Distributed H_∞ Filtering for Polynomial Nonlinear Stochastic Systems in Sensor Networks

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***Abstract*—In this paper, the distributed H_∞ filtering problem is addressed for a class of polynomial nonlinear stochastic systems in sensor networks. For a Lyapunov function candidate whose entries are polynomials, we calculate its first- and second-order derivatives in order to facilitate the use of Itô's differential role. Then, a sufficient condition for the existence of a feasible solution to the addressed distributed H_∞ filtering problem is derived in terms of parameter-dependent linear matrix inequalities (PDLMIs). For computational convenience, these PDLMIs are further converted into a set of sums of squares (SOSs) that can be solved effectively by using the semidefinite programming technique. Finally, a numerical simulation example is provided to demonstrate the effectiveness and applicability of the proposed design approach.**

***Index Terms*—Sensor networks, stochastic systems, polynomial systems, distributed H_∞ filtering, sum of squares, parameter-dependent linear matrix inequalities.**

I. Introduction

FILTERING or state estimation problem has long been one of the fundamental problems in signal processing, communications and control application [1], [14], [18]. The Kalman filtering approach is widely recognized as one of the most effective ways to deal with such estimation problems. In contrast with the classical Kalman filtering approach, the H_∞ filtering technology has the advantage of being able to provide a bound for the worst-case estimation error without the need for knowledge of noise statistics. Therefore, in the past few decades, significant advances have been made in the analysis and synthesis of H_∞ filters, see e.g. [9], [22], [25].

The nonlinearity and stochasticity are arguably two of the main resources in reality that have contributed to the system complexity. As a result, an increasing research attention has been devoted to the H_∞ filtering problem for nonlinear stochastic systems. For example, in [20], the H_∞ filtering problems have been investigated for a general class of discrete-time nonlinear stochastic systems, and a great deal of effort has been devoted in [26] to study the H_∞ filtering problem for continuous-time stochastic systems with a very general form. In these papers, the solutions to the H_∞ filtering problems have been characterized in terms of Hamilton-Jacobi-Isaacs inequalities that are somewhat difficult to solve.

As a well-known fact, there exists a rather general class of nonlinear functions which can be approximated by polynomials via the Taylor expansion centered in one point of interest, and the introduced conservatism (or approximation error) can be reduced by increasing the degree of the polynomials. Instead of working on general nonlinear systems, one could investigate the corresponding polynomial systems with help from the theories of positive polynomial and sum of squares (SOS) expressions [3]. Actually, for many stability issues, one needs to establish positivity of some functions such as the Lyapunov functions. For polynomial functions, such a task can be simplified by testing if the function is a SOS of polynomials. Recently, some researchers have directly formulated the desired solution by means of parameter-dependent linear matrix inequalities (PDLMIs), where the dependence on the parameter is polynomial. These PDLMIs can be solved by utilizing some available SOS solvers.

Sensor networks have recently received increasing interests due to their extensive application in areas such as information collection, environmental monitoring, industrial automation and intelligent buildings [6], [12]. Consequently, the problem of distributed filtering or estimation for sensor networks has gained considerable research attention and some novel distributed filters have been reported, see e.g. [2]. In addition, the consensus-based distributed filtering technology has been developed in parallel to the rapid development of multi-agent consensus control theory, For example, a distributed filter has been introduced in [15] that allows the nodes of a sensor network to track the average of n sensor measurements using an average consensus based distributed filter called consensus filter. The distributed Kalman filtering (DKF) problem considered in [19] has also been based on the average consensus, where the node hierarchy has been used with nodes performing different types of processing and communications.

Looking into the issues discussed above, a thorough literature search reveals that the distributed nonlinear H_∞ filtering problem has so far received very little attention despite its importance in signal processing and sensor networks, and this gives rise to the main motivation for our current investigation. In this paper, we aim to make one of very first few attempts to address the distributed H_∞ filtering problem for a class of polynomial nonlinear stochastic systems that are represented in a state-dependent linear-like form. By choosing a general

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polynomial Lyapunov functional, sufficient conditions are established for the existence of the distributed H_∞ filters, and the desired distributed H_∞ filters can be designed in terms of PDLMIs. When the polynomial system is degenerated to a linear system, it is shown that these PDLMIs can be reduced to the numerically more tractable linear matrix inequalities (LMIs). Then, we proceed to derive the solution to the PDLMIs by solving the problem of the corresponding SOS decomposition with the aid of available SOS solvers. Finally, an illustrative simulation example is provided.

Notation The notation used here is fairly standard except where otherwise stated. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. $\|A\|$ refers to the norm of a matrix A defined by $\|A\| = \sqrt{\text{trace}(A^T A)}$. The notation $X \geq Y$ (respectively, $X > Y$), where X and Y are real symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). Sym$\{A\}$ denotes the symmetric matrix $A + A^T$. I_n represents the identity matrix of dimension n. diag$_n\{A_i\}$ stands for a block-diagonal matrix with the ith diagonal element being A_i and the notation vec$_n\{x_i\}$ denotes $\begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix}$. Moreover, let ($\Omega$, $\mathscr{F}$, $\{\mathscr{F}_t\}_{t\geq 0}$, P) be a complete probability space with a filtration $\{\mathscr{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., it is right continuous and contains all P-null sets). $\mathbb{E}\{x\}$ stands for the expectation of the stochastic variable x with respect to the given probability measure P. Denoted by $L_2([0,\infty),\mathbb{R}^n)$ the space of non-anticipatory square integrable n-dimensional vector-valued stochastic process $f(\cdot) = \{f(t)\}_{t\geq 0}$ with respect to $\{\mathscr{F}_t\}_{t\geq 0}$ with the norm $\|f\|_{L_2} = \{\mathbb{E}\int_0^{+\infty} \|f(t)\|^2 dt\}^{\frac{1}{2}}$. In symmetric block matrices, "$*$" is used as an ellipsis for terms induced by symmetry. Matrices, if they are not explicitly specified, are assumed to have compatible dimensions.

II. Problem Formulation

Consider the following polynomial nonlinear Itô-type stochastic systems (the time variable t is suppressed for simplicity):

$$\begin{cases} dx = f(x)dt + g(x)vdt + f_w(x)dw, \\ \ \ z = m(x), \end{cases} \tag{1}$$

with n sensors modeled by:

$$y_i = l_i(x) + s_i(x)v, \quad i = 1, 2, \cdots, n \tag{2}$$

where $x \in \mathbb{R}^{n_x}$ is the state vector, $z \in \mathbb{R}^{n_z}$ is the signal to be estimated, $y_i \in \mathbb{R}^{n_y}$ is the measurement output measured by sensor i from the plant, w is a standard one-dimensional Brownian motion defined on (Ω, $\mathscr{F}$, P), and $v \in \mathbb{R}^{n_v}$ is the exogenous disturbance input belonging $L_2([0,\infty),\mathbb{R}^{n_v})$.

