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RESEARCH ARTICLE

Total and Minimum Energy Efficiency Tradeoff in Robust Multigroup Multicast Satellite Communications

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Satellite communication is an indispensable part of future wireless communications given its global coverage and long-distance propagation. In satellite communication systems, channel acquisition and energy consumption are two critical issues. To this end, we investigate the tradeoff between the total energy efficiency (TEE) and minimum EE (MEE) for robust multigroup multicast satellite communication systems in this paper. Specifically, under the total power constraint, we investigate the robust beamforming aimed at balancing the TEE-MEE, so as to achieve the balance between the fairness and total performance on the system EE. For this optimization problem, we first model the balancing problem as a nonconvex problem while deriving its approximate closed-form average user rate. Then, the nonconvex problem is handled by solving convex programs sequentially with the help of the semidefinite relaxation and the concave-convex procedure. In addition, depending on the solution rank value, Gaussian randomization and eigenvalue decomposition method are applied to generate the feasible solutions. Finally, simulation results illustrate that the proposed approach can effectively achieve the balance between the TEE and MEE, thus realizing a tradeoff between fairness and system EE performance. It is also indicated that the proposed robust approach outperforms the conventional baselines in terms of EE performance.

Introduction

Compared with previous systems, the fifth generation of mobile communication (5G) has made major breakthroughs in terms of latency, data rate, and mobility. However, existing endeavors are still not enough in response to novel and emerging services [1–4]. To this end, beyond 5G and even sixth generation of mobile communication systems (6G) are put on to the agenda. Although consensus on specific demands of 6G has not been reached, ubiquitous global network coverage, higher data transmission rates, lower propagation delays, and higher energy efficiency (EE) are widely agreed. Under this circumstance, satellite communications that can achieve global coverage and long-distance communications could be an indispensable part of 6G [5–11].

With the rapid increase of energy wastage in wireless systems, green communication technology has attracted extensive attentions [12–15], especially in the satellite communications. The expectation is that the transmission rates can be improved with similar or less power consumptions, so that EE becomes the key performance indicator in the transmission scheme design. Meanwhile, strict requirements are put forward for transmission schemes in improving the EE performance [16,17]. Most of the existing work focused on maximizing the total EE (TEE) or minimum EE (MEE). However, tradeoff between TEE and MEE has not been investigated in the satellite communication systems. Hence, it is of both great theoretical and practical significance for satellite systems to investigate the tradeoff between the TEE and MEE so as to balance the total system performance and the fairness in terms of EE.

In this paper, we investigate the EE optimization for the downlink multibeam geostationary earth orbit (GEO) satellite communication system. In fact, several practical problems need to be considered in the beamforming design of multibeam satellite communication systems. First of all, multibeam satellites are used to generate multiple spot beams to cover different areas in satellite communications. In addition to that, frequency reuse is adopted between spot beams to improve the spectrum efficiency

(SE) performance. Although frequency reuse might introduce serious cochannel interference, precoding at the transmitter side could be used to manage the cochannel interference [18,19]. To this end, we consider the beamforming design to manage the cochannel interference effectively. Besides, in the existing satellite communication standards such as Digital Video Broadcasting - Satellite - Second Generation (DVB-S2) [20] and DVB-S2 Extensions (DVB-S2X) [21], multiple users sharing an identical frame employ the same beamformer, so that the beamforming design can be expressed as a multicast multigroup optimization problem [22]. Furthermore, satellite communication systems rely on the gateway (GW) to obtain the channel state information (CSI), which is required on designing the beamforming. However, due to the long-distance delay, it is infeasible for GW to acquire an accurate CSI at the transmitter (CSIT) [23–26]. Therefore, it is essential to design a robust beamforming scheme with imperfect CSI, especially for multigroup multicast satellite communication systems.

Related Works: In the literature, precoding methods for multibeam satellite communication systems have been widely studied. For instance, authors in [27] studied the frame-based robust precoding method for the multicast multibeam satellite communication systems and proposed a user-clustering algorithm to improve the precoding performance. In [28], the robust precoding problem of multibeam satellite communication maximizing the worst-case outage signal-to-interference-plus-noise ratio (SINR) under the outage and the per-beam power constraints was studied. Moreover, [29] investigated the problem of robust multigroup multicast beamforming for multibeam satellite communication systems with the aim to maximize the MEE. Authors in [30] studied the beamforming problem of multicast multigroup satellites with the objective of maximizing resource efficiency. The above works all used the beamforming design to alleviate the problem of interbeam interference.

In the last few years, there have been many studies on EE optimization. For instance, [31–34] studied the TEE maximization in orthogonal frequency division multiple access (OFDMA) and multiple-input multiple-output (MIMO) systems. In [35], the authors proposed a high-EE-transmission scheme that is suitable for multibeam MIMO satellite systems. The goal of this scheme was to optimize the TEE performance of the MIMO satellite communication systems under the power consumption and total rate constraints. The numerical results show that the TEE in the proposed approach has a marked increase compared with fixed power allocation. In addition, the authors of [36] aimed at maximizing TEE by comprehensively considering the per-user SINR and per-base station transmit power in multicell multicast networks. In order to tackle the above problem, an iterative multicast transmission algorithm with guaranteed convergence was proposed. Besides, the TEE maximization problem under the constraints of total power and quality of service (QoS) in multibeam satellite communication systems was investigated in [37,38]. Although TEE is widely adopted as an essential EE optimization criterion, it ignored the EE fairness on each link. Actually, maximizing the MEE depends on the EE of each link, considering the fairness of the link. In [39], the problem of maximizing the MEE in the multicell cooperative beamforming system was studied, which considers the static and dynamic power consumption. In [40], the power allocation strategies in the spectrum-sharing network were presented, which include maximizing the MEE and harmonic fair EE. In [41], the authors studied the MEE optimization problem in wireless power transfer (WPT)-enabled massive MIMO systems, and an EE power and time allocation algorithm was proposed to obtain the maximum MEE. Moreover, [29] investigated maximizing the MEE of all groups under the total power constraint in the multibeam satellite systems. However, the overall EE performance is not taken into account in the MEE optimization. In the literature, the joint optimization of TEE and MEE exploiting the perfect CSI was studied for the MIMO system in [42], where the fairness and the overall EE performance of the system are both considered. Inspired by [42], we investigate the tradeoff between TEE and MEE in robust multigroup multicast satellite communications.

Contributions: Motivated by the issues mentioned above, we investigate the TEE-MEE tradeoff in robust multigroup multicast satellite communication systems. We summarize the major contributions of this study into three folds:

1. We propose a multiobjective method for EE optimization suitable for satellite communication systems, which consider TEE and MEE simultaneously. Specifically, this method can achieve various tradeoff points on the TEE-MEE plane. This study is of great significance for realizing the tradeoff between fairness and overall EE performance of the satellite system.

2. In the studied satellite communication system, we employ the weighted product of the objectives to handle the optimization problem. Then, we introduce the optimization variables to convert the objective function into an addition form of multiobjectives, which is simpler and easier to handle. In addition, we use a closed-form approximate expression of the rate in the constraints to simplify the optimization procedure.

3. We introduce the semidefinite relaxation (SDR) method, which can convert the above optimization problem into a differential convex (DC) program. Then, via adopting the cocave-convex procedure (CCCP), the locally optimal solution to the nonconvex DC programming is derived by solving the convex programs sequentially. Moreover, depending on the solution rank value, we introduce the Gaussian randomization and eigenvalue decomposition method to generate the feasible solutions.

4. Numerical results indicate that the proposed algorithm can achieve the joint optimization of TEE and MEE, not only considering the fairness but also paying attention to the overall performance of the system. It is also shown that the proposed algorithm is superior to the traditional baselines in terms of the EE performance.

