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Event-Triggered Recursive State Estimation for Stochastic Complex Dynamical Networks under Hybrid Attacks

Yun Chen, Xueyang Meng, Zidong Wang and Hongli Dong

***Abstract*—In this paper, the event-based recursive state estimation problem is investigated for a class of stochastic complex dynamical networks under cyber-attacks. A hybrid cyber-attack model is introduced to take into account both the randomly occurring deception attack and the randomly occurring denial-of-service attack. For the sake of reducing the transmission rate and mitigating the network burden, the event-triggered mechanism is employed under which the measurement output is transmitted to the estimator only when a pre-set condition is satisfied. An upper bound on the estimation error covariance on each node is first derived through solving two coupled Riccati-like difference equations. Then, the desired estimator gain matrix is recursively acquired that minimizes such an upper bound. By means of the stochastic analysis theory, the estimation error is proved to be stochastically bounded with probability 1. Finally, an illustrative example is provided to verify the effectiveness of the developed estimator design method.**

***Index Terms*—Stochastic complex dynamical networks, recursive state estimation, hybrid cyber-attacks, event-triggered protocol, stochastic boundedness.**

I. Introduction

In the past decades, with the ever-increasing research interest on complex network theory and its applications, fruitful results have been published on complex networks [10], [35]. Generally speaking, a typical complex network is composed of a great amount of nodes and edges which represent the individuals and the coupling relationship among individuals, respectively. Due to the fact that the complex network can be used to characterize many real-world networks such as electrical power grids, communication networks, social networks, neural networks and biological networks [1], [7], [31], [49], special attention has been paid on the dynamics analysis of the complex dynamical networks (CDNs) to explore the evolution law of the CDNs. Recently, a great number of research results have been reported on the state estimation (SE), pinning control and synchronization for CDNs [5], [22], [27], [37], [40], [41], [43]. It is worth mentioning that the SE problem for CDNs, which serves as a key role in understanding the network dynamics and fulfilling certain engineering requirements, is a fundamental issue stirring persistent research attention.

Different from the isolated systems, the CDNs exhibit some distinctive features (e.g. large scale and strong coupling) that make the SE problem complicated and difficult. With hope to handle the SE problem for CDNs, some effective methodologies have been developed in literature, see [11], [17], [18], [23], [44]. For example, in [23], [44], the Lyapunov stability theory and the state augmentation technique have been used to deal with the state estimator design issue for CDNs. For nonlinear stochastic CDNs, the extended Kalman filtering (EKF) has been put forward to tackle the covariance-constrained SE problem in [11]. It should be stressed that the estimation algorithms developed in above-mentioned literature are based on the state augmentation technique. Nevertheless, such an augmentation technique brings in certain drawbacks, i.e., the computation complexity would be greatly increased and the coupling relationship among the nodes is required to be known. To avoid these two drawbacks, in [17], [18], a non-augmentation method has been proposed with which the recursive SE for stochastic CDNs with switching topology has been studied. The non-augmentation method proposed in [17], [18] successfully avoids the increase of the computational complexity stemming from augmentation [11].

During the data transmission through the shared communication channels, some network-induced phenomena (e.g. transmission delays, data collision and packet dropout) would inevitably occur due mainly to the limited network bandwidths [3], [9], [11], [24]. In the past few decades, to reduce the consumption of the limited network resources, various communication protocols have been introduced in the estimation/control for large-scale networked systems [8], [19], [24], [32], [42], [50] including complex networks [4], [45], [51], multi-agent systems [36], and sensor networks [38]. From the perspective of reducing the data transmission rate, it has been proven that the event-triggered (ET) protocol is an effective protocol under which the data transmission is permitted only if a prescribed condition is met [15]. Recently, the SE issue for CDNs under the ET protocol has been studied in [12], [30], [34]. Nevertheless, a thorough literature search shows that inadequate attention has been paid on the covariance-constrained recursive SE problem for stochastic CDNs under the ET protocol, which constitutes the first motivation of this

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Yun Chen and Xueyang Meng are with the School of Automation, Hangzhou Dianzi University, Hangzhou 310018, China. (Email: `yunchen@hdu.edu.cn`)

Zidong Wang is with the Department of Computer Science, Brunel University London, Uxbridge, Middlesex, UB8 3PH, United Kingdom. (Email: `Zidong.Wang@brunel.ac.uk`)

Hongli Dong is with the Artificial Intelligence Energy Research Institute, Northeast Petroleum University, Daqing 163318, China; and is also with the Heilongjiang Provincial Key Laboratory of Networking and Intelligent Control, Northeast Petroleum University, Daqing 163318, China. (Email: `shiningdhl@vip.126.com`)

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paper.

In the networked environment, the cyber-attacks occur frequently which pose a serious threat on the network security and the system performance [2], [6], [25], [46]. The widely investigated cyber-attacks include deception attack, denial-of-service (DoS) attack, and replay attack [20], [21], [39]. It is worth noting that, due to the existence of the firewall software, the cyber-attacks cannot always be success, which leads to the randomly occurring cyber-attacks. Up to now, special effort has been devoted to the SE problem under the randomly occurring cyber-attacks and plenty of research results have been available [6], [26], [29]. In the existing literature, it is often the case that only a *single* type of attack has been considered. Nevertheless, in real practice, the attacks are often hybrid. That is, different types of attacks occur alternatively with certain probabilities [20], [21], [39]. Such a hybrid attack can effectively enhance the success possibility of the attack. Note that, the system considered is only an isolated system in most of the existing literature concerning SE problem under the hybrid attacks. When it comes to the CDNs, the relevant results are relatively few, not to mention the simultaneous consideration of the ET protocol. Therefore, another motivation of this paper is to shorten such a gap.

In the stochastic control domain, the mean-square stability and the stability in probability [14], [22], [33] are arguably two representative metrics in the system analysis. In particular, the mean-square stability, as one of most extensively used stability concepts, has obtained persistent research attention due primarily to the close relation with the quadratic Lyapunov function. For example, in [12], the mean-square exponential boundedness analysis of the estimation error has been conducted for stochastic CDNs with randomly occurring sensor delays and random coupling strengths under the ET strategy. In comparison with the mean-square stability, the performance analysis for stochastic systems in probability could provide a milder perspective to characterize system dynamics entering into a bounded domain in probability. Therefore, in this paper, we are going to investigate the boundedness of the estimation error in probability in the SE problem for stochastic CDNs.

To respond to the above discussions, in this paper, the event-based recursive SE issue is considered for a class of stochastic CDNs under hybrid attacks. The difficulties encountered in this paper include: 1) the establishment of a hybrid cyber-attack model to account for the joint influences of random DoS attack and random deception attack; 2) the reduction of the computational complexity resulting from the inversion operation of high-dimensional matrices and the computation of cross-covariance matrices among coupled nodes; 3) the development of an algorithm to calculate the estimator gain matrix (EGM) such that a certain upper bound on the estimation error covariance (EEC) is minimized; and 4) the consideration of the stochastic boundedness of estimation error in the probability sense.

Corresponding to the above-mentioned difficulties, the main contributions of this paper can be outlined in the following three aspects: 1) a unified cyber-attack model is proposed to take into account the joint impact from the randomly occurring hybrid attack which covers the randomly occurring DoS attack and the randomly occurring deception attack as special cases; 2) a non-augmentation technique is applied to circumvent the calculation of the cross-covariance matrices; and 3) by recursively solving two coupled Riccati-like difference equations (RLDEs), the EGM is obtained that minimizes the upper bound on the EEC, and the stochastic boundedness with probability 1 of the estimation error is analyzed.

The remainder of this paper is organized as follows. In Section II, the event-based recursive SE issue is formulated for stochastic CDNs under the hybrid attacks. The main results are given in Section III and the boundedness analysis of the estimation error is conducted in Section IV. A numerical example is given in Section V and the conclusion is drawn in Section VI.

Notation The notations in this paper are standard. For a matrix X, $X>0$ means that X is positive-definite. A^{-1} denotes the inverse of the matrix A and A^T denotes the transpose of the matrix A. tr$\{A\}$ represents the trace of the matrix A. A block-diagonal matrix with diagonal elements $N_1, N_2, \cdots, N_n$ is represented by diag$\{N_1, N_2, \cdots, N_n\}$. $\|x\|$ is the Euclidean norm of a vector x. The probability of the event x is denoted as **Pr**$\{x\}$. $\mathbb{E}\{x\}$ refers to the expectation of a stochastic variable x.

II. Problem Formulation and Preliminaries

Consider the following stochastic CDN with N nodes:

$$\begin{cases} x_{i,z+1} = f(x_{i,z}) + \sum_{j=1}^{N} \omega_{ij}\Gamma x_{j,z} + B_{i,z}w_{i,z} \\ y_{i,z} = s(x_{i,z}) + D_{i,z}v_{i,z} \end{cases} \tag{1}$$

where $x_{i,z} \in \mathrm{R}^n$ $(i = 1, 2, \ldots, N)$ and $y_{i,z} \in \mathrm{R}^m$ denote the state vector and the measurement output of node i, respectively. $f(\cdot) : \mathrm{R}^n \mapsto \mathrm{R}^n$ and $s(\cdot) : \mathrm{R}^n \mapsto \mathrm{R}^m$ are known nonlinear functions. $\Gamma \triangleq \mathrm{diag}\{\gamma_1, \gamma_2, \ldots, \gamma_n\}$ denotes the inner coupling matrix with $\gamma_j \geq 0$ $(j = 1, 2, \ldots, n)$. $W \triangleq [\omega_{ij}]_{N\times N}$ represents the coupling configuration matrix, where ω_{ij} is positive if the node j links with the node i. $w_{i,z}$ and $v_{i,z}$ are mutually independent zero-mean Gaussian white noises with covariances $Q_{i,z} > 0$ and $R_{i,z} > 0$, respectively. $B_{i,z}$ and $D_{i,z}$ are known real matrices with compatible dimensions.

In this paper, it is assumed that the nonlinear functions $f(\cdot) : \mathrm{R}^n \mapsto \mathrm{R}^n$ and $s(\cdot) : \mathrm{R}^n \mapsto \mathrm{R}^m$ are continuously differentiable and satisfy the following Lipschitz conditions:

$$\|f(x) - f(y)\| \leq a_1\|x - y\| \tag{2}$$

$$\|s(x) - s(y)\| \leq b_1\|x - y\| \tag{3}$$

where a_1 and b_1 are known positive scalars.

In practical engineering, from the resource-saving perspective, it is suggested to reduce the communication frequency while preserving a satisfactory system performance. Therefore, in this paper, the ET strategy is employed in the sensor-to-estimator channel. The event-triggering condition on the node i is designed as follows:

$$\varsigma(y_{i,z}, \varpi_{i,z}) \triangleq u_{i,z}^T u_{i,z} - \varpi_{i,z} y_{i,z}^T y_{i,z} > 0 \tag{4}$$

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where $u_{i,z} \triangleq y_{i,s_t^i} - y_{i,z}$ with y_{i,s_t^i} being the latest transmitted measurement. $\varpi_{i,z} > 0$ is a known time-varying scalar.

With the event-triggering condition, the event-triggering instants on node i are determined by

$$s_{t+1}^i \triangleq \inf\{z|\varsigma(y_{i,z}, \varpi_{i,z}) > 0, z > s_t^i\}. \tag{5}$$

On the node i, a zero-order holder is equipped to keep the latest transmitted measurement y_{i,s_t^i} when the event-triggering condition is not satisfied. Hence, the measurement output transmitted from the node i to the estimator i is described as follows:

$$\bar{y}_{i,z} \triangleq y_{i,s_t^i}, z \in \{s_t^i, s_t^i + 1, \ldots, s_{t+1}^i - 1\}.$$

Remark 1: Up to now, in most existing results related to the recursive SE problem for stochastic CDNs, the measurement signal is assumed to be periodically transmitted to the estimator [11], [12], [17], [18], [29], [44], [45]. Nevertheless, for a large-scale CDN, frequent data transmissions would heavily increase the network loads and lead to a waste of the resources. Hence, in this paper, the ET strategy is applied to reduce unnecessary information transmissions [20], [26], [30], [34], [51]. More specifically, it is known from (4) and (5) that the measurement output of node i is transmitted to the estimator if and only if the condition (4) is satisfied. Consequently, the information transmission is reduced and the resource of the communication network is saved.

