Centralized Moving-Horizon Estimation for A Class of Nonlinear Dynamical Complex Networks under Event-Triggered Transmission Scheme

Ming Gao, Zidong Wang*, Li Sheng, Lei Zou and Hongjian Liu

Abstract

This paper is concerned with the problem of event-triggered centralized moving-horizon state estimation for a class of nonlinear dynamical complex networks. An event-triggered scheme is employed to reduce unnecessary data transmissions between sensors and estimators, where the signal is transmitted only when certain condition is violated. By treating sector-bounded nonlinearities as certain sector-bounded uncertainties, the addressed centralized moving-horizon estimation problem is transformed into a regularized robust least-squares problem that can be effectively solved via existing convex optimization algorithms. Moreover, a sufficient condition is derived to guarantee the exponentially ultimate boundedness of the estimation error, and an upper bound of the estimation error is also presented. Finally, a numerical example is provided to demonstrate the feasibility and efficiency of the proposed estimator design method.

Index Terms

Dynamical complex networks, centralized moving-horizon estimation, event-triggered mechanism, sector-bounded nonlinearity, bounded estimation error.

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I. Introduction

Recently, dynamical complex networks (DCNs) have proven to be a persistent focus of research owing to their potential applications in cyber-physical systems, biological networks, power grid networks and social networks, etc [1]–[6]. Generally, the successes of these applications are largely dependent on the true states of the network nodes in DCNs. Due to reasons such as noisy environment, large scale and tight coupling, the state information of DCNs is usually immeasurable or only partially available in practice. As such, the corresponding state estimation problem has become an imperative task in order to acquire the true states of DCNs, and such a task has gained remarkable research attention from the community. For example, the problem of mixed H_2/H_∞ state estimation has been dealt with in [7] for switched DCNs with redundant channels. By means of the variance-constrained approach, the state estimation problem has been investigated in [8] for time-varying DCNs with missing measurements.

It should be noted that the main results [7], [8] have been obtained based on some assumptions on the underlying DCNs that are not always realistic. For instance, the external noises on DCNs are generally required to be energy-bounded or have Gaussian probability distributions so as to facilitate the utilization of the H_∞ filter or Kalman filter algorithms, respectively. However, the noises in real systems might be norm-bounded rather than energy-bounded [9], and the Gaussian characteristics of noises are rarely met in practice. In the presence of norm-bounded noises, the traditional H_∞ and Kalman filter theories are no longer applicable and, in search of an alternative methodology, the moving-horizon estimation (MHE) appears to be particularly suitable to handle norm-bounded non-Gaussian noises whose summation/integral over time could tend to infinity.

The main idea of MHE algorithm is to derive system states by utilizing the moving yet limited information [10], thereby providing a high degree of robustness with respect to plant uncertainties [11], [12] as well as a practical strategy to handle the constraints of systems in estimation [13], [14]. In [13] and [14], the constrained linear and nonlinear MHEs have been presented to handle the inequality constraints on states and noises of systems, respectively. For unconstrained linear and nonlinear systems, the simpler MHE schemes have been proposed in [11], and the existence of an upper bound on the estimation error (EE) has been proved in [15].

At each time step of the MHE, the scheme for linear systems needs the solution to a quadratic program, while most schemes for nonlinear systems require solving a nonlinear programming problem which would lead to heavy online computations [14], [16]–[20]. The problem for designing the MHE scheme for nonlinear systems with less computational load has not been fully examined and remains open. Furthermore, the stability of MHE is a fundamental problem that has been investigated in [11], [13], [18], since MHE calculates the state estimates by using a finite window of data which avoids the computational burden encountered in implementing full-information estimators. In the existing literature, the MHE problem for DCNs with norm-bounded noises has not been adequately investigated, not to mention the case where the nonlinearities are also involved, and this gives rise to the main motivation of this paper.

Due to the advantages including simple installation, low maintenance costs and reduced weight, the networked control systems (NCSs) have found many applications in various domains such as mobile communications, environmental monitoring and smart grids [19]–[34]. Recently, there have been some results on MHE problems for NCSs against network-induced phenomena such as communication link failures [35], quantization effects [36] and packet dropouts [37]. On the other hand, the event-triggered strategy, whose aim is to transmit the signal only when some meaningful events occur, has stirred considerable interest [54]–[56]. Event-triggered design is known to have the merit of saving energy by reducing unnecessary executions over the shared network [38]–[44], and has therefore attracted much effort in the past years with applications to many engineering systems such as DCNs [45], multi-agent systems [46] and sensor networks [47]. Unfortunately, when it comes to the event-triggered MHE problems for DCNs, the corresponding results have been very few due primarily to the lack of appropriate techniques for dealing with the triggering scheme in the context of MHE design. As such, another motivation for our current investigation is to examine the impact of the event-triggered scheme on the centralized MHE design of DCNs.

Summarizing the above discussions, in this paper, we aim to study the event-triggered centralized MHE problem for a class of nonlinear DCNs. The nonlinearity is supposed to satisfy the sector condition, and the triggering condition is based on the absolute error with respect to the measurement. *The novelties of the paper can be summarized as follows. 1) The first attempt is made to study the problem of event-triggered centralized MHE for DCNs with sector-bounded nonlinearities and norm-bounded noises. 2) By treating sector-bounded nonlinearities as certain sector-bounded uncertainties, the centralized moving-horizon (MH) estimator is designed by solving a regularized robust least-squares (RLS) problem. 3) The exponentially ultimate boundedness is analyzed for the EE and an upper bound of the EE is also presented.*

Notations. $A^\dagger \in \mathbb{R}^{n\times m}$ means the Moore-Penrose pseudo inverse of $A \in \mathbb{R}^{m\times n}$. $\|x\|$ stands for the Euclidean norm of a vector while $\|x\|_P = \sqrt{x^TPx}$ where $P > 0$ is a matrix. For a matrix $Q > 0$, $\underline{\varrho}(Q)$ and $\bar{\varrho}(Q)$ respectively indicate the minimum and maximum eigenvalues of Q. Given a time-varying vector $m(s)$, denote $m_{s-N}^s = \mathrm{col}\{m(s-N), m(s-N+1), \dots, m(s)\}$.

II. Problem Formulation and Preliminaries

Consider the following DCN with M nodes:

$$\begin{cases} x_i(s+1) = f(x_i(s)) + \sum_{j=1}^{M} w_{ij}\Lambda x_j(s) + v_i(s) \\ y_i(s) = C_i x_i(s) + \omega_i(s), \quad i = 1,2,\dots,M, \quad s \in \mathbb{N} \end{cases} \tag{1}$$

where $x_i(s) \in \mathbb{R}^n$ and $y_i(s) \in \mathbb{R}^{n_y}$ respectively denotes the system state and measurement output of the i-th node. $v_i(s) \in \mathcal{V} \triangleq \{v_i : \|v_i\| \le v_i^{\max}; v_i \in \mathbb{R}^{n_v}\}$ and $\omega_i(s) \in \mathcal{W} \triangleq \{\omega_i : \|\omega_i\| \le \omega_i^{\max}; \omega_i \in \mathbb{R}^{n_\omega}\}$ represent the system noise and the measurement noise of the i-th node, respectively. $\Lambda = \mathrm{diag}\{\nu_1, \nu_2, \dots, \nu_n\}$ is the inner-coupling matrix, and $W = (w_{ij})_{M\times M}$ is the coupling configuration matrix of the network with $w_{ij} \ge 0$ $(i \ne j)$, and C_i is a constant matrix of appropriate dimension. It is assumed that the nonlinear function $f(\cdot)$ satisfies the sector-bounded condition

$$[f(m) - f(n) - U_1(m-n)]^T[f(m) - f(n) - U_2(m-n)] \le 0, \quad \forall m, n \in \mathbb{R}^n,\ f(0) = 0 \tag{2}$$

where U_1 and U_2 are real matrices of proper dimensions with $U_1 > U_2$.

