DropSPH: ISPH Simulation of Droplet Interactions with a Solid Surface

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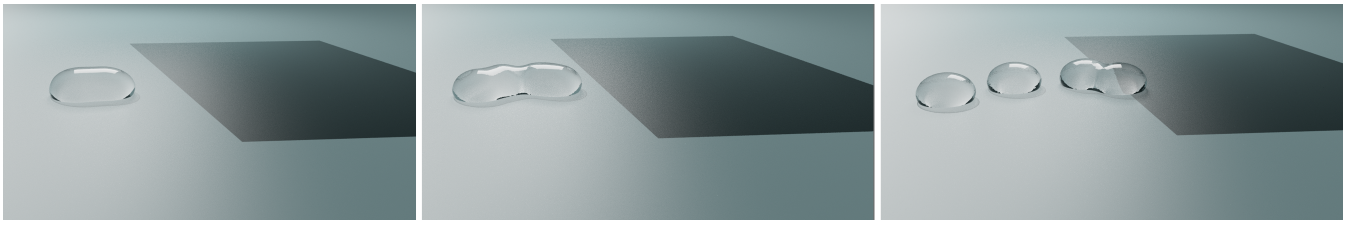

Figure 1: *Droplets impinging upon a flat solid surface, which resulted in a set of deformations. From left to right, spreading, coalescence and break-up can be captured using our model.*

Abstract

We present a physically-based model to simulate droplet behaviours when impacted on a solid surface. Our model creates the coalescence, spreading, and break-up deformations that occur when a liquid droplet collides with a solid surface. We model the attraction-repulsion forces within an improved free surface Incompressible Smoothed Particle Hydrodynamics (ISPH) framework that includes contact force and cohesion force between particles. The results show that our model is effective in simulating several small-scale liquid phenomena.

CCS Concepts

• ***Computing methodologies*** → ***Physical simulation;***

1. Introduction

Simulating fluids using Smoothed Particle Hydrodynamics (SPH) has been an active research topic in graphics for a decade. However, prior research on this topic has mainly focused on large-scale liquid simulations [KBST22]. Despite the advent of SPH on different fronts of liquid simulation, simulating fine-scale, high-detail motions such as those involving droplets interacting with a solid surface are still hard to capture, and remains largely unexplored. Such microscopic-scale simulations can be used in a wide range of applications [JS17], such as special effects in both film and television, driving simulations, 3D printing, as well as physics, engineering, and medical simulations (e.g., respiratory droplet simulations).

In this work, we present a physically-based model for simulating realistic interactions between a droplet and a solid surface within the framework of the ISPH. The basic idea behind this work is generating compelling droplet behaviour upon impact on a solid, which accurately exhibits the physical phenomena of bouncing, spreading, and breaking up. The novelty of our work lies in the use of specifically, a free surface ISPH model to achieve a reduction in the computation time needed for simulating small-scale droplet-solid interactions. The method we use requires significantly less particles to achieve accurate simulations (when compared with e.g. the SPH approach as detailed in [AAT13]). In our method, we employ the improved free surface ISPH model presented in [RNM*19] and the surface tension forces formulated in [AAT13]. The surface tension forces are used to couple the cohesion force and minimise the area of the free surface within the ISPH framework. This would model the inter-particle force as an attraction-repulsion function, which is responsible for causing short-range repulsion and long-range attraction. On the triple line surface tension, a contact force is calculated between the droplet and solid in a way similar in fashion to the one in Breinlinger *et al.,* [BPHK13], and plugged into

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our ISPH model. We demonstrate the efficacy of our model by simulating a droplet impacts upon a solid surface where spread, stretch and break up droplet deformations are realistically captured.

2. Surface tension model in ISPH

The macroscopic phenomena of surface tension appear from attractive forces between particles that experience an unbalanced molecular cohesive force in the boundary regions (which can either be at a liquid-air, liquid-solid, or solid-air interface). Particles in this region resist deformation, and actively minimise the area of a free surface. Therefore, surface tension can be represented in terms of the pressure difference across the surface, according to Laplace's Law,

$$\Delta P = \gamma \cdot \kappa \tag{1}$$

Where ΔP is the pressure difference, γ is the surface tension coefficient, and κ is the surface mean curvature. κ is related to a unit normal $\mathbf{n}$ of the surface as $-\nabla \cdot \mathbf{n}$. The surface tension force can then be formulated as:

$$\mathbf{F}_i^{st} = \gamma_i \kappa_i \mathbf{n}_i \tag{2}$$

Hence, the normal can be calculated by applying the SPH approximation to the gradient of the smoothed colour field as follows:

$$\mathbf{n}_i = \sum_j \frac{m_j}{\rho_j} \nabla W(\| p_i - p_j \|, h), \tag{3}$$

