Sampled-Data-Based Fault-Tolerant Consensus Control for Multi-agent Systems: A Data Privacy Preserving Scheme ⋆

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Abstract

This paper is concerned with the fault-tolerant consensus control problem for a general class of linear continuous-time multi-agent systems (MASs) with the data privacy preserving (DPP) constraints. Rather than employing the traditional cryptographic mechanism, we develop a novel dynamic-quantization-based DPP scheme from a control-theoretic perspective. Under our developed DPP scheme, the sampled, quantized, released, received and recovered data is used to design a fault-tolerant controller. With the adopted controller structure, the consensus of the MASs is guaranteed where the size of the data transmitted among agents can be made finite. A series of numerical examples are given and the comparative analysis is carried out to illustrate the effectiveness of the developed consensus control scheme.

Key words: Data privacy preserving; Dynamic quantization; Sampled data; Multi-agent system; Fault-tolerant Consensus control.

1 Introduction

With rapid development of miniaturizing technologies, many industrial systems of increasing scales fall squarely into the category of multi-agent systems (MASs) with examples including unmanned robots [17, 22], satellites [18,44] and vessels [2,28]). The last decade has seen enormous research attention devoted to dynamic analysis issues with respect to various MASs from both academia and industry [6,10,19]. As a fundamental problem in cooperative control, the consensus control has become an attractive research topic whose main idea is to design appropriate *distributed* controllers in order to achieve consensus on a quantity of interests, see e.g. [3–5, 11, 31, 36] for some recent results.

For MASs, the *distributed* controller relies largely on the distributed data sharing process. Such a process is, unfortunately, often confronted with the phenomenon of information leakage, for example, the illegal acquisition of the individual global positioning system (GPS) data. The information leakage phenomenon, if not properly handled, could lead to unintended consequences and this has therefore triggered many ongoing research initiatives on the data privacy preserving (DPP) technologies. Generally speaking, traditional DPP algorithms are dependent on cryptographic computation which focuses mainly on the data rather than the physical systems (especially dynamic systems).

Recently, some new DPP technologies have been developed in order to make better use of the system dynamics with preserved control-theoretic performance [29]. For example, the perturbation-injection-based scheme (PIS) continuously adds perturbations into the data to be released such that the original data cannot be inferred by adversaries [12, 13, 32], and the dynamic-quantization-based scheme (DQS) generates a series of codewords to conceal the original data through specific transformation and quantization algorithms. In the context of MASs, the DQS has been extensively exploited with many excellent results reported in the literature, see e.g. [21,23,24] for integrator MASs and [20,40] for general linear MASs. Due to the utilization of the transformation/quantization algorithms, the DQS is featured with introducing a new function into MASs to relieve communication congestions and thus reducing the size of the transmitted data. As such, the DQS is selected as the type of DPP schemes to be investigated in this paper.

It is worth mentioning that most existing DQS-related literature has been concerned with discrete-time MASs only, and the corresponding results on continuous-time MASs are very few. One of the most effective methods to bridge the gap between continuous- and discrete-time

⋆ This work was supported in part by the National Natural Science Foundation of China under Grants 61733009 and 61873148, the National Key Research and Development Program of China under Grant 2017YFA0700300, the Natural Science Foundation of Guangdong Province of China under Grant 2018B030311054, the Royal Society of the UK, and the Alexander von Humboldt Foundation of Germany.

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systems is the so-called sampled-data control approach whose main idea is to actuate a continuous-time system via a discrete-time controller [16, 27]. The representative sampling mechanisms include the uniform-sampling method [43], non-uniform sampling approach [48], and event-triggered sampling [25]. A key factor for implementing the sampled-data control lies in the adequate choice of the sampling period as a poor selection could lead to undesired performance degradation or even system instability [14, 15, 26, 35, 45]. As such, it would be practically significant to explore how the sampling period influences the consensus control performance of the MASs under the DQS.

Note that all above-mentioned DPP schemes contribute to the improvement of logical security (including information integrity, confidentiality and availability). Apart from adopting the DPP schemes, another way to improve the logical security is to guarantee the physical security which refers to the protection of tangible items, thereby providing yet another necessary support for logical security. In this sense, the fault-tolerant consensus control (FTCC) has proven to be an effective means to enhance physical security of the MASs with successful applications in a variety of practical systems [1, 8, 37, 41, 42]. Nevertheless, to the best of the authors' knowledge, the FTCC problem for MASs (especially general linear continuous-time MASs) under the framework of DQS remains open yet challenging due mainly to the tight coupling of three factors, namely, the sampling period, the DQS and the fault. To tackle this problem, a degree of freedom should be allocated to the fault-tolerant controller so that the fault-induced effects could be decoupled.

Based on the above discussions, in this paper, we are set to investigate the FTCC problem for continuous-time MASs under the DPP. *The main contributions of this paper are summarized as follows: 1) the fault-induced heterogeneity in the continuous-time MASs is tackled by the dedicatedly designed controller and Lyapunov function; 2) a novel Lyapunov-function-based analysis method is proposed in the sense that the Lyapunov function is not required to decrease along the entire time interval, which helps to obtain a low-conservatism sampling period; and 3) a sufficient condition is derived to ensure the boundedness of the size of the transmitted data..*

The rest of this paper is structured as follows. Section 2 introduces the preliminaries of graph theory and the dynamic-quantization-based DPP scheme. Section 3 provides the main results on consensus and analyzes the size of the transmitted data. Section 4 presents a series of simulation examples and section 5 concludes this paper.

Notations. Let $\mathbf{1}_m$ and $\mathbf{0}_m$ denote the $m\times 1$ column vectors with all ones and all zeros, respectively. $\mathbf{0}_{m\times n}$ stands for the $m\times n$ matrix with all zeros. I_n is a n-dimensional identity matrix. $\mathrm{diag}\{f_0, f_1, \ldots, f_n\}$ represents a diagonal matrix with $f_0, f_1, \ldots, f_n$ as its diagonal elements. $\mathbb{N}$ and $\mathbb{N}^+$ refer to the sets of all natural numbers and positive integers, respectively, i.e., $\mathbb{N} \triangleq \{0, 1, 2, \ldots\}$ and $\mathbb{N}^+ \triangleq \{1, 2, \ldots\}$. Let $\|\cdot\|$ and $\|\cdot\|_\infty$ denote, respectively, the 2-norm and the ∞-norm of a vector or a matrix. The symbol $\otimes$ represents the Kronecker product. Denote the base of the natural logarithm by $\mathbf{e}$. For a given real number x, $\ln(x)$ stands for the natural logarithm of x.

2 Problem Formulation and Preliminaries

2.1 Graph Theory

Given an MAS consisting of N agents, the communication among agents is described by $\mathcal{G} \triangleq (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{\mathcal{V}_1, \mathcal{V}_2, \ldots, \mathcal{V}_N\}$, $\mathcal{E} = \{\mathcal{E}_{ji} | j \in \mathcal{N}_i\}$ and $\mathcal{A} = \left[a_{ij}\right] \in \mathbb{R}^{N\times N}$ denote the set of nodes, the set of edges and the adjacency matrix, respectively. $j \in \mathcal{N}_i$ means that node j is the neighbor of node i. Denote $a_{ij} > 0$ if $\mathcal{E}_{ji} \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. Also, $d_i \triangleq \sum_{j=1}^{N} a_{ij}$ denotes the degree of node i and $d_{\max} \triangleq \max_i d_i$. If $a_{ij} = a_{ji}$, then the graph is said to be an undirected graph. $L = \left[l_{ij}\right] \in \mathbb{R}^{N\times N}$ denotes the Laplacian matrix of the graph $\mathcal{G}$ with $l_{ii} = \sum_{j\neq i} a_{ij},\ l_{ij} = -a_{ij}, i \neq j$.

