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Non-Fragile Dissipative Fuzzy PID Control with Mixed Fading Measurements

Yezheng Wang, Zidong Wang, Lei Zou and Hongli Dong

***Abstract*—This paper is concerned with the extended dissipative fuzzy proportional-integral-derivative (PID) control problem for nonlinear systems subject to controller parameter perturbations over a class of mixed fading channels. The sensors of plant are divided into two groups according to engineering practice, where the individual sensor group transmits the measurements to the controller via a respective communication channel undergoing specific fading effects. Considering the complicated nature of the signal fading with the transmission channels, two stochastic models (i.e. the independent and identically distributed fading model and the Markov fading model) are simultaneously employed to describe the mixed fading effects of the two communication channels corresponding to the two sensor groups. The objective of this paper is to design a non-fragile PID controller such that the closed-loop system is exponentially stable in mean square and extended stochastically dissipative. With the assistance of the Lyapunov stability theory and stochastic analysis method, sufficient conditions are obtained to analyze the system performance. Then, within the established theoretical framework, an iterative optimization algorithm is proposed to design the desired controller parameters by using the convex optimization technique. Finally, two simulation examples are given to verify the effectiveness of the proposed control schemes.**

***Index Terms*—Fuzzy systems, non-fragile control, channel fading, extended dissipativity, PID control.**

I. Introduction

Since its first introduction in [1], the theory of dissipative systems has become a powerful tool to deal with the analysis/synthesis problems in system science. From a systematic perspective of input-output energy, the dissipativity performance is capable of reflecting many fundamental system behaviors such as stability, passivity and H_∞ disturbance rejection/attenuation level. Because the dissipative theory provides a unified and concise framework for studying the system performance, the past decades have witnessed a rapid development of dissipative control/filtering theories and a large number of representative results have been reported in the literature [2]–[5]. Note that, to further extend the application scope of the dissipative theory, a new index called extended dissipativity was first proposed in [6] for continuous-time systems and, by adjusting its weight parameters, this modified index indicates not only the standard dissipativity but also the L_2-L_∞ performance. Since then, the extended-dissipativity-based techniques have attracted considerable research attention and plenty of results have been reported for continuous-time systems [7], [8] and discrete-time systems [9]–[12].

The Takagi-Sugeno (T-S) fuzzy technique is known to be an effective means to deal with control problems for general nonlinear systems. By establishing the T-S fuzzy model, many complex nonlinear functions can be approximated by some linear ones with nonlinear weight parameters subject to any desired approximation accuracy. Such characteristics of local linearity and global nonlinearity facilitate researchers to study the nonlinear control systems by using the T-S fuzzy technique. In the past decades, there has been a rich body of literature concerning the T-S-fuzzy-model-based nonlinear control issues, where multifarious parallel-distributed-compensation (PDC) or non-PDC fuzzy controllers have been designed. To mention a few, those popular fuzzy controllers include fuzzy sliding controllers [13], [14], fuzzy fault-tolerant controllers [15], fuzzy piecewise controllers [16], [17], adaptive fuzzy controllers [18], [19] and modified repetitive fuzzy controllers [20].

Most existing fuzzy controllers are of the proportional type since they are designed based on a proportional relation with respect to the current system states/measurements, which enjoy the convenience in design and implementation. On the other hand, the proportional-integral-derivative (PID) control scheme, which exploits the full information about the past, the current and the future situations of the system dynamics, can find its successful application in almost all industrial systems. For decades, the PID control strategy is known to possess clear engineering background, enhanced robustness, inherent fault-tolerant capability, reliable operation, and concise structure, and has provided engineers with clear guidance on achieving various system performance [21]–[24]. Even with today's popularity of intelligent control, more than 90% practical controllers are still designed in terms of the PID control theory [25], which shows the irreplaceable vitality of the PID-type

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controllers.

Along with the rapid development of automation and modernization, most of industrial systems involve severe nonlinearities, strong coupling, high level of integration and large scale, all of which bring in inevitable difficulties to the controller design issues. Traditional PID controllers, which work well with a linear and time-invariant structure, might not be able to provide satisfactory control performance for complex system. As such, much attention has been paid to the improvement of the applicability of the PID control schemes and, among various improved PID control schemes, the T-S fuzzy PID controller has attracted a particular research interest as it combines the advantages of T-S fuzzy control and PID control techniques. Generally speaking, a T-S fuzzy PID controller is designed based on the obtained T-S fuzzy model, and such a controller is inherently time-varying due to the introduction of the time-varying fuzzy membership functions. Actually, the fuzzy PID control takes advantage of several linear PID controllers working together (under appropriate weights) to give rise to the improvement of the control performance. With respect to the T-S fuzzy PID control problems, some recent works can be found in [26]–[29].

An implicit assumption with almost all existing works concerning the T-S fuzzy PID control problems is that the designed controller can be precisely implemented in practice. Such an assumption is, however, not always true for many reasons such as the round-off error in numerical computation, the finite precision of measuring equipment and the aging/failures of the system components [30]. These limitations on the software/hardware would result in controller parameter perturbations (CPPs) and further degrade the control performance. As such, many researchers make efforts to design the controllers by taking the underlying CPPs into consideration, which is referred to as the non-fragile control problem. Nevertheless, the design of such non-fragile controller is non-trivial in the context of fuzzy PID control because the parameter perturbations may occur *simultaneously* in proportional, integral and derivative terms of all fuzzy submodels of a fuzzy PID controller.

On another research forefront, because of the large scale of the communication network and the quick development of network technology, the networked control systems have received considerable attention from both theorists and engineers. On one hand, introducing the communication network makes it possible to conduct information exchange between system components with high flexibility, low cost, simple installation/maintenance, and few wiring requirements [31]–[34]. On the other hand, due to the distinctive network features (e.g. limited bandwidth, massive data to be processed, open transmission environment, wide distribution and high internal complexity), the system performance would be largely affected by certain network-induced phenomena (e.g. packet dropouts, transmission delays and channel fadings) which, in turn, leads to great challenges in the control tasks.

As a kind of commonly encountered phenomenon in the wireless network communication, the channel fadings have attracted special attention in recent years. Once the channel fading occurs, the amplitude and/or the phase of the transmitted signals would undergo some distortions. Such a less-than-ideal phenomenon, if not properly handled, would directly affect the communication performance between system components and further degrade the system performance. Note that, to reflect the time-varying feature of channel fadings, two effective fading models have been widely employed in the communication and control areas. These two models are the finite-state Markov channel fadings (FSMCFs) and the independent and identically distributed channel fadings (i.i.d.CFs). For the case of FSMCFs, the network (or channel) is thought to have several modes according to the different configurations of the overall physical environment, where a finite-state Markov process is utilized to describe the mode switching. This model is capable of capturing the temporal correlations of channel conditions and has been used to represent many typical communication models such as the Gilbert-Elliott channel model [35]. For the case of i.i.d.CFs, several i.i.d. stochastic variables are employed to reflect the fading level of the network, and this model is capable of describing the communication environment where a set of memoryless parallel independent channels is deployed. A typical example of the i.i.d.CFs is the Erasure channel where the channel coefficients are specialized to the binary value for representing the packet dropouts [36].

So far, much attention has been devoted to the control/filtering problems subject to channel fadings and a great deal of literature has been published on this topic. For example, in [36]–[39], the control problems subject to the i.i.d.CFs have been addressed via some effective methods including fuzzy control, sliding mode control and stochastic control. For FSMCFs, some seminal theoretical results have been reported in [40]–[44] where the controller/filter synthesis and system performance analysis have been discussed in detail.

Note that, in the exiting works concerning control problems under channel fadings, only a single type of fading effect (i.e., either FSMCFs or i.i.d.CFs) has been considered, which is actually based on the implicit assumption that all system information is transmitted via *same* type of communication channels undergoing the *same* type of fadings. Such an assumption is, unfortunately, no longer valid in complicated transmission environments. For example, in heterogeneous cellular networks [45], the macrocell base stations and small base stations are employed together to improve the communication quality, where different types of links exist between user equipment units and base stations. In this case, the signal transmissions in different links would experience different fadings. Such kind of *mixed* fading phenomena also appears in relay-based networks as different paths follow different fading characteristics [46], [47]. The *mixed* fading measurements would greatly complicate the controller design especially for fuzzy PID control problem and, therefore, we are motivated to conduct the current study to deal with mixed channel fading issues.

Summarizing the discussions made so far, we are interested in dealing with the non-fragile fuzzy PID control problems subject to the mixed fading effects. To do this, we are facing two substantial challenges identified as follows: 1) how to establish an appropriate analysis method to explore the extended

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dissipativity for T-S fuzzy systems subject to the coexistence of the FSMCFs, i.i.d.CFs and CPPs? and 2) how to design mode-dependent PID controllers by using faded measurements and observed network modes? Correspondingly, the main contributions of this paper are highlighted from the following three aspects: 1) the dissipative fuzzy PID control problem is, for the first time, investigated for a class of discrete-time T-S fuzzy systems subject to mixed channel fadings and the CPPs; 2) a novel mode-dependent non-fragile fuzzy PID controller is proposed by using the observed network modes; and 3) the desired controller parameters are obtained via solving an optimization problem.

The rest of this paper is arranged as follows. In Section II, the considered nonlinear plant, the signal transmission model, the adopted fuzzy PID controller and the performance index are given in detail. Section III gives the main results concerning the system analysis and controller synthesis. In Section IV, two simulation results and some discussions are presented to verify the obtained theoretical results. Finally, the conclusion is made in Section V.

Notations: In this paper, $\mathbb{R}^n$ represents the n-dimensional Euclidean space. The transposition and trace of a matrix A are denoted by A^T and $\mathrm{tr}(A)$, respectively. $l_2[0,\infty)$ refers to the space of square summable sequences. A block-diagonal matrix D with blocks d_{11}, d_{22}, $\cdots$, d_{nn} in the leading diagonal is described by $D=\mathrm{diag}\{d_{11},d_{22},\cdots,d_{nn}\}$. The symmetric parts in a symmetric matrix are denoted by an asterisk "$*$". I and 0 are used to represent, respectively, the identity matrix and zero matrix of proper dimensions. Given a matrix B, its maximum and minimum eigenvalue are denoted by $\lambda_{\max}(B)$ and $\lambda_{\min}(B)$, respectively. $\mathbb{E}\{\sigma\}$ is the mathematical expectation of the stochastic variable σ. $\Pr\{E_1|E_2\}$ represents the conditional probability of the event E_1 under the event E_2. If two real matrices $E=[e_{ij}]_{m\times n}$ and $F=[f_{ij}]_{m\times n}$ have the same dimensions, then the Hadamard product $E\circ F$ is defined as $E\circ F\triangleq[e_{ij}f_{ij}]_{m\times n}$.

II. Problem Statement and Preliminaries

A. Fuzzy Plants

We consider a kind of nonlinear systems whose sensors are divided into two groups according to the different spatial distribution. The nonlinear systems can be described by the following T-S fuzzy models:

System Rule i: IF $\rho_1(k)$ is W_{i1}, and $\rho_2(k)$ is W_{i2}, and $\cdots$, and $\rho_s(k)$ is W_{is}, THEN

$$\begin{cases} x(k+1)=A_ix(k)+B_iu(k)+E_i\omega(k)\\ y_1(k)=C_1x(k)+F_1\omega(k)\\ y_2(k)=C_2x(k)+F_2\omega(k)\\ z(k)=G_ix(k),\quad i\in\mathbb{I}\triangleq\{1,2,\cdots,r\}\end{cases}\tag{1}$$

where r is the number of fuzzy rules; $W_{i1},\cdots,W_{is}$ are fuzzy sets; $x(k)\in\mathbb{R}^{n_x}$ is the system state; $\rho_i(k)$ $(i=1,2,\cdots,s)$ is the measurable variable; $\rho(k)\triangleq\begin{bmatrix}\rho_1(k) & \rho_2(k) & \cdots & \rho_s(k)\end{bmatrix}^T\in\mathbb{R}^s$ is the premise variable vector; $u(k)\in\mathbb{R}^{n_u}$ is the control input to be designed; $y_1(k)\in\mathbb{R}^{n_1}$ and $y_2(k)\in\mathbb{R}^{n_2}$ are measurement outputs from Sensor Groups I and II, respectively. $z(k)\in\mathbb{R}^{n_z}$ is the controlled output; $\omega(k)\in(\mathbb{R}^{n_\omega},\ l_2[0,+\infty))$ is the energy-bounded external noise (including process noise and measurement noise); A_i, B_i, E_i, C_1, C_2, F_1, F_2 and G_i are real constant matrices of appropriate dimensions.

By using the standard fuzzy inference technique, the fuzzy system (1) can be described by

$$\begin{cases} x(k+1)=\sum_{i=1}^{r}\phi_i(\rho(k))\Big(A_ix(k)+B_iu(k)+E_i\omega(k)\Big)\\ y_1(k)=C_1x(k)+F_1\omega(k)\\ y_2(k)=C_2x(k)+F_2\omega(k)\\ z(k)=\sum_{i=1}^{r}\phi_i(\rho(k))G_ix(k)\end{cases}\tag{2}$$

where $\phi_i(\rho(k))$ $(i\in\mathbb{I})$ is called the normalized membership function calculated by

$$\phi_i(\rho(k))\triangleq\frac{\prod_{j=1}^{s}W_{ij}(\rho_j(k))}{\sum_{i=1}^{r}\prod_{j=1}^{s}W_{ij}(\rho_j(k))}$$

with $0\le W_{ij}(\rho_j(k))\le 1$ being the membership grade of $\rho_j(k)$ in W_{ij}. Meanwhile, for $\forall k\ge 0$, the following properties hold:

$$\phi_i(\rho(k))\ge 0,\quad i\in\mathbb{I},\quad \sum_{i=1}^{r}\phi_i(\rho(k))=1.\tag{3}$$

B. Communication Network

In this paper, the sensors are classified into two groups that are located in two different regions, and the signal transmissions from these two sensor groups to the controller are achieved via two different dedicated communication channels with limited communication capability. For presentation convenience, we label the channel for Sensor Group I (II) as Channel I (II). To account for the diverse transmission environment, two types of channel fading models will be considered in the following.

