

Quantized Distributed Economic Dispatch for Microgrids: Paillier Encryption-Decryption Scheme

Wei Chen, Zidong Wang, Quanbo Ge, Hongli Dong, and Guo-Ping Liu

Abstract—This paper is concerned with the secure distributed economic dispatch (DED) problem of microgrids. A quantized distributed optimization algorithm using the Paillier encryption-decryption scheme is developed. This algorithm is designed to optimally coordinate the power outputs of a collection of distributed generators (DGs) in order to meet the total load demand at the lowest generation cost under the DG capacity limits while ensuring communication efficiency and security. Firstly, to facilitate data encryption and reduce data release, a novel dynamic quantization scheme is integrated into the DED algorithm, through which the effects of quantization errors can be eliminated. Next, utilizing matrix norm analysis and mathematical induction, a sufficient condition is provided to demonstrate that the developed DED algorithm converges precisely to the optimal solution under finite quantization levels (and even the three-level quantization using *sign* transmissions). Moreover, an encryption-decryption scheme is developed based on quantized outputs, which ensures confidential communication by leveraging the homomorphic property of the Paillier cryptosystem. Finally, the effectiveness and superiority of the implemented secure distributed algorithm are confirmed through a simulated example.

Index Terms—Distributed optimization, dynamic quantization, Paillier encryption-decryption communication, economic dispatch, microgrids.

I. INTRODUCTION

With the increasing integration of distributed energy resources (DERs) into modern power grids, a noticeable shift from centralized power generation stations towards a more

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decentralized approach has been observed in recent years [1]–[6]. Amidst this transformation, a paramount challenge is the effective coordination of the power output from each distributed generator (DG) to fulfill the entire load demand. More specifically, due to the intermittent, uncertain and random nature of renewable-based generation resources, DGs often find themselves adjusting their setpoints frequently to maintain a balance between supply and demand within shorter periods, and this situation underscores the pressing need for a rapid dispatch strategy [7]. Furthermore, as the scale of the power grid expands, centralized schemes become ill-equipped to meet the demands of system operation, communication, and computation for numerous DGs spread across vast geographical areas. Addressing this challenge, distributed implementation schemes have been drawing significant interest for their decentralization, autonomy, reliability and scalability [8]–[13].

The distributed economic dispatch (DED) is widely recognized as a cornerstone issue in the energy management of microgrids. The principal aim of DED is to adjust the power output of each DG such that supply-demand balance is achieved at the most cost-effective rate, all while adhering to the DG capacity constraints. Notably, in this setup, each local DG, governed by an agent, determines its local power output using only its own information coupled with information from neighboring agents. To date, a variety of distributed optimization algorithms have been introduced to tackle the ED problem, which encompass methods like the distributed primal-dual algorithm [3], the distributed gradient tracking algorithm [14], the distributed ADMM algorithm [15], and the decentralized exact first-order algorithm (EXTRA) [16], among others.

Within the context of DED algorithms, a local agent communicates and exchanges information with its neighboring agents over open communication networks. In practical engineering scenarios, however, these communication networks are often subjected to bandwidth limitations and potential threats of attacks. Such challenges can significantly impair the performance of the algorithm and might even compromise its convergence. There has been extensive research into addressing the DED problem in these real-world circumstances with studies exploring issues like packet dropouts [17], time delays [18], and cyber-attacks [19], [20]. It is worth noting that the majority of these studies center primarily on the resilience and tolerability of DED algorithms. In contrast, there has been a limited focus on the design of an “active” strategy which aims at enhancing communication efficiency and bolstering communication security. This gap serves as the motivation for

our current study.

Encryption communication, as an active security countermeasure, has been successfully incorporated into various distributed algorithms to ensure confidential transmission and safeguard privacy [9], [21]–[29]. For instance, a secure consensus-based DED algorithm has been introduced in [9] by utilizing the Paillier cryptosystem, and this particular cryptosystem has also been employed in a distributed ADMM algorithm to address the privacy-preserving optimal power flow (OPF) problem [23]. Furthermore, for the alternating current OPF issue, a private distributed management algorithm has been devised in [21], thereby harnessing a distributed primal-dual method combined with a fully homomorphic encryption technique. However, one cannot ignore the inherent complexity of cryptographic algorithms, which tend to consume notable computational and communication resources, potentially restricting their practical application breadth. Given these challenges, it becomes desirable to delve into a more streamlined yet effective encryption scheme that can be effortlessly integrated into DED algorithms.

Cryptography-based techniques are inherently developed drawing upon the foundational principles of number theory. Consequently, the encryption-decryption operation is typically confined to the set of integers [30]. In order to facilitate encryption, signal quantization becomes an essential step. In real-world engineering scenarios, the number of quantization levels is often limited, invariably leading to the introduction of quantization errors [31], [32]. This, in turn, affects the precise convergence of the quantized DED algorithm. Many conventional static quantization schemes propose increasing quantization levels as a means to curb the effects of quantization errors. However, this approach often results in more extensive data release [9], [33]–[35]. Hence, a trade-off materializes between the number of quantization levels and the overall performance of the algorithm within the framework of static quantization. Furthermore, as underscored in [33], the complexity inherent in cryptographic algorithms frequently hinges on the bit length of the encoded outputs. Given these considerations, there is a tangible need to investigate a suitable quantization scheme that can achieve the dual objectives of minimizing data release and nullifying the effects of quantization errors.

Given the preceding discussions, this paper addresses the secure DED problem in microgrids with two substantial challenges identified as follows. 1) *How to design a quantization strategy to deal with the tradeoff between quantization levels and convergence accuracy?* and 2) *How to develop an encryption-decryption scheme to prevent power-sensitive information from being leaked?* We introduce a quantized DED algorithm paired with the Paillier confidential communication scheme, and such a combination ensures optimal power dispatch without the drawbacks of quantization error effects, thereby enhancing communication efficiency and preserving sensitive information concurrently. The primary contributions of our research can be summarized as follows.

- 1) We present a novel quantized DED algorithm. The integration of a dynamic encoding-decoding scheme drastically reduces data release and eliminates the effects of quantization errors, which not only conserves com-

munication resources but also heightens the accuracy of convergence.

- 2) We define a lower bound condition for finite quantization levels. Under this condition, the quantized DED algorithm converges precisely to the optimal solution, thereby bypassing the issue of quantization saturation. Importantly, with the adequate parameter adjustments, the quantized DED algorithm remains viable even under 3-level quantization.
- 3) Building on quantized outputs, we devise an encryption-decryption methodology, which caters to the burgeoning needs for confidential communication and data privacy preservation in microgrids, thereby leveraging the homomorphic attributes of the Paillier cryptosystem.

The rest of this paper is outlined as follows. Section II formulates the ED problem of microgrids. Section III develops a quantized DED algorithm and provides a sufficient condition to ensure the exact convergence of the proposed algorithm under finite quantization levels. A Paillier encryption-decryption scheme is given in Section IV. Section V presents the simulated results. Finally, Section VI concludes this paper.

Notation: \mathbb{R} , \mathbb{Z} , and \mathbb{N}^+ are the sets of real numbers, integers, and positive integers, respectively. $\|\cdot\|_\infty$ is the infinite norm. $\text{col}_N\{x_i\}$ is an N -dimensional column vector with x_i being the i th element. $\text{diag}_N\{a_i\}$ denotes a diagonal matrix with a_i being the i th diagonal element. $\mathbf{1}_N$ is N -dimensional column vector of ones. $\mathbf{0}_{n \times m}$ denotes the $n \times m$ zero matrix. $\{a_1, a_2, \dots, a_N\}^+$ and $\{a_1, a_2, \dots, a_N\}^-$ refer to the largest and smallest value among a_i ($i = 1, 2, \dots, N$), respectively. $\arg \min_x f(x)$ stands for the set of values of x for which the minimum of $f(x)$ is attained.

