

Research on experimental design method of engine system based on D-optimal design

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ABSTRACT

In this paper, a few experimental points were used to calibrate the throttle model, fuel injection model and semi-predictive combustion model of the engine system based on the D-optimal experimental design method. The results show that the number of calibration test points can be reduced by more than 90% on the premise of ensuring the accuracy of each sub-model. At the same time, a set of general calibration experiment design methods for each sub-model in the engine system is established by comparing the distribution of experimental points selected from each sub-model.

Keywords: engine modelling, D-optimal, DOE, model calibration

1 INTRODUCTION

The increasing energy crisis and the continuous improvement of vehicle emission requirements put forward higher requirements on engine thermal efficiency, fuel consumption and pollutant emission and other performance indicators. Engine calibration technology is one of the key technologies to improve engine performance, but the process of traditional calibration completely depends on the bench test, and the demand for test points is usually thousands or more than tens of thousands, which requires a lot of experimental resources, and makes the calibration work cost is high and the cycle is long^[1-2].

The workload of bench calibration can be effectively reduced by establishing a virtual engine model to replace the actual engine for model calibration. However, it is necessary to ensure that the model has a sufficiently high prediction accuracy before using a virtual engine to replace the actual engine for simulation calibration. The improvement of model accuracy depends on a large number of experimental data, which contradicts the purpose of reducing calibration workload by virtual engines. Therefore, how to establish a high-precision engine model based on a small amount of experimental data is an important basis for realizing virtual calibration, reducing cost and shortening cycles.

The objective of optimal experimental design is to obtain a combination of experimental points that can maximize the information matrix. And there are many corresponding evaluation criteria, including Latin Hypercube Sampling, D-optimal, V-optimal and A-optimal, etc. To reduce the experimental points needed for model calibration, Anthony Gullitti et al. ^[3] used the intelligent experimental design system developed by IAV Company, based on the D-optimal experimental design method, and supplemented by the V-optimal experimental design method and space-filling method, and finally completed the model building with a small amount of experimental data (269 experimental points). The ROOT MEAN SQUARE ERROR of the model is less than 2.5%, which has high accuracy. Tianhong Pan et al. ^[4] used the Latin Hypercube

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Sampling algorithm to select only 650 experimental points from 4688381250 experimental points to build the model and the R^2 (coefficient of determination: indicating the degree of overlap between the predicted value and the actual value) of the model was greater than 0.92, which could meet the research requirements.

The above methods can not only ensure the accuracy of the model but also greatly reduce the experimental points required for the model establishment, which can reduce the cost and period of calibration. But these methods are only applicable to the specified engines, and the test points need to be re-selected for other engines, which makes their expansion ability poor. The D-optimal design method has advantages in the application of online calibration and is suitable for a wide range because of its simple calculation and a relatively low requirement on hardware computing capability^[5]. In order to improve the universality of the experimental design scheme among different engines, a study on experimental design based on the D-optimal design method was carried out in this paper. A small number of experimental points were selected to calibrate the throttle model, fuel injection model and semi-predictive combustion model respectively. And a general experimental design method is established by analyzing the distribution of selected experimental points, which greatly reduces the requirement of the number of experimental points for model calibration, and is of great significance for the virtual calibration of engines.

2 METHOD OF MODELLING

2.1 Basic engine parameters

This study is based on a 1.3L GDI (Gasoline Direct Injection) engine with turbocharging and dual VVT. The specific structural parameters are shown in Table 1.

Through the sweep-point experiment, the experimental data of 2600 working points were obtained. The distribution is shown in Figure 1.

Table 1. Basic parameters of the engine.

