DESIGN AND FINITE ELEMENT MODE ANALYSIS OF NONCIRCULAR GEAR

Chao Lin [1], Kai Cheng [2], Datong Qin [1], Caichao Zhu [1], Hua Qiu [3], Xiaohu Ran [1]

1. **The State Key Laboratory of Mechanical Transmission, Chongqing University, China**
2. **School of Engineering and Design, Brunel University, UK**
3. **Department of Mechanical Engineering, Kyushu Sangyo University, Japan**

Abstract

The noncircular gear transmission is an important branch of the gear transmission, it is characterized by its compact structure, good dynamic equilibration and other advantages, and can be used in the automobile, engineering machine, ship, machine tool, aviation and spaceflight field etc. Studying on the dynamics feature of noncircular gear transmission can improve the ability to carry loads of, reduce the vibration and noise of, increase the life of the noncircular gear transmission machine, provides guidance for the design of the noncircular gear, and has significant theories and practical meanings. In this paper, the gear transmission technique is used to studied the design method of the noncircular gear, which contains distribution of teeth on the pitch curve, designs of the tooth tip curve and the tooth root curve, design of the tooth profile curve, the gear system dynamics principle is introduced to establish dynamics model for the noncircular gear; basic theory of finite element and mode analysis method are applied, finite element model for the noncircular gear is established, natural vibration characteristic of the noncircular gear is studied. And the oval gear is taken as an example, the mathematics software MathCAD, the 3D modeling software UG and the finite element software ABAQUS are used to realize precise 3D model of the oval gear. The finite element method is used, the natural vibration characteristic of the oval gear is studied, the main vibration types and natural frequencies of the oval gear and that of the equivalent cylindrical gears are analyzed and compared, the conclusions received reflect the dynamics performance of the oval gear, and solid foundation is laid for dynamics research and engineering application of the oval gear transmission.

Keywords: Noncircular gear, Finite element method, Natural frequency, Natural vibration shape.

1.0 Introduction

The noncircular gear is an important branch of gear transmission, can be used to transmit movement and power between two intersectant axes, is characterized by its compact structure, good dynamic equilibration and other advantages, and can be applied in automobile, engineering machine, ship, machine tool, aviation and space flight field etc. The currently, the studying work of noncircular gear concentrates on geometry modeling, kinematics, machining etc, while that on dynamics is much less. Studying on the dynamics feature of the noncircular gear transmission can improve the ability to carry loads of reduce the vibration and noise, increase the life of the noncircular gear transmission machine, provides guidance for the design of the noncircular gear, and there are significant theories and practical meanings.

2.0 Design of the Noncircular Gear

The pitch curve of the noncircular gear is noncircular, which makes the design of the noncircular gear difficult. The keys of the noncircular gear design are to determine the position on the pitch curve of each tooth, the tooth tip curve, the tooth root curve and the tooth profile curve of the noncircular gear.

First, it give a point on the pitch curve as a beginning point. Then determine the positions for left and right tooth profile of each tooth by calculating arc length according to pitch distance and spiral thickness [1].

2.1 The Tooth Tip Curve and Tooth Root Curve

The tooth tip curve and the tooth root curve of the noncircular gear are normal equal-distance curves of the pitch curve, the normal distances between them and the pitch curve are the tooth addendum and the tooth root height respectively[1], the shown in Fig. 1.

From Fig. 1 the tooth tip curve formula can be written as.

$$r_a = \sqrt{r^2 + h_a^2 + 2rh_a \sin\mu} \tag{1}$$

Where: r_a ——Tooth tip curve radius, r ——Pitch curve radius, h_a ——Tooth addendum,
μ ——Angle between tangential direction and radial direction of a point (P) on pitch curve.

$$\mu = \arctan\frac{r}{\frac{dr}{d\varphi}} \tag{2}$$

$$\theta_a = \varphi - \arcsin\frac{h_a \cos\mu}{r_a} \tag{3}$$

Where: θ_a ——Polar angle of tooth tip curve, φ ——Polar angle of pitch curve.
The tooth root curve formula can be written as.

$$r_f = \sqrt{r^2 + h_f^2 + 2rh_f \sin\mu} \tag{4}$$

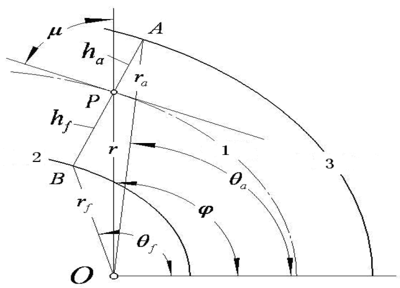
Where: r_f ——Tooth root curve radius, h_f ——Tooth dedendum.

$$\theta_f = \varphi + \arcsin\frac{h_f \cos\mu}{r_f} \tag{5}$$

Where: θ_f ——Polar angle of tooth root curve.

