Distributed Proportional-Integral Fuzzy State Estimation Over Sensor Networks Under Energy-Constrained Denial-of-Service Attacks

Yezheng Wang, Zidong Wang, Lei Zou, Yun Chen, and Dong Yue

***Abstract*—This paper deals with the distributed proportional-integral state estimation problem for nonlinear systems over sensor networks, where a number of spatially distributed sensor nodes are utilized to collect the system information. The signal transmissions among different sensor nodes are realized via their individual channels subject to energy-constrained denial-of-service (EC-DoS) cyber-attacks launched by the adversaries whose aim is to block the node-wise communications. Such EC-DoS attacks are characterized by a sequence of attack starting time-instants and a sequence of attack durations. Based on the measurement outputs of each node, a novel distributed fuzzy proportional-integral estimator is proposed that reflects the topological information of the sensor networks. The estimation error dynamics is shown to be regulated by a switching system under certain assumptions on the frequency and the duration of the EC-DoS attacks. Then, by resorting to the average dwell time method, a unified framework is established to analyze the dynamical behaviors of the resultant estimation error system, and sufficient conditions are obtained to guarantee the stability as well as the weighted H_∞ performance of the estimation error dynamics. Finally, a numerical example is given to verify the effectiveness of the proposed estimation scheme.**

***Index Terms*—Fuzzy systems, distributed state estimation, cyber-attacks, proportional-integral state estimation, sensor networks.**

I. Introduction

A typical sensor network is composed of a large number of sensor nodes deployed in different regions of interest. The battery-powered sensor nodes are capable of sending data according to a given communication topology. Owing to their remarkable information sensing/processing nature, the sensor networks (SNs) have found successful applications in a wide range of areas such as environmental monitoring, traffic control and manufacturing automation [47]. As a fundamental issue in SNs, the distributed state estimation (SE) problem over SNs has received much research attention in the past decade, and many SE algorithms have been proposed in the literature according to specific performance requirements [4], [20], [26]–[28]. To mention a few, such distributed SE methods include the H_∞ SE schemes [15], [47], the Kalman filtering algorithms [9], [23], the set-membership filtering approaches [12], [21], and fuzzy SE strategies [37], [53].

Among the existing SE methods, the Takagi-Sugeno (T-S) fuzzy SE approach is particularly effective for general nonlinear systems with smooth nonlinearities. According to the knowledge of nonlinearities, the T-S fuzzy model can be established by combining several linear submodels connected via the so-called fuzzy membership functions. The T-S fuzzy model possesses concise structure and desired approximation capability for many complex nonlinear functions [39], [40], [42]. Such distinctive advantages make it convenient to design the desired state estimators and, as such, the fuzzy SE problems have recently received increasing research attention [11], [25], [34], [38], [46]. When it comes to SNs, some initial attempts have been made to apply the fuzzy SE technique to nonlinear systems, and some interesting distributed SE algorithms have been developed in the literature [29], [37], [52].

In the past few years, the proportional-integral observer (PIO) design problem has attracted particular research interest with a great many results reported, see e.g. [41] for descriptor systems, [48] for fuzzy systems, [55] for linear time-varying systems, [5] for systems with unknown inputs, and [6] for linear-parameter-varying systems. A typical PIO consists of three parts that work together to achieve the SE task: 1) a copy of the available part of the system to be estimated; 2) a proportional term that represents the current innovation; and 3) an integral term that stands for the historical innovation. With such a structure, both current and historical information of the output estimation error can be utilized to calculate state estimates.

Compared with the traditional proportional-type observer (Luenberger observer), the PIO enjoys more design degrees because of the utilization of the historical innovation, and this would help improving the robustness of the estimation performance. As such, the PIO design problems have been dealt with for many different kinds of linear/nonlinear systems.

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Yezheng Wang is with the College of Electrical Engineering and Automation, Shandong University of Science and Technology, Qingdao 266590, China. (Email: wangyz_sdust@163.com).

Zidong Wang is with the College of Electrical Engineering and Automation, Shandong University of Science and Technology, Qingdao 266590, China; and is also with the Department of Computer Science, Brunel University London, Uxbridge, Middlesex UB8 3PH, United Kingdom. (Email: Zidong.Wang@brunel.ac.uk).

Lei Zou is with the College of Information Science and Technology, Donghua University, Shanghai 201620, China; and is also with the Engineering Research Center of Digitalized Textile and Fashion Technology, Ministry of Education, Shanghai 201620, China. (Email: zouleicup@gmail.com).

Yun Chen is with the School of Automation, Hangzhou Dianzi University, Hangzhou 310018, China. (Email: yunchen@hdu.edu.cn).

Dong Yue is with the Institute of Advanced Technology, Nanjing University of Posts and Telecommunications, Nanjing 210023, China. (Email: medongy@vip.163.com).

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Nevertheless, little research attention has been paid to the distributed PIO design issues over sensor networks, where the network topology comes to play a vitally important role in developing the observer, and this results in substantial complexity in reflecting the spatial information acquired from the SNs. It is, therefore, the major motivation of this paper to look into the issue as how the topology structure influences the overall PIO performance.

In the SNs, the communication between sensor nodes is generally implemented via wireless communication network (WCN) because of its advantages of lengthening the communication distance, improving the communication flexibility and reducing the communication cost. On the other hand, the utilization of the WCN would also bring some new challenges, and one of the key issues in SNs is the security of network since a typically open WCN is vulnerable to cyber-attacks that can be classified into denial-of-service (DoS) attacks, deception attacks and replay attacks. Specifically speaking, the DoS attack can interdict the signal transmissions to cause data missing; the deception attack can maliciously replace the original data with the fake one; and the main idea of the replay attack is to replace the current information by the historical data. Undoubtedly, cyber-attacks pose a great threat leading to deterioration of the system performance or even the instability of the underlying system.

From the adversaries' perspective, it is quite easy to launch the DoS cyber-attacks by overwhelming the online services and rendering them unusable [43]. In the past decade, several mathematical models have been established to describe the DoS behaviors in the context of secure control/filtering problems. For instance, the DoS attack has been modeled in [1] by the Bernoulli-type packet-drop model. In [3], the cyber-attack has been considered in the controller-to-actuator channel, where the DoS attack sequence has been modeled by the Markov process. In most relevant literature, the DoS attacks have been assumed to be randomly occurring with a priori knowledge about the probability distributions. Such an assumption is, unfortunately, sometimes restrictive as the adversaries could arbitrarily launch an attack without having to follow a certain statistical law.

In the seminal work [24], a novel model has been proposed to represent general DoS attacks via using the average dwell time method. In such a model, instead of strict assumptions on the statistical behaviors of the attacks, only some mild constraints have been posed on the attacks' frequency and duration. This kind of attack model is particularly suitable in describing the energy-constrained DoS (EC-DoS) attacks with two modes (sleeping mode for charging and active mode for attacking). Owing to the typically limited attacking energy budget, the EC-DoS attacks have recently received increasing research attention for various cyber-physical systems and a great number of results have appeared in the literature, see [8], [10], [22], [31], [33], [44], to mention just a few.

For the existing results concerning EC-DoS attacks, we have had the following observations: 1) most results have been obtained for centralized state estimation problems over *single* communication channel; 2) for the few results regarding distributed systems, there has been an implicit assumption that all channels are *simultaneously* attacked or not [14], [36], [45]; and 3) the investigated distributed systems under independent EC-DoS attacks have been continuous-time, and the corresponding results are therefore inapplicable to the digital communication scenario [16], [17]. In view of these observations, we conclude that the distributed SE problem has not received adequate attention yet for discrete-time nonlinear systems over SNs under independent EC-DoS attacks, not to mention the consideration of the PIO design. As such, our main motivation is to narrow such a gap.

Summarizing the discussions made thus far, it is of both theoretical importance and practical significance to investigate the distributed PIO design problem over SNs subject to independent EC-DoS attacks. In doing so, some underlying difficulties are identified as follows: 1) how to construct a proper distributed PIO whose structure takes into account both the topology of SNs and the effects induced by EC-DoS attacks? 2) how to analyze the dynamical behavior of the estimation errors under independent EC-DoS attacks? and 3) how to design observer gains to ensure the convergence and the weighted H_∞ performance of the estimation error dynamics? Corresponding to these difficulties, the main contributions of this paper are emphasized as follows: 1) the distributed proportional-integral SE issue is, for the first time, investigated for nonlinear systems over SNs based on the T-S fuzzy framework under the EC-DoS attacks; 2) a novel yet easy-to-implement fuzzy PIO is proposed to achieve SE tasks with desired performance index; and 3) the gain matrices of PIO are calculated via feasible computational algorithms.

The rest of this paper is organized as follows. Section II formulates the problem to be addressed after establishing the model of the considered plant, proposing adequate structure of the PIO, and describing the EC-DoS attacks and the desired SE purposes. Section III gives the main theoretical results, in addition to the discussion on the performance analysis and development of the algorithm for designing the PIO's gains. In Section IV, a numerical example is put forward to verify the effectiveness of the proposed SE scheme. Finally, in Section V, we draw the conclusion of this paper and give future research topics.

Notations: In this paper, $\mathbb{R}^n$ refers to the n-dimensional Euclidean space. $\mathbb{N}_+$ is the set of all positive integers and $\mathbb{N} \triangleq \mathbb{N}_+ \cup \{0\}$. The transposition, inverse and maximum (minimum) eigenvalue of a matrix A are denoted by A^T, A^{-1} and $\lambda_{\max}(A)$ ($\lambda_{\min}(A)$), respectively. $B = \mathrm{diag}\{b_{11}, b_{22}, \cdots, b_{nn}\}$ is used to represent a diagonal-block matrix. The symmetric parts in a symmetric matrix are denoted by an asterisk "$*$". I and 0 are used to represent, respectively, the identity matrix and zero matrix of proper dimensions. For two real matrices E and F that have same dimensions, their Hadamard product is represented by $E \circ F$. $\mathbf{1}_\varrho$ refers to a ϱ-dimension column vector with all elements being 1.

II. Problem Statement and Preliminaries

In this paper, we consider a sensor network having ϱ ($\varrho \in \mathbb{N}_+$) sensor nodes that are distributed in the space in terms of an interconnection topology characterized by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$. Here, $\mathcal{V} \triangleq \{1, 2, \cdots, \varrho\}$ is the set of sensor

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nodes; $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges; and $\mathcal{A} \triangleq [a_{n,j}]_{\varrho \times \varrho}$ is the nonnegative adjacency matrix associated with the edges of the graph, (i.e., $a_{n,j} > 0 \Longleftrightarrow$ edge $(n,j) \in \mathcal{E}$), which means that sensor node n can obtain the information from sensor node j, where $a_{n,n} = 0$ $(n \in \mathcal{V})$. If $(n,j) \in \mathcal{E}$, then node j is called one of the neighbors of node n. For all $n \in \mathcal{V}$, denote $\mathcal{N}_n \triangleq \{j \in \mathcal{V} | (n,j) \in \mathcal{E}\}$ as the set of the neighbors of node n.

A. Fuzzy Plants

Consider a kind of nonlinear dynamic systems described by the following T-S fuzzy systems:

Rule i of the plant: IF $\vartheta_1(k)$ is $W_{i,1}$, and $\vartheta_2(k)$ is $W_{i,2}$, and $\cdots$, and $\vartheta_\iota(k)$ is $W_{i,\iota}$, THEN

$$\begin{cases} x(k+1) = A_i x(k) + E_i \omega(k) \\ \quad z(k) = G_i x(k), \quad i \in \mathcal{T} \triangleq \{1, 2, \cdots, r\} \end{cases} \tag{1}$$

with ϱ sensor nodes modeled by

$$y_n(k) = C_n x(k) + F_n \omega(k), \quad n \in \mathcal{V} \tag{2}$$

where $x(k) \in \mathbb{R}^{n_x}$ is the system state; $z(k) \in \mathbb{R}^{n_z}$ is the signal to be estimated; $y_n(k) \in \mathbb{R}^{n_y}$ is the measurement output of the nth sensor node; $\omega(k) \in \mathbb{R}^{n_\omega}$ is the energy-bounded noises; $\vartheta(k) \triangleq \begin{bmatrix} \vartheta_1(k) & \vartheta_2(k) & \cdots & \vartheta_\iota(k) \end{bmatrix}^T$ is the measurable premise variable vector; $W_{i,1}, \cdots, W_{i,\iota}$ are the fuzzy sets; and A_i, E_i, G_i $(i \in \mathcal{T})$, C_n, F_n $(n \in \mathcal{V})$ are real constant matrices of appropriate dimensions.