The nonlinear functions $f(x)$, $g(x)$, $f_w(x)$, $m(x)$, $l_i(x)$, and $s_i(x)$ ($i = 1, 2, \cdots, n$) are polynomial functions in x, which can be written as the following state-dependent linear-like form:

$$\begin{aligned} &f(x) = F(x)x,\ g(x) = G(x),\ f_w(x) = F_w(x)x, \\ &l_i(x) = L_i(x)x,\ s_i(x) = S_i(x),\ m(x) = M(x)x, \end{aligned} \tag{3}$$

where $F(x) \in \mathbb{R}^{n_x\times n_x}$, $G(x) \in \mathbb{R}^{n_x\times n_v}$, $F_w(x) \in \mathbb{R}^{n_x\times n_x}$, $L_i(x) \in \mathbb{R}^{n_y\times n_x}$, $S_i(x) \in \mathbb{R}^{n_y\times n_v}$ and $M(x) \in \mathbb{R}^{n_z\times n_x}$ are polynomial matrices in x.

In this paper, it is assumed that the n sensor nodes are distributed in space according to a fixed network topology represented by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ of order n with the set of nodes (sensors) $\mathcal{V} = \{1, 2, \cdots, n\}$, set of edges $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$, and an adjacency matrix $\mathcal{A} = [a_{ij}]$. An edge of $\mathcal{G}$ is denoted by (i, j). The adjacency elements associated with the edges of the graph are positive, i.e., $a_{ij} > 0 \iff (i, j) \in \mathcal{E}$. Moreover, $a_{ii} = 1$ for all $i \in \mathcal{V}$. The set of neighbors of node $i \in \mathcal{V}$ plus the node itself is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}\}$. Also, in the sensor network, it is assumed that each sensor node can receive the information from its neighboring nodes according to the given network topology. The information considered here consists of the neighboring measurements and estimates at current time.

The following filter structure is adopted on sensor node i:

$$\begin{cases} d\hat{x}_i = \sum_{j\in\mathcal{N}_i} \hat{K}_{ij} a_{ij} \hat{x}_j dt + \sum_{j\in\mathcal{N}_i} \hat{H}_{ij} a_{ij} y_j dt \\ \ \ \hat{z}_i = \hat{M}_i \hat{x}_i \end{cases} \tag{4}$$

where $\hat{x}_i \in \mathbb{R}^{n_x}$ and $\hat{z}_i \in \mathbb{R}^{n_z}$ are, respectively, the estimates for x and z on the node i, $\hat{K}_{ij} \in \mathbb{R}^{n_x\times n_x}$, $\hat{H}_{ij} \in \mathbb{R}^{n_x\times n_y}$ and $\hat{M}_i \in \mathbb{R}^{n_z\times n_x}$ are filter parameters to be determined. The initial values of filters are $\hat{x}_i(0) = 0$ for all $i = 1, 2, \cdots, n$.

Remark 1: The filter structure in (4) accounts for the communications between the underlying node and its neighboring nodes where the sensor nodes are distributed over a spatial region. Moreover, once all filters parameters are obtained, each filter is able to estimate the system state independently according to (4), which merits the "distributed" feature of the filtering algorithm.

Remark 2: Note that a polynomial can always be written as the state-dependent linear-like form (3). Moreover, considering the issue of easily implementation, in this paper, we adopt the linear time-invariant filter (4) that can be readily designed in practical engineering. In the case that the dynamics of system (1) is fully dominated by the polynomial nonlinearities, an alternate strategy is to construct a filter that includes higher-order approximations of the polynomial system (1) by using the approach of Carleman-linearization (see e.g. [7], [17]) to improve the filtering quality.

Setting $e_i = x - \hat{x}_i$ and $\tilde{z}_i = z - \hat{z}_i$, the following system that governs the filtering error dynamics for the sensor network can be obtained from (1) and (4):

$$\begin{cases} de_i = \Big(F(x) - \sum_{j\in\mathcal{N}_i} \hat{H}_{ij} a_{ij} L_j(x) - \sum_{j\in\mathcal{N}_i} \hat{K}_{ij} a_{ij}\Big) x dt \\ \qquad + \Big(G(x) - \sum_{j\in\mathcal{N}_i} \hat{H}_{ij} a_{ij} S_j(x)\Big) v dt \\ \qquad + \sum_{j\in\mathcal{N}_i} \hat{K}_{ij} a_{ij} e_j dt + F_w(x) x dw \\ \tilde{z}_i = \Big(M(x) - \hat{M}_i\Big) x + \hat{M}_i e_i. \end{cases} \tag{5}$$

Introduce the following notations that will be used in the sequel:

$$\begin{aligned}
&\bar{F}(x) = \mathrm{vec}_n^T\{F^T(x)\}, \quad \bar{G}(x) = \mathrm{vec}_n^T\{G^T(x)\},\\
&\bar{F}_w(x) = \mathrm{vec}_n^T\{F_w^T(x)\},\ \bar{M}(x) = \mathrm{vec}_n^T\{M^T(x)\},\\
&\bar{L}(x) = \mathrm{vec}_n^T\{L_i^T(x)\}, \quad \bar{S}(x) = \mathrm{vec}_n^T\{S_i^T(x)\},\\
&\mathbb{I}_I = \mathrm{vec}_n^T\{I_{n_x}\}, \quad \tilde{M} = \mathrm{diag}_n\{\hat{M}_i\},\\
&e = \mathrm{vec}_n^T\{e_i^T\}, \quad \tilde{z} = \mathrm{vec}_n^T\{\tilde{z}_i^T\}.
\end{aligned} \tag{6}$$

