# The Fluctuating Beckmann Shadowed Fading Model 

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#### Abstract

In this paper, the fundamental statistics of fluctuating Beckmann shadowed (FBS) fading model which is composite of fluctuating Beckmann (FB) and inverse Nakagami- $m$ distributions, are provided. Accordingly, a wide range of composite generalized fading distributions can be obtained as special cases of the proposed distribution, such as, double shadowed $\kappa-\mu$ fading Type I. To this effect, mathematically tractable expressions of both the exact and asymptotic at high signal-to-noise (SNR) regime of the probability density function (PDF), cumulative distribution function (CDF), and generalised-moment generating function (G-MGF) are derived. In addition, the fundamental statistics for integer $m$ and even value of $\mu$ that represent the shadowing of the dominant components and real extension of the number of the multipath clusters, respectively, are given in simple closed-from formats. These statistical characterizations are then used to evaluate the performance of the wireless communications in terms of the outage probability (OP), average bit error probability (ABEP), amount-of-fading (AF), channel quality estimation index (CQEI), error vector magnitude (EVM), average channel capacity (ACC), and effective throughput (ET). Furthermore, the area under the receiver operating characteristics (AUC) curve of the energy detection based spectrum sensing and the secure outage probability (SOP) as well as the lower bound of SOP (SOP ${ }^{L}$ ) of the physical layer are also analysed. The validation of the derived expressions is verified via comparing the numerical results with the Monte-Carlo simulations for different values of the fading parameters.


Index Terms-Fluctuating Beckmann shadowed, fundamental statistics, outage probability, average channel capacity, average bit error probability, effective throughput, energy detection, secure outage probability.

## I. INTRODUCTION

THE shadowing has been widely considered as a largescale fading that may affect on the wireless communications channel at the same time with the small-scale fading, namely, multipath [1]. Hence, many efforts have been dedicated in the state-of-the-art to provide the composite fading distributions. For instance, the authors in [1] derived the probability density function (PDF) of the composite Rayleigh/lognormal fading distribution. Due to the high computational intricacy of the fundamental statistics as well as the performance metrics that is yielded by the log-normal model, the gamma distribution was then used to represent the shadowing impact. Hence, the $K$ distribution was introduced in [2] as composite of Rayleigh/gamma distributions and used in [3] to analyse the outage probability (OP) and average bit error rate (ABER) of differential phase shift keying (DPSK) and minimum shift keying digital modulation schemes. The performance of digital communications systems in terms of the ABER and average channel capacity (ACC) over composite

[^0]of Nakagami- $m /$ gamma model, namely, $K_{G}$ distribution was studied in [4]. The PDF, cumulative distribution function (CDF), and moment generating function (MGF) of Rician shadowed fading conditions were provided in [5] and [6].
Recently, different composite generalised/gamma distributions have been presented in the technical literature to obtain results with better fitting to the empirical measurements than the classical models. In addition, most of the well-known composite fading distributions can be derived from these generalised models via inserting special values for the fading parameters. Accordingly, in [7] and [8], the authors proposed the $\kappa-\mu /$ gamma fading model as a composite of $\kappa-\mu$ fading [9] which is used to model the line-of-sight (LoS) communications scenario and gamma distribution that represents the shadowing impact. The authors in [10] derived the statistics of the $\kappa-\mu$ shadowed fading that represents a unified approach of both the $\kappa-\mu$ and $\eta-\mu$ distributions [11] where the dominant of both the in-phase and quadrature components are shadowed by a Nakagami- $m$ distribution. Hence, the performance of energy detection (ED) based spectrum sensing, error vector magnitude (EVM) and physical layer security over $\kappa-\mu$ shadowed fading channels were studied in [12], [13], and [14], respectively. The $\kappa-\mu$ shadowed fading of [10] was then extended by [15] to obtain the fluctuating Beckmann (FB) fading model that unifies the $\kappa-\mu$ [9] and Beckmann [16], [17] distributions in a single representation (see [15, Table I]). Consequently, the secrecy of the physical layer and effective rate of FB fading were analysed in [18] and [19], respectively. On the other side, the influence of the shadowing on the non-LoS (NLoS) communications condition was represented by the composite of $\eta-\mu /$ gamma fading model [8], [20], [21].

More recently, the inverse gamma (IG) distribution has been given a special attention by several works to model the influence of the shadowing. This is because the inverse distribution provides highly accurate results with simple mathematically tractable statistical properties in comparison with the gamma model [22]. For example, the authors in [23] derived the PDF, CDF, and MGF of both the signal envelope and instantaneous signal-to-noise ratio (SNR) over Fisher-Snedecor $\mathcal{F}$ fading in which the multipath is modelled by a Nakagami- $m$ distribution whereas the shadowing is represented by an inverse Nakagami$m$ distribution. Based on these statistics, the capacity of the channel with different transmission policies, such as, ACC and effective throughput (ET), and physical layer security in terms of the secure outage probability (SOP) and its lower bound $\left(\mathrm{SOP}^{L}\right.$ ) over Fisher-Snedecor $\mathcal{F}$ fading channels, were analysed in [24] and [25], respectively. Furthermore, the behaviour of an energy detector over Fisher-Snedecor $\mathcal{F}$ fading channels was analysed in [26]. The PDF and CDF of both $\kappa-\mu / \mathrm{IG}$ and $\eta-\mu / \mathrm{IG}$ composite fading distributions were given in [27] with practical validations for different communications scenarios.

The applications of these statistical characterisations for the performance analysis of wireless communications systems in terms of the average symbol error rate (ASER), ACC, and ET were investigated in [28], [29]-[30], and [31], respectively.

Based on the empirical advantages of IG distribution in modelling of the large-scale fading, in this paper, the fluctuating Beckmann shadowed (FBS) fading model is suggested as a generalised composite distribution. Moreover, this model reduces to the double shadowed of Rician [32] and $\kappa-\mu$ Type I [33]-[34] fading scenarios after a proper selection for the values of the fading parameters.

The novel contributions of this paper are listed as follows:

- Deriving the PDF, CDF, generalised-MGF (G-MGF) of FBS fading model for both the signal envelope and instantaneous SNR. To this end, novel exact mathematically tractable expressions are obtained in terms of the Lauricella hypergeometric function as well as the Meijer's $G$-function (MGF) of four variables.
- Providing novel simple exact closed-form expressions of the aforementioned statistics via assuming the shadowing severity index $m$ and $\mu$ are integer and even values, respectively.
- The asymptotic expressions of the PDF, CDF, and GMGF at high average SNR regime are also given. These expressions are then used to earn further insights into the impact of the channel parameters on the behaviour of the communications systems .
- Leveraging our new statistical properties, we analyse the OP, ABEP, amount-of-fading (AoF), channel quality estimation index (CQEI), EVM, ACC, and ET of the wireless communications systems. Moreover, the AUC curve of the energy detection based spectrum sensing and the lower bound SOP are studied.
- Capitalising on the above, the asymptotic expressions for all the exact performance metrics at high average SNR values are derived. Accordingly, more insights are obtained when the transmitted signal is subject to double shadowing impacts simultaneously.
Organization: Section II provides some principles about the FB fading distribution. The exact and asymptotic statistical characterizations of FBS fading condition are derived in Section III which is divided into two subsections. These statistics are given for arbitrary values of $\mu$ and $m$ in the Subsection III.A. However, in the Subsection III.B, the PDF, CDF, and G-MGF are presented for even $\mu$ and integer $m$. Section IV demonstrates the special fading models that can be extracted from the FBS fading. The exact and asymptotic expressions of the OP, ABEP, AF, CQEI, EVM, ACC, ET, average AUC of ED, and lower bound of SOP are derived in Section V as applications of FBS fading channel in the performance evaluation of wireless communications systems. In Section VI, the numerical results and Monte Carlo simulations are presented for different shadowing scenarios. Finally, some conclusion remarks about the statistical properties of the proposed model and their applications are highlighted in Section VII.

Notations: $j=\sqrt{-1}, \ln ($.$) is the natural logarithm, \mathbb{E}[$. indicates the expectation process of the random variable (RV), $\operatorname{Var}[$.$] denotes the variance of the \mathrm{RV}, \Gamma($.$) is the Gamma$
function [35, eq. (1.1.1)], $B(.,$.$) is the beta function defined$ in [35, eq. (1.1.43)], $\binom{a}{b}=\frac{a!}{b!(a-b)!}$ is the binomial coefficient [35, eq. (1.1.17)], $(.)_{n}$ is the Pochhammer symbol [35, eq. (1.1.14)], $\psi($.$) is the Euler's digamma function [35, eq.$ (1.1.34)], ${ }_{p} F_{q}(. ;$; . .) represents the confluent hypergeometric function [35, eq. (1.4.1)], $\Phi_{2}^{(4)}$ (.) is the confluent hypergeometric function four variables defined in [35, eq. (1.7.10)], $F_{D}^{(4)}($. is the Lauricella hypergeometric function of four variables presented in [35, eq. (1.7.4)], $\Psi(. ; . ;$.$) is the Tricomi confluent$ hypergeometric function of the second kind defined in [35, eq. (1.3.14)], $G_{c, d}^{a, b}[$.$] is the univariate Meijer's G$-function (MGF) [35, eq. (1.5.1)], and $G_{m, n:[p, q]_{i=1: 4}}^{a, b[c, d]_{i=1: 4}}[$. stands for the Meijer's $G$-function (MGF) of four variables which is a special case of the multivariate Fox's $H$-function (MFHF) that is defined in [36, eq. (A.1)].

## II. Fluctuating Beckmann Fading Model

The received signal envelope over FB fading model, $R^{\mathrm{FB}}$ is expressed as [15, eq. (1)]

$$
\begin{equation*}
R^{\mathrm{FB}}=\sqrt{\sum_{i=1}^{\mu}\left(I_{i}+p_{i} \xi\right)^{2}+\left(Q_{i}+q_{i} \xi\right)^{2}} \tag{1}
\end{equation*}
$$

where $\mu$ is a natural number denoting the number of clusters, $I_{i}$ and $Q_{i}$ are respectively the in-phase and quadrature components of the received signal of the $i$ th cluster which are mutually independent Gaussian random processes with $\mathbb{E}\left[I_{i}\right]=\mathbb{E}\left[Q_{i}\right]=0, p_{i}$ and $q_{i}$ are real numbers that represent the dominant components of the $i$ th cluster with $p^{2}=\sum_{i=1}^{\mu} p_{i}^{2} \neq q^{2}=\sum_{i=1}^{\mu} q_{i}^{2}$ and $\xi$ is a Nakagami- $m$ distributed RV with shape parameter $m$ and $\mathbb{E}\left[\xi^{2}\right]=1$ which accounts for the fluctuation of the LoS component.

Using the PDF of the instantaneous SNR, $\gamma$, over FB fading distribution that is given in [15, eq. (5)] and $f_{R}(r)=$ $\frac{2 r \bar{\gamma}}{\hat{r}^{2}} f_{\gamma}\left(\frac{r^{2} \bar{\gamma}}{\hat{r}^{2}}\right)$ in which $\bar{\gamma}=\mathbb{E}[\gamma]$ is the average $\operatorname{SNR}$ and $\hat{r}=\sqrt{\mathbb{E}\left[\left(R^{\mathrm{FB}}\right)^{2}\right]}$ represents the mean power of the signal, the PDF of the received signal envelope, $R^{\mathrm{FB}}$, is obtained as

$$
\begin{align*}
f_{R^{\mathrm{FB}}}(r)= & \frac{2 \Xi}{\Gamma(\mu) \hat{r}^{2 \mu}} r^{2 \mu-1} \Phi_{2}^{(4)}\left(\frac{\mu}{2}-m, \frac{\mu}{2}-m, m, m ; \mu ;\right. \\
& \left.-\frac{r^{2}}{\hat{r}^{2} \sqrt{\eta \alpha_{2}}},-\frac{r^{2} \sqrt{\eta}}{\hat{r}^{2} \sqrt{\alpha_{2}}},-\frac{r^{2} c_{1}}{\hat{r}^{2}},-\frac{r^{2} c_{2}}{\hat{r}^{2}}\right), \tag{2}
\end{align*}
$$

where $\Xi=\frac{\alpha_{2}^{m-\frac{\mu}{2}}}{\alpha_{1}^{m}}, \alpha_{2}=\frac{4 \eta}{\mu^{2}(1+\eta)^{2}(1+\kappa)^{2}}$ with $\kappa=\frac{p^{2}+q^{2}}{\mu\left(\sigma_{I}^{2}+\sigma_{Q}^{2}\right)}$, $\eta=\frac{\sigma_{I}^{2}}{\sigma_{Q}^{2}}$ with $\sigma_{I}^{2}=\mathbb{E}\left[I_{i}^{2}\right]$ and $\sigma_{Q}^{2}=\mathbb{E}\left[Q_{i}^{2}\right]$, and $c_{1,2}$ are the roots of $\alpha_{1} x^{2}+\beta x+1$ with [15, eqs. (7)/(8)]

$$
\begin{align*}
\alpha_{1} & =\alpha_{2}+\frac{2 \kappa\left(\varrho^{2}+\eta\right)}{m\left(1+\varrho^{2}\right) \mu(1+\eta)(1+\kappa)^{2}} \\
\beta & =-\frac{1}{1+\kappa}\left[\frac{2}{\mu}+\frac{\kappa}{m}\right] \tag{3}
\end{align*}
$$

where $\varrho^{2}=\frac{p^{2}}{q^{2}}$.
When $m$ and $\mu$ are, respectively, integer value and even number, the PDF of $R^{\mathrm{FB}}$ can be derived via substituting [15,

