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Privacy-Preserving Distributed Economic Dispatch of Microgrids Using Edge-Based Additive Perturbations: An Accelerated Consensus Algorithm

Wei Chen, Zidong Wang, Jun Hu, Qing-Long Han, and Guo-Ping Liu

***Abstract*—This paper investigates the privacy-preserving distributed economic dispatch (DED) problem of islanded microgrids. To improve the convergence rate of the DED algorithm, an *accelerated* consensus scheme is adopted by utilizing a short memory. Then, a privacy-preserving strategy is introduced to prevent sensitive information leakage by adding well-designed perturbations into the proposed consensus algorithm at the initial time instant. The primary objective of this paper is to design a privacy-preserving accelerated consensus scheme to achieve a balance between supply and demand at the globally minimized cost while preserving the initial local demand information. By virtue of rigorous algebra manipulation and mathematical induction, a unified framework is established under which the convergence, the optimal convergence rate, and the optimality of the proposed DED algorithm are simultaneously analyzed, and the main results are extended to satisfy the privacy-preserving needs. Furthermore, the proposed privacy-preserving DED algorithm is shown to be resilient against both internal (honest-but-curious) and external eavesdroppers. Finally, the effectiveness of the developed privacy-preserving accelerated consensus algorithm is validated on the IEEE 39-bus power systems.**

***Index Terms*—Microgrids, accelerated consensus algorithm, privacy preservation, distributed economic dispatch, edge-based additive perturbations.**

I. INTRODUCTION

With the rapid advancement of renewable generation technologies, the recent years have witnessed a refreshed impetus to the research on intelligent microgrid due to its clear theoretical significance and practical insights [1]–[6]. As one of the active research topics concerning microgrids, the economic dispatch (ED) problem has received a great deal of attention [7], [8]. In general, the main aim of ED is to develop an optimization algorithm to solve the power allocation problem at minimal generation cost under practical physical constraints. With the nowadays massive access to distributed generators, online calculation and real-time operation become increasingly important especially for microgrids of large scale [9], [10]. In this case, most of the traditional centralized optimization algorithms are no longer effective and therefore the *distributed* implementation of optimization algorithms has recently aroused much research interest in achieving the desired autonomy, flexibility and scalability, see e.g. [11]–[15] and the references therein.

Inspired by the idea of coordinated control of multi-agent systems, a number of consensus-based schemes have been successfully developed to deal with the ED issue of microgrids. For example, a consensus-plus-innovation distributed economic dispatch (DED) algorithm has been proposed in [16] through introducing a feedback mechanism (with a decaying factor) to the consensus update step so as to ensure the optimality and convergence of the algorithm. A consensus-based approach (with a fixed feedback gain) has been developed in [8], and the upper bound of such a gain has been further discussed in [17]. Moreover, a novel consensus-based algorithm has been applied in [18] to address the social welfare problem. In addition, in the context of networked systems, some efforts have been devoted to the investigation of consensus-based DED schemes under network-induced phenomena including packet losses [19], communication delays [9], and cyber-attacks [20]. It should be pointed out that the most of the existing DED approaches focus on convergence and optimality, while little attention is paid to the *dynamic* performance of the optimization algorithm, and this motivates the current investigation.

Convergence rate, which serves as one of the most important dynamic performance indices for the DED algorithm, quantifies the ability to respond to time-varying conditions and even emergencies in practical microgrids [21]. So far, the results on the convergence rate analysis of the DED algorithm have been really scattered. In [17], the convergence rate of the algorithm has been discussed under different topological conditions and the feedback gains have been determined via a large number of simulation tests. The finite- and fixed-time DED algorithms have been developed in [21], [22] whose implementation might be difficult due to the involved nonlinearities and time complexity. In [23], a minimum-time consensus scheme has

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been put forward to solve the ED problem of microgrids via utilizing the historical local information and the normalized kernel of the Hankel matrix, at the cost of occupying a certain amount of storage and computing resources. In addition, a novel consensus-based approach has been proposed in [24] with fast convergence speed for the DED problem of microgrids. Despite the rather fruitful results in the past few years, there is a lack of *quantitative* evaluation of the improved convergence rate in terms of certain appropriate index, and we are therefore encouraged to shorten such a gap.

To implement the DED algorithm, agents are required to exchange data with their neighbors over a typically open communication network, and this may result in the issue of privacy leaks, for example, the adversaries (or rivals) could manipulate the electricity market and even attack microgrids via stealing sensitive information about power grids [3], [25], [26]. To enhance the security of the DED algorithms, some privacy-preserving techniques have been developed in the literature, which can be basically classified into two categories, namely, the cryptography-based [27]–[29] and the noise-injected ones [30]–[33]. The cryptography-based approach, which is known to be computationally expensive, encrypts the transmitted data and constructs the artful information interaction mechanism with help from the algebraic number theory. On the other hand, the noise-injected approach masks the true data via injecting a well-designed noise (or noise sequence) as motivated by the idea of obfuscation. Of course, there are still some other privacy-preserving distributed algorithms such as the state-decomposition technique [34], [35], the information-theoretic method [36], and the hot-pluggable approach [37].

It is worth noting that the privacy-preserving performance may be compromised under the existing noise-injected schemes. For example, though a differentially private scheme can be achieved by inserting the decaying and independent noise sequences over the entire time domain, there is an issue with the overall accuracy and one would have to play the tradeoff between the convergence accuracy and the privacy level [32]. To ensure exact convergence, a series of zero-sum correlated noises can be injected onto the state variables to obfuscate true value [30], [33] but, unfortunately, such a privacy-preserving mechanism is vulnerable to external eavesdroppers. In addition, some additive perturbation signals have been constructed in [31] to prevent the disclosure of sensitive information, but the corresponding privacy-preserving performance may be compromised against the internal eavesdroppers. To this end, there appears to be an urgent need to explore a more effective privacy-preserving scheme that ensures exact convergence of the DED algorithm while preventing privacy disclosure.

In view of the above discussions, we endeavor to further investigate the privacy-preserving DED problem of microgrids in a quantitative way, where the essential difficulties we are going to face lie in the following three aspects: *1) how to develop a suitable DED algorithm to improve the convergence rate without sacrificing computation/communication cost? 2) how to design a privacy-preserving scheme that takes both convergence accuracy and privacy-preserving performance into account? and 3) how to evaluate algorithm performance in terms of convergence, convergence rate, optimality and privacy?* To tackle the above-mentioned difficulties, we are devoted to developing an accelerated consensus scheme with edge-based additive perturbations in order to achieve optimal ED without sensitive information disclosure. Correspondingly, three primary contributions made in this paper are summarized as follows.

1) A novel consensus-based algorithm is proposed to achieve the optimal ED, where the improved convergence rate is quantitatively characterized with help of a dedicatedly defined *asymptotic convergence factor*.
2) With the aid of the algebra analysis technique and the mathematical induction approach, the framework for performance analysis is established in terms of the convergence, the optimal convergence rate, and the optimality of the proposed DED optimization algorithm.
3) A privacy-preserving scheme is designed by adding edge-based perturbations, which is capable of ensuring exact convergence (without perturbation effects) and satisfactory security (against external eavesdroppers and internal honest-but-curious nodes).

The remainder of this paper is arranged as follows. Section II gives some preliminaries on the structure of the microgrid, the optimization problem, the basic DED algorithm, and our research objectives. In Section III, an accelerated consensus scheme is presented and the optimal convergence rate is derived. Section IV introduces a privacy-preserving data exchange scheme, where the convergence and privacy analyses are carried out. Simulation studies are conducted in Section V to illustrate the theoretical results and, finally, Section VI gives some concluding remarks.

Notation: $\mathbb{R}^n$ ($\mathbb{C}^n$) and $\mathbb{R}^{n\times m}$ ($\mathbb{C}^{n\times m}$) refer to, respectively, the n-dimensional real (complex) vector space and the $n\times m$ real (complex) matrix space. $\mathbf{1}_N$ ($\mathbf{0}_N$) denotes an N-dimensional column vector with all ones (zeros). I_N stabds for an identity matrix of N dimensions. A^T means the transpose of matrix A. $|a|$ and $\|a\|_2$ denote, respectively, the modulus and the 2-norm of vector a. $\mathrm{col}_N\{x_i\}$ denotes a column vector with x_i being the ith vector. $\mathrm{diag}_N\{A_i\}$ describes a block-diagonal matrix $\mathrm{diag}\{A_1, A_2, \cdots, A_N\}$.

II. Preliminaries

A. The Microgrid Structure

A typical microgrid consists of physical and cyber systems whose schematic structure is shown in Fig. 1, where the physical system refers to an interconnected electric power grids that contains flexible loads and distributed generators (DGs), and the cyber layer is depicted by a communication network whose main aim is to improve the efficiency of electricity dispatch and promote relative fairness for all participants (including both suppliers and consumers) [7].

In this paper, we assume that each agent (node) is connected with a few loads and a DG, and is able to obtain local supply and demand information. The communication network can be depicted by an undirected graph $\mathscr{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ where $\mathcal{V} = \{1, 2, \cdots, N\}$ is the set of nodes, $\mathcal{E} = \{(i,j)|i,j \in \mathcal{V}\} \subseteq \mathcal{V}\times\mathcal{V}$ is the set of edges, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the

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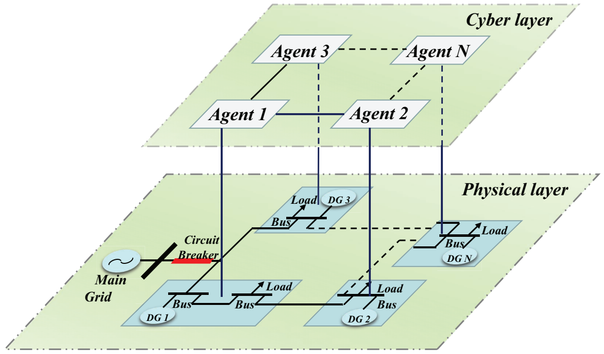

Fig. 1. Microgrid Structure.

adjacency matrix with nonnegative elements, respectively. The adjacent element $a_{ij} = 1$ if and only if the edge $(j,\ i) \in \mathcal{E}$, and otherwise $a_{ij} = 0$. The neighborhood set of node i is denoted as $\mathcal{N}_i = \{j|(i,j) \in \mathcal{E}, i \neq j\}$. The Laplacian matrix is described by $L = \mathcal{D} - \mathcal{A}$ with the degree matrix $\mathcal{D} = \text{diag}_N\{d_i\}$ where $d_i = \sum_{j\in\mathcal{N}_i} a_{ij}$. In addition, the undirected graph $\mathscr{G}$ is assumed to be connected in our paper.

B. The Optimization Problem

In general, the main objective of ED of islanded microgrids is to maintain the global balance between the generation power and the load demand at the least generation cost under practical physical constraints.

