

# CONDENSATION ON A VERTICAL PLATE WITH SINUSOIDAL MICROFINS

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## ABSTRACT

A theoretically-based, approximate solution for condensation on horizontal, micro-finned tubes has been used as a basis to obtain a relation between heat flux and vapour-surface temperature difference for condensation on a vertical plate with sinusoidal microfins. The earlier result was in excellent agreement with experimental data with regard to magnitudes of the heat-transfer coefficient, dependence on fin and tube dimensions and for a range of fluids with widely different properties. The new equation for condensation on a vertical plate with sinusoidal microfins takes account of the vertical gravity force and the lateral surface tension-induced pressure gradient over the curved fin surface. The equation reduces to the Nusselt result when fin height is zero and to simple area-enhancement of the Nusselt result when surface tension is set to zero. Comparisons are made with experimental data for condensation of nitrogen on a vertical surface with sinusoidal microfins.

## 1. INTRODUCTION

During film condensation, and when the condensate surface is curved, a pressure difference  $\sigma/r_c$  exists between the vapour and the condensate. Variation in curvature along the condensate surface results in a pressure gradient  $\frac{d(\sigma/r_c)}{ds}$ . The intensity of the pressure gradient and its effect on the motion of the condensate film depends on the rate of change of curvature along the surface. Investigations of this phenomenon have been largely concerned with condensation on, and in, tubes with microfins. For low fins the condensate surface curvature varies strongly over much of the fin surface and results in

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significant heat transfer enhancement due to local thinning of the condensate film. The present investigation was prompted by recent experimental measurements for condensation of nitrogen on a vertical plate with sinusoidal, low fins.

## 2. ANALYSIS

In a study of condensation on horizontal micro-finned tubes with trapezoidal micro-fins, Rose [1] wrote, on the basis of dimensional analysis, an equation for the effective mean condensate film thickness,  $\delta$ , on each surface of a trapezoidal fin, in terms of gravity and surface tension dimensionless groups:

$$\frac{\delta}{x_1} = \zeta \left( \frac{\mu V}{\Delta \rho g x_2^2}, \frac{\mu V}{\sigma} \right) \quad (1)$$

where  $x_1$  and  $x_2$  are appropriate characteristic lengths,  $V$  is the volume condensation rate per area of surface,  $\Delta \rho = \rho_f - \rho_g$ .

Equation (1) was assumed to take the form

$$\delta = \left( \frac{\mu V}{\frac{A \Delta \rho g}{x_g} + \frac{B \sigma}{x_\sigma^3}} \right)^{1/3} \quad (2)$$

where  $x_g$  and  $x_\sigma$  are appropriate characteristic lengths for the gravity and surface tension driven flows respectively and  $A$  and  $B$  are constants.

To obtain the Nusselt result for condensation on a vertical plate when  $\sigma$  is set to zero  $A = 0.943^4$  and  $x_g = H$ . Following Rose [1],  $x_\sigma$  is the length measured along the fin surface  $l$ . Then, with  $q_s = k \Delta T / \delta$ ,  $V = q_s / (\rho_f h_{fg})$ , Eq. (2) gives:

$$q_s = \left\{ \frac{\rho_f h_{fg} k^3 \Delta T^3}{\mu} \left[ \frac{0.943^4 \Delta \rho g}{H} + \frac{B \sigma}{l^3} \right] \right\}^{1/4} \quad (3)$$

for the heat flux at the surface of the fin.

To obtain the area-enhanced Nusselt heat flux when surface tension is set to zero, the gravity term in Eq. 3 is multiplied by  $(l/p)^4$ . The surface tension term must also be modified so that the Nusselt result is obtained when the fin height  $h$  is zero. To achieve this the surface tension term in Eq. (3) is, somewhat arbitrarily, amended by multiplying by  $\{(l-p)/p\}^n$  where  $n$  is a positive constant. Eq. (3) then becomes:

$$q = \left\{ \frac{\rho_f h_{fg} k^3 \Delta T^3}{\mu} \left[ \frac{0.943^4 \Delta \rho g}{H} \left( \frac{l}{p} \right)^4 + \frac{B\sigma}{l^3} \left( \frac{l-p}{p} \right)^n \right] \right\}^{1/4} \quad (4)$$

with only two disposable constants,  $B$  and  $n$ , in the surface tension term.

When the surface tension is set to zero Eq. (4) gives the Nusselt value multiplied by the area enhancement  $l/p$  and when the fin height is set to zero, and hence  $l = p$ , Eq. (4) gives the Nusselt result. Note that fin height  $h$  does not appear explicitly in Eq. (4) but, for the sinusoidal geometry,  $h$  is a function of  $p$  and  $l$ .