1) FSMCFs: For Sensor Group I with Channel I, the transmitted information via wireless network would undergo the effects of the FSMCFs. Denote the network mode as $\sigma(k)\in\mathbb{L}\triangleq\{1,2,\cdots,\bar{l}\}$ which is a discrete-time Markov stochastic process with the transition probability matrix $\Pi\triangleq[\pi_{ab}]_{a,b\in\mathbb{L}}$ $(\pi_{ab}\in[0,1])$. Then, for $\sigma(k)$, the transition probabilities between modes can be described by

$$\Pr\{\sigma(k+1)=b|\sigma(k)=a\}\triangleq\pi_{ab},\quad\forall a,b\in\mathbb{L}.\tag{4}$$

Under the effects of channel fadings, the signals after being transmitted are of the following form:

$$\bar{y}_1(k)=\Lambda_{\sigma(k)}y_1(k)\tag{5}$$

where $\bar{y}_1(k)$ is the transmitted measurement output of Sensor Group I. The channel fading phenomenon is reflected in $\Lambda_{\sigma(k)}$, which is a stochastic diagonal matrix given by

$$\Lambda_{\sigma(k)}\triangleq\mathrm{diag}\left\{\lambda_{1,\sigma(k)},\lambda_{2,\sigma(k)},\cdots,\lambda_{n_1,\sigma(k)}\right\}\tag{6}$$

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where $0 \le \lambda_{c,\sigma(k)} \le 1$ $(c = 1, 2, \cdots, n_1)$ is a known scalar representing the fading level.

Remark 1: In the finite-state Markov fading model (5), the network is regarded to have finite modes that may result from different configuration of the overall physical environment [41]. In different network mode, the fading level may be different. Furthermore, a Markov stochastic process is employed to describe the mode switching in different time instants that can effectively capture the time-varying features of the transmission environment.

In engineering practice, it is difficult to obtain the accurate modes of the network in a timely manner due to the complicated network environment and the limited measurement capability. In this case, in order to facilitate the design of a mode-dependent controller, we introduce an observed mode signal on the controller side. It is assumed that $\theta(k) \in \bar{\mathbb{L}} \triangleq \{1, 2, \cdots, \bar{s}\}$ is the available mode information for the controller and satisfies

$$\delta_{ef} \triangleq \Pr\{\theta(k) = f | \sigma(k) = e\},\ e \in \mathbb{L},\ f \in \bar{\mathbb{L}} \tag{7}$$

where $\delta_{ef} \in [0, 1]$ and $\sum_{f=1}^{\bar{s}} \delta_{ef} = 1$.

Remark 2: In engineering practice, the mode information $\theta(k)$ can be obtained through several mode detection techniques [41], [44], [48]. The description of the observed mode information with conditional probability (7) is of a general form that covers the following three situations as special cases: a) If $\bar{l} = \bar{s}$ with $\delta_{ee} = 1$ for $\forall e \in \mathbb{L}$, then we have the synchronous case where the real modes are available or the detected modes are completely accurate; b) if $\bar{l} \neq \bar{s}$ with $\delta_{ef} \in (0, 1)$ for $\forall e \in \mathbb{L},\ f \in \bar{\mathbb{L}}$, then we have the asynchronous case where the estimated modes may differ from the actual modes; and c) if $\bar{\mathbb{L}} = \{1\}$, then we come up with the case that no detection scheme is deployed.

2) i.i.d.CFs: For Sensor Group II, it is assumed that the related measurements may experience the i.i.d.CFs. Define $\bar{y}_2(k)$ as the outputs transmitted via channel 2. Then, $\bar{y}_2(k)$ can be described by

$$\bar{y}_2(k) = \Xi(k) y_2(k) \tag{8}$$

where $\Xi(k)$ is a stochastic diagonal matrix with the following structure:

$$\Xi(k) \triangleq \operatorname{diag}\{\xi_1(k), \xi_2(k), \cdots, \xi_{n_2}(k)\} \tag{9}$$

where $\xi_d(k)$ $(d = 1, 2, \cdots, n_2)$ represent the channel coefficients. For each time instant k, $\xi_d(k)$ are the i.i.d. stochastic variables with the following statistical properties:

$$\bar{\xi}_d \triangleq \mathbb{E}\{\xi_d(k)\},$$
$$\xi^*_{cd} \triangleq \mathbb{E}\{(\xi_c(k) - \bar{\xi}_c)(\xi_d(k) - \bar{\xi}_d)\},$$

where, for $\forall c, d = 1, 2, \cdots, n_2$, $\bar{\xi}_d > 0$ and $\xi^*_{cd} > 0$ are known scalars with $\xi^*_{cd} = \xi^*_{dc}$.

To facilitate the later analysis, we denote some auxiliary matrices as follows:

$$\bar{\Xi} \triangleq \operatorname{diag}\{\bar{\xi}_1, \bar{\xi}_2, \cdots, \bar{\xi}_{n_2}\},$$
$$\Upsilon \triangleq \operatorname{diag}\{\xi^*_{11}, \xi^*_{22}, \cdots, \xi^*_{n_2 n_2}\},$$
$$\bar{\Upsilon} \triangleq [\xi^*_{cd}]_{c,d=1,2,\cdots,n_2}.$$

It is easy to see that $\bar{\Xi} > 0$, $\Upsilon > 0$ and $\bar{\Upsilon} \ge 0$.

By defining $\bar{y}(k) \triangleq \begin{bmatrix}\bar{y}_1^T(k) & \bar{y}_2^T(k)\end{bmatrix}^T$ as the whole system measurements after transmitted via the communication network, we have from (5) and (8) that

$$\bar{y}(k) = \big(\bar{C}_{\sigma(k)} + \tilde{C}(k)\big)x(k) + \big(\bar{F}_{\sigma(k)} + \tilde{F}(k)\big)\omega(k) \tag{10}$$

where

$$\bar{C}_{\sigma(k)} \triangleq \begin{bmatrix}\Lambda_{\sigma(k)} C_1 \\ \bar{\Xi} C_2\end{bmatrix}, \quad \tilde{C}(k) \triangleq \begin{bmatrix}0 \\ \tilde{\Xi}(k) C_2\end{bmatrix},$$
$$\bar{F}_{\sigma(k)} \triangleq \begin{bmatrix}\Lambda_{\sigma(k)} F_1 \\ \bar{\Xi} F_2\end{bmatrix}, \quad \tilde{F}(k) \triangleq \begin{bmatrix}0 \\ \tilde{\Xi}(k) F_2\end{bmatrix},$$
$$\tilde{\Xi}(k) \triangleq \operatorname{diag}\{\xi_1(k) - \bar{\xi}_1, \xi_2(k) - \bar{\xi}_2, \cdots, \xi_{n_2}(k) - \bar{\xi}_{n_2}\}.$$

Remark 3: Until now, we have established the signal transmission model for fuzzy system (2). In particular, the effects caused by the complex transmission environment for two sensor groups are characterized by two kinds of channel fadings. In this sense, the considered channel fadings are called to be *mixed* ones, and the proposed mixed fading models (5) and (8) would better reflect the engineering practice, thereby broadening the application scope of our obtained results. For example, if we let $n_1 = n_y$ and $n_2 = 0$ where n_y denotes the dimension of the whole measurement output, then our results reduce to those subject to the single FSMCFs. Similarly, if we let $n_1 = 0$ and $n_2 = n_y$, then our results specialize to those for the single i.i.d.CFs.

C. Fuzzy PID Controller

In this paper, by utilizing the available outputs $\bar{y}(k)$ and the observed mode information $\theta(k)$, we adopt a discrete-type fuzzy PID controller as follows.

Controller Rule j: IF $\bar{\rho}_1(k)$ is $\bar{W}_{j1}$, and $\bar{\rho}_2(k)$ is $\bar{W}_{j2}$, and $\cdots$, and $\bar{\rho}_{\tilde{s}}(k)$ is $\bar{W}_{j\tilde{s}}$, THEN

$$\begin{aligned} u(k) = {} & K^P_{j\theta(k)} \bar{y}(k) + K^I_{j\theta(k)} \sum_{\tau=0}^{k-1} \bar{y}(\tau) \\ & + K^D_{j\theta(k)} \big(\bar{y}(k) - \bar{y}(k-1)\big) \end{aligned} \tag{11}$$

where $K^P_{j\theta(k)}$, $K^I_{j\theta(k)}$ and $K^D_{j\theta(k)}$ $(j \in \{1, 2, \cdots, \bar{r}\} \triangleq \mathbb{I}_1,\ \theta(k) \in \bar{\mathbb{L}})$ are controller gains to be designed.

Taking the phenomenon of the CPPs into account, the controller (11) can be further described by the following compact form:

$$\begin{aligned} u(k) = \sum_{j=1}^{\bar{r}} \varphi_j(\bar{\rho}(k)) \Big(& \big(K^P_{j\theta(k)} + \Delta K^P_{j\theta(k)}(k)\big)\bar{y}(k) \\ & + \big(K^I_{j\theta(k)} + \Delta K^I_{j\theta(k)}(k)\big) \sum_{\tau=0}^{k-1} \bar{y}(\tau) \\ & + \big(K^D_{j\theta(k)} + \Delta K^D_{j\theta(k)}(k)\big)\big(\bar{y}(k) - \bar{y}(k-1)\big) \Big) \end{aligned} \tag{12}$$

where

$$\bar{\rho}(k) \triangleq \begin{bmatrix}\bar{\rho}_1(k) & \bar{\rho}_2(k) & \cdots & \bar{\rho}_{\tilde{s}}(k)\end{bmatrix}^T,$$
$$\varphi_j(\bar{\rho}(k)) \triangleq \frac{\prod_{n=1}^{\tilde{s}} \bar{W}_{jn}(\bar{\rho}_n(k))}{\sum_{j=1}^{\bar{r}} \prod_{n=1}^{\tilde{s}} \bar{W}_{jn}(\bar{\rho}_n(k))},$$

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$$\Delta K^P_{j\theta(k)}(k) \triangleq M^P_{j\theta(k)}\Delta_P(k)N_P,$$
$$\Delta K^I_{j\theta(k)}(k) \triangleq M^I_{j\theta(k)}\Delta_I(k)N_I,$$
$$\Delta K^D_{j\theta(k)}(k) \triangleq M^D_{j\theta(k)}\Delta_D(k)N_D$$

and $\bar{\rho}(k)$ is the premise vector of the controller. It is assumed that $\rho(k)$ and $\bar{\rho}(k)$ are independent of $\xi_a(k)$ ($a = 1, 2, \cdots, n_2$). $\bar{W}_{ij}$ is the fuzzy set. $M^P_{j\theta(k)}$, $M^I_{j\theta(k)}$, $M^D_{j\theta(k)}$, N_P, N_I and N_D are known constant matrices of proper dimensions. $\Delta_P(k)$, $\Delta_I(k)$, $\Delta_D(k)$ are unknown time-varying functions that satisfy [30]:

$$\Delta^T_P(k)\Delta_P(k) \le I,\ \Delta^T_I(k)\Delta_I(k) \le I,\ \Delta^T_D(k)\Delta_D(k) \le I.$$

Remark 4: In the non-fragile fuzzy PID controller (12), the underlying parameter perturbations in proportional, integral and derivative terms of all fuzzy submodels are all taken into account. Obviously, the design of such a controller is more difficult than that of non-fragile state-feedback one or static output-feedback one, where the CPPs only appear in the proportional term [49], [50]. In addition, different from the existing fuzzy PID controllers [26]–[29], the proposed fuzzy controller is not required to share the same premise variables and fuzzy rules with the fuzzy plant. In fact, such a controller can be regarded as a kind of non-PDC one that would increase the design flexibility [51].