Then, the error dynamics governed by (5) can be rewritten as the following compact form

$$\begin{cases} de = \Big(\bar{F}(x) - \bar{H}\bar{L}(x) - \bar{K}\mathbb{I}_I\Big)xdt + \bar{K}edt \\ \qquad + \Big(\bar{G}(x) - \bar{H}\bar{S}(x)\Big)vdt + \bar{F}_w(x)xdw, \\ \tilde{z} = \Big(\bar{M}(x) - \tilde{M}\mathbb{I}_I\Big)x + \tilde{M}e, \end{cases} \tag{7}$$

where

$$\bar{K} = [\hat{K}_{ij}a_{ij}]_{n\times n}, \quad \bar{H} = [\hat{H}_{ij}a_{ij}]_{n\times n} \tag{8}$$

are two sparse matrices satisfying $\bar{K} \in \mathscr{W}_{n_x\times n_x}$ and $\bar{H} \in \mathscr{W}_{n_x\times n_y}$, where $\mathscr{W}_{p\times q}$ is defined as

$$\begin{aligned}\mathscr{W}_{p\times q} = \{\bar{U} = &[U_{ij}] \in \mathbb{R}^{np\times nq} \\ &|U_{ij} \in \mathbb{R}^{p\times q},\ U_{ij} = 0 \ \text{ if } j \notin \mathcal{N}_i\}.\end{aligned} \tag{9}$$

Subsequently, by letting $\eta = \begin{bmatrix} x^T & e^T \end{bmatrix}^T$, the combination of (1) and (7) yields the following augmented system

$$\begin{cases} d\eta = \Big(\mathcal{F}(x)\eta + \mathcal{G}(x)v\Big)dt + \mathcal{F}_w(x)\eta dw, \\ \tilde{z} = \mathcal{M}(x)\eta, \end{cases} \tag{10}$$

where

$$\begin{aligned}
&\mathcal{F}(x) = \begin{bmatrix} F(x) & 0 \\ \bar{F}(x) - \bar{H}\bar{L}(x) - \bar{K}\mathbb{I}_I & \bar{K} \end{bmatrix},\\
&\mathcal{G}(x) = \begin{bmatrix} G(x) \\ \bar{G}(x) - \bar{H}\bar{S}(x) \end{bmatrix}, \mathcal{F}_w(x) = \begin{bmatrix} F_w(x) & 0 \\ \bar{F}_w(x) & 0 \end{bmatrix},\\
&\mathcal{M}(x) = \begin{bmatrix} \bar{M}(x) - \tilde{M}\mathbb{I}_I & \tilde{M} \end{bmatrix}.
\end{aligned} \tag{11}$$

Before proceeding, we introduce the following stability concepts for stochastic system (10).

Definition 1: [8] The zero-solution of the augmented system (10) with $v = 0$ is said to be globally asymptotically stable in probability if (i) for any $\varepsilon > 0$, $\lim_{\eta(0)\to 0} \mathrm{P}\{\sup_{t\geq 0} \|\eta(t)\| > \varepsilon\} = 0$; and (ii) for any initial condition $\eta(0)$, $\mathrm{P}\{\lim_{t\to\infty} \eta(t) = 0\} = 1$.

We are now ready to state the distributed H_∞ filtering problem as follows. In this paper, we are interested in seeking filter parameters $\hat{M}_i \in \mathbb{R}^{n_z\times n_x}$, $\hat{K}_{ij} \in \mathbb{R}^{n_x\times n_x}$, and $\hat{H}_{ij} \in \mathbb{R}^{n_x\times n_y}$ ($i = 1, 2, \cdots, n$, $j \in \mathcal{N}_i$) such that the following two requirements are simultaneously satisfied.

a) The zero-solution of the augmented system (10) with $v = 0$ is globally asymptotically stable in probability.
b) Under the zero-initial condition, the filtering error $\tilde{z}$ satisfies

$$\|\tilde{z}\|_{L_2} < \gamma\|v\|_{L_2} \tag{12}$$

for all nonzero v where $\gamma > 0$ is a given disturbance attenuation level.

III. Main Results

Let us start by dealing with the analysis problem for the stability and H_∞ performance of the polynomial nonlinear stochastic system (10). For this purpose, we select the following Lyapunov function candidate:

$$V(\eta) = \eta^T \mathcal{Q}(\eta)\eta, \tag{13}$$

where $\mathcal{Q}(\eta) \in \mathbb{R}^{d\times d}$ is a symmetrical polynomial matrix in $\eta \in \mathbb{R}^d$ that satisfies $\mathcal{Q}^T(\eta) = \mathcal{Q}(\eta) > 0$ for all η. Here, for notational convenience, we have written $d = (n+1)n_x$.

The following lemma gives the first- and second-order derivatives of the real-value function $V(\eta)$ with respect to the vector η. Note that such derivatives are crucial in using Itô formula for our stochastic analysis.

Lemma 1: Consider the real-valued function $V(\eta)$ defined in (13). The first- and second-order derivatives of the real-value function $V(\eta)$ with respect to the vector $\eta \in \mathbb{R}^d$ are given as follows:

$$\begin{aligned}
V_\eta(\eta) =& 2\eta^T\mathcal{Q}(\eta) + \eta^T\mathcal{D}_{\mathcal{Q}}(\eta)(I_d\otimes\eta)\\
V_{\eta\eta}(\eta) =& 2\mathcal{Q}(\eta) + 2\mathrm{Sym}\{\mathcal{D}_{\mathcal{Q}}(\eta)(I_d\otimes\eta)\}\\
&+ (I_d\otimes\eta^T)\mathcal{W}_{\mathcal{Q}}(\eta)(I_d\otimes\eta)
\end{aligned} \tag{14}$$

where

$$\mathcal{D}_{\mathcal{Q}}(\eta) = \left[\begin{pmatrix} \frac{\partial\mathcal{Q}_{11}}{\partial\eta_1} & \cdots & \frac{\partial\mathcal{Q}_{1d}}{\partial\eta_1} \\ \vdots & & \vdots \\ \frac{\partial\mathcal{Q}_{d1}}{\partial\eta_1} & \cdots & \frac{\partial\mathcal{Q}_{dd}}{\partial\eta_1} \end{pmatrix} \cdots \begin{pmatrix} \frac{\partial\mathcal{Q}_{11}}{\partial\eta_d} & \cdots & \frac{\partial\mathcal{Q}_{1d}}{\partial\eta_d} \\ \vdots & & \vdots \\ \frac{\partial\mathcal{Q}_{d1}}{\partial\eta_d} & \cdots & \frac{\partial\mathcal{Q}_{dd}}{\partial\eta_d} \end{pmatrix}\right]$$