In order to characterize the event triggering mechanism, let the triggering time sequence be $0 \leq k_0 < k_1 < k_2 < \cdots$ and define $y_i^t(s) = y_i(k_m^i)$ for $s \in [k_m^i, k_{m+1}^i)$. The current measurement will be transmitted to the estimator if

$$h_i(y_i^t(s), y_i(s), \delta_i) = (y_i^t(s) - y_i(s))^T (y_i^t(s) - y_i(s)) - \delta_i > 0 \tag{3}$$

where $\delta_i > 0$ is a given threshold. The next triggered instant can be determined by

$$k_{m+1} = \min\{s \in \mathbb{N} | s > k_m,\ h_i(y_i^t(s), y_i(s), \delta_i) > 0\}. \tag{4}$$

Defining $\sigma_i(s) = y_i^t(s) - y_i(s)$, we have

$$y_i^t(s) = y_i(s) + \sigma_i(s) = C_i x_i(s) + \omega_i(s) + \sigma_i(s). \tag{5}$$

Denote

$$\begin{aligned}
x(s) &= [x_1^T(s)\ \ x_2^T(s)\ \ \cdots\ \ x_M^T(s)]^T, \quad F(x(s)) = [f^T(x_1(s))\ \ f^T(x_2(s))\ \ \cdots\ \ f^T(x_M(s))]^T, \\
v(s) &= [v_1^T(s)\ \ v_2^T(s)\ \ \cdots\ \ v_M^T(s)]^T, \quad \omega(s) = [\omega_1^T(s)\ \ \omega_2^T(s)\ \ \cdots\ \ \omega_M^T(s)]^T, \\
\sigma(s) &= [\sigma_1^T(s)\ \ \sigma_2^T(s)\ \ \cdots\ \ \sigma_M^T(s)]^T, \quad y^t(s) = [(y_1^t(s))^T\ \ (y_2^t(s))^T\ \ \cdots\ \ (y_M^t(s))^T]^T, \\
\mathcal{C} &= \mathrm{diag}\{C_1, C_2, \ldots, C_M\}, \quad \mathscr{U}_1 = \mathrm{diag}\{\underbrace{U_1, U_1, \ldots, U_1}_{M}\}, \quad \mathscr{U}_2 = \mathrm{diag}\{\underbrace{U_2, U_2, \ldots, U_2}_{M}\}.
\end{aligned}$$

Then, the system (1) can be rewritten as

$$\begin{cases} x(s+1) = F(x(s)) + (W \otimes \Lambda)x(s) + v(s) \\ \quad y^t(s) = \mathcal{C}x(s) + \omega(s) + \sigma(s), \end{cases} \tag{6}$$

where $\otimes$ represents Kronecker product. In the system (6), the network topology W affects the observability of the nonlinear DCNs deeply. Considering the definition of observability for nonlinear systems in [48], even if each node in DCNs is unobservable, the nonlinear DCN (6) would be locally weakly observable owing to the coupling relation between different nodes.

From (2), one has

$$[F(m) - F(n) - \mathscr{U}_1(m-n)]^T [F(m) - F(n) - \mathscr{U}_2(m-n)] \leq 0,\ \forall m, n \in \mathbb{R}^{nM},\ F(0) = 0. \tag{7}$$

Define $G(x(s)) = F(x(s)) + (W \otimes \Lambda)x(s)$, and it is easy to find that $G(x(s))$ satisfies the following condition

$$[G(m) - G(n) - \mathscr{V}_1(m-n)]^T [G(m) - G(n) - \mathscr{V}_2(m-n)] \leq 0,\ \forall m, n \in \mathbb{R}^{nM},\ G(0) = 0 \tag{8}$$

where $\mathscr{V}_1 = \mathscr{U}_1 + W \otimes \Lambda$ and $\mathscr{V}_2 = \mathscr{U}_2 + W \otimes \Lambda$.

In this paper, the MH strategy will be used to design an event-triggered estimator for DCNs with sector-bounded nonlinearities. More specifically, at any time $s = N, N+1, \ldots,$ the aim is to estimate $x(s-N),\ x(s-N+1),\ \ldots,\ x(s)$ based on the past measurements $\{y^t(s-N),\ y^t(s-N+1),\ \ldots,\ y^t(s)\}$ and a prediction state $\bar{x}(s-N)$ of the state $x(s-N)$. Let $\hat{x}(s-N),\ \hat{x}(s-N+1),\ \ldots,\ \hat{x}(s)$ be the estimates of $x(s-N),\ x(s-N+1),\ \ldots,\ x(s)$, respectively.

Consider the following cost function

$$J(\hat{x}(s-N)) = \|\hat{x}(s-N) - \bar{x}(s-N)\|_P^2 + \sum_{j=s-N}^{s} \left\| y^t(j) - C\hat{x}(j) \right\|^2 \tag{9}$$

where $P > 0$ is the matrix to be designed and N is the moving-horizon size.

For given $\{\bar{x}(s-N), \{y^t(j)\}_{s-N \le j \le k}\}$ and the triggering frequency δ_i ($i = 1, 2, \ldots, M$), the state estimate $\hat{x}(s-N)$ can be derived by suppressing the cost function (9) subject to

$$\begin{cases} \hat{x}(j+1) = G(\hat{x}(j)), \quad j = s-N, s-N+1, \ldots, s-1 \\ \bar{x}(s-N) = G(\hat{x}(s-N-1)), \quad s = N+1, N+2, \ldots. \end{cases} \tag{10}$$

Generally, it is difficult to solve the above centralized MHE problem for nonlinear systems, and an alternative centralized MHE problem will be introduced by extending the robust MHE in [12]. Denoting $u(s) = \hat{x}(s) - \bar{x}(s)$ and according to (8), one has

$$[G(x(s)) - G(\hat{x}(s)) - \mathscr{V}_1 u(s)]^T [G(x(s)) - G(\hat{x}(s)) - \mathscr{V}_2 u(s)] \le 0 \tag{11}$$

which can be rewritten as

$$\begin{aligned} &\left[G(x(s)) - G(\hat{x}(s)) - \left(\frac{\mathscr{V}_1 + \mathscr{V}_2}{2} + \frac{\mathscr{V}_1 - \mathscr{V}_2}{2}\right) u(s)\right]^T \\ &\times \left[G(x(s)) - G(\hat{x}(s)) - \left(\frac{\mathscr{V}_1 + \mathscr{V}_2}{2} - \frac{\mathscr{V}_1 - \mathscr{V}_2}{2}\right) u(s)\right] \le 0. \end{aligned}$$

Denoting $\mathcal{A} = \frac{\mathscr{V}_1 + \mathscr{V}_2}{2}$, $\bar{\mathcal{A}} = \frac{\mathscr{V}_1 - \mathscr{V}_2}{2}$, $\Psi(s) = G(x(s)) - G(\hat{x}(s)) - \mathcal{A}u(s)$, we have $\Psi^T(s)\Psi(s) \le u^T(s)\bar{\mathcal{A}}^T\bar{\mathcal{A}}u(s)$. Then, the sector-bounded nonlinear term $G(x(s)) - G(\hat{x}(s))$ can be written as

$$G(x(s)) - G(\hat{x}(s)) = \mathcal{A}u(s) + \Delta\mathcal{A}(s)\bar{\mathcal{A}}u(s) \tag{12}$$

where $(\Delta\mathcal{A}(s))^T \Delta\mathcal{A}(s) \le I$. Therefore, one has

$$y^t(j) - \mathcal{C}\hat{x}(j) = m(j) + \mathcal{C} \prod_{l=s-N}^{j-1} (\mathcal{A} + \Delta\mathcal{A}(l)\bar{\mathcal{A}}) u(s-N), \quad j = s-N, s-N+1, \ldots, s \tag{13}$$

where

$$m(j) = y^t(j) - \mathcal{C}G^{(j-s+N)}(\bar{x}(s-N)), \tag{14}$$

with $G^{k+1}(\bar{x}(s-N)) = G\big(G^k(\bar{x}(s-N))\big)$, $G^0(\bar{x}(s-N)) = \bar{x}(s-N)$ and $\prod_{l=s-N}^{s-N-1}(\mathcal{A} + \Delta\mathcal{A}(l)\bar{\mathcal{A}}) = 1$.

