strain does extension occur in the fibres themselves and this accounts for the marked change in the modulus of elasticity with elongation (Shah et al) [7]. The increase in stiffness of the ligaments can be seen to act as a protection to the vertebral column as extreme movement is approached.

The function of the spine as a structure and mechanism is maintained and activated by a complex system of muscles. It is interesting to note that the control of the muscles is in advance of engineering achievement e.g. modern avionics, and as yet the simulation of the nerve network and its operation has not been completely successful. The method of muscle function will not be discussed. Figures 8 and 9 show the principal muscles acting on the thoraco-lumbar spine and their origins and insertions are listed below.

(1) <u>Extensors of the spine.</u>

Iliocostalis

Lumborum: Attached by tendon to the sacrum and spinous processes of L_5 - T_{11} and iliac crest. Inserts into the inferior edges of the lower ribs. A powerful muscle.

Iliocostalis

Thoracis: By tendons it is attached to the superior edges of the lower 6 ribs. Inserts into upper 6 ribs.

Longissimus: Blends with iliocostalis at its origin, also attached to transverse processes of all lumbar vertebrae. Inserts by tendon into the transverse processes of the thoracic vertebrae, also into the angle of lower 10 ribs.

Spinalis: By tendons from spinous processes of L_2 - T_{11}. Inserts into S.P. of T_1 - T_6. A thin muscle.

Semi-Spinalis: By tendons from transverse processes of T_{12} - T_1. Inserts into spinous processes of T_7 - C_1. Stronger than the spinalis muscle.

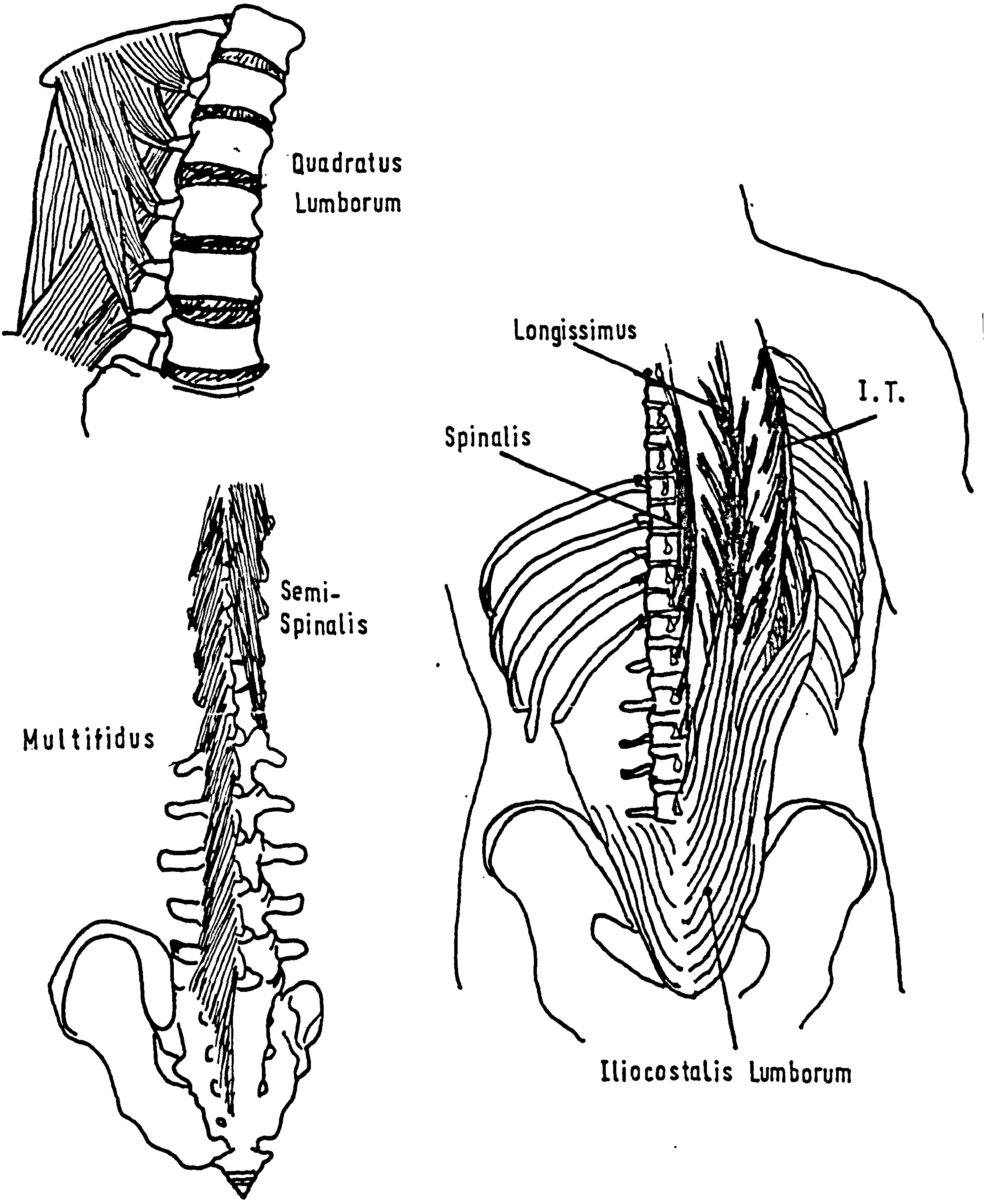


Fig. 8 Extensors Of The Spine

Multifidus: From the sacrum and transverse processes of all vertebrae. Inserts into spinous processes of 3 vertebrae above. This muscle also rotates the column.

Rotores: From transverse processes of all vertebrae to the laminae of the one above. Only rotates the spinal column.

Quadratus Lumborum:

A quadrilateral sheet running between the lowest rib, iliac crest and lumbar transverse processes. Flexes the trunk ipsilaterally.

(2) Flexors of the spine.

Rectus Abdominus:

Two muscles sheets in the abdominal wall on either side of the midline. Attached to the front of the pelvis. Inserts into the costal cartilage of 5th, 6th and 7th ribs.

Transversalis: The deepest layer of lateral abdominal muscles. Forms a complete sheath.

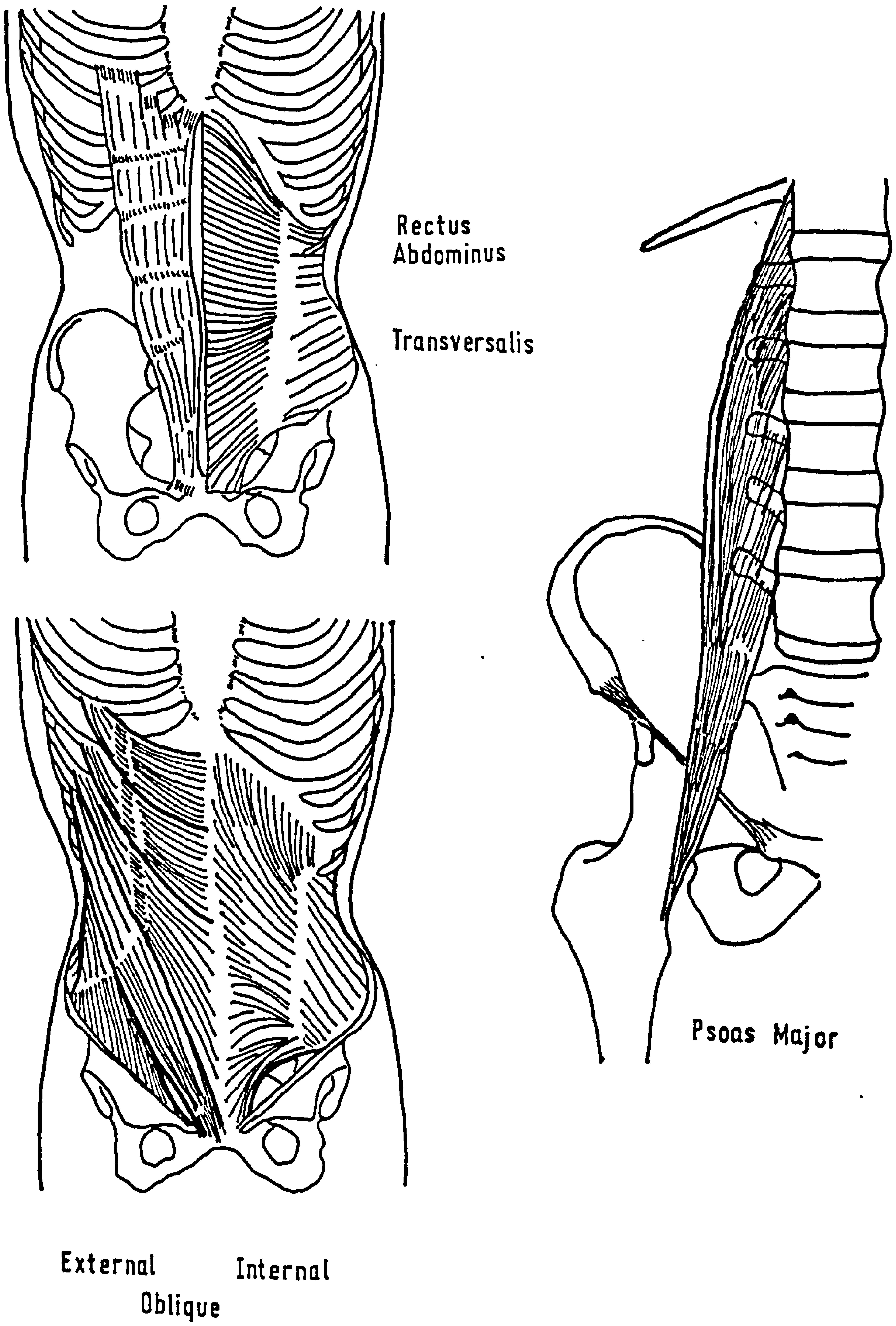


Fig. 9 Flexors Of The Spine

Internal Oblique:	A thin sheet of muscle.
External Oblique:	A stronger sheet running at approx. 90° to the internal oblique.
Psoas Major:	Originates from the vertebral bodies of $L_5 - L_1$. Inserts into the lesser trochanter.

The internal oblique muscle has been found to be active in forward bending and a possible reason for this has been suggested by Robertson [8]. The muscle acts so as to stabilise the pelvis and rotate it in the direction shown in Fig. 10. A similar effect also occurs at the lower thoracic level, and this action can be seen to flatten the lumbar curvature which is essential in a lifting exercise.

It is worth noting that although the abdominal muscles act as flexors of the spine they also assist in alleviating the compressive load on the lumbar vertebrae during lifting. These muscles enable a pressure (above atmospheric) to be maintained in the

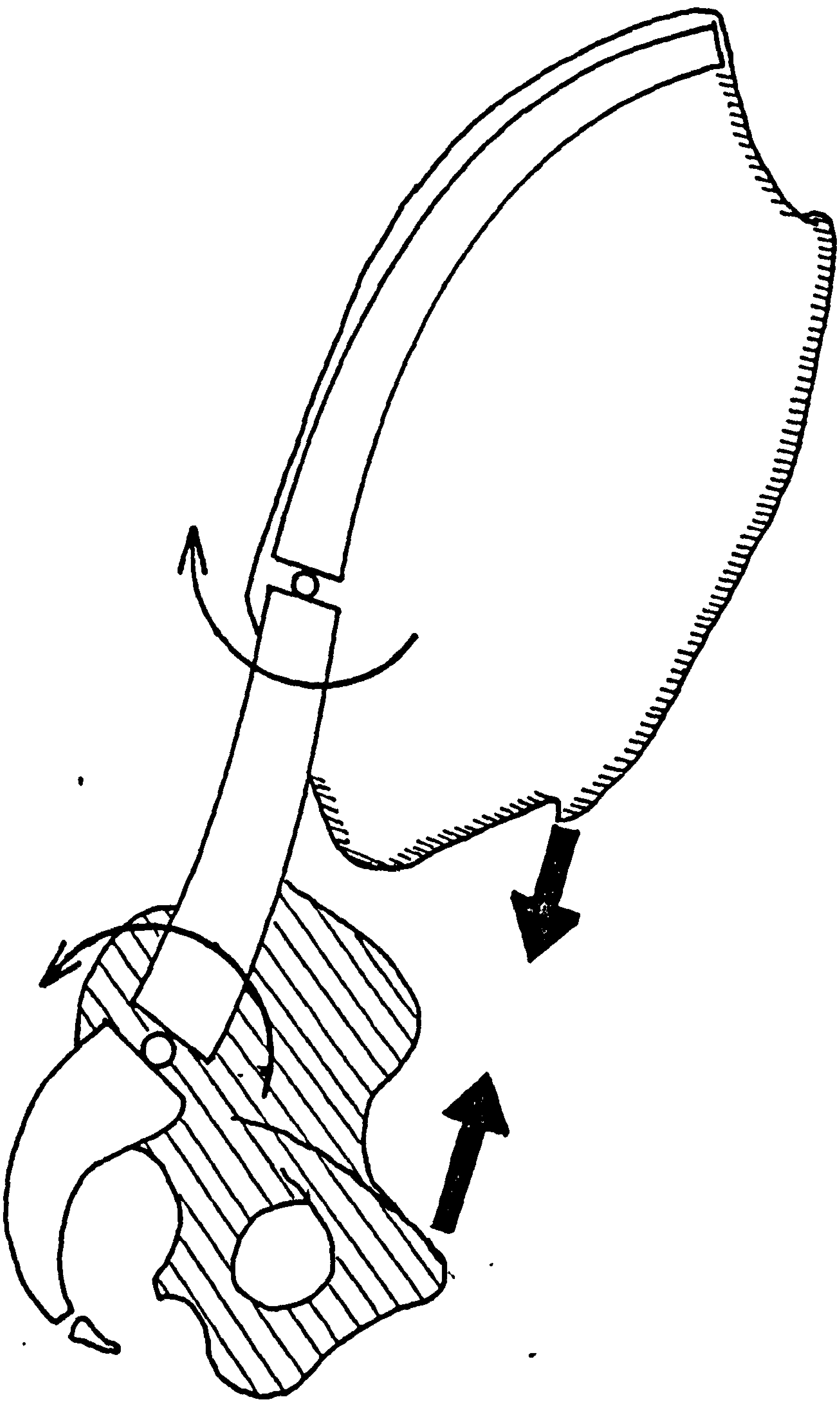


Fig.10 The Action Of Abdominal Muscles

abdomen of up to 18 kPa, although peaks of 53 kPa have been recorded (Eie and Wehn) [9]. The force exerted parallel to the spine on the diaphragm can cause a decrease of up to 30% in the compressive load on the $L_5 - S_1$ disc (Morris et al) [10] Fig. 11. Thus for strenuous lifting activities it is essential to have well-developed abdominal muscles, the belt used by weight-lifters serves as a support for these muscles. The peak internal pressure that is generated can only be maintained for a very short time otherwise cardiovascular disturbances can occur. (Kapandji) [11].

1.2. Spinal Dysfunction.

The structure of the disc has been described earlier and the reader will recall that the annulus fibrosus is made up of laminars of collagen fibre, each adjacent layer having a different orientation. On rotation of one vertebra relative to the next some of the layers will be put into tension while intermediate ones will become slack (Galante) [12], and this sets up wear between the fibres, excessive rotation can even cause the annulus to tear. Obviously the outer layers will suffer first and it is these that possibly possess nerve endings. The tears form a weakness in the annulus and the stress of lifting

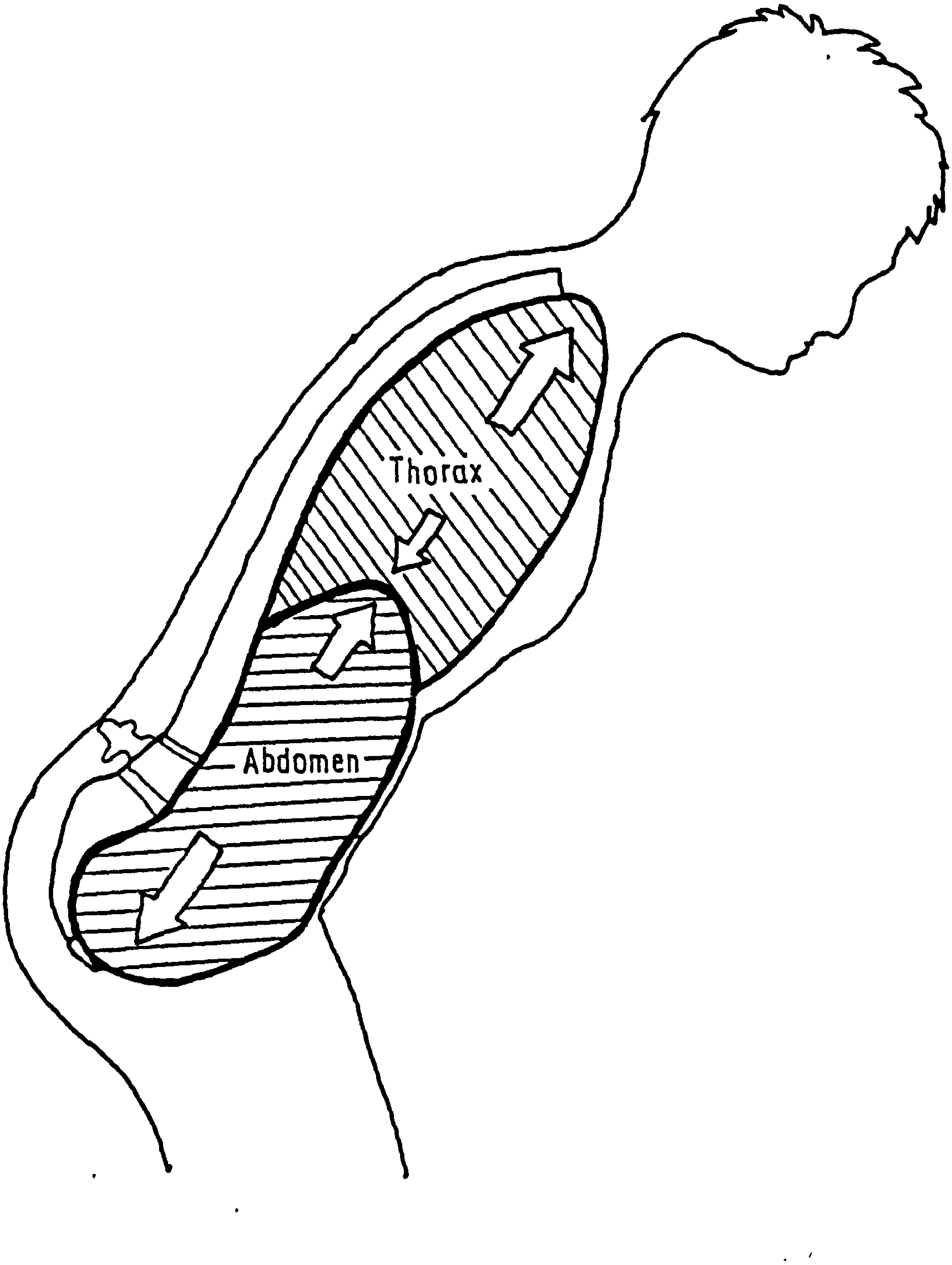


Fig. 11 The Action Of Abdominal Pressure

can cause nuclear material to burst through the disc wall. A further point is that on forced rotation the central nuclear space is decreased (a hollow rubber cylinder will behave in a similar manner) and this causes expulsion of the gelatinous nucleus. The result is a prolapsed intervertebral disc, the exuded material may press against a nerve root causing pain, a misnomer for this type of injury is "slipped disc".

Another factor related to disc injury is the degeneration with age. The change in water content indicates what takes place, the gelatinous nuclear material becomes fibrous and the hydrostatic manner of support is lost. It has been shown (Nachemson) [13] that in a normal disc the nucleus receives 75% of the compressive load and transmits this radially as a pressure load thus putting the annulus into tension. The degeneration of the disc and the loss of hydrostatic actions results in the annulus sustaining more of the compressive load, a narrowing of the disc space and restriction in movement generally follows.

The loss of height in the disc may cause the pedicles to come into contact with the nerve roots and tethering of these sensitive organs can result.

The disc normally can maintain pressures of 0.5 MPa, although higher pressures occur over short periods of time (Nachemson) [14].

There is no blood supply directly to the disc and only the outer layers of the annulus may contain nerve endings. Fluid can flow through the vertebral body and by the pumping action of movement this is the probable source of nourishment for the intervertebral disc (Krämer) [15]. Due to the permeable nature of the disc/vertebra interface, it has been found that the thickness of the disc will vary for a constant load over a long period. During sleep and horizontal rest the discs are able to regain in their original height by imbibition of fluid, a degenerated nucleus has a decreased ability to do this (Hirsch) [16].

The mechanics of injury to the vertebral column are also related to the flow of fluid through the disc and vertebral bone. In severe loading developed quickly by acceleration forces e.g. pilot ejection, it has been found that the anterior part of the vertebral bodies themselves are crushed (generally in the lower thoracic region) (Jones et al) [17], (Delahaye et al) [18], suggesting that the annulus fibrosus and the nucleus pulposus act incompressibly. On slow loading, however, the cartilaginous endplates of the vertebral body crack (called "Schmorl's node").

An explanation for this visualises the disc and the vertebrae as separate fluid compartments (Farfan) [19]. The spongy bone core allows fluid to flow freely, but the cartilaginous endplates and the compact bone cortex create resistance to flow. Thus a high pressure can exist in the disc for a short period of time and it can act as an incompressible compartment. In this situation the endplates act as a diaphragm and the vertebral body becomes the shock-absorber for the spinal column. During impact loading there is sufficient restriction to fluid flow out of the vertebral body for the disc and vertebra to act as two incompressible fluid compartments. Injury occurs by the crushing of the weaker unit i.e. the vertebral body. During slow loading there is sufficient time for pressure release in the vertebral body, but a higher pressure still exists in the disc so the endplate cracks and the equalisation of pressure occurs. Naturally a fractured endplate leads to a degenerated disc. Consider the situation arising with one fractured endplate, Fig. 12.1. The fluid compartment consists of one disc and one vertebral body, by applying the stated theory one would expect endplate (A) to fracture. A similar reasoning applies with regard to the state arising from a ruptured disc and two broken endplates, the fluid compartment now extends over one disc and two vertebrae, Fig. 12.2. and one would expect either (B) or (C) to fracture.

On clinical examination the theory holds, there is a progression from one endplate to the next generally without missing intermediate vertebrae. From the configuration of the spine it can be seen that L_3 is nearly horizontal and therefore accepts high compressive loading and it is often the first to fracture as described. Conversely L_5 is at an angle to the horizontal and some of the load is transmitted by shear, thus the force on the disc is alleviated by the action of the articular facets in weight bearing.

The sequence of degenerative steps in the intervertebral discs can be summarised by the diagram shown in Fig. 13.

Complete or partial shearing of one vertebrae from the next does occur mainly in the cervical and thoracic regions, but this is a result of severe trauma. Hyper-extension or flexion of the spine can

Fig.12 Fracture Of The Vertebral Endplates

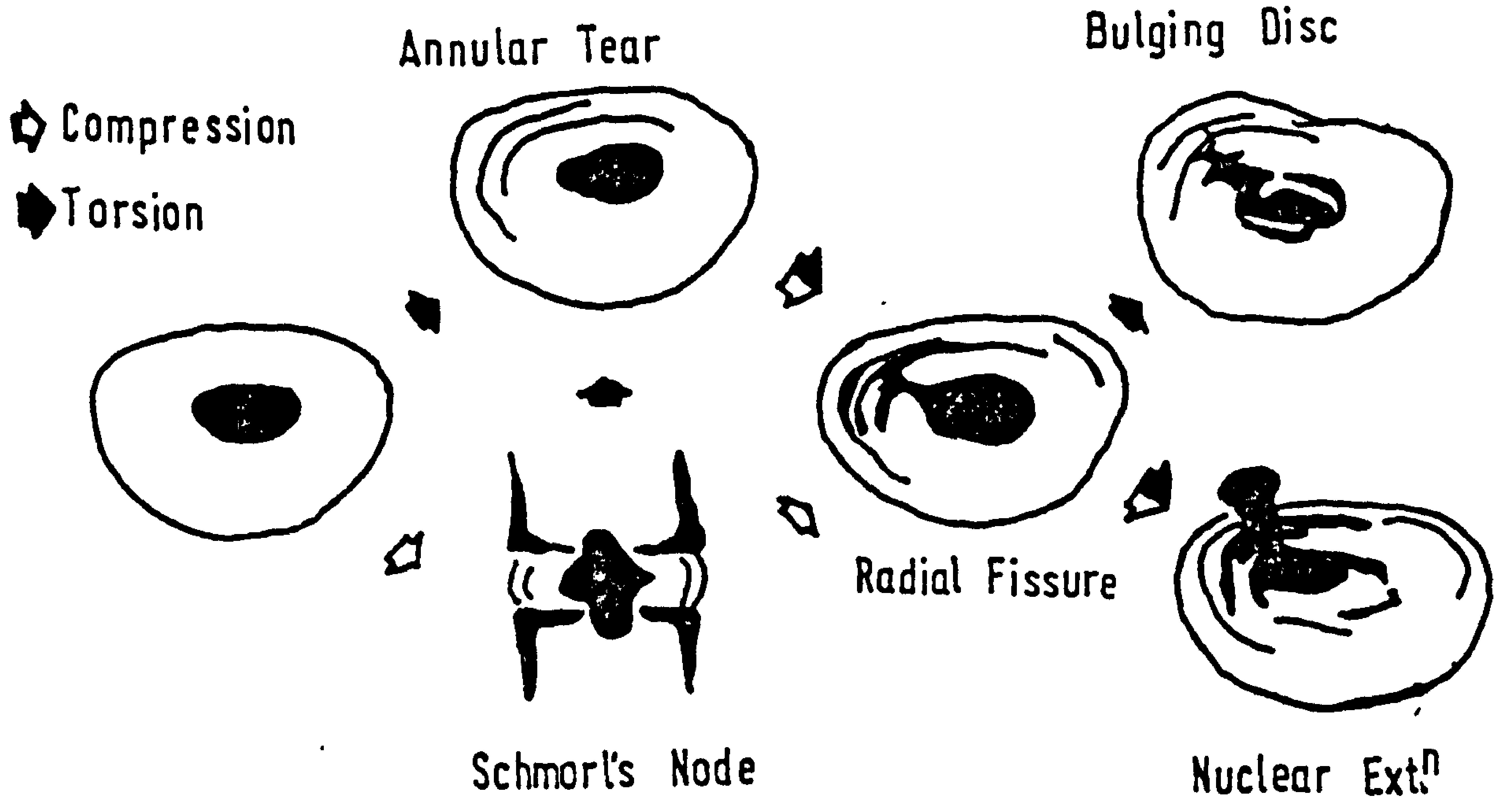


Fig.13 Disc Degeneration Due To Injury (From Farfan [19])

also cause a complete fracture of the vertebral column, this has been particularly prevalent in car accidents in which the occupant has been wearing a lap belt or the seats are without head restraints.' (Markolf and Steidel) [20].

The spine in dysfunction is of prime importance to the surgeon, physician and research worker. Low back pain, for example, is estimated to cost this country at least £100 million per annum in lost working days (Talk Back) [21], and in any one day approximately 50,000 people are off work.

Two further types of abnormality will be described here but a fuller exposition of spinal injury and disease may be found in texts such as Rothman and Simeone. [22].

An example of vertebral fracture can be found as spondylolysis and spondylolisthesis. The former refers to the state in which a crack runs across the laminae effectively removing the inferior articular process and spinous process from the vertebral body, thus the facets are unable to carry load and the shearing force that may be present must be resisted by the discs alone. There is no misplacement of the vertebrae. Very often this fracture is caused by shock-loading e.g. acrobatics, and predominantly occurs in the region L_4 - L_5 and L_5 - S_1 (Tütsch and Ulrich) [23]. Alternatively spondylolysis can have a congenital etiology (Finneson) [24].

Often spondylolisthesis is a development of spondylolysis and in this case describes a state in which the vertebral body has become displaced forward leaving the posterior elements in place (Newman) [25]. The amount of slippage is graded as in Fig. 14. It is remarkable that the spine can still remain stable even with a large amount of slippage. Nerve root compression can occur but often the abnormality goes unnoticed until excessive displacement has taken place. Fig. 15.

In the normal operation of the spine, natural scoliosis occurs whenever the pelvis is tilted but the head remains on the same axis e.g. walking up stairs; this is likened to the coiling of a curved elastic rod (MacConaill and Basmajian) [26]. The deformity "scoliosis" is a structural derangement and has presented a problem in diagnosis and treatment for hundreds of years. See Fig. 16. Hippocrates

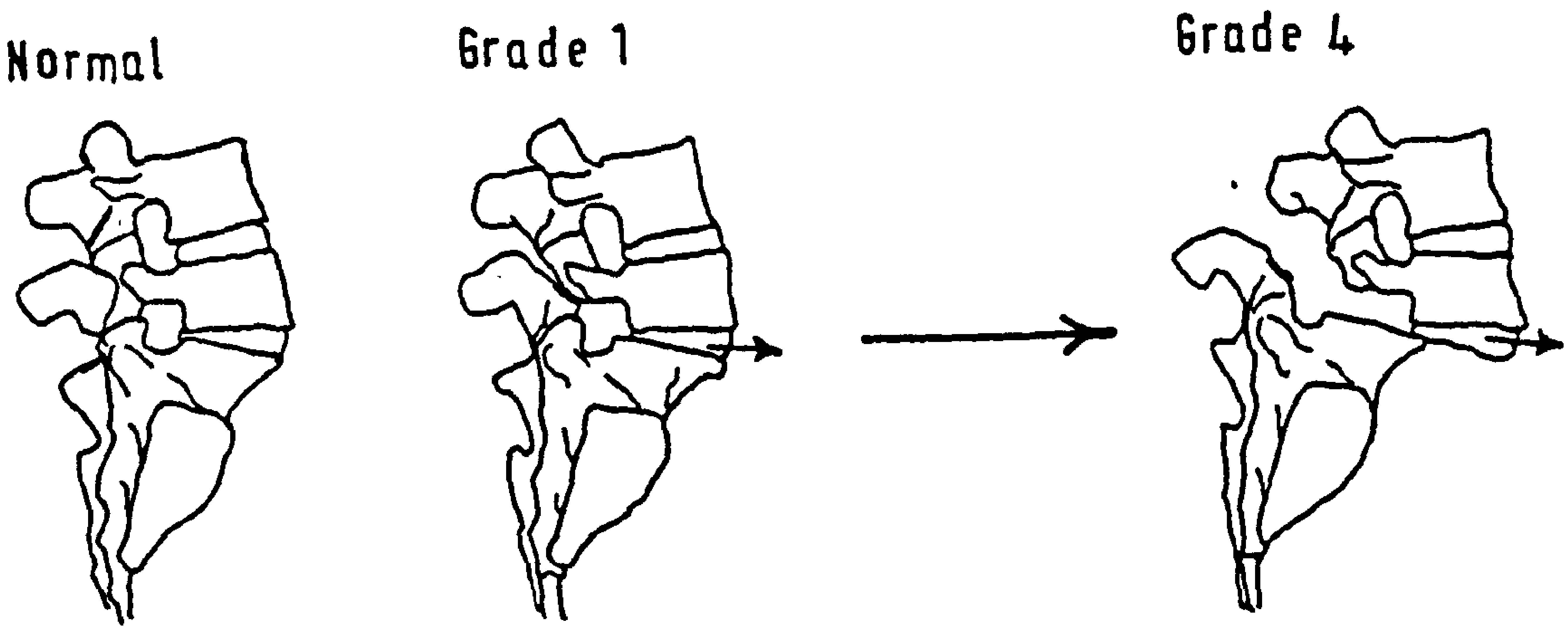
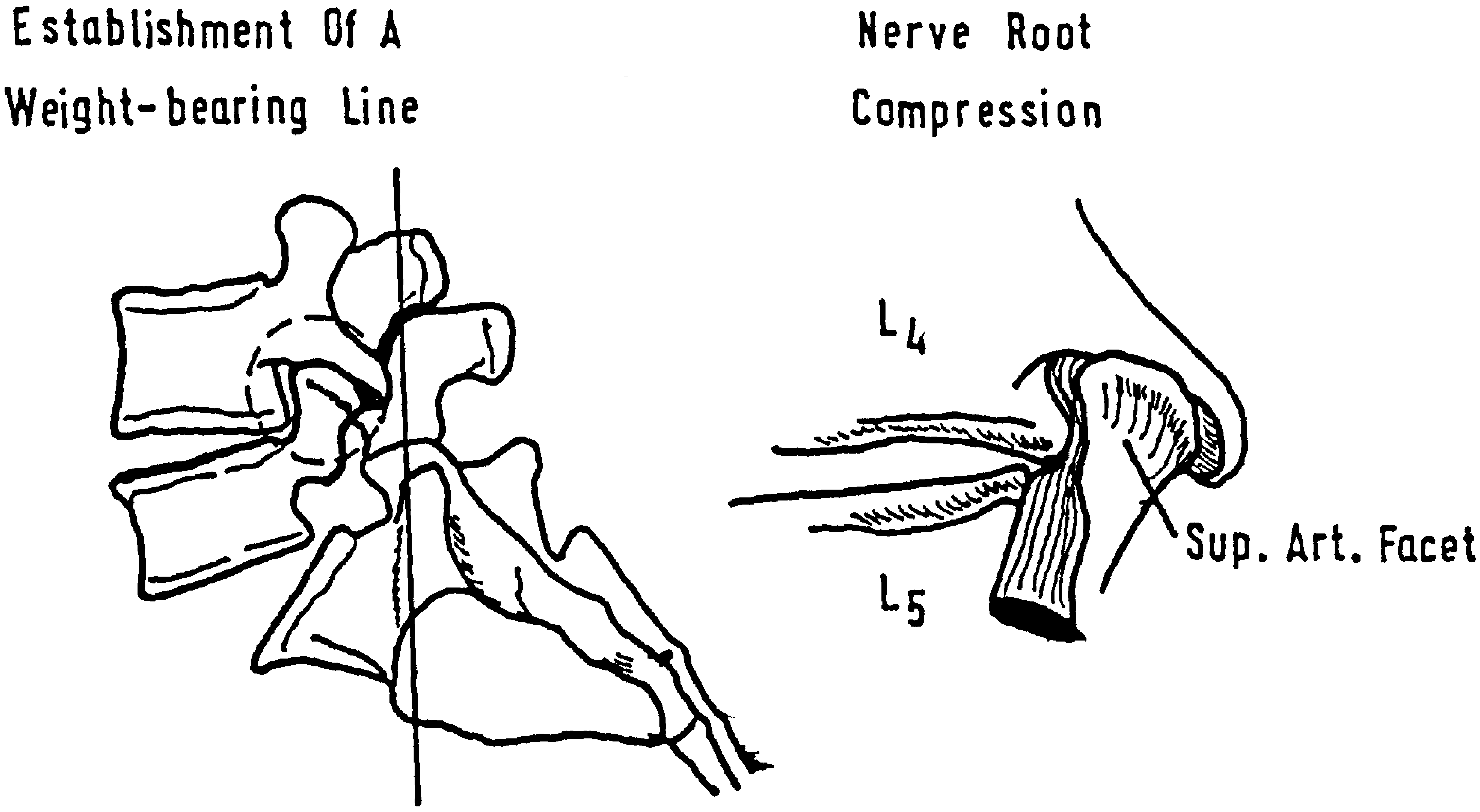
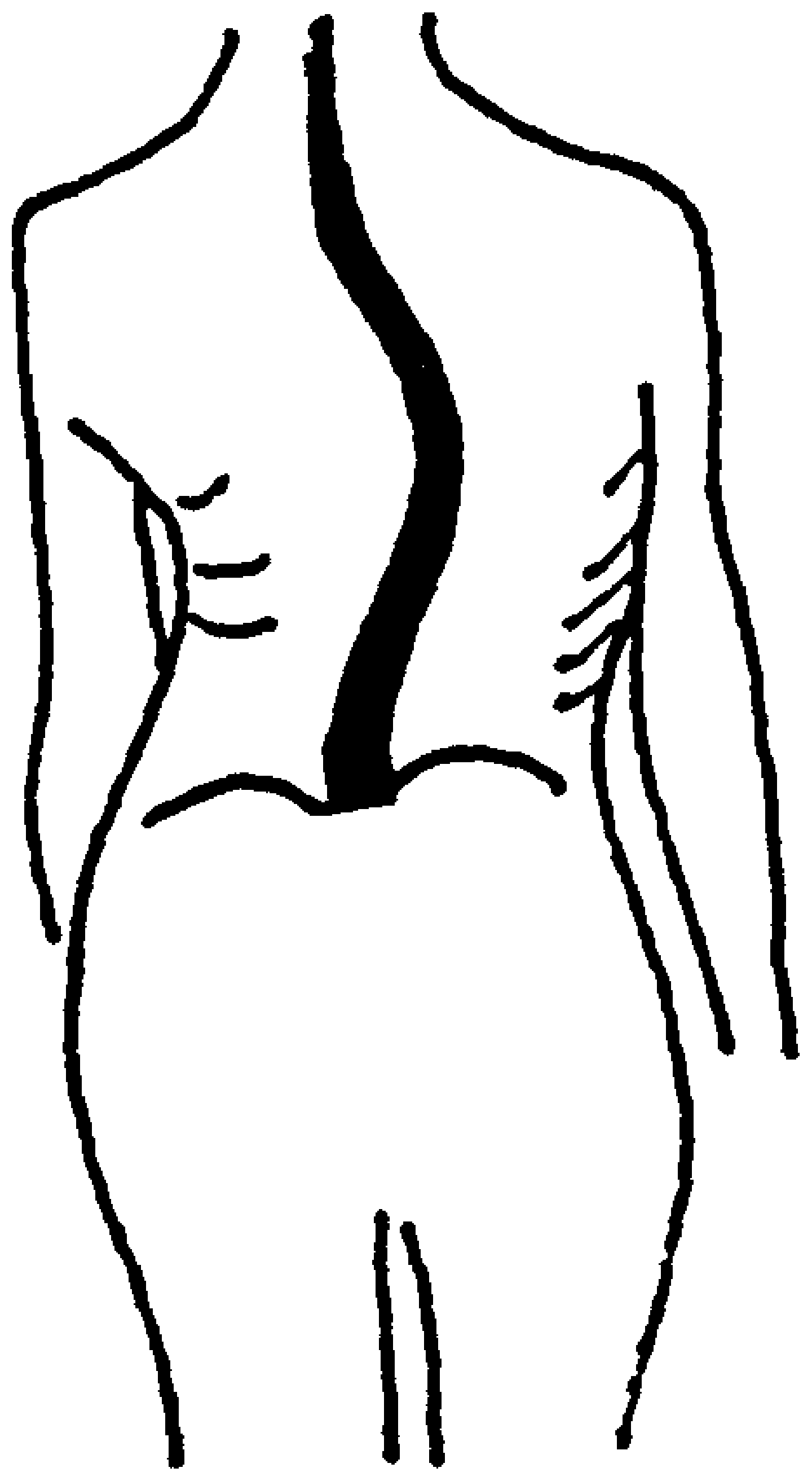


Fig.14 Classification Of Spondylolisthesis


Establishment Of A Weight-bearing Line

Nerve Root Compression

Fig.15 Spondylolisthesis (From Finneson [24])



A C- Configuration Curve

An S Configuration Curve

Fig.16 The Scoliotic Spine

(5th Century B.C.) used this term for a lateral deformity and suggested correction on a frame (Robin) [27]. Idiopathic scoliosis accounts for approximately 80% of scoliotic conditions and as the name suggests its etiology is unknown.

It has been noted that rainbow trout suffering from ascorbic acid deficiency (MacEwen) [28] and animals fed on sweet peas (Nordwall) [29] can develop a scoliosis. In humans the scoliotic spine has been likened to a buckled elastic rod (Lucas) [30], this agrees with the theory that an S-shaped curve is more stable than a C-curve (Steindler) [4]. Others have shown that rotation of the vertebrae is the main cause (Roaf) [31](due to the wedge shape of the vertebral bodies) and this can be linked to muscular imbalance from a disturbed nervous system. A scoliotic curve in infants is often self-correcting (James) [32] whereas a curve developed in adolescents may be progressive. Certainly there is a link between curve development and an excessive growth spurt, this ties in with the result from a computer model that tightening of posterior ligaments can cause a lateral bending and rotation of the column (Schultz and Galante) [33]. But behind all these theories lies the possibility that scoliosis is hereditary and often linked to mental retardation (Wynne-Davies) [34]. Beyond all doubt this deformity has tested the skill and ingenuity of the orthopaedic surgeon.

Frequently evolution and the development of an upright posture in man has been citied as a cause of spinal problems; this is inconsistent with the theory which demands a progression forward, for the spinal column has had sufficient time to change to accommodate the "new" posture (Wood) [35]. This theory also overlooks the fact that many "horizontal" animals suffer back problems, albeit silently. This section closes with two quotes:

Prof. G. L. Stebbins, Univ. of Calif. "...every account of evolution written before 1950 is already or will soon be obsolete".

G. Frederick White, Former President Oberlin Univ. "... Evolution is one tenth bad science and nine tenths bad philosophy".

CHAPTER 2.

Mathematical Modelling.

The description of mathematical models will be given in two sections, one dealing with dynamic and the other with static systems. Differentiation between the two is made on the basis of inclusion or exclusion of acceleration forces.

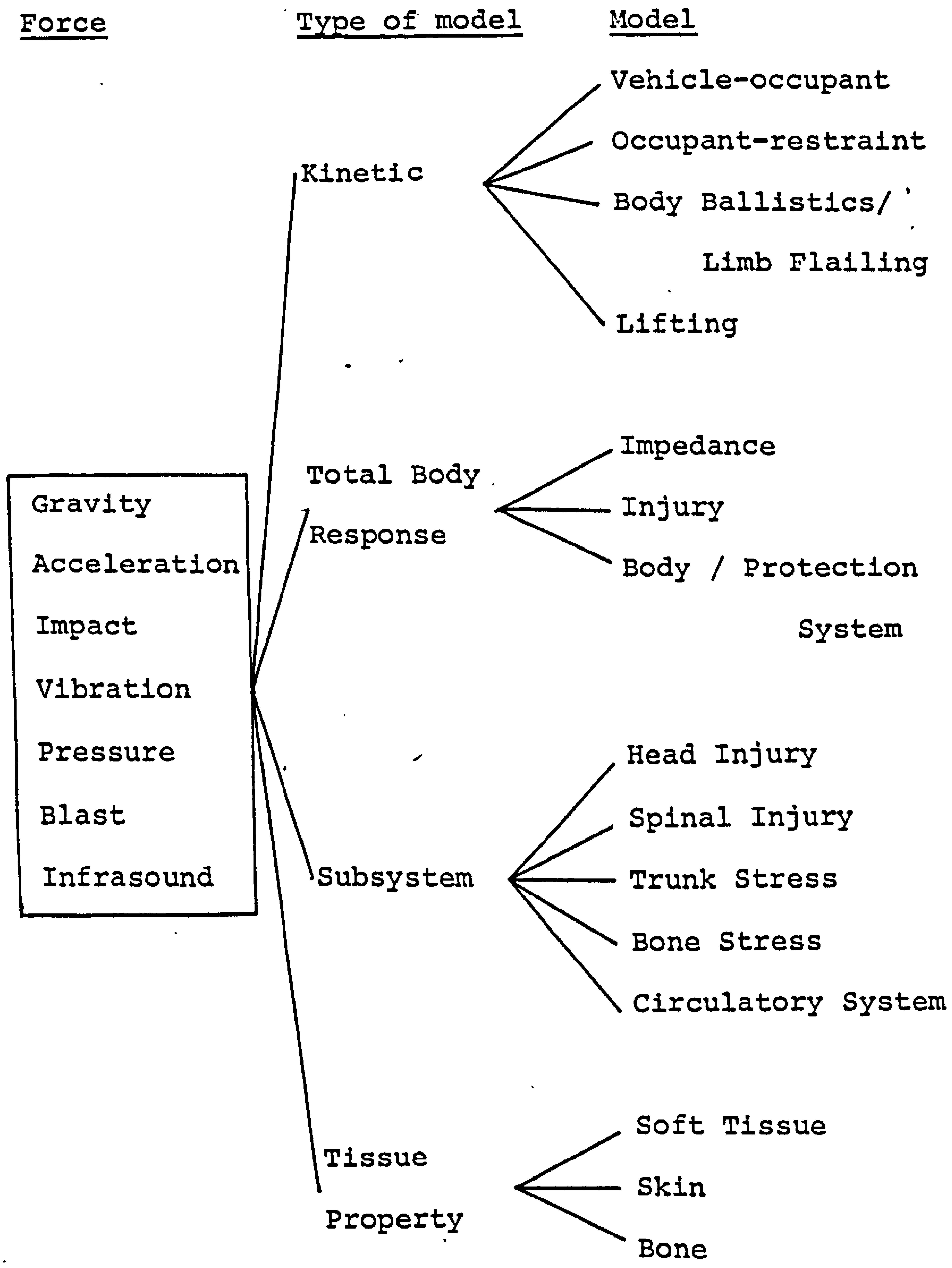
There is an overlap of the two categories at very low levels of acceleration. This area, for example, slow lifting of a weight, has been covered by some static models because the forces due to inertia are small. The dynamic models have been developed using the equations of motion and while they theoretically can handle a static situation, none are sufficiently authentic in anatomical detail to be able to obtain useful results for local areas under simple loading.

A remarkable development has taken place in recent years in the field of biodynamic modelling and many mathematical models have been produced. In part this has been due to a necessity to quantify man-environment interaction to overcome such problems as pilot ejection from aircraft and to simulate occupants in automobile accidents. The main underlying factor, however, has been the existence of inter-disciplinary co-operation.

In the area of static models the application of mathematical analysis to the spine as a structure must have seemed either limited or daunting to the engineer, for it has been through the study of spinal mechanics by the medical profession that an interest in the simulation of the vertebral column has developed. If there has been a failure of engineers to promote this type of research there has also been a lack of assistance from the orthopaedic surgeon in suggesting areas of need.

2.1. Dynamic models.

The mathematical models for the dynamic analysis of the spine can be classified as described overleaf. (von Gierke) [36][37].



The models described overleaf relate to the spinal column or complete body only, and except for 2.1.1.4 they consider the body as a passive system and neglect internal dynamic processes occurring concurrently with the external force spectrum. Breathing and active muscle action are examples of this. The aim of this section is to show a range of analytical methods, it is not intended to be an exhaustive study of models produced.

2.1.1. Kinetic/Kinematic models.

The body segments are characterised by a system of rigid links with appropriate centres of mass and inertial properties. Some of the models can be used to predict motion by supplying joint friction and force as input data. Others are able to forecast joint reaction from explicit movement data, stress levels within a segment are obtained by instantaneously stopping the motion of the model and analysing the static force distribution using the equations of equilibrium.

Examples:

2.1.1.1. McHenry. [38] Fig.17

The model has been used to predict the motion of a body-restraint system of 11 degrees of freedom from an acceleration pulse. Frictional constraints and articulation limitations of the joints are based on empirical fits, however, no attempt has been made to relate calculated joint force with any actual force within the body structure. Elastic behaviour of the joints is also neglected, yet the model has been of use in the evaluation of protective systems e.g. air bag type.

2.1.1.2. Huston and Passerello. [39] Fig. 18

The basis of the model is of an elliptical cylinder to represent the torso, together with a system of frustrums of elliptical cones representing the limbs. The 15 segments are connected either by hinges or ball and socket joints. The model solves the equations of motion, when external forces and relative movement of the limbs are specified, to give the displacement and rotation of the main body. Three examples have been studied, that of simple lifting, underwater swimming and the kicking of a suspended man (parachutist).

An interactive computer graphics version of this type of model has been presented by Boysen et al [40] for symmetric motions of bilateral segments in free fall situations.

2.1.1.3. Wood and Hayes.[41] Fig. 19; Fisher.[42]

The models consist of a series of links, each with centres of gravity and weight to represent

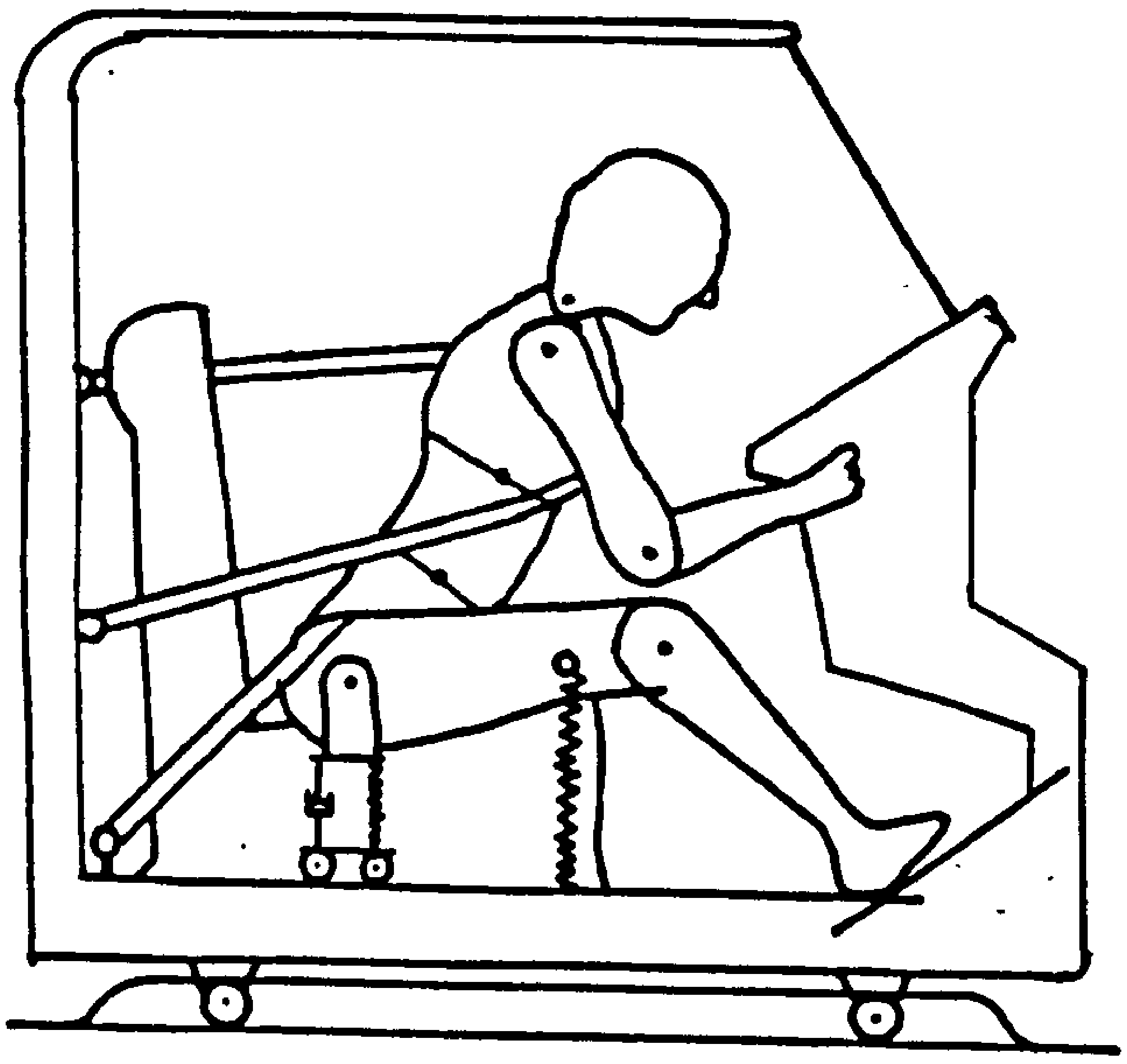


Fig. 17

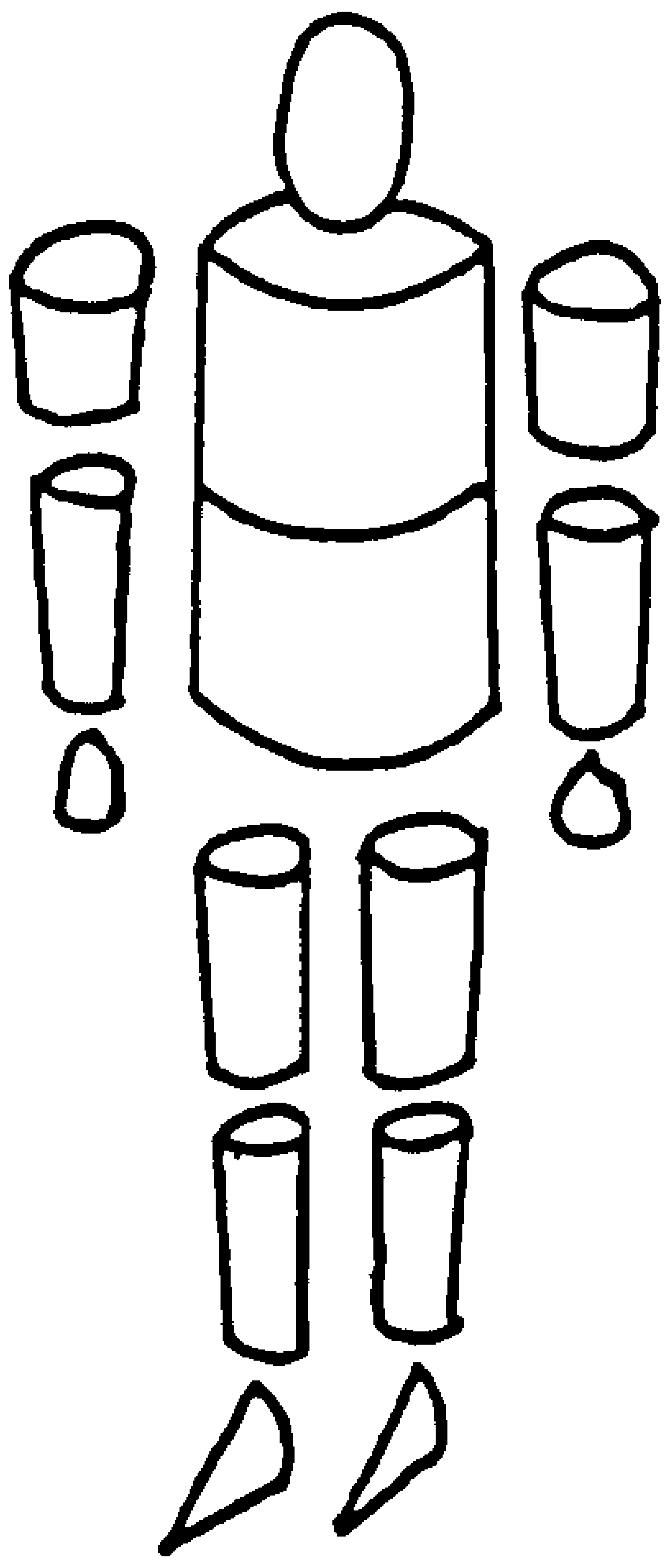


Fig. 18

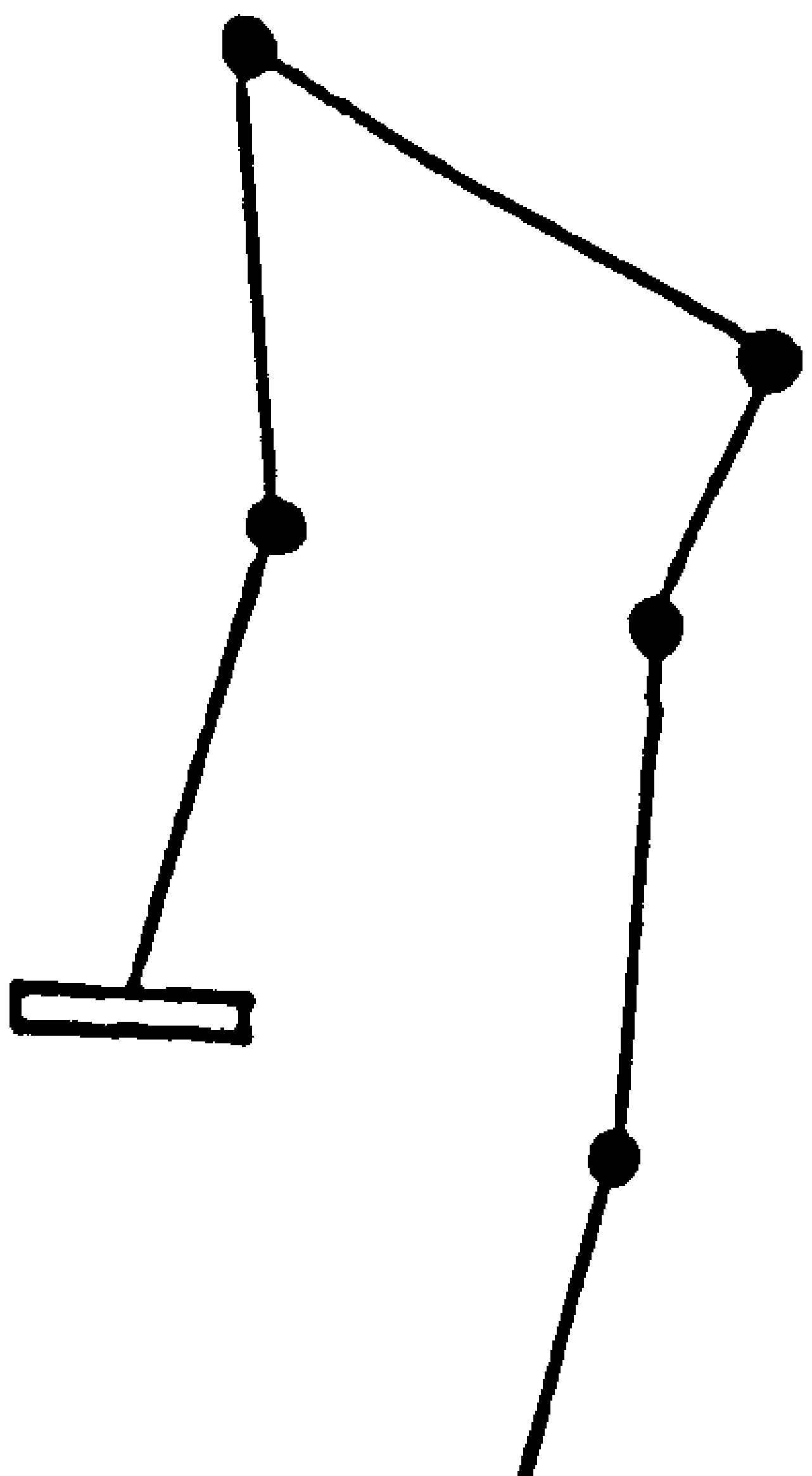


Fig. 19

anatomical segments. Motion of a person performing a lifting task is obtained from cine-photography. Joint reactions are calculated for the dynamic situation and from these the forces acting on the spine are found by static equilibrium in a set position.

2.1.1.4. Hatze. [43]

This model used a mechanical system similar to that of Huston and Passerello (2.1.1.2.) with 17 body segments. The segments are specified by centres of mass and masses and can be of any shape. The joints connect the segments with varying degrees of freedom. For this system the Lagrangian equations of motion are developed. The link mechanical system is acted upon by a variety of non-conservative generalised forces grouped in three categories:

a) passive internal torques produced by tendons and connective tissue which limit the motion,

b) external torques e.g. wind resistance or external friction,

c) generalised muscle torques.

The muscles are visualised as part passive (including viscoelasticity) and part active. Fig. 20. The active element was developed from previous work [44] and allows the muscle to be stimulated in a pattern similar to that obtained from EMG studies. The model has been applied in a simplified form to the right leg and found to work.[45].

2.1.2. Total Body Response Models.

These models are used to gain an understanding of the interaction between various responses or of the overall response of the body. A global view of the body's biodynamic response can be obtained from these models but an assemblage of subsystems would give a more accurate picture. The resonant frequencies of various parts of the body can be suggested and sites of injury anticipated. The body is formed from an arrangement of masses, springs and dampers.

Examples:

2.1.2.1. Coermann et al. [46] Fig. 21

This model consists of a mechanical circuit using a lumped parameter system to represent the mechanical impedance of the human body when exposed to longitudinal vibration (up to 100 Hz). The input force may be applied either to the feet or to the mass of the hips, this simulates the response of a standing and sitting person respectively. This type of model can show clearly the effect on the body response of seat cushions or girdles around the abdomen or differences in body posture.

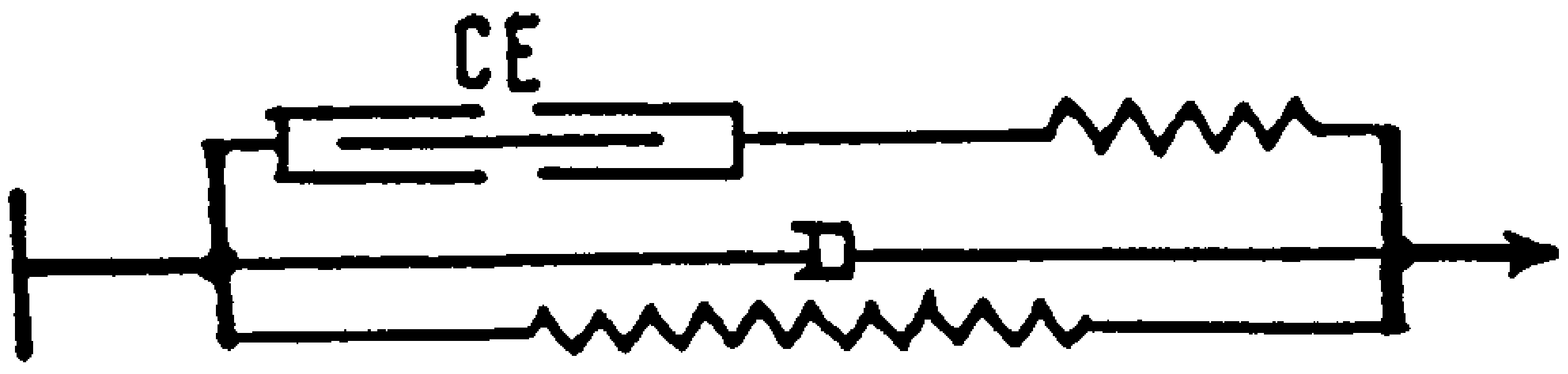


Fig.20 Muscle Model CE Active Part

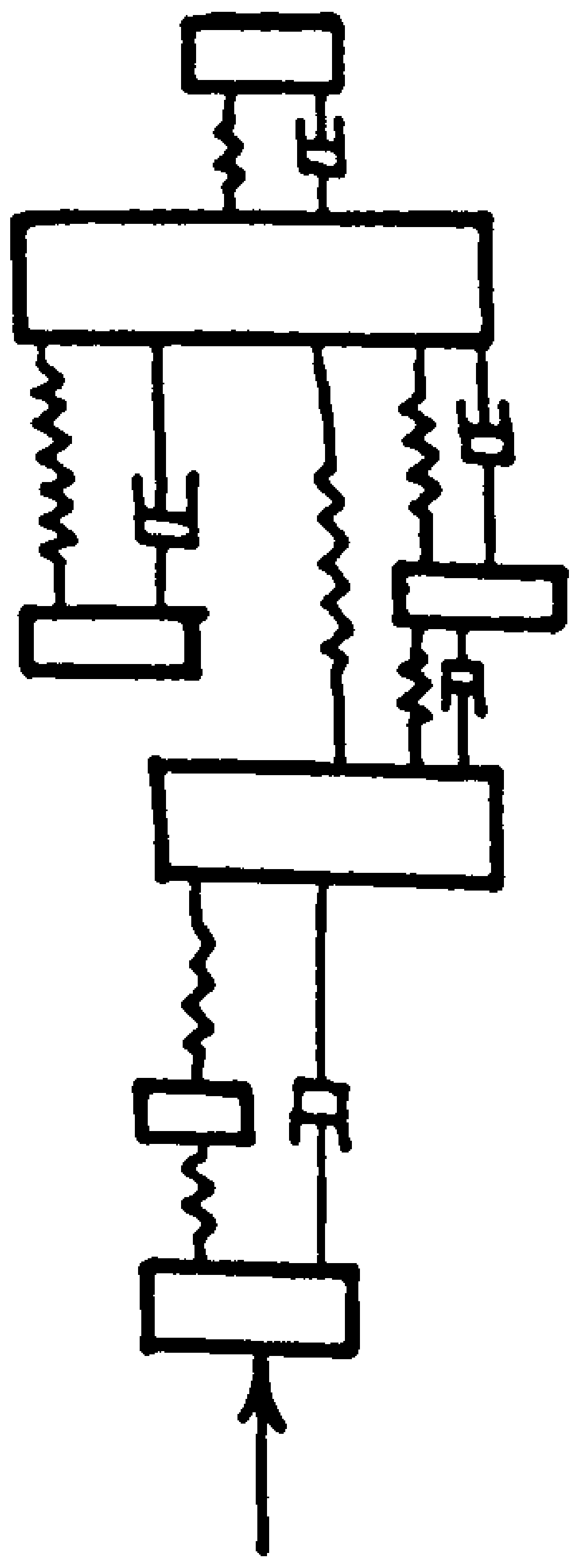


Fig. 21

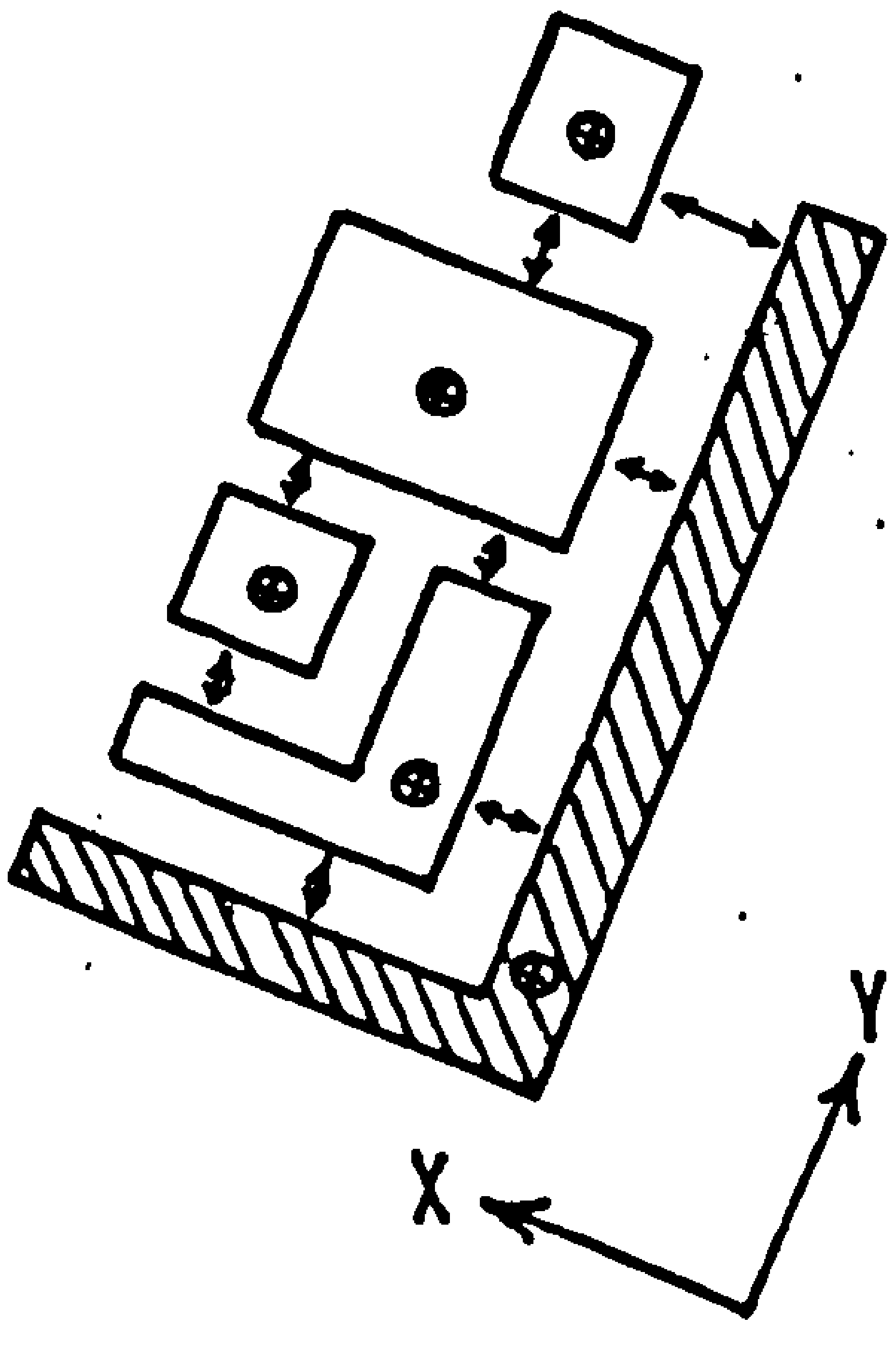


Fig. 22

2.1.2.2. Band. [47] Fig. 22

The model, which has 11 degrees of freedom, has been used to analyse the dynamics of the whole man/ejection seat system. The results predict the forces acting on the buttocks, spine and neck for either a catapult manoeuvre or for rocket powered free flight. Both translation and rotational movement can be handled and the model can also include the aerodynamic drag of the man/seat and of the stabilising drogue.

2.1.2.3. Kaleps et al. [48] Fig. 23

The model has been used to calculate body deformation (spinal compression, pressure in lungs) as a function of external longitudinal forces (impact or vibrational) and pressure loads (blast, acoustic or direct compression) for a frequency range of 1-100 Hz.

2.1.3. Subsystem Models.

Many different model configurations fall within this category, but their uses are twofold: firstly, to predict injury and secondly, to analyse wave propagation along the spine. The mathematical theory behind the models has become both sophisticated and complex, although every effort is made to establish the usefulness of the simulations, many seem to fall short on this account.

Continuum models are helpful in analysing the propogation of a force pulse along the spine and in the most recent versions stress levels may be found at any point along the homogeneous beam-column. But as yet this does not give direct examination of individual vertebrae. The treatment of the vertebral column as a discrete parameter elastic rod i.e. with elastic and rigid elements for the discs and vertebrae respectively, enables forces and moments to be calculated across a rigid segment. Unfortunately, only on the introduction of elastic properties for the vertebrae will the local dynamic stress be calculable. Injury to the vertebral body is a result of high local stress and this aspect is in need of careful study. A number of models have been used to predict the deformation of the vertebral column caused by vertical acceleration and the results show clearly how the vertebral bodies can be subjected to a high local stress causing fracture and crushing of anterior surfaces in the thoracic region.

Examples:

In the examples which follow

2.1.3.1. - 2.1.3.4. are for axial response only.

2.1.3.5. - 2.1.3.10. are for sagittal plane response.

2.1.3.11. - 2.1.3.13. are for full 3-dimensional response.

2.1.3.1. Latham. [49] Fig. 24

Stech [50] and Stech and Payne. [51]

This is a simple single degree of freedom model. In the work of Latham the dynamic response was correlated with experimental data from vibration studies. The overall stiffness of the model developed by Stech was arrived at by adding together the individual spring constants of each vertebra-disc. The resonant frequency of 6 Hz which was calculated agrees with that found by Coermann et al (2.1.2.1.). The obvious limitation of these models is that the accelerations could only be found at the endpoints i.e. head and hips, whereas the area of injury probably lies inbetween.

2.1.3.2. Terry and Roberts. [52] Fig. 25

A viscoelastic rod model which includes damping has been used to simulate the spinal column mathematically. The uniform rod is of a Maxwell-type medium. The model was subjected to a ramp acceleration pulse input.

2.1.3.3. Liu and Murray. [53] Fig. 26

The spine was treated as an elastic rod capped by a rigid mass to represent the head. The lower end of the rod corresponds to the hips and for the case of an acceleration pulse an attempt was made at finding

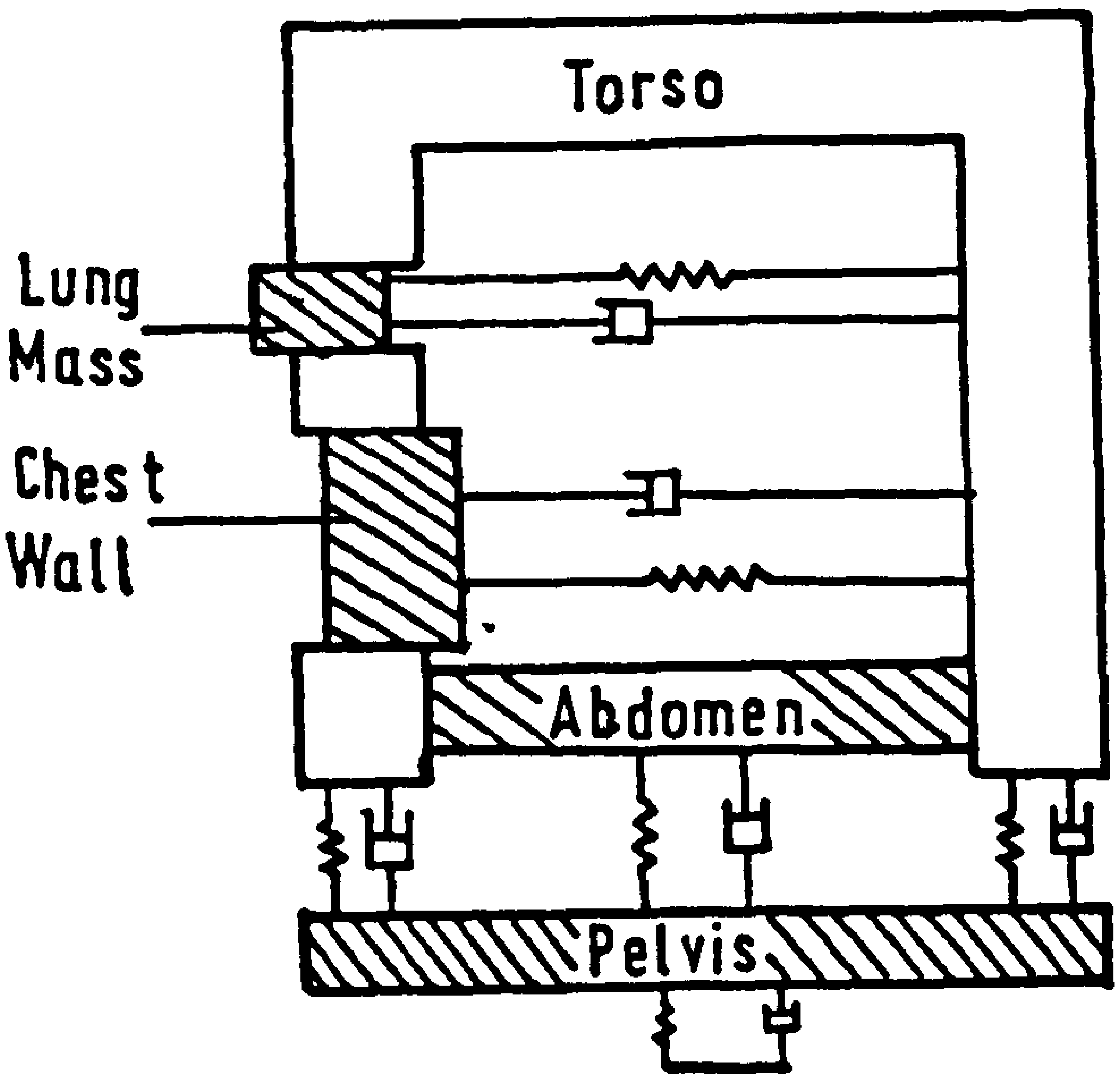


Fig. 23

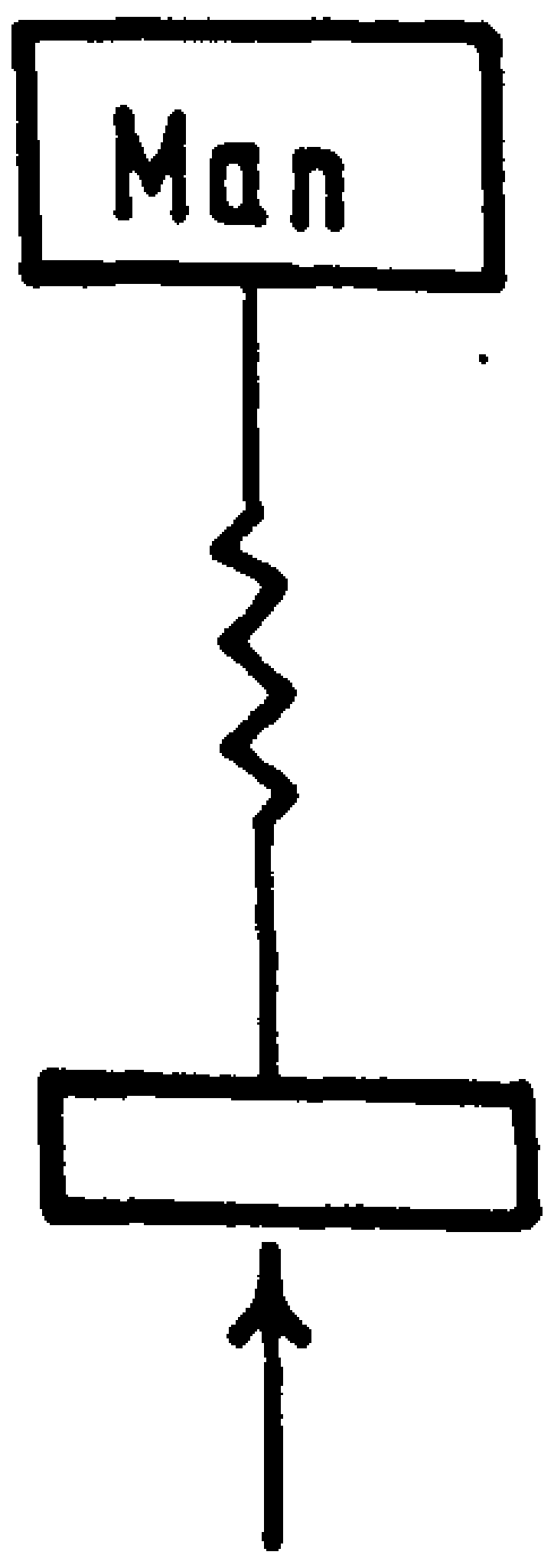


Fig. 24

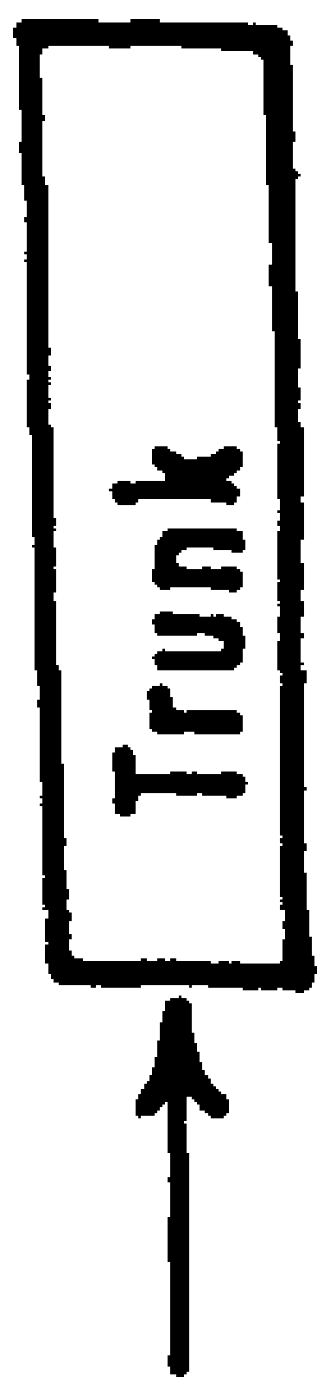


Fig. 25

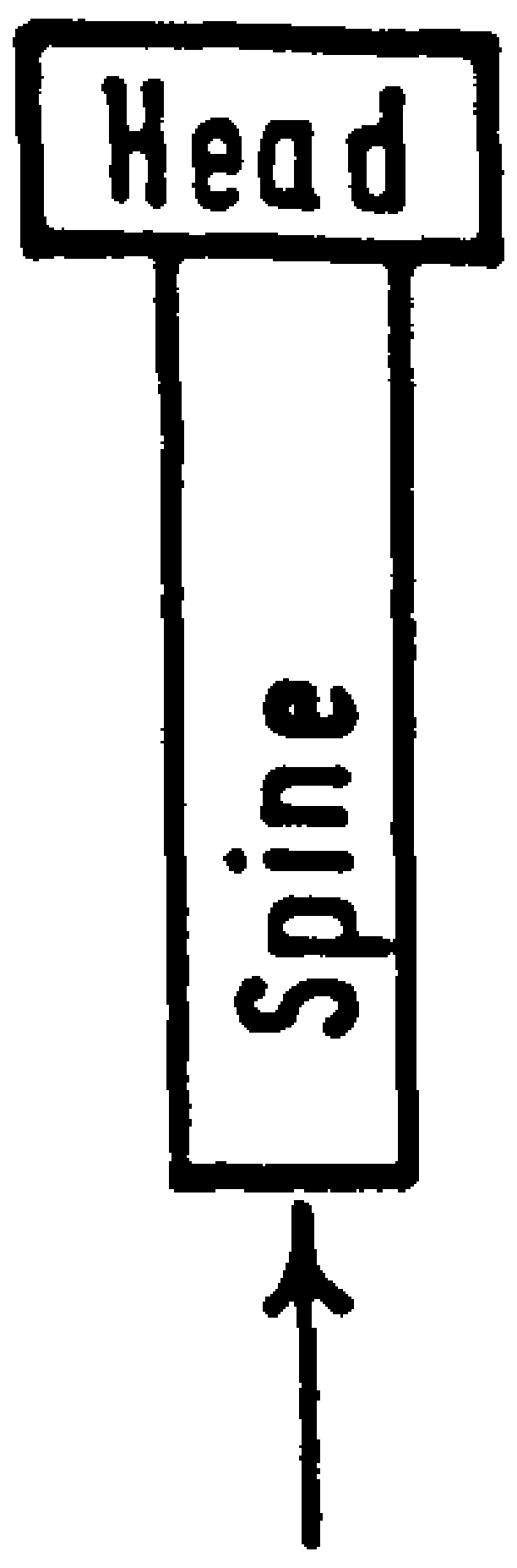


Fig. 26

when and where maximal stress occurred.

2.1.3.4. Toth. [54] Fig. 27

The model was constructed of a number of individual masses to represent the vertebrae in the region T_{12} - L_5 and for the adjoining segments. The choice of spinal areas was made on the basis of the frequency of injury. Using assumed values for the threshold of failure of individual vertebrae the likelihood of structural damage was evaluated. All the masses are connected by springs and dampers.

2.1.3.5. Li, Advani and Lee. [55] Fig. 28

The spine was visualised as a rod subjected to an initial curvature on top of which was a mass representing the head. The equations used, proposed by Hoff [56] and later extended by Sevin [57] to include axial inertia, were solved using an assumed mode method which has been shown to be accurate for low accelerations or short time duration only. [58].

2.1.3.6. Orne and Liu. [59] Fig. 29

A discrete parameter model which takes into account the axial, shear and bending deformation of the disc, the viscoelastic behaviour of the discs, the variable size of the vertebrae and discs, the natural curvature of the spine and the eccentric loading caused by the head and trunk. An acceleration pulse of 10g with a rise time of 14msec was used to simulate pilot ejection. Horizontal impulses were also catered for.

2.1.3.7. Soechting. [60] Fig. 30

Soechting and Paslay. [61]

An elastic rod forms the basis for this model, which also considers the passive effect of the muscles. The muscles were idealised to act along the rod and their stiffness is a function of extension and rate of extension due to the curving of the rod resulting from an acceleration impulse. The overall mass of the system was representative of the torso plus head, although no allowance was made for a variation in mass distribution in the torso, the bending stiffness of the rod was calculated to represent the overall stiffness of the human torso. It was shown that maximum head displacement depended mostly on the muscle properties, the head acceleration and local spinal curvature depended on the bending stiffness of the rod.