Now, let us formalize the optimization problem. For an underlying microgrid with N agents, the ED problem is described as

$$\begin{aligned}\min_{\{P_1^G,\ldots,P_N^G\}} \quad & \sum_{i=1}^{N} C_i\left(P_i^G\right)\\ \text{s.t.} \quad & \sum_{i=1}^{N} P_i^G = \sum_{i=1}^{N} P_i^D = P^D,\\ & P_i^{\min} \leq P_i^G \leq P_i^{\max},\end{aligned} \tag{1}$$

where P_i^D is the local demand, P_i^G is the local generation power, P^D is the total demand satisfying condition $\sum_{i\in\mathcal{V}} P_i^{\min} \leq P^D \leq \sum_{i\in\mathcal{V}} P_i^{\max}$, and the generation cost $C_i(P_i^G)$ of agent i is expressed as the following quadratic function:

$$C_i(P_i^G) = a_i(P_i^G)^2 + b_i P_i^G + c_i. \tag{2}$$

Here, $a_i > 0$, $b_i > 0$, and $c_i > 0$ are the proper cost coefficients. For ease of analysis, the cost function (2) can rewritten as

$$C_i(P_i^G) = \frac{(P_i^G - \psi_i)^2}{2\phi_i} + \iota_i, \tag{3}$$

where $\phi_i = \frac{1}{2a_i}$, $\psi_i = -\frac{b_i}{2a_i}$, and $\iota_i = c_i - \frac{b_i^2}{4a_i}$. Subsequently, define the incremental cost of node i as

$$\lambda_i = \frac{dC_i(P_i^G)}{dP_i^G} = \frac{P_i^G - \psi_i}{\phi_i}. \tag{4}$$

For the physical explanation of the cost function (2), we refer the readers to [7].

C. The Common Consensus-Based ED Algorithm

In order to solve the optimal ED problem (1), the consensus algorithm proposed in [8] is given as follows:

$$\begin{cases} \lambda_{i,k+1} = \lambda_{i,k} + \epsilon_1 \sum\limits_{j\in\mathcal{N}_i} a_{ij}(\lambda_{j,k} - \lambda_{i,k}) + \varepsilon_i s_{i,k},\\ P_{i,k+1}^G = \begin{cases} P_i^{\max}, & \lambda_{i,k+1} \leq \lambda_i^{\max},\\ \phi_i\lambda_{i,k+1} + \psi_i, & \lambda_i^{\min} < \lambda_{i,k+1} < \lambda_i^{\max},\\ P_i^{\min}, & \lambda_{i,k+1} \geq \lambda_i^{\min}, \end{cases}\\ s_{i,k+1} = s_{i,k} + \epsilon_2 \sum\limits_{j\in\mathcal{N}_i} a_{ij}(s_{j,k} - s_{i,k}) - (P_{i,k+1}^G - P_{i,k}^G), \end{cases} \tag{5}$$

where $\lambda_{i,k}, s_{i,k}, (i \in \mathcal{V})$ are, respectively, the incremental cost and the local estimated mismatch between the power generation and the demand, $\varepsilon_i > 0$ is a small gain parameter, $\epsilon_1, \epsilon_2 \in (0, \frac{1}{\max\{d_1,d_2,\cdots,d_N\}+1})$ are the coupling constants, $\lambda_i^{\min} = \frac{P_i^{\min} - \psi_i}{\phi_i}$ and $\lambda_i^{\max} = \frac{P_i^{\max} - \psi_i}{\phi_i}$. Furthermore, the initial value of the consensus algorithm (5) can be set as follows:

$$\begin{cases} P_{i,0}^G \in [P_i^{\min}, P_i^{\max}],\\ \lambda_{i,0} = \frac{P_{i,0}^G - \psi_i}{\phi_i},\\ s_{i,0} = P_i^D - P_{i,0}^G. \end{cases} \tag{6}$$

It should be pointed out that the optimization problem (1) is solved by the consensus algorithm (5) in the following lemma.

Lemma 1: [8] Consider the consensus-based DED algorithm (5) under the condition (6). If the undirected graph $\mathscr{G}$ is connected and the scalar ε_i satisfies $\varepsilon_i \in (0,\ \bar{\varepsilon})$, then the algorithm (5) converges to the optimal solution of the optimization problem (1), i.e.,

$$\lim_{k\to\infty} \lambda_{i,k} = \lambda^\star,\ \lim_{k\to\infty} s_{i,k} = 0,\ \lim_{k\to\infty} P_{i,k}^G = P_i^\star,\ i \in \mathcal{V}. \tag{7}$$

Here, the explicit expressions of $\lambda^\star$ and $P_i^\star$ are written as follows:

$$\begin{cases} \lambda^\star = \dfrac{\sum_{i=1}^N P_i^D - \sum_{i\in\mathcal{V}_1} P_i^\star - \sum_{i\in\mathcal{V}_2}\psi_i}{\sum_{i\in\mathcal{V}_2}\phi_i},\\ P_i^\star = \begin{cases} P_i^{\min}, & \lambda^\star < \lambda_i^{\min},\ i \in \mathcal{V}_1,\\ \phi_i\lambda^\star + \psi_i, & \lambda_i^{\min} \leq \lambda^\star \leq \lambda_i^{\max},\ i \in \mathcal{V}_2,\\ P_i^{\max}, & \lambda^\star > \lambda_i^{\max},\ i \in \mathcal{V}_1, \end{cases} \end{cases} \tag{8}$$

where $\mathcal{V}_1, \mathcal{V}_2$ are the subset of nodes satisfying $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$ and $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$. If node i $(i \in \mathcal{V}_1)$, then the corresponding generator outputs the maximum or minimum power.

Note that the scalar ε_i plays a vital role in ensuring the convergence of the algorithm (5), and its upper bound $\bar{\varepsilon}$ is discussed in *Proposition 2* of [17] by means of a result of the optimal matching distance.

D. The Research Objectives

Before presenting our main objective, two definitions concerning the convergence rate and the privacy are given as follows.

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Definition 1: [39] The *asymptotic convergence factor* is described by

$$r_{\text{asym}} = \sup_{x_0 \neq x^\star} \lim_{k\to\infty} \Big(\frac{\|x_k - x^\star\|_2}{\|x_0 - x^\star\|_2}\Big)^{1/k},$$

where $x^\star = \lim_{k\to\infty} x_k$.

Definition 2: [35] For the topology $\mathscr{G}$ of N nodes, the privacy of all nodes is said to be preserved, if external eavesdroppers and internal honest-but-curious nodes cannot estimate/infer power sensitive information with any accuracy.

We are now in a position to state the research objectives of this paper as follows.

1) Design an accelerated consensus scheme to realize *optimal* power distribution at the *lowest* generation cost with the *fastest* convergence rate for an improved DED algorithm.
2) Develop a privacy-preserving algorithm to prevent the sensitive information from being inferred (or estimated) by internal honest-but-curious agents or external eavesdroppers with any guaranteed accuracy.

Remark 1: It is worth noting that the privacy-sensitive data includes the local demand, the generation parameters, and the generation power, all of which involve significant business information and consuming habits/behaviors, even the safe and reliable operation of power grids [3]. For example, a competitor who aims to pursue more profit may manipulate its power output to upset the electricity market when the individual sensitive data is revealed by eavesdroppers. In addition, if the individual demand is stolen, the burglar might enter into the consumer's home when the residence is empty, resulting in unnecessary property damage or loss. Note that the initial value of the distributed algorithm plays an important role in optimal power dispatch and power-sensitive information preservation in this paper. As a result, it is of practical significance to develop a privacy-preserving DED algorithm to preserve the initial value, which is capable of ensuring the optimal power dispatch without sensitive information disclosure.

Remark 2: Actually, it is much simpler and more efficient to implement the distributed algorithm over undirected graphs than over directed graphs in actual engineering. The undirected graphs are used to stand for the communication networks where communication devices operate in the so-called full-duplex mode [38] that allows communication devices to transmit and receive information, simultaneously. Compared with directed communication networks, the distributed algorithm over undirected graphs does not require any additional information about the nominal out-degrees, and can achieve fast convergence since more information are exchanged.

III. Accelerated Consensus Scheme

In this section, an accelerated consensus algorithm is developed to solve the ED problem (1), and then the optimal convergence rate is derived by taking advantage of the algebraic analysis method. Furthermore, the optimality of the proposed algorithm is analyzed by mathematical induction.

Motivated by the idea of short memory [40], [41], the accelerated DED algorithm is given as follows:

$$\begin{cases} \lambda_{i,k+1} = \alpha\Big(\lambda_{i,k} + \epsilon_1 \sum\limits_{j\in\mathcal{N}_i} a_{ij}(\lambda_{j,k} - \lambda_{i,k}) + \varepsilon_i s_{i,k}\Big) \\ \qquad\quad + (1-\alpha)\lambda_{i,k-1}, \\ P^G_{i,k+1} = \begin{cases} P_i^{\max}, & \lambda_{i,k+1} \leq \lambda_i^{\max}, \\ \phi_i\lambda_{i,k+1} + \psi_i, & \lambda_i^{\min} < \lambda_{i,k+1} < \lambda_i^{\max}, \\ P_i^{\min}, & \lambda_{i,k+1} \geq \lambda_i^{\min}, \end{cases} \\ s_{i,k+1} = \alpha\Big(s_{i,k} + \epsilon_2 \sum\limits_{j\in\mathcal{N}_i} a_{ij}(s_{j,k} - s_{i,k}) \\ \qquad\quad - (P^G_{i,k+1} - P^G_{i,k})\Big) + (1-\alpha)s_{i,k-1}, \end{cases} \tag{9}$$

where $\alpha \in (1,\ 2)$ is an unknown parameter to be designed, and

$$\lambda_{i,-1} = \lambda_{i,0}, \quad s_{i,-1} = s_{i,0}, \quad i \in \mathcal{V}. \tag{10}$$

To obtain our main results, the performance analysis is divided into two parts: the DED algorithm without and with practical constraints.

We first consider the DED algorithm without generation constraints, and the corresponding algorithm (9) can be described by

$$\begin{cases} \lambda_{i,k+1} = \alpha\Big(\lambda_{i,k} + \epsilon_1 \sum\limits_{j\in\mathcal{N}_i} a_{ij}(\lambda_{j,k} - \lambda_{i,k}) + \varepsilon_i s_{i,k}\Big) \\ \qquad\quad + (1-\alpha)\lambda_{i,k-1}, \\ s_{i,k+1} = \alpha\Big((1-\varepsilon_i\phi_i)s_{i,k} + \epsilon_2 \sum\limits_{j\in\mathcal{N}_i} a_{ij}(s_{j,k} - s_{i,k}) \\ \qquad\quad - \epsilon_1\phi_i \sum\limits_{j\in\mathcal{N}_i} a_{ij}(\lambda_{j,k} - \lambda_{i,k})\Big) + (1-\alpha)s_{i,k-1}. \end{cases} \tag{11}$$

For simplicity, let us denote

$$\lambda_k = \text{col}_N\{\lambda_{i,k}\},\ s_k = \text{col}_N\{s_{i,k}\},\ x_k = [\ \lambda_k^T \quad s_k^T\ ]^T,$$
$$\Phi = \text{diag}_N\{\phi_i\},\ \varepsilon = \text{diag}_N\{\varepsilon_i\},\ \varsigma = \mathbf{1}_N^T\Phi\mathbf{1}_N.$$

The compact form of the DED algorithm (11) can be expressed as follows:

$$x_{k+1} = \alpha T x_k + (1-\alpha)x_{k-1}, \tag{12}$$

where

$$T = \begin{bmatrix} A_1 & \varepsilon I \\ \Phi(I_N - A_1) & A_2 - \varepsilon\Phi \end{bmatrix},$$
$$A_1 = I_N - \epsilon_1 L, \quad A_2 = I_N - \epsilon_2 L.$$

Note that A_1 and A_2 are doubly stochastic matrices, which satisfy $A_1\mathbf{1}_N = A_2\mathbf{1}_N = \mathbf{1}_N$ and $\mathbf{1}_N^T A_1 = \mathbf{1}_N^T A_2 = \mathbf{1}_N^T$.

