Cost-Effective Applications of Power Factor Correction for Nonlinear Loads

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***Abstract*—The objective of this paper is to propose a new approach for designing passive *LC* compensators by using the penalty function method as an optimization tool. The performance of the cost-effective passive *LC* compensator for a constant load depends on the appropriate inductor and capacitor selection. Several design methods are reviewed and a novel design methodology is proposed in this paper. By using the proposed method, the designer can quickly find appropriate parameter values to meet the desired circuit performance. Simulated results show that an appropriate combination of the inductor and capacitor selected by the proposed method can meet the desired power-quality requirement. Different cases of design examples are shown in this paper to verify the performance of the proposed design methodology.**

***Index Terms*—Harmonics, power factor, reactive power.**

NOMENCLATURE

R_{LK},X_{LK}	Load resistance and reactance at harmonic number K (in ohms).
G_{LK}, B_{LK}	Load conductance and susceptance at harmonic number K (in ohms).
R_{TK},X_{TK}	Thevenin resistance and reactance at harmonic number K (in ohms).
X_L, X_C	Fundamental inductive and capacitive reactance of the compensator (in ohms).
R	Resistance of the compensator reactor (Ω).
I_S^o	Supply current without compensation (in amps).
I_S	RMS supply current with compensation (in amps).
I_{SK}	Supply current at harmonic number K (in amps).
I_{LK}	Load harmonic current (in amperes).
I_{CK}	Capacitor current at harmonic number K (in amperes).
I_C	RMS capacitor current with compensation (in amperes).
V_L	RMS value of load voltage (in volts).
V_{SK}	Supply voltage at harmonic number K (in volts).
V_{LK}	Load voltage at harmonic number K (in volts).
V_{CR}	Capacitor rated rms voltage (in volts).
V_C	RMS capacitor voltage with compensation (in volts).
V_{CC}	Crest capacitor voltage with compensation (in volts).
PF	Load power factor (in per unit).
d_{PF}	Load displacement power factor (in per unit).
η	Network transmission efficiency (in percentage).
TL	Loss in Thevenin resistance (in kilowatts).

I. INTRODUCTION

POWER-ELECTRONIC loads are rapidly growing in number. Due to their nonlinear nature, they are the major sources of harmonics in electric utility networks.

From the utility customer's point of view, harmonics can be classified as those generated by the customer load itself, which should be compensated locally; and those which are caused by other utility customers whose nonlinear loads have created voltage distortions at a common voltage bus. Distorted bus voltage then causes harmonic currents in neighboring linear compensated loads, particularly if series resonance occurs between Thevenin impedance and compensated load. A customer with such a load needs to minimize the harmonic line currents while simultaneously using a compensator to maintain a high power factor.

Several approaches [1]–[8] discuss the different parameters affecting the economic feasibility of power factor correction. It is shown that the specific costs of the transmission and compensation elements as well as those of the electrical energy and power losses have a decisive influence on the achievable overall economic saving. In some approaches [9]–[13] on the subject to achieving the maximum possible power factor, the effect of harmonic voltages and currents on capacitor and inductor cost has been ignored.

An optimization method for the proper selection of the *LC* compensator values taking into account cost constraints is described for nonlinear loads [14]. It uses the nonlinear model of the loads, which is neglected in [15]. The solution will be obtained with an optimization algorithm based on the reduced gradient technique which will locate maximum power factors by searching along the constant-cost power factor ridges, for the point where the gradient of the transmission loss is zero.

This paper will show that the static *LC* compensator needs to be optimally designed during the compensation of nonlinear loads. The compensator must be designed to meet minimum requirements in terms of power factor and harmonic distortion of the current ITHD and the voltage VTHD while, at the same time, minimizing its total cost. It is assumed that both the equivalent source and load are considered to generate harmonics. Also, it is assumed that the load harmonics are not sufficiently serious to suggest tuned filters, but when combined with source harmonics, the use of a pure capacitive compensator would degrade power factor and overload equipment. Consequently, *LC* compensators are selected [16]. The basic approach to power factor