II. PROBLEM FORMULATION AND PRELIMINARIES

The communication network of DG agents can be modeled by an undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{1, 2, \dots, N\}$ is the vertex set, $\mathcal{E} = \{(i, j) | i, j \in \mathcal{V}\}$ is the edge set, and the pair $(i, j) \in \mathcal{E}$ indicates that agent i and j can exchange information with each other. Denote $\mathcal{N}_i = \{j | (i, j) \in \mathcal{E}, j \neq i\}$ as the neighboring set of node i . In this paper, it is assumed that the undirected graph is connected.

Consider an islanded microgrid consisting of a collection of DGs and electrical loads. The ED problem is to coordinate the power outputs of a group of DGs to meet power supply-demand balance while minimizing the total generation cost by respecting the DG capacity constraints. To be more specific, the ED problem can be modeled as the optimization problem as follows [3], [14], [36]:

$$\begin{aligned} & \arg \min_{\{P_1, \dots, P_N\}} \sum_{i=1}^N F_i(P_i) \\ & \text{subject to } \sum_{i=1}^N P_i = \sum_{i=1}^N P_i^d, \quad \underline{P}_i \leq P_i \leq \bar{P}_i \end{aligned} \quad (1)$$

where P_i , P_i^d are, respectively, the local generation power and local load demand at node i ($i \in \mathcal{V}$), \underline{P}_i , \bar{P}_i refer to the lower and upper power limits, and $F_i(P_i)$ is the local cost function

described by

$$F_i(P_i) = \frac{1}{2}a_i P_i^2 + b_i P_i + c_i, \quad i \in \mathcal{V}, \quad (2)$$

where $a_i > 0$, $b_i, c_i \geq 0$ are the suitable cost function coefficients. Note that the total demand $P^d \triangleq \sum_{i=1}^N P_i^d$ satisfies $\sum_{i=1}^N \underline{P}_i \leq P^d \leq \sum_{i=1}^N \bar{P}_i$.

Define the local incremental cost of node i as

$$\lambda_i = \frac{dF_i(P_i)}{dP_i} = a_i P_i + b_i. \quad (3)$$

The optimal solution to the ED problem (1) is [9], [14]:

$$\begin{cases} \lambda^* = \frac{\sum_{i=1}^N P_i^d - \sum_{i \in \mathcal{V}_1} P_i^* + \sum_{i \in \mathcal{V}_2} b_i/a_i}{\sum_{i \in \mathcal{V}_2} 1/a_i} \\ P_i^* = \begin{cases} \underline{P}_i, & \lambda^* \leq \underline{\lambda}_i, \quad i \in \mathcal{V}_1 \\ \lambda^*/a_i - b_i/a_i, & \underline{\lambda}_i < \lambda^* < \bar{\lambda}_i, \quad i \in \mathcal{V}_2 \\ \bar{P}_i, & \lambda^* \geq \bar{\lambda}_i, \quad i \in \mathcal{V}_1, \end{cases} \end{cases} \quad (4)$$

where $\mathcal{V}_1, \mathcal{V}_2 \subset \mathcal{V}$ satisfy $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$ and $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$, λ^* , P_i^* ($i \in \mathcal{V}$) are the corresponding optimal values, and $\underline{\lambda}_i = a_i \underline{P}_i + b_i$, $\bar{\lambda}_i = a_i \bar{P}_i + b_i$.

The primary objective of this work is threefold: 1) develop a quantized distributed optimization algorithm that ensures exact convergence to the optimal solution (4) without any saturation of the quantizer; 2) establish a sufficient condition for finite quantization levels with an emphasis on minimizing these levels to decrease data release; and 3) introduce a Paillier encryption-decryption scheme that is grounded in quantized outputs and designed to facilitate confidential communication.

III. DISTRIBUTED ALGORITHM AND ITS CONVERGENCE

In this section, a new dynamic quantization scheme is first integrated into the consensus-based optimization algorithm [14], [36]. Then, by resorting to the property of matrix norm and the mathematical induction approach, a sufficient condition is derived to ensure exact convergence of the DED scheme under finite quantization levels without quantization saturation. In particular, the developed algorithm is shown to have exact convergence even with three-level quantization by adjusting proper parameters.

A. The Quantized Distributed Optimization Algorithm

To address the ED problem (1), the basic consensus-based algorithm proposed in [14], [36] is given as follows:

$$\begin{cases} \lambda_{i,k+1} = \lambda_{i,k} + \sum_{j=1}^N l_{ij} (\lambda_{j,k} - \lambda_{i,k}) + \varsigma z_{i,k} \\ z_{i,k+1} = z_{i,k} + \sum_{j=1}^N w_{ij} (z_{j,k} - z_{i,k}) - (P_{i,k+1} - P_{i,k}) \\ P_{i,k+1} = \left\{ \left\{ \frac{\lambda_{i,k+1} - b_i}{a_i}, \underline{P}_i \right\}^+, \bar{P}_i \right\}^-, \quad i \in \mathcal{V}, \end{cases} \quad (5)$$

where $\varsigma > 0$ is a small known scalar whose upper bound $\bar{\varsigma}$ has been discussed in [36], and $z_{i,k}$ is the local estimated mismatch between the supply and demand. Here, l_{ij}, w_{ij} denote, respectively, the (i, j) -th element of weight matrices

$L = [l_{ij}]_N$ and $W = [w_{ij}]_N$ where $l_{ii} = 1 - \sum_{i=1}^N l_{ij}$ and $w_{ii} = 1 - \sum_{i=1}^N w_{ij}$. Note that the matrices L and W are double stochastic. In addition, the initial value is set as

$$P_{i,0} \in [\underline{P}_i, \bar{P}_i], \quad \lambda_{i,0} = a_i P_{i,0} + b_i, \quad z_{i,0} = P_i^d - P_{i,0}. \quad (6)$$

To save the limited network bandwidth, we first introduce a finite-level uniform quantizer $Q_S(x) : \mathbb{R} \rightarrow \mathbb{Z}$ as follows:

$$Q_S(x) = \begin{cases} s & (s - 0.5) < x \leq (s + 0.5) \\ S & x > S + 0.5 \\ -Q_S(-x) & x \leq -0.5. \end{cases}$$

Note that the quantizer $Q_S(x)$ can map the real number $x \in \mathbb{R}$ to the integer $s \in \mathbb{S}$ ($\mathbb{S} = \{\pm s | s = 0, 1, 2, \dots, S\}$) where $S \in \mathbb{N}^+$. The number of quantization levels is $2S + 1$. Moreover, if $|x| \leq S + 0.5$, then the quantizer is not saturated, and the quantization error is subject to $|x - Q_S(x)| \leq 0.5$.

