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Neural-Network-Based Set-Membership Fault Estimation for Two-Dimensional Systems under Encoding-Decoding Mechanism

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***Abstract*—In this paper, the simultaneous state and fault estimation problem is investigated for a class of nonlinear two-dimensional (2-D) shift-varying systems, where the sensors and the estimator are connected via a communication network of limited bandwidth. With the purpose of relieving the communication burden as well as enhancing the transmission security, a new encoding-decoding mechanism is put forward so as to encode the transmitted data with a finite number of bits. The aim of the addressed problem is to develop a neural-network-based set-membership estimator for jointly estimating the system states and the faults, where the estimation errors are guaranteed to reside within an optimized ellipsoidal set. With the aid of the mathematical induction technique and certain convex optimization approaches, sufficient conditions are derived for the existence of the desired set-membership estimator, and the estimator gains as well as the neural network tuning scalars are then presented in terms of the solutions to a set of optimization problems subject to ellipsoidal constraints. Finally, an illustrative example is given to demonstrate the effectiveness of the proposed estimator design method.**

***Index Terms*—Two-dimensional systems, set-membership estimation, fault estimation, neural networks, encoding-decoding mechanism.**

I. Introduction

Two-dimensional (2-D) systems have proven to be particularly suitable in modeling practical processes whose dynamics propagations are bidirectional, see e.g. grid sensor networks, sheet forming, and water stream heating [25], [34], [37], [50]. In general, there are three categories of 2-D state-space models: Attasi model, Fornasini-Marchesini (F-M) first model, and F-M second model [38]. It is worth mentioning that both Attasi and F-M first models can be regarded as special cases of the F-M second model. The system states in the 2-D setting evolve along *two independent* directions and, owing to this distinguishing feature, available paradigms for dealing with 1-D systems are no longer directly applicable to 2-D systems. Accordingly, the past few decades have seen a surge of research interest devoted to the analysis and synthesis issues of various kinds of 2-D systems [2], [15], [41], [42] and the references therein.

In practical engineering, it is often an imperative task to estimate the systems states that evolve over time through collected measurements possibly corrupted by noises or disturbances [26], [33]. Accordingly, the state estimation/filtering issue has long been a popular research topic attracting considerable research interest from both signal processing and control communities, see [3], [6], [24], [28], [32], [47]–[49], [53] for some latest results. Note that, in real-world applications, noises could enter the system in different forms (e.g., stochastic, energy-bounded, amplitude-bounded) and quite a few techniques have been specifically developed to address the individual characteristics of the noises in the context of estimation/filtering. For instance, the celebrated Kalman filtering (KF) and its variants (e.g., extended KF and unscented KF) are well known to be especially efficient for state estimation subject to Gaussian noises based on the minimum variance criterion. For energy-bounded noises, an effective way is to quantify the level of noise attenuation/rejection in the H_∞ sense and then develop the corresponding strategy for state estimation. Recently, the set-membership filtering technique has stirred much attention due to its exceptional capability of coping with the unknown-but-bounded noises [10], [18], [20], [51].

Concerning the state estimation problem for 2-D systems, there have been so far a host of results reported in the literature. Some recent representative works can be summarized as follows. For 2-D systems subject to stochastic noises, the Kalman-type filters have been designed in [36], [46] to accomplish the state estimation task with guaranteed minimum variance. In [19], [44], the H_∞ filtering problems have been addressed for 2-D systems undergoing energy-bounded noises, where the energy-to-energy gain (quantifying the influence from noise to estimation error) is enforced below a pre-specified level. Unfortunately, when facing the so-called *unknown-but-bounded* (UBB) noises, neither KF nor H_∞ filtering algorithms could provide a satisfactory performance. In consideration of UBB noises, an ideal state estimation methodology would be the set-membership (also known as set-valued) algorithm which aims to confine the estimation error within certain allowable regions. To date, very little attention has been paid to the set-membership-estimation (SME) prob-

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lem for 2-D systems, let alone the case where *nonlinearities* and *shift-varying* parameters are also taken into account.

Along with the ever-increasing demand towards reliability/security, fault diagnosis has gradually become pivotal in many branches of practical engineering such as chemical industries and military infrastructures [25]. Specifically, the main tasks of fault diagnosis include fault detection, isolation, and identification [9]. It should be noted that the fault estimation serves as a convenient yet powerful way for the fault diagnosis, and has recently garnered considerable interest with a number of approaches available in the literature including, but are not limited to, the H_∞ estimation approach, the Kalman filter method, the sliding-mode technique and the neural network (NN) based scheme [29], [39].

Among various state/fault estimation methods, the NN-based method has shown distinct advantages in dealing with inherently nonlinear systems due primarily to its excellent learning/approximation capabilities. Nonetheless, it is generally difficult yet time-consuming to tune the weighting factors in training an appropriate NN. To cope with this issue, the SME approach has been applied in [7] to effectively seek the suitable tuning parameters, where the established framework is only applicable to the linear 1-D system and cannot be directly extended to deal with the nonlinear 2-D case. When it comes to the nonlinear *2-D* shift-varying systems, the *NN-based* fault estimation problem has not been studied yet, and this constitutes one of the main motivations of this paper.

As is well known, quantization serves as a fundamental technique in the transmission process of digital data [16], [22]. Up to now, a number of static and dynamic quantization strategies have been proposed and extensively utilized [14], [21], [43], [52]. Among others, the dynamic quantization scheme can offer great flexibility by manipulating certain thresholds/parameters and has therefore attracted particular interest. As a frequently deployed dynamic-quantization-based scheme, the encoding-decoding mechanism (EDM) has been recently put forward to deal with various network-induced issues, where the encoder-decoder pairs is designed based on the zooming in/out quantization strategy [23]. For instance, the EDM has been employed in [17] to solve the distributed consensus problems for multi-agent systems. Moreover, the iterative learning control strategy has been designed in [31] via EDM to investigate the tracking control problem for linear discrete-time systems. Nevertheless, to the best of the authors' knowledge, the *EDM-embedded* joint state and fault estimation (SFE) problem has not been fully examined in the context of nonlinear 2-D shift-varying systems, and this constitutes another motivation of our current research.

In connection with the discussions made so far, in this paper, we concentrate on the NN-based SFE problem for nonlinear 2-D shift-varying systems with the adoption of EDMs. The difficulties we are facing are summarized as follows: 1) how to construct an appropriate EDM for 2-D systems with focus on the inherent characteristics of information propagation along two independent directions? 2) how to design the estimator of suitable form for 2-D systems subject to unknown fault dynamics (UFD) and UBB noises such that the system states and the faults can be jointly estimated? 3) how to quantify the influences on the estimation performance from the EDM and UFD? and 4) how to determine the NN tuning scalars and estimator gains? The main objective of this paper is, therefore, to overcome the listed difficulties by initializing a systematic investigation.

The main contributions of this paper can be highlighted as follows.

- A new EDM is, for the first time, proposed for a class of nonlinear 2-D shift-varying systems with aim to reduce the network communication burden and enhance the signal transmission security, where the proposed EDM depends on two indices on account of the bidirectional evolution of the system dynamics.
- An NN-based set-membership estimator is developed for nonlinear 2-D shift-varying systems subject to UBB noises and UFD, and such an estimator is shown to be capable of jointly estimating system states and faults with guaranteed performance index.
- The feasibility of the developed NN-based SME algorithm is thoroughly analyzed in a rigorous way by resorting to the two-dimensional mathematical induction technique.

The rest of this paper is organized as follows. In Section II, the NN-based set-membership estimator is formulated for nonlinear 2-D shift-varying systems subject to additive faults under dedicatedly introduced EDMs. In Section III, sufficient conditions are derived to ensure the feasibility of the recursive NN-based SME algorithm, and the estimator gains are then obtained by solving a set of optimization problems. Section IV utilizes an illustrative example to demonstrate the effectiveness of the proposed estimator design algorithm. Conclusions are drawn in Section V.

Notations: $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote, respectively, the n-dimensional Euclidean space and the set of all $m\times n$ real matrices. $M>0$ means that M is a positive definite matrix. $\{M_{i,j}\}_{i,j\in\mathbb{N}}$ denotes the set of matrices $\{M_{i^\circ,j^\circ}\,|\,0\le i^\circ\le i, 0\le j^\circ\le j\}$. $\mathrm{diag}\{\cdot\}$ stands for a block-diagonal matrix. In symmetric block matrices, "*" is used as an ellipsis for terms induced by symmetry. I and 0 denote the identity matrix and zero matrix with appropriate dimensions, respectively. The superscript "T" stands for the transpose of a matrix.

II. Problem Formulation and Preliminaries

A. System Model

Consider a nonlinear 2-D shift-varying system described by the following general F-M second model:

$$\begin{aligned} x_{i+1,j+1} = {} & A^{(1)}_{i,j+1}x_{i,j+1} + A^{(2)}_{i+1,j}x_{i+1,j} \\ & + f^{(1)}(x_{i,j+1}) + f^{(2)}(x_{i+1,j}) \\ & + E^{(1)}_{i,j+1}w_{i,j+1} + E^{(2)}_{i+1,j}w_{i+1,j} \end{aligned} \tag{1a}$$

$$y_{i,j} = C_{i,j}x_{i,j} + B_{i,j}\rho_{i,j} + D_{i,j}v_{i,j} \tag{1b}$$

where $i,j\in\mathbf{T}\triangleq[0,\mathcal{T}]$ are horizontal and vertical coordinates with $\mathcal{T}\in\mathbb{N}$; $x_{i,j}\in\mathbb{R}^{n_x}$ and $y_{i,j}\in\mathbb{R}^{n_y}$ are the state vector and the measurement output, respectively; $f^{(1)}(x_{i,j})$ and $f^{(2)}(x_{i,j})$ are known nonlinear functions; $A^{(1)}_{i,j}$, $A^{(2)}_{i,j}$, $B_{i,j}$, $C_{i,j}$, $D_{i,j}$, $E^{(1)}_{i,j}$, and $E^{(2)}_{i,j}$ are known shift-varying matrices

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with appropriate dimensions; $w_{i,j} \in \mathbb{R}^{n_w}$ and $v_{i,j} \in \mathbb{R}^{n_v}$ are the UBB process noise and UBB measurement noise, respectively, which are confined to the following ellipsoidal sets:

$$\Omega^w_{i,j} \triangleq \left\{ w_{i,j} \mid w^{\mathrm{T}}_{i,j} R^{-1}_{i,j} w_{i,j} \le 1 \right\} \tag{2}$$

$$\Omega^v_{i,j} \triangleq \left\{ v_{i,j} \mid v^{\mathrm{T}}_{i,j} S^{-1}_{i,j} v_{i,j} \le 1 \right\} \tag{3}$$

with $R_{i,j}$ and $S_{i,j}$ being known positive definite matrices.

The parameter $\rho_{i,j} \in \mathbb{R}^{n_\rho}$ in (1b) is the fault signal with the following dynamics:

$$\begin{aligned} \rho_{i+1,j+1} = {} & U^{(1)}_{i,j+1}\rho_{i,j+1} + U^{(2)}_{i+1,j}\rho_{i+1,j} \\ & + \mu^{(1)}(\rho_{i,j+1}) + \mu^{(2)}(\rho_{i+1,j}) \end{aligned} \tag{4}$$

where $U^{(1)}_{i,j}$ and $U^{(2)}_{i,j}$ are known shift-varying matrices with appropriate dimensions, and $\mu^{(1)}(\rho_{i,j})$ and $\mu^{(2)}(\rho_{i,j})$ are *unknown* smooth nonlinear functions.

Assumption 1: For $\kappa = 1, 2$, the known nonlinear functions $f^{(\kappa)}(x_{i,j})$ satisfy $f^{(\kappa)}(0) = 0$ and

$$\| f^{(\kappa)}(x_1) - f^{(\kappa)}(x_2) \| \le \| F^{(\kappa)}(x_1 - x_2) \| \tag{5}$$

for all $x_1, x_2 \in \mathbb{R}^{n_x}$, where $F^{(\kappa)}$ $(\kappa = 1, 2)$ are known matrices.

