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Finite-Horizon H_∞ State Estimation for Stochastic Coupled Networks with Random Inner Couplings using Round-Robin Protocol

Yun Chen, Zidong Wang, Licheng Wang and Weiguo Sheng

***Abstract*—This paper is concerned with the problem of finite-horizon H_∞ state estimation for time-varying coupled stochastic networks through the Round-Robin scheduling protocol. The inner coupling strengths of the considered coupled networks are governed by a random sequence with known expectations and variances. For the sake of mitigating the occurrence probability of network-induced phenomena, the communication network is equipped with the Round-Robin protocol that schedules the signal transmissions of the sensors' measurement outputs. By using some dedicated approximation techniques, an uncertain auxiliary system with stochastic parameters is established where the multiplicative noises enter into the coefficient matrix of the augmented disturbances. With the established auxiliary system, the desired finite-horizon H_∞ state estimator is acquired by solving coupled backward Riccati equations, and the corresponding recursive estimator design algorithm is presented that is suitable for online application. The effectiveness of the proposed estimator design method is validated via a numerical example.**

***Index Terms*—Stochastic coupled networks; finite-horizon H_∞ estimation; random inner couplings; Round-Robin protocol; backward Riccati difference equations.**

I. Introduction

With the springing up of the discipline of complexity science over the past few decades, complex networks have become a research hotspot due mainly to their wide range of applications in our daily life. Generally speaking, a typical complex network consists of a large number of nodes and edges which can be used to model individual systems and their interconnections, respectively. Based on this prominent structure, many complex systems can be described by complex network models with examples including nervous systems, computer networks, transportation networks and social networks. Compared with those individual systems, complex networks exhibit features such as strong couplings, inherent nonlinearities as well as large scales that contribute greatly to the complexities in the dynamical behaviors and, therefore, there appears to be an urgent demand in understanding the dynamic evolution of complex networks. In recent years, tremendous research efforts have been devoted to the dynamic analysis issues for complex networks such as stability, synchronization, state estimation and pining control, see e.g. [6], [14]–[16], [18], [22], [26], [30], [31], [45].

It is well recognized that, as an indispensable part of the coupled networks, the coupling strengths have an essential impact on both the topology connection and the dynamics of complex networks. In most of the existing literature, an implicit assumption is that coupling strengths are *deterministic* yet *fixed*. In some practical situations, however, the inner connections between nodes might be uncertain and expose certain switching/random behaviors owing to a variety of reasons such as network congestion, random failures, unknown but sudden changes of the working conditions as well as the unexpected environmental changes. In view of this, particular attention has been paid to the investigation on the impact from the *uncertain/random/switching* couplings on network dynamics [3], [6], [15], [19], [20], [30]. For example, in [22], the coupling strengths have been characterized as a uncertain term and the resulting uncertainties have been dealt with using the H_∞ performance requirements on the filtering error dynamics. The stochastic coupling (either inter-coupling or outer-coupling) connecting strengths has been considered in [26]. In [10], the switching coupling strengths obeying a discrete-time Markov chain have been proposed, whose influences on both the variance constraint and H_∞ performance have been analyzed.

When analyzing the dynamical performance of complex networks, it is often a prerequisite that the state information of all nodes is available especially for certain tasks such as synchronization and consensus. Unfortunately, this is not always possible due to unknown/unpredictable environmental changes or limits of measurement technologies/expenses. Therefore, the state estimation problem of complex networks has become an issue of primary importance that has recently aroused much research interest with many results reported in the literature, see e.g. [26], [31], [39], [40]. It is worth noting that most results on the state estimation of complex networks have

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been concerned with the time-invariant systems. However, in practical situations, the evolution of the networks is very likely to be time-varying with the change of the working environment. As such, the *finite-horizon* state estimation problem for the *time-varying* complex networks, whose main idea is to guarantee a satisfactory *transient* performance over a certain period time, has received some initial research attention [29], [33], [35]. Up to now, several effective approaches/techniques have been developed, e.g. the recursive linear matrix inequality (RLMI) approach [5], [42], the Krein-space theory [21] and the backward coupled recursive Riccati equation (BCRRE) method [26], among which the BCRRE technique has proven to be particularly efficient in facilitating the online applications for nonlinear time-varying systems [8].

With the increase of the network scale, the coupled nodes lead inevitably to a large amount of information exchange demanding a great deal of communication resources that are usually limited. In fact, the inherently limited bandwidth of the communication network is very likely to create obstacles for sustaining the ever-expedited data interactions for a large scale network. To cope with the sparsity of the communication resources, some efficient data transmission strategies have been proposed. For example, with purpose to reduce the communication frequency, the event-triggered strategy has been developed where the information is transmitted only when certain prescribed event is satisfied, see e.g. [11], [15], [27], [32], [44]. Another data scheduling strategy that has recently begun to receive some research attention is the so-called communication protocol whose main idea is to only grant the “selected” data the permission to occupy the communication channel, thereby effectively preventing the undesired data collisions [1]. In general, the commonly deployed communication protocols include the Round-Robin protocol (RRP) [25], [36], [38], [41], [43], the Try-Once-Discard protocol [17] and the random access protocol [48]. Among others, the RRP is a kind of periodic protocols under which all signals are transmitted in a given circular fashion. Due to its structural simplicity and convenient implementation, the RRP has been widely applied in industry with a surge of interest in dynamics analysis (e.g. state estimation) problems for a variety of complex networks, see e.g. [23], [26], [28].

Motivated by the above discussions, in this paper, we like to initiate a systematic investigation on the new yet challenging problem of RRP-based finite-horizon H_∞ state estimation problem for a class of time-varying stochastic coupled networks subject to random inner coupling strengths. The main contributions of this paper can be summarized as follows. 1) The coupled networks are quite comprehensive that involves state-dependent multiplicative noises and random inner coupling connections. 2) The RRP is adopted to schedule the measurement data of the underlying stochastic complex networks. 3) An auxiliary stochastic parameter system is dedicatedly developed so as to facilitate the evaluation of the H_∞ performance over a finite horizon. 4) A novel BCRRE approach is put forward to design the gain parameters of the desired state estimator in a recursive manner.

The remainder of this paper is organized as follows. Section II addresses the problem statement and preliminaries of this paper. The main results are given in Section III. A numerical example is provided in Section IV and the conclusion is drawn in Section V.

Notation: Throughout this paper, the notations used are standard. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. I and 0 are used to indicate an identity matrix and a zero matrix with appropriate dimensions, respectively. $||\cdot||$ designates the Euclidean norm. $\mathbf{E}\{\cdot\}$ denotes the mathematical expectation. $\mathrm{diag}\{\cdots\}$ represents a block diagonal matrix. The Kronecker product of the two matrices A and B is represented as $A\otimes B$. $(A)^\dagger$ is the Moore-Penrose pseudo inverse of matrix A. P^{ij} is the (i,j)-th entry of the matrix P. $l_2[0,T]$ refers to a square summable space over the finite time interval $[0,T]$.

II. Problem Formulation and Preliminaries

A. The Complex Network Model

Consider the following coupled network with N nodes:

$$x_{i,k+1} = f(x_{i,k}) + \sum_{j=1}^{N} r_{ij} G_k x_{j,k} + B_{i,k} v_k + h_i(x_{i,k}) w_k \tag{1}$$

where $x_{i,k}\in\mathbb{R}^n$ ($i\in\mathcal{N}=\{1,2,\cdots,N\}$) is the state vector of node i, $v_k\in\mathbb{R}^{n_v}$ is the bounded exogenous disturbance belonging to $l_2[0,T]$, and w_k is a zero-mean scalar Gaussian random sequence with variance $\mathbf{E}\{w_k^2\}=1$. $B_{i,k}$ is a time-varying matrix with appropriate dimensions. r_{ij} ($i,j\in\mathcal{N}$) is the outer coupling between two nodes i and j. $r_{ij}>0$ ($j\neq i$) indicates that there is information transmission from the node j to node i; otherwise $r_{ij}=0$. We assume in this paper that the information transmission is symmetric and equivalent between two different nodes, i.e., $r_{ij}=r_{ji}$, and the diffusive condition $r_{ii}+\sum_{j=1,j\neq i}^{N} r_{ij}=0$ is satisfied.

The nonlinear vector functions $f(x_{i,k})\in\mathbb{R}^n$ and $h_i(x_{i,k})\in\mathbb{R}^n$ satisfy the following assumption.

Assumption 1: For any vectors $\mu_k,\nu_k\in\mathbb{R}^n$, the followings are true:

$$\begin{aligned}&[f(\mu_k)-f(\nu_k)-F_{1,k}(\mu_k-\nu_k)]^T\\ &\quad\times[f(\mu_k)-f(\nu_k)-F_{2,k}(\mu_k-\nu_k)]\leq 0\end{aligned} \tag{2a}$$

$$\|h_i(\mu_k)-h_i(\nu_k)\|^2\leq\|S_{i,k}(\mu_k-\nu_k)\|^2 \tag{2b}$$

where $f(0)=0$, $h_i(0)=0$, and $F_{1,k},F_{2,k},S_{i,k}$ are known matrices.

