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Non-universal scaling and dynamical feedback in generalized models of financial markets

Dafang Zheng[1,2,*], G. J. Rodgers[1], P. M. Hui[3] and R. D'Hulst[1]

[1] *Department of Mathematical Sciences, Brunel University, Uxbridge, Middlesex UB8 3PH, UK*

[2] *Department of Applied Physics, South China University of Technology, Guangzhou 510641, P.R. China*

[3] *Department of Physics, The Chinese University of Hong Kong, Shatin, New Territories, Hong Kong*

Abstract

We study self-organized models for information transmission and herd behavior in financial markets. Existing models are generalized to take into account the effect of size-dependent fragmentation and coagulation probabilities of groups of agents and to include a demand process. Non-universal scaling with a tunable exponent for the group size distribution is found in the resulting system. We also show that the fragmentation and coagulation probabilities of groups of agents have a strong influence on the average investment rate of the system.

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*e-mail: dafang.zheng@brunel.ac.uk, phdzheng@scut.edu.cn

I. INTRODUCTION

Recently, empirical studies have shown that the price fluctuations measured by the returns in financial markets have a heavy-tailed non-Gaussian distribution [1,2]. Understanding the microscopic mechanism of this market phenomena is a challenging problem that has recently attracted the interest of physicists [3–5]. In a simple percolation model recently proposed by Cont and Bouchaud(CB) [6], it is shown that the *herd behavior* can lead to the desired fat tails for the distributions of price returns in financial markets. Herd means a collective phenomena or a crowd effect, and it assumes that agents do not make decisions independently, but they are usually grouped into clusters of agents sharing the same information and hence making a common decision [7,8]. In the CB model, a random *communication* network of agents is constructed and the group size distribution of agents is linked to the distribution of price returns. It is interesting that in such a simple model the group size distribution of agents has a power law decay with an exponential cut-off, which can be compared with the fat-tailed distribution of price returns in real markets [6]. To capture further such a herd behavior, several extensions of the CB model have been presented, including the fundamentalist influence in the CB model considered by Chang and Stauffer [9], the self-organized dynamical model of Eguíluz and Zimmermann(EZ) [10,11] and the democracy and dictatorship models proposed by D'Hulst and Rodgers [12]. The exponent characterizing the group size distributions in all of these generalized versions was found to be *robust* with the same value of 5/2. Interestingly, however, it was found in a recent work that this scaling behavior can actually be changed [13]. By introducing the size-dependent fragmentation and coagulation rates in the generalized EZ model, the group size distribution of agents was found to be model-dependent with a tunable exponent $\beta(\delta) = 5/2 - \delta$, where δ is the exponent characterizing the power law decay for the probabilities of fragmentation and coagulation of groups of agents in the system [13].

In this work, we will consider how the demand process in the democracy and dictatorship models [12] is affected by the dynamical mechanism of the fragmentation and coagulation of groups of agents in the system. As in Ref. [13], we also introduce a size-dependent fragmentation rate $f_i = s_i^{-\delta}$ and a size dependent coagulation rate $c_{ij} = s_i^{-\delta} s_j^{-\delta}$ to the existing models. We study how the scaling behavior for the group size distribution of agents, together with its character distribution $Q(p)$ [12], depends on the parameter δ in the resulting system. We will discuss both the regimes $0 \leq \delta < 1$ [13] and $\delta \geq 1$.

The outline of this paper is as follows. In section II, we define the models and present both analytical and numerical results for the group size distribution n_s. The character distribution $Q(p)$ of agents is also studied numerically. Section III contains further discussion of the relationship between the average investment rate $\bar{a}$ and the parameter δ. A brief summary of our results and conclusions is also given in this section.