Assumption 2: ([1], [35]) For $\kappa = 1, 2$, the nonlinear functions $\mu^{(\kappa)}(\rho_{i,j})$ are on compact sets $\Omega^{(\kappa)}_\mu$.

Remark 1: It should be noted that Assumption 2 is a standard assumption that ensures the feasibility of the approximation of nonlinear functions $\mu^{(\kappa)}(\rho_{i,j})$ by using neural networks (see e.g., [1], [35]). In practice, such a constraint (on the boundedness of the addressed nonlinear functions) can be guaranteed by the implementation of certain devices such as the signal limiter proposed in [27].

Remark 2: It is seen from the additive fault model (4) that the dynamics of the investigated faults changes bidirectionally along both horizontal and vertical horizons. Apparently, (4) is fairly general that well reflects the engineering practice.

1) If $U^{(1)}_{i,j+1} = I$ and $U^{(2)}_{i+1,j} = \mu^{(1)}(\cdot) = \mu^{(2)}(\cdot) = 0$, we have
$$\rho_{i+1,j+1} = \rho_{i,j+1}$$
representing the *bias fault*.
2) If $U^{(2)}_{i+1,j} = I$ and $U^{(1)}_{i,j+1} = \mu^{(1)}(\cdot) = \mu^{(2)}(\cdot) = 0$, we have
$$\rho_{i+1,j+1} = \rho_{i+1,j}$$
representing the *bias fault*.
3) If $U^{(1)}_{i,j+1} \neq 0$, $U^{(2)}_{i+1,j} \neq 0$, and $\mu^{(1)}(\cdot) = \mu^{(2)}(\cdot) = 0$, we have
$$\rho_{i+1,j+1} = U^{(1)}_{i,j+1}\rho_{i,j+1} + U^{(2)}_{i+1,j}\rho_{i+1,j}$$
representing the *drift fault*.
4) If $U^{(1)}_{i,j+1} \neq 0$, $U^{(2)}_{i+1,j} \neq 0$, $\mu^{(1)}(\cdot) \neq 0$, and $\mu^{(2)}(\cdot) \neq 0$, we have
$$\begin{aligned} \rho_{i+1,j+1} = {} & U^{(1)}_{i,j+1}\rho_{i,j+1} + U^{(2)}_{i+1,j}\rho_{i+1,j} \\ & + \mu^{(1)}(\rho_{i,j+1}) + \mu^{(2)}(\rho_{i+1,j}) \end{aligned}$$
representing the *drift fault with unknown nonlinear dynamics*.

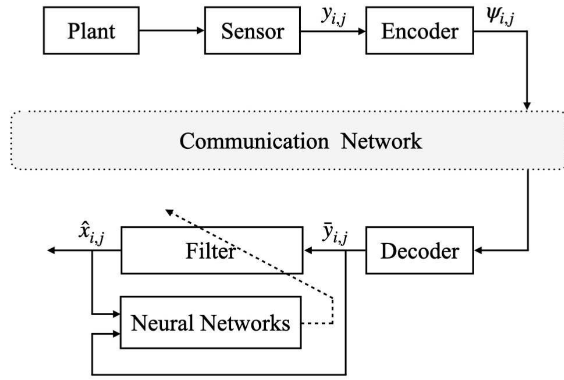

Fig. 1. Block diagram for 2-D systems with EDMs.

Remark 3: Comparing with the rich body of existing results, the fault model (4) investigated in this paper exhibits the following distinguish features: 1) the fault model is fairly general, which covers the bias fault and the drift fault as a special case; and 2) the fault model takes the shift-varying parameter and the unknown nonlinear dynamics into account. Note that the inclusion of the unknown nonlinear dynamics in the presented faults model imposes substantial difficulties on the performance analysis and subsequent design of the desired estimation scheme.

B. Encoding and Decoding Mechanism

In the procedure of data transmission, the signals are processed by the EDM as shown in Fig. 1, where the main steps/principles are described as follows.

Encoding:

The encoding rule is given as

$$\begin{cases} \chi_{0,j} = \chi_{i,0} = 0, \quad \forall\, i, j \in \mathbf{T} \\ \chi_{i,j} = \alpha_{i,j}\psi_{i,j} + \daleth^{(1)}_{i-1,j}\chi_{i-1,j} + \daleth^{(2)}_{i,j-1}\chi_{i,j-1} \\ \psi_{i,j} = \mathfrak{Q}\left\{ \dfrac{1}{\alpha_{i,j}} \left(y_{i,j} - \daleth^{(1)}_{i-1,j}\chi_{i-1,j} - \daleth^{(2)}_{i,j-1}\chi_{i,j-1} \right) \right\} \end{cases} \tag{6}$$

where $\chi_{i,j} \in \mathbb{R}^{n_\chi}$ and $\psi_{i,j} \in \mathbb{R}^{n_\psi}$ are the internal state and the output of the encoder, respectively; $\alpha_{i,j}$ is a known scaling parameter; and $\daleth^{(1)}_{i,j}$ and $\daleth^{(2)}_{i,j}$ are known shift-varying matrices with appropriate dimensions. Here, the uniform quantizer $\mathfrak{Q}$ is characterized by

$$\mathfrak{Q}(\zeta) \triangleq \begin{bmatrix} \mathscr{Q}(\zeta_1) \\ \mathscr{Q}(\zeta_2) \\ \vdots \\ \mathscr{Q}(\zeta_{n_\zeta}) \end{bmatrix} \tag{7}$$

where, for $\hbar = 1, 2, \ldots, n_\zeta$

$$\mathscr{Q}(\zeta_\hbar) \triangleq \begin{cases} 0, & -\frac{\ell}{2} \le \zeta_\hbar < \frac{\ell}{2} \\ \wp\ell, & \frac{(2\wp-1)\ell}{2} \le \zeta_\hbar < \frac{(2\wp+1)\ell}{2} \\ -\mathscr{Q}(-\zeta_\hbar), & \zeta_\hbar < -\frac{\ell}{2} \end{cases} \tag{8}$$

Here, $\zeta \in \mathbb{R}^{n_\zeta}$ is the signal vector, $\zeta_\hbar$ is the $\hbar$th entry of ζ, ℓ is the interval length of the quantization level, $\wp$ is a

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positive integer taking values in $\{1, 2, \ldots, \mathcal{R}\}$, and $2\mathcal{R}+1$ is the number of quantization levels.

Decoding:

The decoding rule is described by

$$\begin{cases} \bar{y}_{0,j} = \bar{y}_{i,0} = 0, \quad \forall\, i, j \in \mathbf{T} \\ \bar{y}_{i,j} = \alpha_{i,j}\psi_{i,j} + \daleth^{(1)}_{i-1,j}\bar{y}_{i-1,j} + \daleth^{(2)}_{i,j-1}\bar{y}_{i,j-1} \end{cases} \tag{9}$$

where $\bar{y}_{i,j} \in \mathbb{R}^{n_y}$ is the output of the decoder.

Letting $e_{i,j} \triangleq \bar{y}_{i,j} - y_{i,j}$ be the decoding error, we acquire

$$\begin{aligned} & e_{i,j} \triangleq \bar{y}_{i,j} - y_{i,j} \\ &= \alpha_{i,j}\psi_{i,j} + \daleth^{(1)}_{i-1,j}\bar{y}_{i-1,j} + \daleth^{(2)}_{i,j-1}\bar{y}_{i,j-1} - y_{i,j} \\ &= \alpha_{i,j}\Big\{\mathfrak{Q}\Big\{\frac{1}{\alpha_{i,j}}\Big(y_{i,j} - \daleth^{(1)}_{i-1,j}\chi_{i-1,j} - \daleth^{(2)}_{i,j-1}\chi_{i,j-1}\Big)\Big\} \\ &\quad - \frac{1}{\alpha_{i,j}}\Big(y_{i,j} - \daleth^{(1)}_{i-1,j}\bar{y}_{i-1,j} - \daleth^{(2)}_{i,j-1}\bar{y}_{i,j-1}\Big)\Big\} \\ &= \alpha_{i,j}\Big\{\mathfrak{Q}\Big\{\frac{1}{\alpha_{i,j}}\Big(y_{i,j} - \daleth^{(1)}_{i-1,j}\chi_{i-1,j} - \daleth^{(2)}_{i,j-1}\chi_{i,j-1}\Big)\Big\} \\ &\quad - \frac{1}{\alpha_{i,j}}\Big(y_{i,j} - \daleth^{(1)}_{i-1,j}\chi_{i-1,j} - \daleth^{(2)}_{i,j-1}\chi_{i,j-1}\Big)\Big\}, \end{aligned} \tag{10}$$

which indicates that the decoding error satisfies

$$\|e_{i,j}\|_\infty \leq \frac{\alpha_{i,j}\ell}{2}.$$

Remark 4: A novel EDM (6)–(9) is, for the first time, constructed for nonlinear 2-D shift-varying systems. Different from the EDM implemented in 1-D systems, the encoding-decoding rules (6) and (9) proposed in our work are designed in accordance with the characteristics of bidirectional evolution of 2-D systems. In general, the new developed EDM possesses the following *advantages*: 1) the quantization input in (6) is

$$\frac{1}{\alpha_{i,j}}(y_{i,j} - \daleth^{(1)}_{i-1,j}\chi_{i-1,j} - \daleth^{(2)}_{i,j-1}\chi_{i,j-1})$$

rather than $y_{i,j}$, which could reduce the bits used for encoding the codeword, thereby promoting the efficiency of utilization of network resources; 2) the parameter $\alpha_{i,j}$ can be dynamically adjusted, which provides extra flexibility in the subsequent estimator design for a better performance; and 3) the security of the transmitted data can be further guaranteed due to the introduction of coefficients $\daleth^{(1)}_{i,j}$ and $\daleth^{(2)}_{i,j}$ in the proposed EDM.

C. NN-Based Set-Membership Estimator

Letting $\vec{x}_{i,j} \triangleq \begin{bmatrix} x^{\mathrm{T}}_{i,j} & \rho^{\mathrm{T}}_{i,j} \end{bmatrix}^{\mathrm{T}}$, one obtains

$$\begin{aligned} \vec{x}_{i+1,j+1} = {}& \mathscr{A}^{(1)}_{i,j+1}\vec{x}_{i,j+1} + \mathscr{A}^{(2)}_{i+1,j}\vec{x}_{i+1,j} \\ & + \vec{f}^{(1)}_{i,j+1} + \vec{f}^{(2)}_{i+1,j} + \vec{\mu}^{(1)}_{i,j+1} + \vec{\mu}^{(2)}_{i+1,j} \\ & + \mathscr{E}^{(1)}_{i,j+1}w_{i,j+1} + \mathscr{E}^{(2)}_{i+1,j}w_{i+1,j} \end{aligned} \tag{11a}$$

$$y_{i,j} = \mathscr{C}_{i,j}\vec{x}_{i,j} + D_{i,j}v_{i,j} \tag{11b}$$

where, for $\kappa = 1, 2$

$$\mathscr{A}^{(\kappa)}_{i,j} \triangleq \begin{bmatrix} A^{(\kappa)}_{i,j} & 0 \\ 0 & U^{(\kappa)}_{i,j} \end{bmatrix}, \quad \vec{f}^{(\kappa)}_{i,j} \triangleq \begin{bmatrix} f^{(\kappa)}(G\vec{x}_{i,j}) \\ 0 \end{bmatrix}$$

$$\vec{\mu}^{(\kappa)}_{i,j} \triangleq \begin{bmatrix} 0 \\ \mu^{(\kappa)}(H\vec{x}_{i,j}) \end{bmatrix}, \; G \triangleq \begin{bmatrix} I & 0 \end{bmatrix}, \; H \triangleq \begin{bmatrix} 0 & I \end{bmatrix}$$

$$\mathscr{E}^{(\kappa)}_{i,j} \triangleq \begin{bmatrix} E^{(\kappa)}_{i,j} \\ 0 \end{bmatrix}, \quad \mathscr{C}_{i,j} \triangleq \begin{bmatrix} C_{i,j} & B_{i,j} \end{bmatrix}.$$

Based on the universal approximation property [13], NNs are used to approximate the unknown nonlinear terms $\vec{\mu}^{(1)}_{i,j+1}$ and $\vec{\mu}^{(2)}_{i+1,j}$ in system (11a) as follows:

$$\begin{aligned} \vec{x}_{i+1,j+1} = {}& \mathscr{A}^{(1)}_{i,j+1}\vec{x}_{i,j+1} + \mathscr{A}^{(2)}_{i+1,j}\vec{x}_{i+1,j} \\ & + \vec{f}^{(1)}_{i,j+1} + \vec{f}^{(2)}_{i+1,j} + W^{(1)}\phi^{(1)}(\vec{x}_{i,j+1}) \\ & + W^{(2)}\phi^{(2)}(\vec{x}_{i+1,j}) + \mathscr{E}^{(1)}_{i,j+1}w_{i,j+1} \\ & + \mathscr{E}^{(2)}_{i+1,j}w_{i+1,j} + \xi^{(1)}_{i,j+1} + \xi^{(2)}_{i+1,j} \end{aligned} \tag{12}$$

where $W^{(1)}$ and $W^{(2)}$ are ideal constant weight matrices of the NNs, $\phi^{(1)}(\cdot)$ and $\phi^{(2)}(\cdot)$ are activation functions, and $\xi^{(1)}_{i,j}$ and $\xi^{(2)}_{i,j}$ are approximation errors.

