Distributed Resilient State Estimation for Cyber-Physical Systems Against Bit Errors: A Zonotopic Set-Membership Approach

Wei Chen, Zidong Wang, Jun Hu, Hongli Dong, and Guo-Ping Liu

Abstract—This paper addresses the distributed resilient state estimation issue for a class of large-scale cyber-physical systems (CPSs) against bit errors. Owing to the reliability of the binary data, a novel binary encoding-decoding scheme is developed to enable digital communication of the CPSs. We consider the situation where the transmitted binary bits may be flipped over noisy communication channels. The primary purpose of this paper is to design a distributed resilient estimation scheme under a bit-flipped detection mechanism to achieve a reliable estimation of the CPSs. First, by means of the Minkowski sum of sets, a novel distributed zonotopic resilient estimation scheme is proposed where the optimal estimator gain is obtained in the minimum F-radius sense. Furthermore, a two-stage bit-flipped detection scheme is developed by taking advantage of 1) the prediction output zonotope and 2) the intersection between the prediction zonotope and the estimation zonotope. Furthermore, in order to improve the algorithm efficiency, a reduction operator is introduced to reduce the order of the zonotope. Finally, the IEEE 30-bus systems are applied to verify theoretical results of the developed distributed resilient estimation algorithm.

Index Terms—Cyber-physical systems (CPSs), distributed resilient estimation, zonotopic set-membership filtering, binary encoding-decoding scheme, bit errors.

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I. INTRODUCTION

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Over the past decade, the cyber-physical systems (CPSs) have become a burgeoning research frontier due to their tremendous application potentials in various fields including power systems, transportation systems, health-care medical systems, and industrial internet of things (IoT) systems [5], [26], [28], [33], [41], [43]. In general, the CPSs are large-scale, interconnected, geographically dispersed, and layered systems, where all coupled subsystems cooperate with each other to perform online monitoring and/or real-time control [15], [16]. In engineering practice, the CPSs consist of physical processes, computational resources as well as communication networks, which constitutes a smart control loop exhibiting adaptability, scalability, resilience, and security (see [6], [12], [14], [17], [38], [39], [47] and the references therein). Owing to its vital importance in science, technology and engineering, the CPSs have been regarded as a core research area such as Industrial Internet in the US and Industry 4.0 in Germany.

State estimation has always been one of the fundamental research problems in signal process and system engineering [36], [44], [48]. In industrial CPSs, the state estimator, which provides an estimate of the internal state of CPSs based on a sequence of available local outputs, is indispensable in a lot of practical applications such as real-time monitoring, remote operation, information security, and fault diagnosis [10], [33], [35], [45]. For example, as the fault diagnosis technologies of manufacturing systems have attracted persistent research attention from both academia and industry in recent years, a general end-to-end diagnosis framework has been developed in [45] for diverse manufacturing applications such as rolling bearing, cutting tool and lithium-ion batteries. Another typical example of the CPSs is the power network enabling integration and coordination between electric and cyber systems, where the state estimator serves as a prerequisite for operation management and feedback control [32].

To some extent, state estimation plays crucial roles in 1) ensuring the safe, efficient and reliable operation of the physical system, 2) lowering the maintenance cost of system equipment, and 3) enhancing the management level of entire systems. In practical applications, the CPSs may be affected by unavoidable external disturbances and unpredictable environment variations, which could result in dramatic degradation of the estimation performance under the framework of traditional static estimation (e.g., the weighted-least-square (WLS) technique [11]). In this regard, it is of essential importance

to develop an effective scheme for CPSs to achieve state estimation of acceptable accuracy.

With the ever-growing system size of modern industrial CPSs, the conventional centralized estimation schemes, which rely mainly on the central processor of limited capacity, might be ineffective in meeting the demand for real-time monitoring and operation of the large-scale CPSs [1], [9], [29]. As a valuable alternative, the distributed estimation strategy has been welcomed in industry whose purpose is to establish multiple local monitoring centers collaborating each other to reach a reliable consensus. So far, some representative distributed estimation/filtering schemes have been developed in [10], [18], [25], [27], [33], [44] for large-scale systems. For example, a novel distributed Kalman-type filtering algorithm with an attack detector has been proposed in [10] for a class of large-scale CPSs.

Different from the pointwise estimation techniques (e.g. Kalman filtering and H_{∞} filtering), the set-membership estimation is an interval-based technique that is particularly suitable in addressing unknown-but-bounded (UBB) noises. So far, the set-membership estimation problem has received an ongoing research attention for various systems, e.g., [36], [48], [51], [52] and the references therein. For example, a parameter-dependent set-membership estimation algorithm has been proposed in [52] for a liner time-varying systems with impulsive measurement outliers where a novel method is proposed to detect the abnormal signal, and the boundedness analysis has been provided to verify the estimation performance. Under the bit rate constraints, a zonotopes-based distributed fusion scheme has been proposed in [51] by the matrix-weighted fusion method, and a new allocation protocol has been designed to improve communication efficiency. It should be pointed out that the set-membership estimation approaches can provide a confidence region to guarantee the safe and credible monitoring boundary of targets, thereby facilitating the reliable monitoring of CPSs under unpredictable environmental changes [15], [16], and this motivates us to carry out the current investigation.

Owing to rapid advances in wireless communication technologies, the communication networks have become an integrated part of CPSs with examples including cognitive radio networks [23], cellular networks [3], and WiMAX [37]. While enjoying the advantages of flexible architecture as well as low installation and maintenance costs, the portable communication infrastructure does bring some networked-induced challenges such as communication delays [19], [31], packet dropout [49], packet disorders [40], and signal distortions [24]. These undesired network-induced phenomena are likely to degrade communication quality and further influence the estimation accuracy of the CPSs. Hence, it is of practical significance to develop a resilient estimation strategy against the randomly occurring network-induced phenomena.

Under the framework of digital communication, the binary encoding-decoding (BED) scheme is gaining an everincreasing popularity in practice whose main idea is to first transform the raw signal is into a finite-length binary bit string which is sent to the receiver via wireless communication channels (see e.g., [24], [42]). It should be pointed out that

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most relevant results on digital communication have been based on the assumption that the transmission is *bit-error-free*. In practical engineering, however, some binary bits might be flipped due to channel noises, and such kind of bit errors may dramatically degrade the communication quality. For the IEEE 802.15.4 type wireless sensor networks (WSNs), the receiver/estimator is overly sensitive to bit errors and corresponding modules cannot maintain the normal connectivity once the biterror rate (BER) exceeds 1% [6]. Up to now, the bit-error issues have received initial research attention from the aspects of statistics and information theory, see e.g., [22], [24], [51]. Unfortunately, the corresponding research from the system theory perspective is far from adequate, and we are therefore set to develop a resilient distributed estimation algorithm with an appropriate bit-flipped detection mechanism for the purpose of reliable target monitoring.