$$\mathcal{W}_{\mathcal{Q}}(\eta) = \begin{bmatrix}
\begin{pmatrix} \frac{\partial^2\mathcal{Q}_{11}}{\partial\eta_1^2} & \cdots & \frac{\partial^2\mathcal{Q}_{1d}}{\partial\eta_1^2} \\ \vdots & & \vdots \\ \frac{\partial^2\mathcal{Q}_{d1}}{\partial\eta_1^2} & \cdots & \frac{\partial^2\mathcal{Q}_{dd}}{\partial\eta_1^2} \end{pmatrix} & \cdots & \begin{pmatrix} \frac{\partial^2\mathcal{Q}_{11}}{\partial\eta_d\partial\eta_1} & \cdots & \frac{\partial^2\mathcal{Q}_{1d}}{\partial\eta_d\partial\eta_1} \\ \vdots & & \vdots \\ \frac{\partial^2\mathcal{Q}_{d1}}{\partial\eta_d\partial\eta_1} & \cdots & \frac{\partial^2\mathcal{Q}_{dd}}{\partial\eta_d\partial\eta_1} \end{pmatrix} \\
\vdots & & \vdots \\
\begin{pmatrix} \frac{\partial^2\mathcal{Q}_{11}}{\partial\eta_1\partial\eta_d} & \cdots & \frac{\partial^2\mathcal{Q}_{1d}}{\partial\eta_1\partial\eta_d} \\ \vdots & & \vdots \\ \frac{\partial^2\mathcal{Q}_{d1}}{\partial\eta_1\partial\eta_d} & \cdots & \frac{\partial^2\mathcal{Q}_{dd}}{\partial\eta_1\partial\eta_d} \end{pmatrix} & \cdots & \begin{pmatrix} \frac{\partial^2\mathcal{Q}_{11}}{\partial\eta_d^2} & \cdots & \frac{\partial^2\mathcal{Q}_{1d}}{\partial\eta_d^2} \\ \vdots & & \vdots \\ \frac{\partial^2\mathcal{Q}_{d1}}{\partial\eta_d^2} & \cdots & \frac{\partial^2\mathcal{Q}_{dd}}{\partial\eta_d^2} \end{pmatrix}
\end{bmatrix} \tag{15}$$

Proof: The proof of this lemma follows from some straightforward algebraic manipulations, and is therefore omitted. ■

In the following theorem, a sufficient condition is derived to guarantee that the requirements a) and b) given in the previous section are simultaneously met.

Theorem 1: Let the filter parameters $\hat{M}_i \in \mathbb{R}^{n_z \times n_x}$, $\hat{K}_{ij} \in \mathbb{R}^{n_x \times n_x}$ and $\hat{H}_{ij} \in \mathbb{R}^{n_x \times n_y}$ $(i = 1, 2, \cdots, n,\ j \in \mathcal{N}_i)$ and the disturbance attenuation level $\gamma > 0$ be given. Then, the zero-solution of the augmented system (10) with $v = 0$ is globally asymptotically stable in probability and the filtering error $\tilde{z}$ satisfies the H_∞ performance constraint (12) for all nonzero exogenous disturbances under the zero-initial condition if, for all $\eta \in \mathbb{R}^d$, there exists a symmetric polynomial matrix $\mathcal{Q}(\eta)$ satisfying

$$\mathcal{Q}(\eta) > 0, \tag{16}$$

$$\begin{bmatrix} \Omega_1(\eta) & \Omega_2(\eta) \\ \Omega_2^T(\eta) & -\gamma^2 I_{n_v} \end{bmatrix} < 0, \tag{17}$$

where

$$\begin{aligned} \Omega_1(\eta) =& \mathrm{Sym}\{\mathcal{Q}(\eta)\mathcal{F}(x)\} + \mathcal{M}^T(x)\mathcal{M}(x) \\ &+ \frac{1}{2}\mathrm{Sym}\{\mathcal{D}_{\mathcal{Q}}(\eta)(I_d \otimes \eta)\mathcal{F}(x)\} \\ &+ \mathcal{F}_w^T(x)\mathcal{R}(\eta)\mathcal{F}_w(x), \\ \Omega_2(\eta) =& \mathcal{Q}(\eta)\mathcal{G}(x) + \frac{1}{2}\mathcal{D}_{\mathcal{Q}}(\eta)(I_d \otimes \eta)\mathcal{G}(x), \\ \mathcal{R}(\eta) =& \mathcal{Q}(\eta) + \mathrm{Sym}\{\mathcal{D}_{\mathcal{Q}}(\eta)(I_d \otimes \eta)\} \\ &+ \frac{1}{2}(I_d \otimes \eta^T)\mathcal{W}_{\mathcal{Q}}(\eta)(I_d \otimes \eta). \end{aligned} \tag{18}$$

Proof: Let us first show that the zero-solution of the nonlinear stochastic system (10) is globally asymptotically stable in probability when $v = 0$. By using Itô's formula, the stochastic differential of $V(\eta)$ defined as (13) along the trajectory of system (10) with $v = 0$ is given by

$$dV(\eta) = \mathscr{L}_{v\equiv 0}V(\eta)dt + V_\eta(\eta)\mathcal{F}_w(x)\eta dw$$

where

$$\mathscr{L}_{v\equiv 0}V(\eta) = V_\eta(\eta)\mathcal{F}(x)\eta + \frac{1}{2}\eta^T\mathcal{F}_w^T(x)V_{\eta\eta}(\eta)\mathcal{F}_w(x)\eta.$$

By using Lemma 1 and noting that $\Omega_1(\eta) < 0$ is implied by (17), one can have

$$\mathscr{L}_{v\equiv 0}V(\eta) = \eta^T\Big(\Omega_1(\eta) - \mathcal{M}^T(x)\mathcal{M}(x)\Big)\eta < 0$$

which indicates that the system (10) with $v = 0$ is globally asymptotically stable in probability based on the Lyapunov stability theory for stochastic systems [8].