Assumption 3: ([45]) For $\kappa = 1, 2$, the ideal weight matrices $W^{(\kappa)}$, the activation function $\phi^{(\kappa)}(\cdot)$, and the approximation errors $\xi^{(\kappa)}_{i,j}$ satisfy the following conditions:

$$\|W^{(\kappa)}\|_F \leq \bar{W}^{(\kappa)}, \quad \|\phi^{(\kappa)}(\cdot)\| \leq \bar{\phi}^{(\kappa)}, \quad \|\xi^{(\kappa)}_{i,j}\| \leq \bar{\xi}^{(\kappa)} \tag{13}$$

where $\bar{W}^{(\kappa)}$, $\bar{\phi}^{(\kappa)}$, and $\bar{\xi}^{(\kappa)}$ are known positive constants.

In this paper, for the nonlinear 2-D shift-varying system (1), an NN-based set-membership estimator is constructed as follows:

$$\begin{aligned} \hat{x}^-_{i+1,j+1} = {}& \mathscr{A}^{(1)}_{i,j+1}\hat{x}_{i,j+1} + \mathscr{A}^{(2)}_{i+1,j}\hat{x}_{i+1,j} \\ & + \hat{f}^{(1)}_{i,j+1} + \hat{f}^{(2)}_{i+1,j} + \hat{W}^{(1)}_{i,j+1}\hat{\phi}^{(1)}_{i,j+1} \\ & + \hat{W}^{(2)}_{i+1,j}\hat{\phi}^{(2)}_{i+1,j} \end{aligned} \tag{14a}$$

$$\begin{aligned} \hat{x}_{i+1,j+1} = {}& \hat{x}^-_{i+1,j+1} + K^{(1)}_{i,j+1}\big(\bar{y}_{i,j+1} - \mathscr{C}_{i,j+1}\hat{x}_{i,j+1}\big) \\ & + K^{(2)}_{i+1,j}\big(\bar{y}_{i+1,j} - \mathscr{C}_{i+1,j}\hat{x}_{i+1,j}\big) \end{aligned} \tag{14b}$$

where, for $\kappa = 1, 2$, $\hat{x}^-_{i,j} \in \mathbb{R}^{n_x}$ is the one-step prediction, $\hat{x}_{i,j} \in \mathbb{R}^{n_x}$ is the estimate of $x_{i,j}$, $\hat{W}^{(\kappa)}_{i,j}$ are the estimates of $W^{(\kappa)}$, $K^{(\kappa)}_{i,j}$ are the estimator gains to be determined and

$$\hat{f}^{(\kappa)}_{i,j} \triangleq \begin{bmatrix} f^{(\kappa)}(G\hat{x}_{i,j}) \\ 0 \end{bmatrix}, \quad \hat{\phi}^{(\kappa)}_{i,j} \triangleq \phi^{(\kappa)}(\hat{x}_{i,j}).$$

The adaptive tuning laws for the NN weights are designed as

$$\begin{aligned} \hat{W}^{(1)}_{i+1,j+1} = {}& \tau^{(1,1)}_{i,j+1}\hat{W}^{(1)}_{i,j+1} + \tau^{(1,2)}_{i,j+1}\mathscr{C}^{\mathrm{T}}_{i,j+1}\big(\bar{y}_{i+1,j+1} \\ & - \mathscr{C}_{i+1,j+1}\hat{x}_{i+1,j+1}\big)(\hat{\phi}^{(1)}_{i,j+1})^{\mathrm{T}} \end{aligned} \tag{15}$$

$$\begin{aligned} \hat{W}^{(2)}_{i+1,j+1} = {}& \tau^{(2,1)}_{i+1,j}\hat{W}^{(2)}_{i+1,j} + \tau^{(2,2)}_{i+1,j}\mathscr{C}^{\mathrm{T}}_{i+1,j}\big(\bar{y}_{i+1,j+1} \\ & - \mathscr{C}_{i+1,j+1}\hat{x}_{i+1,j+1}\big)(\hat{\phi}^{(2)}_{i+1,j})^{\mathrm{T}} \end{aligned} \tag{16}$$

where $\tau^{(1,1)}_{i,j}$, $\tau^{(1,2)}_{i,j}$, $\tau^{(2,1)}_{i,j}$, and $\tau^{(2,2)}_{i,j}$ are positive tuning scalars to be determined. In addition, the persistence of excitation is required to be satisfied. For $\kappa = 1, 2$, the estimation errors between $W^{(\kappa)}$ and $\hat{W}^{(\kappa)}_{i,j}$ are derived as

$$\begin{aligned} & \tilde{W}^{(1)}_{i+1,j+1} \triangleq W^{(1)} - \hat{W}^{(1)}_{i+1,j+1} \\ &= \tau^{(1,1)}_{i,j+1}\tilde{W}^{(1)}_{i,j+1} + (1 - \tau^{(1,1)}_{i,j+1})W^{(1)} - \tau^{(1,2)}_{i,j+1}\mathscr{C}^{\mathrm{T}}_{i,j+1} \end{aligned}$$

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$$
\begin{aligned}
&\quad \times \big(\bar{y}_{i+1,j+1} - \mathscr{C}_{i+1,j+1}\hat{x}_{i+1,j+1}\big)(\hat{\phi}^{(1)}_{i,j+1})^{\mathrm{T}} && (17)\\
&\tilde{W}^{(2)}_{i+1,j+1} \triangleq W^{(2)} - \hat{W}^{(2)}_{i+1,j+1}\\
&= \tau^{(2,1)}_{i+1,j}\tilde{W}^{(2)}_{i+1,j} + (1-\tau^{(2,1)}_{i+1,j})W^{(2)} - \tau^{(2,2)}_{i+1,j}\mathscr{C}^{\mathrm{T}}_{i+1,j}\\
&\quad \times \big(\bar{y}_{i+1,j+1} - \mathscr{C}_{i+1,j+1}\hat{x}_{i+1,j+1}\big)(\hat{\phi}^{(2)}_{i+1,j})^{\mathrm{T}}. && (18)
\end{aligned}
$$

Denoting the estimation error as $\theta_{i,j} \triangleq \vec{x}_{i,j} - \hat{x}_{i,j}$, one obtains

$$
\begin{aligned}
\theta_{i+1,j+1} = {} & (\mathscr{A}^{(1)}_{i,j+1} - K^{(1)}_{i,j+1}\mathscr{C}_{i,j+1})\theta_{i,j+1}\\
& + (\mathscr{A}^{(2)}_{i+1,j} - K^{(2)}_{i+1,j}\mathscr{C}_{i+1,j})\theta_{i+1,j}\\
& + \tilde{W}^{(1)}_{i,j+1}\hat{\phi}^{(1)}_{i,j+1} + \tilde{W}^{(2)}_{i+1,j}\hat{\phi}^{(2)}_{i+1,j}\\
& + \tilde{f}^{(1)}_{i,j+1} + \tilde{f}^{(2)}_{i+1,j} + \mathscr{E}^{(1)}_{i,j+1}w_{i,j+1}\\
& + \mathscr{E}^{(2)}_{i+1,j}w_{i+1,j} - K^{(1)}_{i,j+1}e_{i,j+1}\\
& - K^{(2)}_{i+1,j}e_{i+1,j} - K^{(1)}_{i,j+1}D_{i,j+1}v_{i,j+1}\\
& - K^{(2)}_{i+1,j}D_{i+1,j}v_{i+1,j} + \vec{\xi}^{(1)}_{i,j+1} + \vec{\xi}^{(2)}_{i+1,j} \qquad (19)
\end{aligned}
$$

where, for $\kappa = 1, 2$

$$
\begin{aligned}
&\tilde{f}^{(\kappa)}_{i,j} \triangleq \vec{f}^{(\kappa)}_{i,j} - \hat{f}^{(\kappa)}_{i,j}, \quad \tilde{\phi}^{(\kappa)}_{i,j} \triangleq \phi^{(\kappa)}_{i,j} - \hat{\phi}^{(\kappa)}_{i,j},\\
&\vec{\xi}^{(\kappa)}_{i,j} \triangleq W^{(\kappa)}\tilde{\phi}^{(\kappa)}_{i,j} + \xi^{(\kappa)}_{i,j}.
\end{aligned}
$$

Before giving the main design objectives of this paper, we first present the following assumptions on initial conditions that are of help in the subsequent derivations.

Assumption 4: The evolutions of the estimation error dynamics of the neuron weights $W^{(1)}$ and $W^{(2)}$ are characterized by (17)–(18) whose initial conditions satisfy

$$
\begin{cases}
\mathrm{tr}\big\{(\tilde{W}^{(1)}_{0,j})^{\mathrm{T}}(P^{(1)}_{0,j})^{-1}\tilde{W}^{(1)}_{0,j}\big\} \le 1, & \forall\, j \in \mathbf{T}\\
\mathrm{tr}\big\{(\tilde{W}^{(2)}_{i,0})^{\mathrm{T}}(P^{(2)}_{i,0})^{-1}\tilde{W}^{(2)}_{i,0}\big\} \le 1, & \forall\, i \in \mathbf{T}
\end{cases} \qquad (20)
$$

where $P^{(1)}_{0,j}$ and $P^{(2)}_{i,0}$ are known diagonal positive definite matrices.

Assumption 5: The initial conditions of the state estimation error dynamics described by (19) are given by

$$
\begin{cases}
\theta^{\mathrm{T}}_{0,j}Q^{-1}_{0,j}\theta_{0,j} \le 1, & \forall\, j \in \mathbf{T}\\
\theta^{\mathrm{T}}_{i,0}Q^{-1}_{i,0}\theta_{i,0} \le 1, & \forall\, i \in \mathbf{T}
\end{cases} \qquad (21)
$$

where $Q_{0,j}$ and $Q_{i,0}$ are known positive definite matrices.

The main purpose of this paper is highlighted in twofold as follows.