Denote

$$m_{s-N}^s = \begin{bmatrix} m(s-N) \\ m(s-N+1) \\ \vdots \\ m(s) \end{bmatrix}, \quad \mathcal{F}_N = \begin{bmatrix} \mathcal{C} \\ \mathcal{C}\mathcal{A} \\ \vdots \\ \mathcal{C}\mathcal{A}^N \end{bmatrix}, \quad \mathscr{F}_N\left(\Delta\mathcal{A}_{s-N}^{s-1}\right) = \begin{bmatrix} \mathcal{C} \\ \mathcal{C}(\mathcal{A} + \Delta\mathcal{A}(s-N)\bar{\mathcal{A}}) \\ \vdots \\ \mathcal{C}\prod_{l=1}^{N}(\mathcal{A} + \Delta\mathcal{A}(s-l)\bar{\mathcal{A}}) \end{bmatrix}.$$

Next, we write $\mathscr{F}_N(s)$ instead of $\mathscr{F}_N\left(\Delta\mathcal{A}_{s-N}^{s-1}\right)$ for the sake of brevity. Let $u(s-N) = \hat{x}(s-N) - \bar{x}(s-N)$, and the cost function (9) can be written as

$$\begin{aligned} J(\hat{x}(s-N)) &= \|\hat{x}(s-N) - \bar{x}(s-N)\|_P^2 + \sum_{j=s-N}^{s} \left\| m(j) + \mathcal{C} \prod_{l=s-N}^{j-1} (\mathcal{A} + \Delta\mathcal{A}(l)\bar{\mathcal{A}}) u(s-N) \right\|^2 \\ &= \|u(s-N)\|_P^2 + \left\| m_{s-N}^s + \mathscr{F}_N(s) u(s-N) \right\|^2. \end{aligned} \tag{15}$$

According to the boundedness of $\Delta\mathcal{A}(s)$, there exists a positive definite matrix Υ such that

$$(\mathscr{F}_N(s) - \mathcal{F}_N)^T (\mathscr{F}_N(s) - \mathcal{F}_N) \leq \Upsilon. \tag{16}$$

For simplicity, we choose the matrix $\Upsilon = \rho^2 I$, where $\rho = \max_{\Delta\mathcal{A}_{s-N}^{s-1}} \|\mathscr{F}_N(s) - \mathcal{F}_N\|$. As shown in Proposition 1 of [12], there exists a suitable matrix $\Phi(s)$ such that

$$\|\Phi(s)\| \leq 1, \qquad \mathscr{F}_N(s) - \mathcal{F}_N = \Phi(s)\Upsilon^{\frac{1}{2}}. \tag{17}$$

Therefore, from (15) and (17), we have

$$\begin{aligned} J(\hat{x}(s-N)) &= \|u(s-N)\|_P^2 + \left\| m_{s-N}^k + (\mathcal{F}_N + \mathscr{F}_N(s) - \mathcal{F}_N) u(s-N) \right\|^2 \\ =& J_1(\hat{x}(s-N), \Phi(s)) \triangleq \|u(s-N)\|_P^2 + \left\| m_{s-N}^s + \left(\mathcal{F}_N + \Phi(s)\Upsilon^{\frac{1}{2}}\right) u(s-N) \right\|^2. \end{aligned} \tag{18}$$

The following robust centralized MHE problem is introduced in this paper.

Problem 1: For given $\{\bar{x}(s-N), \{y^t(j)\}_{s-N \leq j \leq s}\}$ and the triggering frequency δ_i $(i = 1, 2, \ldots, M)$, find the optimal estimate $\hat{x}(s-N)$ such that

$$\hat{x}(s-N) = \arg \min_{\hat{x}(s-N)} \max_{\|\Phi(s)\| \leq 1} J_1\left(\hat{x}(s-N), \Phi(s)\right) \tag{19}$$

subject to the constraint (10).

Remark 1: In general, the MHE scheme for nonlinear systems requires the solution to a nonlinear programming problem at each time step, which results in heavy computational loads. The nonlinear function considered in this paper is known as the sector-like condition, which is general and could cover several classes of nonlinearities as special cases. Furthermore, the condition (8) can be transformed into the sector-bounded uncertainty (12). As such, the problem for MHE with sector-bounded nonlinearities is formulated in the form of a regularized RLS problem [49].

Remark 2: For discrete-time linear systems, the MHE problem has been well dealt with in [11], [19], [20] by solving a quadratic program at each time step. For linear systems with sector-bounded uncertainties, the problem of MHE has been studied in [12], and the robustness has been achieved by solving a RLS problem. From Remark 1, the results of this paper can be viewed as an extension of the works in [11], [12] to DCNs with the event-triggered scheme and sector-bounded nonlinearities in some sense.

Our objective of this paper is to design a MH estimator for the DCN (1) by solving *Problem 1* at each time step. Furthermore, the stability properties of the EE will be analyzed.

III. Main Results

In this section, the MH estimator is designed for DCNs with sector-bounded nonlinearities, and the property of the estimator is analyzed. The following lemma is needed in this paper.

Lemma 1: [49] Consider the following regularized RLS problem

$$\min_{u}\max_{\|S\|\leq 1}\left\{\|u\|_P^2+\|(K+\Delta K)u-(L+\Delta L)\|_R^2\right\} \tag{20}$$

where $\Delta K = HSE_K$, $\Delta L = HSE_L$ with H, E_K and E_L being known matrices, and S denoting an arbitrary contraction. Then, problem (20) has a unique global minimum u^* given by

$$u^* = (\hat{P}+K^T\hat{R}K)^{-1}\Big(K^T\hat{R}L+\lambda^* E_K^T E_L\Big) \tag{21}$$

where $\hat{P} = P+\lambda^* E_K^T E_K$ and $\hat{R} = R+RH(\lambda^* I - H^T RH)^{\dagger} H^T R$. The scalar λ^* is determined as

$$\lambda^* = \arg\min_{\lambda\geq\|H^TRH\|}\left\{\|u(\lambda)\|_P^2+\lambda\|E_K u(\lambda)-E_L\|^2+\|Ku(\lambda)-L\|^2_{\hat{R}(\lambda)}\right\} \tag{22}$$

where

$$\begin{aligned}
u(\lambda) &= \Big(\hat{P}(\lambda)+K^T\hat{R}(\lambda)K\Big)^{-1}\Big(K^T\hat{R}(\lambda)L+\lambda E_K^T E_L\Big),\\
\hat{P}(\lambda) &= P+\lambda E_K^T E_K,\\
\hat{R}(\lambda) &= R+RH(\lambda I-H^TRH)^{\dagger}H^TR.
\end{aligned}$$

A. Centralized Moving-Horizon Estimator

This subsection is concerned with the design of the centralized MHE scheme for DCNs based on the actual arrival measurements $\{y^t(s-N),\ y^t(s-N+1),\ \ldots,\ y^t(s)\}$, where $N+1$ is called the length of observations in a moving window from $s-N$ to s. Then, the following theorem can be easily accessible from Lemma 1.