$$
\begin{align*}
& f_{R}(r)=\frac{2 \Xi r^{2 \mu-1}}{B\left(\mu, m_{s}\right)\left[\left(m_{s}-1\right) \hat{r}^{2}\right]^{\mu}} \\
& \times F_{D}^{(4)}\left(\mu+m_{s}, \frac{\mu}{2}-m, \frac{\mu}{2}-m, m, m ; \mu ;-\frac{r^{2}}{\sqrt{\eta \alpha_{2}}\left(m_{s}-1\right) \hat{r}^{2}},-\frac{\sqrt{\eta} r^{2}}{\sqrt{\alpha_{2}}\left(m_{s}-1\right) \hat{r}^{2}},-\frac{c_{1} r^{2}}{\left(m_{s}-1\right) \hat{r}^{2}},-\frac{c_{2} r^{2}}{\left(m_{s}-1\right) \hat{r}^{2}}\right) \tag{7}
\end{align*}
$$

$$
\begin{align*}
& f_{\gamma}(\gamma)=\frac{\Xi \gamma^{\mu-1}}{B\left(\mu, m_{s}\right)\left[\left(m_{s}-1\right) \bar{\gamma}\right]^{\mu}} \\
& \times F_{D}^{(4)}\left(\mu+m_{s}, \frac{\mu}{2}-m, \frac{\mu}{2}-m, m, m ; \mu ;-\frac{\gamma}{\sqrt{\eta \alpha_{2}}\left(m_{s}-1\right) \bar{\gamma}},-\frac{\sqrt{\eta} \gamma}{\sqrt{\alpha_{2}}\left(m_{s}-1\right) \bar{\gamma}},-\frac{c_{1} \gamma}{\left(m_{s}-1\right) \bar{\gamma}},-\frac{c_{2} \gamma}{\left(m_{s}-1\right) \bar{\gamma}}\right) \tag{8}
\end{align*}
$$

$$
\begin{align*}
& F_{\gamma}(\gamma)=\frac{\Xi \gamma^{\mu}}{\mu B\left(\mu, m_{s}\right)\left[\left(m_{s}-1\right) \bar{\gamma}\right]^{\mu}} \\
& \times F_{D}^{(4)}\left(\mu+m_{s}, \frac{\mu}{2}-m, \frac{\mu}{2}-m, m, m ; \mu+1 ;-\frac{\gamma}{\sqrt{\eta \alpha_{2}}\left(m_{s}-1\right) \bar{\gamma}},-\frac{\sqrt{\eta} \gamma}{\sqrt{\alpha_{2}}\left(m_{s}-1\right) \bar{\gamma}},-\frac{c_{1} \gamma}{\left(m_{s}-1\right) \bar{\gamma}},-\frac{c_{2} \gamma}{\left(m_{s}-1\right) \bar{\gamma}}\right)  \tag{10}\\
& \mathcal{M}_{\gamma}^{(n)}(s)=\frac{(10}{\Gamma\left(m_{s}\right)\left[\Gamma\left(\frac{\mu}{2}-m\right) \Gamma(m)\right]^{2}\left[\left(m_{s}-1\right) \bar{\gamma}\right]^{\mu} s^{\mu+n}} G_{2,1:[1,1]_{i=1: 4}^{0,2:[1,1]_{i=1}}\left[\frac{\gamma^{\mu}}{\sqrt{\eta \alpha_{2}}\left(m_{s}-1\right) \bar{\gamma} s}, \frac{1}{\sqrt{\alpha_{2}}\left(m_{s}-1\right) \bar{\gamma} s}, \frac{\sqrt{\eta}}{\left(m_{s}-1\right) \bar{\gamma} s}\right.} \begin{array}{c}
\frac{c_{2}}{\left(m_{s}-1\right) \bar{\gamma} s} \left\lvert\, \begin{array}{c}
\left.1-\mu-m_{s}, 1-\mu-n \left\lvert\,\left(1-\frac{\mu}{2}+m\right)_{i=1: 2}\right.,(1-m)_{i=3: 4}\right] \\
1-\mu
\end{array}(0)_{i=1: 4}\right.
\end{array}
\end{align*}
$$

eq. (10)] into $f_{R}(r)=\frac{2 r \bar{\gamma}}{\hat{r}^{2}} f_{\gamma}\left(\frac{r^{2} \bar{\gamma}}{\hat{r}^{2}}\right)$. Thus, this yields

$$
\begin{equation*}
f_{R^{\mathrm{FB}}}(r)=\frac{2 \Xi}{\hat{r}^{2 \mu}} \sum_{i=1}^{N(m, \mu)} e^{-\frac{\vartheta_{i}}{\hat{r}^{2}} r^{2}} \sum_{l=1}^{\left|\omega_{i}\right|} \frac{\mathcal{A}_{i l}}{\Gamma(l)} r^{2 l-1} \tag{4}
\end{equation*}
$$

where $\omega=\left[m, m, \frac{\mu}{2}-m, \frac{\mu}{2}-m\right], \vartheta=\left[c_{1}, c_{2}, \frac{\mu(1+\eta)(1+\kappa)}{2 \eta}\right.$, $\left.\frac{\mu(1+\eta)(1+\kappa)}{2}\right], N(m, \mu)=2\left[1+\mathbf{u}\left(\frac{\mu}{2}, m\right)\right], \mathbf{u}($.$) is the unit step$ function, and $\mathcal{A}_{i l}$ is calculated by [15, eq.(51)].

## III. Statistical Characterization of the

 Fluctuating Beckmann Shadowed Fading ModelThe received signal envelope over FBS fading distribution, $R$ can be given as

$$
\begin{equation*}
R=\Lambda R^{\mathrm{FB}}=\Lambda \sqrt{\sum_{i=1}^{\mu}\left(I_{i}+p_{i} \xi\right)^{2}+\left(Q_{i}+q_{i} \xi\right)^{2}} \tag{5}
\end{equation*}
$$

where $\Lambda$ is an inverse Nakagami- $m$ RV with shadowing severity index $m_{s}$ and $\mathbb{E}[\Lambda]=1$ and its envelope normalized PDF can be written as

$$
\begin{equation*}
f_{\Lambda}(x)=\frac{2\left(m_{s}-1\right)^{m_{s}}}{\Gamma\left(m_{s}\right) x^{2 m_{s}+1}} e^{-\frac{\left(m_{s}-1\right)}{x^{2}}} \quad m_{s}>1 \tag{6}
\end{equation*}
$$

## A. Exact and Asymptotic Statistical Characterization of FBS Fading Model with Arbitrary $\mu$ and $m$

Theorem 1: Let $r$ follows a FBS distribution with mean $\hat{r}=\sqrt{\left[R^{2}\right]}$, i.e., $r \sim \mathcal{F B S}\left(\hat{r} ; \kappa, \mu, m, \eta, \varrho, m_{s}\right)$ with $\kappa, \mu$, $m, \eta, \varrho, m_{s}, \hat{r}$, and $r \in \mathbb{R}^{+}$, the PDF of the composite signal envelope, $R$, in FBS fading distribution can be expressed as in (7) given at the top of this page.

Proof: See Appendix A-A.
It can be noticed that (7) includes the Lauricella hypergeometric function of four variables, namely, $F_{D}^{(4)}($.$) which is$
not available yet as a built-in function in the popular software packages, such as, Matlab and Mathematica. Therefore, in this paper, this function is efficiently computed via using the Matlab code that can be easily downloaded from [37].

Lemma 1: Let $\gamma \sim \mathcal{F B S}\left(\bar{\gamma} ; \kappa, \mu, m, \eta, \varrho, m_{s}\right)$ in which $\gamma$, $\kappa, \mu, m, \eta, \varrho, m_{s}$, and $\bar{\gamma} \in \mathbb{R}^{+}$where $\bar{\gamma}=\hat{r}^{2} E_{s} / N_{0}$ with $E_{s}$ and $N_{0}$ stand for the energy per symbol and one-sided power spectral density of the additive white Gaussian noise (AWGN) which is statistically independent of the signal, respectively. Then, the exact PDF of $\gamma$ over FBS fading model can be derived as in (8) that is provided at the top of this page and its asymptotic expression at $\bar{\gamma} \rightarrow \infty$ is yielded as

$$
\begin{equation*}
f_{\gamma}^{\infty}(\gamma) \approx \frac{\Xi}{B\left(\mu, m_{s}\right)\left[\left(m_{s}-1\right) \bar{\gamma}\right]^{\mu}} \gamma^{\mu-1} \tag{9}
\end{equation*}
$$

## Proof: See Appendix A-B.

Theorem 2: For $\gamma, \kappa, \mu, m, \eta, \varrho, m_{s}$, and $\bar{\gamma} \in \mathbb{R}^{+}$, the exact and asymptotic CDF of the instantaneous SNR, $\gamma$, over the FBS model can be obtained as in (10) shown at the top of this page and (11), respectively.

$$
\begin{equation*}
F_{\gamma}^{\infty}(\gamma) \approx \frac{\Xi}{\mu B\left(\mu, m_{s}\right)\left[\left(m_{s}-1\right) \bar{\gamma}\right]^{\mu}} \gamma^{\mu} \tag{11}
\end{equation*}
$$

Proof: See Appendix A-C.
Theorem 3: For $s, \kappa, \mu, m, \eta, \varrho, m_{s}$, and $\bar{\gamma} \in \mathbb{R}^{+}$, the G-MGF of the FBS model can be derived as in (12) provided at the top of this page whilst its asymptotic expression is expressed as

$$
\begin{equation*}
\mathcal{M}_{\gamma}^{(n), \infty}(s) \approx \frac{\Gamma(\mu+n) \Xi}{B\left(\mu, m_{s}\right)\left[\left(m_{s}-1\right) \bar{\gamma}\right]^{\mu} s^{\mu+n}} \tag{13}
\end{equation*}
$$

Proof: See Appendix A-D.

## B. Exact and Asymptotic Statistical Characterization of FBS Fading Model with Even $\mu$ and Integer $m$

Theorem 4: When $\mu$ and $m$ are even and integer numbers, respectively, i.e. $\mu \in 2 \mathbb{Z}^{+}$and $m \in \mathbb{Z}^{+}$, and using the same assumptions of Theorem 1, an alternative PDF of $R$ that is given in (7) can be written in simple exact closed-form expression as

$$
\begin{align*}
& f_{R}(r)= \\
& \frac{2 \Xi}{\hat{r}^{2 \mu}} \sum_{i=1}^{N(m, \mu)} \sum_{l=1}^{\left|\omega_{i}\right|} \frac{r^{2 l-1}\left(m_{s}\right)_{l} \mathcal{A}_{i l}}{\Gamma(l)\left(m_{s}-1\right)^{l}\left(1+\frac{\vartheta_{i}}{\left(m_{s}-1\right) \hat{r}^{2}} r^{2}\right)^{m_{s}+l}} . \tag{14}
\end{align*}
$$

Proof: See Appendix B-A.
Lemma 2: Similar to the Lemma 1, but even $\mu$ and integer $m$, the exact PDF of $\gamma$ over FBS fading model can be derived as

$$
\begin{equation*}
f_{\gamma}(\gamma)=\frac{\Xi}{\bar{\gamma}^{\mu}} \sum_{i=1}^{N(m, \mu)} \sum_{l=1}^{\left|\omega_{i}\right|} \frac{\gamma^{l-1}\left(m_{s}\right)_{l} \mathcal{A}_{i l}}{\Gamma(l)\left(m_{s}-1\right)^{l}\left(1+\frac{\vartheta_{i}}{\left(m_{s}-1\right) \bar{\gamma}} \gamma\right)^{m_{s}+l}} . \tag{15}
\end{equation*}
$$

The asymptotic of (15) at $\bar{\gamma} \rightarrow \infty$ is given as

$$
\begin{equation*}
f_{\gamma}^{\infty}(\gamma) \approx \frac{\Xi}{\bar{\gamma}^{\mu}} \sum_{i=1}^{N(m, \mu)} \sum_{l=1}^{\left|\omega_{i}\right|} \frac{\left(m_{s}\right)_{l} \mathcal{A}_{i l}}{\Gamma(l)\left(m_{s}-1\right)^{l}} \gamma^{l-1} . \tag{16}
\end{equation*}
$$

Proof: See Appendix B-B.
Theorem 5: The CDF of $\gamma$ over FBS fading model for even $\mu$ and integer $m$ is expressed as

$$
\begin{align*}
& F_{\gamma}(\gamma)= \\
& \frac{\Xi}{\bar{\gamma}^{\mu}} \sum_{i=1}^{N(m, \mu)} \sum_{l=1}^{\left|\omega_{i}\right|} \frac{\gamma^{l}\left(m_{s}\right)_{l} \mathcal{A}_{i l 2} F_{1}\left(m_{s}+l, l ; l+1 ;-\frac{\vartheta_{i}}{\left(m_{s}-1\right) \bar{\gamma}} \gamma\right)}{\Gamma(l+1)\left(m_{s}-1\right)^{l}} . \tag{17}
\end{align*}
$$

The asymptotic behaviour of the CDF that is given in (17) when $\bar{\gamma}$ tends to $\infty$ can be analysed by

$$
\begin{equation*}
F_{\gamma}^{\infty}(\gamma) \approx \frac{\Xi}{\bar{\gamma}^{\mu}} \sum_{i=1}^{N(m, \mu)} \sum_{l=1}^{\left|\omega_{i}\right|} \frac{\left(m_{s}\right)_{l} \mathcal{A}_{i l}}{\Gamma(l+1)\left(m_{s}-1\right)^{l}} \gamma^{l} \tag{18}
\end{equation*}
$$

Proof: See Appendix B-C.
Theorem 6: The G-MGF of FBS fading distribution when $\mu$ is an even number and $m$ is an integer value can be written as

$$
\begin{align*}
\mathcal{M}_{\gamma}^{(n)}(s)=\frac{\Xi}{\bar{\gamma}^{\mu}} & \sum_{i=1}^{N(m, \mu)} \sum_{l=1}^{\left|\omega_{i}\right|} \frac{(l)_{n}\left(m_{s}\right)_{l} \mathcal{A}_{i l}\left(m_{s}-1\right)^{n}}{\vartheta_{i}^{l+n}} \bar{\gamma}^{l+n} \\
& \times \Psi\left(l+n ; 1-m_{s}+n ; \frac{\left(m_{s}-1\right) \bar{\gamma}}{\vartheta_{i}} s\right) . \tag{19}
\end{align*}
$$

The asymptotic of the G-MGF in (19) can be obtained as

$$
\begin{equation*}
\mathcal{M}_{\gamma}^{(n), \infty}(s) \approx \frac{\Xi}{\bar{\gamma}^{\mu}} \sum_{i=1}^{N(m, \mu)} \sum_{l=1}^{\left|\omega_{i}\right|} \frac{\left(m_{s}\right)_{l}(l)_{n} \mathcal{A}_{i l}}{\left(m_{s}-1\right)^{l} s^{l+n}} \tag{20}
\end{equation*}
$$

Proof: See Appendix B-D.