2.1.3.8. Prasad and King. [62] Fig. 31

Tennyson and King. [63]

The model was based on a discrete parameter

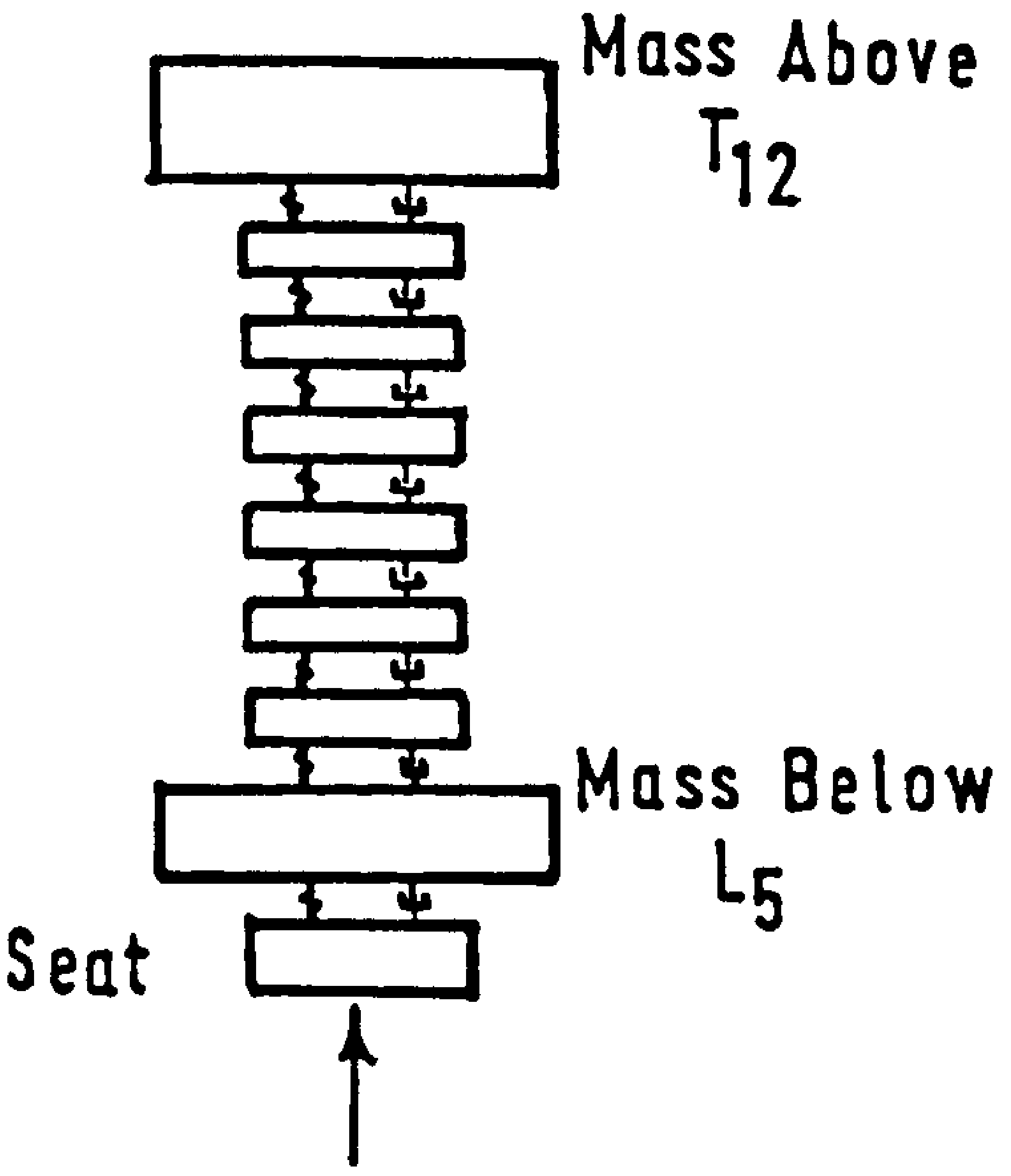


Fig.27

Fig.28

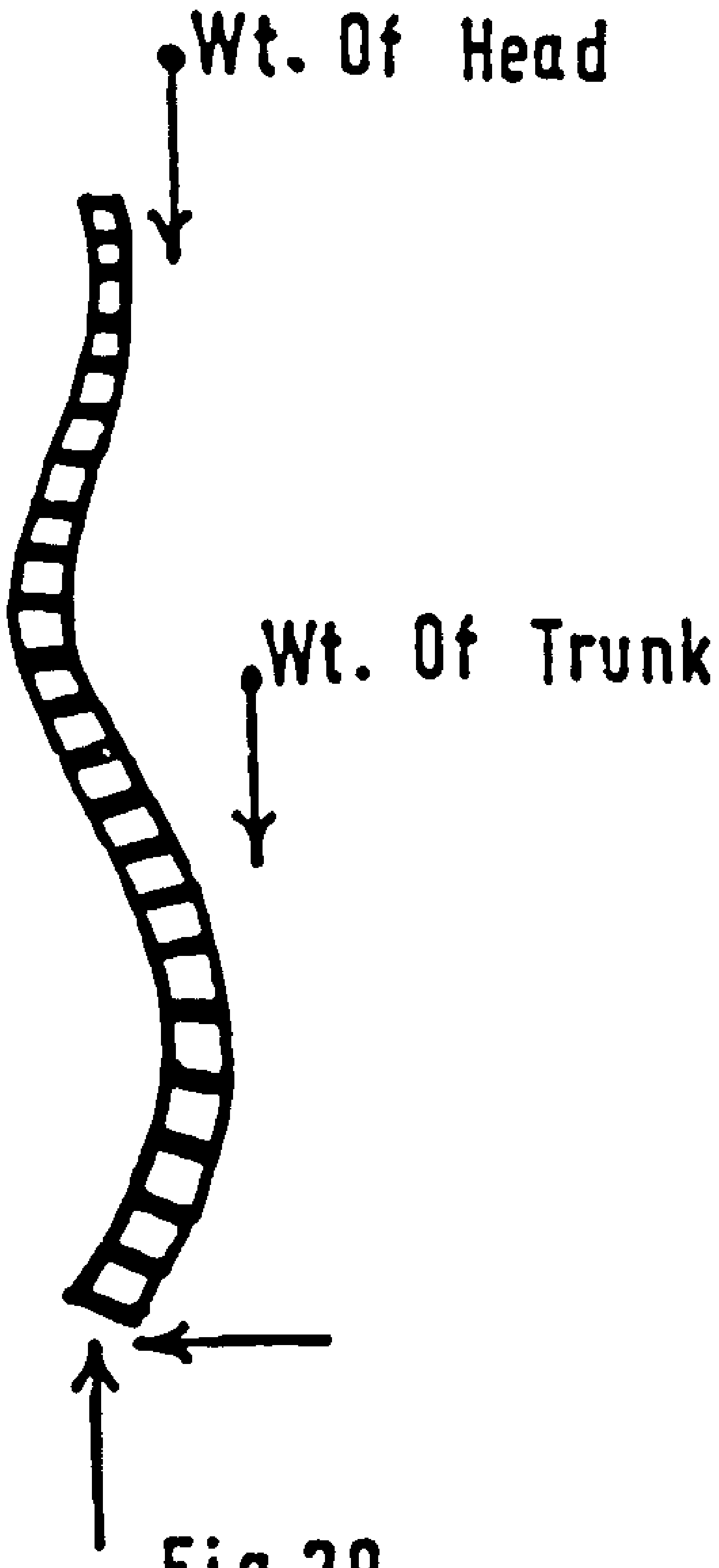


Fig.29

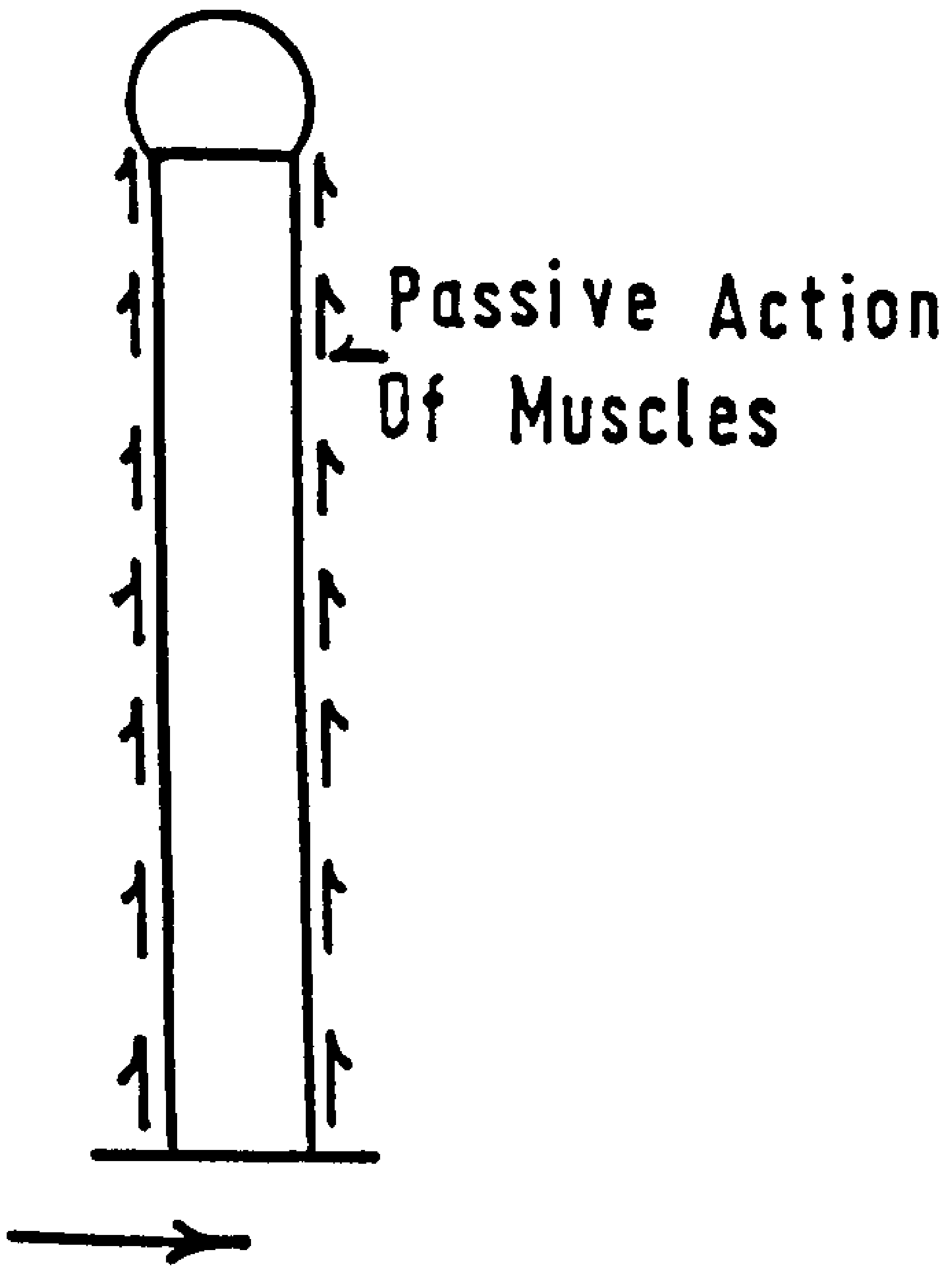


Fig.30

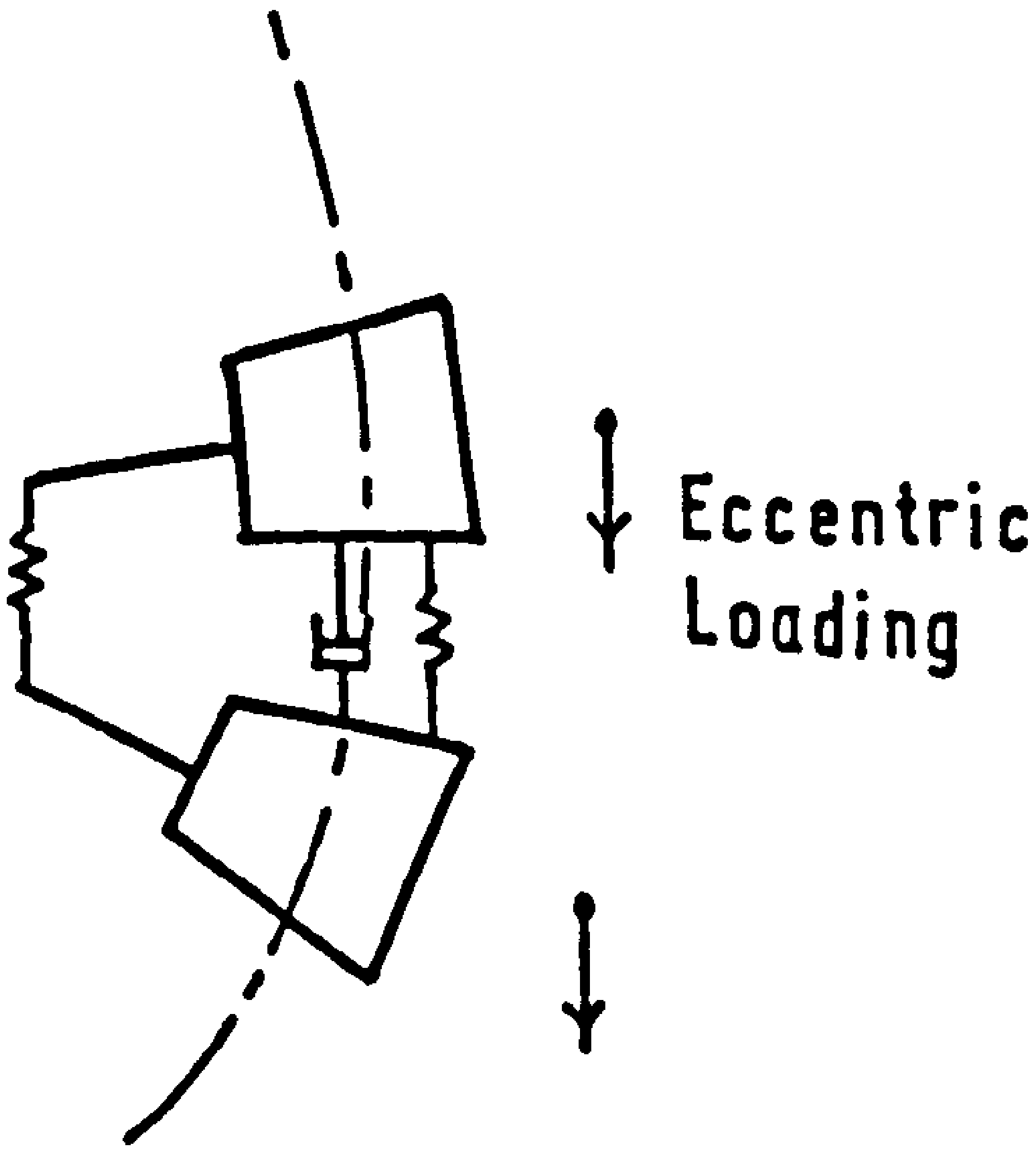


Fig.31

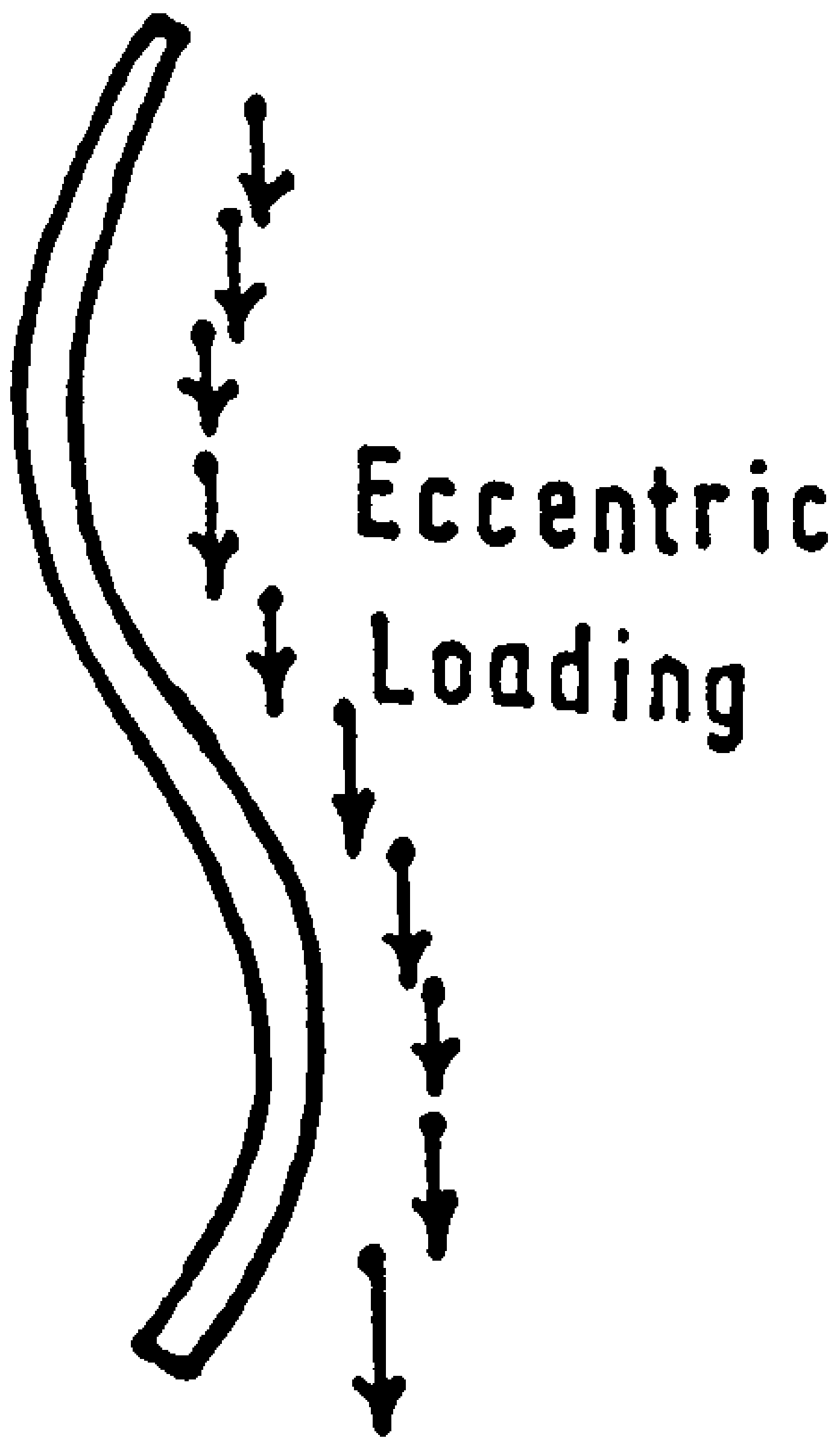


Fig.32

column. The novelty lies in the inclusion of a load path across the articular facets (represented by a spring connection). The discs were visualised as springs and dampers acting to resist axial, shear and rotational movement. Each rigid body carried a portion of the weight of the torso and was aligned to simulate the spinal curvature. The second model also included an attempt at representing muscle action, the forces generated by the muscles were calculated as a linear function of stretch and stretch rate. The attachment of the force generators to individual vertebrae allowed the simulation of separate muscles groups. It was found that in high level acceleration (>10g) the muscles were ineffective in affecting overall spinal column kinematics, although the forces developed significantly increased the vertebral stresses.

2.1.3.9. Cramer, Liu and von Rosenberg.[64] Fig. 32

A curved homogeneous beam model with distributed eccentric loading as in 2.1.3.8. The mathematical equations are solved using finite difference techniques. As the model is based on a continuum structure it was able to calculate stress levels at particular points along the rod. Stress patterns and deflections were produced for a vertical acceleration pulse. The results suggest that vertebral fractures can occur before activation of the restraint system in a pilot ejection situation.

2.1.3.10. Chen. [65] Fig. 33

This is a dynamic version of the static model described in the next section (2.2.4.) A full 3-dimensional geometrical finite element model was used to study the response characteristics of the thoracic wall and ribcage in the simulation of frontal chest impact as in car accidents. The model does take into account the visceral contents of the chest.

Three-Dimensional Response.

2.1.1.11. Rizzi, Whitman and De Silva. [66] Fig.34

The model of a ligamentous spine as a composite of directed curves is developed. The discs are represented by one curved rod of viscoelastic material with fading memory. The vertebrae and ligaments are represented by a second curve of elastic material. By combining these two curves full 3-dimensional movement can be analysed.

2.1.3.12. Panjabi. [67] Fig.35

Suh and Seo. [68]

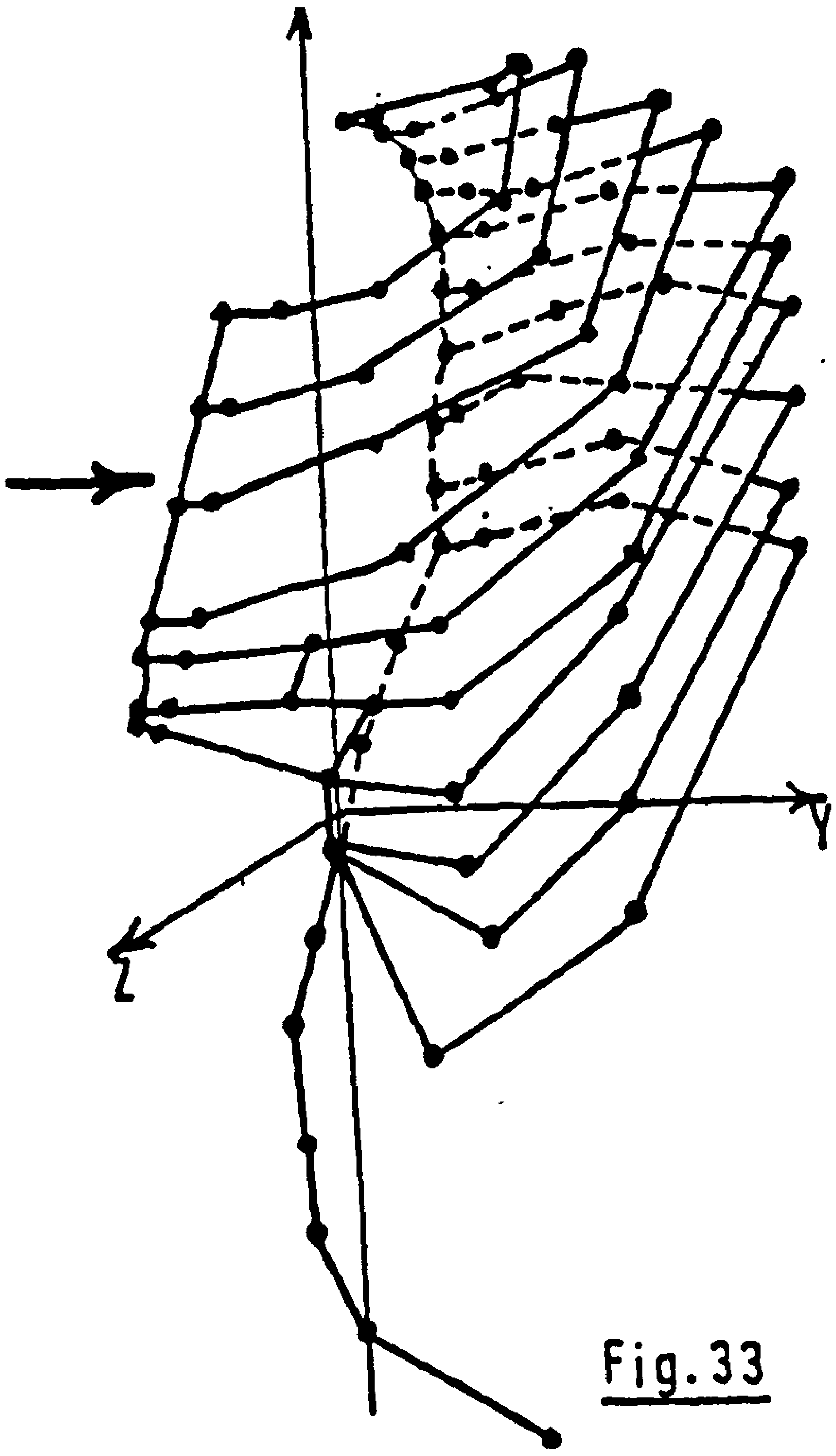
These models consist of a three-dimensional system of masses, springs and dampers. The proposal by Panjabi also includes viscoelastic elements but as yet no practical model exists. The latter model has been used to study the kinematics of the head and neck only.

2.1.3.13. Belytschko, Schwer and Privitzer. [69]Fig.36

This model is a development of the static model described in the next section (2.2.7. [87]). A finite element model to represent the whole spine, head and ribcage is developed. Hydrodynamic elements are also included to simulate the viscera. The model has been used to study pilot ejection and the effect of an eccentric head mass. The axial forces and moments in body segments have been related to stresses and thus sites of potential vertebral injury can be predicted.

2.2. Static Models.

The inter-disciplinary co-operation that has marked the progress of biodynamic modelling has become evident in this area in particular in the work of Schultz et al.



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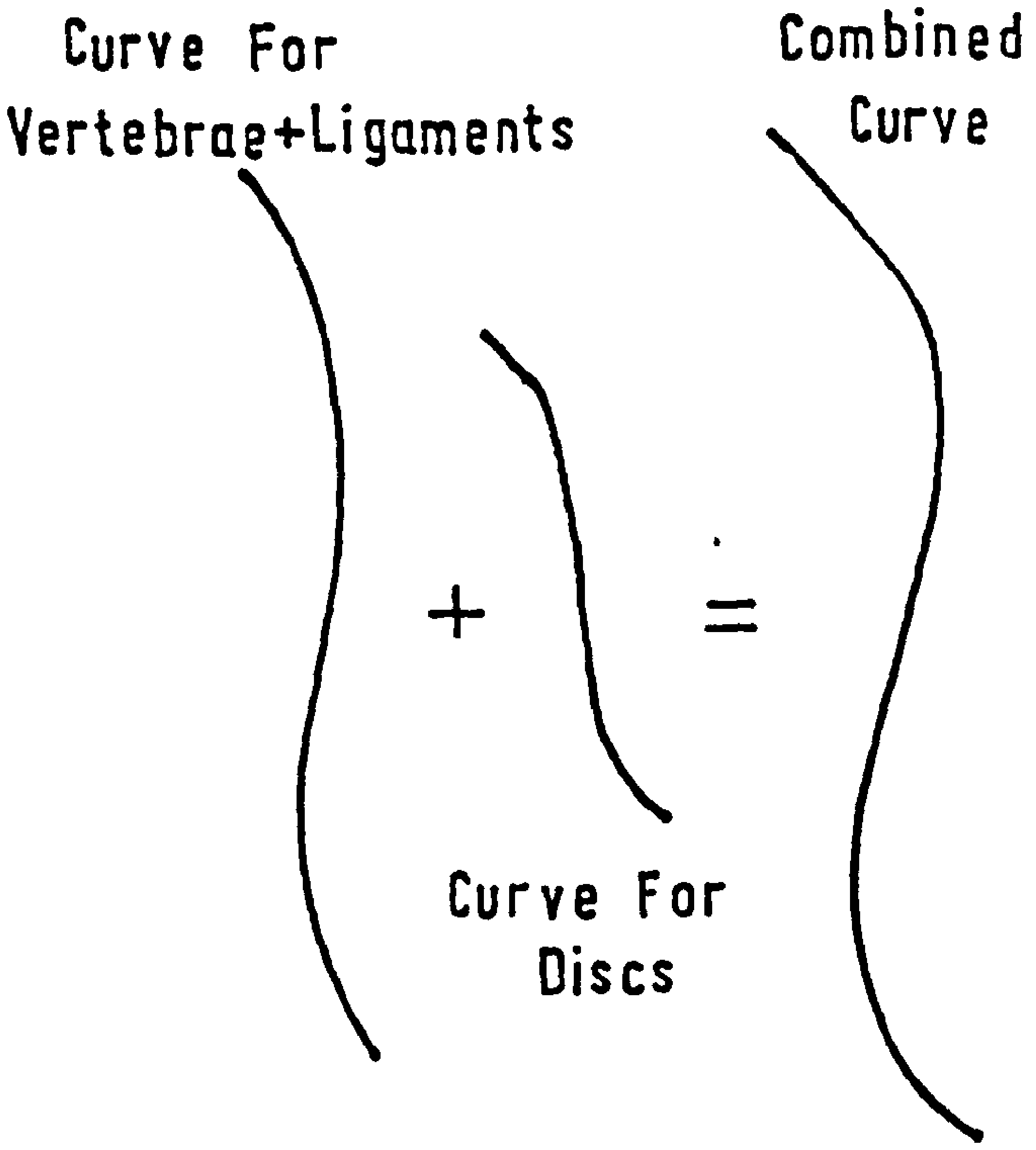

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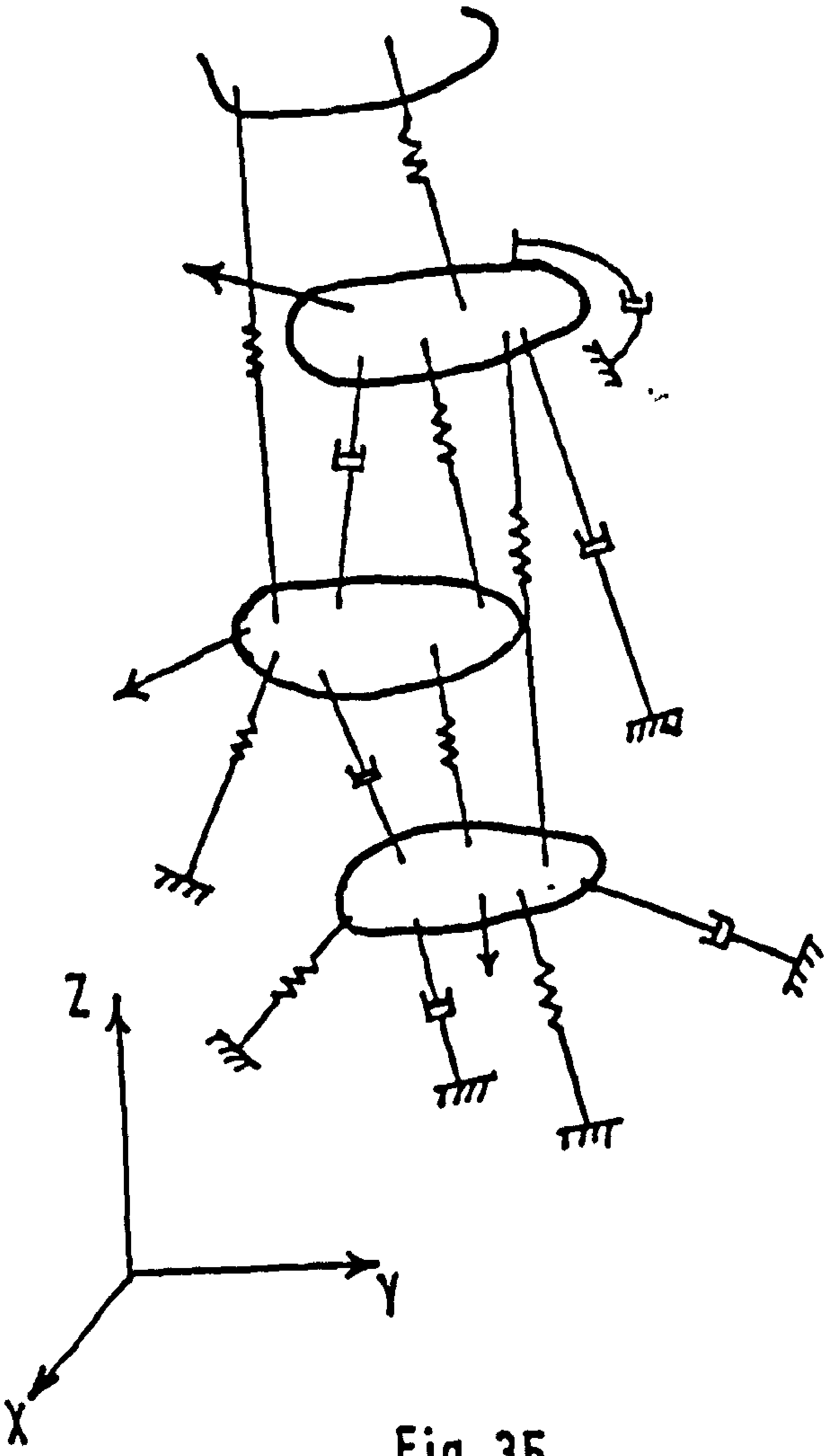

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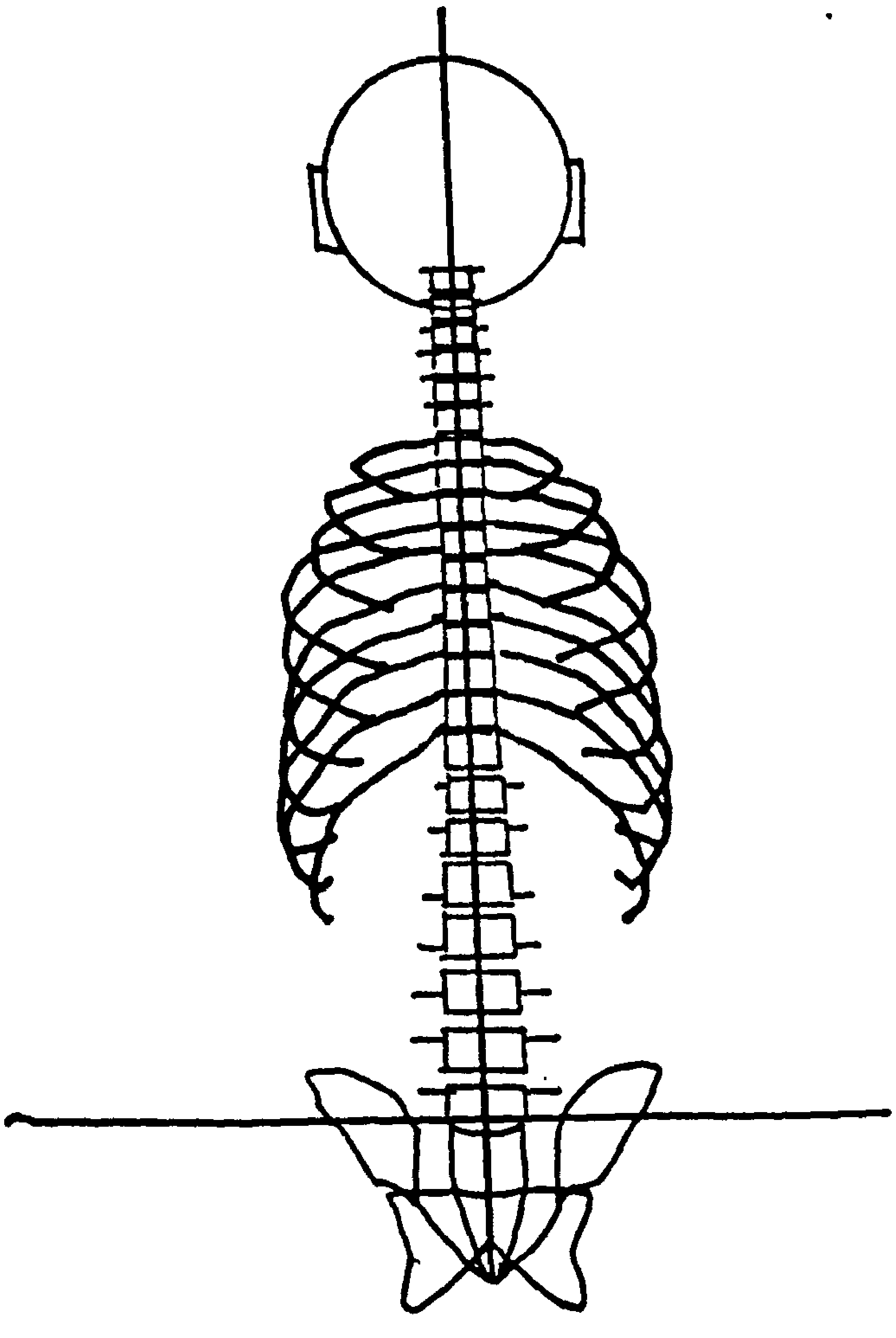


Fig:36

But still much of the work has been produced by independent groups which have sought to quantify ergonomic data or apply a new technique of analysis.

Models of seven groups of researchers will be described in this section:

2.2.1. Chaffin. [70] Fig. 37

This model is a static version of the dynamic simulation presented by Fisher. (2.1.1.3) The body is visualised as a seven element linkage and is used to evaluate situations of lifting, pushing and pulling. The masses and centres of mass of the links were derived from anthropomorphic data. The geometric data for the position of the links are obtained from either a lateral photograph or a body template set to the task position. Joint reaction can then be calculated from the equations of equilibrium relating internal and external applied forces.

In one study the rigid link for the back was replaced by a suitably curved segmented column. An analysis of lifting was carried out, in which the force on the lumbo-sacral joint was calculated with respect to hip rotation, a graph was then produced showing the maximum weight that can be held in the hand for a

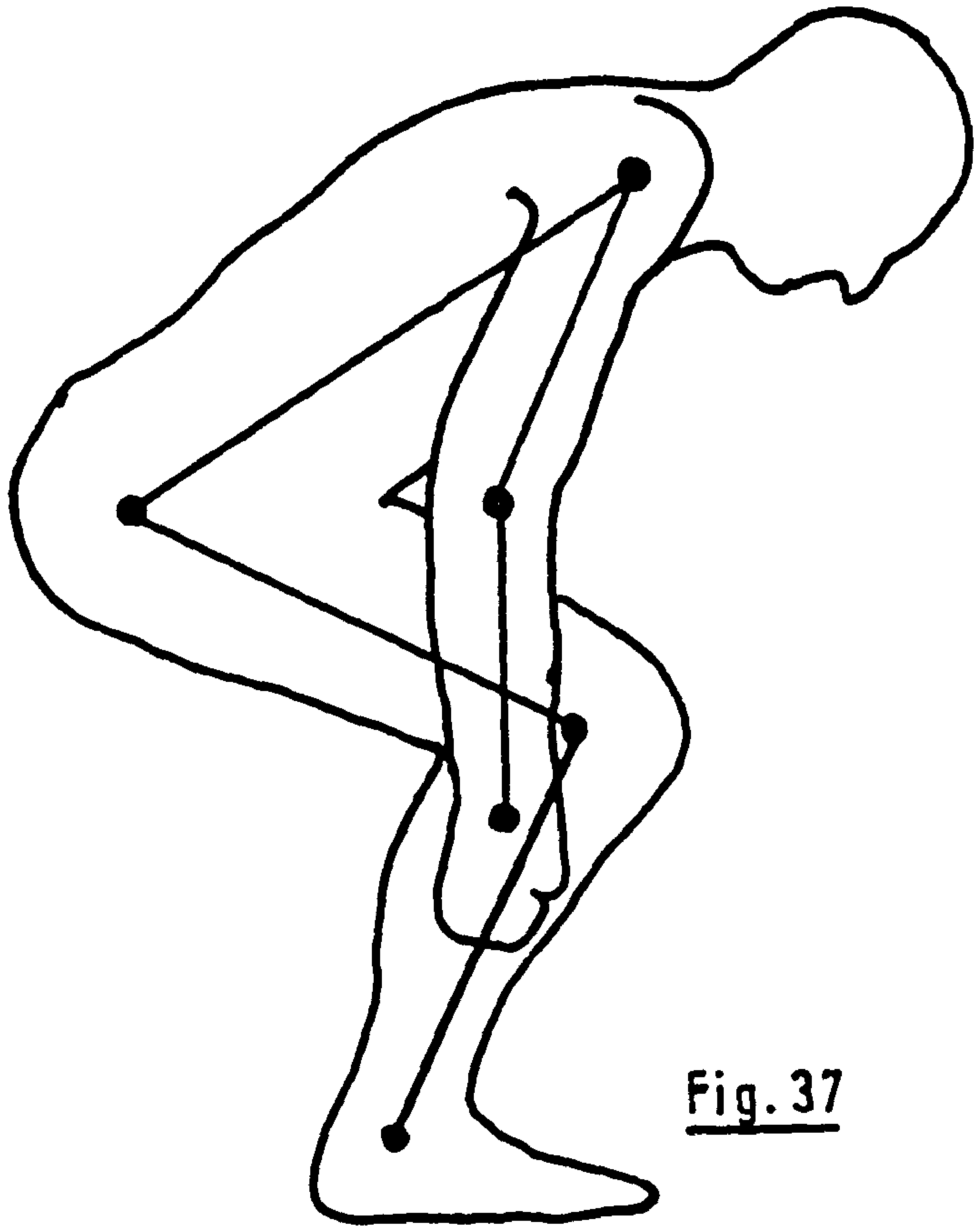
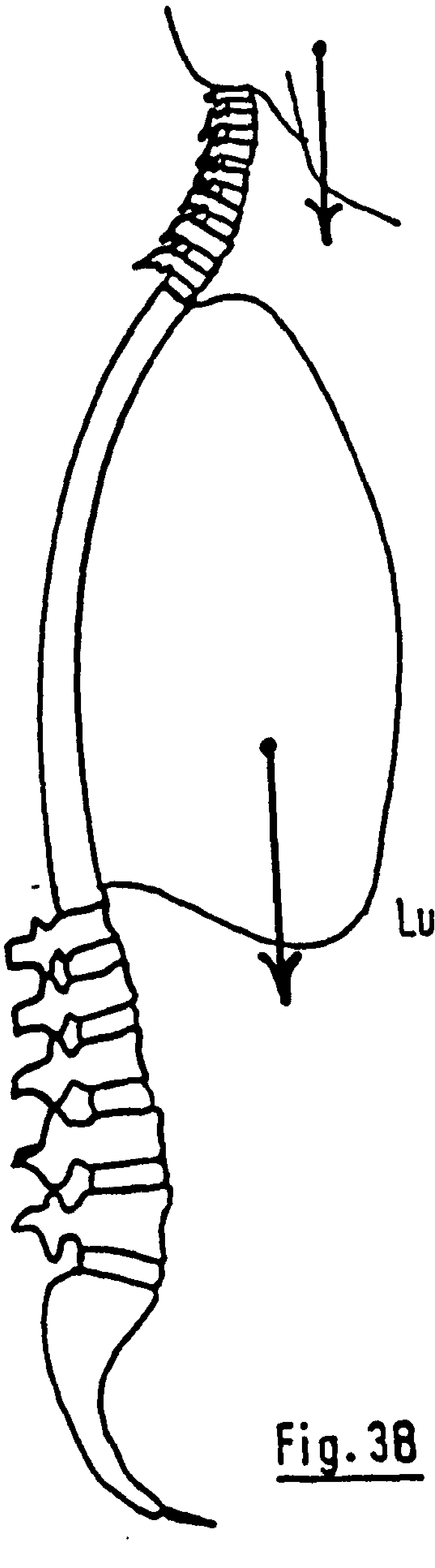


Fig. 37



Lumbar Muscles:
Multifidus
Rotores
Psoas Major
Erector Spinae
Quadratus Lumborum

Fig. 38

specified hip rotation e.g. 0^o hip angle (forward flexion-horizontal) a weight of 445N can be held and this creates a compression force on the $L_5 - S_1$ disc of 5680N. An equation was also proposed relating abdominal pressure to hip rotation, although for the lifting task a set force relief on the spine, due to abdominal pressure, of 25% was used.

A more recent addition to the model's capability has been the facility to plot contours of force exerted by the hands in various positions for pushing, pulling and lifting. This is described by Martin and Chaffin [71], the technique employed uses a binary search procedure working to a criteria of applied maximum muscle torque about the link joints for their relative position, which is derived from Chaffin and Baker. [72]

An example: for a 50th percentile male lifting in the standing position.
Height of hands above ankles 508 mms
Horizontal distance from hands to ankles 762 mms
Then a force of approximately 135N can be lifted.

2.2.2. Seireg and Arvikar. [73] Fig. 38

The theory behind this model is both simple and original. The model of the vertebral column is an extension of a simulation that has been used to analyse the lower extremities of the muscoskeletal system (Seireg and Arvikar) [74]. The lumbar and cervical vertebrae are visualised as individual rigid blocks with the thorax represented as one body. The muscles are specified by vectors acting at their points of attachment, reactions and moments acting on the vertebrae are located at the centre of the vertebral body.

The body posture is defined and the equations of equilibrium developed for each body segment. The number of unknowns exceeds the number of equations, implying a range of solutions are possible. A criterion U is formulated as:

ΣMuscle forces + 4 Σreaction moments + 2 Σtensile reaction at joints.

The minimization of this function locates a suitable answer. The method of minimization used is that of Linear Programming.

The function used was found by comparing a variety of possibilities of equations with EMG studies in previous experiments.

The model's results have been compared to the empirical investigation of intra-discal pressure carried out by Nachemson and found to give good correlation. The values of muscle force and reaction obtained from the model relate to the vertebral geometry and a set posture, thus it would be impossible to ascertain any muscular imbalance or deficiency that may be present in a specific person.

A recent development by Williams and Seireg [75] has been the inclusion of EMG patterns from muscles to act as a comparator for the muscle forces obtained mathematically, if there is a disagreement between the two the geometry of the model is altered until reasonable results occur. The model can in this way be related to a specific person. As yet this has only been tried out on a model jaw.

2.2.3. Farfan and Lamy.[76] Fig. 39

The model developed by this group is concerned with the lumbar region only. The thoracic trunk is considered as a rigid connection. The aim of the model was to calculate the compression and shear forces on the lumbar vertebrae for a person flexing or performing a dead lift. To do this the equations of equilibrium for the vertebrae were established. Included in the forces acting on the spine are those derived from the ligaments, body mass and muscles. The posterior ligaments were taken to act after 40° of flexion had taken place and assumed that they were slack in the upright position. Fourteen muscles were allowed to act on the column with true origins and insertions in the lumbar region. The muscle force generated was calculated as a function K of the cross -sectional area of each muscle. From EMG studies it was established which muscles were in operation in a particular manoeuvre and these were used to balance the moment arm of the mass of the body about the joints. The geometry for the system was established from photographs of a subject to gain the overall configuration and from cadaver cross-sections to gain the vertebral and muscle sizes.

In forward flexion of the spine the first 40° of movement was taken to occur in the spine and after that the movement was obtained by rotating the pelvis. The maximum value for K was found to be 208 KPa for the extensor muscles at the L_5 level also a maximum compression force of 2880 N at L_3. The shear values calculated showed that the value of this force was greatest at L_5 and this is the area in which spondylolisthesis has its highest incidence. The model was also used to study lifts of 133, 580 and 1700N and

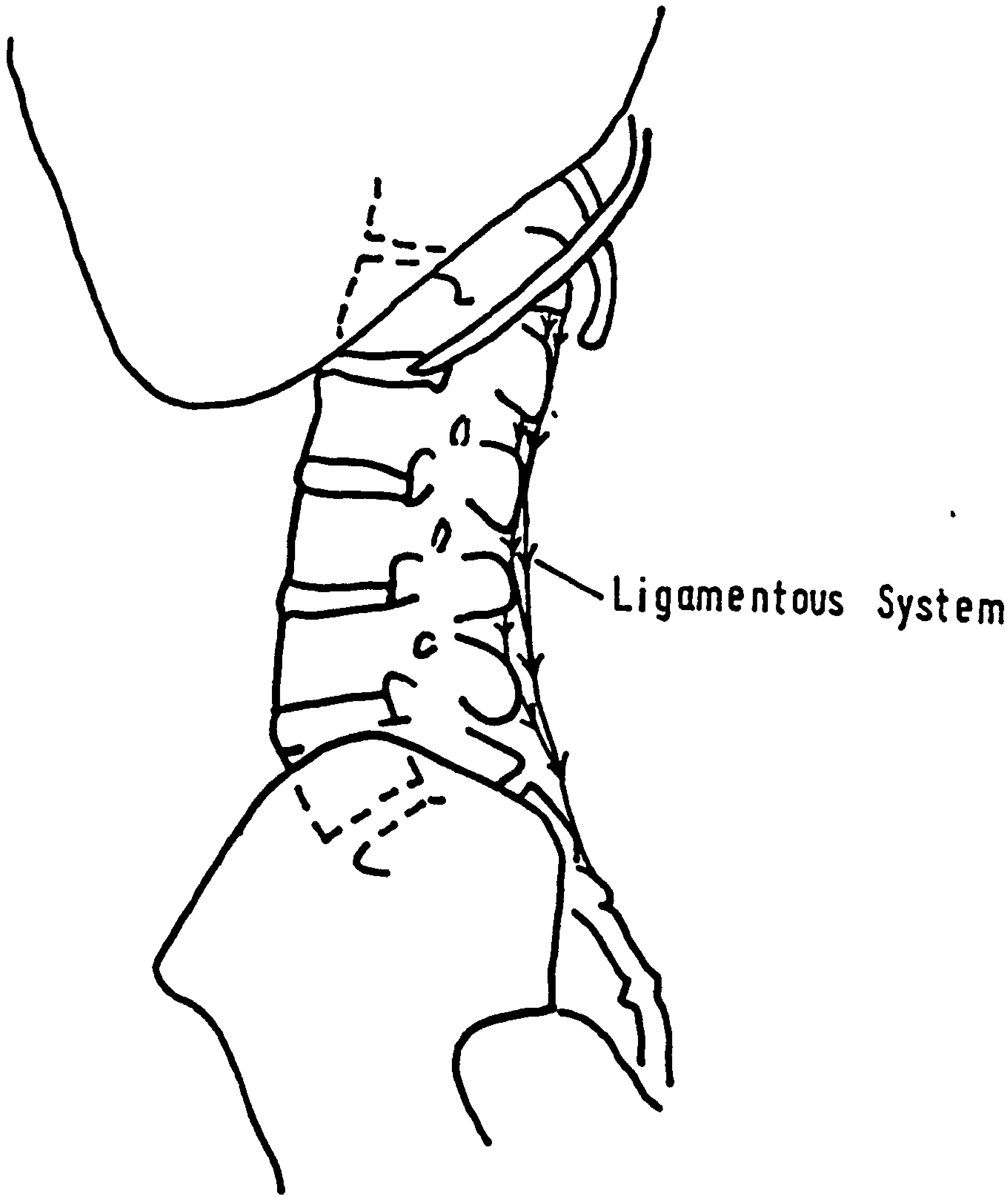


Fig. 39

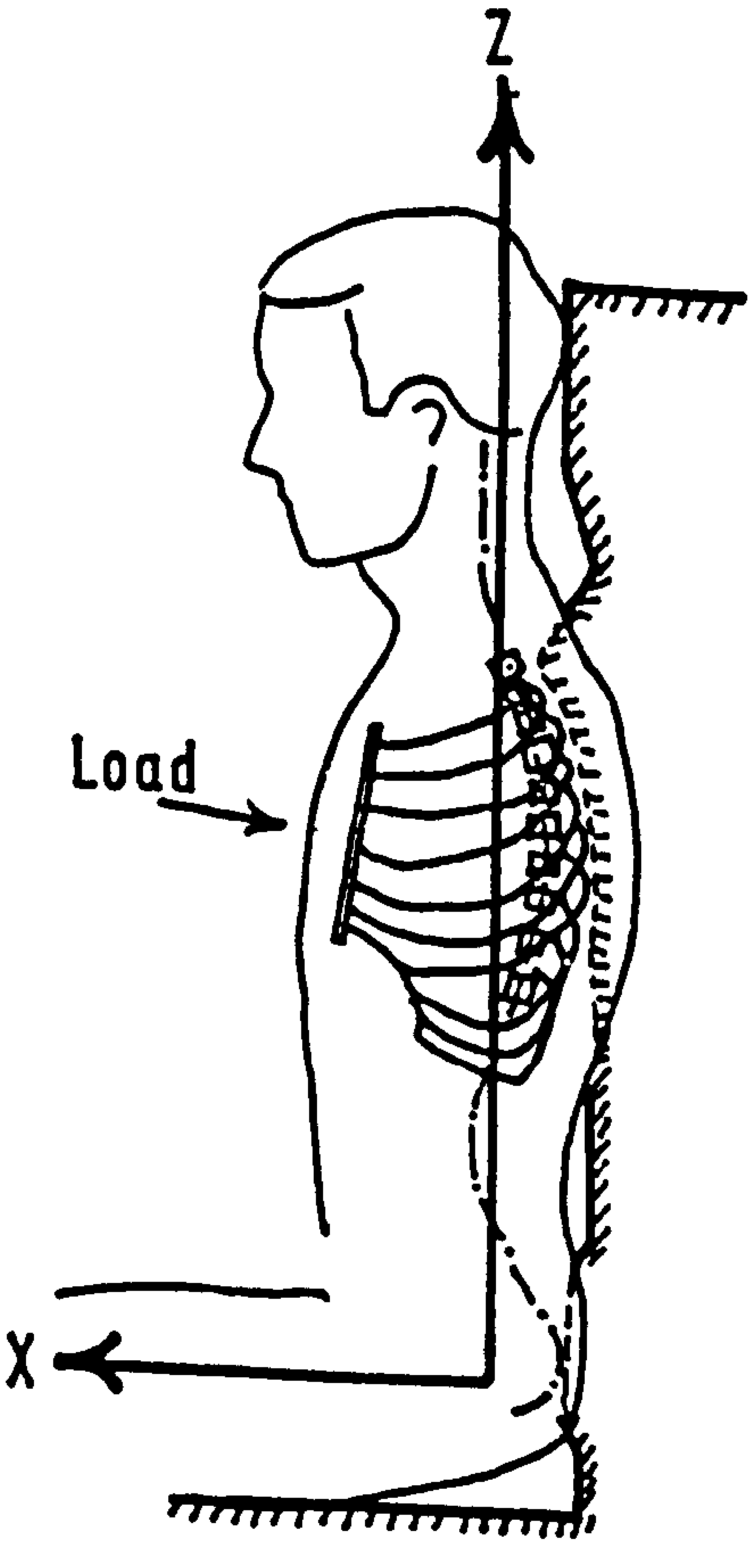


Fig. 40

correspondingly higher levels in force value were obtained although within vertebral, ligamentous and musculature limits.

2.2.4. Roberts and Chen. [77] Fig. 40

The model proposed by this group was probably the first using the "finite element" method of analysis and studies the deformation of the ribcage caused by a load applied to the sternum of a seated person, to simulate the situation of a car accident.

The geometric data for the model was obtained from a small skeleton. Compact bone and cartilage are assumed to be composed of elastic homogeneous and isotropic material, the values for the modulus of elasticity and shear modulus were obtained from Evans [78] . All soft tissue is neglected. The shafts of the ribs are taken as elliptical in cross-section and hollow, the geometry being obtained from their own experiments. The costal cartilage at the end of each rib was modelled as solid and elliptical. Zero flexibility was assumed at the costo-vertebral junction and for the sternal angle. The sternum and vertebrae were taken to be composed of compact bone.

The technique of a finite element analysis demands the decomposition of a structure into various elements, for this model beam type elements were chosen. A vertebra was depicted as possessing a node on the inferior and superior faces of the vertebral body, while the ribs were described by five nodes along their lengths, the nodes are connected by the beam elements. The complete model used 169 nodes to describe the thoracic cage and vertebrae in three-dimensional space.

Using the displacement method of matrix manipulation the vector of nodal displacements was calculated for three loading conditions:

a) Uniform load of 445N over a 152 mm dia. circle centred on the mid sternum line.

b) A uniform line load of 445N along the sternum.

c) A concentrated load acting on the sternum of 445N.

It was noted that the sternum tends to move as a rigid body and a 2 degree of freedom model was proposed to represent this. From the stress analysis of the model for the loading conditions it was found that the highest stresses occurred in the costal cartilage for ribs 1-10 and between the angle and tubercle for ribs 1-7. The values of the stresses were found to be unrealistically high due to the simplifications required in developing the model.

2.2.5 Sundaram and Feng. [79] Fig. 41

This recent model is very similar to that proposed by Roberts and Chen (2.2.4.) and uses geometry and material properties derived in part from their work. Two models are described, one consisting of the ribcage and spinal column while the second includes simulation of the muscles of the thorax and also internal organs. Due to symmetry only half the thorax was represented by the models.

Beam type elements were used to represent the vertebrae, the intervertebral discs, the sacrum and coccyx individually, but the ribs were modelled by a series of 4 beams. The costal cartilage was also represented by beam elements. The sternum was modelled by 6 thin plate elements. In the second model the thoracic muscles were constructed from 135 isoparametric plane stress 3-noded membrane elements and the internal parts (heart and lungs) by 23 isoparametric 8-noded solid elements. The muscles represented include the rhomboids, serratus anterior, pectoralis, intercostals, serratus posterior and transverse thoracis, material properties were assigned to these passive elements for both compression and tension which does not occur in reality. The material properties and geometry for the hollow elliptical cross-section ribs were taken from Roberts and Chen. The cartilage was modelled by equivalent beam elements to take into account the difference in Youngs Modulus for tension and compression, e.g. $E_{tension}$ 24.1 MPa and $E_{comp.}$ = 482 MPa therefore the equivalent E was calculated as 64.4 MPa.

The displacement method of analysis was used to evaluate a number of static loading conditions. For two cases loads of 222.5N was applied to the sternum, (a) adjacent to rib 2, (b) adjacent to rib 6, and for these cases there was a reduction of 20-35% in the posterior movement of the sternum when the internal organs and muscles were included. For other loading cases there was also a reduction in movement when the extra elements were included.

Cartilage tensile stresses reached a maximum at rib 1 for loading case (a) of 10.5 MPa and for case (b) the maximum reached at rib 6 was 11.2 MPa. The ribs were found to attain a maximum stress in the region between the angle and the neck for sternal loadings, e.g. case (b) max. bending stress in rib 4 of 74.9 MPa. It was found that the region between the 3rd and 5th ribs was the site for highest muscle loading especially

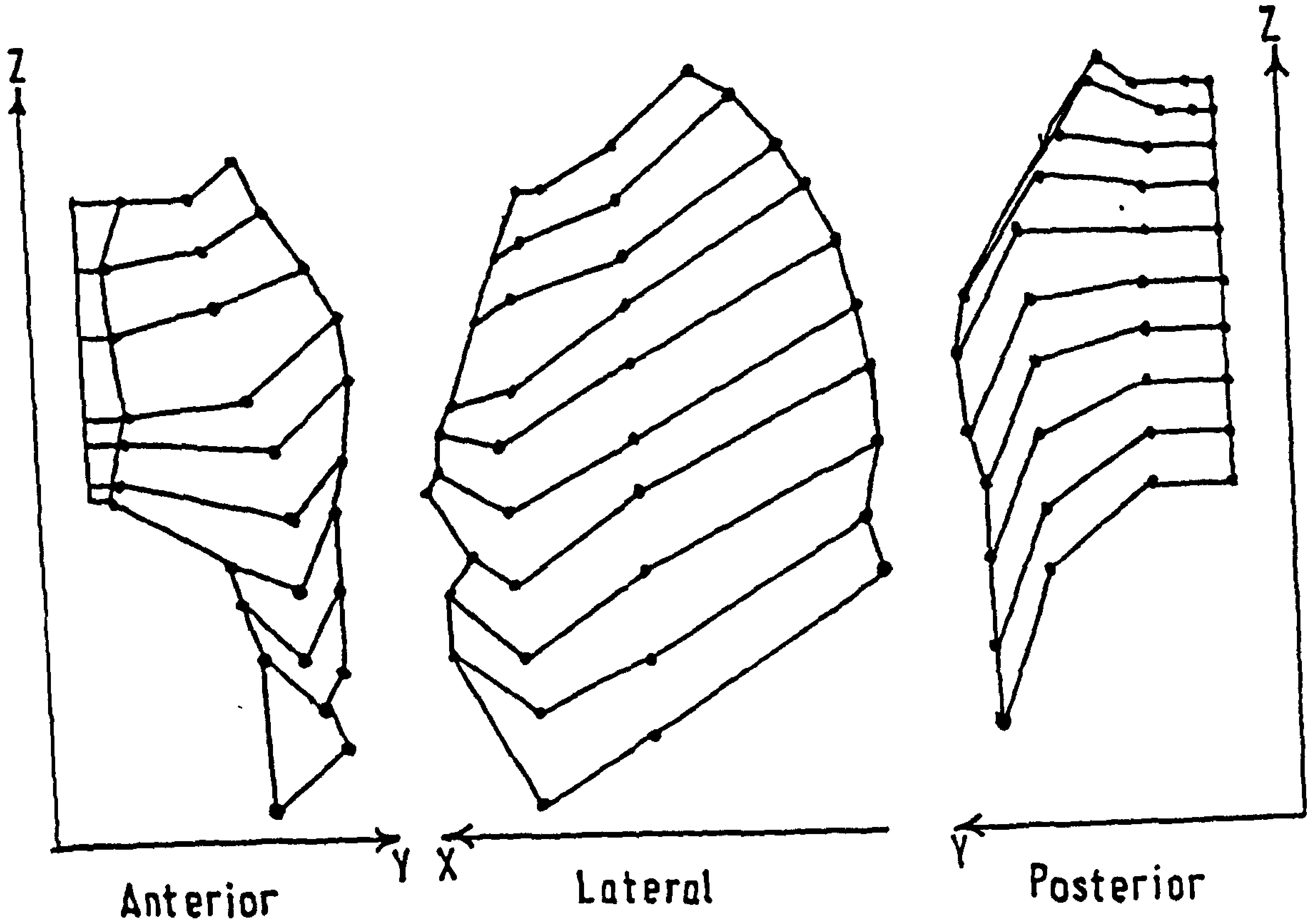


Fig. 41

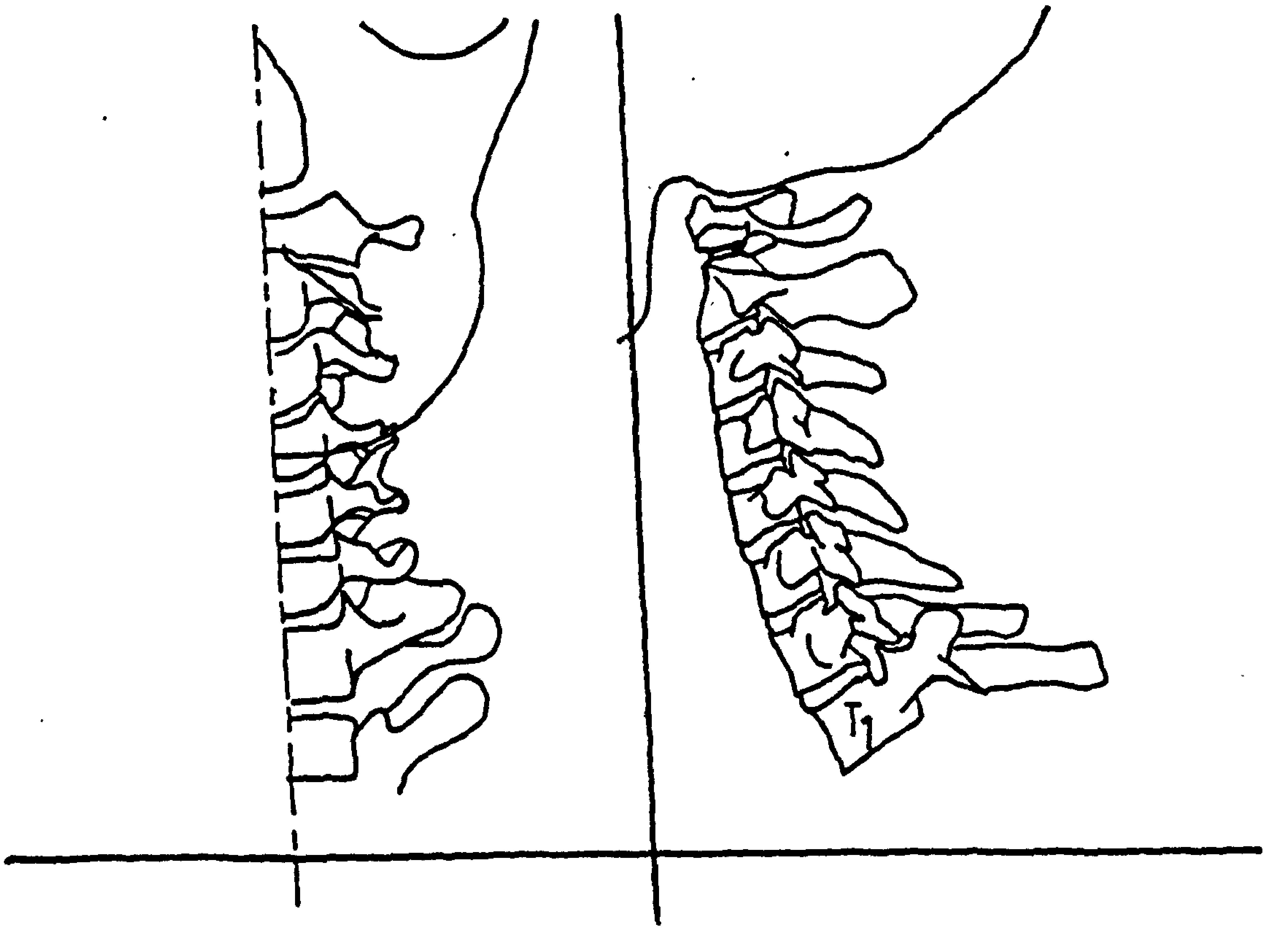


Fig. 42

near the mid-axillary line.

The conclusion from the results was that for loading case (a) there was a possibility of fracture of the first rib and muscle injury overlying ribs 3-6. Injury to the internal organs was also possible but there was little likelihood of sternal injury. For case (b) it was likely that the sternum fractured at the manubrium junction and compression occurred of the heart. The 3rd and 4th ribs in the region of the tubercle and neck and the 6th and 7th in the vicinity of the angle would be subject to fracture.

2.2.6. Hong and Suh. [80] Fig. 42

This model has been used to analyse the functions of individual cervical muscles and to study the effectiveness of these muscles in producing specified movements of the head.

The model consists of a series of rigid bodies (vertebrae) interconnected by deformable elements to represent the intervertebral discs and ligaments. To cope with the large deflections and rotations that are encountered in spinal mechanics, non-linear equations describing the motion explicitly are used and solved for using an iterative method. The Newton-Raphson iterative technique is used, either in its basic form or slightly modified. The elements constituting the model are either spring (axial forces only) or disc elements (having resistance to both forces and moments), these are developed for full 3-dimensional movement.

The analysis is carried out in the following manner:-

1. The basic elasto-static system is set up.
2. The displacement matrix is formulated (non-linear and a function of relative movement and rotation (unknowns)).
3. The displaced system of equations is generated.
4. The equations for the forces in the spring elements is formulated.
5. This is repeated for the disc elements.
6. The 6N non-linear equations for equilibrium are generated from the internal forces of the springs and discs and from the external applied forces.

N is the number of rigid bodies, which represent the "non-deformable" vertebrae.

7. The overall system is formulated by adding the bodies in a group sequential manner e.g. body 2 is added to body 1, then body 3 is added to bodies 1 and 2 and so on. This method aids in the convergence of the iterative process.

8. The Newton-Raphson (basic or modified) is used to solve the equations.

Due to the displacements being calculated directly equations for the non-linearity in material properties for the ligaments (springs) and discs can be included.

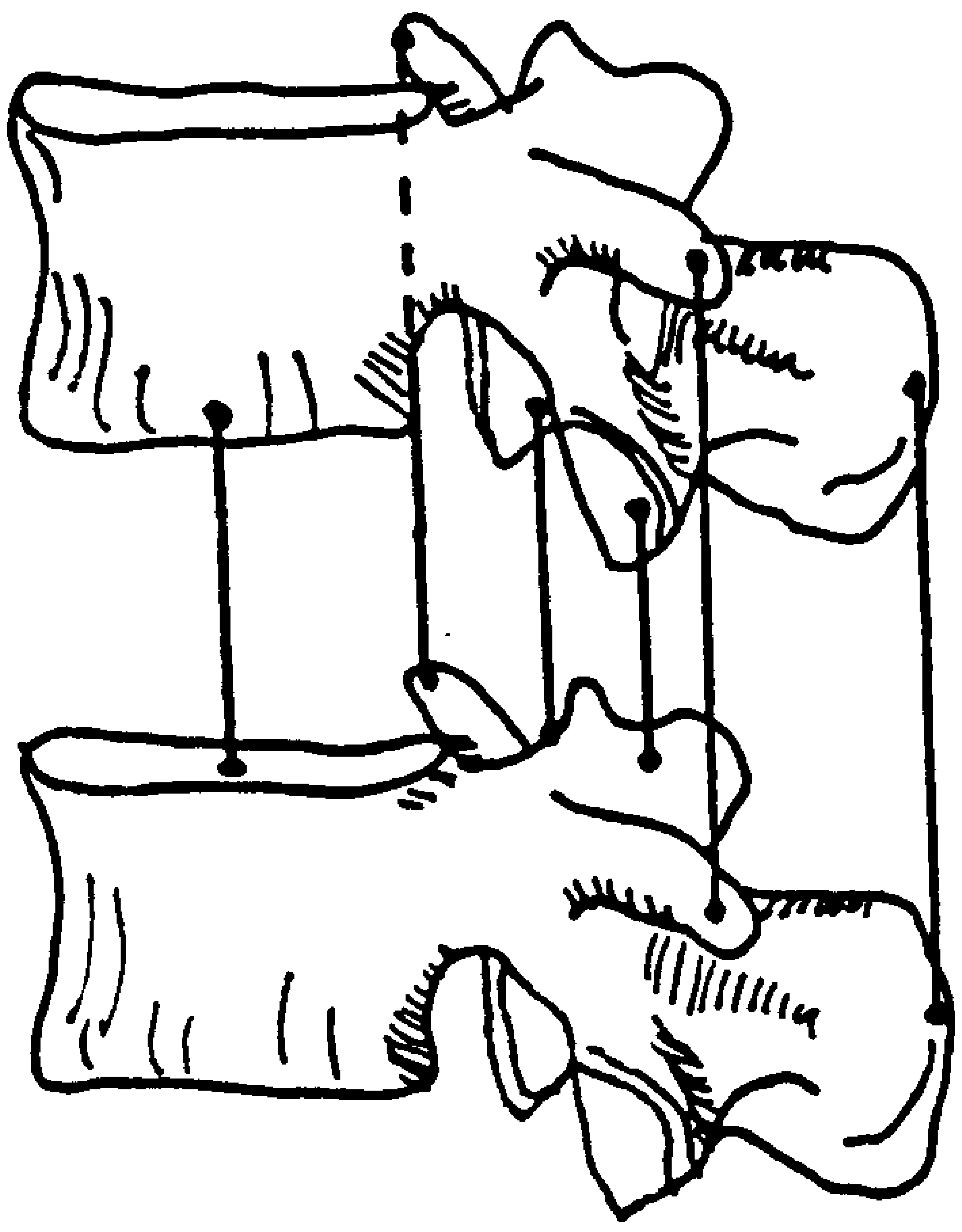
Muscles are attached to the head and vertebrae at their correct points of origin and insertion and are represented by straight lines joining those points. By applying forces along the line of action of the muscles the contribution of each muscle to overall movement of the head can be deduced.

The conclusions arrived at are that the most effective flexor is the sternocleidomastoid, the effective hyperextensors are the trapezius, splenius capitis and semispinalis capitis, the effective rotator to the same side is the splenius capitis and the rotators to the opposite side are the trapezius, multifiders and rotatores. The functions of the rectus capitis posterior, obliquus capitis superior and multifiders are only significant when they are acting not in pairs but in the single-sided mode.

2.2.7. Schultz, Galante, Andriacchi, Belytschko and others. Fig. 43

The work of this group has been orientated toward the understanding of spinal mechanics and the analysis of spinal disorders. To this end the use of computational manipulation, both simple and complex, has proved a success.

The models produced are passive i.e. the action of muscles in producing bending and rotation has been neglected. The first model [33] deals solely with the geometric configuration of the vertebrae. Six points are fixed on the superior surface of each vertebra and six on the inferior surfaces, the co-ordinates of the six points relative to each vertebra are known. The spatial co-ordinates of only the lowest vertebra are known. Six fixed length elements connect the respective pairs of points, thus the body immediately above the first vertebra can also be positioned in space and the global co-ordinates of both bodies are known. The computer is used to position the bodies one over the other for the complete column. To make possible the study of spinal movement the lengths of some of the interconnecting elements are changed. By changing the length of one element it is possible to create a



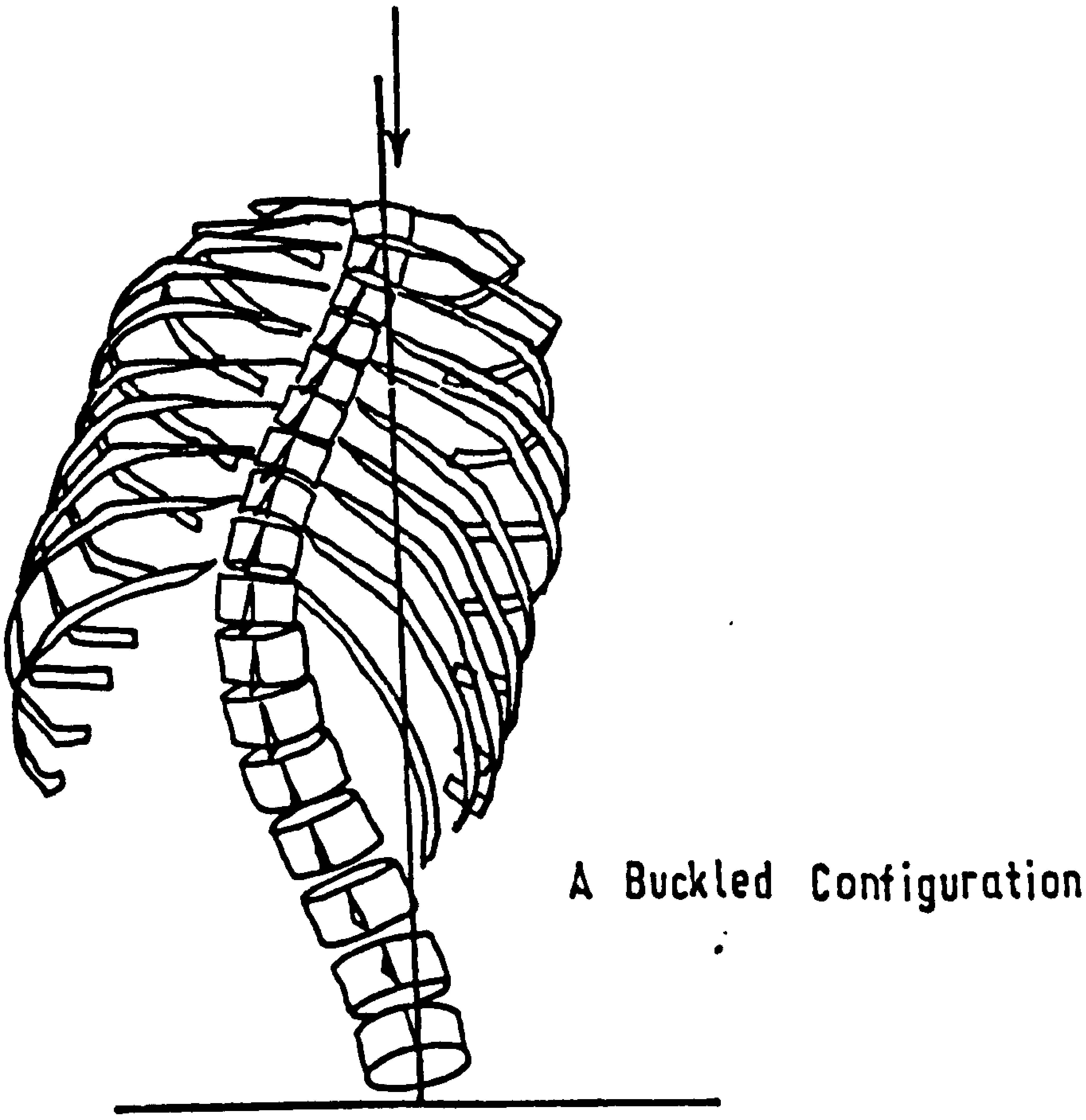


Fig. 43

position in which there is no exact solution fixing the upper vertebra in space, if this is the case a solution is achieved by altering all the elements until a feasible set of equations exists, the alteration of the lengths is carried out to minimize the error that exists between the initial and final set of element lengths. The inter-connecting links are positioned to represent the connecting soft tissues in a human spine. The movement of the spine in flexion, extension and rotation and the interaction between the motions was studied. It was found that the model could represent the movement of the spine using soft tissue deformations that were reasonable. The experiments were compared with published findings in cadaver tests and in-vivo studies.

A further study was made of scoliotic spines [81] to find possible causes of the lateral curves. The model was used in relation to five patients, the lengths of the various elements being changed to fit the simulation's configuration to that of the subjects. The results showed that the cause could lie in the region of the deep back muscles, i.e. changes in the inter-connecting links between the transverse processes and also in the rotator elements could cause a curve to develop. The tips of the spinous processes of the vertebrae always tended to be brought into a straight line by the scoliotic configurations in the model, it also became apparent that mild curves lie within normal motion capabilities.

The second mathematical model [82] was used for a three-dimensional structural analysis of the vertebral column. The vertebrae are idealised as rigid bodies, while the discs and ligaments are represented by deformable elements e.g. bars and beams. The material properties for the elements were obtained from published reports, and trial motion segments ($L_{3/4}$ and $T_{8/9}$) were tested using the model, and the material properties were adjusted so that the model behaved in a similar manner to published findings from cadaver tests. The material properties were taken as quasi-linear and no provision was made for visco-elasticity, although it is possible to include these factors if required. The non-linearities that occur in the system due to large deformation, were treated by a procedure of incremental linearization combined with equilibrium checks. Thus the non-linear characteristics were reduced to a series of small linear steps, and if a state of non-equilibrium was obtained it was corrected by the addition of small external forces. The vertebrae were joined in a similar manner to the previous model, only in this case eight connecting elements were used.

The response to lateral and compressive loads were investigated and compared with the results from the cadaver tests of Lucas and Bresler [83]. Buckling occurs in the ligamentous spine under a compressive load of approximately 20N. A scoliotic spine has also been modelled and this was achieved by adjusting the initial lengths of the elements to match the configuration of the curve. Traction applied to the column was found to reduce the lateral deflection and rotation of the vertebrae. A further study [84] investigated the movement and force deformation properties of complete motion segments. The ligaments and facet joints were found to play a significant role in resisting loading in flexion-extension and torsion, this in particular being due to their lines of action operating at a distance from the centre of rotation.

The techniques for correction of idiopathic scoliosis have also been analysed [85]. Four scoliotic spine configurations were prescribed for treatment with Harrington rod and lateral-force correction. The effect of surgical attack on the discs in reducing their resistance to correction is described. The posterior elements perform an increased role in resisting movement as the discs are progressively cut. Derotation of the vertebrae in the treatment by Harrington rod was shown to be insignificant.

A recent extension to the model has been the inclusion of the ribcage [86]. A model of 234 degrees of freedom was derived. The ribs and sternum were modelled as rigid bodies but stiffnesses were assigned to the costo-vertebral and costo-transverse articulations and costal cartilage. The simulation was validated against experimental results for the deformation of the ribcage. Lateral deflection was found to be in good agreement but in some cases the anterior-posterior displacement was significantly different. (V/5 the results of Agostini et al [87]). The resistance to bending and buckling was found to be increased over the earlier model. With the T_1 vertebra unrestrained the buckling load occurred at 80N. The tractive stiffness of a scoliotic spine was also increased but the buckled configuration of the column resembled that of a scoliosis curve.

This section has briefly described the work performed in connection with the complete vertebral column, other models do exist for the investigation of disc-vertebra properties.

CHAPTER 3.

Material Properties Of The Spinal Column And Of The Connecting Tissues.

The study of the motion of the human spine has been of direct interest to the clinician because of the possible relation between restriction of movement and low back pain. Many studies have been conducted since the early work of Bakke [88] and examples of these are the investigations performed by Lysell [89], White [90], and Rolander [91] on the cervical, thoracic and lumbar portions of the spine in autopsy specimens respectively and of Tanz [92] and Gregersen and Lucas [93] in living subjects.

Unfortunately, experiments carried out on the mechanical properties of the spinal column have been performed with very little uniformity in method or in tabulation of the results; but again the prompting for this type of investigation has come from a need to locate a source for low back pain. A further cause of discrepancy between the findings of different groups has been the use of cadaveric material which has been subjected to differing methods of storage, e.g. on the one hand fresh specimens have been used [94] while on the other, material has been refrigerated for several weeks [95] or embalmed. [96].

A major problem associated with the study of biological material is the range of properties that can exist. Partly this is due to the lack of uniformity between people, and, for example, rather than' specifying a Young's Modulus E based on overall volume a more suitable method would be in ascertaining E for a material of a stated fibre density. Collagen fibre is the main load resistant ingredient in the soft tissue [97] and the content of this material varies from ligament to ligament and from person to person. Also biological substance is dependent upon the health and age of a person, and it has been shown in a number of studies how the mechanical elasticity can decrease with ageing [5],[98].

An important point to take into consideration is this: to what end will the material properties calculated be used? They can be used as a medical indication of the anatomy and biology of the spine or as a numerical value on which the overall kinematics of the body system can be developed. For example, for a mathematical model of the spinal column to be developed it is of prime importance to ascertain the properties of a complete motion segment i.e. two vertebrae with the inclusion of the intervertebral disc between them, and not of individual elements.

Naturally as the anatomical authenticity of the models increase more demand would be placed upon separate elements to have a correct set of mechanical properties.

The chapter will continue by discussing in greater detail the various properties that have been derived from experimental work and this will be done in sections related to the particular aspects relevant to the establishing of a mathematical model.

3.1. Motion Segment Properties.

A motion segment must include the posterior elements of each vertebra, otherwise the test is reduced to a study of the disc/vertebra interface alone. The observation by Hirsch and Nachemson [99] that the posterior elements play no significant role in weight-bearing up to an axial load of 2000N is no excuse for removing them. Indeed that observation is in doubt as the experiments of Hakim and King [100] show that the facets can transmit up to 25% of a statically applied axial load.

The work of a number of researchers is listed overleaf.

Author	Region	Type of load
White (1969) [90]	Thoracic	FE,HPR,L,TC.
Panjabi et al (1976)[101]	"	FE,HPR,L,TC.
Roaf (1960) [102]	Thoracic-Lumbar	FE,HPR,L,TC.
Sonoda (1962) [98]	" "	HPR,TC.
Eie (1966) [103]	" "	FE,TC.
Higgins (1967) [104]	" "	TL.
Snijders (1970) [105]	" "	FE,TC,L.
Moffatt et al (1971)[106]	" "	FE.
Markolf (1972) [107]	" "	FE,HPR,L.
Kazarian (1975) [108]	" "	FE.
Koreska et al (1977) [95]	" "	TC,HPR.
Wyss & Ulrich (1954) [109]	Lumbar	FE,HPR,TC.
Hirsch & Nachemson (1954)[99]	"	TC.
Perey (1957) [110]	"	TC.
Evans & Lissner (1959) [96]	"	FE,TC.L.
Rolander (1966) [91]	"	FE,TC,L.
Smith (1969) [111]	"	TC.
Farfan et al (1970)[112]	"	HPR.
Farfan (1973) [19]	"	TC.
Carlson & Ball (1976)[113]	"	FE, L.
Pope et al (1977) [114]	"	FE,HPR.

FE = Flexion-Extension L = Lateral Bending

HPR = Horizontal Plane Rotation, TC = Tension-Compression

Fig.44 Compression Failure Of Motion Segments

Although the majority of his work was related to the movement and motion coupling of vertebrae, White [90] did suggest spring constants that would determine the relationship between load and deformation. Linkimg this with his mathematical description of the movement of the vertebrae about a helical axis, does give an overall description of the response of each vertebra to forced deformation. However, the assumption of a linear spring constant does limit movement to a particular range.

Rolander [91] conducted experiments on motion segments and studied the effect of posterior fusion. On normal segments the load was applied axially and also with degrees of eccentricity. However, he did not measure overall displacements or rotations directly but attached extensometers at various points, no graph was produced for overall compression but the results shown in Fig. 45 are derived from extensometer measurements of the points on the sides of the inferior and superior vertebral body.

Eie [104] tabulated values of load required to cause vertebral damage and then fracture (in longitudinal compression) for a number of subjects aged between 1½ and 58 years. Damage was taken to occur when blood and tissue fluid exuded from the vertebral body. The range of values recorded for the damage pressure (load/vertebral area) were between the limits of 2.75 - 4.2 MPa. Similar results were obtained in axial compression by Farfan [19]. The first signs of damage were always found to appear in the vertebrae. The data presented by Eie on the resistance to bending merely consists of the moment required to cause a rupture in the lumbar column, the sacrum fixed and increasing loads hung from L_1, and these results are not of any particular use.

This type of experimentation has also been conducted by Smith [109]. In his work the load required for failure in axial compression ranged between 2200 - 9670N and all cases showed some degree of fracture of the vertebral endplate. Perey [110] carried out a dynamic test of motion segments in which he dropped weights on to a preloaded specimen to establish methods of failure of the vertebral endplate. Although Roaf [103] also performed many tests he tabulated very few results, the one graph he does produce shows a load peak of approximately 5790N, which from the text, is the point at which a fracture of the endplate has occurred. It was not stated what size or type of vertebrae to which the graph was related.

The work of Evans and Lissner [96] clearly showed the difference in maximum load that can be achieved by an embalmed specimen as opposed to an unembalmed section. The ranges for these two conditions were 2720 - 6010N and 1290 - 3070N respectively. Their tests also gave values for maximum bending moment that could be attained for a section made up of several embalmed vertebrae, for example, in anterior bending for the entire lumbar region the values ranged between 69.8 - 86.8 Nm, in the lateral direction the values lay between 22.8 - 69.8 Nm.