1) *First*, we aim to design tuning scalars $\tau^{(1,1)}_{i-1,j}$, $\tau^{(1,2)}_{i-1,j}$, $\tau^{(2,1)}_{i,j-1}$, $\tau^{(2,2)}_{i,j-1}$ and NN-based set-membership estimator gains $K^{(1)}_{i-1,j}$, $K^{(2)}_{i,j-1}$ such that the estimation errors $\tilde{W}^{(1)}_{i,j}$, $\tilde{W}^{(2)}_{i,j}$ and $\theta_{i,j}$ are confined to the following sets:

$$
\mathscr{F}^{(1)}_{i,j} \triangleq \Big\{\tilde{W}^{(1)}_{i,j} \,\big|\, g\big(\tilde{W}^{(1)}_{i,j}\big) \le 1\Big\} \qquad (22)
$$

$$
\mathscr{F}^{(2)}_{i,j} \triangleq \Big\{\tilde{W}^{(2)}_{i,j} \,\big|\, g\big(\tilde{W}^{(2)}_{i,j}\big) \le 1\Big\} \qquad (23)
$$

$$
\mathscr{S}_{i,j} \triangleq \Big\{\theta_{i,j} \,\big|\, \theta^{\mathrm{T}}_{i,j}Q^{-1}_{i,j}\theta_{i,j} \le 1\Big\} \qquad (24)
$$

where, for $\kappa = 1, 2$

$$
g\big(\tilde{W}^{(\kappa)}_{i,j}\big) \triangleq \mathrm{tr}\big\{(\tilde{W}^{(\kappa)}_{i,j})^{\mathrm{T}}(P^{(\kappa)}_{i,j})^{-1}\tilde{W}^{(\kappa)}_{i,j}\big\}
$$

with $P^{(1)}_{i,j}$, $P^{(2)}_{i,j}$ being diagonal positive definite matrices and $Q_{i,j}$ being the positive definite matrix.

2) *Second*, based on the obtained results in the first step, we shall determine the optimal values of the tuning scalars and the estimator gains by minimizing the estimation error constraint sets (22)–(24) in the sense of matrix trace.

III. Main Results

In this section, we will design a joint SFE scheme for the addressed nonlinear 2-D shift-varying systems subject to UFD and UBB noises by applying a set-membership approach. Sufficient conditions are established for the existence of the desired estimator that guarantees that both the NN weight estimation error and the state estimation error satisfy the required performance constraints. Then, the desired tuning scalars and estimator gains are obtained by solving the proposed optimization problems.

Lemma 1: Consider the vectors $\chi_{i,j}$ and $\bar{y}_{i,j}$ in (6) and (9). For $\forall\, i, j \in \mathbf{T}$, we have

$$
\chi_{i,j} = \bar{y}_{i,j}. \qquad (25)
$$

Proof: This lemma is proved by the two-dimensional version of mathematical induction, which is conducted via the following two steps.

1) Initial step. From the rules (6) and (9), it is inferred that $\chi_{i,j} = \bar{y}_{i,j}$ is true for $(i,j) \in \{(i^\circ, j^\circ) | i^\circ, j^\circ \in \mathbb{N}, i^\circ + j^\circ = 0\}$.

2) Inductive step. Suppose that $\chi_{i,j} = \bar{y}_{i,j}$ is true for $(i,j) \in \{(i^\circ, j^\circ) | i^\circ, j^\circ \in \mathbb{N}, i^\circ + j^\circ = h\}$ with h being a given positive integer. Then, it remains to prove that $\chi_{i,j} = \bar{y}_{i,j}$ is true for $(i,j) \in \{(i^\circ, j^\circ) | i^\circ, j^\circ \in \mathbb{N}, i^\circ + j^\circ = h+1\}$. In fact, for $(i,j) \in \{(i^\circ, j^\circ) | i^\circ, j^\circ \in \mathbb{N}, i^\circ + j^\circ = h+1\}$, we have

$$
\begin{aligned}
&\bar{y}_{i,j} - \chi_{i,j}\\
&= \alpha_{i,j}\psi_{i,j} + \urcorner^{(1)}_{i-1,j}\bar{y}_{i-1,j} + \urcorner^{(2)}_{i,j-1}\bar{y}_{i,j-1}\\
&\quad - \alpha_{i,j}\psi_{i,j} - \urcorner^{(1)}_{i-1,j}\chi_{i-1,j} - \urcorner^{(2)}_{i,j-1}\chi_{i,j-1}\\
&= \urcorner^{(1)}_{i-1,j}(\bar{y}_{i-1,j} - \chi_{i-1,j}) + \urcorner^{(2)}_{i,j-1}(\bar{y}_{i,j-1} - \chi_{i,j-1})\\
&= 0,
\end{aligned}
$$

which ends the proof. ∎

A. Design of the NN Weight Adaptive Tuning Law

To simplify notations, we denote

$$
\begin{aligned}
&\vec{n} \triangleq n_x + n_\rho, \quad \hat{n} \triangleq 2(n_y + n_w + n_v)\\
&\Gamma_0 \triangleq \mathrm{diag}\{1, 0, 0\}, \quad \Gamma_1 \triangleq \mathrm{diag}\{-1, I, 0\}\\
&\Gamma^{(\kappa)}_2 \triangleq \mathrm{diag}\{-\bar{W}^{(\kappa)}, 0, I\}\\
&\check{\Phi}_{i,j} \triangleq \mathrm{diag}\Big\{\underbrace{\Phi_{i,j}, \Phi_{i,j}, \ldots, \Phi_{i,j}}_{\vec{n}}\Big\}\\
&\Phi_{i,j} \triangleq \bar{y}_{i,j} - \mathscr{C}_{i,j}\hat{x}_{i,j}, \quad \vec{\Phi}_{i,j} = I_{\vec{n}} \otimes \check{\Phi}_{i,j}\\
&\mathscr{W}^{(\kappa)}_{i,j} \triangleq \Big[(\tilde{W}^{(\kappa,1)}_{i,j})^{\mathrm{T}} \quad (\tilde{W}^{(\kappa,2)}_{i,j})^{\mathrm{T}} \quad \cdots \quad (\tilde{W}^{(\kappa,\vec{n})}_{i,j})^{\mathrm{T}}\Big]^{\mathrm{T}}
\end{aligned}
$$

where, for $\kappa = 1, 2$, $\tilde{W}^{(\kappa,\iota)}_{i,j}$ are the ιth column vector of $\tilde{W}^{(\kappa)}_{i,j}$ $(\iota = 1, 2, \ldots, \vec{n})$.

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The following theorem is given to provide a sufficient condition that guarantees that the neuron weights satisfy the constrained sets (22)–(23).

Theorem 1: Consider the nonlinear 2-D shift-varying system (1) and the NN-based set-membership estimator (14). Let the sequence of matrices $\{P^{(1)}_{0,j}\}_{j\in\mathbf{T}}$ and $\{P^{(2)}_{i,0}\}_{j\in\mathbf{T}}$ be given. The estimation errors $\tilde{W}^{(1)}_{i+1,j+1}$ and $\tilde{W}^{(2)}_{i+1,j+1}$ of neuron weights belong to the sets $\mathscr{F}^{(1)}_{i+1,j+1}$ and $\mathscr{F}^{(2)}_{i+1,j+1}$ if there exist tuning scalars $\tau^{(1,1)}_{i,j+1}$, $\tau^{(1,2)}_{i,j+1}$, $\tau^{(2,1)}_{i+1,j}$, $\tau^{(2,2)}_{i+1,j}$, positive scalars $\varepsilon^{(1,1)}_{i,j+1}$, $\varepsilon^{(1,2)}_{i,j+1}$, $\varepsilon^{(2,1)}_{i+1,j}$, $\varepsilon^{(2,2)}_{i+1,j}$ and matrices $P^{(1)}_{i+1,j+1}>0$, $P^{(2)}_{i+1,j+1}>0$ satisfying

$$\begin{bmatrix} -\vec{\Gamma}^{(1)}_{i,j+1} & * \\ \Pi^{(1)}_{i,j+1} & -\mathscr{P}^{(1)}_{i+1,j+1}\end{bmatrix}\le 0 \tag{26}$$

$$\begin{bmatrix} -\vec{\Gamma}^{(2)}_{i+1,j} & * \\ \Pi^{(2)}_{i+1,j} & -\mathscr{P}^{(2)}_{i+1,j+1}\end{bmatrix}\le 0,\quad \forall i,j\in\mathbf{T} \tag{27}$$

where, for $\kappa=1,2$

$$\begin{aligned}
&\bar{\tau}^{(\kappa,1)}_{i,j}\triangleq 1-\tau^{(\kappa,1)}_{i,j},\quad \vec{\Gamma}^{(\kappa)}_{i,j}\triangleq\Gamma_0+\varepsilon^{(\kappa,1)}_{i,j}\Gamma_1+\varepsilon^{(\kappa,2)}_{i,j}\Gamma^{(\kappa)}_2\\
&\Pi^{(1)}_{i,j}\triangleq\begin{bmatrix}-\tau^{(1,2)}_{i,j}\vec{\mathscr{C}}_{i,j}\vec{\Phi}_{i+1,j}\vec{\phi}^{(1)}_{i,j} & \tau^{(1,1)}_{i,j}\Psi^{(1)}_{i,j} & \bar{\tau}^{(1,1)}_{i,j}I\end{bmatrix}\\
&\Pi^{(2)}_{i,j}\triangleq\begin{bmatrix}-\tau^{(2,2)}_{i,j}\vec{\mathscr{C}}_{i,j}\vec{\Phi}_{i,j+1}\hat{\phi}^{(2)}_{i,j} & \tau^{(2,1)}_{i,j}\Psi^{(2)}_{i,j} & \bar{\tau}^{(2,1)}_{i,j}I\end{bmatrix}\\
&\check{\mathscr{C}}_{i,j}\triangleq\mathrm{diag}\{\mathscr{C}^{(1)}_{i,j},\mathscr{C}^{(2)}_{i,j},\ldots,\mathscr{C}^{(\vec{n})}_{i,j}\},\quad \vec{\mathscr{C}}_{i,j}\triangleq I_{\vec{n}}\otimes\check{\mathscr{C}}_{i,j}\\
&\mathscr{P}^{(\kappa)}_{i,j}\triangleq\mathrm{diag}\Big\{\underbrace{P^{(\kappa)}_{i,j},P^{(\kappa)}_{i,j},\ldots,P^{(\kappa)}_{i,j}}_{\vec{n}}\Big\}.
\end{aligned}$$

Here, $(\mathscr{C}^{(\iota)}_{i,j})^{\mathrm{T}}$ is the ιth column vector of $\mathscr{C}_{i,j}$ $(\iota=1,2,\ldots,\vec{n})$.

Proof: This theorem is proved by the two-dimensional version of mathematical induction, which is conducted via the following two steps.

1) Initial step. According to Assumption 4, it is obvious that

$$(\tilde{\mathscr{W}}^{(1)}_{0,j})^{\mathrm{T}}(\mathscr{P}^{(1)}_{0,j})^{-1}\tilde{\mathscr{W}}^{(1)}_{0,j}\le 1,\quad\forall j\in\mathbf{T} \tag{28}$$

$$(\tilde{\mathscr{W}}^{(2)}_{i,0})^{\mathrm{T}}(\mathscr{P}^{(2)}_{i,0})^{-1}\tilde{\mathscr{W}}^{(2)}_{i,0}\le 1,\quad\forall i\in\mathbf{T} \tag{29}$$

are true.