Next, we shall show that the filtering error $\tilde{z}$ satisfies the H_∞ performance constraint (12) under the zero initial condition. Adopting the same Lyapunov function $V(\eta)$ and using Itô's formula again, we can obtain the differential of $V(\eta)$ along the trajectory of system (10) as follows:

$$dV(\eta) = \mathscr{L}_vV(\eta)dt + V_\eta(\eta)\mathcal{F}_w(x)\eta dw \tag{19}$$

where

$$\begin{aligned} \mathscr{L}_vV(\eta) =& V_\eta(\eta)\Big(\mathcal{F}(x)\eta + \mathcal{G}(x)v\Big) \\ &+ \frac{1}{2}\eta^T\mathcal{F}_w^T(x)V_{\eta\eta}(\eta)\mathcal{F}_w(x)\eta. \end{aligned}$$

By integrating (19) from 0 to T with respect to t and taking expectation, one has

$$\mathbb{E}\Big\{V(\eta(T))\Big\} - \mathbb{E}\Big\{V(\eta(0))\Big\} = \mathbb{E}\Big\{\int_0^T \mathscr{L}_vV(\eta(t))dt\Big\}$$

by which, and together with $\eta(0) = 0$ and $V(\eta) \geq 0$, we have from (17) that

$$\begin{aligned} &\mathbb{E}\Big\{\int_0^T \big(\|\tilde{z}(t)\|^2 - \gamma^2\|v(t)\|^2\big)dt\Big\} \\ &= \mathbb{E}\Big\{\int_0^T \big(\|\tilde{z}(t)\|^2 - \gamma^2\|v(t)\|^2 + \mathscr{L}_vV(\eta(t))\big)dt\Big\} \\ &\quad -\mathbb{E}\Big\{V(\eta(T))\Big\} + \mathbb{E}\Big\{V(\eta(0))\Big\} \\ &\leq \mathbb{E}\Big\{\int_0^T \Big(\begin{bmatrix}\eta^T(t) & v^T(t)\end{bmatrix} \\ &\quad \times \begin{bmatrix} \Omega_1(\eta(t)) & \Omega_2(\eta(t)) \\ \Omega_2^T(\eta(t)) & -\gamma^2 I_{n_v} \end{bmatrix}\begin{bmatrix}\eta(t) \\ v(t)\end{bmatrix}\Big)dt\Big\} < 0. \end{aligned}$$

Letting $T \to +\infty$ in the above, the H_∞ performance in (12) follows immediately which ends the proof. ∎

Having conducted the performance analysis in Theorem 1, we are now in a position to deal with the problem of designing distributed H_∞ filters for polynomial nonlinear stochastic systems. Noticing that the matrices $\bar{H}$ and $\bar{K}$ consist of all desired filters parameters independent of variable η, we choose $\mathcal{Q}(\eta)$ as $\mathcal{Q}(\eta) = \mathrm{diag}\{Q(x), P\}$, where $Q(x) \in \mathbb{R}^{n_x \times n_x}$ is a symmetric polynomial matrix in x satisfying $Q^T(x) = Q(x) > 0$ for all x, and $P \in \mathbb{R}^{nn_x \times nn_x}$ is a constant positive definite matrix. Correspondingly, the differential matrices of $Q(x)$ with respect to x defined as the form of (15) are denoted by $D_Q(x)$ and $W_Q(x)$.

By using Schur complement and noting (11), it is easily shown that (17) is equivalent to

$$\begin{bmatrix} \Sigma_1(x) & * & * & * \\ \Sigma_2(x) & P\bar{K} + \bar{K}^TP & * & * \\ \Sigma_3(x) & \bar{G}^T(x)P - \bar{S}^T(x)\bar{H}^TP & -\gamma^2 I_{n_v} & * \\ \Sigma_4(x) & \tilde{M} & 0 & -I_{nn_z} \end{bmatrix} < 0 \tag{20}$$

where

$$\begin{aligned} \Sigma_1(x) =& \mathrm{Sym}\{Q(x)F(x)\} + \bar{F}_w^T(x)P\bar{F}_w(x) \\ &+ \frac{1}{2}\mathrm{Sym}\{D_Q(x)(I_{n_x} \otimes x)F(x)\} \\ &+ F_w^T(x)R(x)F_w(x), \\ \Sigma_2(x) =& P\bar{F}(x) - P\bar{H}\bar{L}(x) - P\bar{K}\mathbb{I}_I, \\ \Sigma_3(x) =& G^T(x)Q(x) + \frac{1}{2}G^T(x)(I_{n_x} \otimes x^T)D_Q^T(x), \\ \Sigma_4(x) =& \bar{M}(x) - \tilde{M}\mathbb{I}_I, \\ R(x) =& Q(x) + \mathrm{Sym}\{D_Q(x)(I_{n_x} \otimes x)\} \\ &+ \frac{1}{2}(I_{n_x} \otimes x^T)W_Q(x)(I_{n_x} \otimes x). \end{aligned} \tag{21}$$

It is observed that, due to the existence of nonlinear terms $P\bar{K}$ and $P\bar{H}$, condition (20) is not an LMI but a BMI (bilinear matrix inequality), which could lead to a nonconvex feasible set. In order to cast it into a solvable LMI, one alternative approach is to take $X = P\bar{K}$ and $Y = P\bar{H}$. To derive the constraints for X and Y, we introduce the following useful lemma.

Lemma 2: Let $P = \text{diag}\{P_1, P_2, \cdots, P_n\}$ with $P_i \in \mathbb{R}^{p\times p}$ $(1 \le i \le n)$ being invertible matrices. If $X = PW$ for $W \in \mathbb{R}^{np\times nq}$, then we have $W \in \mathscr{W}_{p\times q} \Longleftrightarrow X \in \mathscr{W}_{p\times q}$.

Based on Lemma 2, we can obtain the following theorem which shows that the addressed distributed filter design problem is solved for the polynomial nonlinear stochastic system (1) if a parameter-dependent LMI-like inequality is feasible.

Theorem 2: Let the disturbance attenuation level $\gamma > 0$ be given. The distributed H_∞ filtering problem is solved for polynomial nonlinear stochastic system (1) if there exist a symmetric polynomial matrix $Q(x)$, a set of constant positive definite matrices $P_i^T = P_i > 0$ $(i = 1, 2, \cdots, n)$, two constant matrices $X \in \mathscr{W}_{n_x\times n_x}$ and $Y \in \mathscr{W}_{n_x\times n_y}$, and a set of constant matrices $\hat{M}_i$ $(i = 1, 2, \cdots, n)$ such that

$$Q(x) > 0, \tag{22}$$

$$\Upsilon(x) < 0, \tag{23}$$

for all $x \in \mathbb{R}^{n_x}$, where

$$\Upsilon(x) = \begin{bmatrix} \Sigma_1(x) & * & * & * \\ \bar{\Sigma}_2(x) & X + X^T & * & * \\ \Sigma_3(x) & \bar{G}^T(x)P - \bar{S}^T(x)Y^T & -\gamma^2 I_{n_v} & * \\ \Sigma_4(x) & \tilde{M} & 0 & -I_{nn_z} \end{bmatrix},$$

$$\bar{\Sigma}_2(x) = P\bar{F}(x) - Y\bar{L}(x) - X\mathbb{I}_I,$$
$$P = \text{diag}\{P_1, P_2, \cdots, P_n\}, \tag{24}$$

and $\Sigma_1(x)$, $\Sigma_3(x)$, $\Sigma_4(x)$ are defined in (21). Moreover, if (22) and (23) are true, the desired parameters $\hat{M}_i$ $(i = 1, 2, \cdots, n)$ are directly derived, and parameters $\bar{K}$ and $\bar{H}$ are given by

$$\bar{K} = P^{-1}X, \quad \bar{H} = P^{-1}Y. \tag{25}$$

Accordingly, parameters $\hat{K}_{ij}$ and $\hat{H}_{ij}$ $(i = 1, 2, \cdots, n,\ j \in \mathcal{N}_i)$ can be derived from (8).