To facilitate the system analysis and the controller synthesis, we define the following variable:

$$x_I(k) \triangleq \begin{cases} 0, & k = 0 \\ \sum_{\tau=0}^{k-1}\bar{y}(\tau), & k > 0. \end{cases} \tag{13}$$

Then, we have

$$x_I(k+1) = \sum_{\tau=0}^{k}\bar{y}(\tau) = x_I(k) + \bar{y}(k). \tag{14}$$

Considering (2), (12) and (14), we obtain the closed-loop system as follows:

$$\begin{cases} \eta(k+1) = \sum_{i=1}^{r}\sum_{j=1}^{\bar{r}}\phi_i(\rho(k))\varphi_j(\bar{\rho}(k)) \\ \qquad \times \Big(\big(\mathcal{A}^{ij}_{\sigma(k)\theta(k)}(k) + \mathcal{B}^{ij}_{\theta(k)}(k)\tilde{I}\tilde{\Xi}(k)C_2\bar{I}\big)\eta(k) \\ \qquad + \big(\mathcal{E}^{ij}_{\sigma(k)\theta(k)}(k) + \mathcal{B}^{ij}_{\theta(k)}(k)\tilde{I}\tilde{\Xi}(k)F_2\big)\omega(k)\Big) \\ z(k) = \sum_{i=1}^{r}\phi_i(\rho(k))\mathcal{G}_i\eta(k) \end{cases} \tag{15}$$

where

$$\eta(k) \triangleq \begin{bmatrix} x^T(k) & x^T_I(k) & \bar{y}^T(k-1) \end{bmatrix}^T,$$
$$\mathcal{A}^{ij}_{\sigma(k)\theta(k)}(k) \triangleq \begin{bmatrix} \mathcal{A}^{ij11}_{\sigma(k)\theta(k)}(k) & \mathcal{A}^{ij12}_{\theta(k)}(k) & \mathcal{A}^{ij13}_{\theta(k)}(k) \\ \bar{C}_{\sigma(k)} & I & 0 \\ \bar{C}_{\sigma(k)} & 0 & 0 \end{bmatrix},$$
$$\mathcal{A}^{ij11}_{\sigma(k)\theta(k)}(k) \triangleq A_i + B_i\bar{K}_{j\theta(k)}(k)\bar{C}_{\sigma(k)},\ \mathcal{G}_i \triangleq \begin{bmatrix} G_i & 0 & 0 \end{bmatrix},$$
$$\bar{K}_{j\theta(k)}(k) \triangleq K^P_{j\theta(k)} + \Delta K^P_{j\theta(k)}(k) + K^D_{j\theta(k)} + \Delta K^D_{j\theta(k)}(k),$$
$$\mathcal{A}^{ij12}_{\theta(k)}(k) \triangleq B_iK^I_{j\theta(k)} + B_i\Delta K^I_{j\theta(k)}(k),$$
$$\mathcal{A}^{ij13}_{\theta(k)}(k) \triangleq -B_iK^D_{j\theta(k)} - B_i\Delta K^D_{j\theta(k)}(k),$$
$$\mathcal{B}^{ij}_{\theta(k)}(k) \triangleq \begin{bmatrix} B_i\bar{K}_{j\theta(k)}(k) \\ I \\ I \end{bmatrix},\quad \bar{I} \triangleq \begin{bmatrix} I & 0 & 0 \end{bmatrix},$$
$$\mathcal{E}^{ij}_{\sigma(k)\theta(k)}(k) \triangleq \begin{bmatrix} E_i + B_i\bar{K}_{j\theta(k)}(k)\Xi_{\sigma(k)}F \\ \Xi_{\sigma(k)}F \\ \Xi_{\sigma(k)}F \end{bmatrix},\ \tilde{I} \triangleq \begin{bmatrix} 0 \\ I \end{bmatrix}.$$

Before proceeding further, we introduce the following definitions.

Definition 1: [30] The closed-loop system (15) is said to be exponentially mean-square stable if, for $\omega(k) = 0$, there exist constants $s_1 > 0$ and $s_2 \in (0, 1)$ such that

$$\mathbb{E}\left\{\|\eta(k)\|^2\right\} \le s_1 s_2^k \mathbb{E}\left\{\|\eta(0)\|^2\right\}. \tag{16}$$

Definition 2: [9] For given real matrices $S_1 = S_1^T \le 0$, $S_3 = S_3^T > 0$, $S_4 = S_4^T \ge 0$ and arbitrary matrix S_2 satisfying $(\|S_1\| + \|S_2\|)\cdot\|S_4\| = 0$, the fuzzy system (15) is said to be extended stochastically dissipative if, for any $\omega(k) \in l_2[0, +\infty)$, any integer $T > 0$ and under the zero initial condition $\eta(0) = 0$, the following inequality is satisfied:

$$\mathbb{E}\left\{\sum_{k=0}^{T} J(S_1, S_2, S_3, k)\right\} \ge \sup_{0\le k\le T}\mathbb{E}\left\{z^T(k)S_4z(k)\right\} \tag{17}$$

where

$$J(S_1, S_2, S_3, k) \triangleq z^T(k)S_1z(k) + 2z^T(k)S_2\omega(k) + \omega^T(k)S_3\omega(k).$$

Remark 5: The fulfillment of the inequality (17) implies the achievement of the H_∞, l_2-l_∞, passivity and standard dissipativity performance. To be more specific, we have the following:

1) if we set $S_1 = S_2 = 0$, $S_3 > 0$ and $S_4 > 0$, then (17) reduces to the l_2-l_∞ performance index;

2) if we set $S_2 = S_4 = 0$, $S_1 < 0$ and $S_3 > 0$, then (17) reduces to the H_∞ performance index; and

3) if we set $S_4 = 0$, $S_1 \le 0$ and $S_3 > 0$, then (17) reduces to the standard dissipativity/passivity performance index.

In this paper, we aim to design the non-fragile fuzzy PID controller such that the following two requirements are met simultaneously:

R1) the closed-loop system (15) is exponentially mean-square stable in the sense of Definition 1; and

R2) the closed-loop system (15) is extended stochastically dissipative in the sense of Definition 2.

III. Main Results

In order to derive the main results, we first define the following notations and introduce some helpful lemmas.

$$\sum_{i=1}^{r}\sum_{j=1}^{\bar{r}}\phi_i\varphi_j \triangleq \sum_{i=1}^{r}\sum_{j=1}^{\bar{r}}\phi_i(\rho(k))\varphi_j(\bar{\rho}(k)),$$
$$\sum_{i=1}^{r}\sum_{j=1}^{\bar{r}}\sum_{l=1}^{r}\sum_{m=1}^{\bar{r}}\phi_i\varphi_j\phi_l\varphi_m$$

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$$\triangleq \sum_{i=1}^{r}\sum_{j=1}^{\bar{r}}\sum_{l=1}^{r}\sum_{m=1}^{\bar{r}} \phi_i(\rho(k))\varphi_j(\bar{\rho}(k))\phi_l(\rho(k))\varphi_m(\bar{\rho}(k)).$$

Lemma 1: [52] For real matrices Ω_{ij} $(i \in \mathbb{I},\ j \in \mathbb{I}_1)$ and matrix $S > 0$, we have

$$\sum_{i=1}^{r}\sum_{j=1}^{\bar{r}}\sum_{l=1}^{r}\sum_{m=1}^{\bar{r}} \phi_i\varphi_j\phi_l\varphi_m\Omega_{ij}^T S\Omega_{lm} \leq \sum_{i=1}^{r}\sum_{j=1}^{\bar{r}} \phi_i\varphi_j\Omega_{ij}^T S\Omega_{ij}.$$

Lemma 2: [44] For real matrices $A \geq 0$, $B \geq 0$ and $C \geq 0$, if the condition $B \leq C$ is satisfied, then, $A \circ B \leq A \circ C$ holds.

Lemma 3: [30] For real matrices $X = X^T$, M, N with appropriate dimensions and F satisfying $F^TF \leq I$, then, the following inequality holds

$$X + MFN + N^TF^TM^T < 0$$

if and only if there exists a scalar $\mu > 0$ such that

$$\begin{bmatrix} X & * & * \\ \mu M^T & -\mu I & * \\ N & 0 & -\mu I \end{bmatrix} < 0.$$

The following theorem is presented for the analysis of the stability and dissipativity of the closed-loop system (15).

Theorem 1: Consider the fuzzy system (1) and the fuzzy non-fragile PID controller (12) with the given controller parameters and matrices $S_1 = S_1^T \leq 0$, $S_3 = S_3^T > 0$, $S_4 = S_4^T \geq 0$, S_2. Then, the closed-loop system (15) is exponentially mean-square stable and extended stochastically dissipative if there exist matrices $P_t > 0$ and scalars $\alpha_{ijtn} > 0$ (for $\forall i \in \mathbb{I},\ j \in \mathbb{I}_1,\ t \in \mathbb{L}$ and $n \in \bar{\mathbb{L}}$) such that

$$\left(\mathcal{B}_n^{ij}(k)\tilde{I}\right)^T \bar{P}_t\mathcal{B}_n^{ij}(k)\tilde{I} \leq \alpha_{ijtn}I \tag{18}$$

$$\sum_{n=1}^{\bar{s}} \delta_{tn}\left[\left(\bar{\mathcal{A}}_{tn}^{ij}(k)\right)^T \tilde{P}_{tn}^{ij}\bar{\mathcal{A}}_{tn}^{ij}(k) + \bar{S}_t^i\right] < 0 \tag{19}$$

$$-P_t + \mathcal{G}_i^T S_4\mathcal{G}_i < 0 \tag{20}$$

where $\bar{P}_t = \sum_{v=1}^{\bar{l}} \pi_{tv}P_v$ and

$$\bar{\mathcal{A}}_{tn}^{ij}(k) \triangleq \begin{bmatrix}\bar{\mathcal{A}}_{tn}^{1ij}(k) & \bar{\mathcal{A}}_{tn}^{2ij}(k)\end{bmatrix},\quad \bar{\mathcal{A}}_{tn}^{1ij}(k) \triangleq \begin{bmatrix}\mathcal{A}_{tn}^{ij}(k) \\ C_2\bar{I}\end{bmatrix},$$

$$\bar{\mathcal{A}}_{tn}^{2ij}(k) \triangleq \begin{bmatrix}\mathcal{E}_{tn}^{ij}(k) \\ F_2\end{bmatrix},\quad \tilde{P}_{tn}^{ij} \triangleq \begin{bmatrix}\bar{P}_t & 0 \\ 0 & \alpha_{ijtn}\Upsilon_t\end{bmatrix},$$

$$\bar{S}_t^i \triangleq \begin{bmatrix}-P_t - \mathcal{G}_i^TS_1\mathcal{G}_i & * \\ -S_2^T\mathcal{G}_i & -S_3\end{bmatrix}.$$

Proof: Choose a mode-dependent Lyapunov function as follows:

$$V(k,\sigma(k)) = \eta^T(k)P_{\sigma(k)}\eta(k). \tag{21}$$

For $\sigma(k) = t$ $(t \in \mathbb{L})$, by calculating the difference of $V(k,\sigma(k))$ and applying Lemma 1, we have

$$\begin{aligned}
&\Delta V(k,\sigma(k)) \\
&= V(k+1,\sigma(k+1)) - V(k,\sigma(k)) \\
&= \sum_{i=1}^{r}\sum_{j=1}^{\bar{r}}\sum_{l=1}^{r}\sum_{m=1}^{\bar{r}} \phi_i\varphi_j\phi_l\varphi_m \\
&\quad \times \Big[\Big(\mathcal{A}_{t\theta(k)}^{ij}(k) + \mathcal{B}_{\theta(k)}^{ij}(k)\tilde{I}\tilde{\Xi}(k)C_2\bar{I}\Big)\eta(k) \\
&\quad + \Big(\mathcal{E}_{t\theta(k)}^{ij}(k) + \mathcal{B}_{\theta(k)}^{ij}(k)\tilde{I}\tilde{\Xi}(k)F_2\Big)\omega(k)\Big]^T P_{\sigma(k+1)} \\
&\quad \times \Big[\Big(\mathcal{A}_{t\theta(k)}^{lm}(k) + \mathcal{B}_{\theta(k)}^{lm}(k)\tilde{I}\tilde{\Xi}(k)C_2\bar{I}\Big)\eta(k) \\
&\quad + \Big(\mathcal{E}_{t\theta(k)}^{lm}(k) + \mathcal{B}_{\theta(k)}^{lm}(k)\tilde{I}\tilde{\Xi}(k)F_2\Big)\omega(k)\Big] - \eta^T(k)P_t\eta(k) \\
&\leq \sum_{i=1}^{r}\sum_{j=1}^{\bar{r}} \phi_i\varphi_j\Big[\Big(\mathcal{A}_{t\theta(k)}^{ij}(k) + \mathcal{B}_{\theta(k)}^{ij}(k)\tilde{I}\tilde{\Xi}(k)C_2\bar{I}\Big)\eta(k) \\
&\quad + \Big(\mathcal{E}_{t\theta(k)}^{ij}(k) + \mathcal{B}_{\theta(k)}^{ij}(k)\tilde{I}\tilde{\Xi}(k)F_2\Big)\omega(k)\Big]^T P_{\sigma(k+1)} \\
&\quad \times \Big[\Big(\mathcal{A}_{t\theta(k)}^{ij}(k) + \mathcal{B}_{\theta(k)}^{ij}(k)\tilde{I}\tilde{\Xi}(k)C_2\bar{I}\Big)\eta(k) \\
&\quad + \Big(\mathcal{E}_{t\theta(k)}^{ij}(k) + \mathcal{B}_{\theta(k)}^{ij}(k)\tilde{I}\tilde{\Xi}(k)F_2\Big)\omega(k)\Big] - \eta^T(k)P_t\eta(k).
\end{aligned} \tag{22}$$