The dynamic encoding-decoding scheme is constructed by

$$\begin{cases} \hat{\lambda}_{i,k} = h_k r_{i,k}^\lambda + \hat{\lambda}_{i,k-1} \\ \hat{z}_{i,k} = h_k r_{i,k}^z + \hat{z}_{i,k-1} \\ r_{i,k}^\lambda = Q_S\left(\frac{1}{h_k}(\lambda_{i,k} - \hat{\lambda}_{i,k-1})\right) \\ r_{i,k}^z = Q_S\left(\frac{1}{h_k}(z_{i,k} - \hat{z}_{i,k-1})\right) \\ \hat{\lambda}_{i,-1} = \hat{z}_{i,-1} = 0, \end{cases} \quad (7)$$

where $\hat{\lambda}_{i,k}, \hat{z}_{i,k}$ can be regarded as the estimates of $\lambda_{i,k}, z_{i,k}$; $h_k = h_0 \zeta^k$ is the decaying scaling function where $h_0, \zeta \in (0, 1)$ are unknown and to be designed; and $r_{i,k}^\lambda$ and $r_{i,k}^z$ are the quantized outputs.

In the following, the DED algorithm with quantization communication is designed as follows:

$$\begin{cases} \lambda_{i,k+1} = \lambda_{i,k} + \alpha \sum_{j=1}^N l_{ij} (\hat{\lambda}_{j,k} - \hat{\lambda}_{i,k}) + \varsigma z_{i,k} \\ z_{i,k+1} = z_{i,k} + \beta \sum_{j=1}^N w_{ij} (\hat{z}_{j,k} - \hat{z}_{i,k}) - (P_{i,k+1} - P_{i,k}) \\ P_{i,k} = \left\{ \left\{ \frac{\lambda_{i,k} - b_i}{a_i}, \underline{P}_i \right\}^+, \bar{P}_i \right\}^-, \end{cases} \quad (8)$$

where $\alpha, \beta \in (0, 1]$ are the adjustable constants. Note that α, β play an vital role in the analysis of the quantization level.

Denote the estimation error as

$$\hat{e}_{i,k}^\lambda = \hat{\lambda}_{i,k} - \lambda_{i,k}, \quad \hat{e}_{i,k}^z = \hat{z}_{i,k} - z_{i,k} \quad (9)$$

and the quantization error as

$$\begin{cases} e_{i,k}^\lambda = r_{i,k}^\lambda - \frac{1}{h_k}(\lambda_{i,k} - \hat{\lambda}_{i,k-1}), \\ e_{i,k}^z = r_{i,k}^z - \frac{1}{h_k}(z_{i,k} - \hat{z}_{i,k-1}). \end{cases} \quad (10)$$

In light of (7), (9), (10), we have the following relationship between the estimation error and the quantization error:

$$\hat{e}_{i,k}^\lambda = h_k e_{i,k}^\lambda, \quad \hat{e}_{i,k}^z = h_k e_{i,k}^z. \quad (11)$$

In this case, the algorithm (8) can be rewritten as follows:

$$(12) \quad \left\{ \begin{array}{l} \lambda_{i,k+1} = \lambda_{i,k} + \alpha \sum_{j=1}^N l_{ij}(\lambda_{j,k} - \lambda_{i,k}) + \varsigma z_{i,k} \\ \quad + \alpha h_k \sum_{j=1}^N l_{ij}(e_{j,k}^\lambda - e_{i,k}^\lambda) \\ z_{i,k+1} = z_{i,k} + \beta \sum_{j=1}^N w_{ij}(z_{j,k} - z_{i,k}) - (P_{i,k+1} - P_{i,k}) \\ \quad + \beta h_k \sum_{j=1}^N w_{ij}(e_{j,k}^z - e_{i,k}^z) \\ P_{i,k} = \left\{ \left\{ \frac{\lambda_{i,k} - b_i}{a_i}, \underline{P}_i \right\}^+, \bar{P}_i \right\}^- \end{array} \right.$$

B. Convergence Analysis

For the sake of analysis convenience, we denote

$$\lambda_k = \text{col}_N\{\lambda_{i,k}\}, z_k = \text{col}_N\{z_{i,k}\}, P_k = \text{col}_N\{P_{i,k}\},$$

$$e_k^\lambda = \text{col}_N\{e_{i,k}^\lambda\}, e_k^z = \text{col}_N\{e_{i,k}^z\}, \phi = \text{col}_N\left\{\frac{b_i}{a_i}\right\},$$

$$L_\alpha = (1 - \alpha)I_N + \alpha L, L'_\alpha = \alpha(L - I_N),$$

$$W_\beta = (1 - \beta)I_N + \beta W, W'_\beta = \beta(W - I_N),$$

$$\Lambda = \text{diag}_N \left\{ \frac{1}{a_i} \right\}, \frac{1}{a_{\min}} = \left\{ \frac{1}{a_1}, \frac{1}{a_2}, \dots, \frac{1}{a_N} \right\}^+,$$

$$\mathcal{A} = \begin{bmatrix} L_\alpha & \varsigma I_N \\ -\Lambda L'_\alpha & W_\beta - \varsigma \Lambda \end{bmatrix}, \mathcal{B} = \begin{bmatrix} L'_\alpha & \mathbf{0}_{N \times N} \\ -\Lambda L'_\alpha & W'_\beta \end{bmatrix},$$

$$\mathcal{A}' = \begin{bmatrix} L'_\alpha & \varsigma I_N \\ -\Lambda L'_\alpha & W'_\beta - \varsigma \Lambda \end{bmatrix}, \mathcal{B}' = \mathcal{B} - I_{2N},$$

$$\tilde{\mathcal{A}} = \begin{bmatrix} L_\alpha & \varsigma I_N \\ -\tilde{\Lambda} L'_\alpha & W_\beta - \varsigma \Lambda \end{bmatrix}, \tilde{\mathcal{B}} = \begin{bmatrix} L'_\alpha & \mathbf{0}_{N \times N} \\ -\tilde{\Lambda} L'_\alpha & W'_\beta \end{bmatrix},$$

$$\tilde{\Lambda} = \text{diag}\{\tilde{a}_1, \dots, \tilde{a}_N\}, \tilde{a}_i = \begin{cases} 0, & \text{if } P_{i,k} \text{ is saturated} \\ 1/a_i, & \text{otherwise.} \end{cases} \quad (13)$$

It follows from the definition of the infinity norm that

$$\|\mathcal{B}\|_\infty \leq \theta_1 \triangleq \left\{ 2\alpha, \frac{2\alpha}{a_{\min}} + 2\beta \right\}^+,$$

$$\|\mathcal{A}'\|_\infty \leq \theta_2 \triangleq \left\{ 2\alpha + \varsigma, \frac{2\alpha + \varsigma}{a_{\min}} + 2\beta \right\}^+,$$

$$\|\mathcal{B}'\|_\infty \leq \theta_3 \triangleq \left\{ 2\alpha + 1, \frac{2\alpha}{a_{\min}} + 2\beta + 1 \right\}^+. \quad (14)$$

The compact form of (12) is expressed by

$$\lambda_{k+1} = L_\alpha \lambda_k + \varsigma z_k + h_k L'_\alpha e_k^\lambda$$

$$z_{k+1} = W_\beta z_k - (P_{k+1} - P_k) + h_k W'_\beta e_k^z. \quad (15)$$

Assumption 1: There exists a known constant ν satisfying $\nu \geq \{\|\lambda_0\|_\infty, \|z_0\|_\infty\}^+$.