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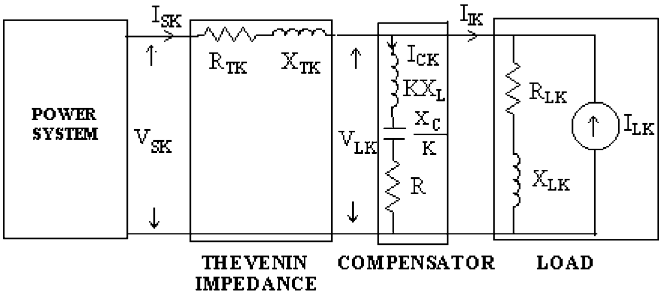

Fig. 1. Single-phase equivalent circuit for Kth harmonic with shunt LC compensator.

correction is first explained, and the cost function expression and analysis are discussed. Then, the optimization algorithm using the penalty function method is presented. Finally, the simulation results are discussed and compared with other published techniques.

II. BASIC APPROACH TO POWER-FACTOR CORRECTION

Fig. 1 is a single-phase equivalent circuit of a bus with LC compensator, experiencing VTHD at harmonic order K because of a voltage source $V_S K$ and harmonic current sources within the load itself $I_L K$. The series reactor X_L has been added to minimize VTHD while X_C has been altered to maintain the power factor at a desired level.

The Thevenin voltage source representing the utility supply and the harmonic current source representing the nonlinear load is

$$\mathrm{v_S(t)} = \sum_{\mathrm{K}} \mathrm{v_{SK}(t)} \tag{1}$$

and

$$\mathrm{i_{LK}(t)} = \sum_{\mathrm{K}} \mathrm{i_{LK(t)}} \tag{2}$$

where K is the order of harmonic present. The Kth harmonic Thevenin source and load impedances are

$$\mathrm{Z_{TK} = R_{TK} + jX_{TK}} \tag{3}$$

and

$$\mathrm{Z_{LK} = R_{LK} + jX_{LK}} \tag{4}$$

or

$$\mathrm{Y_{LK} = G_{LK} - jB_{LK}}. \tag{5}$$

Also, the Kth harmonic LC compensator impedance is

$$\mathrm{Z_{CK} = R + j\left(KX_L - \frac{X_C}{K}\right)}.$$

Finally, this model, Fig. 1, is adequate where VTHD is less than 10% [17].

From the first principle, the following equation can be obtained:

$$\mathrm{V_{SK} = Z_{TK} * I_{SK} + (I_{SK} - I_{LK}) * \frac{(Z_{CK} * Z_{LK})}{(Z_{CK} + Z_{LK}).}}$$

By simplification and substituting the expressions for Z_{TK}, Z_{CK}, and Z_{LK} in the above equation, we can get expressions for I_{SK}, I_{CK}, and V_{LK}.

The approach will be to minimize the compensator cost by adjusting X_C and X_L while keeping the power factor constant. The compensator cost is defined as

$$\mathrm{C = U_C * S_C + U_L * S_L} \tag{6}$$

where U_C, U_L = unit cost of capacitor and inductor ($/kVA), and considered to be constant parameters.

For capacitors and reactors, volt-ampere ratings are defined as [18]

$$\mathrm{S_C} = \left[\sum_{\mathrm{K}} \mathrm{I_{CK}} * \frac{\mathrm{X_C}}{\mathrm{K}}\right]\left[\sum_{\mathrm{K}} \mathrm{I_{CK}^2}\right]^{\frac{1}{2}} \mathrm{kVA} \tag{7}$$

and

$$\mathrm{S_L} = \left[\sum_{\mathrm{K}} \mathrm{I_{CK}} * \mathrm{K} * \mathrm{X_L}\right]\left[\sum_{\mathrm{K}} \mathrm{I_{CK}^2}\right]^{\frac{1}{2}} \mathrm{kVA} \tag{8}$$

where the compensator current I_{CK} is given

$$\begin{aligned}
\mathrm{I_{CK}} &= \frac{\mathrm{V_{SK}(R_{SK} + jX_{LK}) - I_{LK}(R_{TLK} + jX_{TLK})}}{\mathrm{A_{IK} + jA_{JK}}}\\
\mathrm{R_{TLK}} &= \mathrm{R_{TK} * R_{LK} - X_{TK} * X_{LK}}\\
\mathrm{X_{TLK}} &= \mathrm{R_{TK} * X_{LK} + X_{LK} * R_{LK}}\\
\mathrm{A_{IK}} &= \mathrm{R_{TLK} + R(R_{LK} + R_{TK}) - (X_{LK} + X_{TK})}\\
&\quad \times \left(\mathrm{KX_L} - \frac{\mathrm{X_C}}{\mathrm{K}}\right)\\
\mathrm{A_{JK}} &= \mathrm{X_{TLK} + R(R_{LK} + X_{TK}) + (R_{LK} + R_{TK})}\\
&\quad \times \left(\mathrm{KX_L} - \frac{\mathrm{X_C}}{\mathrm{K}}\right).
\end{aligned} \tag{9}$$