The inner coupling of the coupled network (1) is

$$\begin{aligned}G_k&=\mathrm{diag}\{\psi_{1,k},\cdots,\psi_{n,k}\}\cdot\mathrm{diag}\{g_1,\cdots,g_n\}\\ &\triangleq\Psi_k G_0\end{aligned} \tag{3}$$

where $g_q>0$ ($q=1,2,\cdots n$) are known scalars, and $\psi_{q,k}$ are mutually independent random sequences distributed over the intervals $[\underline{u}_q,\bar{u}_q]$ with known scalars $\bar{u}_q\geq\underline{u}_q\geq 0$. The mathematical expectations and variances of $\psi_{q,k}$ are ψ_q and σ_q^2, respectively. Correspondingly, the expectation of the inner coupling matrix G_k is

$$G=\mathbf{E}\{G_k\}=\Psi G_0=\mathrm{diag}\{\psi_1 g_1,\cdots,\psi_n g_n\}. \tag{4}$$

Assumption 2: The random sequences w_k and $\psi_{q,k}$ are mutually independent.

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Remark 1: Both the stochastic noise $w(k)$ and l_2-type external disturbance $v(k)$ are considered in this paper. In contrast with most existing literature, the inner coupling strength matrix G_k in (1) is allowed to change randomly. As pointed out in [22], the inner couplings cannot be exactly known in many real-world coupled networks due to unavoidable variations of the inner connection among subsystems. In fact, the inner connection coefficients of the practical coupled networks are mainly identified through statistic methods and/or measurement technologies, and this would inevitably bring some kind of stochastic perturbations. Furthermore, different from most existing results (see e.g. [22]), the model (3) ensures the inner coupling strengths to be nonnegative and bounded, which reflects the engineering practice closely.

B. Round-Robin Protocol and Estimation Error Dynamics

For the addressed complex network (1), the measurement of the sensor i is described by

$$\tilde{y}_{i,k} = C_{i,k}x_{i,k} + D_{i,k}v_k \tag{5}$$

where $\tilde{y}_{i,k} \in \mathbb{R}^{n_y}$, $v_k \in \mathbb{R}^{n_v}$ is the disturbance as specified in (1), and $C_{i,k}, D_{i,k}$ $(i \in \mathcal{N})$ are time-varying matrices with compatible dimensions.

In this paper, the measurement signals are transmitted through a non-ideal communication channel with limited bandwidth. In order to alleviate the network load and avoid possible network congestion, the RRP is applied to schedule the signal transmission. Based on the RRP, at each time instant, only one node has the access to transmit its information through the shared communication channel. Thus, the real measurement received by the estimator i is

$$y_{i,k} = \begin{cases} \tilde{y}_{i,k}, & \text{mod}(k,N) = i \\ y_{i,k-1}, & \text{otherwise}, \ i \in \mathcal{N} \end{cases} \tag{6}$$

with the initial condition $y_{i,0} = 0$.

Denote

$$y_k \triangleq [y_{1,k}^T \ y_{2,k}^T \cdots y_{N,k}^T]^T, \ \tilde{y}_k \triangleq [\tilde{y}_{1,k}^T \ \tilde{y}_{2,k}^T \cdots \tilde{y}_{N,k}^T]^T.$$

Hence, the measurement updated equation is written as

$$y_k = I_{\sigma_k}\tilde{y}_k + (I - I_{\sigma_k})y_{k-1} \tag{7}$$

where

$$I_{\sigma_k} = \text{diag}\{\underbrace{0,\cdots,0}_{\sigma_k - 1}, I, \underbrace{0,\cdots,0}_{N-\sigma_k}\}$$

with $\sigma_k \in \mathcal{N}$ and $\sigma_k = \text{mod}(k-1, N) + 1$.

Based on (6), the Luenberger-type state estimator for each node is constructed as

$$\hat{x}_{i,k+1} = f(\hat{x}_{i,k}) + \sum_{j=1}^{N} r_{ij}G\hat{x}_{j,k} + L_{i,k}(y_{i,k} - C_{i,k}\hat{x}_{i,k}) \tag{8}$$

where $\hat{x}_{i,k}$ is the estimate of $x_{i,k}$, and $L_{i,k}$ is the time-varying gain matrix to be designed.

By defining the estimation error of the i-th estimator as $e_{i,k} \triangleq x_{i,k} - \hat{x}_{i,k}$, and denoting $x_k \triangleq [x_{1,k}^T \ x_{2,k}^T \cdots x_{N,k}^T]^T$, $\hat{x}_k \triangleq [\hat{x}_{1,k}^T \ \hat{x}_{2,k}^T \cdots \hat{x}_{N,k}^T]^T$ and $e_k \triangleq [e_{1,k}^T \ e_{2,k}^T \cdots e_{N,k}^T]^T$, the estimator (8) can be written as

$$\hat{x}_{k+1} = F_{k,\hat{x}} + (R \otimes G)\hat{x}_k + L_k(y_k - C_k\hat{x}_k) \tag{9}$$

where

$$\begin{aligned} F_{k,\hat{x}} &\triangleq [f^T(\hat{x}_{1,k}) \ f^T(\hat{x}_{2,k}) \cdots f^T(\hat{x}_{N,k})]^T \\ R &\triangleq [r_{ij}]_{N\times N} \\ L_k &\triangleq \text{diag}\{L_{1,k}, L_{2,k}, \cdots, L_{N,k}\} \\ C_k &\triangleq \text{diag}\{C_{1,k}, C_{2,k}, \cdots, C_{N,k}\}. \end{aligned}$$

By setting $f(\chi_{i,k}) = f(x_{i,k}) - f(\hat{x}_{i,k})$ and noticing (1), (7) and (8), we have

$$\begin{aligned} e_{k+1} = &F_{k,e} + [R \otimes (G_k - G) + L_k(I - I_{\sigma_k})C_k]x_k \\ &+ (R \otimes G - L_kC_k)e_k - L_k(I - I_{\sigma_k})y_{k-1} \\ &+ (B_k - L_kI_{\sigma_k}D_k)v_k + H_{k,x}w_k \end{aligned} \tag{10}$$

where

$$\begin{aligned} F_{k,e} &\triangleq [f^T(\chi_{1,k}) \ f^T(\chi_{2,k}) \cdots f^T(\chi_{N,k})]^T \\ B_k &\triangleq [B_{1,k}^T, B_{2,k}^T, \cdots, B_{N,k}^T]^T \\ D_k &\triangleq [D_{1,k}^T, D_{2,k}^T, \cdots, D_{N,k}^T]^T \\ H_{k,x} &\triangleq [h_1^T(x_{1,k}) \ h_2^T(x_{2,k}) \cdots h_N^T(x_{N,k})]^T. \end{aligned}$$

Let the output signal (to be estimated) for the underlying complex network (1) be given by

$$z_{i,k} = E_{i,k}x_{i,k} \tag{11}$$

where the time-varying matrix $E_{i,k}$ is dimensionally compatible. Then, one obtains

$$\begin{aligned} \bar{z}_{i,k} &\triangleq z_{i,k} - \hat{z}_{i,k} = E_{i,k}e_{i,k} \\ &= E_{i,k}x_{i,k} - E_{i,k}\hat{x}_{i,k}. \end{aligned}$$

Setting $\eta_k \triangleq [x_k^T \ y_{k-1}^T \ e_k^T]^T$, $\bar{z}_k \triangleq z_k - \hat{z}_k$, $z_k \triangleq [z_{1,k}^T \ z_{2,k}^T \cdots z_{N,k}^T]^T$ and $\hat{z}_k \triangleq [\hat{z}_{1,k}^T \ \hat{z}_{2,k}^T \cdots \hat{z}_{N,k}^T]^T$, the dynamics of η_k is expressed as

$$\begin{cases} \eta_{k+1} = (\mathcal{A}_k + \Delta_k)\eta_k + \mathcal{F}_k + \mathcal{B}_kv_k + \mathcal{H}_kw_k \\ \bar{z}_k = \mathcal{E}_k\eta_k \end{cases} \tag{12}$$

where

$$\begin{aligned} \mathcal{A}_k &\triangleq \begin{bmatrix} R\otimes G & 0 & 0 \\ I_{\sigma_k}C_k & I_{\sigma_k}^\delta & 0 \\ L_kI_{\sigma_k}^\delta C_k & -L_kI_{\sigma_k}^\delta & R\otimes G - L_kC_k \end{bmatrix} \\ \Delta_k &\triangleq \begin{bmatrix} R\otimes(G_k - G) & 0 & 0 \\ 0 & 0 & 0 \\ R\otimes(G_k - G) & 0 & 0 \end{bmatrix} \\ \mathcal{F}_k &\triangleq \begin{bmatrix} F_{k,x} \\ 0 \\ F_{k,e} \end{bmatrix}, \ \mathcal{H}_k \triangleq \begin{bmatrix} H_{k,x} \\ 0 \\ H_{k,x} \end{bmatrix} \\ \mathcal{B}_k &\triangleq \begin{bmatrix} B_k \\ I_{\sigma_k}D_k \\ B_k - L_kD_k \end{bmatrix} \\ \mathcal{E}_k &\triangleq \begin{bmatrix} 0 & 0 & E_k \end{bmatrix} \\ E_k &\triangleq \text{diag}\{E_{1,k}, E_{2,k}, \cdots, E_{N,k}\} \\ I_{\sigma_k}^\delta &\triangleq I - I_{\sigma_k}. \end{aligned} \tag{13}$$

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Remark 2: The RRP is introduced in this paper to ease the network load and avoid the possible network-induced problems in the shared communication channel. Due to the introduction of I_{σ_k}, the measurement equation (7) and the augmented system (12) exhibit the periodically switching behaviors. However, different from the standard switched systems [5], [7], [37], the switching caused by I_{σ_k} in system (12) is N-periodic in a fixed circle with a constant switching time interval.