It is notable that, although the prevailing network-based communication mechanism greatly facilitates the data exchanges among the components, the potential data safety issue is arisen since the data may subject to malicious cyber-attacks during the transmission. For the purpose of characterizing the cyber-attack as close to the reality as possible, the measurement received by the estimator is modeled as follows

$$\tilde{y}_{i,z} = c_{i,z}(\bar{y}_{i,z} + d_{i,z}\varrho_{i,z}) \tag{6}$$

where $c_{i,z}$ and $d_{i,z}$ are mutually independent Bernoulli distributed variables satisfying the following statistical properties:

$$\mathbb{E}\{c_{i,z}\} = \mathbf{Pr}\{c_{i,z} = 1\} = \bar{c}_i$$
$$\mathbb{E}\{d_{i,z}\} = \mathbf{Pr}\{d_{i,z} = 1\} = \bar{d}_i$$

with $0 \le \bar{c}_i \le 1$ and $0 \le \bar{d}_i \le 1$ being positive scalars.

In (6), $\varrho_{i,z} \triangleq -\bar{y}_{i,z} + \ell_{i,z}$ stands for the deceptive attack signal injected by the hostile attacker, where $\ell_{i,z}$ is a bounded signal with $\ell_{i,z}^T \ell_{i,z} \le \bar{\ell}_i$ and $\bar{\ell}_i$ being a given positive scalar.

Remark 2: It should be mentioned that the measurement model (6) is on the basis of the following engineering insights. 1) From the defenders' viewpoint, most of the engineering systems are equipped with anti-virus software to intercept cyber-attacks. Thus, the attacks cannot always be implemented successfully but occur in a random way; 2) From the attackers' viewpoint, the attack behavior should be complicated enough (e.g. switching the attack behavior among various attack mechanisms) to evade the attack detection and increase the successful ratio. Moreover, the model (6) is quite general that includes the measurement under the randomly occurring DoS attack and the randomly occurring deception attack as special cases. To be more specific, the model (6) is reduced to the measurement model under the randomly occurring DoS attack model when $c_{i,z} = 0$ and the randomly occurring deception attack model when $c_{i,z} = d_{i,z} = 1$. In particular, when $c_{i,z} = 1$ and $d_{i,z} = 0$, the measurement output is successfully transmitted to the estimator without encountering any cyber-attacks.

Remark 3: By setting $d_{i,z} = 0$, (6) reduces to a model for the random DoS attack that has the similar effect as the probabilistic packet loss phenomenon [3], [13], [15], [44], [45], [47]. Actually, both the random DoS attack and the probabilistic packet loss would lead to the measurement missing problems in a probabilistic way. It should be noted that, however, the causes of DoS attack and packet loss are completely different. DoS attack is actively launched by the malicious cyber attackers during communication transmission. On the other hand, the passive impact of packet loss results generally from the conflict between the large amount of transmission data and the limited networks resources.

Based on the received measurement signal $\tilde{y}_{i,z}$, the following state estimator is constructed for the node i at instant $z \in [s_t^i, s_{t+1}^i - 1]$:

$$\begin{aligned}\hat{x}_{i,z+1|z} &= f(\hat{x}_{i,z|z}) + \sum_{j=1}^{N} \omega_{ij}\Gamma\hat{x}_{j,z|z} \\ \hat{x}_{i,z+1|z+1} &= \hat{x}_{i,z+1|z} + K_{i,z+1}[\tilde{y}_{i,z+1} - \bar{c}_i(1-\bar{d}_i)s(\hat{x}_{i,z+1|z})]\end{aligned} \tag{7}$$

where $\hat{x}_{i,z+1|z}$ denotes the one-step prediction of $x_{i,z}$, $\hat{x}_{i,z|z}$ denotes the estimate of $x_{i,z}$ with initial value $\hat{x}_{i,0|0}$, and $K_{i,z+1}$ is the EGM to be determined.

Denote $\tilde{x}_{i,z+1|z} \triangleq x_{i,z+1} - \hat{x}_{i,z+1|z}$ and $\tilde{x}_{i,z+1|z+1} \triangleq x_{i,z+1} - \hat{x}_{i,z+1|z+1}$ as the prediction error and the estimation error, respectively. The prediction error covariance (PEC) and EEC are defined as $P_{i,z+1|z+1} \triangleq \mathbb{E}\{\tilde{x}_{i,z+1|z+1}\tilde{x}_{i,z+1|z+1}^T\}$ and $P_{i,z+1|z} \triangleq \mathbb{E}\{\tilde{x}_{i,z+1|z}\tilde{x}_{i,z+1|z}^T\}$, respectively.

In this paper, the main objective is to develop the state estimator (7) for the stochastic CDN (1) under the ET strategy and the cyber-attacks such that:

- at each time instant z, the EEC has an upper bound $\Phi_{i,z+1|z+1}$, namely, there exists a time-varying positive definite matrix $\Phi_{i,z+1|z+1}$ satisfying

$$P_{i,z+1|z+1} \le \Phi_{i,z+1|z+1};$$

- the upper bound $\Phi_{i,z+1|z+1}$ is minimized at each instant $z+1$ by the desired EGM $K_{i,z+1}$.

To proceed further, several useful lemmas are introduced.

Lemma 1: For any vectors $z_1, z_2 \in \mathrm{R}^n$, $z_1 z_2^T + z_2 z_1^T \le \sigma z_1 z_1^T + \sigma^{-1} z_2 z_2^T$ holds for any scalar $\sigma > 0$.

Lemma 2: [11] The compatibly dimensional matrices B, J, F and G are given with $GG^T \le I$. For a positive definite matrix Y and a positive scalar $\alpha > 0$, if $\alpha^{-1}I - FYF^T > 0$, then one has

$$\begin{aligned}&(B + JGF)Y(B + JGF)^T \\ &\quad \le B(Y^{-1} - \alpha FF^T)^{-1}B^T + \alpha^{-1}JJ^T.\end{aligned}$$

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Lemma 3: [11] For any compatibly dimensional matrices O, P, Z and Q, the following equations hold

$$\frac{\partial \text{tr}(OZP)}{\partial Z}=O^TP^T,\ \frac{\partial \text{tr}(OZ^TP)}{\partial Z}=PO$$
$$\frac{\partial \text{tr}((OZP)Q(OZP)^T)}{\partial Z}=2O^TOZPQP^T.$$

III. Estimator Design

In this section, we are going to solve the design issue of the state estimator (7). First, we will derive upper bounds on the PEC and EEC for each node i. Then, the EGM will be derived which minimizes the upper bound on EEC.

According to (1) and (7), the prediction error $\tilde{x}_{i,z+1|z}$ on node i is calculated as

$$\begin{aligned}\tilde{x}_{i,z+1|z}=&f(x_{i,z})-f(\hat{x}_{i,z|z})\\&+\sum_{j=1}^{N}\omega_{ij}\Gamma(x_{j,z}-\hat{x}_{j,z|z})+B_{i,z}w_{i,z}.\end{aligned}\tag{8}$$

Similarly, the estimation error $\tilde{x}_{i,z+1|z+1}$ is computed as

$$\tilde{x}_{i,z+1|z+1}=\tilde{x}_{i,z+1|z}-K_{i,z+1}[\tilde{y}_{i,z+1}-\bar{c}_i(1-\bar{d}_i)s(\hat{x}_{i,z+1|z})].\tag{9}$$

Applying the Taylor series expansion method to the nonlinear function $f(x_{i,z})$ around $\hat{x}_{i,z|z}$ yields

$$f(x_{i,z})=f(\hat{x}_{i,z|z})+F_{i,z}\tilde{x}_{i,z|z}+o(|\tilde{x}_{i,z|z}|)\tag{10}$$

where $F_{i,z}\triangleq\frac{\partial f(x_{i,z})}{\partial x_{i,z}}|_{x_{i,z}=\hat{x}_{i,z|z}}$ is the coefficient matrix and $o(|\tilde{x}_{i,z|z}|)$ denotes the resulted high-order term. In addition, $o(|\tilde{x}_{i,z|z}|)$ is further represented as

$$o(|\tilde{x}_{i,z|z}|)=A_{i,z}C_{i,z}\tilde{x}_{i,z|z}\tag{11}$$

where $A_{i,z}$ denotes the problem-based scaling matrix, and the unknown time-varying matrix $C_{i,z}$ is used to describe the linearization error of the dynamical model that satisfies $C_{i,z}C_{i,z}^T\leq I$.

In view of (8), (10) and (11), we have

$$\begin{aligned}\tilde{x}_{i,z+1|z}=&(F_{i,z}+A_{i,z}C_{i,z})\tilde{x}_{i,z|z}+\sum_{j=1}^{N}\omega_{ij}\Gamma\tilde{x}_{j,z|z}\\&+B_{i,z}w_{i,z}.\end{aligned}\tag{12}$$

Following the similar line of the treatment for $f(x_{i,z})$, $s(x_{i,z+1})$ is rewritten as follows:

$$s(x_{i,z+1})=s(\hat{x}_{i,z+1|z})+S_{i,z+1}\tilde{x}_{i,z+1|z}+o(|\tilde{x}_{i,z+1|z}|)\tag{13}$$

where $S_{i,z+1}\triangleq\frac{\partial s(x_{i,z+1})}{\partial x_{i,z+1}}|_{x_{i,z+1}=\hat{x}_{i,z+1|z}}$ and

$$o(|\tilde{x}_{i,z+1|z}|)\triangleq G_{i,z+1}H_{i,z+1}\tilde{x}_{i,z+1|z}$$

with $G_{i,z+1}$ being the problem-based scaling matrix, and $H_{i,z+1}$ being an unknown time-varying matrix satisfying $H_{i,z+1}H_{i,z+1}^T\leq I$.

Taking (9) and (13) into consideration, one has

$$\begin{aligned}&\tilde{x}_{i,z+1|z+1}\\=&\tilde{x}_{i,z+1|z}-K_{i,z+1}[c_{i,z+1}(1-d_{i,z+1})u_{i,z+1}\\&+c_{i,z+1}(1-d_{i,z+1})s(x_{i,z+1})-\bar{c}_i(1-\bar{d}_i)s(\hat{x}_{i,z+1|z})\\&+c_{i,z+1}d_{i,z+1}\ell_{i,z+1}+c_{i,z+1}(1-d_{i,z+1})D_{i,z+1}v_{i,z+1}]\\=&\Re_{i,z+1}\tilde{x}_{i,z+1|z}-c_{i,z+1}(1-d_{i,z+1})K_{i,z+1}u_{i,z+1}\\&-[c_{i,z+1}(1-d_{i,z+1})-\bar{c}_i(1-\bar{d}_i)]K_{i,z+1}s(x_{i,z+1})\\&-c_{i,z+1}d_{i,z+1}K_{i,z+1}\ell_{i,z+1}\\&-c_{i,z+1}(1-d_{i,z+1})K_{i,z+1}D_{i,z+1}v_{i,z+1}\end{aligned}\tag{14}$$

where $\Re_{i,z+1}\triangleq I-\bar{c}_i(1-\bar{d}_i)K_{i,z+1}(S_{i,z+1}+G_{i,z+1}H_{i,z+1})$.