Moffatt et al [106] tested the thoraco-lumbar spine, typically T_7 - L_3 in bending in the anterior direction. Their results were tabulated using units of flexural rigidity EI. The average value for the specimens was 2.25 Nm^2, this is similar to the value obtained by Snijders [105] for the spine in ventro-flexion of 2.1 Nm^2. For comparison a steel rod of circular cross-section with a diameter of 4mm will possess an equal degree of flexibility.

From the results discussed so far a simple linear relationship between deformation and load can be derived, but in the work there is little method or consistency and in some cases even incorrect units have been used e.g. pressure should not be measured as pounds or kilograms.

It would be quite in order to say that the early work of Wyss and Ulrich [109] is probably the most comprehensive. They did measure spinal column deformation when subjected to shear as well as the standard tests, unfortunately in some of their experimentation they did remove the posterior elements, and as many of the tests were for complete spinal sections e.g. $L_1 - L_5$ it is difficult to relate them to single motion segments.

Further tests on the spine in compression have been carried out by Higgins [102], Sonoda [98] , Hirsch and Nachemson [99] , Kazarian [108] and Koreska et al [95]. The load deflection graphs of a number of authors are drawn out in Fig. 45. Higgins varied the loading rate and claimed there was a significant difference, but this is not borne out by his results. The paper presented by Koreska et al gives no numerical data on their tests although they were able to note a difference between testing with and without the pedicles attached. A graph showing axial creep was produced for a spine section $T_{12} - L_4$, for an applied load of 736N the creep amounted to 0.1 mm over a 250 sec. period. Hirsch and

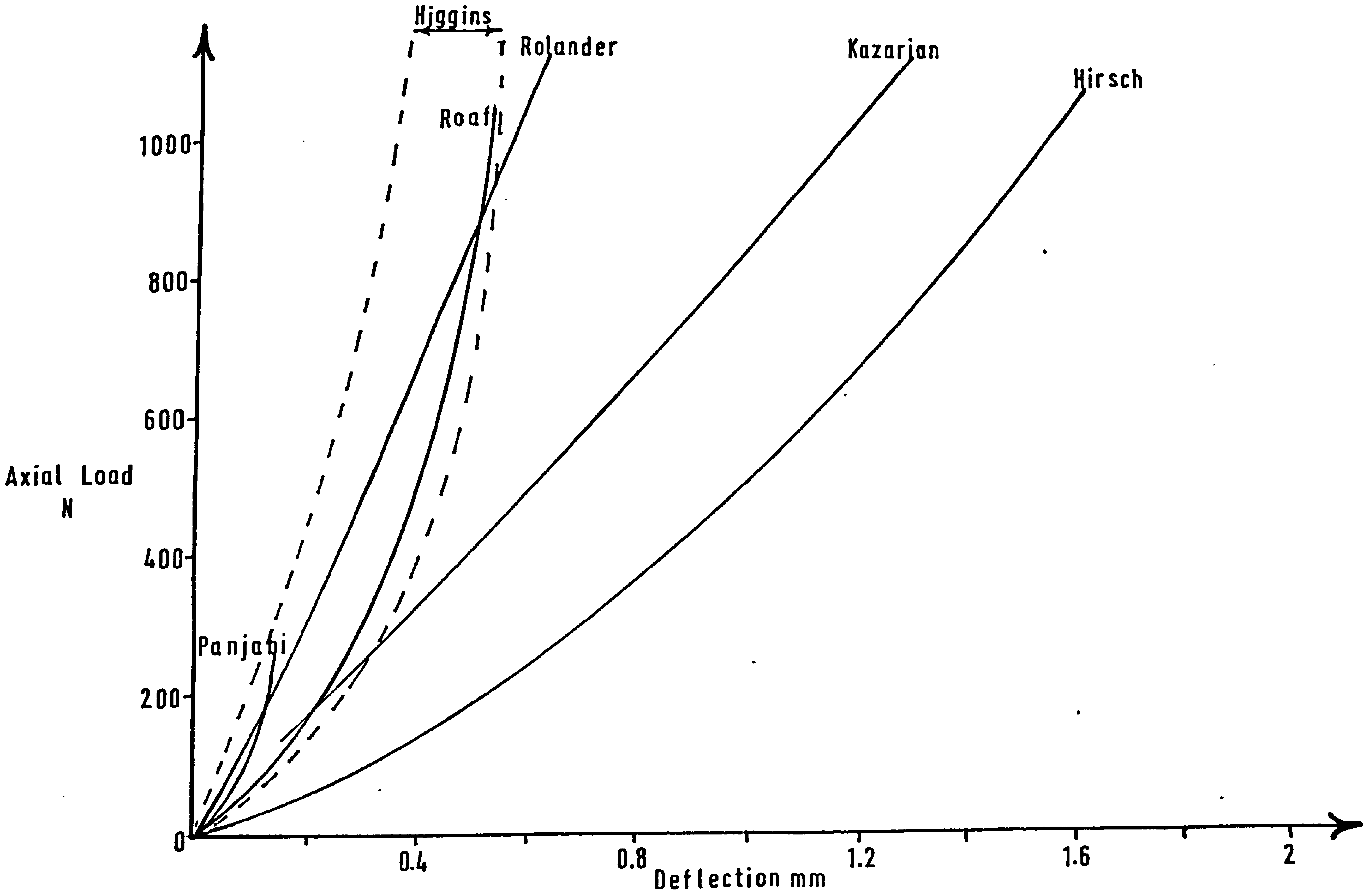


Fig.45 Axial Compression Of Motion Segments

Fig.46 Plane Torsion Of Motion Segments

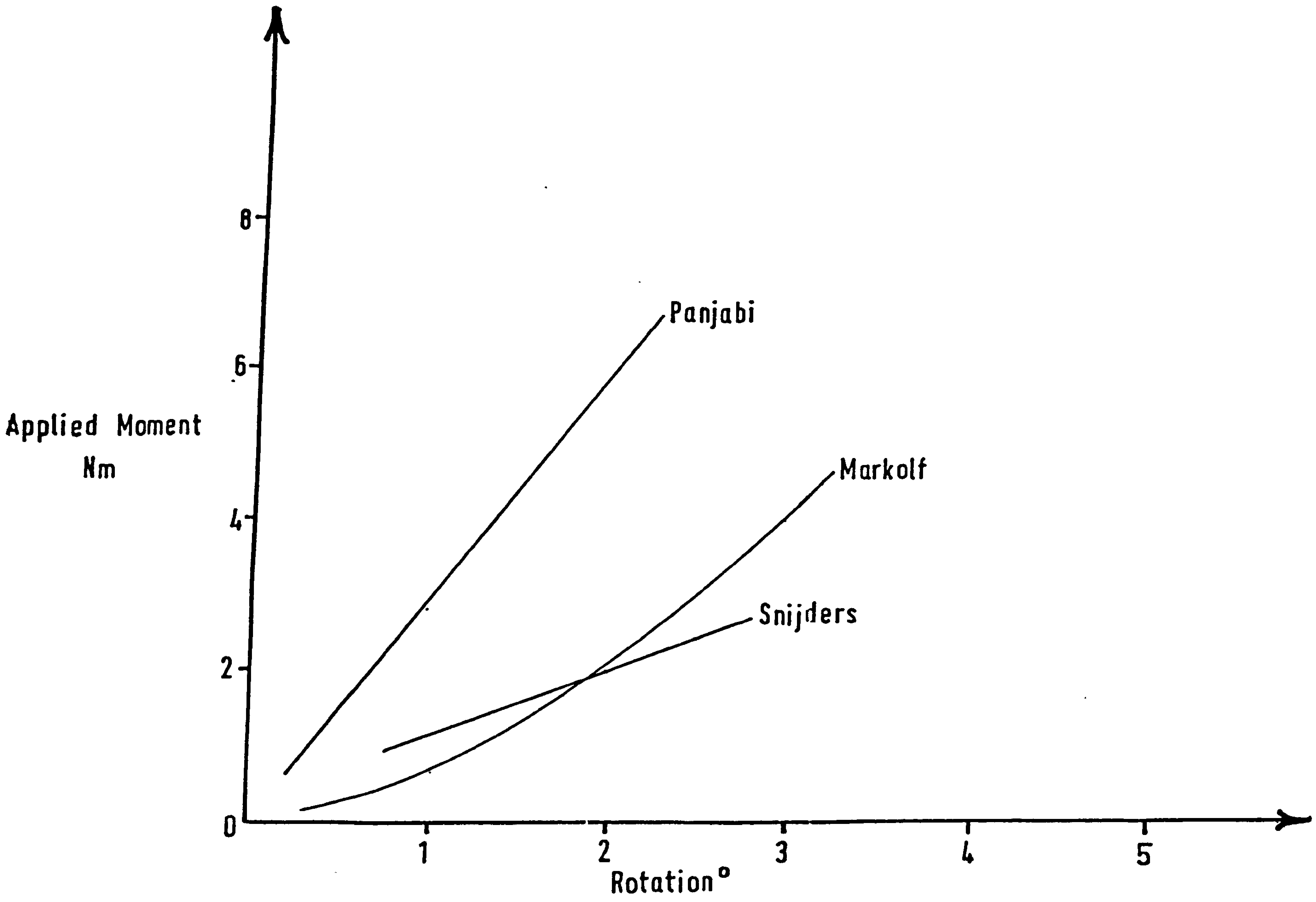


Fig. 47 Rotation Of The Spine In Flexion

Nachemson noted that a relatively stable condition was obtained after 5 - 10 minutes of loading and that the shape of the deflection time curve was independent of the magnitude of the load. They also produced a graph for the 4th lumbar joint (see Fig. 45). The results of Kazarian are similar although from his creep tests a longer time was necessary to obtain a stable condition i.e. greater than 30 minutes for normal discs. The load used by Kazarian was much smaller (93.5N compared with 980N) and this may account for the difference in results. The tests carried out by Sonoda are in general agreement with those described, however he was able to show how the mechanical properties decrease with age.

The flexion-extension tests of Carlson and Ball [113] and of Markolf [107] show good agreement for one spine specimen, their results were 5.7 Nm/deg. and 2.17 - 8.15 Nm/deg. respectively. (The values of Markolf are multiplied by two because of a different application of moments to the column as used by Carlson and Ball). However, the specimens used by Carlson and Ball were embalmed and the general average for bending rigidity was much higher. There was agreement between Carlson and Ball and Panjabi et al [101] in that the spine is stiffest in extension and most flexible in flexion with the value for lateral bending inbetween as shown below.

	Carlson & Ball	Panjabi et al	Markolf
Lateral Bending	100%	100%	100%
Flexion	92%	85%	100%
Extension	112%	109%	112%

This table does not suggest that the absolute values were in agreement between the authors, only ratios obtained within each test are indicated.

All the groups found that the spinal column showed increasing stiffness with increasing angles of rotation. Carlson and Ball did show that the posterior ligaments contribute up to 50% of the stiffness of the spine in anterior-posterior bending, although this was not the case in the work of Markolf, who found very little difference in lateral bending and flexion but an increase in flexibility of about 250% in extension when the posterior facets and processes were removed.

The work of Markolf concerned itself with realistic motions that would be encountered within the spinal column and therefore, the graphs are for small movements and not the results of tests to failure. For an investigation of the segment L_3 - L_4, a linear curve relating applied moment to horizontal plane rotation was obtained, a rotation of 1° required a moment to be applied of 9.9 Nm. The graph produced by Farfan et al [112], which was a test to failure, showed a similar curve over a corresponding range and moment of 10.2 Nm was applied to rotate a lumbar segment through 1°, failure of the joint occurred at 108 Nm. Markolf was also able to show that in torsion the posterior elements contribute up to 3/4 of the stiffness at the L_3 intervertebral joint.

Pope et al [114], in their work on complete cadavers demonstrated the hysteretic behaviour of the body in flexion-extension and also rotation. They also compared the results obtained for rotation with results gained from a live subject. The cadaver was found to be much stiffer, for example, to rotate the live subject through 5° required nearly 5 Nm while the same rotation of the dead specimen needed 10 Nm.

The most useful presentation of data to the engineer has been the tabulation of a flexibility matrix relating force and applied moments to deflection and rotation respectively. This was carried out by Panjabi et al [115] for the thoracic spine. The matrix takes into account the coupling effect of different modes of movement. The average flexibility matrix for all the thoracic vertebrae is shown below:

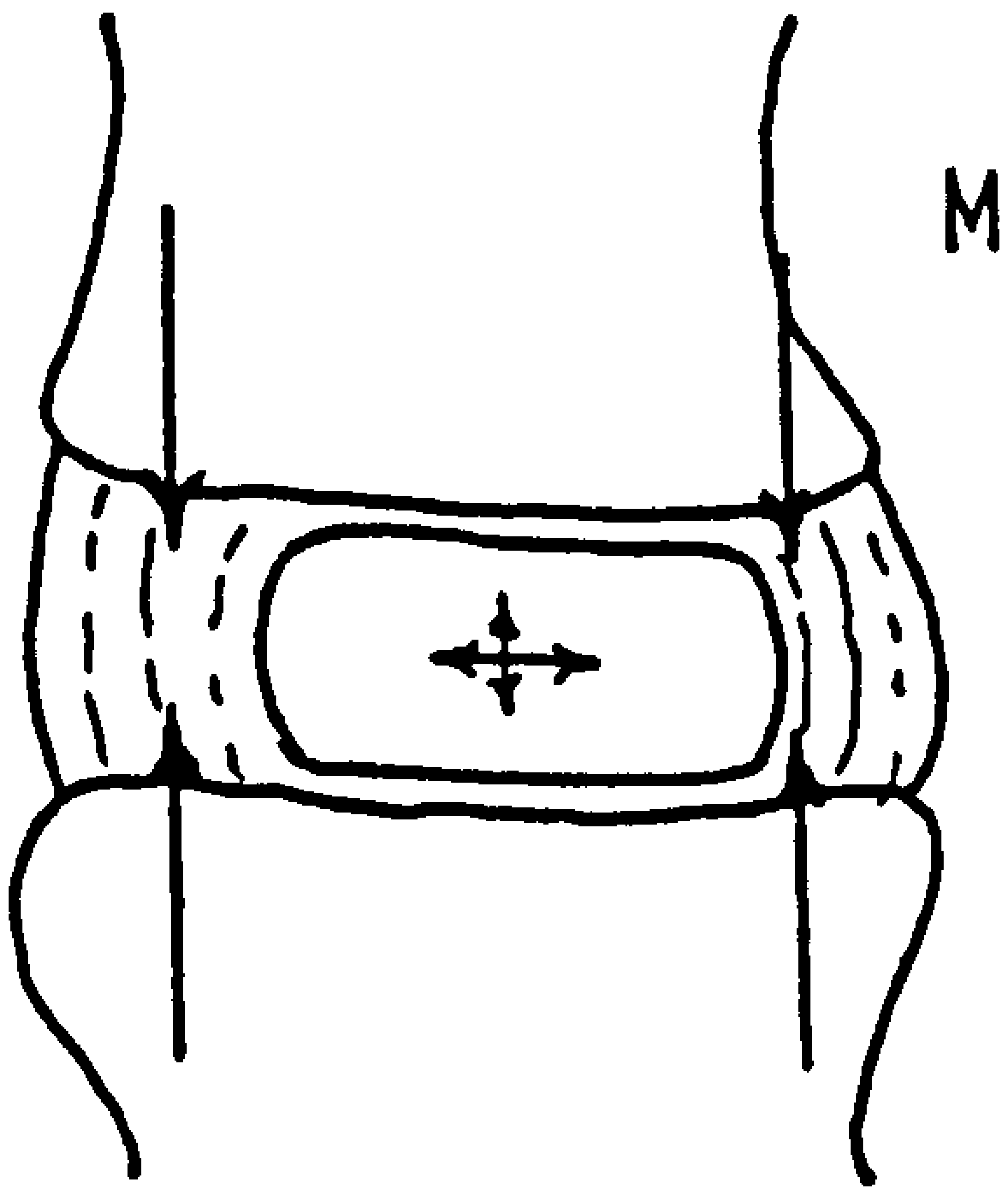
$$10^{-4}\left[\begin{array}{cccccc}108 & 0 & 0 & 0 & -.449 & -.89\\ & 8.12 & 2.2 & .05 & 0 & 0\\ & & 115 & .972 & 0 & 0\\ & & & & & \\ & & & .062 & 0 & 0\\ \text{Symmetrical} & & & & .071 & .0003\\ & & & & & .058\end{array}\right]\begin{array}{l}\text{X-axis}\\ \text{Y-axis}\\ \text{Z-axis}\\ \text{Rotation:}\\ \text{X-axis}\\ \text{Y-axis}\\ \text{Z-axis}\end{array}$$

Units in use are N, Nm, mm and rads.

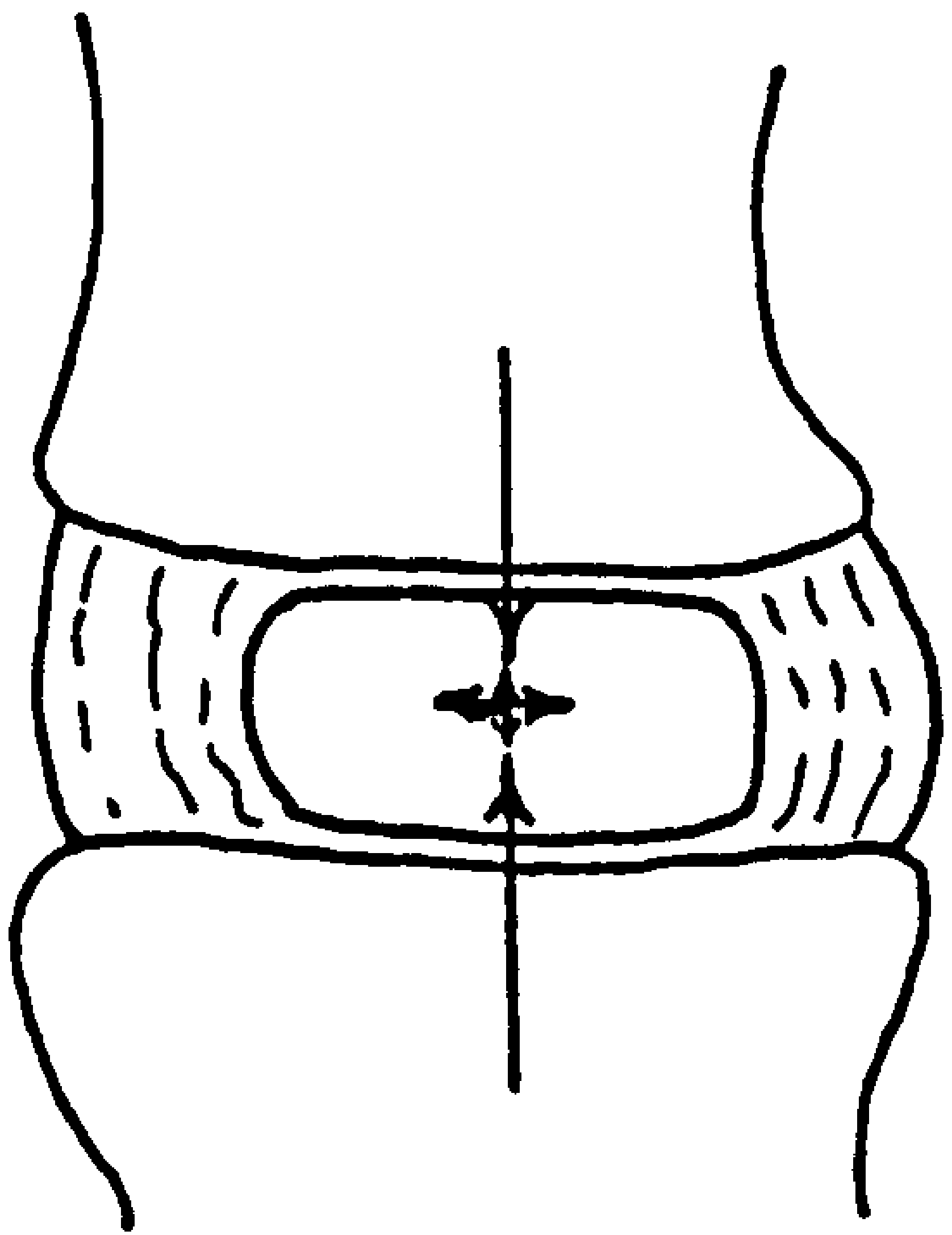
3.2. The Intervertebral Disc.

Research has been carried out into the mechanics of operation of the intervertebral disc, but no final conclusion can yet be drawn. The idea that the hydrostatic action of the nucleus pulposus is principally involved in load bearing must be modified to take into account the work of Markolf and Morris [116]. Their experiments show clearly that the nucleus and the central section of the endplate can be removed but the compressive stiffness and creep and relaxation properties remain virtually unchanged.

A possible solution can be obtained if the load path across the vertebral/disc interface and through the disc itself is "updated". Instead of the nucleus being "pressurised" directly by the vertebral body endplates and the pressure load being distributed radially by putting the annulus fibrosus into tension, the load can be seen to be transmitted via the vertebral body cortex to the annulus fibrosus and it is then the annulus which puts pressure on the nucleus. Naturally the variation inload on the intervertebral joint would cause the nucleus to vary in hydrostatic pressure as has been described by Nachemson and Elfström [117]. Also it was demonstrated by Roaf [103] that the annulus fibrosus bulged only a small amount when loaded but the endplate deflected to a much more noticeable extent, this would agree with the new theory and indicate a load path as shown below in (a). By the method suggested a fracture of the vertebral endplate could also occur, with the disc "bursting" into the vertebral body as described by Jayson et al [118].



(a) Proposed



(b) Previous

Some of the work classified in this section has been mentioned previously, but due to the technique of testing parts, of the experiments are more appropriate here.

Reports of Intervertebral Disc Properties.

Author	Type of test
Galante (1967) [12]	Test on material of annulus fibrosus
Wu & Yao (1976)[119]	" "
Virgin (1951) [120]	Compression test
Wyss & Ulrich (1954)[109]	Comp., rotation and shear
Brown et al (1957) [121]	Comp.
Rolander (1966) [91]	Comp.
Markolf (1972) [107]	Comp. and shear
Farfan (1973) [19]	Rotation
Markolf & Morris (1974)[116]	Comp.
Mathematical simulations	
Farfan et al (1970) [112]	Torsion
Sonnurup (1973) [122]	Compression
Kraus (1973) [123]	Comp. and torsion
Belytschko et al (1974)[124]	Comp. upgraded by Kulak et al (1976) [125] for greater realism
Liu & Ray [126]	Compression

Both Galante [12] and Wu and Yao [119] tested samples from the annulus fibrosus in tension. Wu and Yao obtained an average value for Youngs Modulus E for the fibres in the annulus of 3.78 MPa which is similar to that calculated by Galante. The variation of the values for compliance of the fibres with the orientation of axis of measurement is shown in Fig. 48, derived by Galante. Also the modulus of elasticity was found to vary with radius within the annulus, and an equation defining this has been derived by Sonnurup [122].

$$E_{(r)} = \frac{0.3 \; E_{(ro)}}{1 - 0.7 \frac{r}{ro}} \qquad \text{------} \quad (3\text{-}1)$$

r = radius

ro = internal radius of annulus

The annulus fibrosus is non-linear, inhomogeneous and anisotropic in its physical properties.

Virgin [120] tested single discs attached to thin slices of their inferior and superior vertebrae in compression. He presented load-deflection curves which exhibited various degrees of non-linear behaviour, all were "stiffness increasing", and reported that the disc behaves viscoelastically.

Fig.48 Compliance Of Annulus Fibrosus Material

Brown et al [121] were able to plot curves for the vertical deflection of the disc when subjected to axial compression and also plotted the lateral deflection of the annulus fibrosus. The disc was found to compress rapidly in the initial stages, with the stiffness of the samples ranging from 83 - 1440kN/m, but after the load had reached the region of 890N a stiffness of 2100 - 3500kN/m was obtained.

Rolander [91] and Markolf [107] also tested discs in compression and a graph showing part of the results discussed has been plotted. (Fig. 49).

Wyss and Ulrich [109] tested individual discs and their results are of a similar magnitude for axial and rotational stiffness.

The results of the shear tests performed by Wyss and Ulrich and Markolf [107] are in agreement: for 1mm of shear in the anterior direction of one vertebra between two others held rigidly a force of 500N would be required.

Farfan [19] has shown the contribution of the disc to the overall stiffness of the intervertebral joint in torsion, and from his results a torque of 25Nm would be

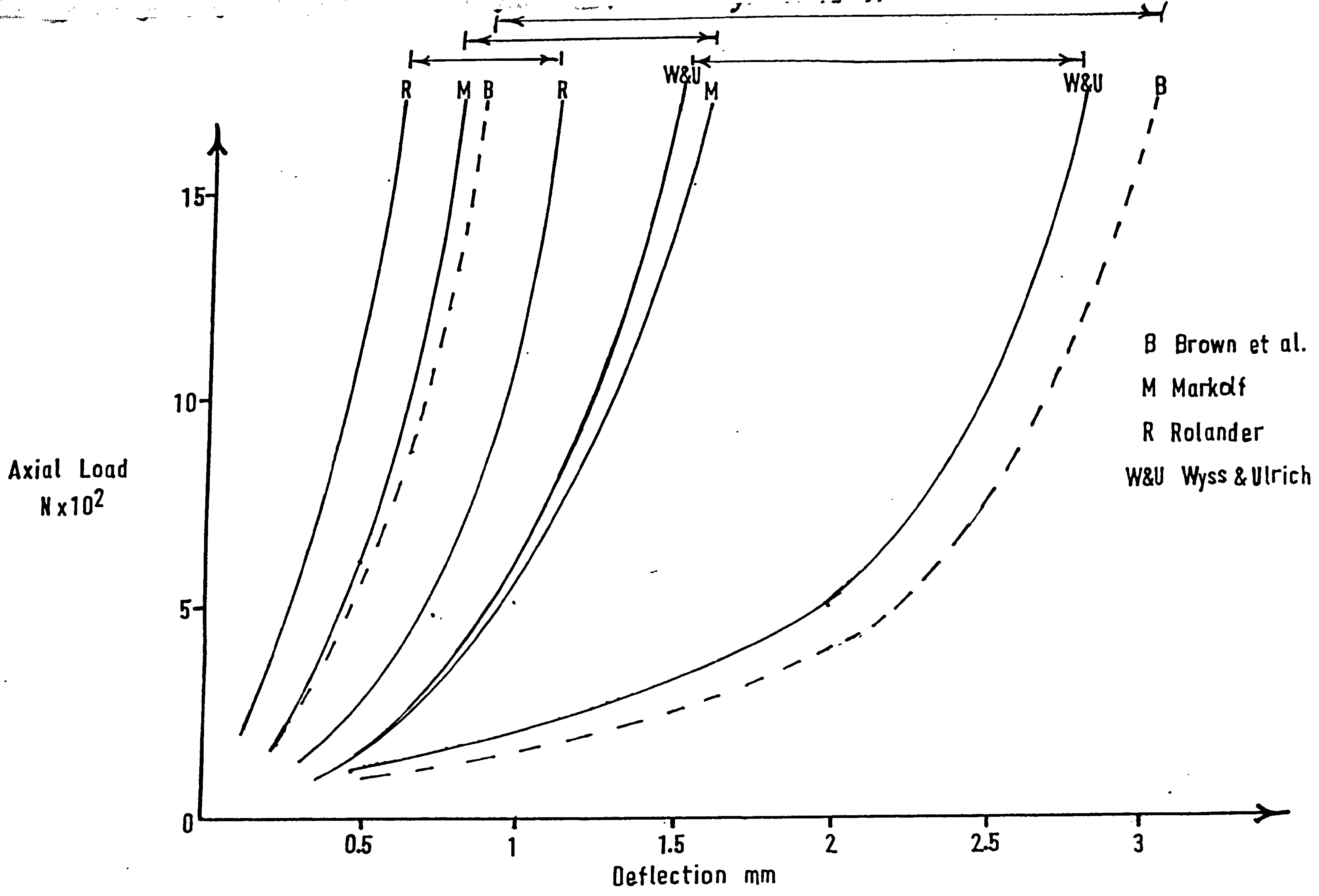


Fig. 49 Axial Compression Of Intervertebral Discs

exerted by the disc for a rotation of 3^{o}.

The mathematical models listed aid one in assessing the effects of different modes of motion but are outside the scope of this chapter.

3.3. The Properties of Ligaments.

The way in which ligament tissue reacts to elongation has been explained by Shah et al [7], who suggest that the "crimped" pattern of the collagen fibres provide the characteristic stress-strain curve as shown in Fig.51. In the initial stages of elongation the crimped section straightens out but further extension stretches the collagen itself. The explanation of the role of collagen and elastin in ligaments provided by Barbenel et al [127] suggests that the elastin ground-substance (approx. 65% dry weight for the ligamentum flavum) holds the patterned collagen, and restores the "crimp" after removal of tension.

Most of the ligaments are able to stretch between 6% and 9% of their original length. The increase in distance measured between the tips of the spinous processes in full forward flexion amounts to 35%, at the level of the facet joints 25% and at the posterior surface of the vertebral body 9%. Thus it would be correct to suggest that most of the posterior ligaments would be slack in the upright position. Nachemson and Evans [5] claim that the ligamentum flavum prestresses the intervertebral joint by 14.7N in the young and this is possible by the highly elastic properties of that ligament.

The results from a number of investigations are listed below.

Ligament	Toe limit strain %	Modulus of elasticity	U.T.S. MN/m^2	Author
Anterior Longitudinal	1.26	12.3 MN/m^2	2.07	Shah et al [7] Tkaczuk [128]
Posterior Longitudinal	2.28	148.3 "	1.67	Shah et al [7] Tkaczuk [128]
Interspinous Ligament	2.8	23.7 "		Shah et al [7]
Ligamentum Flavum	60	98.2 "	7.5	Nachemson & Evans [5]
	60		9.8	Barbenel et al [127]

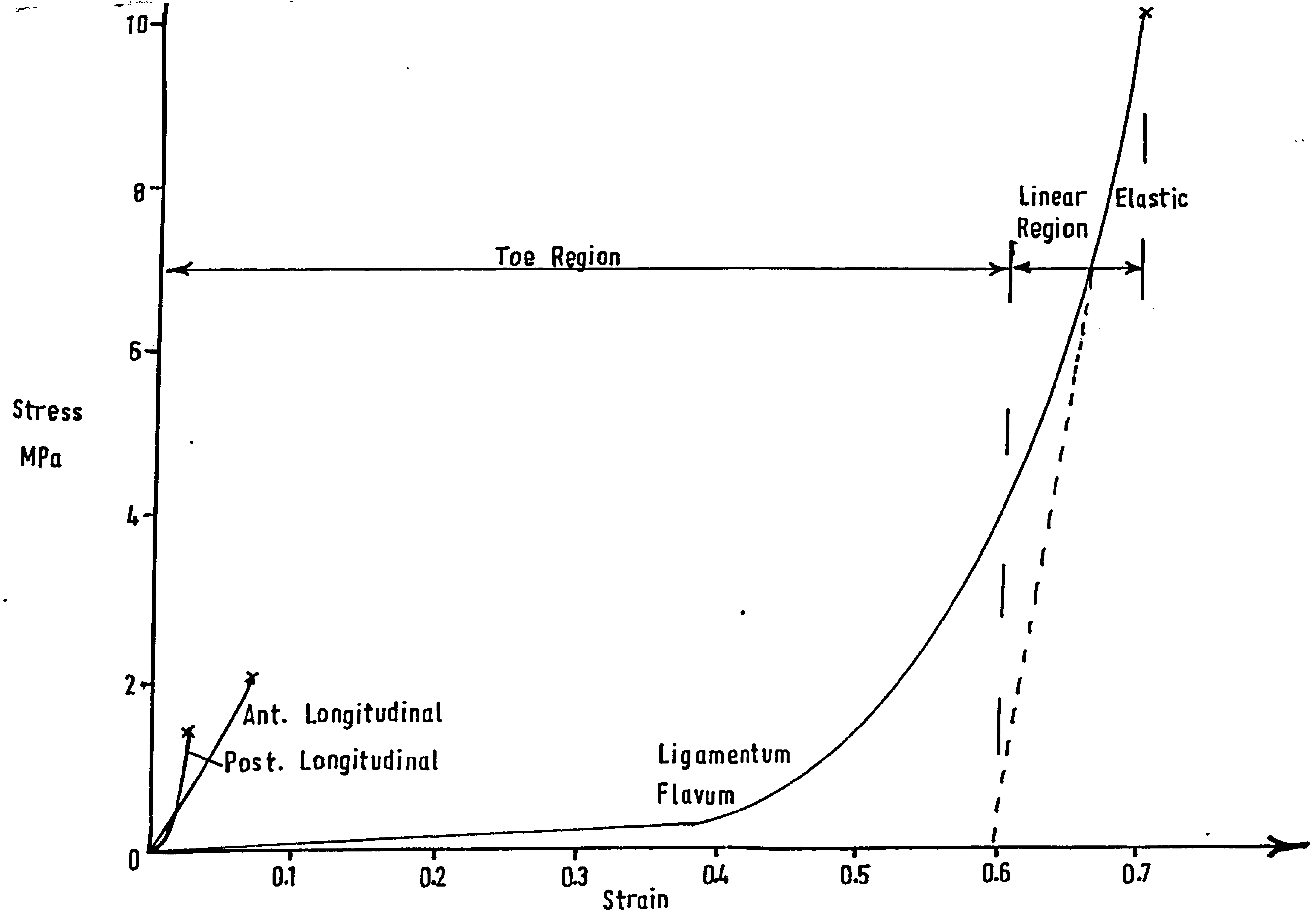


Fig. 51 Typical Stress-Strain Properties For Ligaments

3.4. The Properties of vertebral bone.

The vertebrae are formed from two types of bone, the compact bone cortex and the spongy bone as a "filling" for the vertebrae. Values for their mechanical properties are listed below, these are presented from a sample group of authors.

	Modulus of Elasticity MN/m^2	Ultimate Comp. Strength MN/m^2	Author
Cortical Bone	13720	-	Currey 1970[129]
	16100	-	Evans 1970 [78]
	-	3.54	Farfan 1973 [19]
Cancellous Bone	80	1.7	Yukoo 1952 [130]
	-	3.8	Weaver & Chalmers 1966 [131]
	152	4	McElhaney 1976 [132]
Whole Vertebrae	-	5.6 - 6.8	Göcke 1928 [133]
	39	2.8 - 5.0	Perey 1957 [110]
	-	6.3 - 7.8	Sonoda 1962 [98]
	44	3.1 - 4.4	Hartman 1974[134]

The very low values for Young's Modulus E obtained by Hartman [134] and calculated from the tests of Perey [110] are remarkable when compared with the modulus from cortical and cancellous bone separately, and suggests that the whole philosophy of a tough vertebrae and a spongy soft intervertebral disc, needs to be rethought. Hartman also calculated the mechanical impedance of the disc and vertebrae (based on stress) and suggests that the spine is acoustically tuned (the impedances differ by no more than 10%). Comparing the modulus of elasticity of the vertebrae with that of the intervertebral disc as assigned by Schultz et al [84] shows a ratio of 4 : 1 respectively. The values described above relate to axial compression only, the stiffness of the vertebrae in other modes of movement has not yet been documentated.

In the tests listed the vertebrae were tested fresh and also wet, a dry specimen of bone would present a "stiffness" value at least double that described.

The fatigue characteristics of lumbar vertebrae has been described by Lafferty et al [135]. The test was carried out by applying a horizontal cyclic force to the inferior articular facets while the vertebral body was held rigidly. A force range of 142 - 979N was applied at a frequency of 2Hz. A graph of their results is shown in Fig. 52.

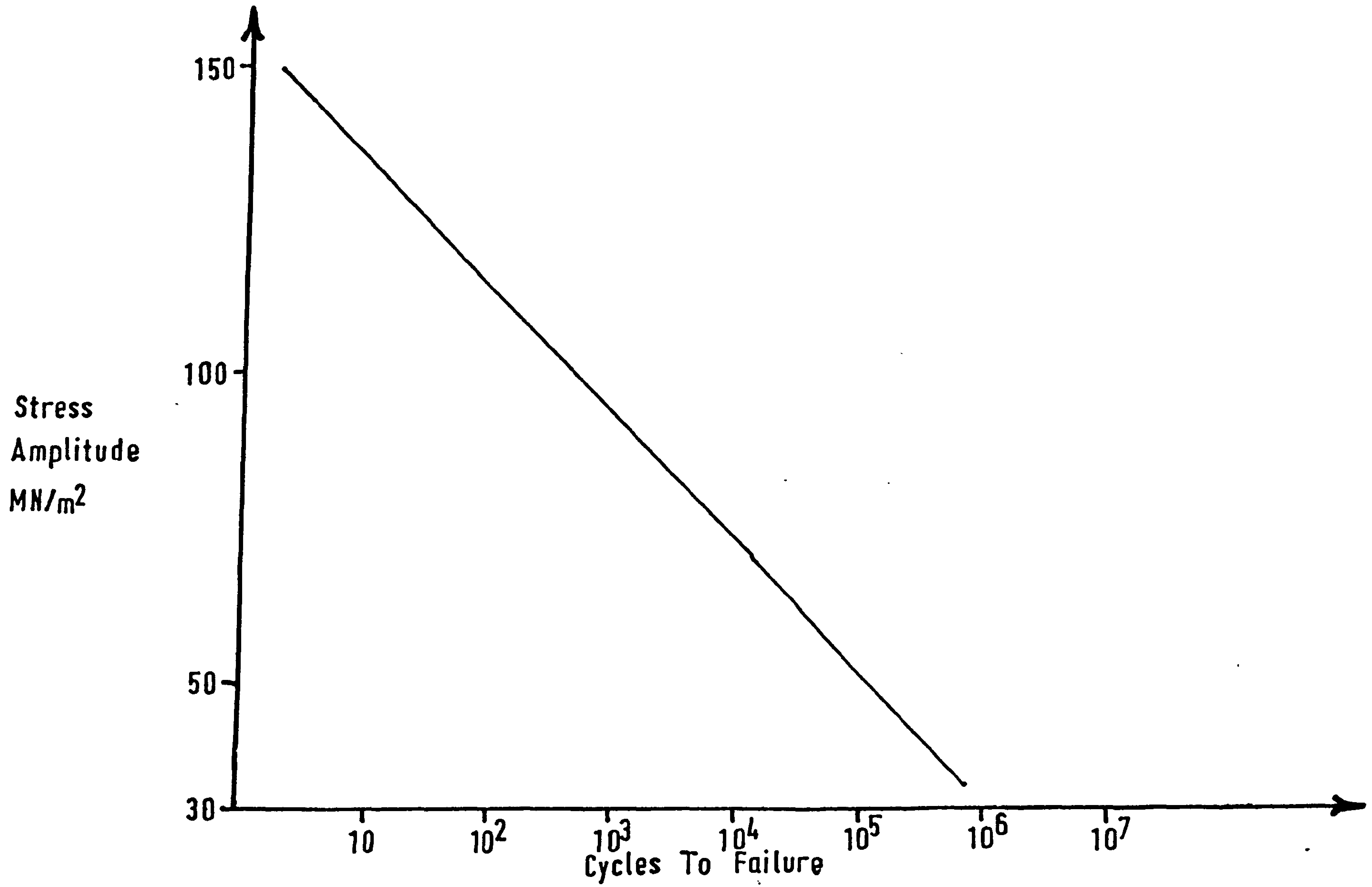


Fig. 52 The Fatique Life Of Human Vertebrae

CHAPTER 4.

A Criticism of Modelling.

This chapter is intended as a brief critical review of the methods of analysis that have been used up to the present, and gives a pointer to the direction which future modelling should take to gain information of use to the understanding of the spinal column, both in the dynamic and static situation.

4.1. Dynamic Modelling.

In the dynamic field of study the known quantity is the input pulse (acceleration) and from this is calculated wave propagation along the column, deformation of the spine from the initial position and an estimate of forces induced at local areas. As the dynamicist is mainly concerned with the response of a body to an external force, the method of analysis as described above would be in the correct order. The link between the input and output data is material property and it is in this area that an improvement must be made.

There.are two main concerns:

4.1.1. The elasticity of the vertebral bone.

As noted in the previous chapter the modulus of elasticity of the vertebral body is in the region of four times that of the intervertebral disc in quasi-static material tests but on closer examination of the dynamic impedance of the two units the difference between the two is less that 10% (Hartman) [135]. This suggests that the spine is acoustically tuned over a large range of stresses. To study wave propagation it is essential to include the material properties of the vertebral body, as yet it has only been in the homogeneous beam column models that an attempt to include this has been made and the work of Rizzi et al [66] was the first to assign independent values for the bone and disc properties. Until the lumped mass models also include the properties for bone they will be limited to the study of gross spinal movements only and the stresses resulting from that motion. It is therefore essential that this type of model is developed to include vertebral properties as it is possible only with the lumped mass systems to have anatomical authenticity.

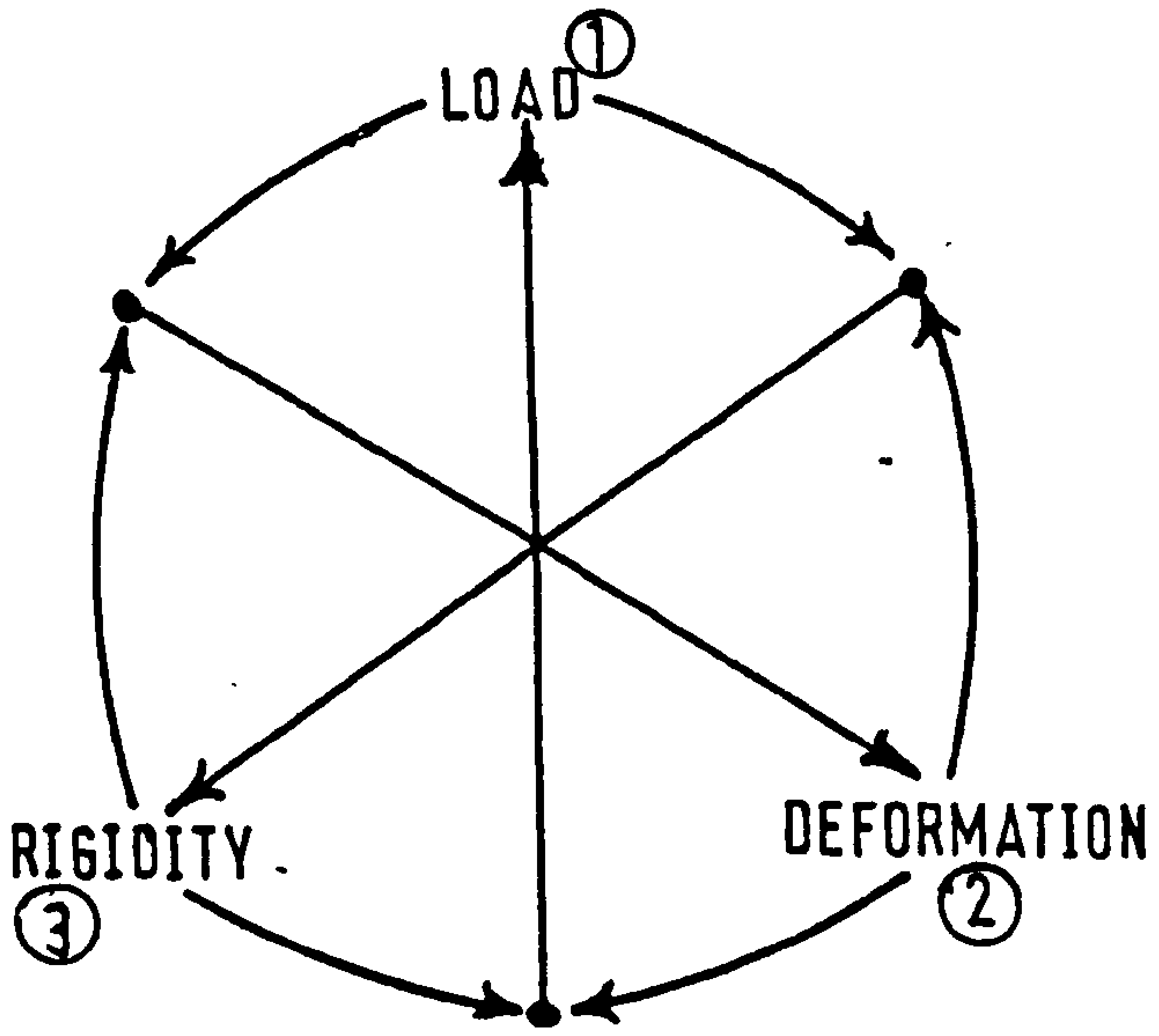
4.1.2. The active role of the muscles.

Both the work of Soechting and Paslay [61] with an homogeneous column and Tennyson and King [63] with a discrete parameter model have included passive muscle action as a function of stretch and stretch rate. The latter model was validated against cadaver experiments and found to agree, this is to be expected as corpses are passive objects, but to what extent the model agrees with live subjects still has to be quantified. It is well known that a person aware of a shock impulse will tense himself for the impact, the effect of self tension has not yet been effectively explored, therefore much more work is needed in this area to be able to predict accurately human responses. The work of Hatze [43] is an exemplary step in the right direction and it would also be hoped that his analysis will be extended to realistic models of the human body and especially the vertebral column. His is the only work to date which includes an active unit within the muscle force generators, the control of this is dependent on stimulation rate and motor unit recruitment (the number of active fibres in the muscle).

Thus it can be seen that further work is required in the field of biodynamic modelling of the spinal column before a definitive model can be produced.

4.2. Static Modelling.

The analysis of static elastic structures is based on the interaction of three criteria. They are load, rigidity and deformation. They are linked in such a way that by knowing two of the conditions it is theoretically possible to calculate the third.



In normal engineering practice the deformation of a system is calculated from a known loading case and structural rigidity (based on material properties and geometry) i.e. 1 + 3 → 2. It has been this method of approach that has been used in the modelling of the spine as an elastic column. Roberts and Chen [77] and Sundaram and Feng [79] built up a system which was acted upon by an external load to simulate a force on the ribcage generated by a collision with an object. Schultz et al [82] applied point loads to the spinal column to cause buckling or at points to simulate the action of a brace or traction. Hong and Suh [80] applied forces to the cervical column as vectors of muscle action to study related movement.

The work of these groups has been useful as mathematical tools but they are not realistic simulation of the behaviour of the spinal column.

It is pointless to draw a comparison with the buckling of a column under the action of a single vertical load and the spine, for there is no equivalent force system in the body to that point load. The muscles of the body are essential to the stability of the vertebral column and they act as one with the structure not as an external applied load. Thus it is quite clear that to omit the muscles and to apply some arbitrary external load to cause deformation is completely erroneous. From this view of the spine the work of Farfan and Lamy [76] should be praised for they did equilibrate muscle force generated with internally generated forces from ligaments and external forces from body mass although their model was for the lumbar region only.

To revert to the discussion on possible methods of solution, in the bioengineering application, the displacement pattern of the spine is often available from radiography and the external loading can be defined (e.g. body mass), but the material properties are seldom known (except from cadaver data) and are always a source of interest to the researcher. Thus an advantageous technique to the solution of structural problems in this field would formulate rigidity from deformation and external load,

i.e. 1 + 2 → 3.

This approach to a solution has been tried and is reported in Chapter 8.

A second approach has also been derived and this forms the basis for the main body of the work carried out in this research and will be discussed in the following chapters. This method makes use of an assumed rigidity, a prescribed deformation and a prescribed external load to calculate an internally generated force pattern which is then equilibrated to vectors to represent the action of individual muscles. Thus the muscles form an intrinsic part of the analysis and there is no need to recourse to a simulation of them as some arbitrary external force to find a solution to the static structural modelling of the human spine.

The physical properties of the vertebral bone are not included in the static models, the error due to this is minimised by prescribing material properties to the intervertebral disc for the complete motion segment (including parts of the superior and inferior vertebrae). This has been done by Schultz et al [84] so that the motion segment properties of the model and experimental data are in general agreement.

To reiterate, there is a necessity in dynamic models to include both the material properties of the vertebral bone and the active simulation of muscle action. In the field of static models it has been the underlying philosophy governing the physics of the models that is in need of reappraisal, the proposed model that follows, (Chapter 9). is an attempt at just that.

CHAPTER 5.

Visualisation of the Human Spine.

In order to produce a model of the human spine it is necessary to be able to obtain accurately the geometry of the vertebrae and column and also to ascertain the vertebral movement from an initial position to a deformed configuration. This chapter describes the methods that are available for obtaining these data. Unfortunately, here, little success has been had in acquiring the in-vivo data and this is due to the limitation imposed upon radiography by the harmful nature of X-rays.

At present there are two non-invasive methods available for viewing the internal structure of the human body, namely ultrasound and radiography (including isotope methods).

5.1. Ultrasound.

Ultrasound [136] has not yet been developed to the extent that an overall picture in a plane orthogonal to the axis along the beam of the instrument can be obtained. Transverse tomography of the spinal column would be of use to this research, if the sum of the cross-sections could be constructed in an axial direction and projected on to a suitable viewing medium. The outline of an object when subjected to ultrasound, which may be sharp, does lack "texture" and if the inside surfaces are to be located e.g. within the spinal canal, then high intensity sound has to be used which can also be harmful to the tissues of the body and here again a limit is set. But possibly the most serious shortcoming is that refraction of the sound beam on striking the bone surface cannot be accounted for. Even for the soft tissue in the body, the density can change by 10% causing severe distortion and this cannot be overcome using ultrasound [137].

5.2. X-ray Scanning Systems.

The use of an X-ray scanning system does seem attractive at the outset, but on closer examination serious drawbacks arise. One of these is the lack of suitable body scanners available in medical institutions. A description of the transverse axial tomographic technique, as applied to a commercial machine, has been published by Lens Van Rijn [138].

EMI have shown on their CT5005 system [139] that the transverse slices can be summed to give a picture along the length of the spine. To do this they used eight contiguous slices at 13mm increments, but the picture developed would be of little use. For an accuracy of 1mm in the data describing the geometry of a supine subject, the cross-sections would have to be imaged at least every 1mm. Thus for an average column this would mean at least 550 separate slices; the radiation dosage would also be very high. The viewing of the reconstructed image on a video screen using a matrix of 320 x 320 picture elements would also make an accuracy of greater than 1mm difficult to achieve. Possibly the most critical drawback is the fact that only a supine position can be accommodated. Flexing of the trunk is impossible with this method of visualisation and is therefore of little use in this research which relies on displacement data for a subject taking up a number of positions.

5.3.. X-ray Screening Systems.

At one stage it was thought possible that the use of X-ray screening machines with image intensifier tubes [140] would provide a solution to the problem of obtaining a series of pictures of the vertebral column for a low radiation dosage. These machines are generally equipped with two methods of viewing when screening namely (a) via a video camera to a video monitor and (b) via large format cine-film. Both these methods were discussed with (a) Brunel Television and

(b) the National Film School respectively, but it was not possible to achieve the resolution on the images required to pick out the geometry of the vertebrae to sufficient accuracy. An additional point is that a TV screen using 625 lines would not be able to give an accuracy better than ± 1mm for the position of specific points. But possibly the most important drawback was the lack of facilities on the X-ray equipment to take either bi-plane or stereo images which would have enabled full 3-dimensional geometry to be created. The correction of the images obtained for magnification using one or other of the two techniques would also have caused a severe problem.

5.4. Single Picture X-rays.

Much time has been spent trying possible methods of obtaining 3-dimensional geometric data from single exposure X-ray images. The most promising solution was the use of stereo plates viewed through a stereo comparator. But the visualisation work had to come to a halt because of the refusal of X-ray units to subject patients to X-ray radiation purely for outside research use. This standpoint is perfectly justifiable although it has meant that the computer model has not been rigorously used on realistic experimental data.

A number of X-ray films have been obtained from the Royal National Orthopaedic Hospital of patients suffering from spinal problems. The X-ray screens used were Agfa Gevaert type SE4, these are made from rare earths and provide the required luminescence at between 1/3 - 1/4 the normal radiation dosage of a fast tungstate screen. The cassettes are placed in a Potter-Bucky stand unit, this incorporates a grid of lead slats which vibrate during exposure. The reason for this is that the beam passing through the body of the patient becomes scattered and this causes a lack of clarity on the X-ray picture. By placing in the path of the beam, between the patient and the X-ray screen, a gridwork of radiation absorbent slats focused to the operational length of the source from the grid, the rays striking the "mask" at angles other than along the axis of the beam will be absorbed thus giving a clearer picture. The grid is made to vibrate so that lines do not occur on the X-ray film due to the thickness of each individual slat.

Anterior-posterior and lateral films were obtained for a boy aged eleven and these have been used in the creation of an initial set of geometry for the spine in the upright position. The author also subjected himself to tests on the feasibility of taking stereo

X-rays at the R.N.O.H.

It became apparent that because the lumbar and thoracic regions are made up of different contents, an X-ray picture of both regions would not give a picture of good resolution. Thus the two regions would have to be dealt with separately. This would mean that for the initial upright position a total of 4 (2 x 2 (stereo)) exposures need to be taken (ant. - post. direction) and a further 4 for the displaced position. (If the person was to bend forward the X-rays for the displaced position would be taken laterally, if bent sideways they would be taken in the ant.-post. direction). The requirement for 8 X-ray exposures for one set of data is ethically unjustifiable.

The stereo pairs obtained from the R.N.O.H. were compared with films produced at Bristol Royal Infirmary for their stereo-comparator equipment and found to be of poorer quality. At Bristol, Ilford Rare Earth screens with Ilford Rapide Films are being used. This point was taken up with Agfa-Gevaert (Brentford) who maintained that they could produce films with greater resolution than we had been obtaining, however, we still await a set of pictures from them.

In recent years, papers have been published showing how accurate 3-dimensional geometry can be obtained from bi-plane X-ray films [141],[142], but the use of specially prepared jigs and accurate location of two X-ray sources at 90° to each other does make these methods unsuitable for use in hospital radiography units which generally are working to a demanding schedule. The rotation between the A-P and lateral exposures taken at the R.N.O.H. was accomplished by turning the patient through 90° (i.e. the equipment was kept stationary) and obviously this is unacceptable for highly accurate analysis of the films.

Much of the complications of taking bi-planes is removed by taking stereo pictures, for this technique the X-ray source has only to be moved a few centimetres between each exposure and this can be done quickly, the limiting factor being the removal and replacement of the film cassette. This method will now be described in more detail.

5.5. Stereo-Radiography.

The method of viewing the spine using stereo images has not been widely accepted by the medical profession in Britain. The reason for this probably lies in the fact that the technique requires high precision if numerical values are to be obtained and also expensive equipment has to be used. For simple viewing of X-ray films the ability to be able to view quickly the anterior-posterior and lateral aspects is probably more beneficial than acquiring a third dimension from a single aspect. Certainly simple measurements, such as those carried out in the evaluation of a scoliotic curve, could not be made easily from a stereo pair. It is only when closer examination is required and sufficient time is available that the viewing of the spinal column in three dimensions becomes useful.

The ultimate would be the reconstruction of the spinal column as a single 3-dimensional image which could be displayed on a monitor screen, possibly by holography. A development has been accomplished by Hierholzer [143], who uses stereo films to produce a three-dimensional model of the spine, using cylinders to represent the vertebrae, which is displayed on a two-dimensional video screen. The visual model can be rotated in space using an on-line computer system.

The apparatus for viewing the stereo X-ray pictures in this research project was made available by Bristol Royal Infirmary. The machine, an StR-3 manufactured by Carl Zeiss (Oberkochen) Ltd., has been described in literature [144],[145].

The reader is requested to peruse such volumes as that written by B. Hallert [146] to gain an understanding of the subject of photogrammetry. That volume also gives a thorough treatment of the development of equations used here. The relevant equations will only be stated. See Fig. 53.

$$X = \frac{b}{b + x' - x''} \cdot x' \qquad \text{-------} (5\text{-}1)$$

$$Y = \frac{b}{b + x' - x''} \cdot y' \qquad \text{-------} (5\text{-}2)$$

note that y' should = y"

$$Z = \frac{b.c}{b + x' - x''} \qquad \text{-------} (5\text{-}3)$$

b is the distance $O_1 - O_2$ which is the base shift.

c is the distance from the film to the focus of the X-ray source.

x', x", y', and y" are the coordinates of the point as seen on the two exposures.

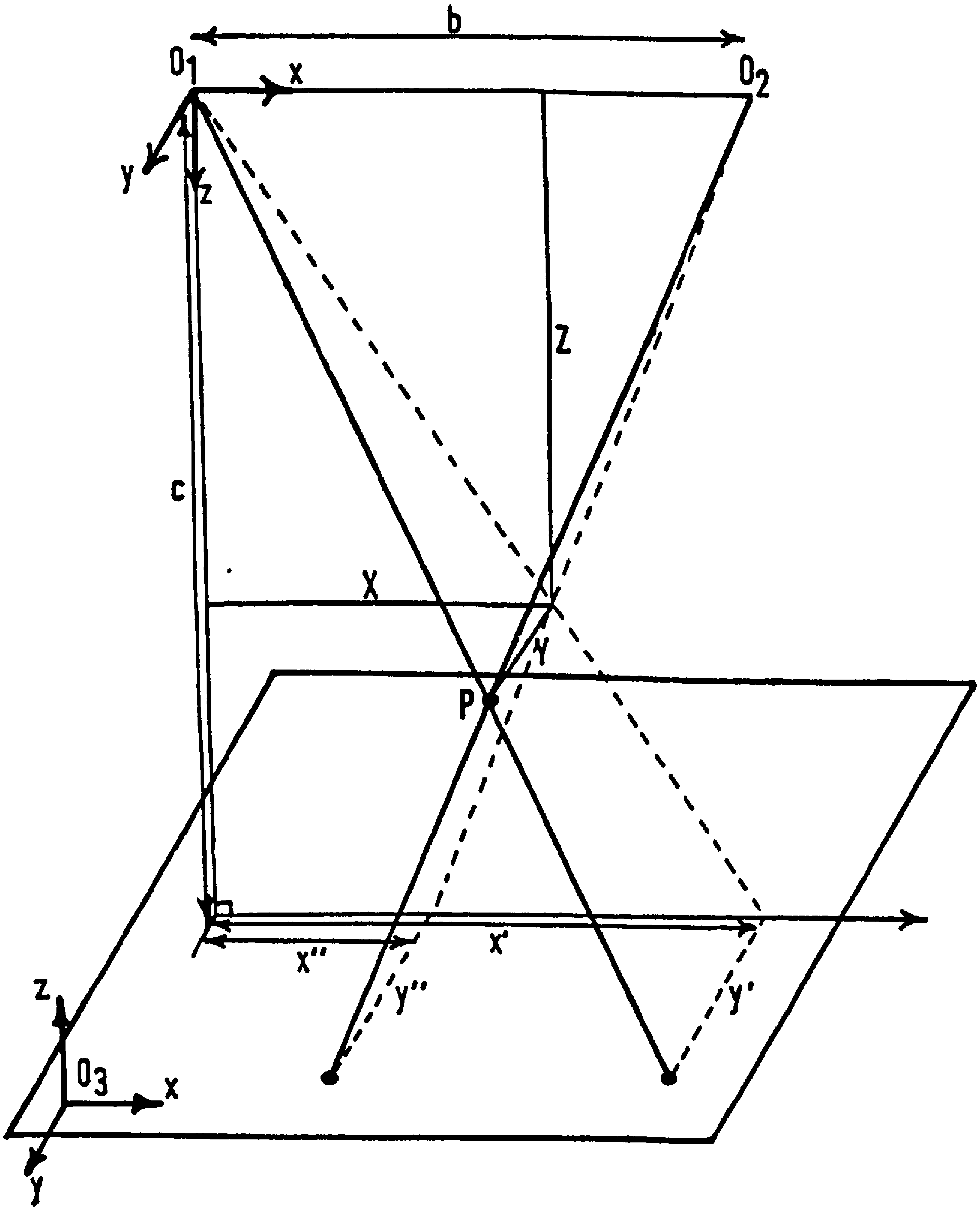


Fig. 53 The Principles Of Stereoscopic Photogrammetry

For use on the stereo-plotter the reference axes have to be shifted from having their origin at the source O_1 to a point of origin lying at O_3 with the Z-axis in the opposite direction. The equations, then become:

$$X = \frac{(K_1 - {}^{b}/2) . (x' - x'') + b . x'}{b + x' - x''} \qquad \text{----}(5-4)$$

$$Y = \frac{K_2 . (x' - x'') + b . y'}{b + x' - x''} \qquad \text{----}(5-5)$$

$$Z = \frac{c. (x' - x'')}{b + x' - x''} \qquad \text{----}(5-6)$$

where K_1 and K_2 are the machine constants for the change in origin of the reference axes.

These equations are pre-programmed into a desk top calculator so that the true co-ordinate readings are read out on paper tape.

The technique of taking the X-rays is simple and is outlined overleaf:

The X-ray tube is aligned to the index lines (either on the film cassette or fixed to the stand). This constitutes the zero position. See Fig. 54.
The tube is then shifted along one axis in a plane parallel to the film. This distance is equal to half the base shift:- typically 3cm for a film-focus distance of 1m. An exposure is made.
A new cassette is inserted and the tube shifted to a position which is the same distance the other side of the zero line as the first and a second exposure is made.

The base shift can be either caudo-cephalad or lateral. From examination of the films produced it was thought that a lateral shift is best. This is because it is possible for the shift in the tube to cause certain points to become masked by other bony structures, this is particularly prevalent in a caudo-cephalad movement. It does depend, however, very much on which points one wishes to analyse, it is essential that a point which is being measured is visible on both exposures.

For a machine of this type it is reasonable to expect an accuracy of between ±0.1 - 0.5 mm [145]

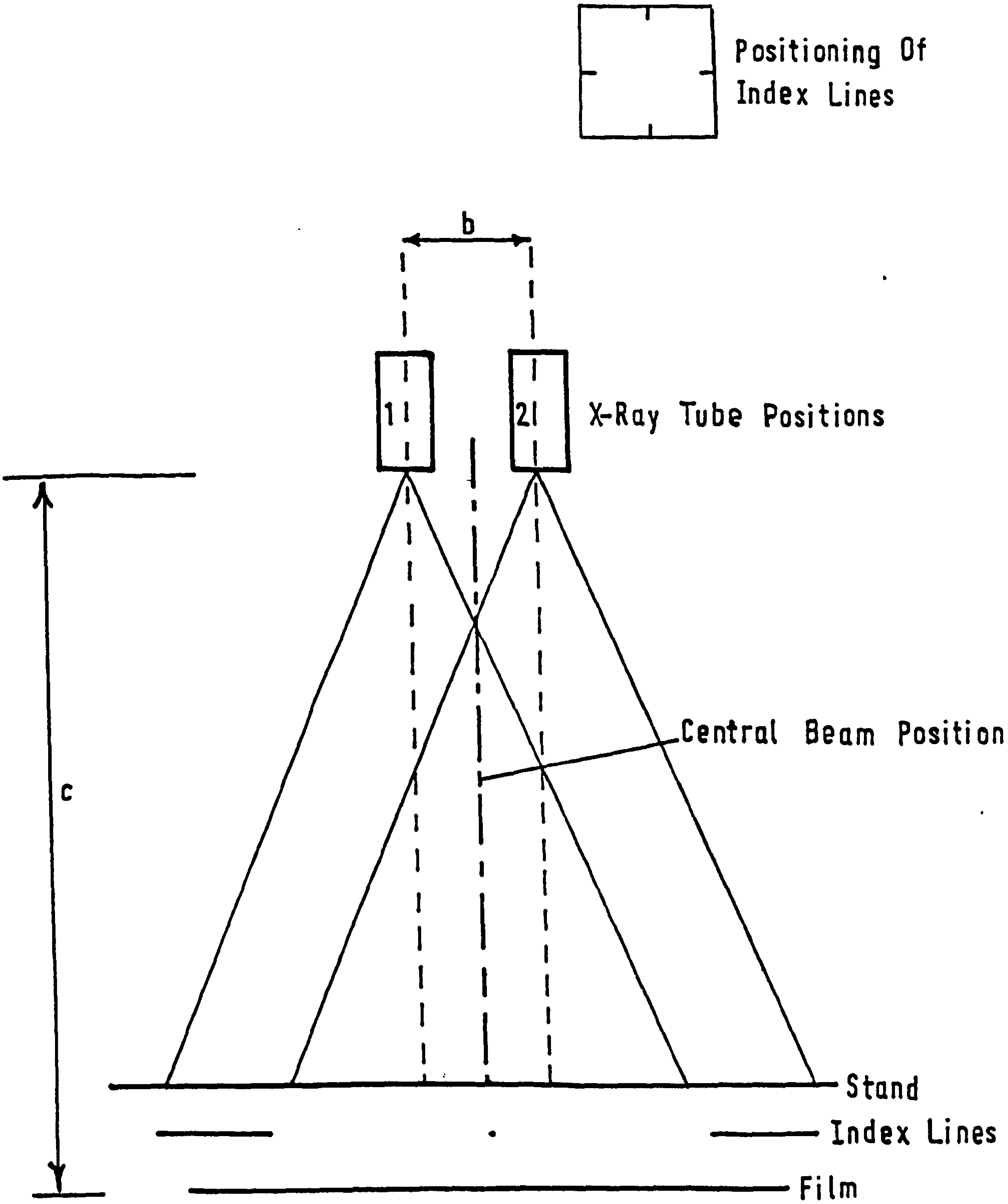


Fig. 54 Diagram Of X-Ray Technique

from X-ray films. An obvious requirement is that the equipment is set up accurately and then used with care. The likelihood is that pronounced errors are a result of poor X-ray alignment when taking the exposures and not due to a lack of ability in viewing the films accurately on the stereo-plotter. If the index lines are positioned on the cassette itself (using a perspex sheet with steel wire embedded in it), as at Bristol Royal Infirmary, then it is necessary for each cassette to be aligned accurately and independently in its holder and with the X-ray tube, otherwise significant errors can arise when the second cassette is inserted and lies off-centre from the original position of the first. It would be better if the index lines were embedded into the "immovable" stand although this is inconvenient when the exposures are taken of a patient lying on a couch. The X-ray equipment used at B.R.I. was not ideal for adjusting the tube to be in an accurate position, the whole gantry from which the tube is hung was of flimsy construction, the equipment available at the R.N.O.H, was substantial in this respect.

Another requirement is that the patient does not move between the two exposures and this presents a problem if there is any form of delay in changing cassettes, it is virtually impossible to achieve this if one is taking 4 pictures (2 x 2 stereo) as recommended earlier.

For viewing on the stereo-plotter the films are mounted on to a photo-carrier which can move in the X and Y direction. The films can also be moved apart or together on independent glass plates, the distance apart of the two plates gives a measure of the Z-direction co-ordinate. The measuring equipment consists of two illuminated markets optically projected on to the films and these are positioned to coincide with the point on the film being observed in all of the three directions by adjusting the photo-carrier.

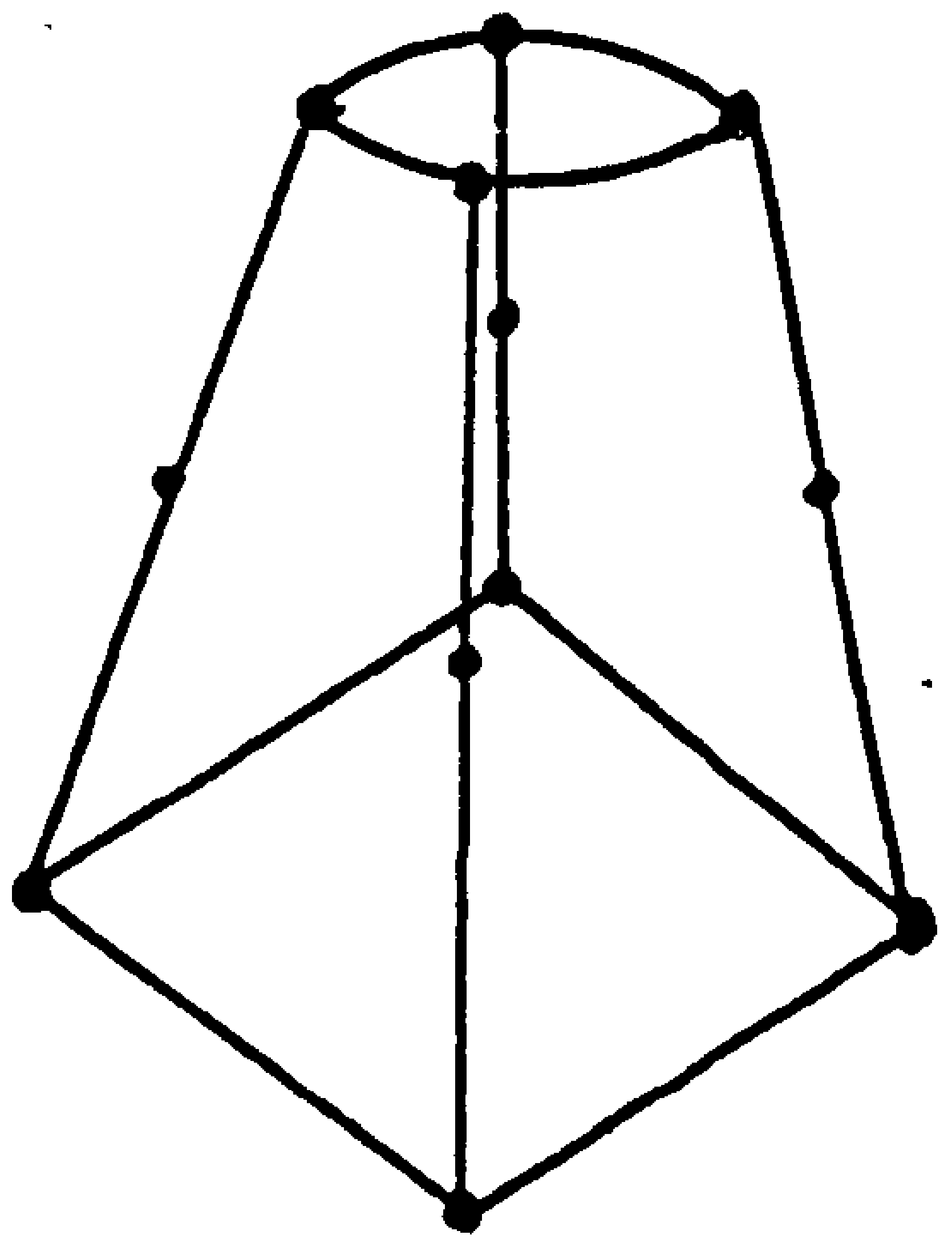
Visually the markers on the two X-rays tend to rise or fall as the plates are moved apart or together. The correct Z co-ordinate is obtained when the markers appear at the same depth as the point under observation.

Viewing of the film through the equipment does require a learning period and is time consuming. Positioning of the optical floating markers on the point to be observed and adjusting the equipment so that they coincide in the Z-direction requires concentration and often it is the case that if one returns to the same point after a rest interval a slightly different reading is obtained.

To look at a specific point and to take measurements of that point is reasonably easy, but flat or curved surfaces cause a severe problem, e.g. to find the depth of the anterior surface of the vertebral body. There is no focus for the eye to correlate with the optical marker, and so achieving a correct depth is very dependent on the visual powers of the operator in observing depth accurately. A particular problem is that a good X-ray contains a surfeit of information, by condensing 3-D on to a 2-D plane, and this tends to confuse the human brain. The radiographic intensity at a particular point is the summation of the opaqueness of all the structures that lie in the path of that particular beam, and hence a stereo pair is limited in its ability to define the particular geometry of these structures. In this respect stereo imaging is much less satisfactory than computerised transverse axial tomography, for example a body scanner would make 18 traverses each at a different angle to the patient, in each cycle the readings from 30 detectors would be measured at least 600 times [139].

A set of binoculars is also fitted so that the image can be magnified 3 x when required and this aids the viewer when searching for hair-line fractures or making accurate measurements, however, the field of view is reduced. A certain amount of eye-strain does occur over prolonged viewing, and this is possibly accentuated by the use of binoculars. In this respect the lighting of the X-ray films, the intensity of the illuminated markers, and the general condition of the room lighting also plays an important part.

To test the accuracy of the stereo plotter a wire model was X-rayed. The model had solder dots at certain points to aid viewing. With this example an accuracy in measuring co-ordinates of better than ±0.3 mm was obtained.



It must be repeated that visualisation work was halted because permission was not granted for the large number of X-ray exposures that were needed.

CHAPTER 6.

A Model Of The Spine Utilizing The Linear Programming Technique.

A number of models have been described in Chapter 2 which consist solely of the equations of equilibrium. Two of these models, 2.2.2. and 2.2.3., include the action of muscle forces. In the first study, the lumbar and cervical regions were described explicitly, the second simulation studies the lumbar region only. Thus a model for the whole spinal system would be an appropriate extension of this earlier work.

In the work of Farfan and Lamy [76] the muscle forces were calculated as functions of their cross-sectional area and an indication as to which particular muscles were in operation was gained from their own EMG work. The muscle forces and ligament tension were used to balance the vertebral segments against the action of bodyweight and external load. The authors used only a few muscles, but each had many points of origin and insertion. If one wanted to look more closely at the operation of the individual muscle strands then their method would require an optimization technique to sort out the levels of activity between the various components, i.e. a muscle may be made up of many strands interconnecting the bony structure, their method would calculate the gross force in the muscle but not define the effect of the individual strands on the system.

Thus it would seem that a model which described the individual muscle links and used an optimization technique from the outset would be advantageous. The model would also have to have the facility to "weight" the relative contributions of the muscles. A model using Linear Programming, similar to that proposed by Seireg and Arvikar [73], would fulfil these requirements.

The model which will be described solves the equations of equilibrium for the spinal column under the action of numerous muscle forces, the intervertebral joint reactions, the intervertebral joint moments caused by the tension in the ligaments and the action of body-weight. The equations formed are underdetermined i.e. there are more variables than equations.

Linear Programming was developed by Dantzig in 1947 for the military logistics of the U.S. Air Force, but technique soon had uses in the fields of economics, operations research and engineering. It has become an essential tool in operation of many companies. It would be true to say that Linear Programming holds a status in

Operations Research equivalent to that of "Finite Element Analysis" in engineering.

A thorough description of linear programming and its applications can be found in standard texts [147], and only a simple outline of the Simplex Method will be given here.

6.1. The Simplex Method.

The problem takes the following form:

$$\text{Optimize} \quad z = \sum_{j=1}^{n} c_j x_j \qquad \text{-----} \quad (8\text{-}1)$$

Subject to the following constraints:

$$\sum_{j=1}^{n} a_{ij} x_j :: b_i \quad \text{for } i=1,\ldots,m$$

$$\text{-----} \quad (8\text{-}2)$$

where :: signifies $\leqslant$, $\geqslant$ or $=$

Thus n is the number of variables

and m is the number of equations.

In the basic method there is a further requirement that

$$x_j \geqslant 0 \qquad \text{for } j=1,\ldots,n \qquad \text{-----} \quad (8\text{-}3)$$

Consider the following example (from Garvin [148]). The constraints imposed on the problem are as follows:

$-x_1 + 3x_2 \leqslant 10$ ----- (8-4)

$x_1 + x_2 \leqslant 6$ ----- (8-5)

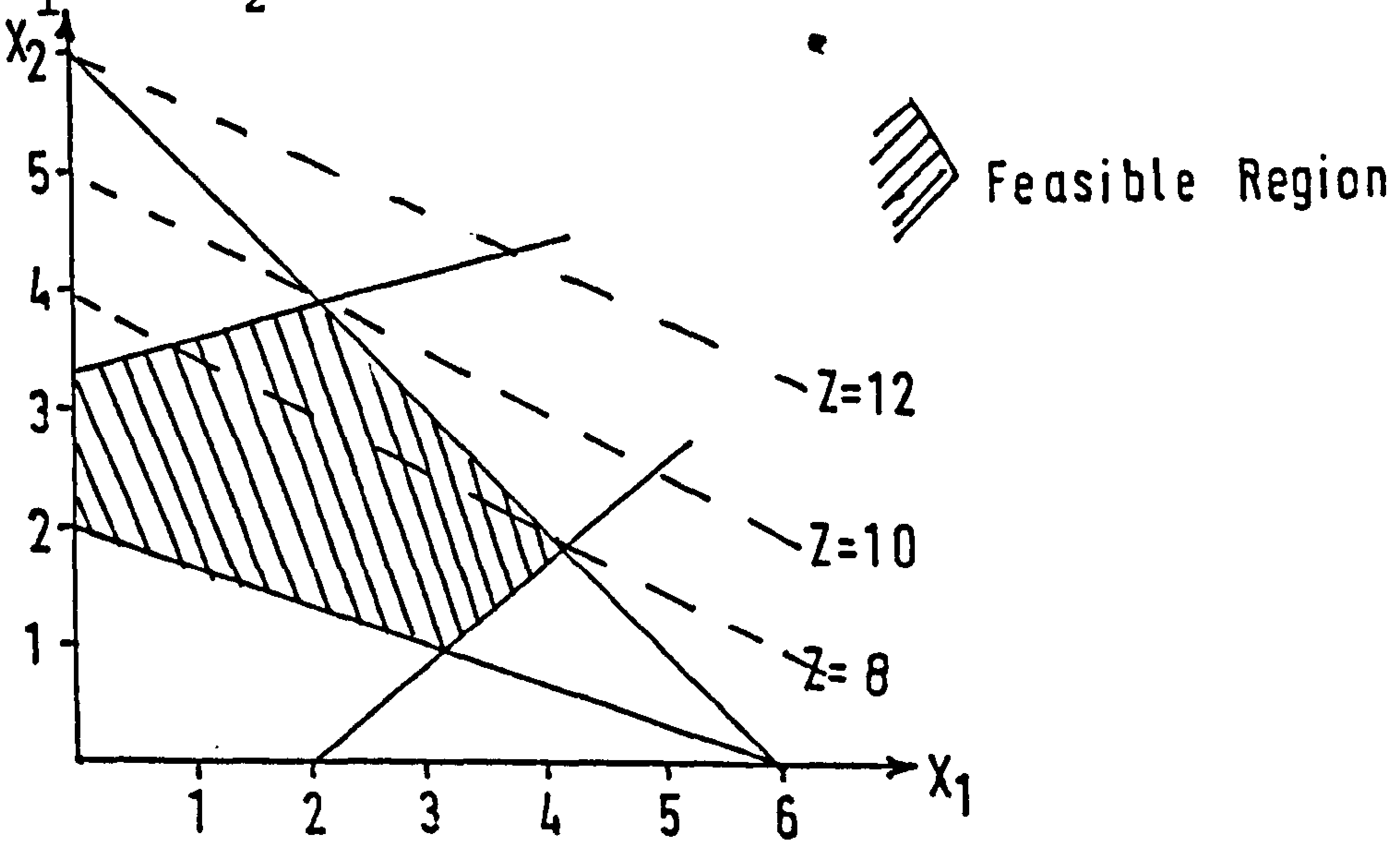
$x_1 - x_2 \leqslant 2$ ----- (8-6)

$x_1 + 3x_2 \geqslant 6$ ----- (8-7)

for $x_1 \geqslant 0$ and $x_2 \geqslant 0$ ----- (8-8)

maximize $x_1 + 2x_2$ ----- (8-9)

From the constraints (8-4) to (8-8) a diagram can be drawn showing the feasible region in which the values for x_1 and x_2 can lie, this is shown below.



The addition of positive slack variables (x_3 - x_6) to the constraints forms equations which can be handled by the technique.

$$-x_1 + 3x_2 + x_3 = 10 \quad \text{-----(8-10)}$$

$$x_1 + x_2 + x_4 = 6 \quad \text{-----(8-11)}$$

$$x_1 + x_2 + x_5 = 2 \quad \text{-----(8-12)}$$

$$x_1 + 3x_2 - x_6 = 6 \quad \text{-----(8-13)}$$

$$x_j \geqslant 0 \quad j=1,2,\ldots,6$$

$$\text{-----(8-14)}$$

Thus 4 equations in 6 unknowns. If two of the variables are arbitrarily set to zero and the four equations solved for the four remaining variables, and this is repeated for all possible combinations, there will be derived fifteen possible solutions. These are shown below:

x_1	0	0	0	0	0	-10	6	2	6	2	8	-2	4	6	3
x_2	0	10/3	6	-2	2	0	0	0	0	4	6	8/3	2	0	1
x_3	10	0	-8	16	4	0	16	12	16	0	0	0	8	16	10
x_4	6	8/3	0	8	4	16	0	4	0	0	-8	16/3	0	0	2
x_5	2	16/3	8	0	4	12	-4	0	-4	4	0	20/3	0	-4	0
x_6	-6	4	12	-12	0	16	0	-4	0	8	20	0	4	0	0
		↑			↑					↑			↑		↑
		A			E					B			C		D

From these only those arrowed fulfil the non-negative criterion. The five acceptable solutions correspond to the apexes of the polygon drawn above. The objective function z is also shown on the diagram.

It can be shown that the optimal feasible solution will occur at one of the corners. The solutions listed above are called basic solutions, the five acceptable solutions are called basic feasible solutions. The corner point which optimizes the objective e.g. point B, is the optimal basic solution.

To start the linear programming routine it is necessary to find a basic solution, from there the procedure progresses to the optimal basic solution.

Consider the following mathematical example:
An initial set of equations can be written

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & 1 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 1 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 & 1 & 0 \\ a_{41} & a_{42} & a_{43} & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_7 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} \text{ ----- } (8\text{-}15)$$

where $x_1 - x_3$ are real variables and $x_4 - x_7$ are the added slack variables.

This can be rearranged

$$x_4 = b_1 - a_{11}x_1 - a_{12}x_2 - a_{13}x_3$$
$$x_5 = b_2 - . \quad . \quad . \quad . \quad . \quad .$$
$$x_6 = b_3 - . \quad . \quad . \quad . \quad . \quad .$$
$$x_7 = b_4 - . \quad . \quad . \quad . \quad . \quad .$$

To start, the equations are reduced by putting the non-unit vectors = 0 i.e. x_1, x_2 and $x_3 = 0$. This corresponds to a basic solution.

But thus it is not an optimum so it is desirable to move to an apex. In this routine one of the columns is made into a unit vector e.g.

$$\left(\begin{array}{cccccccc|c} 1 & a_{12}/a_{11} & & a_{13}/a_{11} & 1/a_{11} & 0 & 0 & 0 & b_1/a_{11} \\ 0 & a_{22} - \frac{a_{21}a_{12}}{a_{11}} & & a_{23} - \frac{a_{21}a_{13}}{a_{11}} & \frac{-a_{21}}{a_{11}} & 1 & 0 & 0 & b_2 - \frac{b_1 a_{21}}{a_{11}} \\ 0 & . & & . & . & 0 & 1 & 0 & . \\ 0 & . & & . & . & 0 & 0 & 1 & . \end{array}\right) \qquad \text{-----}\ (8\text{-}16)$$

This will be the next new corner, more simply written as:

$$\left(\begin{array}{ccccccc|c} 1 & . & . & . & 0 & 0 & 0 & b_1' \\ 0 & . & . & . & 1 & 0 & 0 & b_2' \\ 0 & . & a_{ik}' & . & 0 & 1 & 0 & b_3' \\ 0 & . & . & . & 0 & 0 & 1 & b_4' \end{array}\right) \qquad \text{-----}\ (8\text{-}17)$$

At this point

$x_2 = x_3 = x_4 = 0$ (the non-unit vectors are set

$x_1 = b_1' = b_{1/a_{11}}$ to zero)

$x_5 = b_2'$

$x_6 = b_3'$

$x_7 = b_4'$

In more general terms, for m equations and n variables, in which the kth column is made a unit vector and the rth row contains the pivotal term, this last operation can be explained:

$$\left(\begin{array}{ccccccccccccc|c} a_{11} & a_{12} & \cdot & \cdot & a_{1k} & \cdot & \cdot & a_{1n} & 1 & 0 & \cdot & \cdot & \cdot & b_1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{r1} & a_{r2} & \cdot & \cdot & \boxed{a_{rk}} & \cdot & \cdot & \cdot & 0 & \cdot & \cdot & 1 & \cdot & b_r \\ a_{m1} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_{mn} & \cdot & \cdot & \cdot & \cdot & \cdot & b_m \end{array}\right)$$

----- (8-18)

The first line after manipulation as in (8-16) would become:

$$\left(a_{11} - \frac{a_{r1}a_{1k}}{a_{rk}}, \quad a_{12} - \frac{a_{r2}a_{1k}}{a_{rk}}, \quad 0 \quad \cdot \quad \cdot \quad \middle| \; b_1 - \frac{a_{1k}b_r}{a_{rk}} \right)$$

----- (8-19)

and so on for the other equations.

