This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TNNLS.2021.3055942, IEEE Transactions on Neural Networks and Learning Systems

H_∞ State Estimation for BAM Neural Networks with Binary Mode Switching and Distributed Leakage Delays under Periodic Scheduling Protocol

Fuad E. Alsaadi, Zidong Wang, Yuqiang Luo, Njud S. Alharbi, and Fawaz W. Alsaade

***Abstract*—This paper is concerned with the H_∞ state estimation problem for a class of bidirectional associative memory (BAM) neural networks with binary mode switching, where the distributed delays are included in the leakage terms. A couple of stochastic variables taking values of 1 or 0 are introduced to characterize the switching behavior between the redundant models of the BAM neural network, and a general type of neuron activation function (i.e. the sector-bounded nonlinearity) is considered. In order to prevent the data transmissions from collisions, a periodic scheduling protocol (i.e Round-Robin protocol) is adopted to orchestrate the transmission order of sensors. The purpose of this work is to develop a full-order estimator such that the error dynamics of the state estimation is exponentially mean-square stable and the H_∞ performance requirement of the output estimation error is also achieved. Sufficient conditions are established to ensure the existence of the required estimator by constructing a mode-dependent Lyapunov-Krasovskii functional. Then, the desired estimator parameters are obtained by solving a set of matrix inequalities. Finally, a numerical example is provided to show the effectiveness of the proposed estimator design method.**

***Index Terms*—Artificial neural networks, bidirectional associative memory neural networks, H_∞ state estimation, distributed leakage delays, periodic scheduling protocol.**

I. Introduction

The study on bidirectional associative memory (BAM) neural networks was originated in [25]. The BAM neural network is a widely used artificial neural network (ANN) that is featured by its ability to store a pair of analogue patterns through using the real-time unsupervised learning [28]. As an extension of the unidirectional auto-associator of Hopfield [51], BAM neural networks have found wide applications in a range of areas such as fault diagnosis, model of recognition and cued recall, signal and image processing [1]. Accordingly, the research on BAM neural networks has attracted considerable attention, see e.g. [7], [53] and the references therein.

In many practical applications, structures and parameters of the ANN might suffer from certain abrupt changes due to various reasons such as component and interconnection failures or repairs, variation of environmental factors, large amplitude disturbances, and so forth [11], [45], [48]. These abrupt changes would severely impact the operation of ANNs [12], [13]. In order to protect the ANNs from being compromised by abrupt changes, the switching strategy is often employed to ensure the continuously normal operation of ANNs in real scenarios. With such a strategy, the ANNs would automatically switch to a reductant mode when the abrupt changes take place.

In the applications of ANNs, the information of neuron states is of dominant importance since almost all ANN applications (i.e. optimization and approximation) are dependent on the accurate state information. Unfortunately, the full information of neuron states is not always accessible in practical applications, and this has necessitated the state estimation problem that aims to estimate the state information of ANNs based on available measurements only. Indeed, the state estimation problem of ANNs has been a focus of research in the past decade [6], [30], [39], [47], [50]. For instance, the state estimation issue has been studied in [24] for a class of ANNs with continuous and bounded delays, where certain delay-range-dependent stability conditions have been established. Based upon the passivity theory, which is an effective scheme for stability analysis of nonlinear system, an exponential and passive estimator has been developed in [2] for the delayed Takagi-Sugeno fuzzy Hopfield neural networks with external disturbance. However, to the best of the authors' knowledge, so far, the H_∞ state estimation problem for BAM neural networks with stochastic mode switching has not yet been studied, and this situation constitutes the main motivation of the present investigation.

It is now well known that time-delays have shown their prevalent existence in signal transmission of the ANNs, and a particular kind of time-delays occurs in stabilizing negative feedback terms of neural networks. Such time-delays are

This Project was funded by the research and development office (RDO) at the Ministry of Education, Kingdom of Saudi Arabia, under Grant no (HIQI-2-2019). The authors also, acknowledge with thanks research and development office at King Abdulaziz University (RDO-KAU) for technical support. The work of Y. Luo was supported in part by the National Natural Science Foundation of China under Grant 61903254. (*Corresponding author: Fuad E. Alsaadi.*)

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referred to as the leakage delays [20], [49], which could lead to the deterioration of the system performance or even cause the instability of the system [21], [44]. By now, considerable research attention has been drawn to various analysis issues (e.g. stability analysis [3], [4], [41], [43], passivity analysis and synchronization) of ANNS with leakage delays, see e.g. [9], [18]. For the state estimation issue of ANNs, time delays should be adequately taken into account in the estimator design for the purpose of avoiding undesired oscillation, bifurcation and chaotic attractors. Furthermore, it should be noted that the number of the neurons in an ANN is usually very large, which would lead to a huge amount of measurement data. This is particularly true for the state estimation problem of a BAM neural network with a limited communication channel between the neural network and the remote estimator, in which the considered ANN contains up to billions of neurons in order to deal with some complex design problems.

It is quite common in a networked environment that the state evolutions of certain ANNs need to be closely monitored in a remote way via communication channels with limited bandwidth [29], [34]. In this case, the remote estimators receive measurement outputs of the ANNs sent by a group of sensors through shared communication networks. Such vast data transmissions would lead to heavy burden on the communication channel of limited bandwidth, thereby resulting in communication congestion and consecutive data dropouts [15], [17]. In this case, the utilization of certain communication protocols (e.g. periodic scheduling protocol) serves as an effective method to mitigate the data collision issue over communication channels of limited capacity. The well-known Round-Robin protocol (RRP) is notably one of the most widely employed periodic scheduling protocols and has attracted quite a lot research attention, see e.g. [16], [27], [46].

Although problems of state estimation and stability analysis of BAM neural networks have been heavily discussed in [5], [9], [12], [18], [53] in recent years, no studies on state estimation problems of BAM ANNs with binary mode switching have been reported, not to mention the case where distributed leakage delays and the RRP are involved. Inspired by above discussions, our attention in this paper is paid to the H_∞ state estimation problem for stochastic BAM neural networks with distributed leakage delays where the RRP is adopted to schedule the signal transmission between the neural network and the estimator with hope to make full use of the limited communication bandwidth.

The main contributions of this paper are highlighted as follows: 1) the considered BAM neural networks can switch to another redundant mode when abrupt changes occur, which will thus guarantee the performance of ANNs in case of undergoing abrupt environment changes; 2) the H_∞ state estimation problem is, for the first time, investigated for the stochastic BAM neural networks with distributed leakage delays and RRP scheduling effects; 3) based on the stochastic analysis approach, sufficient conditions are established to guarantee a satisfactory state estimation performance; and 4) the estimator gain matrices are obtained by solving a set of matrix inequalities via standard software package.

The rest of this paper is organized as follows. In Section II, a class of stochastic BAM neural networks with distributed leakage delays and RRP scheduling are introduced and the corresponding H_∞ state estimation problem is formulated. In Section III, sufficient conditions are established that guarantee the existence of the desired estimator parameters, and the estimator parameters are derived in terms of the solution to a set of matrix inequalities. A numerical simulation example is given in Section IV to show the effectiveness and correctness of the proposed state estimation approach. Finally, the conclusion of this work is drawn in Section V.

Notation. The notations used throughout the paper are fairly standard except where otherwise stated. $\mathbb{R}^n$, $\mathbb{R}^{n\times m}$ and $\mathbb{Z}$ $(\mathbb{Z}^+, \mathbb{Z}^-)$ denote, respectively, the n-dimensional Euclidean space, the set of all $n \times m$ real matrices and the set of all integers (nonnegative integers, negative integers). $\|\cdot\|$ refers to the Euclidean norm in $\mathbb{R}^n$. For a scalar $a \in \mathbb{R}$, $|a|$ denotes its absolute value. I_n represents the identity matrix of dimension $n \times n$. The notation $X \geq Y$ (respectively, $X > Y$), where X and Y are symmetric matrices, means that $X-Y$ is positive semi-definite (respectively, positive definite). For a matrix M, M^T and M^{-1} represent its transpose and inverse, respectively. The shorthand $\mathrm{diag}\{M_1,\ M_2, \ldots,\ M_n\}$ denotes a block diagonal matrix with diagonal blocks being the matrices $M_1,\ M_2,\ \ldots,\ M_n$. In symmetric block matrices, the symbol '$*$' is used as an ellipsis for terms induced by symmetry. $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ are the minimum and maximum eigenvalues, respectively. $\mathrm{mod}(a, b)$ represents the unique nonnegative remainder on division of the integer a by the positive integer b. $\delta(a)$ is a binary function which equals to 1 for $a = 0$, and equals to 0 for $a \neq 0$. Moreover, let $(\Omega, \mathscr{F}, \mathrm{Prob})$ be a complete probability space, where Prob, the probability measure, has total mass 1. $\mathbb{E}\{x\}$ and $\mathbb{E}\{x|y\}$ stand for, respectively, the expectation of the stochastic variable x and the expectation of x conditional on y with respect to the given probability measure Prob. Matrices, if they are not explicitly stated, are assumed to have compatible dimensions.

II. Problem Formulation

Consider the following class of stochastic BAM neural networks:

$$\begin{cases} u(k+1) = \big(A+\theta_1(k)C\big)\displaystyle\sum_{d=1}^{\infty}\mu_{1d}u(k-d) \\ \qquad\qquad + \big(V^{(1)}+\theta_1(k)W^{(1)}\big)g\big(v(k)\big), \\ z_1(k) = Mu(k), \\ v(k+1) = \big(B+\theta_2(k)D\big)\displaystyle\sum_{d=1}^{\infty}\mu_{2d}v(k-d) \\ \qquad\qquad + \big(V^{(2)}+\theta_2(k)W^{(2)}\big)f\big(u(k)\big), \\ z_2(k) = Nv(k), \end{cases} \tag{1}$$

where $u(k) \triangleq \begin{bmatrix} u_1(k) & u_2(k) & \cdots & u_n(k)\end{bmatrix}^T \in \mathbb{R}^n$ and $v(k) \triangleq \begin{bmatrix} v_1(k) & v_2(k) & \cdots & v_n(k)\end{bmatrix}^T \in \mathbb{R}^n$ are the neuron state vectors with $u_i(k) \in \mathbb{R}$ and $v_j(k) \in \mathbb{R}$ being the states of the i-th neuron from the neural field F_U and the neural field F_V at time instant $k \in \mathbb{Z}$, respectively. $z_1(k) \in \mathbb{R}^{n_z}$