It is inferred from (12) that the PEC $P_{i,z+1|z}$ is calculated as

$$\begin{aligned}&P_{i,z+1|z}\\&=(F_{i,z}+A_{i,z}C_{i,z})P_{i,z|z}(F_{i,z}+A_{i,z}C_{i,z})^T\\&\quad+\sum_{j=1}^{N}\sum_{l=1}^{N}\omega_{ij}\omega_{il}\Gamma\mathbb{E}\{\tilde{x}_{j,z|z}\tilde{x}_{l,z|z}^T\}\Gamma^T+B_{i,z}Q_{i,z}B_{i,z}^T\\&\quad+\sum_{j=1}^{N}\omega_{ij}\mathbb{E}\{[(F_{i,z}+A_{i,z}C_{i,z})\tilde{x}_{i,z|z}\tilde{x}_{j,z|z}^T\Gamma^T\\&\quad+\Gamma\tilde{x}_{j,z|z}\tilde{x}_{i,z|z}^T(F_{i,z}+A_{i,z}C_{i,z})^T]\}.\end{aligned}\tag{15}$$

Moreover, it is evident from Lemma 1 that

$$\begin{aligned}&\sum_{j=1}^{N}\sum_{l=1}^{N}\omega_{ij}\omega_{il}\Gamma\mathbb{E}\{\tilde{x}_{j,z|z}\tilde{x}_{l,z|z}^T\}\Gamma^T\\&\quad=\frac{1}{2}\sum_{j=1}^{N}\sum_{l=1}^{N}\omega_{ij}\omega_{il}\Gamma\mathbb{E}\{\tilde{x}_{j,z|z}\tilde{x}_{l,z|z}^T+\tilde{x}_{l,z|z}\tilde{x}_{j,z|z}^T\}\Gamma^T\\&\quad\leq\bar{\omega}_i\sum_{j=1}^{N}\omega_{ij}\Gamma P_{j,z|z}\Gamma^T\end{aligned}$$

where $\bar{\omega}_i\triangleq\sum_{l=1}^{N}\omega_{il}$. Also, one derives

$$\begin{aligned}&\sum_{j=1}^{N}\omega_{ij}\mathbb{E}\{[(F_{i,z}+A_{i,z}C_{i,z})\tilde{x}_{i,z|z}\tilde{x}_{j,z|z}^T\Gamma^T\\&\qquad+\Gamma\tilde{x}_{j,z|z}\tilde{x}_{i,z|z}^T(F_{i,z}+A_{i,z}C_{i,z})^T]\}\\&\quad\leq\sum_{j=1}^{N}\omega_{ij}[(F_{i,z}+A_{i,z}C_{i,z})P_{i,z|z}(F_{i,z}+A_{i,z}C_{i,z})^T\\&\qquad+\Gamma P_{j,z|z}\Gamma^T]\\&\quad=\bar{\omega}_i[(F_{i,z}+A_{i,z}C_{i,z})P_{i,z|z}(F_{i,z}+A_{i,z}C_{i,z})^T]\\&\qquad+\sum_{j=1}^{N}\omega_{ij}\Gamma P_{j,z|z}\Gamma^T.\end{aligned}\tag{16}$$

It is concluded from (15)-(16) that

$$\begin{aligned}&P_{i,z+1|z}\\&\leq(1+\bar{\omega}_i)(F_{i,z}+A_{i,z}C_{i,z})P_{i,z|z}(F_{i,z}+A_{i,z}C_{i,z})^T\\&\quad+(1+\bar{\omega}_i)\sum_{j=1}^{N}\omega_{ij}\Gamma P_{j,z|z}\Gamma^T+B_{i,z}Q_{i,z}B_{i,z}^T.\end{aligned}\tag{17}$$

Next, in light of (14), the EEC $P_{i,z+1|z+1}$ is deduced as

$$\begin{aligned}&P_{i,z+1|z+1}\\&=\Re_{i,z+1}P_{i,z+1|z}\Re_{i,z+1}^T+\bar{c}_i(1-\bar{d}_i)K_{i,z+1}\mathbb{E}\{u_{i,z+1}\end{aligned}$$

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$$
\begin{aligned}
&\times u_{i,z+1}^T\}K_{i,z+1}^T + [\bar{c}_i(1-\bar{d}_i) - \bar{c}_i^2(1-\bar{d}_i)^2]K_{i,z+1} \\
&\times \mathbb{E}\{s(x_{i,z+1})s^T(x_{i,z+1})\}K_{i,z+1}^T + \bar{c}_i\bar{d}_iK_{i,z+1} \\
&\times \mathbb{E}\{\ell_{i,z+1}\ell_{i,z+1}^T\}K_{i,z+1}^T + \bar{c}_i(1-\bar{d}_i)K_{i,z+1}D_{i,z+1} \\
&\times R_{i,z+1}D_{i,z+1}^TK_{i,z+1}^T + \Im_{i1,z+1} + \Im_{i1,z+1}^T \\
&+ \Im_{i2,z+1} + \Im_{i2,z+1}^T + \Im_{i3,z+1} + \Im_{i3,z+1}^T \\
&+ \Im_{i4,z+1} + \Im_{i4,z+1}^T + \Im_{i5,z+1} + \Im_{i5,z+1}^T
\end{aligned}
$$

where

$$
\begin{aligned}
\Im_{i1,z+1} \triangleq& -\bar{c}_i(1-\bar{d}_i)\mathbb{E}\{\Re_{i,z+1}\tilde{x}_{i,z+1|z}u_{i,z+1}^TK_{i,z+1}^T\} \\
\Im_{i2,z+1} \triangleq& -\bar{c}_i\bar{d}_i\mathbb{E}\{\Re_{i,z+1}\tilde{x}_{i,z+1}\ell_{i,z+1}^TK_{i,z+1}^T\} \\
\Im_{i3,z+1} \triangleq& -\bar{c}_i^2\bar{d}_i(1-\bar{d}_i)\mathbb{E}\{K_{i,z+1}s(x_{i,z+1})\ell_{i,z+1}^TK_{i,z+1}^T\} \\
\Im_{i4,z+1} \triangleq& [\bar{c}_i(1-\bar{d}_i) - \bar{c}_i^2(1-\bar{d}_i)^2] \\
&\times \mathbb{E}\{K_{i,z+1}s(x_{i,z+1})u_{i,z+1}^TK_{i,z+1}^T\} \\
\Im_{i5,z+1} \triangleq& \bar{c}_i(1-\bar{d}_i)\mathbb{E}\{K_{i,z+1}u_{i,z+1}v_{i,z+1}^TD_{i,z+1}^TK_{i,z+1}^T\}.
\end{aligned}
$$

Taking the nonlinear constraint (3) into account, it is readily seen that

$$
\begin{aligned}
&\mathbb{E}\{s(x_{i,z+1})s^T(x_{i,z+1})\} \\
\le&\mathbb{E}\{\|s(x_{i,z+1})\|^2\}I \\
\le&\mathbb{E}\{b_1^2\|x_{i,z+1}\|^2\}I \\
=&b_1^2\mathrm{tr}\{\mathbb{E}\{x_{i,z+1}x_{i,z+1}^T\}\}I.
\end{aligned}
$$

By using the elementary inequality in Lemma 1, the upper bound on $x_{i,z+1}x_{i,z+1}^T$ is calculated as

$$
\begin{aligned}
&x_{i,z+1}x_{i,z+1}^T \\
=&(\tilde{x}_{i,z+1|z}+\hat{x}_{i,z+1|z})(\tilde{x}_{i,z+1|z}+\hat{x}_{i,z+1|z})^T \\
\le&(1+\sigma_1)\tilde{x}_{i,z+1|z}\tilde{x}_{i,z+1|z}^T \\
&+ (1+\sigma_1^{-1})\hat{x}_{i,z+1|z}\hat{x}_{i,z+1|z}^T
\end{aligned}
$$

where σ_1 is a positive scalar. Then, we have

$$
\begin{aligned}
&\mathbb{E}\{s(x_{i,z+1})s^T(x_{i,z+1})\} \\
\le&b_1^2\mathrm{tr}\{(1+\sigma_1)P_{i,z+1|z}+(1+\sigma_1^{-1})\hat{x}_{i,z+1|z}\hat{x}_{i,z+1|z}^T\}I \\
\triangleq&\Xi_{i1,z+1}I.
\end{aligned}
$$

Recalling the ET mechanism (4) and the definition of $u_{i,z}$, for any $z \in [s_t^i, s_{t+1}^i - 1]$, one has

$$
\begin{aligned}
&\mathbb{E}\{u_{i,z+1}u_{i,z+1}^T\} \\
\le&\mathbb{E}\{u_{i,z+1}^Tu_{i,z+1}\}I \\
\le&\varpi_{i,z}\mathbb{E}\{y_{i,z+1}^Ty_{i,z+1}\}I \\
\le&\varpi_{i,z}(\Xi_{i1,z+1} + \mathbb{E}\{v_{i,z+1}^TD_{i,z+1}^TD_{i,z+1}v_{i,z+1}\})I.
\end{aligned} \tag{18}
$$

Moreover, noting the fact that $\mathrm{tr}(\Gamma_1\Gamma_2) = \mathrm{tr}(\Gamma_2\Gamma_1)$ with Γ_1 and Γ_2 being matrices with compatible dimensions, one obtains

$$
\begin{aligned}
&\mathbb{E}\{v_{i,z+1}^TD_{i,z+1}^TD_{i,z+1}v_{i,z+1}\} \\
=&\mathbb{E}\{\mathrm{tr}\{v_{i,z+1}^TD_{i,z+1}^TD_{i,z+1}v_{i,z+1}\}\} \\
=&\mathbb{E}\{\mathrm{tr}\{D_{i,z+1}v_{i,z+1}v_{i,z+1}^TD_{i,z+1}^T\}\} \\
=&\mathrm{tr}\{D_{i,z+1}R_{i,z+1}D_{i,z+1}^T\}.
\end{aligned} \tag{19}
$$

It is observed from (18) and (19) that

$$
\mathbb{E}\{u_{i,z+1}u_{i,z+1}^T\} \le \varpi_{i,z}\Delta_{i,z+1}I
$$

where $\Delta_{i,z+1} \triangleq \Xi_{i1,z+1} + \mathrm{tr}\{D_{i,z+1}R_{i,z+1}D_{i,z+1}^T\}$.

With the help of Lemma 1, the following inequalities hold:

$$
\begin{aligned}
&\Im_{i1,z+1} + \Im_{i1,z+1}^T \\
\le&\bar{c}_i(1-\bar{d}_i)(\sigma_2\Re_{i,z+1}P_{i,z+1|z}\Re_{i,z+1}^T \\
&+ \sigma_2^{-1}\varpi_{i,z}\Delta_{i,z+1}K_{i,z+1}K_{i,z+1}^T), \\
&\Im_{i2,z+1} + \Im_{i2,z+1}^T \\
\le&\bar{c}_i\bar{d}_i(\sigma_3\Re_{i,z+1}P_{i,z+1|z}\Re_{i,z+1}^T \\
&+ \sigma_3^{-1}K_{i,z+1}\mathbb{E}\{\ell_{i,z+1}\ell_{i,z+1}^T\}K_{i,z+1}^T) \\
\le&\bar{c}_i\bar{d}_i(\sigma_3\Re_{i,z+1}P_{i,z+1|z}\Re_{i,z+1}^T + \sigma_3^{-1}K_{i,z+1}\bar{\ell}_iK_{i,z+1}^T), \\
&\Im_{i3,z+1} + \Im_{i3,z+1}^T \\
\le&\bar{c}_i^2\bar{d}_i(1-\bar{d}_i)K_{i,z+1}(\sigma_4\mathbb{E}\{s(x_{i,z+1})s^T(x_{i,z+1})\} \\
&+ \sigma_4^{-1}\mathbb{E}\{\ell_{i,z+1}\ell_{i,z+1}^T\})K_{i,z+1}^T \\
\le&\bar{c}_i^2\bar{d}_i(1-\bar{d}_i)K_{i,z+1}(\sigma_4\Xi_{i1,z+1} + \sigma_4^{-1}\bar{\ell}_i)K_{i,z+1}^T, \\
&\Im_{i4,z+1} + \Im_{i4,z+1}^T \\
\le&[\bar{c}_i(1-\bar{d}_i) - \bar{c}_i^2(1-\bar{d}_i)^2]K_{i,z+1}(\sigma_5\mathbb{E}\{s(x_{i,z+1}) \\
&\times s^T(x_{i,z+1})\} + \sigma_5^{-1}\varpi_{i,z}\Delta_{i,z+1})K_{i,z+1}^T
\end{aligned}
$$

where σ_i $(i = 2, 3, 4, 5)$ are given positive scalars.