On the basis of (22), it is calculated that

$$\begin{aligned}
&\mathbb{E}\{\Delta V(k,\sigma(k))|\eta(k),\sigma(k)=t\} \\
&\triangleq \mathbb{E}\{V(k+1,\sigma(k+1))|\eta(k),\sigma(k)=t\} - V(k,t) \\
&= \mathbb{E}\{\eta^T(k+1)\bar{P}_t\eta(k+1)|\eta(k),\sigma(k)=t\} - V(k,t) \\
&\leq \sum_{i=1}^{r}\sum_{j=1}^{\bar{r}} \phi_i\varphi_j\sum_{n=1}^{\bar{s}}\delta_{tn} \\
&\quad \times \mathbb{E}\Big\{\Big[\Big(\mathcal{A}_{tn}^{ij}(k) + \mathcal{B}_n^{ij}(k)\tilde{I}\Xi(k)C_2\bar{I}\Big)\eta(k) \\
&\quad + \Big(\mathcal{E}_{tn}^{ij}(k) + \mathcal{B}_n^{ij}(k)\tilde{I}\tilde{\Xi}(k)F_2\Big)\omega(k)\Big]^T \bar{P}_t \\
&\quad \times \Big[\Big(\mathcal{A}_{tn}^{ij}(k) + \mathcal{B}_n^{ij}(k)\tilde{I}\Xi(k)C_2\bar{I}\Big)\eta(k) \\
&\quad + \Big(\mathcal{E}_{tn}^{ij}(k) + \mathcal{B}_n^{ij}(k)\tilde{I}\tilde{\Xi}(k)F_2\Big)\omega(k)\Big] \\
&\quad - \eta^T(k)P_t\eta(k)|\eta(k),\sigma(k)=t\Big\} \\
&= \sum_{i=1}^{r}\sum_{j=1}^{\bar{r}} \phi_i\varphi_j\sum_{n=1}^{\bar{s}}\delta_{tn}\Big[\Big(\mathcal{A}_{tn}^{ij}(k)\eta(k) + \mathcal{E}_{tn}^{ij}(k)\omega(k)\Big)^T \bar{P}_t \\
&\quad \times \Big(\mathcal{A}_{tn}^{ij}(k)\eta(k) + \mathcal{E}_{tn}^{ij}(k)\omega(k)\Big) + \big(C_2\bar{I}\eta(k) + F_2\omega(k)\big)^T \\
&\quad \times \Big(\bar{\Upsilon}_t \circ \Big((\mathcal{B}_n^{ij}(k)\tilde{I})^T\bar{P}_t\mathcal{B}_n^{ij}(k)\tilde{I}\Big)\Big)\big(C_2\bar{I}\eta(k) + F_2\omega(k)\big) \\
&\quad - \eta^T(k)P_t\eta(k)\Big].
\end{aligned} \tag{23}$$

Based on the condition (18), it follows from Lemma 2 that

$$\bar{\Upsilon}_t \circ \Big((\mathcal{B}_n^{ij}(k)\tilde{I})^T\bar{P}_t\mathcal{B}_n^{ij}(k)\tilde{I}\Big) \leq \bar{\Upsilon}_t \circ \alpha_{ijtn}I = \alpha_{ijtn}\Upsilon_t. \tag{24}$$

Then, we have from (23) and (24) that

$$\mathbb{E}\{V(k+1,\sigma(k+1))|\eta(k),\sigma(k)=t\} - V(k,t)$$

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$$\leq \sum_{i=1}^{r}\sum_{j=1}^{\bar{r}}\phi_i\varphi_j\sum_{n=1}^{\bar{s}}\delta_{tn} \times \zeta^T(k)\left[\left(\bar{\mathcal{A}}_{tn}^{ij}(k)\right)^T\tilde{P}_{tn}^{ij}\bar{\mathcal{A}}_{tn}^{ij}(k)+\hat{S}_t\right]\zeta(k) \quad (25)$$

where

$$\zeta(k)\triangleq\begin{bmatrix}\eta^T(k) & \omega^T(k)\end{bmatrix}^T,\quad \hat{S}_t\triangleq\begin{bmatrix}-P_t & 0\\ 0 & 0\end{bmatrix}.$$

Now, to deal with the stability analysis, we let $\omega(k)=0$ and then obtain from (25) that

$$\begin{aligned}&\mathbb{E}\{V(k+1,\sigma(k+1))|\eta(k),\sigma(k)=t\}-V(k,t)\\ &\leq \sum_{i=1}^{r}\sum_{j=1}^{\bar{r}}\phi_i\varphi_j\sum_{n=1}^{\bar{s}}\delta_{tn}\\ &\quad\times\eta^T(k)\left[\left(\bar{\mathcal{A}}_{tn}^{1ij}(k)\right)^T\tilde{P}_{tn}^{ij}\bar{\mathcal{A}}_{tn}^{1ij}(k)-P_t\right]\eta(k)\\ &\triangleq\eta^T(k)\sum_{i=1}^{r}\sum_{j=1}^{\bar{r}}\phi_i\varphi_j\sum_{n=1}^{\bar{s}}\delta_{tn}Q_{ijtn}(k)\eta(k). \end{aligned}\quad (26)$$

Next, it is obtained from (19) that $\sum_{n=1}^{\bar{s}}\delta_{tn}Q_{ijtn}(k)<0$ is satisfied. By further considering the property of membership functions (3), we know that there always exists a sufficiently small scalar $\varrho>0$ such that

$$\sum_{i=1}^{r}\sum_{j=1}^{\bar{r}}\phi_i\varphi_j\sum_{n=1}^{\bar{s}}\delta_{tn}Q_{ijtn}(k)\leq-\varrho I, \quad (27)$$

which yields

$$\begin{aligned}&\mathbb{E}\{V(k+1,\sigma(k+1))|\eta(k),\sigma(k)=t\}-V(k,\sigma(k))\\ &\leq-\varrho\eta^T(k)\eta(k)\\ &=-\varrho\|\eta(k)\|^2.\end{aligned}\quad (28)$$

By taking the mathematical expectation on both sides of (28), we have

$$\mathbb{E}\{V(k+1,\sigma(k+1))-V(k,\sigma(k))\}\leq-\varrho\mathbb{E}\{\|\eta(k)\|^2\}, \quad (29)$$

from which we have for any scalar $\epsilon>0$ that

$$\begin{aligned}&\mathbb{E}\{\epsilon^{k+1}V(k+1,\sigma(k+1))\}-\mathbb{E}\{\epsilon^kV(k,\sigma(k))\}\\ &=\epsilon^{k+1}\mathbb{E}\{V(k+1,\sigma(k+1))-V(k,\sigma(k))\}\\ &\quad+\epsilon^k(\epsilon-1)\mathbb{E}\{V(k,\sigma(k))\}\\ &\leq\big(-\varrho\epsilon^{k+1}+\epsilon^k(\epsilon-1)\bar{\lambda}\big)\mathbb{E}\{\|\eta(k)\|^2\}.\end{aligned}\quad (30)$$

where

$$\bar{\lambda}\triangleq\max\{\lambda_{\max}(P_1),\lambda_{\max}(P_2),\cdots,\lambda_{\max}(P_{\bar{l}})\}.$$

For any integer $N\geq1$, summing up both sides of (30) from $k=0$ to $k=N-1$, we have

$$\begin{aligned}&\sum_{k=0}^{N-1}\big(\mathbb{E}\{\epsilon^{k+1}V(k+1,\sigma(k+1))\}-\mathbb{E}\{\epsilon^kV(k,\sigma(k)\}\big)\\ &\leq\sum_{k=0}^{N-1}\epsilon^k\big((\epsilon-1)\bar{\lambda}-\varrho\epsilon\big)\mathbb{E}\{\|\eta(k)\|^2\}\end{aligned}\quad (31)$$

which implies that

$$\begin{aligned}&\mathbb{E}\{\epsilon^NV(N,\sigma(N))\}-\mathbb{E}\{V(0,\sigma(0))\}\\ &\leq\sum_{k=0}^{N-1}\epsilon^k\alpha(\epsilon)\mathbb{E}\{\|\eta(k)\|^2\}\end{aligned}\quad (32)$$

where

$$\alpha(\epsilon)\triangleq(\epsilon-1)\bar{\lambda}-\varrho\epsilon.$$

Since $\alpha(1)=-\varrho<0$ and $\lim_{b\to+\infty}\alpha(b)=+\infty$, we know that there exists a scalar $\epsilon_0>1$ such that $\alpha(\epsilon_0)=0$. Thus, by considering

$$\begin{aligned}&\mathbb{E}\{V(0,\sigma(0))\}\leq\bar{\lambda}\mathbb{E}\{\|\eta(0)\|^2\},\\ &\mathbb{E}\{V(N,\sigma(N))\}\geq\underline{\lambda}\mathbb{E}\{\|\eta(N)\|^2\}\end{aligned}$$

where

$$\underline{\lambda}\triangleq\min\{\lambda_{\min}(P_1),\lambda_{\min}(P_2),\cdots,\lambda_{\min}(P_{\bar{l}})\},$$

we obtain

$$\mathbb{E}\{\|\eta(N)\|^2\}\leq\frac{\bar{\lambda}}{\underline{\lambda}}\left(\frac{1}{\epsilon_0}\right)^N\mathbb{E}\{\|\eta(0)\|^2\}.$$

By letting $N=k$ and from Definition 1, we know that the closed-loop system is exponentially mean-square stable.

We are now in a position to show the extended stochastic dissipativity of the closed-loop system. By introducing $J(S_1,S_2,S_3,k)$ defined in (17), for the case of $\omega(k)\neq0$, it follows from (25) that

$$\begin{aligned}&\mathbb{E}\{V(k+1,\sigma(k+1))|\eta(k),\sigma(k)=t\}-V(k,t)\\ &-J(S_1,S_2,S_3,k)\\ &\leq\sum_{i=1}^{r}\sum_{j=1}^{\bar{r}}\phi_i\varphi_j\sum_{n=1}^{\bar{s}}\delta_{tn}\\ &\times\zeta^T(k)\left[\left(\bar{\mathcal{A}}_{tn}^{ij}(k)\right)^T\tilde{P}_{tn}^{ij}\bar{\mathcal{A}}_{tn}^{ij}(k)+\bar{S}_t^i\right]\zeta(k).\end{aligned}\quad (33)$$

From (19), the property of membership functions (3) and through the expectation operation, we deduce from (33) that

$$\mathbb{E}\{\Delta V(k,\sigma(k))-J(S_1,S_2,S_3,k)\}<0. \quad (34)$$

Now, let us prove the extended dissipativity of the system (15) in two different cases, i.e., $S_4=0$ and $S_4>0$.

Case 1: If $S_4=0$, then the performance index (17) changes to

$$\mathbb{E}\left\{\sum_{k=0}^{T}J(S_1,S_2,S_3,k)\right\}\geq0. \quad (35)$$

It follows from (34) that

$$\mathbb{E}\{J(S_1,S_2,S_3,k)-\Delta V(k,\sigma(k))\}>0$$

which implies

$$\begin{aligned}&\sum_{k=0}^{T}\mathbb{E}\{J(S_1,S_2,S_3,k)-\Delta V(k,\sigma(k))\}\\ &=\sum_{k=0}^{T}\mathbb{E}\{J(S_1,S_2,S_3,k)\}+\mathbb{E}\{V(0,\sigma(0))\}\end{aligned}$$

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$$- \mathbb{E}\left\{V(T+1, \sigma(T+1))\right\} > 0, \quad \forall T > 0. \tag{36}$$

Since $\mathbb{E}\left\{V(0, \sigma(0))\right\} = 0$ (under the zero initial condition) and $\mathbb{E}\left\{V(T+1, \sigma(T+1))\right\} \geq 0$, we can conclude that (35) is satisfied.

Case 2: If $S_4 > 0$, we have $S_1 = S_2 = 0$ from the constraint $(\|S_1\| + \|S_2\|) \cdot \|S_4\| = 0$. Then, the performance index (17) is converted to

$$\sum_{k=0}^{T} \mathbb{E}\left\{\omega^T(k) S_3 \omega(k)\right\} \geq \sup_{0 \leq k \leq T} \mathbb{E}\left\{z^T(k) S_4 z(k)\right\}. \tag{37}$$

For $0 < k \leq T$, we derive from (34) that

$$\begin{aligned}
&\sum_{\tau=0}^{k-1} \mathbb{E}\left\{\Delta V(\tau) - \omega^T(\tau) S_3 \omega(\tau)\right\} \\
&\quad = \mathbb{E}\left\{V(k)\right\} - \sum_{\tau=0}^{k-1} \mathbb{E}\left\{\omega^T(\tau) S_3 \omega(\tau)\right\} < 0
\end{aligned} \tag{38}$$

which implies

$$\begin{aligned}
\mathbb{E}\left\{V(k, \sigma(k))\right\} =& \mathbb{E}\left\{\eta^T(k) P_{\sigma(k)} \eta(k)\right\} \\
&< \sum_{\tau=0}^{k-1} \mathbb{E}\left\{\omega^T(\tau) S_3 \omega(\tau)\right\} \\
&\leq \sum_{\tau=0}^{T} \mathbb{E}\left\{\omega^T(\tau) S_3 \omega(\tau)\right\}, \quad 0 < k \leq T.
\end{aligned} \tag{39}$$

From (21), we also have

$$\begin{aligned}
&\mathbb{E}\left\{z^T(k) S_4 z(k)\right\} - \mathbb{E}\left\{V(k, \sigma(k))\right\} \\
&= \sum_{i=1}^{r} \sum_{j=1}^{r} \mathbb{E}\left\{\phi_i \phi_j \eta^T(k) \mathcal{G}_i^T S_4 \mathcal{G}_j \eta(k) - \eta^T(k) P_{\sigma(k)} \eta(k)\right\} \\
&\leq \sum_{i=1}^{r} \mathbb{E}\left\{\phi_i \eta^T(k) \big(\mathcal{G}_i^T S_4 \mathcal{G}_i - P_{\sigma(k)}\big) \eta(k)\right\}.
\end{aligned} \tag{40}$$

By utilizing the condition (20), we arrive at

$$\begin{aligned}
\mathbb{E}\left\{z^T(k) S_4 z(k)\right\} <& \mathbb{E}\left\{V(k, \sigma(k))\right\} \\
&< \sum_{\tau=0}^{T} \mathbb{E}\left\{\omega^T(\tau) S_3 \omega(\tau)\right\}, \quad 0 < k \leq T
\end{aligned} \tag{41}$$

and therefore (37) is satisfied. The proof is now complete. ■

Theorem 1 provides a sufficient condition for the analysis of the system stability as well as the extended dissipativity. Based on this, we will deal with the controller design problem in Theorem 2 under the considered performance requirements.