Theorem 1: Given $\{\bar{x}(s-N),\{y^t(j)\}_{s-N\leq j\leq s}\}$, the triggering frequency δ_i $(i=1,2,\ldots,M)$, *Problem 1* has a solution given by

$$\hat{x}(s-N) = \bar{x}(s-N)+\Big(\hat{\mathcal{P}}(\lambda^*)+\mathcal{F}_N^T\hat{\mathcal{R}}(\lambda^*)\mathcal{F}_N\Big)^{-1}\mathcal{F}_N^T\hat{\mathcal{R}}(\lambda^*)m_{s-N}^s \tag{23}$$

where $\hat{\mathcal{P}}(\lambda^*) = \mathcal{P}+\lambda^*\Upsilon$, $\hat{\mathcal{R}}(\lambda^*) = I+(\lambda^* I-I)^{\dagger}$ and the scalar λ^* is the unique solution to the following optimization problem

$$\lambda^* = \arg\min_{\lambda\geq 1}\left\{\|u_{s-N}(\lambda)\|^2_{\mathcal{P}}+\lambda\|\Upsilon^{\frac{1}{2}}u_{s-N}(\lambda)\|^2+\|\mathcal{F}_N u_{s-N}(\lambda)+k_{s-N}^s\|^2_{\hat{\mathcal{R}}(\lambda)}\right\} \tag{24}$$

with

$$\begin{aligned}
u_{s-N}(\lambda) &= \Big(\hat{\mathcal{P}}(\lambda)+\lambda\Upsilon\Big)^{-1}\mathcal{F}_N^T\hat{\mathcal{R}}(\lambda)m_{s-N}^s, \qquad (25)\\
\hat{\mathcal{P}}(\lambda) &= \mathcal{P}+\lambda\rho^2\mathcal{F}_N^T\mathcal{F}_N, \quad \hat{\mathcal{R}}(\lambda) = I+(\lambda I-I)^{\dagger}.
\end{aligned}$$

Proof: Denoting

$$K=\mathcal{F}_N,\quad L=-m_{s-N}^s,\quad H=I,\quad S=\Phi(s),\quad E_K=\Upsilon^{\frac{1}{2}},\quad E_L=0,$$

the cost function (18) can be rewritten as

$$J_1(\hat{x}(s-N),\Phi(s)) = \|u(s-N)\|^2_{\mathcal{P}} + \|(K+\Delta K)u(s-N) - (L+\Delta L)\|^2 \tag{26}$$

where $\Delta K = HSE_K = \Phi(s)\Upsilon^{\frac{1}{2}}$, $\Delta L = 0$. According to Lemma 1, the solution to *Problem 1* is as follows

$$\begin{aligned} u(s-N) &= \big(\hat{\mathcal{P}}(\lambda^*) + K^T\hat{\mathcal{R}}(\lambda^*)K\big)^{-1}\big(K^T\hat{\mathcal{R}}(\lambda^*)L + \lambda^* E_K^T E_L\big) \\ &= -\big(\hat{\mathcal{P}}(\lambda^*) + \mathcal{F}_N^T\hat{\mathcal{R}}(\lambda^*)\mathcal{F}_N\big)^{-1}\mathcal{F}_N^T\hat{\mathcal{R}}(\lambda^*)m^s_{s-N} \end{aligned} \tag{27}$$

where $\hat{\mathcal{P}}(\lambda^*)$ and $\hat{\mathcal{R}}(\lambda^*)$ are the same as defined in (23). Considering $u(s-N) = \bar{x}(s-N) - \hat{x}(s-N)$, we have

$$\hat{x}(s-N) = \bar{x}(s-N) + \Big(\hat{\mathcal{P}}(\lambda^*) + \mathcal{F}_N^T\hat{\mathcal{R}}(\lambda^*)\mathcal{F}_N\Big)^{-1}\mathcal{F}_N^T\hat{\mathcal{R}}(\lambda^*)m^s_{s-N} \tag{28}$$

which is equivalent to (23). From Lemma 1, the scalar λ^* is determined by

$$\begin{aligned} \lambda^* &= \arg\min_{\lambda\geq 1}\Big\{\|u_{s-N}(\lambda)\|^2_{\mathcal{P}} + \lambda\|E_K u_{s-N}(\lambda) - E_L\|^2 + \|K u_{s-N}(\lambda) - L\|^2_{\hat{\mathcal{R}}(\lambda)}\Big\} \\ &= \arg\min_{\lambda\geq 1}\Big\{\|u_{s-N}(\lambda)\|^2_{\mathcal{P}} + \lambda\|\Upsilon^{\frac{1}{2}} u_{s-N}(\lambda)\|^2 + \|\mathcal{F}_N u_{s-N}(\lambda) + k^s_{s-N}\|^2_{\hat{\mathcal{R}}(\lambda)}\Big\} \end{aligned}$$

where $u_{s-N}(\lambda)$ is the same as defined in (25). Therefore, the unique solution $\hat{x}(s-N)$ to (19) can be obtained by (23) with the scalar λ^* in (24). Then, this theorem is proved. ■

Remark 3: If the boundary point $\lambda = 1$ is excluded from the minimization in (24), by following [49], the pseudo-inverse operation in $\hat{\mathcal{R}}(\lambda^*)$ can be solved as $\hat{\mathcal{R}}(\lambda^*) = \frac{\lambda^*}{\lambda^*-1}I$ and (23) can be rewritten as

$$\hat{x}(s-N) = \bar{x}(s-N) + \frac{\lambda^*}{\lambda^*-1}\left(\mathcal{P} + \lambda^*\Upsilon + \frac{\lambda^*}{\lambda^*-1}\mathcal{F}_N^T\mathcal{F}_N\right)^{-1}\mathcal{F}_N^T m^s_{s-N}. \tag{29}$$

The above estimator is time-varying due to the existence of the parameter λ^* which requires the solution to a one-dimensional optimization problem at each time step. If this estimator is infeasible, as done in [49], one can set $\lambda^* = 1+\tau$, where τ can be tuned off-line by simulations. In this case, the solution to *Problem 1* is given by the following approximate MH estimator

$$\hat{x}(s-N) = \bar{x}(s-N) + \Theta(\tau)m^s_{s-N} \tag{30}$$

where $\Theta(\tau) = \frac{1+\tau}{\tau}\left(\mathcal{P} + (1+\tau)\Upsilon + \frac{1+\tau}{\tau}\mathcal{F}_N^T\mathcal{F}_N\right)^{-1}\mathcal{F}_N^T$.

B. Estimation Properties of the Approximate MHE

In this subsection, we will study the boundedness of the EE for the proposed approximate MH estimator (30). Defining $e(k-N) = x(k-N) - \bar{x}(k-N)$ and considering the definition of $m(k-N)$ in (14), one has

$$\begin{aligned} m(s-N) =& \mathcal{C}x(s-N) + \omega(s-N) + \sigma(s-N) - \mathcal{C}\bar{x}(s-N) \\ =& \mathcal{C}\tilde{\mathcal{A}}(s-N-1)e(s-N-1) + \mathcal{C}v(s-N-1) + \omega(s-N) + \sigma(s-N) \end{aligned}$$

where $\tilde{\mathcal{A}}(s-N-1)=\mathcal{A}+\Delta\mathcal{A}(s-N-1)\bar{\mathcal{A}}$. Moreover, for $0\leq j\leq N-1$, we have

$$m(s-j)=\mathcal{C}\prod_{l=j+1}^{N+1}\tilde{\mathcal{A}}(s-l)e(s-N-1)+\mathcal{C}\sum_{l=j+1}^{N+1}\prod_{k=j+1}^{l-1}\tilde{\mathcal{A}}(s-k)v(s-l)+\omega(s-j)+\sigma(s-j)$$

with $\prod_{k=j+1}^{j}\tilde{\mathcal{A}}(s-k)=1$. Therefore, we obtain

$$m_{s-N}^{s}=\mathscr{F}_N(s)\tilde{\mathcal{A}}(s-N-1)e(s-N-1)+\mathscr{H}_N(s)v_{s-N-1}^{s-1}+\omega_{s-N}^{s}+\sigma_{s-N}^{s} \tag{31}$$

where

$$\mathscr{H}_N(s)=\begin{bmatrix}\mathcal{C} & 0 & \cdots & 0\\ \mathcal{C}\tilde{\mathcal{A}}(s-N) & \mathcal{C} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ \mathcal{C}\prod_{l=1}^{N}\tilde{\mathcal{A}}(s-l) & \mathcal{C}\prod_{i=1}^{N-1}\tilde{\mathcal{A}}(s-l) & \cdots & \mathcal{C}\end{bmatrix},\quad v_{s-N-1}^{s-1}=\begin{bmatrix}v(s-N-1)\\ v(s-N)\\ \vdots\\ v(s-1)\end{bmatrix},$$

$$\omega_{s-N}^{s}=\begin{bmatrix}\omega(s-N)\\ \omega(s-N+1)\\ \vdots\\ \omega(s)\end{bmatrix},\quad \sigma_{s-N}^{s}=\begin{bmatrix}\sigma(s-N)\\ \sigma(s-N+1)\\ \vdots\\ \sigma(s)\end{bmatrix}.$$