In (7) and (8), the harmonic voltages are added linearly in the first summation to emphasize the effect of peak (as opposed to rms) voltage on insulation cost.

The compensated PF at the load is given as

$$\begin{aligned}
\mathrm{PF} &= \frac{\mathrm{P_L}}{\mathrm{V_L I_S}}\\
&= \frac{\sum \mathrm{G_{LK} V_{LK}^2}}{\sqrt{\sum \mathrm{I_{SK}^2} \sum \mathrm{V_{LK}^2}}} && (10)\\
\mathrm{I_{SK}} &= \frac{\mathrm{V_{SK}(AR + jBR) + I_{LK}CR}}{\mathrm{A_{IK} + jA_{JK}}} && (11)\\
\mathrm{V_{LK}} &= \frac{\mathrm{V_{SK}(CR) - I_{LK}(DR * ER)}}{\mathrm{A_{IK} + jA_{JK}}} && (12)
\end{aligned}$$

where

$$\begin{aligned}
\mathrm{AR} &= \mathrm{R + R_{LK}}\\
\mathrm{BR} &= \left(\mathrm{X_{LK} + KX_L} - \frac{\mathrm{X_C}}{\mathrm{K}}\right)\\
\mathrm{CR} &= \mathrm{R_{CLK} + jX_{CLK}}\\
\mathrm{DR} &= \mathrm{Z_{CK}}\\
\mathrm{ER} &= \mathrm{R_{TLK} + jX_{TLK}}\\
\mathrm{R_{CLK}} &= \mathrm{RR_{LK} - X_{LK}}\left(\mathrm{KX_L} - \frac{\mathrm{X_C}}{\mathrm{K}}\right)\\
\mathrm{X_{CLK}} &= \mathrm{RX_{LK} + R_{LK}}\left(\mathrm{KX_L} - \frac{\mathrm{X_C}}{\mathrm{K}}\right).
\end{aligned}$$

The additional constraints involved are the effect of supply frequency on the ac resistance, the compensator values, which would create resonant conditions and the manufacturer's standard values for power shunt capacitors.

A. Effect of Supply Frequency on the AC Resistance

In most power systems, one can generally assume that the resistance does not change significantly when studying the effects of harmonics less than the ninth [17]. In this study, it is assumed that

$$\mathrm{R_{TK}} = \mathrm{R_T} \quad \text{and} \quad \mathrm{R_{LK}} = \mathrm{R_L} \tag{13}$$

where R_T is the resistance of the Thevenin impedance at the fundamental frequency, and R_L is the resistance of the load at the fundamental frequency.

B. Resonance Constraint

The expected impedance seen from the Thevenin source is given by

$$\mathrm{Z} = \mathrm{Z_{TK}} + \frac{(\mathrm{R_{CLK}} + \mathrm{jX_{CLK}})}{(\mathrm{Z_{LK}} + \mathrm{Z_{CK}})}. \tag{14}$$

The resonance peaks can be obtained by setting the imaginary part of (14) to zero, resulting in a quadratic equation in X_C and X_L for any given harmonic order K

$$\alpha_1 \left(\mathrm{KX_L} - \frac{\mathrm{X_C}}{\mathrm{K}}\right)^2 + \alpha_2 \left(\mathrm{KX_L} - \frac{\mathrm{X_C}}{\mathrm{K}}\right) + \alpha_3 = 0 \tag{15}$$

where

$$\begin{aligned}\alpha_1 &= \mathrm{X_{TK}} + \mathrm{X_{LK}}\\ \alpha_2 &= \mathrm{R_{LK}^2} + \mathrm{X_{LK}^2} + 2\mathrm{X_{LK}X_{TK}}\\ \alpha_3 &= \mathrm{R^2X_{LK}} + \mathrm{X_{TK}}\left[(\mathrm{R} + \mathrm{R_{LK}})^2 + \mathrm{X_{LK}^2}\right].\end{aligned}$$

The precalculated compensator values for series resonance, by taking the solution of (15) where the square root of the discriminant is positive, are used to subdivide the entire search region into smaller regions. In each region, the total minimums are identified leading to the eventual identification of the global minimum of any of the functions. This makes the optimal LC compensator value not a part of (15).