Fig. 1. The PDF of the signal envelope of FBS fading distribution for $\kappa=$ $2.9, \mu=4, m=2.3, \eta=0.1, \varrho^{2}=0.1, m_{s}=3.8$, and $\hat{r}=0.8$, and some of its special cases that are given in Table I.

It is worth noting that the function MGF of four variables of (12) is also not implemented yet in Matlab and Mathematica. Nevertheless, this function can be readily evaluated via utilising the Matlab code of the MFHF that is presented in [38].

## IV. Special Cases of FBS Fading Model

The derived statistics of the FBS fading model represent the unified expressions for number of well-known fading distributions. For instance, the statistical properties of the FBS fading model become identical with that of the double shadowed $\kappa-\mu$ Type I (example 1) fading [33] and FB [15] fading distributions via inserting $\eta=1$ and $m_{s} \rightarrow \infty$, respectively. Additionally, the FBS reduces to $\kappa-\mu$ shadowed fading [10] via setting $\eta=1$ and $\underline{m_{s}} \rightarrow \infty$. The Beckmann distribution [16] is obtained from the FBS fading model by letting $\underline{\kappa}=K, \underline{\mu}=1, \underline{m} \rightarrow \infty, \underline{\eta}=q, \varrho=r$, and $\underline{m_{s}} \rightarrow \infty$. Similarly, the first order statistics of the $\eta-\mu /$ inverse gamma [27] fading can be deduced from the expressions of Section III via employing $\underline{\hat{r}^{2}}=\frac{m_{s}}{m_{s}-1} \hat{r}^{2}, \underline{\kappa}=0, \underline{\mu}=\mu, \underline{\eta}=\eta$, and $\underline{m_{s}}=m_{s}$. All the fading models that can be extracted from the FBS distribution via selecting certain values for the fading parameters are shown in Table I. As explained in this table, the FBS fading distribution is extremely versatile as it inherit all of the generality of the double shadowed $\kappa-\mu$ Type I (example 1) [33] and FB [15] fading distributions.

Fig. 1 shows the PDF of the signal envelope $R$ of FBS fading distribution for $\kappa=2.9$ (moderate $\operatorname{LoS}$ scenario), $\mu=4, m=2.3$ (mild fluctuation of the LoS component), $\eta=0.1$ (the NLoS components is imbalanced by 10 ), $\varrho^{2}=0.1$ (moderately large power imbalance of the LoS components), $m_{s}=3.8$ (moderate shadowing), and $\hat{r}=0.8$. In this figure, the numerical results and Monte-Carlo simulations are represented by the solid lines and markers, respectively. From Fig. 1, it can be seen that the increasing in $\eta$ of the FBS fading from 0.1 to 1 (double shadowed $\kappa-\mu$ Type I (example 1) [33])

TABLE I
Special Models of the Fluctuating Beckmann Shadowed Fading Distribution

| Fading Models | Fluctuating Beckmann Shadowed Fading Parameters |
| :---: | :---: |
| Double Shadowed $\kappa-\mu$ Type I (example 1) [33] | $\underline{\kappa}=\kappa, \underline{\mu}=\mu, \underline{m}=m, \underline{\eta}=1, \forall \varrho, \underline{m_{s}}=m_{s}$ |
| Double Shadowed Rician Type I (example 1) [32] | $\underline{\kappa}=K, \underline{\mu}=1, \underline{m}=m, \underline{\eta}=1, \forall \varrho, \underline{m_{s}}=m_{s}$ |
| $\kappa-\mu$ Shadowed [10] | $\underline{\kappa}=\kappa, \underline{\mu}=\mu, \underline{m}=m, \underline{\eta}=1, \forall \varrho, \underline{m_{s}} \rightarrow \infty$ |
| Fluctuating Beckmann [15] | $\underline{\kappa}=\kappa, \underline{\mu}=\mu, \underline{m}=m, \underline{\eta}=\eta, \forall \varrho, \underline{m_{s}} \rightarrow \infty$ |
| $\kappa$ - $\mu$ /inverse gamma [27] | $\underline{\hat{r}^{2}}=\frac{m_{s}}{m_{s}-1} \hat{r}^{2}, \underline{\kappa}=\kappa, \underline{\mu}=\mu, \underline{m} \rightarrow \infty, \underline{\eta}=1, \forall \varrho, \underline{m_{s}}=m_{s}$ |
| $\eta$ - $\mu$ /inverse gamma [27] | $\underline{\hat{r}^{2}}=\frac{m_{s}}{m_{s}-1} \hat{r}^{2}, \underline{\kappa} \rightarrow 0, \underline{\mu}=\mu, \underline{\eta}=\eta, \underline{m_{s}}=m_{s}$ |
| $\kappa-\mu$ [9] | $\underline{\kappa}=\kappa, \underline{\mu}=\mu, \underline{m} \rightarrow \infty, \underline{\eta}=1, \forall \varrho, \underline{m_{s}} \rightarrow \infty$ |
| $\eta-\mu$ [9] | $\underline{\kappa} \rightarrow 0, \underline{\mu}=\mu, \underline{\eta}=\eta, \underline{m_{s}} \rightarrow \infty$ |
| Symmetrical $\eta-\kappa$ [39] | $\underline{\kappa}=\kappa, \underline{\mu}=1, \underline{m} \rightarrow \infty, \underline{\eta}=\eta, \varrho=\eta, \underline{m_{s}} \rightarrow \infty$ |
| Asymmetrical $\eta-\kappa$ [40] | $\underline{\kappa}=\kappa, \underline{\mu}=1, \underline{m} \rightarrow \infty, \underline{\eta}=\eta, \varrho \rightarrow 0, \underline{m_{s}} \rightarrow \infty$ |
| Beckmann [16] | $\underline{\kappa}=K, \underline{\mu}=1, \underline{m} \rightarrow \infty, \underline{\eta}=q, \varrho=r, \underline{m_{s}} \rightarrow \infty$ |
| Fisher-Snedecor $\mathcal{F}$ [24] | $\underline{\kappa} \rightarrow 0, \underline{\mu}=m, \underline{\eta}=1, \underline{m_{s}}=m_{s}$ |
| Rician Shadowed [6] | $\underline{\kappa}=K, \underline{\mu}=1, \underline{m}=m, \underline{\eta}=1, \forall \varrho, \underline{m_{s}} \rightarrow \infty$ |
| Nakagami- $n$ (Rice) [41] | $\underline{\kappa}=K, \underline{\mu}=1, \underline{m} \rightarrow \infty, \underline{\eta}=1, \forall \varrho, \underline{m_{s}} \rightarrow \infty$ |
| Nakagami- $q$ (Hoyt) [41] | $\underline{\kappa} \rightarrow 0, \underline{\mu}=1, \underline{\eta}=q, \underline{m_{s}} \rightarrow \infty$ |
| Nakagami-m [41] | $\underline{\kappa} \rightarrow 0, \underline{\mu}=m, \underline{\eta}=1, \underline{m_{s}} \rightarrow \infty$ |
| Rayleigh [41] | $\underline{\kappa} \rightarrow 0, \underline{\mu}=1, \underline{\eta}=1, \underline{m_{s}} \rightarrow \infty$ |
| One-sided Gaussian | $\underline{\kappa} \rightarrow 0, \underline{\mu}=1, \underline{\eta} \rightarrow 0, \underline{m_{s}} \rightarrow \infty$ |

would lead to move the amplitude of the PDF towards its mean value. The reason of this observation that is consistent with the results of [15], refers to the impact of the imbalance of the NLoS components in the case of $\eta \neq 1$ which is more noticeable than $\eta \neq 0.1$. Besides, the amplitude of the PDF of the double shadowed $\kappa-\mu$ Type I slightly increases as well as shifts it to the rightwards and becomes more closer to the mean value when $m_{s}$ approaches to infinity, namely, $\kappa-\mu$ shadowed fading [10]. It is worth remarking that Fig. 1 also includes the PDF of the signal envelope $R$ of double shadowed Rician Type I [32], $\kappa-\mu$ [9], $\eta-\mu$ /inverse gamma [27], and Nakagami$m$ [41] fading distributions which are deduced from the FBS fading model via setting the parameters for specific values as demonstrated in Table I.

## V. Applications of FBS Fading in Performance Evaluation of Wireless Communications

## A. Outage Probability

The OP measures the probability of falling the output SNR under a specific threshold value $\gamma_{t h}$. Mathematically, the OP, $P_{o}$, can be evaluated by [41, eq. (1.4)]

$$
\begin{equation*}
P_{o}=F_{\gamma}\left(\gamma_{t h}\right) \tag{21}
\end{equation*}
$$

where $F_{\gamma}($.$) is provided in (10) and (17) for arbitrary and$ special values of $\mu$ and $m$, respectively.

The asymptotic behaviour of the OP, $P_{o}^{\infty}$, can be studied by (11) and (18). Additionally, the $P_{o}^{\infty}$ may be written in terms of the coding gain, $\mathcal{G}_{c}$, and diversity order, $\mathcal{G}_{d}$, that explains how the slope of the OP increases with $\bar{\gamma}$, i.e., $P_{o}^{\infty} \approx\left(\mathcal{G}_{c} \bar{\gamma}\right)^{-\mathcal{G}_{d}}$. Accordingly, for general values of $\mu$ and $m, \mathcal{G}_{c}=\frac{\left(m_{s}-1\right)}{\gamma_{t h}}\left(\frac{\mu B\left(\mu, m_{s}\right)}{\Xi}\right)^{\frac{1}{\mu}}$ whilst, for even $\mu$ and integer $m, \mathcal{G}_{c}=\left(\Xi \sum_{i=1}^{N(m, \mu)} \sum_{l=1}^{\left|\omega_{i}\right|} \frac{\left(m_{s}\right)_{l} \mathcal{A}_{i l}}{\Gamma(l+1)\left(m_{s}-1\right)^{l}} \gamma^{l}\right)^{-\frac{1}{\mu}}$. However, in both cases, $\mathcal{G}_{d}=\mu$.

## B. Average Bit Error Probability

The ABEP, $\bar{P}_{e}$, can be evaluated by [21, eq. (45)]

$$
\begin{equation*}
\bar{P}_{e}=\frac{a^{b}}{2 \Gamma(b)} \int_{0}^{\infty} \gamma^{b-1} e^{-a \gamma} F_{\gamma}(\gamma) d \gamma \tag{22}
\end{equation*}
$$

where the modulation parameters $a$ and $b$ are defined in Table II.

For general $\mu$ and $m$, the exact expression of the ABEP, $\bar{P}_{e}$, and its asymptotic at high average SNR regime, $\bar{P}_{e}^{\infty}$, can be derived via following the same steps of Appendix A-D. Thus,

$$
\begin{align*}
\bar{P}_{e}= & \frac{\Xi}{2 \Gamma(b) \Gamma\left(m_{s}\right)\left[\Gamma\left(\frac{\mu}{2}-m\right) \Gamma(m)\right]^{2}\left[a\left(m_{s}-1\right) \bar{\gamma}\right]^{\mu}} G_{2,1:[1,1]_{i=1: 4}^{0,2:[1,1]_{i=1: 4}}\left[\frac{1}{a \sqrt{\eta \alpha_{2}}\left(m_{s}-1\right) \bar{\gamma}}, \frac{\sqrt{\eta}}{a \sqrt{\alpha_{2}}\left(m_{s}-1\right) \bar{\gamma}}, \frac{c_{1}}{a\left(m_{s}-1\right) \bar{\gamma}}\right.} \begin{array}{c}
\frac{c_{2}}{a\left(m_{s}-1\right) \bar{\gamma}}\left|\begin{array}{c}
1-\mu-m_{s}, 1-\mu-b \\
-\mu
\end{array}\right|\left(1-\frac{\mu}{2}+m\right)_{i=1: 2},(1-m)_{i=3: 4} \\
(0)_{i=1: 4}
\end{array}
\end{align*}
$$

$$
\mathrm{EVM}=\frac{\Xi\left(\sqrt{\eta \alpha_{2}}\right)^{\mu+\frac{1}{4}}\left(\left(m_{s}-1\right) \hat{r}^{2}\right)^{\frac{1}{4}} \Gamma\left(m_{s}+\frac{1}{4}\right)}{\sqrt{\gamma} \Gamma\left(\frac{1}{4}\right) \Gamma\left(m_{s}\right)\left[\Gamma\left(\frac{\mu}{2}-m\right) \Gamma(m)\right]^{2}} G_{1,1:[1,1]_{i=2: 4}}^{1,1:[1,1]_{i=2: 4}}\left[\eta, c_{1} \sqrt{\eta \alpha_{2}}, c_{2} \sqrt{\eta \alpha_{2}}\left|\begin{array}{c}
1.25-\mu  \tag{30}\\
0.25-\frac{\mu}{2}-m
\end{array}\right| 1-\frac{\mu}{2}+m,(1-m)_{i=3: 4}\right]
$$

TABLE II
Parameters of Different Modulation Formats

| Modulation Format | $a$ | $b$ |
| :---: | :---: | :---: |
| Binary frequency shift keying (BFSK) | 0.5 | 0.5 |
| Binary phase shift keying (BPSK) | 1 | 0.5 |
| Differential BPSK (DBPSK) | 1 | 1 |
| Return-to-zero on-off keying (OOK) | 0.25 | 0.5 |
| Non-return-to-zero OOK | 0.125 | 0.5 |

we have (23) that is shown at the top of the next page and

$$
\begin{equation*}
\bar{P}_{e}^{\infty} \approx \frac{\Xi(b)_{\mu}}{2 \mu B\left(\mu, m_{s}\right)\left[a\left(m_{s}-1\right) \bar{\gamma}\right]^{\mu}} \tag{24}
\end{equation*}
$$

From (24), it is obvious that, for the ABEP, $\mathcal{G}_{c}=a\left(m_{s}-\right.$ 1) $\left(\frac{2 \mu B\left(\mu, m_{s}\right)}{\Xi(b)_{\mu}}\right)^{\frac{1}{\mu}}$ and $\mathcal{G}_{d}=\mu$.

