# SENSITIVITY ANALYSIS MODELLING FOR MICROSCALE MULTIPHYSICS ROBUST ENGINEERING DESIGN

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**ABSTRACT**

Sensitivity Analysis (SA) plays an important role in the development of any practical engineering model. It can help to reveal the sources and mechanisms of variability that provide the key to understanding system uncertainty. SA can also be used to calibrate simulation models for closer agreement with experimental results. Robust Engineering Design (RED) seeks to exploit such knowledge in the search for design solutions that are optimal in terms of performance in the face of variability.

Microscale and multiphysics problems present challenges to modelling due to their complexity, which puts increased demands on computational methods. For example, in developing a model of a piezoelectric actuator, the process of calibration is prolonged by the number of parameters that are difficult to verify with the physical device.

In the approach presented in this paper, normalised sensitivity coefficients are determined directly and accurately using the governing finite element model formulation, offering an efficient means of identifying parameters that affect the output of the model, leading to increased accuracy and knowledge of system performance in the face of variability.

*Keywords: robustness, multiphysics, microscale, sensitivity analysis, normalised sensitivity coefficient*

## 1 INTRODUCTION

Robustness is the quality of a system to maintain its desired characteristic despite fluctuations in component parts or its environment. The robustness of complex engineering systems is determined by many microscale features combined into hierarchies or multiscales [1], particularly in some cases when these features or parameters are at the limits of commercial miniaturization. As statistical variations become relatively more important at small scales, the means of identifying the key features and interactions of robustness in microscale devices is the motivation to investigate a piezoelectric actuator through Finite Element (FE) modelling.

Robust Engineering Design (RED) has focused on design and analysis of experiments, including computer experiments on CAD/CAE simulation models, and also advanced Response Surface Modelling methods, adaptive optimisation methodologies and reliability analysis [2]. Efficient statistical search methods are a central theme of RED with an emphasis on gathering understanding of the parameters investigated, which differentiates it from other methods such as Genetic Algorithms [3]. Parameters affecting the control performance and their levels of influence for a piezoelectric actuator have been determined using orthogonal arrays in a standard Taguchi methodology [4]. This approach requires different configurations of the model to be run in order to determine the influence of the parameters, which has a consequent computational cost and can limit knowledge of relationships to linear or quadratic effects. More efficient combined arrays [5, 6, 7] have proven to be very valuable in computer experiments but methods that require even fewer runs of the simulation are still sought [8]. The application of RED to multiscales is beginning to be addressed [9] and key challenges include in particular, dealing with imprecise model properties due to unreliable material databases. These imprecise model properties lead to uncertainties that are amplified as they are propagated through a series of models at different scales. Since computer models of multiscale multiphysics systems are computationally expensive to run, more efficient robustness analysis techniques are required for RED studies

Sensitivity Analysis (SA) aims to apportion the effects of variation in individual parameters of a model on its output by means of finite differences or derivatives [10]. In the case of Finite Element (FE) models, the corresponding SA study is spatially dependent, and can be evaluated using a modified version of the original governing FE equation solver. By extracting the system equations from the FE model, a set of differential equations to describe the system dynamics can be collated. Based on these equations, SA calculations can be used to approximately model the nature and the sources of output uncertainty during system operation. These calculations will represent a robustness evaluation of the current design and offer a means of identifying potential for improved designs by way of key parameter identification.

In this paper the parameter sensitivities of a piezoelectric actuator are quickly determined by manipulating the stiffness matrix extracted from a multiphysics FE model. The relative scales of features of the piezoelectric actuator model stretch the FE resources and knowledge of parameter sensitivities enables the model to be developed efficiently.

## 2 SENSITIVITY ANALYSIS FE FORMULATION

In the work presented here, Finite Element Sensitivity Analysis (FESA) is performed by differentiating the discretized finite element governing system equations with respect to the parameters of interest. The resulting set of governing and sensitivity differential equations are then solved jointly within the finite element formulation.