2) Inductive step. Supposing that

$$(\tilde{\mathscr{W}}^{(1)}_{i,j+1})^{\mathrm{T}}(\mathscr{P}^{(1)}_{i,j+1})^{-1}\tilde{\mathscr{W}}^{(1)}_{i,j+1}\le 1,\quad\forall i,j\in\mathbf{T} \tag{30}$$

$$(\tilde{\mathscr{W}}^{(2)}_{i+1,j})^{\mathrm{T}}(\mathscr{P}^{(2)}_{i+1,j})^{-1}\tilde{\mathscr{W}}^{(2)}_{i+1,j}\le 1,\quad\forall i,j\in\mathbf{T} \tag{31}$$

are true, we are going to prove that the following inequalities are also true:

$$(\tilde{\mathscr{W}}^{(1)}_{i+1,j+1})^{\mathrm{T}}(\mathscr{P}^{(1)}_{i+1,j+1})^{-1}\tilde{\mathscr{W}}^{(1)}_{i+1,j+1}\le 1 \tag{32}$$

$$(\tilde{\mathscr{W}}^{(2)}_{i+1,j+1})^{\mathrm{T}}(\mathscr{P}^{(2)}_{i+1,j+1})^{-1}\tilde{\mathscr{W}}^{(2)}_{i+1,j+1}\le 1. \tag{33}$$

In fact, it follows from (30)–(31) that there exist $\varpi^{(1)}_{i,j+1}$ and $\varpi^{(2)}_{i+1,j}$ with $\|\varpi^{(1)}_{i,j+1}\|\le 1$ and $\|\varpi^{(2)}_{i+1,j}\|\le 1$ such that

$$\tilde{\mathscr{W}}^{(1)}_{i,j+1}=\Psi^{(1)}_{i,j+1}\varpi^{(1)}_{i,j+1} \tag{34}$$

$$\tilde{\mathscr{W}}^{(2)}_{i+1,j}=\Psi^{(2)}_{i+1,j}\varpi^{(2)}_{i+1,j} \tag{35}$$

where $\Psi^{(1)}_{i,j+1}$ and $\Psi^{(2)}_{i+1,j}$ are factorizations of $\mathscr{P}^{(1)}_{i,j+1}$ and $\mathscr{P}^{(2)}_{i+1,j}$, i.e., $\mathscr{P}^{(1)}_{i,j+1}=\Psi^{(1)}_{i,j+1}(\Psi^{(1)}_{i,j+1})^{\mathrm{T}}$ and $\mathscr{P}^{(2)}_{i+1,j}=\Psi^{(2)}_{i+1,j}(\Psi^{(2)}_{i+1,j})^{\mathrm{T}}$. In accordance with (17)–(18), one has

$$\begin{aligned}\tilde{\mathscr{W}}^{(1)}_{i+1,j+1}=&\,\tau^{(1,1)}_{i,j+1}\tilde{\mathscr{W}}^{(1)}_{i,j+1}-\tau^{(1,2)}_{i,j+1}\vec{\mathscr{C}}_{i,j+1}\vec{\Phi}_{i+1,j+1}\vec{\phi}^{(1)}_{i,j+1}\\&+\bar{\tau}^{(1,1)}_{i,j+1}\mathscr{W}^{(1)}\end{aligned} \tag{36}$$

$$\begin{aligned}\tilde{\mathscr{W}}^{(2)}_{i+1,j+1}=&\,\tau^{(2,1)}_{i+1,j}\tilde{\mathscr{W}}^{(2)}_{i+1,j}-\tau^{(2,2)}_{i+1,j}\vec{\mathscr{C}}_{i+1,j}\vec{\Phi}_{i+1,j+1}\vec{\phi}^{(2)}_{i+1,j}\\&+\bar{\tau}^{(2,1)}_{i+1,j}\mathscr{W}^{(2)}\end{aligned} \tag{37}$$

where, for $\kappa=1,2$

$$\begin{aligned}
&\vec{\phi}^{(\kappa)}_{i,j}\triangleq\mathrm{col}\big\{\check{\phi}^{(\kappa,1)}_{i,j},\check{\phi}^{(\kappa,2)}_{i,j},\ldots,\check{\phi}^{(\kappa,\vec{n})}_{i,j}\big\}\\
&\check{\phi}^{(\kappa,\iota)}_{i,j}\triangleq\mathrm{col}\big\{\hat{\phi}^{(\kappa,\iota)}_{i,j},\hat{\phi}^{(\kappa,\iota)}_{i,j},\ldots,\hat{\phi}^{(\kappa,\iota)}_{i,j}\big\}\\
&\mathscr{W}^{(\kappa)}\triangleq\begin{bmatrix}(W^{(\kappa,1)})^{\mathrm{T}} & (W^{(\kappa,2)})^{\mathrm{T}} & \cdots & (W^{(\kappa,\vec{n})})^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}
\end{aligned}$$

with $\hat{\phi}^{(\kappa,\iota)}_{i,j}$ being the ιth entry of $\hat{\phi}^{(\kappa)}_{i,j}$ and $W^{(\kappa,\iota)}$ being the ιth column vector of $W^{(\kappa)}$ $(\iota=1,2,\ldots,\vec{n})$.

By denoting

$$\eta^{(1)}_{i,j+1}\triangleq\begin{bmatrix}1 & (\varpi^{(1)}_{i,j+1})^{\mathrm{T}} & (\mathscr{W}^{(1)})^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$$

$$\eta^{(2)}_{i+1,j}\triangleq\begin{bmatrix}1 & (\varpi^{(2)}_{i+1,j})^{\mathrm{T}} & (\mathscr{W}^{(2)})^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}},$$

we have from (34)–(37) that

$$\tilde{\mathscr{W}}^{(1)}_{i+1,j+1}=\Pi^{(1)}_{i,j+1}\eta^{(1)}_{i,j+1} \tag{38}$$

$$\tilde{\mathscr{W}}^{(2)}_{i+1,j+1}=\Pi^{(2)}_{i+1,j}\eta^{(2)}_{i+1,j}. \tag{39}$$

On the other hand, according to Assumption 2 and (34)–(35), the following conditions are satisfied

$$\begin{cases}\|\varpi^{(1)}_{i,j+1}\|\le 1,\quad \|\varpi^{(2)}_{i+1,j}\|\le 1\\ \|\mathscr{W}^{(1)}\|\le\bar{W}^{(1)},\quad \|\mathscr{W}^{(2)}\|\le\bar{W}^{(2)}\end{cases} \tag{40}$$

which can be rearranged in terms of $\eta^{(1)}_{i,j+1}$ and $\eta^{(2)}_{i+1,j}$ as follows:

$$\begin{cases}(\eta^{(1)}_{i,j+1})^{\mathrm{T}}\Gamma_1\eta^{(1)}_{i,j+1}\le 0,\ (\eta^{(2)}_{i+1,j})^{\mathrm{T}}\Gamma_1\eta^{(2)}_{i+1,j}\le 0\\ (\eta^{(1)}_{i,j+1})^{\mathrm{T}}\Gamma^{(1)}_2\eta^{(1)}_{i,j+1}\le 0,\ (\eta^{(2)}_{i+1,j})^{\mathrm{T}}\Gamma^{(2)}_2\eta^{(2)}_{i+1,j}\le 0.\end{cases} \tag{41}$$

By applying Schur Complement Lemma [5], it follows from (26)–(27) that

$$-\vec{\Gamma}^{(1)}_{i,j+1}+(\Pi^{(1)}_{i,j+1})^{\mathrm{T}}(\mathscr{P}^{(1)}_{i+1,j+1})^{-1}\Pi^{(1)}_{i,j+1}\le 0 \tag{42}$$

$$-\vec{\Gamma}^{(2)}_{i+1,j}+(\Pi^{(2)}_{i+1,j})^{\mathrm{T}}(\mathscr{P}^{(2)}_{i+1,j+1})^{-1}\Pi^{(2)}_{i+1,j}\le 0. \tag{43}$$

By further utilizing $\mathcal{S}$-procedure [5], it can be derived from (38)–(39), (41)–(43) that

$$(\tilde{\mathscr{W}}^{(1)}_{i+1,j+1})^{\mathrm{T}}(\mathscr{P}^{(1)}_{i+1,j+1})^{-1}\tilde{\mathscr{W}}^{(1)}_{i+1,j+1}\le 1 \tag{44}$$

$$(\tilde{\mathscr{W}}^{(2)}_{i+1,j+1})^{\mathrm{T}}(\mathscr{P}^{(2)}_{i+1,j+1})^{-1}\tilde{\mathscr{W}}^{(2)}_{i+1,j+1}\le 1, \tag{45}$$

which means that the estimation errors $\tilde{W}^{(1)}_{i+1,j+1}$ and $\tilde{W}^{(2)}_{i+1,j+1}$ of neuron weights belong to the sets $\mathscr{F}^{(1)}_{i+1,j+1}$ and $\mathscr{F}^{(2)}_{i+1,j+1}$. Therefore, according to the principle of mathematical induction, the proof is now complete. ■

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B. NN-Based Set-Membership Estimator Design

To facilitate subsequent development, we introduce the following notations:

$$
\begin{aligned}
\Upsilon_{i,j}^{(0)} &\triangleq \mathrm{diag}\{1,\underbrace{0,0,\ldots,0}_{8\vec{n}+\hat{n}}\}\\
\Upsilon_{i,j}^{(1)} &\triangleq \mathrm{diag}\Big\{-1,I,\underbrace{0,0,\ldots,0}_{7\vec{n}+\hat{n}}\Big\}\\
\Upsilon_{i,j}^{(2)} &\triangleq \mathrm{diag}\Big\{-1,0,I,\underbrace{0,0,\ldots,0}_{6\vec{n}+\hat{n}}\Big\}\\
\Upsilon_{i,j}^{(3)} &\triangleq \mathrm{diag}\Big\{-1,0,0,I,\underbrace{0,0,\ldots,0}_{5\vec{n}+\hat{n}}\Big\}\\
\Upsilon_{i,j}^{(4)} &\triangleq \mathrm{diag}\Big\{-1,0,0,0,I,\underbrace{0,0,\ldots,0}_{4\vec{n}+\hat{n}}\Big\}\\
\Upsilon_{i,j}^{(5)} &\triangleq \mathrm{diag}\Big\{-\vec{\xi}^{(1)},\underbrace{0,0,\ldots,0}_{4\vec{n}},I,\underbrace{0,0,\ldots,0}_{3\vec{n}+\hat{n}}\Big\}\\
\Upsilon_{i,j}^{(6)} &\triangleq \mathrm{diag}\Big\{-\vec{\xi}^{(2)},\underbrace{0,0,\ldots,0}_{5\vec{n}},I,\underbrace{0,0,\ldots,0}_{2\vec{n}+\hat{n}}\Big\}\\
\Upsilon_{i,j}^{(7)} &\triangleq \mathrm{diag}\Big\{-\vec{\alpha}_{i,j+1},\underbrace{0,0,\ldots,0}_{6\vec{n}},I,\underbrace{0,0,\ldots,0}_{2\vec{n}+\hat{n}-n_y}\Big\}\\
\Upsilon_{i,j}^{(8)} &\triangleq \mathrm{diag}\Big\{-\vec{\alpha}_{i+1,j},\underbrace{0,0,\ldots,0}_{6\vec{n}+n_y},I,\underbrace{0,0,\ldots,0}_{2\vec{n}+\hat{n}-2n_y}\Big\}\\
\Upsilon_{i,j}^{(9)} &\triangleq \mathrm{diag}\Big\{-1,\underbrace{0,0,\cdots,0}_{6\vec{n}+2n_y},R_{i,j+1}^{-1},\underbrace{0,0,\ldots,0}_{n_w+2(\vec{n}+n_v)}\Big\}\\
\Upsilon_{i,j}^{(10)} &\triangleq \mathrm{diag}\Big\{-1,\underbrace{0,0,\cdots,0}_{6\vec{n}+2n_y+n_w},R_{i+1,j}^{-1},\underbrace{0,0,\ldots,0}_{2(\vec{n}+n_v)}\Big\}\\
\Upsilon_{i,j}^{(11)} &\triangleq \mathrm{diag}\Big\{-1,\underbrace{0,0,\cdots,0}_{6\vec{n}+2(n_y+n_w)},S_{i,j+1}^{-1},0,0,0\Big\}\\
\Upsilon_{i,j}^{(12)} &\triangleq \mathrm{diag}\Big\{-1,\underbrace{0,0,\cdots,0}_{6\vec{n}+\hat{n}-n_v},S_{i+1,j}^{-1},0,0\Big\}\\
\Upsilon_{i,j}^{(13)} &\triangleq \mathrm{diag}\Big\{0,-\vec{\Lambda}_{i,j+1}^{(1)},\underbrace{0,0,\cdots,0}_{5\vec{n}+\hat{n}},I,0\Big\}\\
\Upsilon_{i,j}^{(14)} &\triangleq \mathrm{diag}\Big\{0,0,-\vec{\Lambda}_{i,j+1}^{(2)},\underbrace{0,0,\cdots,0}_{5\vec{n}+\hat{n}},I\Big\}\\
\vec{\alpha}_{i,j} &\triangleq n_y\Big(\frac{\alpha_{i,j}\ell}{2}\Big)^2,\quad \vec{\Lambda}_{i,j}^{(\kappa)} \triangleq \|F^{(\kappa)}G\Lambda_{i,j}\| \ (\kappa=1,2).
\end{aligned}
$$

In order to calculate estimator gains $K_{i,j}^{(1)}$ and $K_{i,j}^{(2)}$ such that the estimation error $\theta_{i,j}$ is confined to the constrained set (23), we present the following theorem.