Based on the aforementioned observations, we endeavor to study the distributed resilient estimation problem with bitflipped detection for a class of large-scale CPSs under BESs in this paper. The technical challenges stem from three aspects listed as follows: 1) how to design a suitable evaluation mechanism to detect the unreliable signal transmission caused by bit errors? 2) how to develop a distributed resilient estimation scheme to mitigate the effects from outliers on the estimation accuracy? and 3) how to develop an efficient filtering algorithm to meet the requirement of real-time monitoring and online detection? In order to handle the above three challenges, we make the following main contributions.

- 1) A novel two-stage set-based evaluation mechanism is proposed to detect the abnormal output by making full use of the prediction output zonotope as well as the intersection of two zonotopes (formed in state prediction step and estimation update).
- 2) Based on the detection results, a prediction-compensated approach is adopted to mitigate the impact from the possible bit errors on the estimation accuracy.
- 3) With the help of the operational rule of the Minkowski sum of sets, the resilient estimation algorithm is developed to ensure that the actual system state is included in certain zonotope with 100% confidence, and the distributed estimator parameter is derived by minimizing the F-radius of its corresponding estimation zonotope at each time instant.
- 4) The validity of the developed distributed resilient scheme is confirmed in the IEEE 30-bus test systems.

The outline of this paper is given as follows. Section II formulates the distributed resilient estimation issue of the CPSs with bit errors. In Section III, a zonotopic set-membership filtering scheme combining with bit-flipped detection is developed, and the optimal estimator parameters are then derived. A practical example is provided in Section IV to verify the feasibility and validity of the designed estimation scheme. Finally, some concluding remarks are drawn in Section V.

Notation: A^T represents the transpose of the matrix A. $||A||_F = \sqrt{\operatorname{Tr}(A^T A)}$ describes the Frobenius norm of matrix $A \in \mathbb{R}^{n \times m}$ where $Tr(\cdot)$ is the trace of the square matrix. Operator cat $\{A_1, A_2, \dots\} = [A_1 \ A_2 \ \dots]$ represents the Copyright © 2023 Institute of Electrical and Electronics Engineers (IEEE). Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works by sending a request to pubs-permissions@ieee.org. See https://journals.ieeeauthorcenter.ieee.org/become-an-ieee-journal-author/publishing-

concatenation of matrices. The symbol " \oplus " is the Minkowski sum. $\mathscr{A}_1 \cap \mathscr{A}_2$ represents the intersection of two sets \mathscr{A}_1 and \mathscr{A}_2 , and \emptyset refers to an empty set.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. Zonotopes

A zonotope is a polytopic set in a given vector space \mathbb{R}^n , which can be regarded as an affine transformation of an unitary hypercube. Specifically, such a set can be characterized as follows:

$$\mathscr{Z} \triangleq \langle \alpha, E \rangle = \left\{ \alpha + \sum_{i=1}^{m} \omega_i e_i : |\omega_i| \le 1 \right\}$$

where the vector $\alpha \in \mathbb{R}^n$ is the center of polytopic set, and $E = \begin{bmatrix} e_1 & e_2 & \cdots & e_m \end{bmatrix} \in \mathbb{R}^{n \times m}$ is the generator matrix with the column vector $e_i \in \mathbb{R}^n$. Furthermore, the dimension and the order of this zonotope \mathscr{Z} are denoted by n and m, respectively. In addition, the *F*-radius of the zonotope $\mathscr{Z} = \langle \alpha, E \rangle$ can be described by the Frobenius norm of matrix E, i.e., $||E||_F$, and the covariation matrix is defined by $P \triangleq EE^T$.

Letting $\mathscr{Z}_1 = \langle \alpha_1, E_1 \rangle$ and $\mathscr{Z}_2 = \langle \alpha_2, E_2 \rangle$ be two zonotopes, the linear transformations on zonotopes are given as follows:

$$\Phi \mathscr{Z}_1 = \langle \Phi \alpha_1, \Phi E_1 \rangle, \ \mathscr{Z}_1 \oplus \mathscr{Z}_2 = \langle \alpha_1 + \alpha_2, \mathsf{cat}\{E_1, E_2\} \rangle$$

where Φ is a matrix of appropriate dimension. Furthermore, given matrix G and vectors $\psi \in \mathscr{Z}_1$, $\sigma \in \mathscr{Z}_2$, one has

$$\chi \triangleq G\psi + \sigma \in G\mathscr{Z}_1 \oplus \mathscr{Z}_2. \tag{1}$$

B. System Description

Generally speaking, the CPSs involve physical devices and cyber resources linked by various communication network mediums. In this paper, the underlying CPSs consist of Nsubsystems, whose physical connections are described by an undirected graph $\mathscr{G} = (\mathcal{V}, \mathcal{E})$ of order N with the node set $\mathcal{V} = \{1, 2, \dots N\}$ and the edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. If $(i, j) \in \mathcal{E}$, there exists a connection between the subsystem i and the subsystem j. The neighborhood set of the node $i \in \mathcal{V}$ is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}\}$.

The dynamics of the *i*-th subsystem is modeled as follows:

$$\begin{aligned}
& (x_{i,t+1} = A_{ii,t}x_{i,t} + \sum_{j \in \mathcal{N}_i} A_{ij,t}x_{j,t} + B_{i,t}u_{i,t} + w_{i,t} \\
& (y_{i,t} = C_{i,t}x_{i,t} + v_{i,t}
\end{aligned}$$

where $x_{i,t} \in \mathbb{R}^{n_x}$, $u_{i,t} \in \mathbb{R}^{n_u}$ and $y_{i,t} \in \mathbb{R}^{n_y}$ are, respectively, the system state, the input and the output of subsystem *i*. $w_{i,t} \in \mathbb{R}^{n_w}$ and $v_{i,t} \in \mathbb{R}^{n_v}$ are the system and measurement noises, respectively. $A_{ii,t}$, $A_{ij,t}$, $B_{i,t}$ and $C_{i,t}$ are system matrices of appropriate dimensions.

Assume that the unknown-but-bounded (UBB) noises $w_{i,t}$ and $v_{i,t}$ belong to the following polytopic sets:

$$\mathscr{W}_i \triangleq \langle 0, Q_i \rangle, \ \mathscr{V}_i \triangleq \langle 0, R_i \rangle, \ i \in \mathcal{V}.$$

Furthermore, without loss of generality, let the initial state $x_{i,0}$ belong to zonotope $\hat{\mathscr{X}}_{i,0} = \langle \hat{x}_{i,0}, \hat{E}_{i,0} \rangle$ with known $\hat{x}_{i,0}$ and $\hat{E}_{i,0}$ for $\forall i \in \mathcal{V}$.

