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Persistence in high frequency financial data: the case of the EuroStoxx 50 futures prices

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ABSTRACT
Differences in the behaviour of asset prices depending on data frequency have not been thoroughly investigated in the literature despite their possible importance. In particular, high-frequency data might contain more information about financial assets because they are updated more rapidly in response to news. This paper explores persistence in high-frequency data (and also daily and monthly ones) in the case of the EuroStoxx 50 futures prices over the period from 2002 to 2018 (720 million trade records) using R/S analysis and the Hurst exponent as a measure of persistence. The results show that persistence is sensitive to the data frequency. More specifically, monthly data are highly persistent, daily ones follow a random walk, and intraday ones are anti-persistent. In addition, persistence varies over time. These findings imply that the Efficient Market Hypothesis (EMH) only holds in the case of daily data, whilst it is possible to make abnormal profits using trading strategies based on reversal strategies at the intraday frequency.

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1. Introduction

Since the seminal paper by Fama (1970), a huge number of studies have examined empirically the Efficient Market Hypothesis (EMH) according to which asset prices should follow a random walk and thus not exhibit long memory, trends, or mean-reversion. Many papers have found instead that (high) persistence is a typical property of financial data (Bariviera, 2017; Caporale et al., 2018; Lo, 1991; Mynhardt et al., 2014; Phillip et al., 2018 and many others). Most of this evidence is based on daily, weekly, or monthly data and suggests that the data frequency matters; for example, Caporale and Gil-Alana (2010) showed (through Monte Carlo experiments and an empirical application to the S&P500) that it affects the estimates of the fractional integration parameter measuring persistence; more precisely, if the true DGP is I (d) with d lying between 0 and 1, d tends to be underestimated at lower frequencies, and this bias tends to be bigger for bigger deviations from 0 or 1. In another paper, Caporale et al. (2019) investigated persistence in financial time series at daily, weekly and monthly frequencies for various financial markets (stock markets, FOREX, commodity markets) over the period from 2000 to 2016 using both R/S analysis and fractional integration; their results indicate that in most cases persistence is higher at lower frequencies, for both returns and their volatility, which is inconsistent with the EMH and implies that abnormal profits can be made by using trading strategies based on trend analysis. More

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evidence of market efficiency is provided by Ozkan (2021), who used the wild bootstrap automatic variance ratio test to analyse the impact of the COVID-19 pandemic on six developed stock markets and found deviations from efficiency only in some periods of the pandemic.

According to Zhang et al. (2022) high-frequency data contain more real-time information about the securities trading process and capture more accurately even tiny changes in the market, and thus analysing them is a very interesting research topic. In addition, Feng et al. (2023) argue that by using high-frequency data risk can be evaluated more precisely.

A few studies have also analysed long-memory behaviour at higher frequencies. For instance, Caporale and Gil-Alana (2013) focused on the US dollar/British exchange rate and found several cases of mean-reverting behaviour when the data are collected every 10 min whilst for even higher frequencies the unit root null cannot be rejected; in other words, persistence tends to be lower at higher frequencies characterised by more noise in price dynamics.

The present study goes further by measuring persistence in the case of data collected at a much higher frequency (namely, micro seconds) using R/S analysis based on the Hurst exponent method; specifically, the series examined is the EuroStoxx 50 futures prices over the period 2002–2018. The analysis is then repeated at lower (monthly and daily) frequencies to examine whether persistence is sensitive to the data frequency, and finally, a sliding-window approach is used to investigate whether it varies over time.

This paper contributes to the existing literature by providing additional evidence on whether or not persistence in asset prices depends on the data frequency. In particular, it shows that the higher the data frequency is, the lower is persistence. To be more specific, monthly data are highly persistent, daily ones follow a random walk, and intraday ones are anti-persistent. Further, persistence is shown to vary over time. On the whole, these findings suggest that the EMH only holds in the case of daily data. The implication for investors is that mean-reverting trading strategies should be adopted at high frequencies whilst trends should be followed at lower ones (weekly, monthly).

The layout of the paper is the following: Section 2 briefly reviews the relevant literature; Section 3 describes the data and outlines the empirical methodology; Section 4 presents the empirical results; Section 5 offers some concluding remarks.