Theorem 2: Consider the fuzzy system (1) and the non-fragile fuzzy PID controller. Let matrices $S_1 = S_1^T \leq 0$, $S_3 = S_3^T > 0$, $S_4 = S_4^T \geq 0$ and S_2 be given. Then, the closed-loop system (15) is exponentially mean-square stable and extended stochastically dissipative if there exist matrices $P_t > 0$, $X_t > 0$, $R_{ijtn} > 0$, K_{jn}^P, K_{jn}^I, K_{jn}^D, scalars $\alpha_{ijtn} > 0$, $\mu_{ijtn}^a > 0$ and $\mu_{ijtn}^b > 0$ satisfying the following conditions for $\forall i \in \mathbb{I},\ j \in \mathbb{I}_1,\ t \in \mathbb{L},\ n \in \bar{\mathbb{L}}$:

$$\begin{bmatrix} \tilde{\Gamma}_{ijtn} & * & * \\ \mu_{ijtn}^a \tilde{M}_{ijtn}^T & -\mu_{ijtn}^a I & * \\ \tilde{N} & 0 & -\mu_{ijtn}^a I \end{bmatrix} < 0 \tag{42}$$

$$\begin{bmatrix} \bar{\Gamma}_{ijtn} & * & * \\ \mu_{ijtn}^b \bar{M}_{ijtn}^T & -\mu_{ijtn}^b I & * \\ \bar{N}_t & 0 & -\mu_{ijtn}^b I \end{bmatrix} < 0 \tag{43}$$

$$\sum_{n=1}^{\bar{s}} \delta_{tn} \left(R_{ijtn} + \bar{S}_t^i\right) < 0 \tag{44}$$

$$-P_t + \mathcal{G}_i^T S_4 \mathcal{G}_i < 0 \tag{45}$$

$$P_t X_t = I \tag{46}$$

where

$$\begin{aligned}
\bar{\Gamma}_{ijtn}^{22} &\triangleq \mathrm{diag}\left\{\bar{X}, -\alpha_{ijtn} \Upsilon_t^{-1}\right\}, \\
\bar{X} &\triangleq \mathrm{diag}\left\{-X_1, -X_2, \cdots, -X_{\bar{l}}\right\}, \\
\bar{\Gamma}_{ijtn} &\triangleq \begin{bmatrix} -R_{ijtn} & * \\ \bar{\Gamma}_{ijtn}^{21} & \bar{\Gamma}_{ijtn}^{22} \end{bmatrix}, \quad \bar{M}_{ijtn} \triangleq \begin{bmatrix} \bar{M}_{tn}^{ij11} & \bar{M}_{tn}^{ij12} \end{bmatrix}, \\
\bar{M}_{tn}^{ij11} &\triangleq \begin{bmatrix} \hat{I}_1 \hat{B}_i M_{jn}^P & \hat{I}_1 \hat{B}_i M_{jn}^D & \hat{I}_1 \hat{B}_i M_{jn}^I \end{bmatrix}, \\
\bar{M}_{tn}^{ij12} &\triangleq \begin{bmatrix} \hat{I}_1 \hat{B}_i M_{jn}^D & \hat{I}_1 \hat{B}_i M_{jn}^P & \hat{I}_1 \hat{B}_i M_{jn}^D \end{bmatrix}, \\
\hat{I}_1 &\triangleq \begin{bmatrix} 0 & 0 & \underbrace{I \quad I \quad \cdots \quad I}_{\bar{l}} & 0 \end{bmatrix}^T, \quad \bar{N}_t \triangleq \begin{bmatrix} \bar{N}_t^{11} \\ \bar{N}_t^{12} \end{bmatrix}, \\
\bar{N}_t^{11} &\triangleq \begin{bmatrix} N_P \bar{C}_t \bar{I}_1 \hat{I}_2 \\ N_D \bar{C}_t \bar{I}_1 \hat{I}_2 \\ N_I \bar{I}_2 \hat{I}_2 \end{bmatrix}, \quad \bar{N}_t^{12} \triangleq \begin{bmatrix} N_D \bar{I}_3 \hat{I}_2 \\ N_P \bar{F}_t \hat{I}_3 \\ N_D \bar{F}_t \hat{I}_3 \end{bmatrix}, \\
\bar{I}_1 &\triangleq \begin{bmatrix} I & 0 & 0 \end{bmatrix}, \quad \bar{I}_2 \triangleq \begin{bmatrix} 0 & I & 0 \end{bmatrix}, \\
\bar{I}_3 &\triangleq \begin{bmatrix} 0 & 0 & -I \end{bmatrix}, \quad \hat{I}_2 \triangleq [I \quad \underbrace{0 \quad 0 \quad \cdots \quad 0}_{\bar{l}+2}] \\
\hat{I}_3 &\triangleq [0 \quad I \quad \underbrace{0 \quad 0 \quad \cdots \quad 0}_{\bar{l}+1}], \\
\bar{B}_n^{ij} &\triangleq \begin{bmatrix} B_i K_{jn}^P + B_i K_{jn}^D \\ I \\ I \end{bmatrix}, \quad \hat{B}_i \triangleq \begin{bmatrix} B_i \\ 0 \\ 0 \end{bmatrix}, \\
\bar{\Gamma}_{ijtn}^{21} &\triangleq \begin{bmatrix} \sqrt{\pi_{t1}} \vec{A}_{tn}^{ij} & \sqrt{\pi_{t1}} \vec{E}_{tn}^{ij} \\ \sqrt{\pi_{t2}} \vec{A}_{tn}^{ij} & \sqrt{\pi_{t2}} \vec{E}_{tn}^{ij} \\ \vdots & \vdots \\ \sqrt{\pi_{t\bar{l}}} \vec{A}_{tn}^{ij} & \sqrt{\pi_{t\bar{l}}} \vec{E}_{tn}^{ij} \\ \alpha_{ijtn} C_2 \bar{I} & \alpha_{ijtn} F_2 \end{bmatrix}, \\
\tilde{M}_{ijtn} &\triangleq \begin{bmatrix} 0 & 0 \\ \sqrt{\pi_{t1}} \hat{B}_i M_{jn}^P & \sqrt{\pi_{t1}} \hat{B}_i M_{jn}^D \\ \sqrt{\pi_{t2}} \hat{B}_i M_{jn}^P & \sqrt{\pi_{t2}} \hat{B}_i M_{jn}^D \\ \vdots & \vdots \\ \sqrt{\pi_{t\bar{l}}} \hat{B}_i M_{jn}^P & \sqrt{\pi_{t\bar{l}}} \hat{B}_i M_{jn}^D \end{bmatrix}, \\
\tilde{N} &\triangleq \begin{bmatrix} N_P \tilde{I} & 0 & 0 & \cdots & 0 \\ N_D \tilde{I} & 0 & 0 & \cdots & 0 \end{bmatrix}, \\
\vec{A}_{tn}^{ij} &\triangleq \begin{bmatrix} \vec{A}_{tn}^{ij11} & B_i K_{jn}^I & -B_i K_{jn}^D \\ \bar{C}_t & I & 0 \\ \bar{C}_t & 0 & 0 \end{bmatrix}, \\
\vec{A}_{tn}^{ij11} &\triangleq A_i + B_i K_{jn}^P \bar{C}_t + B_i K_{jn}^D \bar{C}_t,
\end{aligned}$$

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$$\vec{E}_{tn}^{ij} \triangleq \begin{bmatrix} E_i + B_i K_{jn}^P \bar{F}_t + B_i K_{jn}^D \bar{F}_t \\ \bar{F}_t \\ \bar{F}_t \end{bmatrix},$$

$$\tilde{\Gamma}_{ijtn} \triangleq \begin{bmatrix} -\alpha_{ijtn} I & * & * & \cdots & * \\ \sqrt{\pi_{t1}} \bar{B}_n^{ij} \tilde{I} & -X_1 & * & \cdots & * \\ \sqrt{\pi_{t2}} \bar{B}_n^{ij} \tilde{I} & 0 & -X_2 & \cdots & * \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sqrt{\pi_{t\bar{l}}} \bar{B}_n^{ij} \tilde{I} & 0 & 0 & \cdots & -X_{\bar{l}} \end{bmatrix}.$$

Furthermore, if the conditions (42)-(46) are solvable, then the desired controller gains are obtained by K_{jn}^P, K_{jn}^I and K_{jn}^D directly.

Proof: It is straightforward to see that

$$\begin{aligned} &\left(\mathcal{B}_n^{ij}(k)\tilde{I}\right)^T \bar{P}_t \mathcal{B}_n^{ij}(k)\tilde{I} - \alpha_{ijtn} I \\ &= \begin{bmatrix} \sqrt{\pi_{t1}} \mathcal{B}_n^{ij}(k)\tilde{I} \\ \sqrt{\pi_{t2}} \mathcal{B}_n^{ij}(k)\tilde{I} \\ \vdots \\ \sqrt{\pi_{t\bar{l}}} \mathcal{B}_n^{ij}(k)\tilde{I} \end{bmatrix}^T \tilde{P} \begin{bmatrix} \sqrt{\pi_{t1}} \mathcal{B}_n^{ij}(k)\tilde{I} \\ \sqrt{\pi_{t2}} \mathcal{B}_n^{ij}(k)\tilde{I} \\ \vdots \\ \sqrt{\pi_{t\bar{l}}} \mathcal{B}_n^{ij}(k)\tilde{I} \end{bmatrix} - \alpha_{ijtn} I \\ &\triangleq \Phi_{ijtn}(k). \end{aligned} \tag{47}$$

where

$$\tilde{P} \triangleq \mathrm{diag}\{P_1, P_2, \cdots, P_{\bar{l}}\}.$$

According to the Schur Complement Lemma, $\Phi_{ijtn}(k) < 0$ if and only if the following holds:

$$\begin{aligned} &\begin{bmatrix} -\alpha_{ijtn} I & * & * & \cdots & * \\ \sqrt{\pi_{t1}} \mathcal{B}_n^{ij}(k)\tilde{I} & -P_1^{-1} & * & \cdots & * \\ \sqrt{\pi_{t2}} \mathcal{B}_n^{ij}(k)\tilde{I} & 0 & -P_2^{-1} & \cdots & * \\ \vdots & \vdots & \vdots & \ddots & \\ \sqrt{\pi_{t\bar{l}}} \mathcal{B}_n^{ij}(k)\tilde{I} & 0 & 0 & \cdots & -P_{\bar{l}}^{-1} \end{bmatrix} \\ &= \tilde{\Gamma}_{ijtn}^a + \tilde{M}_{ijtn} \tilde{\Delta}(k) \tilde{N} + \tilde{N}^T \tilde{\Delta}^T(k) \tilde{M}_{ijtn}^T < 0 \end{aligned} \tag{48}$$

where

$$\tilde{\Delta}(k) \triangleq \mathrm{diag}\{\Delta_P(k), \Delta_D(k)\},$$

$$\tilde{\Gamma}_{ijtn}^a \triangleq \begin{bmatrix} -\alpha_{ijtn} I & * & * & \cdots & * \\ \sqrt{\pi_{t1}} \bar{B}_n^{ij} \tilde{I} & -P_1^{-1} & * & \cdots & * \\ \sqrt{\pi_{t2}} \bar{B}_n^{ij} \tilde{I} & 0 & -P_2^{-1} & \cdots & * \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sqrt{\pi_{t\bar{l}}} \bar{B}_n^{ij} \tilde{I} & 0 & 0 & \cdots & -P_{\bar{l}}^{-1} \end{bmatrix}.$$

In terms of Lemma 3 and the condition (42), we obtain $\Phi_{ijtn}(k) < 0$ and thus (18) is satisfied. In addition, by further using the Schur Complement Lemma, we know that

$$\left(\bar{\mathcal{A}}_{tn}^{ij}(k)\right)^T \tilde{P}_{tn}^{ij} \bar{\mathcal{A}}_{tn}^{ij}(k) - R_{ijtn} < 0 \tag{49}$$

if and only if the following inequality holds

$$\begin{aligned} &\begin{bmatrix} -R_{ijtn} & * \\ \vec{\Gamma}_{ijtn}^{21}(k) & \bar{\Gamma}_{ijtn}^{22a} \end{bmatrix} \\ &= \bar{\Gamma}_{ijtn} + \bar{M}_{ijtn} \bar{\Delta}(k) \bar{N}_t + \bar{N}_t^T \bar{\Delta}^T(k) \bar{M}_{ijtn}^T \\ &\triangleq \Psi_{ijtn}(k) < 0 \end{aligned} \tag{50}$$

where

$$\begin{aligned} \bar{\Gamma}_{ijtn}^{22a} &\triangleq \mathrm{diag}\left\{-\tilde{P}^{-1}, -\alpha_{ijtn} \Upsilon_t^{-1}\right\}, \\ \bar{\Delta}(k) &\triangleq \mathrm{diag}\left\{\bar{\Delta}_1(k), \bar{\Delta}_2(k)\right\}, \\ \bar{\Delta}_1(k) &\triangleq \mathrm{diag}\left\{\Delta_P(k), \Delta_D(k), \Delta_I(k)\right\}, \\ \bar{\Delta}_2(k) &\triangleq \mathrm{diag}\left\{\Delta_D(k), \Delta_P(k), \Delta_D(k)\right\}, \end{aligned}$$

$$\vec{\Gamma}_{ijtn}^{21}(k) \triangleq \begin{bmatrix} \sqrt{\pi_{t1}} \mathcal{A}_{tn}^{ij}(k) & \sqrt{\pi_{t1}} \mathcal{E}_{tn}^{ij}(k) \\ \sqrt{\pi_{t2}} \mathcal{A}_{tn}^{ij}(k) & \sqrt{\pi_{t2}} \mathcal{E}_{tn}^{ij}(k) \\ \vdots & \vdots \\ \sqrt{\pi_{t\bar{l}}} \mathcal{A}_{tn}^{ij}(k) & \sqrt{\pi_{t\bar{l}}} \mathcal{E}_{tn}^{ij}(k) \\ \alpha_{ijtn} C_2 \bar{I} & \alpha_{ijtn} F_2 \end{bmatrix}.$$