Before proceeding further, let us introduce the following important lemma.

Lemma 2: If matrices A_1 and A_2 are doubly stochastic and the scalar ε_i satisfies $\varepsilon_i \in (0,\ \bar{\varepsilon})$, then matrix T has a simple eigenvalue 1 and all the remaining $2N-1$ eigenvalues κ_i $(i = 2, 3, \cdots, 2N)$ satisfy $\rho = \max\{|\kappa_2|, |\kappa_3|, \cdots, |\kappa_{2N}|\} < 1$. Moreover, the left and right eigenvectors corresponding to eigenvalue 1 are $v = [\mathbf{1}_N^T \Phi \ \mathbf{1}_N^T]^T/\varsigma$, $u = [\mathbf{1}_N^T \ \mathbf{0}_N^T]^T$, respectively.

Proof: The proof follows the similar line of that of *Lemma 1* in [28], and is therefore skipped here. ■

In light of Definition 1 and Lemma 2, one has that the convergence rate of the common consensus algorithm (5) without generation constraints is ρ. In other words, the convergence rate of accelerated consensus algorithm (11) is ρ when $\alpha = 1$. In what follows, the performance of algorithm (11), in terms of the convergence, the convergence rate and the optimality, is evaluated in the following theorem.

Theorem 1: For the accelerated consensus-based ED algorithm (11) with initial conditions (6), (10), if all DGs have no physical constraints with the connected graph $\mathscr{G}$ and the scalar ε_i satisfies $\varepsilon_i \in (0,\ \bar{\varepsilon})$, then the algorithm (11) converges to the optimal solution (7) of the ED problem (1) with the optimal convergence rate

$$r^\star = \frac{\rho}{1+\sqrt{1-\rho^2}} < \rho < 1 \tag{13}$$

via selecting the following optimal $\alpha^\star$

$$\alpha^\star = \frac{2}{1+\sqrt{1-\rho^2}} \in (1,2). \tag{14}$$

Proof: Let us first analyze the convergence and convergence rate of the proposed distributed ED algorithm.

It follows from Lemma 2 that vectors u, v are the right and left eigenvectors of matrix T corresponding to the eigenvalue 1. Then, we know that there exists an invertible matrix $Q \triangleq \begin{bmatrix} u & q_1 & \cdots & q_{2N-1} \end{bmatrix}$ (with its inverse $Q^{-1} \triangleq \begin{bmatrix} v & r_1 & \cdots & r_{2N-1} \end{bmatrix}^T$) such that $Q^{-1}TQ = \text{diag}\{1, \kappa_2, \cdots, \kappa_{2N}\}$ for vectors $q_i, r_i \in \mathbb{C}^{2N}$ $(i = 1, 2, \cdots, 2N-1)$.

Define the consensus error as $\bar{x}_k = (I - uv^T)x_k$. Based on the fact that $uv^T T = Tuv^T = uv^T uv^T = uv^T$, the dynamics of consensus error system can be described by

$$\begin{aligned} \bar{x}_{k+1} &= \alpha(T - uv^T)x_k + (1-\alpha)(I - uv^T)x_{k-1} \\ &= \alpha(T - uv^T)(x_k - uv^T x_k) + (1-\alpha)\bar{x}_{k-1} \\ &= \alpha(T - uv^T)\bar{x}_k + (1-\alpha)\bar{x}_{k-1}. \end{aligned} \tag{15}$$

Noting that $\text{rank}(uv^T) = \text{rank}(v^T u) = 1$ and $v^T u = 1$, we have that the matrix uv^T has a simple eigenvalue 1 with other eigenvalues being 0. As a result, the matrix $T - uv^T$ can be expressed as $T - uv^T = QJQ^{-1}$ with $J \triangleq \text{diag}\{0, \kappa_2, \cdots, \kappa_{2N}\}$ and its spectral radius is $\rho < 1$.

In what follows, by denoting $\eta_k = \begin{bmatrix} \bar{x}_k^T & \bar{x}_{k-1}^T \end{bmatrix}^T$, we obtain

$$\eta_{k+1} = \Theta \eta_k, \tag{16}$$

where

$$\Theta = \begin{bmatrix} \alpha(T - uv^T) & (1-\alpha)I_{2N} \\ I_{2N} & 0_{2N\times 2N} \end{bmatrix}.$$

Since $Q^{-1}(T - uv^T)Q$ is diagonal, one has

$$\tilde{\Theta} = \text{diag}\{Q^{-1}, Q^{-1}\}\Theta\text{diag}\{Q, Q\}, \tag{17}$$

where $\tilde{\Theta} = \begin{bmatrix} \alpha J & (1-\alpha)I_{2N} \\ I_{2N} & 0_{2N\times 2N} \end{bmatrix}$.

By means of an appropriate permutation of the columns and rows, the matrix $\tilde{\Theta}$ can be transformed into a block diagonal matrix $\tilde{\Theta} = \text{diag}_{2N}\{\tilde{\Theta}_i\}$ where

$$\tilde{\Theta}_i = \begin{bmatrix} \alpha\kappa_i & 1-\alpha \\ 1 & 0 \end{bmatrix} \tag{18}$$

with $\kappa_1 = 0$. Furthermore, the eigenvalues τ_{1_i}, τ_{2_i} of the matrix $\tilde{\Theta}_i$ can be written as

$$\tau_{1_i,2_i} = \frac{\alpha\kappa_i \pm \sqrt{\Delta_{\tilde{\Theta}_i}(\alpha)}}{2}, \tag{19}$$

where $\Delta_{\tilde{\Theta}_i}(\alpha) = \alpha^2\kappa_i^2 - 4\alpha + 4$, and $|\tau_{1_1}| = |\tau_{2_1}| = \sqrt{\alpha - 1}$.

The next step is to minimize $\max_{i\in\mathcal{V}}\{|\tau_{1_i}|, |\tau_{2_i}|\}$ via selecting the optimal α. Note that if $\alpha^2\rho^2 - 4\alpha + 4 \geq 0$, then $\max_{i\in\mathcal{V}}\{|\tau_{1_i}|, |\tau_{2_i}|\} = \frac{\alpha\rho + \sqrt{\alpha^2\rho^2 - 4\alpha + 4}}{2} \triangleq f(\alpha)$. In this case, the definition domain of α is calculated as $\alpha \leq \frac{2}{1+\sqrt{1-\rho^2}} = \alpha^\star \in (1,\ 2)$, and those α satisfying $\alpha \geq \frac{2}{1-\sqrt{1-\rho^2}} \geq 2$ is discarded. As a result, the optimal convergence rate can be written as:

$$r^\star = \min_{\alpha \in (1,\ \alpha^\star]} f(\alpha). \tag{20}$$

It is not difficult to calculate that the function $f(\alpha)$ is monotonically decreasing when $\alpha \in (1, \alpha^\star]$. Hence, the minimum spectral radius of matrix is $r^\star = f(\alpha^\star) = \frac{\rho}{1+\sqrt{1-\rho^2}} < \rho < 1$.

It follows from $\Delta_{\tilde{\Theta}_i}(\alpha^\star) \leq 0$ $(i = 2, 3, \cdots, 2N)$ that

$$|\tau_{1_i}| = |\tau_{2_i}| = \sqrt{\alpha^\star - 1} = \frac{\rho}{1+\sqrt{1-\rho^2}} \in (0, 1). \tag{21}$$

In addition, when $\alpha \in [\alpha^\star,\ 2)$, the spectral radius is expressed as $r = \sqrt{\alpha - 1}$, and its minimum value is also $r^\star$. To sum up, the minimum spectral radius of matrix Θ is $r^\star = \frac{\rho}{1+\sqrt{1-\rho^2}} < 1$ with the optimal $\alpha^\star$, which concludes that the optimal convergence rate is $r^\star$, and therefore $\lim_{k\to\infty} \eta_k = 0$.

In what follows, we are ready to carry out the optimality analysis of the DED algorithm by mathematical induction. Note that $uv^T x_1 = uv^T x_0$ is true. Assume that $uv^T x_k = uv^T x_{k-1}$ holds, one has

$$\begin{aligned} uv^T x_{k+1} &= \alpha uv^T T x_k + (1-\alpha)uv^T x_{k-1} \\ &= \alpha uv^T x_k + (1-\alpha)uv^T x_k \\ &= uv^T x_k, \end{aligned} \tag{22}$$

which further leads to

$$uv^T x_{k+1} = uv^T x_k = \cdots = uv^T x_0 = v^T x_0 u.$$

Hence, the algorithm (11) converges to $\lambda^\star = \lim_{k\to\infty} \lambda_{i,k} = v^T x_0 = \frac{\mathbf{1}_N^T \Theta \lambda_0 + \mathbf{1}_N^T \phi_0}{\varsigma}$, and hence $\lim_{k\to\infty} s_{i,k} = 0$ for $\forall i \in \mathcal{V}$. The proof is now complete. ■

The above theorem shows that the developed accelerated consensus algorithm *without constraints* can converge to the optimal incremental cost $\lambda^\star$ with the optimal convergence rate $r^\star$. Next, we are ready to deal with the general case with generation constraints.

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Theorem 2: For the accelerated consensus-based ED algorithm (9) with initial conditions (6), (10), if communication graph $\mathscr{G}$ is connected and the scalar ε_i satisfies $\varepsilon_i \in (0,\ \bar{\varepsilon})$, then the algorithm (9) converges to the optimal solution (7) of the ED problem (1) with the optimal convergence rate

$$r^\star = \frac{\rho'}{1+\sqrt{1-\rho'^2}} < \rho' < 1$$

via selecting the following optimal $\alpha^\star$

$$\alpha^\star = \frac{2}{1+\sqrt{1-\rho'^2}} \in (1,2).$$

where ρ' is the modulus of the largest eigenvalue of matrix $\tilde{T}$ except 1, and the matrix $\tilde{T}$ is denoted in (25).

