Partial-Nodes-Based State Estimation for Complex Networks with Constrained Bit Rate

Jun-Yi Li, Zidong Wang, Renquan Lu and Yong Xu

***Abstract*—In this paper, the partial-nodes-based (PNB) state estimation issue is investigated for a class of discrete-time complex networks with constrained bit rate and bounded noises. Measurements from only a fraction of nodes in a complex network are acquired and used for state estimation. The communication between sensor nodes and estimators is accomplished over a wireless digital communication network with limited bandwidth. A bit rate constraint model is introduced to reflect the bandwidth allocation rules of partially accessible nodes. A sufficient condition is proposed under which the PNB state estimation error system is guaranteed to be ultimately bounded, and then a bit rate condition assuring a specific estimation performance is presented. The estimator gains are derived by solving two optimization problems in order to ensure two estimation performance metrics (i.e. the smallest ultimate bound and the fastest decay rate). Furthermore, the co-design issue of the bit rate allocation protocol and the estimator gains is addressed by means of particle swarm optimization and linear matrix inequalities. Finally, three numerical simulations are provided to verify the validity of the proposed PNB state estimation approach.**

***Index Terms*—Partial-nodes-based state estimation, constrained bit rate, coding-decoding, co-design problem.**

I. Introduction

Complex networks are large-scale systems consisting of a large number of strongly coupled dynamic units that can be employed to characterize many real-world examples, including but not limited to sensor networks, social networks, power grids, biological networks, and so on [2], [37]. Dynamic units in a complex network often interact with each other through connections, leading to a highly coupled and dynamical network environment. As such, much research enthusiasm has recently been attracted towards dynamical behavior issues of complex networks, e.g. stability, synchronization and state estimation problems [4], [11], [33], [35], [38], [43], [44], [46], [48].

In the dynamic analysis of complex networks, state information plays a vital role due to the fact that it helps to understand the underlying network structure. Unfortunately, the state information is not always accessible due to the vast size of complex networks, strong coupling between nodes, and lack of accurate models. One approach to solve this problem is to use the accessible measurements to obtain an estimate of the network state. Accordingly, the state estimation problem for complex networks has received a great deal of attention in literature (see, [10], [18], [20]). For example, state estimation problems for diverse complex networks have been investigated under different communication protocols [10], [40] and switching topology [16], [20].

To the best of our knowledge, the vast majority of existing studies on state estimation for complex networks have based themselves on an underlying assumption that, all nodes' measurements are accessible, which might be unreasonable in practical applications. For example, it is incredibly expensive to measure all the nodes in a complex network since the number of nodes is often tremendous. Besides, due to the physical limits of the communication network and the operating environment, it is impractical to measure all nodes and then transmit the collected measurements. A malicious Dos attack may also cause some of the measurements to be unobtainable by blocking the transmission channel. Accounting for these problems, a partial-node-based (PNB) state estimation method has been developed in [21] for the first time, where the state estimates of all nodes have been achieved with measurement information only from a portion of the nodes. Although relevant studies have been conducted later in [8], [9], [20], [22], the corresponding literature on PNB state estimation has been scattered, and this constitutes the main motivation of our current study.

For a long time, most state estimation studies for complex networks have paid their attention to analog communication, where measurements are transmitted via the form of analog signals which take continuous values with infinite precision. As opposed to the traditional analog communication scheme, the digital communication scheme is capable of adapting to advanced control equipments and high-quality communication services, and has attracted ever-increasing research interest in the now-popular networked systems [7], [12], [14], [26], [27], [36], [41], [47], [51]. In wireless digital communication, sensors are often subject to sampling, quantization, and coding before transmission. Although sampling and quantization of sensor signals have already received considerable attention in networked control systems (see e.g. [6], [15], [23], [28], [32] and the references therein), the coding procedure has not received much attention, especially in the state estimation field [13], [29]. It should be noted that coding is at the heart

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of analog-to-digital conversion compared to sampling and quantization, and therefore deserves more research attention.

In the digital communication network, the network's bandwidth is usually measured in terms of bit rate which defines the number of bits conveyed through a digital communication network per second. For complex networks with massive nodes, although the total bit rate of the network is large, each node is usually allocated with only a small portion of the total bite rate, and this leads to the inevitable bit rate constraints. The last two decades have seen increasingly rapid advances in the field of control with limited bit rate. Early research in this area has focused on the simplest network topology which consists of a controller and a dynamic system with a feedback loop. Several attempts have been made to reveal the relationship between the minimum bit rate (that ensures various types of stability) and the unstable eigenvalue of the open-loop system [30], [31], [39], [42].

Recently, researchers have shown a persistent research interest in complex systems with network topology and constrained bit rate [5], [17], [19], [45]. The problem of consensus with limited bit rate has been addressed by considering the simultaneous effects of bit rate, network topology, and agent dynamics [45]. In the case of limited bit-rate communication, a quantized observer-based encoding-decoding scheme has been designed to facilitate the distributed coordination of discrete-time multi-agent systems with partially measurable states [17]. In comparison with existing time-triggered consensus strategies, event-triggered strategies with lower bit rate have been proposed in [5], [19] to ensure the asymptotic consensus of multi-agent systems by extracting additional information from the time instances of packet reception. Unfortunately, corresponding literature on state estimation problems for complex networks with constrained bit rate has been scattered, and this constitutes another main motivation of our current study.

Based on the above discussions, it can be concluded that the problem of PNB state estimation for a class of complex networks with constrained bit rate remains open due to the following three main challenges: *1) how to construct a mathematical model to characterize the bandwidth limitation of the complex network with partially accessible nodes? 2) how to quantify the effect of the bit rate on the PNB state estimation performance? and 3) how to develop optimized estimator gains to meet different performance indices and the need for the co-design of the bit rate allocation protocol and the estimator?*

With the encouragement of the discussions conducted so far, we strive to investigate the PNB state estimation for complex networks with constrained bit rate in this paper. The significant contributions of this paper are categorized into three aspects.

1) The PNB state estimation problem under the framework of digital communication networks is tackled by the first attempt, which is more applicable in engineering fields than the conventional PNB state estimation methods under the framework of analog communication networks [9], [20], [21]. Then, a bit rate constraint model is developed, for the first time, to characterize the bandwidth allocation of partially accessible nodes in the complex networks.
2) A sufficient condition is proposed to ensure the ultimate boundedness of the PNB state estimation error system, and a bit rate condition that guarantees the specific state estimation performance is also proposed.
3) The PNB state estimators are designed according to different estimation performance requirements by solving two optimization problems (OPs) and a mixed-integer nonlinear programming (MINP) problem.

Notation: The notation used in this paper is fairly standard. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ represent n dimensional Euclidean space and the set of $n\times m$ real matrices, respectively. $\mathbb{N}$, and $\mathbb{N}^+$ stand for the sets of non-negative integers, and positive integers, respectively. $\text{diag}_N\{A_i\}$ and $\text{col}_N(e_i)$ denote diagonal block matrix $\text{diag}_N\{A_1, A_2, \cdots, A_N\}$ and column vector $\left[e_1^T, e_2^T, \cdots, e_N^T\right]^T$, separately. For any $z\in\mathbb{R}^n$, z^T and $\parallel z\parallel_2$ are its transpose and its Euclidean norm. $\lambda_{\min}\{\mathcal{Q}\}$ ($\lambda_{\max}\{\mathcal{Q}\}$) stands for the minimum (maximum) eigenvalue of $\mathcal{Q}$.

II. Problem Formulation and Preliminaries

A. The system formulation

Consider a complex network with N nodes as follows:

$$\begin{aligned} x_i(k+1) =& A_i x_i(k) + f(x_i(k)) + \sum_{j=1}^{N}\omega_{ij}\Gamma x_j(k) \\ &+ B_i w_i(k),\ i\in\mathcal{V}\triangleq\{1,2,\cdots,N\} \end{aligned} \tag{1}$$

where $x_i(k)\in\mathbb{R}^{n_x}$ represents the system state, and $w_i(k)\in\mathbb{R}^{n_w}$ refers to the system disturbance signal which satisfies $\parallel w_i(k)\parallel_2\leq w_0$. $A_i\in\mathbb{R}^{n_x\times n_x}$ and $B_i\in\mathbb{R}^{n_x\times n_w}$ are known matrices. The nonlinear function $f(\cdot)$ is assumed to satisfy $f(0)=0$ and

$$\begin{aligned} &(f(z_1)-f(z_2)-\bar{u}(z_1-z_2))^T \\ &\quad\times(f(z_1)-f(z_2)-\underline{u}(z_1-z_2))\leq 0 \end{aligned} \tag{2}$$

for $\forall z_1,\ z_2\in\mathbb{R}^{n_x}$, where $\bar{u}$ and $\underline{u}$ are constant known matrices.