### 2.1 Non- geometric parameter sensitivities

In general, it is well known that forces, given by a vector *f*, on a system represented by an FE model are the product of a stiffness matrix, *K*, and a displacement vector, *u*:

$$Ku = f \tag{1}$$

FESA involves the determination of the derivatives of the displacement vector with respect to the system parameters, $\theta$. To compute this derivative, equation (1) is differentiated to give:

$$K\frac{\partial u}{\partial \theta} + \frac{\partial K}{\partial \theta}u = 0 \tag{2}$$

Noting that the force vector *f* is independent of non-geometric parameters. Rearranging (2) gives the matrix vector system:

$$K\frac{\partial u}{\partial \theta} = -\frac{\partial K}{\partial \theta}u \tag{3}$$

Equation (3) is constructed following the assembly and solution of the governing system (1). The sensitivity of the *K* matrix to non-geometric parameters $\theta$ is easily determined from the governing formulation [11], and the solution of (3) determines the sensitivity vector of interest.

### 2.2 Normalised sensitivity coefficients

The sensitivity coefficients determined from Equation (3) represent a linear estimate of the percentage change in a displacement, $u_i$, due to a unit change in a chosen parameter, say $\theta_j$, which will be difficult to compare with the effects of other parameters that differ in physical units and magnitudes. Therefore, a more informative measure of sensitivity will be a *normalised sensitivity coefficient*, given by:

$$\overline{\frac{\partial u_i}{\partial \theta_j}} = \frac{\theta}{u_i}\frac{\partial u_i}{\partial \theta_j} \qquad (4)$$

Here the normalized sensitivity coefficient represents a linear estimate of the percentage change in the output displacement, $u_i$, given a 1% change in $\theta_j$, which is independent of the original system units. Hence normalised sensitivity coefficients are readily comparable with each other and offer a more informative description of parameter importance.

## 3 FE SENSITIVITY ANALYSIS OF A PIEZOELECTRIC ACTUATOR

### 3.1 Model structure

COMSOL was the FE package used in this analysis. The bimorph piezoelectric actuator shown as a meshed FE model in Figure 1 is an assembly of a central ABS stub and an offset piezoelectric wing structure. The inertial forces produced by displacement of the actuated wings produce a driving force at the base of the stub.