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and $z_2(k) \in \mathbb{R}^{n_z}$ are the signals to be estimated. $g\big(v(k)\big) \triangleq \big[g_1^T\big(v_1(k)\big) \quad g_2^T\big(v_2(k)\big) \quad \cdots \quad g_n^T\big(v_n(k)\big)\big]^T \in \mathbb{R}^n$ and $f\big(u(k)\big) \triangleq \big[f_1^T\big(u_1(k)\big) \quad f_2^T\big(u_2(k)\big) \quad \cdots \quad f_n^T\big(u_n(k)\big)\big]^T \in \mathbb{R}^n$ are two vector-valued functions where $f_i(\cdot)$ and $g_i(\cdot)$ $(i = 1, 2, \cdots, n)$ are the neuron activation functions. $V^{(r)} = [v_{ij}^{(r)}]_{n\times n}$ and $W^{(r)} = [w_{ij}^{(r)}]_{n\times n}$ $(r = 1, 2)$ are the connection weight matrices. $A = \mathrm{diag}\{a_1, a_2, \cdots, a_n\}$ $(|a_i| < 1$ for $i = 1, 2, \cdots, n)$ and $B = \mathrm{diag}\{b_1, b_2, \cdots, b_n\}$ $(|b_j| < 1$ for $j = 1, 2, \cdots, n)$ are the state feedback coefficient matrices. C, D, M and N are known constant matrices with appropriate dimensions. $\sum_{d=1}^{\infty} \mu_{id} u(k-d)$ $(i = 1, 2)$ represent the infinitely distributed leakage delays with $0 \leq \mu_{id} \leq 1$ being the convergence constants satisfying $\bar{\mu}_i \triangleq \sum_{d=1}^{\infty} \mu_{id} \leq \sum_{d=1}^{\infty} d\mu_{id} < +\infty$. For any $i \in \{1, 2\}$ and $k \in \mathbb{Z}^+$, $\theta_i(k)$ is a Bernoulli distributed stochastic variable taking values of 1 and 0 with

$$\begin{cases} \mathrm{Prob}\{\theta_1(k) = 1\} = \bar{\theta}_1, & \mathrm{Prob}\{\theta_1(k) = 0\} = 1 - \bar{\theta}_1, \\ \mathrm{Prob}\{\theta_2(k) = 1\} = \bar{\theta}_2, & \mathrm{Prob}\{\theta_2(k) = 0\} = 1 - \bar{\theta}_2. \end{cases}$$

Furthermore, $\{\theta_i(k)\}_{i=1,2;k\in\mathbb{Z}^+}$ are mutually independent random variables.

Remark 1: The well-known BAM neural network has extensive application prospects in the area of pattern recognition. Such a neural network is actually an extension of the unidirectional auto-associator of Hopfield neural network, which implies the BAM neural network is in fact a special recurrent neural network [51]. In the system model (1), the terms $\big(A + \theta_1(k)C\big) \sum_{d=1}^{\infty} \mu_{1d} u(k-d)$ and $\big(B + \theta_2(k)D\big) \sum_{d=1}^{\infty} \mu_{2d} v(k-d)$ are known as the distributed leakage delays, which represent the distributed propagation delays in the leakage terms [20]. As pointed out in [20], the leakage delays have a significant impact on the dynamic analysis problems of neural networks. However, the state estimation problem of neural networks with distributed leakage delays has not gained adequate research attention due mainly to the resulting technical difficulties [5], [20]. In this work, we pay our attention to the infinitely distributed leakage delays, which can be used to effectively describe the feature of the spatial extent in the ANNs [51].

Remark 2: In reality, the structures and parameters of ANNs might undergo abrupt changes due to the variation of environmental factors. Such unexpected abrupt changes would affect the execution of ANNs. In order to alleviate the negative effects, a possible way to protect the service performance of ANNs is to switch the system mode to another redundant one when abrupt changes occur. In this paper, two binary sequences $\theta_1(k)$ and $\theta_2(k)$ are introduced to characterize such a switching behavior according to their distinct values. Apart from the binary mode switching, the multi-mode switching and the Markovian switching can also be used to regulate the aforementioned switching behavior of the ANNs in case of abrupt changes.

In this paper, the vector-valued neuron activation function $g(\cdot)$ and $f(\cdot)$ with $g(0) = f(0) = 0$ are assumed to satisfy the following sector-bounded conditions for any $a, b \in \mathbb{R}^n$:

$$\begin{cases} \big(g(b) - g(a) - R_2(b-a)\big)^T \big(g(b) - g(a) \\ \quad - R_1(b-a)\big) \leq 0, \quad R_1 - R_2 < 0 \\ \big(f(b) - f(a) - S_2(b-a)\big)^T \big(f(b) - f(a) \\ \quad - S_1(b-a)\big) \leq 0, \quad S_1 - S_2 < 0 \end{cases} \tag{2}$$

where R_1, R_2, S_1 and S_2 are constant real matrices of appropriate dimensions. The measurement outputs of the system (1) are of the following form:

$$\begin{cases} x(k) = Eu(k) + G\vartheta(k), \\ y(k) = Fv(k) + H\vartheta(k), \end{cases} \tag{3}$$

where $x(k) \triangleq \big[x_1(k) \quad x_2(k) \quad \cdots \quad x_m(k)\big]^T \in \mathbb{R}^m$ and $y(k) \triangleq \big[y_1(k) \quad y_2(k) \quad \cdots \quad y_m(k)\big]^T \in \mathbb{R}^m$ are the network outputs. E, F, G and H are known constant matrices with appropriate dimensions. $\vartheta(k) \in \mathbb{R}^s$ is the measurement noise belonging to $l_2([0, +\infty); \mathbb{R}^s)$.

In this paper, we assume the sensors are periodically scheduled by the RRP, under which the sensors are allocated with the transmission opportunities one by one with a fixed circular order. In this case, the estimator could only receive partial measurement information at each transmission instant. In order to compensate the received measurement information, a set of zero-order holders (ZOHs) are employed to deal with the update of the received signal for the estimator [40]. Under the effects of RRP and ZOHs, the received signal of the estimator is given as follows:

$$\begin{cases} \vec{x}(k) = \Psi_{\hbar(k)} x(k) + (I_m - \Psi_{\hbar(k)})\vec{x}(k-1), \\ \vec{y}(k) = \Psi_{\hbar(k)} y(k) + (I_m - \Psi_{\hbar(k)})\vec{y}(k-1), \end{cases} \tag{4}$$

where $\vec{x}(k) \triangleq \big[\vec{x}_1(k) \quad \vec{x}_2(k) \quad \cdots \quad \vec{x}_m(k)\big]^T$ and $\vec{y}(k) \triangleq \big[\vec{y}_1(k) \quad \vec{y}_2(k) \quad \cdots \quad \vec{y}_m(k)\big]^T$ are the real received measurements of the estimator corresponding to the sensor outputs $x(k)$ and $y(k)$, respectively. $\Psi_{\hbar(k)} \triangleq \mathrm{diag}\{\delta(\hbar(k) - 1), \delta(\hbar(k) - 2), \cdots \delta(\hbar(k) - m)\}$ is the update matrix with $\hbar(k) = \mathrm{mod}(k-1, m) + 1 \in \{1,\ 2, \cdots, m\}$ being the access token holder by the $\hbar(k)$-th sensor.

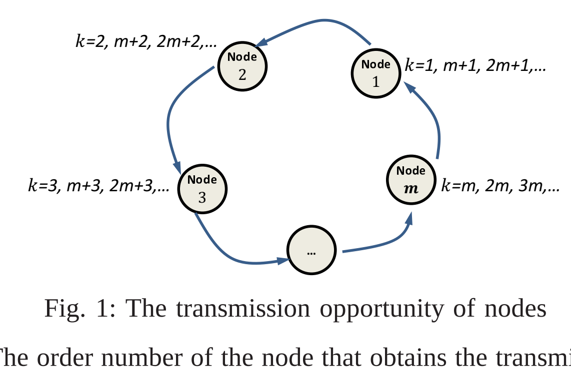

Fig. 1: The transmission opportunity of nodes

The order number of the node that obtains the transmission opportunity at each time instant is described in Fig. 1. One can easily observes from Fig 1 and equation (4) that the information propagation among network nodes is conducted in a fixed circular order. Keeping this in mind and using the zero-order holder strategy, at time k, the measurements arriving at the estimator are given by equation (4) where

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the measurement coefficients where $\Psi_{\hbar(k)} \triangleq \mathrm{diag}\{\delta(\hbar(k)-1), \delta(\hbar(k)-2), \cdots \delta(\hbar(k)-m)\}$ is the protocol-induced update matrix and $\hbar(k) = \mathrm{mod}(k-1, m)+1 \in \{1,\ 2, \cdots, m\}$ is the protocol-induced access token at time k. As a result, protocol-induced coefficients $\Psi_{\hbar(k)}$ and $\hbar(k)$ are introduced to all measurement-related terms (see the estimator (5), error system (6) and Theorems 1–2), adding extra difficulties to the design and analysis of the developed sate estimation approach.

Accounting for the fact that the scheduling process of RRP is independent of the transmitted data, its accuracy or reliability will not be affected by the sensor measurement error. In addition, to prevent the failure of the network in case of faulty nodes, a security strategy is introduced to the network communication where each network node has its prescribed waiting time. If no information is sent from the network node within this waiting time, the node will be discarded and replaced so as to avoid the failure of the whole system.

The following full-order estimator is adopted to estimate the signal $z_1(k)$ and $z_2(k)$:

$$\begin{cases} u_f(k+1) = A_f u_f(k) + K_{f1}\vec{x}(k), \\ x_f(k) = K_{f2}x_f(k-1) + K_{f3}\vec{x}(k), \\ \hat{z}_1 = M_f u_f(k), \\ v_f(k+1) = B_f v_f(k) + L_{f1}\vec{y}(k), \\ y_f(k) = L_{f2}y_f(k-1) + L_{f3}\vec{y}(k), \\ \hat{z}_2 = N_f v_f(k), \end{cases} \tag{5}$$

where $u_f(k) \in \mathbb{R}^n$ and $v_f(k) \in \mathbb{R}^n$ are the estimations of the neuron states $u(k)$ and $v(k)$, respectively. $x_f(k)$ and $y_f(k)$ are the state vectors of the estimator corresponding to $\vec{x}(k)$ and $\vec{y}(k)$, respectively. $z_1(k) \in \mathbb{R}^{n_z}$ and $z_2(k) \in \mathbb{R}^{n_z}$ are the estimator outputs. A_f, B_f, K_{f1}, K_{f2}, K_{f3}, L_{f1}, L_{f2}, L_{f3}, M_f and N_f are the estimator parameters to be designed.