Proof: Recalling the constraint condition $\sum_{i\in\mathcal{V}} P_i^m \le P_D \le \sum_{i\in\mathcal{V}} P_i^M$, it is known from the Karush-Kuhn-Tucker (KKT) necessity condition [42] that there exists at least one generator that is not saturated when supply-demand balance is achieved. If all local power generation is unsaturated at any time instant (i.e., $P_{i,k}^G \in [P_i^{\min},\ P_i^{\max}], \forall k \in \{0,1,\cdots\}, \forall i \in \mathcal{V}$), then the corresponding proof follows readily from Theorem 1. As such, we only investigate the saturated case.

Inspired by [8], denote $\Lambda_k = \sum_{i=1}^N \lambda_{i,k}$ and $S_k = \sum_{i=1}^N s_{i,k}$. In light of the algorithm (11), one has

$$\begin{aligned} \Lambda_{k+1} &= \alpha\Lambda_k + (1-\alpha)\Lambda_{k-1} + \alpha\sum_{i=1}^N \varepsilon_i s_{i,k}, \\ S_{k+1} &= \alpha S_k + (1-\alpha)S_{k-1} - \alpha\sum_{i=1}^N \varepsilon_i\phi_i s_{i,k}. \end{aligned} \quad (23)$$

Without loss of generality, we assume $S_k > 0$. It is observed that Λ_k will increase and the sum of all generation power $\sum_{i=1}^N P_i^G$ will also increase in light of the established relationship in (4). Meanwhile, the mismatch S_k will decrease. After sufficiently long time k_t, if P_{i,k_t}^G is saturated, then $P_{i,k}^G$ stays unchanged for $\forall k > k_t$ since the increment cost $\lambda_{i,k}$ approaches to the same value. Hence, for $k > k_t$, the accelerated algorithm with generation constraints can be written by:

$$x_{k+1} = \alpha\tilde{T}x_k + (1-\alpha)x_{k-1}, \quad (24)$$

where

$$\begin{aligned} &\tilde{T} = \begin{bmatrix} A_1 & \varepsilon I \\ \tilde{\Phi}(I-A_1) & A_2 - \varepsilon\tilde{\Phi} \end{bmatrix}, \\ &\tilde{\Phi} \triangleq \text{diag}\{\tilde{\phi}_1, \tilde{\phi}_2, \cdots, \tilde{\phi}_N\}, \\ &\tilde{\phi}_i \triangleq \begin{cases} 0, & \text{if } i \in \Omega_1(P_{i,k}^G) \cup \Omega_2(P_{i,k}^G) \text{ for } k \ge k_t, \\ \phi_i, & \text{otherwise}, \end{cases} \\ &\Omega_1(P_i^G) \triangleq \{i | P_i^G = P_i^{\max}\}, \\ &\Omega_2(P_i^G) \triangleq \{i | P_i^G = P_i^{\min}\}. \end{aligned} \quad (25)$$

Next, following the similar proof line of *Theorem 1*, the accelerated algorithm can converge to the optimal solution (7) of the optimization problem (1), and the proof is omitted here. ■

IV. Privacy-Preserving Scheme

In this section, the privacy-preserving DED algorithm is first provided by introducing edge-based perturbations at the initial time instant, and then the convergence and optimality of the proposed algorithm are discussed. Furthermore, the privacy analysis is conducted to show the security against external and internal (honest-but-curious) adversaries.

A. Privacy-Preserving DED Algorithm

At the initial time instant, each agent produces some perturbation signals that are injected into edges in the communication graph. To be more specific, agent i generates a group of perturbations $\pi_i^j \in \mathbb{R}$ and transmits $\hat{s}_{i,0}^j = s_{i,0} + \pi_i^j$ to its neighboring node j ($j \in \mathcal{N}_i$). Furthermore, the privacy-preserving accelerated consensus algorithm is designed as:

$$\begin{cases} \lambda_{i,1} = \alpha\Big(\lambda_{i,0} + \epsilon_1 \sum_{j\in\mathcal{N}_i} a_{ij}(\lambda_{j,0} - \lambda_{i,0}) + \varepsilon_i s_{i,0}\Big) \\ \qquad + (1-\alpha)\lambda_{i,-1}, \\ s_{i,1} = \alpha\Big(s_{i,0} + \hat{\epsilon}_2 \sum_{j\in\mathcal{N}_i} a_{ij}(\hat{s}_{j,0}^i - s_{i,0}) + \hat{\epsilon}_2\pi_i^i \\ \qquad - (P_{i,1}^G - P_{i,0}^G)\Big) + (1-\alpha)s_{i,-1}, \end{cases} \quad (26)$$

where $\hat{\epsilon}_2$ is a non-zero constant, $\varepsilon_i \in (0,\ \bar{\varepsilon})$ is a small gain parameter, and $\pi_i^i = -\sum_{j\in\mathcal{N}_i} \pi_j^i$. For $k \ge 1$, all agents transmit their true state information in light of the accelerated consensus algorithm (9). The privacy-preserving DED scheme is summarized in Algorithm 1.

Algorithm 1 Privacy-Preserving DED Algorithm

► **When** $k = 0$:
Step 1: Agent i generates d_i real numbers: $\pi_i^{j_1}$, $\pi_i^{j_2}$, $\cdots$, $\pi_i^{j_{d_i}}$ where $j_1, j_2, \cdots, j_{d_i} \in \mathcal{N}_i$.
Step 2: Agent i computes $\pi_i^i = -\sum_{j\in\mathcal{N}_i} \pi_i^j$.
Step 3: Agent i transmits $\lambda_{i,0}$ and $\hat{s}_{i,0}^j$ to agent j, and receives $\lambda_{j,0}$ and $\hat{s}_{j,0}^i$, where $j \in \mathcal{N}_i$.
Step 4: Agent i updates its state via (26).

► **When** $k \ge 1$:
Step 1: Agent i transmits $\lambda_{i,k}$, $s_{i,k}$ to agent j, and receives $\lambda_{j,k}$, $s_{j,k}$, where $j \in \mathcal{N}_i$.
Step 2: Agent i updates its state via (9).

Theorem 3: Under Algorithm 1, the proposed privacy-preserving accelerated consensus scheme converges to the optimal solution (7) of the optimization problem (1).

Proof: First, define the perturbation matrix as $\Pi = [\pi_i^j]_{N\times N} \in \mathbb{R}^{N\times N}$ where π_i^j is the i-th row and j-th column element of matrix Π, and $\pi_i^j = 0$ if $j \notin \mathcal{N}_i$. Noting that $\pi_i^i = -\sum_{j\in\mathcal{N}_i} \pi_j^i$, one has $\Pi\mathbf{1}_N = \mathbf{0}_N$. Furthermore, the augmented form of (26) can be rewritten as

$$x_1 = \alpha\hat{T}x_0 + (1-\alpha)x_{-1} + \alpha\hat{\epsilon}_2\text{diag}\{\mathbf{0}_{N\times N}, \Pi^T\}\mathbf{1}_{2N}, \quad (27)$$

where

$$\hat{T} = \begin{bmatrix} A_1 & \varepsilon I \\ \Phi(I_N - A_1) & \hat{A}_2 - \varepsilon\Phi \end{bmatrix},$$

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$$A_1 = I_N - \epsilon_1 L, \quad \hat{A}_2 = I_N - \hat{\epsilon}_2 L.$$

Due to $x_0 = x_{-1}$ and $v^T \mathrm{diag}\{\mathbf{0}_{N\times N}, \Pi^T\} = \mathbf{0}_{2N}^T$, one has

$$\begin{aligned} uv^T x_1 &= \alpha uv^T \hat{T} x_0 + (1-\alpha) uv^T x_{-1} \\ &\quad + \alpha \hat{\epsilon}_2 uv^T \mathrm{diag}\{\mathbf{0}_{N\times N}, \Pi^T\} \mathbf{1}_{2N} \\ &= uv^T x_0. \end{aligned}$$

For $k \geq 1$, all agents carry out the distributed algorithm (9). It follows from Theorem 1 that the proposed privacy-preserving distributed algorithm can achieve optimal ED, which ends the proof. ■

The above theorem shows that the privacy-preserving DED algorithm can exactly converge to the optimal solution of ED problem, where the perturbation signals converge to 0 via consensus update due to the well-designed construction of the perturbation matrix.

B. Analysis of Privacy

In this subsection, we shall show that the proposed algorithm can preserve privacy against two types of adversaries [35]: 1) external eavesdroppers who can wiretap communication links of whole network; and 2) honest-but-curious nodes who can learn/infer the state of neighboring nodes but follow the update rules of the DED algorithm.

1) Privacy Preservation Against External Eavesdroppers

Define information set obtained by an external eavesdropper as $\Psi = \{\mathscr{G}, \alpha, \hat{s}_{j,0}^i|_{j\in\mathcal{N}_i}, \lambda_{i,k}|_{k\geq 0}, s_{i,k}|_{k\geq 1}, \forall i \in \mathcal{V}\}$. Then, one has the following theorem.

Theorem 4: Let the conditions in *Theorem 1* be satisfied and the date exchange of all agents follow the privacy-preserving scheme illustrated in Algorithm 1. Then, an external eavesdropper with accessible information set Ψ *cannot* estimate/infer the initial state $s_{i,0}$ $(i \in \mathcal{V})$ with any accuracy.

Proof: Note that the local supply $P_{i,0}^G$ and local demand P_i^D are reflected in $s_{i,0} = P_i^D - P_{i,0}^G$, and hence $s_{i,0}$ is regarded as the privacy value to be preserved. Clearly, Ψ is the only information accessible to the external eavesdropper for the purpose of inferring the initial value s_0, and therefore if Ψ is unchanged under different initial conditions of s_0, then an adversary certainly cannot infer/estimate the initial value s_0. In this case, to prove that the sensitive information $s_0 \triangleq [s_{1,0}, s_{2,0}, \cdots, s_{N,0}]^T$ is not leaked to an external eavesdropper, we provide a privacy-proving approach to show that any arbitrary change of s_0 is indistinguishable to external attackers, i.e., the information set Ψ stays unchanged under arbitrarily different initial value $\bar{s}_0, (\bar{s}_0 \neq s_0)$. To this end, we turn to prove that $\bar{\Psi} = \Psi$ for $\bar{s}_0 \neq s_0$, where $\bar{\Psi}$ is the information set under the initial value $\bar{s}_0$.

To be more specific, the initial conditions, the perturbations and the parameters can be given as follows:

$$\begin{aligned} &\bar{s}_0 = s_0 + \alpha\Delta\epsilon_2(Ls_0 - \Pi^T \mathbf{1}_N), \quad \bar{\lambda}_0 = \lambda_0, \\ &\bar{\pi}_i^j = \pi_i^j + (s_{i,0} - \bar{s}_{i,0}), \qquad \forall (i,j) \in \mathcal{E}, \\ &\bar{\pi}_i^i = -\sum_{j\in\mathcal{N}_i} \bar{\pi}_i^j, \ \bar{\varepsilon}_{i,0} = \frac{\varepsilon_i s_{i,0}}{\bar{s}_{i,0}}, \ \bar{\epsilon}_2 = \hat{\epsilon}_2 + \Delta\epsilon_2, \end{aligned} \tag{28}$$

where $\Delta\epsilon_2 \neq 0$ is an arbitrary constant.