Thus at the new corner

$x'_1 \;=\; 0 \quad i = 1, 2 \,.\,.\,.\, n \quad i \neq k$

$x'_k \;=\; {}^{b}r/a_{rk}$

$x'_{n+1} = b_i - \frac{a_{ik}b_r}{a_{rk}} \quad i = 1, \,.\,.\,.\, m \quad i \neq r$

$$\text{The objective } z' \;=\; \sum_{i=1}^{m+n} c_i x'_1 \qquad \text{-----}\ (8\text{-}20)$$

At the original corner

$x_1 = x_2 \;.\;.\;.\; x_n \;=\; 0$

and $x_{n+i} = b_i \quad i = 1, \,.\,.\,.\,.\, m$

Thus the objective function at the original corner is given by:

$$z = \sum_{i=1}^{m} c_{n+i} b_i \qquad \text{-----}\ (8\text{-}21)$$

at the new corner, from equation (8-20) and substituting in the values for the variables.

$$z' \;=\; \sum_{\substack{i=1 \\ i\neq r}}^{m} \left[c_{n+i} \,.\, (b_i - \frac{b_r a_{ik})}{a_{rk}} \right] + c_k {}^{b}r/a_{rk} \qquad \text{-----}\ (8\text{-}22)$$

After manipulation, the increase in the objective function is:

$$z' - z = \frac{b_r}{a_{rk}} \left[c_k - \sum_{i=1}^{m} c_{n+i} a_{ik} \right] \qquad \text{-----} \; (8\text{-}23)$$

Thus in moving from one corner to the next the equation should be large and positive.

Therefore

$$b_{r/a_{rk}} > 0, \quad a_{rk} > 0 \quad \text{and} \quad b_r \geqslant 0$$

$$c_k - \sum_{i=1}^{m} c_{n+i} a_{ik} \geqslant 0 \qquad \text{-----} \; (8\text{-}24)$$

and $b_{r/a_{rk}}$ must be the least of the ratios $b_{i/a_{ik}}$

----- (8-25)

A simple example will now be considered to show how the mathematics described above can be used.

Maximize $z = 5x + 3y$

Subject to $3x \leqslant 15$

$3x + 2y \leqslant 17$

$4y \leqslant 16$

$x \geqslant 0 \quad y \geqslant 0$

Slack variables, s_1, s_2 and s_3, are added to form equations, these are shown overleaf.

$$\left(\begin{array}{ccccc|c} 3 & 0 & 1 & 0 & 0 & 15 \\ 3 & 2 & 0 & 1 & 0 & 17 \\ 0 & 4 & 0 & 0 & 1 & 16 \end{array}\right)$$

One solution is $x = y = 0$.

Therefore $s_1 = 15$, $s_2 = 17$ and $s_3 = 16$.

The first tableau can be drawn.

Objective c_k		5	3	0	0	0		
		x	y	s_1	s_2	s_3	b	new
s_1	0	③	0	1	0	0	15	← pivotal
s_2	0	3	2	0	1	0	17	term
s_3	0	0	4	0	0	1	16	
$\Sigma c_{n+i}a_{ik}$		0	0	0	0	0	0	← value
$c_k - \Sigma c_{n+i}a_{ik}$		5	3	0	0	0		of the
		↑						objective
								function

The column on the far left of the tableau gives a list of the variables associated with the unit vectors. The neighbouring column is a list of the corresponding objective function values.

If the condition (8-24) is maximized, i.e. the highest column value, then the next pivotal term will lie in the first column.

The condition (8-25) shows that the first row will contain the next pivotal term.

The first column can now be made into a unit vector and the next tableau drawn.

	c_k	5	3	0	0	0	
		x	y	s_1	s_2	s_3	b
x	5	1	0	1/3	0	0	5
s_2	0	0	2	-1	1	0	2
s_3	0	0	4	0	0	1	16
$\Sigma c_{n+i}a_{ik}$		5	0	5/3	0	0	25
$c_k - \Sigma c_{n+i}a_{ik}$		0	3	-5/3	0	0	

The next pivotal element is found to be in the second column and second row. This column is then a unit vector. The third tableau can be drawn.

	c_k	5	3	0	0	0	
		x	y	s_1	s_2	s_3	b
x	5	1	0	1/3	0	0	5
y	3	0	1	-1/2	1/2	0	1
s_3	0	0	0	2	-2	1	12
$\Sigma c_{n+i}a_{ik}$		5	3	1/6	3/2	0	28
$c_k - \Sigma c_{n+i}a_{ik}$		0	0	-1/6	-3/2	0	

There are no further terms in the row $c_k - \Sigma c_{n+i} a_{ik}$ which are positive, therefore there will be no gain in the objective function if one tries to move to another corner. The final result can be easily read from the b column of the last tableau, i.e.

The objective $z = 28$

$x = 5$

and $y = 1$

The example above has only two dimensions and can be solved by hand manipulations, larger models must be solved by digital computer. The model which will be described shortly consists of 380 dimensions and this is impossible to visualise and would be very tedius to solve by hand. The example maximized the value of the objective, a minimization of the objective function can be achieved if the coefficients in the objective are written in the opposite sense i.e. negatively.

The problem, which has been solved as an example of linear programming, started with a system of constraints in which there were fewer variables than constraint equations but this is not a necessary condition in setting up the model. The technique would solve a system of constraints which were underdetermined in the same way. The constraints can also be specified as = or $\geq$ than the equilibrant b, but in this case it is necessary to add both slack and artificial variables. The reason why this must be done can be explained as follows.

Consider the equations (8-10) to (8-14), in this set (8-13) can be seen to have a slack variable with a negative sign. If a tableau were to be drawn for the system a negative unit vector would result and this cannot be dealt with. A further positive artificial variable is added so that a unit vector can be found. The coefficient for the artificial variable in the objective function is -M which is a number very large and positive. As soon as the column for the artificial variable is modified to anything other than a unit vector it is removed from the system. The first simplex tableau for the set of equations would be as follows:

		1	2	0	0	0	0	-M		
		x_1	x_2	x_3	x_4	x_5	x_6	x_7	b	
x_3	0	-1	3	1	0	0	0	0	10	
x_4	0	1	1	0	1	0	0	0	6	
x_5	0	1	-1	0	0	1	0	0	2	
x_7	-M	1	③	0	0	0	-1	1	6	←
$\Sigma c_{n+i}a_{ik}$		-M	-3M	0	0	0	M	-M		
$c_k - \Sigma c_{n+i}a_{ik}$		1+M	1+3M	0	0	0	-M	0		
			↑							

The simplex method requires all the elements to be modified algebraically with each successive iteration even though many of the columns will not be directly used in the solution, e.g. columns s_1, s_2 and s_3 in the earlier example above. The reason that they must be calculated is because they are necessary in the decision as to the next beneficial move in optimizing the objective. This method is very uneconomic in computer time if large systems are handled. To overcome this a Revised Simplex Method has been formulated [149].

6.2. The APEX Package.

The APEX linear programming package is available at the University of London Computer Centre and is suitable for fast solutions of large sized problems. The system has been produced by CDC using the Revised Simplex Method.

The package can be used simply by specifying a single control card, (see Appendix 1), although it also has the versatility to be used as a callable subroutine.

The data for the problem must be specified in MPS format. To do this a separate computer program MJJ2 has been written which creates the information in a suitable form and then transfers this to disc file. The APEX package accesses the disc file directly.

The sequence of inputting data is as follows:

(a) NAME card.

(b) ROW section data.

(c) COLUMN section data.

(d) RHS section data (right-hand side of equations).

(e) BOUNDS section data, the bounds for the values generated for the column variables.

(f) RANGES section data, the ranges set restrictions to the values held by the rows (constraints).

(g) ENDATA card.

6.3. The Data Generating Program. (MJJ2).

The sequence of operations in carrying out an analysis of the spine using linear programming is shown in Fig. 55. The geometry of the spinal column is formulated in the upright position using a set of primary nodes describing the position and orientation of each vertebral body. Secondary nodes give the co-ordinates of the specific points of interest on each vertebra. If a series of positions are to be studied, e.g. forward flexion in four steps, then all that is required is to locate the primary nodes for the deformed positions, the program MJJ3 (listed in Appendix 2) then calculates the positions of the secondary nodes for each movement. The geometric data is stored on file PLOP in randomly accessible format. The data for the linear program is created by program MJJ2 on the CDC7600 machine,

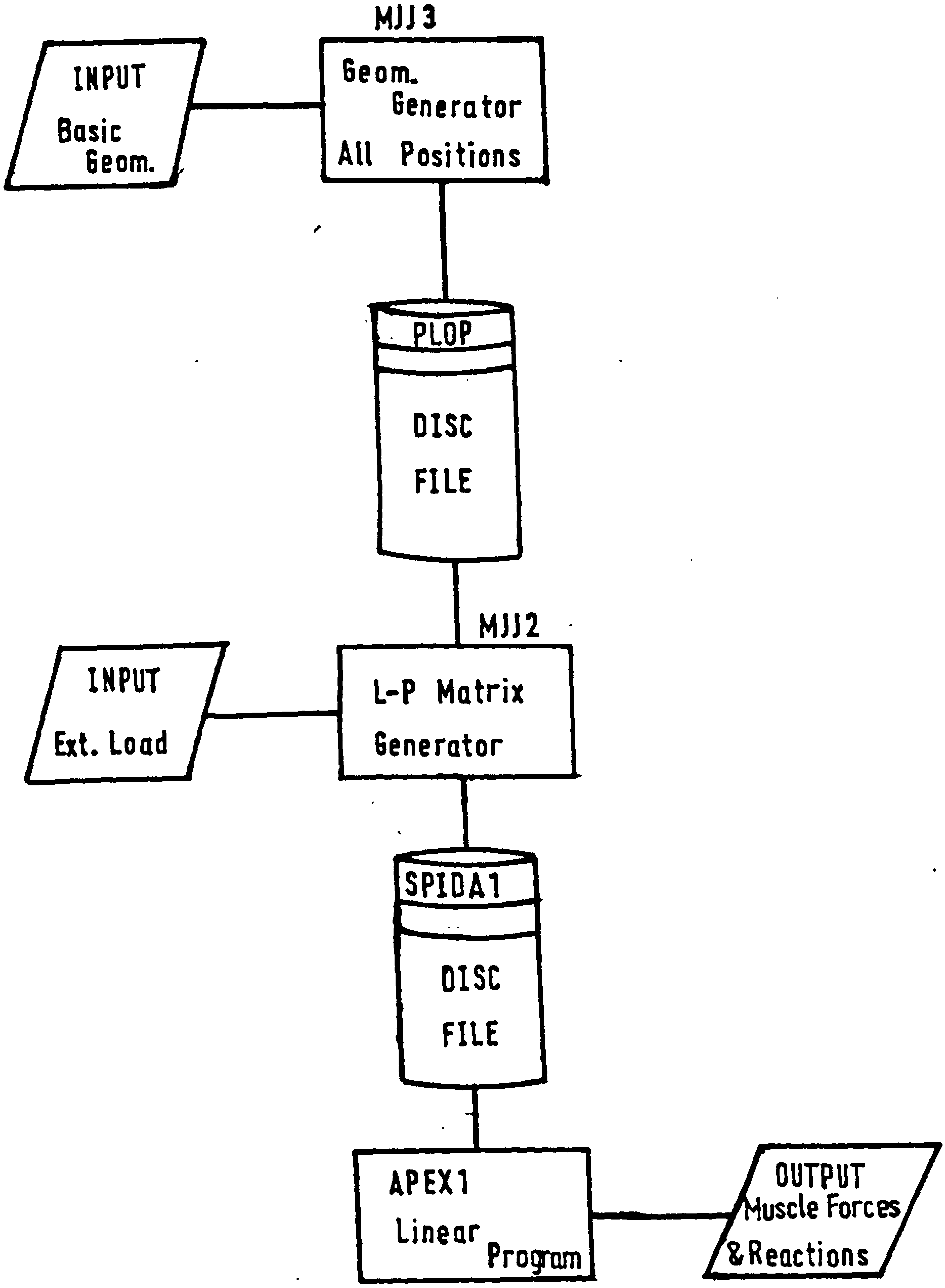


Fig. 55 Flowchart For The L-P Analysis

this catalogued for use on the CDC6600 computer but stored on disc file on the CDC6400.

The computer program MJJ2 will now be discussed in greater detail, for reference it is also listed in Appendix 2.

Input into the program on card is the following information:

MM	the number of vertebrae.
NN	the number of secondary links between the vertebrae.
FF	the number of steps in movement sequence.
FA	the step to be studied.

XAY, YAY, ZAY, XI, YI, ZI are the co-ordinates for the insertion of the Internal Oblique muscle and for the stationary points on the iliac crest respectively.

WT	the bodyweight assigned to each vertebra.
WA	the additional deadweight which is assigned to each vertebra.

All the muscles are specified bilaterally.

Line 6-28. Procedure Angle.

This procedure calculates the cosines of the angles of rotation of the vertebrae, the same procedure is used in the structural analysis program MJJ1, and is more fully explained there. (Chapter 9.2.).

Line 29-49. Procedure Tran.

This procedure calculates the transformation matrix required to rotate the axes as defined in the previous procedure. The rotations are carried out in the YZX sequence. Again this program is the same as that used in program MJJ1 (Chapter 9.3.), it is derived from Beaufait et al [150].

Line 50-111. Procedure Muscle 1.

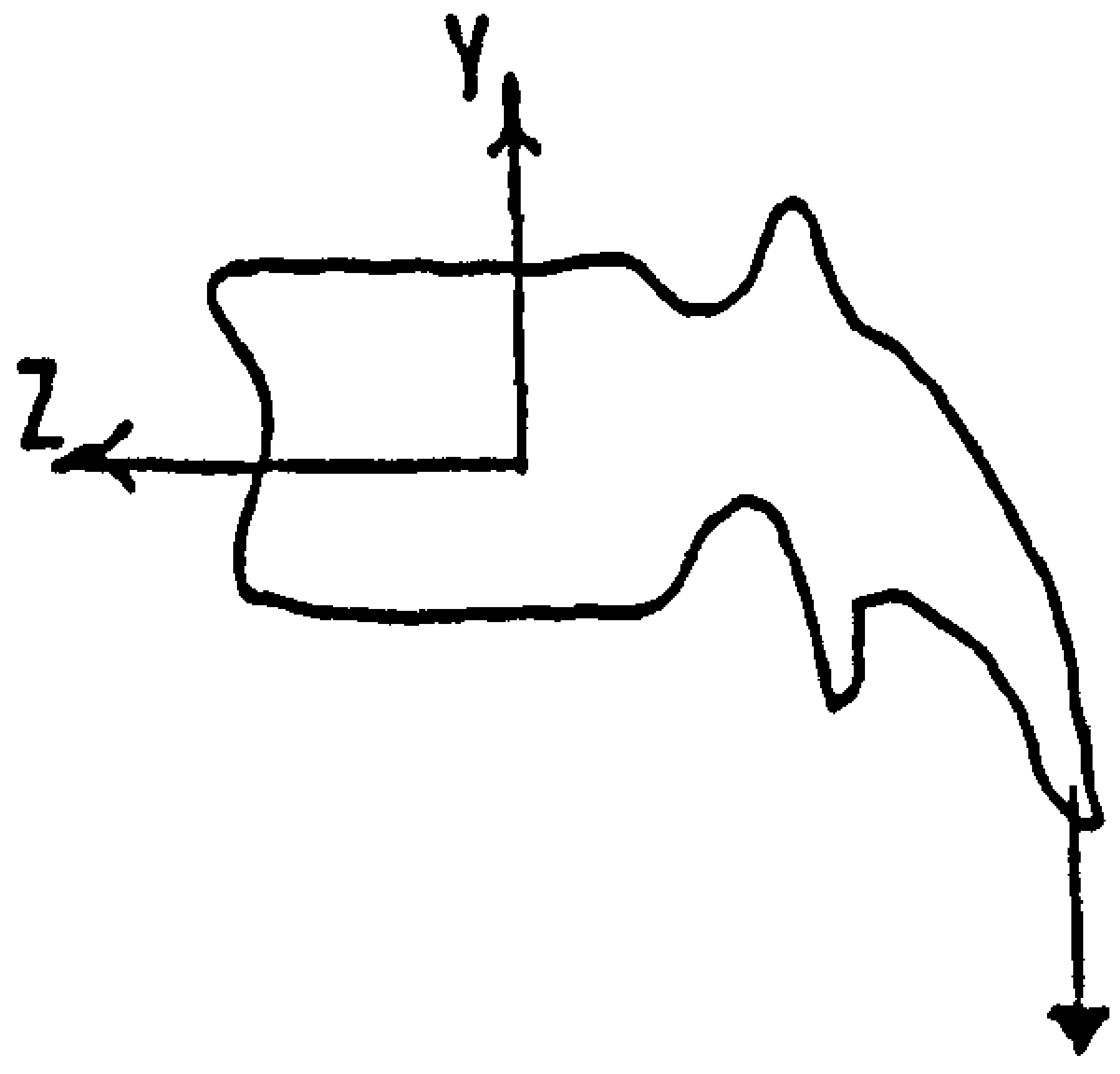
The procedure establishes the force vector of muscles whose origin is either at the tips of the transverse processes of the thoracic vertebrae or the base of the transverse processes of the lumbar vertebrae, and insertion into the spinous processes of other vertebrae. Typically these are the Multifidus and Semi-Spinalis muscles. The moments of force vectors are defined to act around the primary node of the corresponding vertebra.

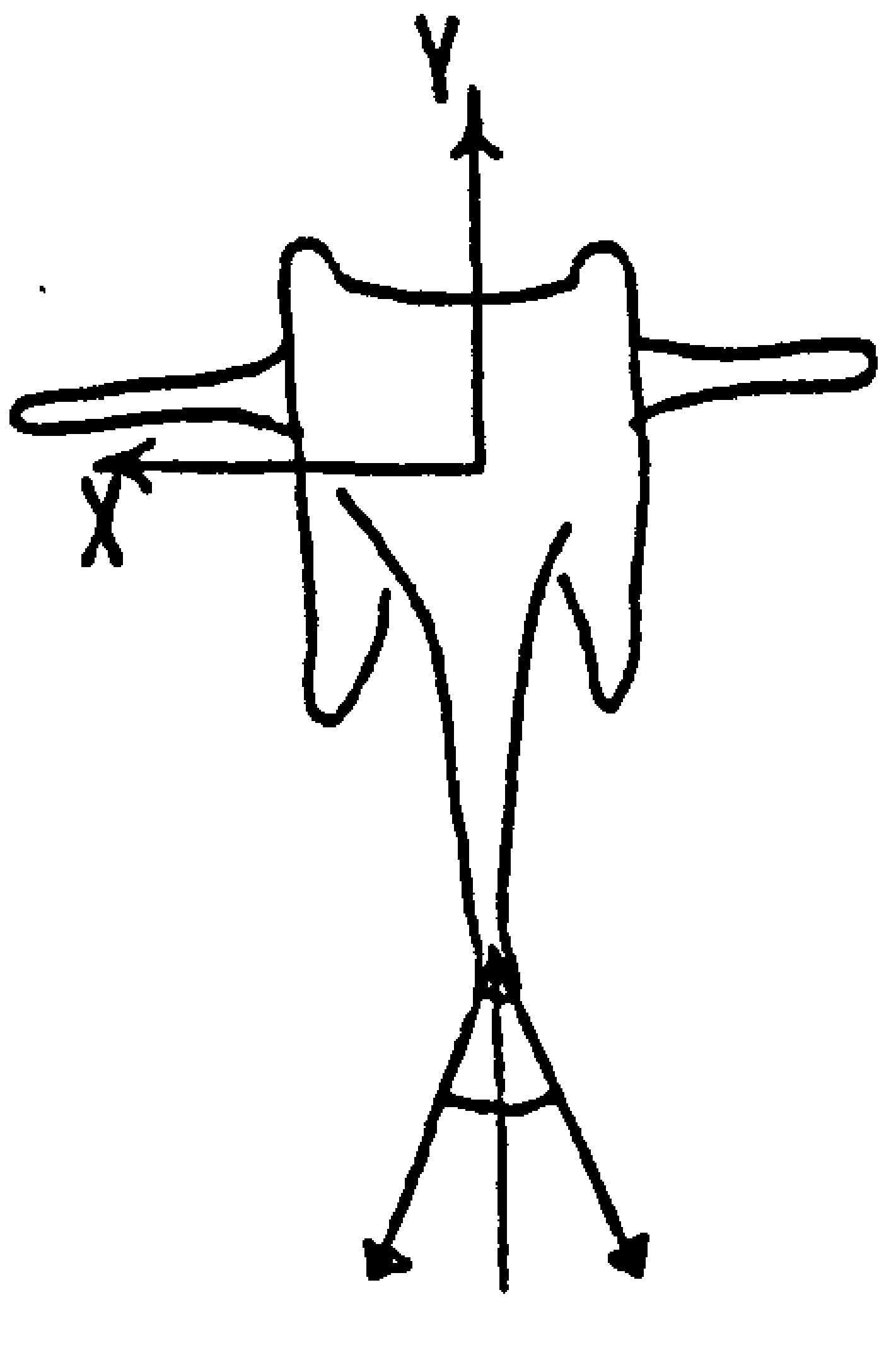
The value RP specifies the number of vertebrae the muscle spans, e.g. the Multifidus muscle spans three vertebrae and the Semi-Spinalis spans five vertebrae. The integer values M and N specify the lowest vertebrae for the origin and insertion of the muscles respectively.

The head is modelled to act about a joint on the superior surface of the first thoracic vertebra. The muscles are required to counteract the mass moment of the head about the local X and Z axes. No moments are taken to be generated about the local Y-axis.

Line 112-173. Procedure Muscle 2.

The procedure calculates the components of the force vector and of the moments about the primary node for the Spinalis muscle. The muscle inserts into the spinous processes of vertebrae $T_1 - T_4$ and has its origin at the spinous process of the vertebrae $T_{11} - L_2$. The angle of insertion and origin into the spinous processes is at 20° to the Y-axis, the inclination of the force vector to the Y-axis is equal to the rotational angle of that vertebra about the X-axis, i.e. if the vertebra is orientated horizontally the angle will be 0° and the muscle will act in a plane parallel to the Y-axis.





Line 174-236. Procedure 3.

This procedure represents the action of the Longissimus Thoracis muscle which has its origin at the sacrum and bases of the transverse processes of the lumbar vertebrae. The insertion is into the tips of the transverse processes of the thoracic vertebrae. The value N defines the lower limit for the points of insertion, in this case the twelfth vertebra.

Line 237-323. Procedure Muscle 4.

The Iliocostalis Thoracis muscle is represented as acting between the angles of the ribs of the first six thoracic vertebrae and the angles of the ribs of the lower six thoracic vertebrae. The Iliocostalis Lumborum has its insertion in the angles of the lower six thoracic vertebrae and origin at the spinous processes of the lumbar vertebrae, a further link accounts for the connection between the lowest rib and the sacrum.

Line 324-356. Procedure Muscle 5.

The action of the Quadratus Lumborum muscle is described in this procedure. The muscle has its origin at points around the iliac crest and inserts into the angle of the lowest rib and the tips of the transverse processes of the upper four lumbar vertebrae.

Line 366-566.

This section inputs the data from the file PLOP in a randomly accessible manner.

Line 577-678.

In this section the procedures are called and generate the matrix of coefficients for the forces and moments developed by the muscles. Only the geometric components of the muscle action are derived and placed in the overall matrix, the magnitude of the forces developed are found by the linear programming routine.

Also in this section are constraints limiting the action of the muscles in the following manner:

KL_{1-2}	The rotational moments about T_1 of the head mass must be balanced by the action of the appropriate muscle forces.
KL_{3-5}	Calculate the reaction components between the head and the first thoracic vertebra.
KL_{4-7}	These two equations maintain an equality of forces developed at the insertions and origins for the Spinalis Thoracis muscle.
KL_{8-13}	As above the first two equations equilibrate the forces produced at the insertions for the Longissimus muscle. The latter four equations maintain the value of the force

generated within the muscle link joining the sacrum and T_{12} to within 10% of the muscle force of the linkage neighbouring it.

KL_{14-65} The individual links forming this muscle, the Iliocostalis Thoracis and Lumborum, are restricted to develop forces within 10% of the average of the rest of the muscle's links.

Line 678-707.

The section formulates the coefficients for the action of the Internal Oblique muscle. The muscle forms a link between the points at the front of the pelvis and points on the front of the ribcage at the ninth rib. This muscle produces forces and moments directly on the first nine thoracic vertebrae.

KL_{66-81} This restricts the individual muscle strands to be equal on their respective sides.

Line 712-731.

The reactions and moments carried between adjacent vertebrae are defined in this section. A reaction or moment is made up of two components, one which is negative and the other positive. Both are entered into the Objective Function in the positive sense so that the modulus, be it for a negative or positive variable, is minimized.

Line 737-776.

This section defines the equilibrant for the set of equations containing muscle vectors and intervertebral reactions. It is formed from the bodyweight and deadweight acting on the spine. Masses are defined to act on each vertebra with an eccentricity to represent a horizontal slice taken through the body with a thickness the same as the height of the included vertebra. This method of assigning masses to each vertebra has been used extensively on dynamic models and would seem more appropriate than a single mass and centre of gravity assigned for the complete trunk. The values for the segmental relative mass and eccentricity are taken from the work of Liu and Wickstrom [151].

The prescribed masses and the curvature of the spinal column can induce moments which cause the spinal column to rotate backwards, pivoting about the L_3 level, this is counteracted by the action of the Internal Oblique muscles.

Line 781-860.

This section outputs the values for the coefficients of the vector components, the reaction components and the values for the equilibrant vector on to paper record. This is used in the checking of the operation of the computer program, the section is often skipped in the normal operation of the program to reduce the amount of paper required and the time taken to print out the large matrix does cause congestion on the line printer.

Line 866-930.

The generated data is transferred to the permanent disc file SPIDA1 in the order suitable for reading into the linear programming package. In this example, the objective function is set to 1 for all the variables, (line 885-888), also there are no sections defining either RANGES or BOUNDS.

6.4. Testing Of The Programs.

The Apex package is well established at U.L.C.C. and the version used in this analysis has had all the known "bugs" corrected. It is therefore assumed that the package functions as defined.

The program MJJ2 has been thoroughly tested by hand. There are no other computer programs which can check the results obtained from this study, but from the symmetry of the forces generated for the muscles from either side of the spine in an upright position, it would indicate that the complete system works together satisfactorily.

CHAPTER 7.

The Use Of The Linear Program Model Of The Spine.

The method of analysing the spinal column as a static structure (statically indeterminate) using the linear program technique has been described in the previous chapter. This chapter will concentrate on the establishing of the Objective Function and the results from a number of studies will be presented.

7.1. The Objective Function.

The Objective Function of the linear programming technique is the linear function which is optimized subject to the constraint equations. The variables included in the Objective Function are the muscle forces, the intervertebral reactions and the moments carried by the intervertebral joint. Each variable can be given a specific weighting factor, this is an indication of the contribution that is made by the variable, in the final solution, relative to the contributions of the other variables.

If the Objective Function is being minimized, then variables which have a large positive value in the Objective will make little contribution to the objective solution. Conversely, those variables which have either a negative or small positive value in the Objective Function will contribute to a larger extent in the solution.

The weighting of the variables in the Objective Function will now be discussed in sections dealing with the three separate groups of variables in the analysis.

7.1.1. The weighting of the muscle variables.

The muscles are limited in their production of force predominantly by their cross-sectional area. The force developed in a muscle is a function of the stimulus input and the number of fibres at a particular cross-section. Thus it would seem that to weight the contribution made by the individual muscles according to their cross-sectional area would be appropriate. The larger the muscle, the larger the cross-sectional area, and therefore, a smaller positive value in the Objective Function is appropriate.

Farfan [19][76], has listed the cross-sectional areas of muscles in the lumbar region and these have been used, in conjunction with an estimate for the muscles of the thoracic region from anatomical diagrams, to give the appropriate weighting factors. The values shown overleaf, which are used throughout these studies, are inversely proportional to the cross-sectional areas of the muscles.

Muscle	Weighting Factor
Multifidus, insertion above T_{12}	12
Multifidus, insertion below T_{12}	5
Semi-Spinalis	8
Spinalis Thoracis	10
Longissimus Thoracis	5
Longissimus Thoracis, origin below T_{12}	6.6
Iliocostalis Thoracis	5
Iliocostalis Lumborum	4
Quadratus Lumborum	7.5
Internal Oblique	1.9

7.1.2. The weighting of the intervertebral reactions.

The variables describing the intervertebral reactions have been given a weighting factor of either 1 or 2.5. Nachemson [14] has estimated the load acting on the third lumbar intervertebral disc from values of intra-discal pressure which he measured using a miniature probe. The load was calculated to be 70kg and 120kg in an upright and 20° forward flexed position respectively. Using the weighting factor of 2.5 it was possible to obtain load values from the computer program within 9% of his results both in the upright and forward flexed position. However, using the unity weighting a more acceptable muscle force distribution was obtained.

7.1.3. The weighting of the intervertebral joint moment reactions.

The moments carried at the intervertebral joints are maintained by the action of the elastic tissues i.e. the intervertebral disc and the ligaments. The moments produced by the elastic elements are a function of the deformation at the joint and the material properties of the tissues. Therefore for a particular movement there will be one specific value for the intervertebral joint reaction at each level. It is not possible with the optimization technique to calculate these reactions exactly.

Weighting factors have been assigned which give values for the intervertebral moments of nearly zero in the upright position but carrying most of the moments in the spinal column in the fully flexed position. It has been widely accepted that in full flexion the muscles of the lumbar back are electrically silent i.e. no active stimulation, thus it must be elastic tissue of the spine which holds it in equilibrium [152][153][154][155].

A series of tests were conducted to ascertain the most effective way of simulating the reaction of the elastic tissue in carrying moments. A moment arm of 15mm was found to be the most suitable for all positions. The weighting factor for the moments was also dependant on the weighting factor used for the reactions, thus for a reaction weighting of 1, the moments were weighted with a value of 12 in the upright position, reducing linearly to 1 in full flexion. Alternatively, with the reactions weighted at 2.5, the corresponding values were 16, reducing to 2 for the reaction moments.

7.2. The Examples Studied.

The geometry of two human spines was used in this series of tests. The first was from a boy aged 11, with no noticeable deformity but suffering from low back pain. The spine length, from S_1 to T_1, was 415mm and the overall weight of the boy was 499N. The second was of a similar boy with a scoliosis curve. The number of tests that have been carried out have been limited by the few X-ray films that were obtainable, see Chapter 5. The tests will be discussed more fully in separate sections.

7.2.1. Forward flexion of the spine.

Both Anterior-Posterior and Lateral X-rays were taken of the first boy in the upright position. A further lateral picture was taken with the subject in a fully flexed position. The co-ordinate geometry of the primary and secondary nodes were obtained from the upright position. From the flexed condition only the positions of the primary nodes were taken. Secondary node positions which could not be seen on the X-rays were obtained from model vertebrae suitably aligned with the column. An arc was drawn for each vertebral movement and this was split into a number of increments, thus the spatial co-ordinates for the primary nodes for a 4 step deflection were developed. The secondary node co-ordinates for the deformed positions were calculated automatically in the computer program MJJ3 which stores the spinal geometry on disc file.

Two examples were analysed:

(a) The incremental forward movement in 4 steps from the upright to a fully flexed position.

(b) The same position as above but carrying a dead load of 550N from the arms.

7.2.2. Lateral flexion of the spine.

The same upright geometry as in 7.2.1. has been used here. The deflected position was artificially created because no X-rays were available for a lateral bend of a patient. Arcs of movement for each vertebra were drawn and the reconstruction for the whole column formed the desired lateral flexion. For the lumbar region a rotation of 2^{o} per vertebra was used, in the thoracic region a 1^{o} rotation was employed. The lateral flexion caused the column to rotate through a total angle of 22^{o}, this is approximately half the rotation for full lateral flexion [11]. Zero motion coupling was assumed in the creation of the deformed geometry i.e. no rotation about a longitudinal axis was accounted for. The change in geometry took place in a lateral plane only, there was no change in the spatial co-ordinates of the spinal column in an anterior-posterior direction.

Only one deflected position was studied because of the limitations in creating the deformed geometry and in defining the reaction of the elastic soft tissues.

7.2.3. A scoliotic configuration of the spine.

A scoliotic spine was also examined, but only in the upright position. The patient was a young boy and the curve, which was of a simple primary type, (C shape), measured 57° on an Anterior-Posterior X-ray. (Cobb measurement).

The spinal column with the aid of three correction devices was also assessed. The devices included a Milwaukee Brace, Halo-Pelvic and Harrington instrumentation. The forces produced by these appliances were superimposed on to the model of the scoliotic spine in the upright position. The same geometry for the column was used in all the cases, the initial X-ray of the patient was without the extra instrumentation.

The Milwaukee Brace is a non-invasive technique used in the correction of moderate scoliotic curves [156]. The forces produced by the various pressure pads have been measured by a number of authors, [157][158][159]. The values of the forces used in this analysis and their points of application are shown in Fig. 56. The brace transfers some of the force to the spinal column through the ribcage and the shoulders, this can only be approximated to in this model as these links are not included.

Halo-Pelvic instrumentation provides a distraction force between the head and the pelvis [160]. The forces in this case are directly transferred to the pelvic bone and to the skull. The distraction acts along the spinal column, this can be simply modelled by adding a vertically acting force to the head. The total force applied, in practice, is up to 180N [161], and this value will be used here.

Harrington instrumentation consists of a solid rod, the ends of which are imbedded into the vertebral bone, it spans the curve on the concave side [162]. The distraction force provided by the rod is used to reduce the curvature and support the spinal column. The forces in the Harrington rods have been measured using intravital wireless telemetry [163], and the value of 200N which will be used here is representative. Fig. 57 shows

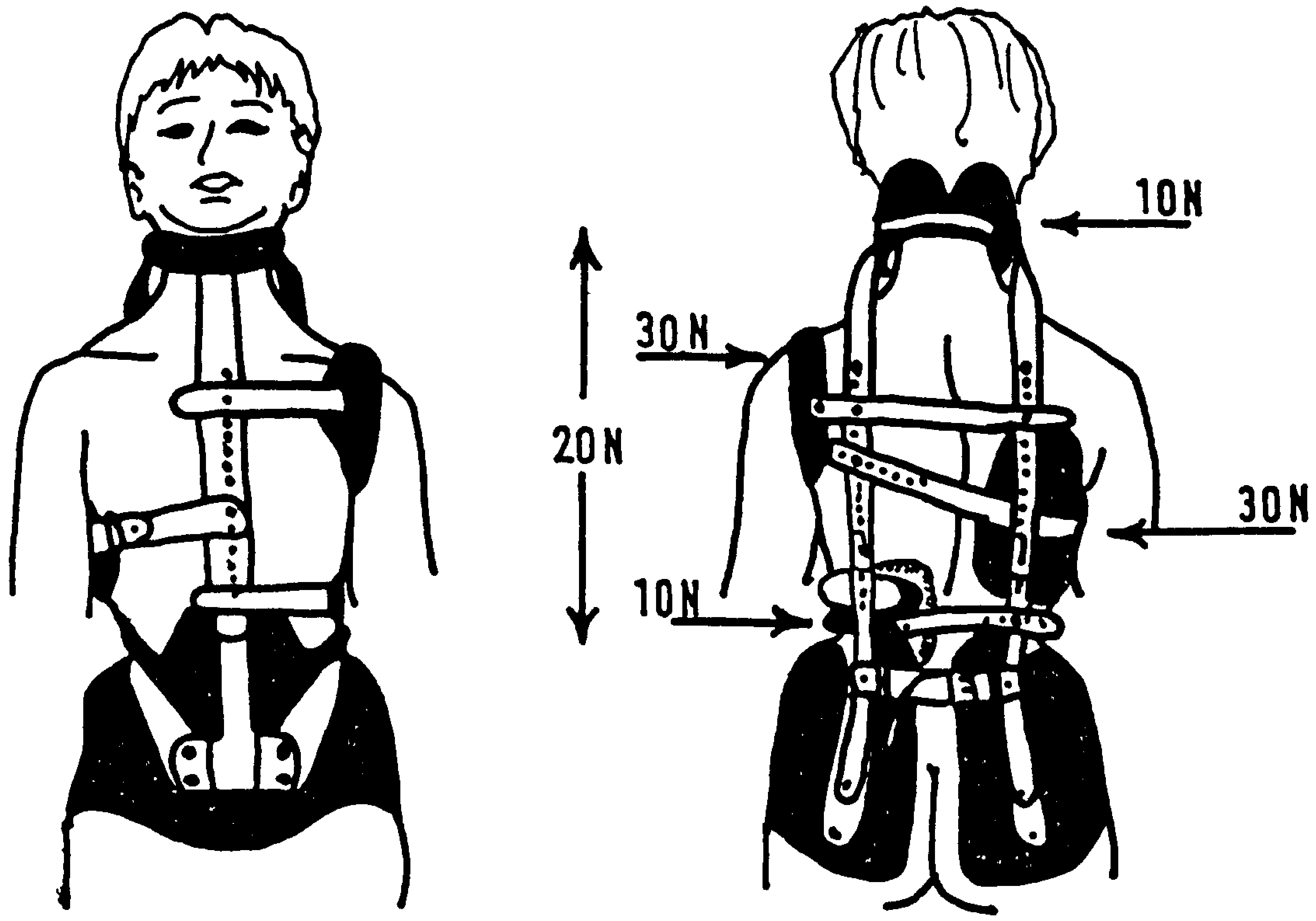


Fig.56 Forces In A Milwaukee Brace

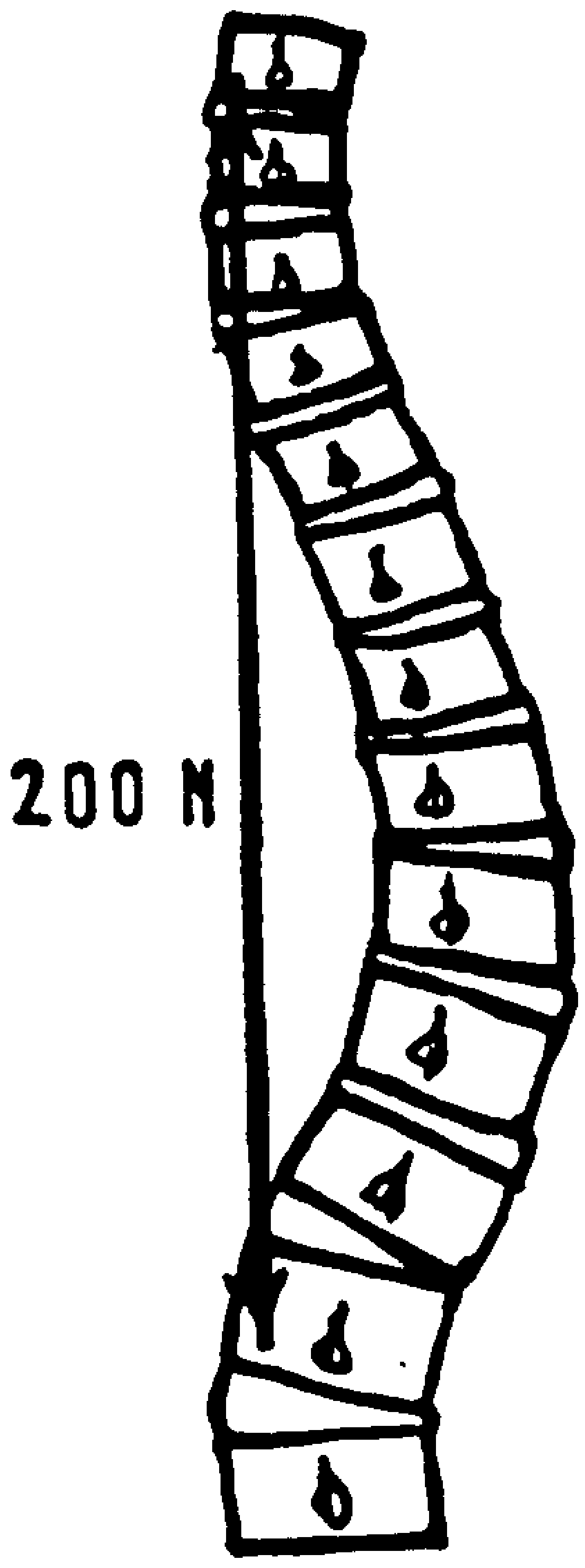


Fig.57 Forces In A Harrington Rod

the application of the extra forces to the spine.

Thus the four following cases are studied:

(a) Scoliotic configuration in the upright position.

(b) with the addition of a Milwaukee Brace.

(c) with Halo-Pelvic distraction.

(d) with Harrington Instrumentation.

7.3. The Results From The Model.

7.3.1. Forward flexion of the spine.

The study was carried out both for a weighting factor for the intervertebral reactions of 1 and 2.5.

Using a weighting factor of 2.5 the intervertebral reaction at L_3 was similar to that found by Nachemson [14]. Figure 58 shows the muscle forces that were derived in that case. Figure 59 shows the intervertebral reactions in the upright position. The values of muscle force obtained for holding a deadweight of 550N are not shown. The same muscle strands were in operation as in the study without the extra load, but the force values were up to 300% higher. The muscles in operation for both cases, with and without the extra dead load, were the Multifidus, the Semi-Spinalis, the Longissimus Thoracis and in the upright position the Internal Oblique. The force distribution was, however, not typical of a

Muscle & Point Of Origin	Force N Incremental Step				
	Upright	1	2	3	4
Multifidus					
T_1	-	-	-	-	-
T_2	-	-	-	-	-
T_3	11.6	28.3	56.6	74.5	-
T_4	-	-	-	-	-
T_5	-	-	-	-	-
T_6	49.8	53.1	98.9	258.8	-
T_7	45.2	33.7	81.3	-	-
T_8	-	-	-	-	-
T_9	-	-	-	-	-
T_{10}	-	-	-	-	-
T_{11}	-	-	-	-	-
T_{12}	46.5	77.5	161.1	-	-
L_1	73.2	113.9	235.8	-	-
L_2	68.6	-	-	-	-
L_3	-	-	-	-	-
L_4	39.5	416.2	1010.3	1834.5	-
L_5	79.3	85.7	219.6	362.0	-
S_1	4.5	43.3	201.6	-	-
S_2	14.3	154.8	-	-	-
S_3	7.1	-	-	-	-
Spinalis Thoracis This muscle is inoperative in all cases.					
Quadratus Lumborum Link between the Iliac Crest and the 12th rib	84.0	-	-	-	-

The dashes indicate zero force.

Fig.58 The Muscle Forces In Forward Flexion (cont.overleaf)
Reaction Weighting 2.5

Muscle & Point Of Origin		Force N Incremental Step			
	Upright	1	2	3	4
Semi-Spinalis					
T_1	-	-	-	-	122.8
T_2	-	-	-	-	-
T_3	-	-	-	-	-
T_4	-	-	-	-	-
T_5	-	-	-	-	-
T_6	-	-	-	-	-
T_7	34.9	49.1	93.3	-	-
T_8	-	-	-	-	-
T_9	-	-	-	-	-
T_{10}	41.3	44.8	84.7	-	-
T_{11}	56.3	-	-	-	-
T_{12}	67.9	187.2	435.5	533.5	-
Longissimus Thoracis					
Insertion T_4	52.2	11.9	-	129.1	-
T_6	16.9	-	-	-	-
T_8	54.2	83.4	91.7	-	-
Origin L_2	123.4	95.3	91.7	129.1	-
Iliocostalis Thoracis & Lumborum					
These muscles are inoperative in all cases.					
Internal Oblique					
Total from 9 strands = 80.2	Only operative in the upright position.				

The muscles are specified bilaterally, only the results from one side are shown here.

Fig.58 The Muscle Forces In Forward Flexion
Reaction Weighting 2.5

Vertebral Level	Reaction Force N			Reaction Moment Nmm		
	X	Y	Z	Rx	Ry	Rz
Above T_1	-	59.2	-5.4	-955.5	-	-
Below T_1	-	78.7	-5.4	-	-	-
" T_2	-	178.9	-10.8	-	-	-
" T_3	-	265.1	-30.0	-	-	-
" T_4	-	401.6	-5.3	-	-	-
" T_5	-	515.5	-20.0	-	-	-
" T_6	-	612.4	-9.8	-	-	-
" T_7	-	699.6	-41.7	-	-	-
" T_8	-	833.3	-46.1	-	-	-
" T_9	-	939.5	-80.3	-	-	-
" T_{10}	-	1009.8	-133.1	-	-	-
" T_{11}	-	1022.2	-190.1	-	-	-
" T_{12}	-	965.4	-193.0	-	-	-
" L_1	-	912.6	-174.7	-	-	-
" L_2	-	726.6	-53.6	-	-	-
" L_3	-	748.9	-56.3	-	-	-
" L_4	-	735.8	-	-	-	-
" L_5	-	651.1	99.2	-	-	-

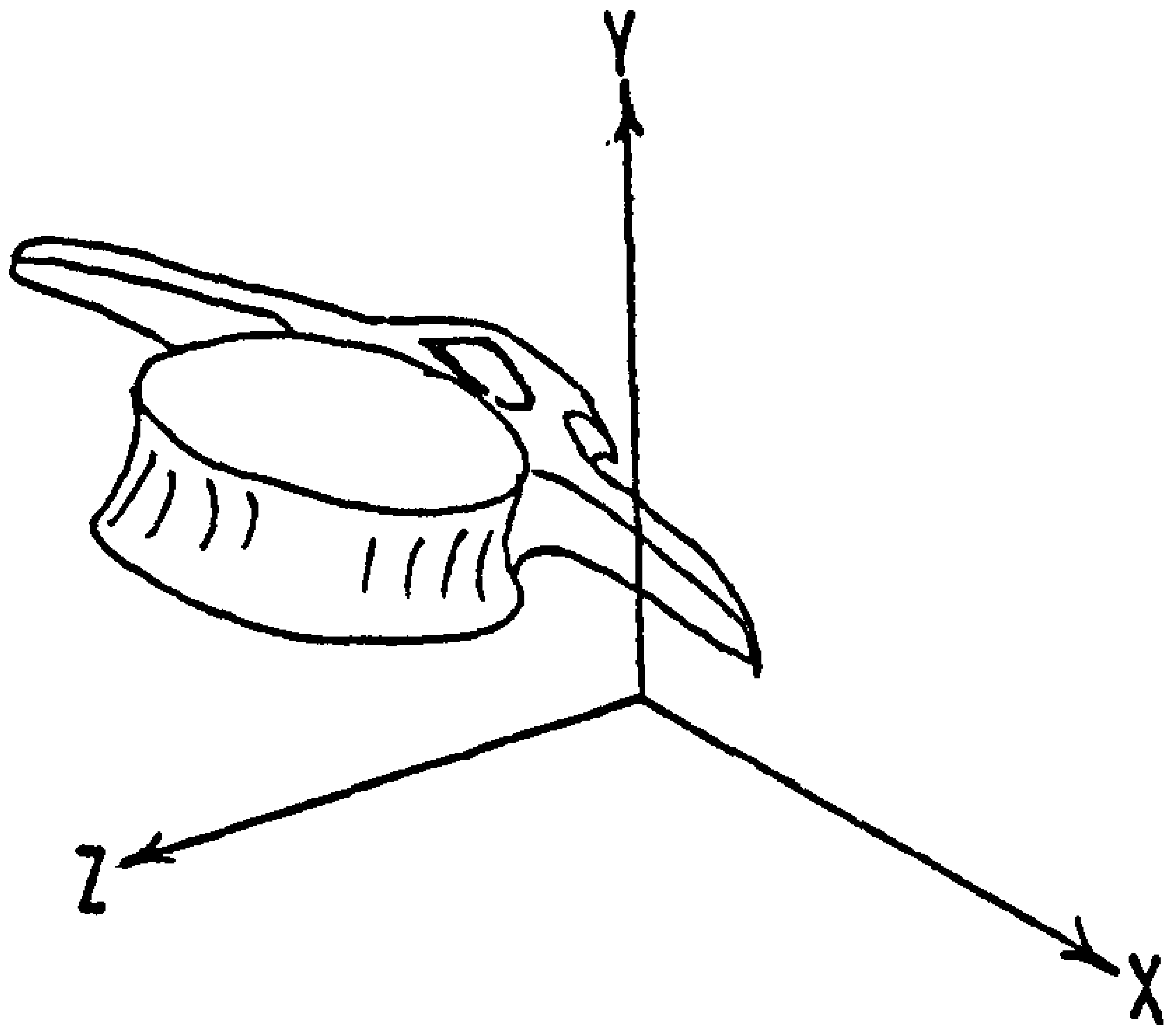


Fig.59 The Intervertebral Reactions In The Upright Position

Reaction Weighting 2.5

real situation, the Multifidus is not the main muscle in operation.

The results for the study using a weighting factor of 1 for the intervertebral reactions are shown in Figures 60, 61 and 62. The Figures 60 and 61 correspond to Figures 58 and 59 respectively for the previous study. Figure 62 tabulates the values of muscle force in carrying the extra dead load of 550N. The muscles in operation, in both cases, included the Multifidus, the Semi-Spinalis, the Longissimus Thoracis, the Iliocostalis Lumborum, the Quadratus Lumborum and the Internal Oblique.

The results of the intervertebral joint reactions and reaction moments are not shown for any of the deformed positions. The reason for this can be simply explained. The values for the intervertebral joint reactions are calculated by the computer program as in (a) of the diagram overleaf. The true situation involves many passive elastic elements each acting at different moment arms from the "pivot" point, as shown in (b). The program does not calculate the individual forces in the ligaments and disc, therefore, it is not possible to find accurately the load on the intervertebral disc alone.

Muscle & Point Of Origin	Force N				
		Incremental Step			
	Upright	1	2	3	4
Multifidus					
T_1	-	-	-	-	-
T_7	7.1	-	-	-	-
T_{12}	34.9	-	-	-	-
L_1	69.9	61.6	119.2	-	-
L_2	9.3	-	-	-	-
L_3	-	-	-	-	-
L_4	50.4	182.5	434.6	900.9	-
L_5	19.8	142.5	386.6	520.8	-
S_1	16.7	134.2	390.8	554.3	-
S_2	28.8	177.8	535.8	-	-
S_3	41.1	-	-	-	-
Semi-Spinalis					
T_1	-	-	-	-	-
T_2	-	-	-	-	-
T_3	-	-	-	-	-
T_4	-	-	-	-	-
T_5	9.3	22.8	45.4	60.2	65.7
T_6	11.2	-	-	-	-
T_7	34.9	49.1	93.3	120.9	-
T_8	36.1	31.1	57.8	74.1	-
T_9	-	11.5	68.8	-	-
T_{10}	48.3	61.9	118.2	155.0	-
T_{11}	-	-	-	-	-
T_{12}	63.4	73.1	145.5	285.3	-
Spinalis Thoracis					
This muscle is inoperative in all cases.					

Fig.60 The Muscle Forces In Forward Flexion (cont.overleaf).
Reaction Weighting 1.0

Muscle & Point Of Origin		Force N Incremental Step				
		Upright	1	2	3	4
Longissimus Thoracis						
Insertion	T_1	-	-	-	-	-
	T_2	-	-	-	-	-
	T_3	-	-	-	-	-
	T_4	44.1	35.1	-	129.1	-
	T_5	-	-	-	-	-
	T_6	67.3	9.1	-	-	-
	T_7	-	-	-	-	-
	T_8	74.5	17.6	33.5	-	-
	T_9	-	-	35.3	25.3	-
	T_{10}	-	-	-	-	-
	T_{11}	-	-	-	-	-
	T_{12}	-	-	-	-	-
Origin	L_1	-	-	-	-	-
	L_2	-	-	-	-	-
	L_3	-	-	-	-	-
	L_4	-	-	-	-	-
	L_5	88.5	29.4	32.8	73.5	-
	Sacrum	97.3	32.4	36.0	80.9	
Iliocostalis Lumborum						
Origin	T_{12}	10.8	82.1	176.9	206.6	-
	L_1	13.2	91.4	185.6	206.6	-
	L_2	13.2	100.2	216.0	230.1	-
	L_3	10.8	82.1	176.9	206.6	-
	L_4	13.2	100.2	216.0	252.3	-
	L_5	10.8	82.1	188.3	252.3	-
	Sacrum	12.1	100.2	216.0	252.3	-

Fig.60 Continued.

Muscle & Point Of Origin		Force N Incremental Step			
	Upright	1	2	3	4
Iliocostalis Thoracis This muscle is inoperative in all cases.					
Quadratus Lumborum					
Insertion Rib T_{12}	-	32.3	71.2	-	-
L_1	-	-	-	-	-
L_2	-	-	-	-	-
L_3	-	-	-	-	-
L_4	-	-	-	-	-
Internal Oblique Total from 9 strands = 80.2. Only operative in the upright position.					

All the muscles are specified bilaterally, only one side is shown here.

Fig.60 The Muscle Forces In Forward Flexion.

Vertebral Level	Reaction Force N			Reaction Moment Nmm		
	X	Y	Z	Rx	Ry	Rz
Above T_1	-	57.8	-0.2	-	-	-
Below T_1	-	98.5	-1.4	-	-	-
" T_2	-	198.7	-6.7	-	-	-
" T_3	-	301.7	-14.9	-	-	-
" T_4	-	432.8	-3.0	-	-	-
" T_5	-	542.2	-14.3	-	-	-
" T_6	-	680.6	-0.4	-	-	-
" T_7	-	768.5	-28.7	-	-	-
" T_8	-	898.2	-28.8	-	-	-
" T_9	-	1008.1	-59.2	-	-	-
" T_{10}	-	1078.2	-111.1	-	-	-
" T_{11}	-	1126.1	-128.3	-	-	-
" T_{12}	-	1177.3	-72.2	-	-	-
" L_1	-	1121.6	-68.2	-	-	-
" L_2	-	1125.9	-76.3	-	-	-
" L_3	-	1149.3	-81.2	-	-	-
" L_4	-	1126.5	-	-	-	-
" L_5	-	995.7	58.1	-	-	-

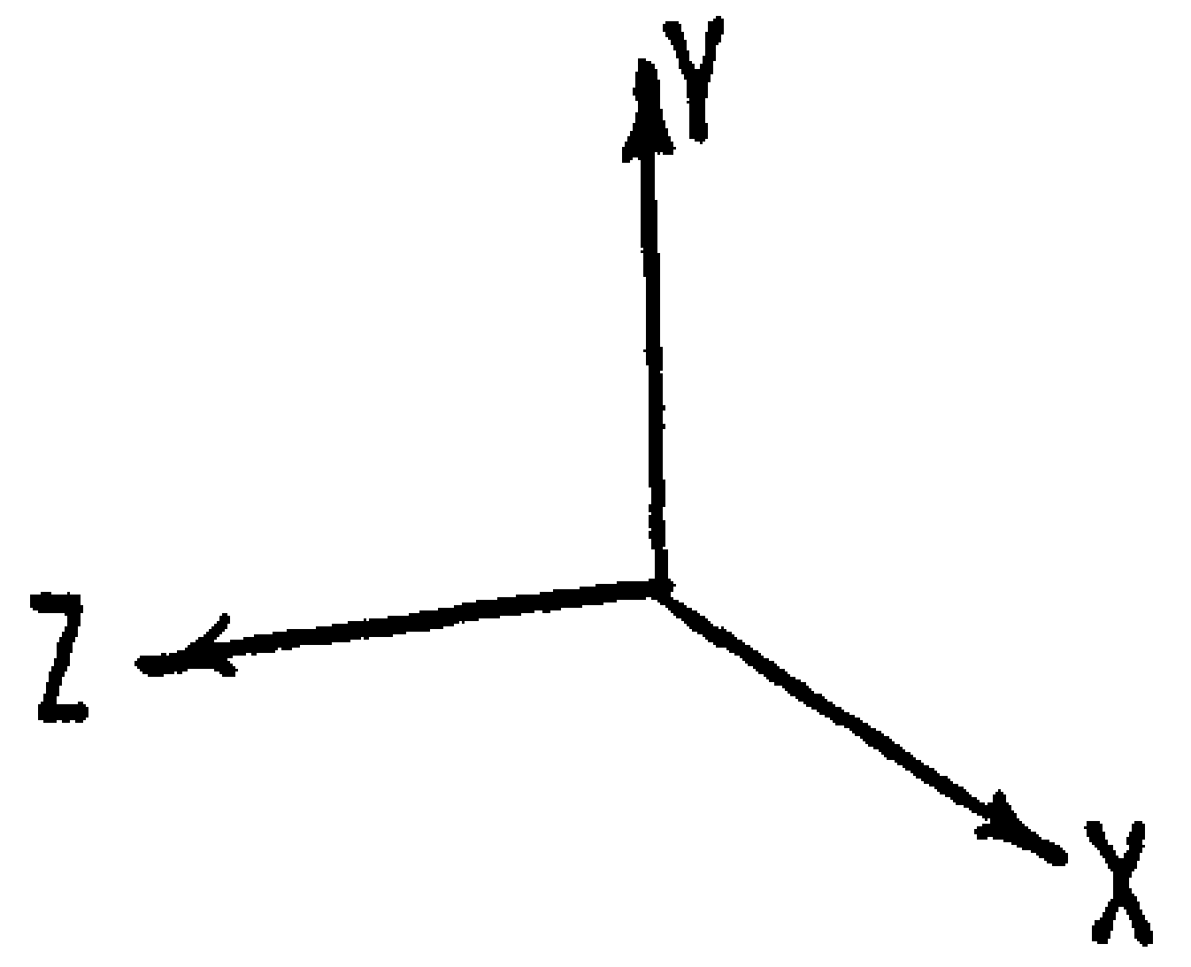


Fig.61 The Intervertebral Reactions In The Upright Position.

Reaction Weighting 1

Muscle & Point Of Origin		Force N			
		Incremental Step			
	Upright	1	2	3	4
Multifidus					
T_1	-	-	-	-	-
-	-	-	-	-	-
T_{12}	112.5	-	-	-	-
L_1	229.7	135.2	278.7	-	-
L_2	-	-	-	-	-
L_3	-	-	-	-	-
L_4	133.7	403.7	1314.9	2796.3	-
L_5	21.8	283.1	1229.7	1798.7	-
S_1	22.1	227.7	1228.2	1905.3	-
S_2	56.2	328.8	1658.9	-	-
S_3	115.6	-	-	-	-
Semi-Spinalis					
T_1	-	-	-	-	-
T_2	-	-	-	-	-
T_3	-	-	-	-	-
T_4	-	-	-	-	-
T_5	9.3	22.7	45.2	60.2	65.6
T_6	11.2	-	-	-	-
T_7	149.7	56.8	105.1	137.7	-
T_8	148.6	49.4	93.6	123.9	-
T_9	-	14.2	117.9	-	-
T_{10}	165.4	118.3	244.4	332.8	-
T_{11}	15.2	-	-	-	-
T_{12}	257.7	145.1	256.1	506.8	-
Spinalis Thoracis					
This muscle is inoperative in all cases.					

Fig.62. The Muscle Forces In Forward Flexion (cont.overleaf)

Reaction Weighting 1 Dead Load 550N

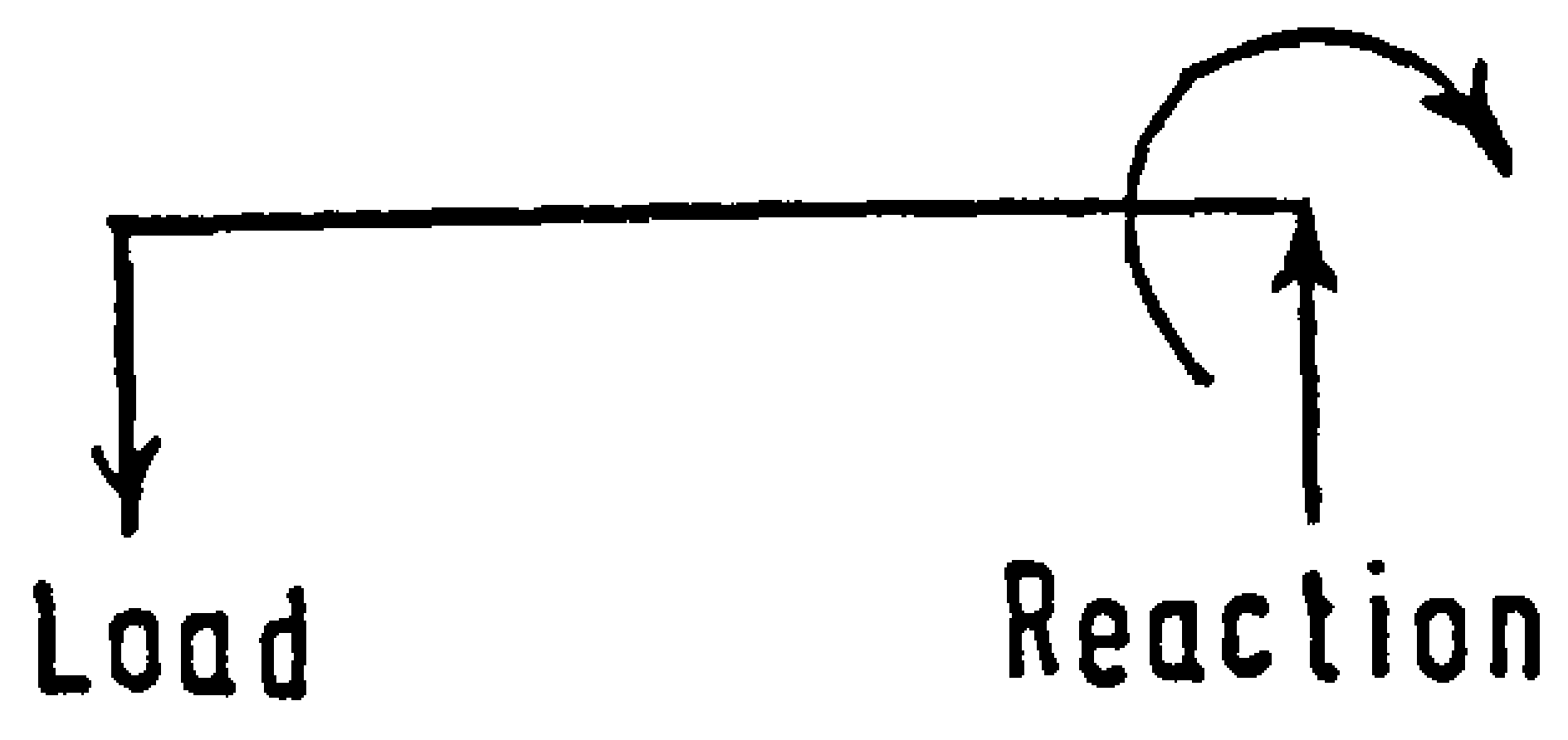
Muscle & Point Of Origin			Force N Incremental Step			
		Upright	1	2	3	4
Longissimus Thoracis						
Insertion	T_1	-	-	-	-	-
	T_2	-	-	-	-	-
	T_3	-	-	-	-	-
	T_4	170.3	78.5	34.5	278.0	-
	T_5	-	-	-	-	-
	T_6	195.7	11.1	-	-	-
	T_7	-	-	-	-	-
	T_8	285.9	28.8	26.1	90.5	-
	T_9	-	-	-	90.2	-
	T_{10}	-	-	-	-	-
	T_{11}	-	-	-	-	-
	T_{12}	-	-	-	-	-
Origin	L_1	-	-	-	-	-
	L_2	-	-	-	-	-
	L_3	-	-	-	-	-
	L_4	-	-	-	-	-
	L_5	310.4	56.4	28.9	218.4	-
	Sacrum	341.5	62.0	31.8	240.3	-
Iliocostalis Lumborum						
Origin	T_{12}	15.7	201.5	573.5	704.4	-
	L_1	19.1	224.4	573.5	704.4	-
	L_2	19.1	246.1	700.2	704.4	-
	L_3	15.7	201.6	573.5	704.4	-
	L_4	19.1	246.1	700.2	860.0	-
	L_5	15.7	201.6	638.6	860.0	-
	Sacrum	17.4	246.1	700.2	860.0	-

Fig.62 Continued.

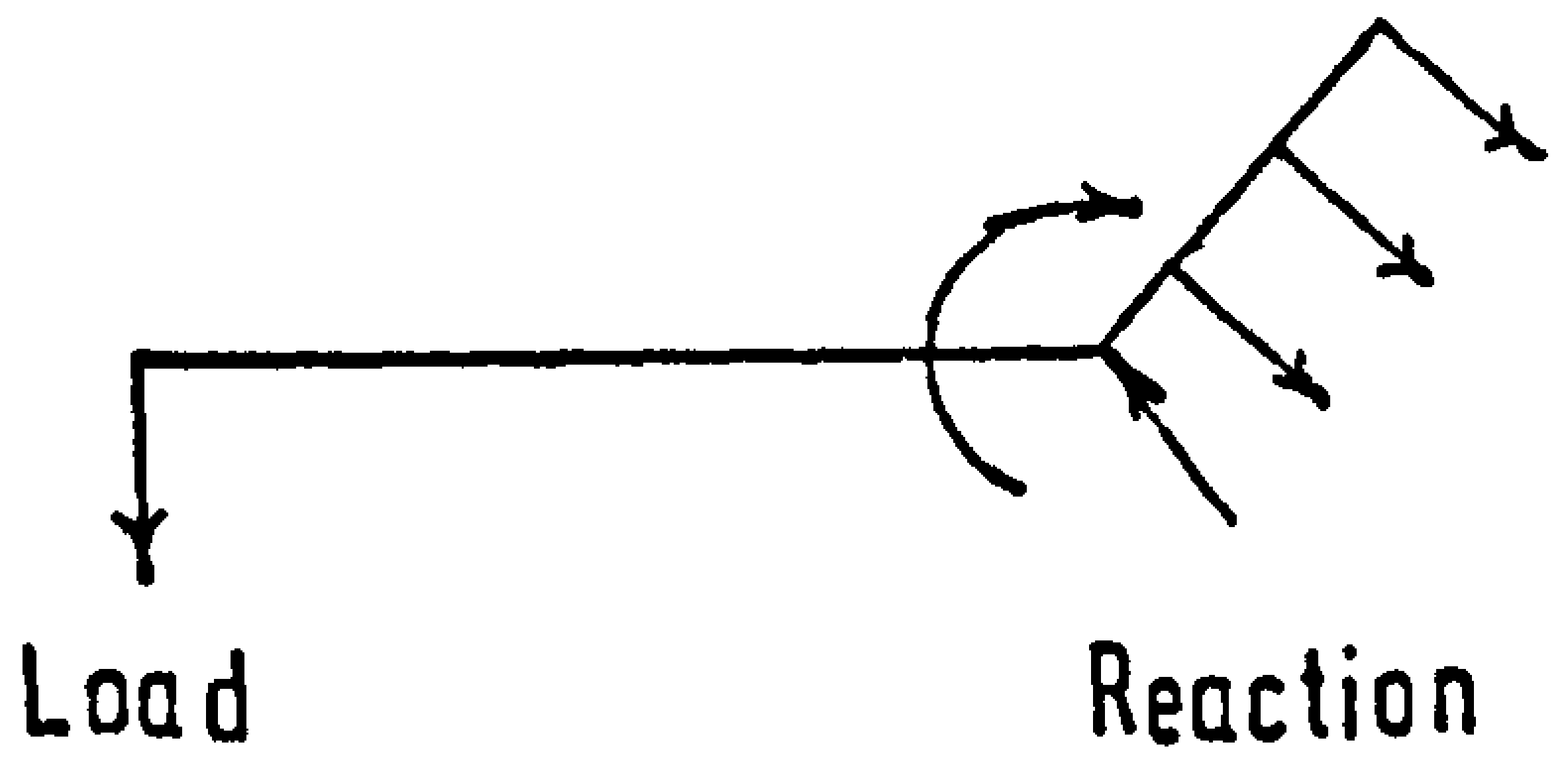
Muscle & Point Of Origin	Force N Incremental Step				
	Upright	1	2	3	4
Iliocostal Thoracis					
This muscle is inoperative in all cases.					
Quadratus Lumborum					
Insertion Rib T_{12}	-	54.2	119.6	-	-
L_1	-	-	-	-	-
L_2	-	-	-	-	-
L_3	-	-	-	-	-
L_4	-	-	-	-	-
Internal Oblique					
Total from 9 strands	478.0	15.4	-	-	-

All the muscles are specified bilaterally, the results from one side only are shown here.

Fig.62 The Muscle Forces In Forward Flexion

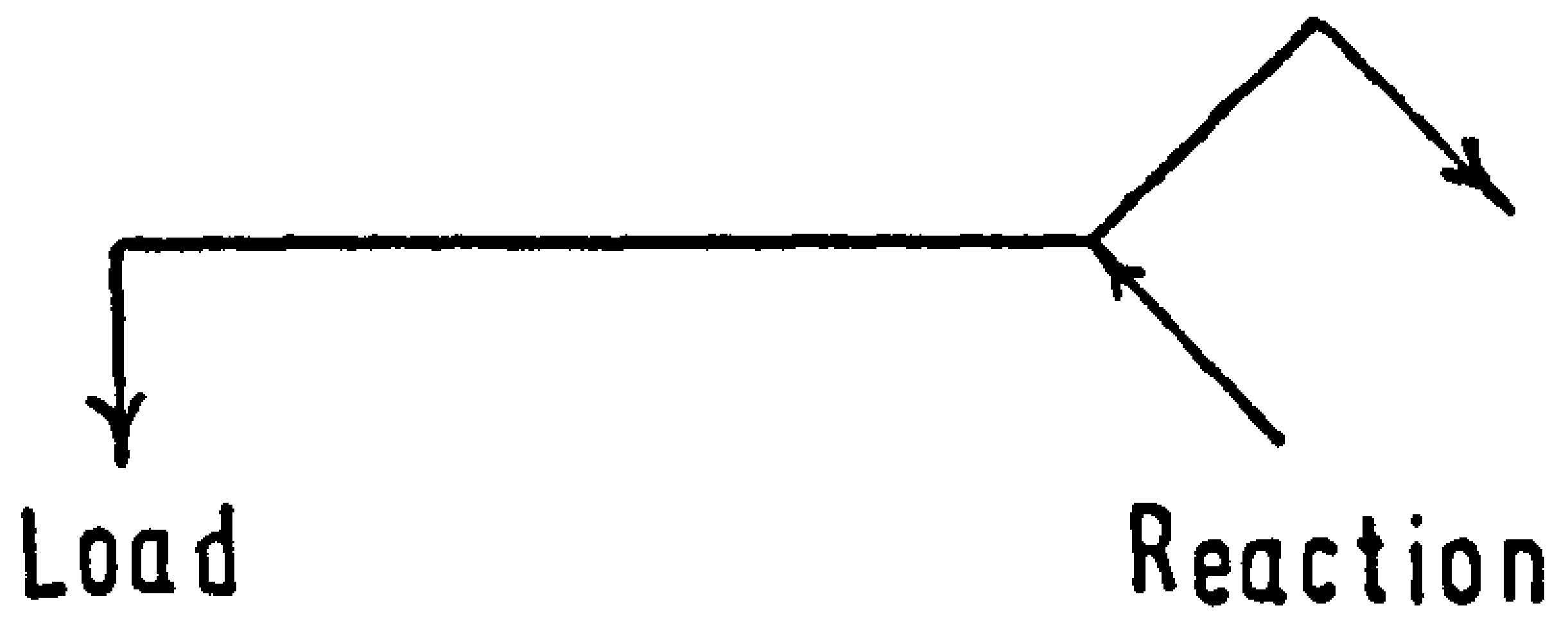


(a)



(b)

However, to check the operation of the computer program a number of intervertebral joint reactions have been recalculated in a more representative manner. To do this it has been assumed that the passive elements act at a mean moment arm of 50mm and no moments are carried at the reaction.



Using values from the study which weighted the reactions at unity, the following results were obtained for the direct compressive load on the L_5 disc in the fully flexed position.

In fully flexed position = 1125N

ditto with 550N deadload = 3820N.

These values assume zero relief due to intra-abdominal pressure. The model described by Morris et al [10] would give equivalent loads of 2260N and 6450N, but they are for a full-grown adult not a young boy as used in this study. The values calculated by Farfan and Lamy [76] for similar positions are 2900N and 11120N respectively and these were obtained for a weight-lifter.

No attempt has been made to relate the compressive load on the L_5 disc to an equivalent pressure, although from the apparent size of the vertebrae on the X-ray films it would be feasible for the pressure from this study to be similar to that for a full-grown adult.

7.3.2. Lateral flexion.

One example is shown for lateral flexion of the spinal column. The results obtained are shown in Fig.63. In this study the "weighting" for the intervertebral reactions was set to unity and for the reaction moments set to 12. All the muscles were in operation to hold this deformed position. Predominantly those acting on the convex side were the Spinalis Thoracis, the Quadratus Lumborum, the Iliocostalis and the Internal Oblique. Those acting on the concave side, to counteract the rotational forces produced by the eccentricity of the masses and the action of the former muscles, were the Multifidus, and the Longissimus Thoracis. The intervertebral reactions in the deformed position are

Muscle	Force N	
& Point Of Origin	Convex side	Concave side
Multifidus		
T_1	-	-
T_2	-	-
T_3	-	-
T_4	-	-
T_5	-	-
T_6	15.0	6.4
T_7	-	3.8
T_8	-	34.1
T_9	-	8.5
T_{10}	-	4.2
T_{11}	-	-
T_{12}	28.0	-
L_1	10.3	-
L_2	-	-
L_3	-	-
L_4	86.3	81.4
L_5	-	245.5
S_1	-	111.3
S_2	-	143.5
S_3	-	143.5
Spinalis Thoracis		
Insertion		
T_3	8.5	-
T_4	41.6	-
Origin		
T_{12}	50.1	-
Quadratus Lumborum		
Insertion		
L_3	163.4	-
L_4	229.1	-

Fig.63 The Muscle Forces For Lateral Flexion. (cont.overleaf)

Muscle & Point Of Origin	Force N Convex side	Concave side
Semi-Spinalis		
T_1	44.3	-
T_2	21.6	-
T_3	14.5	-
T_4	-	-
T_5	-	-
T_6	-	3.2
T_7	1.1	48.5
T_8	-	20.0
T_9	-	-
T_{10}	-	6.4
T_{11}	-	48.9
T_{12}	69.3	-
Iliocostalis Thoracis		
Insertion		
T_1	49.3	-
T_2	40.4	-
T_3	46.5	-
T_4	43.2	-
T_5	40.3	-
T_6	49.3	-
Iliocostalis Lumborum		
Origin		
T_{12}	127.5	-
L_1	146.3	-
L_2	155.6	-
L_3	155.6	-
L_4	151.3	-
L_5	127.5	-
Sacrum	127.5	-

Fig.63 Continued overleaf.

Muscle & Point Of Origin		Force N	
		Convex side	Concave side
Longissimus Thoracis			
Insertion	T_1	-	-
	T_2	-	-
	T_3	-	4.1
	T_4	-	-
	T_5	20.7	-
	T_6	-	-
	T_7	-	-
	T_8	3.0	56.8
	T_9	-	-
	T_{10}	11.1	-
	T_{11}	-	-
	T_{12}	-	127.1
Origin	L_1	-	-
	L_2	-	187.9
	L_3	34.8	-
	L_4	-	-
	L_5	-	-
	Sacrum	-	-
Internal Oblique			
Total from 9 strands		141.4	24.3

Fig.63 The Muscle Forces For Lateral Flexion

shown in Fig. 64.

7.3.2. The scoliotic configuration.

In all the examples described for a scoliotic configuration the objective function for the intervertebral moments was set equal to 6, this allows the intervertebral joints to resist rotational forces developed because of the deformity. Predominantly this is in the region of greatest lateral curvature, between T_6 and T_{12}, typically moments of up to 2250Nmm are recorded in this region.

The forces developed in the muscles for a scoliotic curve without any additional support are shown in Fig. 65. All the muscles are in operation and a similarity with the results for a lateral flexion exists. Here, the Multifidus and Semi-Spinalis muscles are seen to act on the concave side, the Longissimus and Iliocostalis act on the convex side.

The results for the example which included the forces developed in a Milwaukee Brace are shown in Fig. 66. There is very little difference between these results and those without the addition of the brace, although the forces acting are marginally lower and the intervertebral reactions are also slightly reduced.

Vertebral Level	Reaction Force N			Reaction Moment Nmm		
	X	Y	Z	Rx	Ry	Rz
Above T_1	63.5	98.0	10.1			
Below T_1	53.8	136.0	12.5	-	172.2	1542.7
T_2	63.4	239.9	10.6	-	669.9	-
T_3	92.5	348.6	11.5	-	-	-
T_4	119.1	465.5	13.7	-	-	-
T_5	142.1	592.5	3.1	-	-	-
T_6	157.7	703.6	-2.1	-	-	759.1
T_7	144.0	824.3	-37.4	-	-	-
T_8	130.6	961.9	-51.9	-	-	-
T_9	93.7	1110.5	-97.2	-	-	-
T_{10}	41.9	1253.8	-137.3	-	69.1	-
T_{11}	-30.3	1318.2	-166.7	-	-	1676.7
T_{12}	-58.7	1394.9	-216.9	-	-	-
L_1	-15.0	1375.7	-305.1	-	-3725.5	6434.2
L_2	-79.8	1268.9	-320.5	-	-2093.2	643.2
L_3	-10.7	1357.1	-278.4	-	-2178.6	-
L_4	111.2	1354.7	40.7	-	-	-
L_5	198.6	1206.0	262.1	-	-7390.9	3646.3

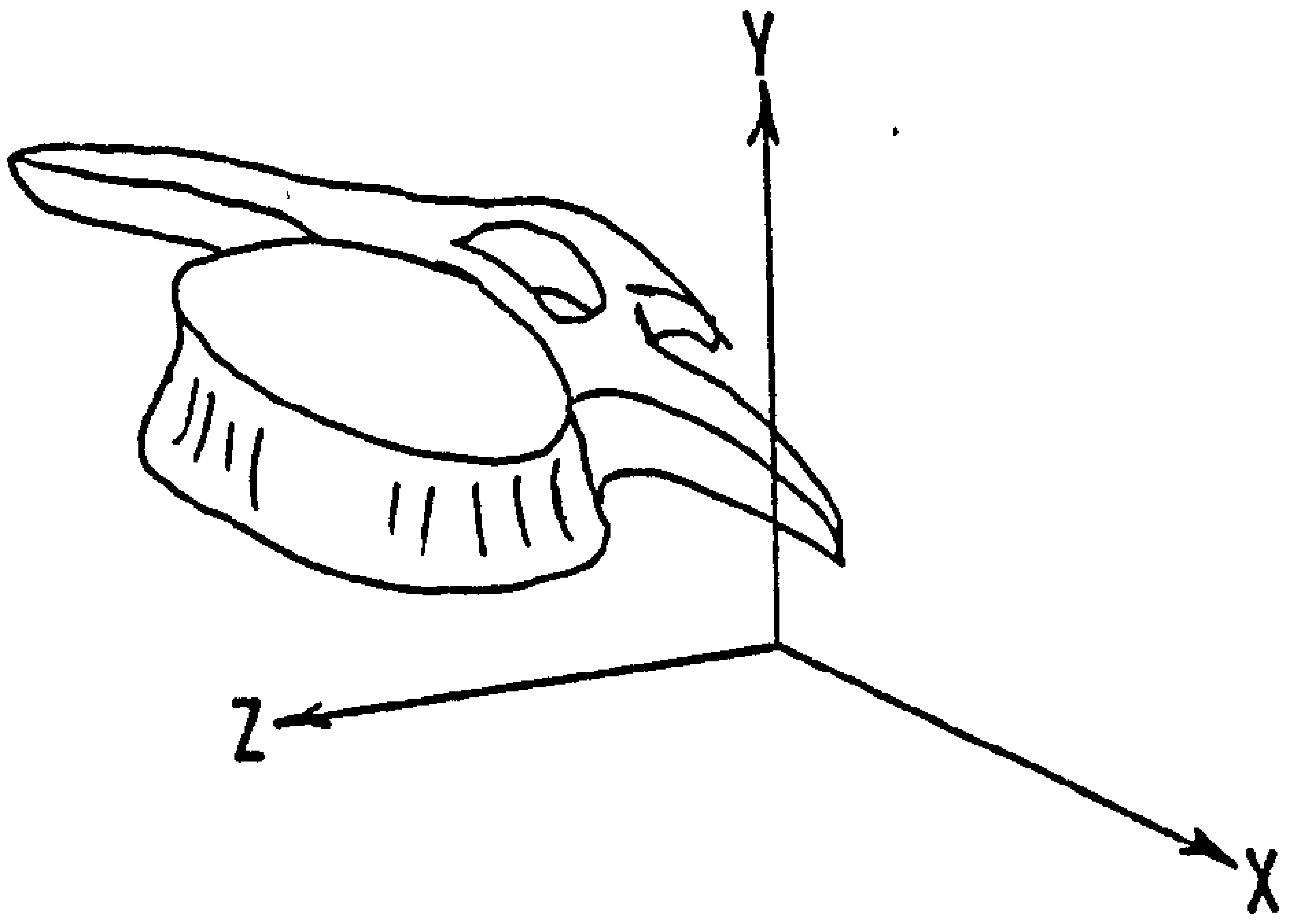


Fig.64. The Intervertebral Reactions In Lateral Flexion

Muscle & Point Of Origin	Force N Concave side	Force N Convex side
Multifidus		
T_5	19.6	-
T_8	5.3	-
T_9	4.1	-
T_{10}	3.2	-
T_{11}	10.3	-
L_4	-	19.7
L_5	-	23.7
S_3	47.1	53.5
Semi-Spinalis		
T_1	-	12.0
T_5	15.5	-
T_6	30.0	-
T_7	13.3	-
T_8	27.3	-
T_9	4.4	-
T_{10}	61.9	-
T_{11}	28.4	-
T_{12}	18.3	-
Spinalis Thoracis		
Insertion		
T_3	3.8	-
T_4	14.1	-
Origin		
T_{11}	17.9	-
Iliocostalis Lumborum		
T_{12}	-	2.8
L_1	-	2.6
L_2	-	2.8
L_3	-	2.8
L_4	-	2.3
L_5	-	2.3
Sacrum	-	2.3

Fig.65 The Muscle Forces For An Upright Scoliosis Curve

(continued overleaf)

Muscle & Point Of Origin		Force N	
		Concave side	Convex side
Longissimus Thoracis			
Insertion	T_1	-	1.6
	T_2	-	4.5
	T_3	-	13.9
	T_4	-	41.0
	T_5	-	2.6
	T_6	-	42.1
	T_7	-	27.9
	T_8	-	39.2
	T_9	-	19.8
	T_{10}	-	66.9
	T_{11}	-	51.8
	T_{12}	-	26.8
Origin	L_1	-	56.4
	L_2	-	136.7
	L_3	-	145.2
Quadratus Lumborum			
Insertion Rib	T_{12}	-	-
	L_1	27.3	-
	L_2	62.4	-
	L_3	132.2	-
	L_4	84.1	-
Internal Oblique			
Total from 9 strands		-	26.8

Fig.65 The Muscle Forces For An Upright Scoliosis Curve

Muscle & Point Of Origin	Force N Concave side	 Convex side
Multifidus		
T_5	18.1	-
T_6	13.6	-
T_9	3.2	-
T_{10}	5.8	-
L_3	-	5.1
L_4	-	20.7
L_5	-	20.8
S_3	52.4	58.0
Semi-Spinalis		
T_1	-	15.5
T_2	-	-
T_3	-	-
T_4	-	-
T_5	15.5	-
T_6	30.0	-
T_7	6.5	-
T_8	8.7	-
T_9	4.2	-
T_{10}	62.7	-
T_{11}	40.5	-
T_{12}	11.8	-
Iliocostalis Thoracis This muscle is inoperative in this example.		
Iliocostalis Lumborum		
T_{12}	-	2.4
L_1	-	2.2
L_2	-	2.4
L_3	-	2.4
L_4	-	2.0
L_5	-	2.0
Sacrum	-	2.0

Fig.66 The Muscle Forces For A Scoliosis Curve
(with a Milwaukee Brace)

cont.overleaf.

Muscle & Point Of Origin		Force N	
		Concave side	Convex side
Longissimus Thoracis			
Insertion	T_1	-	1.7
	T_2	-	9.8
	T_3	-	14.7
	T_4	-	35.8
	T_5	-	1.5
	T_6	-	38.8
	T_7	-	25.5
	T_8	-	29.4
	T_9	-	15.9
	T_{10}	-	65.9
	T_{11}	-	53.6
	T_{12}	-	12.9
Origin	L_1	-	61.0
	L_2	-	114.1
	L_3	-	130.4
Spinalis Thoracis			
Insertion	T_3	2.9	-
	T_4	12.3	-
Origin	T_{11}	15.2	-
Quadratus Lumborum			
Insertion Rib	T_{12}	-	-
	L_1	28.9	-
	L_2	52.3	-
	L_3	126.6	-
	L_4	107.4	-
Internal Oblique			
Total from 9 strands		-	13.9

Fig.66 The Muscle Forces For A Scoliosis Curve (with a Milwaukee Brace)

However, the results for the example including the action of Halo-Pelvic distraction, shown in Fig. 67, are completely reversed. There is a marked reduction in muscle force developed and in this case the Multifidus and Semi-Spinalis predominantly act on the convex side with the Iliocostalis acting on the concave side. This is because the force developed in the external equipment is such as to put the spinal column in tension rather than compression. There is a substantial reduction in the total force developed by the muscles, only the Multifidus, Semi-Spinalis, Iliocostalis, Quadratus Lumborum and Internal Oblique are in operation.