The coupled configuration matrix $\mathcal{W}\triangleq[\omega_{ij}]_{N\times N}$ denotes the topology of the complex network with $\omega_{ij}=\omega_{ji}>0$ if node i is capable of receiving the signal from node j, otherwise $\omega_{ij}=0$. In general, $\mathcal{W}$ is assumed to be symmetric and satisfies $w_{ii}=-\sum_{j=1}^{N}w_{ij}$ for $j\neq i$. The inner-coupling matrix $\Gamma=\text{diag}\{\gamma_1,\gamma_2,\cdots,\gamma_{n_x}\}$ represents connections between different elements of the subsystem, where $\gamma_l\neq 0$ means that the lth component of $x_j(k)$ has an impact on the $x_i(k)$.

B. Measurements of partial nodes under constrained bit rate

The transmission of the measurements in this paper is realized by applying a wireless digital communication network under constrained bit rate. As stated in Section I, for complex networks in practice, only a fraction of the network nodes have access to the corresponding measurement outputs due to the bit rate limitation of the network.

Without loss of generality, it is assumed that we can get access to the measurement output of the first n_0 nodes.

Specifically, the measurements of nodes in this part are given as:

$$y_i(k) = C_i x_i(k) + D_i v_i(k),\ 1 \le i \le n_0 \tag{3}$$

where $y_i(k) \in \mathbb{R}^{n_y}$ denotes the measurement output of the ith node, $v_i(k) \in \mathbb{R}^{n_v}$ refers to the measurement disturbance signal which satisfies $\| v_i(k) \|_2 \le v_0$. $C_i \in \mathbb{R}^{n_y \times n_x}$ and $D_i \in \mathbb{R}^{n_y \times n_v}$ are known matrices.

Data collisions inevitably occur when output information from different nodes passes through the digital communication network with limited bandwidth. A variety of channel allocation protocols are applied to allocate a certain available bit rate to each node to reduce the data collision. The model of bit rate constraint can be expressed as follows,

$$\sum_{i=1}^{n_0} R_i \le R_s \tag{4}$$

where $R_s \in \mathbb{N}^+$ represents the total available bit rate determined by the physical elements, and $R_i \in \mathbb{N}$ indicates the allocated bit rate of node i.

Remark 1: Different media access control (MAC) protocols, such as allocation-based and competition-based MAC protocols, are applied in real networks to reduce data collisions. While competition-based MAC protocols are better at improving the utilization of the network bandwidth in the network environment, the allocation-based MAC protocols have a better effect on the network with massive nodes such as complex networks. In this paper, the static allocation protocol is applied to complex networks [1], [34]. The bit rate constraint model is presented in formula (4).

Remark 2: Due to the limitations of network hardware and communication resource, the total bandwidth of complex networks is often limited. In this case, the number of accessible nodes n_0 directly affects the amount of bit rate that can be allocated to each node. Specifically, the larger the n_0 is, the less bit rate is allocated to each node. Also, less bit rate means lower transmission quality which leads to a worse estimation. Intuitively, if measurements can be extracted from more nodes, i.e., the larger n_0 is, the better estimation results can be achieved. Therefore, the number of nodes providing available measurements has a complex impact on the performance of the state estimation for a complex network. The comprehensive effect of the number of accessible nodes n_0 on the performance of the state estimation will be discussed in Section IV by numerical examples.

C. Coding-decoding procedure under constrained bit rate

To comply with the digital communication fashion, a coding-decoding strategy subject to constrained bit rate condition (4) is presented in this subsection. By coding procedure, the measurement of each accessible node is coded as a string of binary codes selected from the alphabet $\mathbb{A}^{R_i}$ of size 2^{R_i}.

To facilitate the coding-decoding procedure, the following uniform quantizer is brought forward in this paper. For the quantizer of node i, given a scaling parameter $b_i > 0$, the quantization region is identified subsequently by $\mathcal{B}_{b_i} = \{y_i \in \mathbb{R}^{n_y} : |y_i^{(j)}| \le b_i, j = 1, 2, \cdots, n_y\}$, where $y_i^{(j)}$ is the jth element of the vector y_i. By choosing an integer q_i, the hyperrectangles $\mathcal{B}_{b_i}$ will be partitioned into $q_i^{n_y}$ sub-hyperrectangles $I^{i1}_{s^i_1}(b_i) \times I^{i2}_{s^i_2}(b_i) \times \cdots \times I^{in_y}_{s^i_{n_y}}(b_i)$, with $s^i_1, s^i_2, \cdots, s^i_{n_y} \in \{1, 2, \cdots, q_i\}$ and

$$\begin{aligned}
I_1^{ij}(b_i) &\triangleq \left\{ y_i^{(j)} \mid -b_i \le y_i^{(j)} < -b_i + \frac{2b_i}{q_i} \right\} \\
I_2^{ij}(b_i) &\triangleq \left\{ y_i^{(j)} \mid -b_i + \frac{2b_i}{q_i} \le y_i^{(j)} < -b_i + \frac{4b_i}{q_i} \right\} \\
&\vdots \\
I_{q_i}^{ij}(b_i) &\triangleq \left\{ y_i^{(j)} \mid b_i - \frac{2b_i}{q_i} \le y_i^{(j)} \le b_i \right\}.
\end{aligned} \tag{5}$$

For the bit rate constraint model (4) and the uniform quantizers described above, in order to ensure that the information corresponding to each sub-hyperrectangle is uniquely encoded, the maximum number of quantization levels is defined as:

$$q_{im} = \left\lfloor \sqrt[n_y]{2^{R_i}} \right\rfloor \tag{6}$$

where $\lfloor \sqrt[n_y]{2^{R_i}} \rfloor$ describes the maximum integer less than or equal to $\sqrt[n_y]{2^{R_i}}$.

For each $\mathcal{B}_{b_i}$, the center of the hyperrectangle $I^{i1}_{s^i_1}(b_i) \times I^{i2}_{s^i_2}(b_i) \times \cdots \times I^{in_y}_{s^i_{n_y}}(b_i)$ is denoted by

$$\hbar^i_{b_i}\left(s^i_1, s^i_2, \cdots, s^i_{n_y}\right) \triangleq \begin{bmatrix} c_{i1} & c_{i2} & \cdots & c_{in_y} \end{bmatrix}^T \tag{7}$$

with $c_{ij} \triangleq -b_i + \left[\left(\left(2s^i_j - 1\right) b_i\right) / \left\lfloor \sqrt[n_y]{2^{R_i}} \right\rfloor\right]$, $j = 1, 2, \cdots, n_y$. Hence, for any $y_i \in \mathcal{B}_{b_i}$, there exists a certain set of integers $s^i_1, s^i_2, \cdots, s^i_{n_y} \in \{1, 2, \cdots, q_i\}$ such that $y_i \in I^{i1}_{s^i_1} \times I^{i2}_{s^i_2} \times \cdots \times I^{in_y}_{s^i_{n_y}}$, which satisfies the following inequality:

$$\left\| y_i - \hbar^i_{b_i}\left(s^i_1, s^i_2, \cdots, s^i_{n_y}\right) \right\|_2 \le \frac{\sqrt{n_y} b_i}{\left\lfloor \sqrt[n_y]{2^{R_i}} \right\rfloor}. \tag{8}$$

The integers $s^i_1, s^i_2, \cdots, s^i_{n_y} \in \{1, 2, \cdots, q_i\}$ are the components of the codeword in the coding procedure.

Coder for node i under constrained bit rate R_i.

For $y_i(k) \in I^{i1}_{s^i_1}(b_i) \times I^{i2}_{s^i_2}(b_i) \times \cdots \times I^{in_y}_{s^i_{n_y}}(b_i) \subset \mathcal{B}_{b_i}$, the following codeword is generated

$$\mathcal{Y}_i^{R_i}(k) = \left[s^i_1, \cdots, s^i_{n_y}\right]. \tag{9}$$

Decoder for node i.

For the received codeword $\mathcal{Y}_i^{R_i}(k)$, it can be decoded by using the corresponding alphabet $\mathbb{A}^{R_i}$ embedded in the decoder. In addition, the output of the decoder is defined as

$$\hat{y}_i(k) = \hbar^i_{b_i}\left(\mathcal{Y}_i^{R_i}(k)\right) = \hbar^i_{b_i}\left(s^i_1, \cdots, s^i_{n_y}\right). \tag{10}$$

D. Partial-nodes-based state estimation under coding-decoding procedure

Based on the coder (9) and decoder (10) proposed before, the estimator of node i belonging to the first n_0 nodes is capable of receiving the information with the following form at time instant k from the sensor:

$$\begin{aligned}\hat{y}_i(k) &= \hbar^i_{b_i}\left(s^i_1, \cdots, s^i_{n_y}\right)\\ &= \begin{bmatrix} c_{i1} & c_{i2} & \cdots & c_{in_y}\end{bmatrix}^T\\ &= \begin{bmatrix} -b_i + \left\lfloor \left(\left(2s^i_1 - 1\right)b_i\right) / \left\lceil \sqrt[n_y]{2^{R_i}} \right\rceil \right\rfloor \\ -b_i + \left\lfloor \left(\left(2s^i_2 - 1\right)b_i\right) / \left\lceil \sqrt[n_y]{2^{R_i}} \right\rceil \right\rfloor \\ \vdots \\ -b_i + \left\lfloor \left(\left(2s^i_{n_y} - 1\right)b_i\right) / \left\lceil \sqrt[n_y]{2^{R_i}} \right\rceil \right\rfloor \end{bmatrix}.\end{aligned} \tag{11}$$