II. THE MODELS

The models proposed in this paper are generalized versions of the democracy and dictatorship models introduced by D'Hulst and Rodgers [12], which aim to describe not only the information transmission and herd behavior but also the demand process in financial markets. We consider a system with a total of N agents. Initially, each agent is given a microscopic parameter p , which is a random number chosen from a uniform distribution between 0 and 1. Agents are organized into groups sharing the same value of p. Agents belonging to a group have the same opinion and hence make the same decision at a given moment in time. At the beginning of the simulation, all the agents are isolated, i.e. each group has only one agent. At each time step, two agents i and j, with associated number p_i and p_j respectively, are selected at random. A microscopic investment rate is defined as $a_{ij} = |p_i - p_j|$, i.e., with probability a_{ij} agent i and all other agents belonging to his group decide to make a transaction, i.e. to buy or to sell with equal probability. After the transaction the group of agent i is broken up into isolated agents with probability $s_i^{-\delta}$, with $\delta \geq 0$, or the group remains unchanged with probability $1 - s_i^{-\delta}$. Once the group of agent i is fragmented each agent in it will be given a new microscopic parameter p chosen randomly from a uniform distribution between $p_i - R$ and $p_i + R$, with $0 < R < 0.5$. On the other hand, with probability $1 - a_{ij}$ agent i decides not to make a transaction and all the agents in the group of agent i will communicate with the agents in group j, i.e., the two groups of agent i and agent j will combine to form a bigger group with probability $s_i^{-\delta} s_j^{-\delta}$, or remain separate with probability $(1 - s_i^{-\delta} s_j^{-\delta})$. Once the two groups of agent i and agent j are coagulated all agents in both the groups will be given a new common parameter p_{ij}. In the original democracy and dictatorship models, $p_{ij} = (p_i + p_j)/2$ and $p_{ij} = p_i$ for the democracy and dictatorship models, respectively.

In the original democracy and dictatorship models, the group of agent i is fragmented after a transaction, and conversely the two groups of agent i and agent j are coagulated if there is no transaction. In the present work, instead, we introduce a size-dependent fragmentation rate $f_i = s_i^{-\delta}$ and a size-dependent coagulation rate $c_{ij} = s_i^{-\delta} s_j^{-\delta}$ into the systems. It seems reasonable for us to introduce such a generalization in a sense that the size-dependent fragmentation and coagualtion rates can be used to mimic some dynamical properties in real markets, such as the growth and bankruptcy of businesses [14,15]. That is to say, smaller businesses are easier to bankrupt, and, on another hand, larger businesses are more difficult to grow up [16]. So the present models can also be extended to study the size distribution of businesses.

For our present generalized versions we can also write down a master equation for the number $n_s(t)$ of groups with size s [11–13]

$$N \frac{\partial n_s}{\partial t} = -\bar{a} s^{1-\delta} n_s + \frac{(1-\bar{a})}{N} \sum_{r=1}^{s-1} r^{1-\delta} n_r (s-r)^{1-\delta} n_{s-r} - \frac{2(1-\bar{a}) s^{1-\delta} n_s}{N} \sum_{r=1}^{\infty} r^{1-\delta} n_r \qquad (1)$$

for $s > 1$. Here the constant investment rate a which appeared in the original EZ's model [10], as well as in its generalized version [13], has been replaced by an averaged investment rate $\bar{a}$ [12]. Each term on the right hand side of Eq.(1) represents the consequence of a possible action of the agents. The first term describes the dissociation of a group of size s after a transaction is made. The second term represents coagulation of two groups to form a group of size s. The third term represents the combination of a group of size s with another group. For groups of size unity ($s = 1$), we have

$$N \frac{\partial n_1}{\partial t} = \bar{a} \sum_{r=2}^{\infty} r^{2-\delta} n_r - \frac{2(1-\bar{a}) n_1}{N} \sum_{r=1}^{\infty} r^{1-\delta} n_r. \tag{2}$$

Here, the first term comes from the dissociation of groups into isolated agents and the second term describes the combination of a group of size unity with another group. In the steady state, $\frac{\partial n_s}{\partial t} = 0$, we have

$$s^{1-\delta} n_s = \frac{1-\bar{a}}{N\bar{a} + 2(1-\bar{a}) \sum_{r=1}^{\infty} r^{1-\delta} n_r} \sum_{r=1}^{x-1} r^{1-\delta} (s-r)^{1-\delta} n_r n_{s-r} \tag{3}$$

for $s > 1$, and

$$n_1 = \frac{N\bar{a}}{2(1-\bar{a}) \sum_{r=1}^{\infty} r^{1-\delta} n_r} \sum_{r=2}^{\infty} r^{2-\delta} n_s. \tag{4}$$