With the inclusion of the action of Harrington Instrumentation, acting between T_4 and L_3, a more balanced force distribution is obtained, Fig. 68. The Multifidus, the Semi-Spinalis, the Quadratus Lumborum and the Longissimus act on both the convex and concave side of the column. There is a substantial reduction in muscle force in the thoracic region, although the Multifidus is very much more active in the lumbar region.

The value of the objective function is a measure of the total force acting in the system including both the muscles and the intervertebral reactions. On this basis there is a temptation to inversely equate total force value with the effectiveness of the external

Muscle & Point Of Origin	Force N	
	Concave side	Convex side
Multifidus		
T_3	-	14.7
T_4	-	14.8
T_8	-	0.5
L_4	-	69.1
L_5	-	71.6
S_1	18.6	50.0
S_2	70.8	43.4
S_3	76.1	95.9
Semi-Spinalis		
T_2	6.6	-
T_4	-	0.3
T_5	-	11.3
T_8	-	3.9
Quadratus Lumborum		
Insertion L_2	0.2	-
L_3	57.9	-
L_4	51.4	-
Iliocostalis Thoracis		
Insertion T_1	12.1	7.8
T_2	12.1	9.5
T_3	13.2	7.6
T_4	14.8	9.5
T_5	14.8	7.6
Iliocostalis Lumborum		
Origin T_{12}	46.7	-
L_1	57.1	-
L_2	57.1	-
L_3	55.3	-
L_4	53.8	-
L_5	46.7	-
Sacrum	46.7	-
Internal Oblique		
Total from 9 strands	26.7	-

Fig.67 The Muscle Forces For A Scoliosis Curve

(with Halo-Pelvic distraction)

Muscle	Force N	
& Point Of Origin	Concave side	Convex side
Multifidus		
T_5	7.6	-
T_6	0.7	-
T_7	4.7	-
T_{11}	0.3	-
L_1	2.5	-
L_4	-	110.2
L_5	-	117.2
S_1	67.8	101.1
S_2	107.5	144.3
S_3	146.4	192.6
Semi-Spinalis		
T_1	-	12.0
T_5	15.5	-
T_7	-	1.1
T_{10}	22.2	-
T_{11}	0.9	-
T_{12}	4.8	-
Spinalis Thoracis		
Insertion T_3	2.9	-
T_4	8.3	-
Origin L_2	11.2	-
Quadratus Lumborum		
Insertion Rib T_{12}	43.2	79.5
L_1	-	50.5
L_2	-	23.2
L_3	-	-
L_4	1.3	-

Fig.68 The Muscle Forces For A Scoliosis Curve
(with Harrington Instrumentation)

cont.overleaf.

Muscle		Force N
& Point Of Origin	Concave side	Convex side
Iliocostalis Lumborum		
T_{12}	7.9	-
L_1	6.5	-
L_2	7.2	-
L_3	6.5	-
L_4	6.5	-
L_5	7.9	-
Sacrum	7.9	-
Iliocostalis Thoracis		
This muscle is inoperative in this example.		
Internal Oblique		
This muscle is inoperative in this example.		
Longissimus Thoracis		
Insertion T_1	-	-
T_2	-	-
T_3	-	21.4
T_4	-	27.5
T_5	-	-
T_6	0.5	-
T_7	-	-
T_8	5.7	-
T_9	2.9	-
T_{10}	21.9	-
T_{11}	-	9.3
T_{12}	-	-
Origin L_1	-	58.2
L_2	3.1	-
L_3	27.9	-

Fig.68 The Muscle Forces For A Scoliosis Curve (with Harrington Instrumentation).

support, there is no validation for this but the figures are listed below.

	Value of Objective.
Scoliotic curve alone	34317
" " with Milwaukee Brace	30941
" " " Halo-Pelvic Dist.	25824
" " " Harrington Instr.	26124

Both Halo-Pelvic Distraction and Harrington Instrumentation give substantially lower results suggesting that these systems effectively support the vertebral column.

7.4. A Discussion Of The Results.

There is an infinity of ways to forming the Objective Function for the linear programming model, a small change in the Objective can also cause a significant alteration in the results. The results shown in the preceeding sections are typical of what can be obtained using this method, but are not a definitive solution to the examples studied.

There has been no other model of the spinal column musculature proposed which is as complex as this one and it is difficult to assess the accuracy of the results obtained. At present it is also impossible to validate the muscles forces against any experimental data, as only an indication as to the action of a muscle can be gained from EMG recordings.

A severe limitation to the operation of this model has been the difficulty to define the forces generated by the passive elastic tissue of the intervertebral joints. The "weighting" factor does not define explicitly the forces involved at the joints, and therefore, it is possible to have combinations of forces which are unrealistic, e.g. the intervertebral reactions could be zero for all the vertebrae except one which would have a force, there would be no structural reason for this force, but it is there because of the optimization technique requiring to find, say, minimum force in the complete system. A model which defined the intervertebral reactions would be advantageous; to do this a knowledge of the deformation at the joints and the elastic properties of the tissues is required. A model which includes a structural analysis of the passive elastic elements of the spinal column is described in Chapter 9.

7.4.1. EMG results of similar positions.

A number of studies have been carried out in measuring the electrical potential of various muscles of the trunk and back. However, general results from surface electrodes are not sufficiently precise to be of use in comparing with this mathematical model.

(a) Easy standing and forward flexion.

Jonsson [164] using wire electrodes, found activity in the Multifidus muscle at the L_1 and L_4 levels and marked activity at L_2, L_3 and L_5 levels in easy standing. For the same position marked activity at the L_2 level of the Iliocostalis was also found. For the Longissimus, activity was found at all lumbar levels, the highest being in the L_3 region. Donisch and Basmajian [155] placed wire electrodes in the region of the transverso-spinal muscles, they obtained a constant activity in the thoracic region in an upright position but only intermittent electrical potential in the lumbar region. In full forward flexion there was spontaneous electrical silence. Oertengren and Andersson [165] found increasing electrical activity in the back muscles up to full flexion where there was silence. Carlsöö [166] also found activity in the Sacrospinalis muscles which increased with flexion. Pauly [154] studied various movements involving many muscles. In an upright position little activity was found in the Iliocostalis Lumborum or Spinalis muscles. Floyd and Silver [153] found increasing activity in the Erector spinae muscles in forward flexion.

The results from the model in the upright position, as in Figure 60, show the Multifidus muscle in the lumbar region to be in operation, the Semi-Spinalis in the lower thoracic region also active, the Longissimus to be slightly active and the Iliocostalis Lumborum also to be in use. The Internal Oblique was active in the upright position but not thereafter, Floyd and Silver [167] found similar results although Robertson [8] recorded activity in the abdominal muscles during flexion. The reason for this, described on page 11, is to stabilise the pelvis. In the model here the pelvis is a fixed base, therefore the only activity required is to stop the spinal column rotating backwards when upright. The muscles in operation in the model are for static positions only and cannot be expected to agree entirely with tests for a complete dynamic movement. However, the muscles predicted to be in operation by the model are physically feasible, the values of forces derived are open to debate.

(b) Lateral flexion.

Carlsöö [166] recorded marked activity on the convex side in lateral bending but also activity on the concave side. This was also confirmed by Oertengren and Andersson [165]. Pauly [154] states that no activity in the Erector spinae is needed for lateral flexion, this was, however, a dynamic movement and not a holding position.

The model predicts increased activity in the Spinalis Thoracis, the Quadratus Lumborum and Iliocostalis on the convex side. The activity of the Semi-Spinalis and the Longissimus is lessened on the convex side in comparison to the upright position. The Multifidus does show increased activity over the upright position on the concave side in lateral flexion.

(c) EMG findings related to scoliotic conditions have been discussed by Robin [27]. The general conclusion is that there is greater electrical stimulation of the muscles on the convex side, however, there are differing opinions depending on the type of deformity involved and the variety of diseases that can cause it. The results that have been shown from the computer model give values of force that are required to balance the structure against bodyweight and externally applied forces and cannot take into account neurological or other diseases.

A study of the effect of Harrington Instrumentation on the spinal column has been carried out by Schultz and Hirsch [168], using the elastostatic model described in Chapter 2.2.7. They can assess the benefit to reducing the curvature of the deformity by using this technique. Their example showed the technique to be useful for greater curvature, small deformities being better handled by external bracing such as the Milwaukee Brace. Their work cannot, however, be correlated with the results shown here.

CHAPTER 8.

A Technique To Derive The Material Properties For Bar And Beam Element Structures.

It was explained in Chapter 4 how a method of formulating rigidity (based on material properties) from the specification of both deformation and corresponding load would be advantageous. An iteration process to do this has been proposed by Kavanagh [169]. His principle, which was developed for finite element analysis of a material in plane stress, has been extended here to deal with simple structural elements i.e. bars and beams. The displacement method of analysis is used throughout and has been explained by Yettram and Hussain [170].

The process is based upon an iteration procedure which takes the form

$P^{(n+1)} = N.P^{(n)}$ for the n+1th iteration

where N is calculated from $\dfrac{\text{Strain (analytical)}^{(n)}}{\text{Strain (experimental)}}$

As $|\text{Strain (experimental)} - \text{Strain (analytical)}|$ is minimized the value of N tends to 1 and a suitable value for P is obtained.

Consider simple bar and beam elements, which act in pure axial strain, and combined axial strain and bending respectively. In this case Poisson's ratio can be ignored and the material constant sought is simply Young's modulus E.

From Hooke's Law

$$E = \frac{\text{Stress}}{\text{Strain}} \quad \text{i.e.} \quad \frac{\sigma}{\varepsilon} \quad \text{for a bar element}$$

$$\text{or } E = \varepsilon^{-1} . \sigma \qquad \text{-----} \ (8\text{-}1)$$

If the structure consists of k elements then the above equation written in matrix notation becomes:

$$\{E\} = \begin{bmatrix} \varepsilon_1^{-1} & & & & \\ & \varepsilon_2^{-1} & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \varepsilon_k^{-1} \end{bmatrix} . \{\sigma\} \qquad \text{-----} \ (8\text{-}2)$$

or more compactly as

$$\{E\} = [A] \ . \ \{\sigma\} \qquad \text{-----} \ (8\text{-}3)$$

The matrix A is then treated as a constant multiplier derived from the experimental values for strain. This is the denominator in the term N mentioned earlier.

The numerator in N is derived as follows:

An initial guess for Young's modulus E is used to form the stiffness matrix of the structure, after suitable partitioning of the matrix into blocks, the section relating known external load to unknown nodal displacement is inverted. On multiplication with the load vector a set of displacements for the nodes of the system is obtained. From the geometry of the structure, before and after loading, the strain in each element is calculated. On the combination of this matrix with the A matrix the multiplying factor N (Strain(analytical) / Strain(experimental)) is formed and a new value for Young's modulus found. The iteration process is continued by using the new value of E in reformulating the stiffness matrix and then calculating a modified value for strain (analytical). The routine is repeated until the change in E per iteration has reached a suitable low value e.g. 0.01%.

The equations are set out below and a flow chart of the process is shown in Fig. 69 .

Equation of equilibrium for the nth iteration:

$$\{F\} = [ST]^{(n)} \cdot \{D\}^{(n)} \qquad \text{-----} \ (8\text{-}4)$$

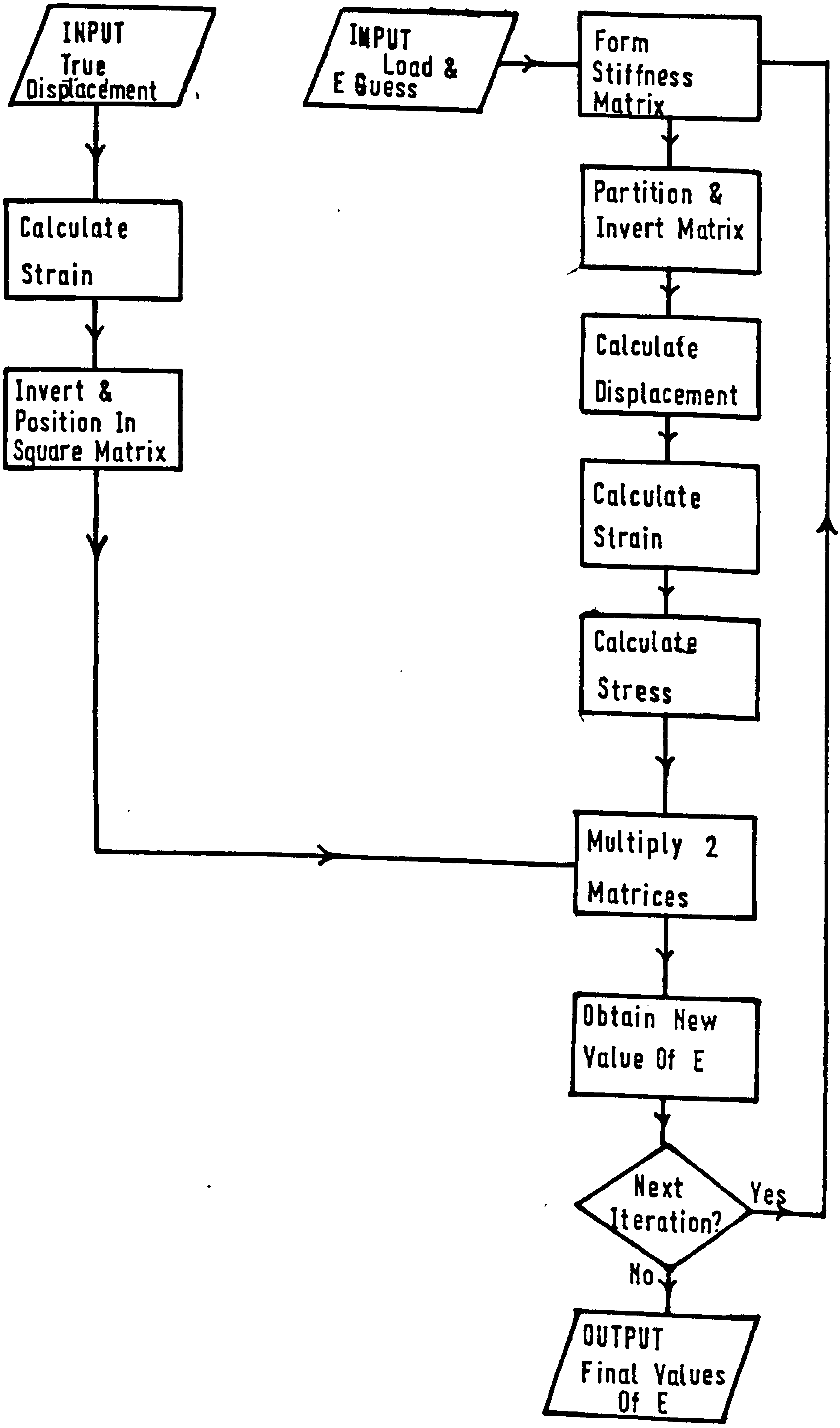


Fig.69 Flowchart For Iteration Procedure

Displacement vector assuming nth set of material constants:

$$\{D\}^{(n)} = [ST^{(n)}]^{-1} . \{F\} \qquad \text{-----} \; (8\text{-}5)$$

Then strain is formulated:

$$\{\varepsilon\}^{(n)} = [DS] . \{D\}^{(n)} \qquad \text{-----} \; (8\text{-}6)$$

And stress:

$$\{\sigma\}^{(n)} = [P]^{(n)} . \{\varepsilon\}^{(n)} \qquad \text{-----} \; (8\text{-}7)$$

The new values for the material constants are given by:

$$[P]^{(n+1)} = [A] . \{\sigma\}^{(n)} \qquad \text{-----} \; (8\text{-}8)$$

$$= [A] . [P]^{(n)} . [DS] . [ST^{(n)}]^{-1} . \{F\} \qquad \text{-----} \; (8\text{-}9)$$

where {F} = load vector {D} = displacement vector

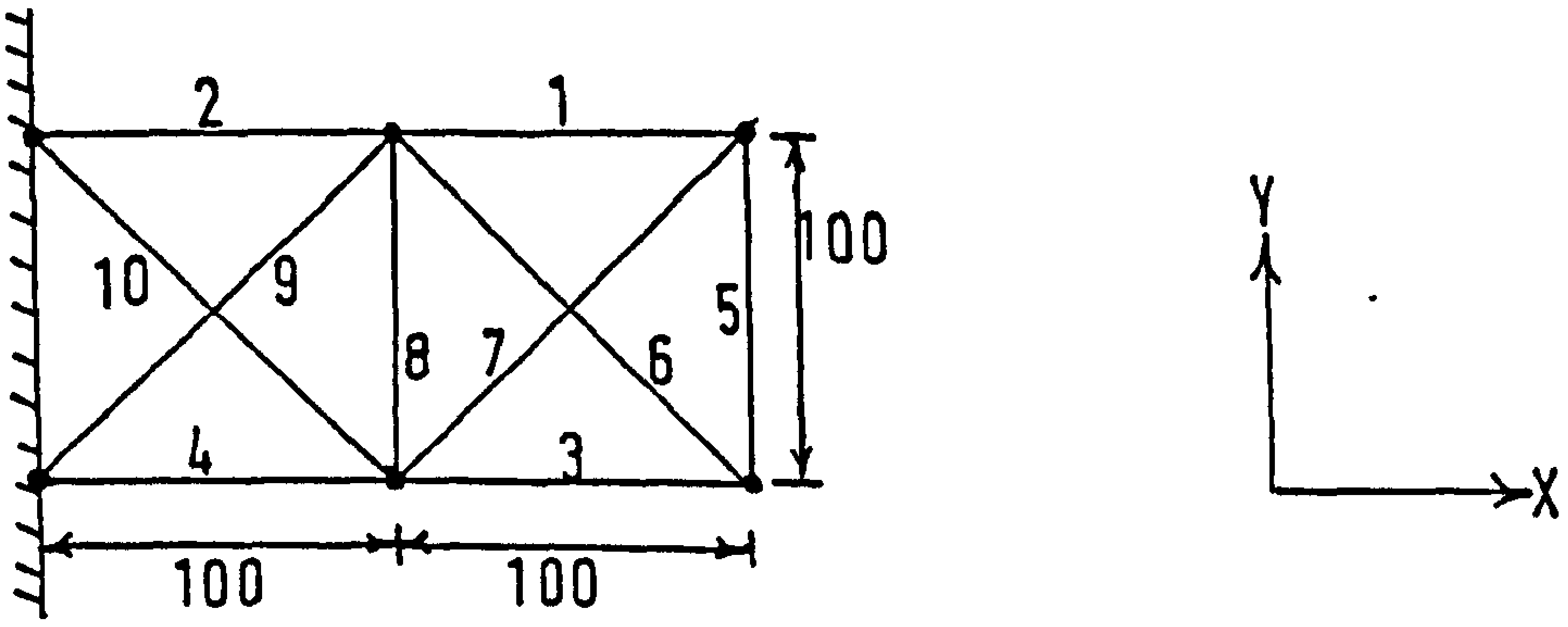
[ST] = stiffness matrix [P] = compliance matrix

[DS] = strain-displacement matrix derived from geometry of the structure.

In all the examples which follow, the experimental values for strain, which formulate the A matrix, were not obtained from true experimental readings but from a separate computer program. The program assumed the correct material properties and calculated the displacement vector accordingly.

8.1. Analysis Of Bar Element Structures.

A framework of pin jointed bar elements (axial load only) was used throughout this series of tests. The geometry of the system was as defined by A.L.Yettram (Brunel University) in a course for Stress and Structural Analysis [171]. The basic configuration of the ten elements is shown below.



The stiffness matrix for an individual bar element is:

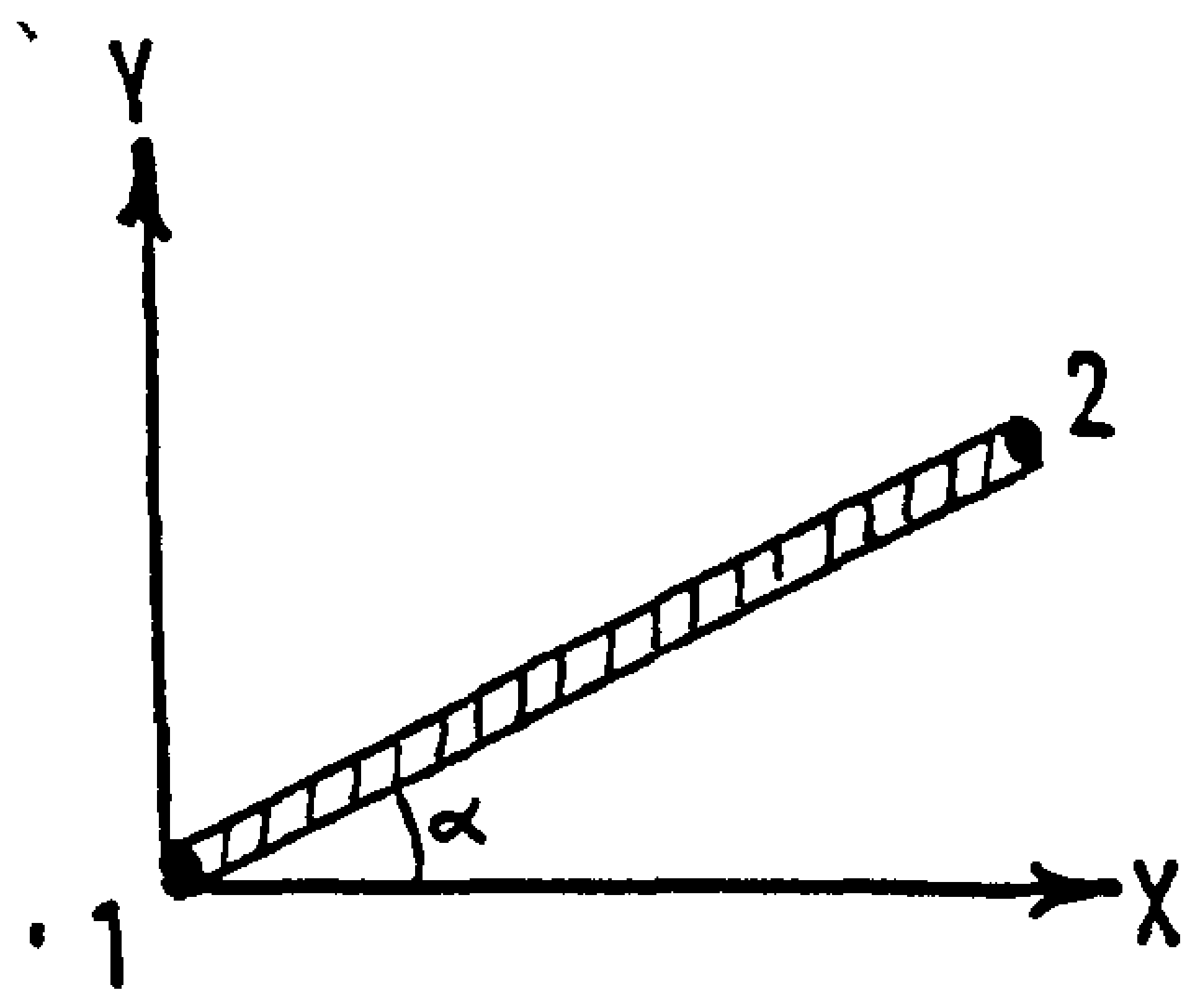
$$\begin{bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \end{bmatrix} = \frac{A.E}{L} \begin{bmatrix} c^2 & cs & -c^2 & -cs \\ & s^2 & -cs & -s^2 \\ & & c^2 & cs \\ \text{SYM.} & & & s^2 \end{bmatrix} \cdot \begin{bmatrix} D_{1x} \\ D_{1y} \\ D_{2x} \\ D_{2y} \end{bmatrix}$$

----- (8-10)

where A is the cross-sectional area of the bar

L is the overall length of the bar $^A/_L = 1$

c = cosα, s = sinα, they are defined by the angle between the bar and the X-axis at node 1, shown overleaf.



The overall stiffness matrix is formed by the summation of the individual element stiffness matrices. The ALGOL computer program used is listed in Appendix 2.

The true displacement of the structure under load was calculated by a separate computer program (using an HP9830 calculator) using the displacement method of analysis.

The data input into the main program is known external load, node displacement (usually to nine significant figures) and a guess for the values of Young's modulus for each element.

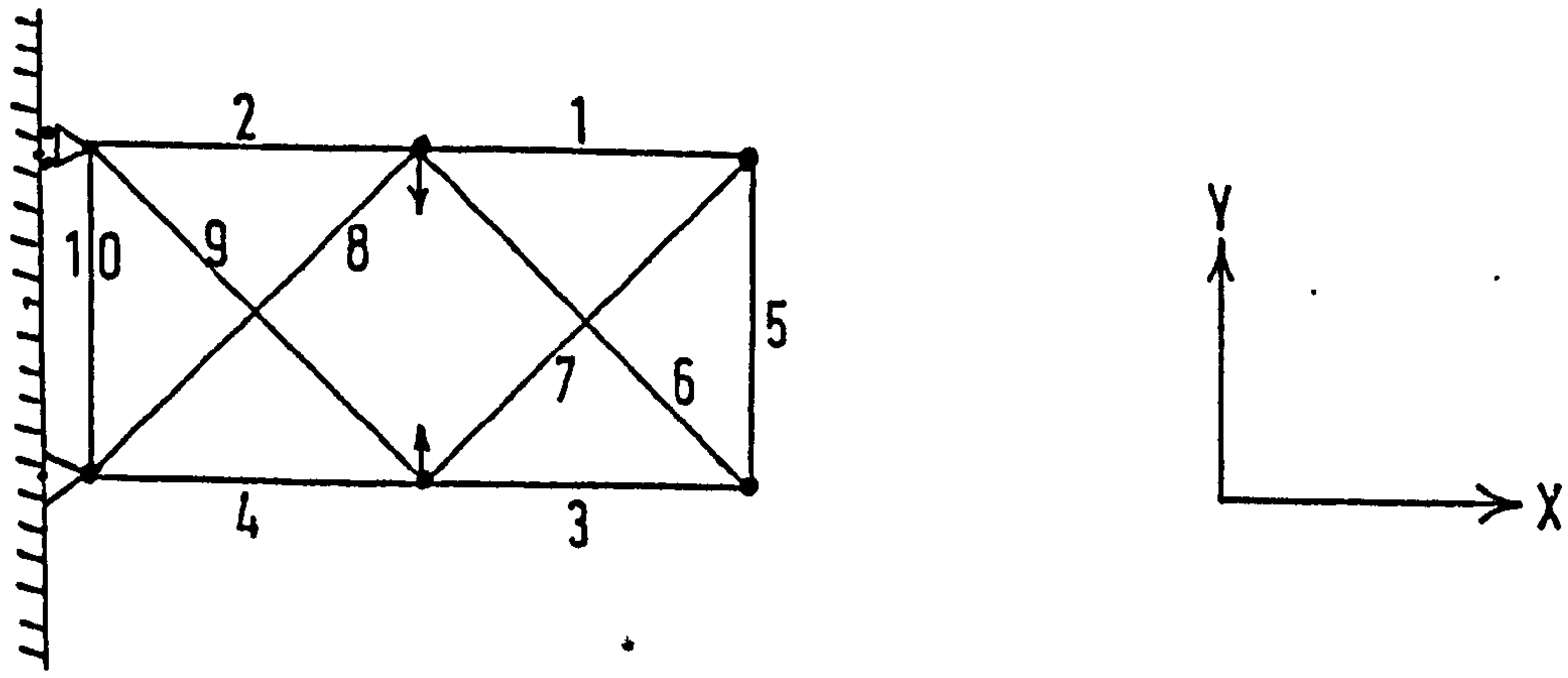
On running the computer it was found that the final values of E were not converging to the correct results, although there was close agreement between true and analytical values of displacement. The final value of E obtained from the iteration routine was found to be dependent on the initial guess (the values were within 5% of the true E). The conclusion is that there is a wide range of member Young's moduli which, when combined into the structure, will yield one particular deformation pattern for a certain configuration of loading. The strain energy calculated for a later example was found to give an overall value the same for both the correct values of E and the offset results, this substantiates the previous conclusion. A later example also showed that symmetry in the system caused problems, which may apply to this structure too.

It had been proposed to simulate muscle action by the application of equal and opposite forces applied to the two nodes between which the muscles acted. To do this, one bar element was removed from the structure (No. 8), the node displacement data, however, was for the system with that element in action. This meant that the structure was acted upon, at the muscle attachment nodes, by an unknown external load. A procedure was added to the computer program to calculate the loads at the nodes in question. This was done using the stiffness matrix in use during the iteration cycle and multiplying it by the displacement vector. The difference between the calculated and input load, at these nodes, represented the force at each end of the muscle. The error in the initial guess for E resulted in an incorrect value for muscle load being obtained. The average value for the node loads was used in describing the overall load field. As the E values, in the iteration routine, approached their correct amount, so the forces at the two nodes tended toward one value as expected. On removing the element from the framework, it was found that the final values of E came to within 3% of their true values (some better than 1%).

For all further tests with a bar framework this configuration was used.

The first tests were carried out with the members having identical values for E.

Study of the results showed that consistently the bay of the frame furthest from the wall produced values of E which were most accurate. (Fig.70). The wall acting as a rigid link at one end of the system was thought to cause this effect. To overcome this problem the framework was modified as shown below:



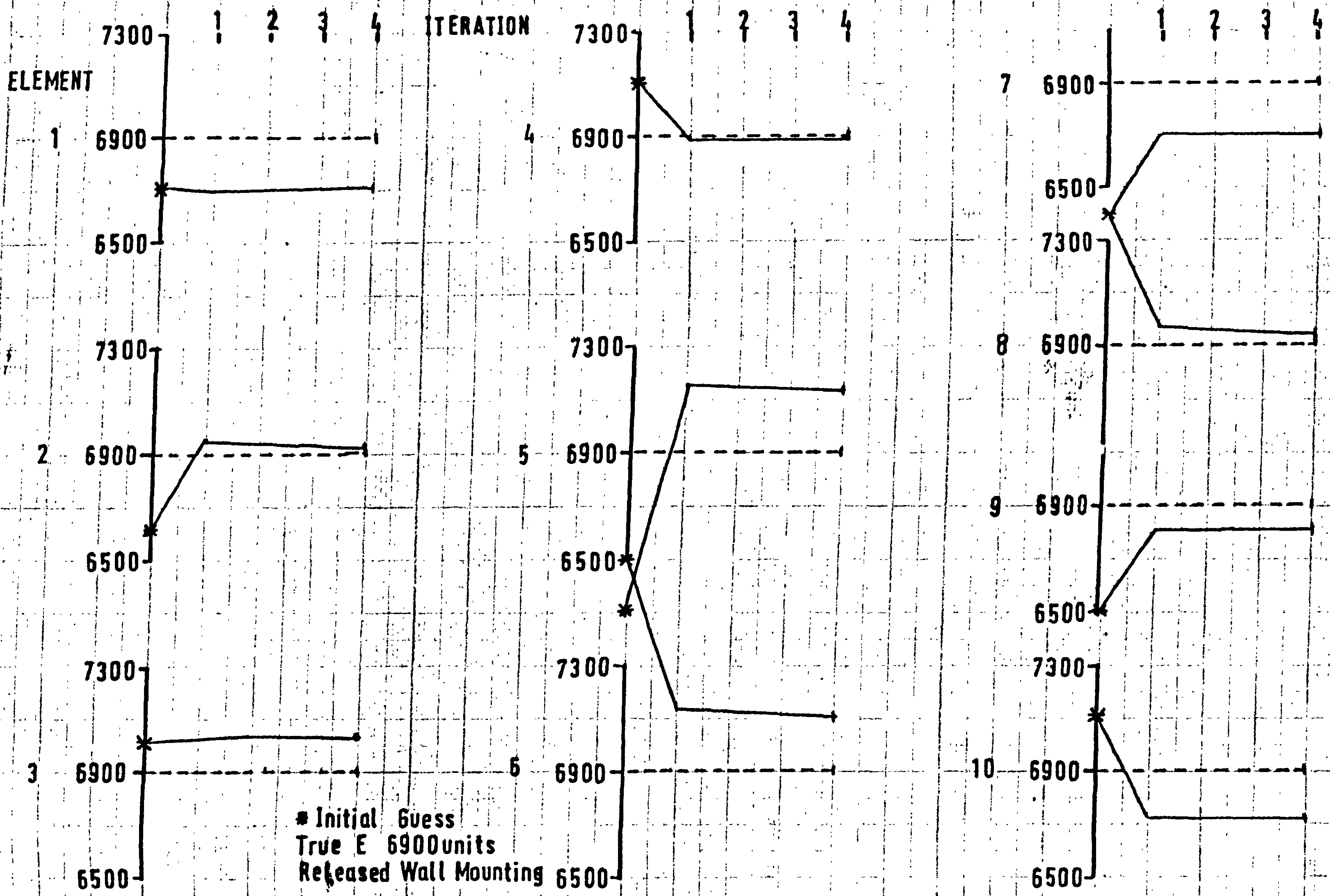


Fig. 70 Variation Of E Per Iteration

One member has been added (No. 10) and the top wall support restrained the system in the X-direction only.

With this arrangement the bay nearest the wall produced results which were best, but no significant increase in accuracy was observed. Fig.71 . is typical of the results obtained. Note: the strains for the members were within 2 x 10^{-5}% of their true values; only E values were markably wrong.

Some of the members were allowed to have different values of E. As before, the results seem to be dependent on the initial guess. Often, if the guess is near the correct answer the final result will be much further out. Fig. 72 shows this. Best results were obtained from an initial 25% offset guess, the final values were within 8% of true and many better than 3%, Fig.73 .

Note: There is rapid movement over the 1st and sometimes 2nd iteration, but very little from then on. The accuracy of the displacement data was increased to 15 significant figures (from 9) but this merely stopped the very slow movement after the 2nd iteration. The values obtained after the 1st or 2nd iterations can be taken as the final results.

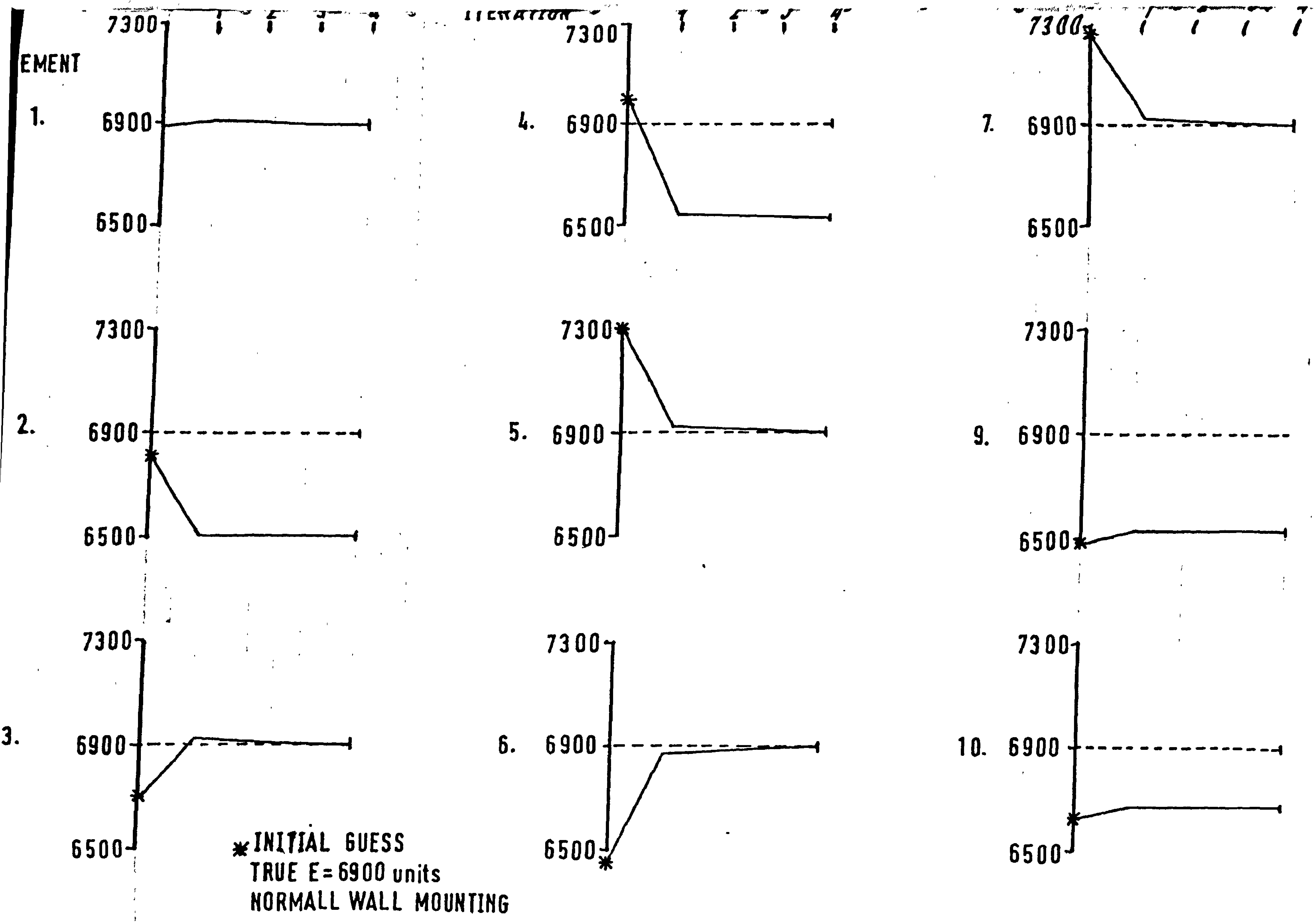


Fig. 71 Variation Of E Per Iteration

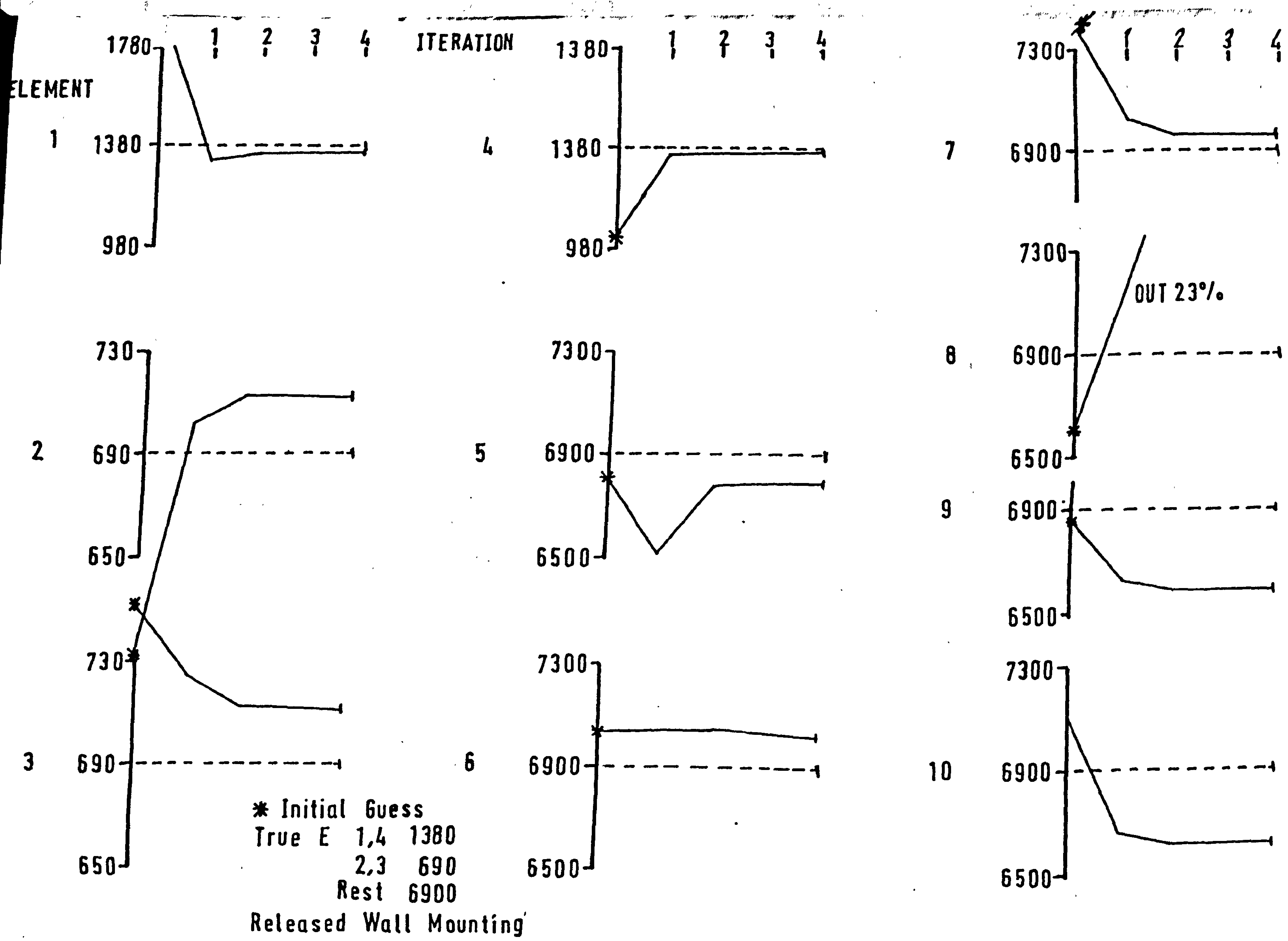
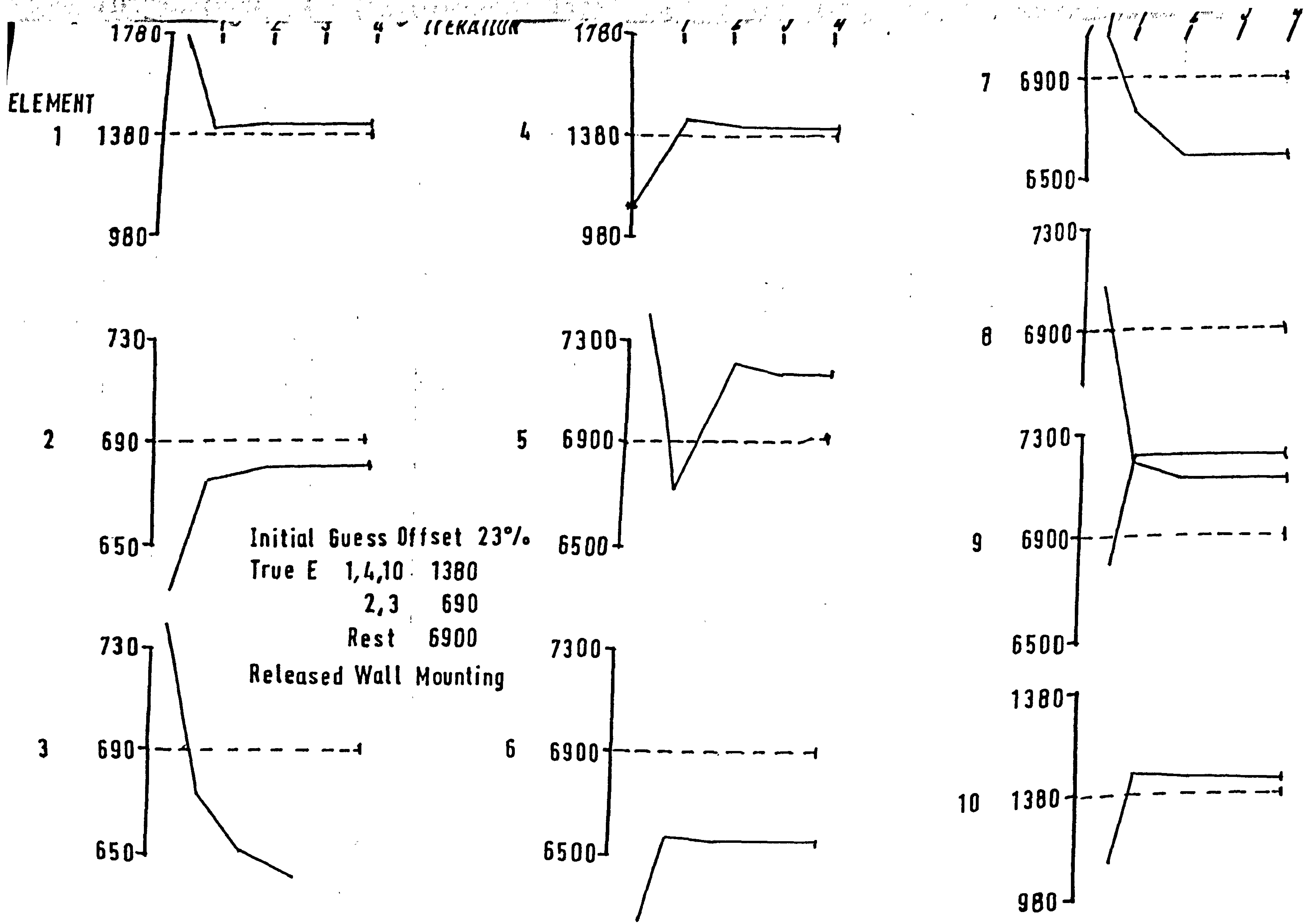

ELEMENT
ITERATION
1
2
3
4
1
1780
1380
980
4
1380
1380
980
7
7300
6900
2
730
690
650
5
7300
6900
6500
8
7300
6900
6500
OUT 23%
9
6900
6500
3
730
690
650
6
7300
6900
6500
10
7300
6900
6500
* Initial Guess
True E 1,4 1380
2,3 690
Rest 6900
Released Wall Mounting

Fig. 72



Initial Guess Offset 23%
True E 1,4,10 1380
2,3 690
Rest 6900
Released Wall Mounting

Fig. 73

	Normal Wall	Released Wall
All elements the same. Stiffness = 6900 N/mm i.e. E = 69GN/m^2	Guess out 10% from true Final values better than 3% Wall bay furthest out XXXXXXXXXX Wild guess (10, 10, 10, etc.) Load out -5% results out 2-3% Load out -10% Mean -9% range 24%	Outside bay out 1st iteration guess gives the same results. does not converge XXXXXXXXXX
Elements 1,4 = 27600 2,3 = 1725 REST = 6900	XXXXXXXXXX	Material too stiff does not work well
Elements 1,4 = 1380 2,3,5 = 690 REST = 6900	Guess out 10% results <5% some <1% Guess out 25% results <15% some 2-4% Load low 10% mean of results -10%	XXXXXXXXXX XXXXXXXXXX XXXXXXXXXX
Elements 1,4,10 = 1380 2,3 = 690 REST = 6900	XXXXXXXXXX XXXXXXXXXX XXXXXXXXXX XXXXXXXXXX	Guess out 25% results out <8% Final iteration guess results to <6% First iteration guess results to <6% Load low 10% results low 8-10%

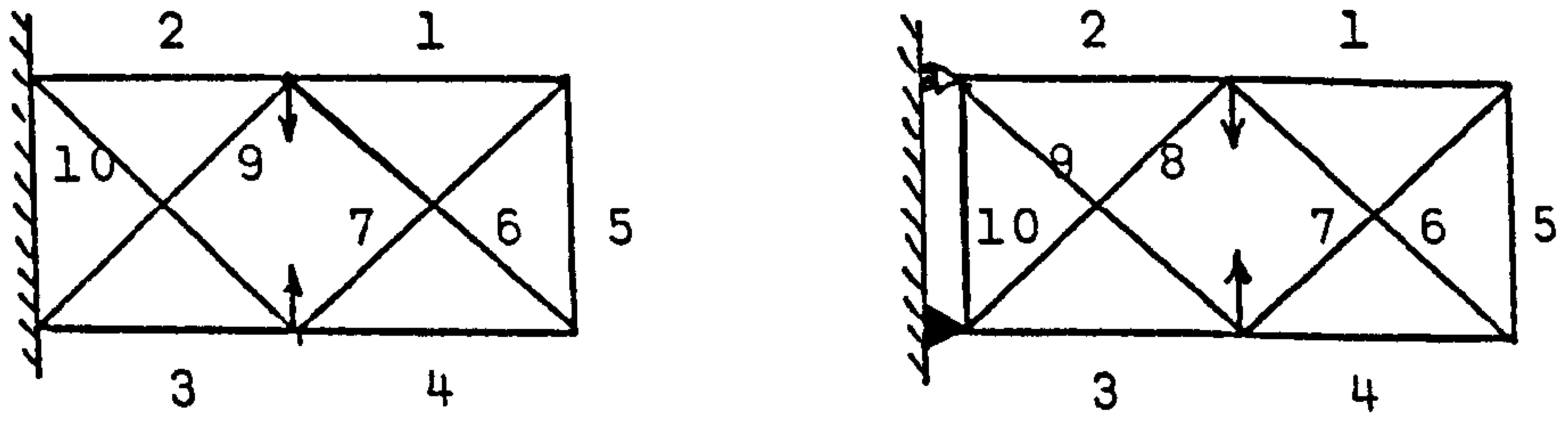


Fig.74. Major Data Changes.

Initial guesses for E were also taken from the results of the first or final iterations from a previous run, but no further improvement in accuracy was forthcoming.

Displacement data to 4 significant figures did not work successfully. It is not known how a range in the values of the material properties affects the iteration process. A less stiff system will not necessarily be more accurate; alternatively a very stiff structure necessitates increased accuracy of the displacement data and computational procedures.

The load accuracy was also varied, i.e. -10%, the results obtained were found to be offset a similar percentage. Isolated cases did not work well and the range in values for the final results was found to be of the order of 20%.

Fig.74 shows a summary of the more important changes made to the structure.

8.2. Analysis Of Beam Element Structures.

(statically determinate)

The beam element is defined as being able to deform in pure bending only. From simple bending theory it can be seen that the correlation between bar and beam members is as follows:

$$\frac{M}{I} \equiv \sigma, \quad \text{curvature} \equiv \varepsilon$$

Therefore the equation

$$\frac{M}{I} = E.\text{curvature} \qquad \text{-----} \ (8\text{-}11)$$

is analagous to $\sigma = E.\varepsilon$

Curvature is thus the condition which is put into the A matrix of the iteration procedure.

The stiffness matrix for a horizontal beam element is shown below (from Przemieniecki [172]).

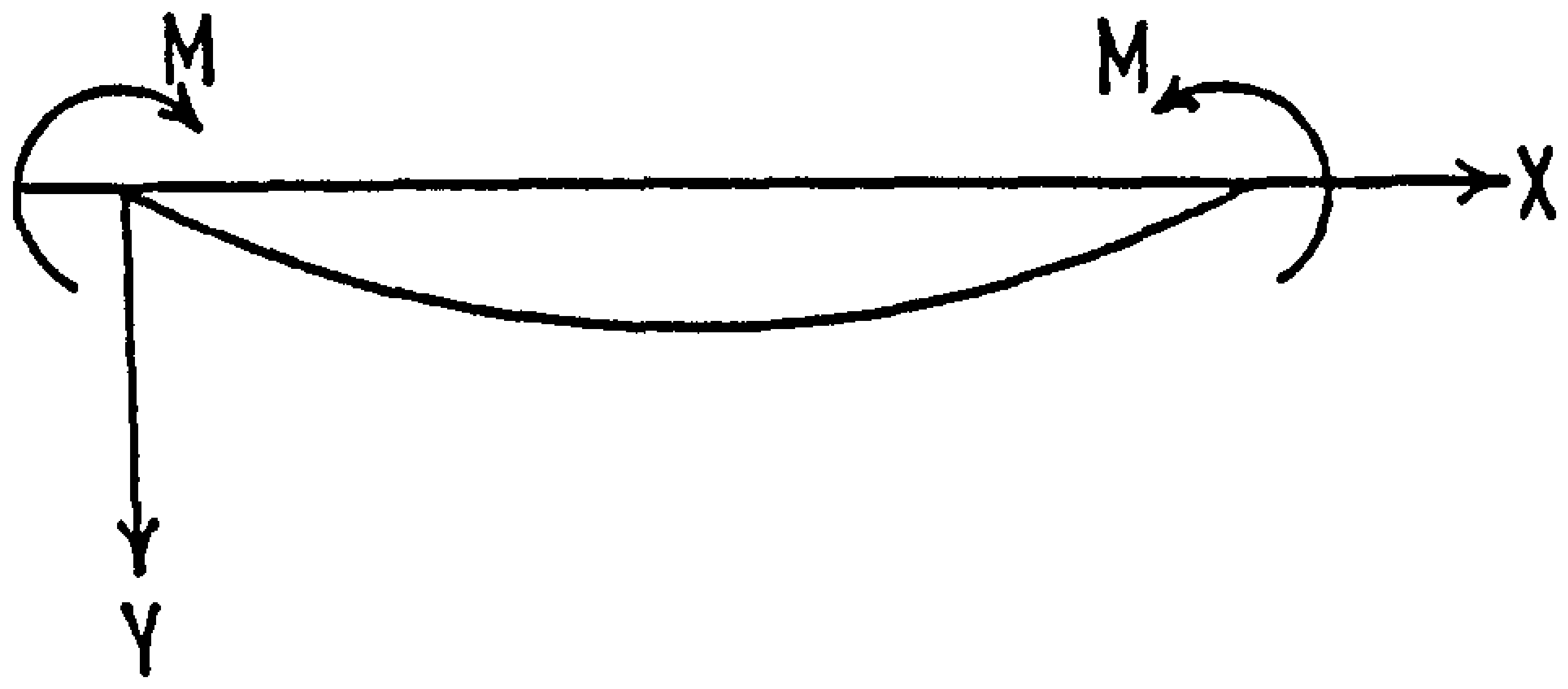
$$\begin{bmatrix} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \end{bmatrix} = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & \frac{-12EI}{L^3} & \frac{6EI}{L^2} \\ & \frac{4EI}{L} & \frac{-6EI}{L^2} & \frac{2EI}{L} \\ & & \frac{12EI}{L^3} & \frac{-6EI}{L^2} \\ \text{SYM.} & & & \frac{4EI}{L} \end{bmatrix} \cdot \begin{bmatrix} D_{1y} \\ \theta_1 \\ D_{2y} \\ \theta_2 \end{bmatrix}$$

----- (8-12)

Where I is the second moment of area, L is the length of the beam, θ_i is the slope at the point i and M is the corresponding applied moment.

When a beam is subjected to a constant bending moment M, it will deform to a circular arc with radius R, (from Gibson [173]). It follows that:

$$\frac{1}{R} = \frac{d_2y}{dx^2} \Bigg/ \left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2} \qquad \text{----- (8-13)}$$



If the deflection is small, then

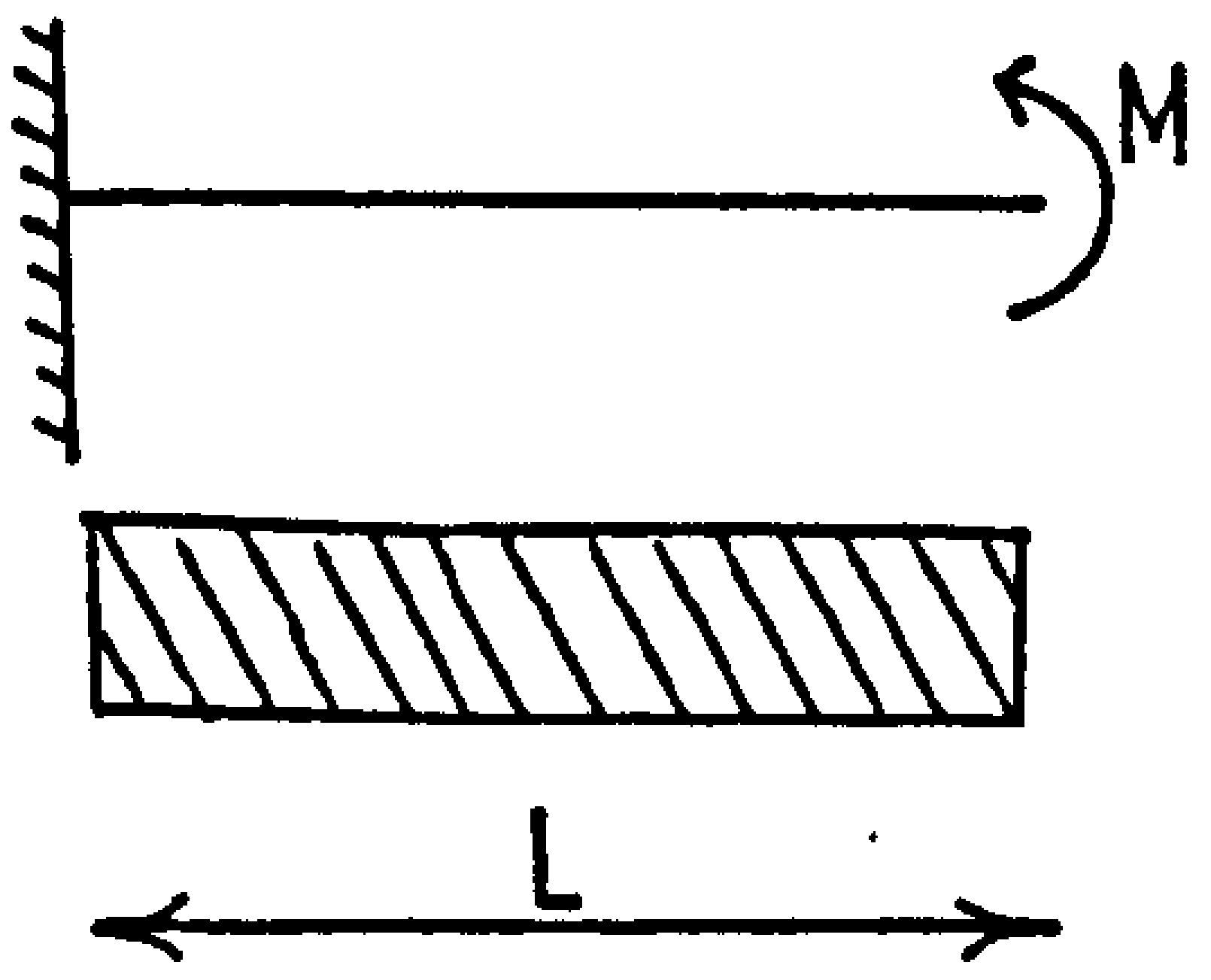
$1 + \left(\frac{dy}{dx}\right)^2$ tends to 1

$$\text{and curvature} = \frac{1}{R} = \frac{d_2y}{dx^2} \qquad \text{----- (8-14)}$$

$$\text{Therefore} \quad M = EI\frac{d_2y}{dx^2} \qquad \text{----- (8-15)}$$

From the moment area theorem it can be shown that the change in slope between two points is equal to the area of the M/EI diagram between those points.

Thus a cantilever with zero slope at the built-in end has a M/EI diagram of

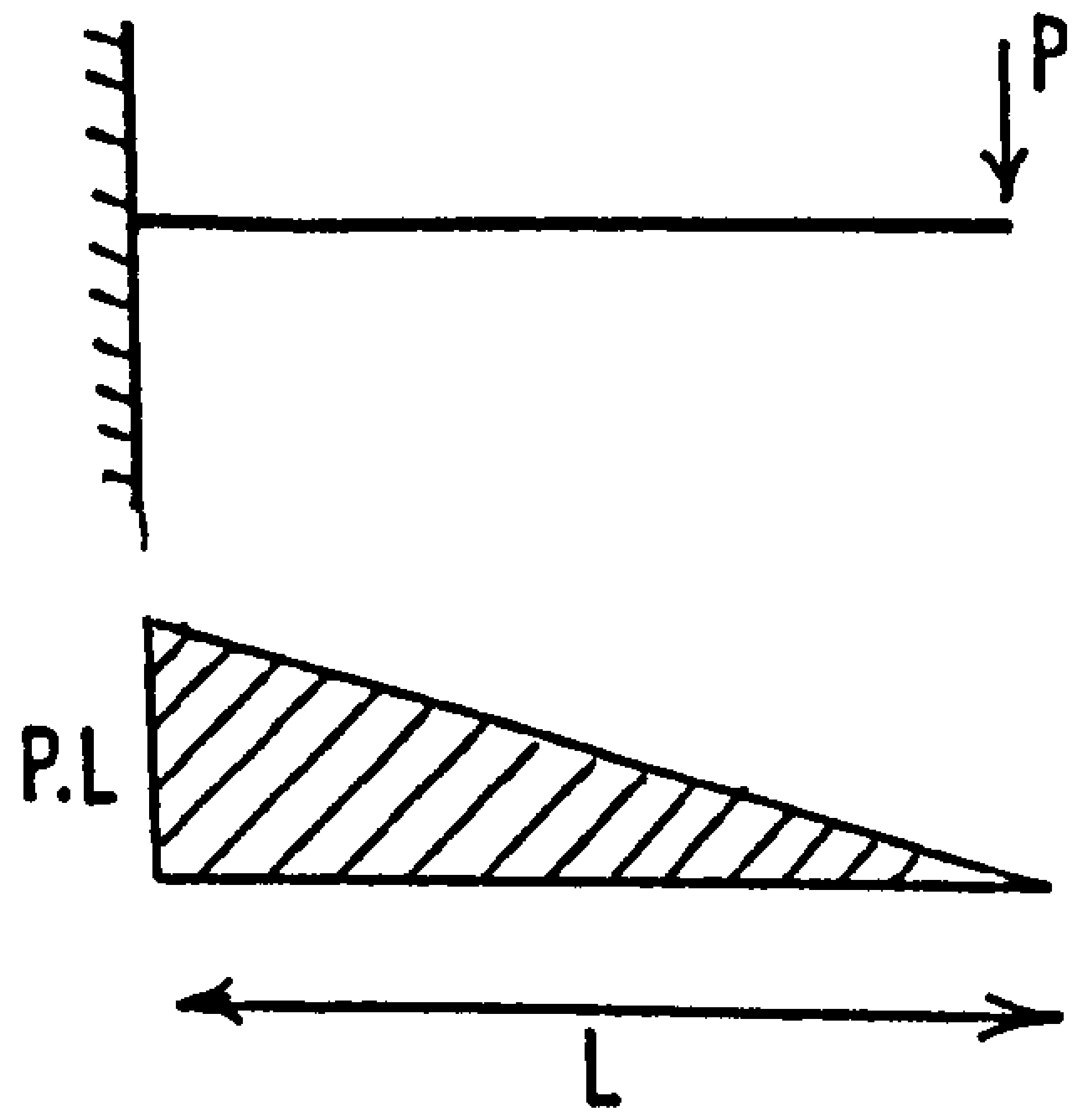


and the area is ML/EI. Therefore the slope at the free end $\theta = \frac{ML}{EI}$, $M = \theta\frac{EI}{L}$. But curvature = $\frac{M}{EI}$, i.e. curvature = $\frac{\theta}{L}$.

Instead of θ the slope of the free end, the end displacement δ can be used in a similar manner. (The moment of the M/EI diagram is equal to the displacement of one end of the beam from the tangent to the slope at the other end).

On joining a series of beams together, however, the displacement of one beam becomes dependent also on the slope of the previous section. Thus, if the slope value is inaccurate, an increase in accuracy of the estimation of curvature using end displacement as well as slope will not necessarily be achieved.

For an end load situation there is a different bending moment distribution, namely:



which shows that the curvature along the beam will vary linearly. The theory used previously can be applied to a particular point, because over an infinitesimal section the curvature of the beam can be assumed to be circular. Thus at the point of application of the load the slope is a maximum and the curvature zero, at the wall the curvature $= \frac{2\theta}{L}$ and the slope is zero. With a single beam cantilever the curvature can only be found at the ends of the elements i.e. node points. Any node can be used in the iteration process provided the value of the curvature is not zero.

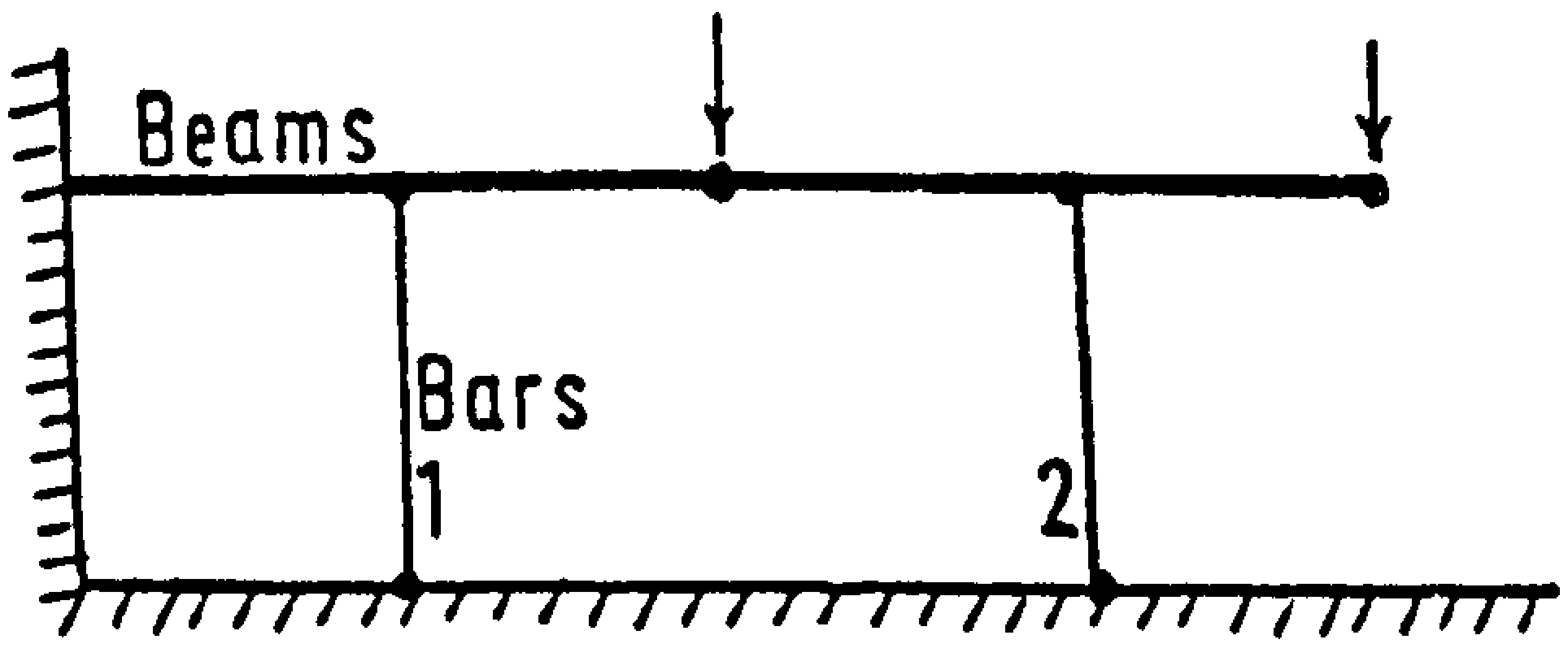
For a single or multi-element cantilever (statically determinate) the iteration technique works perfectly (one iteration), and any sensible guess will converge to the correct result.

Variation of the final E with the offset of the displacement value is shown in Fig. 75 . The Hewlett-Packard calculator has an inbuilt inversion procedure which is not of high accuracy and errors can easily arise when using very stiff or flexible beam elements. A value of $\frac{EI}{L^3}$ = 1 does work well.

8.3. Analysis Of Bar And Beam Element Structures.

A structure, made up of bar and beam, or beam members only, which is statically indeterminate cannot be solved by the iteration process if only the node displacement is known. The bending moment distribution for beam elements has to be known exactly in order to formulate curvature and the A matrix, and this involves having the knowledge of the E values for some of the members.

An example is shown below:



$$I = 100 \qquad \frac{A}{L} = \frac{1}{100} \qquad \frac{EI}{L^3} = 1$$

To find the bending moment diagram the E values for bars 1 and 2 have to be known.

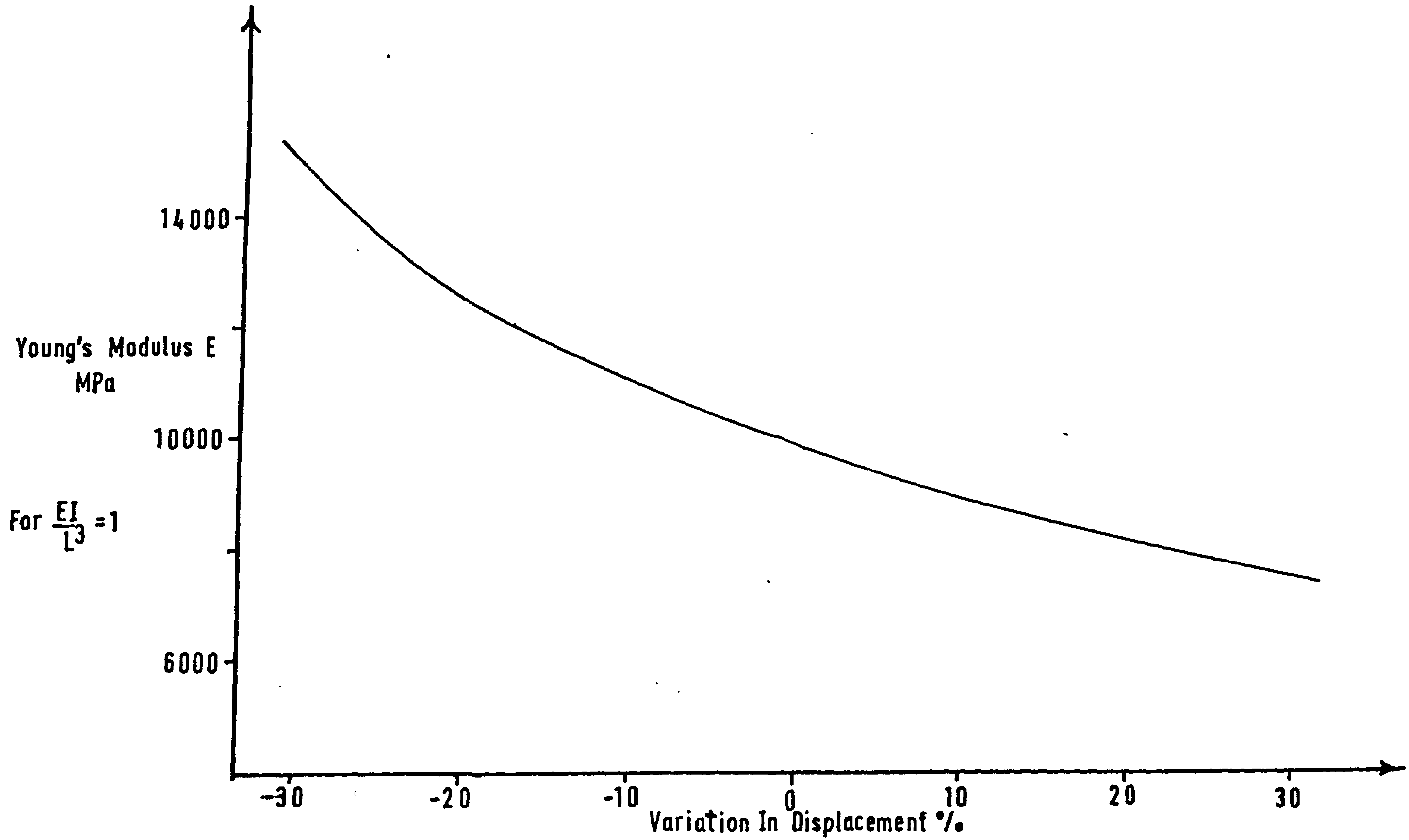
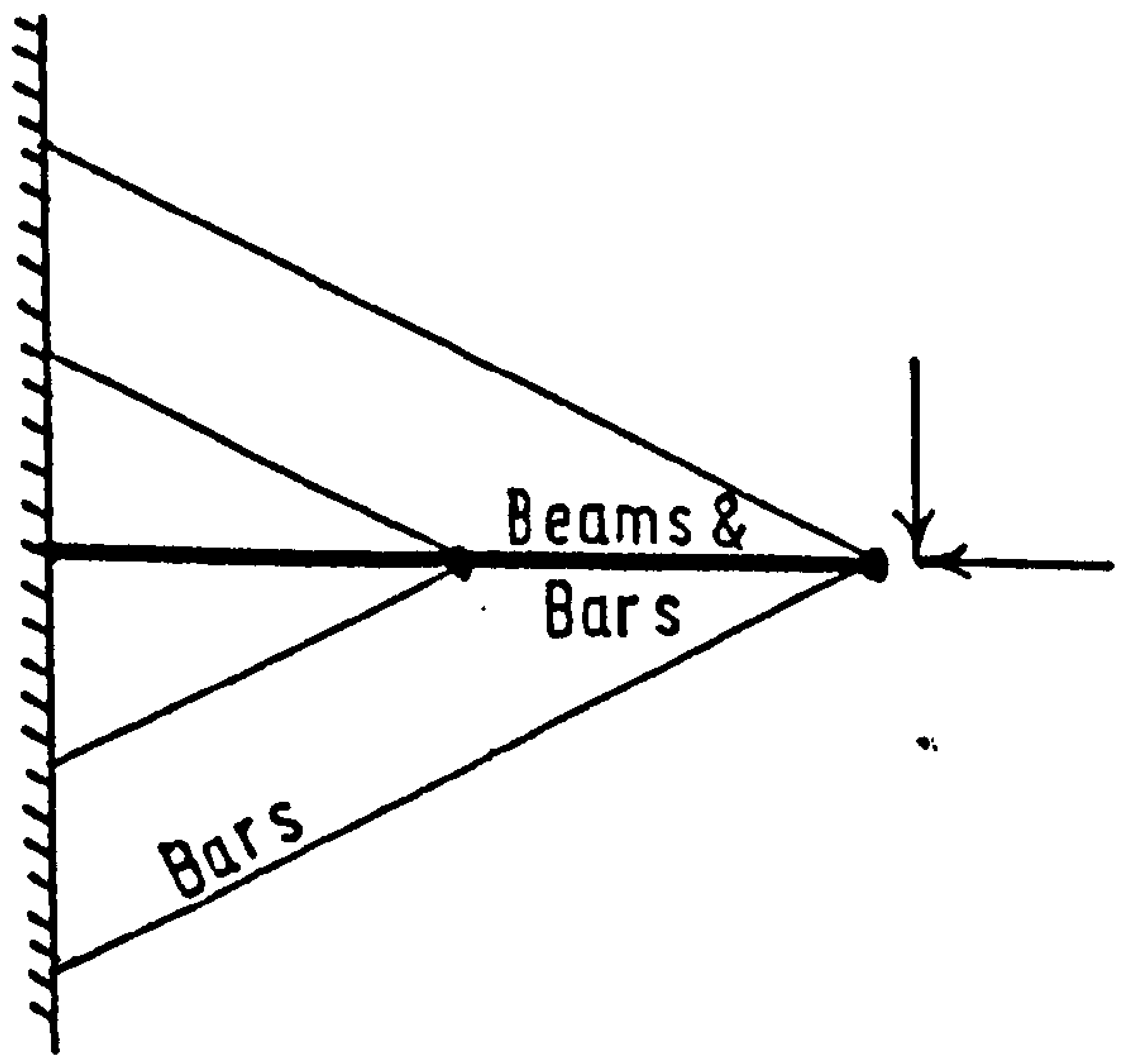


Fig.75 Variation Of E With Displacement Offset

However, it is possible experimentally to analyse this type of structure. The deflection at intermediate points along the beam could be measured and a technique, such as finite difference, could then be used to find the deformation pattern and hence a value of curvature at the nodes.

If curvature can be calculated, the iteration process itself works well for this type of structure, giving accurate answers after two cycles.

A further extension to this work was the inclusion of bar elements co-axial with the beam members, this allows a two-dimensional force system to be applied. An example of this is shown below:



The value of E for the bar element co-axial with the beam can be defined separately from that for the beam. Although the bar element accepts strains along its length the beam element member was assumed to remain at a constant length. The bounds of small deflection theory, which were being used, must not be overstepped by the use of large deformations, e.g. a beam length of 100 units and an end displacement of 10 units set the appropriate limit.

The values for Young's Modulus for all the elements were allowed to change with each iteration but the system did not readily converge to a final value. This was found to be due to the axial load component. For simple systems convergence to a final answer took in excess of 20 cycles. A complex arrangement of members suffered from a failing similar to that of a bar element structure, it would seem that many solutions for the values of E could be found. For a symmetrical system, as in the previous diagram, it was difficult for the process to differentiate between each pair of bar members, i.e.

$$E_1 + E_2 \text{ (analytical)} = E_1 + E_2 \text{ (true)}.$$

$$\text{but} \quad E_1 \text{ (anal.)} \neq E_1 \text{ (true)}.$$

$$\text{and} \quad E_2 \text{ (anal.)} \neq E_2 \text{ (true)}.$$

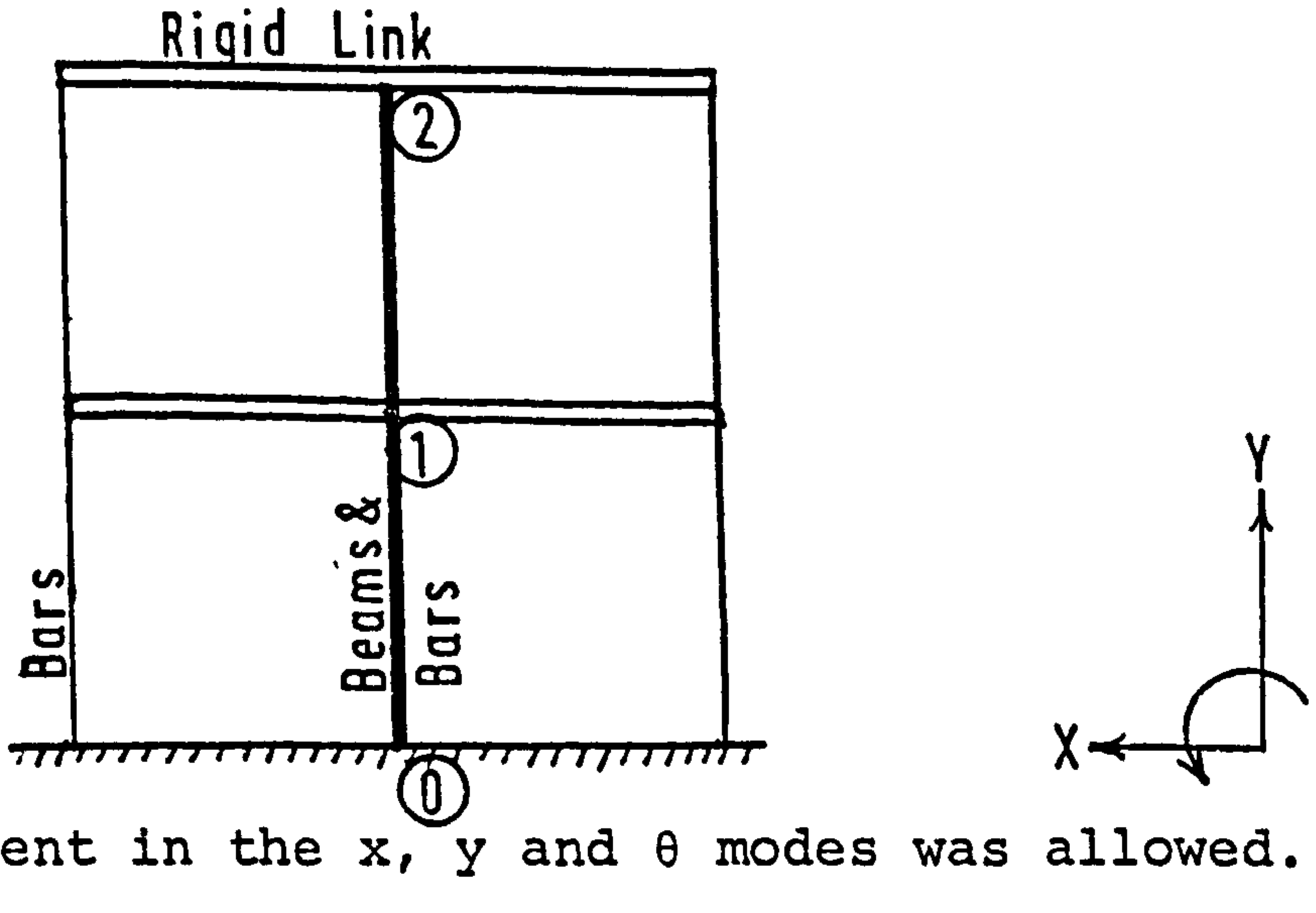
A structure similar to the first sketch but with sloping bar members would work satisfactorily.

Note: High loading with a "soft" material causing a large variation in displacement as E is modified with each cycle is to be avoided, if possible.

8.4. Analysis Of Structures With Rigid Links.

In the simulation of the vertebral column rigid links were to be used in the place of vertebrae. The value of E for the bony structure was assumed to be very much greater than that of the soft tissue surrounding it.

A simple model was defined as shown in the diagram below:



Movement in the x, y and θ modes was allowed.

The stiffness matrix was formed by applying unit deflection of each mode in turn at each node, holding the other nodes stationary. The forces and moments induced by the deflection, when tabulated, form the required matrix of order 9 x 9. Figure 76.

An alternative method is available, nodes can be assigned to the ends of each member and a stiffness matrix made up directly from the element stiffnesses of the order 27 x 27. The number of degrees of freedom can be reduced by finding the correlation between the movement of the primary nodes 1 & 2 and of the supplementary nodes. The transformation:

$$[R]^T[ST][R] \qquad\qquad [R] = \text{correlation matrix}$$

gives the desired stiffness matrix.

The structure was loaded at node 2.

The values of E for the beam and the co-axial bar elements were specified, thus the properties of the four bar members were sought. Without the mode relating axial load to induced rotation and moment to axial movement in the stiffness matrix (marked x), the values of E found for the four outside bars gave the correct overall stiffness for each level, but did not pick out each value correctly.

$$
\left[\begin{array}{c:c:c:c:c:c}
\frac{12E_2I}{L^3} + \frac{12E_5I}{L^3} & 0 & \frac{6E_5I}{L^2} - \frac{6E_2I}{L^2} & -\frac{12E_5I}{L^3} & 0 & \frac{6E_5I}{L^2} \\
\hdashline
 & \frac{AE_1}{L} + \frac{AE_2}{L} + \frac{AE_3}{L} + \frac{AE_4}{L} + \frac{AE_5}{L} + \frac{AE_6}{L} & \left(\frac{AE_6}{L} - \frac{AE_4}{L}\right).100 + \left(\frac{AE_3}{L} - \frac{AE_1}{L}\right).100 & 0 & -\left(\frac{AE_4}{L} + \frac{AE_5}{L} + \frac{AE_6}{L}\right) & \left(\frac{AE_4}{L} - \frac{AE_6}{L}\right).100 \\
\hdashline
 & & \left(\frac{AE_1}{L} + \frac{AE_3}{L} + \frac{AE_4}{L} + \frac{AE_6}{L}\right).100 + 4\left(\frac{E_2I}{L} + \frac{E_5I}{L}\right) & -\frac{6E_5I}{L^2} & \left(\frac{AE_4}{L} - \frac{AE_6}{L}\right).100 & \frac{2E_5I}{L} - \left(\frac{AE_4}{L} + \frac{AE_6}{L}\right).100 \\
\hdashline
 & & & \frac{12E_5I}{L^3} & 0 & -\frac{6E_5I}{L^2} \\
\hdashline
 & \text{SYM.} & & & \frac{AE_4}{L} + \frac{AE_5}{L} + \frac{AE_6}{L} & \left(\frac{AE_6}{L} - \frac{AE_4}{L}\right).100 \\
\hdashline
 & & & & & \frac{4E_5I}{L} + \left(\frac{AE_4}{L} + \frac{AE_6}{L}\right).100
\end{array}\right]
\cdot
\left[\begin{array}{c}
r_{1x} \\ r_{1Y} \\ \theta_1 \\ r_{2x} \\ r_{2Y} \\ \theta_2
\end{array}\right]
$$

Fig.76 Stiffness Matrix, After Partitioning (removal of node O).

$$(E_1 + E_2) \text{ analytical} = (E_1 + E_2) \text{ true.}$$

but E_1 (true) = E_1 (anal.), and E_2 (true) = E_2 (anal.).

This is caused by the symmetry of the system. With the complete stiffness matrix it should have been possible for the iteration procedure to settle on the correct values of E for the bars. Unfortunately, the system proved unstable and no result could be obtained.

If the iteration technique had worked and more elements were included in the structure, it still would have been probable that only the summation of individual stiffness on either side of the beam element could be found. The use of the technique in a model of the spine would, therefore, have been limited.

8.5. Summary.

Points brought to light in this work:

(a) It is necessary to have accurate load data i.e. considering the spine, bodyweight acting at each node must be known (also the cantilever effect of the head and neck).

(b) The technique works for bar element structures, although many solutions seem to exist for one system.

(c) The curvature for the beam members must be known, either from bending moment distribution or experimental data.

(d) Bar and beam element structures can be analysed, if the properties for the beam and co-axial bar are specified.

