Consistency Conditions for Finite-Volume Partition Functions

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Abstract

Using relations from random matrix theory, we derive exact expressions for all n-point spectral correlation functions of Dirac operator eigenvalues in terms of finite-volume partition functions. This is done for both chiral symplectic and chiral unitary random matrix ensembles, which correspond to $SU(N_c \geq 3)$ gauge theories with N_f fermions in the adjoint and fundamental representations, respectively. In the latter case we infer from this an infinite sequence of consistency conditions that must be satisfied by the corresponding finite-volume partition functions.

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The computation of finite-volume partition functions, and in particular the finite-size scaling of Dirac-operator eigenvalue correlations, has been very elegantly phrased in terms of certain random matrix theory distributions [1] which have turned out to be universal [2, 3, 4]. This has led to a highly increased understanding of field theories with spontaneous chiral symmetry breaking (such as QCD) in what is called the mesoscopic scaling regime of finite volumes, a study that in this particular context was initiated by the work of Leutwyler and Smilga [5] (see also ref. [6] for generalizations). The central idea is that a scaling region exists in which the correlations of rescaled Dirac eigenvalues are exactly computable. As the volume is taken to infinity, the scale of magnification is correspondingly increased. There is already evidence from lattice gauge theory simulations [7] that universal, exact, scaling functions are reached in this limit.

Recently, there has been a flurry of activity related to both proofs of universality [2, 4], and in general to the extension of these results to the double-microscopic scaling regime in which both fermion masses and Dirac eigenvalues are rescaled at the same rate in the large-volume limit [3, 8, 9]. One essential observation in this connection is that the relevant Dirac eigenvalue distributions are also computable directly from the field theoretic finite-volume partition functions, without having to go through the random matrix theory formulation [10, 11]. The new ingredient needed is a knowledge of finite-volume partition functions with additional fermion species, the masses of these additional fermions taking the rôles of Dirac eigenvalues in the original theory.

In this paper we shall derive a novel set of relations which provide double-microscopic spectral correlators in terms of suitably extended finite-volume partition functions. Our tool shall again be random matrix theory, but, as before [10, 11], the final expressions will involve only finite-volume partition functions, without reference to random matrix theory. We shall do this for both the cases corresponding to chiral symplectic random matrix theory ensembles ($SU(N_c \geq 3)$ gauge theories coupled to N_f fermions in the adjoint representation), and those corresponding to chiral unitary ensembles ($SU(N_c \geq 3)$ gauge theories coupled to N_f fermions in the fundamental representation). Surprisingly, in the latter case the expressions we get are very different from those one obtains by using factorization of correlation functions in terms of the unitary kernel (which also can be represented directly in terms of extended partition functions). In fact, while the factorization formula shows that all higher correlation functions can be obtained by means of the kernel (which has been shown to be related to the partition function with just two additional fermion species [10]), the new relations involve in this case the partition functions with, for k-point correlation functions, $2k$ additional fermions. This in turn implies highly stringent consistency conditions these finite-volume partition functions must satisfy.

Before we turn to the consistency conditions, we first describe the derivation of new relations between Dirac eigenvalue correlators and finite-volume partition functions. Our starting point is the (chiral) random matrix formulation [1]:

$$\tilde{\mathcal{Z}}_\nu^{(N_f,\beta)}(m_1,\ldots,m_{N_f}) \;=\; \int dW \prod_{f=1}^{N_f} \det\left(iM+m_f\right)\; \exp\left[-\frac{N\beta}{4}\mathrm{tr}\, V(M^2)\right] \;, \tag{1}$$

with

$$M \;=\; \left(\begin{array}{cc} 0 & W^\dagger \\ W & 0 \end{array}\right) \;, \tag{2}$$

and where β from now on labels the matrix ensemble. Thus, $\beta=4$ corresponds to the symplectic ensemble, and $\beta=2$ to the unitary ensemble. The matrices W are rectangular complex matrices of size $N \times (N+\nu)$, and they are integrated over with the Haar measure. The space-time volume V of the finite-volume gauge theory is, in the large-N limit, identified with $2N$. The topological index ν, for convenience always taken to be non-negative here, also counts the number of zero modes of the matrix M.