To facilitate the subsequent analysis, we first establish the following augmented dynamics of the state estimation process:

$$\begin{cases} \xi(k+1) = \bar{A}(k)\xi(k) + \bar{A}_d \sum_{d=1}^{\infty} \mu_{1d}\xi(k-d) \\ \qquad + \overline{V}_1 g\big(\bar{I}\zeta(k)\big) + \bar{G}(k)\vartheta(k) \\ \qquad + \left(\bar{C}\sum_{d=1}^{\infty}\mu_{1d}\xi(k-d) + \overline{W}_1 g\big(\bar{I}\zeta(k)\big)\right)\theta_1(k), \\ \tilde{z}_1(k) = \bar{M}_f\xi(k), \\ \zeta(k+1) = \bar{B}(k)\zeta(k) + \bar{B}_d \sum_{d=1}^{\infty}\mu_{2d}\zeta(k-d) \\ \qquad + \overline{V}_2 f\big(\bar{I}\xi(k)\big) + \bar{H}(k)\vartheta(k) \\ \qquad + \left(\bar{D}\sum_{d=1}^{\infty}\mu_{2d}\zeta(k-d) + \overline{W}_2 f\big(\bar{I}\xi(k)\big)\right)\theta_2(k), \\ \tilde{z}_2(k) = \bar{N}_f\zeta(k), \end{cases} \tag{6}$$

where $\tilde{z}_1(k) \triangleq \hat{z}_1(k) - z_1(k)$ and $\tilde{z}_2(k) \triangleq \hat{z}_2(k) - \hat{z}_1(k)$ are the output state estimation error, and

$$\xi(k) \triangleq \begin{bmatrix} u(k) \\ \vec{x}(k-1) \\ u_f(k) \\ x_f(k-1) \end{bmatrix}, \quad \zeta(k) \triangleq \begin{bmatrix} v(k) \\ \vec{y}(k-1) \\ v_f(k) \\ y_f(k-1) \end{bmatrix},$$

$$\bar{A}(k) \triangleq \begin{bmatrix} 0 & 0 & 0 & 0 \\ \Psi_{\hbar(k)}E & (I_m - \Psi_{\hbar(k)}) & 0 & 0 \\ K_{f1}\Psi_{\hbar(k)}E & K_{f1}(I_m - \Psi_{\hbar(k)}) & A_f & 0 \\ K_{f3}\Psi_{\hbar(k)}E & K_{f3}(I_m - \Psi_{\hbar(k)}) & 0 & K_{f2} \end{bmatrix},$$

$$\bar{A}_d \triangleq \begin{bmatrix} A & 0 & 0 & 0 \\ 0 & 0_{m\times m} & 0 & 0 \\ 0 & 0 & 0_{n\times n} & 0 \\ 0 & 0 & 0 & 0_{m\times m} \end{bmatrix}, \overline{V}_1 \triangleq \begin{bmatrix} V^{(1)} \\ 0_{m\times n} \\ 0_{n\times n} \\ 0_{m\times n} \end{bmatrix},$$

$$\bar{G}(k) \triangleq \begin{bmatrix} 0_{n\times n} & 0 & 0 & 0 \\ \Psi_{\hbar(k)}G & 0_{m\times m} & 0 & 0 \\ K_{f1}\Psi_{\hbar(k)}G & 0 & 0_{n\times n} & 0 \\ K_{f3}\Psi_{\hbar(k)}G & 0 & 0 & 0_{m\times m} \end{bmatrix},$$

$$\bar{C} \triangleq \begin{bmatrix} C & 0 & 0 & 0 \\ 0 & 0_{m\times m} & 0 & 0 \\ 0 & 0 & 0_{n\times n} & 0 \\ 0 & 0 & 0 & 0_{m\times m} \end{bmatrix}, \quad \overline{W}_1 \triangleq \begin{bmatrix} W^{(1)} \\ 0_{m\times n} \\ 0_{n\times n} \\ 0_{m\times n} \end{bmatrix},$$

$$\bar{H}(k) \triangleq \begin{bmatrix} 0_{n\times n} & 0 & 0 & 0 \\ \Psi_{\hbar(k)}H & 0_{m\times m} & 0 & 0 \\ L_{f1}\Psi_{\hbar(k)}H & 0 & 0_{n\times n} & 0 \\ L_{f3}\Psi_{\hbar(k)}H & 0 & 0 & 0_{m\times m} \end{bmatrix},$$

$$\bar{B}(k) \triangleq \begin{bmatrix} 0 & 0 & 0 & 0 \\ \Psi_{\hbar(k)}F & I_m - \Psi_{\hbar(k)} & 0 & 0 \\ L_{f1}\Psi_{\hbar(k)}F & L_{f1}(I_m - \Psi_{\hbar(k)}) & B_f & 0 \\ L_{f3}\Psi_{\hbar(k)}F & L_{f3}(I_m - \Psi_{\hbar(k)}) & 0 & L_{f2} \end{bmatrix},$$

$$\bar{B}_d \triangleq \begin{bmatrix} B & 0 & 0 & 0 \\ 0 & 0_{m\times m} & 0 & 0 \\ 0 & 0 & 0_{n\times n} & 0 \\ 0 & 0 & 0 & 0_{m\times m} \end{bmatrix}, \overline{V}_2 \triangleq \begin{bmatrix} V^{(2)} \\ 0_{m\times n} \\ 0_{n\times n} \\ 0_{m\times n} \end{bmatrix},$$

$$\bar{D} \triangleq \begin{bmatrix} D & 0 & 0 & 0 \\ 0 & 0_{m\times m} & 0 & 0 \\ 0 & 0 & 0_{n\times n} & 0 \\ 0 & 0 & 0 & 0_{m\times m} \end{bmatrix}, \quad \overline{W}_2 \triangleq \begin{bmatrix} W^{(2)} \\ 0_{m\times n} \\ 0_{n\times n} \\ 0_{m\times n} \end{bmatrix},$$

$$\bar{I} \triangleq \begin{bmatrix} I_n & 0_{n\times m} & 0_{n\times n} & 0_{n\times m} \end{bmatrix},$$
$$\bar{M}_f \triangleq \begin{bmatrix} -M & 0_{n_z\times m} & M_f & 0_{n_z\times m} \end{bmatrix},$$
$$\bar{N}_f \triangleq \begin{bmatrix} -N & 0_{n_z\times m} & N_f & 0_{n_z\times m} \end{bmatrix}.$$

Definition 1: The augmented dynamics of state estimation process (6) with $\vartheta(k) \equiv 0$ is said to be *exponentially mean-square stable* if there exist three constants $\alpha > 0$, $\epsilon \in (0,1)$ and $\tau \geq 0$ such that, for $k \in \mathbb{Z}^+$, the following holds:

$$\mathbb{E}\left\{\|\xi(k)\|^2 + \|\zeta(k)\|^2\right\}$$
$$\leq \alpha\epsilon^k \left(\sup_{t\in[-\tau,0]} \mathbb{E}\left\{\|\xi(t)\|^2\right\} + \sup_{t\in[-\tau,0]} \mathbb{E}\left\{\|\zeta(t)\|^2\right\}\right).$$

The objective of this paper is to design a H_∞ estimator of the form (5) for the stochastic BAM neural networks (1) with distributed leakage delays and the measurement outputs (3). More specifically, we aim to obtain the estimator parameters A_f, B_f, K_{fi} and L_{fi} ($i = 1,2,3$) such that the following requirements are simultaneously achieved:

1) the augmented dynamics of the system (6) is *exponentially mean-square stable*; and
2) for the given disturbance attenuation level $\gamma > 0$ with $\vartheta(k) \neq 0$, the output state estimation error $\tilde{z}(k) \triangleq$

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$\begin{bmatrix}\tilde{z}_1^T(k) & \tilde{z}_2^T(k)\end{bmatrix}^T$ satisfies

$$\sum_{k=0}^{\infty}\mathbb{E}\left\{\|\tilde{z}(k)\|^2\right\}\leq\gamma^2\sum_{k=0}^{\infty}\|\vartheta(k)\|^2$$

under the zero initial condition.

III. Main Results

We first present the following lemmas which will be used in the sequel.

Lemma 1: Let $M\in\mathbb{R}^{n\times n}$ be a positive semi-definite matrix. For $x_i\in\mathbb{R}^n$ and constant $a_i>0$ $(i=1,2,\ldots,\infty)$, if the series concerned is convergent, then we have

$$\left(\sum_{i=1}^{\infty}a_ix_i\right)^T M\left(\sum_{i=1}^{\infty}a_ix_i\right)\leq\left(\sum_{i=1}^{\infty}a_i\right)\sum_{i=1}^{\infty}a_ix_i^TMx_i.$$

In this section, our main results will be stated in two theorems. The first theorem provides a set of access-token-dependent conditions such that the augmented dynamics (6) is *exponentially mean-square stable*, and the design of the full-order estimator (5) is discussed in the second theorem. By resorting to the Lyapunov stability theory, the exponential stability condition of the dynamics (6) is derived step by step, and the stochastic analysis technique and matrix theory are employed to facilitate the establishment of the main results.

Theorem 1: Let the estimator gain matrices A_f, K_{fi}, B_f, L_{fi} $(i=1,2,3)$ of the estimator (5) be given. The augmented dynamics of the system (6) is exponentially mean-square stable with the given disturbance attenuation level $\gamma>0$ if there exist positive definite matrices $P_{j\hbar(k)}=\mathrm{diag}\{P_{1,j\hbar(k)},\ P_{2,j\hbar(k)},\ P_{3,j\hbar(k)},\ P_{4,j\hbar(k)}\}$, $P_{j\hbar(k+1)}=\mathrm{diag}\{P_{1,j\hbar(k+1)},\ P_{2,j\hbar(k+1)},\ P_{3,j\hbar(k+1)},\ P_{4,j\hbar(k+1)}\}$ and Q_j $(j=1,2)$ such that the following matrix inequalities hold for all possible token $\hbar(k)\in\{1,\ 2,\cdots,m\}$:

$$\Omega(k)=\begin{bmatrix}\bar{\Omega}_{11} & * & * \\ \bar{\Omega}_{21} & \bar{\Omega}_{22} & * \\ \bar{\Omega}_{31} & \bar{\Omega}_{23} & \bar{\Omega}_{33}\end{bmatrix}<0 \tag{7}$$

where

$$\bar{\Omega}_{11}\triangleq\begin{bmatrix}-\Omega_{11}^k & * & * & * \\ 0 & -\frac{Q_1}{\bar{\mu}_1} & * & * \\ 0 & 0 & -2I_n & * \\ 0 & 0 & \Omega_{43} & -\Omega_{44}^k\end{bmatrix},$$

$$\bar{\Omega}_{21}\triangleq\begin{bmatrix}0 & 0 & 0 & 0 \\ \Omega_{61} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \bar{A}(k) & (\bar{A}_d+\bar{\theta}_1\bar{C}) & (\overline{V}_1+\bar{\theta}_1\overline{W}_1) & 0\end{bmatrix},$$

$$\Omega_{77}^k\triangleq P_{1\hbar(k+1)}^{-1},$$

$$\bar{\Omega}_{22}\triangleq\begin{bmatrix}-\frac{Q_2}{\bar{\mu}_2} & * & * & * \\ 0 & -2I_n & * & * \\ 0 & 0 & -\gamma^2I & * \\ 0 & 0 & \bar{G}(k) & -\Omega_{77}^k\end{bmatrix},$$

$$\bar{\Omega}_{31}\triangleq\begin{bmatrix}0 & 0 & 0 & \bar{B}(k) \\ 0 & \bar{\theta}_1(1-\bar{\theta}_1)\bar{C} & \bar{\theta}_1(1-\bar{\theta}_1)\overline{W}_1 & 0 \\ 0 & 0 & 0 & 0 \\ \bar{M}_f & 0 & 0 & 0 \\ 0 & \bar{N}_f & 0 & 0\end{bmatrix},$$

$$\Omega_{88}^k\triangleq P_{2\hbar(k+1)}^{-1},$$

$$\bar{\Omega}_{32}\triangleq\begin{bmatrix}(\bar{B}_d+\bar{\theta}_2\bar{D}) & (\overline{V}_2+\bar{\theta}_2\overline{W}_2) & \bar{H}(k) & 0 \\ 0 & 0 & 0 & 0 \\ \bar{\theta}_2(1-\bar{\theta}_2)\overline{W}_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0\end{bmatrix},$$

$$\bar{\Omega}_{33}\triangleq\begin{bmatrix}-\Omega_{88}^k & * & * & * & * \\ 0 & -\Omega_{77}^k & * & * & * \\ 0 & 0 & -\Omega_{88}^k & * & * \\ 0 & 0 & 0 & -I_{n_z} & * \\ 0 & 0 & 0 & 0 & -I_{n_z}\end{bmatrix},$$

$$\Omega_{11}^k\triangleq P_{1\hbar(k)}-\bar{\mu}_1Q_1+\bar{I}^TS_2^TS_1\bar{I}+\bar{I}^TS_1^TS_2\bar{I},$$

$$\Omega_{44}^k\triangleq P_{2\hbar(k)}-\bar{\mu}_2Q_2+\bar{I}^TR_2^TR_1\bar{I}+\bar{I}^TR_1^TR_2\bar{I},$$

$$\Omega_{61}\triangleq(S_1+S_2)\bar{I},\quad \Omega_{43}\triangleq\bar{I}^T(R_1+R_2)^T.$$

Proof: Let us first consider the exponential stability for the augmented dynamics (6).

