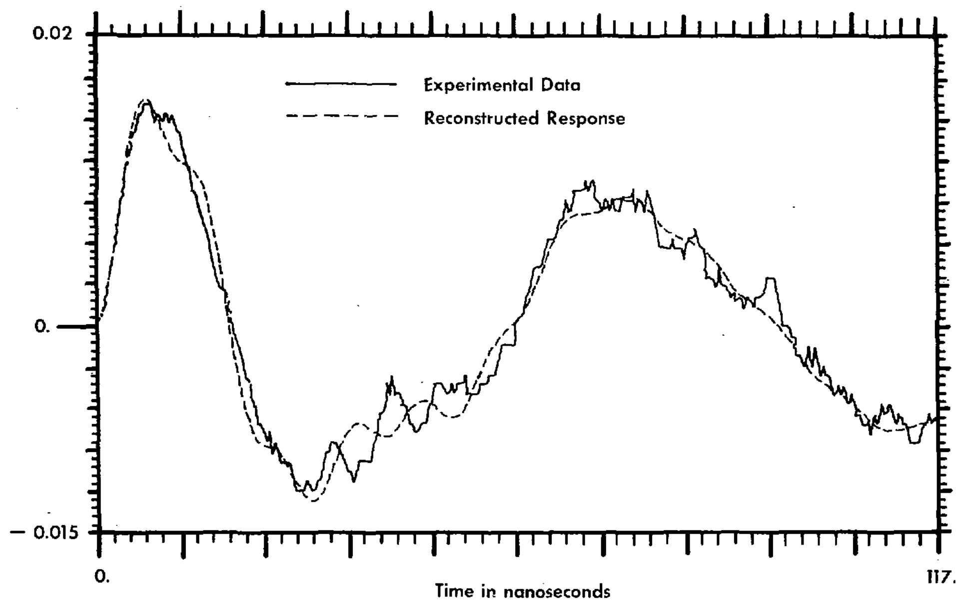

Fig. 2. True response versus reconstructed response of sixth-order system for 10-m long 1-m diameter conducting pipe.

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A Note on the Factorization of Matrices Occurring in Wiener–Hopf Problems

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Abstract—**Simple expressions for the Wiener–Hopf factors of a certain matrix considered by Daniele are given.**

In a recent paper by Daniele [1] the following result for the factorization of matrices was given: if

$$\mathbf{G}(x) = \begin{bmatrix} 1 & a(x) \\ b(x) & 1 \end{bmatrix} = \mathbf{G}_+(x)\,\mathbf{G}_-(x),$$

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where

$$\frac{a(x)}{b(x)} = \frac{n(x)}{p(n)} = r(x),$$

and $n(x)$ and $p(x)$ are entire functions, then

$$G_\pm(x) = \exp\left[\tfrac{1}{2}\ \ln\left(g_\pm(x)\right)\mathbf{I} + \tfrac{1}{2}\ t_\pm(x)\mathbf{T}(x)\right], \tag{1}$$

where

$$\mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad \mathbf{T}(x) = \begin{bmatrix} 0 & n(x) \\ p(x) & 0 \end{bmatrix},$$

$$t(x) \equiv \{n(x)p(x)\}^{-1/2} \ln\left(\frac{1 + \{a(x)b(x)\}^{1/2}}{1 - \{a(x)b(x)\}^{1/2}}\right)$$

$$= t_+(x) + t_-(x),\ g(x) \equiv 1 - a(x)b(x) = g_+(x)g_-(x).$$

The object of this note is to obtain a simpler and more convenient expression for $\mathbf{G}_\pm(x)$ than is given by (1).

Let

$$\Lambda_\pm(x) = \tfrac{1}{2}\ \ln\left(g_\pm(x)\right)\mathbf{I} + \tfrac{1}{2}\ t_\pm(x)\mathbf{T}(x)$$

then (1) can be written as

$$\mathbf{G}_\pm(x) = \exp\left[\Lambda_\pm(x)\right].$$

By means of Sylvester's formula (see Smirnov [2, ch. IV)] we can represent the exponential of the matrix $\Lambda_\pm(x)$ by

$$\mathbf{G}_\pm(x) = e^{\Lambda_\pm(x)}$$

$$= \begin{cases} \left(\dfrac{\lambda_1 e^{\lambda_2} - \lambda_2 e^{\lambda_1}}{\lambda_1 - \lambda_2}\right)\mathbf{I} + \left(\dfrac{e^{\lambda_1} - e^{\lambda_2}}{\lambda_1 - \lambda_2}\right)\Lambda_\pm(x), \\ \qquad \lambda_1 \neq \lambda_2, \\ e^{\lambda}(1-\lambda)\mathbf{I} + e^{\lambda}\Lambda_\pm(x), \qquad \lambda = \lambda_1 = \lambda_2, \end{cases} \tag{2}$$