Let $d_{e,i}(k) \triangleq y_i(k) - \hat{y}_i(k)$ be the decoding error vector of node i, $\bar{d}_e(k) \triangleq \text{col}_{n_0}(d_{e,i}(k))$, and $d_e(k) \triangleq \text{col}(\bar{d}_e(k), 0)$. It follows from (8) and (11) that

$$\|d_{e,i}(k)\|_2 \le \frac{\sqrt{n_y} b_i}{\left\lceil \sqrt[n_y]{2^{R_i}} \right\rceil} \tag{12}$$

$$\|d_e(k)\|_2 = \|\bar{d}_e(k)\|_2 \le \sqrt{\sum_{i=1}^{n_0} \frac{n_y b_i^2}{\left(\left\lceil \sqrt[n_y]{2^{R_i}} \right\rceil\right)^2}}. \tag{13}$$

In terms of the decoded measurements from the first n_0 nodes, the state estimators for the complex network (1) are constructed of the following form

$$\begin{aligned}\hat{x}_i(k+1) =& A_i\hat{x}_i(k) + f(\hat{x}_i(k)) + \sum_{j=1}^{N}\omega_{ij}\Gamma\hat{x}_j(k)\\ &+ K_i(\hat{y}_i(k) - C_i\hat{x}_i(k)),\ i = 1, 2, \cdots, n_0\end{aligned} \tag{14}$$

$$\begin{aligned}\hat{x}_i(k+1) =& A_i\hat{x}_i(k) + f(\hat{x}_i(k)) + \sum_{j=1}^{N}\omega_{ij}\Gamma\hat{x}_j(k)\\ & i = n_0+1, n_0+2, \cdots, N\end{aligned} \tag{15}$$

where $\hat{x}_i(k) \in \mathbb{R}^{n_x}$ is the ith node's state estimate, and $K_i \in \mathbb{R}^{n_x \times n_y}$ is the estimator gain to be designed.

Denoting the estimation error of the ith node as $e_i(k) \triangleq x_i(k) - \hat{x}_i(k)$ and taking the definition of decoding error into account, the corresponding estimation error dynamics of node i is obtained as:

$$\begin{aligned}e_i(k+1) =& A_ie_i(k) + f(e_i(k)) + \sum_{j=1}^{N}\omega_{ij}\Gamma e_j(k) + B_iw_i(k)\\ &- K_i(y_i(k) - d_{e,i}(k) - C_i\hat{x}_i(k))\\ =& A_ie_i(k) + f(e_i(k)) + \sum_{j=1}^{N}\omega_{ij}\Gamma e_j(k) + B_iw_i(k)\\ &- K_iC_ie_i(k) - K_iD_iv_i(k) + K_id_{e,i}(k)\\ & i = 1, 2, \cdots, n_0\end{aligned} \tag{16}$$

$$\begin{aligned}e_i(k+1) =& A_ie_i(k) + f(e_i(k)) + \sum_{j=1}^{N}\omega_{ij}\Gamma e_j(k) + B_iw_i(k)\\ & i = n_0+1, n_0+2, \cdots, N\end{aligned} \tag{17}$$

with $f(e_i(k)) \triangleq f(x_i(k)) - f(\hat{x}_i(k))$.

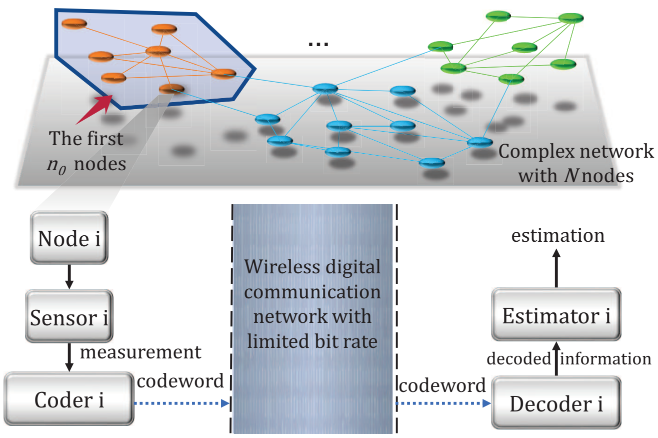

Fig. 1. Schematic of PNB state estimation problem with constrained bit rate

To simplify the symbolic representation, we set

$$\begin{aligned}e(k) &\triangleq \text{col}_N(e_i(k)),\ F(e(k)) \triangleq \text{col}_N(f(e_i(k)))\\ A &\triangleq \text{diag}_N\{A_i\},\ B \triangleq \text{col}_N\{B_i\}\\ \bar{C} &\triangleq \text{diag}_{n_0}\{C_i\},\ C \triangleq \text{diag}\{\bar{C}, 0\}\\ \bar{D} &\triangleq \text{col}_{n_0}\{D_i\},\ D \triangleq \text{col}\{\bar{D}, 0\}\\ \bar{K} &\triangleq \text{diag}_{n_0}\{K_i\},\ K \triangleq \text{diag}\{\bar{K}, 0\}\\ \bar{v}(k) &\triangleq \text{col}_{n_0}(v_i(k)),\ v(k) \triangleq \text{col}_N(\bar{v}_i(k), 0)\\ w(k) &\triangleq \text{col}_N(w_i(k)),\ \mathcal{A} \triangleq (A - KC + \mathcal{W} \otimes \Gamma).\end{aligned} \tag{18}$$

By applying the Kronecker product, the estimation error dynamics (16) is rearranged as the following compact form:

$$\begin{aligned}e(k+1) =& F(e(k)) + \mathcal{A}e(k) - KDv(k)\\ &+ Bw(k) + Kd_e(k).\end{aligned} \tag{19}$$

In order to facilitate the further development of this paper, the following definition is presented to assist in describing the issues to be studied.

Definition 1: The dynamics of the estimation error $e(k)$ [i.e., the solution of system (19)] is said to be exponentially ultimately bounded if there exist constants $\sigma > 0$, $\rho > 0$, and $\phi > 0$, such that

$$\|e(k)\|_2^2 \le \sigma^k\rho + \phi$$

where $\sigma \in [0, 1)$ is the decay rate and ϕ is the asymptotic upper bound (AUB) of $\|e(k)\|_2^2$.

In this paper, the state estimation problem for complex network (1) with constrained bit rate condition (4) is investigated by using decoded measurements from only a part of nodes. The schematic structure is shown in Fig. 1. In the following pages, the sufficient condition will be given to guarantee that the estimation error is exponentially ultimately bounded. The desired estimator gains will be derived by solving some of the OPs proposed in this paper.

III. Main Results

In this section, the ultimate boundedness of PNB state estimation is firstly analyzed. The design issues of the PNB state estimator are then discussed under different estimation performance metrics.

A. The analysis of PNB state estimation

The next theorem will discuss the ultimate boundedness of the PNB state estimation error dynamics (19) and give a sufficient condition to guarantee the ultimate boundedness.

For the sake of simplicity, we denote

$$\begin{aligned}\Phi_{11} \triangleq& \mathrm{diag}\{\Pi_1, \Pi_2\}\\ \Pi_1 \triangleq& \begin{bmatrix} -(1-\gamma)\mathcal{P} - \varepsilon_1 \hat{U}_1 & \varepsilon_1 \hat{U}_2 \\ * & -\varepsilon_1 I \end{bmatrix}\\ \Pi_2 \triangleq& \mathrm{diag}\{-\varepsilon_2 I, -\varepsilon_3 I, -\varepsilon_4 I\}\\ \Phi_{12} \triangleq& \mathrm{col}\left(\mathcal{A}^T\mathcal{P}, \mathcal{P}, -D^T K^T \mathcal{P}, B^T\mathcal{P}, K^T\mathcal{P}\right).\end{aligned} \tag{20}$$

Theorem 1: Under the bit rate condition (4), let the positive integers R_s, R_i ($i = 1, 2, \cdots, n_0$), and the matrix K with appropriate dimensions be given. Then, the dynamics of the PNB state estimation error is ultimately bounded if there exist positive scalars $\varepsilon_1,\ \varepsilon_2,\ \varepsilon_3,\ \varepsilon_4,\ \gamma$, and positive definite matrices $P_i \in \mathbb{R}^{n_x \times n_x}$ ($i = 1, 2, \cdots, N$) such that

$$\Phi_1 = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ * & -\mathcal{P} \end{bmatrix} < 0 \tag{21}$$

with $\mathcal{P} \triangleq \mathrm{diag}_N\{P_i\}$.