We can observe that the proposed algorithm under conditions in (28) can converge to the optimal solution (7), and corresponding proof can follow directly from that of Theorem 3.

Next, let us show that $\bar{\Psi} = \Psi$. Note that $\bar{\pi}_i^j + \bar{s}_{i,0} = \pi_i^j + s_{i,0}$ and $\bar{\lambda}_0 = \lambda_0$, which means that the exchanged information at $k = 0$ is identical under two different initial conditions. Furthermore, in the case that $k \geq 1$, one has

$$\begin{aligned} \Delta s_1 &= s_1 - \bar{s}_1 \\ &= \alpha(I_N - \hat{\epsilon}_2 L)s_0 + (1-\alpha)s_{-1} + \alpha\hat{\epsilon}_2 \Pi^T \mathbf{1}_N \\ &\quad - \alpha(I_N - \bar{\epsilon}_2 L)\bar{s}_0 - (1-\alpha)\bar{s}_{-1} - \alpha\bar{\epsilon}_2 \bar{\Pi}^T \mathbf{1}_N \\ &= (I_N - \alpha\bar{\epsilon}_2 L + \alpha\Delta\epsilon_2 L)s_0 - (I_N - \alpha\bar{\epsilon}_2 L)\bar{s}_0 \\ &\quad + \alpha(\bar{\epsilon}_2 - \Delta\epsilon_2)\Pi^T \mathbf{1}_N - \alpha\bar{\epsilon}_2 \bar{\Pi}^T \mathbf{1}_N \\ &= s_0 - \bar{s}_0 + \alpha\Delta\epsilon_2(Ls_0 - \Pi^T \mathbf{1}_N) \\ &\quad - \alpha\bar{\epsilon}_2(Ls_0 - L\bar{s}_0 - \Pi^T \mathbf{1}_N + \bar{\Pi}^T \mathbf{1}_N) \\ &= 0, \end{aligned} \tag{29}$$

which implies that $\bar{s}_1 = s_1$ and $\bar{\lambda}_1 = \lambda_1$. When $k \geq 2$, the privacy-preserving DED algorithm carries out the identical state update, i.e., $\bar{\lambda}_k = \lambda_k$, $\bar{s}_k = s_k$. Thus, we can conclude that $\bar{\Psi} = \Psi$ under $\bar{s}_0 \neq s_0$, which ends the proof. ■

2) Privacy Preservation Against Honest-But-Curious Agents

Define the information set available for agent i as

$$\begin{aligned} \Omega_i = \big\{&\mathscr{G}, \alpha, \lambda_{j,k}|_{j\in\mathcal{N}_i\cup\{i\}, k\geq 0}, \hat{s}_{j,0}^i|_{j\in\mathcal{N}_i}, s_{j,k}|_{j\in\mathcal{N}_i, k\geq 1}, \\ &s_{i,k}|_{k\geq 0}, \pi_j^j|_{j\in\mathcal{N}_i\cup\{i\}}\big\}. \end{aligned}$$

Theorem 5: Let the conditions in *Theorem 1* be satisfied, and the date exchange of all agents follow the privacy-preserving algorithm summarized in Algorithm 1. Then, an honest-but-curious node i cannot estimate/infer the initial state $s_{j,0}$ $(j \in \mathcal{N}_i)$ of legitimate node j via the obtained information set Ω_i.

Proof: For the convenience of privacy analysis, let us denote the legitimate nodes as $\mathbb{A}$ (Alice) and $\mathbb{B}$ (BoB), and the honest-but-curious node as $\mathbb{E}$ (Eve). Without loss of generality, two cases are investigated as follows:

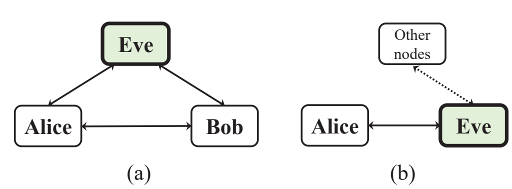

Fig. 2. Two connection configurations

• *Case 1. Agent $\mathbb{A}$ owns at least a legitimate neighboring agent $\mathbb{B}$*

For ease of analysis, let the communication topology be described in Fig. 2(a). Similarity, to show that the privacy of Alice is preserved against the honest-but-curious adversary $\mathbb{E}$, we just verify that $\bar{\Omega}_{\mathbb{E}} = \Omega_{\mathbb{E}}$ holds under $\bar{s}_{\mathbb{A}} \neq s_{\mathbb{A}}$.Specifically, to ensure the optimality of the proposed accelerated consensus algorithm without leaking sensitive information, the initial incremental cost satisfies $\bar{\lambda}_{i,0} = \lambda_{i,0}$ $(i \in \mathcal{V})$, the local demand

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satisfies

$$\bar{P}^D_{\mathbb{A}} \neq P^D_{\mathbb{A}},\ \bar{P}^D_{\mathbb{B}} = P^D_{\mathbb{A}} + P^D_{\mathbb{B}} - \bar{P}^D_{\mathbb{A}},\ \bar{P}^D_{\mathbb{E}} = \bar{P}^D_{\mathbb{E}},$$

and thus the initial power mismatch satisfies

$$\bar{s}_{\mathbb{A}} \neq s_{\mathbb{A}},\ \bar{s}_{\mathbb{B}} = s_{\mathbb{A}} + s_{\mathbb{B}} - \bar{s}_{\mathbb{A}},\ \bar{s}_{\mathbb{E}} = s_{\mathbb{E}}. \tag{30}$$

Furthermore, the perturbations can be selected as

$$\begin{aligned}
&\bar{\pi}^{\mathbb{B}}_{\mathbb{A}} = \pi^{\mathbb{B}}_{\mathbb{A}} + (s_{\mathbb{A},0} - \bar{s}_{\mathbb{A},0}),\ \ \bar{\pi}^{\mathbb{E}}_{\mathbb{A}} = \pi^{\mathbb{E}}_{\mathbb{A}} + (s_{\mathbb{A},0} - \bar{s}_{\mathbb{A},0}),\\
&\bar{\pi}^{\mathbb{A}}_{\mathbb{A}} = \pi^{\mathbb{A}}_{\mathbb{A}} - 2(s_{\mathbb{A},0} - \bar{s}_{\mathbb{A},0}),\ \bar{\pi}^{\mathbb{E}}_{\mathbb{B}} = \pi^{\mathbb{E}}_{\mathbb{B}} + (s_{\mathbb{B},0} - \bar{s}_{\mathbb{B},0}),\\
&\bar{\pi}^{\mathbb{B}}_{\mathbb{B}} = \pi^{\mathbb{B}}_{\mathbb{B}} + \big(\frac{1}{\alpha\hat{\epsilon}_2} - 2\big)(s_{\mathbb{B},0} - \bar{s}_{\mathbb{B},0}),\\
&\bar{\pi}^{\mathbb{A}}_{\mathbb{B}} = \pi^{\mathbb{A}}_{\mathbb{B}} + \big(1 - \frac{1}{\alpha\hat{\epsilon}_2}\big)(s_{\mathbb{B},0} - \bar{s}_{\mathbb{B},0}),\\
&\bar{\pi}^{j}_{\mathbb{E}} = \pi^{j}_{\mathbb{E}}, j = \mathbb{A}, \mathbb{B}, \mathbb{E},
\end{aligned} \tag{31}$$

and the initial gain parameter can be set as

$$\bar{\varepsilon}_{\mathbb{B}} = \frac{\varepsilon_{\mathbb{B}} s_{\mathbb{B},0}}{\bar{s}_{\mathbb{B},0}},\ \bar{\varepsilon}_{\mathbb{A}} = \frac{\varepsilon_{\mathbb{A}} s_{\mathbb{A},0}}{\bar{s}_{\mathbb{A},0}},\ \bar{\varepsilon}_{\mathbb{E}} = \varepsilon_{\mathbb{E}}. \tag{32}$$

It is not difficult to validate that the privacy-preserving DED algorithm can converge to the optimal solution (7) under conditions (30)-(32). Next, we are in position to show that $\bar{\Omega}_{\mathbb{E}} = \Omega_{\mathbb{E}}$.

In light of the established relationship in (31), we have $\bar{\lambda}_{j,0} = \lambda_{j,0}$ and $\hat{s}^{\mathbb{E}}_{j} = \bar{s}_{j,0} + \bar{\pi}^{\mathbb{E}}_{j} = s_{j,0} + \pi^{\mathbb{E}}_{j}$ $(j = \mathbb{A}, \mathbb{B})$, which means that, by using the received information, the node $\mathbb{E}$ cannot tell the difference under different initial conditions at $k = 0$. Subsequently, one obtains

$$\begin{aligned}
\Delta s_{\mathbb{A},1} &= s_{\mathbb{A},1} - \bar{s}_{\mathbb{A},1}\\
&= (\alpha - 2\alpha\hat{\epsilon}_2)\Delta s_{\mathbb{A},0} - \alpha\hat{\epsilon}_2\Delta s_{\mathbb{B},0} + 2\alpha\hat{\epsilon}_2\Delta s_{\mathbb{A},0}\\
&\quad + \alpha\hat{\epsilon}_2\big(\frac{1}{\alpha\hat{\epsilon}_2} - 1\big)\Delta s_{\mathbb{B},0} + (1-\alpha)\Delta s_{\mathbb{A},-1}\\
&= \alpha\Delta s_{\mathbb{A},0} + (1-\alpha)\Delta s_{\mathbb{A},-1} + \Delta s_{\mathbb{B},0}\\
&= \Delta s_{\mathbb{A},0} + \Delta s_{\mathbb{B},0}\\
&= 0.
\end{aligned} \tag{33}$$

Via similar calculation, one has $\Delta s_{\mathbb{B},1} = \Delta s_{\mathbb{E},1} = 0$, which further implies that $\bar{s}_1 = s_1$ and $\bar{\lambda}_1 = \lambda_1$. As a result, $\bar{\Omega}_{\mathbb{E}} = \Omega_{\mathbb{E}}$ holds under different initial conditions, which shows that the proposed algorithm can protect privacy against curious-but-honest neighbors in *Case 1*.

• *Case 2. All neighbors of Alice are honest-but-curious*

Note that multiple honest-but-curious agents can cooperatively estimate/infer the private value of agent $\mathbb{A}$ in collusion with each other, and thus these honest-but-curious agents can be viewed as a node. Hence, without loss of generality, we just consider the case that agent $\mathbb{A}$ has only an honest-but-curious neighboring agent $\mathbb{E}$ as illustrated in Fig. 2(b).