By using the standard fuzzy inference approach, system (1)-(2) can be rewritten in the following global form:

$$\begin{cases} x(k+1) = \sum_{i=1}^{r} h_i(\vartheta(k)) \Big(A_i x(k) + E_i \omega(k) \Big) \\ \quad y_n(k) = C_n x(k) + F_n \omega(k), \quad n \in \mathcal{V}, \\ \quad\quad z(k) = \sum_{i=1}^{r} h_i(\vartheta(k)) G_i x(k) \end{cases} \tag{3}$$

where

$$h_i(\vartheta(k)) \triangleq \frac{\prod_{q=1}^{\iota} W_{i,q}(\vartheta_q(k))}{\sum_{i=1}^{r} \prod_{q=1}^{\iota} W_{i,q}(\vartheta_q(k))}$$

with $0 \leq W_{i,q}(\vartheta_q(k)) \leq 1$ being the membership grade of $\vartheta_q(k)$ in $W_{i,q}$. For $\forall k \in \mathbb{N}$, we have that

$$h_i(\vartheta(k)) \geq 0, \quad i \in \mathcal{T}, \quad \sum_{i=1}^{r} h_i(\vartheta(k)) = 1. \tag{4}$$

B. Communication Network

In this paper, we consider the setting where the information transmission between each sensor node and its neighboring nodes is realized via the WCN consisting of several independent communication channels. The WCN is subject to EC-DoS attacks launched by adversaries who aim at degrading the system performance through maliciously blocking the transmission channels. Due to the independent channels utilized in SNs, the communication channels (from individual sensor nodes to their neighboring nodes) may be compromised by different adversaries.

For the communication channel from sensor node n to its neighboring node j ($(n,j) \in \mathcal{E}$), we let $\{\bar{q}_{n,j}^{(s)}\}_{s=1,2,\cdots}$ and $\{\tau_{n,j}^{(s)}\}_{s=1,2,\cdots}$ be, respectively, the sequence of the starting time-instants of the EC-DoS attack and the sequence of attack durations. Here, $\bar{q}_{n,j}^{(s)}$ and $\tau_{n,j}^{(s)}$ denote, respectively, the sth starting time-instant of the EC-DoS attack and the sth EC-DoS attack duration. Then, the set of the sth EC-DoS attack time-instants can be defined by

$$\mathbb{Q}_{n,j}^{(s)} \triangleq \{\bar{q}_{n,j}^{(s)}, \bar{q}_{n,j}^{(s)} + 1, \cdots, \bar{q}_{n,j}^{(s)} + \tau_{n,j}^{(s)} - 1\}.$$

For any time-instant $k_a < k_b$, let

$$\Phi_{n,j}(k_a, k_b) \triangleq \bigcup_{s \in \mathbb{N}_+} \mathbb{Q}_{n,j}^{(s)} \bigcap \{k_a, k_a + 1, \cdots, k_b\}$$

denote the set of the time-instants on the interval $[k_a, k_b]$ during which the communication channel from sensor node n to node j is subject to EC-DoS attacks.

Different from the traditional single channel with two attack modes (under DoS or not) only, the independent channels utilized in SNs would lead to more complicated attack modes since the attacks on each edge are independent. For this purpose, we define

$$\Gamma(k) \triangleq \{(n,j) \,|(n,j) \in \mathcal{E}, k \in \Phi_{n,j}(0, \infty)\} \tag{5}$$

as the set of channels which are under attack at time-instant k. Obviously, $\Gamma(k) \subseteq \mathcal{E}$ and $\Gamma(k)$ has $2^{|\mathcal{E}|}$ possible constructions, where $|\mathcal{E}|$ denotes the number of elements in edge set $\mathcal{E}$. To be specific, $\Gamma(k) = \varnothing$ means that all channels are safe; $\Gamma(k) = \mathcal{E}$ implies that all channels are under attack; and $\Gamma(k) \subsetneq \mathcal{E}$ with $\Gamma(k) \neq \varnothing$ indicates that partial channels are under attack. For example, assume that a sensor network has three nodes with the communication topology given in Fig. 1 (i.e., $\mathcal{E} = \{(1,3), (2,1), (3,2)\}$), and then $\Gamma(k)$ has $2^3 = 8$ possible constructions listed as follows:

$$\begin{aligned} \Gamma(k) =& \varnothing, \ \Gamma(k) = \{(1,3)\}, \ \Gamma(k) = \{(2,1)\}, \\ \Gamma(k) =& \{(3,2)\}, \ \Gamma(k) = \{(1,3), (2,1)\}, \\ \Gamma(k) =& \{(1,3), (3,2)\}, \ \Gamma(k) = \{(2,1), (3,2)\}, \\ \Gamma(k) =& \{(1,3), (2,1), (3,2)\}. \end{aligned}$$

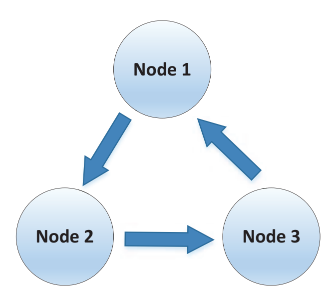

Fig. 1: Topological structure of the sensor network

To facilitate the subsequent estimator design, some rather standard assumptions are adopted from [24] on the EC-DoS frequency and its duration.

Assumption 1: (EC-DoS attack frequency) For any $0 \leq k_a < k_b$, there exist scalars $\varpi_{n,j} > 0$ and $\theta_{n,j} > 1$, $((n,j) \in \mathcal{E})$ such that

$$|\pi_{n,j}(k_a, k_b)| \leq \varpi_{n,j} + \frac{k_b - k_a}{\theta_{n,j}} \tag{6}$$

where $\pi_{n,j}(k_a, k_b) \triangleq \bigcup_{s\in\mathbb{N}_+}\{\bar{q}_{n,j}^{(s)}\}\bigcap\{k_a, \cdots, k_b\}$ is the starting time-instants set of the EC-DoS attack in the channel from node n to node j during the time interval $[k_a, k_b]$, and $|\pi_{n,j}(k_a, k_b)| \in \mathbb{N}$ denotes the total number of the starting time-instants of the EC-DoS attack during the time interval $[k_a, k_b]$.

Assumption 2: (EC-DoS attack duration) For any $0 \leq k_a < k_b$, there exist scalars $m_{n,j} > 0$ and $g_{n,j} > 1$ $((n,j) \in \mathcal{E})$ such that

$$|\Phi_{n,j}(k_a, k_b)| \leq m_{n,j} + \frac{k_b - k_a}{g_{n,j}} \tag{7}$$

where $|\Phi_{n,j}(k_a, k_b)| \in \mathbb{N}$ is the total EC-DoS attack duration in the channel from node n to node j during the time interval $[k_a, k_b]$.

C. Proportional-Integral Observer

By taking fully into account the underlying EC-DoS attacks, we construct the following distributed fuzzy PIO for sensor node n $(n \in \mathcal{V})$:

$$\begin{cases} \hat{x}_n(k+1) = \sum_{v=1}^{r} h_v(\hat{\vartheta}(k))\Big(A_v\hat{x}_n(k) + M_{n,v}\zeta_n(k) \\ \qquad + K_{n,v}\big(y_n(k) - C_n\hat{x}_n(k)\big) \\ \qquad + L_{n,v}\sum_{j\in\mathcal{N}_n}\psi_{n,j}(\Gamma(k)) \\ \qquad \times a_{n,j}\big(y_j(k) - C_j\hat{x}_j(k)\big) \\ \qquad + N_{n,v}\sum_{j\in\mathcal{N}_n}\psi_{n,j}(\Gamma(k))a_{n,j}\zeta_j(k)\Big) \\ \zeta_n(k+1) = S_n\zeta_n(k) + T_n\big(y_n(k) - C_n\hat{x}_n(k)\big) \\ \hat{z}_n(k) = \sum_{v=1}^{r} h_v(\hat{\vartheta}(k))G_v\hat{x}_n(k) \end{cases} \tag{8}$$

where

$$\psi_{n,j}(\Gamma(k)) \triangleq \begin{cases} 0, & \text{if } (n,j) \in \Gamma(k) \\ 1, & \text{otherwise;} \end{cases}$$

$\hat{x}_n(k)$ and $\hat{z}_n(k)$ are, respectively, the estimates of $x(k)$ and $z(k)$ based on the information of node n and its neighboring nodes; $\hat{\vartheta}(k)$ is the premise vector of the observer; and $K_{n,v}$, $L_{n,v}$, $M_{n,v}$, $N_{n,v}$, S_n and T_n are estimator gains to be designed.

Remark 1: The proposed distributed fuzzy PIO, which is inspired by the seminal work [41], can be regarded as a kind of generalized PIO. The main features of the proposed observer are highlighted as fourfold: 1) the information of the neighboring nodes is utilized by considering the topological structure among sensor nodes; 2) the past information with the designed weight is used due to the introduction of the integral term (accumulated-sum term $\zeta_n(k)$); 3) the time-varying scalars $\psi_{n,j}(\Gamma(k))$ are employed to reflect the effects caused by the independent EC-DoS attacks; and 4) considering the large-scale of the SNs, the past output estimation error obtained via sensor node n is only stored in the corresponding observer node n to ease storing burden. The improved structure of our proposed distributed fuzzy PIO enables us to make simultaneous use of the current, the historical and the innovation from neighboring nodes in a unified framework, thereby enhancing the robustness of the desired observer.

By defining $e_n(k) \triangleq x(k) - \hat{x}_n(k)$ and $\tilde{z}_n(k) \triangleq z(k) - \hat{z}_n(k)$, respectively, as the state estimation error and the output estimation error for the node n, we obtain from (3) and (8) that

$$\begin{cases} e_n(k+1) = \sum_{i=1}^{r}\sum_{v=1}^{r} h_i(\vartheta(k))h_v(\hat{\vartheta}(k))\Bigg(\big(A_i - A_v\big)x(k) \\ \qquad + \big(A_v - K_{n,v}C_n\big)e_n(k) \\ \qquad - L_{n,v}\sum_{j\in\mathcal{N}_n} a_{n,j}\psi_{n,j}(\Gamma(k))C_je_j(k) - M_{n,v} \\ \qquad \times \zeta_n(k) - N_{n,v}\sum_{j\in\mathcal{N}_n} a_{n,j}\psi_{n,j}(\Gamma(k))\zeta_j(k) \\ \qquad + \Bigg(E_i - K_{n,v}F_n - L_{n,v}\sum_{j\in\mathcal{N}_n} a_{n,j} \\ \qquad \times \psi_{n,j}(\Gamma(k))F_j\Bigg)\omega(k)\Bigg) \\ \zeta_n(k+1) = S_n\zeta_n(k) + T_n\big(C_ne_n(k) + F_n\omega(k)\big) \\ \tilde{z}_n(k) = \sum_{i=1}^{r}\sum_{v=1}^{r} h_i(\vartheta(k))h_v(\hat{\vartheta}(k))\Big(\big(G_i - G_v\big)x(k) \\ \qquad + G_ve_n(k)\Big). \end{cases} \tag{9}$$