From Lemma 3, $\Psi_{ijtn}(k) < 0$ holds under the condition (43) and the inequality (49) is thus satisfied. Furthermore, under the condition (44), we have

$$\begin{aligned} &\sum_{n=1}^{\bar{s}} \delta_{tn} \left[\left(\bar{\mathcal{A}}_{tn}^{ij}(k)\right)^T \tilde{P}_{tn}^{ij} \bar{\mathcal{A}}_{tn}^{ij}(k) + \bar{S}_t^i \right] \\ &< \sum_{n=1}^{\bar{s}} \delta_{tn} \left(R_{ijtn} + \bar{S}_t^i\right) < 0. \end{aligned} \tag{51}$$

Based on the above analysis, it is clear that the conditions (18)-(20) in Theorem 1 are ensured by the conditions (42)-(46) presented in Theorem 2. Therefore, the proof is complete. ■

Note that the equality (46) in Theorem 2, which results from the consideration of fading channels and the PID control strategy, renders the related conditions *non-convex*, and it is therefore difficult to apply Theorem 2 directly based on the available computing software. As such, in order to facilitate the design of the controller gains, the well-known cone complementarity linearization algorithm will be employed to transform non-convex conditions into strict convex ones via an optimization procedure. By following the similar ideas in [16], our focus is now to solve the optimization problem of minimizing $\mathrm{tr}\left(\sum_{t\in\mathbb{L}} P_t X_t\right)$ subject to LMI constraints (42)-(45) and $\Omega_t \triangleq \begin{bmatrix} P_t & I \\ I & X_t \end{bmatrix} > 0$ for $t \in \mathbb{L}$.

To simplify the presentation when summarizing the controller design algorithm, we denote some auxiliary matrices:

$$\bar{\Phi}_{ijtn} \triangleq \begin{bmatrix} \tilde{\Gamma}_{ijtn}^a & * & * \\ \mu_{ijtn}^a \tilde{M}_{ijtn}^T & -\mu_{ijtn}^a I & * \\ \tilde{N} & 0 & -\mu_{ijtn}^a I \end{bmatrix},$$

$$\bar{\Psi}_{ijtn} \triangleq \begin{bmatrix} \bar{\Gamma}_{ijtn}^a & * & * \\ \mu_{ijtn}^b \bar{M}_{ijtn}^T & -\mu_{ijtn}^b I & * \\ \bar{N}_t & 0 & -\mu_{ijtn}^b I \end{bmatrix}$$

where

$$\bar{\Gamma}_{ijtn}^a \triangleq \begin{bmatrix} -R_{ijtn} & * \\ \bar{\Gamma}_{ijtn}^{21} & \bar{\Gamma}_{ijtn}^{22a} \end{bmatrix}$$

and other internal variables are defined in Theorem 2, (47), (48) and (50).

Finally, we give Algorithm 1 to show the complete process of designing the non-fragile dissipative PID controller over fading channels.

Note that in Algorithm 1, we present the solving process of controller gains under the performance index of dissipativity

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Algorithm 1: Non-fragile Dissipative Fuzzy PID control

Step 1. Set $\iota = 0$. Obtain a set of initial solutions $(P_{t(0)}, X_{t(0)}, K^P_{jn(0)}, K^I_{jn(0)}, K^D_{jn(0)}, R_{ijtn(0)}, \alpha_{ijtn(0)}, \mu^a_{ijtn(0)}, \mu^b_{ijtn(0)})$ by solving (42)-(45) and $\Omega_t > 0$.

Step 2. Solve the problem: $\min \text{ tr} \sum_{t\in\mathbb{L}}(P_t X_{t(\iota)} + P_{t(\iota)} X_t)$ subject to (42)-(45) and $\Omega_t > 0$ to derive an array of feasible solutions $(P_t, X_t, K^P_{jn}, K^I_{jn}, K^D_{jn}, R_{ijtn}, \alpha_{ijtn}, \mu^a_{ijtn}, \mu^b_{ijtn})$. Set $\bar{t} = |\text{tr} \sum_{t\in\mathbb{L}}(P_t X_t) - \bar{l} n_x - 2\bar{l} n_y|$.

Step 3. Substitute the obtained matrix variables $(P_t, X_t, K^P_{jn}, K^I_{jn}, K^D_{jn}, R_{ijtn}, \alpha_{ijtn}, \mu^a_{ijtn}, \mu^b_{ijtn})$ into $\bar{\Phi}_{ijtn} < 0$ and $\bar{\Psi}_{ijtn} < 0$. If these inequalities are satisfied and $\bar{t}$ is less than a small constant number $\upsilon > 0$, then, these obtained variables are solutions we needed. Exit.

Step 4. If $\iota > H$, where H is the maximum number of iterations allowed, exit. Else, set $\iota = \iota + 1$ and go to *Step 2*.

with the given index parameters S_1, S_2, S_3 and S_4. As discussed in Remark 5, such a performance index can be reduced to other commonly used ones (e.g. the well-known H_∞ performance index) by adjusting the index parameters. In order to show the extensibility of this paper, we set $S_2 = S_4 = 0$, $S_1 = -I$ and $S_3 = \gamma^2$ (denoting the disturbance attenuation level), and give Algorithm 2 to deal with the H_∞ fuzzy PID control issue with the minimum disturbance attenuation level by solving an optimization problem. Then, based on Algorithm 2, the optimization strategy for controller gains can be derived.

Algorithm 2: Non-fragile H_∞ Fuzzy PID control

Step 1. Choose a sufficiently large initial $\gamma^2 > 0$, such that there exists a feasible solution to (42)-(45). Set $\gamma^2_{\min} = \gamma^2$.

Step 2. Set $\iota = 0$. Obtain a set of initial solutions $(P_{t(0)}, X_{t(0)}, K^P_{jn(0)}, K^I_{jn(0)}, K^D_{jn(0)}, R_{ijtn(0)}, \alpha_{ijtn(0)}, \mu^a_{ijtn(0)}, \mu^b_{ijtn(0)})$ by solving (42)-(45) and $\Omega_t > 0$.

Step 3. Solve the problem: $\min \text{ tr} \sum_{t\in\mathbb{L}}(P_t X_{t(\iota)} + P_{t(\iota)} X_t)$ subject to (42)-(45) and $\Omega_t > 0$ to derive an array of feasible solutions $(P_t, X_t, K^P_{jn}, K^I_{jn}, K^D_{jn}, R_{ijtn}, \alpha_{ijtn}, \mu^a_{ijtn}, \mu^b_{ijtn})$.

Step 4. Substitute the obtained gain matrices K^P_{jn}, K^I_{jn}, K^D_{jn} into $\bar{\Phi}_{ijtn} < 0$ and $\bar{\Psi}_{ijtn} < 0$. If these inequalities are satisfied, then decrease γ^2 to some extent and set $\gamma^2_{\min} = \gamma^2$. Go to *Step 2*. If $\bar{\Phi}_{ijtn} < 0$ and $\bar{\Psi}_{ijtn} < 0$ are infeasible within the maximum number of iteration that is allowed, then exit. Otherwise, set $\iota = \iota + 1$ and go to *Step 3*.

Remark 6: So far, we have solved the extended dissipative fuzzy control problems with the coexistence of the FSMCFs, i.i.d.CFs and the CPPs. Specifically, we have designed the non-fragile fuzzy PID controller in the presence of the mixed fading effects by 1) developing an appropriate analysis method to explore the extended dissipativity for T-S fuzzy systems and 2) designing mode-dependent PID controllers by using faded measurements and observed network modes. With the assistance of the Lyapunov stability theory and stochastic analysis method, sufficient conditions have been obtained to ensure the stochastic stability as well as the stochastically extended dissipativity. An iterative optimization algorithm has been proposed to design the desired controller parameters by using the convex optimization technique.

Remark 7: Compared with the numerous results concerning the T-S fuzzy control problems, the main novelties of this paper are indicated as follows: 1) the problem investigated is new as the mixed fading effects are considered, for the first time, for general nonlinear systems represented by T-S fuzzy models; and 2) a novel non-fragile fuzzy PID controller is designed with ensured non-fragility and enhanced flexibility, thereby achieving the control tasks under the performance index of extended dissipativity. Furthermore, since we consider a general performance index, our results can be easily extended to control problems under other performance requirements such as the H_∞, l_2-l_∞ and passivity.

IV. Simulation Examples

In this section, one numerical example and one application-motivated example are given to verify the theoretical results obtained in this paper.

A. *Example* 1

Consider a two-rules fuzzy system in the form of (2) whose system matrices and fuzzy membership functions are given by

$$A_1 = \begin{bmatrix} 0.5 & 0.2 \\ 0.3 & 0.9 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.5 & 0.2 \\ 0.3 & 1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.5 \\ 0.9 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} 0.7 \\ 0.5 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0.4 \\ 0.5 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.4 \\ 0.7 \end{bmatrix}, \quad F_1 = 0.2,$$
$$F_2 = 0.5 \quad C_1 = \begin{bmatrix} 0.1 & 0.6 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.6 & 0.4 \end{bmatrix},$$
$$G_1 = \begin{bmatrix} 0.1 & 0.6 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0.4 & -0.1 \end{bmatrix},$$
$$h_1(\rho(k)) = \frac{1 + \sin(x_1(k))}{2}, \quad h_2(\rho(k)) = 1 - h_1(\rho(k)).$$

Considering the complex transmission requirement in practice, two types of channels are used to transmit the measurement output $y_1(k)$ and $y_2(k)$, which would undergo FSMCFs and i.i.d.CFs, respectively. For Channel I, it is assumed that the communication network has two modes called respectively, the “good mode” and the “bad mode” which implies $\sigma(k) \in \{1, 2\}$. The fading coefficients are assumed to be $\lambda_{1,1} = 0.5$ and $\lambda_{1,2} = 0.2$.

The transition probability matrix of this Markov channel is described by

$$\Pi = \begin{bmatrix} 0.4 & 0.6 \\ 0.7 & 0.3 \end{bmatrix}.$$

It is also assumed that on the controller side, two modes can be observed successfully, i.e., $\theta(k) \in \{1, 2\}$ with the conditional probabilities given by

$$\begin{aligned} \Pr\{\theta(k) = 1 | \sigma(k) = 1\} &= 0.4, \\ \Pr\{\theta(k) = 2 | \sigma(k) = 1\} &= 0.6, \\ \Pr\{\theta(k) = 1 | \sigma(k) = 2\} &= 0.5, \\ \Pr\{\theta(k) = 2 | \sigma(k) = 2\} &= 0.5. \end{aligned}$$

For Channel II, the fading coefficient is denoted by $\xi_1(k)$ with the following statistical properties:

$$\begin{aligned} \bar{\xi}_1 &\triangleq \mathbb{E}\{\xi_1(k)\} = 0.5, \\ \xi^*_{11} &\triangleq \mathbb{E}\{(\xi_1(k) - \bar{\xi}_1)(\xi_1(k) - \bar{\xi}_1)\} = 0.1. \end{aligned}$$

Other parameters in the performance index R2) are chosen as $S_1 = -1$, $S_2 = 0.5$, $S_3 = 2$, $S_4 = 0$.

The CPPs in (12) are assumed to be $M^P_{jn} = M^I_{jn} = M^D_{jn} = 0.1$ $(j = 1, 2;\ n = 1, 2)$, $N_P = N_I = N_D = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}$ and $\Delta_P(k) = \Delta_I(k) = \Delta_D(k) = \sin(k)$, where $\Delta_P(k)$, $\Delta_I(k)$ and $\Delta_D(k)$ are unknown for the designer. The aim of this example is to use the obtained theoretical results to design a non-fragile fuzzy PID controller, such that requirements R1) and R2) are satisfied simultaneously.

For simulation purpose, let the external noise as $\omega(k) = \frac{\sin(k)}{k}$, and the initial state as $x(0) = \begin{bmatrix} -0.1 & 0.1 \end{bmatrix}^T$. By setting the simulation run length to be 150 and utilizing the controller gains obtained via Algorithm 1, simulation results are given in Figs. 1-5. Fig. 1 plots the state evolution trajectory of the given fuzzy system without any control strategy, from which we can see that the open-loop system is unstable. Fig. 2 depicts the state trajectory of the closed-loop fuzzy system with the designed non-fragile PID controller. It can be observed that the proposed control scheme performs well under the coexistence of mixed channel fadings and the CPPs. In Figs. 3-4, the measurement outputs and the transmitted outputs of two sensor groups are displayed that show the effects caused by channel fadings.