C. Standard Ratings of the Capacitors

IEEE Std. 18-1992 [19] shows the voltage and reactive power ratings of the capacitors. Each value of the kvar ratings Q_C of the particular voltage is used to calculate the corresponding value of X_{Ci}. This value is then substituted into (16) to become one variable equation in X_L,

Then, the problem becomes

$$\begin{aligned}&\text{Minimize} \quad \mathrm{C(X_{Ci}, X_L)}\\ &\text{Subject to:} \quad \mathrm{PF(X_{Ci}, X_L)} \geq 90\%,\\ &\qquad\qquad \mathrm{X_{Ci}} \text{ and } \mathrm{X_L} \text{ are not a part of } (15).\end{aligned} \tag{16}$$

The objective function and constraints are complicated to be solved directly. Hence, an iterative method is needed to generate the solution. From the experience, the penalty function method is chosen since it requires fewer steps and function evaluations [20], [21].

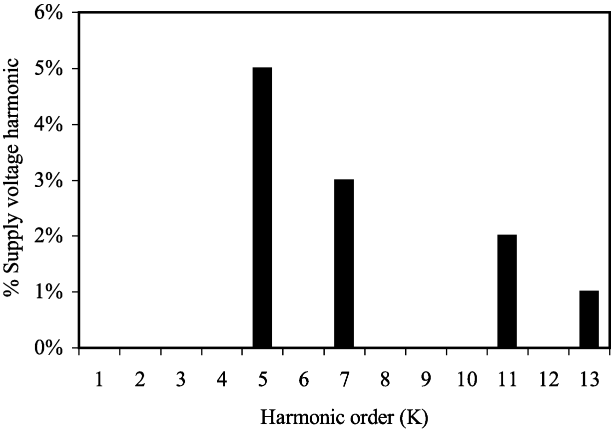

Fig. 2. Harmonic contents of the supply voltage for cases 1, 3, and 5.

Penalty function methods transform the basic optimization problem into an alternative formulation such that numerical solutions are sought by solving a sequence of unconstrained minimization problems. The search algorithm [22] is described in Appendix A.

In the optimization process, the resistance of the compensator reactor has been neglected due to its small value with respect to its fundamental reactance [15].

III. Simulated Examples and Discussion

Five cases of an industrial plant were simulated using the optimization method. The numerical data were primarily taken from an example in [18] where the inductive three-phase load of 5100 kW and 4965 kVAR is connected to a supply bus with voltage 4160 V and 60-Hz frequency. The short-circuit MVA is 150 MVA for cases 1, 2, and 5. Also, it is 80 MVA for cases 3 and 4. The system data for equivalent single-phase mode are

$$\begin{aligned}\text{Thevenin impedance} &= 0.011\,54 + 0.1152\ \Omega \text{ for } 150\ \mathrm{MVA}\\ \text{Thevenin impedance} &= 0.021\,63 + 0.2163\ \Omega \text{ for } 80\ \mathrm{MVA}\\ \text{Load impedance} &= 1.7421 + 1.696\ \Omega.\end{aligned}$$

Either U_L or U_C is 2\$/kVA except in case 5 U_L is 4\$/kVA. 4160-V capacitors are used.

Figs. 2–4 show harmonic contents, arbitrarily selected, of the supply voltage, and the load current.

The advantages of the presented method are explicitly demonstrated in the following tables.

Table I shows that a lower short-circuit capacity, cases 3 and 4, corresponds to a lower cost at the same power factor compared with cases 1 and 2. This to be expected since with higher Thevenin impedance, less harmonic current will flow into the compensated load.

Also, an additional harmonic content, cases 2 and 4, results in a cost increase for obtaining the same power factor compared with cases 1 and 2. This is caused by the increase in compensator rating due to the additional harmonics.