Theorem 1: Given the undirected graph \mathcal{G} , the DED algorithm (8) combined with the dynamic quantization scheme (7)

can converge exactly to the solution (4) of the problem (1) without any saturation of the quantizer if the decaying factor is within the range $\zeta \in (\rho, 1)$ and the quantization level satisfies

$$S \geq \Pi \quad (16)$$

where

$$\Pi = \left\{ \frac{\nu}{h_0} - \frac{1}{2}, \frac{\nu\theta_2}{\zeta h_0} + \frac{\theta_3}{2\zeta} - \frac{1}{2}, \frac{\theta_1\theta_2}{2\zeta(\zeta - \rho)} + \frac{\theta_3}{2\zeta} - \frac{1}{2} \right\}^+$$

with ρ being given in (30), and $\theta_1, \theta_2, \theta_3$ being given in (14).

Proof: The proof consists of the considerations of two cases: 1) the distributed algorithm without DG capacity constraints; and 2) the distributed algorithm with DG capacity constraints.

Case 1: the DED algorithm without constraints.

Denoting $\varphi_k = [\lambda_k^T \ z_k^T]^T$, $e_k = [(e_k^\lambda)^T \ (e_k^z)^T]^T$, it follows from (15) that

$$\varphi_{k+1} = \mathcal{A}\varphi_k + h_k \mathcal{B}e_k \quad (17)$$

where \mathcal{A}, \mathcal{B} are defined in (13).

Following the eigenvalue perturbation approach in [9], [14], [36], [40], the matrix \mathcal{A} has a simple eigenvalue 1, and all the remaining $2N - 1$ eigenvalues are situated within the unit disk. Furthermore, there exist two eigenvectors

$$\mu = \frac{1}{\mathbf{1}_N^T \Lambda \mathbf{1}_N} [\mathbf{1}_N^T \Lambda \ \mathbf{1}_N^T]^T, \nu = [\mathbf{1}_N^T \ \mathbf{0}_N^T]^T$$

satisfying $\mu^T \mathcal{A} = \mu^T$, $\mathcal{A}\nu = \nu$, and $\mu^T \nu = 1$. Note that μ, ν are the left and right eigenvector of matrix \mathcal{A} corresponding to the simple eigenvalue 1. In addition, there exists a transformational matrix $U \in \mathbb{R}^{2N \times 2N}$ of the form $U = [\nu, \nu_2, \dots, \nu_{2N}]$, $U^{-1} = [\mu, \mu_2, \dots, \mu_{2N}]^T$ with $\mu_i, \nu_i \in \mathbb{R}^{2N}$ ($i = 2, 3, \dots, 2N$) such that $U^{-1} \mathcal{A} U = \text{diag}\{1, J\}$ where $J \in \mathbb{R}^{(2N-1) \times (2N-1)}$ is the Jordan block. Based on the eigenvalue distribution of the matrix \mathcal{A} , we have the spectral radius $\rho(J) = \rho_1 < 1$. Noting the matrix $\mathcal{A} - \nu\mu^T = U \text{diag}\{0, J\} U^{-1}$, its spectral radius is $\rho(\mathcal{A} - \nu\mu^T) = \rho(J) = \rho_1$.

On the one hand, denote the consensus error as $\bar{\varphi}_k = (I_{2N} - \nu\mu^T)\varphi_k$. By utilizing the following properties

$$\nu\mu^T \mathcal{A} = \mathcal{A}\nu\mu^T = \nu\mu^T \nu\mu^T = \nu\mu^T,$$

$$\nu\mu^T \mathcal{B} = \mathcal{B}\nu\mu^T = \mathbf{0}_{2N \times 2N}, \quad (18)$$

we have

$$\bar{\varphi}_{k+1} = (\mathcal{A} - \nu\mu^T)\bar{\varphi}_k + h_k \mathcal{B}e_k. \quad (19)$$

Then, it follows from (9) and (11) that

$$\lambda_{k+1} - \hat{\lambda}_k = \lambda_{k+1} - \lambda_k - h_k e_k^\lambda$$

$$= L'_\alpha \lambda_k + \varsigma z_k + h_k L'_\alpha e_k^\lambda - h_k e_k^\lambda$$

$$z_{k+1} - \hat{z}_k = z_{k+1} - z_k - h_k e_k^z$$

$$= W'_\beta z_k - \Lambda L'_\alpha \lambda_k - h_k \Lambda L'_\alpha e_k^\lambda$$

$$- \varsigma \Lambda z_k + h_k W'_\beta e_k^z - h_k e_k^z. \quad (20)$$

Denoting $\psi_k = \frac{1}{h_k} [\lambda_k^T - \hat{\lambda}_{k-1}^T \ z_k^T - \hat{z}_{k-1}^T]^T$, one has

$$\psi_{k+1} = \frac{1}{h_{k+1}} \mathcal{A}' \psi_k + \frac{1}{\zeta} \mathcal{B}' e_k \quad (21)$$

where \mathcal{A}' and \mathcal{B}' are given in (13).

Proceeding further, to ensure that the finite-level quantizer remains unsaturated, we are now poised to prove that the quantization input ψ_k is confined within a defined boundary (i.e., $\sup_{k \geq 0} \|\psi_k\|_\infty \leq S + 0.5$) by means of mathematical induction. For $k = 0$, we have that

$$\|\psi_0\|_\infty = \frac{1}{h_0} \|\varphi_0\|_\infty \leq \frac{v}{h_0} \leq S + 0.5 \quad (22)$$

where v is given in Assumption 1. Furthermore, the quantization error satisfies $\|e_0\|_\infty \leq 0.5$. Then, assume that $\|\psi_s\|_\infty \leq S + 0.5$ and $\|e_s\|_\infty \leq 0.5$ for $\forall s \in \{0, 1, \dots, k\}$.

Denoting $\eta_k = \frac{1}{h_k} \bar{\varphi}_k$, it follows from (19) that

$$\begin{aligned} \eta_k &= \frac{\mathcal{A} - \nu\mu^T}{\zeta} \eta_{k-1} + \frac{\mathcal{B}}{\zeta} e_{k-1} \\ &= \left(\frac{\mathcal{A} - \nu\mu^T}{\zeta} \right)^k \eta_0 + \sum_{s=0}^{k-1} \left(\frac{\mathcal{A} - \nu\mu^T}{\zeta} \right)^{k-1-s} \frac{\mathcal{B}}{\zeta} e_s \\ &\leq \left(\frac{\rho_1}{\zeta} \right)^k \eta_0 + \sum_{s=0}^{k-1} \left(\frac{\rho_1}{\zeta} \right)^{k-1-s} \frac{\mathcal{B}}{\zeta} e_s, \end{aligned} \quad (23)$$

and

$$\begin{aligned} \|\eta_k\|_\infty &\leq \left(\frac{\rho_1}{\zeta} \right)^k \frac{v}{h_0} + \frac{\theta_1}{2\zeta} \sum_{s=0}^{k-1} \left(\frac{\rho_1}{\zeta} \right)^{k-1-s} \\ &= \left(\frac{\rho_1}{\zeta} \right)^k \frac{v}{h_0} + \frac{\theta_1}{2(\zeta - \rho_1)} \left(1 - \left(\frac{\rho_1}{\zeta} \right)^k \right) \\ &\leq \left\{ \frac{v}{h_0}, \frac{\theta_1}{2(\zeta - \rho_1)} \right\}^+. \end{aligned} \quad (24)$$

At the time instant $k + 1$, based on $\mathcal{A}' = \mathcal{A}'(I_{2N} - \nu\mu^T)$ and $\eta_k = \frac{1}{h_k} \bar{\varphi}_k$, it follows from (21) that

$$\psi_{k+1} = \frac{1}{h_{k+1}} \mathcal{A}' \bar{\varphi}_k + \frac{1}{\zeta} \mathcal{B}' e_k = \frac{1}{\zeta} \mathcal{A}' \eta_k + \frac{1}{\zeta} \mathcal{B}' e_k. \quad (25)$$

Furthermore, according to the property of the matrix norm, we can obtain

$$\begin{aligned} \|\psi_{k+1}\|_\infty &\leq \frac{1}{\zeta} \|\mathcal{A}'\|_\infty \|\eta_k\|_\infty + \frac{1}{\zeta} \|\mathcal{B}'\|_\infty \|e_k\|_\infty \\ &\leq \left\{ \frac{v\theta_2}{h_0\zeta}, \frac{\theta_1\theta_2}{2\zeta(\zeta - \rho_1)} \right\}^+ + \frac{\theta_3}{2\zeta} \leq S + 0.5. \end{aligned}$$

Hence, the quantizer $Q_S(x)$ is never saturated under (16).