Define an auxiliary variable:

$$\bar{J}(k) \triangleq \sum_{m=0}^{k} \big(z^T(m)S_1z(m) + 2z^T(m)S_2\omega(m) + \omega^T(m)S_3\omega(m)\big).$$

Then, to verify the robustness of the proposed control strategy, we give Fig. 5 to show the value of $\bar{J}(k)$ under several types of bounded CPPs. From this figure, we can observe that $\bar{J}(k) > 0$ in all cases, reflecting that the considered dissipativity (17) is achieved. In addition, we can also see that three curves in three different cases are almost overlapped (meaning the similar control performance), that tells us the desired robustness against the bounded CPPs. In Fig. 6, we plot $\bar{J}(k)$ under three different energy-bounded noises, from which we can see that $\bar{J}(k) > 0$ always holds. Thus, the desired dissipativity is satisfied. All simulation results verify the effectiveness of the proposed control scheme.

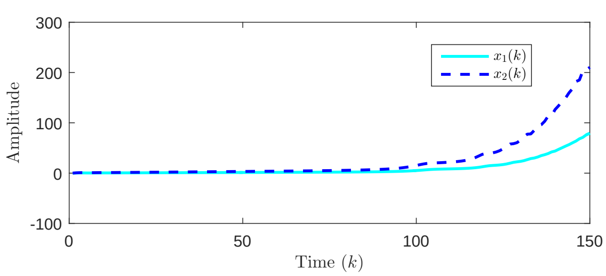

Fig. 1: State trajectory of the open-loop system

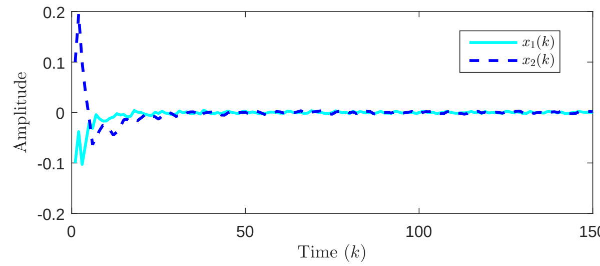

Fig. 2: State trajectory of the closed-loop system

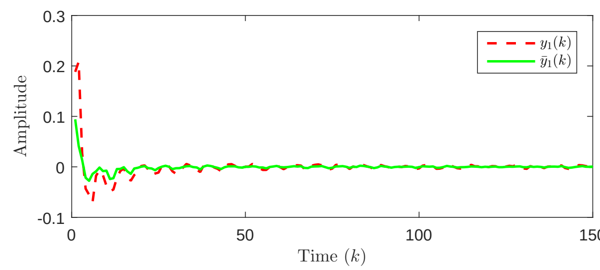

Fig. 3: Measurement output and fading output of Sensor Group I

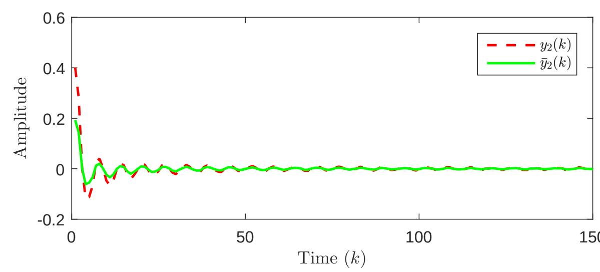

Fig. 4: Measurement output and fading output of Sensor Group II

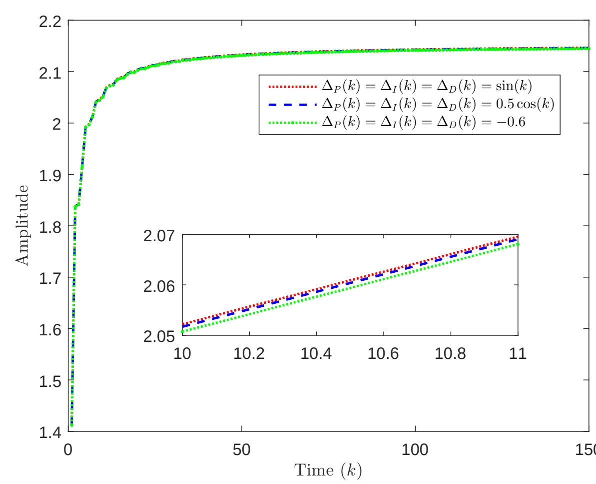

Fig. 5: Value of $\bar{J}(k)$ under different CPPs

B. *Example* 2

In this example, we consider a wireless-network-based truck-trailer control system whose modified model is given as follows [16]:

$$\theta_1(k+1) = \left(1 - \frac{vT}{N}\right)\theta_1(k) + \frac{vT}{n}u(k) + 0.1\omega(k) \quad (52)$$

$$\theta_2(k+1) = \frac{vT}{N}\theta_1(k) + \theta_2(k) + 0.1\omega(k) \quad (53)$$

$$l(k+1) = vT\sin\left(\frac{vT}{2N}\theta_1(k) + \theta_2(k)\right) + l(k) \quad (54)$$

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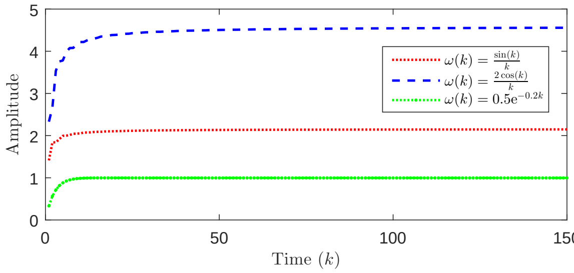

Fig. 6: Value of $\bar{J}(k)$ under different noises

where $\theta_1(k)$ is the angle difference between the truck and the trailer; $\theta_2(k)$ is the angle of the trailer; $l(k)$ is the vertical position of the rear end of the trailer; $u(k)$ is the steering angle; $\omega(k) = \frac{10\sin(k)}{k}$ is the external disturbance; $n = 2.8m$ is the length of the truck; $N = 5.5m$ is the length of the trailer; $T = 2s$ is the sampling time; and $v = -1m/s$ is the constant speed of backing up.

To apply the proposed fuzzy PID control scheme to the above nonlinear system, we need to establish a T-S fuzzy model according to the nonlinear term $\sin(\frac{vT}{2N}\theta_1(k) + \theta_2(k))$. In terms of the key points 0 rad, $\pm\frac{\pi}{6}$ rad, $\pm\pi$ rad and by using the standard fuzzy modeling technique, we can obtain the following discrete-time T-S fuzzy model:

$$x(k+1) = \sum_{i=1}^{3} \phi_i(\rho(k))\Big(A_i x(k) + Bu(k) + E\omega(k)\Big) \quad (55)$$

$$y(k) = Cx(k) + F\omega(k) \quad (56)$$

$$z(k) = Gx(k) \quad (57)$$

where

$$A_1 \triangleq \begin{bmatrix} 1-\frac{vT}{N} & 0 & 0 \\ \frac{vT}{N} & 1 & 0 \\ \frac{v^2T^2}{2N} & vT & 1 \end{bmatrix}, \ A_2 \triangleq \begin{bmatrix} 1-\frac{vT}{N} & 0 & 0 \\ \frac{v}{N} & 1 & 0 \\ \frac{3v^2T^2}{2\pi N} & \frac{3vT}{\pi} & 1 \end{bmatrix},$$

$$A_3 \triangleq \begin{bmatrix} 1-\frac{vT}{N} & 0 & 0 \\ \frac{vT}{N} & 1 & 0 \\ \frac{v^2T}{200N^2} & \frac{v}{100N} & 1 \end{bmatrix}, \ x(k) \triangleq \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} \triangleq \begin{bmatrix} \theta_1(k) \\ \theta_2(k) \\ l(k) \end{bmatrix},$$

$$B \triangleq \begin{bmatrix} \frac{vt}{n} \\ 0 \\ 0 \end{bmatrix}, \quad \rho(k) \triangleq \frac{vT}{2N}x_1(k) + x_2(k), \ E \triangleq \begin{bmatrix} 0.1 \\ 0.1 \\ 0 \end{bmatrix},$$

$$F_1 \triangleq 0.2, \quad F_2 \triangleq 0.5, \quad G \triangleq \begin{bmatrix} 1 & 0.1 & 0 \end{bmatrix}, \ C \triangleq \begin{bmatrix} C_1 \\ C_2 \end{bmatrix},$$

$$C_1 \triangleq \begin{bmatrix} 9 & -2 & 0.03 \end{bmatrix}, \ C_2 \triangleq \begin{bmatrix} 7 & -1 & 0.05 \end{bmatrix},$$

and the normalized membership functions of the plant are displayed in Fig. 7.

The relevant parameters of channel I and channel II are chosen as same as those in Example 1. It is assumed that $\Delta_P(k) = \Delta_I(k) = \Delta_D(k) = 0$. The parameters in performance index are chosen as $S_1 = -1$, $S_2 = 0$, $S_3 = 4$ and $S_4 = 0$. It can be seen that the considered performance index in (2) changes to the following H_∞ one:

$$\mathbb{E}\left\{\sum_{k=0}^{T} z^T(k)z(k)\right\} \leq \mathbb{E}\left\{\gamma^2 \sum_{k=0}^{T} \omega^T(k)\omega(k)\right\} \quad (58)$$

where $\gamma^2 \triangleq S_3$ represents the disturbance attenuation level.

Our aim in this example is to design a fuzzy PID controller with two-rules to back up the truck-trailer with a desired disturbance attenuation level $\gamma^2 = 4$. The membership functions of the controller are shown in Fig. 8. By using the controller gains obtained with the help of Algorithm 1, simulation results are presented in Figs. 9-10. The state evolution of the uncontrolled truck-trailer is given in Fig. 9. The changes of angle and location of the controlled truck-trailer are given in Fig. 10, from which we can see that the closed-loop system is stable under the proposed fuzzy PID controller in the existence of channel fadings.

In Table I, we also give the comparing results of the attained disturbance rejection level $\gamma^* \triangleq \frac{\sum_{s=0}^{k_f} z^T(s)z(s)}{\sum_{s=0}^{k_f} \omega^T(s)\omega(s)}$ under the proposed fuzzy PID controller and the fuzzy P-type one (where k_f denotes the terminal time of simulation). Note that a smaller γ^* implies a higher disturbance attenuation capability. From this table, we can conclude that 1) the attained γ^* in all cases is less than the prescribed $\gamma = 2$, thus showing the desired robustness against energy-bounded noises; and 2) with the assistance of the proposed fuzzy PID controller, the closed-loop system has a better H_∞ performance, therefore verifying the effectiveness of the proposed fuzzy PID control scheme.

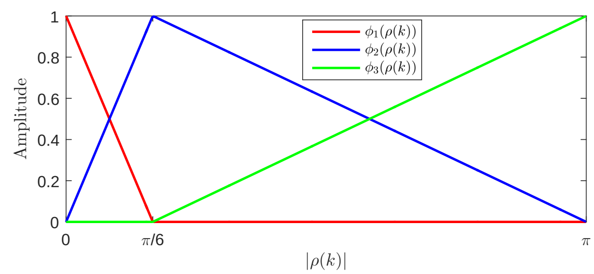

Fig. 7: Membership functions of plant

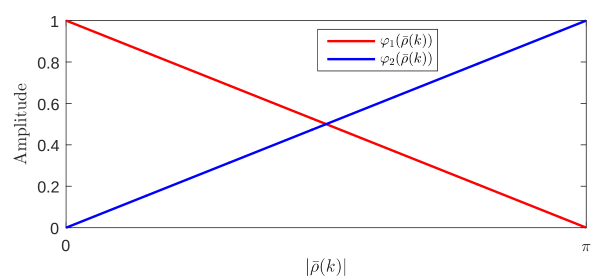

Fig. 8: Membership functions of controller

TABLE I: The Attained γ^* under Fuzzy PID Control and Fuzzy P-type Control Subject to Different Noises

Noise $\omega(k)$	$\frac{\sin k}{k}$	$\frac{5\sin k}{k}$	$\frac{10\sin k}{k}$	$\frac{20\sin k}{k}$
γ^* (fuzzy PID)	0.1677	0.0241	0.0157	0.0110
γ^* (fuzzy P-type)	0.6506	0.0463	0.0202	0.0207

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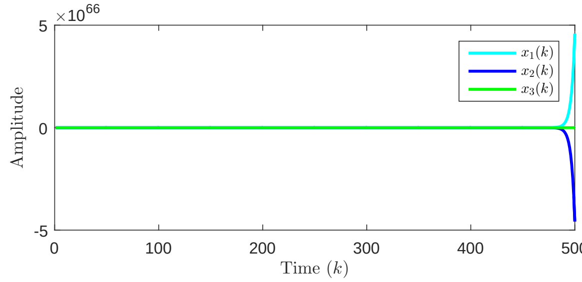

Fig. 9: State trajectory of the open-loop system

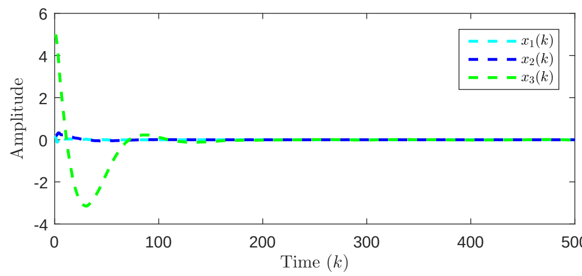

Fig. 10: State trajectory of the closed-loop system

V. Conclusion

In this article, we have addressed the system analysis and PID controller design problems for nonlinear systems with mixed fading effects. To reflect the complex transmission environment, sensors are divided into two groups equipped with two types of specialized wireless channels for transmitting data. Both FSMCFs and i.i.d.CFs have been considered in such a networked transmission environment, and a unified fading model has been established to describe the underlying mixed channel fadings. Then, a non-fragile mode-dependent fuzzy PID controller has been designed based on the observed mode with the improved flexibility. Such a controller is more convenient to be implemented in the practice. With the help of matrix Hadamard product and stochastic analysis method, sufficient conditions have been obtained to check the dissipativity of the system, based on which the controller parameters have been characterized by solving an optimization problem. Finally, two examples have been presented to validate the proposed control schemes. Future research topics include the adaptive fuzzy PID tacking control problems for switched nonlinear systems subject to other network-induced phenomena [18], [19], [31].