Using a generating function method [17], we can derive a self-consistent equation for the number n_s of groups of size s [13]

$$n_s \sim N \left[\frac{4(1-\bar{a})[(1-\bar{a}) + \frac{N\bar{a}}{\sum_{r=1}^{\infty} r^{1-\delta} n_r}]}{[\frac{N\bar{a}}{\sum_{r=1}^{\infty} r^{1-\delta} n_r} + 2(1-\bar{a})]^2} \right]^s s^{-(\frac{5}{2}-\delta)}. \tag{5}$$

For the case of $\delta = 0$, it reduces to the expression for the original democracy and dictatorship models [12]. It indicates that the power law scaling for the distribution of the group number n_s is non-universal, with a tunable exponent $\beta(\delta) = \frac{5}{2} - \delta$ for both the generalized democracy and dictatorship versions. Notice that the power law decay is mediated by an exponential cut-off term in Eq.[5]. For the democracy model, $\bar{a} \approx 0$, which leads to a unit value for the exponential cut off. While, for the dictatorship model, $\bar{a} \approx 0.5$, and the exponential cut off will dominate for larger s [12].

Figure 1 shows our simulation results for the normalized group size distribution n_s/n_1 depending on s, for various values of the parameter δ. In this paper all simulation results are obtained in a time window of $t = 10^5 \sim 10^6$ for a system with $N = 10^4$. Averages are taken over 32 independent runs. For $\delta = 0$, the results for the original democracy and dictatorship models are recovered, i.e., one obtains a power law scaling with exponent $\beta \approx 5/2$ in both the models, but with a dominant exponential cut off for larger s in the dictatorship version.

For $0 \leq \delta \leq 1$, we also have a power law scaling for the group size distribution n_s, but with a tunable exponent $\beta(\delta) = 5/2 - \delta$ in both models. Thus the exponent β of the power law scaling for the group size distribution n_s is no longer so *robust* [10–12], but becomes model dependent and hence non-universal. One finds that the simulation results for the group size distribution n_s in the region of $0 \leq \delta \leq 1$ are in good agreement with the analytic expression given by Eq.[5]. For $\delta > 1$, the scaling behavior of the the group size distribution n_s disappears step by step as δ increases. This indicates that the mean-field analysis of the master equation is invalid and hence the resulting expression in Eq.[5] for the group size distribution n_s is no longer available once the value of the parameter δ exceeds 1. This break down of scaling can be understood qualitatively as the coagulation rate is too small to form large groups of agents for larger δ.

An important feature found in Ref. [12] is that the distributions $Q(p)$ of the p's for the democracy and dictatorship models in the steady state are very different. $Q(p)dp$ is defined as the relative number of the agents associated with a value of the microscopic parameter p inside $(p, p + dp)$. The results for $Q(p)$ are presented in Figure 2, for various values of the parameter δ and a given range of $R = 0.1$. For the democracy model, as shown in Figure 2a, the system is driven towards a coherent state where the distribution $Q(p)$ is Gaussian-like. One finds that the amplitude of spread and hence the height of the peak of $Q(p)$ depends sensitively on δ . For $0 \leq \delta \leq 1$, the peak grows as δ increases. For $\delta > 1$, however, the peak drops as δ increases. This turns out to be that there is a continuous transition for the distribution $Q(p)$ occured at around $\delta = 1$. The distribution $Q(p)$ for the dictatorship model is very different from that obtained in the democracy model. As shown in Figure 2b, a spontaneous segregation into two equal sized populations occurs, with almost one half of the agents associated with a value of p near 0 and the rest near 1. The distribution $Q(p)$ also depends on δ, i.e., $Q(p)$ becomes flat as δ increases. On the other hand, there is no transition for the distribution $Q(p)$ as δ increases, contrary to what is seen in the democracy model.