Note that Alice only communicates with the internal honest-but-curious agent Eve. In light of Algorithm 1, the available information of Eve is

$$\begin{aligned}
\hat{s}^{\mathbb{E}}_{\mathbb{A},0} &= s_{\mathbb{A},0} + \pi^{\mathbb{E}}_{\mathbb{A}},\\
s_{\mathbb{A},1} &= \alpha s_{\mathbb{A},0} + \alpha\hat{\epsilon}_2(\hat{s}^{\mathbb{A}}_{\mathbb{E},0} - s_{\mathbb{A},0}) + \alpha\hat{\epsilon}_2\pi^{\mathbb{A}}_{\mathbb{A}}\\
&\quad + (1-\alpha)s_{\mathbb{A},-1} + \alpha\phi_{\mathbb{A}}(\lambda_{\mathbb{E},0} - \lambda_{\mathbb{A},0}),\\
\pi^{\mathbb{A}}_{\mathbb{A}} &= -\pi^{\mathbb{E}}_{\mathbb{A}}.
\end{aligned}$$

TABLE I
COMPARISON AMONG EXISTING PRIVACY-PRESERVING DED ALGORITHMS.

	ours	[32]	[28]	[33]	[30]
convergence accuracy	✓	×	×	✓	✓
honest-but-curious nodes	✓	✓	✓	✓	✓
external eavesdroppers	✓	✓	✓	×	✓
convergence rate	fast	mid	mid	mid	slow
computational complexity	low	low	high	low	low

Note that $\phi_{\mathbb{A}}$ is unknown to Eve and thus Eve *cannot* infer $s_{\mathbb{A},0}$ via the following equations:

$$s_{\mathbb{A},0} = s_{\mathbb{A},1} - \alpha\hat{\epsilon}_2(\hat{s}^{\mathbb{A}}_{\mathbb{E},0} - \hat{s}^{\mathbb{A}}_{\mathbb{E},0}) - \alpha\phi_{\mathbb{A}}(\lambda_{\mathbb{E},0} - \lambda_{\mathbb{A},0}). \tag{34}$$

Therefore, combing with *Case 1* and *Case 2*, the privacy-preserving property of the developed algorithm is validated against internal honest-but-curious nodes. The proof is complete. ■

Remark 3: In this work, an edge-based additive perturbations are injected into the proposed consensus-based algorithm at the initial time instant for the purpose of privacy preservation. In light of the obtained results in Theorem 2, the convergence rate of the developed algorithm is independent of state variables. As a result, the injected perturbations do not affect the convergence rate of the proposed algorithm. In addition, it should be pointed that the honest-but-curious agent can be regarded as a competitor in microgrids. The competitor may unfairly strike rivals and disrupt market order for more interests by stealing the opponent's power-sensitive information.

Remark 4: In this paper, the edge-based additive perturbations have been injected into the consensus-based DED algorithm to achieve privacy preservation. It is observed in Table I that, in contrast to existing privacy-preserving techniques, our privacy-preserving scheme shows the notable merits from four aspects given as follows: 1) different from the differential privacy technique [32], our approach can exactly converge to the optimal solution to the ED problem (1) due to the well-designed perturbations; 2) in comparison with the homomorphic encryption method [28], the developed privacy-preserving algorithm can be easily implemented due to its simple multiplication and addition operations; 3) unlike the privacy-preserving approach in [33], this work shows a high level of privacy preservation against external eavesdroppers; and 4) compared with [28], [30], [32], [33], our algorithm exhibits the fast convergence. In summary, the developed privacy-preserving DED algorithm covers a wider range of practical applications.

Remark 5: Up to now, we have developed an accelerated consensus algorithm with edge-based additive perturbations to achieve the privacy-preserving optimal ED of microgrids. Compared with the existing results, our work presents the following distinctive superiorities: 1) the developed consensus-based scheme is capable of improving the convergence rate of the DED algorithm without increasing computation/communication cost; 2) the optimal convergence

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rate is accurately obtained by virtue of rigorous algebra analysis according to the defined asymptotic convergence factor; and 3) the adopted privacy-preserving distributed scheme exhibits better system performance in terms of convergence accuracy and security against external eavesdroppers and/or internal honest-but-curious nodes.

V. Simulation Study

In this section, we provide simulated cases to illustrate the validity and superiority of the proposed privacy-preserving DED algorithm.

We adopt the IEEE 39-bus systems with 10 DGs and 18 local loads to validate the obtained results. Assume that there are 10 agents in microgrids, where each agent is connected with a DG and a few local loads. The communication links among all agents are depicted by the red dashed lines as presented in Fig. 3. The parameters of DGs are given in Table II. Set $\varepsilon = 0.03$, $\epsilon_1 = \epsilon_2 = 0.2$.

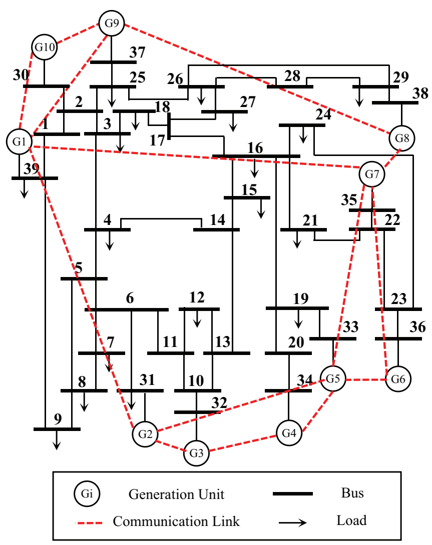

Fig. 3. IEEE 39-Bus system

TABLE II
Parameters of DGs [17]

DGi	a_i	b_i	c_i	P_i^m	P_i^M
$1, 6$	0.105	2.53	78	3.8	40
$2, 7$	0.074	3.17	62	4.2	18
$3, 8$	0.078	3.41	31	8	60
$4, 9$	0.082	4.02	42	5.4	45
$5, 10$	0.094	1.22	51	10	80

Select the local demand P_i^D $(i \in \mathcal{V})$ as $P_1^D = 25KW$, $P_2^D = 10KW$, $P_3^D = 20KW$, $P_4^D = 30KW$, $P_5^D = 35KW$, $P_6^D = 20KW$, $P_7^D = 15KW$, $P_8^D = 20KW$, $P_9^D = 25KW$, $P_{10}^D = 40KW$, and thus the total demand is $P^D = \sum_{i=1}^{10} P_i^D = 240KW$.

A. Case 1: Without Generation Constraints

We first consider the case that the DED algorithm without generation constraints. Select the initial generation power $P_{i,0}^G$ as $P_{1,0}^G = 10KW$, $P_{2,0}^G = 5KW$, $P_{3,0}^G = 8KW$, $P_{4,0}^G = 11KW$, $P_{5,0}^G = 23KW$, $P_{6,0}^G = 12KW$, $P_{7,0}^G = 17KW$, $P_{8,0}^G = 14KW$, $P_{9,0}^G = 23KW$, and $P_{10,0}^G = 34KW$. It calculates $\rho = 0.9640$, and further obtains the optimal convergence rate $r^\star = 0.7614$ with $\alpha^\star = 1.5797$. The test results are shown in Figs. 4-5, It is observed in Fig. 4 that, after a few of time instants, the incremental cost converges to the optimal value $\lambda^\star = 7.031$, the local mismatch approaches 0, local generation power $P_{i,0}^G, i \in \mathcal{V}$ respectively converge to the optimal value $P_i^\star$, and the supply-demand balance is achieved, which verifies the established theoretical results. In addition, Fig. 5 shows the simulation results via employing the common consensus-based ED algorithm [8]. Comparing Fig. 4 with Fig. 5, the proposed accelerated consensus scheme has a significant merit in improving convergence rate.

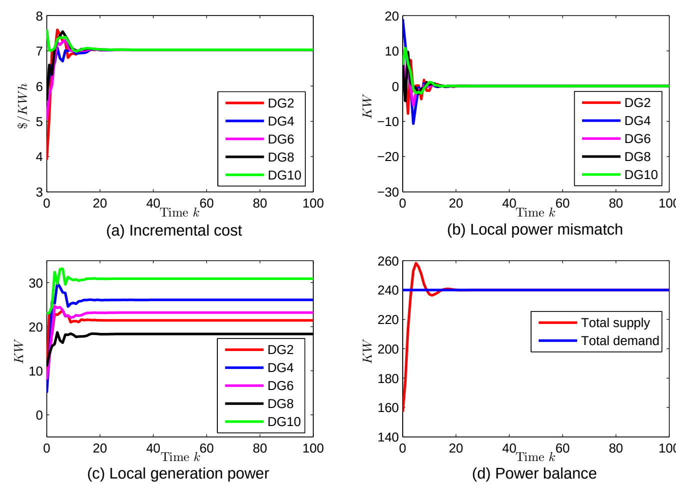

Fig. 4. Convergence of accelerated consensus algorithm without generation constraints

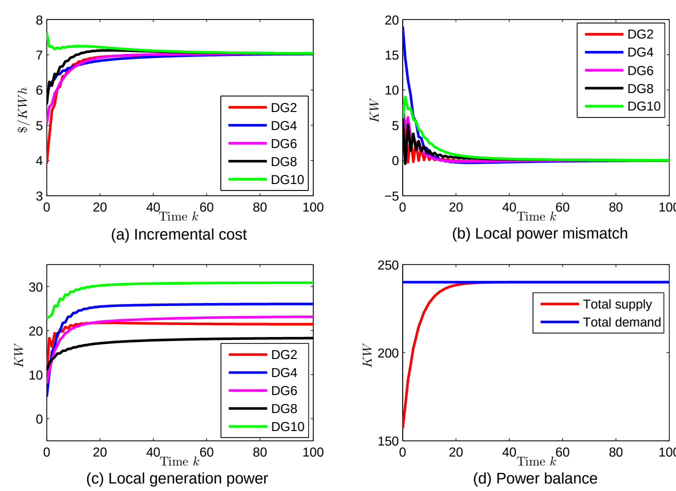

Fig. 5. Convergence of consensus algorithm without generation constraints [8]

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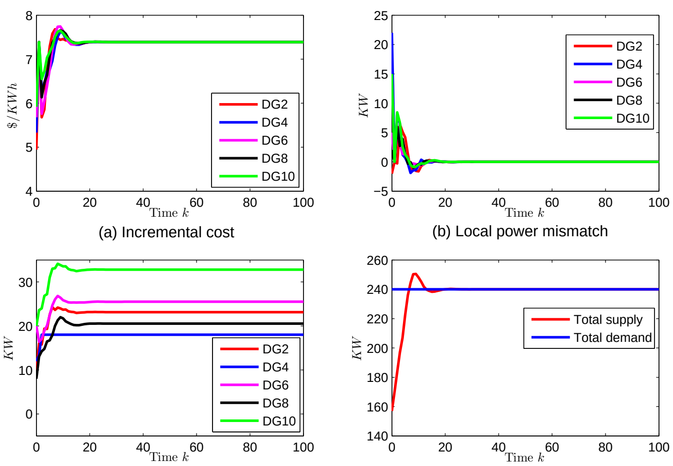
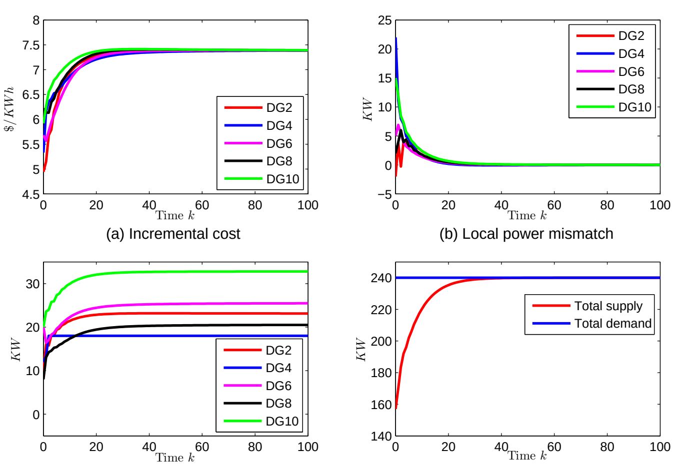
B. Case 2: With Generation Constraints