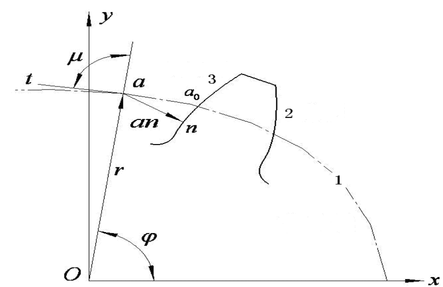
2.2 The Tooth Profile Curve

The tooth profile curve of the cylindrical gear is involute of the basic circle, and can be settled according to the basic circle. The tooth profile curve of the noncircular gear is computed from evolute of tooth profile, and the profile curve of each tooth is different[2]. The tooth profile curve of the noncircular gear can be derived from pitch curve formula by analytic method as shown in Fig. 2.



1——Pitch curve, 2——Root curvem, 3——Tip curve.

Fig. 1. Tip curve and root curve



1——Pitch curve, 2——Left tooth profile, 3——Right tooth profile.

Fig. 2. Tooth profile curve

From Fig. 2 the right tooth profile curve formula of the noncircular gear can be written as.

$$\begin{cases} x_r = r\cos\varphi \mp an\cos(\varphi+\mu+\alpha_n) \\ y_r = r\sin\varphi \mp an\sin(\varphi+\mu+\alpha_n) \end{cases} \tag{6}$$

Where: x_r ——X coordinates value of right tooth profile, y_r ——Y coordinates of right tooth profile, α_n ——Pressure angle of tool, an ——Distance from intersection point between pitch curve and normal of tooth profile to tooth profile along normal direction of tooth profile.

Left tooth profile curve formula of the noncircular gear can be written as.

$$\begin{cases} x_l = r\cos\varphi \pm an\cos(\varphi+\mu-\alpha_n) \\ y_l = r\sin\varphi \pm an\sin(\varphi+\mu-\alpha_n) \end{cases} \tag{7}$$

Where: x_l ——X coordinates of left tooth profile, y_l ——Y coordinates of left tooth profile.

From the gear meshing theory.

$$an = S\cos\alpha_n \tag{8}$$

Where: S ——Arc length on the pitch from point (a) to intersection point (a_0) between the pitch curve and the tooth profile curve.

The tooth profile curve of the noncircular gear can be realized by two methods: 1) The programming, which is difficult to common designer. 2) The equivalent method, which use the involute of the equivalent cylindrical gear to substitute the tooth profile curve of the noncircular gear, and make the model imprecise. All these make the analysis of the noncircular gear difficult. This paper aims at this problem, takes the oval gear as an example, uses the tooth profile curve formula, combines mathematic software MathCAD, software UG and finite element software ABAQUS, and realizes the precise model of the oval gear.

2.3 Oval Gear Modeling

The pitch curve formula of the oval gear can be written as.

$$r = a(1-k^2)/(1-k\cos(n\varphi)) \tag{9}$$

Where: $n = 2$, k——Eccentricity, a——Radius of long axis.

The pitch curve of the oval gear is symmetrical with the X-axis and Y-axis of cartesian coordinate. For the design convenience, the tooth number $Z = 4C + 2$ (C is positive integer), and the sections at long and short axes should be tooth and alveolus respectively. The design steps of oval gear modeling are shown in Fig. 3. The parameters of the oval gear in this paper: The tooth number $Z = 22$, modulus $m = 5mm$, eccentricity $e = 0.1$, tooth addendum $h_a = 5mm$, tooth height $h = 11.25mm$, tooth width $B = 25mm$, radius (length half axles) $a = 54.728\ mm$, inner radius $r_{in} = 25mm$. The two oval gears are same.