Remark 1: The considered physical model (2) is quite general that can be used to describe various practical systems including coupled inverted pendulums mesh [18], [27], multiple maneuvering targets [4], DC microgrids [34], power systems [31] and so on. For instance, in [18], [27], the state x_i consists of the angular position θ_i and the velocity $\dot{\theta}_i$ for inverted pendulums mesh. In [4], the state x_i is formed by the target's position and velocity in X- and Y-axes for maneuvering targets. In [34], the state x_i is composed of the load voltage V_i at common coupling point and the filter current I_{ti} for DC microgrids. In [31], the state x_i comprises the deviation of the frequency Δf_i , the net tie-line power flow ΔP_{tie-i} , generator mechanical output ΔP_{mi} , and valve position ΔP_{vi}^{i} for power systems.

C. BED Scheme with Bit Errors

In order to achieve remote state estimation, as described in Fig. 1, a BED communication strategy is introduced to achieve data transmission from physical subsystem to local estimator. Due to network bandwidth constraints, a group of finite-length bit strings are sent to local estimator via a memoryless binary symmetric channel. In what follows, let us reify this scheme via the following details.

$$\underbrace{\mathcal{Y}_{i,k}}_{\text{Encoder}} \underbrace{\mathcal{R}_{i,k}}_{\text{Encoder}} \xrightarrow{\mathcal{R}_{i,k}}_{\text{symmetric}} \underbrace{\text{Binary}}_{\text{symmetric}} \underbrace{\mathcal{\widetilde{R}}_{i,k}}_{\text{Channel}} \underbrace{\text{Decoder}}_{\text{Decoder}} \underbrace{\tilde{\mathcal{Y}}_{i,k}}_{\text{Channel}}$$

Fig. 1. The BED scheme

Denote the *l*-th row entry (a scalar signal) of the measurement $y_{i,t}$ as $y_{i,t}^l \in \mathbb{R}$ ($l \in \{1, 2, \dots, n_y\}$). Due to the limited measurement range of the device, the signal $y_{i,t}^l$ is assumed to belong to $[\underline{\tau}^l, \overline{\tau}^l]$ at any time instant *t* where $\overline{\tau}^l$ and $\underline{\tau}^l$ are known scalars. For the sake of convenience, a linear transformation of signal $y_{i,t}^l$ can be written as

$$r_{i,t}^{l} = (y_{i,t}^{l} - \underline{\tau}^{l}) / (\overline{\tau}^{l} - \underline{\tau}^{l})$$
(3)

where $r_{i,t}^l \in [0, 1]$ is a normalized signal, and $r_{i,t}^l$ can be written as binary form as follows:

$$r_{i,t}^{l} = \sum_{j=1}^{\infty} 2^{-j} h_{j,t}^{i^{l}}, \tag{4}$$

where $h_{j,t}^{i^l} \in \{0,1\}$ is regarded as the *j*-th binary bit.

Let the capacity of each channel be s + 1 bits. To achieve digital/symbol transmission, the first *s* fixed bits come from (4), that is

$$h_{1,t}^{i^l}, h_{2,t}^{i^l}, \cdots, h_{s,t}^{i^l},$$

and the (s+1)-th auxiliary bit, denoted as $\eta_{s+1,t}^{i^{\iota}} \in \{0,1\}$, is configured by

$$\eta_{s+1,t}^{i^{l}} = \begin{cases} 0, & \bar{\eta}_{s+1,t}^{i^{l}} \in [0, 0.5), \\ 1, & \bar{\eta}_{s+1,t}^{i^{l}} \in [0.5, 1) \end{cases}$$
(5)

where

$$\bar{\eta}_{s+1,t}^{i^l} = 2^s \left(r_{i,t}^l - \sum_{j=1}^s 2^{-j} h_{j,t}^{i^l} \right)$$

is known and determined by both normalized signal $r_{i,t}^l$ and the first encoded s bits.

According to the above descriptions, the signal $r_{i,t}^l$ is encoded into the bit string as follows:

$$\mathcal{R}_{i,t}^{l} \triangleq \{h_{1,t}^{i^{l}}, h_{2,t}^{i^{l}}, \cdots, h_{s,t}^{i^{l}}, \eta_{s+1,t}^{i^{l}}\},$$
(6)

and its corresponding truncated value is expressed by

$$\psi_{i,t}^{l} = \sum_{j=1}^{s} 2^{-j} h_{j,t}^{i^{l}} + 2^{-s} \eta_{s+1,t}^{i^{l}} \triangleq \psi_{k}(r_{i,t}^{l}, s).$$
(7)

Here, $\psi_k(r_{i,t}^l, s)$ is called as a message function, which is employed to discretize the signal $r_{i,t}^l$. It is observed that $\psi_{i,t}^l$ can be described by 2^{s+1} discrete values involved in $\mathcal{D} \triangleq \{i \times 2^{-s} | i = 0, 1, 2, \cdots, 2^s\}$. Furthermore, the truncated error $e_{i,t}^l$ is calculated as $e_{i,t}^l = r_{i,t}^l - \psi_{i,t}^l \leq 2^{-s-1}$, and the quantization error is $q_{i,t}^l = (\overline{\tau}^l - \underline{\tau}^l) e_{i,t}^l$ bounded by $|q_{i,t}^l| \leq M_l$ with $M_l = (\overline{\tau}^l - \underline{\tau}^l) \times 2^{-s-1}$.

Owing to the impact of channel imperfection or noises, some bits in the transmitted binary bit string $\mathcal{R}_{i,t}^l$ may be flipped. In this case, the *received bit string* is represented by:

$$\tilde{\mathcal{R}}_{i,t}^{l} \triangleq \{ \tilde{h}_{1,t}^{i^{l}}, \tilde{h}_{2,t}^{i^{l}}, \cdots, \tilde{h}_{s,t}^{i^{l}}, \tilde{\eta}_{s+1,t}^{i^{l}} \}$$
(8)

where

$$\begin{split} \tilde{\eta}_{s+1,t}^{i^{l}} &\in \{0,1\}, \ \tilde{h}_{j,t}^{i^{l}} \in \{0,1\}, \ j=1,2,\cdots,s, \\ \tilde{\eta}_{s+1,t}^{i^{l}} &= \gamma_{s+1,t}^{i^{l}}(1-\eta_{s+1,t}^{i^{l}}) + (1-\gamma_{s+1,t}^{i^{l}})\eta_{s+1,t}^{i^{l}}, \\ \tilde{h}_{j,t}^{i^{l}} &= \gamma_{j,t}^{i^{l}}(1-h_{j,t}^{i^{l}}) + (1-\gamma_{j,t}^{i^{l}})h_{j,t}^{i^{l}} \end{split}$$

with

$$\gamma_{j,t}^{i^{l}} = \begin{cases} 1, & \text{the } j\text{-th bit is flipped,} \\ 0, & \text{the } j\text{-th bit is not flipped.} \end{cases}$$