Next, we carry out the simulation of the proposed accelerated consensus algorithm with generation constraints. Select the initial generation power $P_{i,0}^G$ as $P_{1,0}^G = 10KW$, $P_{2,0}^G = 12KW$, $P_{3,0}^G = 20KW$, $P_{4,0}^G = 8KW$, $P_{5,0}^G = 20KW$, $P_{6,0}^G = 15KW$, $P_{7,0}^G = 13KW$, $P_{8,0}^G = 18KW$, $P_{9,0}^G = 16KW$, and $P_{10,0}^G = 25KW$. It derives that $\rho = 0.9640$ when $P_{2,k}$ and $P_{7,k}$ are saturated, and further obtains the optimal convergence rate $r^\star = 0.7587$ with $\alpha^\star = 1.5757$. The simulation results are presented in Figs. 6-7. Fig. 6 plots the dynamic evolution of the incremental cost, the local power mismatch, the local generation power, and the global generation power. Note that all the states converge to corresponding optimal solutions. In comparison with [8], our developed algorithm exhibits faster convergence rate, which demonstrates the advantage of the developed algorithm.



(a) Incremental cost (b) Local power mismatch (c) Local generation power (d) Power balance

Fig. 6. Convergence of accelerated consensus algorithm with generation constraints

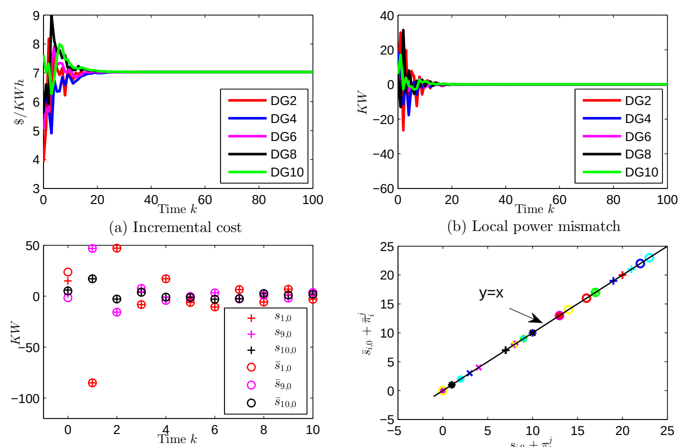


(a) Incremental cost (b) Local power mismatch (c) Local generation power (d) Power balance

Fig. 7. Convergence of consensus algorithm with generation constraints [8]

C. Case 3: Privacy Preservation Against External Eavesdroppers

Now, we are in position to show that the proposed accelerated consensus algorithm with additive perturbations is privacy-preserving against external eavesdroppers. Set $\bar{\epsilon}_2 = 1.1$, $\hat{\epsilon}_2 = 1$. Based on the construction in (28), we set $\bar{s}_0$, $\bar{\pi}_i^j$, $\forall (i,j) \in \mathcal{E}$, and $\bar{\varepsilon}_i = \frac{\varepsilon_i s_{i,0}}{\bar{s}_{i,0}}$ where $i \in \mathcal{V}$. Note that Fig. 8(a)(b) display the evolution of incremental cost and local power mismatch with additive perturbations, which shows the developed privacy-preserving DED algorithm can achieve optimal power allocation at a minimum cost. Fig. 8(c) plots the trajectories of local power mismatch under initial conditions $\bar{s}_0$ and s_0, from which we can see $\bar{s}_k = s_k$, $\forall k \geq 1$. In addition, the transmitted information at $k = 0$ is shown in Fig. 8(d) under initial conditions $\bar{s}_0$ and s_0. It is observed from Fig. 8(c) and Fig. 8(d) that the information sets $\bar{\Psi}$ and Ψ accessible to external eavesdroppers are completely identical, even though all initial values are changed. Hence, the proposed algorithm is privacy-preserving against external eavesdroppers.

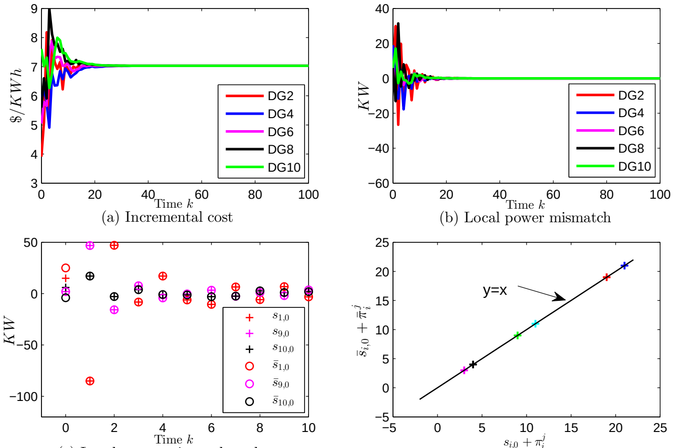


(a) Incremental cost (b) Local power mismatch (c) Local power mismatch under different initial values (d) Transmitted information at $k = 0$: $(s_{i,0} + \pi_i^j, \bar{s}_{i,0} + \bar{\pi}_i^j)$

Fig. 8. Convergence and privacy preservation against external eavesdroppers

D. Case 4: Privacy Preservation Against Honest-But-Curious Agents

In this case, assume that agent 9 (Eve) is an honest-but-curious nodes who can intend to estimate agent 1 (Alice). According to the construction in (23), we select $\bar{P}_1^D = P_1^D + 10 = 35KW$, $\bar{P}_{10}^D = P_{10}^D - 10 = 30KW$, and other local power demand is unchanged. The perturbations and gain parameters are chosen in light of (31)-(32). Fig. 8(a)(b) plot the dynamic of incremental cost and local power mismatch with additive perturbations, which validates the theoretical results derived in *Theorem 3*. Fig. 8(c) shows the trajectories of local power mismatch under initial conditions $\bar{s}_0$ and s_0, where we can see $\bar{s}_k = s_k$, $\forall k \geq 1$. In addition, the transmitted information at $k = 0$ is described in Fig. 8(d) under initial conditions $\bar{s}_0$ and s_0. Combining Fig. 8(c) with Fig. 8(d), the information sets $\Omega_{\mathbb{E}}$ and $\bar{\Omega}_{\mathbb{E}}$ available for honest-but-curious node $\mathbb{E}$ are exactly identical. Therefore, the proposed algorithm is privacy-preserving against internal honest-but-curious nodes.



(a) Incremental cost
(b) Local power mismatch
(c) Local power mismatch under different inital values
(d) Transmitted information at $k = 0$: $(s_{i,0} + \pi_i^j, \bar{s}_{i,0} + \bar{\pi}^j)$ with $i = 1, 9, 10$

Fig. 9. Convergence and privacy preservation against honest-but-curious agents

VI. Conclusion

In this paper, the privacy-preserving DED problem has been investigated for islanded microgrids. A privacy-preserving accelerated consensus algorithm has been proposed to achieve optimal ED with improved convergence rate while preserving privacy information. By taking full advantage of the algebra analysis method and the mathematical induction approach, the optimal parameter has been derived to ensure the convergence, the optimal convergence rate, and the optimality of the proposed accelerated consensus scheme with and without privacy-preserving mechanism. Moreover, the extended analysis of the privacy-preserving performance has exhibited the security against internal and external eavesdroppers. Finally, the simulation cases have been provided to illustrate the validity and superiority of the proposed privacy-preserving DED algorithm. One of the future research topics would be to extend the main results to distributed robust optimization problems with transmission loss and production uncertainty for more general systems such as networked systems [43], [44], communication systems [45], [46], cyber-physical systems [47], power systems [48], [49], multi-agent systems [50], [51], and other complicated systems [52]–[56].

References

[1] X. Chen, S. Hu, Y. Li, D. Yue, C. Dou, and L. Ding, Co-estimation of state and FDI attacks and attack compensation control for multi-area load frequency control systems under FDI and DoS attacks, *IEEE Trans. Smart Grid*, vol. 13, no. 3, pp. 2357–2368, May 2022.

[2] C. Cheng and X. Bai, Robust forecasting-aided state estimation in power distribution systems with event-triggered transmission and reduced mixed measurements, *IEEE Trans. Power Syst.*, vol. 36, no. 5, pp. 4343–4354, Sept. 2021.

[3] A. Mandal, Privacy preserving consensus-based economic dispatch in smart grid systems, in *Proc. Int. Conf. Future Netw. Syst. Security*, 2016, pp. 98–110.

[4] W. Chen, Z. Wang, D. Ding, G. Ghinea, and H. Liu, Distributed formation-containment control for discrete-time multi-agent systems under dynamic event-triggered transmission scheme, *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 53, no. 2, pp. 1308–1319, Feb. 2023.

[5] Z. Zhang, D. Yue, C. Dou, and H. Zhang, Multiagent system-based integrated design of security control and economic dispatch for interconnected microgrid systems, *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 51, no. 4, pp. 2101–2112, Apr. 2021.

[6] J. Zhang, J. Lu, J. Liang, and K. Shi, Privacy-preserving average consensus in multiagent systems via partial information transmission, *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 53, no. 5, pp. 2781–2791, May 2023.

[7] A. J. Wood, B. F. Wollenberg, and G. B. Sheblé, *Power Generation, Operation, and Control.* Hoboken, NJ, USA: Wiley, 2013.

[8] S. Yang, S. Tan, and J.-X. Xu, Consensus based approach for economic dispatch problem in a smart grid, *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 4416–4426, Nov. 2013.

[9] G. Chen and Z. Zhao, Delay effects on consensus-based distributed economic dispatch algorithm in microgrid, *IEEE Trans. Power Syst.*, vol. 33, no. 1, pp. 602–612, Jan. 2018.

[10] B. Qu. Z. Wang, B. Shen, and H. Dong, Distributed state estimation for renewable energy microgrids with sensor saturations, *Automatica*, vol. 131, Sept. 2021, Art. no. 109730.

[11] Y. Li, R. Ren, B. Huang, R. Wang, Q. Sun, D. W. Gao, and H. Zhang, Distributed hybrid-triggering-based secure dispatch approach for smart grid against DoS attacks, *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 53, no. 6, pp. 3574–3587, Jun. 2023.

[12] W. Chen, Z. Wang, D. Ding, and H. Dong, Consensusability of discrete-time multi-agent systems under binary encoding with bit errors, *Automatica*, vol. 133, Nov. 2021, Art. no. 109867.