Written in terms of the eigenvalues λ_i of the hermitian matrix $W^\dagger W$, the partition functions $\tilde{\mathcal{Z}}_\nu^{(N_f,\beta)}$ are (ignoring unimportant overall factors):

$$\tilde{\mathcal{Z}}_\nu^{(N_f,\beta)}(m_1,\ldots,m_{N_f}) \;=\; \prod_{f=1}^{N_f}(m_f^\nu)\int_0^\infty \prod_{i=1}^{N}\left(d\lambda_i\ \lambda_i^{\frac{\beta}{2}\nu+\frac{\beta}{2}-1}\prod_{f=1}^{N_f}(\lambda_i+m_f^2)\ \mathrm{e}^{-\frac{N\beta}{2}V(\lambda_i)}\right)\left|\det_{ij}\lambda_j^{i-1}\right|^\beta \ . \tag{3}$$

We will treat the chiral unitary ($\beta=2$) and the chiral symplectic ($\beta=4$) ensembles in the same fashion in what follows. In principle most of the relations carry over to the chiral orthogonal ($\beta=1$) ensemble (which corresponds to $SU(2)$ gauge theory with N_f fermions in the fundamental representation) as well, but the final steps where we identify finite-volume field theory partition functions require β to be even. For this reason β is restricted to the values 2 and 4 in the following.

Let us define

$$\rho^{(N_f,\nu,\beta)}(\lambda_1,\ldots,\lambda_N;m_1,\ldots,m_{N_f}) \;\equiv\; \frac{1}{\tilde{\mathcal{Z}}_\nu^{(N_f)}(\{m_f\})}\prod_f^{N_f}(m_f^\nu)\prod_i^N w_\beta(\lambda_i)\prod_{j<l}^N|\lambda_j-\lambda_l|^\beta\ , \tag{4}$$

where

$$w_\beta(\lambda) \;=\; \lambda^{\frac{\beta}{2}\nu+\frac{\beta}{2}-1}\prod_f^{N_f}(\lambda+m_f^2)\mathrm{e}^{-\frac{N\beta}{2}V(\lambda)}\ . \tag{5}$$

Definition (4) is proportional to the integrand of the partition function eq. (3). In the last term we have rewritten the Vandermonde determinant in the standard way. All correlation functions with $k<N$ can now be obtained from the density of eq. (4) by integrating out a suitable number of eigenvalues:

$$\begin{aligned}
\rho^{(N_f,\nu,\beta)}(\lambda_1,\ldots,\lambda_k;\{m_f\}) \;&=\; \int_0^\infty\prod_{i=k+1}^N(d\lambda_i)\ \rho^{(N_f,\nu,\beta)}(\lambda_1,\ldots,\lambda_N;m_1,\ldots,m_{N_f})\\
&=\; \frac{1}{\tilde{\mathcal{Z}}_\nu^{(N_f)}(\{m_f\})}\prod_f^{N_f}(m_f^\nu)\prod_i^k w_\beta(\lambda_i)\prod_{j<l}^k|\lambda_j-\lambda_l|^\beta\\
&\quad\times\int_0^\infty\prod_{i=k+1}^N\left(d\lambda_i\ w_\beta(\lambda_i)\prod_{j=1}^k|\lambda_i+(i\sqrt{\lambda_j})^2|^\beta\right)\prod_{k+1\le j<l\le N}|\lambda_j-\lambda_l|^\beta\\
&=\; \prod_i^k\left((i\sqrt{\lambda_i})^{-\beta\nu}w_\beta(\lambda_i)\right)\prod_{j<l}^k|\lambda_j-\lambda_l|^\beta\,\frac{\tilde{\mathcal{Z}}_\nu^{(N_f+\beta k)}(\{m_f\};\{i\sqrt{\lambda_j}\})}{\tilde{\mathcal{Z}}_\nu^{(N_f)}(\{m_f\})}\ .
\end{aligned} \tag{6}$$

In the first step of the calculation we have taken the weight functions $w_\beta(\lambda_{i=1,\ldots,k})$ out of the integral and split the Vandermonde determinant into a prefactor, the additional mass terms[1], and a remaining

[1]This is the precise point where the considerations do not immediately carry over the $\beta=1$ case, since in that case we cannot disregard the absolutely value of the Vandermonde determinant. There is therefore no immediately obvious way of writing it directly in terms of massive partition functions.