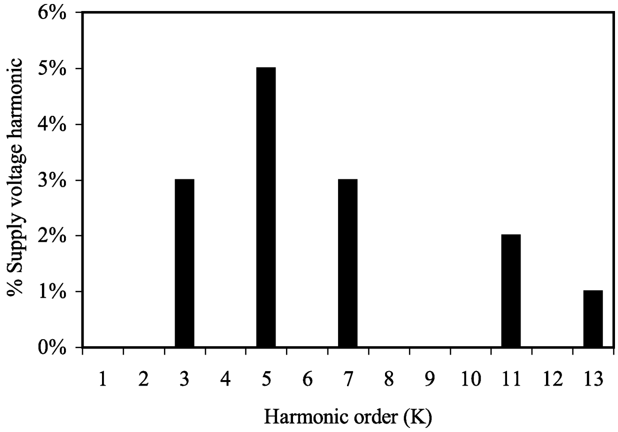

Fig. 3. Harmonic contents of the supply voltage for cases 2 and 4.

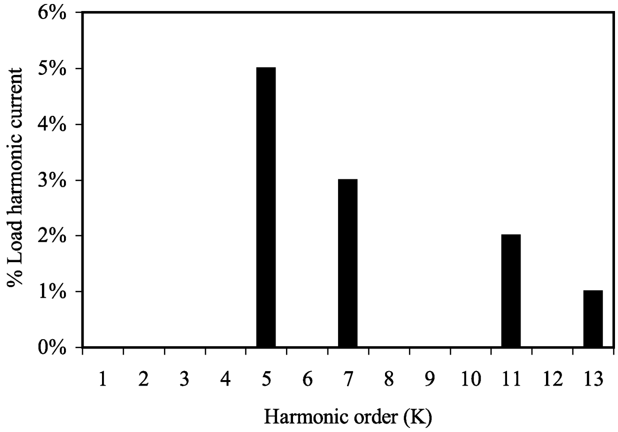

Fig. 4. Harmonic contents of the load current for all cases.

TABLE I
SIMULATED RESULTS FOR DIFFERENT CASES UNDER STUDY

CASE	Q_C	X_L^* (Ω)	PF (%)	C ($)
1	2400	0.7015	90.60	6632
2	2250	1.0896	90.00	7463
3	2550	0.2948	90.26	6154
4	2250	1.0266	90.00	7116
5	2700	0.3038	90.34	7438

Finally, the case of increasing the unit cost of the reactor, case 5, with the same conditions as case 1, has a power factor which requires a higher compensator cost to obtain the same power factor, reflecting the additional reactor cost.

The presented method results in lower supply current, lower transmission loss, higher transmission efficiency, and higher displacement power factor than the uncompensated case. So, the main advantage of the presented method consists in less harmonic power in secondary cables, lines, and switchgear owned by the end user. Tables II and III show the supply current, the transmission loss, the transmission efficiency, and the displacement power factor without and with compensation for different cases under study.

Table IV shows the harmonic contents of the supply current as a percentage of the fundamental current and ITHD with compensation for the different cases under study.

TABLE II
SUPPLY CURRENT, TRANSMISSION LOSS, TRANSMISSION EFFICIENCY, AND DISPLACEMENT POWER FACTOR WITHOUT COMPENSATION

CASE	I_S^o (A)	TL (kW)	η(%)	dPF (%)
1	953.64	10.49	99.34	71.65
2	953.72	10.50	99.34	71.65
3	924.14	18.47	98.77	71.65
4	924.22	18.48	98.77	71.65
5	953.64	10.49	99.34	71.65

TABLE III
SUPPLY CURRENT, TRANSMISSION LOSS, TRANSMISSION EFFICIENCY, AND DISPLACEMENT POWER FACTOR WITH COMPENSATION

CASE	I_S (A)	TL (kW)	η(%)	dPF (%)
1	764.59	6.75	99.59	91.14
2	769.30	6.83	99.58	90.90
3	755.45	12.34	99.22	91.19
4	755.82	12.36	99.22	90.73
5	767.77	6.80	99.59	92.30