Using the same procedure in Appendix B-D, the exact ABEP for even $\mu$ and integer $m$ is obtained as

$$
\begin{align*}
\bar{P}_{e} & =\frac{\Xi}{2 \bar{\gamma}^{\mu}} \sum_{i=1}^{N(m, \mu)} \sum_{l=1}^{\left|\omega_{i}\right|} \frac{(b)_{l}\left(m_{s}\right)_{l} \mathcal{A}_{i l}}{\left[a\left(m_{s}-1\right)\right]^{l} l!} \\
& \times{ }_{3} F_{1}\left(m_{s}+l, l, l+b ; l+1 ;-\frac{\vartheta_{i}}{a\left(m_{s}-1\right) \bar{\gamma}}\right) \tag{25}
\end{align*}
$$

The asymptotic expression of (25) at high average SNR values is expressed as

$$
\begin{equation*}
\bar{P}_{e}^{\infty} \approx \frac{\Xi}{2 \bar{\gamma}^{\mu}} \sum_{i=1}^{N(m, \mu)} \sum_{l=1}^{\left|\omega_{i}\right|} \frac{(b)_{l}\left(m_{s}\right)_{l} A_{i l}}{l\left[a\left(m_{s}-1\right)\right]^{l}} . \tag{26}
\end{equation*}
$$

Similar to (24), $\mathcal{G}_{d}=\mu$ whereas $\mathcal{G}_{c}=$ $\left(\frac{\Xi}{2} \sum_{i=1}^{N(m, \mu)} \sum_{l=1}^{\left|\omega_{i}\right|} \frac{(b)_{l}\left(m_{s}\right)_{l} A_{i l}}{l\left[a\left(m_{s}-1\right)\right]^{l}}\right)^{-\frac{1}{\mu}}$.

## C. Amount of Fading and Channel Quality Estimation Index

The AoF can be computed by [41, eq. (1.27)]

$$
\begin{equation*}
\mathrm{AF}=\frac{\operatorname{Var}[\gamma]}{(\mathbb{E}[\gamma])^{2}}=\frac{\mathbb{E}\left[\gamma^{2}\right]}{(\mathbb{E}[\gamma])^{2}}-1 \tag{27}
\end{equation*}
$$

It can be observed that $\mathbb{E}\left[\gamma^{n}\right]$ can be deduced from the GMGF via inserting $s=0$. However, the G-MGF in both (12) and (19) include $s$ at the denominator. Therefore, we have provided the expression of $\mathbb{E}\left[\gamma^{n}\right]$ for solely the case of even
$\mu$ and integer $m$ via plugging (15) in $\mathbb{E}\left[\gamma^{n}\right]=\int_{0}^{\infty} \gamma^{n} f_{\gamma}(\gamma) d \gamma$ and recalling [42, eq. (3.194.3)]. Hence, this yields

$$
\begin{align*}
\mathbb{E}\left[\gamma^{n}\right]= & \frac{\Xi}{\bar{\gamma}^{\mu}} \sum_{i=1}^{N(m, \mu)} \sum_{l=1}^{\left|\omega_{i}\right|} \frac{\left(m_{s}\right)_{l} \mathcal{A}_{i l}\left(m_{s}-1\right)^{n} \bar{\gamma}^{l+n}}{\Gamma(l) \vartheta_{i}^{l+n}} \\
& \times B\left(n+l, m_{s}-n\right) \quad m_{s}>n \tag{28}
\end{align*}
$$

The CQEI is utilised to obtain insights on the AoF at a certain SNRs. The CQEI is given as

$$
\begin{equation*}
\mathrm{CQEI}=\frac{\operatorname{Var}[\gamma]}{(\mathbb{E}[\gamma])^{3}}=\frac{\mathrm{AF}}{\mathbb{E}[\gamma]} \tag{29}
\end{equation*}
$$

## D. Error Vector Magnitude

EVM is an important performance measure that provides several advantages, such as, identifying some types of the dropping and their sources in the transmission path. Hence, it has become part of wireless communications standards [13].

Lemma 3: When the fading parameters $\mu$ and $m$ are arbitrary numbers, the EVM is given in (30) shown at the top of this page. On the other side, when $\mu \in 2 \mathbb{Z}^{+}$and $m \in \mathbb{Z}^{+}$, the EVM is derived as

$$
\begin{align*}
\mathrm{EVM}=\frac{\Xi}{\sqrt{\gamma} \hat{r}^{2 \mu}} \sum_{i=1}^{N(m, \mu)} & \sum_{l=1}^{\left|\omega_{i}\right|} \frac{\left(m_{s}\right)_{l} \mathcal{A}_{i l} \hat{r}^{2 l}}{\Gamma(l) \vartheta_{i}^{l}}\left(\frac{\vartheta_{i}}{\left(m_{s}-1\right) \hat{r}^{2}}\right)^{1 / 4} \\
& \times B\left(l-0.25, m_{s}-0.25\right) \tag{31}
\end{align*}
$$

Proof: See Appendix C.

## E. Average Channel Capacity

According to Shannon's theorem, the ACC, $C$, can be calculated by

$$
\begin{equation*}
\bar{C}=\frac{1}{\ln (2)} \int_{0}^{\infty} \ln (1+\gamma) f_{\gamma}(\gamma) d \gamma \tag{32}
\end{equation*}
$$

Plugging (15) in (32) and making use of the identities [43, eq. (01.04.26.0003.01)] and [43, eq. (01.02.26.0007.01)] for $\ln (1+x)$ and $(1+x)^{-a}$, respectively, we obtain

$$
\begin{align*}
& \bar{C}=\frac{\Xi}{\ln (2) \Gamma\left(m_{s}\right)} \sum_{i=1}^{N(m, \mu)} \sum_{l=1}^{\left|\omega_{i}\right|} \frac{\mathcal{A}_{i l} \bar{\gamma}^{l-\mu}}{\Gamma(l) \Gamma\left(m_{s}+l\right) \vartheta_{i}^{l}} \\
& \times \int_{0}^{\infty} \gamma^{l-1} G_{2,2}^{1,2}\left[\gamma \left\lvert\, \begin{array}{c}
1,1 \\
1,0
\end{array}\right.\right] G_{1,1}^{1,1}\left[\left.\frac{\vartheta_{i}}{\left(m_{s}-1\right) \bar{\gamma}} \gamma \right\rvert\, \begin{array}{c}
1-m_{s}-l \\
0
\end{array}\right] d \gamma . \tag{33}
\end{align*}
$$

$$
\begin{align*}
& \mathcal{R}=-\frac{1}{A} \log _{2}\left\{\frac{\Xi}{\Gamma(A) \Gamma\left(m_{s}\right)\left[\Gamma\left(\frac{\mu}{2}-m\right) \Gamma(m)\right]^{2}\left[\left(m_{s}-1\right) \bar{\gamma}\right]^{\mu}}\right. \\
& \left.\left.G_{1,1:[1,1]_{i=1: 4}^{1,1:[1,1]_{i=1: 4}}\left[\frac{1}{\sqrt{\eta \alpha_{2}}\left(m_{s}-1\right) \bar{\gamma}}, \frac{\sqrt{\eta}}{\sqrt{\alpha_{2}}\left(m_{s}-1\right) \bar{\gamma}}, \frac{c_{1}}{\left(m_{s}-1\right) \bar{\gamma}}, \frac{c_{2}}{\left(m_{s}-1\right) \bar{\gamma}}\left|\begin{array}{c}
1-\mu-m_{s} \\
A-\mu
\end{array}\right|\left(1-\frac{\mu}{2}+m\right)_{i=1: 2},(1-m)_{i=3: 4}\right.}^{(0)_{i=1: 4}}\right]\right\} . \tag{39}
\end{align*}
$$

With the aid of [43, eq. (07.34.21.0012.01)], the integral of (33) can be expressed in exact closed-form. Consequently, the ACC over FBS fading channel is derived as

$$
\begin{align*}
\bar{C}=\frac{\Xi}{\ln (2) \Gamma\left(m_{s}\right)} & \sum_{i=1}^{N(m, \mu)} \sum_{l=1}^{\left|\omega_{i}\right|} \frac{\mathcal{A}_{i l} \bar{\gamma}^{l-\mu}}{\Gamma(l) \vartheta_{i}^{l}} \\
& \times G_{3,3}^{2,3}\left[\frac{\left(m_{s}-1\right) \bar{\gamma}}{\vartheta_{i}} \left\lvert\, \begin{array}{c}
1-l, 1,1 \\
1, m_{s}, 0
\end{array}\right.\right] . \tag{34}
\end{align*}
$$

At $\bar{\gamma} \rightarrow \infty$, the asymptotic of the ACC that is given in (32), $\bar{C}^{\infty}$, can be evaluated via [21]

$$
\begin{equation*}
\bar{C}^{\infty} \approx \frac{1}{\ln (2)} \int_{0}^{\infty} \ln (\gamma) f_{\gamma}(\gamma) d \gamma \tag{35}
\end{equation*}
$$

Inserting (15) in (35) and employing [44, eq. (2.6.4.7)] with some mathematical manipulations, $\bar{C}^{\infty}$, over FBS fading model is expressed as

$$
\begin{align*}
\bar{C}^{\infty} \approx \frac{\Xi}{\ln (2)} & \sum_{i=1}^{N(m, \mu)} \sum_{l=1}^{\left|\omega_{i}\right|} \frac{\left(m_{s}\right)_{l} \mathcal{A}_{i l} \bar{\gamma}^{l-\mu}}{\Gamma(l) \vartheta_{i}^{l}} B\left(l, m_{s}\right) \\
& \times\left[\ln \left(\frac{\left(m_{s}-1\right) \bar{\gamma}}{\vartheta_{i}}\right)+\psi(l)-\psi\left(m_{s}\right)\right] . \tag{36}
\end{align*}
$$

## F. Effective Throughput

The ET measures the capacity of the channel via taking into consideration the constraints that would cause imperfect the quality-of-service (QoS), such as, system delay [24].

The ET can be calculated by [34, eq. (29)]

$$
\begin{equation*}
\mathcal{R}=-\frac{1}{A} \log _{2}\left\{\int_{0}^{\infty}(1+\gamma)^{-A} f_{\gamma}(\gamma) d \gamma\right\} \tag{37}
\end{equation*}
$$

where $A \triangleq \Theta T B / \ln (2)$ with $\Theta$ represents the delay exponent, $T$ stands for the time, and $B$ denotes the channel's bandwidth.

Substituting (8) into (37) and using the contour integral representation of $F_{D}^{(4)}($.$) s in Appendix A-D, we have the$ following integral in terms of the general values of $\mu$ and $m$

$$
\begin{align*}
& \int_{0}^{\infty} \gamma^{\mu-\sum_{i=1}^{4} t_{i}-1}(1+\gamma)^{-A} d \gamma \stackrel{\left(a_{1}\right)}{=} \\
& \frac{\Gamma\left(\mu-\sum_{i=1}^{4} t_{i}\right) \Gamma\left(A-\mu+\sum_{i=1}^{4} t_{i}\right)}{\Gamma(A)} \tag{38}
\end{align*}
$$

where $a_{1}$ arises after evaluating the integral via employing [42, eq. (3.194.3)] and then invoking the property [42, eq. (8.384.1)].

Now, plugging the result of (38) together with the remaining parts of (8) in (37), the exact expression of the ET over FBS fading channel is yielded after employing the definition of the MGF as shown in (39) given at the top of this page.