C. Approximations of Nonlinear Functions $f(\cdot)$ and $h_i(\cdot)$

It is observed from (12) that two nonlinear vector-valued functions $\mathcal{F}_k$ and $\mathcal{H}_k$ correspond to the nonlinear functions $f(\cdot)$ and $h_i(\cdot)$, respectively. In order to design the H_∞ estimator (8) by means of Riccati-type difference equation method, the nonlinear vector functions $\mathcal{F}_k$ and $\mathcal{H}_k$ are now handled properly as in [9], [21].

Let us consider a time-varying nonlinear scalar-valued function $c(\nu_k)$ satisfying $[c(\nu_k) - a\nu_k][c(\nu_k) + a\nu_k] \leq 0$, which is equivalent to $c^2(\nu_k) \leq a^2\nu_k^2$ or $-a\nu_k \leq c(\nu_k) \leq a\nu_k$, where $a > 0$ is a known scalar. It is easy to verify that there indeed exists a scalar $m_k \in [-1, 1]$ such that $c(\nu_k) = m_k \cdot a\nu_k$ for any $k = 0, 1, 2, \cdots$. Note that the above technical handling process can be directly applied to the nonlinear vector-valued function $f(x_{i,k})$ subject to (2a).

For the nonlinear vector-valued function $f(x_{i,k})$ with $f(0) = 0$, the inequality (2a) can be rewritten as

$$\begin{aligned}&[(f(x_{i,k}) - \Theta_k x_{i,k}) - \Upsilon_k x_{i,k}]^T\\ &\times [(f(x_{i,k}) - \Theta_k x_{i,k}) + \Upsilon_k x_{i,k}] \leq 0\end{aligned}$$

where

$$\Theta_k = \frac{F_{1,k} + F_{2,k}}{2}, \quad \Upsilon_k = \frac{F_{1,k} - F_{2,k}}{2}.$$

Similar to the scalar case discussed previously, there always exists a matrix M_k satisfying $M_k^T M_k \leq I$ such that $f(x_{i,k}) - \Theta_k x_{i,k} = M_k \Upsilon_k x_{i,k}$ or

$$f(x_{i,k}) = \Theta_k x_{i,k} + M_k \Upsilon_k x_{i,k}. \tag{14}$$

Also, one has $f(\chi_{i,k}) = \Theta_k e_{i,k} + M_k \Upsilon_k e_{i,k}$. In view of (2b), the nonlinear function $h_i(x_{i,k})$ can now be rearranged as

$$h_i(x_{i,k}) = \bar{M}_k S_{i,k} x_{i,k} \tag{15}$$

where the matrix $\bar{M}_k$ satisfies $\bar{M}_k^T \bar{M}_k \leq I$.

By introducing the following notations

$$\begin{aligned}\digamma_k &\triangleq \text{diag}\{\underbrace{\Theta_k, \Theta_k, \cdots, \Theta_k}_{N}\}\\ \mathfrak{F}_k &= \text{diag}\{\underbrace{\Upsilon_k, \Upsilon_k, \cdots, \Upsilon_k}_{N}\}\\ S_k &\triangleq \text{diag}\{S_{1,k}, S_{2,k}, \cdots, S_{N,k}\}\\ \mathbf{F}_k &\triangleq \text{diag}\{\digamma_k, 0, \digamma_k\}\\ \mathbb{F}_k &\triangleq \text{diag}\{\mathfrak{F}_k, 0, \mathfrak{F}_k\}\\ \mathbb{H}_k &\triangleq \begin{bmatrix} S_k^T & 0 & S_k^T \end{bmatrix}^T\end{aligned} \tag{16}$$

the nonlinear functions $\mathcal{F}_k$ and $\mathcal{H}_k$ in (12) are expressed as

$$\begin{aligned}\mathcal{F}_k &= (\mathbf{F}_k + M_k \mathbb{F}_k)\eta_k\\ \mathcal{H}_k &= \bar{M}_k \mathbb{H}_k x_k.\end{aligned}$$

Rearranging (12) gives the following auxiliary equation

$$\begin{aligned}\eta_{k+1} &= (\mathcal{A}_k + \mathbf{F}_k + \Delta_k)\eta_k + \mathbf{B}_k \mathbf{v}_k\\ \bar{z}_k &= \mathcal{E}_k \eta_k\end{aligned} \tag{17}$$

where $\mathcal{A}_k, \Delta_k, \mathcal{E}_k$ and $\mathbf{F}_k$ are given in (13) and (16), respectively, and

$$\begin{aligned}\mathbf{B}_k &\triangleq \begin{bmatrix} \mathcal{B}_k & \alpha_k^{-1} I & \beta_k^{-1} I w_k \end{bmatrix}\\ \mathbf{v}_k &\triangleq \begin{bmatrix} v_k^T & \alpha_k (M_k \mathbb{F}_k \eta_k)^T & \beta_k (\bar{M}_k \mathbf{H}_k \eta_k)^T \end{bmatrix}^T\\ \mathbf{H}_k &\triangleq \begin{bmatrix} \mathbb{H}_k & 0 & 0 \end{bmatrix}\end{aligned} \tag{18}$$

with α_k, β_k being tuning scalars to enhance the feasibility of the addressed estimator design problem.

Remark 3: The two nonlinear vector-valued functions $f(\cdot)$ and $h_i(\cdot)$ are transformed to (14) and (15), respectively, based on which the linear difference equation (17) with time-varying parametric uncertainties is obtained. It is noted that (15) is a special case of (14) with $F_{1,k} = -F_{2,k}$, in which the nonlinear function $f(x_{i,k})$ is decomposed into two parts, i.e., the certain term $\Theta_k x_{i,k}$ and the uncertain term $M_k \Upsilon_k x_{i,k}$, respectively.

D. Main Objective and Preliminary Results

In this paper, we aim to construct the state estimator (9) for the network (1) with the random inner coupling (3) and the Round-Robin scheduling protocol (6) such that the resulting augmented estimation error dynamics (12) satisfies the following H_∞ performance index:

$$J_1 = \sum_{k=0}^{T} \mathbf{E}\Big\{ \| \bar{z}_k \|^2 - \gamma^2 \| v_k \|^2 - \gamma^2 \eta_0^T W \eta_0 \Big\} < 0 \tag{19}$$

for a prescribed disturbance attenuation level $\gamma > 0$ over a finite time horizon $[0, T]$, where $W > 0$ is a weighted matrix and η_0 is any given nonzero initial condition.

For the auxiliary system (17), the following performance index is defined

$$\begin{aligned}J_2 = \sum_{k=0}^{T} \mathbf{E}\Big\{ &\| \bar{z}_k \|^2 - \gamma^2 (\| \mathbf{v}_k \|^2 - \| \alpha_k \mathbb{F}_k \eta_k \|^2\\ &- \| \beta_k \mathbf{H}_k \eta_k \|^2) - \gamma^2 \eta_0^T W \eta_0 \Big\}.\end{aligned}$$

From the definitions of $J_1, J_2, \mathbf{v}_k$, it is clear that

$$\begin{aligned}J_1 - J_2 = \sum_{k=0}^{T} \mathbf{E}\Big\{ &\gamma^2 (\| \alpha_k M_k \mathbb{F}_k \eta_k \|^2 - \| \alpha_k \mathbb{F}_k \eta_k \|^2)\\ &+ \gamma^2 (\| \beta_k \bar{M}_k \mathbf{H}_k \eta_k \|^2 - \| \beta_k \mathbf{H}_k \eta_k \|^2) \Big\}.\end{aligned}$$

By employing $M_k^T M_k \leq I$ and $\bar{M}_k^T \bar{M}_k \leq I$, we obtain

$$\begin{aligned}J_1 - J_2 \leq \sum_{k=0}^{T} \mathbf{E}\Big\{ &\gamma^2 (\| M_k \|^2 - I) \| \alpha_k \mathbb{F}_k \eta_k \|^2\\ &+ \gamma^2 (\| \bar{M}_k \|^2 - I) \| \beta_k \mathbf{H}_k \eta_k \|^2 \Big\} \leq 0\end{aligned}$$

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which means that $J_1 < 0$ is ensured by $J_2 < 0$. Hence, in the next section we will concentrate on designing an estimator (8) such that $J_2 < 0$.

Before proceeding further, the following two lemmas are presented.

Lemma 1: ([13]) Let $P = [p_{ij}]_{n\times n}$ be a real matrix and $R = \text{diag}\{r_1, r_2, \cdots, r_n\}$ be a diagonal stochastic matrix. Then, we have

$$\mathbf{E}\{R^T PR\} = \begin{bmatrix} \mathbf{E}\{r_1^2\} & \mathbf{E}\{r_1 r_2\} & \cdots & \mathbf{E}\{r_1 r_n\} \\ \mathbf{E}\{r_2 r_1\} & \mathbf{E}\{r_2^2\} & \cdots & \mathbf{E}\{r_2 r_n\} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{E}\{r_n r_1\} & \mathbf{E}\{r_n r_2\} & \cdots & \mathbf{E}\{r_n^2\} \end{bmatrix} \circ P$$

where $\circ$ is the Hadamard product.

Lemma 2: ([2]) For any zero-mean scalar Gaussian random sequence w_k with prior variance $\mathbf{E}\{w_k^2\} = \sigma^2$, the following holds:

$$\mathbf{E}\{w_k^2 w_l^2\} = \begin{cases} \sigma^4, & \text{if } k \neq l \\ 3\sigma^4, & \text{if } k = l. \end{cases}$$

III. MAIN RESULTS

In this section, a criterion is established to ensure the finite-horizon H_∞ performance constraint $J_2 < 0$ for the auxiliary system (17) based on the BCRRDE approach. Then, the estimator (8) is determined by solving two BCRRDEs. Moreover, a recursive algorithm is provided to compute the gain matrix of the estimator (8).