Vandermonde determinant. In the second step we have disregarded, in the large-N limit, the difference between the integral for $N-k$ eigenvalues and the matrix model partition function of N eigenvalues. It contains βk additional imaginary masses $i\sqrt{\lambda_1}, i\sqrt{\lambda_2}, \ldots, i\sqrt{\lambda_k}$, each of which is β-fold degenerate.

We now go back to the original picture, in which we seek correlators of eigenvalues z_i of the Dirac operator, with $\lambda_i = z_i^2$. We also go to the double-microscopic limit in which $\zeta_i \equiv z_i N 2\pi\rho(0)$ and $\mu_f \equiv m_f N 2\pi\rho(0)$ are kept fixed as $N\to\infty$. All factors of $\exp[-\frac{N\beta}{4}V(\zeta^2)]$ in the measure $w_\beta(\zeta_i^2)$ standing outside the integral in (6) become replaced by unity in this limit, and by identifying $\Sigma = 2\pi\rho(0)$, we can now compare with the field theory finite-volume partition functions. We then obtain the following expression for the density correlators of the scaled ζ_i-variables:

$$\rho_S^{(N_f,\nu,\beta)}(\zeta_1,\ldots,\zeta_k;\mu_1,\ldots,\mu_{N_f}) = C_\beta^{(k)} \prod_i^k \left(\zeta_i^{\beta-1} \prod_f^{N_f} (\zeta_i^2+\mu_f^2) \right) \prod_{j<l}^k |\zeta_j^2-\zeta_l^2|^\beta \times \frac{\mathcal{Z}_\nu^{(N_f+\beta k)}(\mu_1,\ldots,\mu_{N_f};\{i\zeta_1\},\ldots,\{i\zeta_k\})}{\mathcal{Z}_\nu^{(N_f)}(\mu_1,\ldots,\mu_{N_f})}, \quad (7)$$

where each additional mass $i\zeta_j$ is β-fold degenerate. The overall proportionality constant $C_\beta^{(k)}$ is of course not given *a priori*, and has to be fixed by a matching condition. Moreover, the proportionality constant could in principle depend on k, as indicated.

Let us now consider the cases $\beta=4$ and $\beta=2$ separately, beginning with the case of $\beta=4$. As we have explained elsewhere [11], a subset of the spectral correlations derived above also follow from some of the general theorems that have been proven by Mahoux and Mehta [12] using the quaternion formalism. The difficulty there is that the quantity $f_4(\lambda_i,\lambda_j)$, which corresponds to the kernel of the skew-orthogonal polynomials, is now a *quaternion*. It can be represented by a 2×2 matrix. The correlation functions of eigenvalues are then given by quaternion determinants $\det[f_4(\lambda_i,\lambda_j)]_m$ of the kernel $f_4(\lambda_i,\lambda_j)$. We have not been able to express this kernel itself in terms of matrix model (and thus, in the double-microscopic scaling limit, finite-volume field theory) partition functions, but one can easily express the determinant of this kernel in terms of partition functions. For instance, using Theorem 1.2 of ref. [12] we are immediately led to eq. (7) for $k=1$, and also the density-density correlator ($k=2$) can be derived in an analogous way [11]. But the relation (7) is of course far more general.