By using the matrix-augmentation approach, the estimation error dynamics (9) can be rewritten as the following compact form:

$$\begin{cases} \eta(k+1) = \sum_{i=1}^{r}\sum_{v=1}^{r} h_i(\vartheta(k))h_v(\hat{\vartheta}(k)) \\ \qquad \times \Big(\big(\bar{A}_{i,v} + \bar{B}_v(\Gamma(k))\big)\eta(k) \\ \qquad + \big(\bar{E}_{i,v} + \bar{F}_v(\Gamma(k))\big)\omega(k)\Big) \\ \tilde{z}(k) = \sum_{i=1}^{r}\sum_{v=1}^{r} h_i(\vartheta(k))h_v(\hat{\vartheta}(k))\tilde{G}_{i,v}\eta(k) \end{cases} \tag{10}$$

where

$$\eta(k) \triangleq \begin{bmatrix} e^T(k) & x^T(k) & \zeta^T(k) \end{bmatrix}^T,$$
$$e(k) \triangleq \begin{bmatrix} e_1^T(k) & e_2^T(k) & \cdots & e_\varrho^T(k) \end{bmatrix}^T,$$
$$\zeta(k) \triangleq \begin{bmatrix} \zeta_1^T(k) & \zeta_2^T(k) & \cdots & \zeta_\varrho^T(k) \end{bmatrix}^T,$$
$$\tilde{z}(k) \triangleq \mathbf{1}_\varrho \otimes z(k) - \hat{z}(k),$$
$$\hat{z}(k) \triangleq \begin{bmatrix} \hat{z}_1^T(k) & \hat{z}_2^T(k) & \cdots & \hat{z}_\varrho^T(k) \end{bmatrix}^T,$$
$$\bar{A}_{i,v} \triangleq \begin{bmatrix} \Lambda_v^{(1,1)} & \mathbf{1}_\varrho \otimes \big(A_i - A_v\big) & \Lambda_v^{(1,3)} \\ 0 & A_i & 0 \\ \bar{T}\bar{C} & 0 & \bar{S} \end{bmatrix},$$
$$\Lambda_v^{(1,1)} \triangleq \mathrm{diag}\{\hat{A}_{v,1}, \hat{A}_{v,2}, \cdots, \hat{A}_{v,\varrho}\},$$

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$$
\begin{aligned}
\hat{A}_{v,n} &\triangleq A_v - K_{n,v}C_n,\\
\Lambda_v^{(1,3)} &\triangleq \operatorname{diag}\{-M_{1,v}, -M_{2,v}, \cdots, -M_{\varrho,v}\},\\
\bar{T} &\triangleq \operatorname{diag}\{T_1, T_2, \cdots, T_\varrho\},\\
\bar{S} &\triangleq \operatorname{diag}\{S_1, S_2, \cdots, S_\varrho\},\\
\bar{C} &\triangleq \operatorname{diag}\{C_1, C_2, \cdots, C_\varrho\},\\
\bar{B}_v(\Gamma(k)) &\triangleq \begin{bmatrix} \hat{B}_v(\Gamma(k)) & 0 & \vec{B}_v(\Gamma(k)) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\\
\hat{B}_v(\Gamma(k)) &\triangleq -\bar{L}_v\big((\bar{\psi}(\Gamma(k)) \circ \mathcal{A}) \otimes I_{n_y}\big)\bar{C},\\
\bar{\psi}(\Gamma(k)) &\triangleq [\bar{\psi}_{n,j}(\Gamma(k))]_{n,j=1,2,\cdots,\varrho},\\
\bar{\psi}_{n,j}(\Gamma(k)) &\triangleq \begin{cases} \psi_{n,j}(\Gamma(k)), & \text{if } (n,j) \in \mathcal{E} \\ 0, & \text{otherwise}, \end{cases}\\
\vec{B}_v(\Gamma(k)) &\triangleq -\bar{N}_v\big((\bar{\psi}(\Gamma(k)) \circ \mathcal{A}) \otimes I_{n_y}\big),\\
\bar{N}_v &\triangleq \operatorname{diag}\{\tilde{N}_{1,v}, \tilde{N}_{2,v}, \cdots, \tilde{N}_{\varrho,v}\},\\
\bar{L}_v &\triangleq \operatorname{diag}\{\tilde{L}_{1,v}, \tilde{L}_{2,v}, \cdots, \tilde{L}_{\varrho,v}\},\\
\tilde{N}_{n,v} &\triangleq \begin{cases} 0, & \text{if } (n,j) \notin \mathcal{E} \text{ for } \forall j \in \mathcal{V}, \\ N_{n,v}, & \text{otherwise}, \end{cases}\\
\tilde{L}_{n,v} &\triangleq \begin{cases} 0, & \text{if } (n,j) \notin \mathcal{E} \text{ for } \forall j \in \mathcal{V}, \\ L_{n,v}, & \text{otherwise}, \end{cases}\\
\bar{K}_v &\triangleq \operatorname{diag}\{K_{1,v}, K_{2,v}, \cdots, K_{\varrho,v}\},\\
\tilde{G}_{i,v} &\triangleq \begin{bmatrix} \bar{G}_v & \mathbf{1}_\varrho \otimes (G_i - G_v) & 0 \end{bmatrix},\\
\bar{G}_v &\triangleq \operatorname{diag}\{\underbrace{G_v, G_v, \cdots, G_v}_{\varrho}\},\\
\bar{E}_{i,v} &\triangleq \begin{bmatrix} \mathbf{1}_\varrho \otimes E_i - \bar{K}_v\bar{F} \\ E_i \\ \bar{T}\bar{F} \end{bmatrix},\\
\bar{F} &\triangleq \begin{bmatrix} F_1^T, F_2^T, \cdots, F_\varrho^T \end{bmatrix}^T,\\
\bar{F}_v(\Gamma(k)) &\triangleq \begin{bmatrix} -\bar{L}_v\big((\bar{\psi}(\Gamma(k)) \circ \mathcal{A}) \otimes I_{n_y}\big)\bar{F} \\ 0 \\ 0 \end{bmatrix}.
\end{aligned}
$$

The objective of this paper is to investigate the H_∞ distributed PIO design problem over SNs subject to independent EC-DoS attacks such that the following two requirements are satisfied simultaneously:

R1) for $\omega(k) = 0$, the estimation error system (10) is asymptotically stable, i.e., $\lim_{k\to\infty} \eta(k) \to 0$; and

R2) for all nonzero $\omega(k) \in l_2[0,\infty)$ and under zero initial condition, the estimation error $\tilde{z}(k)$ satisfies the following weighted H_∞ performance constraint [7]:

$$
\sum_{k=0}^{\infty} \sigma^k \tilde{z}^T(k)\tilde{z}(k) \le \bar{\gamma}^2 \sum_{k=0}^{\infty} \omega^T(k)\omega(k) \tag{11}
$$

where $0 < \sigma < 1$ and $\bar{\gamma} > 0$ are two scalars.

Remark 2: The weighted H_∞ performance constraint (11), which is also called the exponential H_∞ performance index due to the introduction of the term σ^k, is often used to deal with the disturbance attenuation problems for systems subject to average-dwell-time-related constraints and energy-bounded noises [7], [32], [35]. Note that, if $\sigma \to 1$, then the evaluated performance index reduces to the normal H_∞ performance index over the entire time domain.

Before proceeding further, let define some auxiliary variables for later convenience in deriving our main results. Firstly, we rewrite the edge set $\mathcal{E}$ by arranging its elements according to any-but-fixed order:

$$
\mathcal{E} = \{\mathfrak{e}_1, \mathfrak{e}_2, \cdots, \mathfrak{e}_\varsigma\}
$$

where the term $\mathfrak{e}_{\bar{s}}$ ($\bar{s} = 1, 2, \cdots, \varsigma$) represents the $\bar{s}$th element in $\mathcal{E}$ and $\varsigma \triangleq |\mathcal{E}| \in \mathbb{N}$ refers to the total number of elements in $\mathcal{E}$. Then, we define

$$
\Psi(\Gamma(k)) \triangleq \left(\sum_{\bar{s}=1}^{\varsigma} 2^{\bar{s}-1} c_{\bar{s}} + 1\right) \in \mathcal{D}
$$

where, for $\bar{s} = 1, 2, \cdots, \varsigma$,

$$
c_{\bar{s}} \triangleq \begin{cases} 0, & \text{if } \mathfrak{e}_{\bar{s}} \in \Gamma(k) \\ 1, & \text{otherwise,} \end{cases}
$$

and

$$
\mathcal{D} \triangleq \{1, 2, \cdots, 2^\varsigma\}. \tag{12}
$$

Note that, through the above definitions, a relation is established between the attack construction $\Gamma(k)$ and a newly introduced positive integer $\Psi(\Gamma(k))$ which would be helpful in representing mode-$\Gamma(k)$-dependent variables to be defined later.

III. Main Results

In this section, our purpose is to deal with the performance analysis and observer design issues under the independent EC-DoS attacks.

In the following theorem, sufficient conditions are given that guarantee the desired performance requirements of the estimation error system (10).

Theorem 1: Consider the fuzzy system (3) and the distributed fuzzy PIO (8). Let the observer gains and scalars $0 < \beta_{\Psi(\Gamma)} < 1$, $\mu > 1$, $\gamma > 0$, $0 < \sigma < 1$ be given. Then, the estimation error system (10) is asymptotically stable (when $\omega(k) = 0$) and satisfies the weighted H_∞ performance constraint if, for $\forall \Gamma \subseteq \mathcal{E}$, $(n,j) \in \mathcal{E}$, $\Psi(\Gamma) \in \mathcal{D}$ and $i, v \in \mathcal{T}$, $n \in \mathcal{V}$, there are matrices $P_{n,\Psi(\Gamma)} > 0$, $Q_{\Psi(\Gamma)} > 0$, $R_{n,\Psi(\Gamma)} > 0$, scalars $\kappa_{n,j}$ and $\varepsilon_{n,j}$ such that

$$
\tilde{A}_{i,v}^T(\Gamma) P_{\Psi(\Gamma)} \tilde{A}_{i,v}(\Gamma) + \vec{P}_{i,v,\Psi(\Gamma)} < 0 \tag{13}
$$

$$
P_\nu - \mu P_\epsilon < 0, \quad \nu, \epsilon \in \mathcal{D}, \quad \nu \ne \epsilon \tag{14}
$$

$$
\begin{aligned}
&\ln(1 - \beta_{\Psi(\Gamma)}) + \sum_{(n,j)\in\mathcal{E}} \frac{2\ln\mu}{\theta_{n,j}} \\
&\quad < \sum_{(n,j)\in\Gamma} \kappa_{n,j} + \sum_{(n,j)\notin\Gamma} \varepsilon_{n,j}
\end{aligned} \tag{15}
$$

$$
\kappa_{n,j} - \varepsilon_{n,j} \ge 0 \tag{16}
$$

$$
\sum_{(n,j)\in\mathcal{E}} \left(\frac{\kappa_{n,j} - \varepsilon_{n,j}}{g_{n,j}} + \varepsilon_{n,j}\right) < 0 \tag{17}
$$

$$
\sigma < \mu^{-\bar{\theta}} \tag{18}
$$

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where $\bar{\theta} \triangleq \sum_{(n,j)\in\mathcal{E}} \frac{2}{\theta_{n,j}}$ and

$$\begin{aligned}
\tilde{A}_{i,v}(\Gamma) &\triangleq \begin{bmatrix}\bar{A}_{i,v} + \bar{B}_v(\Gamma) & \bar{E}_{i,v} + \bar{F}_v(\Gamma)\end{bmatrix},\\
\vec{P}_{i,v,\Psi(\Gamma)} &\triangleq \begin{bmatrix}-(1-\beta_{\Psi(\Gamma)})P_{\Psi(\Gamma)} + \tilde{G}_{i,v}^T\tilde{G}_{i,v} & 0\\ 0 & -\gamma^2 I\end{bmatrix},\\
P_{\Psi(\Gamma)} &\triangleq \mathrm{diag}\{\tilde{P}_{\Psi(\Gamma)}, Q_{\Psi(\Gamma)}, \tilde{R}_{\Psi(\Gamma)}\},\\
\tilde{P}_{\Psi(\Gamma)} &\triangleq \mathrm{diag}\{P_{1,\Psi(\Gamma)}, P_{2,\Psi(\Gamma)}, \cdots, P_{\varrho,\Psi(\Gamma)}\},\\
\tilde{R}_{\Psi(\Gamma)} &\triangleq \mathrm{diag}\{R_{1,\Psi(\Gamma)}, R_{2,\Psi(\Gamma)}, \cdots, R_{\varrho,\Psi(\Gamma)}\}.
\end{aligned}$$

Proof: See Appendix A. ■

In Theorem 1, the system performance analysis has been conducted with the help of the switching-system-based theory, based on which the observer gains are designed in the following theorem.