Proof: Define the Lyapunov-like function for the PNB state estimation error system (19) as $V(k) \triangleq e^T(k)\mathcal{P}e(k)$. Then, the difference of $V(k)$ is calculated as:

$$\begin{aligned}&\Delta V(k) = V(k+1) - V(k)\\ =& e^T(k+1)\mathcal{P}e(k+1) - e^T(k)\mathcal{P}e(k)\\ =& [F(e(k)) + \mathcal{A}e(k) - KDv(k) + Bw(k) + Kd_e(k)]^T\mathcal{P}\\ &\times [F(e(k)) + \mathcal{A}e(k) - KDv(k) + Bw(k) + Kd_e(k)]\\ &- e^T(k)\mathcal{P}e(k)\\ =& F^T(e(k))\mathcal{P}F(e(k)) + e^T(k)\mathcal{A}^T\mathcal{P}\mathcal{A}e(k)\\ &+ v^T(k)D^TK^T\mathcal{P}KDv(k) + w^T(k)B^T\mathcal{P}Bw(k)\\ &+ d_e^T(k)K^T\mathcal{P}Kd_e(k) + 2F^T(e(k))\mathcal{P}\mathcal{A}e(k)\\ &- 2F^T(e(k))\mathcal{P}KDv(k) + 2F^T(e(k))\mathcal{P}Bw(k)\\ &+ 2F^T(e(k))\mathcal{P}Kd_e(k) - 2e^T(k)\mathcal{A}^T\mathcal{P}KDv(k)\\ &+ 2e^T(k)\mathcal{A}^T\mathcal{P}Bw(k) + 2e^T(k)\mathcal{A}^T\mathcal{P}Kd_e(k)\\ &- 2v^T(k)D^TK^T\mathcal{P}Bw(k) - 2v^T(k)D^TK^T\mathcal{P}Kd_e(k)\\ &+ 2w^T(k)B^T\mathcal{P}Kd_e(k) - e^T(k)\mathcal{P}e(k).\end{aligned} \tag{22}$$

It is inferred from (2) that

$$\varepsilon_1 \begin{bmatrix} e(k) \\ F(e(k)) \end{bmatrix}^T \begin{bmatrix} \hat{U}_1 & -\hat{U}_2 \\ * & I \end{bmatrix} \begin{bmatrix} e(k) \\ F(e(k)) \end{bmatrix} \le 0. \tag{23}$$

where $\hat{U}_1 \triangleq I_N \otimes U_1$, $\hat{U}_2 \triangleq I_N \otimes U_2$, $U_1 \triangleq (\bar{u}^T\underline{u} + \bar{u}\underline{u}^T)/2$, and $U_2 \triangleq (\bar{u}^T + \underline{u}^T)/2$. Then, taking (22) and (23) into consideration together, one has:

$$\begin{aligned}\Delta V(k) \le& \zeta^T(k)\bar{\Phi}_1\zeta(k) - \gamma V(k) + \varepsilon_2 w^T(k)w(k)\\ &+ \varepsilon_3 v^T(k)v(k) + \varepsilon_4 d_e^T(k)d_e(k)\end{aligned} \tag{24}$$

where

$$\zeta(k) \triangleq \begin{bmatrix} e^T(k) & F^T(e(k)) & v^T(k) & w^T(k) & d_e^T(k) \end{bmatrix}^T$$

$$\bar{\Phi}_1 \triangleq \begin{bmatrix} \bar{\Phi}_{11} & \bar{\Phi}_{12} \\ * & \bar{\Phi}_{22} \end{bmatrix}$$

$$\bar{\Phi}_{11} \triangleq \begin{bmatrix} -(1-\gamma)\mathcal{P} - \varepsilon_1\hat{U}_1 - \mathcal{A}^T\mathcal{P}\mathcal{A} & \mathcal{A}^T\mathcal{P} + \varepsilon_1\hat{U}_2 \\ * & \mathcal{P} - \varepsilon_1 I \end{bmatrix}$$

$$\bar{\Phi}_{12} \triangleq \begin{bmatrix} -\mathcal{A}^T\mathcal{P}KD & \mathcal{A}^T\mathcal{P}B & \mathcal{A}^T\mathcal{P}K \\ -\mathcal{P}KD & \mathcal{P}B & \mathcal{P}K \end{bmatrix}$$

$$\bar{\Phi}_{22} \triangleq \begin{bmatrix} D^TK^T\mathcal{P}KD & -D^TK^T\mathcal{P}B & -D^TK^T\mathcal{P}K \\ * & B^T\mathcal{P}B & B^T\mathcal{P}K \\ * & * & K^T\mathcal{P}K \end{bmatrix} + \Pi_2.$$

By applying the Schur Complement Lemma, it is readily seen from inequality (21) that $\bar{\Phi}_1 < 0$. Then, it follows from the formulas (21) and (24) that

$$\begin{aligned}\Delta V(k) \le& -\gamma V(k) + \varepsilon_2 w^T(k)w(k)\\ &+ \varepsilon_3 v^T(k)v(k) + \varepsilon_4 d_e^T(k)d_e(k).\end{aligned} \tag{25}$$

Denoting $\phi \triangleq \varepsilon_2 N w_0^2 + \varepsilon_3 n_0 v_0^2 + \varepsilon_4 \sum_{i=1}^{n_0} \frac{n_y b_i^2}{\left(\left\lfloor \sqrt[n_y]{2^{R_i}} \right\rfloor\right)^2}$, we have

$$\Delta V(k) \le -\gamma V(k) + \phi. \tag{26}$$

Moreover, for any scalar η, one has

$$\begin{aligned}&\eta^{t+1}V(t+1) - \eta^t V(t)\\ =& \eta^{t+1}(V(t+1) - V(t)) + \eta^t(\eta - 1)V(t)\\ \le& \eta^t(\eta - 1 - \eta\gamma)V(t) + \eta^{t+1}\phi.\end{aligned} \tag{27}$$

Letting $\eta = \bar{\eta} = \frac{1}{1-\gamma}$ and summing both sides of inequality (27) from 0 to $k-1$ in relation to t, we arrive at

$$\bar{\eta}^k V(k) - V(0) \le \frac{\bar{\eta}(1 - \bar{\eta}^k)}{1 - \bar{\eta}}\phi \tag{28}$$

which can be further computed as

$$\begin{aligned}V(k) \le& \frac{V(0)}{\bar{\eta}^k} + \frac{\bar{\eta}(1 - \bar{\eta}^k)}{\bar{\eta}^k(1 - \bar{\eta})}\phi\\ =& (1-\gamma)^k\left(V(0) - \frac{\phi}{\gamma}\right) + \frac{\phi}{\gamma}.\end{aligned} \tag{29}$$

Then, taking the definition of $V(k)$ and (29) into consideration together, one has:

$$\begin{aligned}\|e(k)\|_2^2 \le& \frac{1}{\lambda_{\min}\{\mathcal{P}\}} e^T(k)\mathcal{P}e(k)\\ \le& \frac{(1-\gamma)^k}{\lambda_{\min}\{\mathcal{P}\}}\left(V(0) - \frac{\phi}{\gamma}\right) + \frac{\phi}{\gamma\lambda_{\min}\{\mathcal{P}\}}.\end{aligned} \tag{30}$$

Consequently, recalling Definition 1, it is readily seen that the dynamics of the PNB state estimation error system (19) is exponentially ultimately bounded. Moreover, the AUB of the PNB state estimation error can be calculated by:

$$\frac{\varepsilon_2 N w_0^2 + \varepsilon_3 n_0 v_0^2 + \varepsilon_4 \sum_{i=1}^{n_0} \frac{n_y b_i^2}{\left(\left\lfloor \sqrt[n_y]{2^{R_i}} \right\rfloor\right)^2}}{\gamma\lambda_{\min}\{\mathcal{P}\}}. \tag{31}$$

The proof of this theorem is now complete. ■

The following corollary discusses the bit rate condition that guarantees the desired PNB state estimation performance for a complex network. According to (31), the following sufficient bit rate condition is provided with no need to prove.

Corollary 1: Under the condition mentioned in Theorem 1, the PNB state estimation error system (19) is ultimately bounded with a given AUB ϵ, if there exist a set of bit rates R_i $(i = 1, 2, \cdots, n_0)$ satisfying

$$\sum_{i=1}^{n_0} \frac{n_y b_i^2}{\left(\left\lfloor \sqrt[n_y]{2^{R_i}} \right\rfloor\right)^2} \leq \frac{\gamma\lambda_{min}\{\mathcal{P}\}\epsilon - \varepsilon_2 N w_0^2 - \varepsilon_3 n_0 v_0^2}{\varepsilon_4}. \quad (32)$$

Specifically, when the allocated bit rate are all the same for each node, i.e., $R_1 = R_2 = \cdots = R_{n_0} \triangleq \tilde{R}$, the PNB state estimation error system (19) is ultimately bounded with a given AUB ϵ if

$$\tilde{R} \geq \frac{n_y}{2}\log_2\left(\frac{\varepsilon_4 n_y \sum_{i=1}^{n_0} b_i^2}{\gamma\lambda_{min}\{\mathcal{P}\}\epsilon - \varepsilon_2 N w_0^2 - \varepsilon_3 n_0 v_0^2}\right). \quad (33)$$

Remark 3: The ultimate boundedness is a significant performance index for the estimation error dynamics, which ensures that the estimation error remains within a region nearing the steady state. In this theorem, we propose a sufficient condition to guarantee that the dynamics of the PNB state estimation error system (19) is exponentially ultimately bounded. It is inferred from the result in Theorem 1 that the AUB of the estimation error system (19) is dependent on the system noise $w(k)$, the measurement noise $v(k)$, the number of the accessible nodes in the complex network, and the coding-decoding procedure. Notably, when the bound of the noise, the number of accessible nodes, and the parameters of the coding-decoding procedure are determined, the AUB (31) of the estimation error system depends only on the bit rate. In the specific case, a bit rate condition is subsequently obtained under which the required PNB state estimation performance is satisfied.