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III. DISCUSSION

We have found so far that the size-dependent fragmentation and coagulation rates can have a strong influence not only on the scaling behavior of the group distribution n_s, but also on the character distribution, $Q(p)$, of agents. An agent's microscopic parameter p is actually a characterization of the way the agent is perceived in the market, rather than only the individuality of the agent, and hence the distribution $Q(p)$ determines the decision process in the market [12]. The fact that the distribution $Q(p)$ depends on δ demonstrates that the mechanism of the fragmentation and coagulation of groups presented in this work has a strong effect of feedback on the market, which is also reflected in the dependence of the average investment rate $\bar{a}$ on the value of δ as shown in Figure 3. For the democracy model as shown in Figure 3a, the average investment rate $\bar{a}$ decreases continuously as δ increases for $0 \leq \delta < 1$. For $\delta > 1$, however, $\bar{a}$ increases with δ. $\bar{a}$ has a minimum at around $\delta = 1$. Hence the dependence of the average investment rate $\bar{a}$ on the value of the parameter δ shown in Figure 3a is consistent with the dependence of $Q(p)$ on δ in Figure 2a. For the dictatorship case as shown in Figure 3b, the average investment rate $\bar{a}$ decreases rapidly as δ increases from $\delta = 0$ to $\delta = 2$ and becomes flat as δ goes beyond $\delta \approx 2$. Thus there is no transition in the average investment rate $\bar{a}$. The results found in Figure 3b are also consistent with those in Figure 2b.

We can explain the above observations qualitatively. For the democracy case, it is the competition between the fragmentation and the coagulation of groups that leads to the dependence of the character distribution $Q(p)$, and hence the average investment rate $\bar{a}$, on the parameter δ. That is to say, the smaller the fragmentation rate is, the easier it is to form big groups in the steady state. On the other hand, however, it is difficult to form big groups in the steady state in the case of a small coagulation rate. Hence for a small value of δ, the fragmentation rate is dominant, which results in a decreasing dependence of $\bar{a}$ on δ. When δ exceeds some value, which is about 1 as observed in the simulation, the coagulation rate becomes dominant and hence leads to an increasing dependence of $\bar{a}$

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on δ. For the dictatorship model, the fragmentation rate is no longer dominant in case of small δ due to the large average *effective* fragmentation rate $\bar{f}_{effect} = \bar{a}\bar{f}$. Hence the average investment rate $\bar{a}$ depends only sensitively on the coagulation rate, which leads to a decreasing dependence of $\bar{a}$ on δ for all the values of $\delta \geq 0$.

In summary, we have introduced size-dependent fragmentation and coagulation rates to the democracy and dictatorship models proposed recently by D'Hulst and Rodgers [12]. As in the generalized version of the EZ model [13], non-universal scaling is found in the systems. The exponents characterizing the group size distribution in both democracy and dictatorship models turn out to be model dependent and hence are no longer robust. In the original EZ model [10], as well as in its generalized version [13], an investment rate a is given artificially and is fixed during the whole dynamical trade process. In our present work, however, the microscopic investment rate $a_{ij} = |p_i - p_j|$ is inhomogeneous and is a dynamical parameter governing the demand process of the system. The most interesting feature of the present models is that the average investment rate $\bar{a}$ is strongly influenced by the dynamical mechanism of the fragmention and coagulation of groups of agents. Hence the new models seem to be better at mimicing the information transmission and herd behavior, together with the demand process and the dynamical feedback inherent in real markets.

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FIGURE CAPTIONS

Figure 1: Normalized group size distribution n_s/n_1 as a function of size s on a log-log scale for (a) the generalized democracy model and (b) the generalized dictatorship model for different values of δ obtained by numerical simulations (symbols). The values of δ used in the calculations are: $\delta = 0, 0.30, 0.60, 1.0, 1.3, 1.6$. The solid lines are a guide to the eye corresponding to exponents $\beta = -2.5, -2.2, -1.9, -1.5$ respectively.

Figure 2: Probability distribution $Q(p)$ of the characters p of the agents for (a) the generalized democracy model and (b) the generalized dictatorship model for different values of δ obtained by numerical simulations. The values of δ used in the calculations are: $\delta = 0, 0.30, 0.60, 1.0, 1.3, 1.6$.

Figure 3: Average investment rate as a function of the parameter δ for (a) the generalized democracy model and (b) the generalized dictatorship model.