Theorem 2: Consider the fuzzy system (3) and the distributed fuzzy PIO (8). Let the scalars $0 < \beta_{\Psi(\Gamma)} < 1$, $\mu > 1$, $\gamma > 0$ and $0 < \sigma < 1$ be given. Then, the estimation error system (10) is asymptotically stable (when $\omega(k) = 0$) and satisfies the weighted H_∞ performance constraint if, for $\forall\Gamma \subseteq \mathcal{E}$, $(n,j) \in \mathcal{E}$, $\Psi(\Gamma) \in \mathcal{D}$, $n \in \mathcal{V}$ and $i,\ v \in \mathcal{T}$, there are matrices $P_{n,\Psi(\Gamma)} > 0$, $Q_{\Psi(\Gamma)} > 0$, $R_{n,\Psi(\Gamma)} > 0$, $X_{n,v}^{(1)}$, $X_{n,v}^{(2)}$, $X_{n,v}^{(3)}$, $X_{n,v}^{(4)}$, $X_n^{(5)}$, $X_n^{(6)}$, nonsingular matrices $Y_v^{(1)}$, $Y_{\Psi(\Gamma)}^{(2)}$, $Y^{(3)}$, scalars $\kappa_{n,j}$ and $\varepsilon_{n,j}$ such that (14)-(18) and the following inequalities hold:

$$\begin{bmatrix}\Omega_{i,v,\Psi(\Gamma)}^{(1,1)} & *\\ \Omega_{i,v}^{(2,1)}(\Gamma) & \Omega_{v,\Psi(\Gamma)}^{(2,2)}\end{bmatrix} < 0 \tag{19}$$

where

$$\begin{aligned}
P_{\Psi(\Gamma)} &\triangleq \mathrm{diag}\{\tilde{P}_{\Psi(\Gamma)}, Q_{\Psi(\Gamma)}, \tilde{R}_{\Psi(\Gamma)}\},\\
\tilde{P}_{\Psi(\Gamma)} &\triangleq \mathrm{diag}\{P_{1,\Psi(\Gamma)}, P_{2,\Psi(\Gamma)}, \cdots, P_{\varrho,\Psi(\Gamma)}\},\\
\tilde{R}_{\Psi(\Gamma)} &\triangleq \mathrm{diag}\{R_{1,\Psi(\Gamma)}, R_{2,\Psi(\Gamma)}, \cdots, R_{\varrho,\Psi(\Gamma)}\},\\
Y_v^{(1)} &\triangleq \mathrm{diag}\{Y_{1,v}^{(1)}, Y_{2,v}^{(1)}, \cdots, Y_{\varrho,v}^{(1)}\},\\
Y_{\Psi(\Gamma)}^{(2)} &\triangleq \mathrm{diag}\{Y_{1,\Psi(\Gamma)}^{(2)}, Y_{2,\Psi(\Gamma)}^{(2)}, \cdots, Y_{\varrho,\Psi(\Gamma)}^{(2)}\},\\
Y^{(3)} &\triangleq \mathrm{diag}\{Y_1^{(3)}, Y_2^{(3)}, \cdots, Y_\varrho^{(3)}\},\\
\Omega_{i,v,\Psi(\Gamma)}^{(1,1)} &\triangleq \mathrm{diag}\big\{-(1-\beta_{\Psi(\Gamma)})\tilde{P}_{\Psi(\Gamma)}, -(1-\beta_{\Psi(\Gamma)})Q_{\Psi(\Gamma)},\\
&\quad -(1-\beta_{\Psi(\Gamma)})\tilde{R}_{\Psi(\Gamma)}, -\gamma^2 I\big\} + \mathrm{diag}\big\{\tilde{G}_{i,v}^T\tilde{G}_{i,v}, 0\big\},\\
\Omega_{v,\Psi(\Gamma)}^{(2,2)} &\triangleq \mathrm{diag}\Big\{\tilde{P}_{\Psi(\Gamma)} - Y_v^{(1)} - Y_v^{(1)T},\\
&\quad Q_{\Psi(\Gamma)} - Y_{\Psi(\Gamma)}^{(2)} - Y_{\Psi(\Gamma)}^{(2)T}, \tilde{R}_{\Psi(\Gamma)} - Y^{(3)} - Y^{(3)T}\Big\},\\
\Omega_{i,v}^{(2,1)}(\Gamma) &\triangleq \begin{bmatrix}\bar{\Omega}_v^{(1,1)}(\Gamma) & \bar{\Omega}_{i,v}^{(1,2)} & \bar{\Omega}_v^{(1,3)}(\Gamma) & \bar{\Omega}_{i,v}^{(1,4)}\\ 0 & Y_{\Psi(\Gamma)}^{(2)}A_i & 0 & Y_{\Psi(\Gamma)}^{(2)}E_i\\ \bar{\Omega}^{(3,1)} & 0 & \bar{\Omega}^{(3,3)} & \bar{\Omega}^{(3,4)}\end{bmatrix},\\
\bar{\Omega}_v^{(1,1)}(\Gamma) &\triangleq Y_v^{(1)}\tilde{A}_v - \sum_{s=1}^{\varrho}\bar{I}_s X_{s,v}^{(1)}\tilde{I}_s\bar{C}\\
&\quad - \sum_{s=1}^{\varrho}\bar{I}_s X_{s,v}^{(2)}\tilde{I}_s\big((\bar{\psi}(\Gamma)\circ\mathcal{A})\otimes I_{n_y}\big)\bar{C},\\
\bar{\Omega}_{i,v}^{(1,2)} &\triangleq Y_v^{(1)}\big(1_\varrho \otimes (A_i - A_v)\big),\\
\bar{\Omega}_v^{(1,3)}(\Gamma) &\triangleq -\sum_{s=1}^{\varrho}\bar{I}_s X_{s,v}^{(3)}\tilde{I}_s\\
&\quad - \sum_{s=1}^{\varrho}\bar{I}_s X_{s,v}^{(4)}\tilde{I}_s\big((\bar{\psi}(\Gamma)\circ\mathcal{A})\otimes I_{n_y}\big),\\
\bar{\Omega}^{(3,1)} &\triangleq \sum_{s=1}^{\varrho}\tilde{I}_s^T X_s^{(5)}\tilde{I}_s\bar{C}, \quad \bar{\Omega}^{(3,4)} \triangleq \sum_{s=1}^{\varrho}\tilde{I}_s^T X_s^{(5)}\tilde{I}_s\bar{F},\\
\bar{\Omega}^{(3,3)} &\triangleq \sum_{s=1}^{\varrho}\tilde{I}_s^T X_s^{(6)}\tilde{I}_s,\\
\bar{\Omega}_{i,v}^{(1,4)} &\triangleq Y_v^{(1)}\big(\mathbf{1}_\varrho \otimes E_i\big) - \sum_{s=1}^{\varrho}\bar{I}_s X_{s,v}^{(1)}\tilde{I}_s\bar{F}\\
&\quad - \sum_{s=1}^{\varrho}\bar{I}_s X_{s,v}^{(2)}\tilde{I}_s\big((\bar{\psi}(\Gamma)\circ\mathcal{A})\otimes I_{n_y}\big)\bar{F},\\
\bar{I}_s &\triangleq \big[0\ 0\ \underbrace{I_{n_x}}_{\text{the } 1\times s \text{ block}}\ 0\ \cdots\ 0\big]^T,\\
\tilde{I}_s &\triangleq \big[0\ 0\ \underbrace{I_{n_y}}_{\text{the } 1\times s \text{ block}}\ 0\ \cdots\ 0\big].
\end{aligned}$$

Furthermore, if the above inequalities are solvable, then the observer gains can be calculated by

$$\begin{aligned}
K_{n,v} &= (Y_{n,v}^{(1)})^{-1}X_{n,v}^{(1)}, \quad L_{n,v} = (Y_{n,v}^{(1)})^{-1}X_{n,v}^{(2)},\\
M_{n,v} &= (Y_{n,v}^{(1)})^{-1}X_{n,v}^{(3)}, \quad N_{n,v} = (Y_{n,v}^{(1)})^{-1}X_{n,v}^{(4)},\\
T_n &= (Y_n^{(3)})^{-1}X_n^{(5)}, \quad S_n = (Y_n^{(3)})^{-1}X_n^{(6)}.
\end{aligned}$$

Proof: See Appendix B. ■

Remark 3: So far, we have addressed the distributed fuzzy PIO design problems for nonlinear systems subject to the independent EC-DoS cyber-attacks. First, we have constructed a proper distributed PIO whose structure takes into account both the topology of SNs and the effects induced by EC-DoS attacks. In Theorem 1, we have analyzed the dynamical behavior of the estimation errors under independent EC-DoS attacks and, in Theorem 2, we have further designed observer gains to ensure the convergence and the weighted H_∞ performance of the estimation error dynamics. Note that, in the main results presented in Theorems 1 and 2, all the system parameters and the factors quantifying the effects from EC-DoS cyber-attacks have been adequately included.

Remark 4: Compared with the numerous existing literature about SNs and PIO, the distinctive novelties of our paper are highlighted as follows: 1) the addressed SE problem is new as the effects caused by the independent EC-DoS attacks are, for the first time, analyzed for T-S fuzzy systems over SNs; and 2) the proposed fuzzy PIO is new that exhibits distributed structure and improved flexibility. In addition, by utilizing similar design ideas, our proposed distributed SE algorithm can be easily extended to other large-scale systems such as multi-agent systems and complex dynamical networks.

IV. Simulation Example

In this section, a numerical example and some comparison results are given to show the effectiveness of the proposed SE scheme.

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Consider the following controlled fuzzy system with two fuzzy rules and five sensor nodes:

$$\begin{cases} x(k+1) = \sum_{i=1}^{2} h_i(\vartheta(k))\big(A_i x(k) + B_i u(k) + E_i \omega(k)\big) \\ y_n(k) = C_n x(k) + F_n \omega(k), \quad n = 1,2,3,4,5 \\ z(k) = \sum_{i=1}^{2} h_i(\vartheta(k)) G_i x(k) \end{cases} \tag{20}$$

where

$$A_1 = \begin{bmatrix} 0.1 & 0.4 & 0 \\ 0.2 & 0.8 & 0.1 \\ 0.1 & 0.2 & 0.1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.1 & 0 & 0.1 \\ 0.2 & 0.5 & 0 \\ 0.2 & 0.1 & 0.5 \end{bmatrix},$$
$$E_1 = \begin{bmatrix} 0.1 \\ -0.1 \\ 0.1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} -0.1 \\ 0.1 \\ 0.1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0.9 & 0.7 & 0.5 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} 0.7 & 0.5 & 0.5 \end{bmatrix}, \quad C_3 = \begin{bmatrix} 0.8 & 0.1 & 0.1 \end{bmatrix},$$
$$C_4 = \begin{bmatrix} 0.5 & 0.7 & 0.5 \end{bmatrix}, \quad C_5 = \begin{bmatrix} 0.5 & 0.7 & 0.5 \end{bmatrix},$$
$$F_1 = 0.11, \ F_2 = 0.1, \ F_3 = 0.12,$$
$$G_1 = \begin{bmatrix} 0.2 & 0.2 & 0.1 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0.2 & 0.2 & 0.1 \end{bmatrix},$$
$$u(k) = \begin{bmatrix} 0.321 \\ 0.1 \\ 0.2 \end{bmatrix} y_1(k), \quad h_1(\vartheta(k)) = 1 - \sin^2(x^{(1)}(k)),$$
$$x(k) \triangleq \begin{bmatrix} x^{(1)}(k) \\ x^{(2)}(k) \\ x^{(3)}(k) \end{bmatrix}, \quad h_2(\vartheta(k)) = 1 - h_1(\vartheta(k)),$$
$$F_4 = 0.14, \quad F_5 = 0.2, \quad B_1 = B_2 = \mathrm{diag}\{1,1,1\}.$$

In this example, a sensor network with five sensor nodes is utilized to collect data of the fuzzy system (20). The information exchange among these nodes is conducted based on a fixed communication topology given in Fig. 2, from which we can see that node 2 can obtain data from node 1; node 3 can obtain data from node 2; node 4 can obtain data from node 3; and node 5 can obtain data from node 4. The corresponding adjacency matrix is given as follows:

$$\mathcal{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

There are four channels (labeled as channel (2,1), (3,2), (4,3) and (5,4)) among sensor nodes for information exchange, where EC-DoS attacks (satisfying Assumptions 1-2) would occur which aim at intercepting signal transmissions among sensor nodes. In this example, the EC-DoS attacks on the four channels are first simulated according to Fig. 3.