Organizations: The rest of this paper is organized as follows. System model presents the model of multigroup multicast satellite communication systems. Problem formulation introduces the problem formulation, which is the joint optimization problem of TEE and MEE by considering the total power constraint. Approximate average rate introduces the approximate average rate to acquire the closed-form tight expressions. Subsequently, Semidefinite relaxation and Concave–convex procedure use SDR and CCCP to handle the problem, respectively. Gaussian randomization is employed to obtain the feasible solutions in Gaussian randomization. Simulation Results demonstrates the simulation results and the robustness and effectiveness of the proposed robust algorithm is proved. At last, Conclusion gives a brief conclusion of our research in this paper. In addition, Table 1 lists the abbreviations and acronyms that are utilized in this article for facilitating the understanding of the terminology.

Table 1. Abbreviations and acronyms.

Abbreviation	Terminology
CCCP	Concave-convex procedure
CSIT	Channel state information at the transmitter
GW	Gateway
EE	Energy efficiency
JFI	Jain's fairness index
MIMO	Multiple-input multiple-output
MEE	Minimum energy efficiency
SINR	Signal-to-interference-plus-noise ratio
QoS	Quality of service
SDR	Semidefinite relaxation
TEE	Total energy efficiency
WP	Weighted product

Notations: In this paper, we use bold uppercase and lowercase letters to denote matrices and vectors, respectively. The rest of the notations are shown in Table 2 for clarity.

Table 2. Notation description.

Notation	Description
$(\cdot)^T$	Transpose
$(\cdot)^H$	Hermitian conjugate
$\mathbb{C}^{m\times n}$	The set of complex matrices composed of *m* rows and *n* columns
$\odot$	Hadamard product
$\mathcal{N}(\mathbf{x},\mathbf{Y})$	The real-valued Gaussian distributions with mean vector **x** and covariance matrix **Y**
$\mathcal{CN}(\mathbf{x},\mathbf{Y})$	The complex-valued Gaussian distributions with mean vector **x** and covariance matrix **Y**
$\jmath$	Imaginary unit
$\exp\{\mathbf{x}\}$	The exponential function with *e* as the base and each element in vector **x** as the exponent
$\mathbb{E}\{\cdot\}$	The expectation operator
$\mathrm{diag}(\mathbf{x})$	The diagonal matrix with **x** along its principal diagonal elements
$\mathrm{Tr}(\cdot)$	The matrix trace operator
$\mathbf{X}\succeq\mathbf{0}$	**X** is positive semidefinite
$\|\mathbf{x}\|_2$	The ℓ_2-norm of **x**
$[\mathbf{A}]_{m,n}$	The (m,n)th element of **A**
$\triangleq$	Be defined as
$[x]^+$	$\mathrm{Max}(x,0)$
$\sim$	Be distributed as

Materials and Methods

System Model and Problem Formulation

System model

We consider a downlink multigroup multicast satellite communication system, as depicted in Fig. 1, where N_u single antenna users are simultaneously served by N_t beams. The information data is sent by a GW to the users within the coverage areas of the satellite beams. It is assumed that the feeder link between the GW and satellite is ideal and no intercluster interference is considered [43]. In addition, each beam contains a multicast group, and there are a total of $M=N_\mathrm{t}$ multicast groups. The users are grouped according to the method in [28]. Let $\mathcal{M}=\{1,\ \ldots,M\}$ denote the index set for all groups. The set of indices for users in the *m*th multicast group is represented as $\mathcal{U}_m$ where $m\in\mathcal{M}$, and every user can only belong to one of the multicast groups, i.e., $\mathcal{U}_m\cap\mathcal{U}_n=\varnothing, \forall m,n\in\mathcal{M}, m\neq n$.

The signal received by the *i*th user in group *m* is represented as

$$y_i=\mathbf{h}_i^H\mathbf{w}_m s_m+\underbrace{\sum_{n\neq m}\mathbf{h}_i^H\mathbf{w}_n s_n}_{\text{inter}-\text{group interface}}+n_i, i\in U_m, \tag{1}$$

where $\mathbf{h}_i\in\mathbb{C}^{N_\mathrm{t}\times1}$ and $\mathbf{w}_m\in\mathbb{C}^{N_\mathrm{t}\times1}$ are the forward link beam domain channel vector from all N_t beams to the *i*th user and the beamforming vector for group *m*, respectively. Additionally, s_m is denoted as the transmit signal for all users in group *m* that satisfies $\mathbb{E}\left\{|s_m|^2\right\}=1, \forall m$, and $n_i\sim\mathcal{CN}(0,N_0)$ denotes the additive white Gaussian noise. Note that the transmitted signal energy is normalized, i.e., $\mathbb{E}\left\{|s_m|^2\right\}=1$.

For the *i*th user, its downlink beam domain channel vector is given by [44]

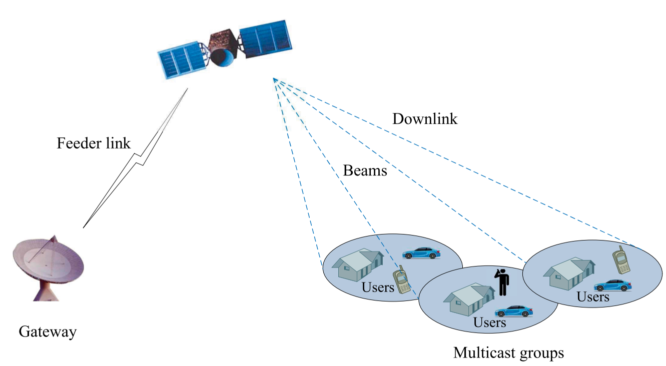

Fig. 1. Model of a downlink multigroup multicast satellite communication system.

$$\mathbf{h}_i=\sqrt{\psi_i}\mathbf{b}_i^{\frac{1}{2}}\odot\mathbf{r}_i^{\frac{1}{2}}\odot\exp\{\jmath\boldsymbol{\theta}_i\}, \tag{2}$$

where ψ_i denotes the scale parameter, $\mathbf{b}_i=[b_{i,1},b_{i,2},\ldots,b_{i,N\mathrm{t}}]^T$ is the N_t-dimensional far-field beam radiation pattern, and the beam gain coefficient from the *j*th feed to the *i*th user is denoted as $b_{i,j}$ [45]. In addition, $\mathbf{r}_i=[r_{i,1},r_{i,2},\ldots,r_{i,N\mathrm{t}}]^T$ denotes the N_t-dimensional rain fading coefficient vector with each element obeying the lognormal random distribution [46]. The phase of the channel is denoted by $\boldsymbol{\theta}_i=[\theta_{i,1},\theta_{i,2},\ldots,\theta_{i,N\mathrm{t}}]^T$ and its elements are uniformly distributed between 0 and 2π and independent of each other [47]. The exact formula of the above channel parameters is detailed as follows.

The large-scale fading coefficient ψ_i can be given as [48]

$$\psi_i = \left(\frac{c}{4\pi f d_0}\right)^2 \frac{G_{r,i}}{\kappa BT}, \tag{3}$$

where the velocity of light is denoted as c, $G_{r,i}$ represents the satellite receive antenna gain of the ith user, f denotes the forward link carrier frequency, d_0 is the height of satellite, κ represents the Boltzmann's constant, bandwidth is represented as B, and the noise temperature at the receiving end is denoted as T. Note that the noise power is normalized by κBT, and N_0 is set to be unit [48]. The beam gain from the jth beam to the ith user can be approximated as [45,49]

$$b_{i,j} = G_T^j \cdot \left(\frac{J_1\left(u_{i,j}\right)}{2u_{i,j}} + 36\frac{J_3\left(u_{i,j}\right)}{u_{i,j}^3}\right)^2, \tag{4}$$

where G_T^j is the gain of transmitting antenna for the jth beam, and the first- and the third-order Bessel functions of the first kind are represented by $J_1(\cdot)$ and $J_3(\cdot)$, respectively. It is noted that $u_{i,j}$ in (4) is expressed as

$$u_{i,j} = 2.07123 \cdot \frac{\sin\varphi_{i,j}}{\sin\varphi_{j,3dB}}, \tag{5}$$

where the off-axis angle between the jth beam and the ith user is expressed by $\varphi_{i,j}$. In addition, $\varphi_{j,3dB}$ is the 3-dB angle for the jth beam, and its value can be regarded as a constant, i.e., $\varphi_{j,3dB} = 0.4^\circ$ [44].