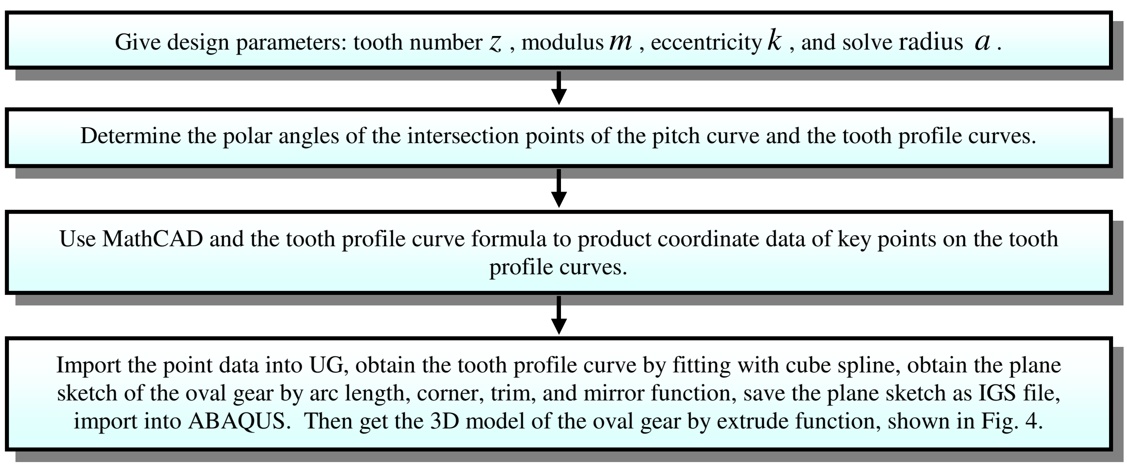


Fig. 3. Design of the oval gear modeling

3.0 Finite Element Model of the Oval Gear

The ABAQUS is one of the most advanced large-scale general finite element software in the world, and has powerful function in big strain, nonlinear (geometry, material and boundary), viscoelastic, dynamic stress, and contact problem fields etc [3].

In this paper, the material of the oval gear is 45 steel, Young's modulus $E = 2.0\times10^{11} N/mm^2$, Poisson's ratio $\mu = 0.3$, and Density $\rho = 7.85\times10^3 kg/m^3$. Create material steel under material module, create the oval gear section, set material steel as property of the oval gear section, and appoint to section of the oval gear.

The boundary condition of finite element model for the oval gear can be set according to the factual working condition. In the meshing process of the oval gears, interference fit is applied between inside surface and axis with spline, the interference fit between axis and the gear can be considered as rigid connection in the finite element model, and the influence of spline is neglected. The tolerance of this simplification is small to dynamic study. In order to reflect the factual condition of gear meshing correctly, the inner surface of the oval gear is restricted, and displacements along X axis, Y axis, Z axis and rotations round with X and Y axis are restricted. In the ABAQUS, the 3D solid unit only has three displacement freedoms. In order to restrict rotation freedom of the oval gear's inner surface, a coupling must be added to couple the inner surface to a point on the center rotating axis of the oval gear, and freedoms of the oval gear's inner surface can be restricted by setting the reference point's freedoms. The boundary condition of the oval gear can be applied to initial step.

The mode is determined by natural property of the gear system, and it is irrespective with outer loads, so it is needless to set the load boundary condition for the oval gear. In course of the meshing, distortion should be reduced farthest, as for the problem that the grids distorts badly, small sized linear reduced integration unit can be used, for the 3D problem, hexahedron unit should be applied utmost, which can get better result with lower cast, the result received form tetrahedron unit is imprecise, so large numbers of units must be applied to get a better result, which makes computing cost increase greatly. According to the principle above, swept meshing technique is applied in the this model, C3D8R unit (8 nodes hexahedron reduced integration unit) is used, and 19600 units and 59433 nodes are received. The finite element model of the oval gear completed is shown in Fig. 5.

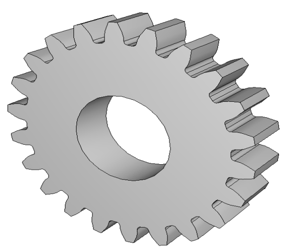


Fig. 4. The 3D model of the oval gear

Fig. 5. The finite element model of the oval gear

4.0 Calculating the Natural Mode and Natural Frequency

The methods of calculating the natural mode and natural frequency. According to the mechanical system dynamics theory and the finite element theory, the movement differential equation of the multi-freedom system can be written as[4].