TABLE IV
HARMONIC CONTENTS OF THE SUPPLY CURRENT AS A PERCENTAGE OF THE FUNDAMENTAL CURRENT AFTER COMPENSATION

CASE	1	2	3	4	5
% I_{S3}	0.0	9.66	0.0	8.52	0.0
% I_{S5}	8.66	7.39	13.40	6.56	19.79
% I_{S7}	4.03	3.84	4.15	3.42	5.53
% I_{S11}	2.32	2.35	1.91	2.09	2.46
% I_{S13}	1.13	1.15	0.88	1.03	1.13
% ITHD	9.89	13.02	14.18	11.52	20.72

TABLE V
HARMONIC CONTENTS OF THE LOAD VOLTAGE AS A PERCENTAGE OF THE FUNDAMENTAL VOLTAGE AFTER COMPENSATION

CASE	1	2	3	4	5
%V_{L3}	0.0	1.97	0.0	1.32	0.0
%V_{L5}	3.91	4.33	0.83	4.18	1.54
%V_{L7}	2.57	2.75	1.57	2.79	1.90
%V_{L11}	1.88	1.99	1.49	2.22	1.54
%V_{L13}	0.99	1.04	0.83	1.21	0.82
% VTHD	5.13	5.94	2.46	5.78	3.00

Table IV shows the harmonic contents of the load voltage as a percentage of the fundamental voltage and VTHD with compensation for the different cases under study.

IEEE Standard 519-1992 [23] lists the harmonic current limits based on the size of the load with respect to the size of the power system to which the load is connected. Also, it shows that the objectives of the nonlinear load harmonic current limits are to limit the maximum individual harmonic voltage to 3% of the fundamental voltage and the total harmonic distortion of the voltage to 5%. Most of the resultant values, Tables IV and V, come out well within standard limits.

TABLE VI
SIMULATED RESULTS FOR DIFFERENT CASES UNDER STUDY USING (18)

CASE	Q_C	X_L^* (Ω)	PF (%)	C ($)
1	2700	0.3038	90.34	6758
2	2400	0.8984	90.17	7805
3	2550	0.2948	90.26	6154
4	2250	1.0533	90.10	7169
5	2700	0.3038	90.34	7438

TABLE VII
SUPPLY CURRENT, TRANSMISSION LOSS, TRANSMISSION EFFICIENCY, AND DISPLACEMENT POWER FACTOR WITH COMPENSATION USING (18)

CASE	I_S (A)	TL (kW)	η(%)	dPF (%)
1	767.77	6.80	99.59	92.30
2	768.58	6.82	99.58	91.70
3	755.45	12.34	99.22	91.19
4	754.59	12.32	99.22	90.80
5	767.77	6.80	99.59	92.30

TABLE VIII
TOTAL HARMONIC DISTORTIONS FOR DIFFERENT CASES UNDER STUDY USING (18)

CASE	1	2	3	4	5
% ITHD	20.72	17.70	14.18	11.17	20.72
% VTHD	2.95	5.50	2.37	5.62	2.95

Now we test the dPF as a constraint in the optimization problem. The compensated dPF at the load is given as

$$\mathrm{dPF} = \frac{\mathrm{P_{L1}}}{\mathrm{V_{L1} I_{S1}}} \tag{17}$$

where $P_{\mathrm{L1}}, V_{\mathrm{L1}}$, and I_{S1} are the fundamental components of the load power, the load voltage, and the supply current.

Then, the problem becomes

$$\begin{aligned} &\text{Minimize} && \mathrm{C(X_{Ci}, X_L)} \\ &\text{Subject to:} && \mathrm{dPF(X_{Ci}, X_L)} \geq 90\%, \\ &&& \mathrm{X_{Ci}} \text{ and } \mathrm{X_L} \text{ are not a part of } (15). \end{aligned} \tag{18}$$

The simulated results are explicitly demonstrated in Tables VI–VIII.

Comparison of the results shows that the general performance of the method, dPF, as a constraint, is satisfactory, providing improvement of the overall performance compared with Tables I, III, IV, and V.

The object of passive LC compensators is to reduce one or more harmonic orders (generally 5, 7, 11). Although cases 2 and 4 have a zero-sequence harmonic, the performance of the system with the suggested compensation is improved. Also, cases 2 and 4 need active filtering or neutral filtering beside the suggested compensation.