Where $W(\cdot)$ is a smoothing kernel used by the SPH framework, and h is the chosen smoothing radius. The m_j denotes the mass and p_j denotes the position of particle j and ρ_j is the density. The surface tension force can then be expressed as follows based on the formulation presented in [AAT13]:

$$\mathbf{F}_i^{st} = K \left(-\gamma_{i,j} m_i \sum_j m_j \frac{(p_i - p_j)}{\| p_i - p_j \|} W(\| p_i - p_j \|, h) \right) + \left(-\gamma_{i,j} m_i \sum_j (\mathbf{n}_i - \mathbf{n}_j) \right), \tag{4}$$

where K is a constant. This formulation includes both the cohesion force and a symmetric force that minimises the surface area. The value of K can be obtained analytically, based on the improved free surface ISPH formulation presented in [RNM*19], where the value of K is found for any interior particle in the initial regular configuration of particles at the free surface. For the choice of W, we use the spline smoothing kernel.

3. Surface tension at the triple line

The contact angle at a triple line of a solid, liquid, and air is determined through a balance of three surface energies, as seen in Young's equation, which means if the contact line at the intersection of these three interfaces reaches equilibrium with no external forces, the surface tensions and contact angle satisfies the following:

$$\gamma_{sa} - (\gamma_{la} \cos\theta_{eq} + \gamma_{ls}) = 0, \tag{5}$$

where $\theta_{eq} (0 < \theta_{eq} < \pi)$ is the stable contact angle, and γ_{ls}, γ_{sa} and γ_{la} are tension coefficients for the liquid-solid, solid-air, and liquid-air interfaces, respectively. When θ_{eq} is small (close to zero), the solid surface is called hydrophilic, and the liquid surface tends to spread flat. The solid surface is called hydrophobic if θ_{eq} is large, and the liquid tends to bead up on the surface. Young's equation is equivalent to setting the contact angle such that the contact force acting on the triple line region is zero, and it is formulated as:

$$F^c = \gamma(\cos\theta_{eq} - \cos\theta_{dyn}), \tag{6}$$

where θ_{dyn} is an arbitrary unbalanced angle, which is balanced when the equilibrium value, $\theta_{dyn} = \theta_{eq}$ is achieved. The contact angle θ_{dyn} can be computed in the simulation using the surface tangent and normal vectors (for more details on this, see Breinlinger *et al.,* [BPHK13]). The force F^c is applied to the ISPH particles in the triple line region, where we apply no-slip conditions on the solid surface. The droplet in our simulations is modelled to have

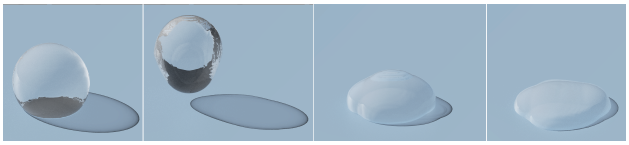

Figure 2: *Our simulation of a droplet impacts upon a solid surface. The sequences from left to right correspond to times $t = 30ms$, $40ms$, $90ms$, and $100ms$.*

density $\rho = 1000\text{kg/m}^3$ with the surface tension is $\gamma = 3\text{N/m}$. Figure 2 shows a simulation of a free falling 2cm droplet of $\approx 3\mathbf{K}$ SPH particles using our method, where the sequence illustrates the droplet dynamics at various stages during the impact, in which the droplet bounces off before spreading on the surface.

4. Preliminary results and discussion

When a droplet impacts a solid surface, it can undergo different types of deformation. Our method captures a range of such droplet deformations, in particular spreading, break up, and coalescence. These deformations can be observed in Figs (1, 2), where Fig (1) illustrates all three types of deformations. Depending on the droplet's parameters and surface conditions, when the droplet impacts upon a solid surface, deformations such as rebounding and splashing happen. The emission of tiny droplets in the impact direction is called impact splashing. In our current work, impact splashing is not captured by the simulation, and as a future direction, we will explore further the possibility to simulate and reproduce phenomena such as impact splashing for varying values of surface roughness.

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