B. Design of PNB state estimator

Based on the analysis of the PNB state estimation error system, the design issue will be addressed by solving two OPs and a MINP problem in order to guarantee certain performance indexes.

OP A: To minimize the ultimate bound of the PNB state estimation error dynamics so as to achieve the best estimation performance under bit rate constraint condition (4) with known total available bit rate R_s and allocated bit rate R_i.

Theorem 2: For PNB state estimation error system (19), let a scalar $\bar{\gamma}$ $(0 < \bar{\gamma} < 1)$ and positive integers R_s, R_i $(i = 1, 2, \cdots, n_0)$ be given. Suppose that there exist four positive scalars $\varepsilon_1,\ \varepsilon_2,\ \varepsilon_3,\ \varepsilon_4$, N positive definite matrices $P_i \in \mathbb{R}^{n_x \times n_x}$ $(i = 1, 2, \cdots, N)$, and n_0 matrices $\mathcal{K}_i \in \mathbb{R}^{n_x \times n_y}$ $(i = 1, 2, \cdots, n_0)$ satisfying

$$\Phi_2 = \begin{bmatrix} \Phi_{21} & \Phi_{22} \\ * & -\mathcal{P} \end{bmatrix} < 0 \quad (34)$$

$$\mathcal{P} \geq I \quad (35)$$

with

$$\begin{aligned} \Phi_{21} &\triangleq \text{diag}\{\Pi_3, \Pi_2\} \\ \mathcal{K} &\triangleq \text{diag}_N\{\bar{\mathcal{K}}, 0\},\ \bar{\mathcal{K}} \triangleq \text{diag}_{n_0}\{\mathcal{K}_i\} \\ \Pi_3 &\triangleq \begin{bmatrix} -\bar{\gamma}\mathcal{P} - \varepsilon_1\hat{U}_1 & \varepsilon_1\hat{U}_2 \\ * & -\varepsilon_1 I \end{bmatrix} \\ \Phi_{22} &\triangleq \text{col}\left(\bar{\mathcal{A}}, \mathcal{P}, -D^T\mathcal{K}^T, B^T\mathcal{P}, \mathcal{K}^T\right) \\ \bar{\mathcal{A}} &\triangleq A^T\mathcal{P} - C^T\mathcal{K}^T + (\mathcal{W}\otimes\Gamma)^T\mathcal{P} \end{aligned} \quad (36)$$

and $\mathcal{P}$ is defined in Theorem 1.

Then, the dynamics of the PNB state estimation error system (19) is ultimately bounded, where the decay rate of the state estimation error $\|e(k)\|_2^2$ is $\bar{\gamma}$. Furthermore, the minimum of the AUB of $\|e(k)\|_2^2$ can be derived by solving the following minimization problem:

$$\min\left\{\varepsilon_2 N w_0^2 + \varepsilon_3 n_0 v_0^2 + \varepsilon_4 \sum_{i=1}^{n_0} \frac{n_y b_i^2}{\left(\left\lfloor \sqrt[n_y]{2^{R_i}} \right\rfloor\right)^2}\right\} \quad (37)$$

subject to the matrix inequality constraints (34) and (35). Moreover, the gains K_i $(i = 1, 2, \cdots, n_0)$ of the estimators can be obtained by (18) and the following formula:

$$K = \mathcal{P}^{-1}\mathcal{K}. \quad (38)$$

Proof: Setting $\gamma = 1 - \bar{\gamma}$, and taking (38) into account, it is readily seen that

$$\Phi_{21} = \Phi_{11},\ \Phi_{22} = \Phi_{12}. \quad (39)$$

Moreover, it is evident from inequality (34) that $\Phi_1 < 0$, which guarantees the ultimate boundedness of the PNB state estimation error system (19).

Following the similar line in Theorem 1, the inequality in terms of $V(k)$ is obtained:

$$V(k) \leq \bar{\gamma}^k\left(V(0) - \frac{\phi}{\gamma}\right) + \frac{\phi}{\gamma} \quad (40)$$

where ϕ is defined in Theorem 1. Then, based on the inequalities (30) and (35), we can derive that

$$\|e(k)\|_2^2 \leq V(k) \leq \bar{\gamma}^k\left(V(0) - \frac{\phi}{1-\bar{\gamma}}\right) + \frac{\phi}{1-\bar{\gamma}}. \quad (41)$$

As such, the asymptotic bound of $\|e(k)\|_2^2$ can be computed by minimizing ϕ, which is equivalent to the condition (37). Then, the proof is complete. ■

OP B: To optimize the decay rate of the PNB state estimation error dynamics for the fastest convergence performance under bit rate constraint (4) with known total available bit rate R_s and allocated bit rate R_i.

Theorem 3: For PNB state estimation error system (19), let positive integers R_s, R_i $(i = 1, 2, \cdots, n_0)$ be given. Suppose that there exist five positive scalars $\varepsilon_1,\ \varepsilon_2,\ \varepsilon_3,\ \varepsilon_4,\ \gamma,\ N+1$ positive definite matrices $P_i \in \mathbb{R}^{n_x \times n_x}$ $(i = 1, 2, \cdots, N)$, $\mathcal{S} \in \mathbb{R}^{Nn_x \times Nn_x}$ and n_0 matrices $\mathcal{K}_i \in \mathbb{R}^{n_x \times n_y}$ $(i = 1, 2, \cdots, n_0)$ satisfying

$$\Phi_3 = \begin{bmatrix} \Phi_{31} & \Phi_{22} \\ * & -\mathcal{P} \end{bmatrix} < 0 \quad (42)$$

$$\mathcal{P} \geq I \tag{43}$$

$$\begin{bmatrix} -\mathcal{S} & \gamma I \\ * & \mathcal{P} - 2I \end{bmatrix} < 0 \tag{44}$$

with

$$\begin{aligned} \Phi_{31} \triangleq& \text{diag}\{\Pi_5, \Pi_2\} \\ \Pi_5 \triangleq& \begin{bmatrix} -\mathcal{P} + \mathcal{S} - \varepsilon_1 \hat{U}_1 & \varepsilon_1 \hat{U}_2 \\ * & -\varepsilon_1 I \end{bmatrix} \end{aligned} \tag{45}$$

and $\mathcal{P}$, Φ_{22}, $\mathcal{K}$, $\bar{\mathcal{K}}$, Π_2 are defined in Theorem 2. Then, the dynamics of the PNB state estimation error system (19) is ultimately bounded. Furthermore, the optimum decay rate of PNB state estimation error $\|e(k)\|_2^2$ can be derived by solving the following maximization problem:

$$\max\{\gamma\} \tag{46}$$

subject to the matrix inequality constraints (42)-(44). Moreover, the gains K_i $(i = 1, 2, \cdots, n_0)$ of the estimators can be derived by taking (18) and the following formula into consideration:

$$K = \mathcal{P}^{-1}\mathcal{K}. \tag{47}$$

Proof: It is evident from the inequality $(\mathcal{P} - I)\mathcal{P}^{-1}(\mathcal{P} - I) \geq 0$ that $-\mathcal{P}^{-1} \leq \mathcal{P} - 2I$. Then, the following inequality is derived in light of (44):

$$\Phi_{33} \triangleq \begin{bmatrix} -\mathcal{S} & \gamma I \\ * & -\mathcal{P}^{-1} \end{bmatrix} \leq \begin{bmatrix} -\mathcal{S} & \gamma I \\ * & \mathcal{P} - 2I \end{bmatrix} < 0. \tag{48}$$

By applying the Schur Complement Lemma to Φ_{33}, one gets $-\mathcal{S} + \gamma^2 \mathcal{P} < 0$. Then, it is inferred from (42) that

$$\begin{bmatrix} \bar{\Phi}_{31} & \Phi_{22} \\ * & -\mathcal{P} \end{bmatrix} < \Phi_3 < 0 \tag{49}$$

with

$$\begin{aligned} \bar{\Phi}_{31} \triangleq& \text{diag}\{\bar{\Pi}_5, \Pi_2\} \\ \bar{\Pi}_5 \triangleq& \begin{bmatrix} -\mathcal{P} + \gamma^2 \mathcal{P} - \varepsilon_1 \hat{U}_1 & \varepsilon_1 \hat{U}_2 \\ * & -\varepsilon_1 I \end{bmatrix}. \end{aligned}$$

Consequently, the dynamics of the PNB state estimation error $e(k)$ is proved to be ultimately bounded on the basis of the Theorem 1.