Noticing $g(0)=f(0)=0$, it follows from (2) that

$$\big(-g(a)+R_2a\big)^T\big(-g(a)+R_1a\big)\leq 0,$$
$$\big(-f(a)+S_2a\big)^T\big(-f(a)+S_1a\big)\leq 0,$$

which implies

$$\begin{cases} g^T\big(\bar{I}\zeta(k)\big)(R_1+R_2)\bar{I}\zeta(k)-g^T\big(\bar{I}\zeta(k)\big)g\big(\bar{I}\zeta(k)\big) \\ \quad -\zeta^T(k)\bar{I}^TR_2^TR_1\bar{I}\zeta(k)\geq 0, \\ f^T\big(\bar{I}\xi(k)\big)(S_1+S_2)\bar{I}\xi(k)-f^T\big(\bar{I}\xi(k)\big)f\big(\bar{I}\xi(k)\big) \\ \quad -\xi^T(k)\bar{I}^TS_2^TS_1\bar{I}\xi(k)\geq 0.\end{cases} \tag{8}$$

Consider the following matrix functional for the system (6):

$$\begin{aligned}V(k)\triangleq\ &\xi^T(k)P_{1\hbar(k)}\xi(k)+\sum_{d=1}^{+\infty}\mu_{1d}\sum_{t=-d}^{-1}\xi^T(k+t)Q_1\\ &\times\xi(k+t)+\zeta^T(k)P_{2\hbar(k)}\zeta(k)\\ &+\sum_{d=1}^{+\infty}\mu_{2d}\sum_{t=-d}^{-1}\zeta^T(k+t)Q_2\zeta(k+t).\end{aligned} \tag{9}$$

Let $\vartheta(k)=0$. Calculating the difference of $V(k)$ along the trajectory of state estimation process (6) and taking the mathematical expectation yield

$$\begin{aligned}&\Delta V(k)\\ =&\mathbb{E}\Big\{V(k+1)-V(k)\Big|\aleph(k)\Big\}\\ =&\mathbb{E}\Big\{\xi^T(k+1)P_{1\hbar(k+1)}\xi(k+1)-\xi^T(k)P_{1\hbar(k)}\xi(k)\\ &+\zeta^T(k+1)P_{2\hbar(k+1)}\zeta(k+1)-\zeta^T(k)P_{2\hbar(k)}\zeta(k)\\ &+\bar{\mu}_1\xi^T(k)Q_1\xi(k)-\sum_{d=1}^{+\infty}\mu_{1d}\xi^T(k-d)Q_1\xi(k-d)\\ &+\bar{\mu}_2\zeta^T(k)Q_2\zeta(k)-\sum_{d=1}^{+\infty}\mu_{2d}\zeta^T(k-d)Q_2\zeta(k-d)\Big|\aleph(k)\Big\},\end{aligned}$$

where $\Theta_t(k)\triangleq\big\{e(k),\ e(k-1),\cdots,e(k-t)\big\}$ and $\aleph(k)\triangleq\bigcup_{t=1}^{\infty}\Theta_t(k)$.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TNNLS.2021.3055942, IEEE Transactions on Neural Networks and Learning Systems

By using Lemma 1, one obtains

$$\begin{aligned}
&\Delta V(k)\\
\leq&\mathbb{E}\Bigg\{\Bigg[\bar{A}(k)\xi(k)+\bar{A}_d\sum_{d=1}^{\infty}\mu_{1d}\xi(k-d)+\overline{V}_1g\big(\bar{I}\zeta(k)\big)\\
&\quad+\bar{\theta}_1\Bigg(\bar{C}\sum_{d=1}^{\infty}\mu_{1d}\xi(k-d)+\overline{W}_1g\big(\bar{I}\zeta(k)\big)\Bigg)\Bigg]^T\\
&\times P_{1\hbar(k+1)}\\
&\times\Bigg[\bar{A}(k)\xi(k)+\bar{A}_d\sum_{d=1}^{\infty}\mu_{1d}\xi(k-d)+\overline{V}_1g\big(\bar{I}\zeta(k)\big)\\
&\quad+\bar{\theta}_1\Bigg(\bar{C}\sum_{d=1}^{\infty}\mu_{1d}\xi(k-d)+\overline{W}_1g\big(\bar{I}\zeta(k)\big)\Bigg)\Bigg]\\
&+\Bigg[\bar{B}(k)\zeta(k)+\bar{B}_d\sum_{d=1}^{\infty}\mu_{2d}\zeta(k-d)+\overline{V}_2f\big(\bar{I}\xi(k)\big)\\
&\quad+\bar{\theta}_2\Bigg(\bar{D}\sum_{d=1}^{\infty}\mu_{2d}\zeta(k-d)+\overline{W}_2f\big(\bar{I}\xi(k)\big)\Bigg)\Bigg]^T\\
&\times P_{2\hbar(k+1)}\\
&\times\Bigg[\bar{B}(k)\zeta(k)+\bar{B}_d\sum_{d=1}^{\infty}\mu_{2d}\zeta(k-d)+\overline{V}_2f\big(\bar{I}\xi(k)\big)\\
&\quad+\bar{\theta}_2\Bigg(\bar{D}\sum_{d=1}^{\infty}\mu_{2d}\zeta(k-d)+\overline{W}_2f\big(\bar{I}\xi(k)\big)\Bigg)\Bigg]\\
&+\bar{\theta}_1(1-\bar{\theta}_1)\Bigg(\bar{C}\sum_{d=1}^{\infty}\mu_{1d}\xi(k-d)+\overline{W}_1g\big(\bar{I}\zeta(k)\big)\Bigg)^T\\
&\times P_{1\hbar(k+1)}\Bigg(\bar{C}\sum_{d=1}^{\infty}\mu_{1d}\xi(k-d)+\overline{W}_1g\big(\bar{I}\zeta(k)\big)\Bigg)\\
&+\bar{\theta}_2(1-\bar{\theta}_2)\Bigg(\bar{D}\sum_{d=1}^{\infty}\mu_{2d}\zeta(k-d)+\overline{W}_2f\big(\bar{I}\xi(k)\big)\Bigg)^T\\
&\times P_{2\hbar(k+1)}\Bigg(\bar{D}\sum_{d=1}^{\infty}\mu_{2d}\zeta(k-d)+\overline{W}_2f\big(\bar{I}\xi(k)\big)\Bigg)\\
&-\xi^T(k)P_{1\hbar(k)}\xi(k)-\zeta^T(k)P_{2\hbar(k)}\zeta(k)\\
&+\bar{\mu}_1\xi^T(k)Q_1\xi(k)-\frac{1}{\bar{\mu}_1}\Bigg(\sum_{d=1}^{+\infty}\mu_{1d}\xi(k-d)\Bigg)^TQ_1\\
&\times\sum_{d=1}^{+\infty}\mu_{1d}\xi(k-d)\\
&+\bar{\mu}_2\zeta^T(k)Q_2\zeta(k)-\frac{1}{\bar{\mu}_2}\Bigg(\sum_{d=1}^{+\infty}\mu_{2d}\zeta(k-d)\Bigg)^TQ_2\\
&\times\sum_{d=1}^{+\infty}\mu_{2d}\zeta(k-d)\Big|\aleph(k)\Bigg\}.
\end{aligned}$$

Keeping the relationship (8) in mind, we have

$$\begin{aligned}
&\Delta V(k)\\
\leq&\mathbb{E}\Bigg\{-\xi^T(k)\big(P_{1\hbar(k)}-\bar{\mu}_1Q_1+\bar{I}^TS_2^TS_1\bar{I}+\bar{I}^TS_1^TS_2\bar{I}\big)\\
&\times\xi(k)-\zeta^T(k)\big(P_{2\hbar(k)}-\bar{\mu}_2Q_2+\bar{I}^TR_2^TR_1\bar{I}\\
&+\bar{I}^TR_1^TR_2\bar{I}\big)\zeta(k)-\frac{1}{\bar{\mu}_1}\Bigg(\sum_{d=1}^{+\infty}\mu_{1d}\xi(k-d)\Bigg)^TQ_1\sum_{d=1}^{+\infty}\mu_{1d}\\
&\times\xi(k-d)-\frac{1}{\bar{\mu}_2}\Bigg(\sum_{d=1}^{+\infty}\mu_{2d}\zeta(k-d)\Bigg)^TQ_2\sum_{d=1}^{+\infty}\mu_{2d}\zeta(k-d)\\
&-2g^T\big(\bar{I}\zeta(k)\big)g\big(\bar{I}\zeta(k)\big)-2f^T\big(\bar{I}\xi(k)\big)f\big(\bar{I}\xi(k)\big)\\
&+2g^T\big(\bar{I}\zeta(k)\big)(R_1+R_2)\bar{I}\zeta(k)\\
&+2f^T\big(\bar{I}\xi(k)\big)(S_1+S_2)\bar{I}\xi(k)\\
&+\Bigg[\bar{A}(k)\xi(k)+\big(\bar{A}_d+\bar{\theta}_1\bar{C}\big)\sum_{d=1}^{\infty}\mu_{1d}\xi(k-d)\\
&+\big(\overline{V}_1+\bar{\theta}_1\overline{W}_1\big)g\big(\bar{I}\zeta(k)\big)\Bigg)\Bigg]^T P_{1\hbar(k+1)}\\
&\times\Bigg[\bar{A}(k)\xi(k)+\big(\bar{A}_d+\bar{\theta}_1\bar{C}\big)\sum_{d=1}^{\infty}\mu_{1d}\xi(k-d)\\
&+\big(\overline{V}_1+\bar{\theta}_1\overline{W}_1\big)g\big(\bar{I}\zeta(k)\big)\Bigg)\Bigg]\\
&+\Bigg[\bar{B}(k)\zeta(k)+\big(\bar{B}_d+\bar{\theta}_2\bar{D}\big)\sum_{d=1}^{\infty}\mu_{2d}\zeta(k-d)\\
&+\big(\overline{V}_2+\bar{\theta}_2\overline{W}_2\big)f\big(\bar{I}\xi(k)\big)\Bigg)\Bigg]^T P_{2\hbar(k+1)}\\
&\times\Bigg[\bar{B}(k)\zeta(k)+\big(\bar{B}_d+\bar{\theta}_2\bar{D}\big)\sum_{d=1}^{\infty}\mu_{2d}\zeta(k-d)\\
&+\big(\overline{V}_2+\bar{\theta}_2\overline{W}_2\big)f\big(\bar{I}\xi(k)\big)\Bigg)\Bigg]\\
&+\bar{\theta}_1(1-\bar{\theta}_1)\Bigg(\bar{C}\sum_{d=1}^{\infty}\mu_{1d}\xi(k-d)+\overline{W}_1g\big(\bar{I}\zeta(k)\big)\Bigg)^T\\
&\times P_{1\hbar(k+1)}\Bigg(\bar{C}\sum_{d=1}^{\infty}\mu_{1d}\xi(k-d)+\overline{W}_1g\big(\bar{I}\zeta(k)\big)\Bigg)\\
&+\bar{\theta}_2(1-\bar{\theta}_2)\Bigg(\bar{D}\sum_{d=1}^{\infty}\mu_{2d}\zeta(k-d)+\overline{W}_2f\big(\bar{I}\xi(k)\big)\Bigg)^T\\
&\times P_{2\hbar(k+1)}\Bigg(\bar{D}\sum_{d=1}^{\infty}\mu_{2d}\zeta(k-d)+\overline{W}_2f\big(\bar{I}\xi(k)\big)\Bigg)\Big|\aleph(k)\Bigg\}.
\end{aligned}$$