For notational simplicity, we denote:

$$\begin{aligned} \mathbb{A}_k &\triangleq \mathcal{A}_k + \mathbf{F}_k \\ \Xi_k &\triangleq \begin{bmatrix} \mathcal{B}_k & \alpha_k^{-1} I & 0 \end{bmatrix} \\ \Gamma_k &\triangleq \begin{bmatrix} 0 & 0 & \beta_k^{-1} I \end{bmatrix} \\ X_k &\triangleq \Xi_k^T P_{k+1} \Xi_k + \Gamma_k^T P_{k+1} \Gamma_k \\ Y_k &\triangleq \Xi_k \Omega_k^{-1} \Xi_k^T + \Gamma_k \Omega_k^{-1} \Gamma_k^T \\ \mathbf{M}_k &\triangleq P_{k+1}(Y_k Q_{k+1} Y_k + 4\Xi_k \Omega_k^{-1} \Gamma_k^T Q_{k+1} \Gamma_k \Omega_k^{-1} \Xi_k^T \\ &\quad + 2\Gamma_k \Omega_k^{-1} \Gamma_k^T Q_{k+1} \Gamma_k \Omega_k^{-1} \Gamma_k^T) P_{k+1} \\ Z_k &\triangleq Q_{k+1} + Q_{k+1} Y_k P_{k+1} + P_{k+1} Y_k Q_{k+1} + \mathbf{M}_k \\ \bar{\mathbf{G}} &\triangleq \begin{bmatrix} R\otimes\bar{G} & 0 & 0 \\ 0 & 0 & 0 \\ R\otimes\bar{G} & 0 & 0 \end{bmatrix} \\ \bar{G} &\triangleq G_0 \text{diag}\{\sigma_1, \sigma_2, \cdots, \sigma_n\} \\ \phi_k &\triangleq L_k \mathbf{C}_k \eta_k \\ \mathcal{I} &\triangleq \begin{bmatrix} 0 & 0 & I \end{bmatrix}^T \\ \mathbf{C}_k &\triangleq \begin{bmatrix} 0 & 0 & -C_k \end{bmatrix}. \end{aligned} \tag{20}$$

A. Estimator Performance Analysis

For the auxiliary system (17), the following theorem is established.

Theorem 1: Consider the coupled network (1) with fixed estimator (8), where the measurement is given by (7). For a given scalar $\gamma > 0$ and a weighted matrix $W > 0$, the auxiliary system (17) achieves the prescribed H_∞ disturbance rejection level γ if there exist a set of positive definite matrices P_k and two sets of positive scalars α_k, β_k $(k = 0, 1, 2, \cdots, T)$ such that the following backward Riccati recursive equations

$$\begin{aligned} P_k =& \mathbb{A}_k^T P_{k+1} \mathbb{A}_k + \bar{\mathbf{G}}^T P_{k+1} \bar{\mathbf{G}} + \mathcal{E}_k^T \mathcal{E}_k + \gamma^2 \alpha_k^2 \mathbb{F}_k^T \mathbb{F}_k \\ &+ \gamma^2 \beta_k^2 \mathbf{H}_k^T \mathbf{H}_k + \mathbb{A}_k^T P_{k+1} \Xi_k \Omega_k^{-1} \Xi_k^T P_{k+1} \mathbb{A}_k \end{aligned} \tag{21}$$

are feasible with $P_{T+1} = 0$ and

$$\begin{aligned} \Omega_k =& \gamma^2 I - X_k > 0 \\ P_0 <& \gamma^2 W. \end{aligned} \tag{22}$$

Proof: See Appendix. ∎

Remark 4: From the proof of Theorem 1, we can see that the random inner coupling matrix G_k and the state-dependent multiplicative stochastic noise $h_i(x_{i,k}) w_k$ pose substantial difficulties on the analysis of the H_∞ performance. By utilizing Lemma 1, the randomness of the matrix G_k is eventually reflected by the term $\bar{\mathbf{G}}^T P_{k+1} \bar{\mathbf{G}}$ in (21). By the decomposition operation in (34), the effects of stochastic noise are reflected by both the last term in (21) and X_k in (22).

We now proceed to design the finite-horizon H_∞ estimator (8) for the network (1) under the worst-case disturbances $\mathbf{v}_k^*$.

Replacing $\mathbf{v}_k$ by $\mathbf{v}_k^* = \Omega_k^{-1} \mathbf{B}_k^T P_{k+1} (\mathcal{A}_k + \mathbf{F}_k) \eta_k$ in (17) and decomposing $\mathcal{A}_k$ into $\mathcal{A}_k = \bar{\mathcal{A}}_k + \mathcal{I} L_k \mathbf{C}_k$, we have

$$\begin{aligned} \eta_{k+1} =& (\bar{\mathbb{A}}_k + \bar{\mathbb{B}}_k \bar{\mathbb{A}}_k + \Delta_k) \eta_k + (\bar{\mathbb{B}}_k + I) \mathcal{I} \phi_k \\ \bar{z}_k =& \mathcal{E}_k \eta_k \end{aligned} \tag{23}$$

where

$$\begin{aligned} \bar{\mathbb{A}}_k \triangleq& \bar{\mathcal{A}}_k + \mathbf{F}_k \\ \bar{\mathcal{A}}_k \triangleq& \begin{bmatrix} R\otimes G & 0 & 0 \\ I_{\sigma_k} C_k & I_{\sigma_k}^\delta & 0 \\ L_k I_{\sigma_k}^\delta C_k & -L_k I_{\sigma_k}^\delta & R\otimes G \end{bmatrix} \\ \mathbb{B}_k \triangleq& \bar{\mathbb{B}}_k \bar{\mathbb{A}}_k + \mathcal{I} L_k \mathbf{C}_k \\ \bar{\mathbb{B}}_k \triangleq& \mathbf{B}_k \Omega_k^{-1} \mathbf{B}_k^T P_{k+1} \\ =& (\Xi_k \Omega_k^{-1} \Xi_k^T + 2\Xi_k \Omega_k^{-1} \Gamma_k^T w_k + \Gamma_k \Omega_k^{-1} \Gamma_k^T w_k^2) P_{k+1}. \end{aligned} \tag{24}$$

Define the following performance index for system (23)

$$\tilde{J} \triangleq \sum_{k=0}^{T} \mathbf{E}\Big\{ \parallel \bar{z}_k \parallel^2 + \parallel \phi_k \parallel \Big\}. \tag{25}$$

In the sequel, we will construct the estimator (8) by minimizing the index $\tilde{J}$ in (25). The corresponding result is presented as follows.

Theorem 2: Consider the coupled network (1) with the measurement (7). Assume that there are two sets of positive definite matrices P_k, Q_k and two sets of positive scalars α_k, β_k $(k = 0, 1, 2, \cdots, T)$ such that, for a given positive scalar $\gamma > 0$ and a weighted matrix $W > 0$, the coupled backward Riccati difference equations (21) and

$$\begin{aligned} Q_k =& \bar{\mathbb{A}}_k^T Z_k \bar{\mathbb{A}}_k + \bar{\mathbf{G}}^T Q_{k+1} \bar{\mathbf{G}} + \mathcal{E}_k^T \mathcal{E}_k + \bar{\mathbb{A}}_k^T (\mathbf{M}_k \\ &+ Q_{k+1} Y_k P_{k+1}) \mathcal{I} L_k \mathbf{C}_k + \mathbf{C}_k^T L_k^T \mathcal{I}^T (\mathbf{M}_k \\ &+ P_{k+1} Y_k Q_{k+1}) \bar{\mathbb{A}}_k - \bar{\mathbb{A}}_k^T (P_{k+1} Y_k + I) \\ &\times Q_{k+1} \mathcal{I} \Psi_k^{-1} \mathcal{I}^T Q_{k+1} (Y_k P_{k+1} + I) \bar{\mathbb{A}}_k \end{aligned} \tag{26}$$

are feasible under the conditions $P_{T+1} = 0$, $Q_{T+1} = 0$, (22) and

$$\Psi_k = \mathcal{I}^T Z_k \mathcal{I} + I > 0. \tag{27}$$

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Then, (8) is the H_∞ estimator of (1) satisfying

$$L_k\mathbf{C}_k = -\Psi_k^{-1}\mathcal{I}^T Q_{k+1}(Y_kP_{k+1}+I)\bar{\mathbb{A}}_k. \tag{28}$$

The values of the given cost functions J_2 and $\tilde{J}$ are, respectively, given as

$$J_2 = \eta_0^T(P_0-\gamma^2 W)\eta_0,\ \ \tilde{J} = \eta_0^T Q_0\eta_0. \tag{29}$$

Proof: See Appendix. ■

Remark 5: It should be noted that, since the augmented disturbance vector $\mathbf{v}_k$ in (18) does not explicitly contain the Gaussian noise w_k which actually exists in $\mathbf{B}_k$, it is reasonable to require that the worst-case disturbance is set as $\mathbf{v}_k \triangleq \mathbf{v}_k^* = \Omega_k^{-1}\mathbf{B}_k^T P_{k+1}(\mathcal{A}_k+\mathbf{F}_k)\eta_k$. In fact, (17) can be viewed as a kind of stochastic parameter systems which have been investigated in the existing literature [8].