The aim of this example is to design a PIO in the form of (8) to estimate system states under the effects of external noises and independent EC-DoS attacks, such that requirements R1) and R2) are satisfied.

For simulation purpose, we first let the energy-bounded external noise be $\omega(k) = 4\cos(k)/k$. Set the simulation run length to be $t_f = 50$ and $\gamma = 1.5$. Then, we construct a PIO in the form of (8) whose gains are obtained by solving linear matrix inequalities presented in Theorem 2. By running Matlab software with the constructed parameters, simulation results are plotted in Figs. 7-10. To be more specific, the evolution trajectory of the original system state and its estimation (obtained by five designed estimator nodes) is displayed in Fig. 7 (about the first element), Fig. 8 (about the second element) and Fig. 9 (about the third element). From these three figures, we can see that the estimated states can track the real system states as time goes on. The evolution trajectory of the estimation error $\tilde{z}_n(k)$ is depicted in Fig. 10 which shows that the estimation error of five nodes is gradually convergent. Furthermore, by simple calculation with $\sigma = 0.9249$ and $\bar{\gamma} = 1.5296$, we have

$$\gamma^* \triangleq \sqrt{\sum_{k=0}^{t_f} \sigma^k \tilde{z}^T(k)\tilde{z}(k)} \Big/ \sqrt{\sum_{k=0}^{t_f} \omega^T(k)\omega(k)}$$
$$= 0.7300 < \bar{\gamma}$$

which implies that the desired weighted H_∞ performance requirement is achieved. It can be seen from these figures and the calculation result that our proposed estimator has a good estimation performance as the estimation requirements are met.

To further check the effects of DoS attacks on estimation performance, we conclude some simulation results in Table I to show the obtained γ^* under four different DoS attacks (launched according to Fig. 3-Fig. 6, respectively). Here, Case 1 represents the situation that attacks in four channels are independent; Case 2 stands for the attacks occurred simultaneously in four channels; Case 3 is corresponding to the situation that the attacks in four channels are activated sequentially; and Case 4 is a combination of Case 2 and Case 3. It can be seen from Table I that the obtained disturbance attenuation levels γ^* in four DoS cases are all less than the prescribed value $\bar{\gamma} = 1.5296$ (i.e., $\gamma^* < \bar{\gamma}$). Thus, the estimation requirement is satisfied.

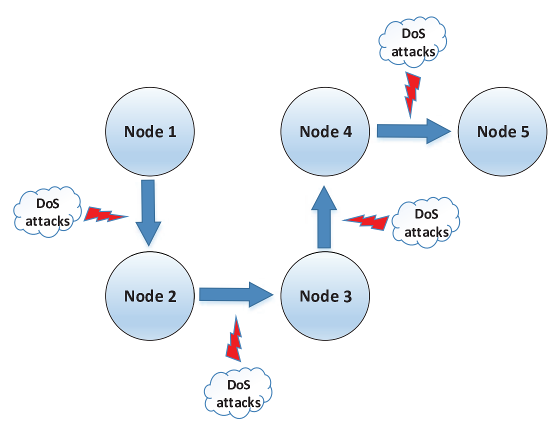

Fig. 2: Topological structure of the sensor network of the simulation example

In addition to the parameter γ^*, the value of accumulated estimation error is another variable to reflect the estimation

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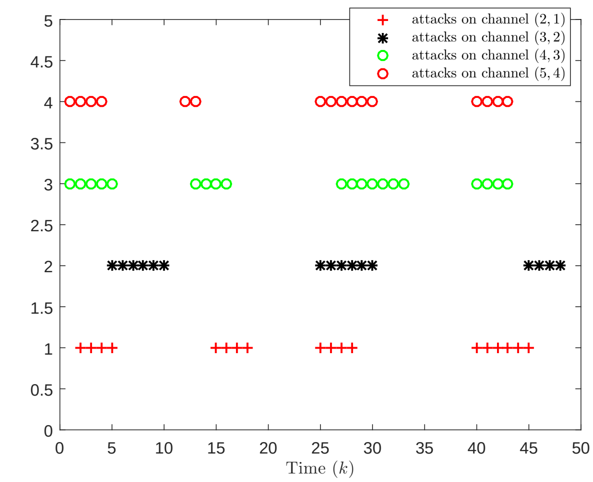

Fig. 3: DoS attacks on four channels (Case 1)

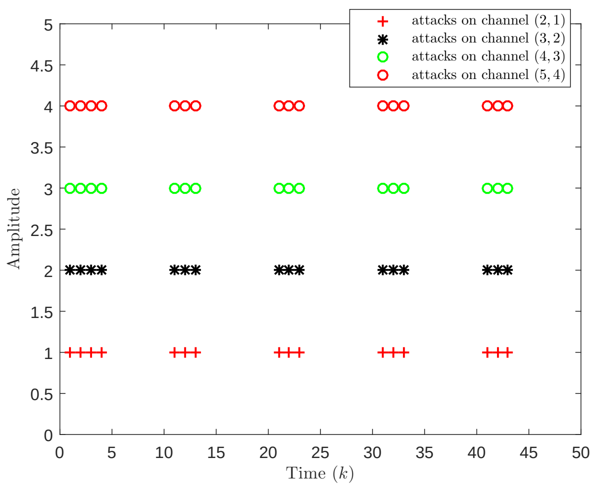

Fig. 4: DoS attacks on four channels (Case 2)

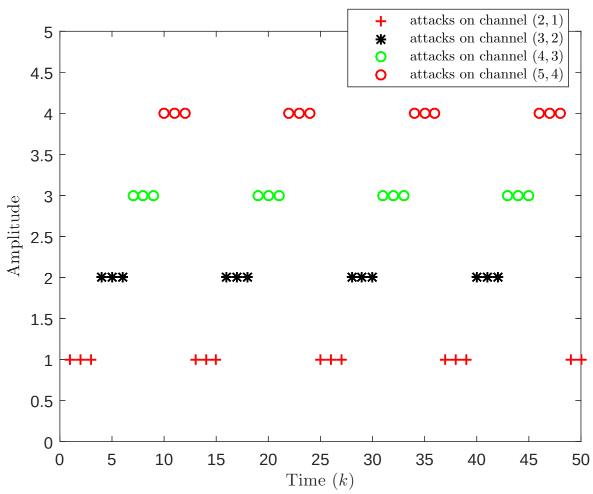

Fig. 5: DoS attacks on four channels (Case 3)

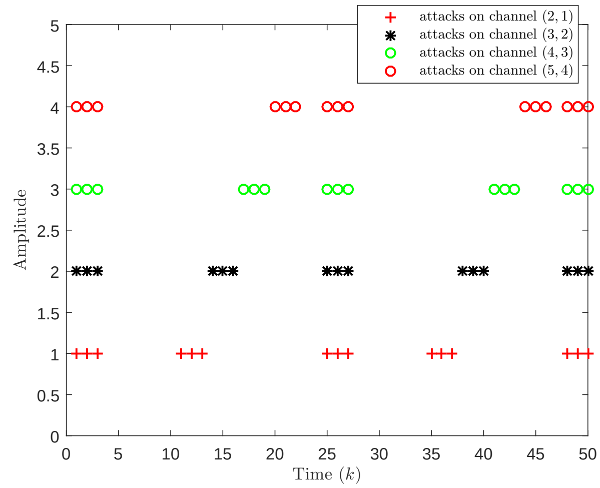

Fig. 6: DoS attacks on four channels (Case 4)

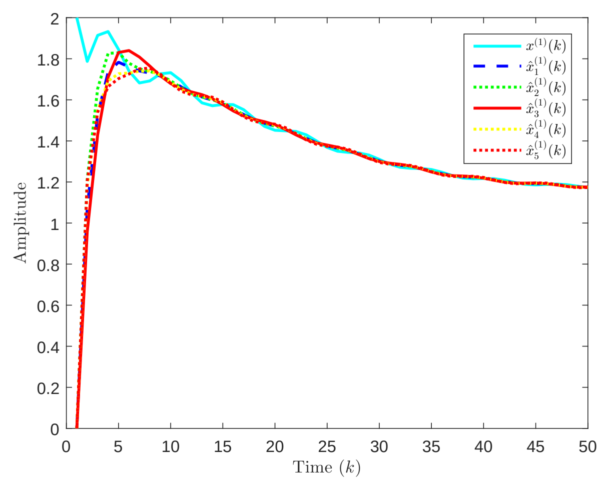

Fig. 7: Dynamical trajectory of state $x^{(1)}(k)$ and its estimation

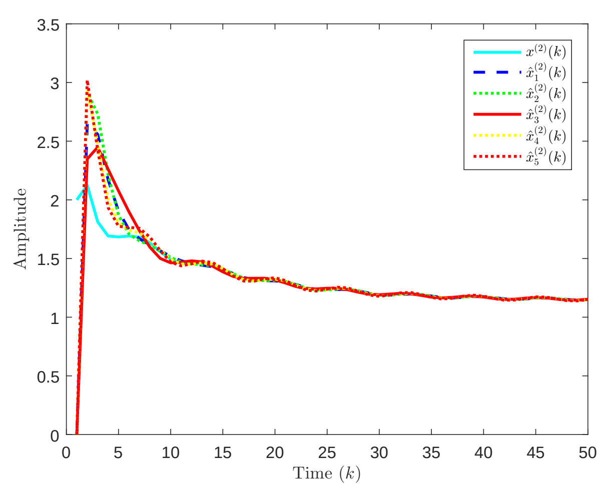

Fig. 8: Dynamical trajectory of state $x^{(2)}(k)$ and its estimation

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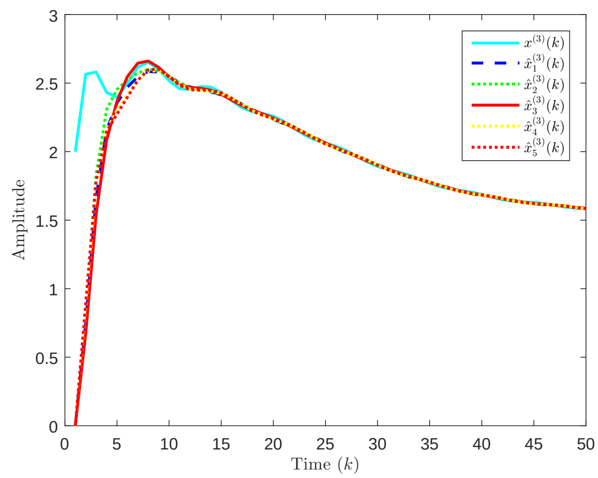

Fig. 9: Dynamical trajectory of state $x^{(3)}(k)$ and its estimation

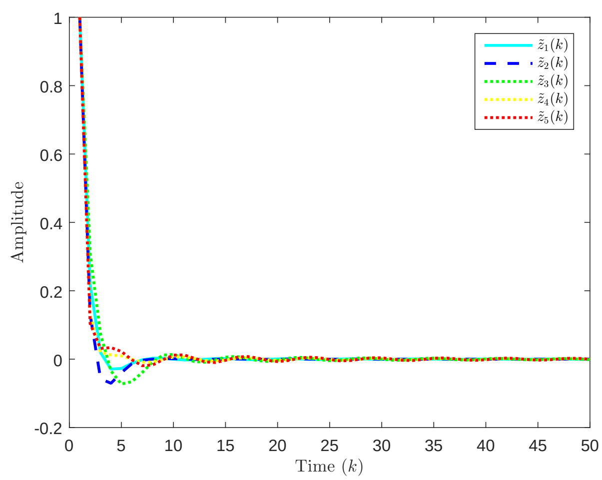

Fig. 10: Dynamical trajectory of estimation error $\tilde{z}_n(k)$, $n = 1, 2, 3, 4, 5$

TABLE I: The Attained γ^* under Different Cases of DoS Attacks

DoS attacks	Case 1	Case 2	Case 3	Case 4
γ^*	0.729967	0.731317	0.729721	0.731304

performance. To display the superiority of the proposed PIO, we define the accumulated estimation error as follows:

$$\tilde{z}_{\text{sum}} \triangleq \sum_{k=0}^{t_f} \tilde{z}^T(k)\tilde{z}(k)$$

where t_f is the terminal time of simulation. Then, Table II lists results of the calculated $\tilde{z}_{\text{sum}}$ with PIO, P-type observer and linear observer (LO) [29], respectively, and the obtained γ^* (with PIO) under different noises. Obviously, a smaller $\tilde{z}_{\text{sum}}$ means a better estimation performance. From Table II, we can conclude that 1) the prescribed weighted H_∞ requirement is achieved; and 2) the proposed PIO can provide a better estimation performance as compared to the P-type observer. All simulation results verify the effectiveness and advantages of the proposed estimation methods.