2. Literature review

Numerous papers have analysed persistence in various financial assets, such as stocks (Lo, 1991; Los & Yalamova, 2006), exchange rates (Da Silva et al., 2007; Kim & Yoon, 2004), commodity prices (Alvarez-Ramirez et al., 2002; Serletis & Rosenberg, 2007), and cryptocurrencies (Bariviera, 2017; Bouri et al., 2016; Caporale et al., 2018; Urquhart, 2016), often providing international evidence (e.g. Jacobsen, 1995; Lento, 2009; Niere, 2013; Zunino et al., 2009). A variety of methods have been applied to estimate persistence: R/S analysis (Caporale et al., 2018; Glenn, 2007; Lento, 2009), fractional integration (Caporale & Gil-Alana, 2013), the generalized Hurst exponent approach (Barunik & Kristoufek, 2010), detrended moving average (Grech & Mazur, 2005), multifractal generalization (Kantelhardt et al., 2002), detrended fluctuation analysis (Taqqu et al., 1995), etc.

Most studies have used daily data (e.g. Niere, 2013; Zunino et al., 2009), considerably fewer have focused on weekly (MacDonald & Taylor, 1992) or monthly (Caporale et al., 2019) ones, and an even smaller number on high-frequency ones. In particular, Andersen and Bollerslev (1997) analysed persistence in 5-min returns in the FOREX and US stock market and found long-memory properties in their volatility, whilst Cotter (2005) reported similar findings in the case of UK futures using data at 5-min intervals; finally, as already mentioned, Caporale and Gil-Alana (2013) examined high frequency data (collected every 1, 2, 3, 5, and 10 min) on the US dollar-British pound spot exchange rate and found lower persistence at higher frequencies; this is consistent with Caporale et al. (2019), who investigated persistence at three different frequencies (daily, weekly and monthly) in various stock, FOREX and commodity markets using both R/S analysis and fractional integration and reached the same conclusion.

According to Shakeel and Srivastava (2021), high-frequency financial time series data exhibit different properties compared to low-frequency ones. In particular, as argued by Zhang et al. (2022), the lower the data frequency is, the more information is lost. In fact Dobrev and Szerszen (2010) find that two years of high-frequency data often suffice to obtain the same level of precision as twenty years of daily data, and Feng et al. (2023) show that the estimation accuracy of the regression coefficient can be significantly improved by using intraday high-frequency data.

It is noteworthy that the highest frequency considered by the papers discussed above is 1 min, none of them investigating persistence at higher frequencies. The present study aims to fill this gap by providing evidence on persistence in the case of data collected at micro seconds as detailed below.

3. Data and methodology

High-frequency data on EuroStoxx 50 futures prices over the period 2002–2018 are used for the empirical analysis; they consist of 720 million trade records being collected every tenth of a second or even more frequently. EuroStoxx 50 futures (the most actively traded EUR-denominated equity index derivatives) give investors exposure to a basket of euro-area blue-chip equities. All trading is done through an electronic limit-order book. Data set includes the following variables: time stamp of the trade, type of the trade record ('B' for a buyer and 'S' for a seller), the size of the trade expressed in number of contracts, and the price at which the trade between buyer and the seller is concluded. Daily and monthly series are also examined. The data source is Eurex Exchange, the leading platform for Eurozone equity and equity index derivatives (https://www.eurex.com/ex-en/).

The measure of persistence used is the Hurst exponent estimated by carrying out R/S analysis. This approach dates back to Hurst (1951) and was extended by Mandelbrot (1972) and Mandelbrot and Wallis (1969). Despite newer methods having been developed since then [such as detrended fluctuation analysis (DFA), multifractal generalization (MF-DFA), stabilogram diffusion analysis (SDA), and additional methods to measure return predictability, such as the wild bootstrap LR test (Ozkan, 2020) as well as others], R/S analysis remains very popular and is very often used for the purpose of estimating persistence in financial data (Danylchuk et al., 2020; Metescu, 2022; Mynhardt et al., 2014; Raimundo & Okamoto, 2018). The rationale for choosing this method is that it is relatively simple and suitable for programming as well as for visual interpretation, and it appears to capture accurately the properties of the data. In particular, it is computationally very attractive since it allows to analyse extremely large data sets consisting of millions of observations as in the present case by implementing rather simple algorithms. Other papers (e.g. Caporale et al., 2019) have also applied alternative methods to estimate persistence at different data frequencies, and their findings are useful for comparison purposes.