For an undirected graph $\mathcal{G}$, if $\mathcal{G}$ is connected, then there exists an orthogonal matrix $T = \left[T_0,\ T_1\right]$ such that $T^T LT = \Lambda = \mathrm{diag}\{0, \Phi\}$, where $T_0 = \sqrt{1/N}\mathbf{1}_N$ and $\Phi = \mathrm{diag}\{\lambda_2, \ldots, \lambda_N\}$ with $0 = \lambda_1 < \lambda_2 \leq \ldots \lambda_{N-1} \leq \lambda_N$ denoting the eigenvalues of L. Obviously, Φ is a positive-definite matrix.

2.2 Problem Formulation

Consider an MAS as follows:

$$\dot{x}_i(t) = Ax_i(t) + B(I - \varrho_i)u_i(t), \tag{1}$$

where $i \in \mathfrak{N} \triangleq \{1, 2, \ldots, N\}$. $x_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^m$ are, respectively, the state variable and the input variable; ϱ_i is defined as $\varrho_i \triangleq \mathrm{diag}\{\varrho_{i1}, \ldots, \varrho_{ih}, \ldots, \varrho_{im}\} \in \mathbb{R}^{m\times m}$, where ϱ_{ih} represents the unknown loss-of-effectiveness (LoE) faults of the h-th actuator of the agent i. Assume that $0 \leq \varrho_{ih} \leq \varrho_{\max} < 1$ with $\varrho_{\max}$ being the known upper bound of ϱ_{ih}.

In this paper, we are interested in designing an FTCC scheme for the MAS described by (1) with hope to preserve the data privacy. As shown in Fig. 1, the continuous-time state of agent j is first sampled, then processed by the dynamic quantization algorithm and finally released. Let s_j represent the released data. After being received by the agent i, s_j is used to extract the useful information first via the recovery algorithm and then design the controller.

The sampled-data-based dynamic quantization algorithm, the data recovery algorithm and the controller are designed as follows.

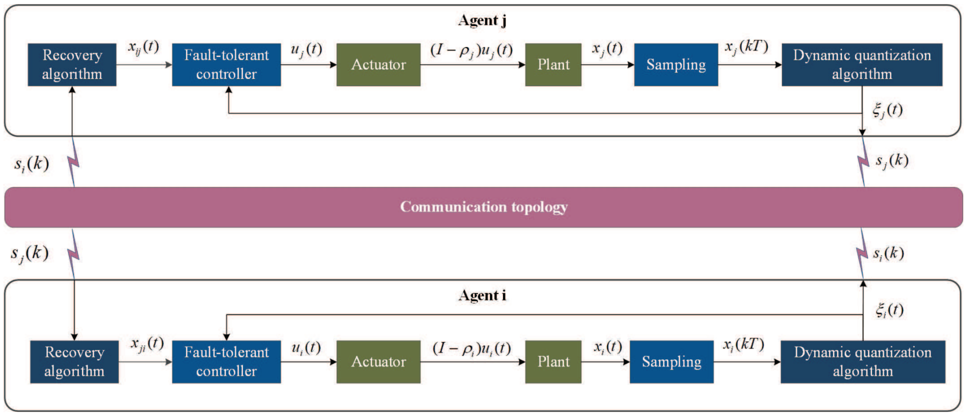

Fig. 1. Dynamic-quantization-based DPP scheme for MASs.

Sampled-data-based dynamic quantization algorithm of agent i:

$$\begin{cases} \dot{\xi}_i(t) = A\xi_i(t), \quad kT < t < (k+1)T,\ k \in \mathbb{N} \\ \xi_i\left((k+1)T\right) = \mathbf{e}^{AT}\xi_i(kT) + g\left((k+1)T\right)s_i(k+1) \\ s_i(k+1) = Q\left(\dfrac{x_i\left((k+1)T\right) - \mathbf{e}^{AT}\xi_i(kT)}{g\left((k+1)T\right)}\right) \\ \xi_i(0) = \mathbf{0}_n, \end{cases} \tag{2}$$

where T represents the sampling period. As shown in Fig. 2, $g(t)$ is a piecewise continuous function serving as a dynamic quantization factor with the following definition:

$$g(t) = \begin{cases} g_0\gamma^k, & t = kk_0T,\ k \in \mathbb{N} \\ g(kk_0T), & kk_0T \le t < (k+1)k_0T \end{cases} \tag{3}$$

where $g_0 > 0$ and $0 < \gamma < 1$. k_0 is a positive integer to be design later.

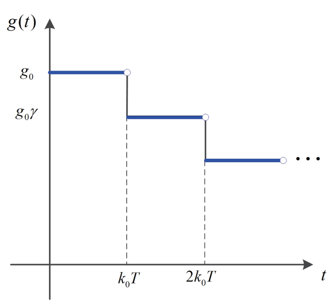

Fig. 2. Dynamic quantization factor $g(t)$.

Given a vector $v = \left[v_1, v_2, \cdots, v_n\right]^T$, the uniform quantizer is defined as $Q(v) \triangleq \left[q(v_1), q(v_2), \cdots, q(v_n)\right]^T$ with

$$q(v_i) = \begin{cases} d\hbar, & (d - \frac{1}{2})\hbar \le v_i < (d + \frac{1}{2})\hbar \\ -q(-v_i), & v_i \le -\frac{1}{2}\hbar, \end{cases} \tag{4}$$

where $d = 0, 1, 2, \ldots, M$ with M as the upper bound. Here, M can be calculated according to

$$M = \max_{i,k} \|s_i(k+1)/\hbar\|_\infty$$

where $\hbar$ is a given quantization parameter and the quantization error satisfies $|v_i - q(v_i)| \le \hbar/2$.

Data recovery algorithm for agent i:

$$\begin{cases} \dot{x}_{ji}(t) = Ax_{ji}(t),\ kT < t < (k+1)T,\ k \in \mathbb{N} \\ x_{ji}\left((k+1)T\right) = \mathbf{e}^{AT}x_{ji}(kT) + g\left((k+1)T\right)s_j(k+1) \\ x_{ji}(0) = \mathbf{0}_n, \quad j \in \mathcal{N}_i, \end{cases} \tag{5}$$

where $x_{ji}(t)$ is the extracted state and $x_{ji}(t) = \xi_j(t)$.

Fault-tolerant controller for agent i:

$$u_i(t) = -cK\sum_{j=1}^{N} a_{ij}(\xi_i(t) - x_{ji}(t)),\ j \in \mathcal{N}_i, \tag{6}$$

where c and K are the coupling gain and the feedback gain, respectively.

By defining the DPP error as $e_i(t) \triangleq \xi_i(t) - x_i(t)$ and the quantization error as

$$\delta_i(k+1) \triangleq s_i(k+1) - \frac{x_i\left((k+1)T\right) - \mathbf{e}^{AT}\xi_i(kT)}{g\left((k+1)T\right)},$$

we have

$$\begin{cases} e_i\left((k+1)T\right) = g\left((k+1)T\right)\delta_i(k+1) \\ e_i(0) = -x_i(0). \end{cases} \tag{7}$$

Denoting

$$x \triangleq \left[x_1^T, x_2^T, \cdots, x_N^T\right]^T,\ e \triangleq \left[e_1^T, e_2^T, \cdots, e_N^T\right]^T,$$
$$\xi \triangleq \left[\xi_1^T, \xi_2^T, \cdots, \xi_N^T\right]^T,\ u \triangleq \left[u_1^T, u_2^T, \cdots, u_N^T\right]^T,$$
$$\varrho \triangleq \mathrm{diag}\{\varrho_1, \varrho_2, \ldots, \varrho_N\},$$

the MAS system can be transformed into:

$$\dot{x}(t) = (I_N \otimes A)x(t) + (I_N \otimes B)(I_{mN} - \varrho)u(t). \tag{8}$$

Before proceeding further, we give the following definition and assumptions.