The chiral unitary ensemble (eq. (7) with $\beta=2$) is actually at present far more interesting, since we in that case already have an alternative description of the same spectral correlators. This is summarized by the master formula for the kernel [10, 11],

$$K_S^{(N_f,\nu)}(\zeta,\zeta';\mu_1,\ldots,\mu_{N_f}) = (-1)^{\nu+[N_f/2]}\sqrt{\zeta\zeta'}\prod_f^{N_f}\sqrt{(\zeta^2+\mu_f^2)(\zeta'^2+\mu_f^2)}\ \frac{\mathcal{Z}_\nu^{(N_f+2)}(\mu_1,\ldots,\mu_{N_f},i\zeta,i\zeta')}{\mathcal{Z}_\nu^{(N_f)}(\mu_1,\ldots,\mu_{N_f})} \quad (8)$$

from which all higher k-point correlation functions follow:

$$\rho_S^{(N_f,\nu)}(\zeta_1,\ldots,\zeta_k;\mu_1,\ldots,\mu_{N_f}) = \det_{1\le a,b\le k} K_S^{(N_f,\nu)}(\zeta_a,\zeta_b;\mu_1,\ldots,\mu_{N_f})\ . \quad (9)$$

A quick glance reveals that these two description (eq. (7) for $\beta=2$, and eq. (9)) are very different for $k\neq 1$. For $k=1$ the two expressions agree up to the overall constant $C_2^{(1)}$, which thus is fixed in that case:

$$C_2^{(1)} = (-1)^{\nu+[N_f/2]}\ . \quad (10)$$

For higher k-point correlation functions, the two alternative descriptions imply non-trivial consistency conditions for the partition functions involved. Surprisingly, we see that these conditions must relate the finite-volume partition functions *with a different number of fermion species* to each other. The relations become particularly transparent if we first analytically continue the additional ("fictitious") fermions masses onto physical values by $\zeta_j \to -i\zeta_j$. Then we immediately obtain the following infinite sequence of consistency conditions:

$$\det_{1\leq a,b\leq k}\left[\sqrt{\zeta_a\zeta_b}\prod_{f=1}^{N_f}\sqrt{(\mu_f^2-\zeta_a^2)(\mu_f^2-\zeta_b^2)}\ \mathcal{Z}_\nu^{(N_f+2)}(\mu_1,\ldots,\mu_{N_f},\zeta_a,\zeta_b)\right] =$$
$$C_2^{(k)}(-1)^{k(\nu+[N_f/2]+1)}\prod_i^k\left(\zeta_i\prod_{f=1}^{N_f}(\mu_f^2-\zeta_i^2)\right)\prod_{j<l}^k|\zeta_j^2-\zeta_l^2|^2\ \frac{\mathcal{Z}_\nu^{(N_f+2k)}(\mu_1,\ldots,\mu_{N_f},\{\zeta_1\},\ldots,\{\zeta_k\})}{\mathcal{Z}_\nu^{(N_f)}(\mu_1,\ldots,\mu_{N_f})^{1-k}} \tag{11}$$

In this case the finite-volume partition functions are thus highly constrained by relations that link theories with N_f+2k fermions to those of N_f+2 and N_f fermions. These relations become quite involved for increasing values of k. There are known exact expressions for the finite-volume partition functions for this case, which corresponds to $SU(N_c\geq 3)$ gauge theories with N_f fermions in the fundamental representation [13]:

$$Z_\nu^{(N_f)}(\mu_1,\ldots,\mu_{N_f}) \ = \ \frac{\det\mathcal{A}(\{\mu_i\})}{\Delta(\mu^2)}\ , \tag{12}$$

where the $N_f\times N_f$ matrix $\mathcal{A}(\{\mu_i\})$ conveniently can be written [10]

$$\mathcal{A}_{ij} \ = \ \mu_i^{j-1}I_{\nu+j-1}(\mu_i)\ , \tag{13}$$

and where $\Delta(\mu^2)$ stands for the Vandermonde determinant of the squared masses μ_i^2. Using this explicit representation, we have verified in a few of the simpler cases that these consistency conditions indeed are satisfied.