The EE system can be derived following from (30) and (31)

$$\begin{aligned}e(s-N)=&\Big(\tilde{\mathcal{A}}(s-N-1)-\Theta(\tau)\mathscr{F}_N(s)\tilde{\mathcal{A}}(s-N-1)\Big)e(s-N-1)+\big(\mathcal{I}-\Theta(\tau)\mathscr{H}_N(s)\big)v_{s-N-1}^{s-1}\\ &-\Theta(\tau)\omega_{s-N}^{s}-\Theta(\tau)\sigma_{s-N}^{s}\end{aligned} \tag{32}$$

where $\mathcal{I}=[I\ \ 0\ \ 0\ \cdots\ 0]$. Furthermore, the system (32) can be rewritten as

$$e(s-N)=\big(\mathcal{B}+\Delta\mathcal{B}(s)\big)e(s-N-1)+\big(\mathcal{D}+\Delta\mathcal{D}(s)\big)\tilde{v}(s) \tag{33}$$

where

$$\begin{aligned}&\mathcal{B}=\mathcal{A}-\Theta(\tau)\mathcal{F}_N\mathcal{A},\quad \Delta\mathcal{B}(s)=\Delta\mathcal{A}(s-N-1)\bar{\mathcal{A}}-\Theta(\tau)\big(\mathscr{F}_N(s)\tilde{\mathcal{A}}(s-N-1)-\mathcal{F}_N\mathcal{A}\big),\\ &\mathcal{D}=\Big[\mathcal{I}-\Theta(\tau)\mathcal{H}_N\ \ -\Theta(\tau)\ \ -\Theta(\tau)\Big],\quad \Delta\mathcal{D}(s)=\Big[-\Theta(\tau)\big(\mathscr{H}_N(s)-\mathcal{H}_N\big)\ \ 0\ \ 0\Big],\\ &\tilde{v}(s)=\begin{bmatrix}v_{s-N-1}^{s-1}\\ w_{s-N}^{s}\\ \sigma_{s-N}^{s}\end{bmatrix},\quad \mathcal{H}_N=\begin{bmatrix}\mathcal{C} & 0 & \cdots & 0\\ \mathcal{C}\mathcal{A} & \mathcal{C} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ \mathcal{C}\mathcal{A}^N & \mathcal{C}\mathcal{A}^{N-1} & \cdots & \mathcal{C}\end{bmatrix}.\end{aligned}$$

By using the similar procedure in (16), one has

$$\Delta\mathcal{B}^T(s)\Delta\mathcal{B}(s)\leq\Upsilon_B,\quad \Delta\mathcal{D}^T(s)\Delta\mathcal{D}(s)\leq\Upsilon_D \tag{34}$$

where Υ_B and Υ_D are known positive definite matrices. The possible choices of Υ_B and Υ_D are $\Upsilon_B=\rho_B^2 I$ and $\Upsilon_D=\rho_D^2 I$, where $\rho_B=\max_{\Delta A_{s-N-1}^{s-1}}\|\Delta\mathcal{B}(s)\|$, $\rho_D=\max_{\Delta A_{s-N}^{s-1}}\|\Delta\mathcal{D}(s)\|$.

Definition 1: The dynamics of the system (33) is exponentially ultimately bounded (EUB) if there exist two scalars $\alpha > 0$ and $\beta \in [0 \;\; 1)$ such that

$$\|e(s)\|^2 < \alpha\beta^s + \gamma \tag{35}$$

where γ represents the upper bound of $\|e(s)\|^2$.

Theorem 2: Consider the DCN (1) and assume that there exist matrices $\mathcal{Q} > 0$, $\mathcal{S} > 0$ and scalars $\epsilon_B > 0$, $\epsilon_D > 0$, $\theta > 0$ satisfying

$$\tilde{\Pi} = \begin{bmatrix} \mathcal{B}^T\mathcal{Q}\mathcal{B} + \mathcal{S} + \epsilon_B\Upsilon_B - \mathcal{Q} & \mathcal{B}^T\mathcal{Q}\mathcal{D} & \mathcal{B}^T\mathcal{Q} & \mathcal{B}^T\mathcal{Q} \\ * & \mathcal{D}^T\mathcal{Q}\mathcal{D} + \epsilon_D\Upsilon_D - \theta I & \mathcal{D}^T\mathcal{Q} & \mathcal{D}^T\mathcal{Q} \\ * & * & \mathcal{Q} - \epsilon_B I & \mathcal{Q} \\ * & * & * & \mathcal{Q} - \epsilon_D I \end{bmatrix} < 0. \tag{36}$$

Then, the EE system (33) is EUB, and the upper bound of the EE can be calculated by

$$\frac{1}{\underline{\varrho}(\mathcal{Q})\mu}\theta(N+1)M(v_*^2 + \omega_*^2 + \delta_*)$$

with

$$\mu = \frac{\underline{\varrho}(\mathcal{S})}{\bar{\varrho}(\mathcal{Q})}.$$

Proof: Construct the following Lyapunov-like function

$$V(s) = e^T(s-N-1)\mathcal{Q}e(s-N-1). \tag{37}$$

Along the EE system (33), the difference of $V(s)$ is calculated as

$$\begin{aligned} V(s+1) - V(s) =& e^T(s-N)\mathcal{Q}e(s-N) - e^T(s-N-1)\mathcal{Q}e(s-N-1) \\ =& \Big[\big(\mathcal{B} + \Delta\mathcal{B}(s)\big)e(s-N-1) + \big(\mathcal{D} + \Delta\mathcal{D}(s)\big)\tilde{v}(s)\Big]^T\mathcal{Q}\Big[\big(\mathcal{B} + \Delta\mathcal{B}(s)\big)e(s-N-1) \\ &+ \big(\mathcal{D} + \Delta\mathcal{D}(s)\big)\tilde{v}(s)\Big] - e^T(s-N-1)\mathcal{Q}e(s-N-1) \\ =& \zeta^T(s)\Pi\zeta(s) \end{aligned} \tag{38}$$

where

$$\zeta(s) = \begin{bmatrix} e(s-N-1) \\ \tilde{v}(s) \\ \Delta\mathcal{B}(s)e(s-N-1) \\ \Delta\mathcal{D}(s)\tilde{v}(s) \end{bmatrix}, \quad \Pi = \begin{bmatrix} \mathcal{B}^T\mathcal{Q}\mathcal{B} - \mathcal{Q} & \mathcal{B}^T\mathcal{Q}\mathcal{D} & \mathcal{B}^T\mathcal{Q} & \mathcal{B}^T\mathcal{Q} \\ * & \mathcal{D}^T\mathcal{Q}\mathcal{D} & \mathcal{D}^T\mathcal{Q} & \mathcal{D}^T\mathcal{Q} \\ * & * & \mathcal{Q} & \mathcal{Q} \\ * & * & * & \mathcal{Q} \end{bmatrix}.$$