In light of the established results in (24), one has

$$\lim_{k \rightarrow \infty} \|\bar{\varphi}_k\|_\infty = \lim_{k \rightarrow \infty} h_k \|\eta_k\|_\infty \leq \sup_{k \geq 0} \|\eta_k\|_\infty \lim_{k \rightarrow \infty} h_k = 0$$

which means that the consensus error approaches exactly to 0 (i.e., $\lim_{k \rightarrow \infty} \bar{\varphi}_k = \mathbf{0}_{2N}$).

Based on the properties in (18), pre-multiplying (17) by matrix $\nu\mu^T$, we can obtain

$$\begin{aligned} \nu\mu^T \varphi_{k+1} &= \nu\mu^T \mathcal{A} \varphi_k + h_k \nu\mu^T \mathcal{B} e_k \\ &= \nu\mu^T \varphi_k \cdots = \nu\mu^T \varphi_0 = \mu^T \varphi_0 \nu, \end{aligned} \quad (26)$$

which implies that

$$\lim_{k \rightarrow \infty} \varphi_k = \lim_{k \rightarrow \infty} \nu\mu^T \varphi_k = \mu^T \varphi_0 \nu. \quad (27)$$

As such, the algorithm (8) with the dynamic quantization scheme (7) can converge to the optimal solution (4) of the problem (1) without quantization error effects.

Case 2: the DED algorithm with constraints.

First, if all DGs operate in the linear region (i.e., $P_{i,k} \in [\underline{P}_i, \bar{P}_i], \forall k \geq 0, i \in \mathcal{V}$), the rest of the proof is exactly the same as that for *Case 1*. As a result, we only consider the case with saturated constraints.

Inspired by [14], [36], we denote $\Theta_k = \sum_{i=1}^N \lambda_{i,k}, Z_k = \sum_{i=1}^N z_{i,k}$, and then have from (15) that

$$\begin{aligned} \Theta_{k+1} &= \Theta_k + \varsigma Z_k, \\ Z_k &= Z_{k-1} - \mathbf{1}_N^T P_k + \mathbf{1}_N^T P_{k-1}, \\ &= Z_0 + \mathbf{1}_N^T P_0 - \mathbf{1}_N^T P_k, \\ &= \sum_{i=1}^N P_i^d - \sum_{i=1}^N P_{i,k}. \end{aligned} \quad (28)$$

Due to $\sum_{i=1}^N \underline{P}_i \leq \sum_{i=1}^N P_i^d \leq \sum_{i=1}^N \bar{P}_i$, there exists at least a DG operating in the unsaturated region. Without loss of generality, we assume $Z_k > 0$. Then, Θ_k will increase and $\mathbf{1}_N^T P_k$ will also increase (according to the monotonicity of the incremental cost) with the generation power in (5). In this process, Z_k will decay. After a period of time K_τ , if P_{i,K_τ} is saturated, then $P_{i,k}$ stays unchanged for $k > K_\tau$. To discuss the dynamic evolution when $k > K_\tau$, the distributed scheme (15) is written as follows:

$$\varphi_{k+1} = \tilde{\mathcal{A}} \varphi_k + h_k \tilde{\mathcal{B}} e_k \quad (29)$$

where $\tilde{\mathcal{A}}, \tilde{\mathcal{B}}$ are given in (13).

Next, denoting $\tilde{\mu} = \frac{1}{\mathbf{1}_N^T \tilde{\Lambda} \mathbf{1}_N} [\mathbf{1}_N^T \tilde{\Lambda} \quad \mathbf{1}_N^T]^T$, the maximum spectral radius is expressed as

$$\rho = \{\rho(\tilde{\mathcal{A}} - \nu\tilde{\mu}^T)\}^+. \quad (30)$$

Following the similar line of *Case 1*, the distributed algorithm (8) with the dynamic communication scheme (7) can converge to the optimal solution (4) of the problem (1) with DG capacity constraints, and the proof is now complete. ■

C. Three-Level Quantization

In this subsection, a sufficient condition is presented to achieve less data release by minimizing the number of quantization levels.

Theorem 2: Under conditions in Theorem 1, the distributed optimization algorithm (8) with three-level quantization (i.e., $S = 1$) converges exactly to the solution (4) of the problem (1) if the following conditions are satisfied:

$$h_0 \in \left[\frac{3v}{2}, \frac{\theta_1}{2v(\zeta - \rho)} \right], \quad 3\theta_2 + \theta_3 \leq 3\zeta. \quad (31)$$

Proof: To achieve the three-level quantization, our aim is to minimize the lower bounds of quantization levels. To be more specific, if we choose the adjustable parameters appropriately such that $\Pi \leq 1$, then we can conclude that the quantizers will never saturate even when $S = 1$.

To this end, recalling (16), we let

$$\frac{v}{h_0} - \frac{1}{2} \leq S = 1,$$

$$\frac{\theta_1\theta_2}{2\zeta(\zeta-\rho)} + \frac{\theta_3}{2\zeta} - \frac{1}{2} \leq \frac{v\theta_2}{\zeta h_0} + \frac{\theta_3}{2\zeta} - \frac{1}{2} \leq S = 1, \quad (32)$$

and can then derive the results in (31), which ends the proof. ■

D. Convergence Rate Analysis

Let us first define the asymptotic convergence factor [38] as

$$o_{asm} = \sup_{z_0 \neq z^*} \lim_{k \rightarrow \infty} \left(\frac{\|z_k - z^*\|_2}{\|z_0 - z^*\|_2} \right)^{1/k}$$

where $z^* = \lim_{k \rightarrow \infty} z_k$.

It follows from the consensus error (19) that

$$\begin{aligned} \bar{\varphi}_{k+1} &= (\mathcal{A} - \nu\mu^T)\bar{\varphi}_k + h_k \mathcal{B}e_k \\ &= (\mathcal{A} - \nu\mu^T)^{k+1}\bar{\varphi}_0 + \sum_{i=0}^k (\mathcal{A} - \nu\mu^T)^{k-i} h_i \mathcal{B}e_i \\ &\leq \rho^{k+1}\bar{\varphi}_0 + \frac{\theta_1 h_0}{2(\rho - \zeta)} (\rho^{k+1} - \zeta^{k+1}) \mathbf{1}_{2N}, \end{aligned}$$

which means that the asymptotic convergence factor is $o_{asm} = \{\rho, \zeta\}^+$. Due to $\zeta \in (\rho, 1)$, one has $o_{asm} = \zeta$, which concludes that the convergence rate is determined by the parameter ζ . It is worth noting that the communication topology \mathcal{G} , the scalar ζ , and the adjustable constants α, β are reflected in the spectral radius of matrix $\mathcal{A} - \nu\mu^T$ (i.e., the value of ρ), and further affect the selection of ζ in light of Theorem 1.