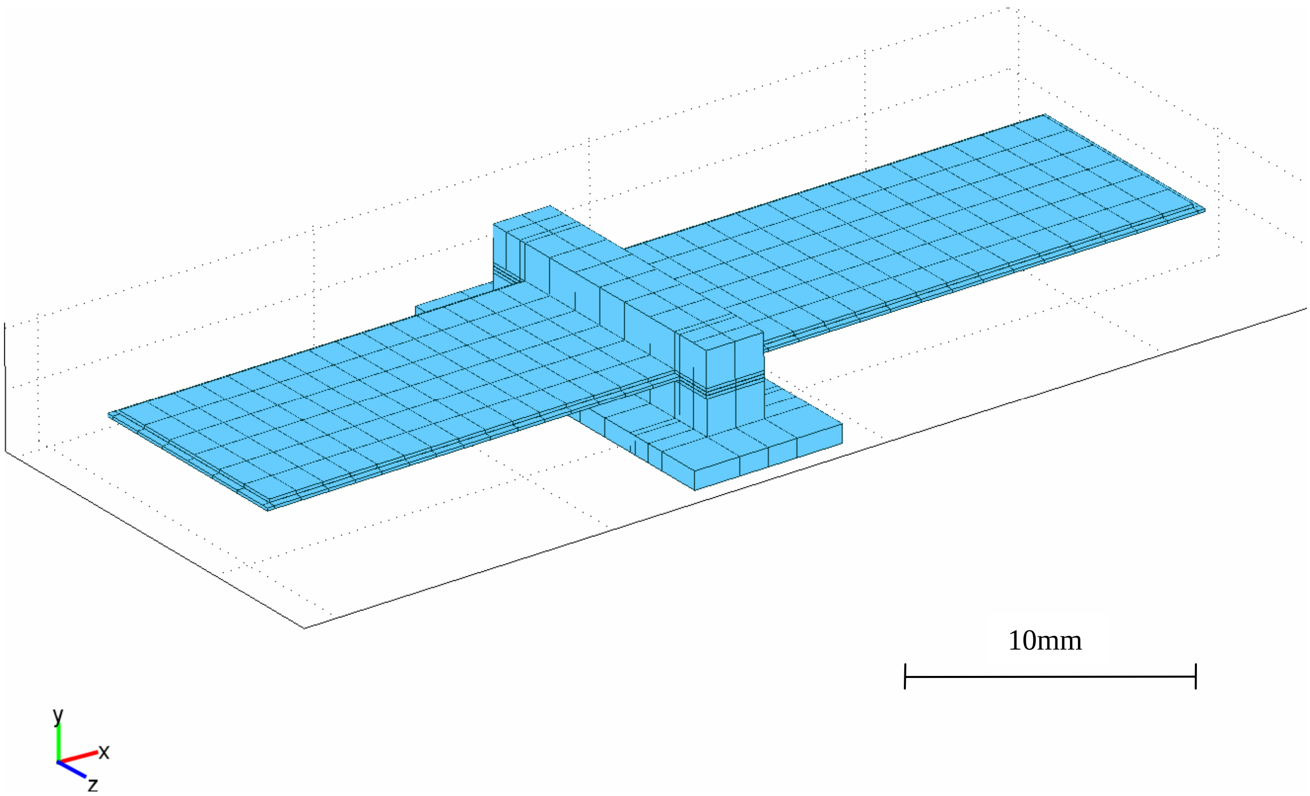


*Figure 1. Piezoelectric actuator*

The wing comprises a 100-micron central brass plate with layers of 100-micron PZT (Lead-Zirconate-Titanate) bonded both sides with epoxy resin 12 microns thick, as shown in Figure 2. The surfaces of the PZT layers have a 5-micron silver electrode coating, which are not included in the FE model other than as boundary conditions.

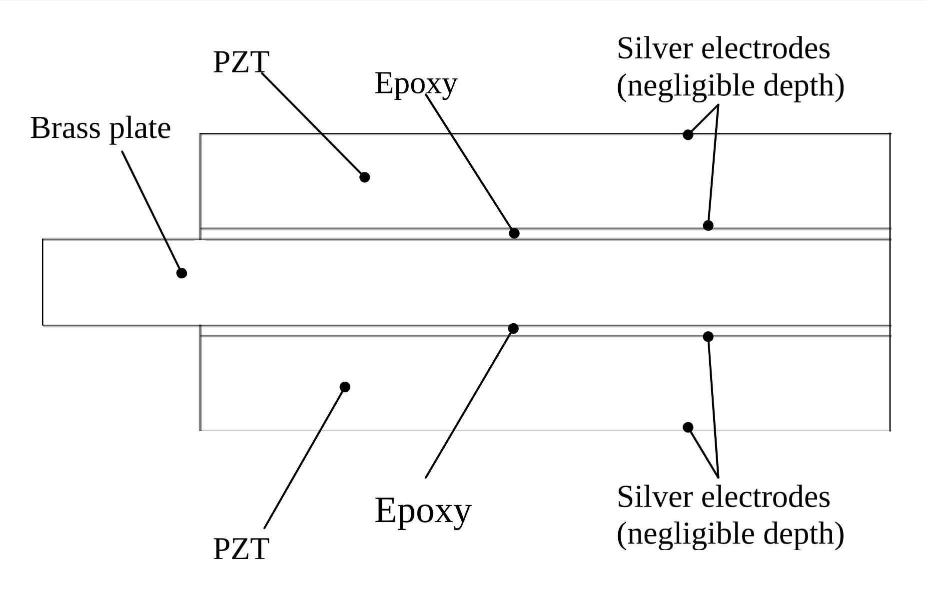


*Figure 2. Piezoelectric actuator wing layer construction*

The silver electrodes connections (not shown) and PZT poling directions (not shown) are such that when excited they work in unison to deflect the plate.