Definition 1 *(Consensus [33]) For the MAS described by* (1)*, if*

$$\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0 \tag{9}$$

holds for any $i, j \in \mathfrak{N}$, then the consensus is said to be reached asymptotically.

Assumption 1 *The undirected graph $\mathcal{G}$ is connected.*

Assumption 2 *There exists a known positive constant χ_0 such that $\|x_i(0)\| \leq \chi_0$, $\forall i \in \mathfrak{N}$.*

The aim of this paper is to design appropriate dynamic quantization and recovery algorithms (2)-(5) as well as fault-tolerant controller (6) to guarantee the consensus of the MAS described by (1).

3 Main Results

In this section, we first provide a novel design method for the sampled-data-based dynamic quantizer and controller to ensure the consensus. Then, we discuss the size of the transmitted data under the structure of the proposed dynamic-quantization-based DPP scheme.

To start with, we make the following augmentations.

By denoting

$$y \triangleq \left[y_1^T, z^T\right]^T \triangleq (T^T \otimes I_n)x$$

where $y_1 \in \mathbb{R}^n$ and $z \in \mathbb{R}^{n(N-1)}$, we have $z = (T_1^T \otimes I_n)x$ and

$$\begin{aligned}\dot{z}(t) &= \big(T_1^T \otimes A - c(T_1^T \otimes B)(I-\varrho)(L\otimes K)\big)x(t) \\ &\quad - c(T_1^T \otimes B)(I-\varrho)(L\otimes K)e(t) \\ &= \big(I_{N-1} \otimes A - c(T_1^T \otimes B)(I-\varrho)(T_1\Phi\otimes K)\big)z(t) \\ &\quad - c(T_1^T \otimes B)(I-\varrho)(L\otimes K)e(t). \end{aligned} \tag{10}$$

The procedure of designing controller gains and other parameters is given as follows:

1) design the coupling gain of the controller (6) such that

$$c \geq \frac{1}{2\lambda_2(1-\varrho_{\max})}, \tag{11}$$

and design the feedback gain as $K = B^T P$ if there exist a positive constant ε and a positive-definite matrix P such that

$$A^T P + PA - PBB^T P + \varepsilon I_n = 0; \tag{12}$$

2) choose the sampling period according to

$$T \leq \min\left\{\frac{\ln(\rho_0)}{\|A\|}, \frac{1}{c\rho_0\rho_1\rho_3}, \frac{\varepsilon}{c\rho_0\rho_1(\varepsilon\rho_3 + \rho_2 a_0\sqrt{\|P\|})}\right\}, \tag{13}$$

where $\rho_0 > 1$, $\rho_1 = \|B\|$, $\rho_2 = \|T_1\Phi^{\frac{1}{2}} \otimes B^T P^{\frac{1}{2}}\|$, $\rho_3 = \|L\otimes K\|$ and $a_0 = 2(1+\eta)c\lambda_N^{\frac{3}{2}}\|PBK\|$ with $\eta > 0$;

3) select a positive integer k_0 satisfying

$$k_0 \geq \frac{\ln(\beta)}{\ln(\Delta_V)}, \tag{14}$$

where

$$\Delta_V = \frac{1}{1+2\eta} + \left(1 - \frac{1}{1+2\eta}\right)\mathbf{e}^{-\frac{\varepsilon(1+2\eta)T}{2(1+\eta)\|P\|}},$$

and

$$\frac{1}{1+2\eta} < \beta < 1.$$

Next, the following lemmas are provided to facilitate the derivation of our main results.

Lemma 1 *For any $k \in \mathbb{N}$, we have*

$$\|e(kT)\| \leq \frac{b(kT)}{\rho_0},$$

where $b(t) = b_0\rho_0\sqrt{Nn}\hbar g(t)$ with $b_0 = \max\left\{\frac{1}{2}, \frac{\chi_0}{\sqrt{n}\hbar g_0}\right\}$.

Proof: For the case of $k = 0$, we have

$$\begin{aligned}\|e(0)\| &= \sqrt{\|x_1(0)\|^2 + \ldots + \|x_N(0)\|^2} \\ &\leq \sqrt{N}\chi_0 \\ &\leq \frac{b(0)}{\rho_0}. \end{aligned} \tag{15}$$

For the case of $k \in \mathbb{N}^+$, we have

$$\begin{aligned}\|e(kT)\| &= \sqrt{\|e_1(kT)\|^2 + \ldots + \|e_N(kT)\|^2} \\ &\leq \frac{\hbar\sqrt{Nn}}{2} g(kT) \\ &\leq \frac{b(kT)}{\rho_0} \end{aligned} \tag{16}$$

which, together with (15), completes the proof.

Lemma 2 *Let a Lyapunov function be given by*

$$V(t) = z^T(t)(\Phi\otimes P)z(t). \tag{17}$$

For any $k_1 \in \mathbb{N}$ and $k_1k_0T \leq t < (k_1+1)k_0T$, the following statements are true.

a) If $V(k_1k_0T) < v_{0,k_1k_0T}^2$, then $V(t) < v_{0,k_1k_0T}^2$.

b) If $V(k_1k_0T) \geq v_{0,k_1k_0T}^2$, then $V(t) \leq V(k_1k_0T)$ and

$$V((k_1+1)k_0T) \leq \max\left\{v_{0,k_1k_0T}^2, \beta V(k_1k_0T)\right\}$$

where c, P, T and k_0 satisfy (11)-(14)*, respectively, and v_{0,k_1k_0T} is a function of $b(k_1k_0T)$ which will be defined later.*

Proof: From (10), we have

$$\begin{aligned}\dot{V}(t) &= 2z^T(t)(\Phi\otimes P)\dot{z}(t) \\ &= 2z^T(t)(\Phi\otimes PA)z(t) \\ &\quad - 2cz^T(t)(\Phi T_1^T \otimes PB)(I-\varrho)(L\otimes K)e(t) \\ &\quad - 2cz^T(t)(\Phi T_1^T \otimes PB)(I-\varrho)(T_1\Phi\otimes K)z(t) \\ &\leq 2z^T(t)(\Phi\otimes PA)z(t)\end{aligned}$$

$$
\begin{aligned}
&+\frac{\varepsilon}{2(1+\eta)}z^T(t)(\Phi\otimes I_n)z(t)\\
&+\frac{2(1+\eta)}{\varepsilon}\left(c\lambda_N^{\frac{2}{3}}\|PBK\|\|e(t)\|\right)^2\\
&-2c\lambda_2(1-\varrho_{\max})z^T(t)(\Phi\otimes PBB^TP)z(t)\\
\le&-\varepsilon z^T(t)(\Phi\otimes I_n)z(t)\\
&+\frac{\varepsilon}{2(1+\eta)}z^T(t)(\Phi\otimes I_n)z(t)\\
&+\frac{2(1+\eta)}{\varepsilon}\left(c\lambda_N^{\frac{2}{3}}\|PB\|\|K\|\|e(t)\|\right)^2\\
=&-\frac{\varepsilon\eta}{1+\eta}z^T(t)(\Phi\otimes I_n)z(t)\\
&-\frac{1}{2\varepsilon(1+\eta)}\Big(\varepsilon^2z^T(t)(\Phi\otimes I_n)z(t)\\
&-\big(2(1+\eta)c\lambda_N^{\frac{2}{3}}\|PBK\|\|e(t)\|\big)^2\Big). \qquad (18)
\end{aligned}
$$

where $\eta>0$.