Note that $(y_{i,s_t^i+1} - y_{i,z+1})v_{i,z+1}^T = 0$ holds for $z = s_t^i$. Moreover, for $z \ne s_t^i$, it is known that

$$
\begin{aligned}
&\mathbb{E}\{K_{i,z+1}u_{i,z+1}v_{i,z+1}^TD_{i,z+1}^TK_{i,z+1}^T\} \\
=&\mathbb{E}\{K_{i,z+1}(y_{i,s_t^i+1} - y_{i,z+1})v_{i,z+1}^TD_{i,z+1}^TK_{i,z+1}^T\} \\
=&\mathbb{E}\{-K_{i,z+1}[s(x_{i,z+1}) + D_{i,z+1}v_{i,z+1}]v_{i,z+1}^TD_{i,z+1}^TK_{i,z+1}^T\} \\
=&\mathbb{E}\{-K_{i,z+1}D_{i,z+1}v_{i,z+1}v_{i,z+1}^TD_{i,z+1}^TK_{i,z+1}^T\} \\
=&-K_{i,z+1}D_{i,z+1}R_{i,z+1}D_{i,z+1}^TK_{i,z+1}^T.
\end{aligned}
$$

Therefore, it is concluded that

$$
\begin{aligned}
&\Im_{i5,z+1} + \Im_{i5,z+1}^T \\
=&-2\bar{c}_i(1-\bar{d}_i)K_{i,z+1}D_{i,z+1}R_{i,z+1}D_{i,z+1}^TK_{i,z+1}^T \le 0.
\end{aligned}
$$

In combination of the above discussions, the upper bound on the EEC $P_{i,z+1|z+1}$ is calculated as follows:

$$
\begin{aligned}
&P_{i,z+1|z+1} \\
\le&n_1\Re_{i,z+1}P_{i,z+1|z}\Re_{i,z+1}^T + n_2K_{i,z+1}\Xi_{i1,z+1}K_{i,z+1}^T \\
&+ n_3\varpi_{i,z}\Delta_{i,z+1}K_{i,z+1}K_{i,z+1}^T + n_4\bar{\ell}_iK_{i,z+1}K_{i,z+1}^T \\
&+ n_5K_{i,z+1}D_{i,z+1}R_{i,z+1}D_{i,z+1}^TK_{i,z+1}^T
\end{aligned} \tag{20}
$$

where

$$
\begin{aligned}
n_1 &\triangleq 1 + \bar{c}_i(1-\bar{d}_i)\sigma_2 + \bar{c}_i\bar{d}_i\sigma_3 \\
n_2 &\triangleq \bar{c}_i(1-\bar{d}_i)[(1-\bar{c}_i(1-\bar{d}_i))(1+\sigma_5) + \bar{c}_i\bar{d}_i\sigma_4] \\
n_3 &\triangleq \bar{c}_i(1-\bar{d}_i)[1 + \sigma_2^{-1} + (1-\bar{c}_i(1-\bar{d}_i))\sigma_5^{-1}] \\
n_4 &\triangleq \bar{c}_i\bar{d}_i[1 + \sigma_3^{-1} + \bar{c}_i(1-\bar{d}_i)\sigma_4^{-1}] \\
n_5 &\triangleq \bar{c}_i(1-\bar{d}_i).
\end{aligned}
$$

In the following theorem, an upper bound on $P_{i,z+1|z+1}$ is presented and the EGM minimized such an upper bound is given.

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Theorem 1: For given positive scalars $\rho_{i,z}$, $\mu_{i,z}$ and $0 < \sigma_6 < 1/N$, if the following two coupled RLDEs:

$$\begin{aligned}&\Phi_{i,z+1|z}\\ \triangleq&(1+\bar{\omega}_i)[F_{i,z}(\Phi^{-1}_{i,z|z}-(1+\sigma_6)\rho_{i,z}I)^{-1}F^T_{i,z}\\ &+\sigma_6^{-1}\rho^{-1}_{i,z}A_{i,z}A^T_{i,z}]+B_{i,z}Q_{i,z}B^T_{i,z}\\ &+(1+\bar{\omega}_i)\sum_{j=1}^{N}\omega_{ij}\Gamma\Phi_{j,z|z}\Gamma^T\end{aligned} \tag{21}$$

and

$$\begin{aligned}&\Phi_{i,z+1|z+1}\\ \triangleq&n_1[(I-\bar{c}_i(1-\bar{d}_i)K_{i,z+1}S_{i,z+1})(\Phi^{-1}_{i,z+1|z}\\ &-(1+\sigma_6)\mu_{i,z+1}I)^{-1}(I-\bar{c}_i(1-\bar{d}_i)K_{i,z+1}S_{i,z+1})^T\\ &+\sigma_6^{-1}\mu^{-1}_{i,z+1}\bar{c}_i^2(1-\bar{d}_i)^2K_{i,z+1}G_{i,z+1}G^T_{i,z+1}K^T_{i,z+1}]\\ &+n_2K_{i,z+1}\Xi_{i1,z+1}K^T_{i,z+1}\\ &+n_3\varpi_{i,z}\Delta_{i,z+1}K_{i,z+1}K^T_{i,z+1}+n_4\bar{\ell}_iK_{i,z+1}K^T_{i,z+1}\\ &+n_5K_{i,z+1}D_{i,z+1}R_{i,z+1}D^T_{i,z+1}K^T_{i,z+1}\end{aligned} \tag{22}$$

with initial condition $P_{i,0|0} \leq \Phi_{i,0|0}$ have positive definite solutions $\Phi_{i,z+1|z}$ and $\Phi_{i,z+1|z+1}$ such that

$$\Phi^{-1}_{i,z|z} > (1+\sigma_6)\rho_{i,z}I \tag{23}$$

and

$$\Phi^{-1}_{i,z+1|z} > (1+\sigma_6)\mu_{i,z+1}I \tag{24}$$

hold, then $\Phi_{i,z+1|z+1}$ is an upper bound on $P_{i,z+1|z+1}$. Moreover, the EGM $K_{i,z+1}$ that minimizes such an upper bound is determined by

$$\begin{aligned}&K_{i,z+1}\\ =&n_1\bar{c}_i(1-\bar{d}_i)[\Phi^{-1}_{i,z+1|z}-(1+\sigma_6)\mu_{i,z+1}I]^{-1}S^T_{i,z+1}\\ &\times[n_1\bar{c}_i(1-\bar{d}_i)S_{i,z+1}(\Phi^{-1}_{i,z+1|z}-(1+\sigma_6)\mu_{i,z+1}I)^{-1}S^T_{i,z+1}\\ &+n_1\bar{c}_i^2(1-\bar{d}_i)^2\sigma_6^{-1}\mu^{-1}_{i,z+1}G_{i,z+1}G^T_{i,z+1}+n_2\Xi_{i1,z+1}I\\ &+n_3\varpi_{i,z}\Delta_{i,z+1}I+n_4\bar{\ell}_iI+n_5D_{i,z+1}R_{i,z+1}D^T_{i,z+1}]^{-1}.\end{aligned} \tag{25}$$

Proof: It follows from Lemma 2 that

$$\begin{aligned}&(F_{i,z}+A_{i,z}C_{i,z})P_{i,z|z}(F_{i,z}+A_{i,z}C_{i,z})^T\\ \leq&F_{i,z}[P^{-1}_{i,z|z}-(1+\sigma_6)\rho_{i,z}I]^{-1}F^T_{i,z}+(1+\sigma_6)^{-1}\rho^{-1}_{i,z}A_{i,z}A^T_{i,z}\\ \leq&F_{i,z}[P^{-1}_{i,z|z}-(1+\sigma_6)\rho_{i,z}I]^{-1}F^T_{i,z}+\sigma_6^{-1}\rho^{-1}_{i,z}A_{i,z}A^T_{i,z}.\end{aligned} \tag{26}$$

Then, using Lemma 3 in [13] and substituting (26) into (17), we have

$$P_{i,z+1|z} \leq \Phi_{i,z+1|z}$$

where $\Phi_{i,z+1|z}$ is given in (21). Furthermore, it is obvious that

$$\begin{aligned}&\Re_{i,z+1}P_{i,z+1|z}\Re^T_{i,z+1}\\ \leq&[I-\bar{c}_i(1-\bar{d}_i)K_{i,z+1}S_{i,z+1}][P^{-1}_{i,z+1|z}-(1+\sigma_6)\mu_{i,z+1}I]^{-1}\\ &[I-\bar{c}_i(1-\bar{d}_i)K_{i,z+1}S_{i,z+1}]^T\\ &+(1+\sigma_6)^{-1}\mu^{-1}_{i,z+1}\bar{c}_i^2(1-\bar{d}_i)^2K_{i,z+1}G_{i,z+1}G^T_{i,z+1}K^T_{i,z+1}\\ \leq&[I-\bar{c}_i(1-\bar{d}_i)K_{i,z+1}S_{i,z+1}][P^{-1}_{i,z+1|z}-(1+\sigma_6)\mu_{i,z+1}I]^{-1}\\ &[I-\bar{c}_i(1-\bar{d}_i)K_{i,z+1}S_{i,z+1}]^T\\ &+\sigma_6^{-1}\mu^{-1}_{i,z+1}\bar{c}_i^2(1-\bar{d}_i)^2K_{i,z+1}G_{i,z+1}G^T_{i,z+1}K^T_{i,z+1}.\end{aligned} \tag{27}$$

Then, it is inferred from (20) and (27) that

$$P_{i,z+1|z+1} \leq \Phi_{i,z+1|z+1}$$

where $\Phi_{i,z+1|z+1}$ is defined in (22).

In terms of Lemma 3, taking the partial derivative of the trace of $\Phi_{i,z+1|z+1}$ with respect to $K_{i,z+1}$ results in

$$\begin{aligned}&\frac{\partial\mathrm{tr}(\Phi_{i,z+1|z+1})}{\partial K_{i,z+1}}\\ =&-2n_1\bar{c}_i(1-\bar{d}_i)[\Phi^{-1}_{i,z+1|z}-(1+\sigma_6)\mu_{i,z+1}I]^{-1}S^T_{i,z+1}\\ &+2n_1\bar{c}_i(1-\bar{d}_i)K_{i,z+1}S_{i,z+1}\\ &\times[\Phi^{-1}_{i,z+1|z}-(1+\sigma_6)\mu_{i,z+1}I]^{-1}S^T_{i,z+1}\\ &+2n_1\bar{c}_i^2(1-\bar{d}_i)^2\sigma_6^{-1}\mu^{-1}_{i,z+1}K_{i,z+1}G_{i,z+1}G^T_{i,z+1}\\ &+2n_2K_{i,z+1}\Xi_{i1,z+1|z}I+2n_3K_{i,z+1}\varpi_{i,z}\Delta_{i,z+1}I\\ &+2n_4K_{i,z+1}\bar{\ell}_iI+2n_5K_{i,z+1}D_{i,z+1}R_{i,z+1}D^T_{i,z+1}.\end{aligned}$$

Letting $\frac{\partial\mathrm{tr}(\Phi_{i,z+1|z+1})}{\partial K_{i,z+1}}=0$, the EGM $K_{i,z+1}$ can be calculated by (25). The proof of this theorem is complete. ■

Remark 4: It is known from the third term of the right-hand side in (22) that the ET mechanism has a major influence on the estimation performance. More specifically, a small threshold $\varpi_{i,z}$ would lead to a small upper bound on the EEC. Conversely, a large $\varpi_{i,z}$ leads to a slow data transmission frequency and a large upper bound on the EEC. Thus, it is of vital importance to choose a suitable threshold to achieve a proper balance between the estimation performance and the resource consumption.

Remark 5: It is well recognized that the augmentation technique serves as an effective tool for the analysis and synthesis of CDNs [11], [45]. Nevertheless, the augmentation technique would unavoidably bring in the inversion operation of high-dimensional matrices and the computation of cross-covariance matrices among coupled nodes. Therefore, inspired by [12], [17], [18], the non-augmentation method is adopted in this paper to handle the event-based recursive SE problem for stochastic CDNs with hybrid attacks, and the difficulties caused by the augmentation technique are avoided.

IV. Boundedness Analysis

In this section, we will analyze the boundedness of the estimation error. First, some preliminaries are presented to facilitate the subsequent boundedness analysis.

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Definition 1: For given positive scalars $\epsilon \leq 1$ and $\theta \leq 1$, if there exists a positive scalar $\varrho(\epsilon,\theta)$ with $\|\tilde{x}_{0|0}\| < \varrho(\epsilon,\theta)$, where $\|\tilde{x}_{0|0}\|$ is the known initial condition, satisfying

$$\mathbf{Pr}\{\|\tilde{x}_{z|z}\| < \epsilon\} \geq 1-\theta,$$

then the estimation error $\tilde{x}_{z|z}$ is said to be stochastically bounded in probability $1-\theta$. Furthermore, if there exists a scalar $\varrho > 0$ with $\|\tilde{x}_{0|0}\| < \varrho$ satisfying

$$\mathbf{Pr}\{\lim_{z\to\infty} \|\tilde{x}_{z|z}\| < \epsilon\} = 1,$$

then $\tilde{x}_{z|z}$ is stochastically bounded in probability 1.

Lemma 4: (Chebyshev inequality [14]) For an arbitrary random variable $\aleph > 0$ with the mean $\mathbb{E}\{\aleph\}$, one has

$$\mathbf{Pr}\{\aleph \geq \sigma\} \leq \frac{\mathbb{E}\{\aleph\}}{\sigma}$$

where σ is a positive scalar.

Assumption 1: The upper bounds of PEC and EEC satisfy $\Phi^{-1}_{i,z+1|z} \leq \frac{1+N}{\lambda}\mu_{i,z}I$ and $\Phi^{-1}_{i,z|z} \leq \frac{1+N}{\lambda}\rho_{i,z}I$, where $N < \lambda < \frac{1+N}{1+\sigma_6}$ and λ is a positive scalar.

Assumption 2: There exist positive scalars ϑ and ξ satisfying $\vartheta \leq \rho_{i,z} \leq \xi$ and $\vartheta \leq \mu_{i,z} \leq \xi$.