For the asymptotic expression of (39), we substitute (9) into (37). Thereafter, with the help of [42, eq. (3.194.3)], the result is

$$
\begin{equation*}
\mathcal{R}^{\infty} \approx-\frac{1}{A} \log _{2}\left\{\frac{\Xi}{B\left(\mu, m_{s}\right)\left[\left(m_{s}-1\right) \bar{\gamma}\right]^{\mu}} B(\mu, A-\mu)\right\} \tag{40}
\end{equation*}
$$

where $A>\mu$.
When the values of $\mu$ and $m$ are respectively limited by even and integer numbers, the exact ET can be derived via inserting (15) in (37). Thus, this yields

$$
\begin{align*}
\int_{0}^{\infty} \gamma^{l-1} & (1+\gamma)^{-A}\left(1+\frac{\vartheta_{i}}{\left(m_{s}-1\right) \bar{\gamma}} \gamma\right)^{-\left(m_{s}+l\right)} d \gamma \stackrel{\left(a_{2}\right)}{=} \\
& \left(\frac{\left(m_{s}-1\right) \bar{\gamma}}{\vartheta_{i}}\right)^{l} B\left(l, A+m_{s}\right) \\
& \times{ }_{2} F_{1}\left(A, l ; A+m_{s}+l ; 1-\frac{\left(m_{s}-1\right) \bar{\gamma}}{\vartheta_{i}}\right) \tag{41}
\end{align*}
$$

where $a_{2}$ follows [42, eq. (3.197.1)].
Now, plugging the result of (41) alongside with the remaining terms of (15) in (37), we have

$$
\begin{align*}
\mathcal{R} & =-\frac{1}{A} \log _{2}\left\{\Xi \sum_{i=1}^{N(m, \mu)} \sum_{l=1}^{\left|\omega_{i}\right|} \frac{\left(m_{s}\right)_{l} \mathcal{A}_{i l} \bar{\gamma}^{l-\mu}}{\Gamma(l) \vartheta_{i}^{l}} B\left(l, A+m_{s}\right)\right. \\
& \left.\times{ }_{2} F_{1}\left(A, l ; A+m_{s}+l ; 1-\frac{\left(m_{s}-1\right) \bar{\gamma}}{\vartheta_{i}}\right)\right\} . \tag{42}
\end{align*}
$$

At high average SNR values, the asymptotic analysis of the ET that is presented in (42) can be deduced via utilising (16), (37), and [42, eq. (3.194.3)] to compute the integral. Accordingly, we obtain

$$
\begin{equation*}
\mathrm{EC}^{\infty} \approx-\frac{1}{A} \log _{2}\left\{\frac{\Xi}{\bar{\gamma}^{\mu}} \sum_{i=1}^{N(m, \mu)} \sum_{l=1}^{\left|\omega_{i}\right|} \frac{\left(m_{s}\right)_{l} \mathcal{A}_{i l} B(l, A-l)}{\Gamma(l)\left(m_{s}-1\right)^{l}}\right\} \tag{43}
\end{equation*}
$$

where $A>l$.

## G. Average AUC of Energy Detection

In the ED, the average $\mathrm{AUC}, \bar{A}$, is employed as an alternative performance metric of the receiver operating characteristic (ROC) curve that depends on both the detection and false alarm probabilities [12].

The average AUC can be computed by [34, eq. (32)]

$$
\begin{equation*}
\bar{A}=1-\sum_{k=0}^{u-1} \sum_{v=0}^{k} \frac{\binom{k+u-1}{k-v}}{2^{k+v+u} v!} \int_{0}^{\infty} \gamma^{v} e^{-\frac{\gamma}{2}} f_{\gamma}(\gamma) d \gamma \tag{44}
\end{equation*}
$$

where $u$ is the time-bandwidth product.

$$
\begin{gather*}
\bar{A}=1-\sum_{k=0}^{u-1} \sum_{v=0}^{k}\binom{k+u-1}{k-v} \frac{2^{-(k+u-\mu)} \Xi}{\Gamma\left(m_{s}\right)\left[\Gamma\left(\frac{\mu}{2}-m\right) \Gamma(m)\right]^{2}\left[\left(m_{s}-1\right) \bar{\gamma}\right]^{\mu} v!} G_{2,1:[1,1]_{i=1: 4}^{0,2}\left[\begin{array}{l}
0,1, l_{i=1} \\
\sqrt{\eta \alpha_{2}}\left(m_{s}-1\right) \bar{\gamma}
\end{array} \frac{2}{\sqrt{\alpha_{2}}\left(m_{s}-1\right) \bar{\gamma}}\right.}^{\left(\frac{2 c_{1}}{\left(m_{s}-1\right) \bar{\gamma}}, \frac{2 c_{2}}{\left(m_{s}-1\right) \bar{\gamma}} \left\lvert\, \begin{array}{c}
1-\mu-m_{s}, 1-\mu-v \left\lvert\,\left(1-\frac{\mu}{2}+m\right)_{i=1: 2}\right.,(1-m)_{i=3: 4} \\
1-\mu
\end{array}\right.,\right.} .
\end{gather*}
$$

$$
\begin{equation*}
\bar{A}=1-\Xi \sum_{k=0}^{u-1} \sum_{v=0}^{k}\binom{k+u-1}{k-v} \frac{1}{2^{k+v+u} v!} \sum_{i=1}^{N(m, \mu)} \sum_{l=1}^{\left|\omega_{i}\right|} \frac{\left(m_{s}\right)_{l}(l)_{v} \mathcal{A}_{i l}\left(m_{s}-1\right)^{v} \bar{\gamma}^{l+v-\mu}}{\vartheta_{i}^{l+v}} \Psi\left(l+v ; v+m_{s}-1 ; \frac{\left(m_{s}-1\right) \bar{\gamma}}{2 \vartheta_{i}}\right) \tag{48}
\end{equation*}
$$

$$
\begin{array}{r}
\mathrm{SOP}^{L}=\frac{\Xi_{D} \Xi_{E}}{\bar{\gamma}_{D}^{\mu_{D}} \bar{\gamma}_{E}^{\mu_{E}}} \sum_{i_{E}=1}^{N_{E}\left(m_{E}, \mu_{E}\right)} \sum_{l_{E}=1}^{\left|\omega_{i_{E}}\right|} \sum_{i_{D}=1}^{N_{D}\left(m_{D}, \mu_{D}\right)} \sum_{l_{D}=1}^{\left|\omega_{i_{D}}\right|} \frac{\theta^{l_{D}}\left(m_{s_{E}}\right)_{l_{E}}\left(m_{s_{D}}\right)_{l_{D}} \mathcal{A}_{i_{E} l_{E}} \mathcal{A}_{i_{D} l_{D}} \bar{\gamma}_{E}^{l_{E}+l_{D}}}{\Gamma\left(m_{s_{E}}+l_{E}\right) \Gamma\left(l_{E}\right) \Gamma\left(m_{s_{D}}+l_{D}\right) \Gamma\left(l_{D}\right) \vartheta_{i_{E}}^{l_{E}+l_{D}}}\left(\frac{m_{s_{E}}-1}{m_{s_{D}}-1}\right) \\
G_{3,3}^{2,3}\left[\left.\frac{\theta \vartheta_{i_{D}}\left(m_{s_{E}}-1\right) \bar{\gamma}_{E}}{\vartheta_{i_{E}}\left(m_{s_{D}}-1\right) \bar{\gamma}_{D}} \right\rvert\, \begin{array}{c}
1-l_{E}-l_{D}, 1-m_{s_{D}}-l_{D}, 1-l_{D} \\
0, m_{s_{E}}-l_{E},-l_{D}
\end{array}\right] \tag{52}
\end{array}
$$

Inserting (8) in (44) and making utilise of the function $F_{D}^{(4)}($.$) in terms of the contour integrals as in (61), the$ following integral is deduced

$$
\begin{align*}
& \int_{0}^{\infty} \gamma^{\mu+v-\sum_{i=1}^{4} t_{i}-1} e^{-\frac{\gamma}{2}} d \gamma \stackrel{\left(a_{3}\right)}{=} \\
& 2^{\mu+v-\sum_{i=1}^{4} t_{i}} \Gamma\left(\mu+v-\sum_{i=1}^{4} t_{i}\right) \tag{45}
\end{align*}
$$

where $a_{3}$ obtains after using [42, eq. (3.381.4)].
Employing the result of (45) as well as the remaining parts of (8) in (44) along with the definition of the multi-variate MGF, the exact average AUC is expressed as in (46) shown at the top of the next page.

At high $\bar{\gamma}$ regime, the average AUC, $\bar{A}^{\infty}$, can be computed via utilising (9) in (44) and recalling [42, eq. (3.381.4)] as

$$
\begin{equation*}
\bar{A}^{\infty} \approx 1-\sum_{k=0}^{u-1} \sum_{v=0}^{k}\binom{k+u-1}{k-v} \frac{2^{-(k+u-\mu)} \Gamma(\mu+v) \Xi}{v!B\left(\mu, m_{s}\right)\left[\left(m_{s}-1\right) \bar{\gamma}\right]^{\mu}} \tag{47}
\end{equation*}
$$

For $\mu \in 2 \mathbb{Z}^{+}$and $m \in \mathbb{Z}^{+}$, the average AUC can be derived via substituting (15) into (44) and invoking [43, eq. (07.33.07.0001.01)]. Therefore, after some mathematical simplifications, we have (48) shown at the top of this page.

The asymptotic behaviour of the average AUC that is provided in (48) can be studied via plugging (16) in (44) and then recalling [42, eq. (3.381.4)]. Thus, this yields

$$
\begin{align*}
& \bar{A}^{\infty} \approx 1-\frac{\Xi}{\bar{\gamma}^{\mu}} \\
& \sum_{k=0}^{u-1} \sum_{v=0}^{k}\binom{k+u-1}{k-v} \frac{1}{v!} \sum_{i=1}^{N(m, \mu)} \sum_{l=1}^{\left|\omega_{i}\right|} \frac{\left(m_{s}\right)_{l}(l)_{v} \mathcal{A}_{i l}}{2^{k+u-l}\left(m_{s}-1\right)^{l}} \tag{49}
\end{align*}
$$

## H. Lower Bound of SoP

The SOP is the probability of dropping the secrecy capacity below a certain secrecy threshold, $R_{s} \geq 0$. Moreover, the
$\mathrm{SOP}^{L}$ is the lower bound of SOP when the instantaneous SNR of the eavesdropper, $\gamma_{E}$, goes to infinity [25].

The $\mathrm{SOP}^{L}$ can be calculated by [18, eq. (20)]

$$
\begin{equation*}
\mathrm{SOP}^{L}=\int_{0}^{\infty} F_{D}\left(\theta \gamma_{E}\right) f_{E}\left(\gamma_{E}\right) d \gamma_{E} \tag{50}
\end{equation*}
$$

where $\theta=\exp \left(R_{s}\right) \geq 1, F_{D}($.$) is the CDF of the instanta-$ neous SNR of the Alice-Bob link, and $f_{E}($.$) is the PDF of$ the instantaneous SNR of the Alice-Eavesdropper link.

In this subsection and for the sake of brevity, the $\mathrm{SOP}^{L}$ is derived when $\mu$ is an even number and $m$ is an integer value.

After substituting (15) and (17) into (50) and then recalling the properties [43, eq. ( 07.23 .26 .0004 .01 )] and [43, eq. ( 01.02 .26 .0007 .01 )] to respectively write the functions ${ }_{2} F_{1}(., . ; .,$.$) and (1+x)^{-a}$ in terms of the single variable MGF, the following integral needs to be evaluated

$$
\begin{gather*}
\int_{0}^{\infty} \gamma_{E}^{l_{E}+i_{D}-1} G_{2,2}^{1,2}\left[\left.\frac{\theta \vartheta_{i_{D}} \gamma_{E}}{\left(m_{s_{D}}-1\right) \bar{\gamma}_{D}} \right\rvert\, \begin{array}{c}
1-m_{s_{D}}-l_{D}, 1-l_{D} \\
0,-l_{D}
\end{array}\right] \\
\quad \times G_{1,1}^{1,1}\left[\frac{\vartheta_{i_{E}} \gamma_{E}}{\left(m_{s_{E}}-1\right) \bar{\gamma}_{E}} \left\lvert\, \begin{array}{c}
\left.1-m_{s_{E}}-l_{E}\right] d \gamma_{E} \stackrel{\left(a_{4}\right)}{=} \\
0
\end{array}\right.\right] \\
\left(\frac{\left(m_{s_{E}}-1\right) \bar{\gamma}_{E}}{\vartheta_{i_{E}}}\right)^{l_{D}+l_{E}} \quad G_{3,3}^{2,3}\left[\left.\frac{\theta \vartheta_{i_{D}}\left(m_{s_{E}}-1\right) \bar{\gamma}_{E}}{\vartheta_{i_{E}}\left(m_{s_{D}}-1\right) \bar{\gamma}_{D}} \right\rvert\,\right. \\
\left.1-l_{E}-l_{D}, 1-m_{s_{D}}-l_{D}, 1-l_{D}\right]  \tag{51}\\
0, m_{s_{E}}-l_{E},-l_{D}
\end{gather*}
$$

where $a_{4}$ follows [43, eq. (07.34.21.0012.01)].
Now, plugging the result of (51) alongside with the remaining terms of the PDF and CDF of (15) and (17), respectively, and performing some straightforward operations, we have the exact expression of the $\mathrm{SOP}^{L}$ as shown in (52) provided at the top of this page.

The asymptotic of the $\mathrm{SOP}^{L}$, $\mathrm{SOP}^{L, \infty}$, when the average SNR of the Bob, $\bar{\gamma}_{D}$, tends to infinity, can be computed via inserting (15) and (18) in (50). Consequently, the following

$$
\begin{equation*}
\operatorname{SOP}^{L, \infty} \approx \frac{\Xi_{D} \Xi_{E}}{\bar{\gamma}_{D}^{\mu_{D}} \bar{\gamma}_{E}^{\mu_{E}}} \sum_{i_{E}=1}^{N_{E}\left(m_{E}, \mu_{E}\right)} \sum_{l_{E}=1}^{\left|\omega_{i_{E}}\right|} \sum_{i_{D}=1}^{N_{D}\left(m_{D}, \mu_{D}\right)} \sum_{l_{D}=1}^{\left|\omega_{i_{D}}\right|} \frac{\theta^{l_{D}}\left(m_{s_{E}}\right)_{L_{E}}\left(m_{s_{D}}\right) l_{l_{D}} \mathcal{A}_{i_{E} l_{E}} \mathcal{A}_{i_{D} l_{D}} \bar{\gamma}_{E}^{l_{E}+l_{D}} B\left(l_{E}+l_{D}, m_{s_{E}}-l_{D}\right)}{\Gamma\left(l_{E}\right) \Gamma\left(l_{D}+1\right) \vartheta_{i_{E}}^{l_{E} l_{D}}\left(m_{s_{E}}-1\right)^{-l_{D}}\left(m_{s_{D}}-1\right)^{l_{D}}} . \tag{54}
\end{equation*}
$$

integral is obtained

$$
\begin{gather*}
\int_{0}^{\infty} \gamma_{E}^{l_{E}+i_{D}-1}\left(1+\frac{\vartheta_{i_{E}}}{\left(m_{s_{E}}-1\right) \bar{\gamma}_{E}}\right)^{-\left(l_{E}+l_{D}\right)} d \gamma_{E} \stackrel{\left(a_{5}\right)}{=} \\
\left(\frac{\left(m_{s_{E}}-1\right) \bar{\gamma}_{E}}{\vartheta_{i_{E}}}\right)^{l_{D}+l_{E}} B\left(l_{E}+i_{D}, m_{s_{E}}-l_{D}\right) \tag{53}
\end{gather*}
$$

where $m_{s_{E}}>l_{D}$ and $a_{5}$ arises after using [42, eq. (3.194.3)].
Substituting (53) together with the remaining terms of (15) and (18) into (50), the result is (54) as shown at the top of this page.