Proof: By setting $P = \text{diag}\{P_1, P_2, \cdots, P_n\}$ and noting $X = P\bar{K}$ and $Y = P\bar{H}$, the inequality (17) follows from (23) immediately, and (16) can be guaranteed by (22) as well as the positive definiteness of matrix P. In addition, from Lemma 2, it follows that $\bar{K} \in \mathscr{W}_{n_x\times n_x}$ and $\bar{H} \in \mathscr{W}_{n_x\times n_y}$. The rest of the proof can be easily accomplished by using Theorem 1. ■

Before we move onto the computational issue of handling PDLMIs obtained in Theorem 2, let us first show that these PDLMIs can be reduced to the numerically more tractable linear matrix inequalities (LMIs) when the polynomial system is degenerated to a linear system. Let the nonlinear system (1) be reduced to a linear system, i.e., $f(x)$, $g(x)$, $f_w(x)$, $m(x)$, $l_i(x)$, and $s_i(x)$ are taken as

$$\begin{aligned} f(x) = Fx, \quad g(x) = G, \quad f_w(x) = F_w x, \\ l_i(x) = L_i x, \quad s_i(x) = S_i, \quad m(x) = Mx. \end{aligned} \tag{26}$$

Choosing the Lyapunov matrix $Q(x)$ as a constant positive definite matrix Q, we obtain the following corollary immediately from Theorem 2.

Corollary 1: Let the disturbance attenuation level $\gamma > 0$ be given. The distributed H_∞ filtering problem is solved for linear stochastic system (1) with (26) if there exist a positive definite matrix $Q^T = Q > 0$, a set of positive definite matrices $P_i^T = P_i > 0$ $(i = 1, 2, \cdots, n)$, two matrices $X \in \mathscr{W}_{n_x\times n_x}$ and $Y \in \mathscr{W}_{n_x\times n_y}$ and a set of matrices $\hat{M}_i$ $(i = 1, 2, \cdots, n)$ such that

$$\begin{bmatrix} \text{Sym}\{QF\} + F_w^T Q F_w + \bar{F}_w^T P \bar{F}_w & * & * & * \\ P\bar{F} - Y\bar{L} - X\mathbb{I}_I & X + X^T & * & * \\ G^T Q & \bar{G}^T P - \bar{S}^T Y^T & -\gamma^2 I_{n_v} & * \\ \bar{M} - \tilde{M}\mathbb{I}_I & \tilde{M} & 0 & -I_{nn_z} \end{bmatrix} < 0, \tag{27}$$

where

$$\bar{F} = \text{vec}_n^T\{F^T\},\ \bar{F}_w = \text{vec}_n^T\{F_w^T\},\ \bar{S} = \text{vec}_n^T\{S_i^T\},$$
$$\bar{G} = \text{vec}_n^T\{G^T\},\ \bar{M} = \text{vec}_n^T\{M^T\},\ \bar{L} = \text{vec}_n^T\{L_i^T\},$$

and $\tilde{M}$ and P are defined in (6) and (24), respectively. Moreover, if (27) is true, the parameters $\hat{M}_i$ $(i = 1, 2, \cdots, n)$ are directly obtained and the parameters $\hat{K}_{ij}$ and $\hat{H}_{ij}$ $(i = 1, 2, \cdots, n,\ j \in \mathcal{N}_i)$ can be derived from (8) and (25).

Let us now discuss the PDLMIs, based on which the solution to the distributed H_∞ filtering synthesis problem is formulated in Theorem 2. In general, solving such PDLMIs involves an infinite set of LMIs and is therefore computationally hard. Fortunately, noting that $\Upsilon(x)$ is actually a polynomial matrix in x, we are motivated to employ the computational method relying on the SOS decomposition of multivariate polynomials to solve (22) and (23). For the convenience of the readers, in what follows, we first introduce some basic notions and necessary foundations on SOS theory.

Definition 2: For $x \in \mathbb{R}^l$, a multivariate polynomial $f(x)$ is a SOS if there exist polynomials $f_1(x), \cdots, f_m(x)$ such that $f(x) = \sum_{i=1}^m f_i^2(x)$.

Remark 3: Obviously, the degree of SOS polynomial is even. In [5], it has been shown that the polynomial with even degree $f(x)$ is a SOS if and only if there exists a positive semidefinite matrix Q such that $f(x) = Z^T(x)QZ(x)$, where $Z(x)$ is a column vector whose entries are all monomials in x with degree no greater than half of that of $f(x)$. Based on this, it is possible to numerically compute a SOS decomposition by using semidefinite programming.

The theory of SOS polynomials can be extended, in a parallel way, for SOS matrix polynomials. A matrix polynomial $F(x) \in \mathbb{R}^{N\times N}$ is SOS if there exist matrix polynomials $F_1(x), \cdots, F_m(x)$ such that $F(x) = \sum_{i=1}^m F_i^T(x)F_i(x)$. As proposed in [4], this can be established with an LMI by using the SMR for matrix polynomials, that is, $F(x)$ is SOS if and only if there exists a positive semidefinite matrix Q such that $F(x) = (Z(x) \otimes I_N)^T Q (Z(x) \otimes I_N)$.

In the following lemma, the SOS decomposition provides a computational relaxation for the nonnegativity of multivariate polynomial matrices.

Lemma 3: [16] Let $F(x)$ be an $N \times N$ symmetric polynomial matrix in $x \in \mathbb{R}^l$. Then, we have the implication: $v^T F(x) v$ is a SOS, where $v \in \mathbb{R}^N \Longrightarrow F(x) \ge 0$ for all $x \in \mathbb{R}^l$.