Parameter	Value
Number of cylinders	4
Number of stroke	4
Bore/mm	76
Stroke/mm	74
Displacement/L	1.342
Maximum compression ratio	10
Injection method	Direct injection
Number of valves	4
Intake mode	VVT

2.2 Throttle model

The throttle can be regarded as a standard throttle valve, and the airflow through the throttle can be treated as a one-dimensional isentropic stable flow from the perspective of fluid mechanics^[6]. The calculation formula is as follows:

$$m_{ub} = A_{eff} \rho_{is} U_{is} = C_D A_R \rho_{is} U_{is} \quad (1)$$

$$\rho_{is} = \begin{cases} \rho_0 (P_r)^{\frac{1}{\gamma}}, & P_r > \frac{2}{\gamma+1} \\ \rho_0 \frac{2}{\gamma+1} \frac{1}{\gamma-1}, & P_r > \frac{2}{\gamma+1} \end{cases} \quad (2)$$

$$U_{is} = \begin{cases} \sqrt{RT_0} \left\{ \frac{2\gamma}{\gamma-1} \left(1 - P_r^{\frac{\gamma-1}{\gamma}} \right) \right\}^{\frac{1}{2}}, & P_r > \frac{2}{\gamma+1} \\ \sqrt{\gamma RT_0} \left\{ \frac{2}{\gamma+1} \right\}^{\frac{1}{2}}, & P_r \leq \frac{2}{\gamma+1} \end{cases} \quad (3)$$

Where, A_{eff} represents the effective circulation area; ρ_{is} represents the upstream stagnation density; U_{is} represents the isentropic density of the throat outlet; C_D represents the flow coefficient; A_R represents the reference flow area; P_r represents the absolute pressure ratio (static pressure at the exit/total pressure at the entrance); R is the gas constant; T_0 represents upstream stagnation temperature; γ is the adiabatic index.

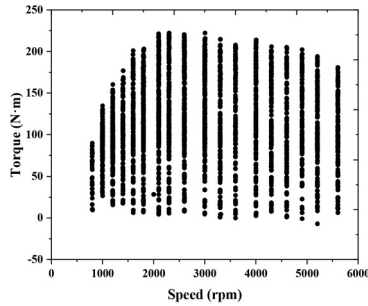


Figure 1. Distribution of 2600 experimental points.

The flow coefficients of each throttle opening were calculated by a one-dimensional isentropic stable flow equation according to the experimental data obtained. Wang Zhuwei et al from the University of Shanghai for Science and Technology have studied the throttle flow coefficient and found that there is a certain variation rule between flow coefficient, throttle angle and pressure ratio^[7]. Therefore, the relationship between them is also studied in this paper, as shown in Figure 2:

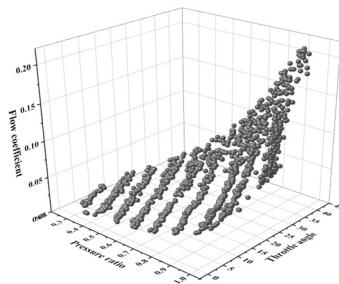


Figure 2. Relationship between throttle flow coefficient and throttle opening and pressure ratio.

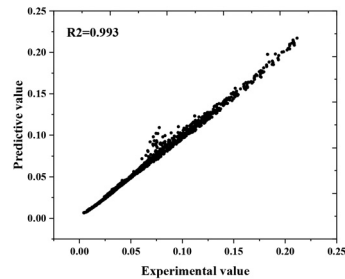


Figure 3. Comparison of simulation results of throttle flow coefficient model.

It can be seen from Figure 2 that the throttle flow coefficient is in smooth plane distribution with the throttle opening and the pressure ratio, and it mainly increases with the increase of the throttle opening. Therefore, a throttle flow coefficient model based on pressure ratio correction is built in this paper, as shown in Equation (4):

$$\begin{cases} C_D = p_0 + p_1\theta + p_2p_r + p_3\theta^2 + p_4\theta p_r + p_5\theta^3 + p_6\theta^2 p_r \\ p_r = \frac{p_{im}}{p_{up_thr}} \end{cases} \quad (4)$$

Where, C_D is the throttle flow coefficient; θ is the throttle opening; p_r is the ratio of the throttle rear-end pressure to the front end pressure; p_{im} is the intake manifold pressure; p_{up_thr} is the throttle front pressure. The accuracy of the model is shown in Figure 3 after the experimental data is brought into the model.