No account has been taken of conduction in the fin. The ratio of thermal conductivity of aluminium, the plate material used in [2], to that of liquid nitrogen, is such that this is negligible compared with experimental uncertainty.

### 3. COMPARISONS WITH EXPERIMENTAL DATA

The experimental data given in [2] are for three different values of fin pitch for a fixed fin height and three values of fin height for a given fin pitch. The relevant dimensions are given in Table 1 and illustrated in Fig. 1.

Figures 2 and 3 compare Eq. (4) with the data when assigning the values  $B = 10$  and  $n = 1/2$  to the constants. These values were chosen by trial and error. In view of the apparent systematic error in the data at low  $\Delta T$  (where the data suggest zero heat flux at  $\Delta T \approx 0.25$  K), no attempt has been made to obtain a “best” fit. Excellent agreement, within the bounds of uncertainty in the data, is seen (except at low  $\Delta T$ ) both with respect to values of heat flux and the trends with varying  $p$  and  $h$ .

Zhu et al [2] give a correlation of their data which, using the present notation, may be written:

$$\frac{q}{q_{Nu}} = 0.011 \left( \frac{h}{H} \right)^{-0.18} \left( \frac{p}{H} \right)^{-0.90} \quad (5)$$

As may be seen, Eq. (5) does not contain surface tension and gives infinite heat flux, rather than the Nusselt result, when  $h$  is set to zero. It is therefore unlikely that this correlation would give correct results for other fluids. In both Eqs. (4) and (5) the enhancement ratio  $q/q_{Nu}$  is independent of  $\Delta T$ . In Fig. 4, the enhancement ratios given by Eqs. (4) and (5) are compared. As anticipated, for fin height zero Eq. (4) gives  $q/q_{Nu}$  equal to 1 while Eq. (5) gives an infinite value. In the experimental range,  $h = 0.3$  mm to  $h = 0.9$  mm, the two results are quite close with Eq. (4) giving values for enhancement ratio around 15% higher than Eq. (5). Note that the correlation of Zhu et al. [2] includes data for two other values of fin pitch as well as the apparently erroneous values at low  $\Delta T$ .

**Table 1:** Fin dimensions used in [2].

$p/\text{mm}$	$h/\text{mm}$	$l/\text{mm}$
1	0.3	1.194
2	0.3	2.107
3	0.3	3.073
1	0.6	1.619
1	0.9	2.126

#### 4. CONCLUSION

Despite the approximation of Eq. (2), and arbitrary adjustment of the surface tension term, Eq. (4) has a theoretical basis and satisfies the conditions that the area-enhanced Nusselt result is obtained when the surface tension set to zero and the Nusselt result is obtained for zero fin height. Together with the good agreement with experimental data for condensation of nitrogen, Eq. (4), perhaps with amendments, might be expected to give satisfactory results for other fluids. The equation should at least provide a sound basis for correlation of new data.

Finally, it may be noted that, while showing that the heat transfer increases as the fin pitch decreases (in line with the experimental data), Eq. (4) gives continuing increase as pitch decreases towards zero. On physical grounds, one might, for very low values of pitch, expect the behaviour to resemble that of a rough surface and ultimately, as the pitch approaches zero, a smooth surface. In reality, towards the root of the fins, the interfin space becomes to some extent, “flooded” and the effective length over which the film is thin is smaller than  $l$ . An appropriate “mean effective length” would be governed by a balance between the transverse surface tension flow and the vertical flow due to gravity and would thus depend on the volume condensation flux and the plate height. In the absence of data for smaller fin pitch, this correction has not been pursued at this stage.

## NOMENCLATURE

$A$	constant in gravity term, see Eq. 2 ( $= 0.943^4$ )
$B$	constant in surface tension term, see Eq. (2)
$g$	specific force of gravity
$H$	plate height
$h$	fin height
$h_{fg}$	specific latent heat of evaporation
$k$	thermal conductivity of condensate
$l$	length measured over the surface of a fin
$n$	constant in surface tension term, see Eq. (4)
$p$	fin pitch
$q$	heat flux based on the smooth surface area, i.e. at the base of the fin
$q_s$	heat flux based on the surface area of a fin