By denoting

$$\phi(k)=\Bigg[\xi^T(k)\quad\Bigg(\sum_{d=1}^{\infty}\mu_{1d}\xi(k-d)\Bigg)^T\quad g^T\big(\bar{I}\zeta(k)\big)\quad\zeta^T(k)\quad\Bigg(\sum_{d=1}^{\infty}\mu_{2d}\zeta(k-d)\Bigg)^T\quad f^T\big(\bar{I}\xi(k)\big)\Bigg]^T,$$

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we have $\Delta V(k) \le \mathbb{E}\left\{\phi(k)^T\Xi(k)\phi(k)\middle|\aleph(k)\right\}$, where

$$\Xi(k) \triangleq \begin{bmatrix} -\Xi_{11} & * & * & * & * & * \\ \Xi_{21} & -\Xi_{22} & * & * & * & * \\ \Xi_{31} & \Xi_{32} & -\Xi_{33} & * & * & * \\ 0 & 0 & \Omega_{43} & -\Xi_{44} & * & * \\ 0 & 0 & 0 & \Xi_{54} & -\Xi_{55} & * \\ \Omega_{61} & 0 & 0 & \Xi_{64} & \Xi_{65} & -\Xi_{66} \end{bmatrix},$$

$$\begin{aligned}
&\Xi_{11} \triangleq \Omega_{11} - \bar{A}^T(k)P_{1\hbar(k+1)}\bar{A}(k),\\
&\Xi_{22} \triangleq \frac{Q_1}{\bar{\mu}_1} - \left(\bar{A}_d + \bar{\theta}_1\bar{C}\right)^T P_{1\hbar(k+1)}\left(\bar{A}_d + \bar{\theta}_1\bar{C}\right)\\
&\quad - \bar{\theta}_1(1-\bar{\theta}_1)\bar{C}^T P_{1\hbar(k+1)}\bar{C},\\
&\Xi_{33} \triangleq 2I - \left(\overline{V}_1 + \bar{\theta}_1\overline{W}_1\right)^T P_{1\hbar(k+1)}\left(\overline{V}_1 + \bar{\theta}_1\overline{W}_1\right)\\
&\quad - \bar{\theta}_1(1-\bar{\theta}_1)\overline{W}_1^T P_{1\hbar(k+1)}\overline{W}_1,\\
&\Xi_{44} \triangleq \Omega_{44} - \bar{B}^T(k)P_{1\hbar(k+1)}\bar{B}(k),\\
&\Xi_{55} \triangleq \frac{Q_2}{\bar{\mu}_2} - \left(\bar{B}_d + \bar{\theta}_2\bar{D}\right)^T P_{1\hbar(k+1)}\left(\bar{B}_d + \bar{\theta}_2\bar{D}\right)\\
&\quad - \bar{\theta}_2(1-\bar{\theta}_2)\bar{D}P_{2\hbar(k+1)}\bar{D},\\
&\Xi_{66} \triangleq 2I - \left(\overline{V}_2 + \bar{\theta}_2\overline{W}_2\right)^T P_{1\hbar(k+1)}\left(\overline{V}_2 + \bar{\theta}_2\overline{W}_2\right)\\
&\quad - \bar{\theta}_2(1-\bar{\theta}_2)\overline{W}_2 P_{2\hbar(k+1)}\overline{W}_2,\\
&\Xi_{21} \triangleq \left(\bar{A}_d + \bar{\theta}_1\bar{C}\right)^T P_{1\hbar(k+1)}\bar{A}(k),\\
&\Xi_{31} \triangleq \left(\overline{V}_1 + \bar{\theta}_1\overline{W}_1\right)^T P_{1\hbar(k+1)}\bar{A}(k),\\
&\Xi_{32} \triangleq \left(\overline{V}_1 + \bar{\theta}_1\overline{W}_1\right)^T P_{1\hbar(k+1)}\left(\bar{A}_d + \bar{\theta}_1\bar{C}\right)\\
&\quad + \bar{\theta}_1(1-\bar{\theta}_1)\overline{W}_1^T P_{1\hbar(k+1)}\bar{C},\\
&\Xi_{54} \triangleq \left(\bar{B}_d + \bar{\theta}_2\bar{D}\right)^T P_{2\hbar(k+1)}\bar{B}(k),\\
&\Xi_{64} \triangleq \left(\overline{V}_2 + \bar{\theta}_2\overline{W}_2\right)^T P_{2\hbar(k+1)}\bar{B}(k),\\
&\Xi_{65} \triangleq \left(\overline{V}_2 + \bar{\theta}_2\overline{W}_2\right)^T P_{2\hbar(k+1)}\left(\bar{B}_d + \bar{\theta}_2\bar{D}\right)\\
&\quad + \bar{\theta}_2(1-\bar{\theta}_2)\overline{W}_2^T P_{2\hbar(k+1)}\bar{D},
\end{aligned}$$

which further imply that

$$\Delta V(k) \le \lambda_{\max}(\Xi(k))\mathbb{E}\left\{\left\|\xi(k)\right\|^2 + \left\|\zeta(k)\right\|^2\middle|\aleph(k)\right\}.$$

On the other hand, by the definition of $V(k)$ in (9), for any sufficiently large integer $\tau > d$, it is obvious that

$$\begin{aligned}
\mathbb{E}\big\{V(k)\big\} \le& \lambda_{\max}(P_{1\hbar(k)})\mathbb{E}\big\{\left\|\xi(k)\right\|^2\big\}\\
&+ \lambda_{\max}(P_{2\hbar(k)})\mathbb{E}\big\{\left\|\zeta(k)\right\|^2\big\}\\
&+ \lambda_{\max}(Q_1)\bar{\mu}_1\sum_{t=-\tau}^{-1}\mathbb{E}\big\{\left\|\xi(k+t)\right\|^2\big\}\\
&+ \lambda_{\max}(Q_2)\bar{\mu}_2\sum_{t=-\tau}^{-1}\mathbb{E}\big\{\left\|\zeta(k+t)\right\|^2\big\}.
\end{aligned}$$

Furthermore, for any given scalar $\mu > 1$, one obtains

$$\begin{aligned}
&\mathbb{E}\big\{\mu^k V(k)\big\}\\
=&\mathbb{E}\left\{V(0) + \sum_{i=0}^{k-1}\left(\mu^{i+1}\Delta V(i) + (\mu-1)\mu^i V(i)\right)\right\}\\
=&\mathbb{E}\left\{V(0) + \sum_{i=0}^{k-1}\mu^i\left(\mu\Delta V(i) + (\mu-1)V(i)\right)\right\}.
\end{aligned}$$

Subsequently, it follows that

$$\begin{aligned}
&\mathbb{E}\big\{\mu^k V(k)\big\}\\
\le&\lambda_{\max}(P_{1\hbar(0)})\mathbb{E}\big\{\left\|\xi(0)\right\|^2\big\} + \lambda_{\max}(P_{2\hbar(0)})\mathbb{E}\big\{\left\|\zeta(0)\right\|^2\big\}\\
&+ \lambda_{\max}(Q_1)\bar{\mu}_1\tau\mathbb{E}\big\{\left\|\xi(t)\right\|^2\big\} + \lambda_{\max}(Q_2)\bar{\mu}_2\tau\mathbb{E}\big\{\left\|\zeta(t)\right\|^2\big\}\\
&+ \lambda_{\max}\big(\mu\Xi(k) + (\mu-1)P_{1\hbar(k)}\big)\sum_{i=0}^{k-1}\mu^i\mathbb{E}\big\{\left\|\xi(k)\right\|^2\big\}\\
&+ \lambda_{\max}\big(\mu\Xi(k) + (\mu-1)P_{1\hbar(k)}\big)\sum_{i=0}^{k-1}\mu^i\mathbb{E}\big\{\left\|\zeta(k)\right\|^2\big\}\\
&+ (\mu-1)\lambda_{\max}(Q_1)\bar{\mu}_1\Bigg(\tau^2\mu^\tau\sup_{j\in\mathbb{N}[-\tau,\,0]}\mathbb{E}\big\{\left\|\xi(k)\right\|^2\big\}\\
&+\tau\mu^\tau\sum_{j=0}^{k-1}\mu^j\mathbb{E}\big\{\left\|\xi(j)\right\|^2\big\}\Bigg)\\
&+ (\mu-1)\lambda_{\max}(Q_2)\bar{\mu}_2\Bigg(\tau^2\mu^\tau\sup_{j\in\mathbb{N}[-\tau,\,0]}\mathbb{E}\big\{\left\|\zeta(k)\right\|^2\big\}\\
&+\tau\mu^\tau\sum_{j=0}^{k-1}\mu^j\mathbb{E}\big\{\left\|\zeta(j)\right\|^2\big\}\Bigg)\\
\le& a_1(\mu)\sup_{j\in\mathbb{N}[-\tau,\,0]}\mathbb{E}\big\{\left\|\xi(j)\right\|^2\big\} + b_1(\mu)\sum_{j=0}^{k}\mu^j\mathbb{E}\big\{\left\|\xi(j)\right\|^2\big\}\\
&+ a_2(\mu)\sup_{j\in\mathbb{N}[-\tau,\,0]}\mathbb{E}\big\{\left\|\zeta(j)\right\|^2\big\}\\
&+ b_2(\mu)\sum_{j=0}^{k}\mu^j\mathbb{E}\big\{\left\|\zeta(j)\right\|^2\big\},
\end{aligned}$$

where

$$\begin{aligned}
a_1(\mu) \triangleq& \lambda_{\max}(P_{1\hbar(0)}) + \lambda_{\max}(Q_1)\bar{\mu}_1\tau\\
&+ (\mu-1)\lambda_{\max}(Q_1)\bar{\mu}_1\tau^2\mu^\tau,\\
a_2(\mu) \triangleq& \lambda_{\max}(P_{2\hbar(0)}) + \lambda_{\max}(Q_2)\bar{\mu}_2\tau\\
&+ (\mu-1)\lambda_{\max}(Q_2)\bar{\mu}_2\tau^2\mu^\tau,\\
b_1(\mu) \triangleq& \mu\lambda_{\max}(\Xi(k)) + (\mu-1)\lambda_{\max}(P_{1\hbar(k)})\\
&+ (\mu-1)\lambda_{\max}(Q_1)\bar{\mu}_1\tau\mu^\tau,\\
b_2(\mu) \triangleq& \mu\lambda_{\max}(\Xi(k)) + (\mu-1)\lambda_{\max}(P_{2\hbar(k)})\\
&+ (\mu-1)\lambda_{\max}(Q_2)\bar{\mu}_2\tau\mu^\tau.
\end{aligned}$$