B. Estimator Design

In this subsection, we will present an algorithm to compute the estimator gain matrix $L_{i,k}$ in (8).

For brevity, the notations

$$W_k \triangleq -\Psi_k^{-1}\mathcal{I}^T Q_{k+1}(Y_kP_{k+1}+I)\bar{\mathbb{A}}_k$$
$$\tilde{I}_i \triangleq \text{diag}\{\underbrace{0,\cdots,0}_{i-1}, I_{n\times n}, \underbrace{0,\cdots,0}_{N-i}\}$$

are adopted In the sequel.

Noticing the form of $\mathbf{C}_k$ in (20), the following equation

$$L_kC_k = \bar{W}_k$$

holds, where $\bar{W}_k \in \mathbb{R}^{Nn\times Nn}$ is the third block element of matrix W_k. Moreover, it can be found that $L_{i,k}C_{i,k} = \bar{W}_k\tilde{I}_i$. Hence, the gain matrix of estimator (8) is computed by using Moore-Penrose pseudo inverse, which is stated in the following Theorem.

Theorem 3: Consider the coupled network (1) the measurement (7). For a given scalar $\gamma > 0$ and a weighted matrix $W > 0$, if there are two sets of positive definite matrices P_k, Q_k with $P_{T+1} = 0, Q_{T+1} = 0$, and two sets of positive scalars α_k, β_k $(k = 0, 1, 2, \cdots, T)$ such that the coupled backward Riccati difference equations (21) and (26) hold under the constraints (22) and (27), then (8) is the H_∞ estimator of (1) with the gain matrix given by

$$L_{i,k} = \bar{W}_k\tilde{I}_iC_{i,k}^{\dagger}. \tag{30}$$

To facilitate the implementation, the following finite-horizon H_∞ estimator (FHHE) design algorithm is provided to recursively compute the gain matrix $L_{i,k}$ based on Theorems 1-3.

Remark 6: In this paper, the problem of finite-horizon H_∞ state estimation is dealt with for a class of time-varying coupled stochastic networks under the Round-Robin scheduling protocol. The underlying network model is quite comprehensive that features stochastic inner coupling strengths and Round-Robin protocol scheduling. Some dedicated approximation techniques are used to establish an uncertain auxiliary system whose coefficient matrix is subject to multiplicative noises, thereby giving rise to a certain stochastic parameter system. Based on such an auxiliary system, we go ahead to design the finite-horizon H_∞ state estimator by solving coupled backward Riccati equations. A recursive algorithm is proposed to calculate the corresponding gain matrix of the H_∞ estimator. Note that, in the conditions of our main results stated in Theorem 3, all the network information (e.g. the nonlinear functions, statistical law about the inner couplings, H_∞ performance index) is reflected, which conforms with the engineering practice.

Algorithm FHHE

Step 1.	Give the scalars $\gamma > 0$, α_k, β_k $(k = 0, 1, \cdots, T)$, weighted matrix $W > 0$, and let $k = T$ and $P_{T+1} = Q_{T+1} = 0$.
Step 2.	Calculate the matrix Ω_k by (22), and solve (27) to obtain the matrix Ψ_k. If $\Psi_k > 0$, then $L_{i,k}$ can be determined by (30), and go to the next step, otherwise go to step 1.
Step 3.	If $\Omega_k > 0$, then P_k and Q_k can be obtained by (21) and (26), respectively, and then go to the next step, else go to step 5.
Step 4.	If $k > 0$, set $k = k-1$ and go to step 2. When $k = 0$, stop.
Step 5.	If anyone in (22) and (27) is violated, then this algorithm is infeasible for given γ, α_k, β_k and W, stop or go to step 1 and start another recursive loop by resetting the values of γ, α_k, β_k and W.

Remark 7: In this paper, a systematic investigation is initiated on the new yet challenging problem of RRP-based finite-horizon H_∞ state estimation problem for a class of time-varying stochastic coupled networks subject to random inner coupling strengths. The main novelties of this paper are outlined as follows: 1) the research problem addressed is new that represents the first of few attempts to deal with the protocol-based state estimation problem for complex networks under hybrid phenomena of stochastic coupling, external disturbances and communication protocols; 2) the concept of finite-horizon H_∞ performance is used to provide a reasonable way in evaluating the transient performance of the stochastic complex networks by means of the disturbance rejection/attenuation capacity; and 3) the developed BCRRE approach is new that offers a recursive algorithm suitable for online applications.

IV. An Illustrative Example

This section provides an illustrative example to show the effectiveness of the state estimation method.

Consider the complex network (1) with three nodes ($N = 3$) over a prescribed finite time horizon $[0, 40]$, i.e., $T = 40$. The outer coupling configuration matrix is given as follows

$$R = \begin{bmatrix} -0.2 & 0.1 & 0.1 \\ 0.1 & -0.2 & 0.1 \\ 0.1 & 0.1 & -0.2 \end{bmatrix}. \tag{31}$$

The time-varying nonlinear functions $f(x_{i,k})$ and $h_i(x_{i,k})$ are chosen as follows

$$f(x_{i,k}) = \begin{bmatrix} 0.2x_{i1,k} + \tanh(0.1x_{i1,k}) \\ 0.15x_{i2,k} + \tanh(0.05x_{i2,k}) \end{bmatrix}$$
$$h_i(x_{i,k}) = \begin{bmatrix} 0.3x_{i1,k} \\ 0.2x_{i2,k} + \tanh(0.1x_{i2,k}) \end{bmatrix}$$

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TABLE I
ESTIMATOR GAIN MATRIX $L_{i,k}$ $(i = 1, 2, 3)$

k	$L_{1,k}$	$L_{2,k}$	$L_{3,k}$
0	-0.1445	0.0881	-0.2679
	-0.0445	-0.0044	-0.1960
1	-0.1436	0.0908	-0.2564
	-0.0452	-0.0066	-0.2037
2	-0.1421	0.0937	-0.2732
	-0.0467	-0.0086	-0.1901
⋮	⋮	⋮	⋮
38	-0.1099	0.0488	-0.2112
	-0.0694	0.0240	-0.2183
39	-0.0859	0.0302	-0.1610
	-0.0861	0.0405	-0.2420
40	0	0	0
	0	0	0

where $x_{i\upsilon,k}$ $(\upsilon = 1, 2)$ is the υ-th element of $x_{i,k}$, and the time-varying matrices $B_{i,k}$ $(i = 1, 2, 3)$ are given as follows

$$B_{1,k} = \begin{bmatrix} 0.6 + 0.3\text{sin}(k) \\ -0.5 \end{bmatrix}$$
$$B_{2,k} = \begin{bmatrix} 0.5 \\ 0.6 + 0.1\text{cos}(0.5k) \end{bmatrix}, \; B_{3,k} = \begin{bmatrix} -0.6 \\ 0.8 \end{bmatrix}.$$

It is easy to verify that

$$F_{1,k} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.15 \end{bmatrix}, \; F_{2,k} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.2 \end{bmatrix}$$
$$S_{i,k} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}.$$

The parameters in (5) and (11) are set as

$$\begin{aligned} C_{1,k} =& [0.3 \quad 0.2], D_{1,k} = 0.5, E_{1,k} = [0.5 \quad 0.5] \\ C_{2,k} =& [-0.2 \quad 0.1], D_{2,k} = 0.5, E_{2,k} = [0.3 \quad 0.4] \\ C_{3,k} =& [0.1 \quad 0.1], D_{3,k} = 0.6, E_{3,k} = [0.4 \quad 0.6]. \end{aligned}$$

Let $\gamma = 0.8$, $\alpha_k = 0.5$, $\beta_k = 0.4$ and $W = I_{15}$. For the inner coupling (3), g_1 and g_2 are, respectively, selected as $g_1 = 0.2$ and $g_2 = 0.3$. $\psi_{1,k}$, $\psi_{2,k}$ and $\psi_{3,k}$ are mutually independent random sequences obeying uniform distribution over the interval $[0.1, 0.9]$. As such, the means and variances of the random sequences $\psi_{i,k}$ are $\psi_1 = \psi_2 = \psi_3 = 0.4$ and $\sigma_1 = \sigma_2 = \sigma_3 = 0.0533$. According to Algorithm FHHE, the values of gain matrix $L_{i,k}$ $(i = 1, 2, 3;\ k = 0, 1, 2, \ldots, 40)$ are recursively calculated and listed in Table I. It should be noted from Algorithm FHHE and Table I that the estimator gains at instant $k = 40$ become zero because of the equality (30) and the preseted values $P_{41} = Q_{41} = 0$.