TABLE II: The Attained γ^* and $\tilde{z}_{\text{sum}}$ under Different Noises

Noise $\omega(k)$	$\frac{4\sin(k)}{k}$	$\frac{4\cos(k)}{k}$	$5e^{-0.1k}$	$\frac{0.2+4\sin(k)}{k}$
γ^*	0.5300	0.7300	0.2098	0.4812
$\tilde{z}_{\text{sum}}$ (PIO)	4.7841	4.8109	4.9701	4.7835
$\tilde{z}_{\text{sum}}$ (P-type)	4.7903	4.8174	4.9743	4.7895
$\tilde{z}_{\text{sum}}$ (LO [29])	13.1444	8.0886	21.7012	15.3565

V. Conclusion

In this paper, we have addressed the distributed SE problems for T-S fuzzy systems. A sensor network with a number of sensor nodes has been employed to measure the information of the plant according to a fixed communication topology. We consider the case that the signal transmissions among sensor n-odes are achieved via WCN with independent channels, whose transmitted data would be corrupted by the EC-DoS attacks. The constrained energy of attacks has been reflected in some standard assumptions on the frequency and duration of attacks. To achieve the desired SE performance, a novel distributed fuzzy PIO has been proposed that can simultaneously use the current and historical innovation with designed weights. With the assistance of the switching-system-based theory, the multiple modes induced by independent EC-DoS attacks have been analyzed, and sufficient conditions have been obtained to check the stability and weighted H_∞ performance of the estimation error system. Finally, our proposed SE scheme has been validated via a numerical example. The future topics include the extension of the results to systems subject to other complex phenomena [2], [13], [18], [19], [30], [49]–[51], [54].

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APPENDIX

A. Proof of Theorem 1

Proof: Choose the following Lyapunov function:

$$V_{\Psi(\Gamma(k))}(k)=\eta^T(k)P_{\Psi(\Gamma(k))}\eta(k). \tag{21}$$

Let k_l $(l\in\mathbb{N},\ k_0=0)$ denote the time-instants at which $\Gamma(k)$ changes (meaning that at least one EC-DoS off/on or on/off transition occurs). For $k\in[k_l,k_{l+1})$, assuming $\Gamma(k)=\Gamma(k_l)=\Gamma$ $(\Gamma\subseteq\mathcal{E})$, we calculate that

$$\begin{aligned}
&V_{\Psi(\Gamma(k))}(k+1)-V_{\Psi(\Gamma(k))}(k)+\beta_{\Psi(\Gamma(k))}V_{\Psi(\Gamma(k))}(k)\\
&=\eta^T(k+1)P_{\Psi(\Gamma(k))}\eta(k+1)\\
&\quad-(1-\beta_{\Psi(\Gamma(k))})\eta^T(k)P_{\Psi(\Gamma(k))}\eta(k)\\
&=\sum_{i=1}^r\sum_{v=1}^r\sum_{\bar{i}=1}^r\sum_{\bar{v}=1}^r h_i(\vartheta(k))h_v(\hat{\vartheta}(k))h_{\bar{i}}(\vartheta(k))h_{\bar{v}}(\hat{\vartheta}(k))\\
&\quad\times\Big(\big(\bar{A}_{i,v}+\bar{B}_v(\Gamma)\big)\eta(k)+\big(\bar{E}_{i,v}+\bar{F}_v(\Gamma)\big)\omega(k)\Big)^TP_{\Psi(\Gamma)}\\
&\quad\times\Big(\big(\bar{A}_{\bar{i},\bar{v}}+\bar{B}_{\bar{v}}(\Gamma)\big)\eta(k)+\big(\bar{E}_{\bar{i},\bar{v}}+\bar{F}_{\bar{v}}(\Gamma)\big)\omega(k)\Big)\\
&\quad-\eta^T(k)\big(1-\beta_{\Psi(\Gamma)}\big)P_{\Psi(\Gamma)}\eta(k)\\
&\le\sum_{i=1}^r\sum_{v=1}^r h_i(\vartheta(k))h_v(\hat{\vartheta}(k))\\
&\quad\times\Big(\big(\bar{A}_{i,v}+\bar{B}_v(\Gamma)\big)\eta(k)+\big(\bar{E}_{i,v}+\bar{F}_v(\Gamma)\big)\omega(k)\Big)^TP_{\Psi(\Gamma)}\\
&\quad\times\Big(\big(\bar{A}_{i,v}+\bar{B}_v(\Gamma)\big)\eta(k)+\big(\bar{E}_{i,v}+\bar{F}_v(\Gamma)\big)\omega(k)\Big)\\
&\quad-\eta^T(k)\big(1-\beta_{\Psi(\Gamma)}\big)P_{\Psi(\Gamma)}\eta(k)\\
&=\sum_{i=1}^r\sum_{v=1}^r h_i(\vartheta(k))h_v(\hat{\vartheta}(k))\\
&\quad\times\begin{bmatrix}\eta(k)\\ \omega(k)\end{bmatrix}^T\left(\begin{bmatrix}\bar{A}^T_{i,v}+\bar{B}^T_v(\Gamma)\\ \bar{E}^T_{i,v}+\bar{F}^T_v(\Gamma)\end{bmatrix}P_{\Psi(\Gamma)}\begin{bmatrix}\bar{A}^T_{i,v}+\bar{B}^T_v(\Gamma)\\ \bar{E}^T_{i,v}+\bar{F}^T_v(\Gamma)\end{bmatrix}^T\right.\\
&\quad\left.+\begin{bmatrix}-(1-\beta_{\Psi(\Gamma)})P_{\Psi(\Gamma)} & 0\\ 0 & 0\end{bmatrix}\right)\begin{bmatrix}\eta(k)\\ \omega(k)\end{bmatrix}.
\end{aligned} \tag{22}$$

We first consider the stability analysis issue by letting $\omega(k)=0$. It follows from (22) that

$$\begin{aligned}
&V_{\Psi(\Gamma(k))}(k+1)-V_{\Psi(\Gamma(k))}(k)+\beta_{\Psi(\Gamma(k))}V_{\Psi(\Gamma(k))}(k)\\
&\le\sum_{i=1}^r\sum_{v=1}^r h_i(\vartheta(k))h_v(\hat{\vartheta}(k))\eta^T(k)\Big(\big(\bar{A}^T_{i,v}+\bar{B}^T_v(\Gamma)\big)\\
&\quad\times P_{\Psi(\Gamma)}\big(\bar{A}_{i,v}+\bar{B}_v(\Gamma)\big)-(1-\beta_{\Psi(\Gamma)})P_{\Psi(\Gamma)}\Big)\eta(k).
\end{aligned}$$

Under condition (13), it is easily checked that

$$V_{\Psi(\Gamma(k))}(k+1)-V_{\Psi(\Gamma(k))}(k)+\beta_{\Psi(\Gamma(k))}V_{\Psi(\Gamma(k))}(k)<0.$$

Thus, we have

$$\begin{aligned}
V_{\Psi(\Gamma(k_l))}(k)&<(1-\beta_{\Psi(\Gamma(k_l))})V_{\Psi(\Gamma(k_l))}(k-1)\\
&<(1-\beta_{\Psi(\Gamma(k_l))})^2V_{\Psi(\Gamma(k_l))}(k-2)\\
&<(1-\beta_{\Psi(\Gamma(k_l))})^3V_{\Psi(\Gamma(k_l))}(k-3)\\
&<\cdots\\
&<(1-\beta_{\Psi(\Gamma(k_l))})^{k-k_l}V_{\Psi(\Gamma(k_l))}(k_l).
\end{aligned}$$

It follows from condition (14) that

$$\begin{aligned}
V_{\Psi(\Gamma(k_l))}(k_l)&=\eta^T(k_l)P_{\Psi(\Gamma(k_l))}\eta(k_l)\\
&<\mu\eta^T(k_l)P_{\Psi(\Gamma(k_{l-1}))}\eta(k_l)=\mu V_{\Psi(\Gamma(k_{l-1}))}(k_l),
\end{aligned}$$

which implies that

$$\begin{aligned}
&V_{\Psi(\Gamma(k_l))}(k)\\
&<(1-\beta_{\Psi(\Gamma(k_l))})^{k-k_l}V_{\Psi(\Gamma(k_l))}(k_l)\\
&<\mu(1-\beta_{\Psi(\Gamma(k_l))})^{k-k_l}V_{\Psi(\Gamma(k_{l-1}))}(k_l)\\
&<\mu(1-\beta_{\Psi(\Gamma(k_l))})^{k-k_l}(1-\beta_{\Psi(\Gamma(k_{l-1}))})V_{\Psi(\Gamma(k_{l-1}))}(k_l-1)\\
&<\mu(1-\beta_{\Psi(\Gamma(k_l))})^{k-k_l}(1-\beta_{\Psi(\Gamma(k_{l-1}))})^2\\
&\quad\times V_{\Psi(\Gamma(k_{l-1}))}(k_l-2)\\
&<\cdots\\
&<\mu(1-\beta_{\Psi(\Gamma(k_l))})^{k-k_l}(1-\beta_{\Psi(\Gamma(k_{l-1}))})^{k_l-k_{l-1}}\\
&\quad\times V_{\Psi(\Gamma(k_{l-1}))}(k_{l-1})\\
&<\cdots\\
&<\mu^{H(k_0,k)}\tilde{\beta}(k_0,k)V_{\Psi(\Gamma(k_0))}(k_0)
\end{aligned}$$

where

$$\tilde{\beta}(k_0,k)\triangleq\big(1-\beta_{\Psi(\Gamma(k_l))}\big)^{k-k_l}\prod_{\vec{i}=0}^{l-1}\big(1-\beta_{\Psi(\Gamma(k_{\vec{i}}))}\big)^{k_{\vec{i}+1}-k_{\vec{i}}},$$

and $H(k_0,k)$ denotes the total switching number of constructions Γ in time interval $[k_0,k]$.

According to [8], [31] and the constraint on DoS frequency (see Assumption 1), we obtain

$$\begin{aligned}
H(k_0,k)&\le 2\sum_{(n,j)\in\mathcal{E}}|\pi_{n,j}(k_0,k)|\\
&\le\sum_{(n,j)\in\mathcal{E}}2\varpi_{n,j}+\sum_{(n,j)\in\mathcal{E}}\frac{2}{\theta_{n,j}}k\\
&\triangleq\bar{\varpi}+\bar{\theta}k
\end{aligned} \tag{23}$$

where

$$\bar{\varpi}\triangleq\sum_{(n,j)\in\mathcal{E}}2\varpi_{n,j},\quad \bar{\theta}\triangleq\sum_{(n,j)\in\mathcal{E}}\frac{2}{\theta_{n,j}}.$$

Since $\mu>1$, we further have

$$\begin{aligned}
\mu^{H(k_0,k)}\tilde{\beta}(k_0,k)&\le\mu^{\bar{\varpi}+\bar{\theta}k}\tilde{\beta}(k_0,k)\\
&=\mathrm{e}^{\bar{\varpi}\ln\mu}\mathrm{e}^{(\bar{\theta}k\ln\mu+\vec{\beta}(0,k))}
\end{aligned}$$

where

$$\vec{\beta}(0,k)\triangleq\sum_{\Gamma\subseteq\mathcal{E}}|\Pi_\Gamma(0,k)|\ln(1-\beta_{\Psi(\Gamma)}),$$

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and $\Pi_\Gamma(0,k)$ is a set denoting the time intervals over which the system is subject to the attack construction Γ in time interval $[0,k]$, and $|\Pi_\Gamma(0,k)|\in\mathbb{N}$ is the total number of elements in set $\Pi_\Gamma(0,k)$. Here, the set $\Pi_\Gamma(0,k)$ is introduced to facilitate the later stability analysis for each attack construction Γ [16].