Following the line similar to the proof of Theorem 1, we obtain the following inequality:

$$V(k) \leq (1 - \gamma^2)^k \left(V(0) - \frac{\phi}{\gamma^2} \right) + \frac{\phi}{\gamma^2} \tag{50}$$

where ϕ is defined in Theorem 1. The decay rate of $\|e(k)\|_2^2$ is thus determined by $1 - \gamma^2$. In this way, the optimum decay rate of PNB state estimation error is obtained by solving the maximization problem (46). The proof is complete. ■

C. The co-design of the PNB estimators and bit rate allocation protocol

In order to minimize the AUB of the PNB state estimation error system (19), the design issue is addressed in Theorem 2 with given bit rate R_s and R_i. It is inferred from (31) that the available bit rate of each node plays an important role in the AUB. On the other hand, the available bit rate is allocated by the MAC protocol, which can be designed in terms of the desired performance. Therefore, the co-design problem involving both the bit rate allocation protocol and the estimator gains will be the focus of the rest of this paper.

In order to address the co-design issue, the following new minimization problem is proposed, according to (4).

Corollary 2: Based on Theorem 2, when the positive integers R_i $(i = 1, 2, \cdots, n_0)$ are variables which need to be determined, the minimum of the AUB of $\|e(k)\|_2^2$ can be derived by solving the following minimization problem:

$$\begin{aligned} \min \quad & \varepsilon_2 N w_0^2 + \varepsilon_3 n_0 v_0^2 + \varepsilon_4 \textstyle\sum_{i=1}^{n_0} \frac{n_y b_i^2}{\left(\left\lceil \sqrt[n_y]{2^{R_i}} \right\rceil\right)^2} \\ s.t. \quad & (4),\ (34),\ (35) \\ & 0 \leq R_i \leq R_s \\ & R_i \in \mathbb{N},\ i = 1, 2, \cdots, n_0. \end{aligned} \tag{51}$$

Moreover, the gains K_i $(i = 1, 2, \cdots, n_0)$ of estimators can be obtained by (18) and the following formula:

$$K = \mathcal{P}^{-1}\mathcal{K}. \tag{52}$$

Proof: The proof is similar to Theorem 2, which is omitted for conciseness. ■

Note that the linearity of the objective function and constraints of OP A lead to the fact that OP A can be efficiently solved by linear matrix inequalities (LMIs). However, the proposed co-design problem in this subsection is a MINP problem which refers to mathematical programming involving the continuous and discrete variables and the nonlinearity in the objective function and constraints [3]. It is difficult to be solved due to the integer constraints of R_i, the matrix inequality constraints (34) and (35), as well as the nonlinear term $\varepsilon_4 \sum_{i=1}^{n_0} \frac{n_y b_i^2}{\left(\left\lceil \sqrt[n_y]{2^{R_i}} \right\rceil\right)^2}$ in the objective function.

To solve such a MINP problem, a combination of the PSO algorithm and the LMI technique is proposed in the subsequent analysis. For the MINP problem (51) with constraints, the first step is to transform (51) into the following form by introducing a penalty function:

$$\begin{aligned} \min \quad & \varepsilon_2 N w_0^2 + \varepsilon_3 n_0 v_0^2 + \varepsilon_4 \textstyle\sum_{i=1}^{n_0} \frac{n_y b_i^2}{\left(\left\lceil \sqrt[n_y]{2^{R_i}} \right\rceil\right)^2} + f_p(R) \\ s.t. \quad & (34),\ (35) \\ & R_i \in \mathbb{N},\ i = 1, 2, \cdots, n_0 \end{aligned} \tag{53}$$

where $f_p(R) = \max\{0, \sum_{i=1}^{n_0} R_i - R_s\}$ is the penalty function with $R = [R_1, R_2, \cdots, R_{n_0}]$. The fitness function of PSO is defined as $\mathbf{F}(R) \triangleq \varepsilon_2 N w_0^2 + \varepsilon_3 n_0 v_0^2 + \varepsilon_4 \sum_{i=1}^{n_0} \frac{n_y b_i^2}{\left(\left\lceil \sqrt[n_y]{2^{R_i}} \right\rceil\right)^2} + f_p(R)$.

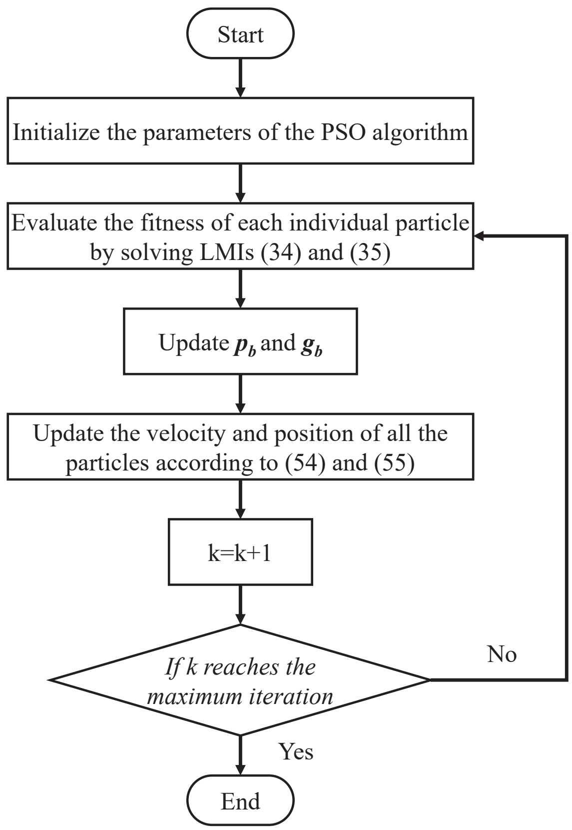

Fig. 2. Flowchart of the PSO-Assisted co-design algorithm

For the optimization problem (53), let $\mathbf{v}_i = [\mathbf{v}_{i,1}, \mathbf{v}_{i,2}, \cdots, \mathbf{v}_{i,n_0}]$ and $\mathbf{R}_i = [\mathbf{R}_{i,1}, \mathbf{R}_{i,2}, \cdots, \mathbf{R}_{i,n_0}]$ represent the velocity and position of the particle i, respectively. The velocity and position updating equations of particle i are given as follows:

$$\begin{aligned}\mathbf{v}_i(t+1) =& \mathbf{w}\mathbf{v}_i(t) + \mathbf{c}_1\mathbf{r}_1\left(\mathbf{p}_i(t) - \mathbf{R}_i(t)\right) \\ &+ \mathbf{c}_2\mathbf{r}_2\left(g_b(t) - \mathbf{R}_i(t)\right) \end{aligned} \tag{54}$$

$$\mathbf{R}_i(t+1) = \mathbf{R}_i(t) + \mathbf{v}_i(t+1) \tag{55}$$

where t is the iteration number, $\mathbf{c}_1$ and $\mathbf{c}_2$ represent the cognitive acceleration coefficient and the social acceleration coefficient, separately, $\mathbf{w}$ refers to the inertia weight, $\mathbf{p}_i$ denotes the historical individual best position (p_b) for particle i, and g_b bespeaks the historical global best position for the entire swarm. Considering that the PSO is used to solve the MINP problem, the initial position and velocity of the particle, as well as the parameters $\mathbf{c}_1$, $\mathbf{c}_2$, and $\mathbf{w}$ are all selected as integers in the algorithm. Moreover, $\mathbf{r}_1$ and $\mathbf{r}_2$ are selected randomly from two integers belonging to 1 or 2 rather than random numbers belonging to $[0, 1]$ in the classical PSO algorithm.

The flowchart of the PSO algorithm proposed in this paper is shown in Fig. 2. The first step is to initialize parameters, including the population size $\mathbf{N}_S$, the maximum number of iterations $\mathbf{N}_I$, $\mathbf{c}_1$, $\mathbf{c}_2$, $\mathbf{w}$, $\mathbf{v}_i$, $\mathbf{R}_i$, and the initial $\mathbf{p}_i$ of each particle. The second step is to evaluate each particle's fitness function $\mathbf{F}(\mathbf{R}_i)$ by solving the LMIs (34) and (35). If the LMIs (34) and (35) are infeasible, then the value of fitness will be artificially assigned a sufficiently large value (10^4 in this paper) to reduce the effect of the corresponding particle on the particle swarm. The third step is to update the p_b by choosing the smaller one between $\mathbf{F}(\mathbf{R}_i)$ and $\mathbf{F}(\mathbf{p}_i)$, and update the g_b by choosing the minimum fitness value in the swarm. Then, according to the updating equations (54) and (55), the velocity and the position of the each particle are updated in the fourth step. Each iteration repeats the process from the second step to the fourth step until the maximum number of iterations is reached. After that, the bit rate allocation protocol can be parameterized according to g_b. Finally, the PNB-based estimators (16) are generated under the gain matrices K_i that are obtained through solving the LMIs (34) and (35).

Remark 4: This paper considers a class of complex networks with limited bandwidth where only partial nodes' measurement outputs are accessible. In order to estimate each node's state more accurately, the problem of appropriately allocating the limited bit rate to a certain number of available nodes is essential. As such, this paper considers the design problems of the bit rate protocol and the estimator parameters simultaneously. Such a co-design problem is further transformed into a MINP problem as shown in (51)and is later solved by means of PSO and the LMIs.