$q_{Nu}$	heat flux given by Nusselt theory
$r_c$	radius of curvature of condensate surface
$s$	distance measured along the condensate surface
$V$	volume condensation rate per area of surface
$x_g$	constant in gravity term, see Eq. (2)
$x_\sigma$	constant in surface tension term, see Eq. (2)
$\Delta\rho$	$\rho_f - \rho_g$
$\Delta T$	vapor-surface temperature difference
$\delta$	effective condensate film thickness
$\mu$	condensate viscosity
$\rho_f$	density of saturated liquid
$\rho_g$	density of saturated vapour
$\sigma$	surface tension

## REFERENCES

- [1] J. W. Rose, An approximate equation for the vapour-side heat-transfer coefficient for condensation on low-finned tubes. *Int. J. Heat Mass Transfer*, **37** (1994) 865–875.
- [2] S. Zhu, X. Zhi, C. Gu, K. Wang, L. Qiu, Enhancing heat transfer performance of nitrogen condensation on a vertical plate with microstructure. *Int. J. Heat Mass Transfer* **172** (2021) 121219.

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Figure 1: Fin dimensions.

Figure 2: Comparison of Eq. (4) with experimental data [2]. Variation of heat flux with fin pitch and constant fin height.

Figure 3: Comparison of Eq. (4) with experimental data [2]. Variation of heat flux with fin height and constant fin pitch.

Figure 4: Dependence of enhancement ratio on fin height for constant fin pitch given by Eqs. (4) and (5).

Figure 1: Fin dimensions.

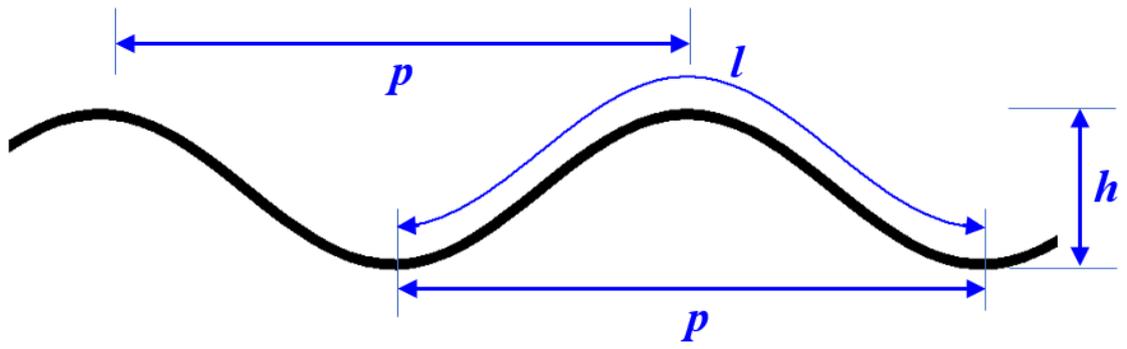


Figure 2: Comparison of Eq. (4) with experimental data [2]. Variation of heat flux with fin pitch and constant fin height.