It is easily known from Assumptions 1-2 and (23)-(24) that the lower bounds and upper bounds of matrices $\Phi_{i,z+1|z}$ and $\Phi_{i,z+1|z+1}$ are guaranteed.

Theorem 2: For given $\tilde{x}_{i,0|0}$, the estimation error $\tilde{x}_{i,z|z}$ of the CDN (1) under the state estimator (7) is bounded in probability 1.

Proof: Denote $\tilde{x}_{z|z} \triangleq [\tilde{x}^T_{1,z|z}, \tilde{x}^T_{2,z|z}, \ldots, \tilde{x}^T_{N,z|z}]^T$ and choose the Lyapunov function as

$$U_z(\tilde{x}_{z|z}) = \sum_{i=1}^{N} \tilde{x}^T_{i,z|z}\Phi^{-1}_{i,z|z}\tilde{x}_{i,z|z} \tag{28}$$

where $\Phi_{i,z|z}$ is an upper bound on the EEC.

It follows from Assumptions 1-2 and (24) that

$$\lambda_1\mathbb{E}\{\|\tilde{x}_{z|z}\|^2\} \leq \mathbb{E}\{U_z(\tilde{x}_{z|z})\} \leq \lambda_2\mathbb{E}\{\|\tilde{x}_{z|z}\|^2\} \tag{29}$$

where $\lambda_1 \triangleq \vartheta(1+\sigma_6)$ and $\lambda_2 \triangleq \frac{\xi(1+N)}{\lambda}$.

According to (12), (14) and (28), it is obvious that

$$\begin{aligned}
&\mathbb{E}\{U_{z+1}(\tilde{x}_{z+1|z+1}|\tilde{x}_{z|z})\}\\
&\leq \sum_{i=1}^{N} n_1(1+\bar{\omega}_i)\mathbb{E}\{\tilde{x}^T_{i,z|z}Y^T_{i,z}\Re^T_{i,z+1}\Phi^{-1}_{i,z+1|z+1}\\
&\quad \times \Re_{i,z+1}Y_{i,z}\tilde{x}_{i,z|z}\} + \sum_{i=1}^{N}\sum_{j=1}^{N} n_1(1+\bar{\omega}_i)\omega_{ij}\\
&\quad \times \mathbb{E}\{\tilde{x}^T_{j,z|z}\Gamma^T\Re^T_{i,z+1}\Phi^{-1}_{j,z+1|z+1}\Re_{i,z+1}\Gamma\tilde{x}_{j,z|z}\}\\
&\quad + \sum_{i=1}^{N} n_2\mathbb{E}\{s^T(x_{i,z+1})K^T_{i,z+1}\Phi^{-1}_{i,z+1|z+1}K_{i,z+1}s(x_{i,z+1})\}\\
&\quad + \sum_{i=1}^{N} n_3\mathbb{E}\{u^T_{i,z+1}K^T_{i,z+1}\Phi^{-1}_{i,z+1|z+1}K_{i,z+1}u_{i,z+1}\}\\
&\quad + \sum_{i=1}^{N} n_4\mathbb{E}\{\ell^T_{i,z+1}K^T_{i,z+1}\Phi^{-1}_{i,z+1|z+1}K_{i,z+1}\ell_{i,z+1}\}\\
&\quad + \sum_{i=1}^{N} \mathbb{E}\{n_1 w^T_{i,z}B^T_{i,z}\Re^T_{i,z+1}\Phi^{-1}_{i,z+1|z+1}\Re_{i,z+1}B_{i,z}w_{i,z}\\
&\quad + n_5 v^T_{i,z+1}D^T_{i,z+1}K^T_{i,z+1}\Phi^{-1}_{i,z+1|z+1}K_{i,z+1}D_{i,z+1}v_{i,z+1}\}
\end{aligned}$$

where $Y_{i,z} \triangleq F_{i,z} + A_{i,z}C_{i,z}$.

Then, from Lemma 2 and (21)-(22), we have

$$\begin{aligned}
\Phi_{i,z+1|z} &\geq (1+\bar{\omega}_i)Y_{i,z}(\Phi^{-1}_{i,z|z} - \rho_{i,z}I)^{-1}Y^T_{i,z}\\
&\geq (1+\bar{\omega}_i)Y_{i,z}\Phi_{i,z|z}Y^T_{i,z}\\
\Phi_{i,z+1|z} &\geq (1+\bar{\omega}_i)\omega_{ij}\Gamma\Phi_{j,z|z}\Gamma^T\\
\Phi_{i,z+1|z} &\geq B_{i,z}Q_{i,z}B^T_{i,z}\\
\Phi_{i,z+1|z+1} &\geq n_1\Re_{i,z+1}(\Phi^{-1}_{i,z+1|z} - \mu_{i,z+1}I)^{-1}\Re^T_{i,z+1}\\
&\geq n_1\Re_{i,z+1}\Phi_{i,z+1|z}\Re^T_{i,z+1}\\
\Phi_{i,z+1|z+1} &\geq n_2K_{i,z+1}\Xi_{i1,z+1}K^T_{i,z+1}\\
\Phi_{i,z+1|z+1} &\geq n_3\varpi_{i,z}\Delta_{i,z+1}K_{i,z+1}K^T_{i,z+1}\\
\Phi_{i,z+1|z+1} &\geq n_4\bar{\ell}_iK_{i,z+1}K^T_{i,z+1}\\
\Phi_{i,z+1|z+1} &\geq n_5K_{i,z+1}D_{i,z+1}R_{i,z+1}D^T_{i,z+1}K^T_{i,z+1}.
\end{aligned}$$

In light of Assumption 1, it is directly obtained that

$$\begin{aligned}
&\sum_{i=1}^{N} n_1(1+\bar{\omega}_i)\mathbb{E}\{\tilde{x}^T_{i,z|z}Y^T_{i,z}\Re^T_{i,z+1}\Phi^{-1}_{i,z+1|z+1}\Re_{i,z+1}Y_{i,z}\tilde{x}_{i,z|z}\}\\
&\leq \sum_{i=1}^{N}(1+\bar{\omega}_i)\mathbb{E}\{\tilde{x}^T_{i,z|z}Y^T_{i,z}\Phi^{-1}_{i,z+1|z}Y_{i,z}\tilde{x}_{i,z|z}\}\\
&\leq \sum_{i=1}^{N}\mathbb{E}\{\tilde{x}^T_{i,z|z}(\Phi^{-1}_{i,z|z} - \rho_{i,z}I)\tilde{x}_{i,z|z}\}\\
&\leq \sum_{i=1}^{N}\frac{1+N-\lambda}{1+N}\mathbb{E}\{\tilde{x}^T_{i,z|z}\Phi^{-1}_{i,z|z}\tilde{x}_{i,z|z}\}\\
&= \frac{1+N-\lambda}{1+N}\mathbb{E}\{U_z(\tilde{x}_{z|z})\}.
\end{aligned}$$

On the other hand, it is easy to know that

$$\begin{aligned}
&\sum_{i=1}^{N}\sum_{j=1}^{N} n_1(1+\bar{\omega}_i)\omega_{ij}\mathbb{E}\{\tilde{x}^T_{j,z|z}\Gamma^T\Re^T_{i,z+1}\Phi^{-1}_{j,z+1|z+1}\\
&\quad \times \Re_{i,z+1}\Gamma\tilde{x}_{j,z|z}\}\\
&\leq \sum_{i=1}^{N}\sum_{j=1}^{N}(1+\bar{\omega}_i)\omega_{ij}\mathbb{E}\{\tilde{x}^T_{j,z|z}\Gamma^T(\Phi^{-1}_{j,z+1|z} - \mu_{i,z}I)\Gamma\tilde{x}_{j,z|z}\}\\
&\leq \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1+N-\lambda}{1+N}(1+\bar{\omega}_i)\omega_{ij}\mathbb{E}\{\tilde{x}^T_{j,z|z}\Gamma^T\Phi^{-1}_{j,z+1|z}\Gamma\tilde{x}_{j,z|z}\}\\
&= N\frac{1+N-\lambda}{1+N}\mathbb{E}\{U_z(\tilde{x}_{z|z})\}
\end{aligned}$$

and

$$\sum_{i=1}^{N} n_3\mathbb{E}\{u^T_{i,z+1}K^T_{i,z+1}\Phi^{-1}_{i,z+1|z+1}K_{i,z+1}u_{i,z+1}\} \leq N.$$

Recalling the relationship $\Phi^{-1}_{i,z+1|z+1} \leq \frac{1+N}{\lambda}\rho_{i,z+1}I$ in Assumption 1, one has

$$\sum_{i=1}^{N} n_2\mathbb{E}\{s^T(x_{i,z+1})K^T_{i,z+1}\Phi^{-1}_{i,z+1|z+1}K_{i,z+1}s(x_{i,z+1})\}$$

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$$\begin{aligned}
&\leq \sum_{i=1}^{N} n_2 \wp_{i,z+1} \mathbb{E}\{s^T(x_{i,z+1}) K_{i,z+1}^T K_{i,z+1} s(x_{i,z+1})\} \\
&= \sum_{i=1}^{N} n_2 \wp_{i,z+1} \mathbb{E}\{\mathrm{tr}\{s^T(x_{i,z+1}) K_{i,z+1}^T K_{i,z+1} s(x_{i,z+1})\}\} \\
&\leq \sum_{i=1}^{N} n_2 \wp_{i,z+1} \mathrm{tr}\{K_{i,z+1} \Xi_{i1,z+1} K_{i,z+1}^T\} \\
&\triangleq \mathcal{M}_1
\end{aligned}$$

where $\wp_{i,z+1} \triangleq \frac{1+N}{\lambda} \rho_{i,z+1}$. Also, one has that

$$\begin{aligned}
&\sum_{i=1}^{N} n_4 \mathbb{E}\{\varrho_{i,z+1}^T K_{i,z+1}^T \Phi_{i,z+1|z+1}^{-1} K_{i,z+1} \varrho_{i,z+1}\} \\
&\quad \leq \sum_{i=1}^{N} n_4 \bar{\ell}_i \wp_{i,z+1} \mathrm{tr}\{K_{i,z+1} K_{i,z+1}^T\} \\
&\quad \triangleq \mathcal{M}_2
\end{aligned}$$

and

$$\begin{aligned}
&\sum_{i=1}^{N} \mathbb{E}\{n_5 v_{i,z+1}^T D_{i,z+1}^T K_{i,z+1}^T \Phi_{i,z+1|z+1}^{-1} K_{i,z+1} D_{i,z+1} v_{i,z+1}\} \\
&\leq \sum_{i=1}^{N} n_5 \wp_{i,z+1} \mathrm{tr}\{K_{i,z+1} D_{i,z+1} R_{i,z+1} D_{i,z+1}^T K_{i,z+1}^T\} \\
&\triangleq \mathcal{M}_3.
\end{aligned}$$

Furthermore, Assumption 1 implies

$$\begin{aligned}
&\sum_{i=1}^{N} \mathbb{E}\{n_1 w_{i,z}^T B_{i,z}^T \Re_{i,z+1}^T \Phi_{i,z+1|z+1}^{-1} \Re_{i,z+1} B_{i,z} w_{i,z}\} \\
&\quad \leq \sum_{i=1}^{N} \mathbb{E}\{w_{i,z}^T B_{i,z}^T \Phi_{i,z+1|z}^{-1} B_{i,z} w_{i,z}\} \\
&\quad \leq \sum_{i=1}^{N} \frac{1+N}{\lambda} \mu_{i,z} \mathbb{E}\{w_{i,z}^T B_{i,z}^T B_{i,z} w_{i,z}\} \\
&\quad = \sum_{i=1}^{N} \frac{1+N}{\lambda} \mu_{i,z} \mathbb{E}\{\mathrm{tr}\{w_{i,z}^T B_{i,z}^T B_{i,z} w_{i,z}\}\} \\
&\quad \triangleq \mathcal{M}_4.
\end{aligned}$$

Based on the above analysis, we have

$$\begin{aligned}
&\mathbb{E}\{U_{z+1}(\tilde{x}_{z+1|z+1}) | \tilde{x}_{z|z}\} \\
&\quad \leq (1+N-\lambda)\mathbb{E}\{U_z(\tilde{x}_{z|z})\} + N + \sum_{l=1}^{4} \mathcal{M}_l \\
&\quad = \delta \mathbb{E}\{U_z(\tilde{x}_{z|z})\} + \mathcal{H}
\end{aligned} \tag{30}$$

where $\delta \triangleq 1+N-\lambda \in (0,1)$ and $\mathcal{H} \triangleq N + \sum_{l=1}^{4} \mathcal{M}_l$.