For notation simplicity, we denote

$$
e_{kT}\triangleq\sup_{kT\le t<(k+1)T}\|e(t)\|,\qquad v_{kT}\triangleq\sup_{kT\le t<(k+1)T}\sqrt{V(t)}.
$$

When $kT\le t<(k+1)T$, it is inferred from (1) and (2) that

$$
e(t)=\left(I_N\otimes\mathbf{e}^{A(t-kT)}\right)e(kT)-\int_{kT}^{t}\left(I_N\otimes\mathbf{e}^{A(t-\tau)}\right)(I_N\otimes B)(I-\varrho)u(\tau)d\tau. \qquad (19)
$$

Noting that

$$
\left\|I_N\otimes\mathbf{e}^{A(t-kT)}\right\|=\|\mathbf{e}^{A(t-kT)}\|\le\mathbf{e}^{\|A\|T}\le\rho_0, \qquad (20)
$$

we have

$$
\|e(t)\|\le\rho_0\|e(kT)\|+\rho_0\rho_1\int_{kT}^{t}\|u(\tau)\|d\tau. \qquad (21)
$$

Since

$$
\int_{kT}^{t}\|u(\tau)\|d\tau\le cT\|L\otimes K\|e_{kT}+\int_{kT}^{t}c\|(L\otimes K)x(\tau)\|d\tau \qquad (22)
$$

and

$$
\begin{aligned}
&\|(L\otimes K)x(t)\|\\
&=\left\|(T_1\Phi T_1^T\otimes B^TP)x(t)\right\|\\
&=\left\|\left(T_1\Phi^{\frac12}\otimes B^TP^{\frac12}\right)\left(\Phi^{\frac12}T_1^T\otimes P^{\frac12}\right)x(t)\right\|\\
&\le\left\|T_1\Phi^{\frac12}\otimes B^TP^{\frac12}\right\|\sqrt{V(t)}, \qquad (23)
\end{aligned}
$$

we obtain

$$
e_{kT}\le\rho_0\|e(kT)\|+c\rho_0\rho_1\rho_3Te_{kT}+c\rho_0\rho_1\rho_2Tv_{kT}. \qquad (24)
$$

By denoting

$$
a_1\triangleq1-c\rho_0\rho_1\rho_3T>0,\qquad a_2\triangleq\frac{c\rho_0\rho_1\rho_2T}{a_1},
$$

and recalling Lemma 1, we have

$$
e_{kT}\le a_2v_{kT}+\frac{b(kT)}{a_1}. \qquad (25)
$$

Note that

$$
V(t)\le z^T(t)(\Phi\otimes I_n)z(t)\|P\|. \qquad (26)
$$

Substituting (25) and (26) into (18) yields

$$
\begin{aligned}
\dot V(t)\le&-\frac{\varepsilon\eta V(t)}{(1+\eta)\|P\|}-\frac{1}{2\varepsilon(1+\eta)}\Bigg(\frac{\varepsilon^2}{\|P\|}\left(V(t)-v_{kT}^2\right)\\
&+\left(\frac{\varepsilon^2}{\|P\|}-a_0^2a_2^2\right)v_{kT}^2-\frac{2a_0^2a_2b(kT)}{a_1}v_{kT}\\
&-\frac{a_0^2}{a_1^2}b^2(kT)\Bigg),\quad kT\le t<(k+1)T. \qquad (27)
\end{aligned}
$$

Defining

$$
f(v_{kT})\triangleq\left(\frac{\varepsilon^2}{\|P\|}-a_0^2a_2^2\right)v_{kT}^2-\frac{2a_0^2a_2b(kT)}{a_1}v_{kT}-\frac{a_0^2}{a_1^2}b^2(kT),
$$

it follows from (13) that

$$
\frac{\varepsilon^2}{\|P\|}-a_0^2a_2^2>0.
$$

Next, denote

$$
v_{0,kT}\triangleq b_Mb(kT),\quad b_M>0.
$$

Then, as shown in Fig. 3, we have that $f(v_{kT})\ge0$ if $v_{kT}\ge v_{0,kT}$.

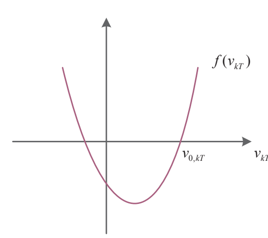

Fig. 3. Curve of $f(v_{kT})$.

We are now ready to prove statements a) and b) in Lemma 2 one by one.

Statement a) Let $V(k_1k_0T)<v_{0,k_1k_0T}^2$. To show that $V(t)<v_{0,k_1k_0T}^2$ holds for any $t\in[k_1k_0T,\ (k_1+1)k_0T)$, we use contradictions.

(1) For the case of $v^2_{k_1k_0T} = V(t')$ where $t' \in (k_1k_0T,\ (k_1k_0+1)T)$, we suppose $V(t') \geq v^2_{0,k_1k_0T}$. Then, $f(v_{k_1k_0T}) \geq 0$ and it follows from (27) that $\dot{V}(t') < 0$. Since $\dot{V}(t)$ is continuous on $t \in [k_1k_0T,\ (k_1+1)k_0T)$, there exists a $t'' \in (k_1k_0T,\ t')$ such that $V(t'') > V(t') = v^2_{k_1k_0T}$, which contradicts the definition of $v_{k_1k_0T}$, i.e., $v_{k_1k_0T} \triangleq \sup_{k_1k_0T \leq t < (k_1+1)k_0T} \sqrt{V(t)}$. Therefore, $V(t') < v^2_{0,k_1k_0T}$, which implies that $V(t) < v^2_{0,k_1k_0T}$ holds for any $t \in [k_1k_0T,\ (k_1k_0+1)T)$.

(2) Consider the case where $v^2_{k_1k_0T} = V((k_1k_0+1)T))$ and $V(t) < V((k_1k_0+1)T))$ hold for any $t \in [k_1k_0T,\ (k_1k_0+1)T)$. Suppose that $V((k_1k_0+1)T)) \geq v^2_{0,k_1k_0T}$ is true. Then, we have $f(v_{k_1k_0T}) \geq 0$. Since $V(t) < V((k_1k_0+1)T))$ and $V(t)$ is continuous on t, we know that there exists a $t' \in (k_1k_0T,\ (k_1k_0+1)T))$ such that $\dot{V}(t'') \geq 0$ for any $t'' \in [t',\ (k_1k_0+1)T)]$, which contradicts (27). Therefore $V((k_1k_0+1)T)) < v^2_{0,k_1k_0T}$, which implies that $V(t) < v^2_{0,k_1k_0T}$ holds for any $t \in [k_1k_0T,\ (k_1k_0+1)T]$.

(3) Note that $b(t) \equiv b(k_1k_0T)$ for any $t \in [k_1k_0T,\ (k_1+1)k_0T)$. Similar to 1) and 2), the results on the interval $t \in [k_1k_0T,\ (k_1k_0+1)T)$ (where $V(t) < v^2_{0,k_1k_0T}$) can be generalized to the interval $t \in [k_1k_0T,\ (k_1+1)k_0T)$. Now, the statement a) in Lemma 2 is proved.

Statement b) Let $V(k_1k_0T) \geq v^2_{0,k_1k_0T}$. First, we aim to show that $V(t) \leq V(k_1k_0T)$ is satisfied for any $t \in [k_1k_0T,\ (k_1+1)k_0T)$.