Theorem 2: Consider the nonlinear 2-D shift-varying system (1), the EDM (6)–(9), and the NN-based set-membership estimator (14). Let the sequences of matrices $\{Q_{0,j}\}_{j\in\mathbf{T}}$ and $\{Q_{i,0}\}_{i\in\mathbf{T}}$ be given. Then, the estimation error $\theta_{i+1,j+1}$ of state belongs to the ellipsoidal set $\mathscr{S}_{i+1,j+1}$ if there exist estimator gains $K_{i,j+1}^{(1)}$, $K_{i+1,j}^{(2)}$, positive scalars $\epsilon_{i,j}^{(m)}$ ($m=1,2,\ldots,14$) and a matrix $Q_{i+1,j+1}>0$ satisfying (26), (27) and

$$
\begin{bmatrix} -\vec{\Upsilon}_{i,j} & * \\ \Xi_{i,j} & -Q_{i+1,j+1} \end{bmatrix} \le 0,\quad \forall i,j\in\mathbf{T} \tag{46}
$$

where

$$
\begin{aligned}
\vec{\Upsilon}_{i,j} &\triangleq \Upsilon_{i,j}^{(0)} + \sum_{m=1}^{14}\epsilon_{i,j}^{(m)}\Upsilon_{i,j}^{(m)}\\
\Xi_{i,j} &\triangleq \begin{bmatrix}0 & \Xi_{i,j}^{(1)} & I & I & \Xi_{i,j}^{(2)} & \Xi_{i,j}^{(3)} & I & I\end{bmatrix}\\
\Xi_{i,j}^{(1)} &\triangleq \begin{bmatrix}\vec{\mathscr{A}}_{i,j+1}^{(1)}\Lambda_{i,j+1} & \vec{\mathscr{A}}_{i+1,j}^{(2)}\Lambda_{i+1,j} & \vec{\Psi}_{i,j+1}^{(1)} & \vec{\Psi}_{i+1,j}^{(2)}\end{bmatrix}\\
\Xi_{i,j}^{(2)} &\triangleq \begin{bmatrix}-K_{i,j+1}^{(1)} & -K_{i+1,j}^{(2)} & \mathscr{E}_{i,j+1}^{(1)} & \mathscr{E}_{i+1,j}^{(2)}\end{bmatrix}\\
\Xi_{i,j}^{(3)} &\triangleq \begin{bmatrix}-K_{i,j+1}^{(1)}D_{i,j+1} & -K_{i+1,j}^{(2)}D_{i+1,j}\end{bmatrix}\\
\vec{\mathscr{A}}_{i,j+1}^{(1)} &\triangleq (\mathscr{A}_{i,j+1}^{(1)} - K_{i,j+1}^{(1)}\mathscr{C}_{i,j+1})\\
\vec{\mathscr{A}}_{i+1,j}^{(2)} &\triangleq (\mathscr{A}_{i+1,j}^{(2)} - K_{i+1,j}^{(2)}\mathscr{C}_{i+1,j}).
\end{aligned}
$$

Proof: This theorem will be again proved by using the two-dimensional version of mathematical induction that consists of the following two steps.

1) Initial step. It is known immediately from Assumption 5 that

$$
\theta_{i,j}^{\mathrm{T}}Q_{i,j}^{-1}\theta_{i,j} \le 1 \tag{47}
$$

is true for $(i,j)\in\{(i,j)|i,j\in\mathbf{T}, i=0 \text{ or } j=0\}$.

2) Inductive step: Letting

$$
\theta_{i^\circ,j^\circ}^{\mathrm{T}}Q_{i^\circ,j^\circ}^{-1}\theta_{i^\circ,j^\circ} \le 1 \tag{48}
$$

be true for $(i^\circ,j^\circ)\in\{(i,j+1),(i+1,j)\}$, we need to show that

$$
\theta_{i+1,j+1}^{\mathrm{T}}Q_{i+1,j+1}^{-1}\theta_{i+1,j+1} \le 1 \tag{49}
$$

is also true.

For $(i^\circ,j^\circ)\in\{(i,j+1),(i+1,j)\}$, it is easy to verify from (48) that there exist $\varsigma_{i,j+1}$ and $\varsigma_{i+1,j}$ (with $\|\varsigma_{i,j+1}\|\le 1$ and $\|\varsigma_{i+1,j}\|\le 1$) such that

$$
\begin{cases} \theta_{i,j+1} = \Lambda_{i,j+1}\varsigma_{i,j+1} \\ \theta_{i+1,j} = \Lambda_{i+1,j}\varsigma_{i+1,j} \end{cases} \tag{50}
$$

where $\Lambda_{i,j+1}$ and $\Lambda_{i+1,j}$ are factorizations of $Q_{i,j+1}$ and $Q_{i+1,j}$, respectively, i.e., $Q_{i,j+1} = \Lambda_{i,j+1}\Lambda_{i,j+1}^{\mathrm{T}}$ and $Q_{i+1,j}=\Lambda_{i+1,j}\Lambda_{i+1,j}^{\mathrm{T}}$.

On the other hand, it follows from Theorem 1 that

$$
\begin{cases} (\tilde{W}_{i,j+1}^{(1)}\hat{\phi}_{i,j+1}^{(1)})^{\mathrm{T}}(P_{i,j+1}^{(1)})^{-1}(\tilde{W}_{i,j+1}^{(1)}\hat{\phi}_{i,j+1}^{(1)}) \le \breve{\phi}_{i,j+1}^{(1)} \\ (\tilde{W}_{i+1,j}^{(2)}\hat{\phi}_{i+1,j}^{(2)})^{\mathrm{T}}(P_{i+1,j}^{(2)})^{-1}(\tilde{W}_{i+1,j}^{(2)}\hat{\phi}_{i+1,j}^{(2)}) \le \breve{\phi}_{i+1,j}^{(2)} \end{cases} \tag{51}
$$

which means

$$
\begin{cases} (\tilde{W}_{i,j+1}^{(1)}\hat{\phi}_{i,j+1}^{(1)})(\tilde{W}_{i,j+1}^{(1)}\hat{\phi}_{i,j+1}^{(1)})^{\mathrm{T}} \le \vec{P}_{i,j+1}^{(1)} \\ (\tilde{W}_{i+1,j}^{(2)}\hat{\phi}_{i+1,j}^{(2)})(\tilde{W}_{i+1,j}^{(2)}\hat{\phi}_{i+1,j}^{(2)})^{\mathrm{T}} \le \vec{P}_{i+1,j}^{(2)} \end{cases} \tag{52}
$$

with

$$
\begin{aligned}
&\breve{\phi}_{i,j}^{(1)} \triangleq \frac{1}{n_\phi}(\hat{\phi}_{i,j}^{(1)})^{\mathrm{T}}(\hat{\phi}_{i,j}^{(1)}),\quad \breve{\phi}_{i,j}^{(2)} \triangleq \frac{1}{n_\phi}(\hat{\phi}_{i,j}^{(2)})^{\mathrm{T}}(\hat{\phi}_{i,j}^{(2)})\\
&\vec{P}_{i,j}^{(1)} \triangleq \breve{\phi}_{i,j}^{(1)}P_{i,j}^{(1)},\quad \vec{P}_{i,j}^{(2)} \triangleq \breve{\phi}_{i,j}^{(2)}P_{i,j}^{(2)}.
\end{aligned}
$$

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Similarly, from (52), there also exist $\vec{\varpi}^{(1)}_{i,j+1}$ and $\vec{\varpi}^{(2)}_{i+1,j}$ (with $\|\vec{\varpi}^{(1)}_{i,j+1}\| \leq 1$ and $\|\vec{\varpi}^{(2)}_{i+1,j}\| \leq 1$) such that

$$\begin{cases} \tilde{W}^{(1)}_{i,j+1}\hat{\phi}^{(1)}_{i,j+1} = \vec{\Psi}^{(1)}_{i,j+1}\vec{\varpi}^{(1)}_{i,j+1} \\ \tilde{W}^{(2)}_{i+1,j}\hat{\phi}^{(2)}_{i+1,j} = \vec{\Psi}^{(2)}_{i+1,j}\vec{\varpi}^{(2)}_{i+1,j} \end{cases} \tag{53}$$

where $\vec{\Psi}^{(1)}_{i,j+1}$ and $\vec{\Psi}^{(2)}_{i+1,j}$ are factorizations of $\vec{P}^{(1)}_{i,j+1}$ and $\vec{P}^{(2)}_{i+1,j}$, respectively, i.e., $\vec{P}^{(1)}_{i,j+1} = \vec{\Psi}^{(1)}_{i,j+1}(\vec{\Psi}^{(1)}_{i,j+1})^{\mathrm{T}}$ and $\vec{P}^{(2)}_{i+1,j} = \vec{\Psi}^{(2)}_{i+1,j}(\vec{\Psi}^{(2)}_{i+1,j})^{\mathrm{T}}$.

Letting

$$\vartheta_{i,j} \triangleq \begin{bmatrix} 1 & (\vartheta^{(1)}_{i,j})^{\mathrm{T}} & (\vartheta^{(2)}_{i,j})^{\mathrm{T}} & (\vartheta^{(3)}_{i,j})^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}},$$

in view of (50) and (53), we rewrite (19) as

$$\theta_{i+1,j+1} = \Xi_{i,j}\vartheta_{i,j} \tag{54}$$

where

$$\begin{aligned} \vartheta^{(1)}_{i,j} &\triangleq \mathrm{col}\{\varsigma_{i,j+1}, \varsigma_{i+1,j}, \vec{\varpi}^{(1)}_{i,j+1}, \vec{\varpi}^{(2)}_{i+1,j}\} \\ \vartheta^{(2)}_{i,j} &\triangleq \mathrm{col}\{\xi^{(1)}_{i,j+1}, \xi^{(2)}_{i+1,j}, e_{i,j+1}, e_{i+1,j}, w_{i,j+1}, w_{i+1,j}\} \\ \vartheta^{(3)}_{i,j} &\triangleq \mathrm{col}\{v_{i,j+1}, v_{i+1,j}, \tilde{f}^{(1)}_{i,j+1}, \tilde{f}^{(2)}_{i+1,j}\}. \end{aligned}$$

According to Assumption 3, (2), (3), (50) and (53), the following conditions are satisfied

$$\begin{cases} \|\varsigma_{i,j+1}\| \leq 1, \quad \|\varsigma_{i+1,j}\| \leq 1 \\ \|\vec{\varpi}^{(1)}_{i,j+1}\| \leq 1, \quad \|\vec{\varpi}^{(2)}_{i+1,j}\| \leq 1 \\ \|\xi^{(1)}_{i,j+1}\| \leq \bar{\xi}^{(1)}, \quad \|\xi^{(2)}_{i+1,j}\| \leq \bar{\xi}^{(2)} \\ \|e_{i,j+1}\| \leq \vec{\alpha}_{i,j+1}, \quad \|e_{i+1,j}\| \leq \vec{\alpha}_{i+1,j} \\ w^{\mathrm{T}}_{i,j+1}R^{-1}_{i,j+1}w_{i,j+1} \leq 1 \\ w^{\mathrm{T}}_{i+1,j}R^{-1}_{i+1,j}w_{i+1,j} \leq 1 \\ v^{\mathrm{T}}_{i,j+1}S^{-1}_{i,j+1}v_{i,j+1} \leq 1 \\ v^{\mathrm{T}}_{i+1,j}S^{-1}_{i+1,j}v_{i+1,j} \leq 1 \end{cases} \tag{55}$$

which, in terms of $\vartheta_{i,j}$, can be further rewritten as

$$\vartheta^{\mathrm{T}}_{i,j}\Upsilon^{(\bar{m})}_{i,j}\vartheta_{i,j} \leq 0, \quad \bar{m} = 1, 2, \ldots, 12 \tag{56}$$

with $\bar{\xi}^{(1)} \triangleq 2\bar{W}^{(1)}\bar{\phi}^{(1)} + \bar{\varepsilon}^{(1)}$ and $\bar{\xi}^{(2)} \triangleq 2\bar{W}^{(2)}\bar{\phi}^{(2)} + \bar{\varepsilon}^{(2)}$.