(e) A rigid link in the system does tend to cause problems. It is probably not possible to find the stiffness of individual elements meeting at one point. A symmetrical structure should be avoided if possible.

(f) If muscles were simulated by equal and opposite forces, the technique would register changes in load at the nodes, but not individual muscle forces if many were acting per node.

All these factors indicate that severe difficulties would be met if the technique described above were to be applied to a mathematical simulation of the spine. Such a model would comprise of 17 rigid links, each interconnected by elastic elements representing 8-9 major ligaments, the intervertebral disc and numerous muscle forces (170+). Furthermore, in order to bring in the required data for curvature, the quality and quantity of the experimental deflection data required could not possibly be obtained. In view of these difficulties, a more direct method, rather than this iterative technique, is more likely to lead to practical results.

The following chapter describes a novel method for formulating a model of the human spine.

CHAPTER 9.

A Static Model Of The Spinal Column.

The mathematical model proposed in Chapter 4 is described in more detail here. Two numerical techniques are applied to the solution of the problem, namely linear programming (described in Chapter 6) and a structural analysis based on the Displacement method.

The sequence of using the computer program generated is as follows:

(a) Linear programming is used for the spine involving 171 muscles and the reaction forces and torques between the vertebral bodies. The objective function is minimized, and this is equivalent to finding the minimum total static force required to hold the structure in equilibrium.

(b) The structural analysis program calculates the force vector applied at each vertebra to hold the spine in a prescribed deformed position. The analysis assumes that in the initial position there is no residual strain in the system. Thus it can be seen that this program calculates the change in reactions between the vertebrae due to the change from the initial position to the deformed geometry.

(c) Combining the change in reaction, from (b), with the initial calculated reaction, from (a), gives the reaction between the vertebrae in the deformed state. The structural analysis accepts as input data the deformation of the spinal column, obtained from X-ray pictures. Thus if the person X-rayed is standing in the "bent" position and is consciously tensing his muscles the effect of the "tenseness" will be included in the applied forces needed to hold the spine. Alternatively a relaxed state will be noticeable by less deformation of the elastic tissues. The ability to "sense" this difference in posture is dependent on the accuracy available in obtaining the deflected spinal geometry.

(d) A modified linear program is used for the deformed geometry of the spine in which the reaction forces and torques are known. This program primarily calculates the muscle forces acting on the system. The linear program in the first instance solves the equations of equilibrium for the static system. The structural analysis program calculates the forces resulting from a prescribed deformation and this is also in equilibrium, if the bounds of small deflection theory are observed. Thus on combining the two values a third state of equilibrium is formed which can be solved by linear programming.

The flow diagram for this sequence of mathematical programs is shown in Fig. 77.

9.1. The Linear Programming.

As the linear programming has been described earlier for a system in which muscle forces and intervertebral joint reactions and torques are unknowns, it is only necessary to re-establish the underlying methodology here. Namely:

(a) The muscles are employed in balancing the structure against the action of gravity on the body mass.

(b) The body mass is assumed to act at each vertebra with a value equal to the mass of the transverse section which is cut across the body corresponding to that vertebra and with

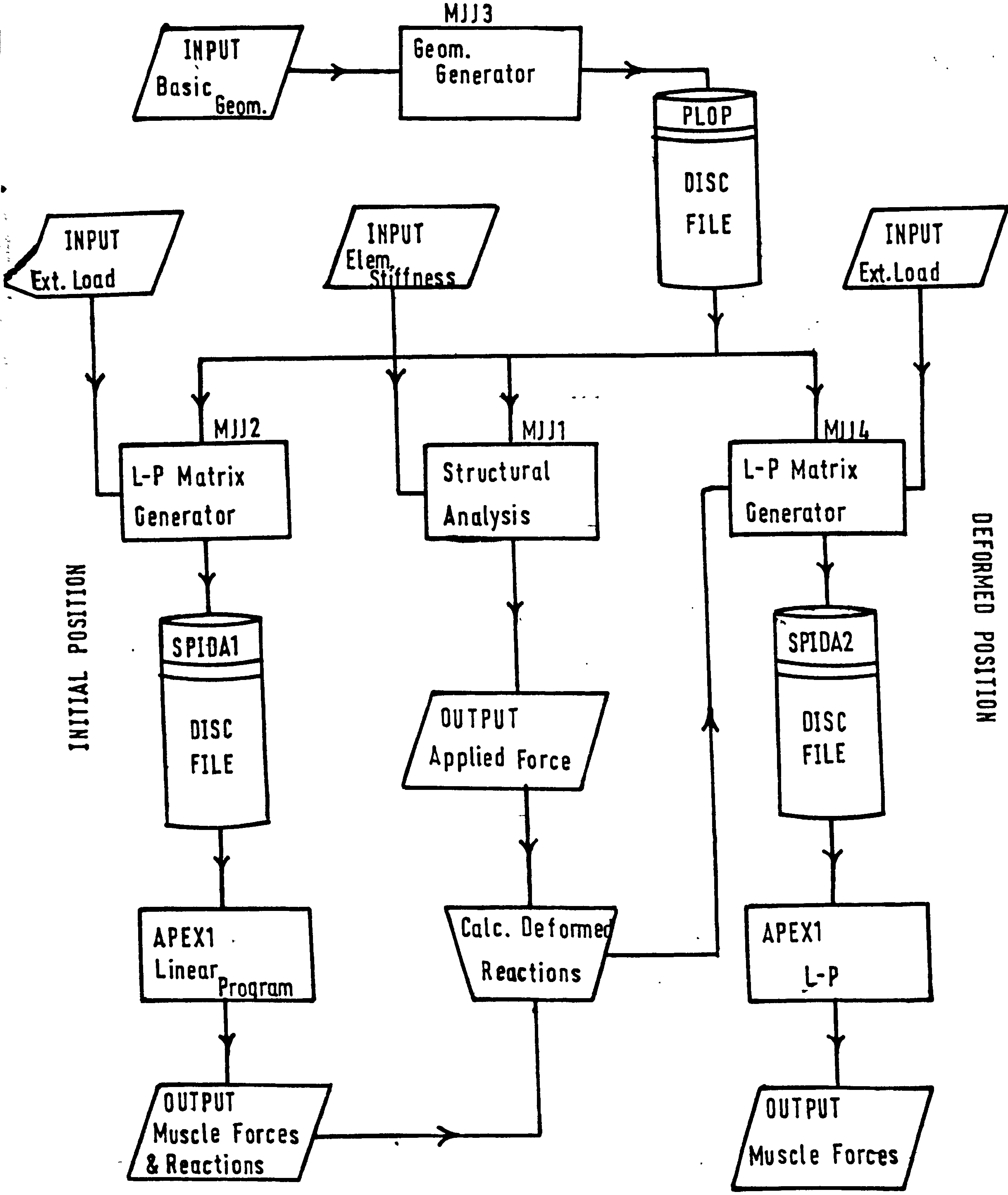


Fig.77 Flowchart For Computer Model Of The Spine

a depth equal to the longitudinal height of the vertebra and disc. [151].

(c) The muscles are given points of origin and insertion as derived from textbook anatomy.

(d) The muscles are given appropriate weighting factors in the objective function of the linear program and the analysis finds a solution which is the minimum for a linear function of the total force in the structure.

9.2. The Structural Analysis.

The structural analysis will be described in greater detail. This program deals solely with the elastic deformation that takes place in the intervertebral joints in moving from an initial position to a deflected geometry. The assumptions made are as follows:

(a) The vertebrae are assumed to be rigid links, with their geometry obtained from X-ray pictures.

(b) The material properties used in the elastic elements conform to the overall motion segment properties so that the error due to assuming rigid vertebral bodies is minimized (see Chapter 4, page 80). The individual properties for the intervertebral discs and ligaments were obtained from the work of Schultz et al [84].

(c) The material properties of the elements are linearly elastic. It is possible with the program to perform the displacement in a number of steps and each step could have different elastic properties to conform to a non-linear curve.

(d) There are no viscoelastic elements in the structure. This in part in due to the lack of data which could directly be used in this type of model, although it would be feasible to reduce the forces calculated by the program by a percentage to account for any relaxation in the system. However, the effect of viscoelasticity would be very small because the spine in its upright position would have reached a stable situation by the time the X-rays were taken and the effect of bending from this configuration would introduce a negligible effect (the bent position is only maintained for a few seconds to take the X-ray). This subject has been briefly mentioned in Chapter 3, pages 61-62.

(e) There is no residual strain in the elastic elements for the structure in its initial position.

(f) All the elastic elements are connected between neighbouring vertebrae i.e. a deformable link does not "jump" vertebrae thereby missing out those in between. It is only the supra-spinous ligament which, of the links represented, does miss out vertebrae in the in-vivo situation and this is only in the L_4 - L_5 - S_1 region. The effect of this on the overall motion segment properties is small.

The spinal column is idealised as rigid links connected together by bar and beam elements representing the ligaments and intervertebral discs respectively. All the bar elements are able to accept axial tension, but only those between the articular facets can take compression forces and this is to represent the action of those joints. The beam element interconnecting the vertebrae is able to resist axial, rotatory and shear forces. The active muscles which are considered as force generators alone do not play a part in this analysis which is concerned solely with the passive structure.

The computer program calculates the forces and moments which are required to hold each rigid link in a prescribed position (after deflection) acting against the passive elastic forces generated in the deformable links.

In matrix notation:

$$\{F\} = [ST] . \{D\}$$

{F} is the applied force vector.

{D} is the displacement vector.

[ST] is the stiffness matrix formed from the elastic properties of the elements.

As mentioned earlier, there are no elements joining vertebrae but missing out an intermediate one, thus it is possible to consider the movement of one vertebra relative to the next. The program does just this and can, therefore, accommodate large displacements of the spine while working to the criterion of small deflection theory between adjacent vertebrae. It is possible to deform the spine its fullest extent in flexion and simulate this in four incremental steps and not go outside the generally accepted limits of small deflection theory. For each step the geometry of the elastic elements is recalculated and for each step of incremental linearization the elements are assumed to have zero residual strain. By a summation of the forces and moments at each vertebra for the number of steps taken a total value is arrived at which is the force required to hold the vertebrae in that position. This method greatly reduces both the time of calculation and the core size required on the computer.

A typical example involving 17 rigid links with 7 bar and 1 beam element between each link would require 12k words working space and for a 4 step displacement take 15 secs execution time. The program itself uses 44k words of core space.

One primary node is assigned to each vertebra and this describes completely the movement in space of the vertebral body. The soft tissue is connected to secondary nodes on the vertebra and the movement of these is directly related to that of the primary node. Thus the stiffness matrix for the link joining the primary nodes is formulated from the element matrices defined for the secondary nodes.

9.3. The Structural Analysis Program.

The computer program will now be described and a flow chart for this work is shown in Fig.78 . For reference the program is listed in Appendix 2.

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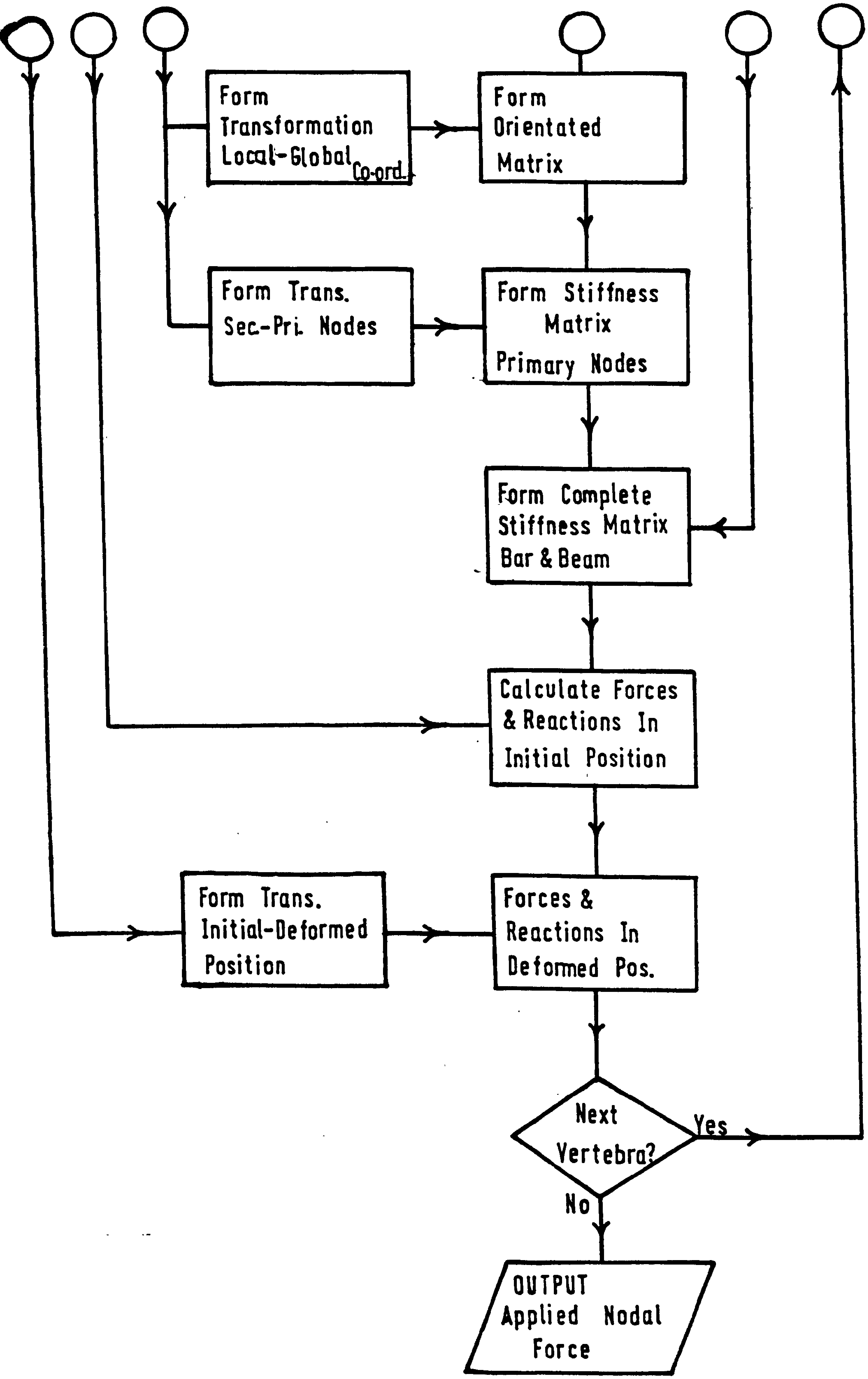


Fig.78 Flowchart For Structural Analysis

The procedures RO, ANGLE, TRAN and TRANS are subroutines which are called into use in the procedure SPINAL. The procedure SPINAL constitutes the main body of the program and is called into operation the same number of times as the number of steps in the movement.

Lines 2 - 64, Procedure RO.

This procedure transforms the stiffness matrices of the elements as related to the secondary nodes to the stiffness matrix operating between the primary nodes.

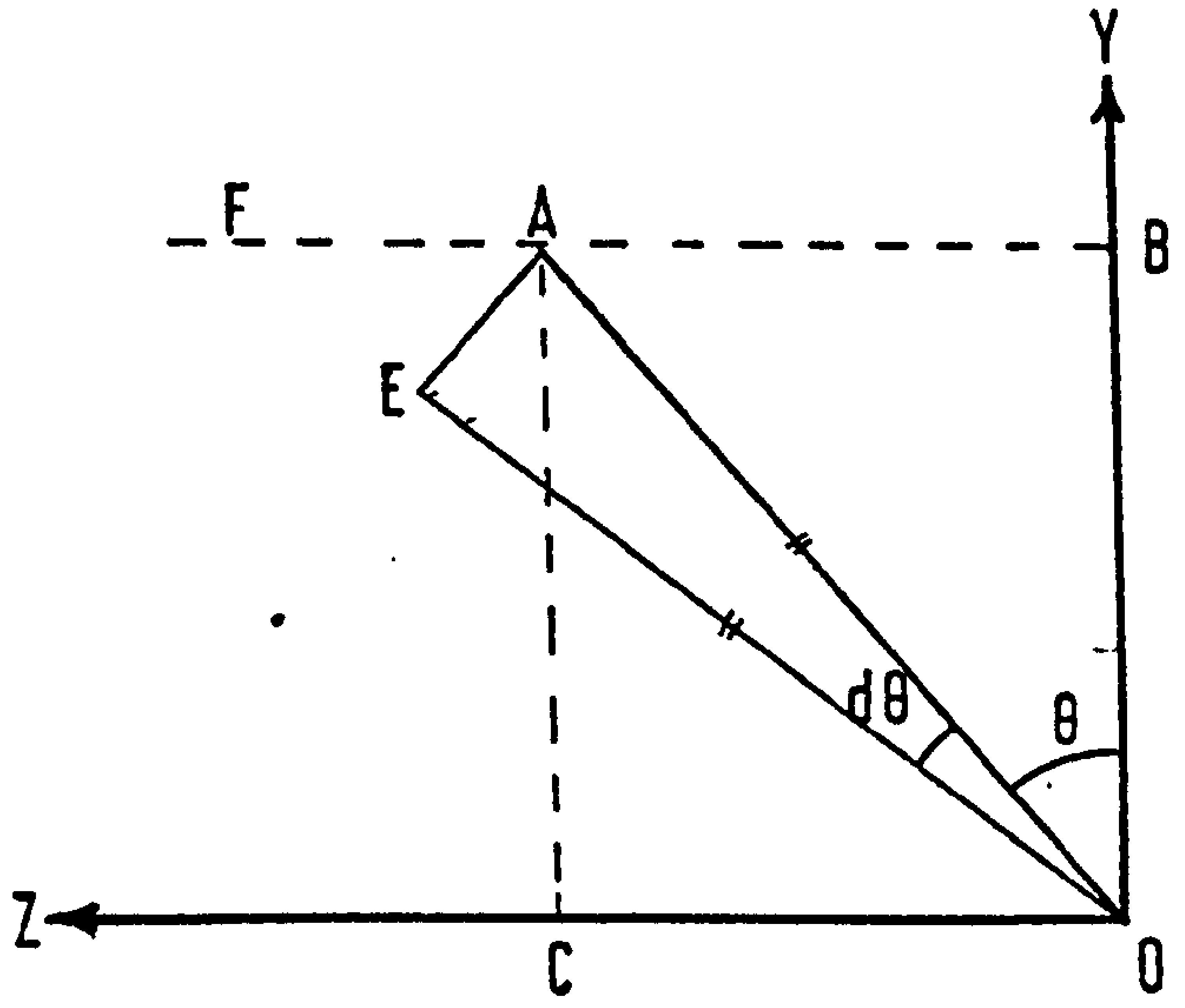
The displacement vectors are related thus:

$$\{D_s\} = [R] \cdot \{D_p\}$$

The subscripts s and p refer to the secondary and primary nodes respectively. R is the transformation matrix and RT its transpose. The integer values MF and NB define the superior or inferior ends of the elastic element and whether the element is a bar or a beam respectively. The integer Q defines whether it is the superior or inferior vertebra which is being transformed.

Translational movement is the same at the secondary and primary nodes. But rotation at the primary node will also cause an extra translation at the secondary node.

Consider rotation about the X axis:



Let OA = OE = L.

Then movement in the Y direction = $-L.d\theta.\sin(\theta + \frac{d\theta}{2})$

----- (9-1)

and movement in the Z direction = $L.d\theta.\cos(\theta + \frac{d\theta}{2})$

----- (9-2)

Similarly for rotation about the Y axis:

movement in the X direction = $L.d\phi.\cos(\phi + \frac{d\phi}{2})$

----- (9-3)

movement in the Z direction = $-L.d\phi.\sin(\phi + \frac{d\phi}{2})$

----- (9-4)

For rotation about the Z axis:

movement in the X direction = $-L.d\psi.\sin(\psi + \frac{d\psi}{2})$

----- (9-5)

movement in the Y direction = $L.d\psi.\cos(\psi + \frac{d\psi}{2})$

----- (9-6)

The equations (9-1 to 9-6) can be expanded, for example:

$$\sin(\theta + \frac{d\theta}{2}) = \sin\theta.\cos(\frac{d\theta}{2}) + \cos\theta.\sin(\frac{d\theta}{2})$$

but $\sin\theta = AB/AO$ and $\cos\theta = OB/AO$

AB is also the Z-axis co-ordinate for the secondary node in the initial position. OB is the Y-axis co-ordinate for the secondary node.

Therefore the movement in the Y direction for rotation about the X-axis can be written:

$$-d\theta.(\underline{Z}.\cos(\frac{d\theta}{2}) + \underline{Y}.\sin(\frac{d\theta}{2}))$$

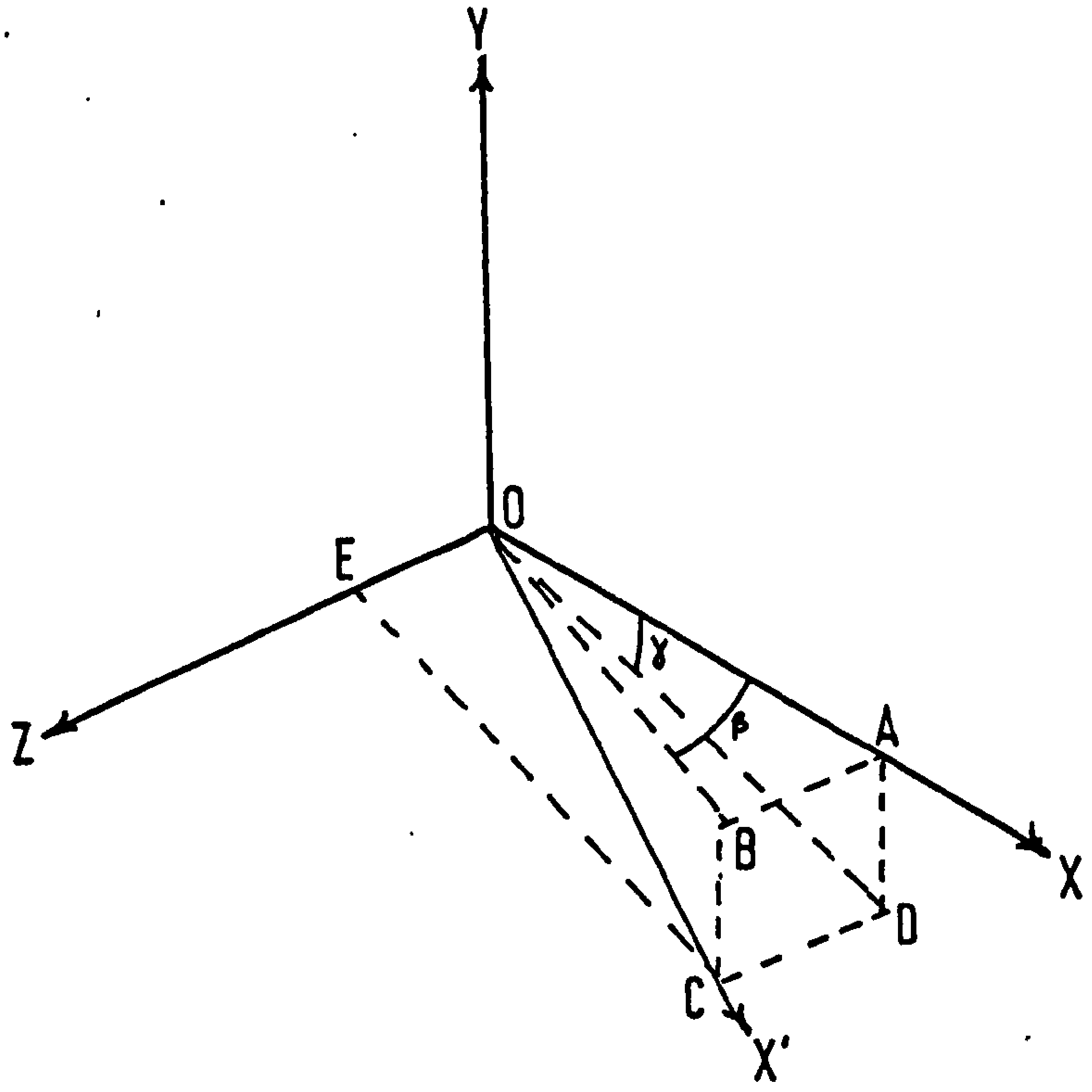
where $\underline{Z}$ and $\underline{Y}$ are the initial axis co-ordinates. As the rotation is known, a unique position for the secondary node, after rotation, can be obtained. The other equations (9-2 to 9-6) are developed in a similar way.

The coefficients linking the translation and rotation of the two nodes are stored in the R matrix.

Lines 65 - 87, Procedure Angle.

The procedure calculates the cosines of the angles of rotation to be used in the transformation of the spatial axes. The cosines are applicable only for a

YZX sequence transformation. ALPHA, BETA and GAMMA are the values of the angles of rotation when projected on to the plane of the original axes. These are the angles as would be seen on an X-ray picture.



The cosine of the angle between the axes X and X' is given by

$CX = OA/OC$

The value AL calculates the length OC

$OC^2 = OB^2 + BC^2 = OB^2 + AD^2$

Let $\beta = A\hat{O}B$ and $\gamma = A\hat{O}D$

Then $AL = OA\surd(\tan^2\beta + 1/\cos^2\gamma)$

Therefore $CX = 1/AL$ ----- (9-7)

The cosine of the angle between the axes Y and X' is CY

$$CY = BC/OC = AD/OC$$

$$= |\tan\gamma/OC| = |\tan\gamma/AL| \quad \text{-----} \quad (9\text{-}8)$$

The XZ plane is defined as 0° rotation, therefore if the rotation is clockwise, i.e. -ve the angle required will be $90^{\circ}+$, and the cosine will be negative.

The cosine of the angle between the Z and X' axes is CZ.

$$CZ = OE/OC = AB/OC$$

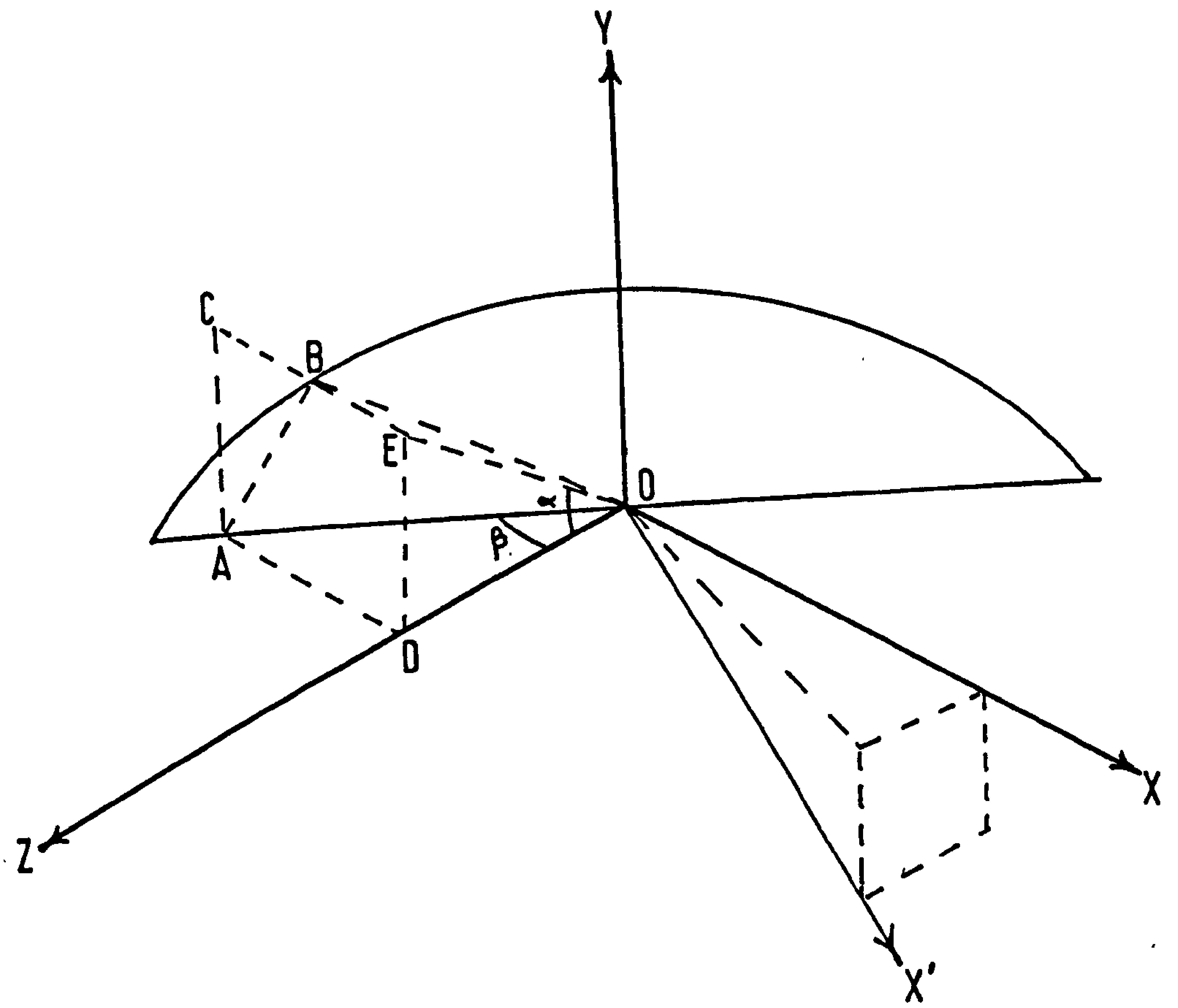
$$AB = OA.\tan\beta$$

$$OC = \surd(OD^2 + CD^2)$$

$$CZ = |\tan\beta/\surd((1/\cos\beta)^2 + (\tan\gamma)^2) \quad \text{-----} \quad (9\text{-}9)$$

If the rotation is greater than zero i.e. counter-clockwise then the angle will be greater than 90° and CZ will be -ve.

The rotation of the axes when viewed looking down the X' axis is defined by the sine and cosine of the movement, CRO and SRO respectively.



The cosine CRO is evaluated as OA/OB

$\cos\beta = OD/OA$ and $\tan\alpha = ED/OD = AC/OD$

Therefore $AB = AC/\cos\gamma = OD.\tan\alpha/\cos\gamma$

$$OB^2 = AB^2 + OA^2$$

$$= OD^2.\tan^2\alpha/\cos^2\gamma + OD^2/\cos^2\beta$$

$$CRO = \frac{OD.\cos\beta}{OB}$$

$$= \frac{1}{\sqrt{}(\tan^2\alpha.\cos^2\beta/\cos\gamma + 1)} \quad \text{-----} \quad (9\text{-}10)$$

But CRO cannot be greater than 1, if by computational error it is, then it is set = 1.

The value for SRO is calculated as

$$SRO = \sqrt{}(1 - CRO^2) \quad \text{-----} \quad (9\text{-}11)$$

The sine is negative when the rotation is clockwise.

Lines 88 - 108, Procedure TRAN.

The procedure is derived from Beaufait et al [150]. It forms a matrix C which is the transformation matrix for changing from one set of spatial axes to another. The matrix is formulated for the YZX sequence transformation, the values for CX, CY, CZ, CRO and SRO are input from the ANGLE procedure.

Lines 109 - 128, Procedure TRANS.

This procedure performs in a similar manner to TRAN, the transformation matrix is formulated for a ZYX sequence rotation. The cosines of the angles of rotation are defined in the main program.

Lines 130 - 599, Procedure SPINAL.

This is the main procedure which performs the structural analysis for the system for each increment of movement.

The integer values MM and NN define the number of vertebrae and the number of bar elements between the rigid links (representing the vertebrae) respectively. Also input into this section are the stiffnesses of the bar elements K and of the beam elements KK (6 per element).

Line 150.

This is the beginning of the structural assembly and repeats from M = 1, the uppermost vertebra (T_1), to M = MM, the lowermost vertebra (L_5). The corresponding links lie below the superior vertebra, e.g. M = 3, the link between T_3 and T_4 is being considered.

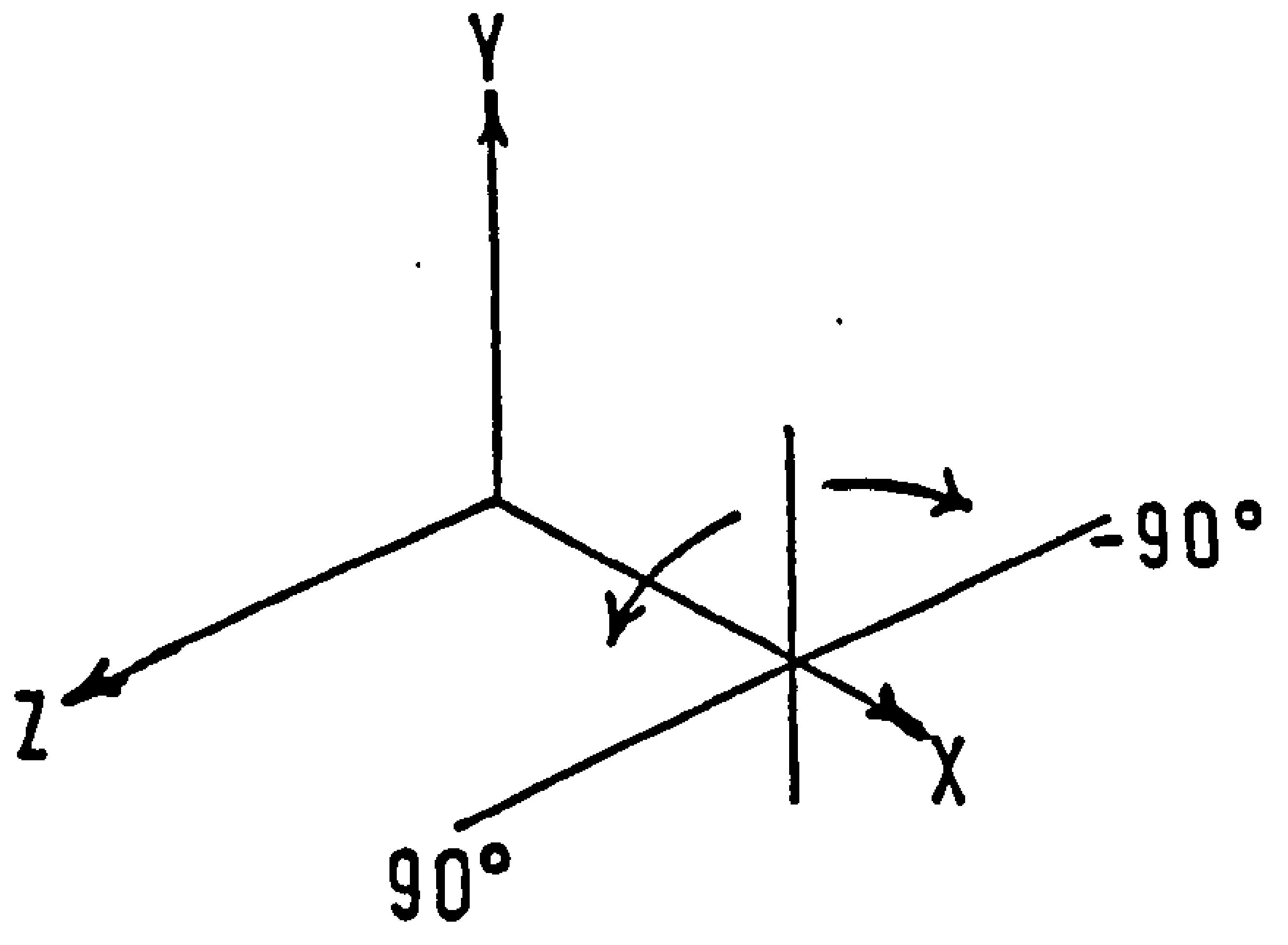
The values ALPHA, BETA and GAMMA store the angular rotations of the superior vertebra and ALPHA2, BETA2 and GAMMA2 store the rotations of the inferior vertebra. The relative rotation of the superior to the inferior vertebra is stored in vector DA. The values of ALPHA2, BETA2 and GAMMA2 are used in the procedure ANGLE, the transformation matrix C is then produced by procedure TRAN. The relative rotations stored in DA are then modified by the transformation and the resultant values stored in D, this is the true relative rotation of the vertebrae. This is done so that the apparent angles viewed on the X-ray plate in the original axis system can be used as the true rotation of the superior vertebra relative to the inferior vertebra in the initial position.

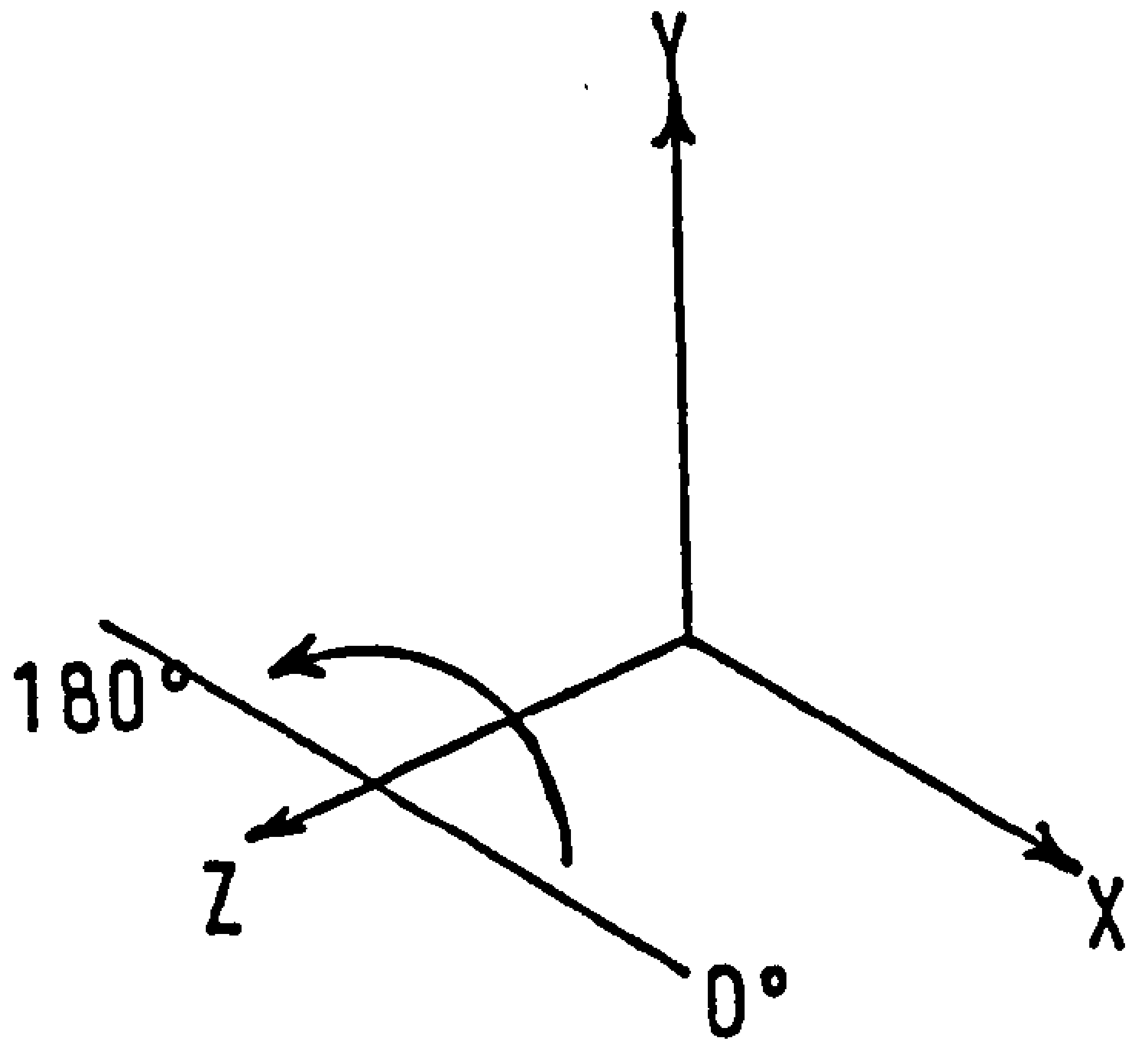
Line 162 - 208.

This section calculates the displacement of the superior vertebra relative to the inferior in its original position. The geometric length L between the primary nodes is calculated for the initial undeformed position.

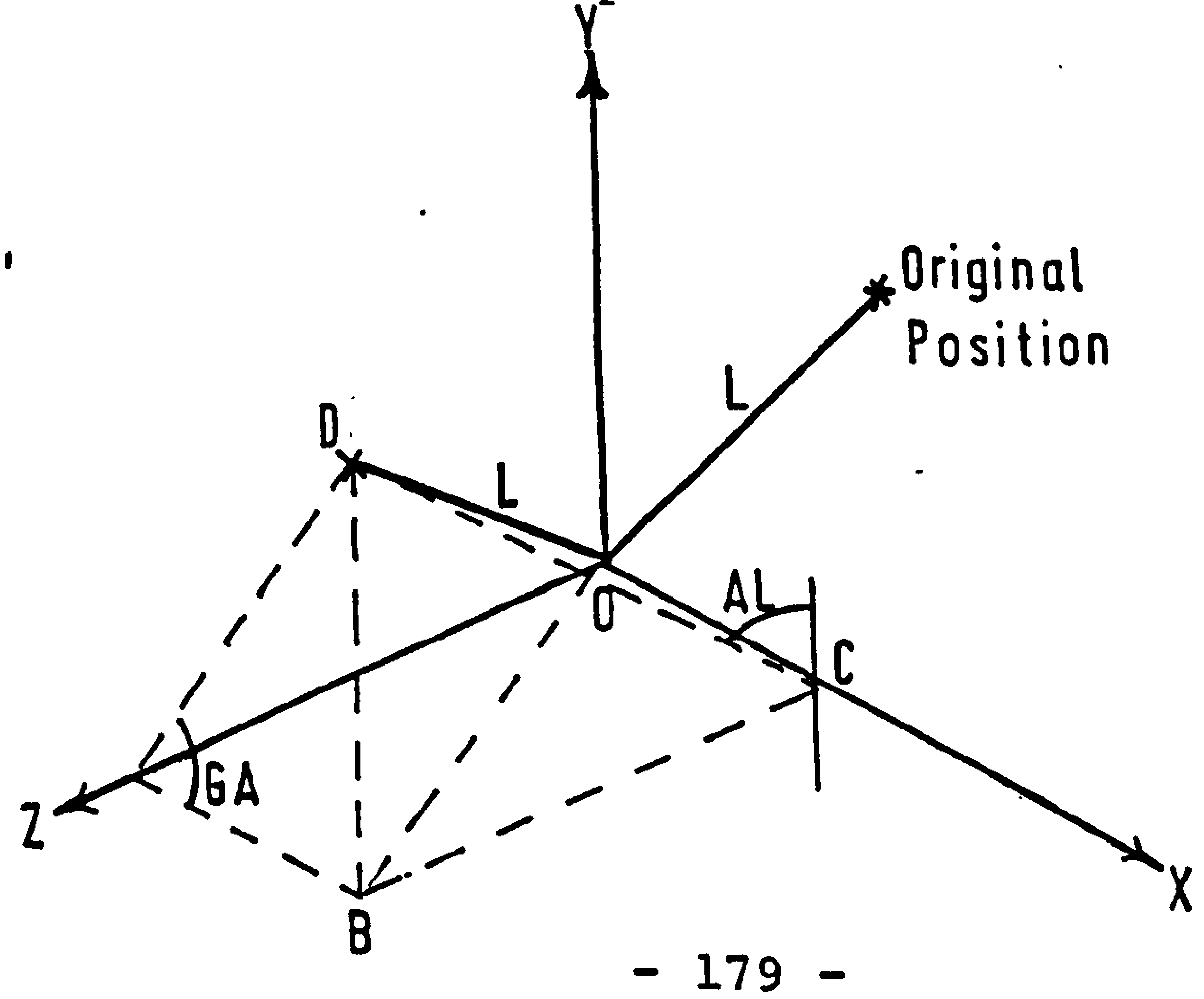
The angle of rotation, about the X axis, that the primary node makes with the inferior is recorded as ALP.

Similarly GAM is the angle for rotation about the Z axis. The program is set to accept values of ALP between 90° and -90° and values of GAM between 0° and 180°.





With two angles and one length the primary node of the superior vertebra is fixed in space relative to the inferior node. Adding to these angles the apparent rotations of the inferior vertebra will give the undeformed position of the superior vertebra relative to the deformed orientation of the inferior vertebra. The coordinates of the superior vertebra can now be calculated, these are MOVX, MOVY and MOVZ. This is then the position of the superior vertebra if no deformation had occurred at that level although the inferior vertebra has been rotated to its deformed position.



MOVY is the Y co-ordinate = DB

$$OD^2 = OB^2 + DB^2$$

$$L^2 = AB^2 + BC^2 + DB^2$$

$$\tan(GA) = DB/AB \quad \text{therefore } AB = DB/\tan(GA)$$

$$\tan(AL) = BC/DB \quad \text{therefore } BC = DB.\tan(AL)$$

$$L^2 = DB^2/\tan^2(GA) + DB^2.\tan^2(AL) + DB^2$$

Therefore

$$MOVY = \frac{L}{\sqrt{(1/\tan^2(GA) + 1/\cos^2(AL))}} \quad \text{-----} \quad (9\text{-}12)$$

The other co-ordinates can be easily calculated:

$$MOVX = \tan(GA).MOVY \quad \text{-----} \quad (9\text{-}13)$$

$$MOVZ = \tan(AL).MOVY \quad \text{-----} \quad (9\text{-}14)$$

If the rotation GA ± 90° then the X co-ord. will be -ve

AL ± 90° then the Y co-ord. will be -ve

AL ± 0° then the Z co-ord. will be -ve

Lines 201 - 203.

The co-ordinates MOVX, MOVY and MOVZ added to the displaced co-ordinates of the inferior vertebra give the position of the superior vertebra prior to deformation of that level.

The deformation that has taken place at that level is calculated from the actual position of the superior vertebra minus the anticipated position i.e. for the X axis.

$$\text{displacement} = XPD_m - (MOVX + XPD_{m+1})$$

This calculated displacement is related to the deformed position of the inferior vertebra. The structure is assembled in the initial position therefore, it is necessary to rotate back the co-ordinate axes an equal amount to the forward deformation rotation. The YZX transformation is used and the new values for the relative displacement stored in vector D. The values held are the true displacement of the primary node of the superior vertebra relative to the initial position of the inferior vertebral primary node.

Line 216.

This is the beginning of the loop which sets up the stiffness matrix for the bar elements. This repeats NN times (i.e. the number of bar elements per level).

Lines 221 - 226.

XPOS1-2, YPOS1-2 and ZPOS1-2 store the distances between the secondary nodes and the primary node for that vertebra. This will be used when finding the new co-ordinates of the displaced secondary nodes.

Lines 231 - 238.

PB, QB and RB store the geometric distances between the two ends of the bar element, and L calculates its length. AB, BB and CB are the direction cosines of the bar relative to the X, Y and Z axes.

Lines 243 - 267.

The matrix relating secondary node movement to that of the primary node is formulated using the procedure RO. The R matrix is also used to calculate the true distance of the secondary node from the primary node in the deformed positon, and this is restored as XPOS1-2, YPOS1-2 and ZPOS1-2. The vector DC is formed which holds the displacement of the secondary node.

Lines 273 - 286.

This section checks whether the element will be put into tension or compression. The values PB, QB and RB hold the co-ordinate distances between the two ends of the bar element. The deformed length of the element LC is then calculated from PB, QB and RB. The value of LC is compared with the bar's original length. If the bar is found to be compressed the program can move straight on to the next element, thereby removing the force contribution of the link from the system. If certain elements can take compression forces it can easily be arranged for the program not to skip that load bearing bar e.g. lines 277 and 278.

The value FO is the calculated force within the element from the change in its overall length and this is used to give an indication of the load that is carried, it is not necessarily equal to the force calculated in the overall stiffness matrix.

Lines 291 - 302.

The stiffness matrix for the bar elements is formulated as in Przemieniecki [172].

Lines 307 - 317.

The stiffness matrix joining the secondary nodes is multiplied by the transformation matrix R and its transpose RT to form the matrix relating the primary nodes.

$$[ST_p] = [RT] . [ST_s] . [R]$$

The subscripts p and s refer to primary and secondary nodes respectively. The new stiffness matrix is placed in STC which is used to store the total assemblage stiffness matrix for the bar and beam elements forming that particular elastic link.

The program then returns for the next bar element for this vertebral level. After completing the stiffness matrix for the bar elements the program continues to the single beam.

Lines 321 - 432.

This entire section deals with the setting up of the stiffness matrix for the beam element. The section is in six parts:

(a) Lines 334 - 340.

The direction cosines for the beam elements are established and held in CX, CY and CZ.

(b) Lines 349 - 364.

The beam is orientated in space using the ZYX transformation. For the sake of simplicity there is assumed to be zero rotation about the axis along the length of the element.

(c) Lines 369 - 381.

The stiffness matrix is assembled (from Przemieniecki [172]), from the elastic properties read into the program. The analysis can handle both slender and thick beams, to simulate the intervertebral disc thick beams are used.

(d) Lines 386 - 397.

The stiffness matrix is transformed from the local to global co-ordinates.

(e) Lines 402 - 420.

The R matrix and its transpose RT are formed, they relate the movement of the secondary node to that of the primary node. The matrix is much the same as that for the bar elements except that rotatory terms for the orientation of the secondary nodes are included. XBOS1-2, YBOS1-2 and ZBOS1-2 hold values for the co-ordinate distances of the secondary nodes from the primary node after deformation has taken place. This is used to obtain the co-ordinates of the displaced secondary nodes, which is required if there are a number of steps.

(f) Lines 425 - 432.

The stiffness matrix for the beam element joining the primary nodes is formulated by multiplying together the matrices shown below:

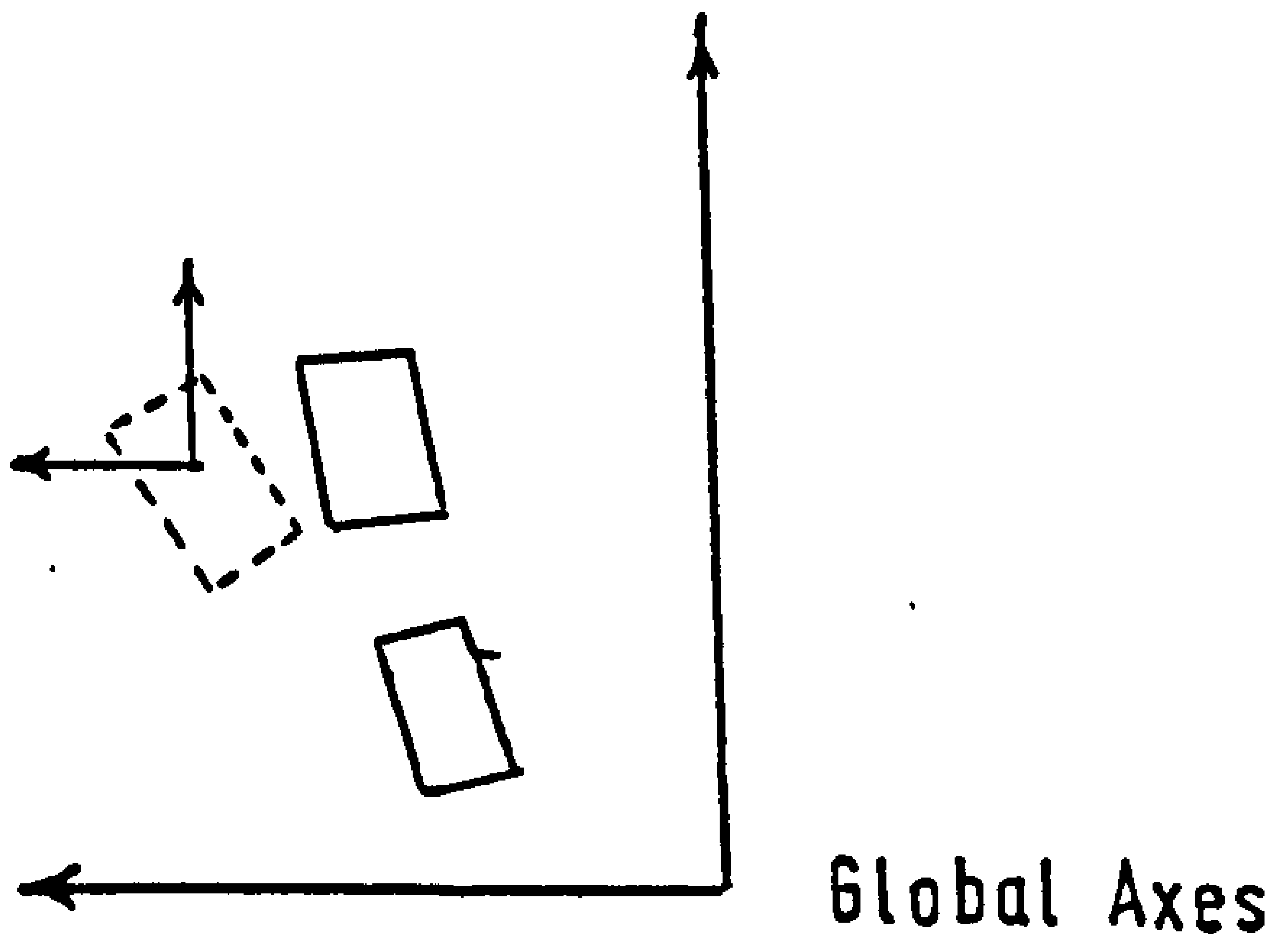
$$[RT] \cdot [ST_s] \cdot [R]$$

The final matrix is held in STC.

The program has now calculated the complete stiffness matrix relating the superior to the inferior primary node, it continues by finding the applied force at each primary node.

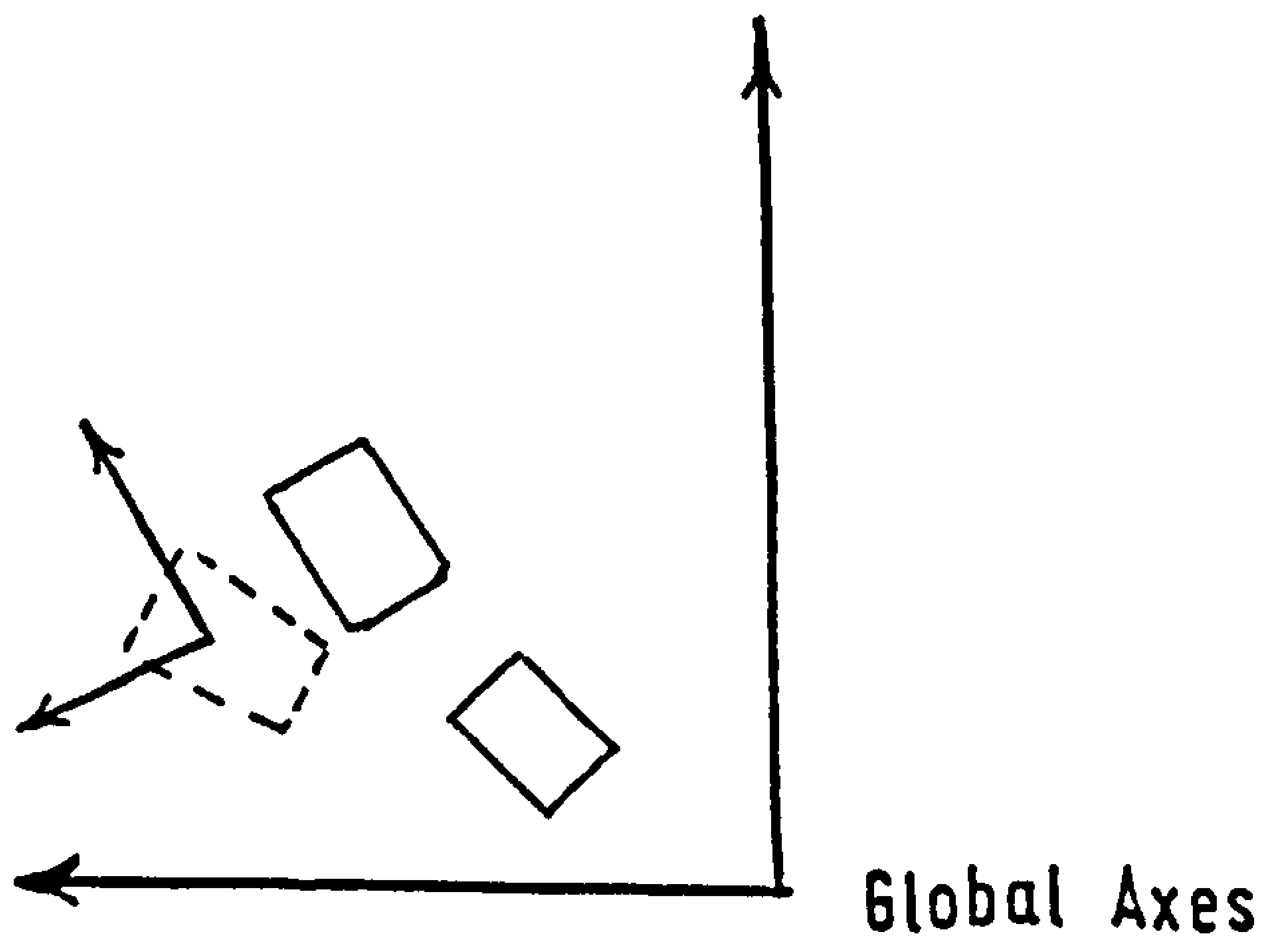
Lines 437 - 459.

The forces and reactions are calculated by multiplying together the stiffness matrix STC and the displacement vector D. The values so derived refer to the deformation of the superior vertebra relative to the inferior. The inferior vertebra is in its initial position.



Lines 463 - 483.

The vertebrae are in fact in a deformed position corresponding to that shown below:



Thus it is essential to calculate the components of the forces in the orientation of the global axes. This is done using the YZX transformation based on the deflected position of the inferior vertebra.

Lines 488 - 539.

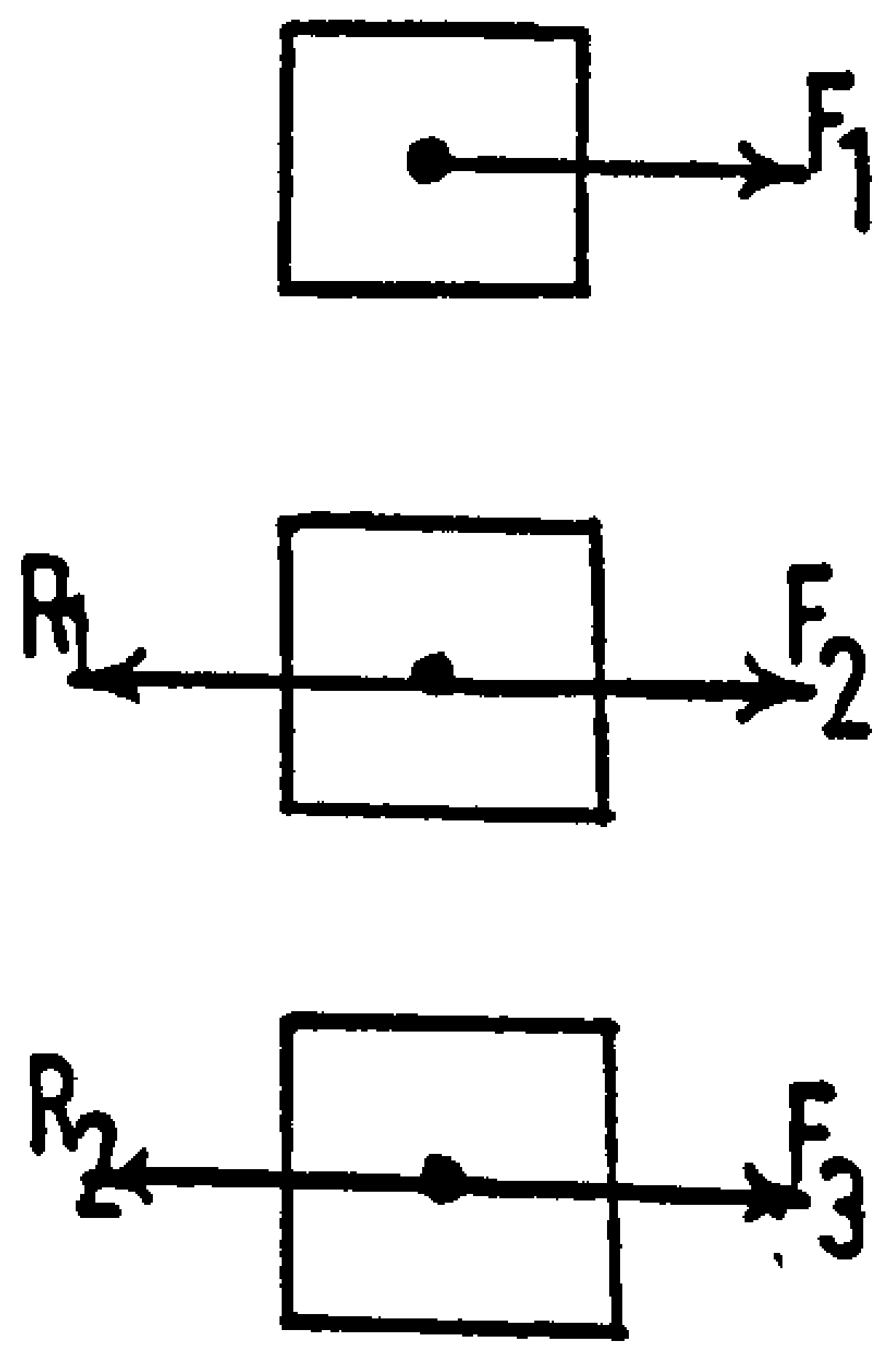
The co-ordinates of the deformed position of the secondary nodes are found by combining the displaced position of the primary node with the distances of the secondary nodes from the primary node as calculated in Lines 418 - 420.

Lines 544 - 570.

The forces and moments calculated for one vertebra relative to the one below are transformed as described above. The separate vertebral units are combined to form the complete spinal column and therefore the applied forces and moments FT at each vertebra are derived.

e.g. $FT_2 = F_2 - R_1$

$FT_3 = F_3 - R_2$ and so on.



F and R are the forces and reactions at each vertebra respectively.

Lines 600 - 793.

This section forms the calling program. The bulk of this, lines 606 - 719, input the co-ordinates for the secondary and primary nodes from the disc file PLOP in a randomly accessible manner. The program requires data for all the positions of the primary nodes but only the initial position for the secondary nodes.

Lines 742 - 789.

This section rearranges part of the data to be used in the next deformation step.

Line 793. End.

9.4. Testing The Structural Analysis Program.

The program has been tested extensively by hand and also on simple elastic structures of bar and beam elements. The simple structures were also orientated at several different angles to verify that the program would accommodate the rotations expected from data of the bending of the spine.

The values for the applied force as generated by this program have been checked against a package called NEWPAC, which was devised by British Rail to analyse structures [174]. To simulate rigid links in this package material properties of very high magnitude were used. The results obtained from these two programs were in good agreement, often less than 2% error was achieved.

9.5. The Modified Linear Program.

The computer program MJJ4 generates the data which is to be used in the APEX linear programming package. This forms the final step of the analysis of the spine as outlined in the flow chart, Fig.77. The program MJJ4 is very similar to the program MJJ2 and this has been described in Chapter 6. The difference is that the reaction forces and moments acting between the vertebrae, which are unknown in MJJ2, are known in this program. Therefore the variables which are minimized relate to the muscle forces only. The values for the known reactions are input on card and are added to the bodyweight and deadweight which form the equilibrant for the system of equations of equilibrium.

This program has been tested by comparing with the program MJJ2. To do this the reactions derived from MJJ2 are used in the second linear program which is set up for the same initial geometry. The muscle forces produced by the two programs were identical, showing that they worked in a similar manner.

CHAPTER 10.

The Use Of The Complete Model.

The computer programs related to this model have been described in their separate functions and it therefore remains to be shown how they function together. The flow chart shown as Fig.77 may give the impression that the combination of programs was unwieldy. In operation this has proved not to be so. The use of disc files to interconnect the programs has meant that the transfer of large amounts of data has not involved manual operation and therefore a good turn round of jobs can be obtained. Indeed in some cases the greatest chore has been for the computer link operator to get consecutive programs in the correct order!

The basic geometry of the spine is input into the first program MJJ3 on card. From this is generated the disc file PLOP which holds all the required data for all the displacement steps. The program MJJ3 generates much of these data internally. Using random accessible disc storage the three programs MJJ1, MJJ2 and MJJ4 can access the file PLOP directly. These programs are run on the CDC 7600 machine at the University of London Computer Centre. The data files SPIDA1 and SPIDA2, containing the matrices to be handled by the linear program, are catalogued via the CDC 6600 machine and stored on the

6400 computer. The APEX linear program package can access these files directly on the CDC 6600 machine.

The only manual link in the system is in the evaluation of the reaction forces that are required for the deformed position linear program.

10.1. The Examples Studied.

The geometry of the spine to be used in this study was obtained from an 11 year old boy, the same subject as used in the analysis described in Chapter 7.

Both the Anterior-Posterior and Lateral X-ray plates were obtained for the initial upright position. A further lateral picture was taken for a forward flexion of the subject down to the horizontal. The co-ordinate geometry of the primary and secondary nodes were derived from the upright position. From the flexed picture were taken the positions of the primary nodes only. The computer program MJJ3 is able to calculate the secondary node co-ordinates from the displaced position of the primary nodes.

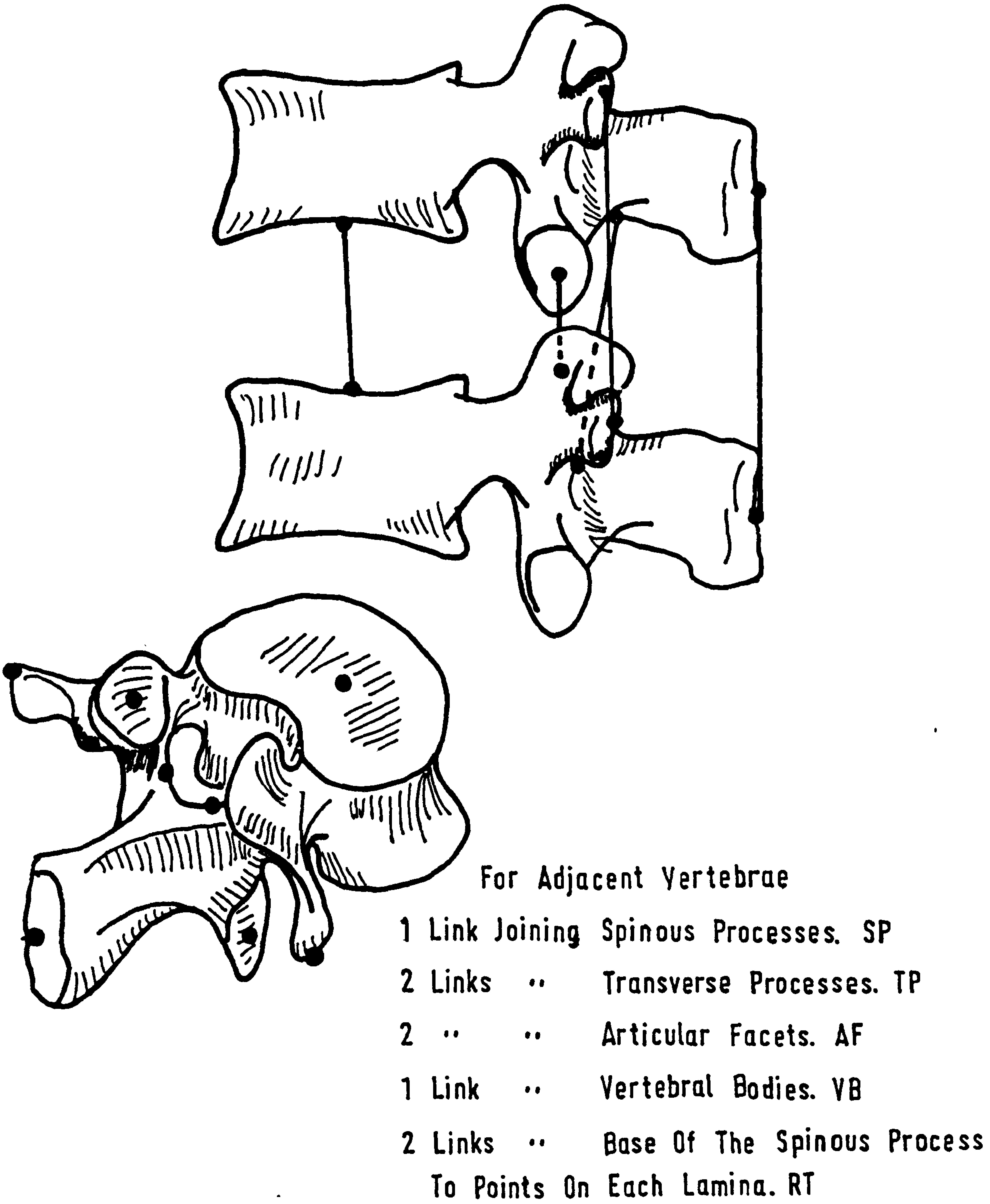
The full forward flexion was split into a series of incremental steps, in the initial examples a 4 step displacement was used. The first increment of movement was used extensively to test the sensitivity of the overall model. In these examples no allowance was made for the distortion on the X-ray films, this is acceptable for the single step but would cause errors in the displacement data for the full forward flexion. The reason is that in the second case points which initially appear in one area of the film are, in the displaced picture, in a different region and thus would be seriously affected by the change in distortion across the film. A corrected set of co-ordinates was calculated and used in a 3 increment deflection.

Thus the following three cases were studied:

(a) Single small increment of movement.
(b) Full 4 step forward flexion.
(c) Full 3 step forward flexion, with distortion correction.

10.2. The Deformable Element Material Properties.

The rigid links are connected together, in the structural analysis, by 7 bar elements and 1 beam element. This is similar to the model proposed by Schultz et al [84] and the material properties for these elements are taken from the work of that group. The connection for these elements are shown in Fig.79.



For Adjacent Vertebrae

1 Link Joining Spinous Processes. SP

2 Links ·· Transverse Processes. TP

2 ·· ·· Articular Facets. AF

1 Link ·· Vertebral Bodies. VB

2 Links ·· Base Of The Spinous Process To Points On Each Lamina. RT

Fig.79 The Elastic Elements Joining The Vertebrae

Summarized they are as follows:

(a) 1 beam element to represent the intervertebral disc. The beam is able to resist full 3-dimensional movement.

(b) 1 bar element joining the tips of the spinous processes, to represent the inter- and supraspinous ligaments (SP).

(c) 2 bar elements joining the tips of the transverse processes, to represent the intertransverse ligaments (TP).

(d) 2 bar elements joining the inferior articular facets of the superior vertebra to the superior articular facets of the inferior vertebra, they represent the action of the intervertebral joints (AF).

(e) 2 bar elements joining the laminae of the inferior vertebra to points near the base of the spinous process of the superior vertebra, and these primarily represent the role of the ligamentum flavum (RT).

In all, 17 movable vertebrae were represented (T_1 - L_5); the sacrum was assumed to act as an immovable base.

The material properties for the beams representing the discs were altered according to the difference in height between the discs of the model of Schultz et al and the discs of the present study. The same effective diameter for the discs was assumed as in their model.

The properties of all the elements are listed in Appendix 4.

10.3. The Results From The Model.

Three typical examples have been selected from the cases studied and they are outlined below to give an indication of the numerical values that were derived.

(a) The analysis of a single small step in forward flexion.

In this case the displacements for the vertebrae were the results of successive adjustments to obtain a deformation which would seemingly give suitable forces in the structural analysis. The linear programming in the initial position did not include reaction moments carried by the intervertebral joint. Therefore the muscles alone balance the complete structure against bodyweight. The objective function in this case was set to unity for all variables.

In this study the muscles in operation in the upright position were the Multifidus, Semi-Spinalis and the Internal Oblique, the forces produced are shown in Fig. 80 . The reactions between the vertebrae are listed in Fig. 81 . The structural analysis program produced values for the forces required to deform the spine as shown in Fig. 82 . From these two sets of numerical values can be derived the reactions between the vertebrae in the deformed position. The data input into the second linear program for the known reactions was in the form:

Reaction on Inf. surface of vertebra-
Reaction on Superior surface of the
vertebral body.

The APEX package failed to obtain a complete solution, 19 of the equations were found to be infeasible, i.e. the muscle force vectors in the system could not equal the equilibrant of the particular equations. The values for those equations which were not satisfied were often in excess of 200% in error. The package does give a print out of the muscle forces in operation at the time when it halted and these are shown in Fig. 83 . It must be emphasized that the forces listed are not the correct final result, and there is no immediate way of knowing how close these results are to the set of values which would have been obtained had the program been able to reach a complete solution.

Muscle & Points Of Origin	Force N
Multifidus	
T_3	4.7
T_4	18.8
T_5	39.8
T_6	46.1
T_7	58.2
T_8	75.9
T_9	74.5
T_{10}	84.9
T_{11}	97.6
T_{12}	102.5
L_1	97.5
L_2	97.4
L_3	73.1
L_4	51.4
L_5	41.3
S_1	26.1
S_2	17.5
S_3	5.1
Semi-spinalis	
T_1	10.6
Internal Oblique	
Total force from 9 strands	80.2

All the muscles act bilaterally, only one side shown here.

Fig. 80. Muscle Forces In The Upright Position.

Vertebral Level	Reaction Force N		
	X	Y	Z
above T_1	-	65.7	-
below T_1	-	99.4	-6.3
" T_2	-	198.3	-16.5
" T_3	-	290.6	-37.0
" T_4	-	380.3	-57.9
" T_5	-	470.9	-80.4
" T_6	-	559.1	-81.9
" T_7	-	638.2	-99.8
" T_8	-	716.1	-112.9
" T_9	-	790.1	-162.5
" T_{10}	-	839.5	-209.1
" T_{11}	-	834.6	-268.7
" T_{12}	-	776.2	-284.6
" L_1	-	699.5	-258.4
" L_2	-	617.8	-197.8
" L_3	-	571.2	-118.7
" L_4	-	547.5	-45.9
" L_5	-	513.5	6.7

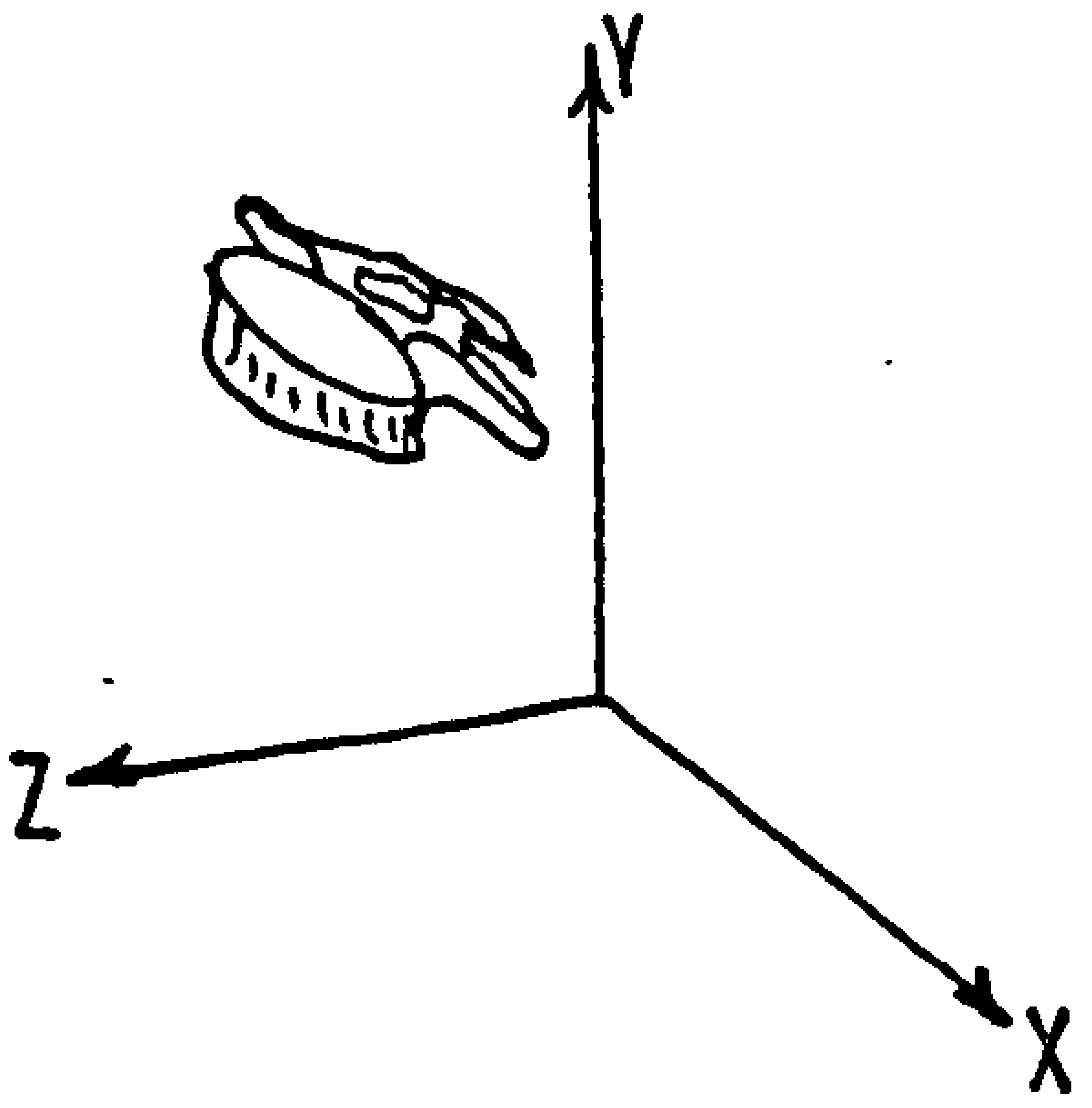


Fig. 81. The Intervertebral Reactions In The Upright Position.

Vertebral Level	Applied Force N			Applied Moment Nmm		
	X	Y	Z	Rx	Ry	Rz
T_1	-	-56.1	-9.7	133.8	-	-
T_2	-	-50.9	-13.2	-203.0	-	-
T_3	-	-43.4	7.2	-153.3	-	-
T_4	-	-27.1	-5.3	-380.8	-	-
T_5	-	-25.5	-18.1	-70.9	-	-
T_6	-	-21.6	-8.3	1148.8	-	-
T_7	-	-19.4	-10.6	1007.8	-	-
T_8	-	-16.5	-1.6	676.8	-	-
T_9	-	-22.5	-6.5	513.4	-	-
T_{10}	-	-22.7	-22.4	2012.3	-	-
T_{11}	-	-14.7	-32.1	3505.7	-	-
T_{12}	-	-15.8	-25.0	5496.7	-	-
L_1	-	-20.1	-24.6	6783.7	-	-
L_2	-	-23.1	-15.7	8273.6	-	-
L_3	-	-22.5	-26.3	8867.2	-	-
L_4	-	-7.9	0.4	5650.5	-	-
L_5	-	-0.3	-73.9	2561.8	-	-

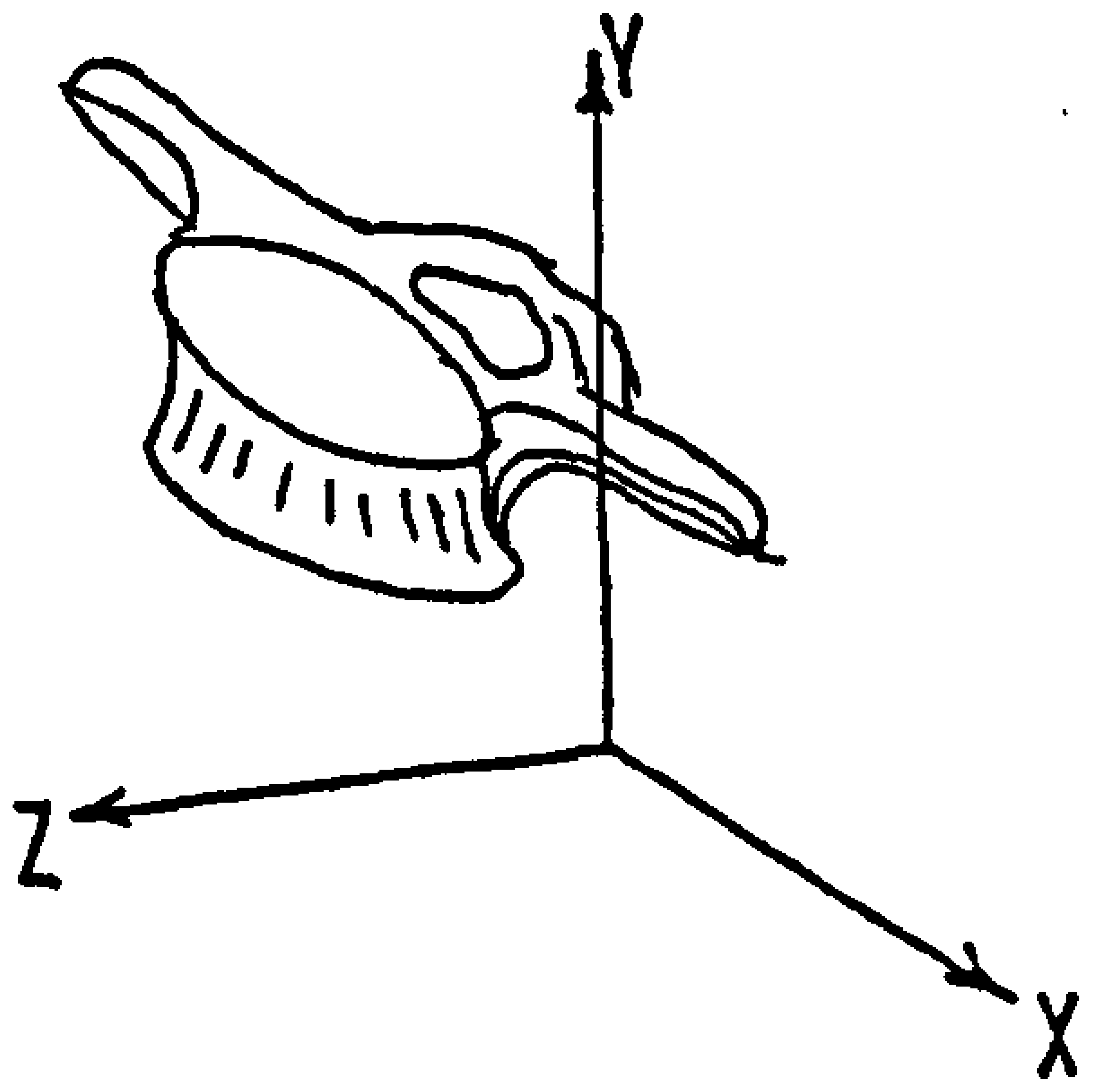


Fig.82. The Forces Required To Deform The Spine.

Muscle & Point Of Origin	Force N
Multifidus	
T_1	34.9
-	-
T_4	30.3
T_5	16.8
T_6	71.9
T_7	23.4
T_8	54.7
T_9	77.6
T_{10}	59.4
T_{11}	84.3
T_{12}	102.9
L_1	31.4
L_2	-
L_3	1.1
L_4	8.2
L_5	-
S_1	56.2
Semi-spinalis	
T_3	5.0
T_4	7.1
T_5	-
T_6	29.8
T_7	-
T_8	8.3
T_9	-
T_{10}	13.2

Fig. 83 The Muscle Forces In A Deformed Position (cont.)

Muscle	Force N
Spinalis	
Insertion T_1	10.2
T_3	47.1
Origin T_{11}	1.1
L_2	56.2
Longissimus Thoracis	
Insertion T_2	3.5
T_5	41.4
T_6	28.7
T_7	7.2
T_{11}	9.4
T_{12}	89.6
Origin L_1	136.2
L_3	0.6
L_4	41.0
L_5	0.9
Sacrum	1.0
Iliocostalis Lumborum	
Origin L_1	17.5
L_2	17.5
L_3	17.5
L_4	21.4
L_5	19.5
Sacrum	21.4
Quadratus Lumborum	
Insertion L_1	236.5
L_2	60.1
L_4	45.5

Fig.83. The Muscle Forces In A Deformed Position.

(b) The analysis of a 4 step full flexion.

The displacement of the vertebrae, in this test, were as taken from the X-ray films and not modified. The initial linear program was the same as described above and in Fig. 80 and Fig. 81. The structural analysis produced results for the four steps and the summation of these increments is shown in Fig. 84. The final linear program stopped in a similar manner to the example above. The number of infeasible constraints was 70. The error in those equations which were not satisfied, i.e. the difference between the anticipated equilibrant derived from the preceding programs and the values obtained from the muscle force vectors in this program, was vast.

This does indicate that the manipulation of the geometric data in the first study was worthwhile in reducing the error in the results from the structural analysis program. This points to the fact that an increase in accuracy of the geometric data would yield substantially better results. To rely on the results from manipulated data would be both pointless as well as unethical. The modifying of the data is also very time-consuming and for movement in which motion coupling occurs it would be very difficult to obtain a realistic deformation from an educated guess.