By means of (29) and (30), it is easy to see

$$\begin{aligned}
\mathbb{E}\{\|\tilde{x}_{z|z}\|^2\} &\leq \frac{\mathbb{E}\{U_z(\tilde{x}_{z|z})\}}{\lambda_1} \\
&\leq \frac{\delta \mathbb{E}\{U_{z-1}(\tilde{x}_{z-1|z-1})\} + \mathcal{H}}{\lambda_1} \\
&\quad \vdots \\
&\leq \frac{\lambda_2 \mathbb{E}\{\|\tilde{x}_{0|0}\|^2\}\delta^z}{\lambda_1} + \frac{\mathcal{H}\sum_{i=0}^{z-1}\delta^i}{\lambda_1} \\
&= \beta \mathbb{E}\{\|\tilde{x}_{0|0}\|^2\}\delta^z + \gamma
\end{aligned} \tag{31}$$

where $\beta \triangleq \frac{\lambda_2}{\lambda_1}$ and $\gamma \triangleq \frac{\mathcal{H}\sum_{i=0}^{z-1}\delta^i}{\lambda_1}$ are positive scalars. Moreover, (29) implies

$$\mathbf{Pr}\{\|\tilde{x}_{z|z}\| \geq \upsilon\} \leq \mathbf{Pr}\{U_z(\tilde{x}_{z|z}) \geq \lambda_1 \upsilon^2\}$$

and

$$\mathbf{Pr}\{\|\tilde{x}_{z|z}\| - \lambda_1\gamma \geq \upsilon\} \leq \mathbf{Pr}\{U_z(\tilde{x}_{z|z}) - \lambda_1\gamma \geq \lambda_1 \upsilon^2\}$$

for any positive scalar υ. Then, in view of Lemma 4 and (31), we obtain that

$$\begin{aligned}
\mathbf{Pr}\{\|\tilde{x}_{z|z}\| - \lambda_1\gamma \geq \upsilon\} &\leq \frac{\mathbb{E}\{U_z(\tilde{x}_{z|z})\} - \lambda_1\gamma}{\lambda_1 \upsilon^2} \\
&\leq \beta \mathbb{E}\{\|\tilde{x}_{0|0}\|^2\} \frac{\delta^z}{\upsilon^2}.
\end{aligned}$$

By denoting $\upsilon \triangleq \sqrt{\beta \mathbb{E}\{\|\tilde{x}_{0|0}\|^2\}}\delta^{\frac{z}{4}}$ and $\epsilon_0 \triangleq \lambda_1\gamma = \mathcal{H}\sum_{i=0}^{z-1}\delta^i$, we immediately derive that

$$\mathbf{Pr}\{\|\tilde{x}_{z|z}\| \geq \epsilon_0 + \upsilon\} \leq \delta^{\frac{z}{2}}$$

and

$$\mathbf{Pr}\{\|\tilde{x}_{z|z}\| < \epsilon_0 + \upsilon\} > 1 - \delta^{\frac{z}{2}}.$$

Noting $\upsilon \to 0$ as $z \to \infty$, we have

$$\mathbf{Pr}\{\lim_{z\to\infty} \|\tilde{x}_{z|z}\| < \epsilon\} = 1$$

where $\epsilon \triangleq \frac{\mathcal{H}}{1-\delta}$.

From Definition 1, it is concluded that the estimation error $\tilde{x}_{i,z|z}$ is stochastically bounded in probability 1. The proof is complete. ■

Remark 6: Different from [12], [17] where the estimation error has been proved to be mean-square exponentially bounded, we guarantee that the estimation error is stochastically bounded in probability 1, which helps us to understand the random feature of the estimation performance.

Remark 7: In this paper, the problem of ET recursive SE is dealt with for stochastic CDNs under hybrid attacks. With the help of the non-augmentation vector method and the stochastic analysis method, the local estimator is designed for each node based on the solutions to two RLDEs. Moreover, the estimation error is proved to be stochastically bounded in probability 1. Compared with the existing literature, the distinct features of this paper can be summarized as follows: 1) the underlying network model is quite comprehensive that characterizes the hybrid attacks and the event-triggering scheme in a unified framework; 2) a hybrid attack model is proposed to account for the joint influence of DoS attack

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and deception attack on the estimation performance; and 3) a non-augmentation technique is used to establish the EES for each node, thereby avoiding the inversion operation of high-dimensional matrices and the computation of cross-covariance matrices among coupled nodes.

V. An Illustrative Example

In this section, we will verify the effectiveness of the proposed estimator design method by an illustrative example. Consider the stochastic CDN (1) with four nodes, where the matrix $W = (\omega_{ij})_{4\times 4}$ is set as

$$\omega_{ij} = \begin{cases} -0.3 & i = j \\ 0.1 & i \neq j \end{cases}.$$

The matrix Γ is set as $\Gamma = 0.2I_2$ with $\gamma_1 = \gamma_2 = 0.2$.

The covariances of $w_{i,z}$ and $v_{i,z}$ are $Q_{1,z} = 0.4$, $Q_{2,z} = 0.2$, $Q_{3,z} = 0.2$, $Q_{4,z} = 0.4$, $R_{1,z} = 0.5$, $R_{2,z} = 0.6$, $R_{3,z} = 0.6$, and $R_{4,z} = 0.5$, respectively. The parameter matrices $B_{i,z}$ and $D_{i,z}$ are chosen as

$$B_{1,z} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \ B_{2,z} = \begin{bmatrix} 0.1 \\ 0.15 \end{bmatrix}$$
$$B_{3,z} = \begin{bmatrix} 0.15 \\ 0.1 \end{bmatrix}, \ B_{4,z} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$$
$$D_{i,z} = 0.1 \ (i = 1, 2, 3, 4).$$

The nonlinear functions $f(x_{i,z})$ and $s(x_{i,z})$ are

$$f(x_{i,z}) = \begin{bmatrix} 0.9\sin(x^1_{i,z}) + 0.4\sin(x^2_{i,z}) \\ 0.7\sin(x^1_{i,z}) + 0.45\sin(x^2_{i,z}) \end{bmatrix}$$
$$s(x_{i,z}) = 0.1x^1_{i,z} + 0.1x^2_{i,z}$$

where $x_{i,z} \triangleq [x^1_{i,z} \quad x^2_{i,z}]^T$ denotes the state vector. Note that the above nonlinear function $s(x_{i,z})$ satisfies the nonlinear constraint (3), and $\|s(x_{i,z})\| = 0.1(x^1_{i,z}+x^2_{i,z}) \leq 0.1\sqrt{2}\|x_{i,z}\|$ with $b_1 = 0.1\sqrt{2}$.

The matrices $F_{i,z}$, $A_{i,z}$, $S_{i,z}$ and $G_{i,z}$ are

$$F_{i,z} = \begin{bmatrix} 0.9\cos(\hat{x}^1_{i,z|z}) & 0.4\cos(\hat{x}^2_{i,z|z}) \\ 0.7\cos(\hat{x}^1_{i,z|z}) & 0.45\cos(\hat{x}^2_{i,z|z}) \end{bmatrix}$$
$$A_{1,z} = \begin{bmatrix} 0.02 & 0 \\ 0 & 0.02 \end{bmatrix}, A_{2,z} = \begin{bmatrix} 0.02 & 0 \\ 0 & 0.02 \end{bmatrix}$$
$$A_{3,z} = \begin{bmatrix} 0.03 & 0 \\ 0 & 0.03 \end{bmatrix}, A_{4,z} = \begin{bmatrix} 0.03 & 0 \\ 0 & 0.03 \end{bmatrix}$$
$$S_{i,z} = [0.1 \quad 0.1], G_{i,z} = [0.01 \quad 0.01]$$

where $\hat{x}_{i,z|z} \triangleq [\hat{x}^1_{i,z|z} \quad \hat{x}^2_{i,z|z}]^T$ is the estimate.

The event-triggering threshold in (4) is given with the following form:

$$\varpi_{i,z} = \varpi_{i0} + \frac{\varpi_{i1}}{z}$$

where $\varpi_{10} = 0.2$, $\varpi_{20} = 0.2$, $\varpi_{30} = 0.3$, $\varpi_{40} = 0.15$ and $\varpi_{i1} = 0.1 \ (i = 1, 2, 3, 4)$.

The deception signal is given as $\ell_{i,z} = 0.2 + 0.2\sin(x_{i,z})$. It is easy to see $\ell^T_{i,z}\ell_{i,z} = 0.04(1 + \sin(x_{i,z}))^2 \leq 0.16$, and we then obtain $\bar{\ell}_i = 0.16$.

The rest of the parameters are selected as $\bar{c}_i = 0.9$, $\bar{d}_i = 0.2$ $\rho_{1,z} = \rho_{2,z} = \rho_{3,z} = \rho_{4,z} = 0.002$, $\mu_{1,z} = \mu_{2,z} = \mu_{3,z} = \mu_{4,z} = 0.001$, $\sigma_1 = 0.1$, $\sigma_2 = 0.2$, $\sigma_3 = 0.3$, $\sigma_4 = 0.4$, $\sigma_5 = 0.5$ and $\sigma_6 = 0.1$.

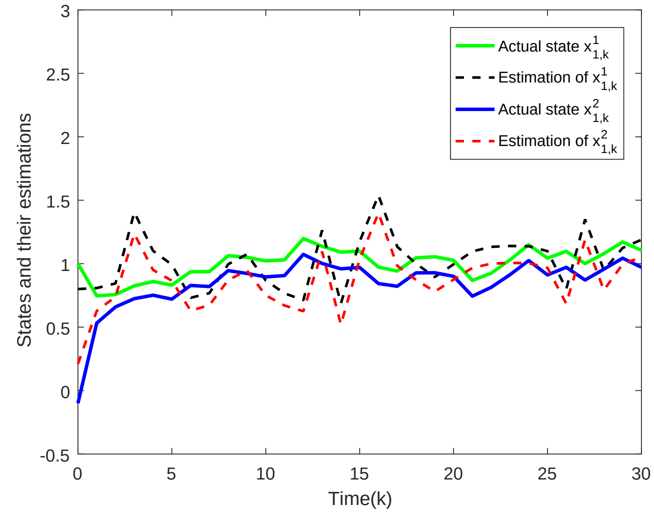

Fig. 1: State and its estimate on node 1 for CDN (1)

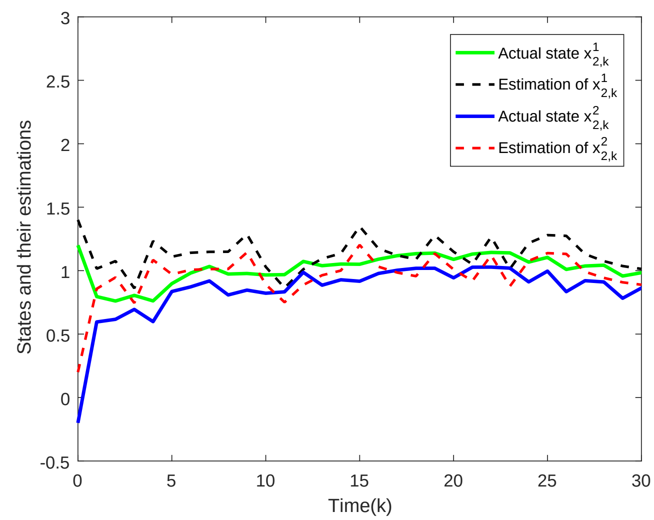

Fig. 2: State and its estimate on node 2 for CDN (1)

The initial conditions of the states and the estimates are given as follows:

$$x_{1,0} = [1.00 \quad -0.1]^T, x_{2,0} = [1.20 \quad -0.20]^T$$
$$x_{3,0} = [1.40 \quad -0.6]^T, x_{4,0} = [1.60 \quad -0.48]^T$$
$$\hat{x}_{1,0|0} = [0.80 \quad 0.21]^T, \hat{x}_{2,0|0} = [1.40 \quad 0.20]^T$$
$$\hat{x}_{3,0|0} = [1.60 \quad 0.20]^T, \hat{x}_{4,0|0} = [1.80 \quad 0.25]^T$$
$$\Phi_{1,0|0} = 4I_2, \Phi_{2,0|0} = 6I_2, \Phi_{3,0|0} = 9I_2, \Phi_{4,0|0} = 10I_2.$$

By recursively solving (21) and (22), we obtain the desired estimator gain for each node. The corresponding state trajectories of four nodes and their estimates are shown in Figs. 1-4.