Adding the zero term

$$\begin{aligned} &\epsilon_B e^T(s-N-1)\Delta B^T(s)\Delta\mathcal{B}(s)e(s-N-1) - \epsilon_B e^T(s-N-1)\Delta\mathcal{B}^T(s)\Delta\mathcal{B}(s)e(s-N-1) \\ &+\epsilon_D\tilde{v}^T(s)\Delta\mathcal{D}^T(s)\Delta\mathcal{D}(s)\tilde{v}(s) - \epsilon_D\tilde{v}^T(s)\Delta\mathcal{D}^T(s)\Delta\mathcal{D}(s)\tilde{v}(s) + \theta\tilde{v}^T(s)\tilde{v}(s) - \theta\tilde{v}^T(s)\tilde{v}(s) \\ &+\mu V(s) - \mu V(s) = 0 \end{aligned}$$

to the right side of (38) and considering (34), we obtain

$$\begin{aligned}
&V(s+1)-V(s)\\
=&\zeta^T(s)\Pi\zeta(s)+\epsilon_D\tilde{v}^T(s)\Delta\mathcal{D}^T(s)\Delta\mathcal{D}(s)\tilde{v}(s)-\epsilon_D\tilde{v}^T(s)\Delta\mathcal{D}^T(s)\Delta\mathcal{D}(s)\tilde{v}(s)+\theta\tilde{v}^T(s)\tilde{v}(s)-\theta\tilde{v}^T(s)\tilde{v}(s)\\
&+\epsilon_B e^T(s-N-1)\Delta\mathcal{B}^T(s)\Delta\mathcal{B}(s)e(s-N-1)-\epsilon_B e^T(s-N-1)\Delta\mathcal{B}^T(s)\Delta\mathcal{B}(s)e(s-N-1)\\
&+\mu V(s)-\mu V(s)\\
\le&\zeta^T(s)\Pi\zeta(s)+\epsilon_D\tilde{v}^T(s)\Upsilon_D\tilde{v}(s)+\theta\tilde{v}^T(s)\tilde{v}(s)+\epsilon_B e^T(s-N-1)\Upsilon_B e(s-N-1)\\
&-\epsilon_D\tilde{v}^T(s)\Delta\mathcal{D}^T(s)\Delta\mathcal{D}(s)\tilde{v}(s)-\epsilon_B e^T(s-N-1)\Delta\mathcal{B}^T(s)\Delta\mathcal{B}(s)e(s-N-1)-\theta\tilde{v}^T(s)\tilde{v}(s)\\
&+\mu V(s)-\mu V(s)\\
\le&\zeta^T(s)\tilde{\Pi}\zeta(s)-\mu V(s)+\theta\tilde{v}^T(s)\tilde{v}(s) \qquad (39)
\end{aligned}$$

where $\mathcal{S}=\mu\mathcal{Q}$ in $\tilde{\Pi}$ in the formula (36).

Denoting

$$v_*=\max_{1\le i\le M} v_i^{\max},\quad \omega_*=\max_{1\le i\le M}\omega_i^{\max},\quad \delta_*=\max_{1\le i\le M}\delta_i$$

and considering (36), it follows from (39) that

$$V(s+1)-V(s)<-\mu V(s)+\theta(N+1)M(v_*^2+\omega_*^2+\delta_*). \qquad (40)$$

For any scalar $\xi>0$, one has

$$\begin{aligned}
\xi^{s+1}V(s+1)-\xi^sV(s)&=\xi^{s+1}(V(s+1)-V(s))+\xi^s(\xi-1)V(s)\\
&<\xi^{s+1}\big[-\mu V(s)+\theta(N+1)M(v_*^2+\omega_*^2+\delta_*)\big]+\xi^s(\xi-1)V(s)\\
&=\xi^s(\xi-\xi\mu-1)V(s)+\xi^{s+1}\theta(N+1)M(v_*^2+\omega_*^2+\delta_*). \qquad (41)
\end{aligned}$$

Setting $\xi=\xi_\circ=\frac{1}{1-\mu}$ and taking the sum on both sides of (41) from 0 to $s-1$, one has

$$\xi^sV(s)-V(0)<\frac{\xi_\circ(1-\xi_\circ^s)}{1-\xi_\circ}\theta(N+1)M(v_*^2+\omega_*^2+\delta_*) \qquad (42)$$

which implies that

$$\begin{aligned}
\xi_\circ^sV(s)<&V(0)+\frac{\xi_\circ(1-\xi_\circ^s)}{1-\xi_\circ}\theta(N+1)M(v_*^2+\omega_*^2+\delta_*)\\
=&V(0)+\frac{\xi_\circ}{1-\xi_\circ}\theta(N+1)M(v_*^2+\omega_*^2+\delta_*)-\frac{\xi_\circ^{s+1}}{1-\xi_\circ}\theta(N+1)M(v_*^2+\omega_*^2+\delta_*). \qquad (43)
\end{aligned}$$

Therefore, we have

$$V(s)<\xi_\circ^{-s}\left[V(0)-\frac{1}{\mu}\theta(N+1)M(v_*^2+\omega_*^2+\delta_*)\right]+\frac{1}{\mu}\theta(N+1)M(v_*^2+\omega_*^2+\delta_*). \qquad (44)$$

Moreover, it can be found that

$$\|e(s-N-1)\|^2<\frac{\xi_\circ^{-s}}{\underline{\varrho}(\mathcal{Q})}\left[V(0)-\frac{1}{\mu}\theta(N+1)M(v_*^2+\omega_*^2+\delta_*)\right]+\frac{1}{\underline{\varrho}(\mathcal{Q})\mu}\theta(N+1)M(v_*^2+\omega_*^2+\delta_*). \qquad (45)$$

According to Definition 1, the EE system (33) is EUB. The proof is complete. ■

Remark 4: In Theorem 2, the stability property of the approximate MH estimator (30) is proved, and the EE system (33) is EUB if the LMI (36) is feasible. Moreover, it can be found that the upper bound of the EE is proportional to the window length $N+1$, the number of coupled nodes M, the maximum values of bounded noises v_*, ω_* and the maximum trigger threshold δ_*.

Remark 5: Recently, the problem of state estimation for DCNs has attracted increasing research interest in the existing literature. For example, the recursive state estimators have been designed in [4] and [8] for DCNs with Gaussian noises. The problem of H_∞ state estimation has been investigated in [7] and [45] for DCNs with energy-bounded noises. However, the above methodologies cannot be applied to state estimation of DCNs with norm-bounded noises, and this problem is solved by employing the centralized MHE approach in this paper.

Remark 6: In [12] and [17], the stabilities of the optimal and approximate MH estimators are discussed under the assumption that the system is quadratically stable. Due to the nonlinear and time-varying properties, the stability of the optimal MH estimator (29) cannot be analyzed by using the similar way as in Theorem 2. However, during the proof of Theorem 2, we do not require the stability of the system, which shows that the results in this paper are less conservative than those in [12] and [17].

Remark 7: In recent years, there has been a large literature on distributed MHE strategies, extending the works in [11], [12] to large-scale networked systems. For example, the problem of distributed MHE has been considered for two-time-scale nonlinear systems [50] and nonlinear constrained systems [51]. Different from these references, in this paper, the MH estimator is designed in a centralized way rather than a distributed way. Actually, the centralized state estimation problem for DCNs has gained ongoing research interest during the past several decades [52], [53]. For the centralized state estimation problem, the impact of the network topology on the system dynamics is reflected through the coupling configuration matrix and the inner-coupling matrix by using the augmentation approach. One of our future research topics is the study of distributed MHE for nonlinear DCNs with the event-triggered scheme.