TABLE IX
SIMULATED RESULTS FOR DIFFERENT CASES UNDER STUDY USING (19)

CASE	Q_C	X_L^* (Ω)	PF (%)	C ($)
1	2700	0.3038	90.34	6758
2	2700	0.4421	90.64	7579
3	2400	0.6708	90.64	6449
4	2250	1.0533	90.10	7169
5	2700	0.3038	90.34	7438

TABLE X
SUPPLY CURRENT, TRANSMISSION LOSS, TRANSMISSION EFFICIENCY, AND DISPLACEMENT POWER FACTOR WITH COMPENSATION USING (19)

CASE	I_S (A)	TL (kW)	η(%)	dPF (%)
1	767.77	6.80	99.59	92.30
2	765.02	6.75	99.59	92.72
3	750.55	12.19	99.23	91.06
4	754.59	12.32	99.22	90.80
5	767.77	6.80	99.59	92.30

TABLE XI
TOTAL HARMONIC DISTORTIONS AND CAPACITOR LIMITS FOR DIFFERENT CASES UNDER STUDY USING (19)

CASE	1	2	3	4	5
% ITHD	20.72	20.81	8.39	11.17	20.72
% VTHD	2.95	6.48	4.93	5.62	2.95
% V_C	103.41	106.38	157.82	140.20	103.41
%V_{CC}	112.34	123.19	255.83	225.40	112.34

Capacitors shall be capable of continuous operation provided that none of the following limitations are exceeded [19]: V_C not exceeding 110 of V_{CR}, and V_{CC} not exceeding $1.2 * \sqrt{2}$ of V_{CR} including harmonics but excluding transients. To make the design method guarantee that the voltage across the capacitor will not exceed the permissible maximum level, the problem becomes

$$\begin{aligned} &\text{Minimize} && \mathrm{C(X_{Ci}, X_L)} \\ &\text{Subject to:} && \mathrm{dPF(X_{Ci}, X_L)} \geq 90\%, \\ &&& \mathrm{X_{Ci}} \text{ and } \mathrm{X_L} \text{ are not a part of } (15) \\ &&& \mathrm{V_C(X_C)} \leq 110\% \\ &&& \mathrm{V_{CC}(X_C)} \leq 120\sqrt{2}\%. \end{aligned} \tag{19}$$

The simulated results are explicitly demonstrated in Tables IX–XI.

Comparison of the results shows that the general performance of the method is satisfactory, providing improvement of the overall performance compared with Tables VI and VII. Table XI shows that the resultant values come out well within standard limits except cases 3 and 4. If the resultant values are greater than standard limits, it is a good idea [18] to use capacitors with a higher voltage rating. For cases 3 and 4, 6640-V capacitors are used for a 4160-V application. In this case, the limits become 98.8 and 87.77 for V_{C}, and 160.16 and 141.11 for V_{CC}.

Finally, the presented method includes the improvement in the accuracy of the solution and in the ability of it to guarantee convergence to the optimal solutions as illustrated by the examples taken from existing publications.

IV. COMPARISON WITH OTHER OBJECTIVE FUNCTIONS

To verify the effectiveness of the proposed LC compensator design approach, a comparison will be made to two published approaches [24], [25].

The approach of [24] is to minimize VTHD on the load by adjusting X_C and X_L while keeping the dPF constant and construing the cost of the compensating circuit to a series of fixed values. This can be accomplished by minimizing a total cost function TC, which represents a combination of VTHD and C

$$\mathrm{TC} = \mathrm{C_f} * (\mathrm{VTHD})^2 + \mathrm{C} \tag{20}$$

where $\mathrm{C_f}$ is an arbitrary factor to convert $(VTHD)^2$ to cost. Then, the problem becomes

$$\begin{aligned} &\text{Minimize} && \mathrm{TC}(\mathrm{X_{Ci}}, \mathrm{X_L}) \\ &\text{Subject to:} && \mathrm{PF}(\mathrm{X_{Ci}}, \mathrm{X_L}) \geq 90\%, \\ & && \mathrm{X_{Ci}} \text{ and } \mathrm{X_L} \text{ are not a part of } (15). \end{aligned} \tag{21}$$