Remark 5: This paper investigates the state estimation problem for a class of discrete-time complex networks based on partial nodes' measurements. According to an allocation-based MAC, a bit rate constraint model is first introduced to reflect the bandwidth limitation of complex networks with n_0 accessible nodes. Under the proposed bit rate constraint model, Theorem 1 provides a sufficient condition to guarantee the ultimate boundedness of the PNB state estimation error dynamics. Two optimization problems focusing on different estimation performances are presented to design the desired state estimators. Furthermore, a co-design problem that incorporates the bit rate allocation protocol and the estimator parameters is proposed to improve the estimation accuracy and such a problem is well addressed by resorting to the PSO and LMI techniques.

Remark 6: The state estimation problem for discrete-time complex networks has attracted extensive research attention and abundant literature has been collected. This paper is more innovative compared to the established literature in the following ways: 1) the proposed bit rate constraint model is new in terms of portraying the extent of communication bandwidth constraints and the bandwidth allocation rules for part of the nodes in complex networks; 2) the bit rate condition established in this paper is new due to the fact that it reveals the relationship between the specific PNB state estimation performance and the bit rate; 3) the design algorithms for PNB state estimator gains are new which meet the needs for different performances of the estimation error system.

IV. Numerical Example

In this section, three numerical simulations are carried out to illustrate the effectiveness of the PNB state estimator proposed in this paper for the complex network (1) with constrained bit rate.

The complex network (1) with five nodes is considered in this section with the following parameters:

$$A_1 = \begin{bmatrix} 0.12 & 0.06 \\ 0.12 & 0.18 \end{bmatrix}, \ A_2 = \begin{bmatrix} 0.04 & 0.02 \\ 0.04 & 0.06 \end{bmatrix}$$
$$A_3 = \begin{bmatrix} 0.08 & 0.12 \\ 0.12 & 0.12 \end{bmatrix}, \ A_4 = \begin{bmatrix} 0.01 & 0.03 \\ 0.02 & 0.03 \end{bmatrix}$$
$$A_5 = \begin{bmatrix} 0.04 & 0.04 \\ 0.06 & 0.02 \end{bmatrix}, \ B_1 = \begin{bmatrix} 0.02 \\ 0.02 \end{bmatrix}, \ B_2 = \begin{bmatrix} 0.02 \\ 0.01 \end{bmatrix}$$
$$B_3 = \begin{bmatrix} 0.06 \\ 0.02 \end{bmatrix}, \ B_4 = \begin{bmatrix} 0.03 \\ 0.02 \end{bmatrix}, \ B_5 = \begin{bmatrix} 0.01 \\ 0.02 \end{bmatrix}.$$

The coupling configuration matrix is assumed as the following form:

$$\mathcal{W} = \begin{bmatrix} -0.60 & 0.20 & 0.20 & 0.10 & 0.10 \\ 0.20 & -0.50 & 0.10 & 0.10 & 0.10 \\ 0.20 & 0.10 & -0.80 & 0.20 & 0.30 \\ 0.10 & 0.10 & 0.20 & -0.50 & 0.10 \\ 0.10 & 0.10 & 0.30 & 0.10 & -0.50 \end{bmatrix} \quad (56)$$

and the inner-coupling matrix is assumed to be an identity diagonal matrix.

The nonlinear function is supposed to satisfy the following forms:

$$f(x_i(k)) = \begin{bmatrix} 0.6\tanh(0.2x_{i1}(k)) + 0.7x_{i1}(k) \\ 0.3\tanh(0.5x_{i2}(k)) + 0.6x_{i2}(k) \end{bmatrix}.$$

Then, it can be seen that the nonlinear function satisfies the sector bounded condition (2) with

$$\bar{u}_1 = \begin{bmatrix} 0.82 & 0 \\ 0 & 0.75 \end{bmatrix}, \ \underline{u}_1 = \begin{bmatrix} 0.7 & 0 \\ 0 & 0.6 \end{bmatrix}.$$

Assume that the measurements of the first three nodes can be obtained and possess the following parameters:

$$C_1 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \ C_2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \ C_3 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$
$$D_1 = \begin{bmatrix} -0.03 \\ -0.04 \end{bmatrix}, \ D_2 = \begin{bmatrix} -0.03 \\ -0.02 \end{bmatrix}, \ D_3 = \begin{bmatrix} -0.02 \\ -0.01 \end{bmatrix}.$$

The following three cases are given to verify the effectiveness of the method proposed in this paper.

***Case 1:* PNB state estimation effects of OP A and OP B.**

This case discusses the effects of OP A and OP B on different performance indexes of the developed PNB state estimators. The total available bit rate of the complex network is set to be 32 bps, and the available bit rate of each accessible node is allocated as $R_1 = R_2 = R_3 = 10$ bps by an average allocation protocol (AAP). The scaling parameters of each quantizer are chosen as $b_1 = 0.6$, $b_2 = 0.7$, $b_3 = 0.6$. Especially, the $\bar{\gamma}$ in Theorem 2 is set to be 0.96. Then, by applying Theorem 2 and Theorem 3, the estimator gain of each node can be obtained, respectively.

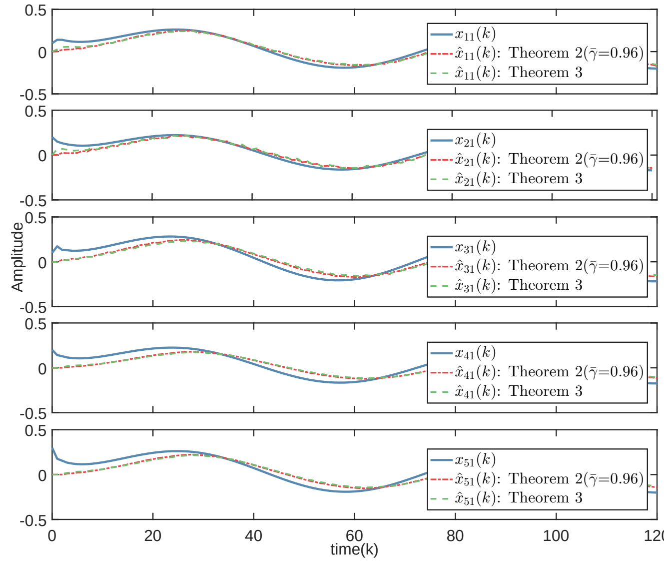

Fig. 3. The trajectories of $x_{i1}(k)$ and $\hat{x}_{i1}(k)$ ($i \in \{1, 2, 3, 4, 5\}$)

TABLE I
THE COMPARISON BETWEEN OP A AND OP B

Total bit rate R_s(bps)	**OP A** ($\bar{\gamma} = 0.96$)	**OP B**
Setting-like time	8	6
Upper bound of the trajectory	0.1763	0.1923

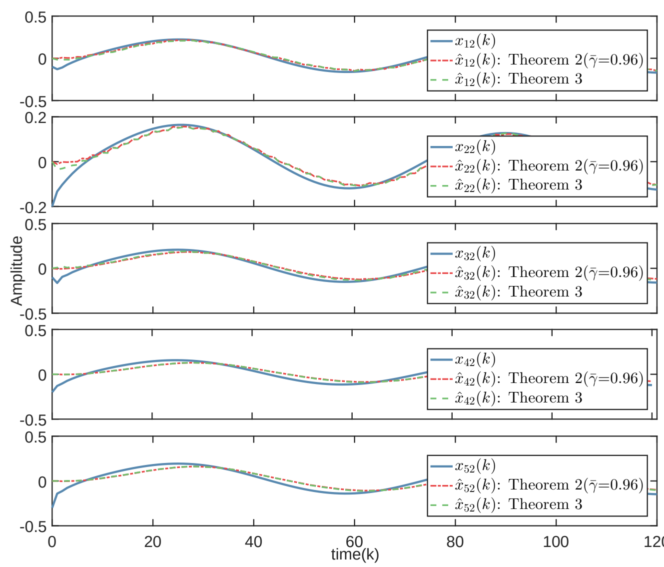

Fig. 4. The trajectories of $x_{i2}(k)$ and $\hat{x}_{i2}(k)$ ($i \in \{1, 2, 3, 4, 5\}$)

The simulation results are plotted in Figs. 3-4 and Table I. Fig. 3 and Fig. 4 sketch the trajectories of the first and the second component of the state and their estimates, respectively. It can be observed from Figs. 3-4 that the estimator of each node performs well in estimating the state trajectories. The trajectories of the estimation error dynamics subject to OP A and OP B are displayed in Fig. 5. In terms of OP A and OP B, the upper bound of the estimation error and the time required for the error dynamics to reach and remain within

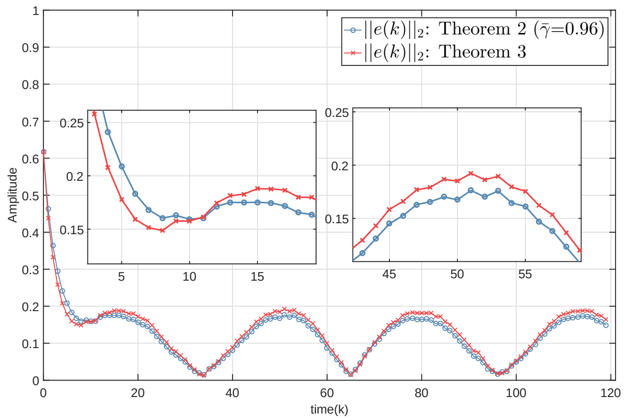

Fig. 5. The trajectories of PNB estimation error $\| e(k) \|_2$ subject to OP A and OP B

the "steady-state region" ("settling-like times") are shown in Table I. From Fig. 5 and Table I, we can see that OP A exhibits better performance in reducing the upper bound of estimation errors while OP B leads to a low decay rate.