Remark 1: It should be pointed out that, in the dynamic quantization scheme (7), the signal $(\lambda_{i,k} - \hat{\lambda}_{i,k-1})$ can be regarded as ‘‘prediction error’’. In general, the size of the prediction error is much less than that of the original variable, thereby resulting in fewer bit condition. Furthermore, it follows from (9) and (11) that $\lambda_{i,k} = \hat{\lambda}_{i,k} - h_k e_{i,k}^\lambda$. Intuitively speaking, if the quantizer is not saturated, then $\hat{\lambda}_{i,k}$ can accurately estimate $\lambda_{i,k}$ due to the exponentially decaying function h_k . As a result, the proposed quantized DED algorithm can achieve exact convergence without quantization error effects and this is true even under three-level quantization by adjusting proper parameters.

IV. PAILLIER ENCRYPTION-DECRYPTION SCHEME

In this section, we begin by presenting an overview of the Paillier cryptosystem. Subsequently, by leveraging the homomorphic property of the encryption algorithm, we delineate a confidential interaction protocol.

To ensure secure communication and safeguard data privacy, we adopt the Paillier cryptosystem, recognized for its robust security. This system has gained wide acclaim in data communication and signal processing domains. In essence, the Paillier cryptosystem can be segmented into three distinct phases: 1) generate a pair of public and private keys (K^p, K^s) ; 2) encrypt the plaintext m as the ciphertext $n = \mathbb{E}(m)$; and 3) decrypt the ciphertext n as the plaintext $m = \mathbb{D}(n)$ (see [30], [37], [40] for more details). Furthermore, the Paillier cryptosystem has the following homomorphic properties:

$$\mathbb{E}(m_1 + m_2) = \mathbb{E}(m_1)\mathbb{E}(m_2), \quad (\mathbb{E}(m))^s = \mathbb{E}(sm). \quad (33)$$

It is worth noting that the above properties play an essential role in sensitive information preservation and communication security.

Remark 2: Several key aspects of the Paillier cryptosystem warrant emphasis here. 1) Within the framework of the Paillier cryptosystem, the public key can be openly disseminated for encryption purposes, whereas the private key is discreetly reserved for decryption, which means that any node can encrypt the plaintext using the public key, but decryption of the corresponding ciphertext is exclusively reserved for nodes possessing the appropriate private key. 2) All operations within the Paillier cryptosystem are conducted within the realm of integer numbers. Consequently, signal quantization is an indispensable prerequisite prior to the encryption process. 3) The computational complexity of the encryption algorithm is inherently governed by the bit length of both the public key and the plaintext (that is, the quantized output). The selection of the public key’s bit length is contingent upon the value of the plaintext as detailed in [30]. Given these considerations, the proposed quantization-centric encryption-decryption approach offers significant advantages, particularly in terms of reduced computational complexity and minimized data dissemination.

In the following, denote

$$\begin{aligned} \delta r_{ij,k}^\lambda &= l_{ij}(r_{j,k}^\lambda - r_{i,k}^\lambda), \quad \delta r_{ij,k}^z = w_{ij}(r_{j,k}^z - r_{i,k}^z), \\ \delta \hat{\lambda}_{ij,k} &= l_{ij}(\hat{\lambda}_{j,k} - \hat{\lambda}_{i,k}), \quad \delta \hat{z}_{ij,k} = w_{ij}(\hat{z}_{j,k} - \hat{z}_{i,k}). \end{aligned}$$

The quantized distributed algorithm (8) is rewritten as

$$\begin{cases} \lambda_{i,k+1} = \lambda_{i,k} + \alpha \sum_{j=1}^N \delta \hat{\lambda}_{ij,k} + \varsigma z_{i,k} \\ z_{i,k+1} = z_{i,k} + \beta \sum_{j=1}^N \delta \hat{z}_{ij,k} - (P_{i,k+1} - P_{i,k}) \\ P_{i,k} = \left\{ \left\{ \frac{\lambda_{i,k} - b_i}{a_i}, \underline{P}_i \right\}^+, \bar{P}_i \right\}^-, \end{cases} \quad (34)$$

where

$$\begin{aligned} \delta \hat{\lambda}_{ij,k} &= h_k \delta r_{ij,k}^\lambda + \delta \hat{\lambda}_{ij,k-1}, \\ \delta \hat{z}_{ij,k} &= h_k \delta r_{ij,k}^z + \delta \hat{z}_{ij,k-1}, \\ \delta \hat{\lambda}_{ij,-1} &= \delta \hat{z}_{ij,-1} = 0. \end{aligned} \quad (35)$$

Furthermore, the weights can be constructed as follows:

$$l_{ij} = l_{i \rightarrow j} l_{j \rightarrow i}, \quad w_{ij} = w_{i \rightarrow j} w_{j \rightarrow i}, \quad (36)$$

where

$$\begin{aligned} l_{i \rightarrow j} &= (1 + \{d_i, d_j\})^{-\frac{1}{2}} \sum_{s=1}^l 2^{s-l-1} \kappa_s^\lambda, \\ w_{i \rightarrow j} &= (1 + \{d_i, d_j\})^{-\frac{1}{2}} \sum_{s=1}^l 2^{s-l-1} \kappa_s^z, \end{aligned} \quad (37)$$

with $\kappa_s^\lambda, \kappa_s^z \in \{0, 1\}$ being random variables, l being the length of bit string, d_i being the number of neighboring nodes of agent i . Moreover, $\mathcal{K}^\lambda \triangleq \{\kappa_l^\lambda, \kappa_{l-1}^\lambda, \dots, \kappa_1^\lambda\}$, $\mathcal{K}^z \triangleq \{\kappa_l^z, \kappa_{l-1}^z, \dots, \kappa_1^z\}$ are the binary bit string.

Now, the homomorphically encrypted communication scheme can be summarized in Algorithm 1.

Algorithm 1 Homomorphically Encrypted Communication

► **Initialization:**

Node i generates a pair of public and private keys (K_i^p, K_i^s) , and random variables $l_{i \rightarrow j}$ and $w_{i \rightarrow j}$ via (37).

► **Encryption:**

Node i obtains the quantized outputs $r_{i,k}^\lambda, r_{i,k}^z$ via (7), and encrypts $-r_{i,k}^\lambda, -r_{i,k}^z$ as $\mathbb{E}(-r_{i,k}^\lambda), \mathbb{E}(-r_{i,k}^z)$ by utilizing the public key K_i^p .

► **Communication:** (the flow $i \rightarrow j \rightarrow i$)

Step 1: Node i transmits ciphertexts $\mathbb{E}(-r_{i,k}^\lambda), \mathbb{E}(-r_{i,k}^z)$ and the public key K_i^p to the node j .