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where λ_1 and λ_2 are the eigenvalues of $\Lambda_\pm(x)$. These eigenvalues are given by the roots of the equation

$$(\lambda - \tfrac{1}{2} \ln (g_\pm(x)))^2 - \tfrac{1}{4} p(x)n(x)t_\pm(x) = 0,$$

that is

$$\lambda_1 = \tfrac{1}{2} \ln (g_\pm(x)) + \tfrac{1}{2} \{p(x)n(x)\}^{1/2}\, t_\pm(x), \qquad (3)$$

$$\lambda_2 = \tfrac{1}{2} \ln (g_\pm(x)) - \tfrac{1}{2} \{p(x)n(x)\}^{1/2}\, t_\pm(x). \qquad (4)$$

We shall not bother with the trivial case where $\lambda_1 = \lambda_2 = \lambda$. Thus substituting the distinct eigenvalues (3) and (4) into (2) gives after some algebraic simplification

$$G_\pm(x) = \sqrt{g_\pm(x)} \cdot \begin{bmatrix} \cosh [\tfrac{1}{2} \{n(x)p(x)\}^{1/2} t_\pm(x)] & \{n(x)/p(x)\}^{1/2} \sinh [\tfrac{1}{2} \{n(x)p(x)\}^{1/2} t_\pm(x)] \\ \{p(x)/n(x)\}^{1/2} \sinh [\tfrac{1}{2} \{n(x)\}^{1/2} t_\pm(x)] & \cosh [\tfrac{1}{2} \{n(x)p(x)\}^{1/2} t_\pm(x)] \end{bmatrix}.$$

It is easy to check that $G(x) = G_+(x)G_-(x)$ by direct multiplication.

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Multiyear Slant-Path Rain Fade Statistics at 28.56 GHz for Wallops Island, VA

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Abstract—**Multiyear rain fade statistics at 28.56 GHz have been compiled for the region of Wallops Island, VA, covering the time periods April 1, 1977–March 31, 1978, and September 1, 1978–August 31, 1979. The 28.56-GHz attenuations were derived by monitoring the beacon signals from the Comstar geosynchronous satellite, D_2, during the first year, and satellite, D_3, during the second year. Comparisons are made of yearly, monthly, and time of day fade statistics for the first, second, and combined years. Although considerable year to year variations in exceedance times exist for the monthly and time of day fade statistics, the overall fade distributions for the individual years showed relatively small differences. For example, comparing the second year fades relative to those of the first year at equal percentages of time, less than 20 percent rms deviation was found. The year to year variations of rain rate distributions are also examined and show consistently small differences. The resultant fade distribution at 28.56 GHz for Wallops Island, VA, are compared with that arrived at using a prediction method which is a recent refinement of the International Radio Consultative Committee (CCIR) global model, and an rms decibel deviation of less than 14 percent was noted.**

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I. INTRODUCTION

As future earth–satellite communications systems are being planned at frequencies above 10 GHz, the designers of such systems desire a knowledge of rain fade statistics for establishing link transmitter and receiver parameter requirements. In particular, they are interested in fade statistics established from meaningful data bases, namely those statistics calculated over a multiyear data period [1]. They are also interested in the year to year variations of these statistics for purposes of establishing margins of uncertainties in their design criteria.

In this communication we describe fade statistics derived from the monitoring of a 28.56-GHz beacon signal emanating from the Comstar geosynchronous satellite [2] over two noncontiguous years. These data were acquired at the NASA facility at Wallops Island, VA (37° 51′ 16.8″ N latitude, 75° 30′ 48.4″ W longitude), approximately 180 km southeast of Washington, DC, off the mid-Atlantic coast. During the first year period the receiving antenna pointed in the direction of satellite D_2 (95° W longitude) with an azimuth and elevation of 210.0° and 41.6°, respectively. During the second year the receiving antenna pointed towards the satellite, D_3 (87° W longitude) with an azimuth and elevation of 198.3° and 44.5°, respectively. The switch to the second satellite was a result of the beacon turnoff on D_2.

The experimental configuration as well as the results associated with the first year's data base (April 1, 1977–March 31, 1978) were previously described by Goldhirsh [3], [4]. We examine here the multiyear variations associated with the second year's data base (September 1, 1978–August 31, 1979), as well as the combined years' results. Specifically, the yearly variations of the yearly, monthly, and time of day fade distributions as well as the yearly variations of the rain rate distributions are presented.

II. CUMULATIVE FADE DISTRIBUTIONS

In Fig. 1 are given the fade distributions for the first, second, and combined years' data bases. Although a csc θ scaling would have resulted in a five-percent reduction of the second year's fades when normalized to the path angle of the first year, this scaling was considered sufficiently small and was not implemented for the combined years' case. The fades range from 3 to 25 dB, and the exceedance times range approximately from one percent (87.6 h) to 0.06 percent (5.25 h). Above 25 dB, the fade statistics appeared noisy (exceedance times smaller than 5 h) and are not shown. Although the two yearly distributions are close to one another, the second year's exceedance times are somewhat larger than the first up to 24 dB. The average ratio of percentages down to 25 dB is approximately 1.20 (average of larger percentages divided by smaller ones) and the rms deviation

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