The exogenous disturbance is $v(k) = 2\cos(k)e^{-0.02k}$. The initial conditions of the measurements, system states and their estimations are given as $y_{1,0} = y_{2,0} = y_{3,0} = 0$, $x_{1,0} = [0.1 \quad -0.1]^T$, $x_{2,0} = [0.2 \quad 0.1]^T$, $x_{3,0} = [0.1 \quad 0.2]^T$, $\hat{x}_{1,0} = [-0.1 \quad 0.1]^T$, $\hat{x}_{2,0} = [0.1 \quad -0.1]^T$ and $\hat{x}_{3,0} = [-0.1 \quad 0.1]^T$. The responses of $x_{i,k}$ $(I = 1, 2, 3)$ and their estimates are illustrated in Figs. 1-3. The output estimation errors $\bar{z}_{i,k}$ $(i = 1, 2, 3)$ are shown in Fig. 4. It is evident

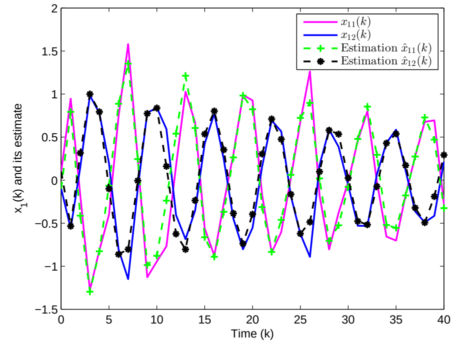

Fig. 1. $x_1(k)$ and its estimates

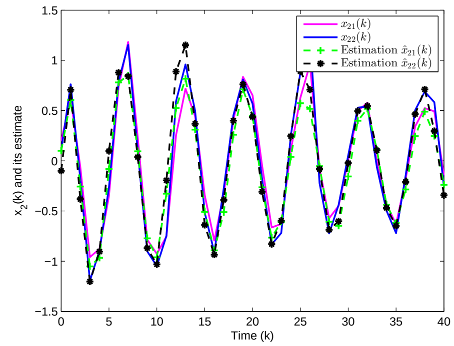

Fig. 2. $x_2(k)$ and its estimates

from Fig. 4 that the designed estimator achieves satisfactory performance over the prescribed finite horizon $[0,\ 40]$.

V. CONCLUSIONS

In this paper, the finite-Horizon H_∞ state estimation problem has been investigated for a class of stochastic coupled networks subject to random inner coupling variations and the Round-Robin scheduling protocol. In order to reduce the network communication burden, the Round-Robin transmission protocol has been applied to schedule the measurement signals from the sensors to the estimators. A novel linearization technique has been developed to deal with the system nonlinearities. Also, for the purpose of facilitating the theoretical

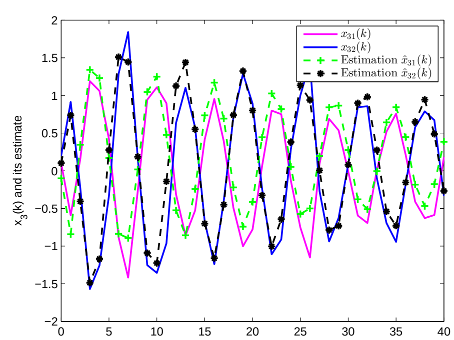

Fig. 3. $x_3(k)$ and its estimates

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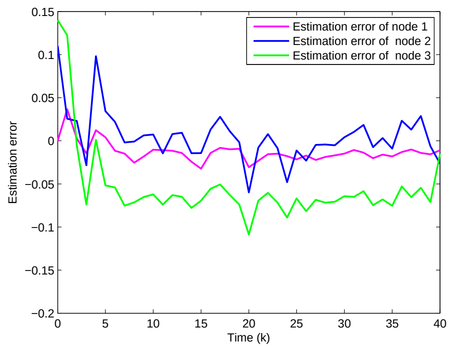

Fig. 4. The output estimation errors

analysis, some ingenious system transformation operations have been adopted such that the multiplicative noise term is included in the coefficient matrix rather than the augmented disturbance vector. By employing the backward Riccati difference equation technique, the desired finite-horizon H_∞ state estimators have been determined by recursively solving a set of coupled difference equations. The effectiveness of the state estimation algorithm has been finally verified by a numerical example. Future research topics would be to extend the main results to more complicated systems with different performance specifications (e.g. variance-constrained performance, mixed H_2/H_∞ index). Similarly, we can also investigate the finite-horizon H_∞ state estimation problems for stochastic coupled networks based on event-triggered mechanism and other communication scheduling protocols [4], [6], [10], [24], [35], [46]–[50].

APPENDIX

A. Proof of Theorem 1

Proof: Define the following index

$$\bar{J}_k \triangleq \mathbf{E}\{\eta_{k+1}^T P_{k+1}\eta_{k+1} - \eta_k^T P_k \eta_k\}. \tag{32}$$

Along the dynamics of auxiliary system (17), the index (32) is calculated by

$$\begin{aligned}\bar{J}_k =& \mathbf{E}\Big\{\eta_k^T[(\mathcal{A}_k + \mathbf{F}_k)^T P_{k+1}(\mathcal{A}_k + \mathbf{F}_k) \\ &+ \Delta_k^T P_{k+1}\Delta_k + 2(\mathcal{A}_k + \mathbf{F}_k)^T P_{k+1}\Delta_k - P_k]\eta_k \\ &+ 2\eta_k^T(\mathcal{A}_k + \mathbf{F}_k + \Delta_k)^T P_{k+1}\mathbf{B}_k\mathbf{v}_k \\ &+ \mathbf{v}_k^T\mathbf{B}_k^T P_{k+1}\mathbf{B}_k\mathbf{v}_k\Big\}.\end{aligned} \tag{33}$$

By virtue of $\mathbf{E}\{\Delta_k\} = 0$ and denoting $\mathbb{A}_k \triangleq \mathcal{A}_k + \mathbf{F}_k$, (33) is simplified as

$$\begin{aligned}\bar{J}_k =& \mathbf{E}\Big\{\eta_k^T[\mathbb{A}_k^T P_{k+1}\mathbb{A}_k + \Delta_k^T P_{k+1}\Delta_k - P_k]\eta_k \\ &+ 2\eta_k^T\mathbb{A}_k^T P_{k+1}\mathbf{B}_k\mathbf{v}_k + \mathbf{v}_k^T\mathbf{B}_k^T P_{k+1}\mathbf{B}_k\mathbf{v}_k\Big\}.\end{aligned}$$

In addition, one further calculates

$$\begin{aligned}&\mathbf{E}\Big\{\eta_k^T\Delta_k^T P_{k+1}\Delta_k\eta_k\Big\} \\ =& \mathbf{E}\Big\{\eta_k^T \mathrm{diag}\{\tilde{R}_k^T(P_{k+1}^{11} + 2P_{k+1}^{13} + P_{k+1}^{33})\tilde{R}_k, 0, 0\}\eta_k\Big\}\end{aligned}$$

where $\tilde{R}_k = R\otimes(G_k - G)$ and $P_{k+1}^{\mu\nu}$ is the (μ,ν)-th entry of matrix P_{k+1} $(\mu,\nu = 1,2,3)$.

Applying Lemma 1 and the mutual independence of $\psi_{q,k}$ $(q = 1,2,\cdots,n)$ in (3), one observes that

$$\mathbf{E}\Big\{\eta_k^T\Delta_k^T P_{k+1}\Delta_k\eta_k\Big\} = \mathbf{E}\Big\{\eta_k^T\bar{\mathbf{G}}^T P_{k+1}\bar{\mathbf{G}}\eta_k\Big\}.$$

It is obvious that $\mathbf{B}_k$ in (18) is decomposed into two parts as follows

$$\begin{aligned}\mathbf{B}_k =& \begin{bmatrix} \mathcal{B}_k & \alpha_k^{-1}I & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & \beta_k^{-1}I \end{bmatrix} w_k \\ =& \Xi_k + \Gamma_k w_k\end{aligned} \tag{34}$$

where the first and second parts indicate in deterministic and stochastic settings, respectively.

It follows from the statistical characteristics of w_k that

$$\mathbf{E}\Big\{2\eta_k^T\mathbb{A}_k^T P_{k+1}\mathbf{B}_k\mathbf{v}_k\Big\} = \mathbf{E}\Big\{2\eta_k^T\mathbb{A}_k^T P_{k+1}\Xi_k\mathbf{v}_k\Big\}$$

and

$$\mathbf{E}\Big\{\mathbf{v}_k^T\mathbf{B}_k^T P_{k+1}\mathbf{B}_k\mathbf{v}_k\Big\} = \mathbf{E}\Big\{\mathbf{v}_k^T X_k\mathbf{v}_k\Big\}$$

where X_k is given in (20). Then, one derives

$$\begin{aligned}\bar{J}_k =& \mathbf{E}\Big\{\eta_k^T[\mathbb{A}_k^T P_{k+1}\mathbb{A}_k + \bar{\mathbf{G}}^T P_{k+1}\bar{\mathbf{G}} - P_k]\eta_k \\ &+ 2\eta_k^T\mathbb{A}_k^T P_{k+1}\Xi_k\mathbf{v}_k + \mathbf{v}_k^T X_k\mathbf{v}_k\Big\}.\end{aligned}$$

By adding the following zero term

$$\begin{aligned}&\| \bar{z}_k \|^2 - \gamma^2(\| \mathbf{v}_k \|^2 - \| \alpha_k\mathbb{F}_k\eta_k \|^2 - \| \beta_k\mathbf{H}_k\eta_k \|^2) \\ -& \| \bar{z}_k \|^2 + \gamma^2(\| \mathbf{v}_k \|^2 - \| \alpha_k\mathbb{F}_k\eta_k \|^2 - \| \beta_k\mathbf{H}_k\eta_k \|^2)\end{aligned}$$

to $\bar{J}_k$ results in

$$\begin{aligned}\bar{J}_k =& \mathbf{E}\Big\{\eta_k^T[\mathbb{A}_k^T P_{k+1}\mathbb{A}_k + \bar{\mathbf{G}}^T P_{k+1}\bar{\mathbf{G}} - P_k \\ &+ \mathcal{E}_k^T\mathcal{E}_k + \gamma^2\alpha_k^2\mathbb{F}_k^T\mathbb{F}_k + \gamma^2\beta_k^2\mathbf{H}_k^T\mathbf{H}_k]\eta_k \\ &+ 2\eta_k^T\mathbb{A}_k^T P_{k+1}\Xi_k\mathbf{v}_k - \mathbf{v}_k^T(\gamma^2 I - X_k)\mathbf{v}_k - \| \bar{z}_k \|^2 \\ &+ \gamma^2(\| \mathbf{v}_k \|^2 - \| \alpha_k\mathbb{F}_k\eta_k \|^2 - \| \beta_k\mathbf{H}_k\eta_k \|^2)\Big\}.\end{aligned} \tag{35}$$