Corresponding to the encoding process (3)-(7), the bit string $\tilde{R}_{i,t}^{l}$ is transformed into

$$\tilde{\psi}_{i,t}^{l} = \sum_{j=1}^{s} 2^{-j} \tilde{r}_{j,t}^{i^{l}} + 2^{-s} \tilde{\eta}_{s+1,t}^{i^{l}} \triangleq \psi_{t}(\tilde{r}_{i,t}^{l},s), \qquad (9)$$

and the *received signal* $\tilde{y}_{i,t}^l$ is written by

$$\tilde{y}_{i,t}^{l} = (\overline{\tau}^{l} - \underline{\tau}^{l})\tilde{\psi}_{i,t}^{l} + \underline{\tau}^{l}.$$
(10)

In addition, the augmented information at the receiver side is expressed by

$$\tilde{y}_{i,t} = \left[\tilde{y}_{i,t}^{1}, \tilde{y}_{i,t}^{2}, \cdots, \tilde{y}_{i,t}^{l}, \cdots, \tilde{y}_{i,t}^{n_{y}} \right]^{T}.$$
 (11)

D. Problem To Be Addressed

This paper deals with the distributed resilient estimation issue for interconnected distributed systems (1) undergoing UBB disturbances, quantization errors and possible bit errors under known control input. The considered set-based distributed estimator, which includes the local prediction zonotope $\widehat{\mathscr{X}}_{i,t}$, the local estimation zonotope $\widehat{\mathscr{X}}_{i,t}$, and the bit-flipped detector $\vartheta_{i,t} \in \{0, 1\}$, is described by

$$\begin{cases} \bar{\mathscr{X}}_{i,t+1} = f_i(\hat{\mathscr{X}}_{i,t}, \hat{\mathscr{X}}_{j,t}, u_{i,t}), \ j \in \mathcal{N}_i, \\ \hat{\mathscr{X}}_{i,t+1} = g_i(\bar{\mathscr{X}}_{i,t+1}, \vartheta_{i,t+1}, \tilde{y}_{i,t+1}), \ i \in \mathcal{V}. \end{cases}$$
(12)

Consequently, the primary objectives of this work are listed as follows.

- Design an appropriate distributed estimator (12) such that all possible state variable x_{i,t} resides in the zono-topes *X*_{i,t} and *X*_{i,t}, simultaneously.
- 2) Develop an effective bit-flipped detection scheme, that is, design an indicator function $\vartheta_{i,t}$, which depends on the local prediction zonotope $\hat{\mathcal{X}}_{i,t}$ and the local estimation zonotope $\hat{\mathcal{X}}_{i,t}$, such that the possible bit errors are perceived and filtered in a timely manner.
- 3) Derive an optimal recursive estimator parameter in the minimum *F*-radius sense, i.e.,

$$K_{i,t} = \arg\min_{K_{i,t}} \|\hat{\mathscr{X}}_{i,t}\|_F, i \in \mathcal{V}.$$
 (13)

III. MAIN RESULTS

In this section, the prediction and estimation zonotopes (combining with a bit-flipped detector) are first given to ensure the real system states to be within these two sets. Then, the distributed zonotopic set-membership filtering algorithm is developed via minimizing the F-radius of the local estimation zonotope. In addition, a reduced-order operation is introduced to reduce computation burden.

To alleviate the performance degradation of the distributed estimation accuracy caused by the bit errors, we adopt the resilient zonotopic set-membership estimation scheme for the CPSs (2). First, the local prediction zonotope $\hat{\mathscr{X}}_{i,t+1} = \langle \bar{x}_{i,t+1}, \bar{E}_{i,t+1} \rangle$ is given by

$$\bar{x}_{i,t+1} = A_{ii,t}\hat{x}_{i,t} + \sum_{j \in \mathcal{N}_i} A_{ij,t}\hat{x}_{j,t} + B_{i,t}u_{i,t}$$
 (14a)

and

$$\bar{E}_{i,t+1} = \begin{bmatrix} A_{ii,t}\hat{E}_{i,t} & \operatorname{cat}_{j\in\mathcal{N}_i}\{A_{ij,t}\hat{E}_{j,t}\} & Q_i \end{bmatrix}$$
(14b)

where $\bar{x}_{i,t+1}$ is the one-step prediction of $x_{i,t}$, $\hat{x}_{i,t}$ is the estimated state of $x_{i,t}$, and $\hat{E}_{i,t}$, $\bar{E}_{i,t+1}$ are the estimation generator matrix and the prediction generator matrix, respectively.

Next, the estimation zonotope $\hat{\mathscr{X}}_{i,t+1} = \langle \hat{x}_{i,t+1}, \hat{E}_{i,t+1} \rangle$ is described by

$$\hat{x}_{i,t+1} = \bar{x}_{i,t+1} + \vartheta_{i,t+1} K_{i,t+1} (\tilde{y}_{i,t+1} - C_{i,t+1} \bar{x}_{i,t+1})$$
(15a)

and

$$E_{i,t+1} = \begin{bmatrix} L_{i,t+1}\bar{E}_{i,t+1} & \vartheta_{i,t+1}K_{i,t+1}M & -\vartheta_{i,t+1}K_{i,t+1}R_i \end{bmatrix}$$
(15b)

where $K_{i,t+1}$ is the local estimator gain to be designed, $L_{i,t+1} \triangleq I - \vartheta_{i,t+1}K_{i,t+1}C_{i,t+1}$, and $M \triangleq \operatorname{diag}_{n_y}\{M_l\}$. Here, the indicator function $\vartheta_{i,t+1}$ is introduced to detect the abnormal value caused by bit errors, which is designed by the following two steps.

1) After the prediction in (14), carry out the following detection:

$$\vartheta_{i,t+1} = \begin{cases} 1, & \tilde{y}_{i,t+1} \in \mathscr{Y}_{i,t+1} \\ 0, & \tilde{y}_{i,t+1} \notin \mathscr{Y}_{i,t+1} \end{cases}$$
(16)

where $\mathscr{Y}_{i,t+1} \triangleq C_{i,t+1} \overline{\mathscr{X}}_{i,t+1} \oplus \mathscr{Y}_{i,t} \oplus \langle 0, -M \rangle$ is the prediction output zonotope.