Remark 1. It is worth noting that the amplitudes of the multibeam satellite channel vector are related to several constant coefficients over the coherence time interval of interest [48], including the far-field beam radiated pattern, the scale parameter, the rainfall attenuation, the gaseous absorption, and the cloud attenuation [50,51]. The latter 2 coefficients can be negligible in Ka band compared with the rainfall attenuation due to its relatively slow variation [48]. Therefore, they can be omitted in the satellite channel model in (2). On the other hand, due to the existence of some time-varying factors, the channel phases change rapidly. For example, atmospheric absorption, fog, and rain can cause severe variations of the time-varying phase on satellite channel elements [43]. Therefore, these time-varying phase components can be modeled as those in [43,48].

Considering the amplitude invariance within the coherence time interval, we pay our attention to modeling the phase uncertainty [48]. At time t_0, the ith user returns the channel estimation result to the GW. After the propagation and processing delays, the satellite receives CSIT through the feeder link and performs precoding at time t_1 [43]. To facilitate the analysis, we model the channel phase of the ith user at t_1 as

$$\boldsymbol{\theta}_i\left(t_1\right) = \boldsymbol{\theta}_i\left(t_0\right) + \mathbf{e}_i, \tag{6}$$

where the channel phase error is expressed as $\mathbf{e}_i = [e_{i,1}, e_{i,2}, \dots e_{i,Nt}]^T$, which satisfies $\mathbf{e}_i \sim \mathcal{N}\left(\mathbf{0}, \sigma_i^2 \mathbf{I}\right)$, and σ_i^2 denotes the variance of the phase error [43,48,52]. Let $\overline{\mathbf{h}}_i$ and $\mathbf{h}_i$ be the estimated channel at instant t_0 and the actual channel at instant t_1, respectively. Then, the actual channel $\mathbf{h}_i$ will be [23]

$$\mathbf{h}_i = \overline{\mathbf{h}}_i \odot \mathbf{q}_i = \operatorname{diag}\left(\overline{\mathbf{h}}_i\right)\mathbf{q}_i, \tag{7}$$

where $\mathbf{q}_i$ is the error vector associated with the channel phase error which can be expressed as

$$\mathbf{q}_i \triangleq \exp\left\{j\mathbf{e}_i\right\}. \tag{8}$$

Given $\mathbf{Q}_i \triangleq \mathbf{q}_i\mathbf{q}_i^H$, the correlation matrix of $\mathbf{q}_i$ can be represented as

$$\mathbf{A}_i = \mathbb{E}\left\{\mathbf{q}_i\mathbf{q}_i^H\right\} = \mathbb{E}\left\{\mathbf{Q}_i\right\}. \tag{9}$$

It is ready to obtain the elements of autocorrelation matrix $\mathbf{A}_i$ as follows [27]

$$\left[\mathbf{A}_i\right]_{m,n} = \begin{cases} 1, & m = n, \\ \exp\left\{-\sigma_i^2\right\}, & m \neq n. \end{cases} \tag{10}$$

One can see that compared to the random variable of channel phase error, the value of matrix $\mathbf{A}_i$ is relatively fixed. Therefore, we can handle the problem caused by channel phase uncertainty by seeking the expectation of the channel phase error, i.e., by using the value of matrix $\mathbf{A}_i$.

Problem formulation

According to the signal model in (1), we can obtain the SINR of the ith user in group m as follows [23]

$$\mathrm{SINR}_{i,m} \triangleq \frac{\left|\mathbf{w}_m^H\mathbf{h}_i\right|^2}{\sum_{n\neq m}\left|\mathbf{w}_n^H\mathbf{h}_i\right|^2 + N_0}, \quad \forall i \in \mathcal{U}_m, m, n \in \mathcal{M}. \tag{11}$$

According to the above section, the accurate satellite channel vector $\mathbf{h}_i$ is troublesome to acquire. Therefore, the robust beamformer design based on expectation of accurate channel vector is introduced next. Let $R_{i,m}$ denote the data rate of the ith user in group m, which can be modeled as

$$R_{i,m} \triangleq \mathbb{E}\left\{\log_2\left(1 + \mathrm{SINR}_{i,m}\right)\right\}. \tag{12}$$

It is worth noting that, in order to ensure robustness, the rate function in (12) is the average value related to the satellite channel phase uncertainty $\mathbf{q}_i$. Then, the multicast rate of the users in group m can be denoted by

$$R_m \triangleq \min_{i\in\mathcal{U}_m} R_{i,m} = \min_{i\in\mathcal{U}_m} \mathbb{E}\left\{\log_2\left(1+\mathrm{SINR}_{i,m}\right)\right\}. \tag{13}$$

According to the multicast rate of each group, the sum rate of all multicast groups is expressed as

$$R_{\mathrm{tot}} \triangleq \sum_{m=1}^M R_m = \sum_{m=1}^M \min_{i\in\mathcal{U}_m} \mathbb{E}\left\{\log_2\left(1+\mathrm{SINR}_{i,m}\right)\right\}. \tag{14}$$

In what follows, we describe the power consumption model of the transmitter side. Especially, the power consumption of the mth group is modeled as

$$P_m \triangleq \xi_m \|\mathbf{w}_m\|_2^2 + P_{0,m}, \tag{15}$$

where $\xi_m \geq 1$ represents the inefficiency of the power amplifier, $\xi_m \|\mathbf{w}_m\|_2^2$ is the power radiated by the transmitting antenna array to group m, and the basic power consumption for group

m is expressed as $P_{0,m}$ [29]. Then, the total power consumption of all groups is modeled as

$$P_{\text{tot}} \triangleq \sum_{m=1}^{M} P_m = \sum_{m=1}^{M} \left(\xi_m \|\mathbf{w}_m\|_2^2 + P_{0,m} \right). \tag{16}$$

With the multigroup multicast rate and power consumption, we can get the EE of group m as [29]

$$\text{EE}_m \triangleq B\frac{R_m}{P_m} = B\frac{\min_{i \in U_m} \mathbb{E}\left\{\log_2\left(1+\text{SINR}_{i,m}\right)\right\}}{\xi_m\|\mathbf{w}_m\|_2^2 + P_{0,m}}, \tag{17}$$

$\mathbb{E}$ where B represents the bandwidth. The system TEE can be denoted as

$$\text{EE}_{\text{tot}} \triangleq B\frac{R_{\text{tot}}}{P_{\text{tot}}} = B\frac{\sum_{m=1}^{M} \min_{i \in U_m} \mathbb{E}\left\{\log_2\left(1+\text{SINR}_{i,m}\right)\right\}}{\sum_{m=1}^{M} \left(\xi_m\|\mathbf{w}_m\|_2^2 + P_{0,m}\right)}. \tag{18}$$

Unlike conventional beamforming design which merely considers maximization of the TEE or MEE, we focus on balancing the tradeoff between TEE and MEE, which can be seen as a multiobjective optimization problem. Specifically, we introduce a new metric, namely, the weighted product (WP) of TEE and MEE, which is given by [42]

$$F_{\text{WP}} = \text{EE}_{\text{tot}}^{\beta} \left(\min_{1 \le m \le M} \text{EE}_m \right)^{1-\beta}, \tag{19}$$

where β and $1-\beta$ respectively represent the precedence weight of TEE and MEE ($0 \le \beta \le 1$). Note that we can obtain the tradeoff points between TEE and MEE by changing the value of β.
In the following part, we aim to maximize the weighted product of TEE and MEE under the total power constraint, so as to realize the tradeoff between fairness and the overall EE performance of the satellite system. Therefore, the robust beamforming design can be formulated as

$$\mathcal{F}_1: \quad \max_{\{\mathbf{w}_m\}_{m=1}^{M}} \quad F_{\text{WP}} = \text{EE}_{\text{tot}}^{\beta} \left(\min_{1 \le m \le M} \text{EE}_m \right)^{1-\beta} \tag{20A}$$

$$\text{s. t.} \sum_{m=1}^{M} \|\mathbf{w}_m\|_2^2 \le P, \tag{20B}$$

where P represents the transmit power budget.