In [25], the objective function to be maximized in the optimal compensation is the profit function TP, which can be described as the amount a customer can gain annually by using an LC compensator to reduce the power losses in the supply network while the active power of the load is nearly constant. This quantity is C subtracted from the discount offered by the utility

$$\mathrm{TP} = \mathrm{C_f} * \left(\mathrm{I_S^{o2}} - \mathrm{I_S^2}\right) - \mathrm{C} \tag{22}$$

where I_S^o and I_S are, respectively, the magnitudes of source current without and with compensation, and C_f is the utility discount factor. Then, the problem becomes

$$\begin{aligned} &\text{Minimize} && \mathrm{TP}(\mathrm{X_{Ci}}, \mathrm{X_L}) \\ &\text{Subject to:} && \mathrm{PF}(\mathrm{X_{Ci}}, \mathrm{X_L}) \geq 90\%, \\ & && \mathrm{X_{Ci}} \text{ and } \mathrm{X_L} \text{ are not a part of } (15). \end{aligned} \tag{23}$$

The simulated results are explicitly demonstrated in Table XII.

Table XII shows that the general performance of the presented method is satisfactory compared with the other reported methods taken from existing publications.

TABLE XII
COMPARISON OF SIMULATED RESULTS FOR CASE 1 WITH DIFFERENT OBJECTIVE FUNCTIONS

EQUATION	Q_C	X_L^* (Ω)	PF (%)	C ($)
(21)	2700	0.3038	90.34	6758
(23)	2400	0.8279	90.99	6971
	I_S (A)	TL (kW)	η(%)	dPF (%)
(21)	767.77	6.80	99.59	92.30
(23)	761.16	6.87	99.59	91.50
	% VTHD		% ITHD	
(21)	2.59		20.72	
(23)	5.24		9.33	

V. CONCLUSION

A method is presented for finding the optimum fixed LC combination to minimize its cost at a load bus while holding the power factor at a desired value and constraining the compensator values which would create resonant conditions and the manufacturers' standard values for power shunt capacitors.

The presented method performs two major tasks: to produce certain level of reactive power, and to provide a low impedance path for harmonic currents that must be confined. Both requirements listed have a strong impact on the total cost of LC compensator.

The results and examples presented should serve as a guideline only. In practice, the system parameter data may be uncertain. However, the proposed method may still be applied by finding the power factor curves for a variety of system conditions commensurate with those normally encountered and selecting the capacitor and reactor for the best average performance.

Ongoing research effort consists of the modification and application of this method to take into account load profiles, other constraints (for example, the allowable overloads for capacitor imposed by standard or recommendations), and probability density function of several indices as well known in recent international standards; for example, IEC 1000-3-6.

APPENDIX A

VI. PENALTY FUNCTION METHOD ALGORITHM

Let the basic optimization problem, with inequality constraints, be of the form (16). This problem is converted into an unconstrained minimization problem by constructing a function of the form

$$\mathrm{f(X_L)} = \mathrm{C(X_C, X_L)} + \sum_{\mathrm{m}} \mu_{\mathrm{m}} \left(\max\left[0, \mathrm{g_m(X_C, X_L)}\right]\right) \tag{A.1}$$

where g_m is some function of the constraints, μ_m is a positive constant known as the penalty parameter, and $max[0, g_m(X_C, X_L)]$ is the commonly used form of the penalty parameter, which is the second part of (A.1).

Step 1) Start with an initial feasible point X_C^0, X_L^0 satisfying all of the constraints with a strict inequality sign. Start with an initial value of $\mu_1 > 0$. Set $J = 1$.

Step 2) Minimize $f(X_L)$, (20), by using any of the unconstrained minimization methods and obtain the solution X_C^*, X_L^*. The golden section search method [2] can be applied for obtaining the optimal X_L^*.

Step 3) Test whether the solution X_C^*, X_L^* is the optimum solution of the original problem. The algorithm will stop when a feasible point will be reached or when the relative change in the objective function is small.

$$\varepsilon < 10^{-6}. \tag{A.2}$$

If X_C^*, X_L^* is found to be optimum, terminate the process. Otherwise, go to the next step.

Step 4) Find the value of the next penalty parameter as

$$\mu_m^{(J+1)} = \beta \mu_m^{(J)} \tag{A.3}$$

where $\beta < 1$.

Step 5) Set the new value of $J = J+1$, take the new starting point as X_C^*, X_L^*, and go to step 2.

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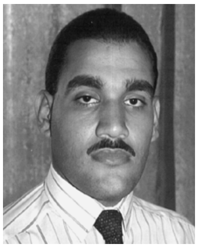

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