In Fig. 5, the triggering time sequences on four nodes are exhibited, from which we can find that only a small ratio of measurement signals are transmitted to the estimator while the estimation performance is still preserved.

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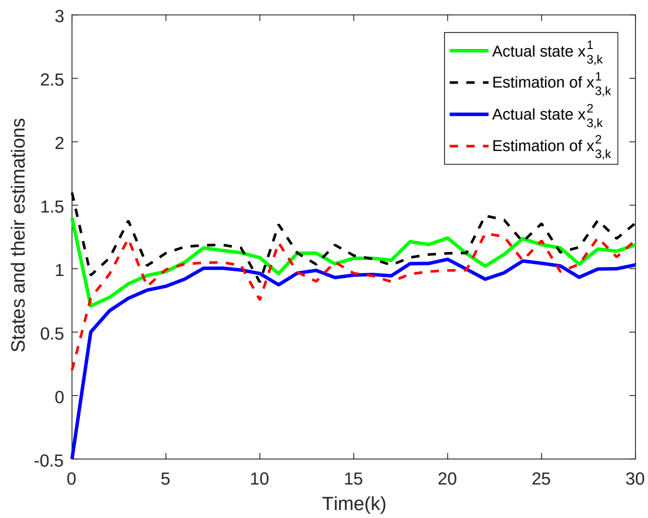

Fig. 3: State and its estimate on node 3 for CDN (1)

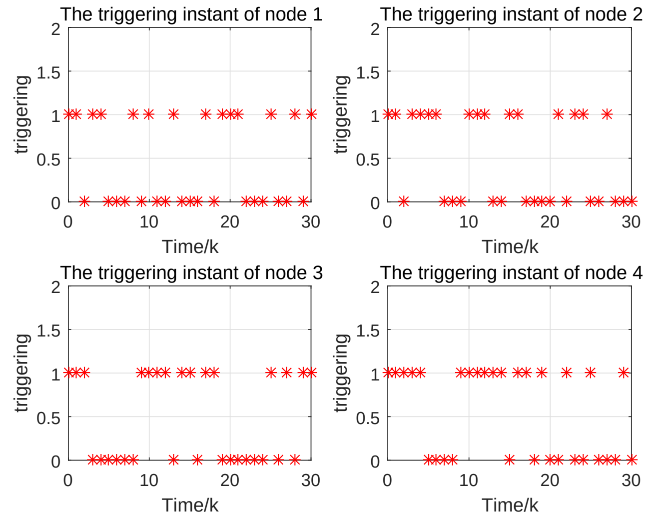

Fig. 5: The triggering instants on four nodes

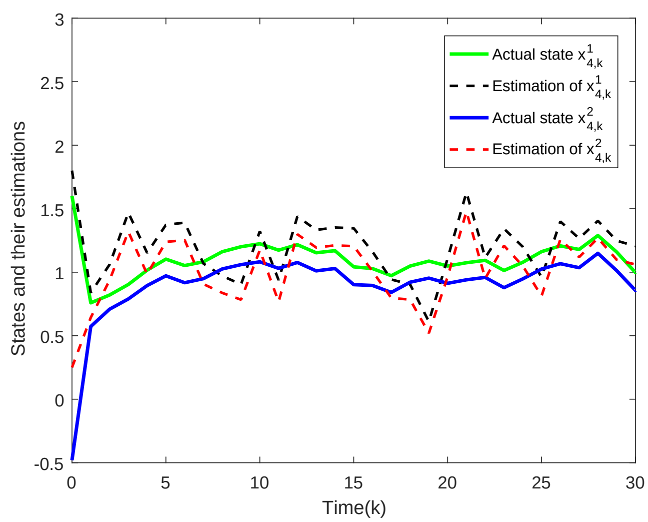

Fig. 4: State and its estimate on node 4 for CDN (1)

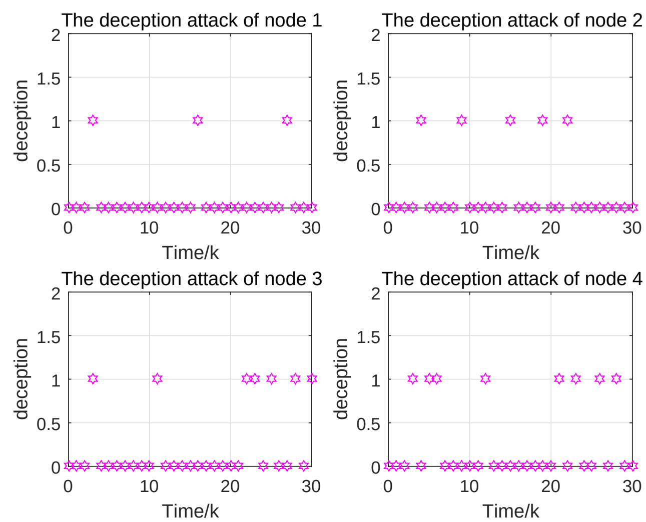

Fig. 6: The deception attacks on four nodes

Figs. 6-7 show the occurrence of the deception attack and the DoS attack, respectively. It follows from Fig. 8 that when the deception attack or the DoS attack occurs, the estimation errors undergo notably changes until the normal measurements are received by the estimators again.

For node i, the mean estimation error (MEE) is defined as

$$\mathrm{MEE}_{i,z} = \ln\left\{\frac{1}{M}\sum_{t=1}^{M} e_{i,z}^{(t)}\right\}$$

where $M = 500$ is the number of Monte Carlo simulations and $e_{i,z}^{(t)}$ is the Euclidean norm of the estimation error in the t-th test. From the MEE on the four nodes depicted in Fig. 9, we confirm that the proposed ET state estimator performs well under cyber-attacks.

VI. Conclusion

The event-based recursive SE problem has been studied in this paper for stochastic CDNs under hybrid attacks. For the sake of alleviating network burdens and reducing energy consumption, the ET protocol has been applied to regulate the data transmissions from the sensor to the estimator. Based on the non-augmentation technique, a recursive state estimator has been designed separately for each node by solving two coupled RLDEs. A certain upper bound on the EEC has been derived and then minimized by choosing appropriate EGM. Moreover, the estimation error has been proved to be stochastically bounded with probability 1. Finally, a numerical example has been conducted to demonstrate the validity of the presented estimator design scheme. In the future, we will be devoted to investigating the recursive state estimation issue for stochastic CDNs with switching topology [28] under the dynamical event-triggered strategy [16], [48].

References

[1] M. S. Ali, M. Usha, Z. Orman, and S. Arik, "Improved result on state estimation for complex dynamical networks with time varying delays

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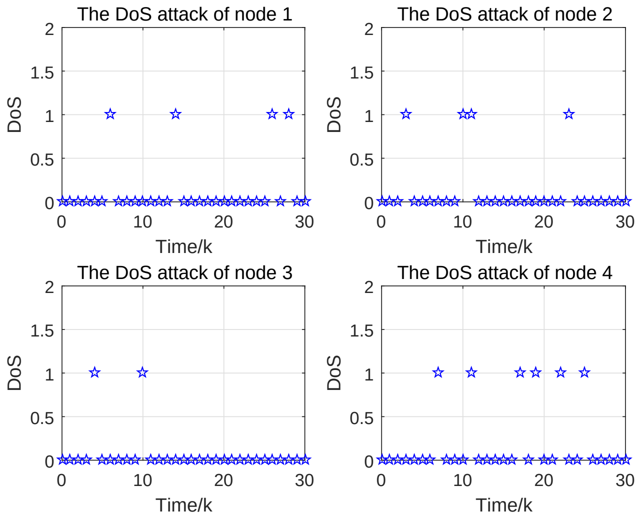

Fig. 7: The DoS attacks on four nodes

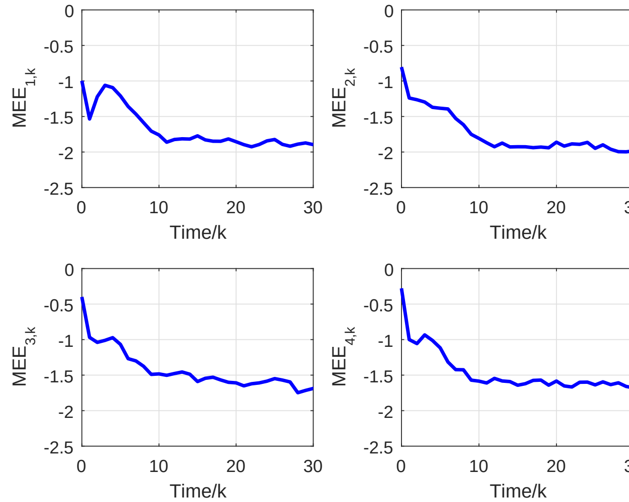

Fig. 9: The MEE on four nodes

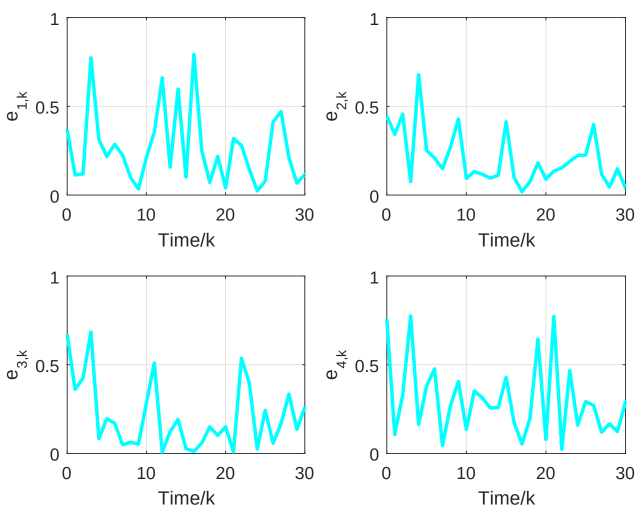

Fig. 8: The Euclidean norm of estimation errors on four nodes

and stochastic sampling via sampled-data control," *Neural Networks*, vol. 114, pp. 28–37, 2019.

[2] G. K. Befekadu, V. Gupta and P. J. Antsaklis, "Risk-sensitive control under Markov modulated denial-of-service (DoS) attack strategies," *IEEE Transactions on Automatic Control*, vol. 60, pp. 3299–3304, Mar. 2015.

[3] Y. Chen, Z. Chen, Z. Chen and A. Xue, "Observer-based passive control of non-homogeneous Markov jump systems with random communication delays," *International Journal of Systems Science*, vol. 51, no. 6, pp. 1133–1147, 2020.

[4] Y. Chen, Z. Wang, L. Wang and W. Sheng, "Finite-horizon H_∞ state estimation for stochastic coupled networks with random inner couplings using Round-Robin protocol," *IEEE Transactions on Cybernetics*, vol. 51, no. 3, pp. 1204-1215, Mar. 2021.

[5] Y. Chen, Z. Wang, J. Hu and Q.-L. Han, "Synchronization control for discrete-time-delayed dynamical networks with switching topology under actuator saturations," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 32, no. 5, pp. 2040–2053, 2021.

[6] D. Ding, Z. Wang, D. W. C. Ho and G. Wei, "Distributed recursive filtering for stochastic systems under uniform quantizations and deception attacks through sensor networks," *Automatica*, vol. 78, pp. 231–240, Apr. 2017.

[7] F. Dörfler and F. Bullo, "Synchronization in complex networks of phase oscillators: A survey," *Automatica*, vol. 50, no. 6, pp. 1539–1564, Jun. 2014.

[8] W. P. M. H. Heemels, K. H. Johansson and P. Tabuada, "An introduction to event-triggered and self-triggered control," In: *Procedings of 51st IEEE Conference on Decision and Control*, pp. 3270–3285, 2012.

[9] J. P. Hespanha, P. Naghshtabrizi and Y. Xu, "A survey of recent results in networked control systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 138–162, Jan. 2007.

[10] J. M. Hofman, A. Sharma and D. J. Watts, "Prediction and explanation in social systems," *Science*, vol. 355, pp. 486–488, Feb. 2017.

[11] J. Hu, Z. Wang, S. Liu and H. Gao, "A variance-constrained approach to recursive state estimation for time-varying complex networks with missing measurements," *Automatica*, vol. 64, pp. 155–162, Feb. 2016.

[12] J. Hu, G. Liu, H. Zhang and H. Liu, "On state estimation for nonlinear dynamical networks with random sensor delays and coupling strength under event-based communication mechanism," *Information Sciences*, vol. 511, pp. 265–283, Feb. 2020.

[13] J. Hu, Z. Wang, H. Gao and L. K. Stergioulas, "Extended Kalman filtering with stochastic nonlinearities and multiple missing measurements," *Automatica*, vol. 48, no. 9, pp. 2007–2015, Sept. 2012.