2) After the measurement update in (15), determine the indicator function $\vartheta_{i,t+1}$ by

$$\vartheta_{i,t+1} = \begin{cases} 1, & \bar{\mathscr{X}}_{i,t+1} \cap \hat{\mathscr{X}}_{i,t+1} \neq \emptyset \\ 0, & \bar{\mathscr{X}}_{i,t+1} \cap \hat{\mathscr{X}}_{i,t+1} = \emptyset. \end{cases}$$
(17)

Remark 2: In this paper, the proposed set-based bit-flipped detection scheme contains two steps. The first step conducts a *preliminary* detection by evaluating whether the received measurement information is contained in the prediction output zonotope. Specifically, if $\tilde{y}_{i,t+1} \notin \mathscr{Y}_{i,t+1}$, then the transmitted signal is unreliable and the indicator function is chosen as $\vartheta_{i,t+1} = 0$; otherwise, $\vartheta_{i,t+1} = 1$. In light of the operational rule of the Minkowski sum of sets, one has $x_{i,t+1} \in \hat{\mathcal{X}}_{i,t+1}$ and $x_{i,t+1} \in \bar{\mathscr{X}}_{i,t+1}$, and further derives $x_{i,t+1} \in (\hat{\mathscr{X}}_{i,t+1} \cap$ $\bar{\mathscr{X}}_{i,t+1}$). As such, the second step serves as a *refined* detection in order to evaluate whether there exists an intersection between the prediction zonotope and the estimation zonotope. To be more specific, if $\mathscr{X}_{i,t+1} \cap \mathscr{X}_{i,t+1} = \emptyset$, then the obtained zonotope is unreliable and the received signal suffers from bit error effects. In this case, the update step is replaced by the prediction step for next operation. For more details, we refer the readers to Algorithm 1.

Remark 3: In industrial systems, the data detection provides an alarm operation once the given tolerance level is exceeded, and hence plays an ultimately significant role in achieving the safe and reliable operation of CPSs (see e.g., [30], [46]). In particular, a general framework has been proposed in [46] for discovering cyber-physical systems directly from data, which involves the identification of physical systems as well as the inference of transition logics of cyber systems. In existing literature, the typical detection strategies include Bayesian detection with binary hypothesis [20], χ^2 detection [46], and likelihood ratio detection [21]. It should be pointed out that most of existing detection approaches cannot ensure 100% confidence for systems with stochastic noises obeying a certain distribution. In order to overcome such a drawback, we develop a set-based detection scheme such that all actual system states are guaranteed to be constrained in a specified state space for a general case where the system noise is UBB. In this case, the small abnormal signals, that can be regarded as the admissible noises, would not deteriorate the filtering performance even if these abnormal signals are involved in the filtering process.

In what follows, a theorem is provided to show that the setbased resilient estimation scheme can contain all actual states $x_{i,t}$ with 100% confidence.

Theorem 1: Let the initial condition $x_{i,0} \in \hat{\mathscr{X}}_{i,0}$ hold for $\forall i \in \mathcal{V}$. For the proposed distributed resilient estimator (14)-(15) with the bit-flipped detector (16)-(17), each subsystem state resides in the corresponding prediction zonotope and estimation zonotope, i.e., $x_{i,t} \in \tilde{\mathscr{X}}_{i,t}$ and $x_{i,t} \in \hat{\mathscr{X}}_{i,t}$ at any time instant t.

Proof: The mathematical induction approach is adopted to prove this theorem.

For k = 0, $x_{i,0} \in \hat{\mathscr{X}}_{i,0}$ holds. Assume that $x_{i,t} \in \hat{\mathscr{X}}_{i,t}$ holds at time instant t. Since the process noise $w_{i,t}$ belongs to the zonotope $\mathscr{W}_{i,t}$ and the local control input $u_{i,t}$ is known, we obtain from the operation rule (1) and system dynamics (2) that

$$x_{i,t+1} \in A_{ii,t} \langle \hat{x}_{i,t}, \hat{E}_{i,t} \rangle \oplus \sum_{j \in \mathcal{N}_i} A_{ij,t} \langle \hat{x}_{j,t}, \hat{E}_{j,t} \rangle$$
$$\oplus B_{i,t} \langle u_{i,t}, 0 \rangle \oplus \langle 0, Q_i \rangle.$$
(18)

By resorting to the operation rule of the Minkowski sum of sets, we derive that $x_{i,t+1} \in \overline{\mathscr{X}}_{i,t+1}$ where $\overline{\mathscr{X}}_{i,t+1}$ is given in (14a)-(14b).

Note that

$$x_{i,t+1} = L_{i,t+1}x_{i,t+1} + \vartheta_{i,t+1}K_{i,t+1}\tilde{y}_{i,t+1} + \vartheta_{i,t+1}K_{i,t+1}q_{i,t+1} - \vartheta_{i,t+1}K_{i,t+1}v_{i,t+1}$$
(19)

where $q_{i,t+1} \triangleq \begin{bmatrix} q_{i,t+1}^1 & q_{i,t+1}^2 & \cdots & q_{i,t+1}^{n_y} \end{bmatrix}^T$. Due to the facts that $x_{i,t+1} \in \tilde{\mathscr{X}}_{i,t+1} = \langle \bar{x}_{i,t+1}, \bar{E}_{i,t+1} \rangle$,

Due to the facts that $x_{i,t+1} \in \mathscr{X}_{i,t+1} = \langle \bar{x}_{i,t+1}, E_{i,t+1} \rangle$, $q_{i,t+1} \in \langle 0, M \rangle$, $v_{i,t+1} \in \langle 0, R_i \rangle$, and $\tilde{y}_{i,t+1}$ is a known vector, we have

$$x_{i,t+1} \in L_{i,t+1} \langle \bar{x}_{i,t+1}, \bar{E}_{i,t+1} \rangle \oplus \vartheta_{i,t+1} K_{i,t+1} \langle \tilde{y}_{i,t+1}, 0 \rangle \oplus \vartheta_{i,t+1} K_{i,t+1} \langle 0, M \rangle \oplus (-\vartheta_{i,t+1} K_{i,t+1}) \langle 0, R_i \rangle.$$
(20)

Recalling the operation rule of Minkowski sum, one has that $x_{i,t+1} \in \hat{\mathscr{X}}_{i,t+1}$ where specific description of $\hat{\mathscr{X}}_{i,t+1}$ are provided in (15a)-(15b).

The above theorem shows that the proposed zonotopic estimation scheme can contain all actual states for any subsystem at any time instant. Now, let us carry out the local estimator design by minimizing the trace of the estimation covariation matrix at any time instant t.