Next, we proceed to handle the nonlinear terms $\tilde{f}^{(1)}_{i,j+1}$ and $\tilde{f}^{(2)}_{i+1,j}$ in system (19). According to Assumption 1 and (50), we have

$$\begin{aligned} \|\tilde{f}^{(1)}_{i,j+1}\| =& \ \|\vec{f}^{(1)}_{i,j+1} - \hat{f}^{(1)}_{i,j+1}\| \\ =& \ \|f^{(1)}(G\vec{x}_{i,j+1}) - f^{(1)}(G\hat{x}_{i,j+1})\| \\ \leq& \ \|F^{(1)}G\theta_{i,j+1}\| \\ =& \ \|\vec{\Lambda}^{(1)}_{i,j+1}\varsigma_{i,j+1}\| \end{aligned} \tag{57}$$

$$\begin{aligned} \|\tilde{f}^{(2)}_{i+1,j}\| =& \ \|\vec{f}^{(2)}_{i+1,j} - \hat{f}^{(2)}_{i+1,j}\| \\ =& \ \|f^{(2)}(G\vec{x}_{i+1,j}) - f^{(2)}(G\hat{x}_{i+1,j})\| \\ \leq& \ \|F^{(2)}G\theta_{i+1,j}\| \\ =& \ \|\vec{\Lambda}^{(2)}_{i+1,j}\varsigma_{i+1,j}\| \end{aligned} \tag{58}$$

which, in terms of $\vartheta_{i,j}$, are expressed as

$$\vartheta^{\mathrm{T}}_{i,j}\Upsilon^{(13)}_{i,j}\vartheta_{i,j} \leq 0 \tag{59}$$

$$\vartheta^{\mathrm{T}}_{i,j}\Upsilon^{(14)}_{i,j}\vartheta_{i,j} \leq 0. \tag{60}$$

Subsequently, by applying Schur Complement Lemma [5], it follows from (46) that

$$-\vec{\Upsilon}_{i,j} + \Xi^{\mathrm{T}}_{i,j}Q^{-1}_{i+1,j+1}\Xi_{i,j} \leq 0. \tag{61}$$

By further resorting to $\mathcal{S}$-procedure [5], it can be derived from (54), (56), (59)–(61) that (49) is true for $(i+1, j+1)$, which means that the estimation error $\theta_{i+1,j+1}$ of state belongs to the ellipsoidal set $\mathscr{S}_{i+1,j+1}$. Therefore, according to the principle of mathematical induction, the proof is now complete. ■

C. Optimization Problem

Theorems 1–2 outline principles of seeking tuning scalars in NN weight tuning laws and NN-based set-membership estimator gains. It should be noted that neither of the schemes provides an optimal solution. In what follows, a corollary is presented to determine the tuning scalars and the estimator gains via optimizing the constraint sets in the sense of matrix trace.

Corollary 1: Consider the nonlinear 2-D shift-varying system (1), the EDM (6)–(9), and the NN-based set-membership estimator (14). Let the sequences of matrices $\{P^{(1)}_{0,j}\}_{j\in\mathbf{T}}$, $\{P^{(2)}_{i,0}\}_{j\in\mathbf{T}}$, $\{Q_{0,j}\}_{j\in\mathbf{T}}$ and $\{Q_{i,0}\}_{i\in\mathbf{T}}$ be given. The constraint sets $\mathscr{F}^{(1)}_{i+1,j+1}$, $\mathscr{F}^{(2)}_{i+1,j+1}$ and $\mathscr{S}_{i+1,j+1}$ are minimized in the sense of matrix trace if there exist tuning scalars $\tau^{(1,1)}_{i,j+1}$, $\tau^{(1,2)}_{i,j+1}$, $\tau^{(2,1)}_{i+1,j}$, $\tau^{(2,2)}_{i+1,j}$ and estimator gains $K^{(1)}_{i,j+1}$, $K^{(2)}_{i+1,j}$ such that the following optimization problem (**OP**) is feasible:

$$\mathbf{OP}: \min_{\substack{\tau^{(1,1)}_{i,j+1}, \tau^{(1,2)}_{i,j+1}, \tau^{(2,1)}_{i+1,j}, \\ \tau^{(2,2)}_{i+1,j}, K^{(1)}_{i,j+1}, K^{(2)}_{i+1,j}}} \mathrm{tr}\,(\mathscr{M}_{i+1,j+1}) \quad \text{subject to } (26), (27), (46) \tag{62}$$

where $\mathscr{M}_{i,j} \triangleq \varpi_1 P^{(1)}_{i,j} + \varpi_2 P^{(2)}_{i,j} + \varpi_3 Q_{i,j}$ and ϖ_1, ϖ_2, ϖ_3 are positive scalars satisfying $\sum_{\nu=1}^{3}\varpi_\nu = 1$.

For the purpose of numerical calculation, we describe the estimator design procedure in Algorithm 1, which is based on the recursive linear matrix inequality (RLMI) approach.

Algorithm 1: NN-based SME algorithm

Input: System initial conditions $\hat{W}^{(1)}_{0,j}$, $\hat{W}^{(2)}_{i,0}$, $\vec{x}_{0,j}$, $\vec{x}_{i,0}$, $\hat{x}_{0,j}$, $\hat{x}_{i,0}$.
Output: $\tau^{(1,1)}_{i,j}$, $\tau^{(1,2)}_{i,j}$, $\tau^{(2,1)}_{i,j}$, $\tau^{(2,2)}_{i,j}$, $K^{(1)}_{i,j}$, $K^{(2)}_{i,j}$.

for i=1:$\mathcal{T}$ **do**
  for j=1:$\mathcal{T}$ **do**
    Compute the tuning parameters $\tau^{(1,1)}_{i,j}$, $\tau^{(1,2)}_{i,j}$, $\tau^{(2,1)}_{i,j}$, $\tau^{(2,2)}_{i,j}$ and the estimator gains $K^{(1)}_{i,j}$, $K^{(2)}_{i,j}$ by solving the **OP** from Corollary 1;
    Compute the estimated state by (14);
    Update the NN weights by (15)–(16);
return $\tau^{(1,1)}_{i,j}$, $\tau^{(1,2)}_{i,j}$, $\tau^{(2,1)}_{i,j}$, $\tau^{(2,2)}_{i,j}$, $K^{(1)}_{i,j}$, $K^{(2)}_{i,j}$;

Remark 5: So far, the SFE problem has been solved for the addressed nonlinear 2-D shift-varying system. Note that, in comparison to the rich body of existing literature on fault estimation problems, our results exhibit the following distinguishing features: 1) the addressed SFE problem is new that represents one of the first few attempts to cope with nonlinear 2-D shift-varying systems with EDMs and UBB noises by utilizing the NN-based SME algorithm; 2) the proposed EDM is new, which is designed under the 2-D framework and is capable of dealing with dynamics evolving along both horizontal and vertical coordinates; and 3) the two-dimensional version of the mathematical induction method is utilized to examine the feasibility of the developed NN-based SME algorithm that confines the estimation error to an optimized ellipsoidal set.

Remark 6: This paper launches a systematic investigation on the SFE problem for a class of nonlinear 2-D shift-varying systems in the context of networked systems with certain engineering-oriented complexities (i.e., EDMs, UFD, and UBB noises). By exploiting a common consideration for several up-to-date approaches such as set-membership estimation method, two-dimensional version of the mathematical induction approach, and NN approximation method, the addressed problem has been thoroughly examined and the desired parameters (i.e., NN tuning scalars and estimator gains) have been formulated in terms of the solutions to a set of optimization problems. Within the established framework, it is not difficult to extend our results to more general systems with more complicated dynamics with more complex network-induced phenomena.

IV. Illustrative Example

In this section, the effectiveness of the proposed NN-based SME algorithm is verified by the Darboux equation which can model several industrial processes such as water stream heating and gas absorption. In practical applications, the 2-D systems are inevitably suffering from environmental chances (e.g., temperature fluctuations and harmonic vibration), and accordingly, the parameters of the 2-D systems may be affected to some extent. The system parameters are taken from [36] as

$$A_{i,j}^{(1)} = \begin{bmatrix} 0.3 & -0.1\sin(i+j) \\ 0.2 & 0.1 \end{bmatrix}$$
$$A_{i,j}^{(2)} = \begin{bmatrix} 0.1+0.15\mathrm{e}^{-3i} & 0 \\ 0.2 & 0.2 \end{bmatrix}$$
$$B_{i,j} = \begin{bmatrix} 0.2 \\ 0.05+0.1\cos(2i) \end{bmatrix}$$
$$C_{i,j} = \begin{bmatrix} 1 & 0.5+0.15\sin(i+j) \end{bmatrix},\ D_{i,j} = 0.3-0.1\mathrm{e}^{-2i}$$
$$E_{i,j}^{(1)} = \begin{bmatrix} 0.3 \\ 0.25\mathrm{e}^{-3i} \end{bmatrix}, \qquad E_{i,j}^{(2)} = \begin{bmatrix} 0.1+0.1\sin(i) \\ 0.35 \end{bmatrix}$$
$$U_{i,j}^{(1)} = 0.1+0.15\mathrm{e}^{-3i}, \qquad U_{i,j}^{(2)} = 0.1+0.05\cos(j)$$
$$f^{(1)}(x_{i,j}) = 0.1\begin{bmatrix} |x_{i,j}^{(1)}| & |x_{i,j}^{(2)}| \end{bmatrix}^{\mathrm{T}}$$
$$f^{(2)}(x_{i,j}) = 0.15\begin{bmatrix} |x_{i,j}^{(1)}| & |x_{i,j}^{(2)}| \end{bmatrix}^{\mathrm{T}}$$
$$\mu^{(1)}(\rho_{i,j}) = 0.25\cos(\rho_{i,j}),\ \mu^{(2)}(\rho_{i,j}) = 0.25\sin(\rho_{i,j}).$$

TABLE III
Tuning scalars $\tau_{i,j}^{(1,1)}$

i \ j ($\tau_{i,j}^{(1,1)}$)	1	2	$\cdots$	20
1	0.2732	0.2651	$\cdots$	0.2066
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
20	0.2891	0.2901	$\cdots$	0.2103

TABLE IV
Tuning scalars $\tau_{i,j}^{(1,2)}$

i \ j ($\tau_{i,j}^{(1,2)}$)	1	2	$\cdots$	20
1	0.5011	0.4986	$\cdots$	0.4306
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
20	0.5161	0.5007	$\cdots$	0.4138