Vertebral Level	Applied Force N			Applied Moment Nmm		
	X	Y	Z	Rx	Ry	Rz
T_1	-	1949.3	-226.6	7953.8	-	-
T_2	-	-2665.1	-686.4	22152.2	-	-
T_3	-	573.8	2036.9	23302.3	-	-
T_4	-	-74.2	-2869.4	17049.3	-	-
T_5	-	-61.6	1667.4	-6737.3	-	-
T_6	-	682.5	-2148.6	-41667.3	-	-
T_7	-	-4923.0	631.9	-75079.5	-	-
T_8	-	2908.5	523.0	-97177.1	-	-
T_9	-	-461.6	-1179.8	-74214.9	-	-
T_{10}	-	708.2	-2727.1	-28922.0	-	-
T_{11}	-	-894.8	-1634.8	-25678.4	-	-
T_{12}	-	59.3	-669.1	-29872.6	-	-
L_1	-	613.3	2121.9	-2953.5	-	-
L_2	-	3088.0	946.7	-51976.2	-	-
L_3	-	-707.8	1711.9	-151463.1	-	-
L_4	-	-2293.0	-3521.6	-145119.9	-	-
L_5	-	-1776.5	-32.6	-76827.8	-	-

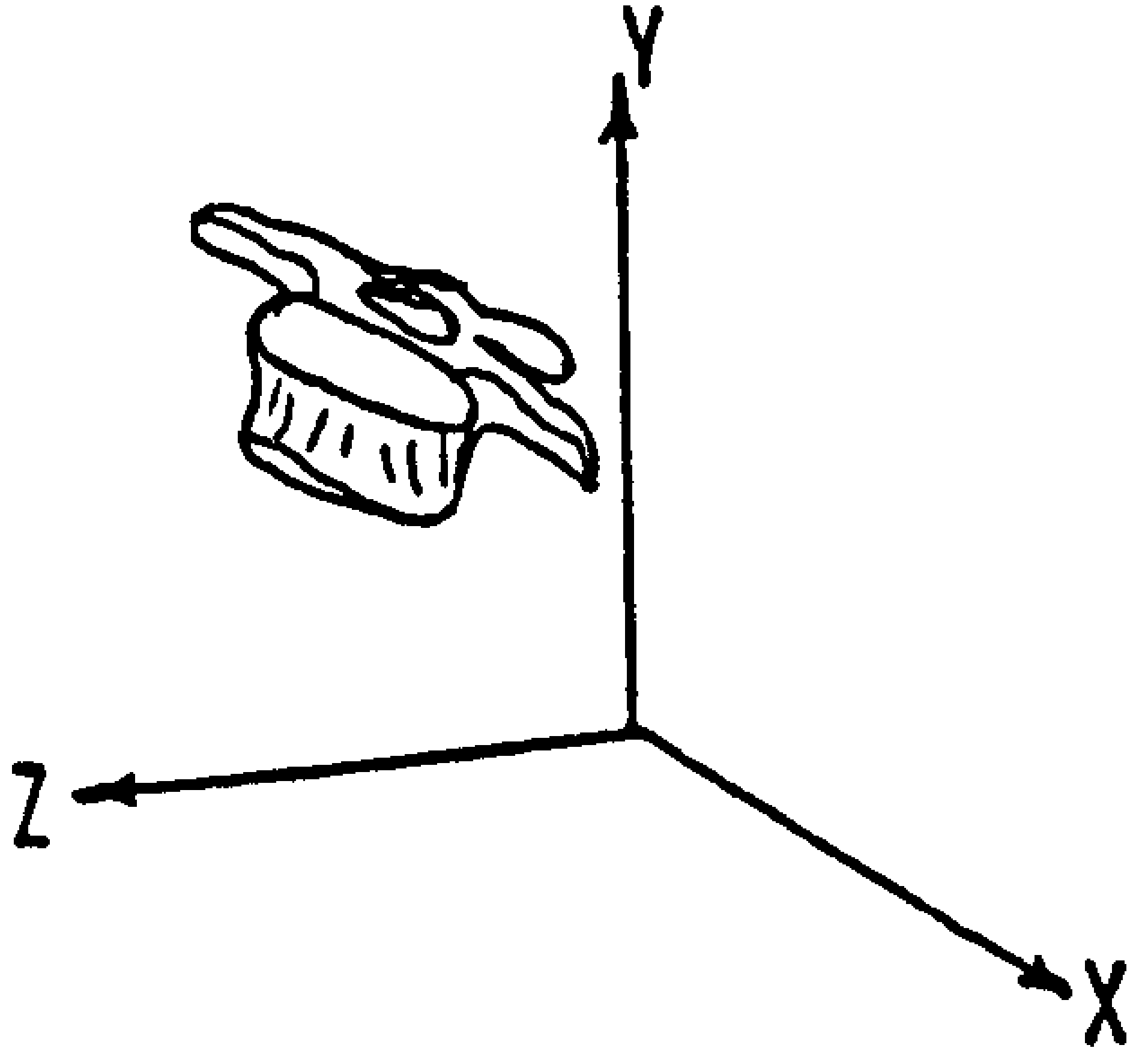


Fig.84 The Forces Required To Deform The Spine.

(Full Flexion)

(c) The analysis of a single step in forward flexion.

This study used the same displaced geometry as the first example. Thus the structural analysis results would be the same as shown in Fig.82 . The difference in this case was the use of an initial linear program which included the action of the intervertebral joints in carrying reaction moments, and the weighting of the objective function for the muscles as used in the model described in Chapter 7.1.1.

The results from the linear program showed that the intervertebral joints carried most of the moments produced in the spinal column with only a few of the muscle links functioning. The muscles carrying load included the Multifidus, the Semi-Spinalis, the Quadratus Lumborum, and the Internal Oblique. The values for the forces and the intervertebral reactions are shown in Fig.85 and Fig.86 respectively. As before, no complete solution was obtained for the second linear program, in this case 46 infeasible constraints existed at the cessation of the package.

The reason for including this example is to demonstrate that varying the relative contributions of the muscles and intervertebral joint reactions in the initial linear program but keeping the same structural analysis results would not give an equilibrium state which could be solved in the deformed position, i.e. if

Muscle & Point Of Origin	Force N
Multifidus	
L_4	4.0
L_5	3.9
S_1	4.5
S_2	4.9
S_3	3.2
Semi-spinalis	
T_5	9.3
T_{12}	5.8
Quadratus Lumborum From Iliac Crest to the 12th rib	15.3
Internal Oblique	
Total from 9 strands	80.2

All the muscles act bilaterally, the results of one side alone are shown here.

Fig.85. Muscle Forces In The Upright Position

Vertebral Level	Reaction Force N			Reaction Moment Nmm		
	X	Y	Z	Rx	Ry	Rz
Above T_1	-	57.8	-0.2	-955.6	-	-
Below T_1	-	77.3	-0.2	-3372.3	-	-
T_2	-	111.9	1.4	-3612.6	-	-
T_3	-	147.4	3.0	-3877.3	-	-
T_4	-	182.0	4.6	-4900.8	-	-
T_5	-	199.7	6.4	-4249.8	-	-
T_6	-	225.2	8.0	-3749.7	-	-
T_7	-	262.4	6.1	-4272.4	-	-
T_8	-	288.9	7.7	-4309.8	-	-
T_9	-	316.0	9.3	-4046.3	-	-
T_{10}	-	342.7	10.9	-1867.9	-	-
T_{11}	-	353.8	10.9	-	-	-
T_{12}	-	379.6	-	-	-	-
L_1	-	398.9	-5.3	-	-	-
L_2	-	418.4	10.2	-	-	-
L_3	-	440.7	-12.7	-	-	-
L_4	-	459.5	-6.0	-	-	-
L_5	-	475.4	0.6	-	-	-

Fig.86. The Intervertebral Reactions In The Upright Position

the relative contributions are altered it may be thought that a particular set of values may be found which will allow a solution to be found for the deformed position, this is not the case, rather the opposite is likely to be true, if the structural analysis produced forces consistent with the muscle system then many different initial linear program values could be used to give a final solution in the deformed state.

The first example described, which came the closest to finding a solution to the complete model, was further modified to accept ranges within which the values for the equilibrants for the equations were allowed to float. A range of 40% i.e. ±20% on all the equations would allow a solution to be found, but there is no way, in this case, to ascertain the error that would be present in the values for the muscle forces in the deformed position.

10.4. A Discussion Of The Results From The Model.

The first linear program functioned as expected and the structural analysis gave results which were accurate with regard to the input displacement, but on combining the reactions of the first and the applied forces of the second program to give the intervertebral reactions for the deformed position and using the values in a second linear program, no solution could be obtained. The simple reason for this is that the values substituted into the final linear program were inconsistent with the force vectors representing the action of the muscles, i.e. the prescribed deflection of the spine cannot be achieved with the specified muscle system.

The reasons for this are possibly:

(a) The prescribed deflection input into the structural analysis program was inaccurate and because of this the values for the applied forces required to deform the spine were also inaccurate.

(b) The assemblage of elastic elements with the given material properties is inconsistent with the real situation, and thus gives false values for the applied forces.

(c) The muscle system superimposed on to the model in the linear programming section is "unrealistic".

The predominant factor causing the linear programming not to find a solution consistent with the input data was the poor accuracy of the input displacement. The present accuracy of the geometry from the X-ray films is in the range ±4mm, the required accuracy would be <±0.1mm. Thus there is a requirement to find an accurate method of measuring the co-ordinate geometry of the spine.

As no experimental work has been carried out with regard to ascertaining the force-deflection characteristics of the intervertebral joint it is not possible to improve on point (b) outlined above.

The 171 force vectors representing the action of the muscles are sufficient to find solutions to most movements, although a problem may exist if gross coupled motion is prescribed e.g. full lateral flexion plus rotation.

10.5. A Method To Aid The Functioning Of The Model.

The second linear program is written in the form of equations of equilibrium represented in matrix notation as

$$[A]\{X\} = -\{W\} - \{R\} \qquad \text{-----} \; (10\text{-}1)$$

where [A] is the matrix holding the coefficients describing the action of the force vectors.

{X} is the force vector.

{W} is the vector of external applied force e.g. bodyweight

{R} is the vector of intervertebral reactions and moments.

The right-hand side of the equation is known and specified as input. A solution is sought for the vector X. The equals sign specifies the solution in a very rigid manner, if some degree of slack is introduced into the equation then a solution to the whole system of equations is more easily found.

Consider one equation from the complete system:

$$a_1x_1 + a_2x_2 + \ldots\ldots\ldots + a_nx_n = b \qquad \text{-----} \ (10\text{-}2)$$

The introduction of a further variable z thus:

$$a_1x_1 + a_2x_2 + \ldots\ldots\ldots + a_nx_n - z < b \qquad \text{-----} \ (10\text{-}3)$$

$$-a_1x_1 - a_2x_2 + \ldots\ldots\ldots - a_nx_n + z > -b \qquad \text{-----} \ (10\text{-}4)$$

$$\text{for } z > 0$$

This enables the z variable to be a measure of the error of the equation. As z would also appear in the objective function, the error, which it represents, would also be minimized. This type of modification can be introduced to many of the equations to give a degree of flexibility to the system.

The APEX program, when it can find no solution due to the infeasibility of the equations, does give a print out which gives the values of error of the equations in question from the input data. Thus it can be seen which equations are causing the most problem and they can be rectified by the above method.

In the present model the values of error for the equations were often many orders of magnitude greater than the specified values for the equilibrant of the equations, and therefore, not suitable for treatment. By artificial manipulation of the displacement data, thus altering the values of the applied force calculated in the structural analysis, a much smaller error was obtainable in the linear programming and the method of assistance outlined above would have been applicable. But it would have been pointless to merely produce artificial data for the sake of gaining results.

No further work has been carried out on this model of the human spine. If more accurate displacement data can be gained then it would be advantageous to continue and find solutions to the equations.

CHAPTER 11.

Conclusions.

The major conclusion from this work is that the model of the static spine, which includes muscle forces and also the passive elasticity of the spinal column, described in Chapters 9 and 10, is a viable proposition if data of an adequate quality are available. Although it has not been possible here to obtain usable values for the muscle forces acting on the spinal column, when geometric data for the displacement of the spine are obtained and the model re-evaluated, it will be possible to see more clearly the strengths and weaknesses in the mathematical technique put forward.

A model of the spine which calculates the material properties of the structural elements and the applied forces from the imposed external load and the deflection of the structure using the iterative technique proposed by Kavanagh [169] is unlikely to give any useful result. (this is described in Chapter 8).

The model of the spine using the Linear Programming technique alone, described in Chapters 6 and 7, does give an indication of the muscle forces required to hold the upper body in various positions. Solving the set of equations of equilibrium (statically indeterminate structure) by this method does function well. The limitation imposed by this method, e.g. the non-inclusion of passive elastic structural elements and the minimization of total force in the system alone, cannot be overcome, and consequently, this limits the accuracy of the results derived.

11.1. The General Conclusions.

A résumé of the general conclusions formed through this work will be given here.

The points 1 - 7 relate to obtaining material properties and geometric data.

1. Although there have been many tests conducted on the material properties of the spinal components, little consistency in their method or tabulation of results makes it very difficult to obtain values for specific elements to be used in a general model with sufficient confidence. At present perhaps the most suitable way is to carry out one's own material tests, particularly for individual ligaments, e.g. intertransverse, supraspinous and capsular ligaments which have been ignored by previous investigators.

2. If the geometric data for the displacement of the spinal column is to be obtained from X-ray plates then a very high resolution is required on the films.

For the structural analysis of the spine, an accuracy of at least ±0.1mm is required.

3. Taking of the X-rays using a bi-plane system requires special jigs if high accuracy is to be obtained. This is unsuitable when the pictures are to be taken in a radiography unit of a hospital because of the inconvenience to the medical staff. The alternative is to use stereo films which are measured on a stereo-comparator. This method can yield a high level of accuracy, but also requires careful setting up, which may be unsuitable for a medical establishment in routine clinical work.

4. Even if high resolution X-ray films are obtainable, it is extremely difficult to fix in space, using the stereo technique, points on a curved surface or those masked by bone or on a surface which is along the axis of view. Thus points on the laminae or on the surfaces of the vertebral body are difficult to locate.

5. The use of an X-ray scanning system does seem promising if the cross-sectional slices could be summed to give an accurate picture on a plane orthogonal to the plane of the transverse slice.

6. The visualisation work was halted due to the restriction placed on taking X-rays for non-clinical use. This follows generally current U.K. practice based on ethical considerations.

7. Ultrasound cannot be used to obtain geometric data due to:
   (a) The change in density of the body tissues causes a change in refraction which distorts the picture.
   (b) Using harmless low level intensity it is not possible to penetrate the vertebral bone, thus points which are in the shadow of the vertebra cannot be viewed.

The following points 8 - 10 are an appraisal of present modelling methods.

8. In the field of dynamic modelling there are two predominant factors which limit the authenticity of the simulation. These are:
   (a) The omission of the active muscle action.
   (b) The non-inclusion of the material properties of the vertebral bone in discrete parameter models.

9. The structural analysis of the spine as a static system has generally taken the form similar to a study of an engineering situation, i.e. the deflection of the structure is calculated from the material

properties of the elements and the applied external load. This type of analysis completely ignores the action of the muscles, as they cannot be included into the passive system, and consequently the models lack authenticity.

10. A more suitable way of analysing the elastic structure of the spine is to calculate the applied load which causes the system of passive elements to take up a prescribed deformation. The applied force can be broken down into components which represent the action of the individual muscles. A minimization technique can be used to find the forces acting in the strands of each muscle.

The following points, 11 - 12, refer to the simulation of the spinal column including the muscles and the passive elasticity of the spine.

11. The method of studying the spinal column as described in 10. was followed using finite element and linear programming technique. (Chapters 9 and 10). It was possible to obtain results only in the undeformed upright position, no feasible solution was obtained for a deflected geometry. This could have been due to the following reasons:

    (a) The prescribed deflection input into the structural analysis program was insufficiently precise and because of this the values of applied forces required to deform the spine were also imprecise.

(b) The assemblage of elastic elements simulating the passive structure with the given material properties was based on simplified data which could be inconsistent with the real situation, and thus false values for the applied forces may have been obtained.

(c) The muscle system superimposed on to the model in the linear programming section was an idealisation.

Of these points, (a) is the most likely to have caused the failure to obtain useful results.

12. The lack of suitable accurate X-ray pictures has meant that the model has not been fully validated and the points above cannot be fully explored. However, very recently some data have been obtained from the University of Vermont [175]. This is probably the best available at present and is described in Appendix 5.

The points 13 - 17, which follow, all relate to the iterative technique proposed by Kavanagh [169].

13. Using the iterative technique proposed by Kavanagh to derive the material properties of the elastic

elements and the muscle forces acting, it was found that a prime requirement for all the tests was accurate load data,i.e.accurate bodyweight values.

14. The technique works well for a sample bar element structure, although many separate solutions seem to exist for one structural and load system.

15. A beam or bar and beam element structure cannot be analysed if it is either statically indeterminate or the curvatures of the beam elements unknown.

16. The muscles can be simulated by equal and opposite forces acting at the nodes. The technique would register the changes in load at the nodes, but the muscle forces cannot be deduced from these values if more than one muscle were acting at each node.

17. A system which included rigid links in the structure was found not to give useful results.

The final points 18 - 22 all refer to the model of the spine based on the equations of equilibrium alone.

18. The model of the spine which used the equations of equilibrium alone, (Chapters 6 and 7), was found to work well. A solution to the statically indeterminate system was found using Linear Programming as a suitable

minimization technique. The overriding drawback with this method is that the forces developed in the passive elastic tissue e.g. ligaments, cannot be accurately specified. The intervertebral reactions were, however, included in the system and minimized subject to the Objective Function of the Linear Programming.

19. A suitable objective function was found to be:
    (a) Weighting the muscles inversely proportional to their cross-sectional area.
    (b) Linearly reducing the weighting for the intervertebral reaction moments in flexion from the upright to the fully bent position, this changed the contribution of the intervertebral joint from zero in the initial position to taking all the moments in the fully flexed position.

20. In forward flexion, the muscles predominantly in action were the Multifidus, the Semi-Spinalis, the Longissimus Thoracis and the Iliocostalis Lumborum. The forces in these muscles increased with flexion up to full flexion when they were silent. Adding a deadweight of 550N, carried by the arms, increased the forces by up to 300%.

21. In lateral flexion the Multifidus and the Longissimus Thoracis were found to act mainly on the concave side, whereas the Iliocostalis Thoracis and Lumborum acted on the convex side. The Semi-Spinalis was found to be quite evenly balanced. The Quadratus Lumborum, the Spinalis Thoracis and the Internal Oblique were also mainly active on the convex side.

22. A study of scoliotic configuration showed a complex system of forces, the Multifidus, Semi-Spinalis, Spinalis Thoracis and the Quadratus Lumborum were found to act on the concave side while the Iliocostalis Lumborum, Longissimus Thoracis and Internal Oblique acted on the convex side. The addition of spinal supports did noticeably reduce the muscular forces present. A complete reversal of the muscle pattern from that above was found on the inclusion of Halo-Pelvic distraction, this was because the forces generated in the apparatus were sufficient to put the column into tension. The effectiveness of the various types of mechanical assistance in supporting the spine was in the following order:
(a) Most effective - Halo-Pelvic Distraction.
(b) - Harrington Instrumentation.
(c) - Milwaukee Brace.

11.2. Further Research.

The most important area of study to be continued with is in the obtaining of accurate geometrical data for the movement of the spinal column from an initial to a deformed position. This aspect is crucial to the working of the structural analysis described in Chapter 9. An improvement must be made in the quality of the geometric data before a rigorous validation of the complete model can be made. If these data can be obtained then the order of precedence of other areas would be as follows:

(a) Further work to improve the quality of the material property data.

(b) The inclusion of a more realistic and complex system of muscles.

(c) Co-ordinated work between the spinal model and EMG studies.

(d) The inclusion of the ribcage.

(e) Obtaining more realistic bodyweight data.

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APPENDIX 1.

The Control Cards For Running The Computer Programs.

The control cards listed below enable the computer programs, referred to in Chapters 6 and 9, to be run on the CDC machines at the University of London Computer Centre.

Program MJJ1.

```
JOB(GREM108,J6,T20,M7600)
ATTACH(OLDPL,MJJ1,ID=GREM108)
UPDATE(Q,P=OLDPL,C=COMPILE)
ATTACH(PLOP)
RFL(50000)
ALGOL(I=COMPILE,S=2,D)
RFL(50000,L=15)
LGO(C,P=Z,D=4,S)
```

Program MJJ2.

```
JOB(GREM108,J6,T20,M7600)
ATTACH(OLDPL,MJJ2,ID=GREM108)
UPDATE(Q,P=OLDPL,C=COMPILE)
ATTACH(PLOP)
RFL(60000)
ALGOL(I=COMPILE,S=2,D)
RFL(60000,L=170)
FILE(SPIDA1,RT=Z,BT=C,MRL=80)
```

```
LGO(C,P=Z,D=4,S)
REWIND(SPIDA1)
CATALOG(SPIDA1,SPIDA1,ID=GREM108,CY=1,ST=PFS)
```

Program MJJ3.

```
JOB(GREM108,J3,M7600)
ATTACH(OLDPL,MJJ3,ID=GREM108)
UPDATE(Q,P=OLDPL,C=COMPILE)
REQUEST(PLOP,PF)
RFL(40000)
ALGOL(I=COMPILE,S=2,D)
RFL(40000,L=20)
LGO(C,P=Z,D=4,S)
REWIND(PLOP)
CATALOG(PLOP)
```

Program MJJ4.

This program uses similar control cards to program MJJ2, the main difference lies in the creation of a file called SPIDA2.

Program to run the APEX package.

```
JOB(GREM108,J9,T100,M6600)
ATTACH(SPIDA1,SPIDA1,ID=GREM108,CY=1)
PURGE(SPIDA1)
COPYCR(SPIDA1,TAPE1)
ATTACH(APEX,APEXIA,ID=PUBLIC)
```

```
   RFL(70000)
   REWIND(TAPE1)
→  APEX(SOLVE,MIN,L,EQ)  ←
```

A similar set of control cards runs the APEX package when finding a solution to the data presented on file SPIDA2, except the appropriate control cards refer to this file instead of SPIDA1. The arrowed line controls the operation of the APEX linear program. MIN indicates minimization, L and EQ give a listing of equations, rows, columns, bounds and ranges.

APPENDIX 2.

Listing Of The Computer Programs.

The main computer programs used in this study are listed here. They are all written so as to be run at the University of London Computer Centre using ALGOL 60 level 4.

The listings are in this order:

(a) The Reversal program for a bar element structure. Referred to in Chapter 8.1.

(b) The program MJJ1, which is a structural analysis of the spine. This is described in Chapter 9.3.

(c) The program MJJ2, this generates the data which is fed into the linear programming package. Both muscle forces and intervertebral reactions are unknowns in this program. The program is described in Chapter 6.3.

(d) The program MJJ3, which generates the geometric data which is used in the programs MJJ1, MJJ2 and MJJ4, see Fig.77 . The data input into this program is described in Appendix 3.

(a) The Reversal program for a bar element structure is listed overleaf.

```
'BEGIN'
'COMMENT'

ITERATION TO FIND MATERIAL CONSTANTS FOR ALY FRAME  ELEMENT 8 REMOVED
AND REPLACED BY EQUAL AND OPPOSITE FORCES;

'INTEGER'I,J,K,L,M,N;
'REAL''ARRAY'E,S,R,ETA[1:10],D[1:12],X,INVERSEA[1:12,1:12],
ST[1:12,1:12],STR,EN[1:10],C[1:10],
B[1:12,1:24],REFORM[1:10,1:10],ALOAD[1:8];
'REAL' RS,PIVOT,TT,AL;
INARRAY(60,D);
INARRAY(60,E);
INARRAY(60,ALOAD);
OUTPUT(61,'('2/')');
OUTARRAY(61,D);
OUTPUT(61,'('2/')');
OUTARRAY(61,E);
'COMMENT'

EVALUATION AND REARRANGEMENT OF STRAIN MATRIX;

AL:=(300*SQRT(2));
ETA[1]:=((SQRT((D[1]-D[3]+300)↑2+(D[2]-D[4])↑2))-300)/300;
ETA[2]:=((SQRT((D[3]+300)↑2+D[4]↑2))-300)/300;
ETA[3]:=((SQRT((D[7]-D[9]+300)↑2+(D[8]-D[10])↑2))-300)/300;
ETA[4]:=((SQRT((D[9]+300)↑2+D[10]↑2))-300)/300;
ETA[5]:=((SQRT((D[1]-D[7])↑2+(D[2]-D[8]+300)↑2))-300)/300;
ETA[6]:=(((SQRT((D[7]-D[3]+300)↑2+(D[4]-D[8]+300)↑2))-AL )*SQRT(2))/600;
ETA[7]:=(((SQRT((D[1]-D[9]+300)↑2+(D[2]-D[10]+300)↑2))-AL )*
SQRT(2))/600;
ETA[8]:=(((SQRT((D[3]+300)↑2+(D[4]+300)↑2))-AL )*SQRT(2))/600;
ETA[9] :=(((SQRT((D[9]+300)↑2+(D[10]-300)↑2))-AL )*SQRT(2))/600;
'FOR' I:=1 'STEP' 1 'UNTIL' 9 'DO'
REFORM[I,J]:=0;
'FOR' J:=1 'STEP' 1 'UNTIL' 9 'DO'
'FOR' I:=1 'STEP' 1 'UNTIL' 9 'DO'
REFORM[I,I]:=1/ETA[I];
OUTPUT(61,'('2/')');
OUTARRAY(61,ETA);
OUTPUT(61,'('2/')');
OUTARRAY(61,REFORM);
'COMMENT'

FORMATION OF STIFFNESS MATRIX;

L:=0;
RESTART:
L:=L+1;
'FOR' I:=1 'STEP' 1 'UNTIL' 12'DO'
'BEGIN'
'FOR' J:=1'STEP' 1 'UNTIL' 12 'DO'
ST[I,J]:=0;
'END';
ST[1,1]:=E[1]+E[7]/2;
ST[2,2]:=E[5]+E[7]/2;
ST[3,3]:=E[1]+E[2]+E[6]/2+E[8]/2;
ST[4,4]:=E[6]/2+E[8]/2;
ST[5,5]:=E[2]+E[9]/2;
```

```
ST[7,7]:=E[3]+E[6]/2;
ST[8,8]:=E[5]+E[6]/2;
ST[9,9]:=E[3]+E[4]+E[9]/2+E[7]/2;
ST[10,10]:=E[9]/2+E[7]/2;
ST[11,11]:=E[4]+E[8]/2;
ST[1,3]:=ST[3,1]:=-E[1];
ST[3,5]:=ST[5,3]:=-E[2];
ST[7,9]:=ST[9,7]:=-E[3];
ST[9,11]:=ST[11,9]:=-E[4];
ST[2,8]:=ST[8,2]:=-E[5];
ST[9,10]:=ST[10,9]:=E[7]/2-E[9]/2;
ST[1,2]:=ST[2,1]:=E[7]/2;
ST[1,9]:=ST[1,10]:=ST[2,9]:=ST[2,10]:=ST[9,1]:=ST[10,1]:=
ST[9,2]:=ST[10,2]:=-E[7]/2;
ST[11,12]:=ST[12,11]:=ST[12,12]:=E[8]/2;
ST[3,11]:=ST[3,12]:=ST[4,12]:=ST[4,11]:=ST[11,3]:=ST[12,3]:=
ST[11,4]:=ST[12,4]:=-E[8]/2;
ST[3,4]:=ST[4,3]:=E[8]/2-E[6]/2;
ST[3,7]:=ST[4,8]:=ST[7,8]:=ST[7,3]:=ST[8,4]:=ST[8,7]:=-E[6]/2;
ST[3,8]:=ST[4,7]:=ST[8,3]:=ST[7,4]:=E[6]/2;
ST[5,6]:=ST[5,9]:=ST[6,10]:=ST[6,5]:=ST[9,5]:=ST[10,6]:=-E[9]/2;
ST[5,10]:=ST[6,9]:=ST[10,5]:=ST[9,6]:=ST[6,6]:=E[9]/2;
OUTPUT(61,'('2/')');
OUTARRAY(61,ST);
R[4]:=R[10]:=0;
'FOR' J:=1 'STEP' 1 'UNTIL' 12 'DO'
'BEGIN'
R[4]:=R[4]+ST[4,J]*D[J];
R[10]:=R[10]+ST[10,J]*D[J];
'END';
ALOAD[4]:=-(ABS(R[4])+ABS(R[10]))/2;
ALOAD[8]:=(ABS(R[4])+ABS(R[10]))/2;
OUTPUT(61,'('2/')');
OUTARRAY(61,ALOAD);
'COMMENT'

PROCEDURE TO INVERT MATRIX ST AND PUT IN MATRIX INVERSEA;

'FOR'I:=5'STEP'1'UNTIL'8'DO'
'FOR' J:=1'STEP'1'UNTIL'12'DO'
ST[I,J]:=ST[I+2,J];
'FOR'J:=5'STEP'1'UNTIL'8'DO'
'FOR'I:=1'STEP'1'UNTIL'8'DO'
ST[I,J]:=ST[I,J+2];
N:=8;
M:=2*N;
'FOR' I:=1 'STEP' 1 'UNTIL' N 'DO'
'BEGIN'
'FOR' J:=1 'STEP' 1 'UNTIL' N 'DO'
B[I,J]:=ST[I,J];
'FOR' J:=N+1 'STEP' 1 'UNTIL' M 'DO'
'IF' J=N+I 'THEN' B[I,J]:=1.0 'ELSE' B[I,J]:=0;
'END';
'FOR' I:=1 'STEP' 1 'UNTIL' N 'DO'
'BEGIN'
PIVOT:=B[I,I];
'FOR' J:=I+1 'STEP' 1 'UNTIL' N 'DO'
'IF' ABS(PIVOT)<ABS(B[J,I])'THEN'
'BEGIN'
```

```
'FOR' K:=1 'STEP' 1 'UNTIL' M 'DO'
'BEGIN'
TT:=B[I,K];
B[I,K]:=B[J,K];
B[J,K]:=TT;
'END';
PIVOT:=B[I,I];
'END';
'FOR' K:=M 'STEP' -1 'UNTIL' I 'DO'
B[I,K]:=B[I,K]/B[I,I];
'FOR' J:=I+1 'STEP' 1 'UNTIL' N 'DO'
'FOR' K:=M 'STEP' -1 'UNTIL' I 'DO'
B[J,K]:=B[J,K]-B[I,K]*B[J,I];
'END';
'FOR' J:=1 'STEP' 1 'UNTIL' N 'DO'
X[N,J]:=B[N,N+J];
'FOR' I:=N-1 'STEP' -1 'UNTIL' 1 'DO'
'BEGIN'
'FOR' J:=1 'STEP' 1 'UNTIL' N 'DO'
X[I,J]:=B[I,N+J];
'FOR' K:=N 'STEP' -1 'UNTIL' I+1 'DO'
'FOR' J:=1 'STEP' 1 'UNTIL' N 'DO'
X[I,J]:=X[I,J]-B[I,K]*X[K,J];
'END';
'FOR' I:=1 'STEP' 1 'UNTIL' N 'DO'
'FOR' J:=1 'STEP' 1 'UNTIL' N 'DO'
INVERSEA[I,J]:=X[I,J];
OUTPUT(61,'('2/')');
OUTARRAY(61,INVERSEA);
'COMMENT'

SOLUTION OF C=INVERSEA * LOAD   C IS THE K ITERATION DISPLACEMENT;

 'FOR'I:=1'STEP'1'UNTIL'8'DO'
 'BEGIN'
 RS:=0;
 'FOR'J:=1'STEP'1'UNTIL'8'DO'
 RS:=RS+(INVERSEA[I,J]*ALOAD[J]);
 C[I]:=RS;
 'END';
 OUTPUT(61,'('2/')');
 OUTARRAY(61,C);
 STR[1]:=((SQRT((C[1]-C[3]+300)↑2+(C[2]-C[4])↑2))-300)/300;
 STR[2]:=((SQRT((C[3]+300)↑2+C[4]↑2))-300)/300;
 STR[3]:=((SQRT((C[5]-C[7]+300)↑2+(C[6]-C[8])↑2))-300)/300;
 STR[4]:=((SQRT((C[7]+300)↑2+C[8]↑2))-300)/300;
 STR[5]:=((SQRT((C[1]-C[5])↑2+(C[2]-C[6]+300)↑2))-300)/300;
 STR[6]:=(((SQRT((C[5]-C[3]+300)↑2+(C[4]-C[6]+300)↑2))-AL)*SQRT(2))/600;
 STR[7]:=(((SQRT((C[1]-C[7]+300)↑2+(C[2]-C[8]+300)↑2))-AL)*SQRT(2))/600;
 STR[8]:=(((SQRT((C[3]+300)↑2+(C[4]+300)↑2))-AL)*SQRT(2))/600;
 STR[9]:=(((SQRT((C[7]+300)↑2+(C[8]-300)↑2))-AL)*SQRT(2))/600;
 OUTPUT(61,'('2/')');
 OUTARRAY(61,STR);
 'COMMENT'

 SOLUTION FOR K ITERATION OF MAT CONSTANTS;

 'FOR' I:=1 'STEP' 1 'UNTIL' 9 'DO'
 S[I]:=E[I]*STR[I];
```

```
'FOR' K:=1 'STEP' 1 'UNTIL' M 'DO'
'BEGIN'
TT:=B[I,K];
B[I,K]:=B[J,K];
B[J,K]:=TT;
'END';
PIVOT:=B[I,I];
'END';
'FOR' K:=M 'STEP' -1 'UNTIL' I 'DO'
B[I,K]:=B[I,K]/B[I,I];
'FOR' J:=I+1 'STEP' 1 'UNTIL' N 'DO'
'FOR' K:=M 'STEP' -1 'UNTIL' I 'DO'
B[J,K]:=B[J,K]-B[I,K]*B[J,I];
'END';
'FOR' J:=1 'STEP' 1 'UNTIL' N 'DO'
X[N,J]:=B[N,N+J];
'FOR' I:=N-1 'STEP' -1 'UNTIL' 1 'DO'
'BEGIN'
'FOR' J:=1 'STEP' 1 'UNTIL' N 'DO'
X[I,J]:=B[I,N+J];
'FOR' K:=N 'STEP' -1 'UNTIL' I+1 'DO'
'FOR' J:=1 'STEP' 1 'UNTIL' N 'DO'
X[I,J]:=X[I,J]-B[I,K]*X[K,J];
'END';
'FOR' I:=1 'STEP' 1 'UNTIL' N 'DO'
'FOR' J:=1 'STEP' 1 'UNTIL' N 'DO'
INVERSEA[I,J]:=X[I,J];
OUTPUT(61,'('2/')');
OUTARRAY(61,INVERSEA);
'COMMENT'

SOLUTION OF C=INVERSEA * LOAD   C IS THE K ITERATION DISPLACEMENT;

 'FOR'I:=1'STEP'1'UNTIL'8'DO'
 'BEGIN'
 RS:=0;
 'FOR'J:=1'STEP'1'UNTIL'8'DO'
 RS:=RS+(INVERSEA[I,J]*ALOAD[J]);
 C[I]:=RS;
 'END';
 OUTPUT(61,'('2/')');
 OUTARRAY(61,C);
 STR[1]:=((SQRT((C[1]-C[3]+300)↑2+(C[2]-C[4])↑2))-300)/300;
 STR[2]:=((SQRT((C[3]+300)↑2+C[4]↑2))-300)/300;
 STR[3]:=((SQRT((C[5]-C[7]+300)↑2+(C[6]-C[8])↑2))-300)/300;
 STR[4]:=((SQRT((C[7]+300)↑2+C[8]↑2))-300)/300;
 STR[5]:=((SQRT((C[1]-C[5])↑2+(C[2]-C[6]+300)↑2))-300)/300;
 STR[6]:=(((SQRT((C[5]-C[3]+300)↑2+(C[4]-C[6]+300)↑2))-AL)*SQRT(2))/600;
 STR[7]:=(((SQRT((C[1]-C[7]+300)↑2+(C[2]-C[8]+300)↑2))-AL)*SQRT(2))/600;
 STR[8]:=(((SQRT((C[3]+300)↑2+(C[4]+300)↑2))-AL)*SQRT(2))/600;
 STR[9]:=(((SQRT((C[7]+300)↑2+(C[8]-300)↑2))-AL)*SQRT(2))/600;
 OUTPUT(61,'('2/')');
 OUTARRAY(61,STR);
 'COMMENT'

 SOLUTION FOR K ITERATION OF MAT CONSTANTS;

 'FOR' I:=1 'STEP' 1 'UNTIL' 9 'DO'
 S[I]:=E[I]*STR[I];
```

```
'FOR' I:=1 'STEP' 1 'UNTIL' 9 'DO'
'BEGIN'
EN[I]:=0;
'FOR' J:=1 'STEP' 1 'UNTIL' 9 'DO'
EN[I]:=EN[I]+(REFORM[I,J]*S[J]);
'END';
OUTPUT(61,'('2/')');
OUTPUT(61,'('/,20B,'('L=')',3D')',L);
'IF'L'EQUAL'10'THEN''GOTO'EXCEL;
'FOR' I:=1 'STEP' 1 'UNTIL' 9 'DO'
'IF' ABS(E[I]-EN[I]) 'GREATER' 0,00001*E[I] 'THEN''GO TO' REPEAT;
'GO TO' EXCEL;
REPEAT:
'FOR' I:=1 'STEP' 1 'UNTIL' 9 'DO'
E[I]:=ABS(EN[I]);
OUTPUT(61,'('2/')');
OUTARRAY(61,E);
'GO TO' RESTART;
EXCEL:
OUTPUT(61,'('2/')');
OUTARRAY(61,EN);
'END';
```

(b) The program MJJ1 is listed overleaf.

```
"BEGIN"
"PROCEDURE" RO(Q,N,NN,X,XB,XP,Y,YB,YP,Z,ZB,ZP,D,R,RT,MF,NB);
"VALUE" Q,N,NN,MF,NB;
"INTEGER" Q,N,NN,MF,NB;
"REAL""ARRAY" X,Y,Z,XB,YB,ZB,XP,YP,ZP,D,R,RT;
"BEGIN"
"REAL""ARRAY" XD,YD,ZD[1:2];
"INTEGER" ML,I,J,NC,ND;
"COMMENT"

        FORMS DISPLACEMENT TRANSFORMATION MATRIX R;

"FOR" I:=1 "STEP" 1 "UNTIL" 12 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 12 "DO"
R[I,J]:=RT[I,J]:=0;
ND:=(Q*1)-1;
"IF" NB=1 "THEN"
"BEGIN"
R[4,7]:=R[5,8]:=R[6,9]:=RT[7,4]:=RT[8,5]:=RT[9,6]:=1;
ML:=(((Q*NN)-NN)*2)+(N*2)-2;
XD[1]:=X[ML+MF]-XP[MF+ND];
YD[1]:=Y[ML+MF]-YP[MF+ND];
ZD[1]:=Z[ML+MF]-ZP[MF+ND];
"IF" MF=1 "THEN"
"BEGIN"
XD[2]:=X[ML+2]-XP[Q+1];
YD[2]:=Y[ML+2]-YP[Q+1];
ZD[2]:=Z[ML+2]-ZP[Q+1];
"END";
"END";
"IF" NB=2 "THEN"
"BEGIN"
R[4,4]:=R[5,5]:=R[6,6]:=R[7,7]:=R[8,8]:=R[9,9]:=R[10,10]:=R[11,11]:=
R[12,12]:=RT[4,4]:=RT[5,5]:=RT[6,6]:=RT[7,7]:=RT[8,8]:=RT[9,9]:=
RT[10,10]:=RT[11,11]:=RT[12,12]:=1;
ML:=(Q*2)-2;
XD[1]:=XB[ML+MF]-XP[MF+ND];
YD[1]:=YB[ML+MF]-YP[MF+ND];
ZD[1]:=ZB[ML+MF]-ZP[MF+ND];
"IF" MF=1 "THEN"
"BEGIN"
XD[2]:=XB[ML+2]-XP[Q+1];
YD[2]:=YB[ML+2]-YP[Q+1];
ZD[2]:=ZB[ML+2]-ZP[Q+1];
"END";
"END";
R[1,1]:=R[2,2]:=R[3,3]:=RT[1,1]:=RT[2,2]:=RT[3,3]:=1;
R[1,5]:=RT[5,1]:=(ZD[1]*COS(D[5]/2)-XD[1]*SIN(D[5]/2));
R[1,6]:=RT[6,1]:=-(YD[1]*COS(D[6]/2)+XD[1]*SIN(D[6]/2));
R[2,4]:=RT[4,2]:=-(ZD[1]*COS(D[4]/2)+YD[1]*SIN(D[4]/2));
R[2,6]:=RT[6,2]:=(XD[1]*COS(D[6]/2)-YD[1]*SIN(D[6]/2));
R[3,4]:=RT[4,3]:=(YD[1]*COS(D[4]/2)-ZD[1]*SIN(D[4]/2));
R[3,5]:=RT[5,3]:=-(XD[1]*COS(D[5]/2)+ZD[1]*SIN(D[5]/2));
"IF" MF=1 "THEN"
"BEGIN"
NC:=(NB*3)-3;
R[4+NC,11]:=RT[11,4+NC]:=ZD[2];
R[4+NC,12]:=RT[12,4+NC]:=-YD[2];
R[5+NC,10]:=RT[10,5+NC]:=-ZD[2];
```

```
R[5+NC,12]:=RT[12,5+NC]:=XD[2];
R[6+NC,10]:=RT[10,6+NC]:=YD[2];
R[6+NC,11]:=RT[11,6+NC]:=-XD[2];
"END";
"END" OF PROCEDURE RO;
"PROCEDURE" ANGLE(CX,CY,CZ,CRO,SRO,ALPHA,BETA,GAMMA,PI);
"VALUE" ALPHA,BETA,GAMMA,PI;
"REAL" CX,CY,CZ,CRO,SRO,ALPHA,BETA,GAMMA,PI;
"BEGIN"
"REAL" AL;
"COMMENT"

      FIND COSINES OF ANGLES OF ROTATION;

AL:=SQRT((SIN(BETA)/COS(BETA))'2+1/(COS(GAMMA))'2);
CX:=1/AL;
CY:=ABS((SIN(GAMMA)/COS(GAMMA))/AL);
"IF" GAMMA<0 "THEN" CY:=-CY;
CZ:=ABS(SIN(BETA)/COS(BETA))/SQRT((1/COS(BETA))'2+(SIN(GAMMA)/
COS(GAMMA))'2);
"IF" BETA>0 "THEN" CZ:=-CZ;
"IF" ALPHA=PI/2 "THEN" CRO:=1 "ELSE"
CRO:=1/SQRT(((((SIN(ALPHA)/COS(ALPHA))'2)*(COS(BETA))'2)/
(COS(GAMMA))'2)+1);
"IF" CRO>1 "THEN" CRO:=1;
SRO:=SQRT(1-CRO'2);
"IF" ALPHA<0 "THEN" SRO:=-SRO;
"END" OF PROCEDURE ANGLE;
"PROCEDURE" TRAN(C,CX,CY,CZ,CRO,SRO);
"VALUE" CX,CY,CZ,CRO,SRO;
"REAL" CX,CY,CZ,CRO,SRO;
"REAL""ARRAY" C;
"BEGIN"
"REAL" CA;
"COMMENT"

      YZX COORDINATE TRANSFORMATION;

CA:=SQRT(CX'2+CZ'2);
C[1,1]:=CX;
C[1,2]:=CY;
C[1,3]:=CZ;
C[2,1]:=((-CX*CY*CRO)-(CZ*SRO))/CA;
C[2,2]:=CA*CRO;
C[2,3]:=((-CY*CZ*CRO)+(CX*SRO))/CA;
C[3,1]:=((CX*CY*SRO)-(CZ*CRO))/CA;
C[3,2]:=-CA*SRO;
C[3,3]:=((CY*CZ*SRO)+(CX*CRO))/CA;
"END" OF PROCEDURE TRAN;
"PROCEDURE" TRANS(C,CX,CY,CZ,SRO,CRO);
"VALUE" CX,CY,CZ,SRO,CRO;
"REAL" CX,CY,CZ,SRO,CRO;
"REAL""ARRAY" C;
"BEGIN"
"REAL" CA;
"COMMENT"

      ZYX COORDINATE TRANSFORMATION;

```

```
CA:=SQRT(CX'2+CY'2);
C[1,1]:=CX;
C[1,2]:=CY;
C[1,3]:=CZ;
C[2,1]:=((-CX*CZ*SRO)-(CY*CRO))/CA;
C[2,2]:=((-CY*CZ*SRO)+(CX*CRO))/CA;
C[2,3]:=CA*SRO;
C[3,1]:=((-CX*CZ*CRO)+(CY*SRO))/CA;
C[3,2]:=((-CY*CZ*CRO)-(CX*SRO))/CA;
C[3,3]:=CA*CRO;
"END" OF  PROCEDURE TRANS;
"PROCEDURE" SPINAL(MM,NN,X,Y,Z,XB,YB,ZB,XP,YP,ZP,AP,BP,GP,XPD,YPD,ZPD,
          APD,BPD,GPD,XR,XR2,XRB,XRB2,K,KB,FZ);
"VALUE" MM,NN;
"INTEGER" MM,NN;
"REAL""ARRAY" X,Y,Z,XB,YB,ZB,XP,YP,ZP,AP,BP,GP,XPD,YPD,ZPD,APD,
          BPD,GPD,XR,XR2,XRB,XRB2,K,KB,FZ;
"BEGIN"
"INTEGER" I,II,J,JJ,M,MA,MB,MC,MF,N,NB,NO,Q;
"REAL" KK,STA,PB,QB,RB,AB,BB,CB,PX,RX,QX,CX,CY,CZ,CA,PI,L,LC,FO,
      SRO,CRO,ALPHA,BETA,GAMMA,ALPHA2,BETA2,GAMMA2,ROTA,SSTA,RTA,RRTA,
      ROTB,ROTG,ALP,BET,GAM,AL,BE,GA,MOVX,MOVY,MOVZ;
"REAL""ARRAY" CT,CTT,ST,STC,STP,STT,R,RT[1:12,1:12],D,DC,KA[1:6],DA[1:3]
      ,C[1:3,1:3],F,FT,RE[1:6*MM],XPOS1,YPOS1,ZPOS1,XPOS2,YPOS2,
      ZPOS2[1:MM*NN],XBOS1,YBOS1,ZBOS1,XBOS2,YBOS2,ZBOS2[1:MM];
"FOR" I:=1 "STEP" 1 "UNTIL" MM*6 "DO"
F[I]:=FT[I]:=RE[I]:=0;
"COMMENT"

       CALCULATES NODAL FORCE REQUIRED TO DISPLACE THE SPINE;

"FOR" M:=1 "STEP" 1 "UNTIL" MM "DO"
"BEGIN"
PI:=3.1415929;
OUTPUT(61,"("2/,30B,"("****************************************************
**")"")")");
OUTPUT(61,"("4/,30B,"("RIGID LINK NUMBER")",4Z")",M);
"COMMENT"

      FORM DISPLACEMENTS OF PRIMARY NODES

       OBTAIN RELATIVE ROTATION OF VERTEBRAL BODIES;

"FOR" I:=1 "STEP" 1 "UNTIL" 6 "DO"
D[I]:=DC[I]:=0;
ALPHA:=APD[M]-AP[M];
BETA:=BPD[M]-BP[M];
GAMMA:=GPD[M]-GP[M];
ALPHA2:=APD[M+1]-AP[M+1];
BETA2:=BPD[M+1]-BP[M+1];
GAMMA2:=GPD[M+1]-GP[M+1];
DA[1]:=ALPHA-ALPHA2;
DA[2]:=BETA-BETA2;
DA[3]:=GAMMA-GAMMA2;
ANGLE(CX,CY,CZ,CRO,SRO,ALPHA2,BETA2,GAMMA2,PI);
TRAN(C,CX,CY,CZ,CRO,SRO);
"FOR" I:=1 "STEP" 1 "UNTIL" 3 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 3 "DO"
D[I+3]:=D[I+3]+C[I,J]*DA[J];
```

```
L:=SQRT((XP[M+1]-XP[M])'2+(YP[M+1]-YP[M])'2+(ZP[M+1]-ZP[M])'2);
"IF" ZP[M]-ZP[M+1]=0 "THEN" ALP:=0 "ELSE"
"IF" YP[M]-YP[M+1]=0 "THEN" ALP:=PI/2 "ELSE"
ALP:=ARCTAN((ZP[M]-ZP[M+1])/(YP[M]-YP[M+1]));
"IF" YP[M]-YP[M+1]=0 "THEN" GAM:=0 "ELSE"
"IF" XP[M]-XP[M+1]=0 "THEN" GAM:=PI/2 "ELSE"
GAM:=ARCTAN((YP[M]-YP[M+1])/(XP[M]-XP[M+1]));
"IF" GAM<0 "THEN" GAM:=PI-GAM;
OUTPUT(61,"("2/,10B,-ZD,3D,10B,-ZD,3D,10B,-ZD,3D")",ALP,BET,GAM);
AL:=ALP+ALPHA2;
BE:=BET+BETA2;
GA:=GAM+GAMMA2;
OUTPUT(61,"("2/,10B,-ZD,3D,10B,-ZD,3D,10B,-ZD,3D")",AL,BE,GA);
"IF" AL=PI/2 "THEN" MOVY:=0 "ELSE"
"IF" GA=0 "THEN" MOVY:=0 "ELSE"
"IF" GA=PI/2 "THEN" MOVY:=ABS(L*COS(AL)) "ELSE"
MOVY:=L/SQRT((1/COS(AL))'2+(COS(GA)/SIN(GA))'2);
"IF" MOVY>L "THEN" MOVY:=L;
MOVX:=ABS((COS(GA)/SIN(GA))*MOVY);
MOVZ:=ABS(MOVY*SIN(AL)/COS(AL));
"IF" ABS(GA)>PI/2 "THEN" MOVX:=-MOVX;
"IF" ABS(AL)>PI/2 "THEN" MOVY:=-MOVY;
"IF" AL<0 "THEN" MOVZ:=-MOVZ;
DA[1]:=XPD[M]-(MOVX+XPD[M+1]);
DA[2]:=YPD[M]-(MOVY+YPD[M+1]);
DA[3]:=ZPD[M]-(MOVZ+ZPD[M+1]);
"FOR" I:=1 "STEP" 1 "UNTIL" 3 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 3 "DO"
D[I]:=D[I]+C[I,J]*DA[J];
OUTPUT(61,"("2/,15B,"("ARRAY D")",/")");
OUTARRAY(61,D);
"FOR" I:=1 "STEP" 1 "UNTIL" 12 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 12 "DO"
STC[I,J]:=CT[I,J]:=CTT[I,J]:=R[I,J]:=RT[I,J]:=STT[I,J]:=0;
"COMMENT"

      START OF CYCLE FOR BAR ELEMENTS;

"FOR" N:=1 "STEP" 1 "UNTIL" NN "DO"
"BEGIN"
MA:=((M*NN)-NN)*2;
MB:=(N*2)-2;
KK:=K[N+MA/2];
XPOS1[N+MA/2]:=X[MA+MB+1]-XP[M];
YPOS1[N+MA/2]:=Y[MA+MB+1]-YP[M];
ZPOS1[N+MA/2]:=Z[MA+MB+1]-ZP[M];
XPOS2[N+MA/2]:=X[MA+MB+2]-XP[M+1];
YPOS2[N+MA/2]:=Y[MA+MB+2]-YP[M+1];
ZPOS2[N+MA/2]:=Z[MA+MB+2]-ZP[M+1];
"COMMENT"

      CALCULATE DIRECTION COSINES;

MC:=MA+MB+1;
PB:=X[MC+1]-X[MC];
QB:=Y[MC+1]-Y[MC];
RB:=Z[MC+1]-Z[MC];
L:=SQRT(PB'2+QB'2+RB'2);
AB:=PB/L;
```

```
BB:=QB/L;
CB:=RB/L;
"COMMENT"

     FORM DISPLACEMENT TRANSFORMATION;

"FOR" I:=1 "STEP" 1 "UNTIL" 12 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 12 "DO"
ST[I,J]:=STP[I,J]:=0;
"IF" M>1 "THEN"
"BEGIN"
RO(M-1,N,NN,X,XB,XP,Y,YB,YP,Z,ZB,ZP,D,R,RT,2,1);
"FOR" I:=1 "STEP" 1 "UNTIL" 3 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 6 "DO"
DC[I]:=DC[I]+R[I,J]*D[J];
MB:=(M*NN)-NN;
XPOS2[N+MB-NN]:=XPOS2[N+MB-NN]+DC[1]-D[1];
YPOS2[N+MB-NN]:=YPOS2[N+MB-NN]+DC[2]-D[2];
ZPOS2[N+MB-NN]:=ZPOS2[N+MB-NN]+DC[3]-D[3];
"END";
"FOR" I:=1 "STEP" 1 "UNTIL" 3 "DO"
DC[I]:=0;
RO(M,N,NN,X,XB,XP,Y,YB,YP,Z,ZB,ZP,D,R,RT,1,1);
"FOR" I:=1 "STEP" 1 "UNTIL" 3 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 6 "DO"
DC[I]:=DC[I]+R[I,J]*D[J];
MB:=(M*NN)-NN;
XPOS1[N+MB]:=XPOS1[N+MB]+DC[1]-D[1];
YPOS1[N+MB]:=YPOS1[N+MB]+DC[2]-D[2];
ZPOS1[N+MB]:=ZPOS1[N+MB]+DC[3]-D[3];
"COMMENT"

     CHECK IF ELEMENTS ARE IN TENSION OR COMPRESSION

     IF IN COMPRESSION SET ST=0;

PB:=X[MC+1]-(X[MC]+(DC[1]*(-AB)));
QB:=Y[MC+1]-(Y[MC]+(DC[2]*(-BB)));
RB:=Z[MC+1]-(Z[MC]+(DC[3]*(-CB)));
LC:=SQRT(PB'2+QB'2+RB'2);
"IF" N=4 "THEN""GO TO" NORETURN;
"IF" N=5 "THEN""GO TO" NORETURN;
"IF" LC<L "THEN""GO TO" RETURN;
NORETURN:
FO:=KK*(LC-L);
OUTPUT(61,"("2/,9B,"("BAR ELEMENT")",2B,3Z,5B,"("LENGTH")",8B,
"("FORCE")",10B,"("DIRECTION COSINES")"")",N);
OUTPUT(61,"("/,28B,-2ZD,3D,4B,-4ZD,3D,7B,-ZD,3D,7B,-ZD.3D,7B,-ZD.3D")",
L,FO,AB,BB,CB);
FO:=0;
"COMMENT"

     FORM LOCAL STIFFNESS MATRIX;

ST[1,1]:=ST[4,4]:=KK*(AB'2);
ST[2,2]:=ST[5,5]:=KK*(BB'2);
ST[3,3]:=ST[6,6]:=KK*(CB'2);
ST[1,2]:=ST[2,1]:=ST[4,5]:=ST[5,4]:=KK*AB*BH;
ST[1,3]:=ST[3,1]:=ST[4,6]:=ST[6,4]:=KK*AB*CB;
```

```
ST[2,3]:=ST[3,2]:=ST[5,6]:=ST[6,5]:=KK*BB*CB;
ST[1,4]:=ST[4,1]:=-KK*(AB'2);
ST[2,5]:=ST[5,2]:=-KK*(BB'2);
ST[3,6]:=ST[6,3]:=-KK*(CB'2);
ST[1,5]:=ST[5,1]:=ST[2,4]:=ST[4,2]:=-KK*AB*BB;
ST[1,6]:=ST[6,1]:=ST[3,4]:=ST[4,3]:=-KK*AB*CB;
ST[2,6]:=ST[6,2]:=ST[3,5]:=ST[5,3]:=-KK*BB*CB;
"COMMENT"

      FORM STIFFNESS MATRIX FOR PRIMARY NODES;

"FOR" I:=1 "STEP" 1 "UNTIL" 6 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 12 "DO"
"FOR" JJ:=1 "STEP" 1 "UNTIL" 6 "DO"
STP[I,J]:=STP[I,J]+ST[I,JJ]*R[JJ,J];
"FOR" I:=1 "STEP" 1 "UNTIL" 12 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 12 "DO"
"FOR" JJ:=1 "STEP" 1 "UNTIL" 6 "DO"
STC[I,J]:=STC[I,J]+RT[I,JJ]*STP[JJ,J];
RETURN:
"FOR" I:=1 "STEP" 1 "UNTIL" 6 "DO"
DC[I]:=0;
"END";
"COMMENT"

      PROCEDURE BEAM;

MB:=(M*2)-2;
XBOS1[M]:=XB[MB+1]-XP[M];
YBOS1[M]:=YB[MB+1]-YP[M];
ZBOS1[M]:=ZB[MB+1]-ZP[M];
XBOS2[M]:=XB[MB+2]-XP[M+1];
YBOS2[M]:=YB[MB+2]-YP[M+1];
ZBOS2[M]:=ZB[MB+2]-ZP[M+1];
"COMMENT"

      CALCULATE DIRECTION COSINES FOR ZYX TRANSFORMATION;

PX:=XB[MB+2]-XB[MB+1];
QX:=YB[MB+2]-YB[MB+1];
RX:=ZB[MB+2]-ZB[MB+1];
L:=SQRT(PX'2+QX'2+RX'2);
CX:=PX/L;
CY:=QX/L;
CZ:=RX/L;
OUTPUT(61,"("2/,9B,"("BEAM ELEMENT")",9B,"("LENGTH")",23B,"("DIRECTION
COSINES")"")");
OUTPUT(61,"("/,28B,-2ZD.3D,22B,-ZD.3D,7B,-ZD.3D,7B,-ZD.3D")",L,CX,CY,CZ
);
"COMMENT"

      FORM TRANSFORMATION MATRIX;

SRO:=0;
CRO:=1.0;
TRANS(C,CX,CY,CZ,SRO,CRO);
"FOR" I:=1 "STEP" 1 "UNTIL" 12 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 12 "DO"
CT[I,J]:=CTT[I,J]:=ST[I,J]:=R[I,J]:=RT[I,J]:=STP[I,J]:=STT[I,J]:=0;
```

```
"FOR" II:=1 "STEP" 1 "UNTIL" 4 "DO"
"BEGIN"
NO:=(II*3)-3;
"FOR" JJ:=1 "STEP" 1 "UNTIL" 3 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 3 "DO"
CT[JJ+NO,J+NO]:=C[JJ,J];
"END";
"FOR" I:=1 "STEP" 1 "UNTIL" 12 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 12 "DO"
CTT[J,I]:=CT[I,J];
"COMMENT"

      FORM STIFFNESS MATRIX FOR SECONDARY NODES;

MA:=(M*6)-6;
ST[1,1]:=ST[7,7]:=KB[MA+1];
ST[1,7]:=ST[7,1]:=-KB[MA+1];
ST[4,4]:=ST[10,10]:=KB[MA+2];
ST[4,10]:=ST[10,4]:=-KB[MA+2];
ST[2,2]:=ST[3,3]:=ST[8,8]:=ST[9,9]:=KB[MA+3];
ST[2,8]:=ST[8,2]:=ST[3,9]:=ST[9,3]:=-KB[MA+3];
ST[2,6]:=ST[6,2]:=ST[2,12]:=ST[12,2]:=ST[5,9]:=ST[9,5]:=ST[9,11]:=
ST[11,9]:=KB[MA+4];
ST[3,5]:=ST[5,3]:=ST[3,11]:=ST[11,3]:=ST[6,8]:=ST[8,6]:=
ST[8,12]:=ST[12,8]:=-KB[MA+4];
ST[5,5]:=ST[6,6]:=ST[11,11]:=ST[12,12]:=KB[MA+5];
ST[5,11]:=ST[11,5]:=ST[6,12]:=ST[12,6]:=KB[MA+6];
"COMMENT"

      FORM DIRECTED STIFFNESS MATRIX;

"FOR" I:=1 "STEP" 1 "UNTIL" 12 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 12 "DO"
"FOR" JJ:=1 "STEP" 1 "UNTIL" 12 "DO"
STT[I,J]:=STT[I,J]+ST[I,JJ]*CT[JJ,J];
"FOR" I:=1 "STEP" 1 "UNTIL" 12 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 12 "DO"
"BEGIN"
STA:=0;
"FOR" JJ:=1 "STEP" 1 "UNTIL" 12 "DO"
STA:=STA+CTT[I,JJ]*STT[JJ,J];
ST[I,J]:=STA;
"END";
"COMMENT"

      FORM DISPLACEMENT TRANSFORMATION;

"IF" M>1 "THEN"
"BEGIN"
RO(M-1,N,NN,X,XB,XP,Y,YB,YP,Z,ZB,ZP,D,R,RT,2,2);
"FOR" I:=1 "STEP" 1 "UNTIL" 3 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 6 "DO"
DC[I]:=DC[I]+R[I,J]*D[J];
XBOS2[M-1]:=XBOS2[M-1]+DC[1]-D[1];
YBOS2[M-1]:=YBOS2[M-1]+DC[2]-D[2];
ZBOS2[M-1]:=ZBOS2[M-1]+DC[3]-D[3];
"END";
"FOR" I:=1 "STEP" 1 "UNTIL" 3 "DO"
DC[I]:=0;
```

```
RO(M,N,NN,X,XB,XP,Y,YB,YP,Z,ZB,ZP,D,R,RT,1,2);
"FOR" I:=1 "STEP" 1 "UNTIL" 3 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 6 "DO"
DC[I]:=DC[I]+R[I,J]*D[J];
XBOS1[M]:=XBOS1[M]+DC[1]-D[1];
YBOS1[M]:=YBOS1[M]+DC[2]-D[2];
ZBOS1[M]:=ZBOS1[M]+DC[3]-D[3];
"COMMENT"

      FORM STIFFNESS MATRIX FOR PRIMARY NODES;

"FOR" I:=1 "STEP" 1 "UNTIL" 12 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 12 "DO"
"FOR" JJ:=1 "STEP" 1 "UNTIL" 12 "DO"
STP[I,J]:=STP[I,J]+ST[I,JJ]*R[JJ,J];
"FOR" I:=1 "STEP" 1 "UNTIL" 12 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 12 "DO"
"FOR" JJ:=1 "STEP" 1 "UNTIL" 12 "DO"
STC[I,J]:=STC[I,J]+RT[I,JJ]*STP[JJ,J];
"COMMENT"

      CALCULATE FORCE AT EACH NODE IN TURN;

MA:=(M*6)-6;
"FOR" II:=1 "STEP" 1 "UNTIL" 6 "DO"
"BEGIN"
STA:=0;
"FOR" I:=1 "STEP" 1 "UNTIL" 6 "DO"
STA:=STA+STC[II,I]*D[I];
F[MA+II]:=STA;
"END";
"FOR" II:=7 "STEP" 1 "UNTIL" 12 "DO"
"BEGIN"
STA:=0;
"FOR" I:=1 "STEP" 1 "UNTIL" 6 "DO"
STA:=STA+STC[II,I]*D[I];
RE[MA+II-6]:=STA;
"END";
"END";
OUTPUT(61,"("2/")");
OUTPUT(61,"("15B,"("ARRAY F ")"")");
OUTARRAY(61,F);
OUTPUT(61,"("2/")");
OUTPUT(61,"("15B,"("ARRAY RE ")"")");
OUTARRAY(61,RE);
"COMMENT"

      MODIFY TO DEFLECTED POSITION   YZX TRANSFORMATION;

"FOR" I:=1 "STEP" 1 "UNTIL" MM*NN*3 "DO"
XR[I]:=XR2[I]:=0;
"FOR" I:=1 "STEP" 1 "UNTIL" MM*3 "DO"
XRB[I]:=XRB2[I]:=0;
"FOR" M:=1 "STEP" 1 "UNTIL" MM "DO"
"BEGIN"
ALPHA:=APD[M+1]-AP[M+1];
BETA:=BPD[M+1]-BP[M+1];
GAMMA:=GPD[M+1]-GP[M+1];
ANGLE(CX,CY,CZ,CRO,SRO,ALPHA,BETA,GAMMA,PI);
```

```
TRAN(C,CX,CY,CZ,CRO,SRO);
"FOR" I:=1 "STEP" 1 "UNTIL" 6 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 6 "DO"
CT[I,J]:=CTT[I,J]:=0;
"FOR" II:=1 "STEP" 1 "UNTIL" 2 "DO"
"BEGIN"
NO:=(II*3)-3;
"FOR" I:=1 "STEP" 1 "UNTIL" 3 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 3 "DO"
CT[I+NO,J+NO]:=C[I,J];
"END";
"COMMENT"

      FIND NEW CO-ORDS OF SECONDARY NODES;

MA:=(M*NN*3)-NN*3;
NO:=MA/3;
"FOR" I:=1 "STEP" 1 "UNTIL" NN "DO"
"BEGIN"
MB:=MA+(I*3)-3;
"FOR" J:=1 "STEP" 1 "UNTIL" 3 "DO"
XR[MB+J]:=C[1,J]*XPOS1[I+NO]+C[2,J]*YPOS1[I+NO]+C[3,J]*ZPOS1[I+NO];
XR[MB+1]:=XR[MB+1]+XPD[M];
XR[MB+2]:=XR[MB+2]+YPD[M];
XR[MB+3]:=XR[MB+3]+ZPD[M];
"IF" M>1 "THEN"
"BEGIN"
MB:=MA+(I*3)-(NN*3)-3;
"FOR" J:=1 "STEP" 1 "UNTIL" 3 "DO"
XR2[MB+J]:=C[1,J]*XPOS2[I+NO-NN]+C[2,J]*YPOS2[I+NO-NN]+
C[3,J]*ZPOS2[I+NO-NN];
XR2[MB+1]:=XR2[MB+1]+XPD[M];
XR2[MB+2]:=XR2[MB+2]+YPD[M];
XR2[MB+3]:=XR2[MB+3]+ZPD[M];
"END";
"IF" M=MM "THEN"
"BEGIN"
MB:=(MM-1)*NN*3+(I*3)-3;
MC:=((MM-1)*NN*2)+(I*2);
XR2[MB+1]:=X[MC];
XR2[MB+2]:=Y[MC];
XR2[MB+3]:=Z[MC];
"END";
"END";
MB:=(M*3)-3;
"FOR" J:=1 "STEP" 1 "UNTIL" 3 "DO"
XRB[MB+J]:=C[1,J]*XBOS1[M]+C[2,J]*YBOS1[M]+C[3,J]*ZBOS1[M];
XRB[MB+1]:=XRB[MB+1]+XPD[M];
XRB[MB+2]:=XRB[MB+2]+YPD[M];
XRB[MB+3]:=XRB[MB+3]+ZPD[M];
"IF" M>1 "THEN"
"BEGIN"
MB:=(M*3)-6;
"FOR" J:=1 "STEP" 1 "UNTIL" 3 "DO"
XRB2[MB+J]:=C[1,J]*XBOS2[M-1]+C[2,J]*YBOS2[M-1]+C[3,J]*ZBOS2[M-1];
XRB2[MB+1]:=XRB2[MB+1]+XPD[M];
XRB2[MB+2]:=XRB2[MB+2]+YPD[M];
XRB2[MB+3]:=XRB2[MB+3]+ZPD[M];
"END";
```

```
"IF" M=MM "THEN"
"BEGIN"
MB:=(M*3)-3;
MC:=MM*2;
XRB2[MB+1]:=XB[MC];
XRB2[MB+2]:=YB[MC];
XRB2[MB+3]:=ZB[MC];
"END";
"COMMENT"

      CALCULATE APPLIED FORCES AT EACH NODE;

MA:=(M*6)-6;
"FOR" I:=1 "STEP" 1 "UNTIL" 6 "DO"
"BEGIN"
STA:=RTA:=0;
"FOR" J:=1 "STEP" 1 "UNTIL" 6 "DO"
STA:=STA+CT[J,I]*F[MA+J];
C[1,I]:=STA;
"FOR" J:=1 "STEP" 1 "UNTIL" 6 "DO"
RTA:=RTA+CT[J,I]*RE[MA+J];
CTT[1,I]:=RTA;
"END";
"FOR" I:=1 "STEP" 1 "UNTIL" 6 "DO"
"BEGIN"
F[MA+I]:=C[1,I];
RE[MA+I]:=CTT[1,I];
"END";
"END";
"FOR" I:=1 "STEP" 1 "UNTIL" 6 "DO"
FT[I]:=F[I];
"IF" MM<2 "THEN""GO TO" MISS;
"FOR" M:=1 "STEP" 1 "UNTIL" MM-1 "DO"
"BEGIN"
MA:=(M*6)-6;
"FOR" I:=1 "STEP" 1 "UNTIL" 6 "DO"
FT[MA+I+6]:=F[MA+I+6]+RE[MA+I];
"END";
MISS:
OUTPUT(61,"("2/,30B,"("*********************************************
***")"")");
"FOR" I:=1 "STEP" 1 "UNTIL" MM "DO"
"BEGIN"
MA:=(I*6)-6;
OUTPUT(61,"("2/,10B,"("APPLIED FORCE AT NODE")",3Z")",I);
OUTPUT(61,"("2/,10B,-5ZD.5D,10B,-5ZD.5D,10B,-5ZD.5D,/,15B,-5ZD.5D,10B,
-5ZD.5D,10B,-5ZD.5D")",FT[MA+1],FT[MA+2],FT[MA+3],FT[MA+4],FT[MA+5],
FT[MA+6]);
"END";
"GO TO" NOOUT;
OUTPUT(61,"("4/,40B,"(" COORDINATES OF DISPLACED VERTEBRAE")"")");
"FOR" I:=1 "STEP" 1 "UNTIL" MM "DO"
"BEGIN"
OUTPUT(61,"("2/,10B,"("VERTEBRAL LEVEL")",3Z")",I);
OUTPUT(61,"("2/,10B,"("SECONDARY NODES BAR ELEMENTS")"")");
MB:=(I*3*NN)-3*NN;
"FOR" J:=1 "STEP" 3 "UNTIL" 3*NN-1 "DO"
OUTPUT(61,"("2/,10B,-2ZD.3D,10B,-2ZD.3D,10B,-2ZD.3D,2/,10B,-2ZD.3D,10B,
-2ZD.3D,10B,-2ZD.3D")",XR[MB+J],XR[MB+J+1],XR[MB+J+2],XR2[MB+J],
```

```
XR2[MB+J+1],XR2[MB+J+2]);
OUTPUT(61,"("2/,10B,"("SECONDARY NODES BEAM ELEMENTS")"")");
MA:=(I*3)-2;
OUTPUT(61,"("2/,10B,-2ZD.3D,10B,-2ZD.3D,10B,-2ZD.3D,2/,10B,-2ZD.3D,10B,
-2ZD.3D,10B,-2ZD.3D")",XRB[MA],XRB[MA+1],XRB[MA+2],XRB2[MA],XRB2[MA+1],
XRB2[MA+2]);
"END";
NOOUT;
"FOR" I:=1 "STEP" 1 "UNTIL" 6*MM "DO"
FZ[I]:=FZ[I]+FT[I];
"END" OF PROCEDURE SPINAL;
"INTEGER" MM,MA,MB,NN,I,J,FF,FI,P;
INPUT(60,"("N")",MM,NN,FF);
"BEGIN"
"REAL""ARRAY"X,Y,Z[1:2*MM*NN],XB,YB,ZB[1:2*MM],K[1:MM*NN],KB,FZ[1:6*MM],
      XP,YP,ZP,AP,BP,GP,XPD,YPD,ZPD,APD,BPD,GPD[1:MM+1],XPC,YPC,ZPC,
      APC,BPC,GPC[1:FF+1,1:MM+1],XR,XR2[1:MM*NN*3],XRB,XRB2[1:MM*3];
"PROCEDURE" XLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I;
"FOR" I:=1 "STEP" 1 "UNTIL" 2*MM*NN "DO"
ELT(X[I]);
"END" OF PROCEDURE XLIST;
"PROCEDURE" YLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I;
"FOR" I:=1 "STEP" 1 "UNTIL" 2*MM*NN "DO"
ELT(Y[I]);
"END" OF PROCEDURE YLIST;
"PROCEDURE" ZLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I;
"FOR" I:=1 "STEP" 1 "UNTIL" 2*MM*NN "DO"
ELT(Z[I]);
"END" OF PROCEDURE ZLIST;
"PROCEDURE" XBLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I;
"FOR" I:=1 "STEP" 1 "UNTIL" MM*2 "DO"
ELT(XB[I]);
"END" OF PROCEDURE XBLIST;
"PROCEDURE" YBLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I;
"FOR" I:=1 "STEP" 1 "UNTIL" MM*2 "DO"
ELT(YB[I]);
"END" OF PROCEDURE YBLIST;
"PROCEDURE" ZBLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I;
"FOR" I:=1 "STEP" 1 "UNTIL" MM*2 "DO"
ELT(ZB[I]);
"END" OF PROCEDURE ZBLIST;
```

```
"PROCEDURE" XPLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I,J;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" MM+1 "DO"
ELT(XPC[I,J]);
"END" OF PROCEDURE XPLIST;
"PROCEDURE" YPLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I,J;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" MM+1 "DO"
ELT(YPC[I,J]);
"END" OF PROCEDURE YPLIST;
"PROCEDURE" ZPLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I,J;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" MM+1 "DO"
ELT(ZPC[I,J]);
"END" OF PROCEDURE ZPLIST;
"PROCEDURE" APLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I,J;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" MM+1 "DO"
ELT(APC[I,J]);
"END" OF PROCEDURE APLIST;
"PROCEDURE" BPLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I,J;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" MM+1 "DO"
ELT(BPC[I,J]);
"END" OF PROCEDURE BPLIST;
"PROCEDURE" GPLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I,J;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" MM+1 "DO"
ELT(GPC[I,J]);
"END" OF PROCEDURE GPLIST;
P:=1;
FETCHLIST(63,P,XLIST);
P:=MM*NN*2+1;
FETCHLIST(63,P,YLIST);
P:=MM*NN*4+1;
FETCHLIST(63,P,ZLIST);
P:=MM*NN*6+1;
FETCHLIST(64,P,XBLIST);
P:=MM*NN*6+MM*2+1;
FETCHLIST(64,P,YBLIST);
P:=MM*NN*6+MM*4+1;
```

```
FETCHLIST(64,P,ZBLIST);
P:=MM*NN*6+MM*6+1;
FETCHLIST(65,P,XPLIST);
P:=MM*NN*6+MM*6+(MM+1)*(FF+1)*1+1;
FETCHLIST(65,P,YPLIST);
P:=MM*NN*6+MM*6+(MM+1)*(FF+1)*2+1;
FETCHLIST(65,P,ZPLIST);
P:=MM*NN*6+MM*6+(MM+1)*(FF+1)*3+1;
FETCHLIST(65,P,APLIST);
P:=MM*NN*6+MM*6+(MM+1)*(FF+1)*4+1;
FETCHLIST(65,P,BPLIST);
P:=MM*NN*6+MM*6+(MM+1)*(FF+1)*5+1;
FETCHLIST(65,P,GPLIST);
"GO TO" NOUT;
"FOR" I:=1 "STEP" 1 "UNTIL" MM "DO"
"BEGIN"
OUTPUT(61,"("2/,10B,"("VERTEBRAL LEVEL")",3Z")",I);
OUTPUT(61,"("2/,10B,"("SECONDARY NODES BAR ELEMENTS")"")");
MA:=(I*2*NN)-2*NN;
"FOR" J:=1 "STEP" 1 "UNTIL" 2*NN "DO"
OUTPUT(61,"("2/,10B,-2ZD.3D,10B,-2ZD.3D,10B,-2ZD.3D")",X[MA+J],Y[MA+J],
Z[MA+J]);
OUTPUT(61,"("2/,10B,"("SECONDARY NODES BEAM ELEMENTS")"")");
MB:=(I*2)-2;
"FOR" J:=1 "STEP" 1 "UNTIL" 2 "DO"
OUTPUT(61,"("2/,10B,-2ZD.3D,10B,-2ZD.3D,10B,-2ZD.3D")",XB[MB+J],
YB[MB+J],ZB[MB+J]);
"END";
"COMMENT"

      THE ANALYSIS IS CARRIED OUT IN FF NUMBER OF STEPS;

NOUT:
INARRAY(60,KB);
INARRAY(60,K);
"FOR" FI:=1 "STEP" 1 "UNTIL" FF "DO"
"BEGIN"
"FOR" I:=1 "STEP" 1 "UNTIL" MM+1 "DO"
"BEGIN"
XP[I]:=XPC[FI,I];
YP[I]:=YPC[FI,I];
ZP[I]:=ZPC[FI,I];
AP[I]:=APC[FI,I];
BP[I]:=BPC[FI,I];
GP[I]:=GPC[FI,I];
XPD[I]:=XPC[FI+1,I];
YPD[I]:=YPC[FI+1,I];
ZPD[I]:=ZPC[FI+1,I];
APD[I]:=APC[FI+1,I];
BPD[I]:=BPC[FI+1,I];
GPD[I]:=GPC[FI+1,I];
"END";
OUTPUT(61,"("2/,10B,"("PRIMARY NODES OF RIGID BODIES")"")");
"FOR" I:=1 "STEP" 1 "UNTIL" MM+1 "DO"
OUTPUT(61,"("2/,5B,3Z,2B,-2ZD.3D,10B,-2ZD.3D,10B,-2ZD.3D,10B,-2ZD.3D,
10B,-2ZD.3D,10B,-2ZD.3D")",I,XP[I],YP[I],ZP[I],AP[I],BP[I],GP[I]);
OUTPUT(61,"("2/,10B,"("DEFLECTED PRIMARY NODES")"")");
"FOR" I:=1 "STEP" 1 "UNTIL" MM+1 "DO"
OUTPUT(61,"("2/,5B,3Z,2B,-2ZD.3D,10B,-2ZD.3D,10B,-2ZD.3D,10B,-2ZD.3D,
```

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10B,-2ZD.3D,10B,-2ZD,3D")",I,XPD[I],YPD[I],ZPD[I],APD[I],BPD[I],GPD[I]);
SPINAL(MM,NN,X,Y,Z,XB,YB,ZB,XP,YP,ZP,AP,BP,GP,XPD,YPD,ZPD,APD,BPD,
                 GPD,XR,XR2,XRB,XRB2,K,KB,FZ);
"FOR" J:=1 "STEP" 1 "UNTIL" NN*MM "DO"
"BEGIN"
MA:=(J*2)-1;
MB:=(J*3)-2;
X[MA]:=XR[MB];
X[MA+1]:=XR2[MB];
Y[MA]:=XR[MB+1];
Y[MA+1]:=XR2[MB+1];
Z[MA]:=XR[MB+2];
Z[MA+1]:=XR2[MB+2];
"END";
"FOR" J:=1 "STEP" 1 "UNTIL" MM "DO"
"BEGIN"
MA:=(J*2)-1;
MB:=(J*3)-2;
XB[MA]:=XRB[MB];
XB[MA+1]:=XRB2[MB];
YB[MA]:=XRB[MB+1];
YB[MA+1]:=XRB2[MB+1];
ZB[MA]:=XRB[MB+2];
ZB[MA+1]:=XRB2[MB+2];
"END";
"END";
OUTARRAY(61,FZ);
"END";
"END";
```

```
CHANNEL,60=INPUT,P80
CHANNEL,61=OUTPUT,P136,PP60
CHANNEL,63=PLOP,W,L102
CHANNEL,64=PLOP,W,L34
CHANNEL,65=PLOP,W,L72
```

(c) The program MJJ2 is listed overleaf.