IV. A Numerical Example

In this section, a numerical example is provided to show the effectiveness of the event-triggered MH estimator for the DCN (1) with sector-bounded nonlinearities. Assume that the DCN (1) is composed of four nodes with $\Lambda = \mathrm{diag}\{0.4, 0.6\}$ and

$$W = \begin{bmatrix} -0.2 & 0 & 0.1 & 0.1 \\ 0 & -0.1 & 0.1 & 0 \\ 0.1 & 0.1 & -0.3 & 0.1 \\ 0.1 & 0 & 0.1 & -0.2 \end{bmatrix}.$$

The nonlinear function is set as

$$f(x_i(s)) = \begin{bmatrix} 0.96x_i^{(1)}(s) + 0.08x_i^{(2)}(s) + 0.02\sin\left(x_i^{(1)}(s)\right)x_i^{(1)}(s) \\ 0.09x_i^{(1)}(s) + 0.85x_i^{(2)}(s) + 0.01\cos\left(x_i^{(2)}(s)\right)x_i^{(1)}(s) + 0.03\cos\left(x_i^{(2)}(s)\right)x_i^{(2)}(s) \end{bmatrix}$$

where $x_i(s) = \left[x_i^{(1)}(s) \;\; x_i^{(2)}(s) \right]^T$ $(i = 1, 2, 3, 4)$ with $x_i^{(k)}(s)$ $(k = 1, 2)$ representing the k-th entry of the vector $x_i(s)$. It is easily verified that $f(x_i(s))$ satisfies the condition (2) with

$$U_1 = \begin{bmatrix} 0.98 & 0.08 \\ 0.1 & 0.88 \end{bmatrix}, \qquad U_2 = \begin{bmatrix} 0.94 & 0.08 \\ 0.08 & 0.82 \end{bmatrix}.$$

The parameters C_i and the noises $v_i(s)$, $\omega_i(s)$ $(i = 1, 2, 3, 4)$ are as follows:

$$C_1 = [1 \;\; 1], \quad C_2 = [0.5 \;\; 1], \quad C_3 = [1 \;\; 0.8], \quad C_4 = [0.8 \;\; 0.7],$$

$$v_1(s) = 0.3\cos(s) \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \;\; v_2(s) = 0.3n(s) \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \;\; v_3(s) = 0.4n(s) \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \;\; v_4(s) = 0.3\sin(s) \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

$$\omega_1(s) = 0.5\sin(s), \;\; \omega_2(s) = 0.3\cos(s), \;\; \omega_3(s) = 0.4n(s), \;\; \omega_4(s) = 0.3n(s)$$

where $n(s)$ is a stochastic variable which obeys uniform distribution over $[0, 1]$. Moreover, the initial conditions of the DCN are chosen as

$$x_1(0) = [0.4 \;\; 0.2]^T, \quad x_2(0) = [0.2 \;\; 0.2]^T, \quad x_3(0) = [0.2 \;\; 0.4]^T, \quad x_4(0) = [0.3 \;\; 0.3]^T.$$

Case 1. In this case, the window length is set as $N+1 = 6$, the weight matrix $P = I$ and the threshold $\delta_i = 1$ $(i = 1, 2, 3, 4)$. According to Theorem 1, the MH estimator can be derived in the form of (23). A particle swarm optimization (PSO) algorithm is employed to solve the optimization problem (24) at each time step, and the optimal MH estimator (29) is obtained. Moreover, an approximate MH estimator (30) is presented by choosing the parameter $\lambda^* = 2$.

To make our simulation nontrivial, the DCN under consideration is set to be unstable. The state trajectories, estimates from MHE and estimates from approximate MHE for all nodes in the networks are depicted in Figs. 1-8, respectively. From Theorem 2, we obtain that the upper bound of the EE is 12.1707. The EE is described in Fig. 9, which shows the effectiveness of the MHE and approximate MHE. Furthermore, it can be found from Fig. 9 that the MHE performs better than the approximate MHE resulting from the optimization of λ at every step. In Fig. 10, the triggering time of node i is depicted, and the triggering frequency is decreased.

TABLE I

THE AVERAGE EE INDEX χ AND THE AVERAGE TRIGGERED RATIO κ WITH DIFFERENT THRESHOLD δ

δ	1	2	3	4	6
χ	3.3828	3.8880	4.4699	5.1253	6.0699
κ	25.32%	18.43%	13.62%	10.9%	9.46%

Case 2. Next, the impacts of the threshold δ_i $(i = 1, 2, 3, 4)$ on the estimation performances will be analyzed. The EE index is defined as $\chi = \frac{1}{N}\sum_{s=0}^{N} \|e(s)\|^2$, and the triggered ratio κ is defined as the number of triggered instants over the total number of time points. In this case, we set $N = 150$ and $\delta_i = \delta$ $(i = 1, 2, 3, 4)$. By conducting 50 independent simulation trials, χ and κ are shown in Table I, which implies that a larger δ would lead to a worse EE index while reduce the data transmission in a networked environment. In order to achieve a trade-off between the accuracy guaranteeing and the energy-saving via event-triggering, the threshold δ should be properly fine-tuned in practical systems.

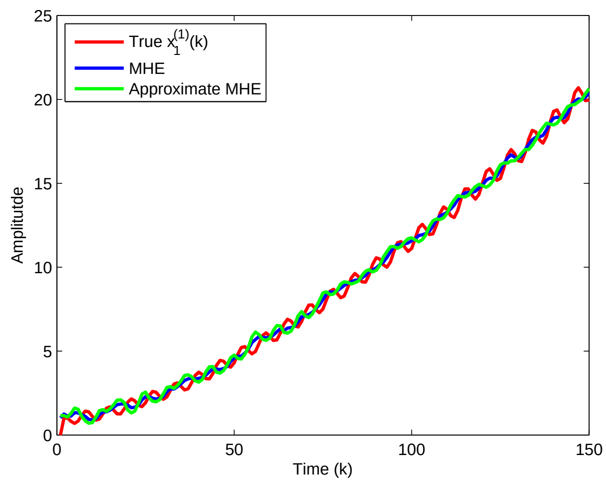

Fig. 1. The state trajectory of $x_1^{(1)}(s)$ and its estimates.

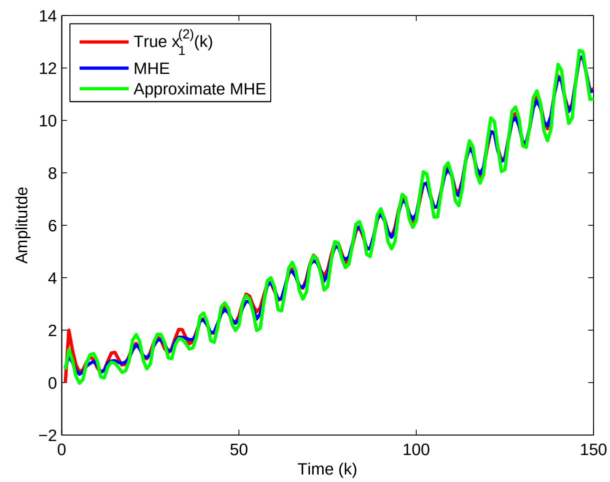

Fig. 2. The state trajectory of $x_1^{(2)}(s)$ and its estimates.

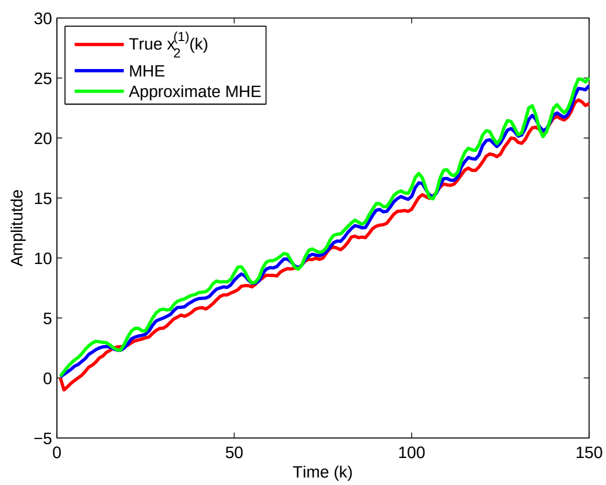

Fig. 3. The state trajectory of $x_2^{(1)}(s)$ and its estimates.