Similar to the proof of *Statement a*, by constructing a contradiction, we can easily derive $V(t) \leq V(k_1k_0T)$ and, more specifically, $V(t) < V(k_1k_0T)$ for any $t \in (k_1k_0T,\ (k_1+1)k_0T)$. This further implies that, if there exists a $t' \in [k_1k_0T,\ (k_1+1)k_0T)$ such that $V(t') = v^2_{0,k_1k_0T}$, then $V(t) < v^2_{0,k_1k_0T}$ holds for any $t \in (t',\ (k_1+1)k_0T)$. Thus, one has $V\left((k_1+1)k_0T\right) \leq v^2_{0,k_1k_0T}$ due to the continuity of $V(t)$.

If $V(t) > v^2_{0,k_1k_0T}$ holds for all $t \in [k_1k_0T,\ (k_1+1)k_0T)$, then it follows from (27) that

$$\begin{aligned}\dot{V}(t) &\leq -\frac{\varepsilon\eta V(t)}{(1+\eta)\|P\|} - \frac{\varepsilon^2\left(V(t)-v^2_{kT}\right)}{2\varepsilon(1+\eta)\|P\|}\\ &= \frac{\varepsilon}{2(1+\eta)\|P\|}v^2_{kT} - \frac{\varepsilon(1+2\eta)}{2(1+\eta)\|P\|}V(t) \qquad (28)\end{aligned}$$

is true for any $kT \leq t < (k+1)T$, where $k \in \mathbb{N}$ and $k_1k_0 \leq k \leq (k_1+1)k_0 - 1$.

Since $V(t)$ is continuous, it is seen from (28) that $V(t)$ keeps decreasing during the interval $[k_1k_0T,\ (k_1+1)k_0T)$. Consequently, we have

$$V\left((k_1+1)k_0T\right) \leq (\Delta_V)^{k_0} V(k_1k_0T) \qquad (29)$$

which, together with (14), finally leads to

$$V\left((k_1+1)k_0T\right) \leq \beta V(k_1k_0T). \qquad (30)$$

Up to now, Statement b) has been proved to be true and the proof is complete.

Lemma 3 *For any* $k_1 \in \mathbb{N}^+$, *the Lyapunov function* (17) *satisfies*

$$V(k_1k_0T) \leq \max\left\{\mathcal{H}(k_1,\gamma,\beta)v^2_{0,0},\ \beta^{k_1}V(0)\right\}, \qquad (31)$$

where

$$\mathcal{H}(k_1,\gamma,\beta) \triangleq \max\Big\{(\gamma^2)^{k_1-1}, (\gamma^2)^{k_1-2}\beta, \ldots, \gamma^2\beta^{k_1-2}, \beta^{k_1-1}\Big\}.$$

Proof: By virtue of Lemma 2, we have

$$\begin{aligned}V(k_0T) &\leq \max\left\{v^2_{0,0}, \beta V(0)\right\}\\ &= \max\left\{\mathcal{H}(1,\gamma,\beta)v^2_{0,0}, \beta V(0)\right\}, \qquad (32)\end{aligned}$$

and further obtain

$$\begin{aligned}V(2k_0T) &\leq \max\left\{v^2_{0,k_0T}, \beta V(k_0T)\right\}\\ &\leq \max\left\{v^2_{0,k_0T}, \beta v^2_{0,k_0T}, \beta^2 V(0)\right\}\\ &= \max\left\{\mathcal{H}(2,\gamma,\beta)v^2_{0,0}, \beta^2 V(0)\right\}. \qquad (33)\end{aligned}$$

Along this line, (31) can be derived through iterative calculations and the proof is then complete.

Theorem 1 *Under Assumptions 1 and 2, the consensus of agents* (1) *can be reached by the controller* (6) *and the sampled-data-based dynamic quantization as well as data recovery algorithms* (2)-(5) *if the controller gains and relevant parameters are designed according to* (11)-(14).

Proof: It follows from Lemma 3 that

$$\begin{aligned}\lim_{k_1\to\infty} V(k_1k_0T) \leq \max\Big\{&\lim_{k_1\to\infty}\mathcal{H}(k_1,\gamma,\beta)v^2_{0,0},\\ &\lim_{k_1\to\infty}\beta^{k_1}V(0)\Big\}. \qquad (34)\end{aligned}$$

Moreover, it is obvious that $\lim_{k_1\to\infty}\mathcal{H}(k_1,\gamma,\beta) = 0$ holds for the case of $\beta = \gamma^2$. For the case of $\beta \neq \gamma^2$, we have

$$\begin{aligned}\lim_{k_1\to\infty}\mathcal{H}(k_1,\gamma,\beta) &\leq \lim_{k_1\to\infty}\sum_{l=0}^{k_1-1}(\gamma^2)^{k_1-l-1}\beta^l,\\ &= \lim_{k_1\to\infty}\frac{(\gamma^2)^{k_1}-\beta^{k_1}}{\gamma^2-\beta}\\ &= 0, \qquad (35)\end{aligned}$$

which, together with $\lim_{k_1\to\infty}\beta^{k_1}V(0) = 0$, yields $\lim_{k_1\to\infty}V(k_1k_0T) = 0$. Finally we get $\lim_{t\to\infty}z(t) = \mathbf{0}_{n(N-1)}$.

Noting $z(t) = \mathbf{0}_{n(N-1)}$, one has

$$x(t) = (T\otimes I_n)y(t) = \sqrt{\frac{1}{N}}\left[y_1^T(t), y_1^T(t), \cdots, y_1^T(t)\right]^T,$$

which means that the consensus is reached. The proof is complete.

We are now ready to analyze the size of the transmitted data $\|s_i(k+1)\|_\infty$ ($k \in \mathbb{N}$), which reflects the requirement on the network bandwidth. Based on Theorem 1, we obtain the following result.

Theorem 2 *Let the dynamic-quantization-based DPP scheme described in Theorem 1 be given. If $\beta \leq \gamma^2$, then the size of the transmitted data among agents is bounded, that is, $\|s_i(k+1)\|_\infty$ is bounded for any $i \in \mathfrak{N}$ and $k \in \mathbb{N}$.*

Proof: For $\forall i \in \mathfrak{N}$, at time instant $(k+1)T$ where $k_1 k_0 T \leq kT < (k+1)T \leq (k_1+1)k_0 T$ with $k_1 \in \mathbb{N}$ and $k \in \mathbb{N}^+$, we obtain

$$\begin{aligned}
&\left\| \frac{x_i\left((k+1)T\right) - \mathbf{e}^{AT}\xi_i(kT)}{g\left((k+1)T\right)} \right\| \\
&\leq \rho_0 \left\| \frac{x_i(kT) - \left(\mathbf{e}^{AT}\xi_i\left((k-1)T\right)\right) + g(kT)s_i(k)\right)}{g\left((k+1)T\right)} \right\| \\
&\quad + \left\| \frac{\int_{kT}^{(k+1)T} \left(I_N \otimes \mathbf{e}^{A((k+1)T-\tau)}B\right)(I-\varrho)u(\tau)d\tau}{g\left((k+1)T\right)} \right\| \\
&\leq \frac{g(kT)\rho_0}{g\left((k+1)T\right)} \left\| \frac{x_i(kT) - \mathbf{e}^{AT}\xi_i\left((k-1)T\right)}{g(kT)} - s_i(k) \right\| \\
&\quad + \frac{\rho_0\rho_1}{g\left((k+1)T\right)} \int_{kT}^{(k+1)T} \|u(\tau)\| \, d\tau \\
&\leq \frac{\rho_0 \hbar \sqrt{n}}{\gamma} + \frac{\rho_0\rho_1}{g\left((k+1)T\right)} \int_{kT}^{(k+1)T} \|u(\tau)\| \, d\tau. \qquad (36)
\end{aligned}$$

From (22), (23) and (25), we have

$$\begin{aligned}
&\int_{kT}^{(k+1)T} \|u(\tau)\| \, d\tau \\
&\leq (ca_2\rho_3 T + c\rho_2 T)v_{kT} + \frac{c\rho_3 T}{a_1} b(kT). \qquad (37)
\end{aligned}$$