As can be seen from Figure 3, R^2 of the flow coefficient calculated by the model is 0.993. This indicates that the established throttle flow coefficient model has high accuracy.

2.3 Fuel injection model

The accuracy of fuel injection mass measurement was ensured by receiving the signal of injection pulse width transmitted by the ECU. And the main characteristic of the injector is the injection flow characteristic, which expresses the relationship among rail pressure, injection pulse width and injection volume.

The relationship between injection volume and injection pulse width under different rail pressures can be obtained through the analysis of engine injection data, as shown in Figure 4. It can be seen from Figure 4 that there is a linear relationship between injection volume and injection pulse width under fixed rail pressure. And the injection model is:

$$T_{inj} = kQ_f + t \quad (5)$$

Where, T_{inj} represents the injection pulse width, Q_f represents the injection volume of a single cycle, k represents the slope, and t represents the intercept.

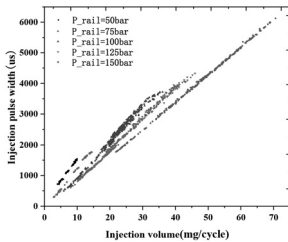


Figure 4. Relationship between injection pulse width and injection volume.

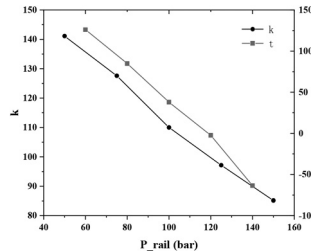


Figure 5. Relationship between slope and intercept and rail pressure.

As the pressure P_{rail} changes, the slope K and the intercept T will also change. The changing relationship between slope k and intercept t with rail pressure is shown in Figure 5, and a good linear relationship between them is shown in Figure 5. And the

regression model of slope k and intercept t with rail pressure (injection flow coefficient model) is established:

$$k = m_0 \times P_{\text{rail}} + m_1 \quad (6)$$

$$t = n_0 \times P_{\text{rail}} + n_1 \quad (7)$$

The model accuracy is shown in Figure 6 after the experimental data is brought into the model:

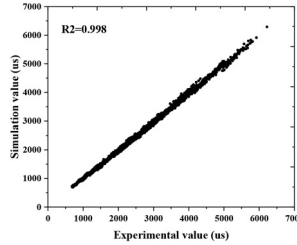


Figure 6. Comparison of simulation results of injection pulse width.

It can be seen from Figure 6 that the R^2 of injection pulse width calculated by the model is 0.998, which indicates that the established injection model has high accuracy.

2.4 Semi-predictive combustion model

The Wiebe combustion model is usually chosen for combustion calculation when the virtual engine is used instead of the real engine for simulation calculation [8]. However, CA_{50} and CA_{10-90} are unknown before the experiment and cannot be directly used as the input of the combustion model. Therefore, the Wiebe combustion model alone cannot be used to predict the heat release rate of combustion under variable working conditions. In this research, a semi-predictive combustion model is established, which enables the virtual engine model to predict the values of CA_{50} and CA_{10-90} under variable operating conditions, and it can predict the values of CA_{50} and CA_{10-90} according to the operating conditions of the engine. The formula is shown as follows:

$$\theta_{50} = a_0 + a_1\lambda^2 + a_2\lambda + a_3p_{im}^2 + a_4p_{im} + \frac{a_5}{T^2} + a_6\theta_{spark}^2 + \frac{a_7}{RPM^2} + a_8IVO + a_9RGF^2 \quad (8)$$

$$CA_{10-90} = b_0 + b_1\theta_{50} + b_2\theta_{50}^2 + b_3\theta_{spark}^2 + b_4\theta_{spark} + b_5p_{im}^2 + b_6\lambda^2 + \frac{b_7}{RPM^2} + b_8RGF^2 \quad (9)$$

$$CA_{50} = \theta_{50} + \theta_{spark} \quad (10)$$

Where, CA_{10-90} is the duration of the Wiebe combustion curve, CA_{50} is the Angle "anchoring" the Wiebe curve to TDC, λ is the excess air coefficient; p_{im} is the intake manifold pressure (bar); T is the inlet air temperature (K); θ_{spark} is the ignition advance Angle; RPM is the engine speed (r/min); IVO is the opening phase of the intake valve; RGF is the residual exhaust gas rate in the cylinder.