The process noise $w_{i,j}$ and the measurement noise $v_{i,j}$ are selected as $w_{i,j} = 0.5\sin(0.1(i+j))$ and $v_{i,j} = 0.3\cos(0.1(i+2j))$, whose weighting matrices are chosen as $R_{i,j} = 0.3I$ and $S_{i,j} = 0.2I$. The scaling parameter is selected as $\alpha_{i,j} = 0.8$ and $\daleth_{i,j}^{(1)} = \daleth_{i,j}^{(2)} = 0.5I$. The activation function vectors are constructed as

$$\phi^{(\kappa)}(\vec{x}_{i,j}) = 0.3\begin{bmatrix} \tanh(\vec{x}_{i,j}^{(1)}) & \tanh(\vec{x}_{i,j}^{(2)}) & \tanh(\vec{x}_{i,j}^{(3)}) \end{bmatrix}^{\mathrm{T}}.$$

Let the initial conditions be given as

$$\begin{cases} \hat{W}_{0,j}^{(1)} = 1.5I, & \forall j \in [0\ 10] \\ \hat{W}_{0,j}^{(1)} = 0, & \forall j \in [11\ 20] \\ \hat{W}_{i,0}^{(2)} = 2I, & \forall i \in [0\ 10] \\ \hat{W}_{i,0}^{(2)} = 0, & \forall i \in [11\ 20]. \end{cases}$$
$$\begin{cases} \vec{x}_{0,j} = \vec{x}_{i,0} = \begin{bmatrix} 1 & 1.4 & 0.5 \end{bmatrix}^{\mathrm{T}}, & \forall i,j \in [0\ 10] \\ \vec{x}_{0,j} = \vec{x}_{i,0} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}, & \forall i,j \in [11\ 20] \\ \hat{x}_{0,j} = \hat{x}_{i,0} = \begin{bmatrix} 0.9 & 1 & 0.3 \end{bmatrix}^{\mathrm{T}}, & \forall i,j \in [0\ 10] \\ \hat{x}_{0,j} = \hat{x}_{i,0} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}, & \forall i,j \in [11\ 20]. \end{cases}$$

According to Algorithm 1, the estimator gains and the tuning scalars can be calculated recursively as listed in Tables I–VI (only partial results are shown here due to limited space). The simulation results are presented in Figs. 2–9. Among them, Figs. 2–7 plot the system state $x_{i,j}$, the system fault $\rho_{i,j}$, and the estimation error $\theta_{i,j}$, which show the effectiveness of the proposed NN-based SME algorithm. Figs. 8–9 depict the evolution of $\|\hat{W}_{i,j}^{(1)}\|$ and $\|\hat{W}_{i,j}^{(2)}\|$ according to the designed adaptive tuning laws, respectively.

V. Conclusion

This paper has addressed the SFE problem for a class of nonlinear 2-D shift-varying systems subject to additive faults and UBB noises. A new EDM has been designed for 2-D systems, where the zooming-in/out-based encoder and decoder have been utilized to improve the communication efficiency. An NN-based SME algorithm has been developed for 2-D systems to confine the estimation error to an optimized

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TABLE I
ESTIMATOR GAINS $K_{i,j}^{(1)}$

$K_{i,j}^{(1)}$ j / i	1	2	$\cdots$	20
1	$[0.2282\ 0.2049\ 0.1139]^{\mathrm{T}}$	$[0.2701\ 0.1965\ 0.1311]^{\mathrm{T}}$	$\cdots$	$[0.3074\ 0.2144\ 0.0157]^{\mathrm{T}}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
20	$[0.2081\ 0.1534\ 0.1401]^{\mathrm{T}}$	$[0.1903\ 0.1407\ 0.1228]^{\mathrm{T}}$	$\cdots$	$[0.2597\ 0.1891\ 0.0224]^{\mathrm{T}}$

TABLE II
ESTIMATOR GAINS $K_{i,j}^{(2)}$

$K_{i,j}^{(2)}$ j / i	1	2	$\cdots$	20
1	$[0.1508\ 0.2566\ 0.0998]^{\mathrm{T}}$	$[0.1622\ 0.3014\ 0.1327]^{\mathrm{T}}$	$\cdots$	$[0.1004\ 0.2589\ 0.1199]^{\mathrm{T}}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
20	$[0.0901\ 0.2442\ 0.1194]^{\mathrm{T}}$	$[0.1197\ 0.2865\ 0.1272]^{\mathrm{T}}$	$\cdots$	$[0.1061\ 0.2485\ 0.0831]^{\mathrm{T}}$

TABLE V
TUNING SCALARS $\tau_{i,j}^{(2,1)}$

$\tau_{i,j}^{(2,1)}$ j / i	1	2	$\cdots$	20
1	0.2465	0.2507	$\cdots$	0.2726
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
20	0.1387	0.1514	$\cdots$	0.2955

TABLE VI
TUNING SCALARS $\tau_{i,j}^{(2,2)}$

$\tau_{i,j}^{(2,2)}$ j / i	1	2	$\cdots$	20
1	0.3726	0.4265	$\cdots$	0.4716
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
20	0.4597	0.4013	$\cdots$	0.4026

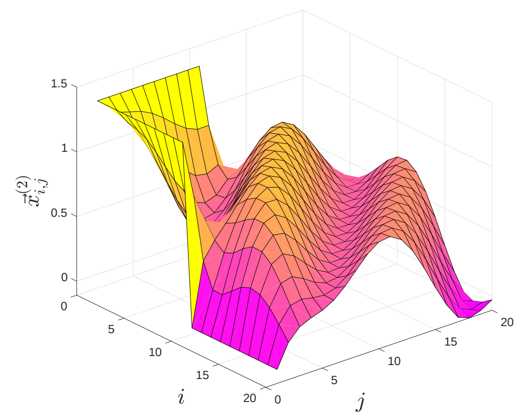

Fig. 3. The system state $x^{(2)}$.

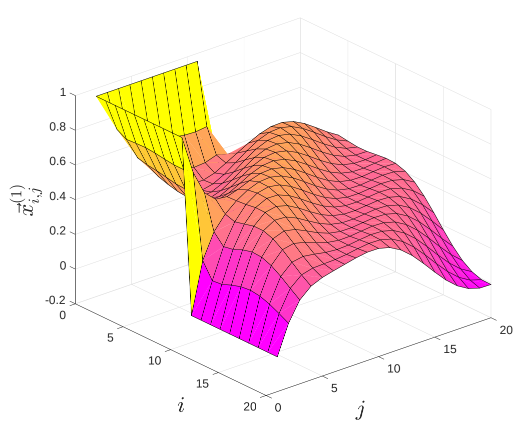

Fig. 2. The system state $x^{(1)}$.

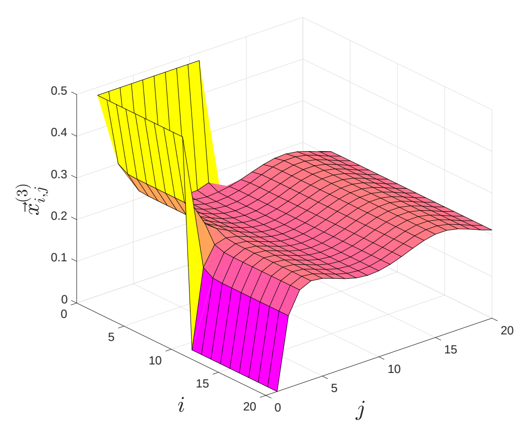

Fig. 4. The system fault ρ.

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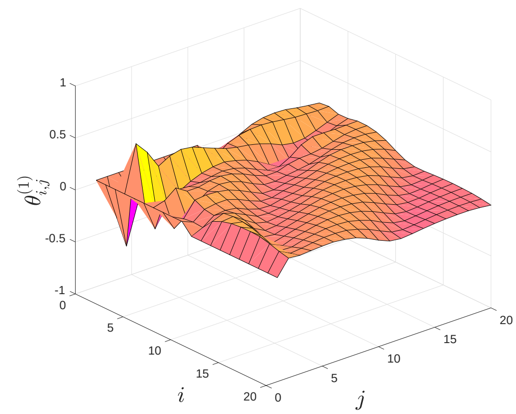

Fig. 5. The estimation error $\theta^{(1)}$.

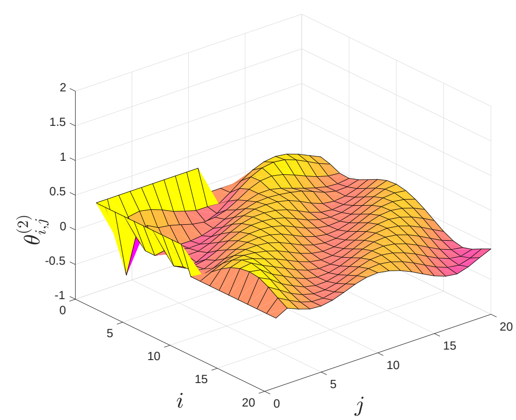

Fig. 6. The estimation error $\theta^{(2)}$.

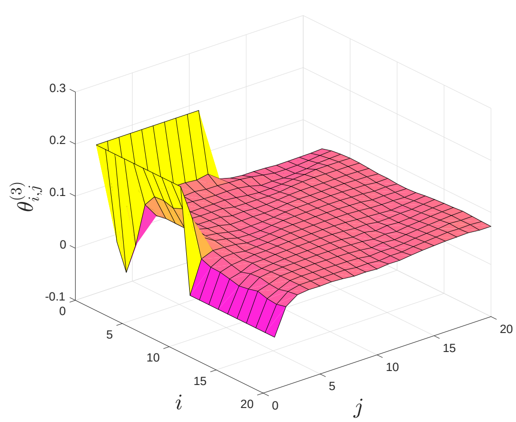

Fig. 7. The estimation error $\theta^{(3)}$.

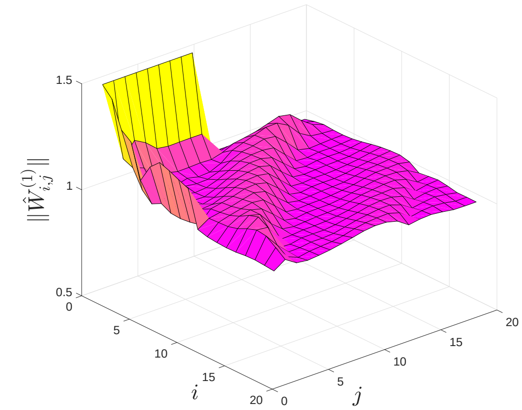

Fig. 8. The weight matrix $\hat{W}^{(1)}$.

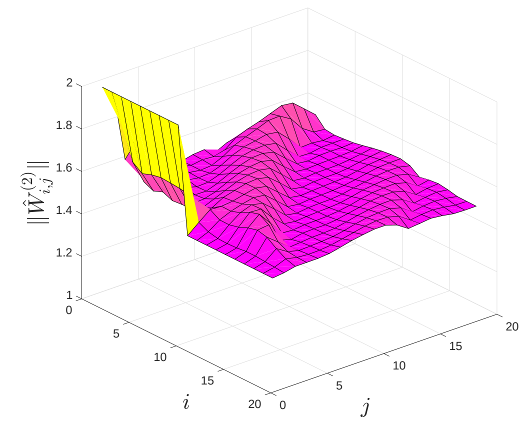

Fig. 9. The weight matrix $\hat{W}^{(2)}$.

ellipsoidal set. By utilizing the two-dimensional mathematical induction approach, the feasibility of the proposed estimation algorithm has been examined, and the desired gains can be computed by solving a series of optimization problems. Finally, a simulation example has been provided to verify the usefulness of the proposed estimator design method. Further research topics include the extension of the main results to more complex systems, such as fuzzy systems [12], [40], positive systems [11] and complex networks [8], [30].

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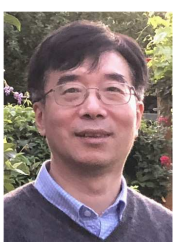

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