Recalling Lemmas 2 and 3, for the case of $k_1 \in \mathbb{N}^+$, we obtain

$$\begin{aligned}
v_{kT} &\leq \max\left\{ \sqrt{V(k_1k_0T)}, v_{0,k_1k_0T} \right\} \\
&\leq \max\left\{ \sqrt{\mathcal{H}(k_1,\gamma,\beta)v_{0,0}^2, \beta^{k_1}V(0)}\,, b_M b(k_1k_0T) \right\}. \qquad (38)
\end{aligned}$$

In view of $\beta \leq \gamma^2$, we have

$$\begin{aligned}
v_{kT} &\leq \max\left\{ \sqrt{(\gamma^2)^{k_1-1}v_{0,0}^2, \beta^{k_1}V(0)}\,, b_M b(k_1k_0T) \right\} \\
&= \max\left\{ \gamma^{k_1-1}b_M b(0), \sqrt{\beta^{k_1}V(0)}\,, b_M b(k_1k_0T) \right\} \\
&= \max\left\{ \gamma^{k_1-1}b_M b_0 \rho_0 \sqrt{Nn}\hbar g_0, \sqrt{\beta^{k_1}V(0)} \right\}, \qquad (39)
\end{aligned}$$

which, together with (36) and (37), yields

$$\begin{aligned}
&\left\| \frac{x_i\left((k+1)T\right) - \mathbf{e}^{AT}\xi_i(kT)}{g\left((k+1)T\right)} \right\| \\
&\leq c\rho_0\rho_1 T(a_2\rho_3 + \rho_2) \max\left\{ \frac{b_M b_0 \rho_0 \sqrt{Nn}\hbar}{\gamma^2}, \frac{\sqrt{V(0)}}{g_0\gamma} \right\} \\
&\quad + \frac{c b_0 \rho_0^2 \rho_1 \rho_3 T \sqrt{Nn}\hbar + \rho_0\sqrt{n}\hbar}{\gamma}. \qquad (40)
\end{aligned}$$

Then, it follows from Assumption 2 and $\|s_i(k+1)\|_\infty \leq \|s_i(k+1)\|$ that $\|s_i(k+1)\|_\infty$ is bounded for any $k \in \mathbb{N}^+$.

Based on the above analysis, it can be directly proved that $\|s_i(k+1)\|_\infty$ is bounded for the case of $k = 0$. The proof is complete now.

Remark 1 *It follows from Theorem 1 that $\lim_{t\to\infty} e(t) = \mathbf{0}_{nN}$ and $\lim_{t\to\infty} z(t) = \mathbf{0}_{n(N-1)}$, which together with*

$$\begin{aligned}
\dot{y}(t) = &\left(I_N \otimes A - c(T^T \otimes B)(I-\varrho)(LT \otimes K)\right)y(t) \\
&- c(T^T \otimes B)(I-\varrho)(L \otimes K)e(t)
\end{aligned}$$

leads to $\dot{y}_1(t) = Ay_1(t)$. Then we have

$$\begin{aligned}
y_1(t) &= \mathbf{e}^{At}y_1(0) \\
&= \mathbf{e}^{At}(T_0^T \otimes I_n)x(0) \\
&= \mathbf{e}^{At}\sqrt{\frac{1}{N}}\sum_{i=1}^N x_i(0).
\end{aligned}$$

Finally we arrive at

$$\lim_{t\to\infty} x_i(t) = \lim_{t\to\infty} \sqrt{\frac{1}{N}}y_1(t) = \frac{1}{N}\lim_{t\to\infty} \mathbf{e}^{At}\sum_{i=1}^N x_i(0),$$

which implies that the consensus point is determined by the matrix A. If A is Hurwitz, that is, all the eigenvalues of A are on the left half s-plane, then $\lim_{t\to\infty} x_i(t) = \mathbf{0}_n$. If $A = \mathbf{0}_{n\times n}$, e.g. the single-integrator MASs, then $\lim_{t\to\infty} x_i(t) = \frac{1}{N}\sum_{i=1}^N x_i(0)$, that is, the average consensus can be achieved.

Remark 2 *In the context of consensus, it is known from Theorem 1 that the selection of β is independent of γ which, together with (14), implies that k_0 can be arbitrary positive integer. However, some combinations of β and γ might not be able to guarantee the boundedness of the size of the transmitted data. In view of this, we provide a sufficient condition in Theorem 2, i.e., $\beta \leq \gamma^2$, to ensure the boundedness as can be illustrated in the simulation part.*

Remark 3 *Suppose that the global information of the graph $\mathcal{G}$ is unknown a priori, which means that the exact value of λ_i ($i = 2, \ldots, N$) is unknown. In this case, based on*

$$2\left(1 - cos(\pi/N)\right)d(\mathcal{G}) \leq \lambda_2 \leq \lambda_N \leq 2d_{\max} \qquad (41)$$

where $d(\mathcal{G})$ represents the edge connectivity of the graph, we can further derive a totally distributed controller with parameters c and T modified as

$$c \geq \frac{1}{4\left(1 - cos(\pi/N)\right)(1-\varrho_{\max})d(\mathcal{G})}, \qquad (42)$$

$$\begin{aligned}
T \leq \min\Bigg\{ &\frac{ln(\rho_0)}{\|A\|}\,,\ \frac{1}{c\rho_0\rho_1\rho_3}\,, \\
&\frac{\varepsilon}{c\rho_0\rho_1(\varepsilon\rho_3 + \rho_2 a_0\sqrt{\|P\|})} \Bigg\} \qquad (43)
\end{aligned}$$

where $\rho_0 > 1$, $\rho_1 = \|B\|$, $\rho_2 = 2d_{\max}\|B^T P^{\frac{1}{2}}\|$, $\rho_3 = 2d_{\max}\|K\|$ and $a_0 = 4c(1+\eta)d_{\max}\|PBK\|$. Note that $d(\mathcal{G})$ stands for the minimal number of edges whose absence would result in the loss of the graph connectivity [9].

Until now, we have established a dynamic-quantization-based DPP scheme. Through the introduction of DQS, the size of the transmitted data can be reduced highly. Considering that the quantization and recovery algorithms may be obtained by potential eavesdroppers, in what follows, we would like to provide two complementary schemes to further enhance the security of shared data.

Complementary scheme I. In case that some agents may be eavesdroppers, we provide a possible remedy over undirected graphs based on the proposed scheme to ensure that the state of all agents is kept secret from their neighbors.

Note that the contribution of agent j in the controller (6), i.e., $c_{ij}(t) \triangleq a_{ij}(\xi_i(t) - x_{ji}(t))$, can be calculated according to

$$\begin{cases} \dot{c}_{ij}(t) = Ac_{ij}(t), \ kT < t < (k+1)T, \ k \in \mathbb{N} \\ c_{ij}((k+1)T) = \mathbf{e}^{AT} c_{ij}(kT) + g((k+1)T) \\ \qquad\qquad \times a_{ij}(s_i(k+1) - s_j(k+1)) \\ c_{ij}(0) = \mathbf{0}_n, \quad j \in \mathcal{N}_i, \end{cases} \tag{44}$$

which implies that the value of $a_{ij}(\xi_i(t) - x_{ji}(t))$ can be derived from the value of $a_{ij}(s_i(k+1) - s_j(k+1))$. In view of this, the Paillier encryption algorithm (PEA) can be exploited as a complementary scheme to further protect the data $x_{ji}(t)$. The PEA has a notable feature that the product of two ciphertexts is decrypted into the sum of their corresponding plaintexts [34].