One of the surprising consequences of the connection to random matrix theory is that the finite-volume field theory partition functions can be used directly to compute the universal double-microscopic limits of those orthogonal polynomials that are associated with the random matrix technique [11]. While there at the moment is no obvious interpretation of these orthogonal polynomials in field theory terms, it is interesting to note that the connection between the kernel

$$\begin{aligned}K_S^{(N_f,\nu)}(\zeta,\zeta';\{\mu_f\}) \ &= \ C_2(\zeta\zeta')^{\nu+\frac{1}{2}}\prod_f^{N_f}\sqrt{(\zeta^2+\mu_f^2)(\zeta'^2+\mu_f^2)}\frac{1}{\zeta^2-\zeta'^2}\\ &\times\ \left[P_{N-1}^{(N_f,\nu)}(\zeta^2;\{\mu_f\})P_N^{(N_f,\nu)}(\zeta'^2;\{\mu_f\})-P_N^{(N_f,\nu)}(\zeta^2;\{\mu_f\})P_{N-1}^{(N_f,\nu)}(\zeta'^2;\{\mu_f\})\right]\end{aligned} \tag{14}$$

and these orthogonal polynomials provide us with yet more consistency conditions that must be imposed on the finite-volume partition functions. We have already reproduced the relation (8) between the kernel and the partition functions. We now compare this with the corresponding relation for the double-microscopic limit of the orthogonal polynomials [11],

$$P_N^{(N_f,\nu)}(\zeta^2;\mu_1,\ldots,\mu_{N_f}) \ = \ C_3(-1)^N(i\zeta)^{-\nu}\ \frac{\mathcal{Z}_\nu^{(N_f+1)}(\mu_1,\ldots,\mu_{N_f},i\zeta)}{\mathcal{Z}_\nu^{(N_f)}(\mu_1,\ldots,\mu_{N_f})}\ , \tag{15}$$

where the normalization constant C_3 is left unspecified.[2] Inserting eq. (15) into eq. (14) and expanding in $1/N$ we obtain the following set of consistency conditions

$$\mathcal{Z}_\nu^{(N_f+2)}(\{\mu_f\},\zeta,\zeta') = \frac{C}{(\zeta'^2-\zeta^2)\mathcal{Z}_\nu^{(N_f)}(\{\mu_f\})} \times \left[\left((\sum_f^{N_f}\mu_f\partial_{\mu_f}+\zeta\partial_\zeta)\mathcal{Z}_\nu^{(N_f+1)}(\{\mu_f\},\zeta)\right)\mathcal{Z}_\nu^{(N_f+1)}(\{\mu_f\},\zeta') - (\zeta\leftrightarrow\zeta')\right] \tag{16}$$

where we have again rotated back to real fermion masses. There is yet another relation from random matrix models among the orthogonal polynomials themselves, which relates the polynomials with N_f massive flavors to those with N_f+1 [3]. Surprisingly enough this relation leads to precisely the same consistency conditions eq. (16). One can fix the proportionality constant by tracing it back to the matching condition between the double-microscopic spectral density (the kernel evaluated at coincident points), but we leave it here unspecified since the proportionality of the left and right hand sides of eq. (16) already gives a highly non-trivial series of conditions. Using the explicit expression eq. (12) we have verified in the first few cases that the relations of eq. (16) are satisfied. Taking the consistency conditions eqs. (11) and (16) together the finite-volume partition functions for theories with N_f+2k fermions are now given only in terms of those of N_f+1 and N_f fermions.

The results presented above trivially carry over from the chiral unitary ensemble to the ordinary unitary ensemble, which has been conjectured to describe $SU(N_c \geq 3)$ gauge theories with an *even* number of fermions N_f in (2+1) dimensions [1]. The partition function of that ensemble is

$$\tilde{\mathcal{Z}}^{(N_f)}(m_1,\ldots,m_{N_f}) = \int dM \prod_{f=1}^{N_f}\det(M+im_f)\ \exp(-N\mathrm{tr}\,V(M^2)) , \tag{17}$$

where the integration is over the Haar measure of hermitian $N\times N$ matrices M, and where masses are grouped into pairs of opposite signs:

$$\mathrm{diag}(m_1,m_2,\ldots,m_{N_f/2},-m_1,-m_2,\ldots,-m_{N_f/2}) .$$

In terms of the eigenvalues λ_i of the hermitian matrix M this gives:

$$\tilde{\mathcal{Z}}^{(N_f)}(m_1,\ldots,m_{N_f}) = \int_{-\infty}^{\infty}\prod_{i=1}^{N}\left(d\lambda_i\prod_{f=1}^{N_f/2}(\lambda_i^2+m_f^2)\ \mathrm{e}^{-NV(\lambda_i^2)}\right)\left|\det_{ij}\lambda_j^{i-1}\right|^2 , \tag{18}$$

where we again have ignored all irrelevant overall factors.

We then immediately get an analogous sequence of consistency conditions for the finite-volume partition functions of this ensemble. The kernel in this case follows from the master formula [11]

$$K_S^{(N_f)}(\zeta,\zeta';\mu_1,\ldots,\mu_{N_f}) = \frac{1}{2\pi}\prod_f^{N_f/2}\sqrt{(\zeta^2+\mu_f^2)(\zeta'^2+\mu_f^2)}\ \frac{\mathcal{Z}^{(N_f+2)}(\mu_1,\ldots,\mu_{N_f},i\zeta,i\zeta')}{\mathcal{Z}^{(N_f)}(\mu_1,\ldots,\mu_{N_f})} . \tag{19}$$

[2] This overall constant simply specifies the normalization of the polynomials, and we easily fix it once we choose the prescription (monic, or otherwise). However, there is no need to fix this constant here.

The analogue of eq. (11) therefore becomes

$$\det_{1\leq a,b\leq k}\left[\prod_{f=1}^{N_f/2}\sqrt{(\mu_f^2-\zeta_a^2)(\mu_f^2-\zeta_b^2)}\ \mathcal{Z}^{(N_f+2)}(\mu_1,\ldots,\mu_{N_f},\zeta_a,\zeta_b)\right]=$$

$$\tilde{C}^{(k)}(2\pi)^k\prod_i^k\left(\prod_{f=1}^{N_f/2}\left(\mu_f^2-\zeta_i^2\right)\right)\prod_{j<l}^k|\zeta_j-\zeta_l|^2\frac{\mathcal{Z}^{(N_f+2k)}(\mu_1,\ldots,\mu_{N_f},\{\zeta_1\},\ldots,\{\zeta_k\})}{\mathcal{Z}^{(N_f)}(\mu_1,\ldots,\mu_{N_f})^{1-k}}\ . \qquad (20)$$

The representation of the involved finite-volume field theory partition functions in terms of group manifold integrals was given by Verbaarschot and Zahed in the third paper of ref. [1], and explicitly worked out in ref. [3] (for N_f even). A relation similar to eq. (16) between partition functions with odd and even N_f can be worked out as well. The corresponding orthogonal polynomials in terms of partition functions have been given in ref. [11].

To conclude: We have extended the analysis of refs. [10, 11] to the case of higher k-point correlation functions of Dirac eigenvalues in terms of finite-volume partition functions for $SU(N_c\geq 3)$ gauge theories coupled to N_f fermions in both the fundamental and adjoint representations. For the case of adjoint fermions, relations for higher k-point spectral correlators in terms of finite-volume partition functions are new. For the case of fundamental fermions, we have made the derivation without going through the factorization formalism based on the (chiral) unitary kernels. This has allowed us to establish infinite sequences of consistency conditions for the involved partition functions. We have also shown how a different sequence of consistency conditions arise from comparing the expression for the orthogonal polynomials with that of the kernel. This new set of consistency conditions involves both the partition functions themselves, and their derivatives. All of these relations share the remarkable property of being easily derived on the basis of the connection to random matrix theory, while their origin in proper field theory terms remain obscure at present. It is a challenge to explain these relations at the level of effective Lagrangians in the finite-volume "mesoscopic" scaling regime.

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