[13] Y. Zhang, L. Zou, Y. Wang, and Y.-A. Wang, Estimator design for complex networks with encoding decoding mechanism and buffer-aided strategy: A partial-nodes accessible case, *ISA Trans.*, vol. 127, pp. 68-79, Aug. 2022.

[14] X. He, Y. Zhao, and T. Huang, Optimizing the dynamic economic dispatch problem by the distributed consensus-based ADMM approach, *IEEE Trans. Indus. Informat.*, vol. 16, no. 5, pp. 3210–3221, May 2020.

[15] H. Li, Q. Lü, X. Liao, and T. Huang, Accelerated convergence algorithm for distributed constrained optimization under time-varying general directed graphs, *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 50, no. 7, pp. 2612–2622, Jul. 2020.

[16] Z. Chen, X. Yu, W. Xu, and G. Wen, Modeling and control of islanded DC microgrid clusters with hierarchical event-triggered consensus algorithm, *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 68, no. 1, pp. 376–386, Jan. 2021.

[17] R. Wang, Q. Li, B. Zhang, and L. Wang, Distributed consensus based algorithm for economic dispatch in a microgrid, *IEEE Trans. Smart Grid*, vol. 10, no. 4, pp. 3630–3640, Jul. 2019.

[18] H. Pourbabak, J. Luo, T. Chen, and W. Su, A novel consensus-based distributed algorithm for economic dispatch based on local estimation of power mismatch, *IEEE Trans. Smart Grid*, vol. 9, no. 6, pp. 5930–5942, Nov. 2018.

[19] J. Wu, T. Yang, D. Wu, K. Kalsi, and K. H. Johansson, Distributed optimal dispatch of distributed energy resources over lossy communication networks, *IEEE Trans. Smart Grid*, vol. 8, no. 6, pp. 3125–3137, Nov. 2017.

[20] C. Zhao, J. He, P. Cheng, and J. Chen, Analysis of consensus-based distributed economic dispatch under stealthy attacks, *IEEE Trans. Indus. Electron.*, vol. 64, no. 6, pp. 5107–5117, Jun. 2017.

[21] G. Chen and Z. Guo, Initialization-free distributed fixed-time convergent algorithms for optimal resource allocation, *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 52, no. 2, pp. 845-854, Feb. 2022.

[22] S. Mao, Z. Dong, P. Schultz, Y. Tang, K. Meng, Z. Y. Dong, and F. Qian, A finite-time distributed optimization algorithm for economic dispatch in smart grids, *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 51, no. 4, pp. 2068–2079, Apr. 2021.

[23] T. Yang, D. Wu, Y. Sun, and J. Lian, Minimum-time consensus-based approach for power system applications, *IEEE Trans. Indus. Electron.*, vol. 63, no. 2, pp. 1318–1328, Feb. 2016.

[24] Z. Tang, D. J. Hill, and T. Liu, A novel consensus-based economic dispatch for microgrids, *IEEE Trans. Smart Grid*, vol. 9, no. 4, pp. 3920–3922, Jul. 2018.

[25] Y. Xiong and Z. Li, Privacy-preserved average consensus algorithms with edge-based additive perturbations, *Automatica*, vol. 140, 2022, Art. No. 110223.

[26] Z. Zhang, S. Yang, W. Xu and K. Di, Privacy-preserving distributed ADMM with event-triggered communication, *IEEE Trans. Neural Netw. Learni. Syst.*, doi: 10.1109/TNNLS.2022.3192346.

[27] W. Chen, L. Liu, and G.-P. Liu, Privacy-preserving distributed economic dispatch of microgrids: A dynamic quantization based consensus scheme with homomorphic encryption, *IEEE Trans. Smart Grid*, vol. 14, no. 1, pp. 701–713, Jan. 2023.

[28] Y. Yan, Z. Chen, V. Varadharajan, M. J. Hossain, and G. E. Town, Distributed consensus-based economic dispatch in power grids using the Paillier cryptosystem, *IEEE Trans. Smart Grid*, vol. 12, no. 4, pp. 3493–3502, Jul. 2021.

[29] A. B. Alexandru, K. Gatsis, Y. Shoukry, S. A. Seshia, P. Tabuada, and G. J. Pappas, Cloud-based quadratic optimization with partially homomorphic encryption, *IEEE Trans. Autom. Control*, vol. 66, no. 5, pp. 2357–2364, May 2021.

[30] S. Mao, Y. Tang, Z. Dong, K. Dong, K. Meng, Z. Y. Dong, and F. Qian, A privacy preserving distributed optimization algorithm for economic dispatch over time-varying directed networks, *IEEE Trans. Indus. Informat.*, vol. 17, no. 3, pp. 1689–1701, Mar. 2021.

[31] N. Rezazadeh and S. S. Kia, Privacy preservation in a continuous-time static average consensus algorithm over directed graphs. In *Proc. Amer. Control Conf.*, 2018, vol. 6, pp. 5890–5895.

[32] A. Wang, W. Liu, T. Dong, X. Liao, and T. Huang, DisEHPPC: Enabling heterogeneous privacy-preserving consensus-based scheme for economic dispatch in smart grids, *IEEE Trans. Cybern.*, vol. 52, no. 6, pp. 5124–5135, Jun. 2022.

[33] C. Zhao, J. Chen, J. He, and P. Cheng, Privacy-preserving consensus-based energy management in smart grids, *IEEE Trans. Signal Process.*, vol. 66, no. 23, pp. 6162–6176, Dec. 2018.

[34] X. Chen, L. Huang, K. Ding, S. Dey, and L. Shi, Privacy-preserving push-sum average consensus via state decomposition, *IEEE Trans. Autom. Control*, vol. 68, no. 12, pp. 7974–7981, Dec. 2023.

[35] W. Chen, Z. Wang, H. Dong, J. Mao, and G.-P. Liu, Privacy-preserving distributed economic dispatch of microgrids over directed graphs via state decomposition: A fast consensus-based algorithm, *IEEE Trans. Indus. Informat.*, doi:10.1109/TII.2023.3321027.

[36] N. Gupta, J. Katz, and N. Chopra, Information-theoretic privacy in distributed average consensus, 2018. [Online]. arXiv:1809.01794.

[37] I. L. D. Ridgley, R. A. Freeman, and K. M. Lynch, Private and hot-pluggable distributed averaging, *IEEE Control Syst. Lett.*, vol. 4, no. 4, pp. 988–993, Oct. 2020.

[38] A. Grami, *Introduction to Digital Communications*, Cambridge, MA, USA: Academic Press, 2015, pp. 506–509.

[39] L. Xiao and S. Boyd, Fast linear iterations for distributed averaging, *Syst, Control Lett.*, vol. 53, pp. 65–78, Sept. 2004.

[40] E. Kokiopoulou and P. Frossard, Polynomial filtering for fast convergence in distributed consensus, *IEEE Trans. Signal Process.*, vol. 57, no. 1, pp. 342–354, Jan. 2009.

[41] B. N. Oreshkin, M. J. Coates, and M. G. Rabbat, Optimization and analysis of distributed averaging with short node memory, *IEEE Trans. Signal Process.*, vol. 58, no. 5, pp. 2850-2865, May 2010.

[42] D. P. Bertsekas, *Nonlinear Programming*. Belmont, MA, USA: Athena Scientific, 1999.

[43] R. Caballero-Águila, A. Hermoso-Carazo, and J. Linares-Pérez, Networked distributed fusion estimation under uncertain outputs with random transmission delays, packet losses and multi-packet processing, *Signal Process.*, vol. 156, pp. 71–83, 2019.

[44] X. Wang, Y. Sun, and D. Ding, Adaptive dynamic programming for networked control systems under communication constraints: a survey of trends and techniques, *Int. J. Netw. Dyn. Intell.*, vol. 1, no. 1, pp. 85–98, Dec. 2022.

[45] D. Ciuonzo, V. Carotenuto, and A. De Maio, On multiple covariance equality testing with application to SAR change detection, *IEEE Trans. Signal Process.*, vol. 65, no. 19, pp. 5078–5091, 2017.

[46] S. Wang, Z. Wang, H. Dong, B. Shen, and Y. Chen, Quadratic filtering for discrete time-varying non-Gaussian systems under binary encoding schemes, *Automatica*, vol. 158, art. no. 111268, Dec. 2023.

[47] Q. Cui, K. Liu, Z. Ji, and W. Song, Sampling-data-based distributed optimisation of second-order multi-agent systems with PI strategy, *Int. J. Syst. Sci.*, vol. 54, no. 6, pp. 1299–1312, 2023.

[48] Z. Wu, E. Tian, and H. Chen, Covert attack detection for LFC systems of electric vehicles: A dual time-varying coding method, *IEEE/ASME Trans. Mechatronics*, vol. 28, no. 2, pp. 681–691, Apr. 2023.

[49] H. Zhang, D. Yue, C. Dou, and G. P. Hancke, Two-layered hierarchical optimization strategy with distributed potential game for interconnected hybrid energy systems, *IEEE Trans. Cybern.*, vol. 53, no. 9, pp. 5436–5447, Sept. 2023.

[50] W. Chen, Z. Wang, J. Hu, and G.-P. Liu, Differentially private average consensus with logarithmic dynamic encoding-decoding scheme, *IEEE Trans. Cybern.*, vol. 53, no. 10, pp. 6725–6736, Oct. 2023.

[51] W. Li, Y. Niu, and Z. Cao, Event-triggered sliding mode control for multi-agent systems subject to channel fading, *Int. J. Syst. Sci.*, vol. 53, no. 6, pp. 1233–1244, 2022.

[52] W. Chen, Z. Wang, D. Ding, X. Yi, and Q.-L. Han, Distributed state estimation over wireless sensor networks with energy harvesting sensors, *IEEE Trans. Cybern.*, vol. 53, no. 5, pp. 3311–3324, May 2023.

[53] Y. Wang, H.-J. Liu, and H.-L. Tan, An overview of filtering for sampled-data systems under communication constraints, *Int. J. Netw. Dyn. Intell.*, vol. 2, no. 3, art. no. 100011, Sept. 2023.

[54] A. Rajagopal and S. Chitraganti, State estimation and control for networked control systems in the presence of correlated packet drops, *Int. J. Syst. Sci.*, vol. 54, no. 11, pp. 2352–2365, 2023.

[55] Z.-H. Pang, T. Mu, F.-Y. Hou, Y. Shi, and J. Sun, Two networked predictive control methods for output tracking of networked systems with plant-model mismatch, *Int. J. Syst. Sci.*, vol. 54, no. 10, pp. 2073–2088, 2023.

[56] G. He, M. Hu, and Z. Shen, Consensus of switched multi-agents system with cooperative and competitive relationship, *Syst. Sci. Control Eng.*, vol. 11, no. 1, art. no. 2192008, 2023.

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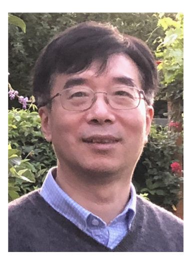

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