Here, we know $a_1(1) > 0$ and $a_2(1) > 0$ from the positive definiteness of the matrices $P_{j\hbar(k)}$ and Q_j $(j = 1, 2)$, and $b_1(1) < 0$ and $b_2(1) < 0$ from $\Xi(k) < 0$. By further denoting

$$\overline{\Xi}(k) \triangleq \begin{bmatrix} -\overline{\Xi}_{11} & * & * & * & * & * & * \\ \Xi_{21} & -\Xi_{22} & * & * & * & * & * \\ \Xi_{31} & \Xi_{32} & -\Xi_{33} & * & * & * & * \\ 0 & 0 & \Omega_{43} & -\overline{\Xi}_{44} & * & * & * \\ 0 & 0 & 0 & \Xi_{54} & -\Xi_{55} & * & * \\ \Omega_{61} & 0 & 0 & \Xi_{64} & \Xi_{65} & -\Xi_{66} & * \\ \overline{\Xi}_{71} & \overline{\Xi}_{72} & \overline{\Xi}_{73} & \overline{\Xi}_{74} & \overline{\Xi}_{75} & \overline{\Xi}_{76} & -\overline{\Xi}_{77} \end{bmatrix},$$

$$\begin{aligned}
&\overline{\Xi}_{11} \triangleq \Omega_{11} - \bar{A}^T(k)P_{1\hbar(k+1)}\bar{A}(k) + \bar{M}_f^T\bar{M}_f,\\
&\overline{\Xi}_{44} \triangleq \Omega_{44} - \bar{B}^T(k)P_{1\hbar(k+1)}\bar{B}(k) + \bar{N}_f^T\bar{N}_f,
\end{aligned}$$

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$$\begin{aligned}
&\overline{\Xi}_{71} \triangleq \bar{G}^T(k)P_{1\hbar(k+1)}\bar{A}(k),\\
&\overline{\Xi}_{72} \triangleq \bar{G}^T(k)P_{1\hbar(k+1)}\big(\bar{A}_d+\bar{\theta}_1\bar{C}\big),\\
&\overline{\Xi}_{73} \triangleq \bar{G}^T(k)P_{1\hbar(k+1)}\big(\overline{V}_1+\bar{\theta}_1\overline{W}_1\big),\\
&\overline{\Xi}_{74} \triangleq \bar{H}^T(k)P_{1\hbar(k+1)}\bar{B}(k),\\
&\overline{\Xi}_{75} \triangleq \bar{H}^T(k)P_{1\hbar(k+1)}\big(\bar{B}_d+\bar{\theta}_2\bar{D}\big),\\
&\overline{\Xi}_{76} \triangleq \bar{H}^T(k)P_{1\hbar(k+1)}\big(\overline{V}_2+\bar{\theta}_2\overline{W}_2\big),\\
&\overline{\Xi}_{77} \triangleq -\bar{G}^T(k)P_{1\hbar(k+1)}\bar{G}(k)-\bar{H}^T(k)P_{2\hbar(k+1)}\bar{H}(k)+\gamma^2,
\end{aligned}$$

we obtain $\overline{\Xi}(k) < 0$ from $\Omega(k) < 0$ in the help of Schur Complement, which indicates $\Xi(k) < 0$ because $\Xi(k)$ is a principal submatrix of $\overline{\Xi}(k)$. Moreover, noting that $a_1(1) > 0$, $a_2(1) > 0$, $b_1(1) < 0$ and $b_2(1) < 0$, we have

$$\begin{aligned}
&\mathbb{E}\big\{\|\xi(k)\|^2\big\}+\mathbb{E}\big\{\|\zeta(k)\|^2\big\}\\
&\le -\mu^{-k}\frac{\max\{a_{1(z)},a_{2(z)}\}}{\max\{b_1(z),b_2(z)\}}\Bigg(\sup_{j\in\mathbb{N}[-\tau,\,0]}\Big(\mathbb{E}\big\{\|\xi(j)\|^2\big\}\\
&\quad+\mathbb{E}\big\{\|\zeta(j)\|^2\big\}\Big)-\mathbb{E}\big\{z^kV(k)\big\}\Bigg),
\end{aligned}$$

which indicates that the state estimation process (6) is *exponentially stable* in the mean-square sense.

We are now in the position to deal with the H_∞ performance analysis for the state estimation process (6). Based upon the aforementioned stability analysis, it follows that

$$\begin{aligned}
&\sum_{k=0}^{\infty}\mathbb{E}\left\{V(k+1)-V(k)+\tilde{z}^T(k)\tilde{z}(k)-\gamma^2\vartheta^T(k)\vartheta(k)\right\}\\
\le&\mathbb{E}\Bigg\{-\xi^T(k)\big(P_{1\hbar(k)}-\bar{\mu}_1Q_1\\
&+\bar{I}^TS_2^TS_1\bar{I}+\bar{I}^TS_1^TS_2\bar{I}\big)\xi(k)\\
&-\zeta^T(k)\big(P_{2\hbar(k)}-\bar{\mu}_2Q_2+\bar{I}^TR_2^TR_1\bar{I}+\bar{I}^TR_1^TR_2\bar{I}\big)\zeta(k)\\
&-\frac{1}{\bar{\mu}_1}\left(\sum_{d=1}^{+\infty}\mu_{1d}\xi(k-d)\right)^TQ_1\sum_{d=1}^{+\infty}\mu_{1d}\xi(k-d)\\
&-\frac{1}{\bar{\mu}_2}\left(\sum_{d=1}^{+\infty}\mu_{2d}\zeta(k-d)\right)^TQ_2\sum_{d=1}^{+\infty}\mu_{2d}\zeta(k-d)\\
&-2g^T\big(\bar{I}\zeta(k)\big)g\big(\bar{I}\zeta(k)\big)-2f^T\big(\bar{I}\xi(k)\big)f\big(\bar{I}\xi(k)\big)\\
&+2g^T\big(\bar{I}\zeta(k)\big)(R_1+R_2)\bar{I}\zeta(k)\\
&+2f^T\big(\bar{I}\xi(k)\big)(S_1+S_2)\bar{I}\xi(k)\\
&+\Bigg[\bar{A}(k)\xi(k)+\big(\bar{A}_d+\bar{\theta}_1\bar{C}\big)\sum_{d=1}^{\infty}\mu_{1d}\xi(k-d)\\
&+\big(\overline{V}_1+\bar{\theta}_1\overline{W}_1\big)g\big(\bar{I}\zeta(k)\big)+\bar{G}(k)\vartheta(k)\Bigg]^TP_{1\hbar(k+1)}\\
&\times\Bigg[\bar{A}(k)\xi(k)+\big(\bar{A}_d+\bar{\theta}_1\bar{C}\big)\sum_{d=1}^{\infty}\mu_{1d}\xi(k-d)\\
&+\big(\overline{V}_1+\bar{\theta}_1\overline{W}_1\big)g\big(\bar{I}\zeta(k)\big)+\bar{G}(k)\vartheta(k)\Bigg]\\
&+\Bigg[\bar{B}(k)\zeta(k)+\big(\bar{B}_d+\bar{\theta}_2\bar{D}\big)\sum_{d=1}^{\infty}\mu_{2d}\zeta(k-d)\\
&+\big(\overline{V}_2+\bar{\theta}_2\overline{W}_2\big)f\big(\bar{I}\xi(k)\big)+\bar{H}(k)\vartheta(k)\Bigg]^TP_{2\hbar(k+1)}\\
&\times\Bigg[\bar{B}(k)\zeta(k)+\big(\bar{B}_d+\bar{\theta}_2\bar{D}\big)\sum_{d=1}^{\infty}\mu_{2d}\zeta(k-d)\\
&+\big(\overline{V}_2+\bar{\theta}_2\overline{W}_2\big)f\big(\bar{I}\xi(k)\big)+\bar{H}(k)\vartheta(k)\Bigg]\\
&+\bar{\theta}_1(1-\bar{\theta}_1)\left(\bar{C}\sum_{d=1}^{\infty}\mu_{1d}\xi(k-d)+\overline{W}_1g\big(\bar{I}\zeta(k)\big)\right)^T\\
&\times P_{1\hbar(k+1)}\left(\bar{C}\sum_{d=1}^{\infty}\mu_{1d}\xi(k-d)+\overline{W}_1g\big(\bar{I}\zeta(k)\big)\right)\\
&+\bar{\theta}_2(1-\bar{\theta}_2)\left(\bar{D}\sum_{d=1}^{\infty}\mu_{2d}\zeta(k-d)+\overline{W}_2f\big(\bar{I}\xi(k)\big)\right)^T\\
&\times P_{2\hbar(k+1)}\left(\bar{D}\sum_{d=1}^{\infty}\mu_{2d}\zeta(k-d)+\overline{W}_2f\big(\bar{I}\xi(k)\big)\right)\\
&+\xi^T(k)\bar{M}_f^T\bar{M}_f\xi(k)\\
&+\zeta^T(k)\bar{N}_f^T\bar{N}_f\zeta(k)-\gamma^2\vartheta^T(k)\vartheta(k)\Big|\aleph(k)\Bigg\}\\
=&\mathbb{E}\left\{\overline{\phi}(k)^T\Xi(k)\overline{\phi}(k)\big|\aleph(k)\right\},
\end{aligned}$$

where $\overline{\phi}(k) \triangleq \begin{bmatrix}\phi^T(k) & \vartheta^T(k)\end{bmatrix}^T$. Considering $V(k) \ge 0$ for any $\vartheta(k) \ne 0$ and the zero initial condition, one immediately has

$$\begin{aligned}
&\sum_{k=0}^{\infty}\mathbb{E}\left\{\tilde{z}^T(k)\tilde{z}(k)-\gamma^2\vartheta^T(k)\vartheta(k)\right\}\\
\le&\sum_{k=0}^{\infty}\mathbb{E}\left\{V(k+1)-V(k)+\tilde{z}^T(k)\tilde{z}(k)-\gamma^2\vartheta^T(k)\vartheta(k)\right\}\\
\le&0,
\end{aligned}$$

and the proof is complete. ■

In Theorem 1, sufficient conditions, which are dependent on the access token $\hbar(k)$, are established to ensure the exponential stability of the dynamics of estimation process (6). Here, it should be emphasized that the disturbance attenuation level γ given in this paper is a prescribed constant which might not be optimal. In case of seeking the optimal disturbance attenuation level for the proposed estimation approach, the gevp solver of the LMI toolbox in Matlab can be used. Moreover, since the matrix inequalities (7) are nonlinear due to the existence of the term $P^{-1}_{1\hbar(k+1)}$, which brings much difficulty for numerical solutions. The following theorem provides a method to deal with the design issue of the suggested estimator (5).