The Hurst exponent (H) lies in the interval [0, 1]. Data are persistent when $H > 0.5$. Data where Hurst exponent is equal or close to 0.5 can be defined as random. $H < 0.5$ is typical for anti-persistent data.

In brief, the algorithm for the R/S analysis is constructed as follows. First, a sub-period range is calculated as the difference between the maximum and minimum data values within the analysed sub-period. Then the average value for each sub-period is divided by the corresponding standard deviation. Next, the length of the sub-period is increased and the calculation is repeated until the size of the sub-period is equal to that of the original series. As a result, a set of ratios (computed as the average value of range divided by the associated standard deviation) for all the sub-periods is obtained. Next regression analysis is carried out using these data, the slope of the regression being an estimate of the Hurst exponent. More details are provided below.

1. A time series of length M is transformed into one of length $N = M - 1$ with prices converted into log returns:

$$N_i = \log\left(\frac{R_{t+1}}{R}\right), t = 1, 2, 3, \ldots (M - 1) \tag{1}$$

2. This period is divided into A sub-periods with length n, such that $A_n = N$, then each sub-period is defined as I_a for $a = 1, 2, 3.. ., A$. Each element I_a is represented as N_k with $k = 1, 2, 3.. ., N$. For each I_a with length n the average e_a is defined as:

$$e_a = \frac{1}{n}\sum_{k=1}^{n} N_{k,a}, k = 1,2,3,\ldots N, a = 1,2,3\ldots, \mathrm{A}. \quad (2)$$

3. Accumulated deviations $X_{k,a}$ from the average e_a for each sub-period I_a are defined as:

$$X_{k,a} = \sum_{i=1}^{k}(N_{i,a} - e_a). \quad (3)$$

The range is defined as the maximum index $X_{k,a}$ minus the minimum $X_{k,a}$, within each sub-period (I_a):

$$R_{Ia} = \max(X_{k,a}) - \min(X_{k,a}), 1 \le k \le n.. \quad (4)$$

4. The standard deviation S_{Ia} is calculated for each sub-period I_a:

$$S_{Ia} = \left(\left(\frac{1}{n}\right)\sum_{k=1}^{n}(N_{k,a} - e_a)^2\right)^{0,5}. \quad (5)$$

5. The Rescaled Range (R_{Ia}/S_{Ia}) is defined as the range (R_{Ia}) divided by the standard deviation (S_{Ia}) for each sub-period I_a (in step 2 above, one obtains adjacent sub-periods of length n). The average R/S for length *n* is defined as:

$$(R/S)_n = (1/A)\sum_{i=1}^{A}(R_{Ia}/S_{Ia}). \quad (6)$$

6. The length *n* is increased to the next higher level, $(M - 1)/n$, and must be an integer number. In this case, we use *n*-indexes that include the initial and end points of the time series, and Steps 1–6 are repeated until $n = (M - 1)/2$.
7. Using least squares the following regression is run: $\log(R/S) = \log(c) + H\log(n)$, *where t*he slope of the regression (*H*) is an estimate of the Hurst exponent.

To investigate the issue of persistence in the data the following procedure is used. Hurst exponent is calculated for a 'data window', where the number of observations is less than for the full sample. The first value of the Hurst exponent is calculated for the first window which starts at the beginning of the data set; then the next value is obtained by shifting the window by a given number of observations. This sliding-window approach is applied for each of the following windows. This generates a data set with time-varying Hurst exponents.

In our application, we analyse returns computed as follows:

$$\mathrm{R_i} = \left(\frac{\mathrm{Close_i}}{\mathrm{Open_i}} - 1\right) \times 100\%, \quad (7)$$

where R_i–returns on the *i-th* period in percentage terms; $Open_i$–open price on the *i-th* period; $Close_i$–close price on the *i-th* period.

4. Empirical results

Descriptive statistics for the R/S analysis in the case of the high-frequency data collected at intervals of microseconds are presented in Table 1.