Theorem 2: For the considered distributed CPSs (2) with unreliable measurement (11) caused by possible bit errors, the estimator gain $K_{i,t+1}$ in the minimum *F*-radius sense is designed by

$$K_{i,t+1} = \bar{P}_{i,t+1}C_{i,t+1}^{T}(C_{i,t+1}\bar{P}_{i,t+1}C_{i,t+1}^{T} + R_{i}R_{i}^{T} + MM^{T})^{-1}$$
(21)

where $\bar{P}_{i,t+1} = \bar{E}_{i,k+1} \bar{E}_{i,t+1}^T$.

Proof: First, define a cost function $\mathcal{J}_{i,t} = \|\mathscr{X}_{i,t}\|_F^2$. With the help of the property of matrix trace, one has the following relationship:

$$\mathcal{J}_{i,t+1} = \operatorname{Tr}\{\hat{E}_{i,t+1}^T \hat{E}_{i,t+1}\} = \operatorname{Tr}\{\hat{E}_{i,t+1} \hat{E}_{i,t+1}^T\} = \operatorname{Tr}\{\hat{P}_{i,t+1}\}.$$

Then, the prediction covariation matrix $\bar{P}_{i,t}$ and estimation covariation $\hat{P}_{i,t}$ can be expressed by

$$\bar{P}_{i,t+1} = \bar{E}_{i,t+1} \bar{E}_{i,t+1}^{T} \\
= A_{ii,t} \hat{P}_{i,t} A_{ii,t}^{T} + \sum_{j \in \mathcal{N}_{i}} A_{ij,t} \hat{P}_{j,t} A_{ij,t}^{T} \\
+ Q_{i} Q_{i}^{T}$$
(22a)

and

$$\hat{P}_{i,t+1} = \hat{E}_{i,t+1}\hat{E}_{i,t+1}^{T}
= L_{i,t+1}\bar{P}_{i,t+1}L_{i,t+1}^{T} + \vartheta_{i,t+1}K_{i,t+1}MM^{T}K_{i,t+1}^{T}
+ \vartheta_{i,t+1}K_{i,t+1}R_{i}R_{i}^{T}K_{i,t+1}^{T}.$$
(22b)

To minimize the cost index $\mathcal{J}_{i,t} = \|\hat{\mathscr{X}}_{i,t}\|_F^2$, we take the partial derivative of $\operatorname{Tr}\{\hat{P}_{i,t+1}\}$ (with respect to $K_{i,t+1}$) and obtain

$$\frac{\partial \text{Tr}\{\bar{P}_{i,t+1}\}}{\partial K_{i,t+1}} = -2\bar{P}_{i,t+1}C_{i,t+1}^T + 2K_{i,t+1}(C_{i,t+1}P_{i,t+1}) \times C_{i,t+1}^T + R_iR_i^T + MM^T).$$
(23)

Letting $\partial \text{Tr}\{\hat{P}_{i,t+1}\}/\partial K_{i,t+1} = 0$, we directly derive the optimal estimator gain (21) in the minimum *F*-radius sense. The proof is now complete.

Remark 4: The set-based estimation has been extensively investigated in recent years, and the corresponding schemes can be roughly divided into categories, namely, the interval observer schemes and the set-membership schemes. The main idea of the former category is to adopt two observers to generate upper and lower bounds of actual states, which is usually subjected to some rather strict design conditions. Different from the interval observer schemes, the set-membership approaches take full advantage of a predefined geometrical set within which the actual state resides. So far, there are many traditional set descriptions (to evaluate the estimation performance) which include, but are not limited to, zonotopes [13], [50], ellipsoids [14], [52], and parallelotopes [7]. Among them, the proposed zonotopic approach is achieved in the minimum F-radius sense and carried out in a recursive manner. Hence, the zonotopic approach has merits in excellent estimation accuracy and low calculation complexity.

So far, we have developed the distributed resilient estimation scheme of CPSs under the BED scheme with bit errors via a zonotopic set-membership filtering approach, where a twostep bit-flipped detection mechanism is adopted to reduce the effects of bit errors, and the corresponding distributed estimator gain is obtained by minimizing F-radius of the estimation zonotope. It should be noted that the order of the zonotope is significantly increased because of the operation (14b), and this adds a large burden to the computation over time. As such, there is a practical need to look for an appropriate method to reduce/limit the order of the zonotope.

Motivated by [8], a reduction operator, denoted by $\Phi(\mathscr{Z})$, is employed such that the inclusion relationship $\Phi(\mathscr{Z}) \supseteq \mathscr{Z}$ holds and the maximum order of the operator $\Phi(\mathscr{Z})$ is less than that of the zonotope $\mathscr{Z} \triangleq \langle c, E \rangle$.

Let us now present the specific construction method for order reduction as follows. First, sort the columns of generation

matrix E on decreasing vector norm and generate matrix E^* :

$$E^* = \begin{bmatrix} e_1^* & \cdots & e_i^* & \cdots & e_m^* \end{bmatrix}, \ \|e_i^*\| \ge \|e_{i+1}^*\|.$$
 (24)

Letting the maximum order of the operator $\Phi(\mathscr{Z})$ is κ ($\kappa > n$), the operator $\Phi(\mathscr{Z})$ can be expressed by

$$\Phi(\mathscr{Z}) = \begin{cases} \langle c, E \rangle, & m \le \kappa, \\ \langle c, E^* \rangle, & m > \kappa \end{cases}$$
(25)

where

$$E^{*} \triangleq [E^{*}_{\uparrow} \Psi(E^{*}_{\downarrow})],$$

$$E^{*}_{\uparrow} \triangleq \begin{bmatrix} e_{1}^{*} & e_{2}^{*} & \cdots & e_{\kappa-n}^{*} \end{bmatrix},$$

$$E^{*}_{\downarrow} \triangleq \begin{bmatrix} e_{\kappa-n+1}^{*} & e_{\kappa-n+2}^{*} & \cdots & e_{m}^{*} \end{bmatrix}.$$

and $\Psi(E^*_{\downarrow}) \in \mathbb{R}^{n \times n}$ is denoted by

$$\Psi(E_{\downarrow}^{*}) = \operatorname{diag}\left\{\sum_{i=\kappa-n+1}^{m} |e_{1,i}^{*}|, \cdots, \sum_{i=\kappa-n+1}^{m} |e_{n,i}^{*}|\right\}$$
(26)

with $e_{t,i}^*$ being the *t*-th element of vector e_i^* .

With the help of above order-reduced operation, a suboptimal distributed estimator parameter is given as follows.