Theorem 2: Under the RRP and the assumption (2), there exists an H_∞ exponential estimator of the form (5) for the original stochastic BAM neural networks (1) such that the augmented dynamics of the system (6) is exponentially mean-square stable with the given disturbance attenuation level $\gamma > 0$ if there exist positive definite matrices $P_{1,jl} \in \mathbb{R}^{n\times n}$, $P_{2,jl} \in \mathbb{R}^{m\times m}$, $P_{3,jl} \in \mathbb{R}^{n\times n}$, $P_{4,jl} \in \mathbb{R}^{m\times m}$, Q_j $(j = 1,2,\ l = 1,\ 2,\cdots,m)$ and matrices $\mathcal{A}_{fl}$, $\mathcal{K}_{fil}$, $\mathcal{B}_{fl}$, $\mathcal{L}_{fil}$ $(i = 1,2,3)$ such that the following linear matrix inequalities (LMIs) hold:

$$\Omega(l,r) < 0\quad\begin{cases} l\in\{1,\ 2,\cdots,m-1\}, & r=l+1,\\ l=m, & r=1,\end{cases}\tag{10}$$

where

$$P_{jl} \triangleq \mathrm{diag}\{P_{1,jl},\ P_{2,jl},\ P_{3,jl},\ P_{4,jl}\},$$

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$$\Omega(l,r) \triangleq \begin{bmatrix} \check{\Omega}_{11} & * & * \\ \check{\Omega}_{21} & \check{\Omega}_{22} & * \\ \check{\Omega}_{31} & \check{\Omega}_{32} & \check{\Omega}_{33} \end{bmatrix}, \ \check{\Omega}_{21} \triangleq \begin{bmatrix} 0 & 0 & 0 & 0 \\ \Omega_{61} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \widetilde{A}^{l,r} & \widetilde{A}_d^r & \widetilde{V}_1^r & 0 \end{bmatrix},$$

$$\check{\Omega}_{11} \triangleq \begin{bmatrix} -\Omega_{11}^l & * & * & * \\ 0 & -\frac{Q_1}{\bar{\mu}_1} & * & * \\ 0 & 0 & -2I_n & * \\ 0 & 0 & \Omega_{43} & -\Omega_{44}^l \end{bmatrix},$$

$$\widetilde{G}^{l,r} \triangleq \begin{bmatrix} 0 & 0 & 0 & 0 \\ P_{2,1r}\Psi_l G & 0 & 0 & 0 \\ \mathcal{K}_{f1r}\Psi_l G & 0 & 0 & 0 \\ \mathcal{K}_{f3r}\Psi_l G & 0 & 0 & 0 \end{bmatrix},$$

$$\check{\Omega}_{22} \triangleq \begin{bmatrix} -\frac{Q_2}{\bar{\mu}_2} & * & * & * \\ 0 & -2I_n & * & * \\ 0 & 0 & -\gamma^2 I & * \\ 0 & 0 & \widetilde{G}^{l,r} & -\widetilde{\Omega}_{77}^r \end{bmatrix},$$

$$\check{\Omega}_{31} \triangleq \begin{bmatrix} 0 & 0 & 0 & \widetilde{B}^{l,r} \\ 0 & \widetilde{C}^r & \widetilde{W}_1^r & 0 \\ 0 & 0 & 0 & 0 \\ \bar{M}_f & 0 & 0 & 0 \\ 0 & \bar{N}_f & 0 & 0 \end{bmatrix},$$

$$\check{\Omega}_{32} \triangleq \begin{bmatrix} \widetilde{B}_d^r & \widetilde{V}_2^r & \widetilde{H}^{l,r} & 0 \\ 0 & 0 & 0 & 0 \\ \widetilde{D}^r & \widetilde{W}_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\check{\Omega}_{33} \triangleq \mathrm{diag}\{-\widetilde{\Omega}_{88}^r, -\widetilde{\Omega}_{77}^r, -\widetilde{\Omega}_{88}^r, -I_{n_z}, -I_{n_z}\},$$

$$\Omega_{11}^l \triangleq P_{1l} - \bar{\mu}_1 Q_1 + \bar{I}^T S_2^T S_1 \bar{I} + \bar{I}^T S_1^T S_2 \bar{I},$$

$$\Omega_{44}^l \triangleq P_{2l} - \bar{\mu}_2 Q_2 + \bar{I}^T R_2^T R_1 \bar{I} + \bar{I}^T R_1^T R_2 \bar{I},$$

$$\widetilde{\Omega}_{77}^r \triangleq P_{1r}, \ \widetilde{\Omega}_{88}^r \triangleq P_{2r}, \ \widetilde{A}_d^r \triangleq P_{1r}\big(\bar{A}_d + \bar{\theta}_1 \bar{C}\big),$$

$$\widetilde{V}_1^r \triangleq P_{1r}\big(\overline{V}_1 + \bar{\theta}_1 \overline{W}_1\big),$$

$$\widetilde{B}_d^r \triangleq P_{2r}\big(\bar{B}_d + \bar{\theta}_2 \bar{D}\big), \ \widetilde{V}_2^r \triangleq P_{2r}\big(\overline{V}_2 + \bar{\theta}_2 \overline{W}_2\big),$$

$$\widetilde{C}^r \triangleq \bar{\theta}_1(1-\bar{\theta}_1)P_{1r}\bar{C}, \ \widetilde{W}_1^r \triangleq \bar{\theta}_1(1-\bar{\theta}_1)P_{1r}\overline{W}_1,$$

$$\widetilde{D}^r \triangleq \bar{\theta}_2(1-\bar{\theta}_2)P_{2r}\bar{D}, \ \widetilde{W}_2^r \triangleq \bar{\theta}_2(1-\bar{\theta}_2)P_{2r}\overline{W}_2,$$

$$\widetilde{A}^{l,r} \triangleq \begin{bmatrix} 0 & 0 & 0 & 0 \\ P_{2,1r}\Psi_l E & P_{2,1r}(I_m - \Psi_l) & 0 & 0 \\ \mathcal{K}_{f1r}\Psi_l E & \mathcal{K}_{f1r}(I_m - \Psi_l) & \mathcal{A}_{fr} & 0 \\ \mathcal{K}_{f3r}\Psi_l E & \mathcal{K}_{f3r}(I_m - \Psi_l) & 0 & \mathcal{K}_{f2r} \end{bmatrix},$$

$$\widetilde{B}^{l,r} \triangleq \begin{bmatrix} 0 & 0 & 0 & 0 \\ P_{2,2r}\Psi_l F & P_{2,2r}(I_m - \Psi_l) & 0 & 0 \\ \mathcal{L}_{f1r}\Psi_l F & \mathcal{L}_{f1r}(I_m - \Psi_l) & \mathcal{B}_{fr} & 0 \\ \mathcal{L}_{f3r}\Psi_l F & \mathcal{L}_{f3r}(I_m - \Psi_l) & 0 & \mathcal{L}_{f2r} \end{bmatrix},$$

$$\widetilde{H}^{l,r} \triangleq \begin{bmatrix} 0 & 0 & 0 & 0 \\ P_{2,2r}\Psi_l H & 0 & 0 & 0 \\ \mathcal{L}_{f1r}\Psi_l H & 0 & 0 & 0 \\ \mathcal{L}_{f3r}\Psi_l H & 0 & 0 & 0 \end{bmatrix}.$$

In this case, the gains of the desired H_∞ exponential estimator are given as follows $(i = 2, 3)$:

$$\begin{aligned} A_f &= P_{3,1r}^{-1}\mathcal{A}_{fr}, \quad & K_{f1} &= P_{3,1r}^{-1}\mathcal{K}_{f1r}, \\ K_{fi} &= P_{4,1r}^{-1}\mathcal{K}_{fir}, \quad & B_f &= P_{3,2r}^{-1}\mathcal{B}_{fr}, \\ L_{f1} &= P_{3,2r}^{-1}\mathcal{L}_{f1r}, \quad & L_{fi} &= P_{4,2r}^{-1}\mathcal{L}_{fir}. \end{aligned} \tag{11}$$

Proof: According to the relationship (11), one has $\mathcal{A}_{fr} = P_{3,1r}A_f$, $\mathcal{K}_{f1r} = P_{3,1r}K_{f1}$, $\mathcal{K}_{fir} = P_{4,1r}K_{fi}$ $(i = 2, 3)$, $\mathcal{B}_{fr} = P_{3,2r}B_f$, $\mathcal{L}_{f1r} = P_{3,2r}L_{f1}$, $\mathcal{L}_{fir} = P_{4,2r}L_{fi}$ $(i = 2, 3)$. Consequently, it is easy to deduce $\Omega(k) < 0$ in (7) by performing a congruence transformation

$$\digamma \triangleq \mathrm{diag}\{I_{8(n+m)+2n}, \ P_{1r}^{-1}, \ P_{2r}^{-1}, \ P_{1r}^{-1}, \ P_{2r}^{-1}, \ I_{n_z}, \ I_{n_z}\}$$

to the condition (10), which completes the proof. ■

Remark 3: It can be seen from the condition (10) that the nonlinear term $P_{1\hbar(k+1)}^{-1}$ is successfully eliminated by using the matrix transformation technique. Actually, it can be verified that the established condition (10) is equivalent to the condition (7) by replacing l and r with $\hbar(k)$ and $\hbar(k+1)$, respectively. Meanwhile, the estimator gains are characterized by a low-hanging inverse transformation (11). Feasibility of the estimator design problem can be checked by the solvability of the LMIs (10).

IV. Illustrative Example

The main purpose of this section is to examine the validity of the achieved results in the previous section, and the performance of the H_∞ exponential estimator (5) under the RRP scheduling.