Theorem 3: Let the disturbance attenuation level $\gamma > 0$ be given. Suppose that, for the nonlinear stochastic system (1), there exist a symmetric polynomial matrix $Q(x)$, a set of constant positive definite matrices $P_i^T = P_i > 0$ $(i = 1, 2, \cdots, n)$, two constant matrices $X \in \mathscr{W}_{n_x \times n_x}$ and $Y \in \mathscr{W}_{n_x \times n_y}$, a set of constant matrices $\hat{M}_i$ $(i = 1, 2, \cdots, n)$, and two positive constant scalars $\epsilon_1 > 0$ and $\epsilon_2 > 0$ such that the following expressions

$$\nu_1^T (Q(x) - \epsilon_1 I_{n_x}) \nu_1, \tag{28}$$

$$-\begin{bmatrix} \nu_1^T & \nu_2^T \end{bmatrix} (\Upsilon(x) + \epsilon_2 I_{d+nn_z+n_v}) \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} \tag{29}$$

are sums of squares, where ν_1 and ν_2 are arbitrary vectors with appropriate dimension, and $\Upsilon(x)$ is defined in (24). Then, the distributed H_∞ filtering problem is solvable. In this case, the desired parameters $\hat{M}_i$ $(i = 1, 2, \cdots, n)$ are directly obtained, and parameters $\hat{K}_{ij}$ and $\hat{H}_{ij}$ $(i = 1, 2, \cdots, n,\ j \in \mathcal{N}_i)$ can be derived from (8) and (25).

Proof: By Lemma 3, it follows from (28) and (29) that $Q(x) > 0$ and $\Upsilon(x) < 0$, respectively. Therefore, the proof of Theorem 3 follows directly from Theorem 2. ■

It is shown in Theorem 3 that the PDLMIs in Theorem 2 can be transformed into a set of SOSs that can be solved effectively by using the semidefinite programming technique.

IV. An Illustrative Example

To demonstrate the applicability of the proposed filtering techniques, in this example, we consider the localization problem of Unmanned Aerial Vehicles (UAVs) [13]. For the purpose of model simplicity, we consider the movement of UAV in a beeline only. The dynamic model of a UAV is usually a nonlinear system containing some monomials. Moreover, the Itô-type stochastic perturbations are inevitable in practical engineering that should also been taken into account. Reserving the monomials and linearizing the other nonlinearities, we can obtain the dynamic model of the UAV as follows:

$$\begin{cases} ds = (-s + 0.2132\alpha + 0.1521 s^2 \alpha + 0.01 v) dt - 0.1123 s dw \\ d\alpha = (-0.5000\alpha - 0.1018\alpha^3 + 0.01 v) dt \\ \qquad\qquad + (0.2182 s \alpha^2 - 0.1231\alpha) dw, \end{cases} \tag{30}$$

where s is the position and α is the ground speed of the UAV. The signal to be estimated is chosen as $z = s + \alpha$.

It is assumed that the measurements of the UAV are measured by the following three sensors: $y_1 = -s + 0.1v$, $y_2 = -\alpha + 0.1v$ and $y_3 = s + \alpha + 0.1v$, where the networked topology is represented by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ with the set of nodes $\mathcal{V} = \{1, 2, 3\}$, set of edges $\mathcal{E} = \{(1,1), (1,3), (2,1), (2,2), (3,2), (3,3)\}$ and the adjacency matrix $\mathcal{A} = [a_{ij}]_{3\times 3}$ where adjacency elements $a_{ij} = 1$ when $(i, j) \in \mathcal{E}$; otherwise, $a_{ij} = 0$.

To employ the distributed filtering scheme proposed in this paper, we denote $x = \begin{bmatrix} s & \alpha \end{bmatrix}^T$ and then rewrite the system (30) and the sensor model, respectively, into the following state-dependent linear-like forms:

$$\begin{cases} dx = F(x) x dt + G(x) v dt + F_w(x) x dw \\ z = M(x) x \end{cases} \tag{31}$$

and

$$y_i = L_i(x) x + S_i(x) v, \quad i = 1, 2, \cdots, n, \tag{32}$$

where

$$F(x) = \begin{bmatrix} -1 & 0.2132 + 0.1521 s^2 \\ 0 & -0.5000 - 0.1018\alpha^2 \end{bmatrix}, G(x) = \begin{bmatrix} 0.01 \\ 0.01 \end{bmatrix},$$
$$F_w(x) = \begin{bmatrix} -0.1123 & 0 \\ 0.2182\alpha^2 & -0.1231 \end{bmatrix}, \quad L_1(x) = \begin{bmatrix} -1 & 0 \end{bmatrix},$$
$$L_2(x) = \begin{bmatrix} 0 & -1 \end{bmatrix}, L_3(x) = \begin{bmatrix} 1 & 1 \end{bmatrix}, M(x) = \begin{bmatrix} 1 & 1 \end{bmatrix},$$
$$S_1(x) = S_2(x) = S_3(x) = 0.1.$$

The H_∞ performance level is taken as $\gamma = 0.1$ and the values of ϵ_1 and ϵ_2 are fixed at 10^{-5}. We choose YALMIP and SeDuMi as SOS and SDP solvers, respectively. We choose $Q(x)$ as a symmetric polynomial matrix of degree 2 and solve the sums of squares (28)-(29) to obtain the variables $Q(x)$, P_1, P_2, P_3, X, and Y as shown in Appendix.

Then, by (8) and (25), all parameters of the desired distributed filters can be derived as follows:

$$K_{11} = \begin{bmatrix} -98.1172 & -9.2805 \\ 5.7581 & -25.4391 \end{bmatrix}, \ H_{11} = \begin{bmatrix} -160.5274 \\ 6.6549 \end{bmatrix},$$
$$K_{13} = \begin{bmatrix} 63.0614 & -8.4844 \\ -5.2445 & 23.6384 \end{bmatrix}, \quad H_{13} = \begin{bmatrix} 187.2951 \\ 0.6733 \end{bmatrix},$$
$$K_{21} = \begin{bmatrix} 97.2006 & 10.4094 \\ 0.2031 & 20.9471 \end{bmatrix}, \quad H_{21} = \begin{bmatrix} -123.5114 \\ 21.2000 \end{bmatrix},$$
$$K_{22} = \begin{bmatrix} -109.3737 & 1.9398 \\ 2.0041 & -25.0969 \end{bmatrix}, \ H_{22} = \begin{bmatrix} 123.7794 \\ -25.4859 \end{bmatrix},$$
$$K_{32} = \begin{bmatrix} 76.6447 & -15.4559 \\ -7.4080 & 20.0689 \end{bmatrix}, \quad H_{32} = \begin{bmatrix} -26.8159 \\ -26.8263 \end{bmatrix},$$
$$K_{33} = \begin{bmatrix} -86.1482 & 3.3113 \\ 5.1804 & -26.3714 \end{bmatrix}, \quad H_{33} = \begin{bmatrix} 87.0761 \\ 23.1480 \end{bmatrix},$$
$$\hat{M}_1 = \begin{bmatrix} 0.1000 & 0.0923 \end{bmatrix}, \quad \hat{M}_2 = \begin{bmatrix} 0.1000 & 0.0926 \end{bmatrix},$$
$$\hat{M}_3 = \begin{bmatrix} 0.1000 & 0.0925 \end{bmatrix}.$$