TEE-MEE Tradeoff

Problem $\mathcal{F}_1$ is challenging because of the difficulty in calculating the explicit achievable multicast rate function. In order to tackle this problem, we first use an explicit tight approximation of the ergodic average rate to reduce the optimization complexity. Then, the resultant nonconvex problem is handled by solving convex programs sequentially with the help of SDR and CCCP. Finally, Gaussian randomization or eigenvalue decomposition is adopted to acquire the beamformers.

Approximate average rate

Firstly, we convert problem $\mathcal{F}_1$ into an equivalent problem that is easier to handle. In order to address the minimization operation involved in the rate expression in (13), we introduce some auxiliary variables $\{\alpha_m\}_{m=1}^{M}$, and then problem (20) can be equivalently converted into

$$\mathcal{F}_2: \max_{\{\mathbf{w}_m\}_{m=1}^{M}, \{\alpha_m\}_{m=1}^{M}} F_{\text{WP}} = \left(\frac{\sum_{m=1}^{M} \alpha_m}{\sum_{m=1}^{M} \left(\xi_m \| \mathbf{w}_m \|_2^2 + P_{0,m}\right)} \right)^{\beta} \cdot \left(\min_{1 \le m \le M} \frac{\alpha_m}{\xi_m \| \mathbf{w}_m \|_2^2 + P_{0,m}} \right)^{1-\beta} \tag{21A}$$

$$\text{s. t. } R_{i,m} \ge \alpha_m, \forall i \in \mathcal{U}_m, m, n \in \mathcal{M}, \tag{21B}$$

$$\sum_{m=1}^{M} \|\mathbf{w}_m\|_2^2 \le P. \tag{21C}$$

Note that the bandwidth B is omitted in the objective function of $\mathcal{F}_2$ without loss of generality.
The objective function of $\mathcal{F}_2$ is in the form of a weighted product, and we adopt a variable transformation approach [42] to handle $\mathcal{F}_2$. In particular, we introduce auxiliary variables 2^u and 2^v, and problem $\mathcal{F}_2$ in (21) can be converted to the following problem through some mathematical operations (note that the maximization of x is equivalent to the maximization of $\log_2 x$):

$$\mathcal{F}_3: \max_{\{\mathbf{w}_m\}_{m=1}^{M}, \{\alpha_m\}_{m=1,u,v}^{M}} f = \log_2 F_{\text{WP}} = \beta u + (1-\beta) v \tag{22A}$$

$$\text{s. t. } R_{i,m} \ge \alpha_m, \forall i \in \mathcal{U}_m, m, n \in \mathcal{M}, \tag{22B}$$

$$\frac{\sum_{m=1}^{M} \alpha_m}{\sum_{m=1}^{M} \left(\xi_m\|\mathbf{w}_m\|_2^2 + P_{0,m}\right)} \ge 2^u, \forall m \in \mathcal{M}, \tag{22C}$$

$$\frac{\alpha_m}{\xi_m\|\mathbf{w}_m\|_2^2 + P_{0,m}} \ge 2^v, \forall m \in \mathcal{M}, \tag{22D}$$

$$\sum_{m=1}^{M} \|\mathbf{w}_m\|_2^2 \le P. \tag{22E}$$

It can be noted that the average rate in constraint (22B) does not allow an explicit expression because of the involved expectation operations. The traditional Monte Carlo approach has high computational complexity and large memory requirements, which might be impractical for satellite communication systems. Therefore, we employ an approximate function of the average rate hereafter, which is given by [53,54]

$$R_{i,m} \approx \bar{R}_{i,m} \triangleq \log_2\left(1 + \frac{\mathbb{E}\left\{ \left|\mathbf{w}_m^H \mathbf{h}_i\right|^2 \right\}}{\mathbb{E}\left\{ \sum_{n \ne m} \left|\mathbf{w}_n^H \mathbf{h}_i\right|^2 \right\} + N_0} \right). \tag{23}$$

Note that the approximation in (23) has been shown to be tight via theoretical analysis and numerical results [53,55]. In addition, in Simulation Results, simulation results further prove the effectiveness of proposed solution. We will adopt $\overline{R}_{i,m}$ instead of $R_{i,m}$ in the following optimization procedure.

Semidefinite relaxation

We note that problem (22) is still difficult to tackle. To this end, we invoke an efficient approximation method called as SDR. Then we transform the optimization variables $\{\mathbf{w}_m\}_{m=1}^{M}$ into $\{\mathbf{W}_m \triangleq \mathbf{w}_m\mathbf{w}_m^H\}_{m=1}^{M}$. Note that $\mathbf{W}_m = \mathbf{w}_m\mathbf{w}_m^H$ for some $\mathbf{w}_m \in \mathbb{C}_{\mathrm{t}}^{N\times 1}$ if and only if $\mathbf{W}_m \succeq \mathbf{0}$ and rank$(\mathbf{W}_m) = 1$. Therefore, according to the property of trace and expectation, the above approximate rate can be rewritten as

$$
\begin{aligned}
\overline{R}_{i,m} &= \log_2\left(1+\frac{\mathbb{E}\{\mathrm{Tr}(\mathbf{H}_i\mathbf{W}_m)\}}{\mathbb{E}\{\sum_{n\neq m}\mathrm{Tr}(\mathbf{H}_i\mathbf{W}_n)\}+N_0}\right)\\
&= \log_2\left(1+\frac{\mathrm{Tr}(\mathbb{E}\{\mathbf{H}_i\mathbf{W}_m\})}{\sum_{n\neq m}\mathrm{Tr}(\mathbb{E}\{\mathbf{H}_i\mathbf{W}_n\})+N_0}\right)\\
&= \log_2\left(1+\frac{\mathrm{Tr}(\mathbf{H}_i'\mathbf{W}_m)}{\sum_{n\neq m}\mathrm{Tr}(\mathbf{H}_i'\mathbf{W}_n)+N_0}\right),
\end{aligned} \tag{24}
$$

where $\mathbf{H}_i \in \mathbb{C}^{N_t\times N_t}$ represents the instantaneous channel correlation matrix

$$
\mathbf{H}_i \triangleq \mathbf{h}_i\mathbf{h}_i^H = \mathrm{diag}(\overline{\mathbf{h}}_i)\mathbf{Q}_i\mathrm{diag}\left(\overline{\mathbf{h}}_i^H\right), \tag{25}
$$

and $\mathbf{H}_i'$ represents the long term channel correlation matrix

$$
\begin{aligned}
\mathbf{H}_i' &= \mathbb{E}\{\mathbf{H}_i\} = \mathrm{diag}(\overline{\mathbf{h}}_i)\mathbb{E}\{\mathbf{Q}_i\}\mathrm{diag}\left(\overline{\mathbf{h}}_i^H\right)\\
&= \mathrm{diag}(\overline{\mathbf{h}}_i)\mathbf{A}_i\mathrm{diag}\left(\overline{\mathbf{h}}_i^H\right),
\end{aligned} \tag{26}
$$

where $\mathbf{Q}_i$ denotes the channel phase uncertainty. Likewise, the transmitting power of each group is given by $\|\mathbf{w}_m\|_2^2 = \mathrm{Tr}(\mathbf{W}_m)$.