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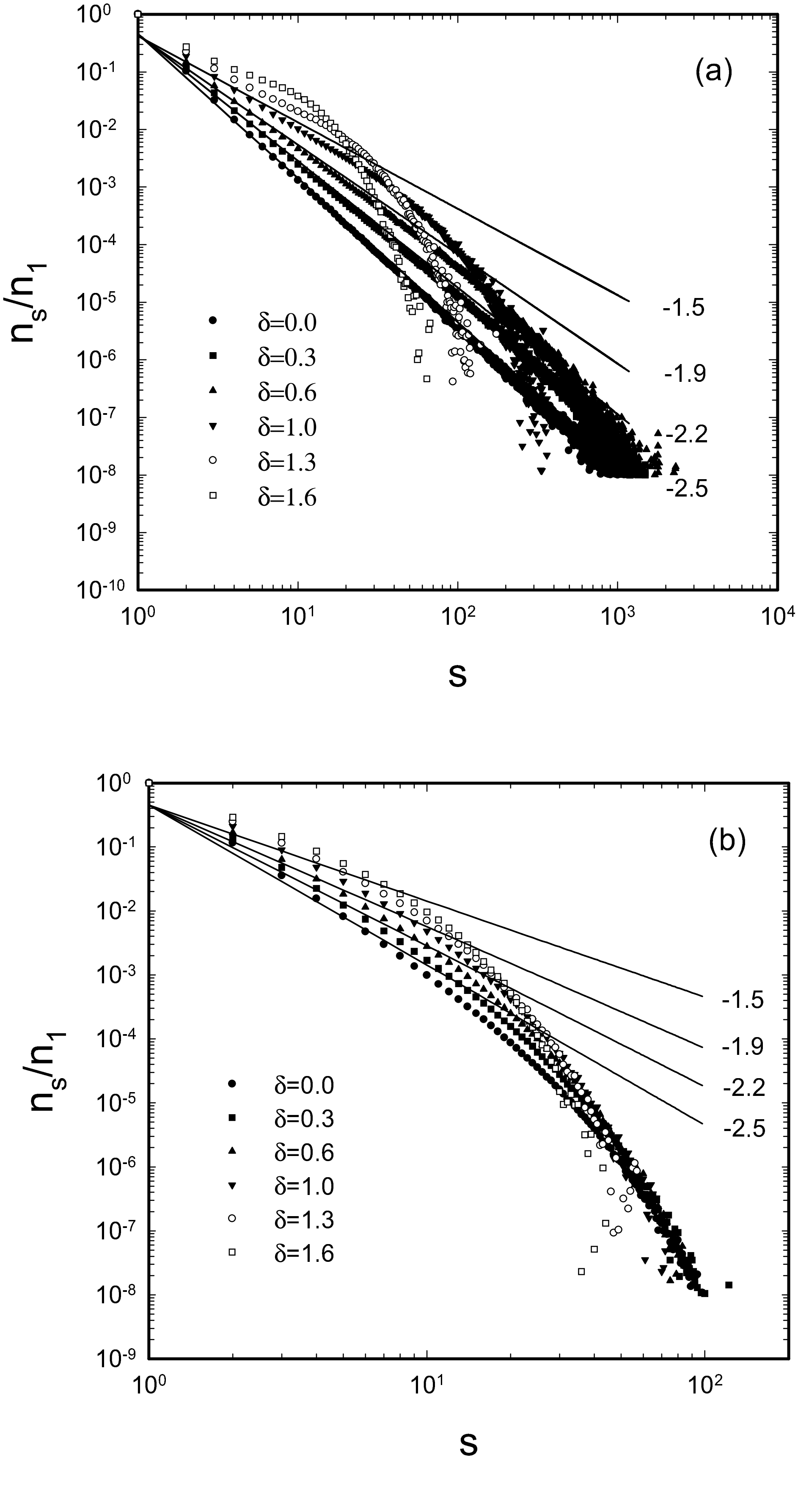
(a)
n_s/n_1
s
δ=0.0
δ=0.3
δ=0.6
δ=1.0
δ=1.3
δ=1.6
-1.5
-1.9
-2.2
-2.5
(b)
n_s/n_1
s
δ=0.0
δ=0.3
δ=0.6
δ=1.0
δ=1.3
δ=1.6
-1.5
-1.9
-2.2
-2.5