Consider a three-neuron stochastic BAM neural network of the form (1), which is configured by the following (here only partial matrices are listed out to save space)

$$W_2 = \begin{bmatrix} 0.32 & 0 & 0.28 \\ -0.04 & 0.04 & 0.43 \\ 0.27 & 0 & 0.15 \end{bmatrix}, \ C = \begin{bmatrix} -0.21 & 0.02 & 0 \\ -0.13 & 0.04 & 0.02 \\ 0.01 & 0 & 0.07 \end{bmatrix},$$

$$E = \begin{bmatrix} -0.43 & 0 & 0.02 \\ -0.01 & -0.27 & 0 \\ -0.04 & 0.01 & 0 \end{bmatrix}, \ H = \begin{bmatrix} -0.37 & 0 & -0.42 \\ -0.03 & 0 & -0.09 \\ 0.09 & 0 & 0.03 \end{bmatrix},$$

$$M = \begin{bmatrix} -0.41 & 0.21 & 0.09 \\ 0.21 & 0.11 & 0.02 \end{bmatrix}, \ V_2 = \begin{bmatrix} 0 & 0.03 & 0.06 \\ 0.02 & 0 & -0.08 \\ 0.03 & 0.07 & 0 \end{bmatrix}.$$

The nonlinearities $g(\cdot)$ and $f(\cdot)$ are set as

$$\begin{aligned} g\big(v_1(k)\big) &= 0.03v_1(k) - \tanh\big(0.06v_1(k)\big), \\ f\big(u_1(k)\big) &= 0.12u_1(k) - \tanh\big(0.01u_1(k)\big), \\ g\big(v_2(k)\big) &= 0.25v_2(k) - \tanh\big(0.13v_2(k)\big), \\ f\big(u_2(k)\big) &= -0.15u_2(k) - \tanh\big(0.13u_2(k)\big), \\ g\big(v_3(k)\big) &= 0.01v_3(k) - \tanh\big(0.02v_3(k)\big), \\ f\big(u_3(k)\big) &= -0.04u_3(k) - \tanh\big(0.03u_3(k)\big). \end{aligned}$$

It is obvious that the adopted nonlinearities satisfy the condition (2) with $R_1 = \mathrm{diag}\{-0.03, \ 0.12, \ -0.01\}$, $R_2 = \mathrm{diag}\{0.03, \ 0.25, \ 0.01\}$, $S_1 = \mathrm{diag}\{0.12, \ -0.15, \ -0.04\}$ and $S_2 = \mathrm{diag}\{0.13, \ -0.02, \ -0.01\}$. We take $\mu_{1d} = 0.42 \times 0.7^d$, $\mu_{2d} = 0.6 \times 0.7^d$ and $\gamma = 1.4353$. Then it can be seen that $\bar{\mu}_1 = 1.4$ and $\bar{\mu}_2 = 1.2$.

With the help of the LMI toolbox in Matlab software, we can obtain the solution to the matrix inequality $\Omega(l,r) < 0$ in Theorem 2 and, accordingly, the desired estimator gains of H_∞ exponential estimator (5) are characterized as follows:

$$A_f = \begin{bmatrix} 0.0078 & -0.0000 & -0.0004 \\ 0.0002 & 0.0049 & 0.0000 \\ 0.0007 & -0.0002 & -0.0000 \end{bmatrix},$$

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$$B_f = \begin{bmatrix} -0.0058 & 0.0000 & 0.0003 \\ -0.0001 & -0.0037 & -0.0000 \\ -0.0005 & 0.0001 & -0.0000 \end{bmatrix},$$

$$K_{f1} = \begin{bmatrix} 0.0077 & 0.0000 & 0.0017 \\ 0.0002 & 0.0049 & -0.0016 \\ 0.0007 & -0.0002 & -0.0021 \end{bmatrix},$$

$$K_{f2} = \begin{bmatrix} 0.0078 & -0.0000 & -0.0004 \\ 0.0002 & 0.0049 & 0.0000 \\ 0.0007 & -0.0002 & -0.0000 \end{bmatrix},$$

$$K_{f3} = \begin{bmatrix} 0.0077 & 0.0000 & 0.0017 \\ 0.0002 & 0.0049 & -0.0016 \\ 0.0007 & -0.0002 & -0.0021 \end{bmatrix},$$

$$L_{f1} = \begin{bmatrix} -0.0132 & -0.0010 & 0.0018 \\ 0.0018 & -0.0055 & -0.0007 \\ -0.0053 & 0.0029 & -0.0035 \end{bmatrix},$$

$$L_{f2} = \begin{bmatrix} -0.0058 & 0.0000 & 0.0003 \\ -0.0001 & -0.0037 & -0.0000 \\ -0.0005 & 0.0001 & -0.0000 \end{bmatrix},$$

$$L_{f3} = \begin{bmatrix} -0.0132 & -0.0010 & 0.0018 \\ 0.0018 & -0.0055 & -0.0007 \\ -0.0053 & 0.0029 & -0.0035 \end{bmatrix},$$

$$M_f = \begin{bmatrix} -0.1550 & -0.0250 & 0.1550 \\ 0.4150 & 0.0350 & 0.7100 \end{bmatrix},$$

$$N_f = \begin{bmatrix} 0.3850 & -0.1450 & -0.1450 \\ -0.3650 & -0.1000 & -0.3700 \end{bmatrix}.$$

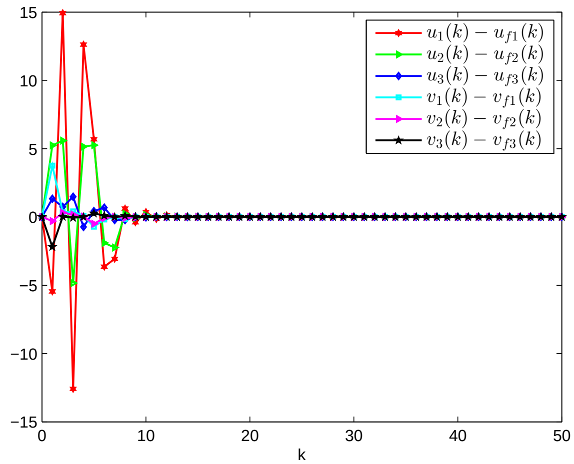
Let the disturbance input be

$$\vartheta(k) = \begin{cases} 0.6, & \text{if } 1 \le k \le 15, \\ 0, & \text{otherwise.} \end{cases}$$

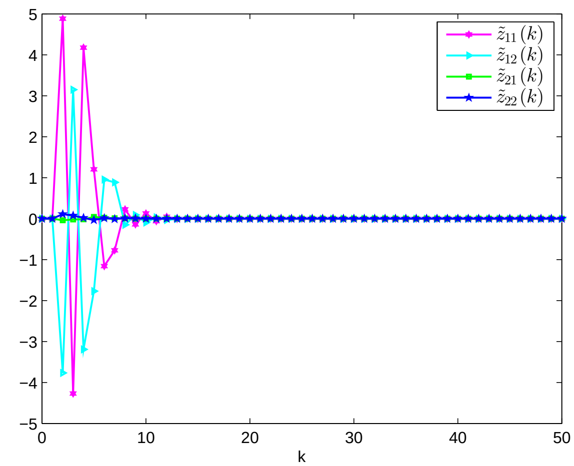
Consequently, the state estimation results of the discussed estimator can be simulated in the Matlab platform. Figs. 2 are the state responses of the estimation errors of the neuron states $u(k) - u_f(k)$ and $v(k) - v_f(k)$, and the state estimation errors $\tilde{z}_1$ and $\tilde{z}_2$, respectively. It is confirmed from the simulation that the designed H_∞ exponential estimator performs well.

Letting $C = A$ and $B = D$, we investigate the effect of the binary mode switching on the system performance. Figs. 2–3 show the state estimation errors of the proposed algorithm with and without mode switching, from which one observes that, the estimation speed of the algorithm with mode switching is slower than the estimation speed of the algorithm without mode switching. In addition, the estimation speed of the considered system without the RRP can be revealed by simply setting $\Psi_{\hbar(k)} = I_m$.

Remark 4: Although state estimation problems for BAM neural networks have gained much research interest among researchers,existing results can be hardly applied to large-scale networks with communication constraints. In comparison with the developed estimation approach in [8], our proposed H_∞ state estimation problem is capable of handling the difficulties brought by the simultaneous presence of the limited communication bandwidth, the leakage delays and the stochastic switching between redundant models, resulting in more accurate and precise estimation results on the neuron states. In the absence of the leakage delays, stochastic switching and communication constraints, the main results obtained in this paper will degenerate to that in [8].



(a) $u(k) - u_f(k)$ and $v(k) - v_f(k)$



(b) $\tilde{z}(k)$

Fig. 2: State estimation error

V. Conclusions

In this paper, we have developed a systematic framework for H_∞ state estimation problem with communication-constrained measurements. The presences of the distributed delays in the leakage terms and the stochastic mode switching have been simultaneously considered. The well-known RRP has been used to avoid the possible transmission collisions subjected to limited bandwidth. The purpose of this paper is to estimate the output signal of the BAM neural networks through the received measurements. By applying stochastic analysis means, convex optimization techniques, and Lyapunov stability theory, sufficient conditions have been derived to ensure the existence of the discussed H_∞ estimator in terms of matrix inequalities. Then, the desired estimator parameters are calculated by solving a set of LMIs. Finally, an illustrative example has been provided to demonstrate the effectiveness of the proposed state estimation method.

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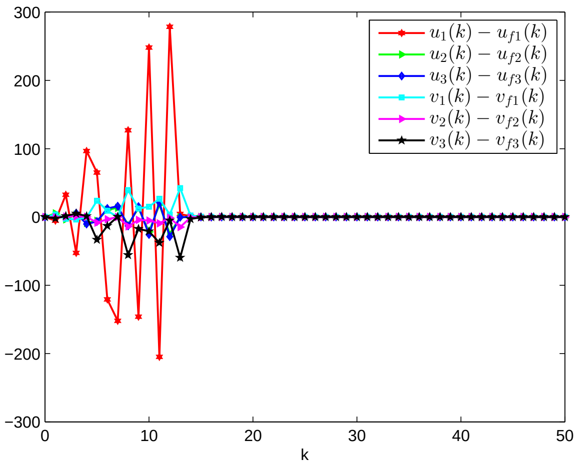
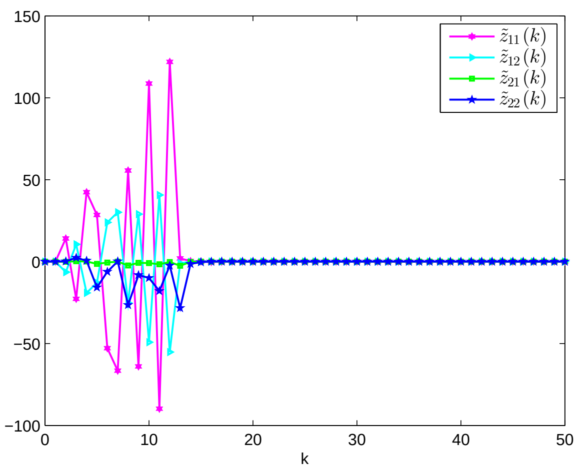


(a) $u(k)-u_f(k)$ and $v(k)-v_f(k)$



(b) $\tilde{z}(k)$

Fig. 3: State estimation error without mode switching

Further research topics include the extension of the main results to

- Fusion estimation problems for delayed ANNs [19], [35].
- Moving-horizon estimation problems for BAM neural networks with network-induced phenomena [54], [56].
- The improvement of the state estimation performance by using some latest optimization algorithms [37], [38].
- State estimation problems under various communication protocols, e. g. the random access protocol and try-once-discard protocol [23], [33], [42], [55].
- State estimation problems for BAM neural networks with engineering-oriented complexities, e.g. uncertain parameters, correlated missing measurements, random packet losses, censored measurements and event-triggered mechanism [10], [14], [22], [26], [31], [32], [36], [52].

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This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TNNLS.2021.3055942, IEEE Transactions on Neural Networks and Learning Systems

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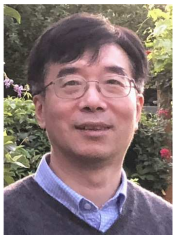
This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TNNLS.2021.3055942, IEEE Transactions on Neural Networks and Learning Systems



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