Through the above transformations and some mathematical operations, problem (22) is relaxed as follows

$$
\mathcal{F}_4: \max_{\{\mathbf{W}_m\}_{m=1}^M, \{\alpha_m\}_{m=1}^M, u, v} \quad f = \log_2 F_{\mathrm{WP}} = \beta u + (1-\beta)v \tag{27A}
$$

$$
\mathrm{s.t.}\ \overline{R}_{i,m} \geq \alpha_m,\ \forall i \in \mathcal{U}_m, m, n \in \mathcal{M}, \tag{27B}
$$

$$
\log_2\left(\sum_{m=1}^M \alpha_m\right) - \log_2\left(\sum_{m=1}^M \left(\xi_m \mathrm{Tr}(\mathbf{W}_m) + P_{0,m}\right)\right) \geq u, \forall m \in \mathcal{M}, \tag{27C}
$$

$$
\log_2\alpha_m - \log_2\left(\xi_m\mathrm{Tr}(\mathbf{W}_m) + P_{0,m}\right) \geq v, \forall m \in \mathcal{M}, \tag{27D}
$$

$$
\sum_{m=1}^M \mathrm{Tr}(\mathbf{W}_m) \leq P, \tag{27E}
$$

$$
\mathbf{W}_m \succeq \mathbf{0}, \forall m \in \mathcal{M}. \tag{27F}
$$

It is worth noting that problem $\mathcal{F}_4$ is easier to deal with, because compared with problem $\mathcal{F}_3$, the nonconvex constraint rank $(\mathbf{W}_m) = 1$ is relaxed in problem $\mathcal{F}_4$.

Concave–convex procedure

To make the notations more concise, we denote $\mathbf{W} \triangleq \{\mathbf{W}_1, \ldots, \mathbf{W}_M\}$, and then problem $\mathcal{F}_4$ can be rewritten as

$$
\mathcal{F}_5: \max_{\{\alpha_m\}_{m=1}^M, \mathbf{W}, u, v} \quad f = \log_2 F_{\mathrm{WP}} = \beta u + (1-\beta)v \tag{28A}
$$

$$
\mathrm{s.t.}\ f_m(\mathbf{W}) - g_m(\mathbf{W}) \geq \alpha_m,\ \forall i \in \mathcal{U}_m, m, n \in \mathcal{M}, \tag{28B}
$$

$$
\log_2\left(\sum_{m=1}^M \alpha_m\right) - m_m(\mathbf{W}) \geq u, \forall m \in \mathcal{M}, \tag{28C}
$$

$$
\log_2\alpha_m - y_m(\mathbf{W}) \geq v,\ \forall m \in \mathcal{M}, \tag{28D}
$$

$$
(27\mathrm{e}), (=27\mathrm{f}). \tag{28E}
$$

where

$$
f_m(\mathbf{W}) = \log_2\left(\mathrm{Tr}\left(\mathbf{H}_i' \sum_{m=1}^M \mathbf{W}_m\right) + N_0\right), \tag{29A}
$$

$$
g_m(\mathbf{W}) = \log_2\left(\mathrm{Tr}\left(\mathbf{H}_i' \sum_{n\neq m} \mathbf{W}_n\right) + N_0\right), \tag{29B}
$$

$$
m_m(\mathbf{W}) = \log_2\left(\sum_{m=1}^M \left(\xi_m\mathrm{Tr}(\mathbf{W}_m) + P_{0,m}\right)\right), \tag{29C}
$$

$$
y_m(\mathbf{W}) = \log_2\left(\xi_m\mathrm{Tr}(\mathbf{W}_m) + P_{0,m}\right). \tag{29D}
$$

We can observe that $f_m(\mathbf{W})$, $g_m(\mathbf{W})$, $m_m(\mathbf{W})$, and $y_m(\mathbf{W})$ are all concave over $\mathbf{W}$, and then problem $\mathcal{F}_5$ is a DC programming. Thus, we utilize the CCCP, a powerful heuristic method, to handle this DC problem. The basic thought lies in replacing $g_m(\mathbf{W})$, $m_m(\mathbf{W})$, and $y_m(\mathbf{W})$ with their first-order Taylor expansions $\triangle g_m(\mathbf{W})$, $\triangle m_m(\mathbf{W})$, and $\triangle y_m(\mathbf{W})$. This method has been applied in some previous literatures, and its effectiveness has been verified in [52,56–58]. Then, the iteration is carried out until convergence [59], and the constraints in (28) can be re-expressed as

$$f_m(\mathbf{W}) - \triangle g_m^{(\lambda)}(\mathbf{W}) \geq \alpha_m, \tag{30A}$$

$$\log_2\left(\sum_{m=1}^{M} \alpha_m\right) - \triangle m_m^{(\lambda)}(\mathbf{W}) \geq u, \tag{30B}$$

$$\log_2 \alpha_m - \triangle y_m^{(\lambda)}(\mathbf{W}) \geq v, \tag{30C}$$

where

$$\triangle g_m^{(\lambda)}(\mathbf{W}) = g_m\left(\mathbf{W}^{(\lambda)}\right) + \sum_{a \neq m} \mathrm{Tr}\left\{\left(\frac{\partial g_m\left(\mathbf{W}^{(\lambda)}\right)}{\partial \mathbf{W}_a}\right)^T \left(\mathbf{W}_a - \mathbf{W}_a^{(\lambda)}\right)\right\}, \tag{31A}$$

$$\triangle m_m^{(\lambda)}(\mathbf{W}) = m_m\left(\mathbf{W}^{(\lambda)}\right) + \sum_{m=1}^{M} \mathrm{Tr}\left\{\left(\frac{\partial m_m\left(\mathbf{W}^{(\lambda)}\right)}{\partial \mathbf{W}_m}\right)^T \left(\mathbf{W}_m - \mathbf{W}_m^{(\lambda)}\right)\right\}, \tag{31B}$$

$$\triangle y_m^{(\lambda)}(\mathbf{W}) = y_m\left(\mathbf{W}^{(\lambda)}\right) + \mathrm{Tr}\left\{\left(\frac{\partial y_m\left(\mathbf{W}^{(\lambda)}\right)}{\partial \mathbf{W}_m}\right)^T \left(\mathbf{W}_m - \mathbf{W}_m^{(\lambda)}\right)\right\}, \tag{31C}$$

$\mathbf{W}^{(\lambda)}$ refers to an aggregation of variables $\left\{\mathbf{W}_1^{(\lambda)}, \ldots, \mathbf{W}_M^{(\lambda)}\right\}$, and the iteration index is denoted by λ. Furthermore, the derivatives in Eq. 31 are calculated as

$$\frac{\partial g_m\left(\mathbf{W}^{(\lambda)}\right)}{\partial \mathbf{W}_a} = \frac{\mathbf{H}_i'^T}{\left(\mathrm{Tr}\left(\mathbf{H}_i' \sum_{n \neq m} \mathbf{W}_n^{(\lambda)}\right) + N_0\right)\ln 2}, \tag{32A}$$

$$\frac{\partial m_m\left(\mathbf{W}^{(\lambda)}\right)}{\partial \mathbf{W}_m} = \frac{\xi_m \mathbf{I}}{\left(\sum_{m=1}^{M}\left(\xi_m \mathrm{Tr}\left(\mathbf{W}_m^{(\lambda)}\right) + P_{0,m}\right)\right)\ln 2}, \tag{32B}$$

$$\frac{\partial y_m\left(\mathbf{W}^{(\lambda)}\right)}{\partial \mathbf{W}_m} = \frac{\xi_m \mathbf{I}}{\left(\xi_m \mathrm{Tr}\left(\mathbf{W}_m^{(\lambda)}\right) + P_{0,m}\right)\ln 2}. \tag{32C}$$

Employing the above transformations, the problem in Eq. 31 can be transformed into a series of optimization subproblems, which is expressed as

$$\mathcal{F}_6: \max_{\mathbf{W}, \{\alpha_m\}_{m=1}^{M}, u, v} f = \log_2 F_{\mathrm{WP}} = \beta u + (1-\beta) v \tag{33A}$$

$$\mathrm{s.t.}\ f_m(\mathbf{W}) - \triangle g_m^{(\lambda)}(\mathbf{W}) \geq \alpha_m, \forall i \in \mathcal{U}_m, m, n \in \mathcal{M}, \tag{33B}$$

$$\log_2\left(\sum_{m=1}^{M} \alpha_m\right) - \triangle m_m^{(\lambda)}(\mathbf{W}) \geq u, \forall m \in \mathcal{M}, \tag{33C}$$

$$\log_2 \alpha_m - \triangle y_m^{(\lambda)}(\mathbf{W}) \geq v, \forall m \in \mathcal{M}, \tag{33D}$$

$$(27\mathrm{E}), (27\mathrm{F}). \tag{33E}$$

Remark 2. Note that problem $\mathcal{F}_6$ is convex and can be handled utilizing classical convex optimization methods. If the ranks of $\boldsymbol{W}_m^*$, $\forall m$, to problem $\mathcal{F}_6$ are all equal to one, the corresponding $\boldsymbol{W}_m^*$, $\forall m$, will be a feasible solution to problem $\mathcal{F}_2$. Then, the robust multicast beamformers can be obtained.