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{p(t)\} \tag{10}$$

Where $[M]$——Mass matrix, $\{\ddot{u}\}$——Acceleration matrix, $[C]$——Damping matrix, $\{\dot{u}\}$——Velocity matrix, $[K]$——Stiffness matrix, $\{u\}$——Displacement matrix, $\{p(t)\}$——Outer load matrix.

When the damping force is neglected and the system is free of load, the movement differential equation of undamped free vibration system can be written as[4].

$$([K]-\omega^2[M])\{\phi\}=0 \tag{11}$$

Where ω ——Frequencies of system, $\{\phi\}$——Eigenvector of system.

The LANCZOS method and the subspace iterative method are provided to calculate eigenvalue. When the system has many freedoms and plentiful characteristic modes are requested, it is much quicker by applying the LANCZOS method, while few characteristic modes (< 20) are requested, it is much quicker by using subspace iterative method. In this paper, the LANCZOS method is applied.

5.0 Analytical Result

The structure vibration can be expressed as linear combination of each order natural vibration shape, while lower order vibration shape has big influence on structure vibration, and play a decisive role in structure's dynamic character. First 5 to 10 orders are needed only when mode analysis.

In order to explain the dynamic character of the oval gear contrastively, the finite element models of equivalent cylindrical gears (0 degree, 30 degree, 60 degree, 90 degree) are established, and the natural vibrations and natural frequencies are calculated and analyzed. The 1st, 2nd, 3rd, 5th, 7th, 10th mode vibration shapes are shown in Fig. 6 (a) ~ (f), and the vibration shapes and natural frequencies are shown in Table 1.

The Table 1 and Fig. 7 show that the natural frequencies of the oval gear lie between the corresponding order's natural frequencies of the big section (0 degree) and that of the small section (90 degree), they are bigger than that of the big section and smaller than that of the small section. The natural frequency increases with the order increases. The vibration shapes of the oval gear are same with that of equivalent cylindrical gear, but the orders arisen are different. Compared with the cylindrical gear, the frequencies to each order of the oval gear is different obviously, while the frequencies to each order of the cylindrical gear may be same or similar. The reason is that the cylindrical gear is symmetrical with the rotating center absolutely, while the oval gear is symmetrical with the rotating center incompletely.

The Table 1 and Fig. 6 show that the 3rd, 4th and 6th mode vibration shapes are same, the 5th and 9th mode vibration shapes are same, and the 7th and 8th mode vibration shapes are same. The main differences lie in that the vibration directions of each tooth are different.

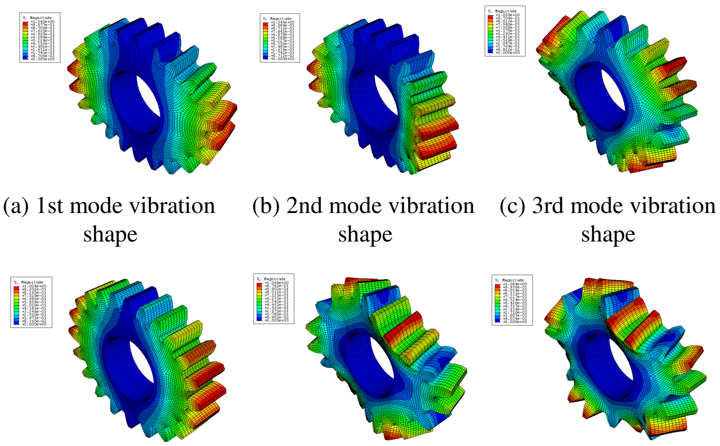
The Fig. 8 shows that the natural frequencies to each order of the equivalent cylindrical gear increase while the polar angle of the oval gear increases and the pitch curve radius decrease. The compared with cylindrical gear, the distance among the amplitudes of each tooth to each mode of the oval gear is quite big. For example, for the 2nd SZ mode, the amplitude of the tooth about the big section (0 degree) is quite big, while the tooth about the small section doesn't vibrate basically.