Inspired by [7], we rewrite a_{ij} as $a_{ij} = a_i \times a_j$ with a_i and a_j generated by agents i and j, respectively. Recalling the parameter design rules (12), (14) and (41)-(43), we have that c, K, T and k_0 depend mainly on λ_N and λ_2, i.e., the maximum and minimum eigenvalues of L, and have no direct relationship with the exact value of each element of L. This implies that the dynamic-quantization-based DPP scheme is robust against certain changes in communication topologies. Therefore, given two appropriate values λ_N and λ_2, we can find parameters α_1 and α_2 such that $\lambda_2 I \leq \alpha_1 L < \alpha_2 L \leq \lambda_N I$ holds. Consequently, $a_i(k+1)$ and $a_j(k+1)$ can be chosen from the range $\left[\sqrt{-\alpha_1 l_{ij}}, \ \sqrt{-\alpha_2 l_{ij}}\,\right]$. Based on this, we design the complementary scheme, as shown in Algorithm 1.

Complementary scheme II. Consider that every agent in the MAS is trustworthy whereas there may still exist eavesdroppers around the MAS. In this case, Algorithm 1 can be simplified to Algorithm 2.

We only need to transmit the first packet at time $k = 0$ using PEA to ensure the initial state is secure. Then, let the dynamic-quantization-based DPP scheme take over, which is sufficient to protect the state information of the MAS since the initial inferred error of the eavesdropper will lead to an exponentially growing estimation error.

Algorithm 1 Complementary scheme I

(1) **Preparation**: At time instant $t = 0$, agent i generates a public key $\mathbb{K}_i^P$ and a private key $\mathbb{K}_i^S$, and sends the public key $\mathbb{K}_i^P$ to agent j, $j \in \mathcal{N}_i$;

(2) **At time instant** $k+1$, $k \in \mathbb{N}$:

(2.1) Agents i and j choose $a_i(k+1)$ and $a_j(k+1)$, respectively;

(2.2) According to $\mathbb{K}_i^P$, agents i and j encrypt $s_i(k+1)$ and $-s_j(k+1)$ into $\mathbb{E}(s_i(k+1))$ and $\mathbb{E}(-s_j(k+1))$, respectively;

(2.3) Agent i sends $\mathbb{E}(s_i(k+1))$ to agent j;

(2.4) Agent j receives $\mathbb{E}(s_i(k+1))$ and calculates $\mathbb{E}(s_i(k+1)) \times \mathbb{E}(-s_j(k+1))$, which equals to $\mathbb{E}(s_i(k+1) - s_j(k+1))$;

(2.5) Agent j gets $\left(\mathbb{E}\left(s_i(k+1) - s_j(k+1)\right)\right)^{a_j(k+1)}$, which equals to $\mathbb{E}\big(a_j(k+1)(s_i(k+1) - s_j(k+1))\big)$;

(2.6) Agent j sends $\mathbb{E}\big(a_j(k+1)(s_i(k+1) - s_j(k+1))\big)$ to agent i;

(2.7) According to $\mathbb{K}_i^S$, agent i decrypts the received information $\mathbb{E}\big(a_j(k+1)(s_i(k+1) - s_j(k+1))\big)$;

(2.8) Agent i finally obtains $a_i(k+1) \times a_j(k+1)(s_i(k+1) - s_j(k+1))$.

Algorithm 2 Complementary scheme II

(1) **Preparation**: At time instant $t = 0$, agent i generates a public key $\mathbb{K}_i^P$ and a private key $\mathbb{K}_i^S$, and sends the public key $\mathbb{K}_i^P$ to agent j, $j \in \mathcal{N}_i$;

(2) **At time instant** $k = 0$:

(2.1) Agent j encrypts $s_j(1)$ into $\mathbb{E}(s_j(1))$ according to $\mathbb{K}_i^P$, and sends $\mathbb{E}(s_j(1))$ to agent i;

(2.2) According to $\mathbb{K}_i^S$, agent i decrypts the received information $\mathbb{E}(s_j(1))$;

(2.3) Agent i estimates $x_{ji}(T)$ according to (5);

(3) **At time instant** $k > 0$:

(3.1) Agent j sends $s_j(k+1)$ to agent i;

(3.2) Agent i estimates $x_{ji}((k+1)T)$ according to (5).

Remark 4 *So far, we have solved the FTCC problem for a general class of linear continuous-time multi-agent systems (MASs) under the data privacy preserving (DPP) scheme. In comparison to the existing literature on MASs with the DPP constraints, the main results developed in this paper exhibit the following three distinctive merits: 1) the proposed dynamic-quantization-based DPP scheme is new in that the system dynamics is taken into account when the MASs suffer from possible faults; 2) the developed sampled-data strategy is new because the sampling period is purposely designed based on a Lyapunov-function-analysis method; and 3) the derived sufficient condition is new in the sense of ensuring the boundedness of the size of the transmitted data. One of our fu-*

ture research topics is to study the FTCC problem with network-induced effects [46, 47].

4 Simulations

Consider a network consisting of four agents [30] described by (1) with

$$A = \begin{bmatrix} 0.2 & 1 & 0 \\ -0.1 & -0.5 & 0.1 \\ 0.1 & -0.1 & -0.2 \end{bmatrix}, \; B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix},$$

where A has an unstable eigenvalue 0.0949. The undirected topology among agents is shown in Fig. 4 with

$$L = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 2 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

and $\lambda_2 = 1$, $\lambda_N = 4$.

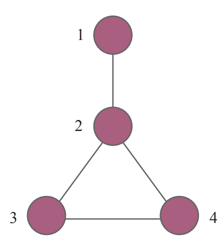

Fig. 4. Topology of the multi-agent system.

Case 1: The fault-free case with $\varrho_{\max} = 0$.

Choosing $\rho_0 = 1.02$, $\varepsilon = 0.1$, $\beta = 0.81$, $\gamma = 0.9$, $\eta = 0.5$, $c = 0.5001$, $\hbar = 1$, and $g_0 = 0.5$, we can design that $K = \begin{bmatrix} 0.4129 & 0.4599 & 0.1636 \end{bmatrix}$. The parameters can be set as $T = 0.01s$ and $k_0 = 739$ according to (13)-(14). The initial states are given as $x_1(0) = \begin{bmatrix} 10, & 0, & 0 \end{bmatrix}^T$, $x_2(0) = \begin{bmatrix} 20, & 2, & 1 \end{bmatrix}^T$, $x_3(0) = \begin{bmatrix} 3, & 0, & 3 \end{bmatrix}^T$ and $x_4(0) = \begin{bmatrix} 4, & 1, & 2 \end{bmatrix}^T$.

Denoting $x_i^e(t) \triangleq \|x_i(t) - \frac{1}{N}\sum_{j=1}^{N} x_j(t)\|$ and $s_{\max}(k) \triangleq \max_i \|s_i(k)\|_\infty$, we obtain the simulation results as shown in Figs. 5-7. Fig. 5 indicates that the consensus of the MAS is indeed asymptotically reached. The chattering in Fig. 6 is caused by the non-zero function $g(t)$ and will be eliminated as $g(t)$ approaches zero. Fig. 7 shows that the size of the transmitted data is bounded under the condition $\beta = \gamma^2$ as proved in Theorem 2. To show the impact of different β on the size of the transmitted data, we conduct the following simulation with $\beta = 0.9998$. The other parameters are set the same as above. By choosing $T = 0.01s$ and $k_0 = 1$, we obtain the simulation results as shown in Figs. 8-10. The consensus can be asymptotically reached. However, as pointed out in Remark 2, the size of the transmitted data will tend to infinity eventually. Therefore, parameters β and γ should be selected appropriately and Theorem 2 provides such a feasible solution.