## VI. Numerical and Simulation Results

In this section, the influence of the shadowing parameter $m_{s}$ on the derived performance metrics over FBS fading channel is studied via comparing the numerical results with the Monte Carlo simulations which are obtained by $10^{6}$ realizations. In all figures, the theoretical, simulation and asymptotic results are represented by the markers, solid and dashed lines, respectively. Three different scenarios which are heavy, moderate, and light shadowing with $m_{s}=1.5, m_{s}=5.5$, and $m_{s}=50$, respectively, are analysed. Additionally, in all figures, excluding the special cases, $\kappa=2, \mu=1.5, m=2.5$, $\eta=0.1$, and $\varrho^{2}=0.2$. Furthermore, all the figures include the result of some special cases of the FBS fading, such as, FB, Rayleigh, $\kappa-\mu$, and double Rician shadowed, that are given in Table I. ${ }^{1}$

Figs. 2 and 3 illustrate the OP for $\gamma_{t h}=0 \mathrm{~dB}$ and $\gamma_{t h}=10$ dB and ABEP for BPSK and BFSK modulation schemes, respectively, versus the average $\mathrm{SNR}, \bar{\gamma}$. As expected, both the OP and ABEP diminish when $m_{s}$ increases. This improvement in the system performance refers to the reduction in the impact of the shadowing on the total received signal power. For example, at $\gamma_{t h}=10 \mathrm{~dB}$ and $\mathrm{SNR}=20 \mathrm{~dB}$ (fixed), the OP for $m_{s}=1.5$ is approximately 0.1545 whereas the values of the OP for $m_{s}=5.5$ and $m_{s}=50$ are nearly 0.0433 and 0.0318 , respectively. Similarly, when SNR is constant at 20 dB , the ABEP of BPSK with $m_{s}=50$ is roughly $27.75 \%$ less than that of $m_{s}=5.5$. Besides, for all cases of $m_{s}$ of Fig. 2, the OP of $\gamma_{t h}=0 \mathrm{~dB}$ is lower than that of $\gamma_{t h}=10 \mathrm{~dB}$ which is consistent with the results of [41]. This is because the range of the average SNR of $\gamma_{t h}=0 \mathrm{~dB}$ is smaller than that of $\gamma_{t h}=10 \mathrm{~dB}$ which would lead to decrease the probability that depends on the comparison $\bar{\gamma} \leq \gamma_{t h}$. Additionally, Figs. 2 and 3 also plot the OP and ABEP for Rayleigh fading channel. From both figures and after a certain value of average SNR, it can be noticed that the values of the OP and ABEP of Rayleigh fading are higher than that of the FBS fading model

[^1]

Fig. 2. OP versus $\bar{\gamma}$ for different values of $m_{s}$ and $\gamma_{t h}$.


Fig. 3. ABEP versus $\bar{\gamma}$ for different values $m_{s}$, BFSK and BPSK schemes.
with $m_{s}=1.5$. This change in the behaviour is due to the effect of the parameter $\mu$ that increases with the high value of $\bar{\gamma}$. This fact proves the correctness of our derived expressions of the diversity order of the OP and ABEP which depend on $\mu$ that is 1 for Rayleigh and 1.5 for other cases.

Fig. 4 depicts the EVM versus instantaneous SNR, $\gamma$, for $\hat{r}=1$ and $\hat{r}=0.5$. As anticipated, the EVM reduces when $\hat{r}$ and/or $m_{s}$ increase. The reason for the influence of $\hat{r}$ refers to the increasing in the mean value of the received signal power whereas the explanation for the impact of $m_{s}$ on the EVM is the same as that of the OP and ABEP. For instance, at $\gamma=-6$ dB and $m_{s}=5.5$, the EVM is decreased by approximately


Fig. 4. EVM versus $\gamma$ for different values of $m_{s}$ and $\hat{r}$.


Fig. 5. CQEI versus $\bar{\gamma}$ for different values of $m_{s}$.

## $29.25 \%$ when $\hat{r}$ changes from 0.5 to 1 .

The CQEI versus $\bar{\gamma}$ is shown in Fig. 5 for $m_{s}=2.1$, $m_{s}=6.1$, and $m_{s}=50$. It is clear that as $m_{s}$ decreases, the CQEI increases. This is because the diminishing in $m_{s}$ would lead to increase the AF. From Fig. 5 and at $\bar{\gamma}=15$ dB (fixed), when $m_{s}$ reduces from 6.1 to 2.1 , the CQEI is increased by nearly $44.83 \%$. For further validation of the derived expressions, the curves of the CQEI of $\kappa-\mu$ and Nakagami-1.5 fading conditions are also presented in Fig. 5. It can be noticed that $\kappa-\mu$ has better performance than Nakagami-1.5. This superiority is the result of the influence of the parameter $\kappa$ which is 2 for $\kappa-\mu$ fading and $\kappa \rightarrow 0$ for Nakagami-1.5 fading.

Fig. 6 portrays the ACC versus $\bar{\gamma}$ for $\mu=2$ and $m=3$. Like the OP, ABEP, EVM, and CQEI, the ACC improves with the increasing in $m_{s}$ and/or $m$ and for the same reasons that have been explained before. For example, when $m_{s}=1.5$ and $\bar{\gamma}=20 \mathrm{~dB}$ are fixed, the values of the ACC of $m=3$ and $m=1$ are approximately 5.195 and 4.732 , respectively.


Fig. 6. ACC versus $\bar{\gamma}$ for different values of $m_{s}$ and $m$.


Fig. 7. ET versus $\bar{\gamma}$ for different values of $m_{s}$ and $A$.

The ACC of $\kappa-\mu$ fading model is also given in Fig. 6 for the validation as well as comparison purposes.

In Fig. 7, the ET versus $\bar{\gamma}$ is plotted for $A=3.5$ and $A=4.5$. Similar to the previous performance measures, the ET enhances when the shadowing changes form heavy to moderate and light. As one can observe, that moving from $m_{s}=1.5$ to $m_{s}=5.5$ and $m_{s}=50,4.5 \mathrm{~dB}$ and 6 dB are respectively needed to obtain ET $=4 \mathrm{bits} / \mathrm{s} / \mathrm{Hz}$. Moreover, it is interesting to notice that the moving from $A=4.5$ to $A=3.5,4.36 \mathrm{~dB}$ and 4.37 dB are roughly required to achieve $\mathrm{ET}=5 \mathrm{bits} / \mathrm{s} / \mathrm{Hz}$ for $m_{s}=5.5$ and $m_{s}=50$, respectively. The decreasing in $A$ is yielded due to the diminishing in the delay exponent of the received signal. These observations are matched with the results reported in [46] and [47]. Additionally, Fig. 5 plots the ET for the double Rician shadowed Type I fading model which is extracted from the FBS fading model via inserting $K=\kappa$ ( $K$ stands for the Rician factor), $\mu=1, \eta=1, m=2.5$, and $m_{s}=1.5$. This curve shows the degradation in the performance of an ET due


Fig. 8. Average CAUC versus $\bar{\gamma}$ for different values of $m_{s}$ and $u$.


Fig. 9. $\operatorname{SOP}^{L}$ versus $\bar{\gamma}$ for different values of $m_{s}$ and $\bar{\gamma}_{E}$.
to the diminishing in $\mu$ from $\mu=1.5$ to $\mu$ which confirms the correctness of our derived expression.

Fig. 8 portrays the complementary AUC (CAUC) which equals to $1-\bar{A}$ versus $\bar{\gamma}$ for different $m_{s}$ and $u$. From this figure, it is obvious that the CAUC reduces, i.e., $\bar{A}$ becomes better, when $m_{s}$ increases and/or $u$ decreases. The reason for the impact of $m_{s}$ has been comprehensively explained in the analysis of the results of Figs. 2-7 whereas the influence of $u$ refers to the faster increasing in the average detection probability in comparison with the false alarm probability. At constant $u=5$ and $\bar{\gamma}=25 \mathrm{~dB}$, the CAUC of $m_{s}=1.5$ is approximately $79.79 \%$ lower than that of $m_{s}=5.5$. At the same value of $\bar{\gamma}$ and $m_{s}=50$, the variation in $u$ from 5 to 2 decreases the CAUC by nearly $58.21 \%$. It is worth interesting that the CAUC of FB fading condition is also presented in Fig. 8 via using $m_{s}=50$, i.e., $m_{s} \rightarrow \infty$.

The $\mathrm{SOP}^{L}$ versus $\bar{\gamma}$ is depicted in Fig. 9 for different scenarios of the shadowing and values of $\bar{\gamma}_{E}$. The parameters of both main and wiretap channels are supposed to be arbitrarily
distributed variates via using $\left(\kappa_{l}, \mu_{l}, m_{l}, \eta_{l}, \varrho_{l}^{2}\right)=(2,2,1,0.1$, 0.2 ) with $l \in\{D, E\}, m_{s_{E}}=5.5$, and $m_{s_{D}}=1.5,5.5,50$. From this figure, a clear improvement in the secrecy behaviour is noticed when $m_{s_{D}}, \bar{\gamma}_{D}$ and/or $\bar{\gamma}_{E}$ become high, large, and/or low, respectively. Intuitively, the high increasing in $\bar{\gamma}_{D}$ refers to good quality of the main communication channel whereas the high decreasing in $\bar{\gamma}_{E}$ means a large degradation in the quality of the Alice-Eavesdropper link, namely, wiretap channel. To obtain $10^{-2}$ of $\mathrm{SOP}^{L}$ at constant $\bar{\gamma}_{E}=5 \mathrm{~dB}$, it requires nearly $36 \mathrm{~dB}, 32 \mathrm{~dB}$, and 31 dB , respectively, for $m_{s}=1.5, m_{s}=5.5$, and $m_{s}=50$. In addition, when $m_{s}=1.5$ and $\bar{\gamma}_{D}=25 \mathrm{~dB}$ are fixed, the $\mathrm{SOP}^{L}$ of $\bar{\gamma}_{E}=5$ and $\bar{\gamma}_{E}=-5$ are $8.712 \times 10^{-2}$ and $1.199 \times 10^{-2}$, respectively. For comparison purpose, the $\mathrm{SOP}^{L}$ over FB fading model that has been studied in [18] is also provided in Fig. 9.

In all figures, it can be seen that an excellent agreement between the numerical results and their Monte Carlo simulation counterparts which affirms the validation of our derived expressions. Besides, all the asymptotic at high average SNR and exact results are perfectly matched which represents another verification for the correctness of our analysis.

## VII. Conclusions

This paper was devoted to provide the statistical properties of a new composite fading model, namely, FBS fading which is a composite of fluctuating Beckmann/inverse Nakagami-m distributions. To be specific, the PDF, CDF, and G-MGF of the received signal envelope as well as the instantaneous SNR were derived for two different cases. In the first case in which $\mu$ and $m$ were assumed to be arbitrary values, the fundamental statistics were derived in novel exact mathematically tractable closed-form expressions in terms of the Lauricella hypergeometric and Fox's $H$ functions of four variables. On the contrary, in the second case, simple exact closed-from PDF, CDF, and G-MGF were obtained via assuming even and integer numbers for $\mu$ and $m$, respectively. Additionally, to gain further insights into the system behaviour, the asymptotic expressions of the statistical characterizations at high average SNR values were provided for both cases of the fading parameters. These statistics were then employed to evaluate the performance of the wireless communications systems over FBS fading channels. In particular, the OP, ABEP, EVM, CQEI, ACC, ET, AUC of ED, $\mathrm{SOP}^{L}$ were analysed for different scenarios of shadowing impact, namely, heavy, moderate, and light. From all the numerical results that are compared with the Monte-Carlo simulations, one can notice that the performance becomes better when $m_{s}$ increases. Furthermore, all the figures have included the results for some special models of the FBS fading channel, such as, FB, $\kappa-\mu$, and Rayleigh, that are given in Table I for validation and comparison purposes. The expressions of this paper can be used for several scenarios of composite double shadowed multipath fading channels due to their including two fading parameters for the shadowing which are $m$ and $m_{s}$.