Moreover, invoking the completing-the-squares technique leads to

$$\begin{aligned}&2\eta_k^T\mathbb{A}_k^T P_{k+1}\Xi_k\mathbf{v}_k - \mathbf{v}_k^T(\gamma^2 I - X_k)\mathbf{v}_k \\ =& (\mathbf{v}_k^*)^T\Omega_k\mathbf{v}_k^* - (\mathbf{v}_k - \mathbf{v}_k^*)^T\Omega_k(\mathbf{v}_k - \mathbf{v}_k^*)\end{aligned}$$

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where $\mathbf{v}_k^* = \Omega_k^{-1}\Xi_k^T P_{k+1}\mathbb{A}_k\eta_k$ and $\Omega_k = \gamma^2 I - X_k$. Subsequently, we arrive at

$$\begin{aligned}\bar{J}_k =&\mathbf{E}\Big\{\eta_{k+1}^T P_{k+1}\eta_{k+1} - \eta_k^T P_k\eta_k\Big\}\\ =&\mathbf{E}\Big\{\eta_k^T[\mathbb{A}_k^T P_{k+1}\mathbb{A}_k + \bar{\mathbf{G}}^T P_{k+1}\bar{\mathbf{G}} + \mathcal{E}_k^T\mathcal{E}_k + \gamma^2\alpha_k^2\mathbb{F}_k^T\mathbb{F}_k\\ &+ \gamma^2\beta_k^2\mathbf{H}_k^T\mathbf{H}_k + \mathbb{A}_k^T P_{k+1}\Xi_k\Omega_k^{-1}\Xi_k^T P_{k+1}\mathbb{A}_k - P_k]\eta_k\\ &- (\mathbf{v}_k - \mathbf{v}_k^*)^T\Omega_k(\mathbf{v}_k - \mathbf{v}_k^*) - \parallel \bar{z}_k \parallel^2\\ &+ \gamma^2(\parallel \mathbf{v}_k \parallel^2 - \parallel \alpha_k\mathbb{F}_k\eta_k \parallel^2 - \parallel \beta_k\mathbf{H}_k\eta_k \parallel^2)\Big\}.\end{aligned} \tag{36}$$

Summing both sides of (36) from 0 to T and noticing (21), it is easy to see that

$$\begin{aligned}&\mathbf{E}\Big\{\eta_{T+1}^T P_{T+1}\eta_{T+1} - \eta_0^T P_0\eta_0\Big\}\\ =&\sum_{k=0}^{T}\mathbf{E}\Big\{-(\mathbf{v}_k - \mathbf{v}_k^*)^T\Omega_k(\mathbf{v}_k - \mathbf{v}_k^*) - \parallel \bar{z}_k \parallel^2\\ &+ \gamma^2(\parallel \mathbf{v}_k \parallel^2 - \parallel \alpha_k\mathbb{F}_k\eta_k \parallel^2 - \parallel \beta_k\mathbf{H}_k\eta_k \parallel^2)\Big\}.\end{aligned} \tag{37}$$

On the basis of (20), we deduce that

$$\begin{aligned}J_2 =&\sum_{k=0}^{T}\mathbf{E}\Big\{\parallel \bar{z}_k \parallel^2 - \gamma^2(\parallel \mathbf{v}_k \parallel^2 - \parallel \alpha_k\mathbb{F}_k\eta_k \parallel^2\\ &- \parallel \beta_k\mathbf{H}_k\eta_k \parallel^2) - \gamma^2\eta_0^T W\eta_0\Big\}\\ =&\mathbf{E}\Big\{-\eta_{T+1}^T P_{T+1}\eta_{T+1} - \eta_0^T(\gamma^2 W - P_0)\eta_0\\ &- \sum_{k=0}^{T}(\mathbf{v}_k - \mathbf{v}_k^*)^T\Omega_k(\mathbf{v}_k - \mathbf{v}_k^*)\Big\}.\end{aligned} \tag{38}$$

With the help of $P_{T+1} = 0$, $\Omega_k > 0$ and $P_0 < \gamma^2 W$, it is obvious that $J_2 < 0$ holds and this completes the proof of this theorem. ■

B. Proof of Theorem 2

Proof: Introduce a performance index as

$$\tilde{J}_k = \mathbf{E}\{\eta_{k+1}^T Q_{k+1}\eta_{k+1} - \eta_k^T Q_k\eta_k\}. \tag{39}$$

Noticing (23), (24) and $\mathbf{E}\{\Delta_k\} = 0$, the above index is computed as

$$\begin{aligned}\tilde{J}_k =&\mathbf{E}\Big\{\eta_k^T[(\bar{\mathbb{A}}_k + \bar{\mathbb{B}}_k\bar{\mathbb{A}}_k + \Delta_k)^T Q_{k+1}(\bar{\mathbb{A}}_k + \bar{\mathbb{B}}_k\bar{\mathbb{A}}_k + \Delta_k)\\ &- Q_k]\eta_k + \phi_k^T\mathcal{I}^T(\bar{\mathbb{B}}_k + I)^T Q_{k+1}(\bar{\mathbb{B}}_k + I)\mathcal{I}\phi_k\\ &+ 2\eta_k^T(\bar{\mathbb{A}}_k + \bar{\mathbb{B}}_k\bar{\mathbb{A}}_k + \Delta_k)^T Q_{k+1}(\bar{\mathbb{B}}_k + I)\mathcal{I}\phi_k\Big\}\\ =&\mathbf{E}\Big\{\eta_k^T[(\bar{\mathbb{A}}_k + \bar{\mathbb{B}}_k\bar{\mathbb{A}}_k)^T Q_{k+1}(\bar{\mathbb{A}}_k + \bar{\mathbb{B}}_k\bar{\mathbb{A}}_k)\\ &+ \Delta_k^T Q_{k+1}\Delta_k - Q_k]\eta_k + 2\eta_k^T(\bar{\mathbb{A}}_k + \bar{\mathbb{B}}_k\bar{\mathbb{A}}_k)^T Q_{k+1}\\ &(\bar{\mathbb{B}}_k + I)\mathcal{I}\phi_k + \phi_k^T\mathcal{I}^T(\bar{\mathbb{B}}_k + I)^T Q_{k+1}(\bar{\mathbb{B}}_k + I)\mathcal{I}\phi_k\Big\}.\end{aligned} \tag{40}$$

As in the proof of Theorem 1, it is easy to observe that

$$\mathbf{E}\Big\{\eta_k^T\Delta_k^T Q_{k+1}\Delta_k\eta_k\Big\} = \mathbf{E}\Big\{\eta_k^T\bar{\mathbf{G}}^T Q_{k+1}\bar{\mathbf{G}}\eta_k\Big\}$$

where $\bar{\mathbf{G}}$ is given in Theorem 1.

It follows from Lemma 2 that $\mathbf{E}\{w_k\} = \mathbf{E}\{w_k^3\} = 0$, $\mathbf{E}\{w_k^2\} = 1$, $\mathbf{E}\{w_k^4\} = 3$ and

$$\begin{aligned}&\mathbf{E}\Big\{\eta_k^T[(\bar{\mathbb{A}}_k + \bar{\mathbb{B}}_k\bar{\mathbb{A}}_k)^T Q_{k+1}(\bar{\mathbb{A}}_k + \bar{\mathbb{B}}_k\bar{\mathbb{A}}_k)]\eta_k\Big\}\\ =&\mathbf{E}\Big\{\eta_k^T\bar{\mathbb{A}}_k^T[Q_{k+1} + 2Q_{k+1}\bar{\mathbb{B}}_k + \bar{\mathbb{B}}_k^T Q_{k+1}\bar{\mathbb{B}}_k]\bar{\mathbb{A}}_k\eta_k\Big\}\\ =&\mathbf{E}\Big\{\eta_k^T\bar{\mathbb{A}}_k^T[Q_{k+1} + 2Q_{k+1}Y_k P_{k+1} + \bar{\mathbb{B}}_k^T Q_{k+1}\bar{\mathbb{B}}_k]\bar{\mathbb{A}}_k\eta_k\Big\}\end{aligned} \tag{41}$$

where Y_k is given in (20), and the last term of the above equality is shown as

$$\begin{aligned}&\mathbf{E}\Big\{\eta_k^T\bar{\mathbb{A}}_k^T\bar{\mathbb{B}}_k^T Q_{k+1}\bar{\mathbb{B}}_k\bar{\mathbb{A}}_k\eta_k\Big\}\\ =&\mathbf{E}\Big\{\eta_k^T\bar{\mathbb{A}}_k^T P_{k+1}(\Xi_k\Omega_k^{-1}\Xi_k^T Q_{k+1}\Xi_k\Omega_k^{-1}\Xi_k^T\\ &+ 4\Xi_k\Omega_k^{-1}\Gamma_k^T Q_{k+1}\Gamma_k\Omega_k^{-1}\Xi_k^T\\ &+ 3\Gamma_k\Omega_k^{-1}\Gamma_k^T Q_{k+1}\Gamma_k\Omega_k^{-1}\Gamma_k^T\\ &+ 2\Gamma_k\Omega_k^{-1}\Gamma_k^T Q_{k+1}\Xi_k\Omega_k^{-1}\Xi_k^T)P_{k+1}\bar{\mathbb{A}}_k\eta_k\Big\}\\ \triangleq&\mathbf{E}\{\eta_k^T\bar{\mathbb{A}}_k^T\mathbf{M}_k\bar{\mathbb{A}}_k\eta_k\}\end{aligned} \tag{42}$$