Corollary 1: For the considered distributed CPSs (2) with unreliable measurement (11) caused by possible bit errors, the estimator gain $K_{i,t+1}^{\star}$ under the order-reduced operation (25) is given by

$$K_{i,t+1}^{\star} = \bar{P}_{i,t+1}^{\star} C_{i,t+1}^{T} (C_{i,t+1} \bar{P}_{i,t+1}^{\star} C_{i,t+1}^{T} + R_{i} R_{i}^{T} + M M^{T})^{-1}$$
(27)

where $\bar{P}_{i,t+1}^{\star} = \bar{E}_{i,t+1}^{\star} (\bar{E}_{i,t+1}^{\star})^T$ and the matrix $\bar{E}_{i,t+1}^{\star}$ is obtained via operator $\Psi(\tilde{\mathscr{X}}_{i,t+1})$.

Proof: The proof of Corollary 1 is similar to that of Theorem 2, except that we need to add order-reduced operation $\Psi(\bar{\mathscr{X}}_{i,t+1})$ before calculating the prediction covariation matrix, and the proof is thus is skipped here for brevity.

The distributed resilient estimation scheme can be summarized in Algorithm 1.

Remark 5: Up to now, the distributed resilient estimation issue has been comprehensively investigated for a class of large-scale CPSs under BED scheme with possible bit errors. In contrast to the existing results, our distributed resilient estimation shows the distinctive advantages in the following three aspects: 1) the model of the digital communication over noisy network channels is roundly characterized including BED scheme and possible bit-flipped phenomenon; 2) the addressed resilient estimation issue is new where a bit-flipped detection is embedded into zonotopic set-membership estimator to attenuate the negative influence from unreliable communication caused by possible flipped bit errors on estimation accuracy; and 3) the developed distributed estimation algorithm only depends on local information by taking full advantage of the operation rules of zonotope, which satisfies the requirement of scalability for online application.

Algorithm 1 Distributed resilient estimation

- Step 1. Set initial state $x_{i,0}$, zonotope \mathscr{W}_i , \mathscr{V}_i , initial estimation zonotope $\hat{\mathscr{X}}_{i,0} = \langle \hat{x}_{i,0}, \hat{E}_{i,0} \rangle$, and obtain the estimation covariation matrix $\hat{P}_{i,0}$ where $i \in \mathcal{V}$. Set k = 0.
- Step 2. Compute the prediction generator matrix $\overline{E}_{i,t+1}$ via (14b) and the prediction center vector $\overline{x}_{i,t+1}$ via (14a). If the order of the generator matrix $\overline{E}_{i,t+1}$ exceeds κ , perform the order-reduced operation via (25). Compute the prediction covariation matrix $\overline{P}_{i,t+1}$ via (22a),
- Step 3. Detect $\tilde{y}_{i,t+1}$ via (16). If $\tilde{y}_{i,t+1} \in \mathscr{Y}_{i,t+1}$, go to next step; otherwise carry out $\hat{\mathscr{X}}_{i,t+1} \leftarrow \tilde{\mathscr{X}}_{i,t+1}$ and jump to Step 6.
- Step 4. Compute the distributed estimator $K_{i,t}$ via (21), the estimation generator matrix $\hat{E}_{i,t+1}$ via (15b), the estimation covariation matrix $\hat{P}_{i,t+1}$ via (22b), and the estimation center vector $\hat{x}_{i,t+1}$ via (14b), respectively.
- Step 5. Operate the bit-flipped detection via (17). If $\hat{\mathcal{X}}_{i,t+1} \cap \hat{\mathcal{X}}_{i,t+1} = \emptyset$, carry out $\hat{\mathcal{X}}_{i,t+1} \leftarrow \hat{\mathcal{X}}_{i,t+1}$, else turn to next step.
- Step 6. If $t \le t_{\text{max}}$, set t = t + 1 and go back to Step 2; otherwise go to Step 7.
- Step 7. Stop.

IV. SIMULATION RESULTS

In this paper, the feasibility of the developed distributed resilient estimation scheme is validated in the standard IEEE 30-bus power networks. As shown in Fig. 2, the power networks have 6 generator unit, whose connection relationship is depicted by red dashed lines. The corresponding undirected edges can be written as: (1,2), (1,4), (2,3), (3,4), (4,5), (3,5), (3,6), (5,6).



Fig. 2. IEEE 30-bus power networks.

Consider the distributed resilient estimation issue of the large-scale power networks. The dynamics of the *i*-th sub-

TABLE I Basic Nomenclatures of Each Subsystem

Notations	Nomenclature
Δf_{i}	The frequency deviation
ΔJ_{i}	The frequency deviation.
ΔP_{l_i}	The load deviation.
ΔP_{t_i}	The turbine valve position deviation.
ΔP_{r_i}	The deviation of the load reference set-point.
ΔP_{n_i}	The net tie-line power flow.
ΔP_{g_i}	The generator mechanical power deviation.
R_i	The speed droop coefficient.
D_i	The equivalent damping coefficient.
T_{t_i}	The time constant of turbine.
T_{g_i}	The time constant of governor.
$\check{H_i}$	The equivalent inertia constant.
T_{ij}	The synchronizing power coefficient.
β_i	The frequency bias.

system, adopted from [23], is modeled by

$$\begin{cases} \dot{x}'_{i}(k) = A'_{ii}x'_{i}(k) + \sum_{j \in \mathcal{N}_{i}} A'_{ij}x'_{j}(k) + B'_{i}u'_{i}(k) + L'_{i}\Delta P_{l_{i}} \\ y'_{i}(k) = C'_{i}x'_{i}(k) \end{cases}$$
(28)

where the subsystem state $x'_i(k)$ is specified by

 $x_i'(k) = \begin{bmatrix} \Delta f_i & \Delta P_{g_i} & \Delta P_{t_i} & \Delta P_{n_i} \end{bmatrix}^T,$

 $u'_i(k) = \Delta P_{r_i}$ is the input, and y'_i is the output. Furthermore, the system matrices in the dynamic equation (28) can be described by

where the description of symbols is given in Table I.

In what follows, the power networks (28) can be discretized as

$$\begin{cases} x_{i,t+1} = A_{ii}x_{i,t} + \sum_{j \in \mathcal{N}_i} A_{ij}x_{j,t} + B_i u_{i,t} + L_i \Delta P_{l_i} \\ y_{i,t} = C_i x_{i,t} + v_{i,t} \end{cases}$$
(29)

where $A_{ii} = e^{A'_{ii}\tau}$, $A_{ij} = e^{A'_{ij}\tau} - I_n$, $B'_i = \int_0^\tau e^{A'_{ii}t}B'_i dt$, $C_i = C'_i$, $L_i = \int_0^\tau e^{A'_{ii}t}L'_i dt$, τ is the sampling period, and $v_{i,t}$ is the UBB noise possibly introduced by modeling error or external disturbance.