## 3.2 Model parameters

Two forms of the piezoelectric constitutive equations can be used by COMSOL, namely stress-charge and strain-charge. The choice for the purposes of this study is arbitrary as once the material parameters have been specified one formulation can easily be derived from the other. Thus, for the stress-charge formulation we have two coupled equations:

$$T = C_E S + e^t E \tag{5}$$

$$D = eS + \varepsilon_S E \tag{6}$$

Equation (5) represents the mechanical domain, where stress, $T$, is related to the product of a stiffness coefficient $C_E$ and the strain $S$; plus the piezoelectric coupling coefficient, $e$, with the electric field, $E$. Also, $C_E$ is the equivalent of Young's modulus for the PZT (under a constant electric field hence the suffix $_E$), which is anisotropic and therefore has different coefficients of stiffness in each direction.

Equation (6) represents the electrical domain, where electric flux density, D, is related to the product of the piezoelectric coupling coefficient, $e$, and applied strain, $S$; plus the electrical permittivity, $\varepsilon_S$, at constant strain, with the applied electric field, $E$.

Many parameters are hidden within the terms of Equation (5) and Equation (6) due to the anisotropic nature of the PZT material. Table 1 shows the set of 11 parameters selected for the SA from the large number of parameters available in the FE model.

*Table 1. Experimental parameters*

| | Parameter | Nominal Value |
|---|---|---|
| 1 | Brass Young's modulus | 120e9 N/m$^2$ |
| 2 | Brass density | 8800 kg/m$^3$ |
| 3 | ABS Young's modulus | 2.1e9 N/m$^2$ |
| 4 | ABS density | 1600 kg/m$^3$ |
| 5 | Epoxy Young's modulus | 3e9 N/m$^2$ |
| 6 | Epoxy density | 1200 kg/m$^3$ |
| 7 | PZT density | 7500 kg/m$^3$ |
| 8 | $e_{31}$ and $e_{32}$ | -6.623 C/m$^2$ |
| 9 | $e_{33}$ | 23.2403 C/m$^2$ |
| 10 | $C_{E11}$ and $C_{E22}$ | 1.27e11 N/m$^2$ |
| 11 | $C_{E33}$ | 1.17e11 N/m$^2$ |

The convention used in Table 1 is that the 3-direction is normal to the plane of the brass plate and the 1-direction and 2-direction are in the plane of the plate. (x and z directions, respectively, in Fig. 1).

Therefore the $e_{31}$ coupling coefficient linking a stress in the 1-direction to an applied field in the 3 direction; likewise $e_{32}$ links a stress in the 2-direction to an applied field in the 3 direction. $e_{31}$ and $e_{32}$ are assumed equal due to the fact that the material properties are constant in the plane of the PZT layer but vary normal to the plate (in the direction of the applied electric field), which is $e_{33}$.

$C_{E11}$ is the Young's modulus equivalent for a direct stress in the 1-direction, i.e. linking the stress in the 1-direction to an applied strain in the 1-direction. Similarly, $C_{E22}$ is the Young's modulus equivalent for a direct stress in the 2-direction and are considered equal in the plane of the PZT. $C_{E33}$ is the Young's modulus equivalent for a direct stress in the 3-direction.

Elsewhere, bulk material properties are selected as candidates because they could be easily changed if found to be significant. The parameters are then employed at their nominal values.

### 3.3 Results and Analysis

Figure 3 shows the static deflection of the wings due to an applied dc electric current, which will produce zero force at the stub as inertial forces are only produced under dynamic excitation. Therefore the deflections at A and B will be used here as the objective function in applying the FESA approach.