As can be seen, the mean values are in the range [0.35–0.40], which implies the presence of anti-persistence in the data. However, there is also evidence of time variation in the Hurst exponent; this is apparent from Figure 1, which displays the results from the dynamic analysis for the high-frequency data based on a sliding-window approach.

To examine whether the differences between the estimated values of the Hurst exponent for different years are statistically significant we perform *t*-tests (see Table 2); these confirm that indeed they are in the majority of cases, which implies that the degree of market efficiency changes over time.

Table 1. Descriptive statistics for the R/S analysis.

Year/parameter	Mean	Standard deviation	Interval	Min	Max	Count
2002	0.35	0.06	0.33	0.12	0.45	167
2003	0.39	0.06	0.37	0.08	0.45	219
2004	0.37	0.05	0.29	0.15	0.44	113
2005	0.35	0.08	0.42	0.03	0.45	187
2006	0.35	0.08	0.43	0.04	0.47	254
2007	0.35	0.09	0.45	0.01	0.45	233
2008	0.36	0.08	0.43	0.03	0.46	233
2009	0.37	0.07	0.39	0.06	0.45	229
2010	0.37	0.08	0.44	0.02	0.46	252
2011	0.37	0.08	0.46	0.03	0.49	235
2012	0.38	0.08	0.42	0.05	0.47	229
2013	0.39	0.08	0.41	0.08	0.49	247
2014	0.38	0.09	0.43	0.04	0.47	252
2015	0.40	0.09	0.46	0.04	0.50	186
2016	0.40	0.09	0.49	0.02	0.51	251
2017	0.39	0.09	0.46	0.04	0.50	247
2018	0.39	0.10	0.43	0.05	0.48	238
All	0.37	0.08	0.50	0.01	0.51	3772

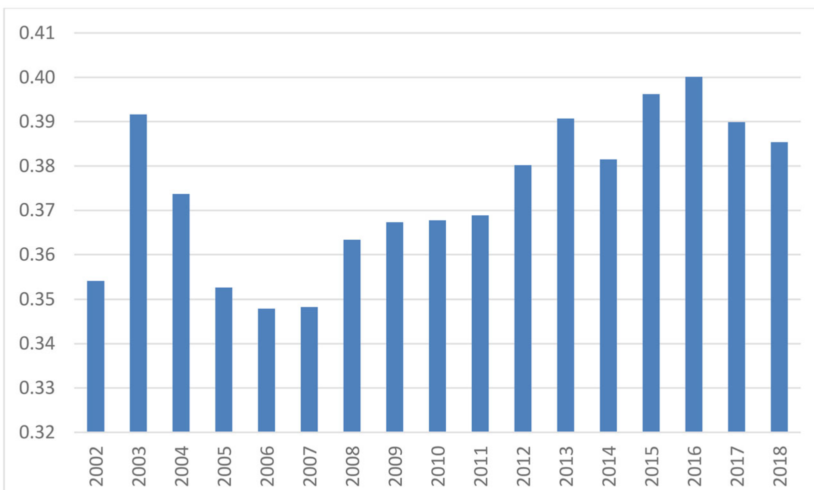

Figure 1. Hurst exponent during 2002–2018.

Table 2. *t*-Tests for differences in the R/S analysis results.

Year/parameter	*t*-Test	Difference is statistically significant
2002	4.00	Yes
2003	3.79	Yes
2004	0.15	No
2005	3.70	Yes
2006	5.40	Yes
2007	4.21	Yes
2008	1.98	Yes
2009	1.43	No
2010	1.29	No
2011	1.02	No
2012	1.10	No
2013	3.13	Yes
2014	1.25	No
2015	3.12	Yes
2016	4.39	Yes
2017	2.69	Yes
2018	1.71	No

Next, we replicate the dynamic analysis using daily data (see Figure 2); this is not feasible in the case of monthly data as the sample (12 observations in each case) would be too small.