[14] R. Khasminskii, *Stochastic Stability of Differential Equations (2nd Edition)*, Springer, Heidelberg, 2010.

[15] Q. Li, B. Shen, Z. Wang and W. Sheng, "Recursive distributed filtering over sensor networks on Gilbert-Elliott channels: A dynamic event-triggered approach," *Automatica*, vol. 113, art. no. 108681, 2020.

[16] Q. Li, Z. Wang, N. Li, and W. Sheng, "A dynamic event-triggered approach to recursive filtering for complex networks with switching topologies subject to random sensor failures," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 31, no. 10, pp. 4381-4388, Oct. 2020.

[17] W. Li, Y. Jia and J. Du, "State estimation for stochastic complex networks with switching topology," *IEEE Transactions on Automatic Control*, vol. 62, no. 12, pp. 6377–6384, Dec. 2017.

[18] W. Li, Y. Jia and J. Du, "Non-augmented state estimation for nonlinear stochastic coupling networks," *Automatica*, vol. 78, pp. 119–122, Apr. 2017.

[19] X. Li, H. Dong, Z. Wang and F. Han, "Set-membership filtering for state-saturated systems with mixed time-delays under weighted try-once-discard protocol," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 66, no. 2, pp. 312–316, Feb. 2019.

[20] J. Liu, Y. Gu, L. Zha, Y. Liu and J. Cao, "Event-triggered H_∞ load frequency control for multiarea power systems under hybrid cyber attacks," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 49, no. 8, pp. 1665–1678, Aug. 2019.

[21] J. Liu, M. Yang, E. Tian, J. Cao and S. Fei, "Event-based security control for state-dependent uncertain systems under hybrid-attacks and its application to electronic circuits," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 66, no. 12, pp. 4817–4828, Dec. 2019.

[22] X. Liu, D. W. C. Ho, Q. Song and W. Xu, "Finite/fixed-time pinning

synchronization of complex networks with stochastic disturbances," *IEEE Transactions on Cybernetics*, vol. 49, no. 6, pp. 2398–2403, Jun. 2019.

[23] Y. Liu, Z. Wang, J. Liang and X. Liu, "Synchronization and state estimation for discrete-time complex networks with distributed delays," *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, vol. 38, no. 5, pp. 1314–1325, Oct. 2008.

[24] J. Mao, Y. Sun, X. Yi, H. Liu and D. Ding, Recursive filtering of networked nonlinear systems: A survey, *International Journal of Systems Science*, vol. 52, no. 6, pp. 1110–1128, 2021.

[25] Y. Mo, J. Hespanha and B. Sinopoli, "Robust detection in the presence of integrity attacks," In: *Proceedings of the American Control Conference*, Montreal, 2012, pp. 3541–3546.

[26] R. Pan, Y. Tan, D. Du and S. Fei, "Adaptive event-triggered synchronization control for complex networks with quantization and cyber-attacks," *Neurocomputing*, vol. 382, no. 21, pp. 249–258, Mar. 2020.

[27] M. Porfiri and M. Bernardo, "Criteria for global pinning-controllability of complex networks," *Automatica*, vol. 44, no. 12, pp. 3100–3106, Dec. 2008.

[28] J. Qin, Q. Ma, X. Yu, and L. Wang, "On synchronization of dynamical systems over directed switching topology: An algebraic and geometric perspective," *IEEE Transactions on Automatic Control*, vol. 65, no. 12, pp. 5083-5098, Dec. 2020.

[29] B. Shen, Z. Wang, D. Wang and Q. Li, "State-saturated recursive filter design for stochastic time-varying nonlinear complex networks under deception attacks," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 31. no. 10, pp. 3788-3800, Oct. 2020.

[30] L. Sheng, Z. Wang, L. Zou and F. E. Alsaadi, "Event-based H_∞ state estimation for time-varying stochastic dynamical networks with state- and disturbance-dependent noises," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 28, no. 10, pp. 2382–2394, Oct. 2017.

[31] O. Sporns, "The human connectome: A complex network," *Annals of the New York Academy of Sciences*, vol. 1224, no. 1, pp. 109–125, Jan. 2011.

[32] M. Tabbara and D. Nesic, "Input-output stability of networked control systems with stochastic protocols and channels," *IEEE Transactions on Automatic Control*, vol. 53, no. 5, pp. 1160–1175, Jun. 2008.

[33] H. Tan, B. Shen, K. Peng and H. Liu, Robust recursive filtering for uncertain stochastic systems with amplify-and-forward relays, *International Journal of Systems Science*, vol. 51, no. 7, pp. 1188–1199, 2020.

[34] L. Wang, Z. Wang, T. Huang and G. Wei, "An event-triggered approach to state estimation for a class of complex networks with mixed time delays and nonlinearities," *IEEE Transactions on Cybernetics*, vol. 46, no. 11, pp. 2497–2508, Nov. 2016.

[35] D. J. Watts and S. H. Strogatz, "Collective dynamics of 'small-world' networks," *Nature*, vol. 393, no. 6684, pp. 440–442, Jun. 1998.

[36] W. Xu, D. W. C. Ho, J. Zhong and B. Chen, "Event/self-triggered control for leader-following consensus over unreliable network with DoS attacks," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 30, no. 10, pp. 3137–3149, Oct. 2019.

[37] Y. Xu, R. Lu, H. Peng, K. Xie and A. Xue, "Asynchronous dissipative state estimation for stochastic complex networks with quantized jumping coupling and uncertain measurements," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 28, no. 2, pp. 268–277, Feb. 2017.

[38] Y. Xu, R. Lu, P. Shi, H. Li and S. Xie, "Finite-time distributed state estimation over sensor networks with Round-Robin protocol and fading channels," *IEEE Transactions on Cybernetics*, vol. 48, no. 1, pp. 336–345, Jan. 2018.

[39] Y. Xu, J.-Y. Li, R. Lu, C. Liu and Y. Wu, "Finite-horizon $l_2 - l_\infty$ synchronization for time-varying Markovian jump neural networks under mixed-type attacks: Observer-based case," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 30, no. 6, pp. 1695–1704, Jun. 2019.

[40] S. Yang, Z. Guo, and J. Wang, "Global synchronization of multiple recurrent neural networks with time delays via impulsive interactions," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 28, no. 7, pp. 1657–1667, 2016.

[41] X. Yang, J. Lam, D. W. C. Ho and Z. Feng, "Fixed-time synchronization of complex networks with impulsive effects via nonchattering control," *IEEE Transactions on Automatic Control*, vol. 62, no. 11, pp. 5511–5521, 2017.

[42] X. Yin and D. Yue, "Event-triggered tracking control for heterogeneous multi-agent systems with Markov communication delays," *Journal of the Franklin Institute*, vol. 350, no. 5, pp. 1312–1334, 2013.

[43] D. Yue and H. Li, "Synchronization stability of continuous/discrete complex dynamical networks with interval time-varying delays," *Neurocomputing*, vol. 73, no. 4–6, pp. 809–891, 2010.

[44] D. Zhang, Q. Wang, D. Srinivasan, H. Li and L. Yu, "Asynchronous state estimation for discrete-time switched complex networks with communication constraints," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 29, no. 5, pp. 1732–1746, May 2018.

[45] H. Zhang, J. Hu, H. Liu, X. Yu and F. Liu, "Recursive state estimation for time-varying complex networks subject to missing measurements and stochastic inner coupling under random access protocol," *Neurocomputing*, vol. 346, pp. 48–57, Jun. 2019.

[46] H. Zhang, P. Cheng, L. Shi and J. Chen, "Optimal DoS attack scheduling in wireless networked control system," *IEEE Transactions on Control Systems Technology*, vol. 24, no. 3, pp. 843–852, May 2016.

[47] L. Liu, L. Ma, J. Zhang and Y. Bo, "Distributed non-fragile set-membership filtering for nonlinear systems under fading channels and bias injection attacks," *International Journal of Systems Science*, vol. 52, no. 6, pp. 1192–1205, 2021.

[48] G. Zhao and C.-C. Hua, "A hybrid dynamic event-triggered approach to consensus of multi-agent systems with external disturbances," *IEEE Transactions on Automatic Control*, early access, doi: 10.1109/TAC.2020.3018437.

[49] Z. Zhao, Z. Wang, L. Zou and J. Guo, Set-membership filtering for time-varying complex networks with uniform quantisations over randomly delayed redundant channels, *International Journal of Systems Science*, vol. 51, no. 16, pp. 3364–3377, 2020.

[50] L. Zou, Z. Wang, Q.-L. Han and D. Zhou, "Moving horizon estimation for networked time-delay systems under Round-Robin protocol," *IEEE Transactions on Automatic Control*, vol. 64, no. 12, pp. 5191-5198, Dec. 2019.

[51] L. Zou, Z. Wang, H. Gao and X. Liu, "Event-triggered state estimation for complex networks with mixed time delays via sampled data information: The continuous-time case," *IEEE Transactions on Cybernetics*, vol. 45, no. 12, pp. 2804–2815, Dec. 2015.

Yun Chen was born in Zhejiang, China. He received the B.E. degree in thermal engineering in 1999 from Central South University of Technology (Central South University), Changsha, China, and the M.E. degree in engineering thermal physics in 2002 and Ph.D. degree in control science and engineering in 2008, both from Zhejiang University, Hangzhou, China.

From August 2009 to August 2010, he was a visiting fellow with the School of Computing, Engineering and Mathematics, University of Western Sydney, Australia. From December 2016 to December 2017, he was an academic visitor with the Department of Mathematics, Brunel University London, UK. In 2002, he joined Hangzhou Dianzi University, China, where he is currently a Professor. His research interests include stochastic and hybrid systems, robust control and filtering.

Xueyang Meng was born in Jiangsu, China, in 1995. He received the B.E. degree from Luoyang Institute of Science and Technology, Luoyang, China, in 2017. He is currently pursuing the Ph.D. degree in control science and engineering with Hangzhou Dianzi University, Hangzhou, China. His research interests include networked control systems and state estimation.

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Zidong Wang (SM'03-F'14) was born in Jiangsu, China, in 1966. He received the B.Sc. degree in mathematics in 1986 from Suzhou University, Suzhou, China, and the M.Sc. degree in applied mathematics in 1990 and the Ph.D. degree in electrical engineering in 1994, both from Nanjing University of Science and Technology, Nanjing, China.

He is currently Professor of Dynamical Systems and Computing in the Department of Computer Science, Brunel University London, U.K. From 1990 to 2002, he held teaching and research appointments in universities in China, Germany and the UK. Prof. Wang's research interests include dynamical systems, signal processing, bioinformatics, control theory and applications. He has published more than 600 papers in international journals. He is a holder of the Alexander von Humboldt Research Fellowship of Germany, the JSPS Research Fellowship of Japan, William Mong Visiting Research Fellowship of Hong Kong.

Prof. Wang serves (or has served) as the Editor-in-Chief for *International Journal of Systems Science*, the Editor-in-Chief for *Neurocomputing*, the Editor-in-Chief for *Systems Science & Control Engineering*, and an Associate Editor for 12 international journals including IEEE Transactions on Automatic Control, IEEE Transactions on Control Systems Technology, IEEE Transactions on Neural Networks, IEEE Transactions on Signal Processing, and IEEE Transactions on Systems, Man, and Cybernetics-Part C. He is a Member of the Academia Europaea, a Member of the European Academy of Sciences and Arts, an Academician of the International Academy for Systems and Cybernetic Sciences, a Fellow of the IEEE, a Fellow of the Royal Statistical Society and a member of program committee for many international conferences.

Hongli Dong received the Ph.D. degree in control science and engineering from the Harbin Institute of Technology, Harbin, China, in 2012.

From 2009 to 2010, she was a Research Assistant with the Department of Applied Mathematics, City University of Hong Kong, Hong Kong. From 2010 to 2011, she was a Research Assistant with the Department of Mechanical Engineering, The University of Hong Kong, Hong Kong. From 2011 to 2012, she was a Visiting Scholar with the Department of Information Systems and Computing, Brunel University London, London, U.K. From 2012 to 2014, she was an Alexander von Humboldt Research Fellow with the University of Duisburg–Essen, Duisburg, Germany. She is currently a Professor with the Artificial Intelligence Energy Research Institute, Northeast Petroleum University, Daqing, China. She is also the Director of the Heilongjiang Provincial Key Laboratory of Networking and Intelligent Control, Daqing. Her current research interests include robust control and networked control systems.

Dr. Dong is a very active reviewer for many international journals.