```
LGOL-60 VERSION 4,1 LEVEL 07 AT PSR 0433

"BEGIN"
"COMMENT"

        PROGRAM TO SET UP THE MATRICES TO BE USED IN THE LINEAR PROGRAM;

"PROCEDURE" ANGLE(CX,CY,CZ,CRO,SRO,ALPHA,BETA,GAMMA,PI);
"VALUE" ALPHA,BETA,GAMMA,PI;
"REAL" CX,CY,CZ,CRO,SRO,ALPHA,BETA,GAMMA,PI;
"BEGIN"
"REAL" AL;
"COMMENT"

        FIND COSINES OF ANGLES OF ROTATION;

AL:=SQRT((SIN(BETA)/COS(BETA))'2+1/(COS(GAMMA))'2);
CX:=1/AL;
CY:=ABS((SIN(GAMMA)/COS(GAMMA))/AL);
"IF" GAMMA<0 "THEN" CY:=-CY;
CZ:=ABS(SIN(BETA)/COS(BETA))/SQRT((1/COS(BETA))'2+(SIN(GAMMA)/
COS(GAMMA))'2);
"IF" BETA>0 "THEN" CZ:=-CZ;
"IF" ALPHA=PI/2 "THEN" CRO:=1 "ELSE"
CRO:=1/SQRT(((((SIN(ALPHA)/COS(ALPHA))'2)*(COS(BETA))'2)/
(COS(GAMMA))'2)+1);
"IF" CRO>1 "THEN" CRO:=1;
SRO:=SQRT(1-CRO'2);
"IF" ALPHA<0 "THEN" SRO:=-SRO;
"END" OF PROCEDURE ANGLE;
"PROCEDURE" TRAN(C,CX,CY,CZ,CRO,SRO);
"VALUE" CX,CY,CZ,CRO,SRO;
"REAL" CX,CY,CZ,CRO,SRO;
"REAL""ARRAY" C;
"BEGIN"
"REAL" CA;
"COMMENT"

        YZX COORDINATE TRANSFORMATION;

CA:=SQRT(CX'2+CZ'2);
C[1,1]:=CX;
C[1,2]:=CY;
C[1,3]:=CZ;
C[2,1]:=((-CX*CY*CRO)-(CZ*SRO))/CA;
C[2,2]:=CA*CRO;
C[2,3]:=((-CY*CZ*CRO)+(CX*SRO))/CA;
C[3,1]:=((CX*CY*SRO)-(CZ*CRO))/CA;
C[3,2]:=-CA*SRO;
C[3,3]:=((CY*CZ*SRO)+(CX*CRO))/CA;
"END" OF PROCEDURE TRAN;
"PROCEDURE" MUSCLE1(XT,YT,ZT,XS,YS,ZS,XP,YP,ZP,XA,YA,ZA,RXA,RYA,RZA,
        M,MZ,MD,N,RP,KL,XXX,YYY,ZZZ);
"VALUE" M,MD,N,RP;
"INTEGER" M,MZ,MD,N,RP;
"REAL" XXX,YYY,ZZZ;
"REAL""ARRAY" XT,YT,ZT,XS,YS,ZS,XP,YP,ZP,XA,YA,ZA,RXA,RYA,RZA,KL;
"COMMENT"

        TRANSVERSOSPINALIS MUSCLE GROUP AND MOMENTS FOR HEAD;

```

```
"BEGIN"
"INTEGER" I,J,K,MA,MB,MC;
"REAL" L,XBA,YBA,ZBA;
"FOR" I:=1 "STEP" 1 "UNTIL" M "DO"
"BEGIN"
MA:=(I-1)*2;
"FOR" J:=1 "STEP" 1 "UNTIL" 2 "DO"
"BEGIN"
"IF" I<N+1 "THEN"
"BEGIN"
MB:=I+5;
MC:=J+MA+(RP*2);
K:=MC+MD;
XBA:=XS[MB]-XP[I];
YBA:=YS[MB]-YP[I];
ZBA:=ZS[MB]-ZP[I];
L:=SQRT((XS[MB]-XT[MC])'2+(YS[MB]-YT[MC])'2+(ZS[MB]-ZT[MC])'2);
XA[I,K]:=(XT[MC]-XS[MB])/L;
YA[I,K]:=(YT[MC]-YS[MB])/L;
ZA[I,K]:=(ZT[MC]-ZS[MB])/L;
RXA[I,K]:=(YA[I,K]*(-ZBA))+ZA[I,K]*YBA;
RYA[I,K]:=(XA[I,K]*ZBA)+ZA[I,K]*(-XBA);
RZA[I,K]:=(XA[I,K]*(-YBA))+YA[I,K]*XBA;
MZ:=K;
"END";
MB:=I+5-RP;
MC:=J+MA;
K:=MC+MD;
L:=SQRT((XS[MB]-XT[MC])'2+(YS[MB]-YT[MC])'2+(ZS[MB]-ZT[MC])'2);
XBA:=XT[MC]-XP[I];
YBA:=YT[MC]-YP[I];
ZBA:=ZT[MC]-ZP[I];
XA[I,K]:=(XS[MB]-XT[MC])/L;
YA[I,K]:=(YS[MB]-YT[MC])/L;
ZA[I,K]:=(ZS[MB]-ZT[MC])/L;
RXA[I,K]:=(YA[I,K]*(-ZBA))+ZA[I,K]*YBA;
RYA[I,K]:=(XA[I,K]*ZBA)+ZA[I,K]*(-XBA);
RZA[I,K]:=(XA[I,K]*(-YBA))+YA[I,K]*XBA;
"IF" I<RP+1 "THEN"
"BEGIN"
XBA:=XT[MC]-XXX;
YBA:=YT[MC]-YYY;
ZBA:=ZT[MC]-ZZZ;
KL[1,K]:=(YA[I,K]*ZBA)-ZA[I,K]*YBA;
KL[2,K]:=(XA[I,K]*(-YBA))+YA[I,K]*XBA;
KL[3,K]:=-XA[I,K];
KL[4,K]:=-YA[I,K];
KL[5,K]:=-ZA[I,K];
"END";
"END";
"END";
"END" OF PROCEDURE MUSCLE1;
"PROCEDURE" MUSCLE2(XS,YS,ZS,XP,YP,ZP,XA,YA,ZA,RXA,RYA,RZA,AAP,ABP,
        AGP,MD,K);
"VALUE" MD;
"INTEGER" MD,K;
"REAL""ARRAY" XS,YS,ZS,XP,YP,ZP,XA,YA,ZA,RXA,RYA,RZA,AAP,ABP,AGP;
"COMMENT"
```

```
           SPINALIS THORACIS MUSCLE;

     "BEGIN"
     "REAL""ARRAY" C[1:3,1:3];
     "INTEGER" MA,MB,I,J;
     "REAL" XBA,YBA,ZBA,AA,BB,CC,CX,CY,CZ,CRO,SRO,PI;
     PI:=3.14159;
     "FOR" I:=1 "STEP" 1 "UNTIL" 4 "DO"
     "BEGIN"
     ANGLE(CX,CY,CZ,CRO,SRO,AAP[I],ABP[I],AGP[I],PI);
     TRAN(C,CX,CY,CZ,CRO,SRO);
     MA:=(I*2)-2;
     "FOR" J:=1 "STEP" 1 "UNTIL" 2 "DO"
     "BEGIN"
     K:=MD+J+MA;
     XBA:=XS[I+5]-XP[I];
     YBA:=YS[I+5]-YP[I];
     ZBA:=ZS[I+5]-ZP[I];
     AA:=0,342;
     "IF" J=2 "THEN" AA:=-AA;
     BB:=-0,93969;
     CC:=0;
     XA[I,K]:=AA*C[1,1]+BB*C[1,2]+CC*C[1,3];
     YA[I,K]:=AA*C[2,1]+BB*C[2,2]+CC*C[2,3];
     ZA[I,K]:=AA*C[3,1]+BB*C[3,2]+CC*C[3,3];
     RXA[I,K]:=(YA[I,K]*(-ZBA))+ZA[I,K]*YBA;
     RYA[I,K]:=(XA[I,K]*ZBA)+ZA[I,K]*(-XBA);
     RZA[I,K]:=(XA[I,K]*(-YBA))+YA[I,K]*XBA;
     "END";
     "END";
     MB:=K;
     "FOR" I:=1 "STEP" 1 "UNTIL" 4 "DO"
     "BEGIN"
     ANGLE(CX,CY,CZ,CRO,SRO,AAP[I+10],ABP[I+10],AGP[I+10],PI);
     TRAN(C,CX,CY,CZ,CRO,SRO);
     MA:=(I*2)-2;
     "FOR" J:=1 "STEP" 1 "UNTIL" 2 "DO"
     "BEGIN"
     K:=J+MA+MB;
     XBA:=XS[I+15]-XP[I+10];
     YBA:=YS[I+15]-YP[I+10];
     ZBA:=ZS[I+15]-ZP[I+10];
     AA:=0,342;
     "IF" J=2 "THEN" AA:=-AA;
     BB:=0.93969;
     CC:=0;
     XA[I+10,K]:=AA*C[1,1]+BB*C[1,2]+CC*C[1,3];
     YA[I+10,K]:=AA*C[2,1]+BB*C[2,2]+CC*C[2,3];
     ZA[I+10,K]:=AA*C[3,1]+BB*C[3,2]+CC*C[3,3];
     RXA[I+10,K]:=(YA[I+10,K]*(-ZBA))+ZA[I+10,K]*YBA;
     RYA[I+10,K]:=(XA[I+10,K]*ZBA)+ZA[I+10,K]*(-XBA);
     RZA[I+10,K]:=(XA[I+10,K]*(-YBA))+YA[I+10,K]*XBA;
     "END";
     "END";
     "END" OF PROCEDURE MUSCLE2;
     "PROCEDURE" MUSCLE3(XT,YT,ZT,XP,YP,ZP,XA,YA,ZA,RXA,RYA,RZA,MD,M,N,K);
     "VALUE" M,N,MD;
     "INTEGER" M,N,MD,K;
     "REAL""ARRAY" XT,YT,ZT,XP,YP,ZP,XA,YA,ZA,RXA,RYA,RZA;
```

```
"COMMENT"

        LONGISSIMUS THORACIS MUSCLE GROUP;

"BEGIN"
"INTEGER" I,J,MA,MC;
"REAL" L,XBA,YBA,ZBA;
"FOR" I:=1 "STEP" 1 "UNTIL" N "DO"
"BEGIN"
MA:=(I*2)-2;
"IF" I<N+1 "THEN"
"FOR" J:=1 "STEP" 1 "UNTIL" 2 "DO"
"BEGIN"
K:=MA+J+MD;
MC:=J+MA;
XBA:=XT[MC]-XP[I];
YBA:=YT[MC]-YP[I];
ZBA:=ZT[MC]-ZP[I];
L:=SQRT((XT[MC]-XT[MC+2])'2+(YT[MC]-YT[MC+2])'2+(ZT[MC]-ZT[MC+2])'2);
XA[I,K]:=(XT[MC+2]-XT[MC])/L;
YA[I,K]:=(YT[MC+2]-YT[MC])/L;
ZA[I,K]:=(ZT[MC+2]-ZT[MC])/L;
RXA[I,K]:=(YA[I,K]*(-ZBA))+ZA[I,K]*YBA;
RYA[I,K]:=(XA[I,K]*ZBA)+ZA[I,K]*(-XBA);
RZA[I,K]:=(XA[I,K]*(-YBA))+YA[I,K]*XBA;
"END";
"IF" I>N "THEN"
"FOR" J:=1 "STEP" 1 "UNTIL" 2 "DO"
"BEGIN"
K:=MA+J+MD;
MC:=J+MA;
XBA:=XT[MC]-XP[I];
YBA:=YT[MC]-YP[I];
ZBA:=ZT[MC]-ZP[I];
L:=SQRT((XT[MC]-XT[MC-2])'2+(YT[MC]-YT[MC-2])'2+(ZT[MC]-ZT[MC-2])'2);
XA[I,K]:=(XT[MC-2]-XT[MC])/L;
YA[I,K]:=(YT[MC-2]-YT[MC])/L;
ZA[I,K]:=(ZT[MC-2]-ZT[MC])/L;
RXA[I,K]:=(YA[I,K]*(-ZBA))+ZA[I,K]*YBA;
RYA[I,K]:=(XA[I,K]*ZBA)+ZA[I,K]*(-XBA);
RZA[I,K]:=(XA[I,K]*(-YBA))+YA[I,K]*XBA;
"END";
"END";
I:=12;
"FOR" J:=1 "STEP" 1 "UNTIL" 2 "DO"
"BEGIN"
MC:=J+22;
XBA:=XT[MC]-XP[I];
YBA:=YT[MC]-YP[I];
ZBA:=ZT[MC]-ZP[I];
L:=SQRT((XT[MC]-XT[MC+16])'2+(YT[MC]-YT[MC+16])'2+(ZT[MC]-ZT[MC+16])'
XA[I,K+J]:=(XT[MC+16]-XT[MC])/L;
YA[I,K+J]:=(YT[MC+16]-YT[MC])/L;
ZA[I,K+J]:=(ZT[MC+16]-ZT[MC])/L;
RXA[I,K+J]:=(YA[I,K+J]*(-ZBA))+ZA[I,K+J]*YBA;
RYA[I,K+J]:=(XA[I,K+J]*ZBA)+ZA[I,K+J]*(-XBA);
RZA[I,K+J]:=(XA[I,K+J]*(-YBA))+YA[I,K+J]*XBA;
"END";
"END" OF PROCEDURE MUSCLE3;
```

```
"PROCEDURE" MUSCLE4(XAR,YAR,ZAR,XS,YS,ZS,XA,YA,ZA,RXA,RYA,RZA,
        XT,YT,ZT,MD,ME,KK,XP,YP,ZP,XI,YI,ZI);
"VALUE" MD;
"INTEGER" MD,ME,KK;
"REAL""ARRAY" XAR,YAR,ZAR,XS,YS,ZS,XA,YA,ZA,RXA,RYA,RZA,
        XT,YT,ZT,XP,YP,ZP,XI,YI,ZI;
"COMMENT"

        ILIOCOSTALIS THORACIS AND LUMBORUM MUSCLE GROUP;

"BEGIN"
"INTEGER" MA,MB,MC,I,II,J,K;
"REAL" L,LC,XBA,YBA,ZBA;
"FOR" I:=1 "STEP" 1 "UNTIL" 6 "DO"
"BEGIN"
MA:=(I*2)-2;
"FOR" J:=1 "STEP" 1 "UNTIL" 2 "DO"
"BEGIN"
MC:=J+MA;
K:=J+MA+MD;
XBA:=XAR[MC]-XP[I];
YBA:=YAR[MC]-YP[I];
ZBA:=ZAR[MC]-ZP[I];
L:=SQRT((XAR[MC]-XAR[MC+6])'2+(YAR[MC]-YAR[MC+6])'2+(ZAR[MC]-ZAR[MC+6]
)'2);
XA[I,K]:=(XAR[MC+6]-XAR[MC])/L;
YA[I,K]:=(YAR[MC+6]-YAR[MC])/L;
ZA[I,K]:=(ZAR[MC+6]-ZAR[MC])/L;
RXA[I,K]:=(YA[I,K]*(-ZBA))+ZA[I,K]*YBA;
RYA[I,K]:=(XA[I,K]*ZBA)+ZA[I,K]*(-XBA);
RZA[I,K]:=(XA[I,K]*(-YBA))+YA[I,K]*XBA;
II:=I+6;
XBA:=XAR[MC+6]-XP[II];
YBA:=YAR[MC+6]-YP[II];
ZBA:=ZAR[MC+6]-ZP[II];
XA[II,K]:=-XA[I,K];
YA[II,K]:=-YA[I,K];
ZA[II,K]:=-ZA[I,K];
 RXA[II,K]:=(YA[II,K]*(-ZBA))+ZA[II,K]*YBA;
 RYA[II,K]:=(XA[II,K]*ZBA)+ZA[II,K]*(-XBA);
 RZA[II,K]:=(XA[II,K]*(-YBA))+YA[II,K]*XBA;
 "END";
 "END";
 ME:=K;
 "FOR" I:=7 "STEP" 1 "UNTIL" 12 "DO"
 "BEGIN"
 MA:=(I*2)-2;
 MB:=I+10;
 "FOR" J:=1 "STEP" 1 "UNTIL" 2 "DO"
 "BEGIN"
 K:=MA+J+MD;
 MC:=J+MA;
 XBA:=XAR[MC]-XP[I];
 YBA:=YAR[MC]-YP[I];
 ZBA:=ZAR[MC]-ZP[I];
 L:=SQRT((XAR[MC]-XS[MB])'2+(YAR[MC]-YS[MB])'2+(ZAR[MC]-ZS[MB])'2);
 XA[I,K]:=(XS[MB]-XAR[MC])/L;
 YA[I,K]:=(YS[MB]-YAR[MC])/L;
 ZA[I,K]:=(ZS[MB]-ZAR[MC])/L;
```

```
RXA[I,K]:=(YA[I,K]*(-ZBA))+ZA[I,K]*YBA;
RYA[I,K]:=(XA[I,K]*ZBA)+ZA[I,K]*(-XBA);
RZA[I,K]:=(XA[I,K]*(-YBA))+YA[I,K]*XBA;
"IF" I=12 "THEN"
"BEGIN"
LC:=SQRT((XAR[MC]-XI[J+10])'2+(YAR[MC]-YI[J+10])'2+(ZAR[MC]-
ZI[J+10])'2);
KK:=K+2;
XA[I,KK]:=(XI[J+10]-XAR[MC])/LC;
YA[I,KK]:=(YI[J+10]-YAR[MC])/LC;
ZA[I,KK]:=(ZI[J+10]-ZAR[MC])/LC;
RXA[I,KK]:=(YA[I,KK]*(-ZBA))+ZA[I,KK]*YBA;
RYA[I,KK]:=(XA[I,KK]*ZBA)+ZA[I,KK]*(-XBA);
RZA[I,KK]:=(XA[I,KK]*(-YBA))+YA[I,KK]*XBA;
"END";
II:=I+5;
XBA:=XS[II]-XP[II];
YBA:=YS[II]-YP[II];
ZBA:=ZS[II]-ZP[II];
XA[II,K]:=-XA[I,K];
YA[II,K]:=-YA[I,K];
ZA[II,K]:=-ZA[I,K];
RXA[II,K]:=(YA[II,K]*(-ZBA))+ZA[II,K]*YBA;
RYA[II,K]:=(XA[II,K]*ZBA)+ZA[II,K]*(-XBA);
RZA[II,K]:=(XA[II,K]*(-YBA))+YA[II,K]*XBA;
"END";
"END";
"END" OF PROCEDURE MUSCLE4;
"PROCEDURE" MUSCLES(XTT,YTT,ZTT,XI,YI,ZI,XP,YP,ZP,XA,YA,ZA,
        RXA,RYA,RZA,MD,K);
"VALUE" MD;
"INTEGER" MD,K;
"REAL""ARRAY" XTT,YTT,ZTT,XI,YI,ZI,XP,YP,ZP,XA,YA,ZA,RXA,RYA,RZA;
"COMMENT"

        QUADRATUS LUMBORUM MUSCLE;

"BEGIN"
"INTEGER" I,II,J,MA,MC;
"REAL" L,XBA,YBA,ZBA;
"FOR" I:=1 "STEP" 1 "UNTIL" 5 "DO"
"BEGIN"
MA:=(I*2)-2;
II:=I+11;
"FOR" J:=1 "STEP" 1 "UNTIL" 2 "DO"
"BEGIN"
MC:=J+MA;
K:=J+MA+MD;
XBA:=XTT[MC]-XP[II];
YBA:=YTT[MC]-YP[II];
ZBA:=ZTT[MC]-ZP[II];
L:=SQRT((XTT[MC]-XI[MC])'2+(YTT[MC]-YI[MC])'2+(ZTT[MC]-ZI[MC])'2);
XA[II,K]:=(XI[MC]-XTT[MC])/L;
YA[II,K]:=(YI[MC]-YTT[MC])/L;
ZA[II,K]:=(ZI[MC]-ZTT[MC])/L;
RXA[II,K]:=(YA[II,K]*(-ZBA))+ZA[II,K]*YBA;
RYA[II,K]:=(XA[II,K]*ZBA)+ZA[II,K]*(-XBA);
RZA[II,K]:=(XA[II,K]*(-YBA))+YA[II,K]*XBA;
"END";
```

```
"END";
"END" OF PROCEDURE MUSCLES5;
"INTEGER" I,II,J,JJ,K,KK,M,MM,MA,MD,ME,MZ,N,NN,P,RP,F,FA,FF;
"REAL" XXX,YYY,ZZZ,L,MC,XBA,YBA,ZBA;
INPUT(60,"("N")",MM,NN,FF,FA);
"BEGIN"
"REAL""ARRAY" XP,YP,ZP[1:MM],XS,YS,ZS[1:MM+5],XT,YT,ZT[1:2*(MM+3)]
       ,XTT,YTT,ZTT[1:10],XI,YI,ZI[1:12],XAR,YAR,ZAR[1:24],
       XA,YA,ZA,RXA,RYA,RZA[1:MM,1:385],KL[1:90,1:200],FT[1:6*MM],
       AAP,ABP,AGP,WA[1:MM],WT,ECX,ECZ[1:MM+1],RH[1:190],
        XAY,YAY,ZAY[1:2];
"PROCEDURE" XPLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I;
"FOR" I:=1 "STEP" 1 "UNTIL" MM "DO"
ELT(XP[I]);
"END" OF PROCEDURE XPLIST;
"PROCEDURE" YPLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I;
"FOR" I:=1 "STEP" 1 "UNTIL" MM "DO"
ELT(YP[I]);
"END" OF PROCEDURE YPLIST;
"PROCEDURE" ZPLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I;
"FOR" I:=1 "STEP" 1 "UNTIL" MM "DO"
ELT(ZP[I]);
"END" OF PROCEDURE ZPLIST;
"PROCEDURE" APLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I;
"FOR" I:=1 "STEP" 1 "UNTIL" MM "DO"
ELT(AAP[I]);
"END" OF PROCEDURE APLIST;
"PROCEDURE" BPLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I;
"FOR" I:=1 "STEP" 1 "UNTIL" MM "DO"
ELT(ABP[I]);
"END" OF PROCEDURE BPLIST;
"PROCEDURE" GPLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I;
"FOR" I:=1 "STEP" 1 "UNTIL" MM "DO"
ELT(AGP[I]);
"END" OF PROCEDURE GPLIST;
"PROCEDURE" XSLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" J;
"FOR" J:=1 "STEP" 1 "UNTIL" 22 "DO"
ELT(XS[J]);
```

```
"END" OF PROCEDURE XSLIST;
"PROCEDURE" YSLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" J;
"FOR" J:=1 "STEP" 1 "UNTIL" 22 "DO"
ELT(YS[J]);
"END" OF PROCEDURE YSLIST;
"PROCEDURE" ZSLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" J;
"FOR" J:=1 "STEP" 1 "UNTIL" 22 "DO"
ELT(ZS[J]);
"END" OF PROCEDURE ZSLIST;
"PROCEDURE" XTLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" J;
"FOR" J:=1 "STEP" 1 "UNTIL" 40 "DO"
ELT(XT[J]);
"END" OF PROCEDURE XTLIST;
"PROCEDURE" YTLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" J;
"FOR" J:=1 "STEP" 1 "UNTIL" 40 "DO"
ELT(YT[J]);
"END" OF PROCEDURE YTLIST;
"PROCEDURE" ZTLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" J;
"FOR" J:=1 "STEP" 1 "UNTIL" 40 "DO"
ELT(ZT[J]);
"END" OF PROCEDURE ZTLIST;
"PROCEDURE" XARLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" J;
"FOR" J:=1 "STEP" 1 "UNTIL" 24 "DO"
ELT(XAR[J]);
"END" OF PROCEDURE XARLIST;
"PROCEDURE" YARLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" J;
"FOR" J:=1 "STEP" 1 "UNTIL" 24 "DO"
ELT(YAR[J]);
"END" OF PROCEDURE YARLIST;
"PROCEDURE" ZARLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" J;
"FOR" J:=1 "STEP" 1 "UNTIL" 24 "DO"
ELT(ZAR[J]);
"END" OF PROCEDURE ZARLIST;
"PROCEDURE" XTTLIST(ELT);
"PROCEDURE" ELT;
```

```
"BEGIN"
"INTEGER" J;
"FOR" J:=1 "STEP" 1 "UNTIL" 10 "DO"
ELT(XTT[J]);
"END" OF PROCEDURE XTTLIST;
"PROCEDURE" YTTLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" J;
"FOR" J:=1 "STEP" 1 "UNTIL" 10 "DO"
ELT(YTT[J]);
"END" OF PROCEDURE YTTLIST;
"PROCEDURE" ZTTLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" J;
"FOR" J:=1 "STEP" 1 "UNTIL" 10 "DO"
 ELT(ZTT[J]);
 "END" OF PROCEDURE ZTTLIST;
 "PROCEDURE" EXLIST(ELT);
 "PROCEDURE" ELT;
 "BEGIN"
 "INTEGER" J;
 "FOR" J:=1 "STEP" 1 "UNTIL" MM+1 "DO"
 ELT(ECX[J]);
 "END" OF PROCEDURE EXLIST;
 "PROCEDURE" EZLIST(ELT);
 "PROCEDURE" ELT;
 "BEGIN"
 "INTEGER" J;
 "FOR" J:=1 "STEP" 1 "UNTIL" MM+1 "DO"
 ELT(ECZ[J]);
 "END" OF PROCEDURE EZLIST;
 "PROCEDURE" XXLIST(ELT);
 "PROCEDURE" ELT;
 "BEGIN"
 ELT(XXX);
 "END" OF PROCEDURE XXLIST;
 "PROCEDURE" YYLIST(ELT);
 "PROCEDURE" ELT;
 "BEGIN"
 ELT(YYY);
 "END" OF PROCEDURE YYLIST;
 "PROCEDURE" ZZLIST(ELT);
 "PROCEDURE" ELT;
 "BEGIN"
 ELT(ZZZ);
 "END" OF PROCEDURE ZZLIST;
 P:=(MM*6)*(NN+1)+(FA*(MM+1))+1;
 FETCHLIST(65,P,XPLIST);
 P:=(MM*6)*(NN+1)+(MM+1)*((FF+1)*1+FA)+1;
 FETCHLIST(65,P,YPLIST);
 P:=(MM*6)*(NN+1)+(MM+1)*((FF+1)*2+FA)+1;
 FETCHLIST(65,P,ZPLIST);
 P:=(MM*6)*(NN+1)+(MM+1)*((FF+1)*3+FA)+1;
 FETCHLIST(65,P,APLIST);
 P:=(MM*6)*(NN+1)+(MM+1)*((FF+1)*4+FA)+1;
 FETCHLIST(65,P,BPLIST);
 P:=(MM*6)*(NN+1)+(MM+1)*((FF+1)*5+FA)+1;
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```
FETCHLIST(65,P,GPLIST);
P:=MM*NN*6+MM*6+(MM+1)*(FF+1)*6+FA*22+1;
FETCHLIST(66,P,XSLIST);
P:=MM*NN*6+MM*6+(MM+1)*(FF+1)*6+(FF+1)*22*1+(FA*22)+1;
FETCHLIST(66,P,YSLIST);
P:=MM*NN*6+MM*6+(MM+1)*(FF+1)*6+(FF+1)*22*2+(FA*22)+1;
FETCHLIST(66,P,ZSLIST);
P:=MM*NN*6+MM*6+(MM+1)*(FF+1)*6+(FF+1)*22*3+(FA*40)+1;
FETCHLIST(67,P,XTLIST);
P:=MM*NN*6+MM*6+(MM+1)*(FF+1)*6+(FF+1)*(22*3+40*1)+(FA*40)+1;
FETCHLIST(67,P,YTLIST);
P:=MM*NN*6+MM*6+(MM+1)*(FF+1)*6+(FF+1)*(22*3+40*2)+(FA*40)+1;
FETCHLIST(67,P,ZTLIST);
P:=MM*NN*6+MM*6+(MM+1)*(FF+1)*6+(FF+1)*(22*3+40*3)+(FA*24)+1;
FETCHLIST(68,P,XARLIST);
P:=(MM*6)*(NN+1)+(MM+1)*(FF+1)*6+(FF+1)*(66+120+24*1)+(FA*24)+1;
FETCHLIST(68,P,YARLIST);
P:=(MM*6)*(NN+1)+(MM+1)*(FF+1)*6+(FF+1)*(66+120+24*2)+(FA*24)+1;
FETCHLIST(68,P,ZARLIST);
P:=(MM*6)*(NN+1)+(MM+1)*(FF+1)*6+(FF+1)*(66+120+24*3)+(FA*10)+1;
FETCHLIST(69,P,XTTLIST);
P:=(MM*6)*(NN+1)+(MM+1)*(FF+1)*6+(FF+1)*(66+120+72+10*1)+(FA*10)+1;
FETCHLIST(69,P,YTTLIST);
P:=(MM*6)*(NN+1)+(MM+1)*(FF+1)*6+(FF+1)*(66+120+72+10*2)+(FA*10)+1;
FETCHLIST(69,P,ZTTLIST);
P:=(MM*6)*(NN+1)+(FF+1)*((MM+1)*6+66+120+72+30)+(FA*(MM+1))+1;
FETCHLIST(70,P,EXLIST);
P:=(MM*6)*(NN+1)+(FF+1)*((MM+1)*7+66+120+72+30)+(FA*(MM+1))+1;
FETCHLIST(70,P,EZLIST);
P:=(MM*6)*(NN+1)+(FF+1)*((MM+1)*8+66+120+72+30)+1+(FA*1);
FETCHLIST(71,P,XXLIST);
P:=(MM*6)*(NN+1)+(FF+1)*((MM+1)*8+66+120+72+30+1)+1+(FA*1);
FETCHLIST(71,P,YYLIST);
P:=(MM*6)*(NN+1)+(FF+1)*((MM+1)*8+66+120+72+30+2)+1+(FA*1);
FETCHLIST(71,P,ZZLIST);
"FOR" I:=1 "STEP" 1 "UNTIL" MM "DO"
AGP[I]:=AGP[I]-1.571;
INPUT(60,"("N")",XAY[1],XAY[2],YAY[1],YAY[2],ZAY[1],ZAY[2]);
INARRAY(60,XI);
INARRAY(60,YI);
INARRAY(60,ZI);
INARRAY(50,WT);
INARRAY(60,WA);
"FOR" I:=1 "STEP" 1 "UNTIL" MM "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 385 "DO"
XA[I,J]:=YA[I,J]:=ZA[I,J]:=RXA[I,J]:=RYA[I,J]:=RZA[I,J]:=0;
MD:=0;
RP:=3;
M:=N:=MM;
MUSCLE1(XT,YT,ZT,XS,YS,ZS,XP,YP,ZP,XA,YA,ZA,RXA,RYA,RZA,M,MZ,MD,N,RP,
      KL,XXX,YYY,ZZZ);
MD:=MD+1;
OUTPUT(61,"("2/,15B,"("MULTIFIDUS MUSCLE  FROM    ")",4Z,
"("   TO   ")",4Z")",MD,MZ);
MD:=MZ;
RP:=5;
M:=12;
N:=7;
MUSCLE1(XT,YT,ZT,XS,YS,ZS,XP,YP,ZP,XA,YA,ZA,RXA,RYA,RZA,M,MZ,MD,N,RP,
```

```
    KL,XXX,YYY,ZZZ);
MD:=MD+1;
OUTPUT(61,"("2/,15B,"("SEMI SPINALIS MUSCLE THORACIS AND CERVICIS F
")",4Z,"("   TO   ")",4Z")",MD,MZ);
MD:=MZ+1;
KL[1,MD]:=100;
KL[3,MD]:=(SIN(AGP[1]));
KL[4,MD]:=-ABS(COS(AGP[1]));
MUSCLE2(XS,YS,ZS,XP,YP,ZP,XA,YA,ZA,RXA,RYA,RZA,AAP,ABP,AGP,MD,K);
"FOR" J:=1 "STEP" 2 "UNTIL" 7 "DO"
"BEGIN"
KL[6,MD+J]:=1;
KL[6,MD+J+8]:=-1;
KL[7,MD+J+1]:=1;
KL[7,MD+J+9]:=-1;
"END";
MD:=MD+1;
OUTPUT(61,"("2/,15B,"("SPINALIS THORACIS MUSCLE   FROM  ")",4Z,"("   T
   ")",4Z")",MD,K);
MD:=K;
M:=MM;
N:=12;
MUSCLE3(XT,YT,ZT,XP,YP,ZP,XA,YA,ZA,RXA,RYA,RZA,MD,M,N,K);
"FOR" J:=1 "STEP" 2 "UNTIL" 24 "DO"
"BEGIN"
KL[8,MD+J]:=1;
KL[9,MD+J+1]:=1;
"IF" J>11 "THEN"
"BEGIN"
KL[8,MD+J+12]:=-1;
KL[9,MD+J+13]:=-1;
"END";
"END";
KL[10,MD+33]:=-0.9;
KL[10,MD+35]:=1;
KL[11,MD+33]:=1.1;
KL[11,MD+35]:=-1;
KL[12,MD+34]:=-0.9;
KL[12,MD+36]:=1;
KL[13,MD+34]:=1.1;
KL[13,MD+36]:=-1;
OUTPUT(61,"("2/,15B,"("LONGISSIMUS THORACIS  FROM   ")",4Z,
"("   TO   ")",4Z")",MD+1,K+2);
MD:=K+2;
MUSCLE4(XAR,YAR,ZAR,XS,YS,ZS,XA,YA,ZA,RXA,RYA,RZA,XT,YT,ZT,MD,ME,KK,
      XP,YP,ZP,XI,YI,ZI);
"FOR" I:=1 "STEP" 4 "UNTIL" 21 "DO"
"BEGIN"
"FOR" JJ:=1 "STEP" 2 "UNTIL" 11 "DO"
"BEGIN"
F:=JJ+MD;
KL[I+13,F]:=-0.15;
KL[I+14,F]:=0.183;
KL[I+15,F+1]:=-0.15;
KL[I+16,F+1]:=0.183;
"END";
II:=MD+1+((I-1)/2);
KL[I+13,II]:=0.85;
KL[I+14,II]:=-0.816;
```

```
KL[I+15,II+1]:=0.85;
KL[I+16,II+1]:=-0.816;
"END";
"FOR" I:=1 "STEP" 4 "UNTIL" 25 "DO"
"BEGIN"
"FOR" JJ:=1 "STEP" 2 "UNTIL" 13 "DO"
"BEGIN"
F:=JJ+MD+12;
KL[I+37,F]:=-0.1286;
KL[I+38,F]:=0.157;
KL[I+39,F+1]:=-0.1286;
KL[I+40,F+1]:=0.157;
"END";
II:=MD+13+((I-1)/2);
KL[I+37,II]:=0.8714;
KL[I+38,II]:=-0.843;
KL[I+39,II+1]:=0.8714;
KL[I+40,II+1]:=-0.843;
"END";
OUTPUT(61,"("2/,15B,"("ILIOCOSTALIS THORACIS  FROM   ")",4Z,
"("   TO   ")",4Z")",MD+1,ME);
ME:=ME+1;
OUTPUT(61,"("2/,15B,"("ILIOCOSTALIS LUMBORUM  FROM   ")",4Z,
"("   TO   ")",4Z")",ME,KK);
MD:=KK;
MUSCLES5(XTT,YTT,ZTT,XI,YI,ZI,XP,YP,ZP,XA,YA,ZA,RXA,RYA,RZA,MD,K);
MD:=MD+1;
OUTPUT(61,"("2/,15B,"("QUADRATUS LUMBORUM  FROM   ")",4Z,"("   TO   ":
4Z")",MD,K);
"FOR" J:=1 "STEP" 1 "UNTIL" 9 "DO"
"BEGIN"
"FOR" I:=1 "STEP" 1 "UNTIL" 2 "DO"
"BEGIN"
L:=SQRT((XAY[I]-110)'2+(YAY[I]-70)'2+(ZAY[I]-133)'2);
"IF" I=2 "THEN""BEGIN"
L:=SQRT((XAY[I]+110)'2+(YAY[I]-70)'2+(ZAY[I]-133)'2);
"END";
MC:=(J*2)-2+I+153;
XBA:=XAY[I]-XP[J+1];
YBA:=YAY[I]-YP[J+1];
ZBA:=ZAY[I]-ZP[J+1];
XA[J+1,MC]:=(110-XAY[I])/L;
"IF" I=2 "THEN" XA[J+1,MC]:=(-110-XAY[I])/L;
YA[J+1,MC]:=(70-YAY[I])/L;
ZA[J+1,MC]:=(133-ZAY[I])/L;
RXA[J+1,MC]:=(YA[J+1,MC]*(-ZBA))+ZA[J+1,MC]*YBA;
RYA[J+1,MC]:=(XA[J+1,MC]*ZBA)+ZA[J+1,MC]*(-XBA);
RZA[J+1,MC]:=(XA[J+1,MC]*(-YBA))+YA[J+1,MC]*XBA;
"END";
"END";
J:=154;
KL[66,J]:=KL[67,J+1]:=KL[68,J+2]:=KL[69,J+3]:=KL[70,J+4]:=KL[71,J+5]
:=KL[72,J+6]:=KL[73,J+7]:=KL[74,J+8]:=KL[75,J+9]:=KL[76,J+10]
:=KL[77,J+11]:=KL[78,J+12]:=KL[79,J+13]:=KL[80,J+14]:=KL[81,J+15]:=1;
KL[66,J+2]:=KL[67,J+3]:=KL[68,J+4]:=KL[69,J+5]:=KL[70,J+6]:=KL[71,J+7
:=KL[72,J+8]:=KL[73,J+9]:=KL[74,J+10]:=KL[75,J+11]:=KL[76,J+12]
:=KL[77,J+13]:=KL[78,J+14]:=KL[79,J+15]:=KL[80,J+16]:=KL[81,J+17]:=-1
OUTPUT(61,"("2/,15B,"("INTERNAL OBLIQUE  FROM     154   TO   171")"")
"COMMENT"
```

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            ESTABLISH REACTION COEFFICIENTS;

     MD:=172;
     XA[1,MD]:=YA[1,MD+2]:=ZA[1,MD+4]:=KL[3,MD+1]:=
     KL[4,MD+3]:=KL[5,MD+5]:=-1;
     XA[1,MD+1]:=YA[1,MD+3]:=ZA[1,MD+5]:=KL[3,MD]:=KL[4,MD+2]:=KL[5,MD+4]:=
     MD:=177;
     "FOR" I:=1 "STEP" 1 "UNTIL" MM "DO"
     "BEGIN"
     K:=(I*6)-5+MD;
     MC:=(I*6)-5+MD+102;
     P:=1;
     "FOR" J:=0 "STEP" 1 "UNTIL" 1 "DO"
     "BEGIN"
     "IF" J=1 "THEN" P:=-P;
     XA[I+J,K]:=YA[I+J,K+2]:=ZA[I+J,K+4]:=P;
     XA[I+J,K+1]:=YA[I+J,K+3]:=ZA[I+J,K+5]:=-P;
     RXA[I,MC]:=RYA[I,MC+2]:=RZA[I,MC+4]:=15;
     RXA[I,MC+1]:=RYA[I,MC+3]:=RZA[I,MC+5]:=-15;
     "IF" I=MM "THEN""GO TO" POOP;
     "END";
     "END";
     "COMMENT"

            FORM RHS OF SYSTEM OF EQUATIONS;

     POOP;
     MC:=0;
     "FOR" J:=1 "STEP" 1 "UNTIL" MM "DO"
     "BEGIN"
     YA[J,385]:=WT[J+1]+WA[J];
     RXA[J,385]:=0;
     RZA[J,385]:=0;
     RXA[J,385]:=-WT[1]*(ECZ[1]+ZP[1]-ZP[J]);
     RZA[J,385]:=WT[1]*(ECX[1]+XP[1]-XP[J]);
     "IF" J>1 "THEN"
     "FOR" I:=2 "STEP" 1 "UNTIL" J "DO"
     "BEGIN"
     RXA[J,385]:=RXA[J,385]-(WT[I]*(ECZ[I]+ZP[I-1]-ZP[J]))-(WA[I-1]*
     (ZP[I-1]-ZP[J]));
     RZA[J,385]:=RZA[J,385]+(WT[I]*(ECX[I]+XP[I-1]-XP[J]))+(WA[I-1]*
     (XP[I-1]-XP[J]));
     "END";
     "IF" RXA[J,385]>0 "THEN"
     "BEGIN"
     "IF" MC<RXA[J,385] "THEN"
     "BEGIN"
     MC:=RXA[J,385];
     II:=J;
     "END";
     RXA[J,385]:=0;
     KL[82,230]:=MC;
     "END";
     RXA[J,385]:=RXA[J,385]-(WT[J+1]*ECZ[J+1]);
     RZA[J,385]:=RZA[J,385]+(WT[J+1]*ECX[J+1]);
     "END";
     "FOR" J:=1 "STEP" 1 "UNTIL" 9 "DO"
     "FOR" I:=1 "STEP" 1 "UNTIL" 2 "DO"
```

```
"BEGIN"
MC:=(J*2)-2+I+153;
YBA:=YAY[I]-YP[II];
ZBA:=ZAY[I]-ZP[II];
KL[82,MC]:=(YA[J+1,MC]*(-ZBA))+ZA[J+1,MC]*YBA;
"END";
KL[1,200]:=-WT[I]*(ECZ[I]+ZP[I]-ZZZ);
KL[2,200]:=-WT[I]*(ECX[I]+XP[I]-XXX);
KL[4,200]:=WT[I];
"COMMENT"

      CONDENSED OUTPUT TO LINE PRINTER;

"GO TO" NOOUT;
"FOR" J:=1 "STEP" 5 "UNTIL" 385 "DO"
"BEGIN"
OUTPUT(61,"("2/")");
OUTPUT(61,"("26B,3Z,16B,3Z,16B,3Z,16B,3Z,16B,3Z")",J,J+1,J+2,J+3,J+4);
"FOR" I:=1 "STEP" 1 "UNTIL" MM "DO"
"BEGIN"
"FOR" JJ:=0 "STEP" 1 "UNTIL" 5 "DO"
"IF" XA[I,J+JJ]"NOT EQUAL"0 "THEN""GO TO" CONTINUE1;
"GO TO" PASS1;
CONTINUE1:
OUTPUT(61,"("2/")");
OUTPUT(61,"("5B,"("X    ")",3Z,10B,-2ZD.5D,10B,-2ZD.5D,10B,-2ZD.5D,10B,
-2ZD,5D,10B,-2ZD.5D")",I,XA[I,J],XA[I,J+1],XA[I,J+2],XA[I,J+3],XA[I,J+
));
PASS1:
"FOR" JJ:=0 "STEP" 1 "UNTIL" 5 "DO"
"IF" YA[I,J+JJ]"NOTEQUAL"0 "THEN" "GO TO" CONTINUE2;
"GO TO" PASS2;
CONTINUE2:
OUTPUT(61,"("2/")");
OUTPUT(61,"("5B,"("Y    ")",3Z,10B,-2ZD.5D,10B,-2ZD.5D,10B,-2ZD.5D,10B,
-2ZD.5D,10B,-2ZD.5D")",I,YA[I,J],YA[I,J+1],YA[I,J+2],YA[I,J+3],YA[I,J+
));
PASS2:
"FOR" JJ:=0 "STEP" 1 "UNTIL" 5 "DO"
"IF" ZA[I,J+JJ]"NOT EQUAL"0 "THEN""GO TO" CONTINUE3;
"GO TO" PASS3;
CONTINUE3:
OUTPUT(61,"("2/")");
OUTPUT(61,"("5B,"("Z    ")",3Z,10B,-2ZD.5D,10B,-2ZD.5D,10B,-2ZD.5D,10B,
-2ZD,5D,10B,-2ZD.5D")",I,ZA[I,J],ZA[I,J+1],ZA[I,J+2],ZA[I,J+3],ZA[I,J+
));
PASS3:
"FOR" JJ:=0 "STEP" 1 "UNTIL" 5 "DO"
"IF" RXA[I,J+JJ]"NOT EQUAL"0 "THEN""GO TO" CONTINUE4;
"GO TO" PASS4;
CONTINUE4:
OUTPUT(61,"("2/")");
OUTPUT(61,"("5B,"("RX   ")",3Z,10B,-2ZD.5D,10B,-2ZD.5D,10B,-2ZD.5D,10B,
-2ZD.5D,10B,-2ZD.5D")",I,RXA[I,J],RXA[I,J+1],RXA[I,J+2],RXA[I,J+3],
RXA[I,J+4]);
PASS4:
"FOR" JJ:=0 "STEP" 1 "UNTIL" 5 "DO"
"IF" RYA[I,J+JJ]"NOT EQUAL"0 "THEN""GO TO" CONTINUE5;
"GO TO" PASS5;
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CONTINUE5:
OUTPUT(61,"("2/")");
OUTPUT(61,"("5B,"("RY  ")",3Z,10B,-2ZD.5D,10B,-2ZD.5D,10B,-2ZD.5D,10B,
-2ZD.5D,10B,-2ZD.5D")",I,RYA[I,J],RYA[I,J+1],RYA[I,J+2],RYA[I,J+3],
RYA[I,J+4]);
PASS5:
"FOR" JJ:=0 "STEP" 1 "UNTIL" 5 "DO"
"IF" RZA[I,J+JJ]"NOT EQUAL"0 "THEN""GO TO" CONTINUE6;
"GO TO" PASS6;
CONTINUE6:
OUTPUT(61,"("2/")");
OUTPUT(61,"("5B,"("RZ  ")",3Z,10B,-2ZD.5D,10B,-2ZD.5D,10B,-2ZD.5D,10B,
-2ZD.5D,10B,-2ZD.5D")",I,RZA[I,J],RZA[I,J+1],RZA[I,J+2],RZA[I,J+3],
RZA[I,J+4]);
PASS6:
"END";
"IF" J<200 "THEN"
"FOR" I:=1 "STEP" 1 "UNTIL" 90 "DO"
"BEGIN"
"FOR" JJ:=0 "STEP" 1 "UNTIL" 5 "DO"
"IF" KL[I,J+JJ]"NOT EQUAL"0 "THEN""GO TO" CONTINUE7;
"GO TO" PASS7;
CONTINUE7:
OUTPUT(61,"("2/")");
OUTPUT(61,"("5B,"("KL  ")",3Z,10B,-2ZD.5D,10B,-2ZD.5D,10B,-2ZD.5D,10B,
-2ZD.5D,10B,-2ZD.5D")",I,KL[I,J],KL[I,J+1],KL[I,J+2],KL[I,J+3],KL[I,J+
);
PASS7:
"END";
OUTPUT(61,"("2/")");
OUTPUT(61,"("30B,"("**************************************************
*********")"")");

"END";
"COMMENT"

      OUTPUT TO DISC FILE ON 6600  FOR INPUT TO L-P PACKAGE;

NOOUT:
OUTPUT(62,"("/,"("NAME")",10B,"("SPINAL")"")");
OUTPUT(62,"("/,"("ROWS")"")");
OUTPUT(62,"("/,B,"("N")",2B,"("OBJ")"")");
"FOR" I:=1 "STEP" 1 "UNTIL" 17 "DO"
"BEGIN"
OUTPUT(62,"("/,B,"("E")",2B,"("X")",DD")",I);
OUTPUT(62,"("/,B,"("E")",2B,"("Y")",DD")",I);
OUTPUT(62,"("/,B,"("E")",2B,"("Z")",DD")",I);
OUTPUT(62,"("/,B,"("E")",2B,"("RX")",DD")",I);
OUTPUT(62,"("/,B,"("E")",2B,"("RY")",DD")",I);
OUTPUT(62,"("/,B,"("E")",2B,"("RZ")",DD")",I);
"END";
"FOR" I:=1 "STEP" 1 "UNTIL" 9 "DO"
OUTPUT(62,"("/,B,"("E")",2B,"("KL")",DD")",I);
"FOR" I:=10 "STEP" 1 "UNTIL" 65 "DO"
OUTPUT(62,"("/,B,"("G")",2B,"("KL")",DD")",I);
"FOR" I:=66 "STEP" 1 "UNTIL" 82 "DO"
OUTPUT(62,"("/,B,"("E")",2B,"("KL")",DD")",I);
OUTPUT(62,"("/,"("COLUMNS")"")");
"FOR" J:=1 "STEP" 1 "UNTIL" 381 "DO"
```

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"BEGIN"
MC:=1;
OUTPUT(62,"("/,4B,DDD,7B,"("OBJ")",7B,ZD.D")",J,MC);
"FOR" I:=1 "STEP" 1 "UNTIL" 17 "DO"
"BEGIN"
"IF" XA[I,J]"NOT EQUAL" 0 "THEN"
OUTPUT(62,"("/,4B,DDD,7B,"("X")",DD,7B,-2ZD.5D")",J,I,XA[I,J]);
"IF" YA[I,J]"NOT EQUAL" 0 "THEN"
OUTPUT(62,"("/,4B,DDD,7B,"("Y")",DD,7B,-2ZD.5D")",J,I,YA[I,J]);
"IF" ZA[I,J]"NOT EQUAL" 0 "THEN"
OUTPUT(62,"("/,4B,DDD,7B,"("Z")",DD,7B,-2ZD.5D")",J,I,ZA[I,J]);
"IF" RXA[I,J]"NOT EQUAL" 0 "THEN"
OUTPUT(62,"("/,4B,DDD,7B,"("RX")",DD,6B,-2ZD.5D")",J,I,RXA[I,J]);
"IF" RYA[I,J]"NOT EQUAL" 0 "THEN"
OUTPUT(62,"("/,4B,DDD,7B,"("RY")",DD,6B,-2ZD.5D")",J,I,RYA[I,J]);
"IF" RZA[I,J]"NOT EQUAL" 0 "THEN"
OUTPUT(62,"("/,4B,DDD,7B,"("RZ")",DD,6B,-2ZD.5D")",J,I,RZA[I,J]);
"END";
"IF" J<200 "THEN"
"FOR" I:=1 "STEP" 1 "UNTIL" 82 "DO"
"IF" KL[I,J]"NOT EQUAL" 0 "THEN"
OUTPUT(62,"("/,4B,DDD,7B,"("KL")",DD,6B,-2ZD.5D")",J,I,KL[I,J]);
"END";
OUTPUT(62,"("/,"("RHS")"")");
J:=385;
"FOR" I:=1 "STEP" 1 "UNTIL" 17 "DO"
"BEGIN"
"IF" XA[I,J]"NOT EQUAL" 0 "THEN"
OUTPUT(62,"("/,4B,DDD,7B,"("X")",DD,7B,-5ZD.3D")",J,I,XA[I,J]);
"IF" YA[I,J]"NOT EQUAL" 0 "THEN"
OUTPUT(62,"("/,4B,DDD,7B,"("Y")",DD,7B,-5ZD.3D")",J,I,YA[I,J]);
"IF" ZA[I,J]"NOT EQUAL" 0 "THEN"
OUTPUT(62,"("/,4B,DDD,7B,"("Z")",DD,7B,-5ZD.3D")",J,I,ZA[I,J]);
"IF" RXA[I,J]"NOT EQUAL" 0 "THEN"
OUTPUT(62,"("/,4B,DDD,7B,"("RX")",DD,6B,-5ZD.3D")",J,I,RXA[I,J]);
"IF" RYA[I,J]"NOT EQUAL" 0 "THEN"
OUTPUT(62,"("/,4B,DDD,7B,"("RY")",DD,6B,-5ZD.3D")",J,I,RYA[I,J]);
"IF" RZA[I,J]"NOT EQUAL" 0 "THEN"
OUTPUT(62,"("/,4B,DDD,7B,"("RZ")",DD,6B,-5ZD.3D")",J,I,RZA[I,J]);
"END";
J:=200;
"FOR" I:=1 "STEP" 1 "UNTIL" 82 "DO"
"IF" KL[I,J]"NOT EQUAL" 0 "THEN"
OUTPUT(62,"("/,4B,"("385")",7B,"("KL")",DD,6B,-5ZD.3D")",I,KL[I,J]);
OUTPUT(62,"("/,"("ENDATA")"")");
"END";
"END";
```

```
CHANNEL,60=INPUT,P80
CHANNEL,61=OUTPUT,P136,PP60
CHANNEL,62=SPIDA1,P80
CHANNEL,65=PLOP,W,L17
CHANNEL,66=PLOP,W,L22
CHANNEL,67=PLOP,W,L40
CHANNEL,68=PLOP,W,L24
CHANNEL,69=PLOP,W,L10
CHANNEL,70=PLOP,W,L18
CHANNEL,71=PLOP,W,L1
```

(d) The program MJJ3 is listed overleaf.

```
"BEGIN"
"REAL""ARRAY" XSE,YSE,ZSE[1:5],XTE,YTE,ZTE[1:15],XS,YS,ZS[1:22],
      XT,YT,ZT[1:40],XTT,YTT,ZTT[1:10],XAR,YAR,ZAR[1:24],C[1:3,1:3];
"REAL" ALPHA,BETA,GAMMA,PI,CRO,SRO,CX,CY,CZ,ECY;
"INTEGER" MM,NN,FF,I,J,F,MA,MB,P;
INPUT(60,"("N")",MM,NN,FF);
"BEGIN"
"REAL""ARRAY" XSC,YSC,ZSC[1:FF+1,1:22],XTC,YTC,ZTC[1:FF+1,1:40],
      XARC,YARC,ZARC[1:FF+1,1:24],XTTC,YTTC,ZTTC[1:FF+1,1:10],
      X,Y,Z[1:MM*NN*2],XB,YB,ZB[1:MM*2],XPC,YPC,ZPC,APC,BPC,
      GPC,ECX,ECZ[1:FF+1,1:MM+1],ECC[1:MM+1],XXX,YYY,ZZZ[1:FF+1,1:1];
"PROCEDURE" ANGLE(CX,CY,CZ,CRO,SRO,ALPHA,BETA,GAMMA,PI);
"VALUE" ALPHA,BETA,GAMMA,PI;
"REAL" CX,CY,CZ,CRO,SRO,ALPHA,BETA,GAMMA,PI;
"BEGIN"
"REAL" AL;
"COMMENT"

      FIND COSINES OF ANGLES OF ROTATION;

AL:=SQRT((SIN(BETA)/COS(BETA))'2+1/(COS(GAMMA))'2);
CX:=1/AL;
CY:=ABS((SIN(GAMMA)/COS(GAMMA))/AL);
"IF" GAMMA<0 "THEN" CY:=-CY;
CZ:=ABS(SIN(BETA)/COS(BETA))/SQRT((1/COS(BETA))'2+(SIN(GAMMA)/
COS(GAMMA))'2);
"IF" BETA>0 "THEN" CZ:=-CZ;
"IF" ALPHA=PI/2 "THEN" CRO:=1 "ELSE"
CRO:=1/SQRT((((SIN(ALPHA)/COS(ALPHA))'2)*(COS(BETA))'2)/
(COS(GAMMA))'2)+1);
"IF" CRO>1 "THEN" CRO:=1;
SRO:=SQRT(1-CRO'2);
"IF" ALPHA<0 "THEN" SRO:=-SRO;
"END" OF PROCEDURE ANGLE;
"PROCEDURE" TRAN(C,CX,CY,CZ,CRO,SRO);
"VALUE" CX,CY,CZ,CRO,SRO;
"REAL" CX,CY,CZ,CRO,SRO;
"REAL""ARRAY" C;
"BEGIN"
"REAL" CA;
"COMMENT"

      YZX COORDINATE TRANSFORMATION;

CA:=SQRT(CX'2+CZ'2);
C[1,1]:=CX;
C[1,2]:=CY;
C[1,3]:=CZ;
C[2,1]:=((-CX*CY*CRO)-(CZ*SRO))/CA;
C[2,2]:=CA*CRO;
C[2,3]:=((-CY*CZ*CRO)+(CX*SRO))/CA;
C[3,1]:=((CX*CY*SRO)-(CZ*CRO))/CA;
C[3,2]:=-CA*SRO;
C[3,3]:=((CY*CZ*SRO)+(CX*CRO))/CA;
"END" OF PROCEDURE TRAN;
"PROCEDURE" XLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I;
```

```
"FOR" I:=1 "STEP" 1 "UNTIL" 2*MM*NN "DO"
ELT(X[I]);
"END" OF PROCEDURE XLIST;
"PROCEDURE" YLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I;
"FOR" I:=1 "STEP" 1 "UNTIL" 2*MM*NN "DO"
ELT(Y[I]);
"END" OF PROCEDURE YLIST;
"PROCEDURE" ZLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I;
"FOR" I:=1 "STEP" 1 "UNTIL" 2*MM*NN "DO"
ELT(Z[I]);
"END" OF PROCEDURE ZLIST;
"PROCEDURE" XBLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I;
"FOR" I:=1 "STEP" 1 "UNTIL" MM*2 "DO"
ELT(XB[I]);
"END" OF PROCEDURE XBLIST;
"PROCEDURE" YBLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I;
"FOR" I:=1 "STEP" 1 "UNTIL" MM*2 "DO"
ELT(YB[I]);
"END" OF PROCEDURE YBLIST;
"PROCEDURE" ZBLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I;
"FOR" I:=1 "STEP" 1 "UNTIL" MM*2 "DO"
ELT(ZB[I]);
"END" OF PROCEDURE ZBLIST;
"PROCEDURE" XPLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I,J;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" MM+1 "DO"
ELT(XPC[I,J]);
"END" OF PROCEDURE XPLIST;
"PROCEDURE" YPLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I,J;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" MM+1 "DO"
ELT(YPC[I,J]);
"END" OF PROCEDURE YPLIST;
"PROCEDURE" ZPLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I,J;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
```

```
"FOR" J:=1 "STEP" 1 "UNTIL" MM+1 "DO"
ELT(ZPC[I,J]);
"END" OF PROCEDURE ZPLIST;
"PROCEDURE" APLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I,J;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" MM+1 "DO"
ELT(APC[I,J]);
"END" OF PROCEDURE APLIST;
"PROCEDURE" BPLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I,J;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" MM+1 "DO"
ELT(BPC[I,J]);
"END" OF PROCEDURE BPLIST;
"PROCEDURE" GPLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I,J;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" MM+1 "DO"
ELT(GPC[I,J]);
"END" OF PROCEDURE GPLIST;
"PROCEDURE" XSLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I,J;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 22 "DO"
ELT(XSC[I,J]);
"END" OF PROCEDURE XSLIST;
"PROCEDURE" YSLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I,J;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 22 "DO"
ELT(YSC[I,J]);
"END" OF PROCEDURE YSLIST;
"PROCEDURE" ZSLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I,J;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 22 "DO"
ELT(ZSC[I,J]);
"END" OF PROCEDURE ZSLIST;
"PROCEDURE" XTLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I,J;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 40 "DO"
ELT(XTC[I,J]);
"END" OF PROCEDURE XTLIST;
```

```
"PROCEDURE" YTLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I,J;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 40 "DO"
ELT(YTC[I,J]);
"END" OF PROCEDURE YTLIST;
"PROCEDURE" ZTLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I,J;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 40 "DO"
ELT(ZTC[I,J]);
"END" OF PROCEDURE ZTLIST;
"PROCEDURE" XARLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I,J;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 24 "DO"
ELT(XARC[I,J]);
"END" OF PROCEDURE XARLIST;
"PROCEDURE" YARLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I,J;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 24 "DO"
ELT(YARC[I,J]);
"END" OF PROCEDURE YARLIST;
"PROCEDURE" ZARLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I,J;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 24 "DO"
ELT(ZARC[I,J]);
"END" OF PROCEDURE ZARLIST;
"PROCEDURE" XTTLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I,J;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 10 "DO"
ELT(XTTC[I,J]);
"END" OF PROCEDURE XTTLIST;
"PROCEDURE" YTTLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I,J;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 10 "DO"
ELT(YTTC[I,J]);
"END" OF PROCEDURE YTTLIST;
"PROCEDURE" ZTTLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
```

```
"INTEGER" I,J;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" 10 "DO"
ELT(ZTTC[I,J]);
"END" OF PROCEDURE ZTTLIST;
"PROCEDURE" EXLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I,J;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" MM+1 "DO"
ELT(ECX[I,J]);
"END" OF PROCEDURE EXLIST;
"PROCEDURE" EZLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I,J;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
"FOR" J:=1 "STEP" 1 "UNTIL" MM+1 "DO"
ELT(ECZ[I,J]);
"END" OF PROCEDURE EZLIST;
"PROCEDURE" XXLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
ELT(XXX[I,1]);
"END" OF PROCEDURE XXLIST;
"PROCEDURE" YYLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
ELT(YYY[I,1]);
"END" OF PROCEDURE YYLIST;
"PROCEDURE" ZZLIST(ELT);
"PROCEDURE" ELT;
"BEGIN"
"INTEGER" I;
"FOR" I:=1 "STEP" 1 "UNTIL" FF+1 "DO"
ELT(ZZZ[I,1]);
"END" OF PROCEDURE ZZLIST;
"COMMENT"

        THIS PROGRAM STORES ON RANDOM ACCESSIBLE DISC FILE THE

        GEOMETRY OF THE SPINE IN THE INITIAL AND DEFORMED POSITIONS,

        NOT STORED ARE MATERIAL PROPS AND XI YI ZI;

 INARRAY(60,XSE);
 INARRAY(60,YSE);
 INARRAY(60,ZSE);
 INARRAY(60,XTE);
 INARRAY(60,YTE);
 INARRAY(60,ZTE);
 INARRAY(60,XAR);
 INARRAY(60,YAR);
 INARRAY(60,ZAR);
```

```
INARRAY(60,X);
INARRAY(60,Y);
INARRAY(60,Z);
INARRAY(60,XB);
INARRAY(60,YB);
INARRAY(60,ZB);
INARRAY(60,XPC);
INARRAY(60,YPC);
INARRAY(60,ZPC);
INARRAY(60,APC);
INARRAY(60,BPC);
INARRAY(60,GPC);
INPUT(60,"("N")",ECY);
INARRAY(60,ECC);
INARRAY(60,XXX);
INARRAY(60,YYY);
INARRAY(60,ZZZ);
"FOR" I:=1 "STEP" 2 "UNTIL" 24 "DO"
"BEGIN"
MA:=(I-1)/2;
XT[I]:=X[1+(MA*(NN*2))];
XT[I+1]:=X[5+(MA*(NN*2))];
YT[I]:=Y[1+(MA*(NN*2))];
YT[I+1]:=Y[5+(MA*(NN*2))];
ZT[I]:=Z[1+(MA*(NN*2))];
ZT[I+1]:=Z[5+(MA*(NN*2))];
"END";
"FOR" I:=25 "STEP" 1 "UNTIL" 40 "DO"
"BEGIN"
XT[I]:=XTE[I-24];
YT[I]:=YTE[I-24];
ZT[I]:=ZTE[I-24];
"END";
"FOR" I:=25 "STEP" 2 "UNTIL" 32 "DO"
"BEGIN"
MA:=(I-1)/2;
XTT[I-22]:=X[1+(MA*(NN*2))];
XTT[I-21]:=X[5+(MA*(NN*2))];
YTT[I-22]:=Y[1+(MA*(NN*2))];
YTT[I-21]:=Y[5+(MA*(NN*2))];
ZTT[I-22]:=Z[1+(MA*(NN*2))];
ZTT[I-21]:=Z[5+(MA*(NN*2))];
"END";
XTT[1]:=XAR[23];
XTT[2]:=XAR[24];
YTT[1]:=YAR[23];
YTT[2]:=YAR[24];
ZTT[1]:=ZAR[23];
ZTT[2]:=ZAR[24];
"FOR" I:=1 "STEP" 1 "UNTIL" MM "DO"
"BEGIN"
MA:=(I-1);
XS[I+5]:=X[3+(MA*(NN*2))];
YS[I+5]:=Y[3+(MA*(NN*2))];
ZS[I+5]:=Z[3+(MA*(NN*2))];
"END";
"FOR" I:=1 "STEP" 1 "UNTIL" 5 "DO"
"BEGIN"
XS[I]:=XSE[I];
```

```
YS[I]:=YSE[I];
ZS[I]:=ZSE[I];
"END";
"FOR" I:=1 "STEP" 1 "UNTIL" 22 "DO"
"BEGIN"
XSC[1,I]:=XS[I];
YSC[1,I]:=YS[I];
ZSC[1,I]:=ZS[I];
"END";
"FOR" I:=1 "STEP" 1 "UNTIL" 40 "DO"
"BEGIN"
XTC[1,I]:=XT[I];
YTC[1,I]:=YT[I];
ZTC[1,I]:=ZT[I];
"END";
"FOR" I:=1 "STEP" 1 "UNTIL" 24 "DO"
"BEGIN"
XARC[1,I]:=XAR[I];
YARC[1,I]:=YAR[I];
ZARC[1,I]:=ZAR[I];
"END";
"FOR" I:=1 "STEP" 1 "UNTIL" 10 "DO"
"BEGIN"
XTTC[1,I]:=XTT[I];
YTTC[1,I]:=YTT[I];
ZTTC[1,I]:=ZTT[I];
"END";
"FOR" I:=1 "STEP" 1 "UNTIL" 5 "DO"
"BEGIN"
XS[I]:=XS[I]-XPC[1,1];
YS[I]:=YS[I]-YPC[1,1];
ZS[I]:=ZS[I]-ZPC[1,1];
"END";
"FOR" I:=6 "STEP" 1 "UNTIL" 22 "DO"
"BEGIN"
XS[I]:=XS[I]-XPC[1,I-5];
YS[I]:=YS[I]-YPC[1,I-5];
ZS[I]:=ZS[I]-ZPC[1,I-5];
"END";
"FOR" I:=1 "STEP" 2 "UNTIL" 34 "DO"
"FOR" J:=0 "STEP" 1 "UNTIL" 1 "DO"
"BEGIN"
MA:=(I+1)/2;
XT[I+J]:=XT[I+J]-XPC[1,MA];
YT[I+J]:=YT[I+J]-YPC[1,MA];
ZT[I+J]:=ZT[I+J]-ZPC[1,MA];
"END";
"FOR" I:=1 "STEP" 2 "UNTIL" 24 "DO"
"FOR" J:=0 "STEP" 1 "UNTIL" 1 "DO"
"BEGIN"
MA:=(I+1)/2;
XAR[I+J]:=XAR[I+J]-XPC[1,MA];
YAR[I+J]:=YAR[I+J]-YPC[1,MA];
ZAR[I+J]:=ZAR[I+J]-ZPC[1,MA];
"END";
"FOR" I:=1 "STEP" 2 "UNTIL" 10 "DO"
"FOR" J:=0 "STEP" 1 "UNTIL" 1 "DO"
"BEGIN"
MA:=(I+1)/2+11;
```

```
XTT[I+J]:=XTT[I+J]-XPC[1,MA];
YTT[I+J]:=YTT[I+J]-YPC[1,MA];
ZTT[I+J]:=ZTT[I+J]-ZPC[1,MA];
"END";
"FOR" I:=1 "STEP" 1 "UNTIL" MM+1 "DO"
"BEGIN"
ECX[1,I]:=0;
ECZ[1,I]:=ECC[I];
"END";
PI:=3.1415929;
"FOR" F:=2 "STEP" 1 "UNTIL" FF+1 "DO"
."FOR" I:=1 "STEP" 1 "UNTIL" MM "DO"
 "BEGIN"
 ALPHA:=APC[F,I]-APC[1,I];
 BETA:=BPC[F,I]-BPC[1,I];
 GAMMA:=GPC[F,I]-GPC[1,I];
 ANGLE(CX,CY,CZ,CRO,SRO,ALPHA,BETA,GAMMA,PI);
 TRAN(C,CX,CY,CZ,CRO,SRO);
 "IF" I=1 "THEN"
 "BEGIN"
 ECX[F,I]:=C[1,1]*ECX[1,1]+C[2,1]*ECY+C[3,1]*ECZ[1,1];
 ECZ[F,I]:=C[1,3]*ECX[1,1]+C[2,3]*ECY+C[3,3]*ECZ[1,1];
 XXX[F,1]:=C[1,1]*(XXX[1,1]-XPC[1,1])+C[2,1]*(YYY[1,1]-YPC[1,1])
 +C[3,1]*(ZZZ[1,1]-ZPC[1,1])+XPC[F,1];
 YYY[F,1]:=C[1,2]*(XXX[1,1]-XPC[1,1])+C[2,2]*(YYY[1,1]-YPC[1,1])
 +C[3,2]*(ZZZ[1,1]-ZPC[1,1])+YPC[F,1];
 ZZZ[F,1]:=C[1,3]*(XXX[1,1]-XPC[1,1])+C[2,3]*(YYY[1,1]-YPC[1,1])
 +C[3,3]*(ZZZ[1,1]-ZPC[1,1])+ZPC[F,1];
 "FOR" J:= 1 "STEP" 1 "UNTIL" 5 "DO"
 "BEGIN"
 XSC[F,J]:=C[1,1]*XS[J]+C[2,1]*YS[J]+C[3,1]*ZS[J]+XPC[F,I];
 YSC[F,J]:=C[1,2]*XS[J]+C[2,2]*YS[J]+C[3,2]*ZS[J]+YPC[F,I];
 ZSC[F,J]:=C[1,3]*XS[J]+C[2,3]*YS[J]+C[3,3]*ZS[J]+ZPC[F,I];
 "END";
 "END";
 XSC[F,I+5]:=C[1,1]*XS[I+5]+C[2,1]*YS[I+5]+C[3,1]*ZS[I+5]+XPC[F,I];
 YSC[F,I+5]:=C[1,2]*XS[I+5]+C[2,2]*YS[I+5]+C[3,2]*ZS[I+5]+YPC[F,I];
 ZSC[F,I+5]:=C[1,3]*XS[I+5]+C[2,3]*YS[I+5]+C[3,3]*ZS[I+5]+ZPC[F,I];
 MA:=(I*2)-2;
 "FOR" J:=1 "STEP" 1 "UNTIL" 2 "DO"
 "BEGIN"
 XTC[F,MA+J]:=C[1,1]*XT[MA+J]+C[2,1]*YT[MA+J]+C[3,1]*ZT[MA+J]+XPC[F,I];
 YTC[F,MA+J]:=C[1,2]*XT[MA+J]+C[2,2]*YT[MA+J]+C[3,2]*ZT[MA+J]+YPC[F,I];
 ZTC[F,MA+J]:=C[1,3]*XT[MA+J]+C[2,3]*YT[MA+J]+C[3,3]*ZT[MA+J]+ZPC[F,I];
 "IF" I<13 "THEN"
 "BEGIN"
 XARC[F,MA+J]:=C[1,1]*XAR[MA+J]+C[2,1]*YAR[MA+J]+C[3,1]*ZAR[MA+J]+
 XPC[F,I];
 YARC[F,MA+J]:=C[1,2]*XAR[MA+J]+C[2,2]*YAR[MA+J]+C[3,2]*ZAR[MA+J]+
 YPC[F,I];
 ZARC[F,MA+J]:=C[1,3]*XAR[MA+J]+C[2,3]*YAR[MA+J]+C[3,3]*ZAR[MA+J]+
 ZPC[F,I];
 "END";
 "IF" I>11 "THEN"
 "BEGIN"
 MB:=(I*2)-24;
 "IF" I<17 "THEN"
 "BEGIN"
 XTTC[F,MB+J]:=C[1,1]*XTT[MB+J]+C[2,1]*YTT[MB+J]+C[3,1]*ZTT[MB+J]
```

```
+XPC[F,I];
YTTC[F,MB+J]:=C[1,2]*XTT[MB+J]+C[2,2]*YTT[MB+J]+C[3,2]*ZTT[MB+J]
+YPC[F,I];
ZTTC[F,MB+J]:=C[1,3]*XTT[MB+J]+C[2,3]*YTT[MB+J]+C[3,3]*ZTT[MB+J]
+ZPC[F,I];
"END";
"END";
"END";
ECX[F,I+1]:=C[1,1]*ECX[1,I+1]+C[3,1]*ECZ[1,I+1];
ECZ[F,I+1]:=C[1,3]*ECX[1,I+1]+C[3,3]*ECZ[1,I+1];
"END";
"FOR" F:=2 "STEP" 1 "UNTIL" FF+1 "DO"
"FOR" I:=35 "STEP" 1 "UNTIL" 40 "DO"
"BEGIN"
XTC[F,I]:=XTC[1,I];
YTC[F,I]:=YTC[1,I];
ZTC[F,I]:=ZTC[1,I];
"END";
P:=1;
STORELIST(63,P,XLIST);
P:=MM*NN*2+1;
STORELIST(63,P,YLIST);
P:=MM*NN*4+1;
STORELIST(63,P,ZLIST);
P:=MM*NN*6+1;
STORELIST(64,P,XBLIST);
P:=MM*NN*6+MM*2+1;
STORELIST(64,P,YBLIST);
P:=MM*NN*6+MM*4+1;
STORELIST(64,P,ZBLIST);
P:=MM*NN*6+MM*6+1;
STORELIST(65,P,XPLIST);
P:=(MM*6)*(NN+1)+(FF+1)*((MM+1)*1)+1;
STORELIST(65,P,YPLIST);
P:=(MM*6)*(NN+1)+(FF+1)*((MM+1)*2)+1;
STORELIST(65,P,ZPLIST);
P:=(MM*6)*(NN+1)+(FF+1)*((MM+1)*3)+1;
STORELIST(65,P,APLIST);
P:=(MM*6)*(NN+1)+(FF+1)*((MM+1)*4)+1;
STORELIST(65,P,BPLIST);
P:=(MM*6)*(NN+1)+(FF+1)*((MM+1)*5)+1;
STORELIST(65,P,GPLIST);
P:=(MM*6)*(NN+1)+(FF+1)*((MM+1)*6)+1;
STORELIST(66,P,XSLIST);
P:=(MM*6)*(NN+1)+(FF+1)*((MM+1)*6+22)+1;
STORELIST(66,P,YSLIST);
P:=(MM*6)*(NN+1)+(FF+1)*((MM+1)*6+44)+1;
STORELIST(66,P,ZSLIST);
P:=(MM*6)*(NN+1)+(FF+1)*((MM+1)*6+66)+1;
STORELIST(67,P,XTLIST);
P:=(MM*6)*(NN+1)+(FF+1)*((MM+1)*6+66+40)+1;
STORELIST(66,P,YTLIST);
P:=(MM*6)*(NN+1)+(FF+1)*((MM+1)*6+66+80)+1;
STORELIST(67,P,ZTLIST);
P:=(MM*6)*(NN+1)+(FF+1)*((MM+1)*6+66+120)+1;
STORELIST(68,P,XARLIST);
P:=(MM*6)*(NN+1)+(FF+1)*((MM+1)*6+66+120+24)+1;
STORELIST(68,P,YARLIST);
P:=(MM*6)*(NN+1)+(FF+1)*((MM+1)*6+66+120+48)+1;
```

```
STORELIST(68,P,ZARLIST);
P:=(MM*6)*(NN+1)+(FF+1)*((MM+1)*6+66+120+72)+1;
STORELIST(69,P,XTTLIST);
P:=(MM*6)*(NN+1)+(FF+1)*((MM+1)*6+66+120+72+10)+1;
STORELIST(69,P,YTTLIST);
P:=(MM*6)*(NN+1)+(FF+1)*((MM+1)*6+66+120+72+20)+1;
STORELIST(69,P,ZTTLIST);
P:=(MM*6)*(NN+1)+(FF+1)*((MM+1)*6+66+120+72+30)+1;
STORELIST(65,P,EXLIST);
P:=(MM*6)*(NN+1)+(FF+1)*((MM+1)*7+66+120+72+30)+1;
STORELIST(65,P,EZLIST);
P:=(MM*6)*(NN+1)+(FF+1)*((MM+1)*8+66+120+72+30)+1;
STORELIST(71,P,XXLIST);
P:=(MM*6)*(NN+1)+(FF+1)*((MM+1)*8+66+120+72+30+1)+1;
STORELIST(71,P,YYLIST);
P:=(MM*6)*(NN+1)+(FF+1)*((MM+1)*8+66+120+72+30+2)+1;
STORELIST(71,P,ZZLIST);
OUTPUT(61,"("2/")");
OUTARRAY(61,XSC);
OUTPUT(61,"("2/")");
OUTARRAY(61,YSC);
OUTPUT(61,"("2/")");
OUTARRAY(61,ZSC);
OUTPUT(61,"("2/")");
OUTARRAY(61,XTC);
OUTPUT(61,"("2/")");
OUTARRAY(61,YTC);
OUTPUT(61,"("2/")");
OUTARRAY(61,ZTC);
OUTPUT(61,"("2/")");
OUTARRAY(61,XARC);
OUTPUT(61,"("2/")");
OUTARRAY(61,YARC);
OUTPUT(61,"("2/")");
OUTARRAY(61,ZARC);
OUTPUT(61,"("2/")");
OUTARRAY(61,XTTC);
OUTPUT(61,"("2/")");
OUTARRAY(61,YTTC);
OUTPUT(61,"("2/")");
OUTARRAY(61,ZTTC);
"END";
"END";
```

```
CHANNEL,60=INPUT,P80
CHANNEL,61=OUTPUT,P136,PP60
CHANNEL,63=PLOP,W,L102
CHANNEL,64=PLOP,W,L34
CHANNEL,65=PLOP,W,L90
CHANNEL,66=PLOP,W,L110
CHANNEL,67=PLOP,W,L200
CHANNEL,68=PLOP,W,L120
CHANNEL,69=PLOP,W,L50
CHANNEL,71=PLOP,W,L5
```

APPENDIX 3.

The Data Input Into The Program MJJ3.

This computer program accepts data for the three-dimensional geometry of the spine. Co-ordinate data must be specified for all the steps of movement for the primary nodes which define the position and orientation of the vertebral bodies. The secondary nodes need only to be defined for the initial position, the co-ordinates for the deformed positions are calculated within the program.

The program is set up to handle the data for the structural analysis program MJJ1 and the data generating programs for the APEX package MJJ2 and MJJ4. This is not a general program, it is set up to accept data in a specific order and cannot be readily changed for different types of mathematical model.

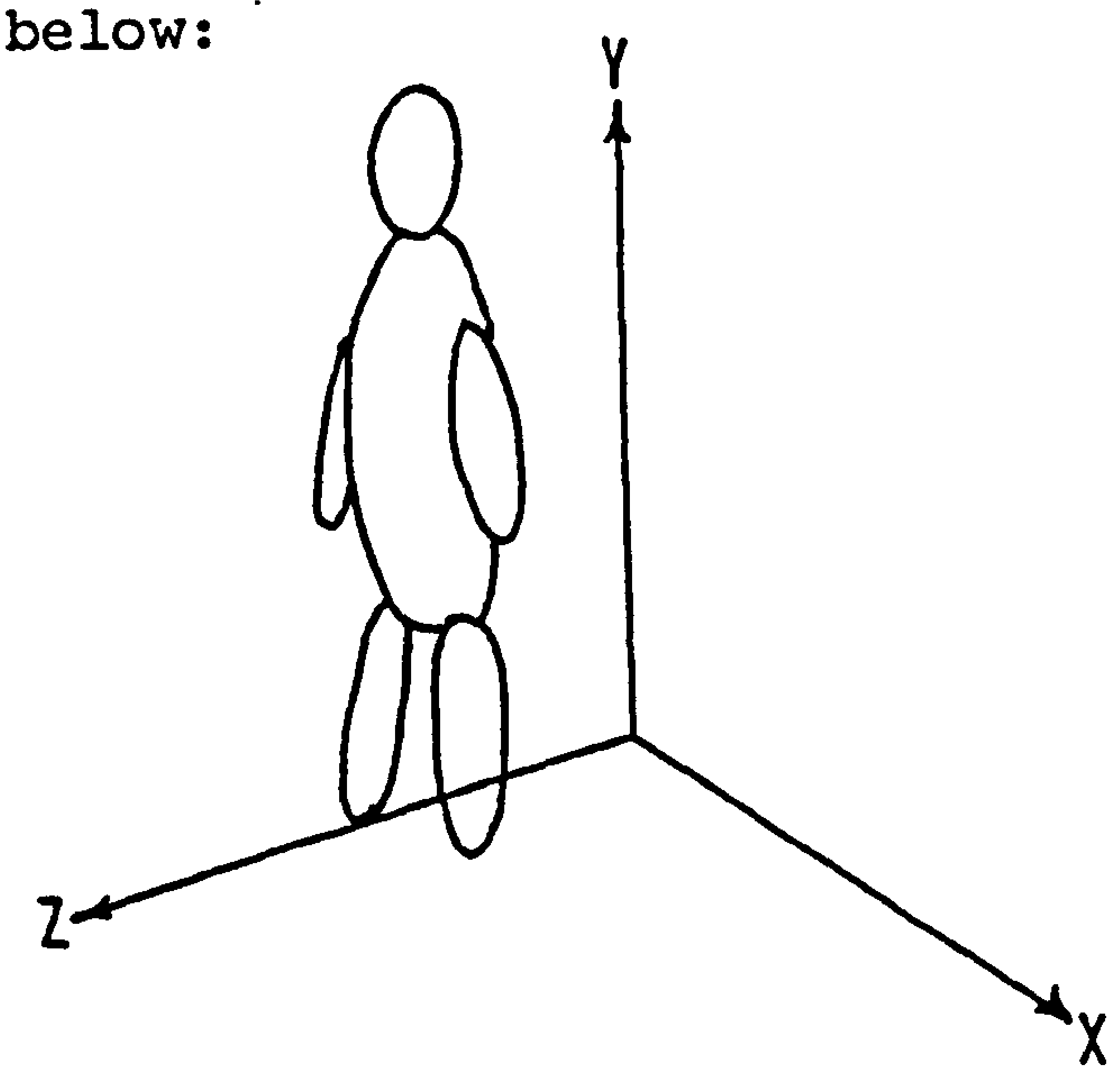
Described below is the sequence in which the data must be input into the program on card.

MM specifies the number of vertebrae, typically 17.

NN specifies the number of bar links between the vertebrae, typically 7.

FF specifies the number of incremental steps in the movement, typically between 1 and 4.

The spatial axes are orientated so that the Y-axis points vertically upwards, the Z-axis is in the direction Posterior-Anterior and the X-axis is positioned laterally as shown below:



The first arrays input the co-ordinates of the spinous processes of the vertebrae C_3 - C_7. XSE, YSE and ZSE are of a field length of 5 and relate to the X, Y and Z axes respectively.

The arrays XTE, YTE and ZTE are of length 16 and hold the co-ordinates for the base of the transverse processes and for corresponding points on the sacrum for the vertebrae L_1 - S_3. The points are specified as first one side of the spine then the other, then again for the next vertebra and so on. Typically the X-axis positive side is first.

The arrays XAR, YAR and ZAR hold the co-ordinates for the angles of the ribs 1-12. Each has a field length of 24, the order in which they are input is similar to above.

The arrays X, Y and Z hold the main body of co-ordinates for the secondary nodes in their initial position. They are specified in the following order:

(a) The values for the tips of the first transverse processes (X-axis positive), for the superior and inferior vertebrae, this corresponds to the TP elements of the structural analysis.

(b) The superior and inferior values for the tips of the spinous processes of the superior and inferior vertebrae, corresponds to the SP elements.

(c) The superior and inferior values for the second transverse processes (sup. and inf. vertebrae).

(d) The values for the first articular facets for the superior and inferior vertebrae. This corresponds to the AF elements.

(e) The second set of values for the second set of articular facets.

(f) The first set of values for the link joining the base of the spinous process of the superior vertebra to the lamina of the inferior, this corresponds to the RT elements.

(g) The second set of values for the other link is specified.

When this is complete for the first vertebra, the data is continued for the second and so on down to S_1. The co-ordinates for some of the points are repeated in the levels for adjacent vertebrae; e.g. the points for the transverse processes will first be specified as the inferior co-ordinate of the link from the upper vertebra, for the next vertebra it will be written as the superior co-ordinate and so on. This also applies for the spinous processes. The reason for this is that one point is located and this acts both as an origin and insertion for the links above and below.

The arrays XB, YB and ZB hold the co-ordinate values for the superior and inferior ends of the beam element connecting the vertebrae from T_1 - S_1. Typically the points are positioned in the centre of the vertebral endplate.

The arrays XPC, YPC, ZPC, APC, BPC, GPC hold the values for the position and orientation of the primary nodes for the vertebrae T_1 - S_1. APC, BPC and GPC are the rotational angles about the X, Y and Z axes respectively and are specified in radians. These values are input for the initial position first and then for the next increment of movement and so on.

The value ECY stores the Y-axis distance from the primary node of the first thoracic vertebra to the centre of mass of the head.

The vector ECC holds the values for the eccentricity of the masses assigned to each vertebra T_1 - L_5 and the head. The distance is measured in the Z-axis direction from the centre of mass of the segment to the primary node of the vertebra [151]. The eccentricity of the head, which is read in first, is specified as the distance, in the Z direction, from the centre of mass for the head to the primary node of the first thoracic vertebra.

The values XXX, YYY and ZZZ are the co-ordinates of the pivot point positioned on the superior endplate of the first thoracic vertebra around which the moments for the mass of the head are calculated. It is around this point that the muscles must balance the mass moment generated. The head is positioned in space relative to the position and orientation of the first thoracic vertebra.

APPENDIX 4.

The Material Properties Of The Bar And Beam Elements.

The material properties for the bar and beam elements which are used in the structural analysis of the spine are listed here. The stiffnesses for the beam elements, which are derived from Schultz et al [84], are modified to account for the difference in height of the discs which may exist between their model and the one described in this work.

The Bar Elements.

To represent the SP links joining the spinous processes of the vertebrae:
The values vary linearly from T_1 to L_5, 4.9 - 9.7 N/mm.

To represent the TP links joining the tips of the transverse processes of the vertebrae:
The values vary linearly from T_1 to L_5, 9.8 - 19.6 N/mm.

To represent the RT links joining the base of the spinous process of the superior vertebrae to the laminae of the inferior vertebrae:
The values vary linearly from T_1 to L_5, 14.7 - 24.5 N/mm.

To represent the AF links joining the articular facets of the vertebrae, these also take into account the kinematic constraints of the facets and are orientated in an Ant.-Post. direction in the thoracic region changing to a lateral orientation in the lumbar region:

The values are constant for all vertebrae, 49 N/mm.

The Beam Elements.

The stiffness for the beam elements are derived for thick beams, as described in Przemieniecki [172].

$K_1 = EA/AL$ The axial stiffness of the beam. N/mm.

$K_2 = GJ/L$ The torsional stiffness of the beam. Nmm/rad.

$K_3 = \frac{12EI}{(1+\Phi)L^3}$ The shear stiffness of the beam. N/mm.

$K_4 = \frac{6EI}{(1+\Phi)L^2}$

$K_5 = \frac{(4+\Phi)EI}{(1+\Phi)L}$ The bending stiffness for both lateral and forward flexion. Nmm/rad.

$K_6 = \frac{(2-\Phi)EI}{(1+\Phi)L}$

where $\Phi = \frac{12EI}{GAL^2}$

The symbols A, E, G, I, J and L take on their usual significance.

Vertebra	K_1	K_2	K_3	K_4	K_5	K_6
T_1	773	22073	671	1341	21402	-16038
T_2	811	20274	739	1630	28920	-21577
T_3	883	23544	818	1806	37853	-29727
T_4	906	25898	809	1985	47134	-37208
T_5	881	27825	777	2096	50806	-39277
T_6	1413	47088	1272	2495	80251	-70271
T_7	1472	58860	1400	2747	98101	-87112
T_8	1655	66218	1469	28823	119899	-108370
T_9	1153	53791	1080	3177	88114	-69052
T_{10}	1365	63676	1292	3487	110465	-91288
T_{11}	1390	74120	1033	3648	93999	-67738
T_{12}	1745	96946	986	4112	87626	-52673
L_1	1427	107018	800	4314	83924	-36472
L_2	1678	134201	942	4618	94673	-48491
L_3	1561	124886	865	4877	90724	-34640
L_4	1672	131369	928	5233	84665	-24488
L_5	1783	145911	1299	6052	82291	-24793

APPENDIX 5.

Additional X-Ray Data.

Immediately prior to the completion of this series of studies additional X-ray data were obtained from the University of Vermont [175]. These data are probably comparable to the best currently available in accuracy, but still are insufficient to fully test the computer simulation presented here.

Stereo photogrammetry is used to measure the spatial co-ordinates of points on the vertebrae. A carefully calibrated jig is also used to obtain accurate origin and reference points. A computer is used to compare the two X-ray films and give the accurate spatial co-ordinates. The accuracy of the results is better than ±1mm for a number of distinct landmarks on the vertebrae.

However, the major shortcoming of the data was that it was restricted to the lumbar region alone, specifically L_1 - L_4. The points on the vertebrae which were measured included the following:

(a) The centre of the superior vertebral body endplate.

(b) The centre of the inferior vertebral body endplate.

(c) The centroid of the vertebral body.

(d) The tip of the spinous process.

(e) & (f) The tips of the transverse processes.

All these points were measured for all the deflected positions.

Three movements were studied using three subjects. They are as follows:

(i) Lateral Bending.

The positions measured were for the neutral initial position, extreme left, middle left, middle right and extreme right lateral flexion.

(ii) Axial Rotation.

Similar to above the positions measured were neutral, extreme left, medium left, medium right and extreme right axial rotation.

(iii) Forward Flexion.

The initial upright and four further increments of forward flexion were measured. The maximum total flexion was approx. 30° at T_1.

For the inclusion into the critical structural analysis of the spine a further 8 points per vertebra are required (see Chapter 10.2.). These would be for the articular facets and points on the laminae and at the base of the spinous process. For use in the linear programming section still further points are required but these do not require the same degree of accuracy.

The structural analysis program is designed to accept data for any number of vertebrae provided that a fixed base is also included. Thus a requirement is to include the co-ordinates of some of the points on S_1, any co-ordinate movement of the fixed base between incremental positions can therefore be removed from the data for the movable points. The data, which stop at L_4, cannot be easily analysed with the structural analysis program in its present state. Obtaining the applied force required to hold the spine in a deformed position against the action of the passive elastic elements can only be done for L_1 - L_3 and a severe anomaly would occur for the L_4 - L_5 interconnection.

With a degree of modification the knowledge of the intervertebral reactions in the deformed position of only a section of the spine can be included into the linear programming.

If points had been included for L_5 and S_1, it may have been possible to gain results which would have shown more clearly the requirements for the complete functioning of the model of the static spine described in Chapters 9 and 10.