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## Appendix A

## Proofs of Statistical Characterization of FBS <br> Fading Model with Arbitrary $\mu$ and $m$

## A. Proof of Theorem 1

Based on (5), the PDF of $R$ can be determined by the product of $\Lambda$ and $R^{\mathrm{FB}} \mathrm{RVs}$ as

$$
\begin{equation*}
f_{R}(r)=\int_{0}^{\infty} \frac{1}{y} f_{R^{\mathrm{FB}}}(y) f_{\Lambda}\left(\frac{r}{y}\right) d y \tag{55}
\end{equation*}
$$

Substituting (2) and (6) into (55) along with the change $z=y^{2}$, we have

$$
\begin{gather*}
f_{R}(r)=\frac{2 \Xi\left(m_{s}-1\right)^{m_{s}}}{\Gamma\left(m_{s}\right) \Gamma(\mu) r^{2 m_{s}+1} \hat{r}^{2 \mu}} \\
\times \int_{0}^{\infty} z^{\mu+m_{s}-1} e^{-\frac{\left(m_{s}-1\right) z}{r^{2}}} \Phi_{2}^{(4)}\left(\frac{\mu}{2}-m, \frac{\mu}{2}-m, m, m ; \mu\right. \\
\left.\quad-\frac{z}{\hat{r}^{2} \sqrt{\eta \alpha_{2}}},-\frac{z \sqrt{\eta}}{\hat{r}^{2} \sqrt{\alpha_{2}}},-\frac{z c_{1}}{\hat{r}^{2}},-\frac{z c_{2}}{\hat{r}^{2}}\right) d z \tag{56}
\end{gather*}
$$

After invoking [35, eq. (1.i), p. 259] to solve the integral of (56) and invoking the identity [42, eq. (8.384.1)], the result is (7) and this completes the proof.

## B. Proof of Lemma 1

The exact expression of the PDF of the instantaneous SNR, $\gamma$, over FBS fading channel that is given in (8) can be derived via plugging (7) in [41, eq. (2.3)] which is given as

$$
\begin{equation*}
f_{\gamma}(\gamma)=\frac{\hat{r}}{2 \sqrt{\gamma \bar{\gamma}}} f_{R}\left(\sqrt{\frac{\hat{r}^{2} \gamma}{\bar{\gamma}}}\right) \tag{57}
\end{equation*}
$$

When $\bar{\gamma}$ approaches to infinity, the function $F_{D}^{(4)}(.) \approx 1$ [10]. Using this fact in (8), we obtain (9) and this finishes the proof.

## C. Proof of Theorem 2

The CDF of $\gamma$ over FBS fading condition can be derived by

$$
\begin{equation*}
F_{\gamma}(\gamma)=\int_{0}^{\gamma} f_{\gamma}(\gamma) d \gamma \tag{58}
\end{equation*}
$$

Inserting (8) in (58) and using the infinite series representation of the function $F_{D}^{(4)}($.$) [35, eq. (1.7.4)], this yields$

$$
\begin{align*}
& F_{\gamma}(\gamma)=\frac{\Xi}{B\left(\mu, m_{s}\right)\left[\left(m_{s}-1\right) \bar{\gamma}\right]^{\mu}} \sum_{t_{1}, \cdots, t_{4}=0}^{\infty} \frac{(\mu+m)_{\sum_{i=1}^{4} t_{i}}}{(\mu)_{\sum_{i=1}^{4} t_{i}}} \\
& \frac{(\mu / 2-m)_{t_{1}}(\mu / 2-m)_{t_{2}}(m)_{t_{3}}(m)_{t_{4}}}{t_{1}!\cdots t_{4}!} \int_{0}^{\gamma} \gamma^{\mu-\sum_{i=1}^{4} t_{i}-1} d \gamma \\
& \left(-\frac{1}{\sqrt{\eta \alpha_{2}}\left(m_{s}-1\right) \bar{\gamma}}\right)^{t_{1}}\left(-\frac{\sqrt{\eta}}{\sqrt{\alpha_{2}}\left(m_{s}-1\right) \bar{\gamma}}\right)^{t_{2}} \\
& \left(-\frac{c_{1}}{\left(m_{s}-1\right) \bar{\gamma}}\right)^{t_{3}}\left(-\frac{c_{2}}{\left(m_{s}-1\right) \bar{\gamma}}\right)^{t_{4}} \tag{59}
\end{align*}
$$

The integral of (59) can be directly evaluated. After that, recalling the property $\Gamma(x+y)=\Gamma(x)(x)_{y}$ [35, eq. (1.1.15)], performing some mathematical manipulations and then using [35, eq. (1.7.4)], (10) is deduced.

The asymptotic of (10) that is given in (11) can be derived via following the same methodology of (9). Hence, the proof is accomplished.

## D. Proof of Theorem 3

The G-MGF of $n$-th moment can be computed by [17, eq. (2)]

$$
\begin{equation*}
\mathcal{M}_{\gamma}^{(n)}(s)=\mathbb{E}\left[\gamma^{n} e^{-\gamma s}\right]=\int_{0}^{\infty} \gamma^{n} e^{-s \gamma} f_{\gamma}(\gamma) d \gamma \tag{60}
\end{equation*}
$$

Plugging (8) in (60) and rewriting $F_{D}^{(4)}($.$) in terms of four$ Barnes-type contour integrals via utilising [45, eq. (1.10.16)], we have

$$
\begin{align*}
& \mathcal{M}_{\gamma}^{(n)}(s)=\frac{\Xi}{\Gamma\left(m_{s}\right)\left[\Gamma\left(\frac{\mu}{2}-m\right) \Gamma(m)\right]^{2}\left[\left(m_{s}-1\right) \bar{\gamma}\right]^{\mu}(2 \pi j)^{4}} \int_{\mathcal{T}_{1}} \\
& \ldots \int_{\mathcal{T}_{4}} \frac{\Gamma\left(\mu+m_{s}-\sum_{i=1}^{4} t_{i}\right)\left\{\prod_{i=1}^{2} \Gamma\left(\frac{\mu}{2}-m-t_{i}\right) \Gamma\left(t_{i}\right)\right\}}{\Gamma\left(\mu-\sum_{i=1}^{4} t_{i}\right)} \\
& \left\{\prod_{i=3}^{4} \Gamma\left(m-t_{i}\right) \Gamma\left(t_{i}\right)\right\} \int_{0}^{\infty} \gamma^{n+\mu-\sum_{i=1}^{4} t_{i}-1} e^{-s \gamma} d \gamma \\
& \left(\frac{1}{\sqrt{\eta \alpha_{2}}\left(m_{s}-1\right) \bar{\gamma}}\right)^{-t_{1}}\left(\frac{\sqrt{\eta}}{\sqrt{\alpha_{2}}\left(m_{s}-1\right) \bar{\gamma}}\right)^{-t_{2}} \\
& \left(\frac{c_{1}}{\left(m_{s}-1\right) \bar{\gamma}}\right)^{-t_{3}}\left(\frac{c_{2}}{\left(m_{s}-1\right) \bar{\gamma}}\right)^{-t_{4}} d t_{1} \cdots d t_{4} \tag{61}
\end{align*}
$$

where $\mathcal{T}_{i}$ is the suitable contour in the $t$-plane from $\nu-i \infty$ to $\nu+i \infty$ with $\nu$ is a constant value.

With the aid of [42, eq. (3.381.4)], the linear integral of (61) can be calculated in exact closed-form expression. Thereafter, making employ of the definition of the MFHF that is given in [36, eq. (A.1)] and its relation with the MGF, the result is (12).

To obtain (13), we first plug (9) in (60). Then, with the help of [42, eq. (3.381.4)], the proof is completed.

## Appendix B

## Proofs of Statistical Characterization of FBS Fading Model with Even $\mu$ and Integer $m$

## A. Proof of Theorem 4

When $\mu$ is an even number and $m$ is an integer value, the PDF of $R$ can be also derived via using (55). To this end, we first substitute (6) and (4) into (55). Thus, we have

$$
\begin{align*}
f_{R}(r)= & \frac{4 \Xi\left(m_{s}-1\right)^{m_{s}}}{\Gamma\left(m_{s}\right) \hat{r}^{2 \mu} r^{2 m_{s}-1}} \sum_{i=1}^{N(m, \mu)} \sum_{l=1}^{\left|\omega_{i}\right|} \frac{\mathcal{A}_{i l}}{\Gamma(l)} \\
& \times \int_{0}^{\infty} y^{2\left(m_{s}+l\right)-1} e^{-\left(\frac{\vartheta_{i}}{\hat{r}^{2}}+\frac{\left(m_{s}-1\right)}{r^{2}}\right) y^{2}} d y \tag{62}
\end{align*}
$$

Using the substitution $z=y^{2}$ in (62) and then evaluating the integral via invoking [42, eq. (3.381.4)], (14) is deduced.

## B. Proof of Lemma 2

An alternative expression of the PDF of $\gamma$ over FBS fading model that is presented in (8) can be obtained via plugging (14) in (57). Consequently, the result is (15).

At $\bar{\gamma} \rightarrow \infty$, the function $\left(1+\frac{\vartheta_{i}}{\left(m_{s}-1\right) \bar{\gamma}} \gamma\right)^{m_{s}+l}$ of (15) tends to unity. Hence, we get (16) and this ends the proof.

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## C. Proof of Theorem 5

The CDF of $\gamma$ over FBS fading model when $\mu \in 2 \mathbb{Z}$ and $m \in \mathbb{Z}$ can be derived via inserting (15) in (58). Accordingly, we have

$$
\begin{align*}
F_{R}(\gamma)= & \frac{\Xi}{\bar{\gamma}^{\mu}} \sum_{i=1}^{N(m, \mu)} \sum_{l=1}^{\left|\omega_{i}\right|} \frac{\left(m_{s}\right)_{l} \mathcal{A}_{i l}}{\Gamma(l)\left(m_{s}-1\right)^{l}} \\
& \times \int_{0}^{\gamma} \gamma^{l-1}\left(1+\frac{\vartheta_{i}}{\left(m_{s}-1\right) \bar{\gamma}} \gamma\right)^{-\left(m_{s}+l\right)} d \gamma \tag{63}
\end{align*}
$$

After invoking [42, eq. (3.194.1)] to calculate the integral of (63), the result is (17).

The asymptotic of (17) at high average SNR regime can be easily deduced via employing the fact that ${ }_{2} F_{1}(a, b ; c ; 0) \approx 1$. Consequently, we obtain (18) and this ends the proof.

## D. Proof of Theorem 6

To derive the exact G-MGF for the special values of $\mu$ and $m$, we first insert (15) in (60). Thus, this obtains

$$
\begin{align*}
& \mathcal{M}_{\gamma}^{(n)}(s)=\frac{\Xi}{\bar{\gamma}^{\mu}} \sum_{i=1}^{N(m, \mu)} \sum_{l=1}^{\left|\omega_{i}\right|} \frac{\left(m_{s}\right)_{l} \mathcal{A}_{i l}}{\Gamma(l)\left(m_{s}-1\right)^{l}} \\
& \times \int_{0}^{\infty} \gamma^{n+l-1}\left(1+\frac{\vartheta_{i}}{\left(m_{s}-1\right) \bar{\gamma}} \gamma\right)^{-\left(m_{s}+l\right)} e^{-s \gamma} d \gamma \tag{64}
\end{align*}
$$

Now, with the aid of [35, eq. (1.3.14)], the integral of (64) can be solved in exact closed-form as in (19).

Plugging (16) in (60) and utilising [42, eq. (3.381.4)] as well as the identity [35, eq. (1.1.15)], the expression of the asymptotic of the G-MGF at high average SNR values is yielded as in (20) which completes the proof.

## Appendix C <br> Proof of Lemma 3

The EVM over fading channel can be evaluated by [13]

$$
\begin{equation*}
\mathrm{EVM}=\int_{0}^{\infty} \frac{f_{R}(r)}{\sqrt{r \gamma}} d r \tag{65}
\end{equation*}
$$

Substituting (7) into (65) together with the variable change $z=r^{2}$ and four Barnes-type contour integrals representation of the function $F_{D}^{(4)}($.$) , this obtains$
$\mathrm{EVM}=\frac{\Xi}{\sqrt{\gamma} \Gamma\left(m_{s}\right)\left[\Gamma\left(\frac{\mu}{2}-m\right) \Gamma(m)\right]^{2}\left[\left(m_{s}-1\right) \hat{r}^{2}\right]^{\mu}(2 \pi j)^{4}}$
$\int_{\mathcal{T}_{1}} \cdots \int_{\mathcal{T}_{4}} \frac{\Gamma\left(\mu+m_{s}-\sum_{i=1}^{4} t_{i}\right)\left\{\prod_{i=1}^{2} \Gamma\left(\frac{\mu}{2}-m-t_{i}\right) \Gamma\left(t_{i}\right)\right\}}{\Gamma\left(\mu-\sum_{i=1}^{4} t_{i}\right)}$
$\left\{\prod_{i=3}^{4} \Gamma\left(m-t_{i}\right) \Gamma\left(t_{i}\right)\right\} \int_{0}^{\infty} z^{\mu-\sum_{i=1}^{4} t_{i}-\frac{5}{4}} d z$
$\left(\frac{1}{\sqrt{\eta \alpha_{2}}\left(m_{s}-1\right) \hat{r}^{2}}\right)^{-t_{1}}\left(\frac{\sqrt{\eta}}{\sqrt{\alpha_{2}}\left(m_{s}-1\right) \hat{r}^{2}}\right)^{-t_{2}}$
$\left(\frac{c_{1}}{\left(m_{s}-1\right) \hat{r}^{2}}\right)^{-t_{3}}\left(\frac{c_{2}}{\left(m_{s}-1\right) \hat{r}^{2}}\right)^{-t_{4}} d t_{1} \cdots d t_{4}$

Rewriting (66) in terms of three contour integrals to yield

$$
\begin{aligned}
& \mathrm{EVM}=\frac{\Xi}{\sqrt{\gamma} \Gamma\left(m_{s}\right)\left[\Gamma\left(\frac{\mu}{2}-m\right) \Gamma(m)\right]^{2}\left[\left(m_{s}-1\right) \hat{r}^{2}\right]^{\mu}(2 \pi j)^{3}} \\
& \int_{\mathcal{T}_{2}} \cdots \int_{\mathcal{T}_{4}} \Gamma\left(\frac{\mu}{2}-m-t_{2}\right) \Gamma\left(t_{2}\right)\left\{\prod_{i=3}^{4} \Gamma\left(m-t_{i}\right) \Gamma\left(t_{i}\right)\right\} \\
& \int_{0}^{\infty} z^{\mu-\sum_{i=2}^{4} t_{i}-\frac{5}{4}} G_{2,2}^{1,2}\left[\left.\frac{z}{\sqrt{\eta \alpha_{2}}\left(m_{s}-1\right) \hat{r}^{2}} \right\rvert\,\right.
\end{aligned}
$$

$$
\left.\begin{array}{c}
1-\frac{\mu}{2}+m, 1-\mu-m_{s}+\sum_{i=2}^{4} t_{i} \\
0,1-\mu+\sum_{i=2}^{4} t_{i}
\end{array}\right] d z\left(\frac{\sqrt{\eta}}{\sqrt{\alpha_{2}}\left(m_{s}-1\right) \hat{r}^{2}}\right)^{-t_{2}}
$$

$$
\begin{equation*}
\left(\frac{c_{1}}{\left(m_{s}-1\right) \hat{r}^{2}}\right)^{-t_{3}}\left(\frac{c_{2}}{\left(m_{s}-1\right) \hat{r}^{2}}\right)^{-t_{4}} d t_{2} \cdots d t_{4} \tag{67}
\end{equation*}
$$

Solving the inner integral via using [36, eq. (2.9)] and then employing [36, eq. (A.1)] and its mathematical relation with the MGF, the EVM is expressed in exact closed-form as in (30).