Figure 1

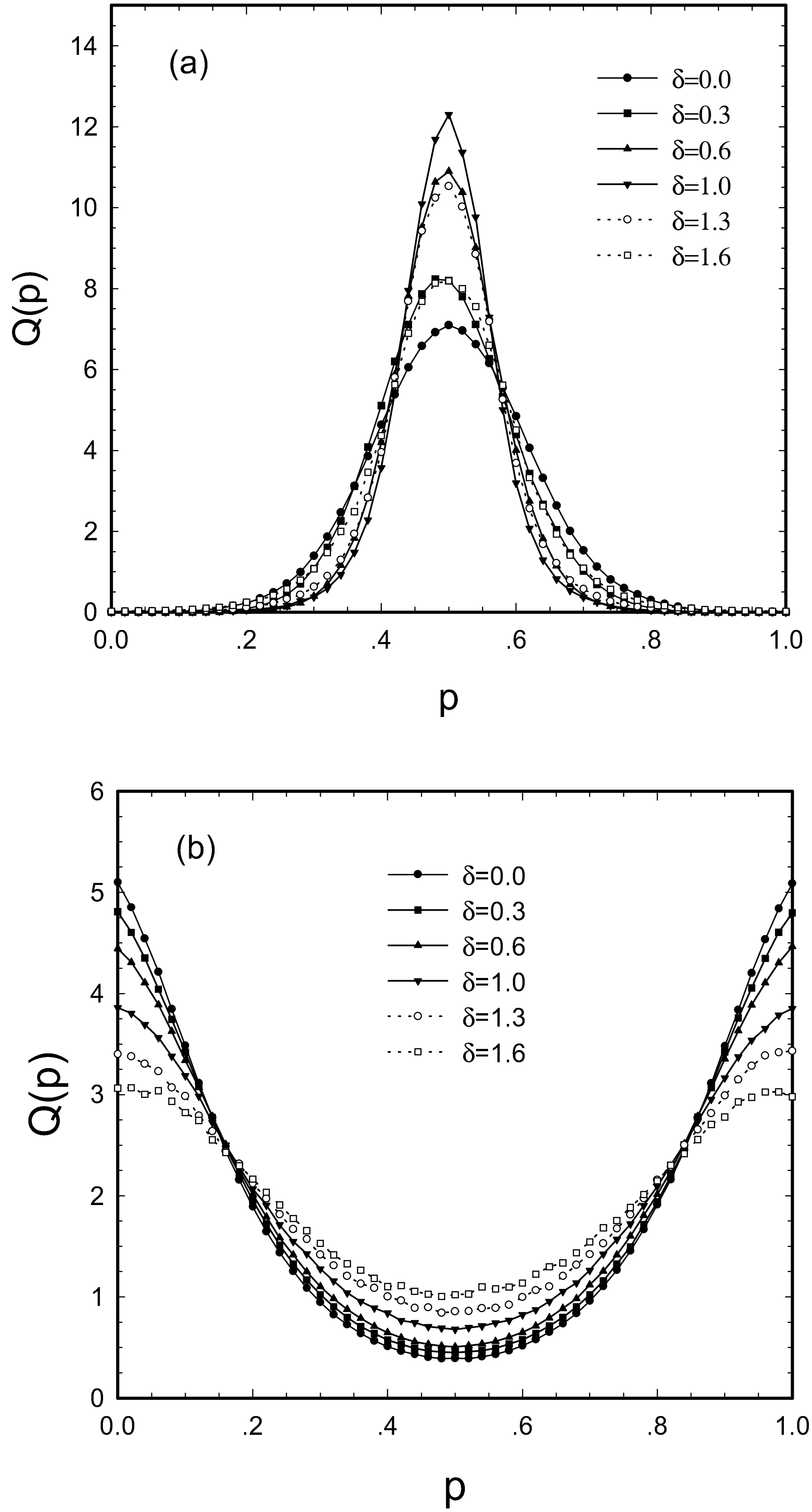
(a)
δ=0.0
δ=0.3
δ=0.6
δ=1.0
δ=1.3
δ=1.6
Q(p)
p
0
2
4
6
8
10
12
14
0.0
.2
.4
.6
.8
1.0
(b)
δ=0.0
δ=0.3
δ=0.6
δ=1.0
δ=1.3
δ=1.6
Q(p)
p
0
1
2
3
4
5
6
0.0
.2
.4
.6
.8
1.0