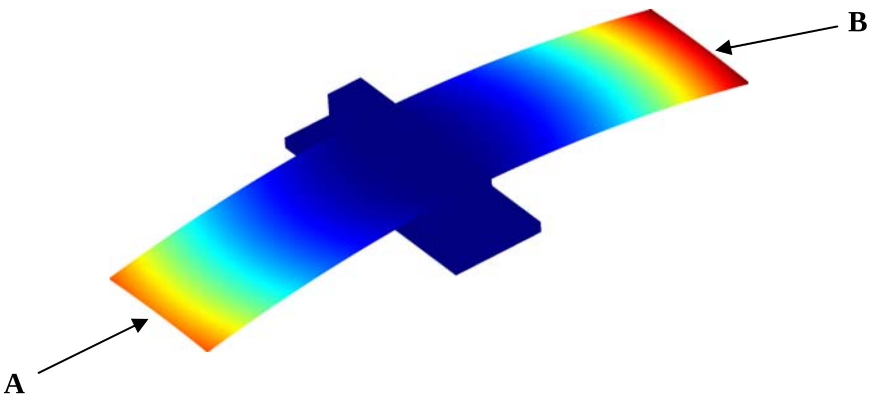


*Figure 3. Measurement of static deflection for offset actuator*

The displacement, $u$, in Equation 5 will be represented by the Root Mean Square value of the two tip measurements, A and B, in order to accommodate asymmetric behaviour caused by the offset position of the stub.

Table 2 shows the RMS values from the Sensitivity Analysis both for a conventional Finite Difference (FD) approach and the FESA approach. In both approaches the effect on displacement of a 1% change in parameter values.

*Table 2. Sensitivity Analysis – relative effect on output*

| | **Parameters** | **Finite Difference** | **FESA** |
|---|---|---|---|
| 1 | Brass Young's modulus | -5.5277e-002 | -5.8761e-002 |
| 2 | Brass density | 0 | 3.9314e-017 |
| 3 | ABS Young's modulus | -2.1112e-002 | -2.4199e-002 |
| 4 | ABS density | 0 | 7.1480e-018 |
| 5 | Epoxy Young's modulus | -2.2681e-004 | 5.8656e-005 |
| 6 | Epoxy density | 0 | 5.3610e-018 |
| 7 | PZT density | 0 | 3.3506e-017 |
| 8 | $e_{31}$ and $e_{32}$ | 2.6095e-001 | 2.6108e-001 |
| 9 | $e_{33}$ | 6.8113e-001 | 6.8129e-001 |
| 10 | $C_{E11}$ and $C_{E22}$ | -1.4553e+000 | -1.4785e+000 |
| 11 | $C_{E33}$ | -1.7425e+000 | -1.7794e+000 |

It is evident from Table 2 that the numerical error differences between FD and FESA are small, particularly comparing the sensitivities of the various material densities, which we would expect to have little effect in the absence of inertial forces. These numerical errors are of a much lower order than the key parameters identified consistently for both approaches.

# 4 DISCUSSION

## 4.1 Results

The results of the SA confirm the prominent role of the dielectric properties that are expected in this simple static case, as it is the fundamental function of the PZT layer. From Figure 4 it can be seen which parameters will adjust the static deflection and which to leave alone, as considerable time is spent in tuning the parameters in order for the model to better approximate reality. The efficiency gain in this simple deflection case will be amplified when SA is applied to refining the FE model for a modal analysis.