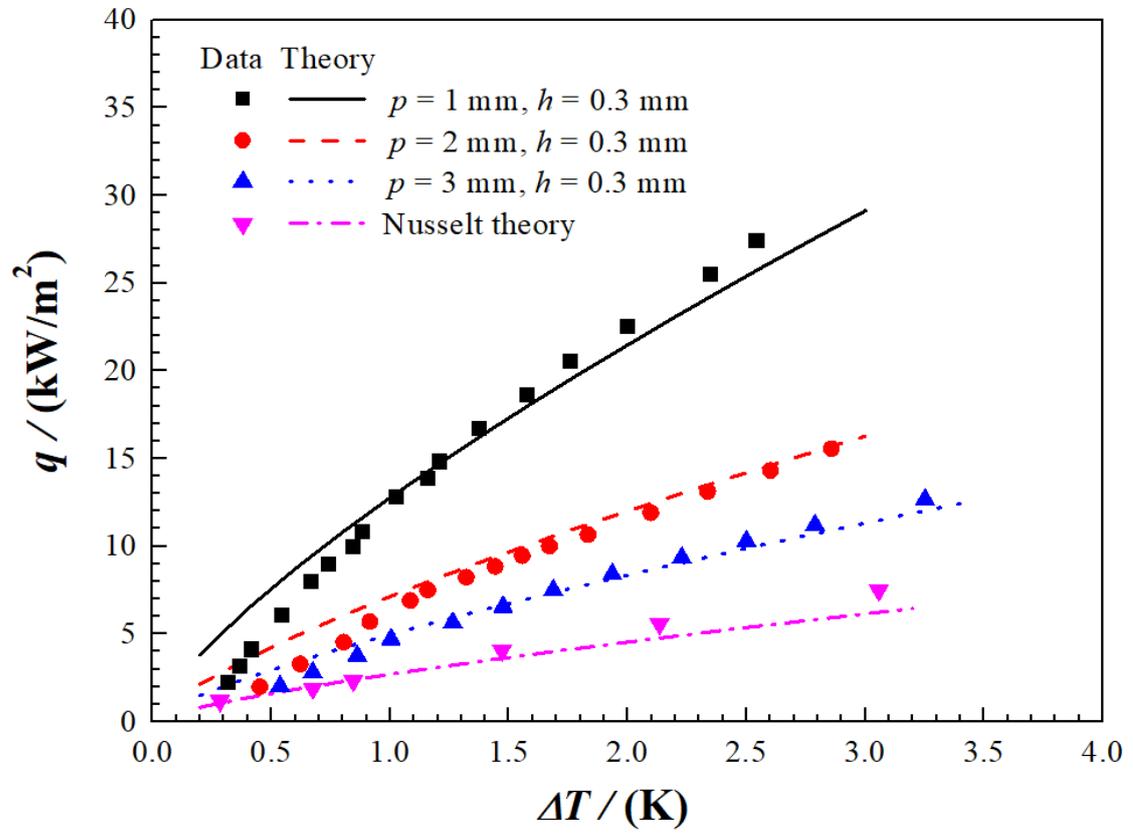


Figure 3: Comparison of Eq. (4) with experimental data [2]. Variation of heat flux with fin height and constant fin pitch.

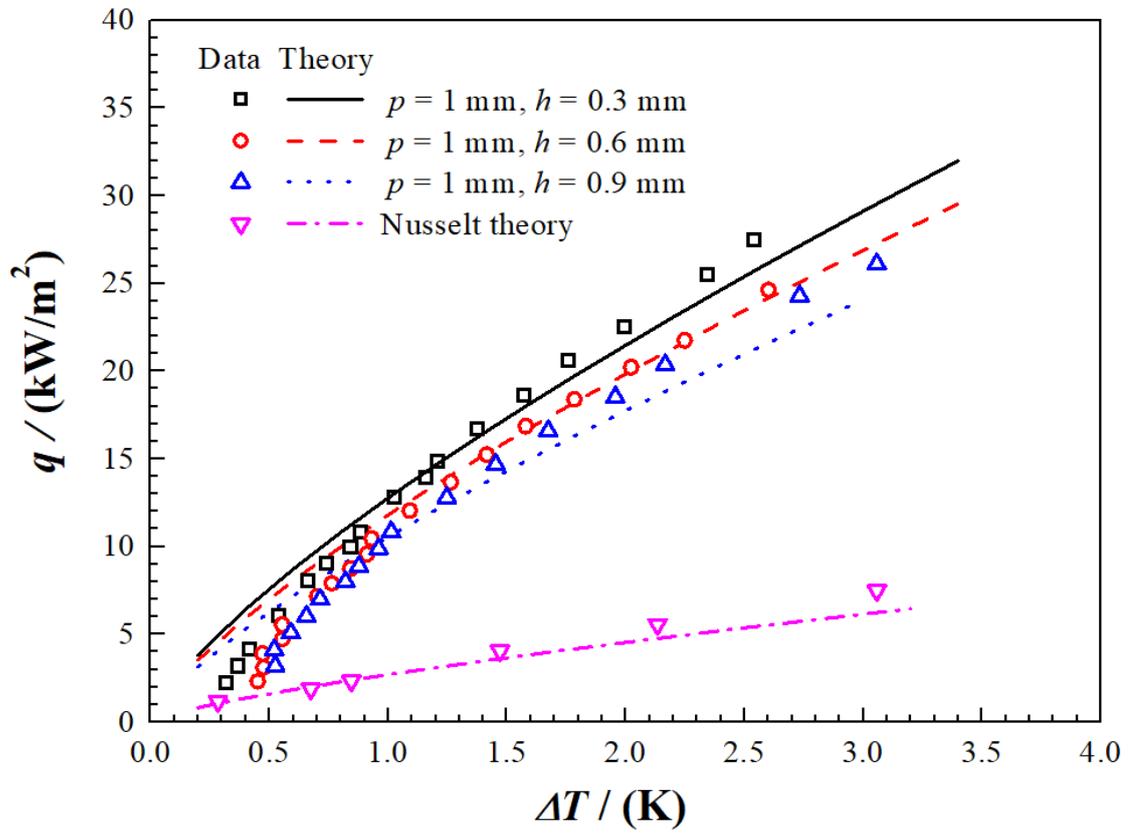


Figure 4: Dependence of enhancement ratio on fin height for constant fin pitch given by Eqs. (4) and (5).