Gaussian randomization

When the ranks of $\mathbf{W}_m^*$, $\forall m$, obtained in problem $\mathcal{F}_6$ are not all one, we adopt the Gaussian randomization method to handle the rank issue. Note that the Gaussian randomization approach has been proved to be quasi-optimal in several practical scenarios [60].

We detail the Gaussian randomization procedure for our problem as follows. Firstly, the eigenvalue decomposition of $\mathbf{W}_m^*$ is given by

$$\mathbf{W}_m^* = \mathbf{U}\boldsymbol{\Sigma}\mathbf{U}^H, \tag{34}$$

then the candidate Gaussian vectors can be calculated as [61]

$$\widehat{\mathbf{w}}_m = \mathbf{U}\boldsymbol{\Sigma}^{1/2}\mathbf{v}_m, \tag{35}$$

where $\mathbf{U}$ and $\boldsymbol{\Sigma}$ are calculated from (34), and $\mathbf{v}_m \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$ is a complex Gaussian random vector. We obtain several candidate Gaussian vectors by repeating the above step.

Then, for a specific Gaussian realization $\left\{\widehat{\mathbf{w}}_m\right\}_{m=1}^{M}$, the corresponding power control problem for reallocating the transmit power among the candidate beamforming vectors can be expressed as [62]

$$\mathcal{S}_1: \max_{\{p_m\}_{m=1}^{M}, \{\alpha_m\}_{m=1}^{M}, u, v} f = \log_2 F_{\mathrm{WP}} = \beta u + (1-\beta) v \tag{36A}$$

$$\mathrm{s.t.}\ \mathbb{E}\left\{\log_2\left(1 + \frac{\left|\widehat{\mathbf{w}}_m^H \mathbf{h}_i\right|^2 p_m}{\sum_{n \neq m}\left|\widehat{\mathbf{w}}_n^H \mathbf{h}_i\right|^2 p_n + N_0}\right)\right\} \geq \alpha_m, \forall i \in \mathcal{U}_m, m, n \in \mathcal{M}, \tag{36B}$$

$$\frac{\sum_{m=1}^{M} \alpha_m}{\sum_{m=1}^{M}\left(\xi_m \|\mathbf{w}_m\|_2^2 p_m + P_{0,m}\right)} \geq 2^u, \forall m \in \mathcal{M}, \tag{36C}$$

$$\frac{\alpha_m}{\xi_m \|\mathbf{w}_m\|_2^2 p_m + P_{0,m}} \geq 2^v, \forall m \in \mathcal{M}, \tag{36D}$$

$$\sum_{m=1}^{M} \| \mathbf{w}_m \|_2^2 p_m \leq P, \tag{36E}$$

where p_m denotes the power scaling factor of group $\mathcal{U}_m$. Denoting $\boldsymbol{p} \triangleq \{p_m\}_{m=1}^{M}$ and using the approximation function of the average rate in (23), we can further express problem $\mathcal{S}_1$ as follows:

$$\mathcal{S}_2: \max_{\boldsymbol{p}, \{\alpha_m\}_{m=1}^{M}, u, v} f = \log_2 F_{\mathrm{WP}} = \beta u + (1-\beta) v \tag{37A}$$

$$\text{s.t.} f_m^-(\boldsymbol{p}) - g_m^-(\boldsymbol{p}) \geq \alpha_m, \forall i \in \mathcal{U}_m, m, n \in \mathcal{M}, \tag{37B}$$

$$\log_2\left(\sum_{m=1}^{M} \alpha_m\right) - m_m^-(\boldsymbol{p}) \geq u, \forall m \in \mathcal{M}, \tag{37C}$$

$$\log_2 \alpha_m - y_m^-(\boldsymbol{p}) \geq v, \forall m \in \mathcal{M}, \tag{37D}$$

$$\sum_{m=1}^{M} \|\mathbf{w}_m\|_2^2 p_m \leq P, \tag{37E}$$

where

$$f_m^-(\boldsymbol{p}) = \log_2\left(\sum_{m=1}^{M} \mathbb{E}\left\{\left|\hat{\mathbf{w}}_m^H \mathbf{h}_i\right|^2\right\} p_m + N_0\right), \tag{38A}$$

$$g_m^-(\boldsymbol{p}) = \log_2\left(\sum_{n \neq m} \mathbb{E}\left\{\left|\hat{\mathbf{w}}_n^H \mathbf{h}_i\right|^2\right\} p_n + N_0\right), \tag{38B}$$

$$m_m^-(\boldsymbol{p}) = \log_2\left(\sum_{m=1}^{M} \left(\xi_m \|\mathbf{w}_m\|_2^2 p_m + P_{0,m}\right)\right), \tag{38C}$$

$$y_m^-(\boldsymbol{p}) = \log_2\left(\xi_m \|\mathbf{w}_m\|_2^2 p_m + P_{0,m}\right). \tag{38D}$$

Based on the CCCP method, problem $\mathcal{S}_2$ is further converted into

$$\mathcal{S}_3: \max_{\boldsymbol{p}, \{\alpha_m\}_{m=1}^{M}, u, v} f = \log_2 F_{\mathrm{WP}} = \beta u + (1-\beta) v \tag{39A}$$

$$\text{s.t.} f_m^-(\boldsymbol{p}) - \triangle g_{m,(\lambda)}^-(\boldsymbol{p}) \geq \alpha_m, \forall i \in \mathcal{U}_m, m, n \in \mathcal{M}, \tag{39B}$$

$$\log_2\left(\sum_{m=1}^{M} \alpha_m\right) - \triangle m_{m,(\lambda)}^-(\boldsymbol{p}) \geq u, \forall m \in \mathcal{M}, \tag{39C}$$

$$\log_2 \alpha_m - \triangle y_{m,(\lambda)}^-(\boldsymbol{p}) \geq v, \forall m \in \mathcal{M}, \tag{39D}$$

$$\sum_{m=1}^{M} \|\mathbf{w}_m\|_2^2 p_m \leq P, \tag{39E}$$

where $\boldsymbol{p}_{(\lambda)} \triangleq \{p_{m,(\lambda)}\}_{m=1}^{M}$

$$\triangle g_{m,(\lambda)}^-(\boldsymbol{p}) = g_m^-(\boldsymbol{p}_{(\lambda)}) + \sum_{a \neq m} \left(\frac{\partial g_m^-(\boldsymbol{p}_{(\lambda)})}{\partial p_a}\right)^T (p_a - p_{a,(\lambda)}), \tag{40A}$$

$$\triangle m_{m,(\lambda)}^-(\boldsymbol{p}) = m_m^-(\boldsymbol{p}_{(\lambda)}) + \sum_{m=1}^{M} \left(\frac{\partial m_m^-(\boldsymbol{p}_{(\lambda)})}{\partial p_m}\right)^T (p_m - p_{m,(\lambda)}), \tag{40B}$$

$$\triangle y_{m,(\lambda)}^-(\boldsymbol{p}) = y_m^-(\boldsymbol{p}_{(\lambda)}) + \left(\frac{\partial y_m^-(\boldsymbol{p}_{(\lambda)})}{\partial p_m}\right)^T (p_m - p_{m,(\lambda)}). \tag{40C}$$