where

$$\begin{aligned}\mathbf{M}_k =&P_{k+1}(\Xi_k\Omega_k^{-1}\Xi_k^T Q_{k+1}\Xi_k\Omega_k^{-1}\Xi_k^T\\ &+ 4\Xi_k\Omega_k^{-1}\Gamma_k^T Q_{k+1}\Gamma_k\Omega_k^{-1}\Xi_k^T\\ &+ 3\Gamma_k\Omega_k^{-1}\Gamma_k^T Q_{k+1}\Gamma_k\Omega_k^{-1}\Gamma_k^T\\ &+ 2\Gamma_k\Omega_k^{-1}\Gamma_k^T Q_{k+1}\Xi_k\Omega_k^{-1}\Xi_k^T)P_{k+1}\\ =&P_{k+1}(Y_k Q_{k+1}Y_k + 4\Xi_k\Omega_k^{-1}\Gamma_k^T Q_{k+1}\Gamma_k\Omega_k^{-1}\Xi_k^T\\ &+ 2\Gamma_k\Omega_k^{-1}\Gamma_k^T Q_{k+1}\Gamma_k\Omega_k^{-1}\Gamma_k^T)P_{k+1}.\end{aligned}$$

Then, we obtain

$$\begin{aligned}&\mathbf{E}\Big\{\eta_k^T[(\bar{\mathbb{A}}_k + \bar{\mathbb{B}}_k\bar{\mathbb{A}}_k)^T Q_{k+1}(\bar{\mathbb{A}}_k + \bar{\mathbb{B}}_k\bar{\mathbb{A}}_k)]\eta_k\Big\}\\ =&\mathbf{E}\Big\{\eta_k^T\bar{\mathbb{A}}_k^T[Q_{k+1} + 2Q_{k+1}Y_k P_{k+1} + \mathbf{M}_k]\bar{\mathbb{A}}_k\eta_k\Big\}\\ \triangleq&\mathbf{E}\Big\{\eta_k^T\bar{\mathbb{A}}_k^T Z_k\bar{\mathbb{A}}_k\eta_k\Big\}.\end{aligned} \tag{43}$$

Similarly, we have

$$\begin{aligned}&\mathbf{E}\Big\{2\eta_k^T(\bar{\mathbb{A}}_k + \bar{\mathbb{B}}_k\bar{\mathbb{A}}_k)^T Q_{k+1}(\bar{\mathbb{B}}_k + I)\mathcal{I}\phi_k\Big\}\\ =&\mathbf{E}\Big\{2\eta_k^T\bar{\mathbb{A}}_k^T Q_{k+1}(\bar{\mathbb{B}}_k + I)\mathcal{I}\phi_k\\ &+ 2\eta_k^T\bar{\mathbb{A}}_k^T\bar{\mathbb{B}}_k^T Q_{k+1}(\bar{\mathbb{B}}_k + I)\mathcal{I}\phi_k\Big\}\\ =&\mathbf{E}\Big\{2\eta_k^T\bar{\mathbb{A}}_k^T Q_{k+1}(Y_k P_{k+1} + I)\mathcal{I}\phi_k\\ &+ 2\eta_k^T\bar{\mathbb{A}}_k^T(\mathbf{M}_k + P_{k+1}Y_k Q_{k+1})\mathcal{I}\phi_k\Big\}\end{aligned} \tag{44}$$

and

$$\begin{aligned}&\mathbf{E}\bigg\{\phi_k^T\mathcal{I}^T(\bar{\mathbb{B}}_k+I)^TQ_{k+1}(\bar{\mathbb{B}}_k+I)\mathcal{I}\phi_k\bigg\}\\ =&\mathbf{E}\bigg\{\phi_k^T\mathcal{I}^T[Q_{k+1}+2Q_{k+1}Y_kP_{k+1}+\mathbf{M}_k]\mathcal{I}\phi_k\bigg\}\\ \triangleq&\mathbf{E}\bigg\{\phi_k^T\mathcal{I}^TZ_k\mathcal{I}\phi_k\bigg\}.\end{aligned}\tag{45}$$

Consequently, we have

$$\begin{aligned}\tilde{J}_k=&\mathbf{E}\bigg\{\eta_k^T[\bar{\mathbb{A}}_k^TZ_k\bar{\mathbb{A}}_k+\bar{\mathbf{G}}^TQ_{k+1}\bar{\mathbf{G}}-Q_k+\mathcal{E}_k^T\mathcal{E}_k\\&+\bar{\mathbb{A}}_k^T(\mathbf{M}_k+Q_{k+1}Y_kP_{k+1})\mathcal{I}L_k\mathbf{C}_k\\&+\mathbf{C}_k^TL_k^T\mathcal{I}^T(\mathbf{M}_k+P_{k+1}Y_kQ_{k+1})\bar{\mathbb{A}}_k]\eta_k\\&+2\eta_k^T\bar{\mathbb{A}}_k^TQ_{k+1}(Y_kP_{k+1}+I)\mathcal{I}\phi_k\\&+\phi_k^T(\mathcal{I}^TZ_k\mathcal{I}+I)\phi_k-\parallel\bar{z}_k\parallel^2-\parallel\phi_k\parallel^2\bigg\}.\end{aligned}\tag{46}$$

Denoting

$$\begin{aligned}\Psi_k=&\mathcal{I}^TZ_k\mathcal{I}+I\\ \phi_k^*=&\Psi_k^{-1}\mathcal{I}^TQ_{k+1}(Y_kP_{k+1}+I)\bar{\mathbb{A}}_k\eta_k\end{aligned}\tag{47}$$

we have

$$\begin{aligned}&2\eta_k^T\bar{\mathbb{A}}_k^TQ_{k+1}(Y_kP_{k+1}+I)\mathcal{I}\phi_k+\phi_k^T(\mathcal{I}^TZ_k\mathcal{I}+I)\phi_k\\ =&(\phi_k+\phi_k^*)^T\Psi_k(\phi_k+\phi_k^*)-(\phi_k^*)^T\Psi_k\phi_k^*.\end{aligned}$$

Thus, $\tilde{J}_k$ becomes as

$$\begin{aligned}\tilde{J}_k=&\mathbf{E}\bigg\{\eta_k^T[\bar{\mathbb{A}}_k^TZ_k\bar{\mathbb{A}}_k+\bar{\mathbf{G}}^TQ_{k+1}\bar{\mathbf{G}}+\mathcal{E}_k^T\mathcal{E}_k-Q_k\\&+\bar{\mathbb{A}}_k^T(\mathbf{M}_k+Q_{k+1}Y_kP_{k+1})\mathcal{I}L_k\mathbf{C}_k\\&+\mathbf{C}_k^TL_k^T\mathcal{I}^T(\mathbf{M}_k+P_{k+1}Y_kQ_{k+1})\bar{\mathbb{A}}_k\\&-\bar{\mathbb{A}}_k^T(P_{k+1}Y_k+I)^TQ_{k+1}\mathcal{I}\\&\times\Psi_k^{-1}\mathcal{I}^TQ_{k+1}(Y_kP_{k+1}+I)\bar{\mathbb{A}}_k]\eta_k\\&+(\phi_k+\phi_k^*)^T\Psi_k(\phi_k+\phi_k^*)-\parallel\bar{z}_k\parallel^2-\parallel\phi_k\parallel^2\bigg\}.\end{aligned}\tag{48}$$

According to (26) and (48), it follows readily that

$$\begin{aligned}&\mathbf{E}\bigg\{\eta_{T+1}^TQ_{T+1}\eta_{T+1}-\eta_0^TQ_0\eta_0\bigg\}\\ =&\sum_{k=0}^{T}\mathbf{E}\bigg\{(\phi_k+\phi_k^*)^T\Psi_k(\phi_k+\phi_k^*)\parallel\bar{z}_k\parallel^2-\parallel\phi_k\parallel^2\bigg\}.\end{aligned}\tag{49}$$

Combining (25), (39) and $Q_{T+1}=0$ leads to

$$\tilde{J}=\sum_{k=0}^{T}\mathbf{E}\bigg\{(\phi_k+\phi_k^*)^T\Psi_k(\phi_k+\phi_k^*)\bigg\}+\eta_0^TQ_0\eta_0\tag{50}$$

and we see immediately that $\phi_k=-\phi_k^*$, or

$$L_k\mathbf{C}_k=-\Psi_k^{-1}\mathcal{I}^TQ_{k+1}(Y_kP_{k+1}+I)\bar{\mathbb{A}}_k\tag{51}$$

minimizes the performance index $\tilde{J}$ as $\eta_0^TQ_0\eta_0$. Meanwhile, from (38) with $P_{T+1}=0$, the cost of J_2 is determined by $\eta_0^T(P_0-\gamma^2W)\eta_0$. The proof of this theorem is thus fulfilled. ∎

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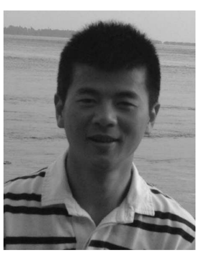

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