In the simulation, the exogenous disturbance input is selected as $v(t) = \exp(-t/200) \times n(t)$ where $n(t)$ is uniformly distributed over $[-2.5, 2.5]$. Simulation results are presented in Figs. 1-4. Fig. 1 plots the output $z(t)$ and its estimates from the filters 1, 2, and 3. Fig. 2 shows the estimation error $\tilde{z}_i(t)$ $(i = 1, 2, 3)$. The actual state response $s(t)$ and its estimates from the filters 1, 2, and 3 are depicted in Fig.3, and the actual state response $\alpha(t)$ and its estimates from the filters 1, 2, and 3 are plotted in Fig. 4. Under the zero-initial condition, the L_2-norms of the filtering error $\tilde{z}$ and the external disturbance v are computed as 1.2735 and 13.3673, respectively, which confirm that the H_∞ performance constraint (12) is well achieved.

V. Conclusions

In this paper, we have made an attempt to investigate the distributed H_∞ filtering problem for a class of polynomial nonlinear stochastic systems represented in a state-dependent linear-like form. By choosing a general polynomial Lyapunov functional, sufficient conditions have been established for the existence of the distributed H_∞ filters, and the desired distributed H_∞ filters have been designed in terms of PDLMIs.

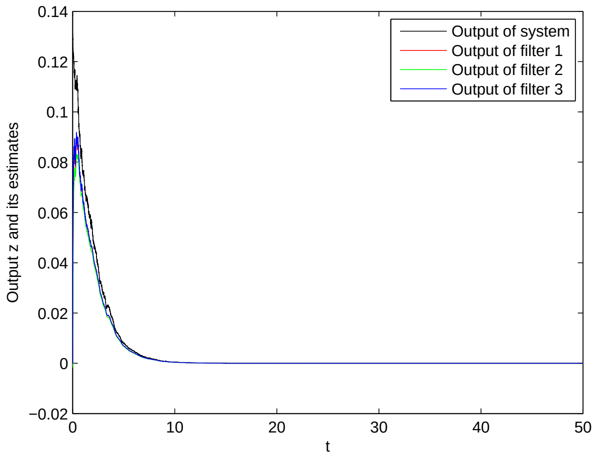

Fig. 1. Output $z(t)$ and its estimates.

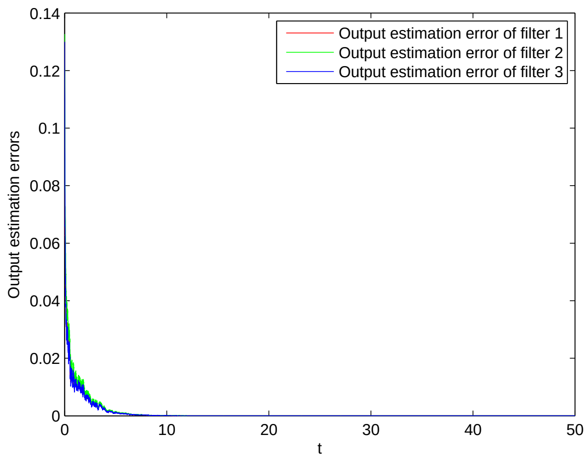

Fig. 2. Filtering errors $\tilde{z}_i(t)$, i=1,2,3.

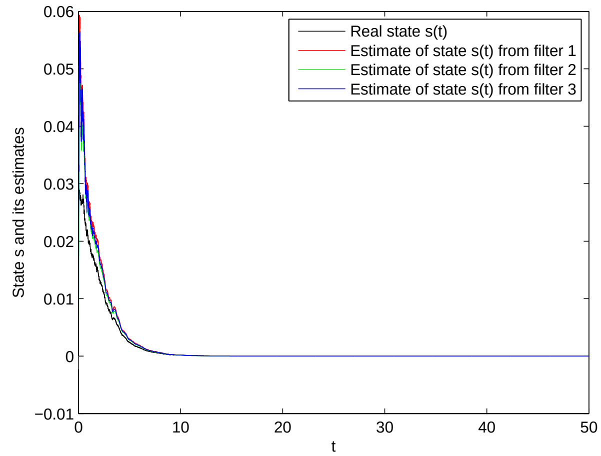

Fig. 3. State $s(t)$ and its estimates.

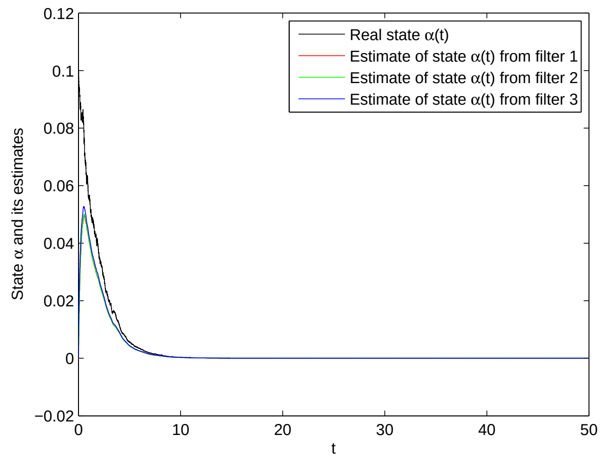

Fig. 4. State $\alpha(t)$ and its estimates.

As a by-product, when the polynomial system is degenerated to a linear system, it has been shown that these PDLMIs can be reduced to the numerically more tractable linear matrix inequalities (LMIs). Then, we have derived the solution to the PDLMIs by solving the problem of the corresponding SOS decomposition with the aid of available SOS solvers. Further research topics include the the analysis of polynomial nonlinear filter for the polynomial nonlinear system and the study on the performance of polynomial filter.

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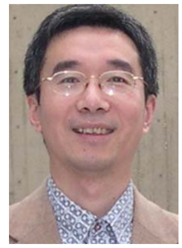

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