In addition, the derivatives in Eq. 40 are given by

$$\frac{\partial g_m^-(\boldsymbol{p}_{(\lambda)})}{\partial p_a} = \frac{\mathbb{E}\left\{\left|\hat{\mathbf{w}}_a^H \mathbf{h}_i\right|^2\right\}}{\left(\sum_{n \neq m} \mathbb{E}\left\{\left|\hat{\mathbf{w}}_n^H \mathbf{h}_i\right|^2\right\} p_{n,(\lambda)} + N_0\right) \ln 2}, \tag{41A}$$

$$\frac{\partial m_m^-(\boldsymbol{p}_{(\lambda)})}{\partial p_m} = \frac{\xi_m \|\mathbf{w}_m\|_2^2}{\left(\sum_{m=1}^{M} \left(\xi_m \|\mathbf{w}_m\|_2^2 p_m + P_{0,m}\right)\right) \ln 2}, \tag{41B}$$

$$\frac{\partial y_m^-(\boldsymbol{p}_{(\lambda)})}{\partial p_m} = \frac{\xi_m \|\mathbf{w}_m\|_2^2}{\left(\xi_m \|\mathbf{w}_m\|_2^2 p_m + P_{0,m}\right) \ln 2}. \tag{41C}$$

We address problem $\mathcal{S}_3$ using the above procedure iteratively and get $\{p_m\}_{m=1}^{M}$, thereby obtaining the corresponding candidate beamformer vectors $\mathbf{w}_m^\star = \sqrt{p_m} \hat{\mathbf{w}}_m, \forall m$. Among all feasible candidates $\mathbf{w}_m^\star$, the value that maximizes the objective function is selected as the final beamforming vector for group $\mathcal{U}_m$. In summary, the whole approach for the considered problem is described in Algorithm 1.

Algorithm 1 Robust Beamforming for TEE-MEE Trade-off

Input: Initial feasible beamformer correlation matrices $\left\{\mathbf{W}_m^{(0)}\right\}_{m=1}^{M}$.

Output: The beamformer vectors $\{\mathbf{w}_m^*\}_{m=1}^{M}$.

Initialization: iteration threshold ϵ and index $\lambda = 0$.
repeat
    Handling the convex optimization problem $\mathcal{F}_6$ with $\mathbf{W}_m^{(\lambda)}$ to obtain $\mathbf{W}_m^{\mathrm{temp}}, \alpha_m^{\mathrm{temp}}, u^{\mathrm{temp}}$ and v^{temp}.
    Calculate $f^{(\lambda+1)} = \beta u^{\mathrm{temp}} + (1-\beta) v^{\mathrm{temp}}$.
    Update $\left\{\mathbf{W}_m^{(\lambda+1)}\right\}_{m=1}^{M} = \{\mathbf{W}_m^{\mathrm{temp}}\}_{m=1}^{M}$, and set $\lambda = \lambda + 1$.
until $|f^{(\lambda)} - f^{(\lambda-1)}| \leq \epsilon$.
Set $\{\mathbf{W}_m^*\}_{m=1}^{M} = \left\{\mathbf{W}_m^{(\lambda)}\right\}_{m=1}^{M}$.
Determine whether $\{\mathbf{W}_m^*\}_{m=1}^{M}$ are all unit-rank or not. If the ranks equal to one for all groups, adopt the eigenvalue decomposition to acquire the beamformers $\{\mathbf{w}_m^*\}_{m=1}^{M}$ from $\{\mathbf{W}_m^*\}_{m=1}^{M}$. Otherwise, apply the Gaussian randomization approach.

Results and Discussion

In this section, we discuss the performance of the proposed robust algorithm in multigroup multicast satellite communication systems through numerical simulations. It is assumed that the variances of channel phase errors are the same for different users as $\sigma_i^2 = \sigma^2$. The basic power consumptions and inefficiency of the power amplifier are the same for different groups, respectively, which are expressed as $\xi_m = \xi$, $P_{0,m} = P_0$. The number of users in group $\mathcal{U}_m$ is set to be $N_m = 5, \forall m$, and the number of beams N_t is set to be 7. The simulation results are based on 10^6 channel realizations. Table 3 lists the values for the simulation parameters [27,29].

In Fig. 2, the approximate rate in (22) is compared with the exact ergodic rate in (12) for the different parameter settings. One can find that under different power and channel phase error variances, the exact average rate of the Monte Carlo-based simulation and approximation rate of our deductions match closely, which indicates the validness of our derivations. Figure 3 depicts the convergence curves of Algorithm 1 for some typical system settings. It can be observed that the objective function has a fast convergence speed and usually converges within about 5 iterations for the given simulation parameters. Simulation results also indicate that using more antennas can improve the EE performance. This is because with the increase of the number of antennas, the beam becomes narrower and the interference between beams decreases. Then, the better EE performance can be obtained.

The TEE-MEE tradeoff curves attained by Algorithm 1 under different priority weights are demonstrated in Fig. 4. We can observe that all the TEE-MEE points of the proposed approach lie on or above the line where TEE = MEE as $\mathrm{EE_{tot}} \geq \min_{1 \leq m \leq M} \mathrm{EE}_m$. In addition, we adopt the baseline approach in which the obsolete CSI is directly utilized, known as the conventional approach. The estimated channel is used as the actual channel without taking into account the channel uncertainty of the user in the conventional approach [27]. It can be observed that the proposed robust beamforming design approach outperforms the conventional one.

Table 3. Simulation parameters.

Parameter	Value
Orbit altitude	$d_0 = 3.6 \times 10^4$ (km)
Boltzmann's constant	$\kappa = 1.38 \times 10^{-23}$ (Joule/K)
Carrier frequency	$f = 20$ (GHz)
Hexagonal beam length	250 (km)
Satellite antenna gain	$G_T^j = 38$ (dBi)
Noise bandwidth	$B = 50$ (MHz)
Receiver gain to noise temperature	$G_{r,i}/T = 15$ (dB/K)
Rain fading variance	1.63 (dB)
Rain fading mean	−2.6 (dB)
3-dB angle	$\theta_{3dB} = 0.4^\circ$
Power amplifier inefficiency	$\xi = 2$
Per-group basic power	$P_0 = 40$ (dBm)

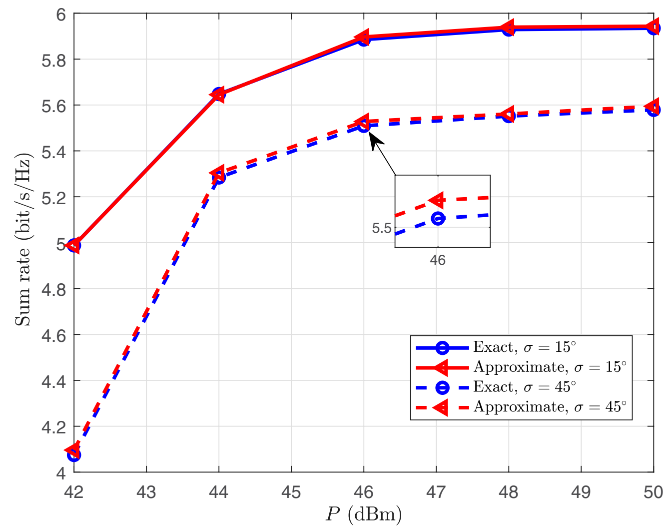

Fig. 2. The exact average rate in (12) versus the approximate rate in (23) for different setup parameters.

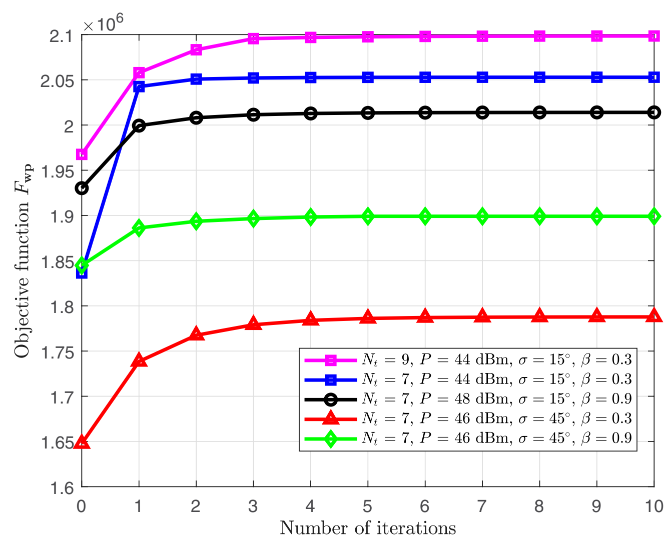

Fig. 3. Convergence trajectories of Algorithm 1 versus the number of iterations for different system setup parameters.