References

[1] J. Willems, Dissipative dynamical systems, part 1: general theory; part 2: linear systems with quadratic supply rate, *Archive for Rational Mechanics and Analysis*, vol. 45, pp. 321-393, 1972.

[2] Z. Feng, J. Lam and H. Gao, α-Dissipativity analysis of singular time-delay systems, *Automatica*, vol. 47, no. 11, pp. 2548-2552, Nov. 2011.

[3] V. Ghanbari, P. Wu and P. J. Antsaklis, Large-scale dissipative and passive control systems and the role of star and cyclic symmetries, *IEEE Transactions on Automatic Control*, vol. 61, no. 11, pp. 3676-3680, Nov. 2016.

[4] M. A. Regaieg, M. Kchaou, J. Bosche, A. E.-Hajjaji and M. Chaabane, Robust dissipative observer-based control design for discrete-time switched systems with time-varying delay, *IEEE Control Theory and Applications*, vol. 13, no. 18, pp. 3026-3039, Dec. 2019.

[5] Z. Wang, J. Lam, L. Ma, Y. Bo and Z. Guo, Variance-constrained dissipative observer-based control for a class of nonlinear stochastic systems with degraded measurements, *Journal of Mathematical Analysis and Applications*, vol. 377, no. 2, pp. 645-658, May 2011.

[6] B. Zhang, W. X. Zheng and S. Xu, Filtering of Markovian jump delay systems based on a new performance index, *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 60, no. 5, pp. 1250-1263, May 2013.

[7] R. Manivannan, R. Samidurai, J. Cao, A. Alsaedi and F. E. Alsaadi, Design of extended dissipativity state estimation for generalized neural networks with mixed time-varying delay signals, *Information Sciences*, vol. 424, pp. 175-203, Jan. 2018.

[8] J. Xia, G. Chen and W. Sun, Extended dissipative analysis of generalized Markovian switching neural networks with two delay components, *Neurocomputing*, vol. 260, pp. 275-283, Oct. 2017.

[9] M. Chen, H. Shen and F. Li, On dissipative filtering over unreliable communication links for stochastic jumping neural networks based on a unified design method, *Journal of the Franklin Institute-Engineering and Applied Mathematics*, vol. 353, no. 17, pp. 4583-4601, Nov. 2016.

[10] H. Shen, S. Huo, J. Cao and T. Huang, Generalized state estimation for Markovian coupled networks under Round-Robin protocol and redundant channels, *IEEE Transactions on Cybernetics*, vol. 49, no. 4, pp. 1292-1301, Apr. 2019.

[11] H. Shen, Y. Zhu, L. Zhang and J. H. Park, Extended dissipative state estimation for Markov jump neural networks with unreliable links, *IEEE Transactions on Neural Networks and Learning Systems*, vol. 28, no. 2, pp. 346-358, Feb. 2017.

[12] J. Wang, L. Shen, J. Xia, Z. Wang and X. Chen, Asynchronous dissipative filtering for nonlinear jumping systems subject to fading channels, *Journal of the Franklin Institute-Engineering and Applied Mathematics*, vol. 357, no. 1, pp. 589-605, Jan. 2020.

[13] W. Ji, J. Qiu and H. R. Karimi, Fuzzy-model-based output feedback sliding-mode control for discrete-time uncertain nonlinear systems, *IEEE Transactions on Fuzzy Systems*, vol. 28, no. 8, pp. 1519-1530, Aug. 2020.

[14] M. A. Khanesar, O. Kaynak, S. Yin and H. Gao, Adaptive indirect fuzzy sliding mode controller for networked control systems subject to time-varying network-induced time delay, *IEEE Transactions on Fuzzy Systems*, vol. 23, no. 1, pp. 205-214, Feb. 2015.

[15] J.-X. Zhang and G.-H. Yang, Adaptive fuzzy fault-tolerant control of uncertain Euler-Lagrange systems with process faults, *IEEE Transactions on Fuzzy Systems*, vol. 28, no. 10, pp. 2619-2630, Oct. 2020.

[16] J. Qiu, G. Feng and H. Gao, Asynchronous output-feedback control of networked nonlinear systems with multiple packet dropouts: T-S fuzzy affine model-based approach, *IEEE Transactions on Fuzzy Systems*, vol. 19, no. 6, pp. 1014-1030, Dec. 2011.

[17] H. Zheng, W.-B. Xie, H.-K. Lam and L. Wang, Membership-function-dependent stability analysis and local controller design for T-S fuzzy systems: a space-enveloping approach, *Information Sciences*, vol. 548, pp. 233-253, Feb. 2021.

[18] S. Li, C. K. Ahn and Z. Xiang, Adaptive fuzzy control of switched nonlinear time-varying delay systems with prescribed performance and unmodeled dynamics, *Fuzzy Sets and Systems*, vol. 371, pp. 40-60, Sept. 2019.

[19] M. Li, S. Li, C. K. Ahn and Z. Xiang, Adaptive fuzzy event-triggered command-filtered control for nonlinear time-delay systems, *IEEE Transactions on Fuzzy Systems*, DOI:10.1109/TFUZZ.2021.3052095, 2021.

[20] R. Sakthivel, P. Selvaraj and B. Kaviarasan, Modified repetitive control design for nonlinear systems with time delay based on T-S fuzzy model, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 50, no. 2, pp. 646-655, Feb. 2020.

[21] K. H. Ang, G. Chong and Y. Li, PID control system analysis, design, and technology, *IEEE Transactions on Control Systems Technology*, vol. 13, no. 4, pp. 559-576, Jul. 2005.

[22] J. Liu, J. Lam, M. Shen and Z. Shu, Non-fragile multivariable PID controller design via system augmentation, *International Journal of Systems Science*, vol. 48, no. 10, pp. 2168-2181, 2017.

[23] C. Zhao and L. Guo, PID controller design for second order nonlinear uncertain systems, *Science China-Information Sciences*, vol. 60, no. 2, Feb. 2017.

[24] C. Zhao and L. Guo, Control of nonlinear uncertain systems by extended PID, *IEEE Transactions on Automatic Control*, DOI: 10.1109/TAC.2020.3030876, 2020.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI10.1109/TFUZZ.2022.3165691, IEEE Transactions on Fuzzy Systems

[25] H. Zhang, Y. Shi and A. S. Mehr, Robust static output feedback control and remote PID design for networked motor systems, *IEEE Transactions on Industrial Electronics*, vol. 58, no. 12, pp. 5396-5405, Dec. 2011.

[26] K. Cao, X. Gao, H.-K. Lam and A. Vasilakos, H_∞ fuzzy PID control synthesis for T-S fuzzy systems, *IET Control Theory and Applications*, vol. 10, no. 6, pp. 607-616, Apr. 2016.

[27] C.-S. Chiu and T.-S. Chiang, H_∞ output-feedback fuzzy proportional-integral control of fully delayed input/output systems, *ISA Transactions*, vol. 66, pp. 22-31, Jan. 2017.

[28] X. Lv, J. Fei and Y. Sun, Fuzzy PID controller design for uncertain networked control systems based on T-S fuzzy model with random delays, *International Journal of Fuzzy Systems*, vol. 21, no. 2, pp. 571-582, Mar. 2019.

[29] Y. Wang, Z. Wang, L. Zou and H. Dong, Multiloop decentralized H_∞ fuzzy PID-like control for discrete time-delayed fuzzy systems under dynamical event-triggered schemes, *IEEE Transactions on Cybernetics*, DOI:10.1109/TCYB.2020.3025251, 2020.

[30] D. Zhao, Z. Wang, G. Wei and X. Liu, Non-fragile H_∞ state estimation for recurrent neural networks with time-varying delays: on proportional-integral observer design, *IEEE Transactions on Neural Networks and Learning Systems*, DOI:10.1109/TNNLS.2020.3015376, 2020.

[31] M. S. Mahmoud and M. M. Hamdan, Fundamental issues in networked control systems, *IEEE/CAA Journal of Automatica Sinica*, vol. 5, no. 5, pp. 902-922, Sept. 2018.

[32] L. Zou, Z. Wang and D. Zhou, Moving horizon estimation with non-uniform sampling under component-based dynamic event-triggered transmission, *Automatica*, vol. 120, art. no. 109154, Oct. 2020.

[33] L. Zou, Z. Wang, J. Hu and D. Zhou, Moving horizon estimation with unknown inputs under dynamic quantization effects, *IEEE Transactions on Automatic Control*, vol. 65, no. 12, pp. 5368-5375, Dec. 2020.

[34] G. C. Walsh, H. Ye and L. G. Bushnell, Stability analysis of networked control systems, *IEEE Transactions on Control Systems Technology*, vol. 10, no. 3, pp. 438-446, May 2002.

[35] E. N. Gilbert, Capacity of a burst-noise channel, *Bell System Technical Journal*, vol. 39, pp. 1253-1265, Sept. 1960.

[36] N. Elia, Remote stabilization over fading channels, *Systems and Control Letters*, vol. 54, no. 3, pp. 237-249, Mar. 2005.

[37] N. Xiao, L. Xie and L. Qiu, Feedback stabilization of discrete-time networked systems over fading channels, *IEEE Transactions on Automatic Control*, vol. 57, no. 9, pp. 2176-2189, Sept. 2012.

[38] L. Xu, Y. Mo, L. Xie and N. Xiao, Mean square stabilization of linear discrete-time systems over power-constrained fading channels, *IEEE Transactions on Automatic Control*, vol. 62, no. 12, pp. 6505-6512, Dec. 2017.

[39] Y. Yang, Y. Niu and Z. Zhang, Dynamic event-triggered sliding mode control for interval Type-2 fuzzy systems with fading channels, *ISA Transactions*, vol. 110, pp. 53-62, Apr. 2021.

[40] J. Liu, N. Elia and S. Tatikonda, Capacity-achieving feedback schemes for Gaussian finite-state Markov channels with channel state information, *IEEE Transactions on Information Theory*, vol. 61, no. 7, pp. 3632-3650, Jul. 2015.

[41] D. E. Quevedo, A. Ahlén and K. H. Johansson, State estimation over sensor networks with correlated wireless fading channels, *IEEE Transactions on Automatic Control*, vol. 58, no. 3, pp. 581-593, Mar. 2013.

[42] N. Xiao, Y. Niu and L. Xie, State feedback stabilization over finite-state fading channels, *Asian Journal of Control*, vol. 18, no. 3, pp. 1052-1061, May 2016.

[43] L. Xu, L. Xie and N. Xiao, Mean-square stabilization over Gaussian finite-state Markov channels, *IEEE Transactions on Control of Network Systems*, vol. 5, no. 4, pp. 1830-1840, Dec. 2018.

[44] J. Song, Z. Wang, Y. Niu and H. Dong, Genetic-algorithm-assisted sliding-mode control for networked state-saturated systems over hidden Markov fading channels, *IEEE Transactions on Cybernetics*, DOI: 10.1109/TCYB.2020.2980109.

[45] Z. H. Abbas, A. Ullah, G. Abbas, F. Muhammad and F. Y. Li, Outage probability analysis of user-centric SBS-based HCNets under hybrid Rician/Rayleigh fading, *IEEE Communications Letters*, vol. 24, no. 2, pp. 297-301, Feb. 2020.

[46] J. M. Moualeu, T. M. N. Ngatched and D. B. d. Costa, Sequential relay selection in D2D-enabled cellular networks with outdated CSI over mixed fading channels, *IEEE Wireless Communications Letters*, vol. 8, no. 1, pp. 245-248, Feb. 2019.

[47] H. K. Sahu and P. R. Sahu, Quadrature space shift keying performance with dual-hop AF relay over mixed fading, *International Journal of Communication Systems*, vol. 32, no. 11, art. no. e3969, Jul. 2019.

[48] P. Sadeghi, R. A. Kennedy, P. B. Rapajic and R. Shams, Finite-state Markov modeling of fading channels: a survey of principles and applications, *IEEE Signal Processing Magazine*, vol. 25, no. 5, pp. 57-80, Sept. 2008.

[49] J. Cheng, Y. Shan, J. Cao and J. H. Park, Nonstationary control for T-S fuzzy Markovian switching systems with variable quantization density, *IEEE Transactions on Fuzzy Systems*, vol. 29, no. 6, pp. 1375-1385, Jun. 2021.

[50] J. Cheng, Y. Wang, J. H. Park, J. Cao and K. Shi, Static output feedback quantized control for fuzzy Markovian switching singularly perturbed systems with deception attacks, *IEEE Transactions on Fuzzy Systems*, DOI:10.1109/TFUZZ.2021.3052104, 2021.

[51] C. Zhang, H.-K. Lam, J. Qiu, C. Liu and Q. Chen, A new design of membership-function-dependent controller for T-S fuzzy systems under imperfect premise matching, *IEEE Transactions on Fuzzy Systems*, vol. 27, no. 7, pp. 1428-1440, Jul. 2019.

[52] X. Guan and C.-L. Chen, Delay-dependent guaranteed cost control for T-S fuzzy systems with time delays, *IEEE Transactions on Fuzzy Systems*, vol. 12, no. 2, pp. 236-249, Apr. 2004.

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