For the system addressed above, in this paper, the sampling period is $\tau = 1s$, the range of system output is [-1, 3] (i.e., $\underline{\tau}^l = -1, \ \overline{\tau}^l = 3, \ l = 1, 2, \cdots, n^y$), the channel capacity is set as m = 8 bits, the maximum order of the operator is $\kappa = 10$, the noise generation matrices are denoted as $Q_i = 0.4I$, $R_i = 0.1$, and other system parameters are given in Table II. Without loss of generality, the bit error is assumed to be

TABLE II Parameter Setups

Paramotors	The values of the <i>i</i> -th area					
1 al alleters	1	2	3	4	5	6
D_i	1	0.7	0.9	0.9	0.7	0.86
H_i	10	12	10	8	8	10
R_i	0.05	0.05	0.0625	0.08	0.05	0.05
T_{t_i}	0.3	0.65	0.4	0.3	0.6	0.8
T_{g_i}	0.1	0.1	0.1	0.1	0.1	0.1
T_{ij}	0.05	0.05	0.05	0.05	0.05	0.05

 TABLE III

 ESTIMATOR PARAMETERS OF SUBSYSTEMS.

t	1	2	3		
$K_{1,t}$	[0.0428]	[0.0428]	[0.0428]		
	-0.5948	-0.5950	-0.5952		
	-0.8031	-0.8031	-0.8028	•••	
	0.0157	0.0159	0.0160		
$K_{2,t}$	[0.0427]	0.0425	0.0425		
	-0.4299	-0.3936	-0.3937		
	-0.7853	-0.7855	-0.7856		
	0.0151	0.0149	0.0147		
$K_{3,t}$	0.0423	0.0428	0.0428		
	-0.5069	-0.4143	-0.4143		
	-0.6493	-0.6359	-0.6358		
	0.0501	0.0305	0.0306		
$K_{4,t}$	[0.0423]	0.0428	0.428		
	-0.5069	-0.4144	-0.4143		
	-0.6493	-0.6359	-0.6358		
	0.0502	0.0304	0.0306		
$K_{5,t}$	[0.0436]	[0.0436]	0.0436		
	-0.2605	-0.2606	-0.2607		
	-0.5027	-0.5028	-0.5030		
	0.0211	0.0209	0.0207		
$K_{6,t}$	0.0431	0.0427	0.0428		
	-0.3287	-0.4690	-0.3435		
	-0.7990	-0.8160	-0.7913		
	0.0101	0.0234	0.0145		

random with the probability $\mathbb{P}\{\gamma_{j,t}^{i^l} = 1\} = 0.05$ and the input is set as $u_{i,t} = 0$.

The load deviations ΔP_{l_i} $(i \in \mathcal{V})$ are regarded as external disturbances which are chosen as follows:

$$\begin{split} P_{l_1} = & 0.4\sin(1.1t)(2\text{Ra}-1), \ P_{l_2} = 0.4\cos(1.5t)(2\text{Ra}-1), \\ P_{l_3} = & 0.4\sin(1.3t)(2\text{Ra}-1), \ P_{l_4} = 0.4\cos(1.2t)(2\text{Ra}-1), \\ P_{l_5} = & 0.4\sin(1.4t)(2\text{Ra}-1), \ P_{l_6} = 0.4\cos(1.7t)(2\text{Ra}-1), \end{split}$$

where "Ra" is a stochastic variable subjecting to uniform distribution U(0, 1).

The measurement noises $v_i, (i \in \mathcal{V})$ are taken as follows:

$$v_{1,t} = 0.1\sin(1.7t)(2\mathbf{Ra} - 1), v_{2,t} = 0.1\cos(2t)(2\mathbf{Ra} - 1), v_{3,t} = 0.1\sin(2t)(2\mathbf{Ra} - 1), v_{4,t} = 0.1\cos(1.8t)(\mathbf{Ra} - 1), v_{5,t} = 0.1\sin(2.1t)(2\mathbf{Ra} - 1), v_{6,t} = 0.1\cos(t)(2\mathbf{Ra} - 1).$$

With the help of Algorithm 1, the desired estimator parameters $K_{i,t}$ $(i \in \mathcal{V})$ are given in Table III.

The simulation results are presented in Figs. 3-10 to show to substantiate the proposed distributed resilient filtering algorithm. Figs. 3-8 depict the true trajectories and their estimates for subsystems 1-6, respectively, where the blue dotted line is the estimation center and two red lines are the upper and lower bound. It is observed that all actual states resides in the estimation zonotopes, and the sharp variations of bounds mean that the bit-flipped detection is triggered, and the predictioncompensated mechanism is adopted to mitigate the effects of bit errors. Furthermore, a comparison with the normal setmembership is given as follows. Figs. 9-10 show the state trajectories of subsystems 1, 4 and their estimates by the proposed distributed estimation scheme without the bit-flipped detector. By comparing Figs. 3, 6 with Figs. 9-10, the proposed resilient scheme can achieve the reliable estimation for the large-scale CPSs with abnormal measurements. Overall, numerical simulation confirms that the developed distributed resilient algorithm can ensure all actual states residing in their estimation zonotopes, which show the perfect estimation performance.



Fig. 3. Actual state of subsystem 1 and its estimate.



Fig. 4. The actual state of subsystem 2 and its estimate.

V. CONCLUSIONS

In this paper, a zonotopic distributed resilient algorithm has been developed to solve the distributed state estimation issue for a class of large-scale CPSs under the BED scheme with bit errors. A uniform model has been established to comprehensively describe the digital communication process including encoding, bit flipping and decoding. To mitigate the impact from the unreliable signal transmission caused by possible bit errors, a novel distributed resilient estimation scheme with a



Fig. 5. The actual state of subsystem 3 and its estimate.



Fig. 6. The actual state of subsystem 4 and its estimate.

bit-flipped detection mechanism has been proposed by virtue of the zonotopic set-membership approach, where the bit errors can be detected by a two-step set-based evaluation mechanism. Moreover, the optimal distributed estimator has been designed by minimizing the F-radius of the local estimation zonotope. In addition, a simple but effective reduction operation has been given to improve computational efficiency for online application. Finally, a typical framework of the IEEE 30-bus system has been tested to illustrate the feasibility and validity of the developed distributed resilient estimation scheme. Future research topics would be the extension of our results to other more complicated systems (e.g. automated vehicles [15], [16] and microgrids [32]).

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Fig. 7. The actual state of subsystem 5 and its estimate.



Fig. 8. The actual state of subsystem 6 and its estimate.

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Fig. 9. The state trajectory of subsystem 1 and its estimate without bit-flipped detection.



Fig. 10. The state trajectory of subsystem 4 and its estimate without bit-flipped detection.

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