For even $\mu$ and integer $m$, the EVM is derived via inserting (14) in (65) and making use of $z=r^{2}$ as follows

$$
\begin{align*}
\mathrm{EVM}= & \frac{2 \Xi}{\sqrt{\gamma} \hat{r}^{2 \mu}} \sum_{i=1}^{N(m, \mu)} \sum_{l=1}^{\left|\omega_{i}\right|} \frac{\left(m_{s}\right)_{l} \mathcal{A}_{i l}}{\Gamma(l)\left(m_{s}-1\right)^{l}} \\
& \times \int_{0}^{\infty} z^{l-\frac{5}{4}}\left(1+\frac{\vartheta_{i}}{\left(m_{s}-1\right) \hat{r}^{2}} z\right)^{m_{s}+l} d z \tag{68}
\end{align*}
$$

Recalling [42, eq. (3.194.3)] to compute the integral of (68), (31) is obtained and this finishes the proof.

## REFERENCES

[1] F. Hansen and F. I. Meno, "Mobile fading-Rayleigh and lognormal superimposed," IEEE Trans. Veh. Technol., vol. 26, no. 4, pp. 332-335, Nov. 1977.
[2] A. Abdi and M. Kaveh, " $K$ distribution: An appropriate substitute for Rayleigh-Lognormal distribution in fading-shadowing wireless channels," Electron. Lett., vol. 34, pp. 851852, Apr. 1998.
[3] A. Abdi and M. Kaveh, "Comparison of DPSK and MSK bit error rates for K and Rayleigh-Lognormal fading distributions," IEEE Commun. Lett., vol. 4, no. 4, pp. 122-124, Apr. 2000.
[4] P. S. Bithas, N. C. Sagias, P. T. Mathiopoulos, G. K. Karagiannidis, and A. A. Rontogiannis, "On the performance analysis of digital communications over generalized- $K$ fading channels," IEEE Commun. Lett., vol. 10, no. 5, pp. 353-355, May 2006.
[5] A. Abdi,W. C. Lau, M.-S. Alouini, and M. Kaveh, "A new simple model for land-mobile satellite channels: First- and second-order statistics," IEEE Trans. Wireless Commun., vol. 2, no. 3, pp. 519-528, May 2003.
[6] J. F. Paris, "Closed-form expressions for Rician shadowed cumulative distribution function," Electron. Lett., vol. 46, pp. 952-953, Jun. 2010.
[7] P. C. Sofotasios and S. Freear, "On the $\kappa$ - $\mu$ /gamma composite distribution: A generalized multipath/shadowing fading model," SBMO/IEEE MTT-S Int. Microwave and Optoelectronics Conf. (IMOC), Oct. 2011, pp. 390-394.
[8] H. Al-Hmood and H. S. Al-Raweshidy, "Unified modeling of composite $\kappa-\mu /$ gamma, $\eta-\mu /$ gamma, and $\alpha-\mu /$ gamma fading channels using a mixture gamma distribution with applications to energy detection," IEEE Antennas Wireless. Propag. Lett., vol. 16, pp. 104-108, Apr. 2017.
[9] M. Yacoub, , "The $\kappa-\mu$ distribution and the $\eta-\mu$ distribution," IEEE Antennas Propag. Mag., vol. 49, no. 1, pp. 68-81, Feb. 2007.
[10] J. F. Paris, "Statistical characterization of $\kappa-\mu$ shadowed fading," IEEE Trans. Veh. Technol., vol. 63, no. 2, pp. 518-526, Feb. 2014.
[11] L. Moreno-Pozas, F.J. Lopez-Martinez, J.F. Paris, and E. Martos-Naya, "The $\kappa-\mu$ shadowed fading model: Unifying the $\kappa-\mu$ and $\eta-\mu$ distributions," IEEE Trans. Veh. Technol., vol. 65, no. 12, pp. 9630-9641, Dec. 2016.
[12] H. Al-Hmood, and H. Al-Raweshidy, "Analysis of energy detection with diversity receivers over non-identically distributed $\kappa-\mu$ shadowed fading channels," Electron. Lett., vol. 53, no. 2, pp. 83-85, Jan. 2017.

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[13] S. Parthasarathy, S. Kumar, R. K. Ganti, S. Kalyani and K. Giridhar, "Error vector magnitude analysis in generalized fading with co-channel interference," IEEE Trans. Commun., vol. 66, no. 1, pp. 345-354, Jan. 2018.
[14] H. Al-Hmood and H. S. Al-Raweshidy, "Exact closed-form capacity and outage probability of physical layer security in $\kappa-\mu$ shadowed fading channels," IET Commun.,vol. 13, no. 19, pp. 3235-3243, Oct. 2019.
[15] P. Ramirez-Espinosa, F. J. Lopez-Martinez, J. F. Paris, M. D. Yacoub, and E. Martos-Naya, "An extension of the $\kappa-\mu$ shadowed fading model: statistical characterization and applications," IEEE Trans. Veh. Technol., vol. 67, no. 5, pp. 3826-3837, May 2018.
[16] P. Beckmann, "Statistical distribution of the amplitude and phase of a multiply scattered field," J. Res. Nat. Bureau Standards-D. Radio Propag., vol. 66, no. 3, pp. 231-240, 1962.
[17] J. P. Peña-Martín, J. M. Romero-Jerez and F. J. LopezMartinez, "Generalized MGF of Beckmann fading with applications to wireless communications performance analysis," IEEE Trans. Commun., vol. 65, no. 9, pp. 3933-3943, Sept. 2017.
[18] H. Al-Hmood and H. Al-Raweshidy, "Performance analysis of physicallayer security over fluctuating Beckmann fading channels," IEEE Access, vol. 7, pp. 119541-119556, Aug. 2019.
[19] H. Al-Hmood and H. Al-Raweshidy, "Effective rate analysis over fluctuating Beckmann fading channels," arXiv preprint, Mar. 2019, https://arxiv.org/abs/1903.07026
[20] P. C. Sofotasios and S. Freear, "On the $\eta-\mu /$ gamma and the $\lambda-\mu /$ gamma multipath/shadowing distributions," Australasian Telecommun. Net. and Appl. Conf. (ATNAC), Nov. 2011, pp. 1-6.
[21] J. Zhang, M. Matthaiou, Z. Tan, and H. Wang, "Performance analysis of digital communication systems over composite $\eta-\mu /$ gamma composite fading channels," IEEE Trans. Veh. Technol., vol. 61, no. 7, pp. 31143124, Sep. 2012.
[22] P. Ramírez-Espinosa and F. J. López-Martínez, "Composite fading models based on inverse gamma shadowing: Theory and validation," IEEE Trans. Wireless Commun., vol. 20, no. 8, pp. 5034-5045, Aug. 2021.
[23] S. K. Yoo, S. L. Cotton, P. C. Sofotasios, M. Matthaiou, M. Valkama, and G. K. Karagiannidis, "The Fisher-Snedecor $\mathcal{F}$ distribution: A simple and accurate composite fading model," IEEE Commun. Lett., vol. 21, no. 7, pp. 1661-1664, Jul. 2017.
[24] S. K. Yoo et al., "A comprehensive analysis of the achievable channel capacity in $\mathcal{F}$ composite fading channels," IEEE Access, vol. 7, pp. 34078-34094, Feb. 2019.
[25] L. Kong and G. Kaddoum, "On physical layer security over the FisherSnedecor $\mathcal{F}$ wiretap fading channels," IEEE Access, vol. 6, pp. 3946639472, July 2018.
[26] H. Al-Hmood, "Performance of cognitive radio systems over $\kappa-\mu$ shadowed with integer $\mu$ and Fisher-Snedecor $\mathcal{F}$ fading channels," Int. Conf. Eng. Technol. and Appl. (IICETA), Sep. 2018, pp. 130-135.
[27] S. K. Yoo, N. Bhargav, S. L. Cotton, P. C. Sofotasios, M. Matthaiou, M. Valkama, and G. K. Karagiannidis, "The $\kappa-\mu /$ inverse gamma and $\eta$ $\mu /$ inverse gamma composite fading models: Fundamental statistics and empirical validation," IEEE Trans. Commun., vol. 69, no. 8, pp. 55145530, Aug. 2021.
[28] P. C. Sofotasios et al., "Error analysis of wireless transmission over generalized multipath/shadowing channels," IEEE Wireless Commun. and Net. Conf. (WCNC), June 2018, pp. 1-6.
[29] _-, "Ergodic capacity analysis of wireless transmission over generalized multipath/shadowing channels," IEEE 87th Veh. Technol. Conf. (VTC Spring), July 2018, pp. 1-5.
[30] ——, "Capacity analysis under generalized composite fading conditions," Int. Conf. on Advanced Commun. Technol. and Net. (CommNet), May 2018, pp. 1-10.
[31] S. L. Cotton, P. C. Sofotasios, S. Muhaidat and G. K. Karagiannidis, "Effective capacity analysis over generalized composite fading channels," IEEE Access, vol. 8, pp. 123756-123764, June 2020.
[32] N. Simmons, C. R. N. da Silva, S. L. Cotton, P. C. Sofotasios, and M. D. Yacoub, "Double shadowing the Rician fading model," IEEE Wireless Commun. Lett., vol. 8, no. 2, pp. 344-347, Apr. 2019.
[33] N. Simmons, C. R. N. D. Silva, S. L. Cotton, P. C. Sofotasios, S. K. Yoo, and M. D. Yacoub, "On shadowing the $\kappa-\mu$ fading model," IEEE Access, vol. 8, pp. 120513-120536, June 2020.
[34] H. Al-Hmood and H. S. Al-Raweshidy, "Unified composite distribution with applications to double shadowed $\kappa-\mu$ fading channels," IEEE Trans. Veh. Technol., vol. 70, no. 7, pp. 7182-7186, July 2021.
[35] H. M. Srivastava, and H. L. Manocha, A treatise on generating functions, Wiley, New York, 1984.
[36] A. M. Mathai, R. K. Saxena, and H. J. Haubold, The H-function: theory and applications, Springer Science \& Business Media, 2009.
[37] MATLAB Code for Computing $F_{D}^{(n)}().$. Accessed: Aug. 15, 2022. [Online]. Available: http://faculty.smu.edu/rbutler/
[38] H. Chergui, M. Benjillali, and M.-S. Alouini, "Rician $K$-factorbased analysis of XLOS service probability in 5G outdoor ultra-dense networks," IEEE Wireless Commun. Lett., vol. 8, no. 2, pp. 428-431, Apr. 2019.
[39] M. D. Yacoub, G. Fraidenraich, H. B. Tercius, and F. C. Martins, "The symmetrical $\eta-\kappa$ distribution: A general fading distribution," IEEE Trans. Broadcast., vol. 51, no. 4, pp. 504-511, Dec. 2005.
[40] _-, "The asymmetrical $\eta-\kappa$ distribution," J. Commun. Inf. Syst., vol. 20, no. 3, pp. 182-187, 2005.
[41] M. K. Simon and M.-S. Alouini, Digital Communications Over Fading Channels. New York, NY, USA: Wiley, 2005.
[42] I. S. Gradshteyn, and I. M. Ryzhik, Table of Integrals, Series and Products, 7th edition. Academic Press Inc., 2007.
[43] Wolfram Research, Inc., Accessed: Aug. 15, 2022. [Online]. Available: http://functions.wolfram.com/id
[44] A. P. Prudnikov, Y. A. Brychkov, and O. I. Marichev, Integrals and Series: Elementary Functions, 4th edition. Gordon \& Breach Sci. Publ., New York, 1998, vol. 1.
[45] A. Mathai and H. J. Haubold, Special Functions for Applied Scientists. Springer, New York, 2008.
[46] J. Zhang, Z. Tan, H. Wang, Q. Huang, and L. Hanzo, "The effective throughput of MISO systems over $\kappa-\mu$ fading channels," IEEE Trans. Veh. Technol., vol. 63, no. 2, pp. 943-947, Feb. 2014.
[47] H. Al-Hmood and H. S. Al-Raweshidy, "On the effective rate and energy detection based spectrum sensing over $\alpha-\eta-\kappa-\mu$ fading channels," IEEE Trans. Veh. Technol., vol. 69, no. 8, pp. 9112-9116, Aug. 2020.


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[^1]:    ${ }^{1}$ It is worth mentioning that the impact of the other fading parameters, namely, $\kappa, \mu, \eta, m$, and $\varrho$, on the performance of FB fading condition has been extensively explained in the technical literature (please refer to [15], [18], and [19]). Therefore, in this work, we have focused on the index $m_{s}$.