Step 2: Node j encrypts $r_{j,k}^\lambda, r_{j,k}^z$ as $\mathbb{E}(r_{j,k}^\lambda), \mathbb{E}(r_{j,k}^z)$ by utilizing the received public key K_i^p , and calculates the (weighted) differences as follows:

$$\begin{aligned} \mathbb{E}(r_{j,k}^\lambda - r_{i,k}^\lambda) &= \mathbb{E}(r_{j,k}^\lambda) \mathbb{E}(-r_{i,k}^\lambda), \\ \mathbb{E}(r_{j,k}^z - r_{i,k}^z) &= \mathbb{E}(r_{j,k}^z) \mathbb{E}(-r_{i,k}^z), \end{aligned}$$

and

$$\begin{aligned} \mathbb{E}(l_{j \rightarrow i}(r_{j,k}^\lambda - r_{i,k}^\lambda)) &= (\mathbb{E}(r_{j,k}^\lambda - r_{i,k}^\lambda))^{l_{j \rightarrow i}}, \\ \mathbb{E}(w_{j \rightarrow i}(r_{j,k}^z - r_{i,k}^z)) &= (\mathbb{E}(r_{j,k}^z - r_{i,k}^z))^{w_{j \rightarrow i}}. \end{aligned}$$

Step 3: Node j returns $\mathbb{E}(l_{j \rightarrow i}(r_{j,k}^\lambda - r_{i,k}^\lambda)), \mathbb{E}(w_{j \rightarrow i}(r_{j,k}^z - r_{i,k}^z))$ to node i .

► **Decryption and Update:**

Step 1: Node i decrypts ciphertexts $\mathbb{E}(l_{j \rightarrow i}(r_{j,k}^\lambda - r_{i,k}^\lambda)), \mathbb{E}(w_{j \rightarrow i}(r_{j,k}^z - r_{i,k}^z))$ as $l_{j \rightarrow i}(r_{j,k}^\lambda - r_{i,k}^\lambda), w_{j \rightarrow i}(r_{j,k}^z - r_{i,k}^z)$ via utilizing the private key K_i^s .

Step 2: Node i multiplies $l_{j \rightarrow i}(r_{j,k}^\lambda - r_{i,k}^\lambda), w_{j \rightarrow i}(r_{j,k}^z - r_{i,k}^z)$ with $l_{i \rightarrow j}, w_{i \rightarrow j}$ to get weighted differences $\delta r_{ij,k}^\lambda, \delta r_{ij,k}^z$, respectively; calculates $\delta \hat{\lambda}_{ij,k}, \delta \hat{z}_{ij,k}$ via (35) and then updates the state via (34).

It should be highlighted that the Paillier encryption-decryption transmission offers robust protection against external adversaries who might attempt to intercept the communication link. As illustrated in Algorithm 1, it becomes clear that, without access to the secret key, an eavesdropper is incapacitated in terms of decrypting the conveyed information. Another potential threat arises from malevolent attackers who might endeavor to introduce spurious data, thereby jeopardizing the convergence efficacy of the DED algorithm. One fundamental countermeasure to such attacks involves the deployment of digital signatures. These serve as a deterrent to external attackers, enabling the recipient to identify any potential alterations made during the entire communication cycle, see [33] for more details.

Let us conduct a succinct analysis to demonstrate the privacy preservation measures against honest-but-curious nodes who might attempt to infer neighboring sensitive data based on the information they have accessed. Regarding the information flow from node i to node j , the latter, without access to the private key K_i^p , is rendered incapable of decrypting the received messages $\mathbb{E}(-r_{i,k}^\lambda), \mathbb{E}(-r_{i,k}^z)$ owing to the intrinsic security of the Paillier algorithm. Moreover, weights l_{ij} and w_{ij} are unknown to node i according to the construction of

random weights (36), (37). In this regard, node i is precluded from making any reasonable inferences about $\hat{\lambda}_{j,k}, \hat{z}_{j,k}$ by

$$\hat{\lambda}_{j,k} = \delta \hat{\lambda}_{ij,k} / l_{ij} + \hat{\lambda}_{i,k}, \quad \hat{z}_{j,k} = \delta \hat{z}_{ij,k} / w_{ij} + \hat{z}_{i,k}, \quad (38)$$

where $\delta \hat{\lambda}_{ij,k}, \delta \hat{z}_{ij,k}$ can be obtained via (35). Furthermore, agent i cannot estimate $\lambda_{j,k}$ and $z_{j,k}$ via

$$\lambda_{j,k} = \hat{\lambda}_{j,k} - h_k e_{j,k}^\lambda, \quad z_{j,k} = \hat{z}_{j,k} - h_k e_{j,k}^z. \quad (39)$$

Therefore, the privacy of agent j is preserved.

Remark 3: In microgrids, the power-sensitive information involves the generation power, the cost function parameters, and the local load demand [39], which is reflected in the internal state of the DED algorithm. It should be underlined that the power-sensitive information plays an essential role in ensuring the normative order of the power market, and the safe and reliable operation of power grids.

Remark 4: So far, we have developed a quantized DED algorithm with the Paillier encryption-decryption scheme to achieve energy management of microgrids at the lowest economic cost without exposing sensitive information. Compared with the previous work [9], [40], this paper offers the following two distinctive features: 1) the proposed quantization scheme is general, which is independent of the distributed algorithm structure; and 2) the developed quantized DED algorithm provides more freedom in flexible quantization levels (even for the three-level quantization), thereby achieving less data transmission. In summary, our paper gives a rather general dynamic quantization scheme to achieve less data release with largely reduced computation and communication burden in the Paillier encryption-decryption process.

V. NUMERICAL EXAMPLE

In this section, an example is provided to verify the effectiveness and superiority of the obtained theoretical results on the IEEE 39-bus test system.

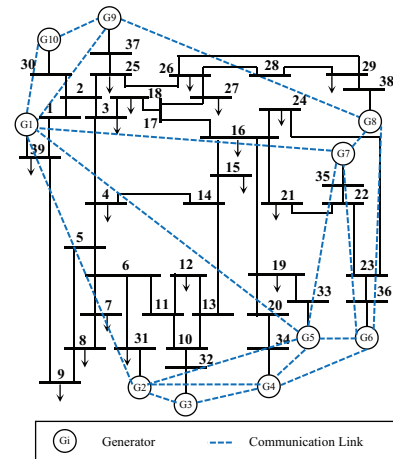


Fig. 1. IEEE 39-bus test system

There are 10 DGs and 18 local loads in the IEEE 39-bus test system. Assume that each DG is controlled by an agent, and the communication network is depicted by blue dashed lines

TABLE I
PARAMETERS OF DGs [36]

DG i	a_i	b_i	c_i	P_i^m	P_i^M
1,6	0.21	2.53	78	3.8	40
2,7	0.148	3.17	62	4.2	18
3,8	0.156	3.41	31	8	60
4,9	0.164	4.02	42	5.4	45
5,10	0.188	1.22	51	10	80

in Fig. 1. The DG parameters are provided in Table I [36]. The length of bit string is 16, and the simulated software is MATLAB (R2023a) in a PC of 2.60 GHz Intel Core CPU i9-13900U and 16GB RAM.

The local demand P_i^d is set as $25kW$, $25kW$, $25kW$, $25kW$, $25kW$, $25kW$, $25kW$, $25kW$, $25kW$, and $15kW$. The total demand $\sum_{i=1}^{10} P_i^d = 240kW$. The local power supply $P_{i,0}$ is chosen as $25kW$, $25kW$, $25kW$, $25kW$, $25kW$, $25kW$, $25kW$, $25kW$, $25kW$, and $15kW$. In light of the established results in (4), the optimal solutions are $\lambda^* = 7.39$, and $P_1^* = P_6^* = 23.14kW$, $P_2^* = P_7^* = 18kW$, $P_3^* = P_8^* = 25.51kW$, $P_4^* = P_9^* = 20.54kW$, and $P_5^* = P_{10}^* = 32.81kW$, respectively.