The model accuracy is shown in Figure 7 after the experimental data is brought into the model. It can be seen from Figure 7 that the R^2 values of CA_{50} and CA_{10-90} calculated by the combustion model are 0.96 and 0.87, respectively. Due to the complexity of the combustion model, it is difficult to make an accurate prediction of CA_{10-90} . However, this accuracy can meet the basic requirements of the combustion model, and there is still a large space to further improve the prediction accuracy of CA_{10-90} .

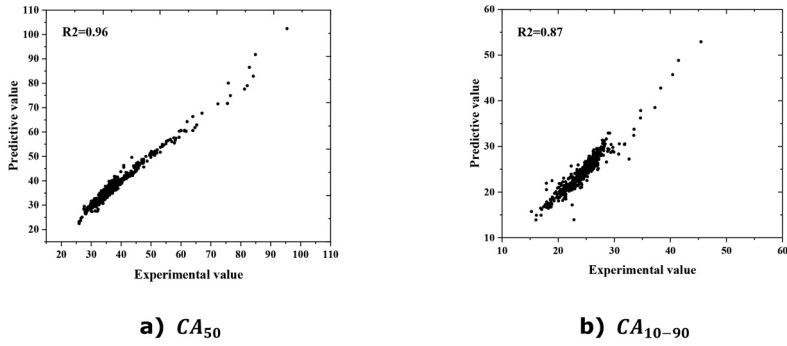


Figure 7. Comparison of simulation results of the combustion model.

3 SELECTION OF FEATURE SAMPLE POINTS

3.1 D-optimal design method based on Bayesian Modification

Experimental design refers to the rational arrangement of experiments and the use of fewer experimental points to obtain relatively ideal model calibration results, so as to reduce the calibration cycle and cost^[9]. The experimental design methods include classical design method, space-filling design method and optimal design method. Among them, the optimal design method can properly establish the design criteria that can reflect the purpose of the experiment, and obtain the optimal design scheme to arrange the experiment, so as to save the experiment cost. The optimal design method is usually adopted if you have a deep understanding of the engine and the optimal model has been selected^[10].

Regression models can generally be expressed as the following matrix ^[11]:

$$y = X\beta + \epsilon \quad (11)$$

Where y is the output; β is the parameter to be estimated; X is the coefficient matrix. The least-square estimation of this model is $\hat{\beta} = X^T X^{-1} X^T y$, and $X^T X$ is required to be non-degenerate. Model of the covariance matrix of the $\text{cov} \hat{\beta} = \sigma^2 X^T X^{-1}$, and sigma error factors by experiment. The matrix $M = X^T X$ is the information matrix that contains the information of the model and the experimental points.

D-optimal design refers to maximizing the determinant value of the information matrix ($X^T X$), which is as follows:

$$|X^{*T} X^*| = \max |X^T X| \quad (12)$$

In some cases, the sample points obtained according to the D-optimal design may have duplicate values, which not only wastes computational resources but also has no help to the fitting of model coefficients. Therefore, the Bayesian Modification (BM) method proposed by Dumouchel and Jones is adopted to add coefficient terms of higher-order at the end of the established regression model, which can effectively solve the problem of repeated points ^[12].

The process of D-optimal design based on Bayesian modification includes data standardization, determination of candidate point matrix, selection of regression model, determination of experimental points, the establishment of information matrix and optimization design. In this study, it is programmed to quickly obtain the experimental design scheme according to the Fedorov algorithm, as shown in Figure 8.