Considering the fact of $\sum_{\Gamma\subseteq\mathcal{E}}|\Pi_\Gamma(0,k)|=k$ and condition (15), we derive

$$\begin{aligned}
&\bar\theta k\ln\mu+\vec\beta(0,k)\\
&=\sum_{\Gamma\subseteq\mathcal{E}}\Bigg(\ln(1-\beta_{\Psi(\Gamma)})+\sum_{(n,j)\in\mathcal{E}}\frac{2\ln\mu}{\theta_{n,j}}\Bigg)|\Pi_\Gamma(0,k)|\\
&<\sum_{\Gamma\subseteq\mathcal{E}}\Bigg(\sum_{(n,j)\in\Gamma}\kappa_{n,j}+\sum_{(n,j)\notin\Gamma}\varepsilon_{n,j}\Bigg)|\Pi_\Gamma(0,k)|\\
&=\sum_{(n,j)\in\mathcal{E}}\Bigg(\kappa_{n,j}\sum_{\Gamma\subseteq\mathcal{E},(n,j)\in\Gamma}|\Pi_\Gamma(0,k)|\\
&\quad+\varepsilon_{n,j}\sum_{\Gamma\subseteq\mathcal{E},(n,j)\notin\Gamma}|\Pi_\Gamma(0,k)|\Bigg).
\end{aligned}$$

Considering the following equalities:

$$\begin{aligned}
\sum_{\Gamma\subseteq\mathcal{E},(n,j)\in\Gamma}|\Pi_\Gamma(0,k)|&=|\Phi_{n,j}(0,k)|,\\
\sum_{\Gamma\subseteq\mathcal{E},(n,j)\notin\Gamma}|\Pi_\Gamma(0,k)|&=k-|\Phi_{n,j}(0,k)|,
\end{aligned}$$

we have from (16) that

$$\begin{aligned}
&\bar\theta k\ln\mu+\vec\beta(0,k)\\
&<\sum_{(n,j)\in\mathcal{E}}\big(\kappa_{n,j}|\Phi_{n,j}(0,k)|+\varepsilon_{n,j}k\\
&\quad-\varepsilon_{n,j}|\Phi_{n,j}(0,k)|\big)\\
&\le\sum_{(n,j)\in\mathcal{E}}(\kappa_{n,j}-\varepsilon_{n,j})m_{n,j}\\
&\quad+\sum_{(n,j)\in\mathcal{E}}\Bigg(\frac{\kappa_{n,j}-\varepsilon_{n,j}}{g_{n,j}}+\varepsilon_{n,j}\Bigg)k\\
&\triangleq\bar m+\bar g k
\end{aligned}$$

where

$$\begin{aligned}
\bar m&\triangleq\sum_{(n,j)\in\mathcal{E}}(\kappa_{n,j}-\varepsilon_{n,j})m_{n,j},\\
\bar g&\triangleq\sum_{(n,j)\in\mathcal{E}}\Bigg(\frac{\kappa_{n,j}-\varepsilon_{n,j}}{g_{n,j}}+\varepsilon_{n,j}\Bigg).
\end{aligned}$$

To this end, we conclude that

$$V_{\Psi(\Gamma(k))}(k)<\mathrm{e}^{\bar\varpi\ln\mu+\bar m}\mathrm{e}^{\bar g k}V_{\Psi(\Gamma(0))}(0).$$

Furthermore, note the following inequalities

$$\begin{aligned}
\underline{\lambda}\eta^T(k)\eta(k)&\le\eta^T(k)P_{\Psi(\Gamma(k))}\eta(k),\\
\eta^T(0)P_{\Psi(\Gamma(0))}\eta(0)&\le\bar\lambda\eta^T(0)\eta(0)
\end{aligned}$$

where, for $\forall\Psi(\Gamma)\in\mathcal{D}$ (with the definition of $\mathcal{D}$ being given in (12)),

$$\underline{\lambda}\triangleq\min\{\lambda_{\min}(P_{\Psi(\Gamma)})\},\quad\bar\lambda\triangleq\max\{\lambda_{\max}(P_{\Psi(\Gamma)})\}.$$

Then, it follows that

$$\|\eta(k)\|^2<\frac{\bar\lambda}{\underline{\lambda}}\mathrm{e}^{\bar\varpi\ln\mu+\bar m}\mathrm{e}^{\bar g k}\|\eta(0)\|^2$$

which, together with condition (17), leads to $\bar g<0$, implying $0<\mathrm{e}^{\bar g}<1$, and therefore we know immediately that $\eta(k)\to0$ as $k\to\infty$. Thus, the estimation error system (10) is asymptotically stable.

We are now in a position to check the weighted H_∞ performance of system (10). For this purpose, we define

$$J(k)\triangleq\tilde z^T(k)\tilde z(k)-\gamma^2\omega^T(k)\omega(k).$$

For $k\in[k_l,k_{l+1})$ with $\Gamma(k_l)=\Gamma$ ($\Gamma\subseteq\mathcal{E}$), we calculate that

$$\begin{aligned}
&V_{\Psi(\Gamma(k_l))}(k+1)-(1-\beta_{\Psi(\Gamma(k_l))})V_{\Psi(\Gamma(k_l))}(k)+J(k)\\
&=\sum_{i=1}^r\sum_{v=1}^r\sum_{\bar i=1}^r\sum_{\bar v=1}^r h_i(\vartheta(k))h_v(\hat\vartheta(k))h_{\bar i}(\vartheta(k))h_{\bar v}(\hat\vartheta(k))\\
&\quad\times\Big(\big(\bar A_{i,v}+\bar B_v(\Gamma)\big)\eta(k)+\big(\bar E_{i,v}+\bar F_v(\Gamma)\big)\omega(k)\Big)^TP_{\Psi(\Gamma)}\\
&\quad\times\Big(\big(\bar A_{\bar i,\bar v}+\bar B_{\bar v}(\Gamma)\big)\eta(k)+\big(\bar E_{\bar i,\bar v}+\bar F_{\bar v}(\Gamma)\big)\omega(k)\Big)\\
&\quad-\eta^T(k)(1-\beta_{\Psi(\Gamma)})P_{\Psi(\Gamma)}\eta(k)+J(k)\\
&\le\sum_{i=1}^r\sum_{v=1}^r\sum_{\bar i=1}^r\sum_{\bar v=1}^r h_i(\vartheta(k))h_v(\hat\vartheta(k))h_{\bar i}(\vartheta(k))h_{\bar v}(\hat\vartheta(k))\\
&\quad\times\Big(\tilde A_{i,v}^T(\Gamma)P_{\Psi(\Gamma)}\tilde A_{\bar i,\bar v}(\Gamma)+\vec P_{i,v,\Psi(\Gamma)}\Big)\\
&\le\sum_{i=1}^r\sum_{v=1}^r h_i(\vartheta(k))h_v(\hat\vartheta(k))\\
&\quad\times\Big(\tilde A_{i,v}^T(\Gamma)P_{\Psi(\Gamma)}\tilde A_{i,v}(\Gamma)+\vec P_{i,v,\Psi(\Gamma)}\Big).
\end{aligned}$$

In terms of the condition (13), it can be derived that

$$V_{\Psi(\Gamma(k))}(k+1)-(1-\beta_{\Psi(\Gamma(k))})V_{\Psi(\Gamma(k))}(k)+J(k)<0,$$

which implies that

$$\begin{aligned}
&V_{\Psi(\Gamma(k))}(k)<(1-\beta_{\Psi(\Gamma(k_l))})^{k-k_l}V_{\Psi(\Gamma(k))}(k_l)\\
&\quad-\sum_{p=k_l}^{k-1}(1-\beta_{\Psi(\Gamma(k_l))})^{k-p-1}J(p)\\
&<\mu(1-\beta_{\Psi(\Gamma(k_l))})^{k-k_l}V_{\Psi(\Gamma(k_{l-1}))}(k_l)\\
&\quad-\sum_{p=k_l}^{k-1}(1-\beta_{\Psi(\Gamma(k_l))})^{k-p-1}J(p)\\
&<\mu(1-\beta_{\Psi(\Gamma(k_l))})^{k-k_l}\Bigg((1-\beta_{\Psi(\Gamma(k_{l-1}))})^{k_l-k_{l-1}}\\
&\quad\times V_{\Psi(\Gamma(k_{l-1}))}(k_{l-1})\\
&\quad-\sum_{p=k_{l-1}}^{k_l-1}(1-\beta_{\Psi(\Gamma(k_{l-1}))})^{k_l-p-1}J(p)\Bigg)\\
&\quad-\sum_{p=k_l}^{k-1}(1-\beta_{\Psi(\Gamma(k_l))})^{k-p-1}J(p)\\
&<\cdots\\
&<\mu^{H(k_0,k)}(1-\beta_{\Psi(\Gamma(k_l))})^{k-k_l}\times\cdots\times(1-\beta_{\Psi(\Gamma(k_0))})^{k_1-k_0}
\end{aligned}$$

$$
\begin{aligned}
&\times V_{\Gamma(k_0)}(k_0) - \mu^{H(k_0,k)}(1-\beta_{\Psi(\Gamma(k_l))})^{k-k_l} \times \cdots \\
&\times (1-\beta_{\Psi(\Gamma(k_0))})^{k_1-k_0} \sum_{p=k_0}^{k_1-1} (1-\beta_{\Psi(\Gamma(k_0))})^{k_1-p-1} J(p) \\
&- \cdots - \sum_{p=k_l}^{k-1} (1-\beta_{\Psi(\Gamma(k_l))})^{k-p-1} J(p).
\end{aligned}
$$

Under the zero initial condition $V_{\Psi(\Gamma(0))}(0) = 0$ and using the fact $V_{\Psi(\Gamma(k))}(k) \geq 0$, we obtain from the above inequality that

$$
\begin{aligned}
&\mu^{H(k_0,k)}(1-\beta_{\Psi(\Gamma(k_l))})^{k-k_l} \times \cdots \times (1-\beta_{\Psi(\Gamma(k_0))})^{k_1-k_0} \\
&\times \sum_{p=k_0}^{k_1-1} (1-\beta_{\Psi(\Gamma(k_0))})^{k_1-p-1} J(p) + \cdots \\
&+ \sum_{p=k_l}^{k-1} (1-\beta_{\Psi(\Gamma(k_l))})^{k-p-1} J(p) < 0. \qquad (24)
\end{aligned}
$$

By multiplying both sides of (24) by $\mu^{-H(0,k)}$ $(\mu > 1)$, one has

$$
\begin{aligned}
&\sum_{p=0}^{k-1} \mu^{-H(0,p)} \vec{\beta}_{\max}^{k-p-1} \tilde{z}^T(p)\tilde{z}(p) \\
&\quad < \sum_{p=0}^{k-1} \mu^{-H(0,p)} \vec{\beta}_{\min}^{k-p-1} \gamma^2 \omega^T(p)\omega(p) \qquad (25)
\end{aligned}
$$

where

$$
\begin{aligned}
&\vec{\beta}_{\max} \triangleq 1 - \bar{\beta}, \quad \vec{\beta}_{\min} \triangleq 1 - \underline{\beta}, \\
&\quad \bar{\beta} \triangleq \max\{\beta_{\Psi(\Gamma)}\}, \quad \underline{\beta} \triangleq \min\{\beta_{\Psi(\Gamma)}\}, \quad \forall \Psi(\Gamma) \in \mathcal{D}.
\end{aligned}
$$