As can be seen, the only similarity between the two sets of estimates is the instability of persistence in both cases. This is confirmed by the very low correlation coefficient between them (−0.06). It is also

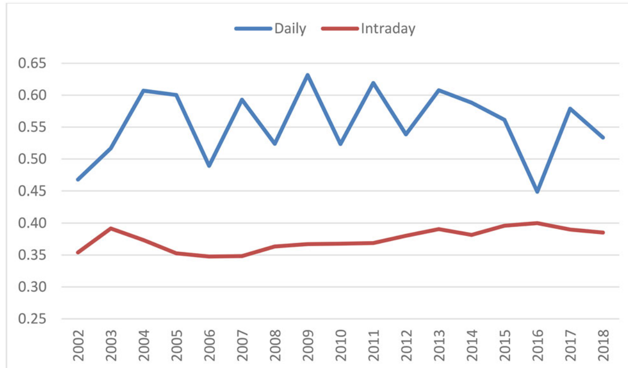

Figure 2. Dynamics of the Hurst exponent during 2002–2018: daily and intraday data.

Table 3. R/S analysis results for the cases of daily and high-frequency data.

Year	Daily	Intraday	Difference, %
2002	0.47	0.35	−32%
2003	0.52	0.39	−32%
2004	0.61	0.37	−62%
2005	0.60	0.35	−70%
2006	0.49	0.35	−41%
2007	0.59	0.35	−70%
2008	0.52	0.36	−44%
2009	0.63	0.37	−72%
2010	0.52	0.37	−42%
2011	0.62	0.37	−68%
2012	0.54	0.38	−42%
2013	0.61	0.39	−56%
2014	0.59	0.38	−54%
2015	0.56	0.40	−42%
2016	0.45	0.40	−12%
2017	0.58	0.39	−48%
2018	0.53	0.39	−38%

Table 4. Hurst exponent values for different data frequencies, 2002–2018.

Data frequency	Hurst
Month	0.72
Day	0.54
Intraday	0.37

apparent that persistence in daily data is much higher (see Table 3): the higher data frequency is characterised by a lower Hurst exponent and by anti-persistence rather than persistence.

Table 4 reports the estimates of the Hurst exponent for the whole sample at the three frequencies considered (monthly, daily, and intraday)—these confirm that the behaviour of the series is very different at different frequencies. More specifically, monthly returns are the most persistent. This implies that they contain information about their future values, and thus autoregressive models can be estimated to predict them and develop strategies to 'beat' the market. By contrast, intraday data are anti-persistent, which suggests that contrarian trading strategies should be applied.

5. Conclusions

This paper uses the Hurst exponent (calculated by means of R/S analysis) to explore the long-memory properties of high-frequency financial data for the case of EuroStoxx 50 futures prices over the period 2002–2018. The aim of the analysis is to establish whether or not persistence is sensitive to the data frequency (intraday, daily, monthly) and whether or not it varies over time. Although such issues had already been examined in some earlier studies (see, e.g. Caporale & Gil-Alana, 2010), none had used data collected at microseconds as the present one does.

Our findings indicate that the series exhibits very different properties at different frequencies. More specifically, the higher the data frequency is, the lower is persistence. Monthly data are highly persistent, daily ones follow a random walk, and intraday ones are anti-persistent. These results are in line with those obtained by Caporale et al. (2019) for stock markets, FOREX, and commodity markets. Lower persistence at higher frequencies for the case of FOREX was detected by Caporale and Gil-Alana (2013). The dynamic R/S analysis also shows that persistence varies over time. These results are consistent with those reported by Mynhardt et al. (2014) for different stock market indices and currency pairs. The implication of these results is that the EMH only holds in the case of daily returns. In particular, in the case of intraday data, it appears to be possible for traders, and more specifically scalpers (who enter and exit financial markets quickly, usually within seconds, using high levels of leverage to place large-sized trades in the hopes of achieving greater profits from minuscule price changes) to make abnormal profits by adopting mean-reverting trading strategies (sell after price increases, buy after price declines).

A limitation of this paper is the fact that it uses a single approach (R/S analysis) to estimate persistence. Additional methods, such as fractional integration, DFA, MF-DFA, SDA, etc. are now available and could be applied in future papers. Moreover, owing to limited data availability, our analysis has only been carried out for the EuroStoxx 50 futures prices. To check the robustness of our findings, it would be interesting to investigate the same issues for other types of assets and countries, and also to examine in greater depth any possible instabilities in price behaviour.

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