The Fig. 6 and Table 1 show that the main vibration shapes of the oval gear is the DZ mode and YZ mode, while radial vibration is quite small. So the DZ mode and YZ mode is the vibration shape which is most possible to arouse resonance of the oval gear. In design of the oval gear transmission system, the natural vibration shapes and natural frequencies should be considered adequately, the working frequency should keeps away from the natural frequencies to avoid resonance.

Table 1: Natural frequencies and vibration shapes of the oval gear and the equivalent gears

Model	Order	1	2	3	4	5	6	7	8	9	10
Oval gear	Frequency	12170	12235	14896	15179	16739	18203	19274	20348	22882	23925
	Type	DZ1	SZ	DZ2	DZ2	YZ	DZ2	DZ3	DZ3	YZ	DZ4
0^0	Frequency	11000	11000	11377	11719	11720	15275	15277	17004	17007	20343
	Type	DZ1	DZ1	SZ	DZ2	DZ2	DZ3	DZ3	YZ	YZ	DZ4
30^0	Frequency	12834	12835	13202	13516	13523	16969	16979	18457	18474	21982
	Type	DZ1	DZ1	SZ	DZ2	DZ2	DZ3	DZ3	YZ	YZ	DZ4
60^0	Frequency	17084	17085	17411	17699	17701	20802	20806	21646	21652	25485
	Type	DZ1	DZ1	SZ	DZ2	DZ2	DZ3	DZ3	YZ	YZ	DZ4
90^0	Frequency	19622	19692	19693	20997	20999	24472	24485	24542	24543	29489
	Type	SZ	DZ1	DZ1	DZ2	DZ2	YZ	YZ	DZ3	DZ3	DZ3

DZ1—1st folio vibration,DZ2—2nd folio vibration,SZ—Bevel vibration,DZ3—3rd folio vibration,DZ4—4th folio vibration,YZ—Circle vibration



(a) 1st mode vibration shape (b) 2nd mode vibration shape (c) 3rd mode vibration shape

(d) 5th mode vibration shape (e) 7th mode vibration shape (f) 10th mode vibration shape

Fig. 6. Mode vibration shapes of the oval gear

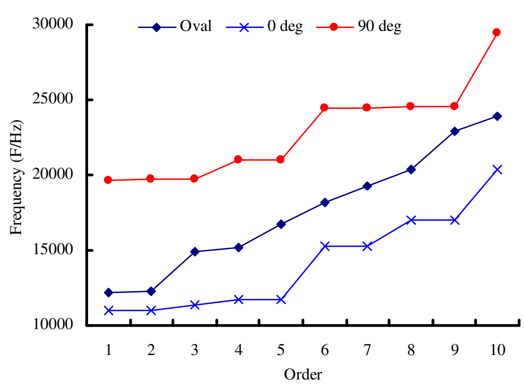


Fig. 7. Relation of frequencies between the oval gear and equivalent gear

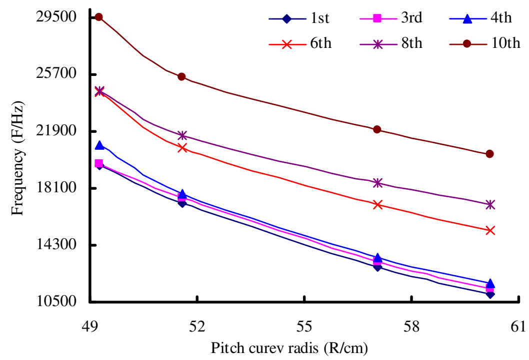


Fig. 8. Relation between frequencies and radius

6.0 Conclusion

- In this paper, the design method of the noncircular gear is studied by using gear transmission technique, the mathematics software MathCAD, the 3D solid modeling software and the finite element software are combined to realize precise model of the oval gear, and a solid foundation is laid for analysis of the oval gear.
- The gear system dynamics principle is introduced to establish dynamics model for the noncircular gear.
- The basic theory of finite element and mode analysis method are applied, the finite element model for the noncircular gear is established, and natural vibration characteristic of the noncircular gear is studied.
- The finite element method is used, the natural vibration characteristic of the oval gear is studied, the main vibration shapes and natural frequencies of the oval gear and that of the equivalent cylindrical gears are analyzed and compared, the conclusions received reflect the dynamics performance of the oval gear, and solid foundation is laid for dynamics research and engineering application of the oval gear.

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