The test results are depicted in Figs. 2-5. In Fig. 2, the incremental cost $\lambda_{i,k}$ and the local power $P_{i,k}$ approach to the corresponding optimal value λ^* , P_i^* , the estimated mismatch approaches 0, and the supply-demand balance is guaranteed. Fig. 3 shows quantized outputs and estimation errors. It is observed that the 3-level quantizer is never saturated, and estimation errors approach 0. Fig. 4 plots the received encrypted messages. Compared with the modular bits of the Paillier cryptosystem is 64 in [9], and 32 in [40], the modular bits of this work can be selected as 16 due to the reduced size of quantization outputs, and thus the magnitude of the encrypted broadcast messages are much smaller than that of [9], [40]. As a result, the developed algorithm largely improves communication efficiency. The total simulation time is 20.42s for 1200 instants of 10 nodes. It implies that the average time for each control instant is 1.70ms (much less than [9], [40]), which is applicable for constrained low-cost microprocessors. Overall, unlike the existing results [9], [40], our results require fewer bits and less simulation time.

Next, to highlight well convergence performance, three DED algorithms are compared as follows: 1) ADED: the adopted DED algorithm; 2) DPD: the distributed primal and dual algorithm [1]; and 3) DDA: the distributed dual averaging method [41]. The evolutions of the total mismatch (i.e., $\Delta P_k \triangleq \sum_{i=1}^N (P_i^d - P_{i,k})$) are plotted in Fig. 5. It is observed that the adopted algorithm can achieve fast convergence at a geometric rate. The simulated results show that the proposed quantized DED scheme with Paillier encryption-decryption communication can converge to the optimal solution exactly while ensuring data transmission efficiency and security.

VI. CONCLUSION

In this study, we have introduced a refined quantized distributed optimization algorithm combined with the Paillier

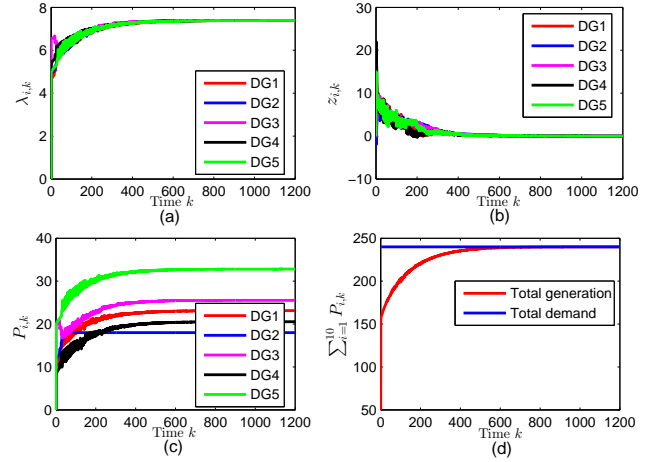


Fig. 2. Simulated results of the proposed scheme. (a) Incremental cost $\lambda_{i,k}$; (b) Estimated mismatch $z_{i,k}$; (c) Local power $P_{i,k}$; (d) Power balance.

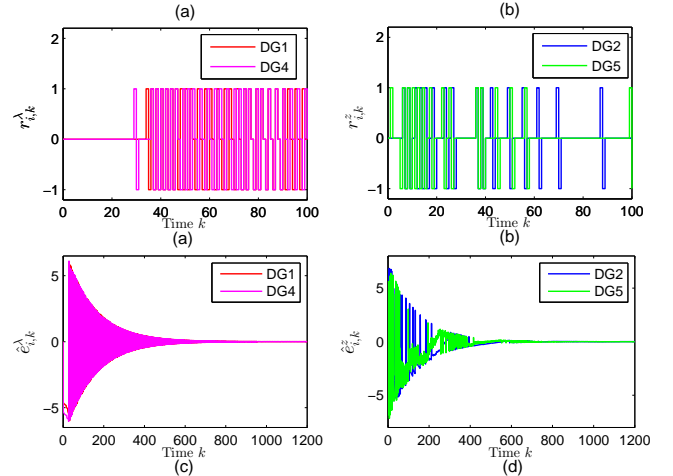


Fig. 3. Simulated results of the proposed scheme. (a) Quantization output $r_{i,k}^\lambda$; (b) Quantization output $r_{i,k}^z$; (c) Estimation error $e_{i,k}^\lambda$; (d) Estimation error $e_{i,k}^z$.

encryption scheme to ensure secure DED in microgrids. Leveraging a finite-level uniform quantizer, we have devised a dynamic encoding-decoding strategy to optimize communication and encryption processes. Through matrix norm properties and mathematical induction, we have established conditions for exact algorithm convergence without quantization errors, and this is true even under three-level quantization. Furthermore, by tapping into the Paillier algorithm's attributes, we have enhanced data security and privacy. Simulations have validated the superiority and efficacy of our approach. Future directions would be the extensions of resilient distributed energy management problems of microgrids with the power flow and thermal constraints over time-varying directed graphs [3], [25], [41]–[48].

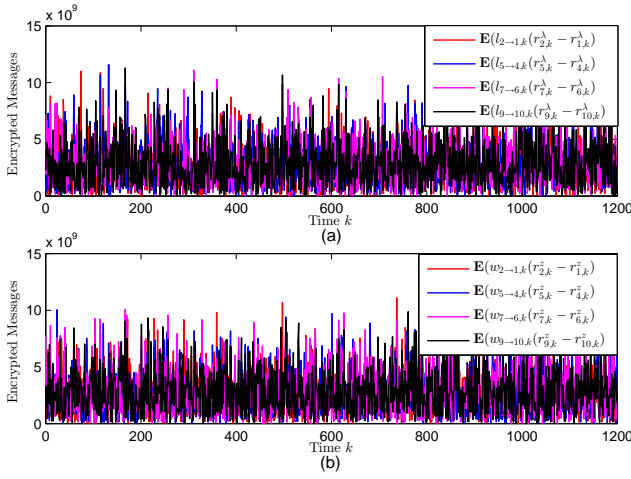


Fig. 4. Test results of the proposed algorithm. (a) Encrypted messages $\mathbb{E}(l_{j \rightarrow i}(r_{j,k}^\lambda - r_{i,k}^\lambda))$; (b) Encrypted messages $\mathbb{E}(w_{j \rightarrow i}(r_{j,k}^\lambda - r_{i,k}^\lambda))$.

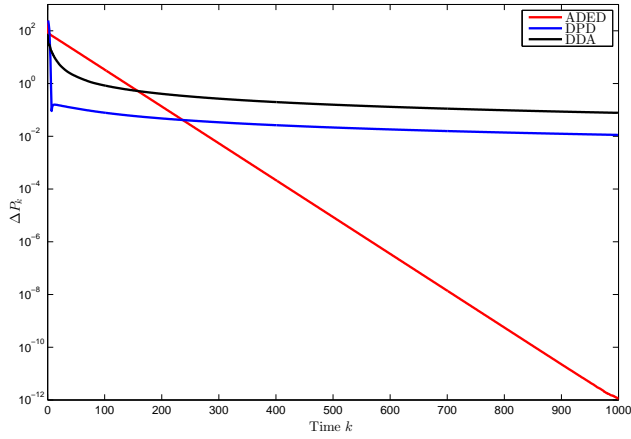


Fig. 5. The dynamic evolutions of ΔP_k via different algorithms.

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