As an important evaluation index of the sample-data-based method, the sampling period T reflects the sampling rate of the sensor and the frequency of communication. The choice of T is a critical issue and, in general, we prefer to choose a larger sampling period while preserving consensus. Compared with the method in [30], the method proposed in this paper leads to a larger sampling period as shown in Table 1, where sampling periods are calculated with varying ε. It is worth noting that both results are derived based on the assumption that the graph information is known in advance.

Table 1
Comparative analysis of the sampling period.

ε	$\max T$ (s) in [30]	$\max T$ (s) in this paper
0.001	0.0011	0.0034
0.01	0.0041	0.0131
0.1	0.0040	0.0167
1	0.00084	0.0088
10	0.00007	0.0024

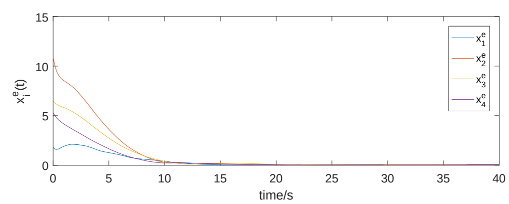

Fig. 5. Trajectories of $x_i^e(t)$ with $\beta = 0.81$ in Case 1.

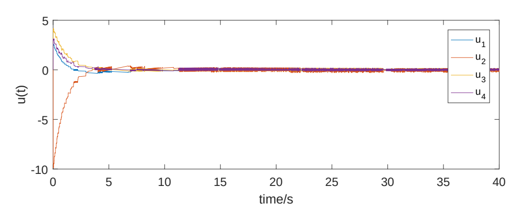

Fig. 6. Trajectories of $u_i(t)$ with $\beta = 0.81$ in Case 1.

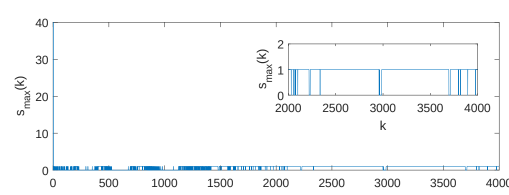

Fig. 7. Trajectory of $s_{\max}(k)$ with $\beta = 0.81$ in Case 1.

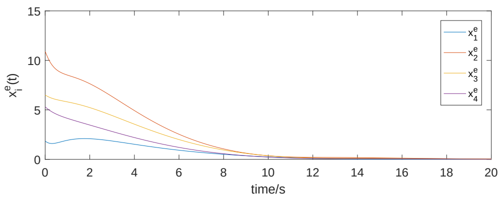

Fig. 8. Trajectories of $x_i^e(t)$ with $\beta = 0.9998$ in Case 1.

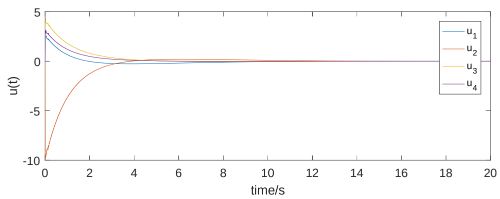

Fig. 9. Trajectories of $u_i(t)$ with $\beta = 0.9998$ in Case 1.

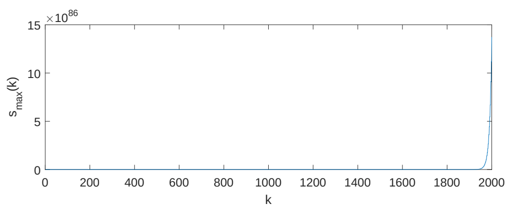

Fig. 10. Trajectory of $s_{\max}(k)$ with $\beta = 0.9998$ in Case 1.

Case 2: The faulty case with $\varrho_{\max} = 0.3$.

Choosing $\rho_0 = 1.02$, $\varepsilon = 0.1$, $\beta = 0.902$, $\gamma = 0.95$, $\eta = 0.1$, $c = 0.7144$, $\hbar = 1$, and $g_0 = 0.1$, we come up with $K = \begin{bmatrix} 0.4129 \; 0.4599 \; 0.1636 \end{bmatrix}$ according to (12). The parameters can be designed as $T = 0.01s$ and $k_0 = 1326$ according to (13)-(14). The initial states are $x_1(0) = \begin{bmatrix} 2, & 0, & 0 \end{bmatrix}^T$, $x_2(0) = \begin{bmatrix} 10, & 2, & 1 \end{bmatrix}^T$, $x_3(0) = \begin{bmatrix} 3, & 0, & 3 \end{bmatrix}^T$ and $x_4(0) = \begin{bmatrix} 4, & 1, & 2 \end{bmatrix}^T$. Suppose that $\varrho = \text{diag}\{0.3, 0.1, 0.2, 0.15\}$. As shown in Figs. 11-13, the proposed fault-tolerant consensus controller can tolerate the LoE faults of actuators satisfying $\varrho \le \varrho_{\max}$ and the size of the transmitted data is indeed bounded.

Recall the design rules (11)-(14). For any given $0 \le \varrho_{\max} < 1$, we know that there always exist a matrix K and parameters c, T and k_0 such that the consensus is reached. However, with the increase of $\varrho_{\max}$, the value of $\min T$ will decrease (as shown in Table 2), which means that more effort is needed to tolerate more serious faults. Note that $\min T$ provided in Table 2 is calculated by choosing the same parameters as in the case of $\varrho_{\max} = 0.3$.

5 Conclusion

In this paper, we have proposed a novel DPP scheme (including a dynamic quantization algorithm and a data recovery algorithm) for a class of continuous-time MASs corrupted by faults. A kind of sampled-data-based controller is employed to deal with such continuous-time MASs and one of our main contributions is the development of a novel Lyapunov-function-based analysis method. In addition, we have derived a sufficient condition to ensure that the size of the transmitted data is bounded, which helps relieve the computational burden and saving communication bandwidth. As a future research topic, the integration of DPP technologies and systems dynamics deserves further investigation especially in the presence of disturbances, dishonest nodes [13] and multi-rate sampling [38, 39].

Table 2
Values of max T towards different $\varrho_{\max}$.

$\max T$ (s)	$\varrho_{\max}$	c
0.0145	0.2	0.6251
0.0112	0.3	0.7144
0.0082	0.4	0.8334
0.0057	0.5	1.0001
0.0037	0.6	1.2501
0.0021	0.7	1.6668
0.0009	0.8	2.5001

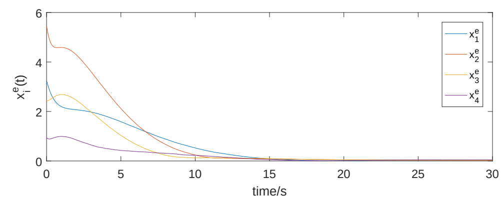

Fig. 11. Trajectories of $x_i^e(t)$ in Case 2.

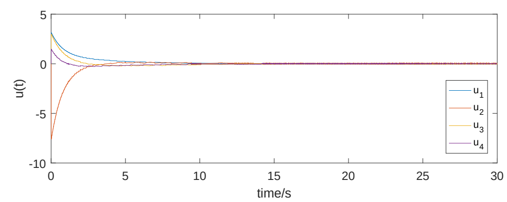

Fig. 12. Trajectories of $u_i(t)$ in Case 2.

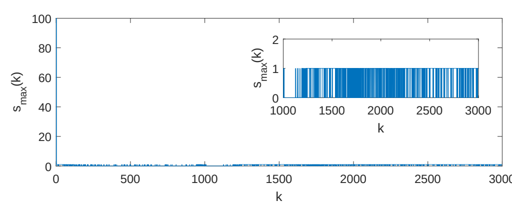

Fig. 13. Trajectory of $s_{\max}(k)$ in Case 2.

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