Figure 5 compares the influence of the weighting factor β on the fairness of the power allocation under different power constraints. In terms of fairness evaluation of different users' EEs, we adopt Jain's fairness index (JFI) given by $\mathcal{J} = \frac{\left(\sum_{m=1}^{M} \mathrm{EE}_m\right)^2}{N_t \sum_{m=1}^{M} \mathrm{EE}_m^2}$ with $1 \leq \mathcal{J} \leq 1$ [42]. Generally speaking, the closer JFI value is to 1, the fairer the power is allocated across users. Numerical results show that JFI decreases with the increase of the weighting factor β. When $\beta < 0.7$, JFI = 1, which indicates that the fairness of the system EE is guaranteed.

In Fig. 6, we compare the performance of the proposed robust beamforming design approach with the conventional one for different system setups in terms of the weighted product of TEE and MEE. Note that the conventional approach does

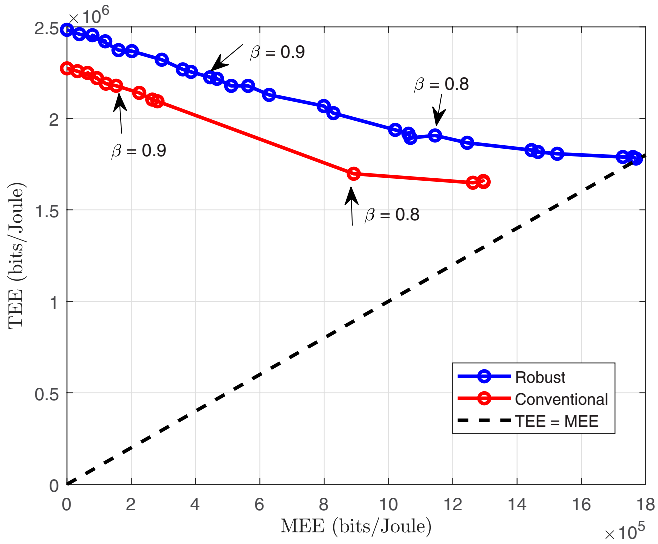

Fig. 4. TEE-MEE tradeoff curves of the proposed and conventional approaches with $P = 46$ dBm, $\sigma = 45$.

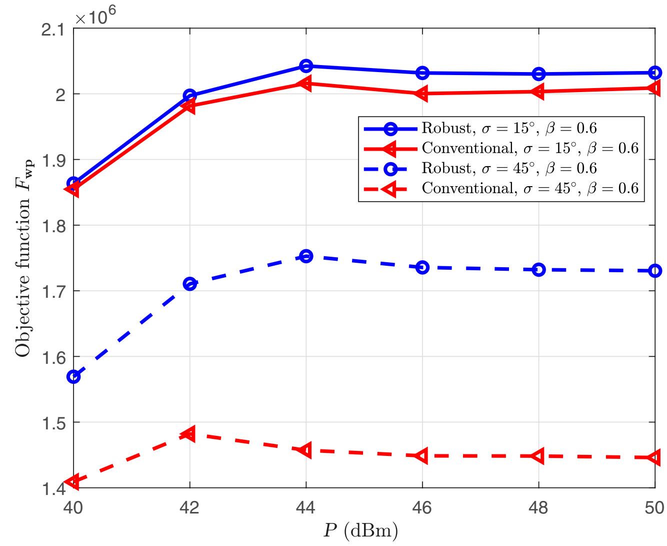

Fig. 6. Comparison between the proposed and conventional approaches in terms of the weighted product of TEE and MEE.

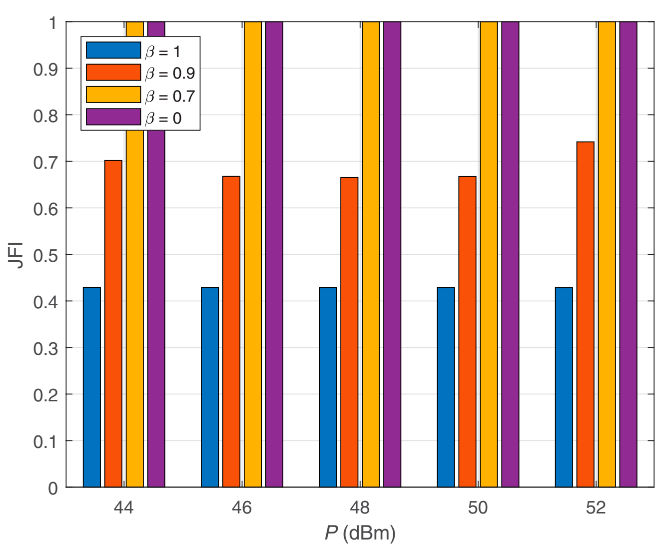

Fig. 5. JFI versus the power budget for different priority weights.

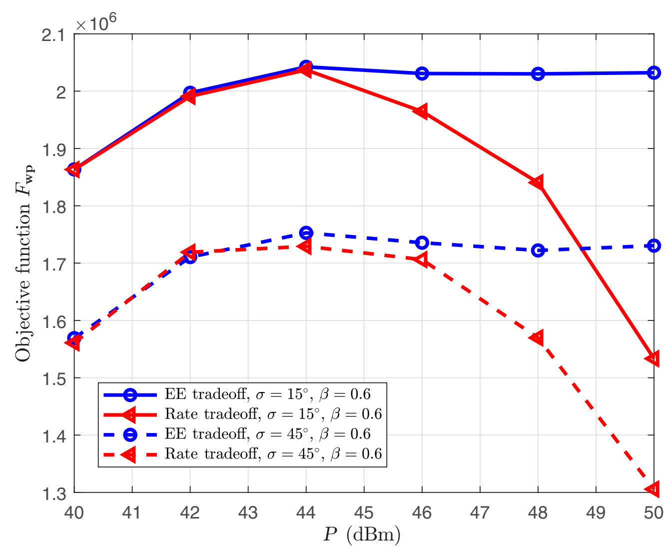

Fig. 7. Comparison of EE performance in terms of FWP of the EE tradeoff approach and the rate tradeoff approach with $\beta = 0.6$.

not consider the uncertainty of the channel phase errors. The numerical results indicate that the proposed robust approach can effectively offset the impact of the channel phase errors and provide nonnegligible performance gains. In addition, as the channel phase error increases, the performance gain of the proposed robust beamforming design approach over the conventional one becomes more notable.

We compare the EE performance in terms of F_{WP} between the EE tradeoff and the rate tradeoff approaches in Fig. 7. It can be noted that the performance of F_{WP} is close for the 2 approaches in the lower power region, which indicates that transmission with all power budget is nearly energy efficient. In the large power region, the EE performance in terms of F_{WP} of the EE tradeoff approach outperforms that of the rate tradeoff approach. This is because there exists a threshold of transmit power for maximizing F_{WP}, and any excess power will reduce F_{WP}.

Conclusion

We studied the EE optimization for robust multigroup multicast satellite communication systems in this paper, aiming at the beamforming design that balances the TEE-MEE tradeoff. We used the approximate average rate with a closed form to convert the original optimization problem into an easier-to-handle form. The nonconvex problem was tackled by solving convex programs sequentially with the help of the SDR and the CCCP. Finally, we acquired the final beamforming vectors by adopting Gaussian randomization or the eigenvalue decomposition method. Numerical results illustrated that the proposed robust beamforming design approach could effectively balance the tradeoff between the TEE and MEE, realizing the overall consideration of the fairness and total system EE performance. In addition, the proposed robust algorithm could outperform the conventional baselines in terms of the EE performance.

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Data Availability

All data needed to evaluate the conclusions of the study are presented in the paper.

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