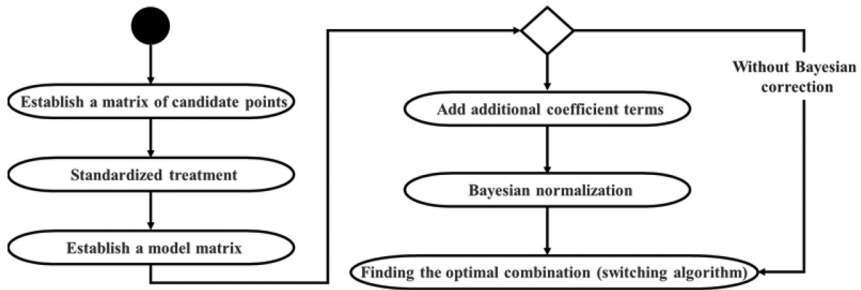


Figure 8. Implementation flow of D-optimal design.

3.2 Application of D-optimal in the throttle model

The throttle flow coefficient model based on pressure ratio modification is experimentally designed according to the principle of D-optimal design. The input of the model is throttle opening θ and the ratio of throttle rear-end pressure to front end pressure p_r , and the number of target samples selected is set as 20. The distribution of the final selected sample points is shown in Figure 9.

It can be seen from Figure 9 that the sample points are mainly distributed on the edge of the interval composed of throttle opening and pressure ratio, and only one point is about the centre of the interval. According to the geometric feature analysis of the selected points, the points at the edge cover the whole interval and the points with sharp shape changes on the edge are called feature points. The overall shape of the surface is mainly affected by these feature points because the throttle flow coefficient, θ and pressure ratio p_r are quadratic functions and the shape of the surface is smooth.

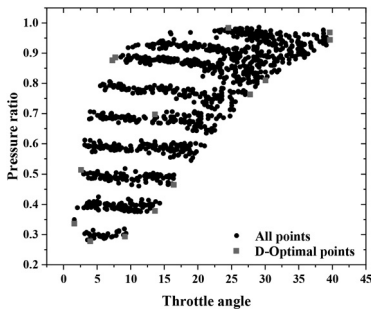


Figure 9. Distribution of sample points in the throttle flow coefficient model.

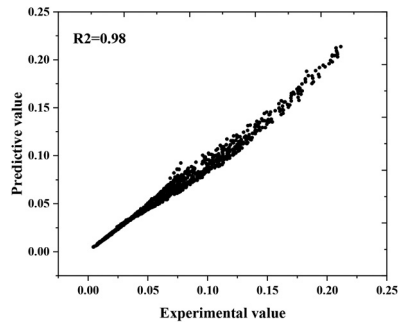


Figure 10. Comparison of experimental and predicted throttle flow coefficient results.

The 20 selected sample points were substituted into the throttle flow coefficient model for solving, and the values of each coefficient in the model could be obtained, as shown in Table 2:

Table 2. Fitting results of throttle flow coefficient model coefficients.

parameter	value	parameter	value
p_0	6.90e-3	p_4	5.10e-3
p_1	-6.20e-5	p_5	5.50e-6
p_2	-1.37e-2	p_6	-3.50e-4
p_3	14.00e-4		

Next, the prediction accuracy of the model is verified. The remaining non-pressurized conditions (throttle opening less than 90 degrees, about 1300 experimental points) were substituted into the model for testing, and the results were shown in Figure 10. It can be seen from Figure 10 that the predicted value of the throttle flow coefficient is basically consistent with the experimental value, and its R^2 is 0.98.

It can be seen that the throttle flow coefficient model based on the calibration of 20 sample points can accurately predict the throttle flow coefficient under different throttle opening and pressure ratios under all non-pressurized conditions, which can achieve the purpose of using a small number of experimental points to calibrate the model.

3.3 Application of D-optimal in the fuel injection model

Similarly, the injection model is designed experimentally according to the principle of D-optimal design. The inputs of the model are injection pressure and injection volume, and the number of target samples selected is set as 16. The distribution of the final selected sample points is shown in Figure 11.