Recalling (23), one obtains

$$
\begin{aligned}
&\sum_{p=0}^{k-1} \mu^{-\bar{\varpi}} \mu^{-\bar{\theta} p} \vec{\beta}_{\max}^{k-p-1} \tilde{z}^T(p)\tilde{z}(p) \\
&\quad \leq \sum_{p=0}^{k-1} \mu^{-H(0,p)} \vec{\beta}_{\max}^{k-p-1} \tilde{z}^T(p)\tilde{z}(p) \\
&\quad < \sum_{p=0}^{k-1} \mu^{-H(0,p)} \vec{\beta}_{\min}^{k-p-1} \gamma^2 \omega^T(p)\omega(p) \\
&\quad \leq \sum_{p=0}^{k-1} \vec{\beta}_{\min}^{k-p-1} \gamma^2 \omega^T(p)\omega(p). \qquad (26)
\end{aligned}
$$

Summing up both sides of (26) from $k = 1$ to $k = \infty$, we arrive at

$$
\begin{aligned}
&\sum_{k=1}^{\infty} \sum_{p=0}^{k-1} \mu^{-\bar{\theta} p} \vec{\beta}_{\max}^{k-p-1} \tilde{z}^T(p)\tilde{z}(p) \\
&\quad < \sum_{k=1}^{\infty} \sum_{p=0}^{k-1} \vec{\beta}_{\min}^{k-p-1} \mu^{\bar{\varpi}} \gamma^2 \omega^T(p)\omega(p). \qquad (27)
\end{aligned}
$$

Note that (27) can be rewritten by

$$
\begin{aligned}
&\sum_{p=0}^{\infty} \mu^{-\bar{\theta} p} \tilde{z}^T(p)\tilde{z}(p) \sum_{k=p+1}^{\infty} \vec{\beta}_{\max}^{k-p-1} \\
&\quad < \sum_{p=0}^{\infty} \mu^{\bar{\varpi}} \gamma^2 \omega^T(p)\omega(p) \sum_{k=p+1}^{\infty} \vec{\beta}_{\min}^{k-p-1}. \qquad (28)
\end{aligned}
$$

Since $0 < \beta_{\Psi(\Gamma)} < 1$ for $\forall \Psi(\Gamma) \in \mathcal{D}$, we know directly from (18) and (28) that

$$
\sum_{k=0}^{\infty} \sigma^k \tilde{z}^T(k)\tilde{z}(k) < \bar{\gamma}^2 \sum_{k=0}^{\infty} \omega^T(k)\omega(k)
$$

where $\bar{\gamma} \triangleq \sqrt{\frac{\bar{\beta}}{\underline{\beta}} \mu^{\bar{\varpi}}}\gamma$. The proof is complete now. ∎

B. Proof of Theorem 2

Proof: By means of the Schur Complement Lemma, we know that (13) holds if and only if the following holds:

$$
\begin{bmatrix} \vec{P}_{i,v,\Psi(\Gamma)} & * \\ \tilde{A}_{i,v}(\Gamma) & -P_{\Psi(\Gamma)}^{-1} \end{bmatrix} < 0. \qquad (29)
$$

Then, pre- and post-multiplying the matrix in (29) by $\mathrm{diag}\{I,\ Y_v^{(1)},\ Y_{\Psi(\Gamma)}^{(2)},\ Y^{(3)}\}$ and its transposition, respectively, we obtain the following matrix

$$
\begin{bmatrix} \vec{P}_{i,v,\Psi(\Gamma)} & * \\ \bar{Y}_{v,\Psi(\Gamma)} \tilde{A}_{i,v}(\Gamma) & -\bar{Y}_v P_{\Psi(\Gamma)}^{-1} \bar{Y}_v^T \end{bmatrix} \qquad (30)
$$

where $\bar{Y}_{v,\Psi(\Gamma)} \triangleq \mathrm{diag}\{Y_v^{(1)},\ Y_{\Psi(\Gamma)}^{(2)},\ Y^{(3)}\}$.

Let

$$
\begin{aligned}
&X_{n,v}^{(1)} = Y_{n,v}^{(1)} K_{n,v}, \quad X_{n,v}^{(2)} = Y_{n,v}^{(1)} L_{n,v}, \\
&X_{n,v}^{(3)} = Y_{n,v}^{(1)} M_{n,v}, \quad X_{n,v}^{(4)} = Y_{n,v}^{(1)} N_{n,v}, \\
&X_n^{(5)} = Y_n^{(3)} T_n, \quad X_n^{(6)} = Y_n^{(3)} S_n.
\end{aligned}
$$

Consider the following terms:

$$
\begin{aligned}
Y_v^{(1)} \bar{K}_v &= \mathrm{diag}\{Y_{1,v}^{(1)} K_{1,v},\ Y_{2,v}^{(1)} K_{2,v},\ \cdots,\ Y_{\varrho,v}^{(1)} K_{\varrho,v}\} \\
&= \sum_{s=1}^{\varrho} \bar{I}_s X_{s,v}^{(1)} \tilde{I}_s, \\
Y_v^{(1)} \bar{L}_v &= \mathrm{diag}\{Y_{1,v}^{(1)} \tilde{L}_{1,v},\ Y_{2,v}^{(1)} \tilde{L}_{2,v},\ \cdots,\ Y_{\varrho,v}^{(1)} \tilde{L}_{\varrho,v}\} \\
&= \sum_{s=1}^{\varrho} \bar{I}_s X_{s,v}^{(2)} \tilde{I}_s, \\
Y_v^{(1)} \bar{M}_v &= \mathrm{diag}\{Y_{1,v}^{(1)} M_{1,v},\ Y_{2,v}^{(1)} M_{2,v},\ \cdots,\ Y_{\varrho,v}^{(1)} M_{\varrho,v}\} \\
&= \sum_{s=1}^{\varrho} \bar{I}_s X_{s,v}^{(3)} \tilde{I}_s, \\
Y_v^{(1)} \bar{N}_v &= \mathrm{diag}\{Y_{1,v}^{(1)} \tilde{N}_{1,v},\ Y_{2,v}^{(1)} \tilde{N}_{2,v},\ \cdots,\ Y_{\varrho,t}^{(1)} \tilde{N}_{\varrho,v}\} \\
&= \sum_{s=1}^{\varrho} \bar{I}_s X_{s,v}^{(4)} \tilde{I}_s, \\
Y^{(3)} \bar{T} &= \mathrm{diag}\{Y_1^{(3)} T_1,\ Y_2^{(3)} T_2,\ \cdots,\ Y_{\varrho}^{(3)} T_{\varrho}\} \\
&= \sum_{s=1}^{\varrho} \tilde{I}_s^T X_s^{(5)} \tilde{I}_s, \\
Y^{(3)} \bar{S} &= \mathrm{diag}\{Y_1^{(3)} S_1,\ Y_2^{(3)} S_2,\ \cdots,\ Y_{\varrho}^{(3)} S_{\varrho}\} \\
&= \sum_{s=1}^{\varrho} \tilde{I}_s^T X_s^{(6)} \tilde{I}_s.
\end{aligned}
$$

With the help of the well-known matrix inequality

$$-YP^{-1}Y^T \leq P - Y - Y^T$$

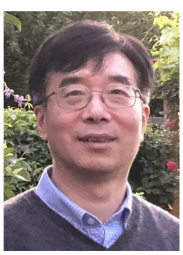
where $P > 0$ and Y is a real matrix, it can be concluded from (19) that

$$\begin{bmatrix} \vec{P}_{i,v,\Psi(\Gamma)} & * \\ \bar{Y}_{v,\Psi(\Gamma)}\tilde{A}_{i,v}(\Gamma) & -\bar{Y}_v P^{-1}_{\Psi(\Gamma)} \bar{Y}_v^T \end{bmatrix} \leq \begin{bmatrix} \Omega^{(1,1)}_{i,v,\Psi(\Gamma)} & * \\ \Omega^{(2,1)}_{i,v,\Psi(\Gamma)} & \Omega^{(2,2)}_{v,\Psi(\Gamma)} \end{bmatrix} < 0.$$

Therefore, the proof is complete. ■

Yezheng Wang received the B.Eng. degree in automation from the Qilu University of Technology, Jinan, China, in 2017. He is currently pursuing the Ph.D. degree in control theory and control engineering with the College of Electrical Engineering and Automation, Shandong University of Science and Technology, Qingdao, China. Since February 2023, he has been a Visiting Ph.D. Student with the Department of Computer Science, Brunel University London, Uxbridge, U.K. His current research interests include the control and filtering for T-S fuzzy system.

Mr. Wang is a very active reviewer for many international journals.



Zidong Wang (SM'03-F'14) received the B.Sc. degree in mathematics in 1986 from Suzhou University, Suzhou, China, and the M.Sc. degree in applied mathematics in 1990 and the Ph.D. degree in electrical engineering in 1994, both from Nanjing University of Science and Technology, Nanjing, China.

He is currently a Professor of Dynamical Systems and Computing in the Department of Computer Science, Brunel University London, U.K. From 1990 to 2002, he held teaching and research appointments in universities in China, Germany and the U.K. Prof. Wang's research interests include dynamical systems, signal processing, bioinformatics, control theory and applications. He has published more than 700 papers in international journals. He is a holder of the Alexander von Humboldt Research Fellowship of Germany, the JSPS Research Fellowship of Japan, William Mong Visiting Research Fellowship of Hong Kong.

Prof. Wang serves (or has served) as the Editor-in-Chief for *International Journal of Systems Science*, the Editor-in-Chief for *Neurocomputing*, the Editor-in-Chief for *Systems Science & Control Engineering*, and an Associate Editor for 12 international journals including IEEE Transactions on Automatic Control, IEEE Transactions on Control Systems Technology, IEEE Transactions on Neural Networks, IEEE Transactions on Signal Processing, and IEEE Transactions on Systems, Man, and Cybernetics-Part C. He is a Member of the Academia Europaea, a Member of the European Academy of Sciences and Arts, an Academician of the International Academy for Systems and Cybernetic Sciences, a Fellow of the IEEE, a Fellow of the Royal Statistical Society and a member of program committee for many international conferences.

Lei Zou received the Ph.D. degree in control science and engineering in 2016 from Harbin Institute of Technology, Harbin, China.

He is currently a Professor with the College of Information Science and Technology, Donghua University, Shanghai, China. From October 2013 to October 2015, he was a Visiting Ph.D. Student with the Department of Computer Science, Brunel University London, Uxbridge, U.K. His research interests include control and filtering of networked systems, moving-horizon estimation, state estimation subject to outliers, and secure state estimation.

Prof. Zou serves (or has served) as an Associate Editor for *IEEE/CAA Journal of Automatica Sinica*, *Neurocomputing*, *International Journal of Systems Science*, and *International Journal of Control, Automation and Systems*, a Senior Member of *IEEE*, a Member of *Chinese Association of Automation*, a Regular Reviewer of *Mathematical Reviews*, and a very active reviewer for many international journals.

Yun Chen was born in Zhejiang Province, China, in 1976. He received the B.E. degree in thermal engineering in 1999 from Central South University of Technology (now Central South University), Changsha, China, and the M.E. degree in engineering thermal physics in 2002 and Ph.D. degree in control science and engineering in 2008, both from Zhejiang University, Hangzhou, China. He is currently a Professor at Hangzhou Dianzi University, Hangzhou, China.

Dong Yue is currently a professor and dean of the Institute of Advanced Technology and College of Automation & AI, Nanjing University of Posts and Telecommunications, Nanjing, China. He is the Chair of IEEE IES Technical Committee on NCS and Applications and Chair of IEEE PES Smart Grid & Emerging Technologies Satellite Committee-China. He has served as the Associate Editor of IEEE Industrial Electronics Magazine, IEEE Transactions on Industrial Informatics, IEEE Transactions on Systems, Man and Cybernetics: Systems, IEEE Transactions on Neural Networks and Learning Systems, Journal of the Franklin Institute and International Journal of Systems Science, the Guest Editor of Special Issue on New Trends in Energy Internet: Artificial Intelligence-based Control, Network Security and Management, IEEE Transactions on Systems, Man, and Cybernetics: Systems. He is a Fellow of the IEEE for his contribution to network-based control and its applications to power systems. Up to now, he has published more than 250 papers in international journals and 4 books in Springer. He holds more than 90 patents. His research interests include analysis and synthesis of networked control systems, multi-agent systems, optimal control of power systems, and internet of things.