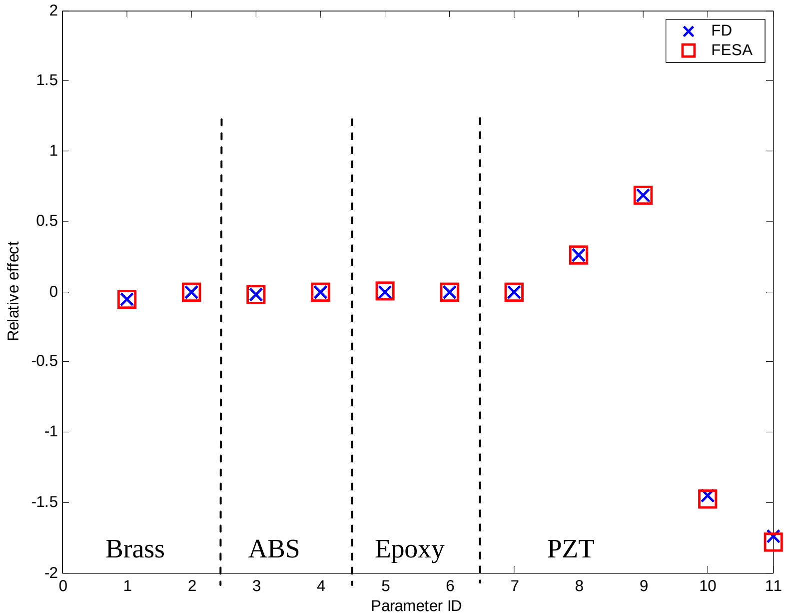


*Figure 4. Relative sensitivity of parameters*

In practice, the initial parameter values used in an FE model of a piezoelectric actuator are quite difficult to match precisely to the physical device. FESA steers updating of the model towards values that improve model validation without laborious testing of each parameter individually. However, a better match in behaviour has to be balanced against the model being representative of the device.

The dominance of the PZT parameter effects over those of other parameters is clearly shown but more importantly for this paper is the consistent agreement between the FESA direct approach and the more computationally expensive FD approach. FESA assesses local sensitivities and as such should only be used to adjust the model parameters slightly. Large changes of parameter values will require FESA to be reapplied.

Implementing the FESA approach described in this paper for a modal analysis of the model, such as eigenfrequency or frequency response analysis, will involve more sophisticated measures of performance. In a dynamic analysis we would expect other parameters in Table 1 to have a more significant effect than for this static analysis. There are also issues of non-unique solutions and inevitably techniques of multiobjective analysis that become relevant, which add to the computational burden. This FESA approach will incorporate micro-scale geometry in future work.

### 4.2 RED of microscale multiphysics systems

At the macroscopic level, the piezoelectric actuator is simply a vibrating wing fixed to a stub that produces a force to excite a sound panel. At the microscopic level the wing comprises the layers described in Figure 2 in which coupling behaviour determines the behaviour of the device. Adapting a view of multiscale RED from [9] in the context of this microscale investigation, we can consider two scales of the device as shown in Figure 5.