Figure 2

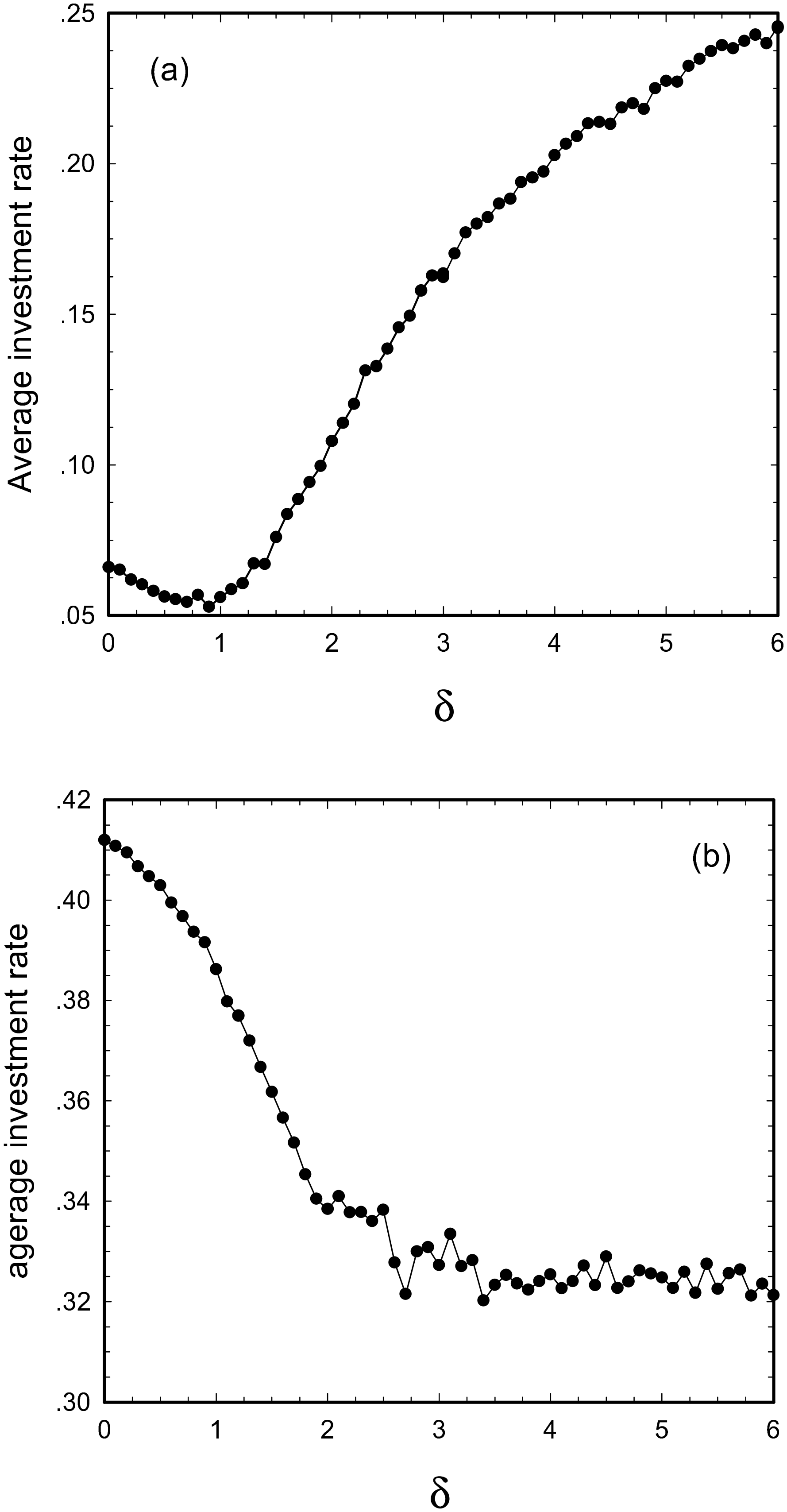
(a)
Average investment rate
.25
.20
.15
.10
.05
0
1
2
3
4
5
6
δ
(b)
agerage investment rate
.42
.40
.38
.36
.34
.32
.30
0
1
2
3
4
5
6
δ

Figure 3