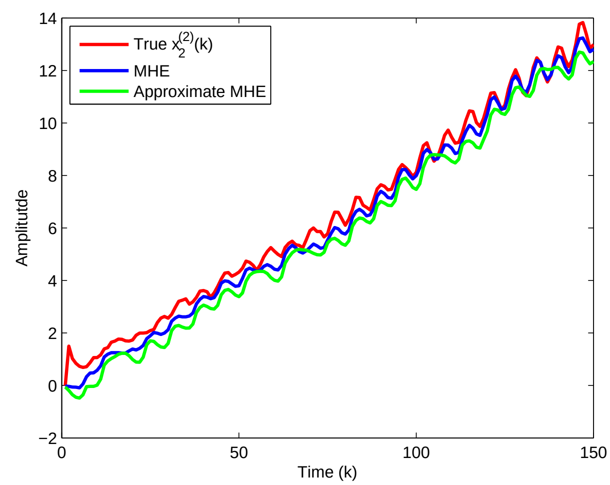

Fig. 4. The state trajectory of $x_2^{(2)}(s)$ and its estimates.

V. Conclusion

In this paper, the event-triggered centralized MHE problem has been investigated for a class of nonlinear DCNs. The event-triggered scheme has been introduced to reduce the information exchange frequency between the sensors and estimators. The MH estimator has been derived by minimizing a quadratic cost function, which can be dealt with by solving a regularized RLS problem. Subsequently, the convergence property of the estimator error has been analyzed, and the upper bound of the EE has been presented. Finally, an illustrative example has been given to highlight the effectiveness of the proposed design strategy. Further research topics contain the study of fault estimation for DCNs and the investigation of distributed MHE for nonlinear DCNs with the event-triggered scheme.

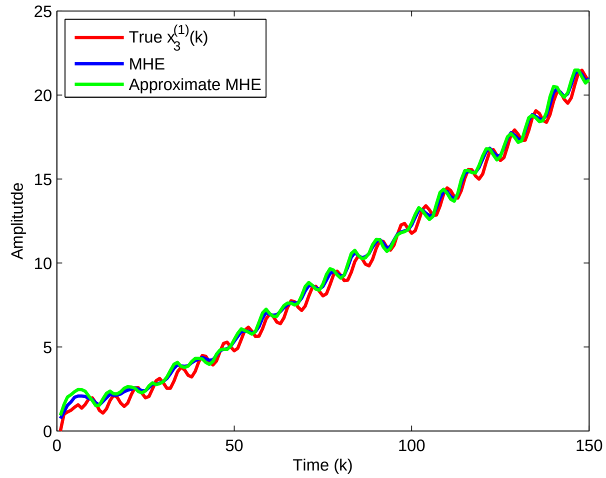

Fig. 5. The state trajectory of $x_3^{(1)}(s)$ and its estimates.

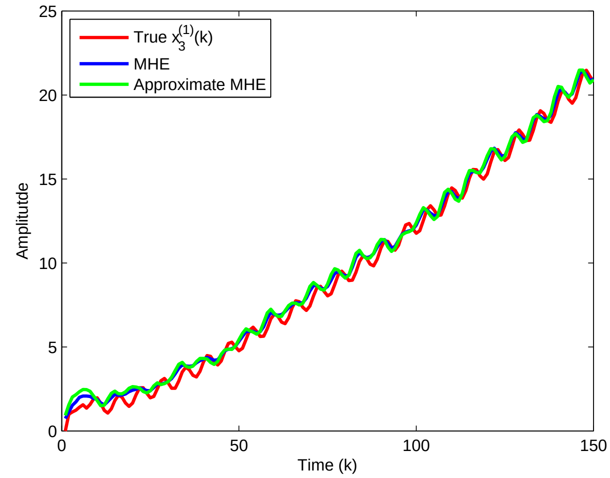

Fig. 6. The state trajectory of $x_3^{(2)}(s)$ and its estimates.

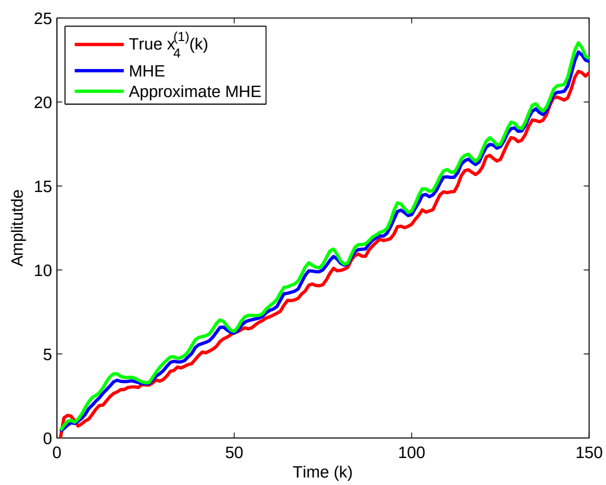

Fig. 7. The state trajectory of $x_4^{(1)}(s)$ and its estimates.

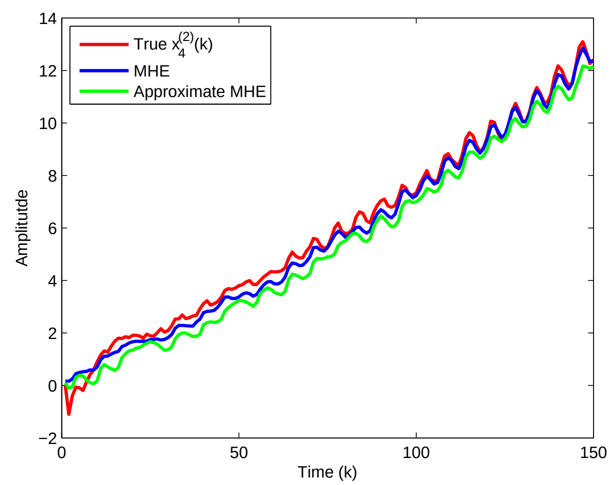

Fig. 8. The state trajectory of $x_4^{(2)}(s)$ and its estimates.

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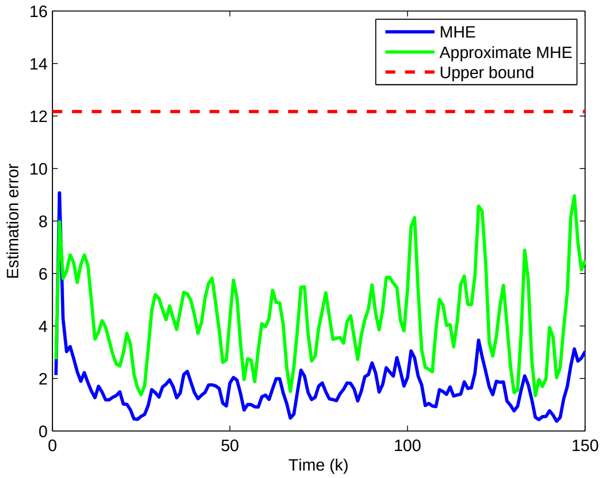

Fig. 9. The EE $\|e(s)\|^2$.

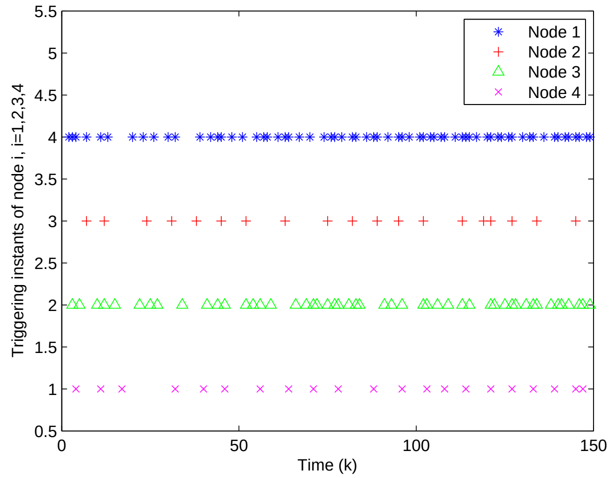

Fig. 10. The triggering instants of node i ($i = 1, 2, 3, 4$).

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