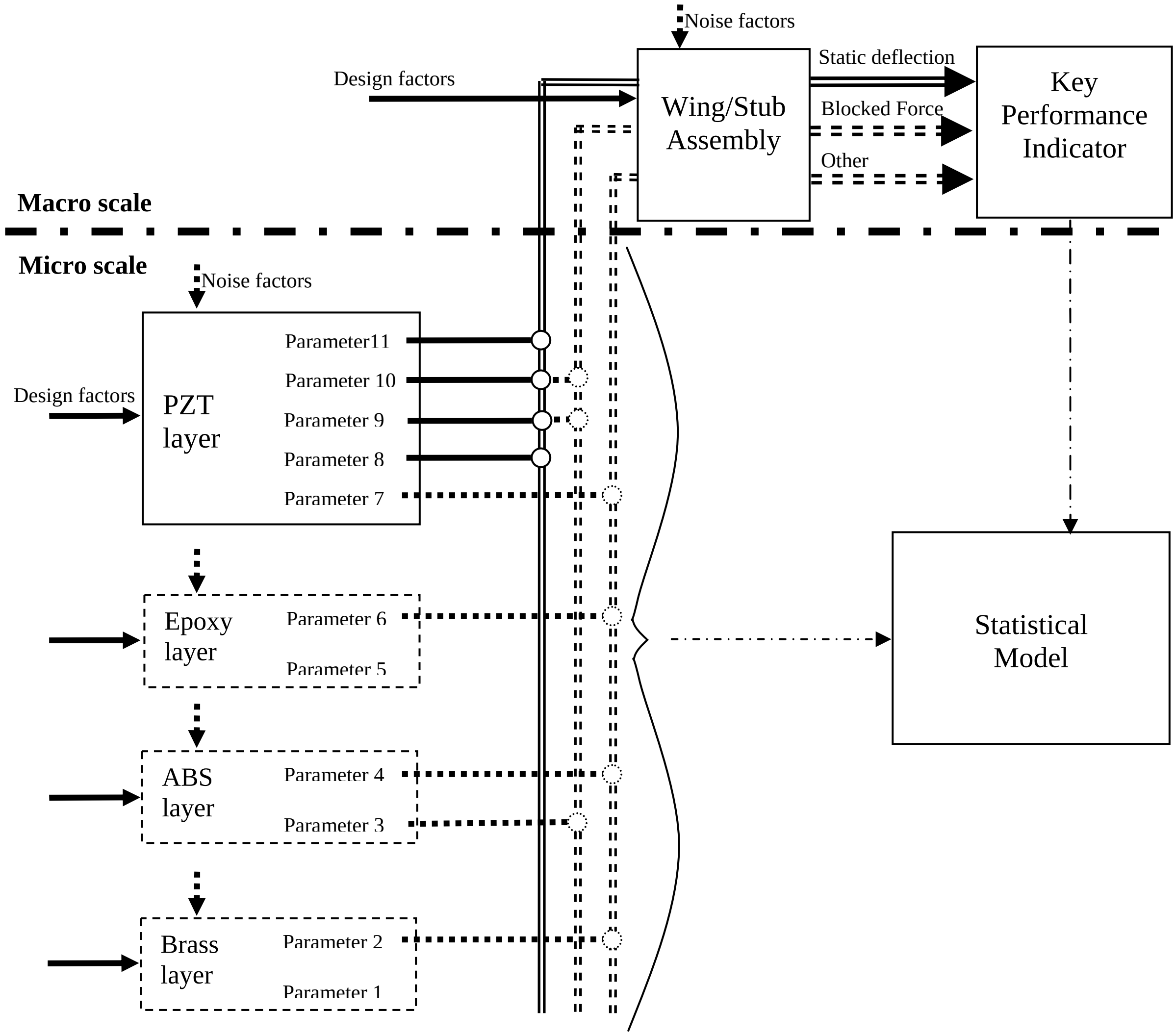


*Figure 5. Schematic two-level RED representation of piezoelectric actuator*

From the FESA results for static deflection of the wing, only the PZT constituent layer is identified in Figure 5 as having significant parameters. If FESA were also applied for the blocked force produced by the wing/stub assembly and additional parameters-response links identified as illustrated. Then a map of the design problem begins to build that indicates how sensitivity propagates through the system for all responses of interest but it does not show that due to coupling and multiobjectives the net effect will be to link parameters together. The FESA method will make large savings in the high computational cost of obtaining the sensitivities in this context, which can then be invested in building a statistical model of a Key Performance Indicator for optimisation, an area we intend to develop further.

## 5 CONCLUSIONS

The FESA method for the efficient calculation of the sensitivity of design performance with respect to changes in design parameter values has been described. The method has been demonstrated for a piezoelectric actuator design example, highlighting relevant sensitivity characteristics.

Parameters are likely to be coupled in microscale multiphysics devices, which makes separate analysis of component parts less meaningful. Therefore, it is the contention of the Authors that the use of SA methods in FE models is of increasing importance for microscale multiphysics applications.

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