The 16 selected experimental points were brought into the injection model to solve the parameters, and other experimental points were used to verify the accuracy. The model accuracy is shown in Figure 12:

It can be seen from Figure 12 that the predicted value of injection pulse width is the same as the experimental value, and its R^2 is 0.997.

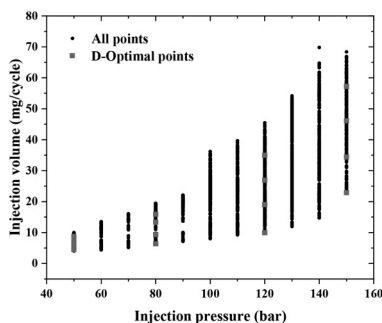


Figure 11. Distribution of sample points in the injection model.

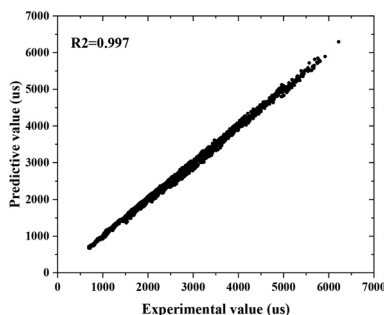


Figure 12. Comparison of experimental and predicted injection pulse width results.

3.4 Application of D-optimal in the semi-predictive combustion model

Similarly, the semi-predictive combustion model is designed experimentally according to the principle of D-optimal design. And the number of target samples selected is set to 54. The selected experimental points are put into the model to solve the model parameters, and the model accuracy is shown in Figure 13:

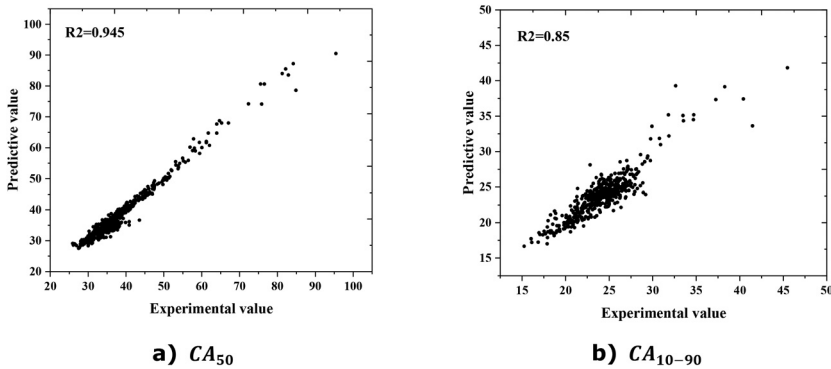


Figure 13. Comparison of the experimental and predicted values for CA_{50} and CA_{10-90} in the combustion model.

It can be seen from Figure 13 that the predicted values of CA_{50} and CA_{10-90} in the combustion model are basically consistent with the experimental values, and their R^2 are 0.94 and 0.85, respectively.

4 GENERAL EXPERIMENTAL SCHEME

4.1 Design of experimental scheme

The above sample points are targeted at the engine in this study, which may not apply to engines of different types. In order to improve the expansion ability of the experimental design scheme, this paper summarizes a general experimental design scheme by analyzing the selection rules of experimental points under different models.

This paper takes the throttle model as an example to study the general experimental scheme. In the throttle model, the experimental points selected based on the D-optimal method are mainly distributed on the edge of the interval composed of throttle opening θ and pressure ratio p_r . This range is not the same for different engines. In this study, a universal experimental design scheme applicable to different engines is summarized to solve this problem by analyzing the distribution characteristics of selected experimental points and combining the feasibility of real bench experiment operation, as follows:

- (1) Turn off the turbocharger and set the throttle opening to the minimum allowable opening θ_{\min} (such as 1 degree). When the engine speed is gradually increased from idle speed to the maximum allowable speed, the minimum and maximum values of the throttle gate rear pressure ratio are recorded, and these two values are recorded as feature points under the θ_{\min} opening degree.
- (2) A certain number of throttle opening points are selected evenly within the throttle opening range. And the selection points can be denser when the throttle opening is small, and sparser when the throttle opening is largely

based on the characteristics of the throttle. In this study, the throttle opening is adjusted to 3 degrees, 5 degrees, 7 degrees, 10 degrees, 15 degrees, 20 degrees, 25 degrees, 30 degrees, 35 degrees and 40 degrees respectively, and the rotating speed is adjusted according to Step (1) to obtain the corresponding feature points under each opening. Besides, when the throttle opening is adjusted to 15 degrees and the ratio of the rear end of the throttle to the front end pressure is 0.7 by adjusting the engine speed, the working point is also the feature point.

- (3) The flow coefficients of all the above feature points were calculated and substituted into the throttle flow characteristic model for a solution.
- (4) Verify the predictive ability of the model. In addition, several experiments were carried out (changing the throttle opening and the pressure ratio) to compare the throttle flow coefficient with the predicted value of the model.

It indicates that the flow coefficient model has a high predictive ability and the experimental design scheme can be used for virtual calibration of the intake model if the error is small. On the contrary, these verification points can also be added as feature points, and steps (3) and (4) can be repeated until the verification results meet the accuracy requirements if the error is large.

4.2 Verification of the experimental scheme

Next, a 1.4L cylinder direct injection gasoline engine was used to test the versatility of the above experimental design. The engine previously obtained 600 test points through sweeping points. According to the above experimental design scheme, 20 sample points were selected and substituted into the throttle flow characteristic model for the solution, and then the remaining 580 test points were used for verification.

The validation results are shown in Figure 14. It can be seen that the shape of the theoretical value and the predicted value of the flow coefficient is basically the same, and its R^2 is 0.972.

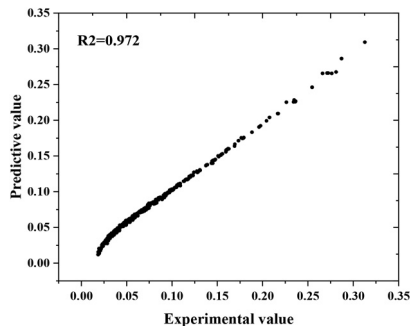


Figure 14. Comparison of experimental and predicted throttle flow coefficient results.

The above research shows that the experimental design of throttle flow characteristics is suitable for different gasoline engines, and accurate throttle flow characteristics can be obtained through a small number of test points calibration.

Similarly, according to the point selection rules of the fuel injection model and semi-predictive combustion model, the accuracy of the 1.4L engine model was verified by a similar experimental design scheme, and their R^2 are 0.99, 0.93 and 0.84, respectively.

5 CONCLUSION

In this study, the throttle model, fuel injection model and semi-predictive combustion model were established respectively through the correlation analysis among variables, and the key experimental points were selected for these three models through the experimental design method based on D- optimal. According to the distribution law of selected experimental points, the respective experimental schemes were designed, and their versatility was verified. Finally, the following conclusions were drawn:

- (1) The established throttle flow coefficient model, injection pulse width model and semi-predictive combustion model have high prediction accuracy and the R^2 values of the model prediction results and the experimental results are 0.993, 0.998, 0.96 and 0.87, respectively.
- (2) Based on the D-optimal experimental design method, 20, 16 and 54 experimental points were selected to identify the model parameters of the throttle model, fuel injection model and semi-predictive combustion model, and the R^2 values of the models were 0.98, 0.997, 0.945 and 0.85 respectively, which can reduce the number of test points needed for calibration by more than 90% while ensuring the accuracy of the model.
- (3) By analyzing the distribution of selected experimental points of each model, a general experimental scheme was designed and verified in another engine. The verification results show that the R^2 values of the model can reach 0.972, 0.99, 0.93 and 0.84 respectively when the number of experimental points is reduced by more than 90%, which proves the feasibility of the general experimental scheme.

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