Case 2: **Effects of different allocation protocols on AUB.**

TABLE II
THE AUBS SUBJECT TO DIFFERENT BIT RATE ALLOCATION PROTOCOLS

Quantizer Parameters	Protocol	Allocation of Bit Rate	AUB
$b_1 = 0.6$, $b_2 = 0.9$, $b_3 = 1.2$	AAP	$R_1 = 21$, $R_2 = 21$, $R_3 = 21$	0.3181
	PSO-OAP	$R_1 = 20$, $R_2 = 22$, $R_3 = 22$	0.3148
$b_1 = 0.6$, $b_2 = 2.2$, $b_3 = 2.6$	AAP	$R_1 = 21$, $R_2 = 21$, $R_3 = 21$	0.3415
	PSO-OAP	$R_1 = 19$, $R_2 = 22$, $R_3 = 23$	0.3275

In this case, simulations are conducted under different allocation protocols (i.e. PSO-based optimal allocation protocol (PSO-OAP) and AAP) to analyze their respective effect on the AUB. In the AAP, the total available bit rate is evenly allocated to each node without considering its characteristics. In comparison, PSO-OAP assigns the bit rate to each node subject to the optimized results obtained by Corollary 2.

The total available bit rate of the digital communication network is set to be 64 bps. The decay rate $\bar{\gamma}$ in OP A is set to be 0.96. Moreover, two sets of scaling parameters are selected as shown in Table II to demonstrate the generality of the effectiveness of PSO-OAP and to discuss the effect of different parameters b_i on the estimation performance. Then, the values of AUB can be obtained by solving OP A subject to the AAP and the MINP in (51), respectively. The corresponding results are shown in Table II, from which we can find the following two observations: 1) PSO-OAP performs better than AAP in reducing the AUB of the PNB estimation error system; 2) the larger the difference between the parameters b_i ($i = 1, 2, 3$), the more pronounced the advantage of PSO-OAP in reducing AUB of the PNB estimation error system.

Case 3: **The comprehensive effect of the number of accessible nodes and the bit rate on PNB estimation errors.**

This case aims to discuss the comprehensive effect of the number of accessible nodes and the total available bit rate R_s on the estimation error through numerical simulation. The accessible nodes are selected to be the first i ($i = 1, 2, 3, 4, 5$) nodes, respectively. The parameters of the measurements are set to be:

$$C_1 = C_2 = C_3 = C_4 = C_5 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$D_1 = D_2 = D_3 = D_4 = D_5 = \begin{bmatrix} -0.03 \\ -0.04 \end{bmatrix}.$$

The total available bit rate R_s are set to be 16 bps and 32 bps, separately, and the available bit rate of accessible nodes is allocated by the AAP. The scaling parameters of each quantizer are chosen as $b_1 = 0.6$, $b_2 = 0.7$, $b_3 = 0.6$, $b_4 = 0.9$, $b_5 = 0.8$. The $\bar{\gamma}$ is set to be 0.96 for OP A. Then, a set of simulations are conducted with different number of accessible nodes and different total available bit rate.

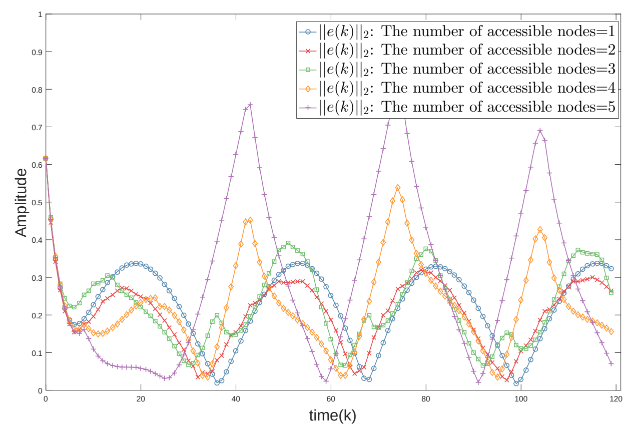

Fig. 6. The trajectories of estimation error $\| e(k) \|_2$ with i accessible nodes and $R_s = 16$ bps ($i = 1, 2, 3, 4, 5$)

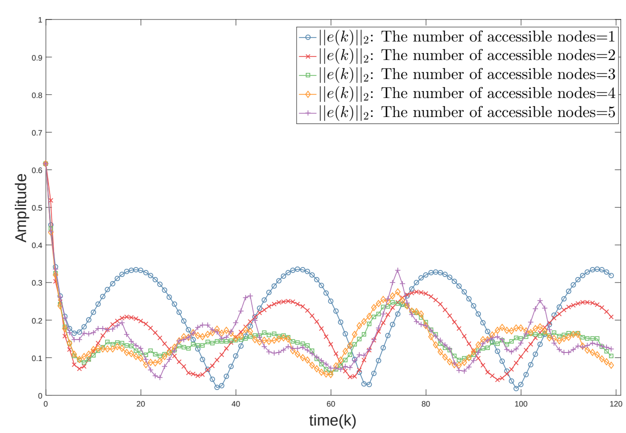

Fig. 7. The trajectories of estimation error $\| e(k) \|_2$ with i accessible nodes and $R_s = 32$ bps ($i = 1, 2, 3, 4, 5$)

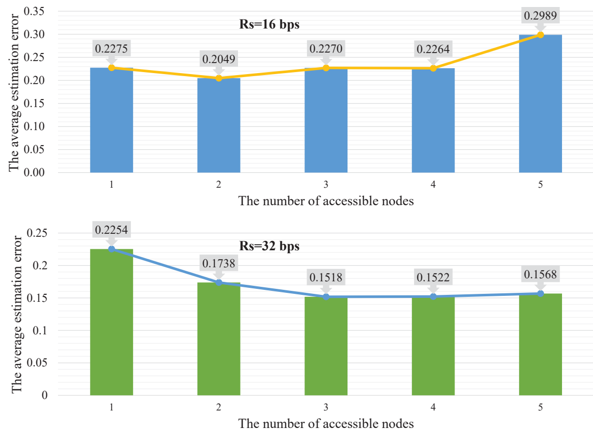

Fig. 8. The average PNB estimation error $\bar{e}(k)$ with $R_s = 16$ bps and $R_s = 32$ bps

The corresponding simulation results are plotted in Figs. 6-8, which indicate that the number of nodes that provide available measurements outputs has a significant effect on the state estimation of a complex network. Under the conditions of $R_s = 16$ and $R_s = 32$, the estimation error $\| e(k) \|_2$ with i ($i = 1, 2, 3, 4, 5$) accessible nodes are plotted in Figs. 6-7. The average estimation error is denoted as $\bar{e}(k) = \| e(k) \|_2 / n$, where n is the simulation run length (120 in this paper). Then, in terms of different available bit rate R_s, the average estimation error $\bar{e}(k)$ with i ($i = 1, 2, 3, 4, 5$) accessible nodes are shown in Fig. 8. From Figs. 6-8, we can find the following three observations: 1) larger bit rate leads to a smaller estimation error; 2) when the bit rate is at a low level ($R_s = 16$), the number of accessible nodes n_0 is not as large as necessary, which is natural since the larger the n_0 is, the less bit rate is allocated to each node and the larger the decoding error; and 3) when the bit rate is set to $R_s = 32$ and the number of accessible nodes is less than 5, the larger the number of accessible nodes, the better the estimation performance, which can be explained intuitively since more near-perfect measurements yield more available information for state estimation.

V. Conclusions

In this paper, the PNB state estimation problem has been investigated for a class of discrete-time complex networks with constrained bit rate. A bit rate constraint model has been proposed to describe the bandwidth allocation of partially accessible nodes in a complex network. A sufficient condition has been constructed under which the estimation error system is ultimately bounded. Then, a bit rate condition that guarantees a particular PNB state estimation performance has been developed. In order to ensure two different estimation performance indices, two OPs have been resolved to obtain the optimized estimators. Moreover, the co-design issue of the bit rate allocation protocol and the estimator gains has been settled. Three illustrative numerical cases have been provided to illustrate the feasibility and effectiveness of our results. Further research topics include 1) the PNB state estimation with other estimation techniques such as the moving horizon estimation [49], [50] and 2) improving the estimation performance by using some effective optimization strategies [24], [25].

References

[1] A. S. Althobaiti and M. Abdullah, Medium access control protocols for wireless sensor networks classifications and cross-layering, *Procedia Computer Science*, vol. 65, pp. 4–16, 2015.

[2] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D. Hwang, Complex networks : Structure and dynamics, *Physics Reports*, vol. 424, pp. 175–308, 2006.

[3] M. R. Bussieck and A. Pruessner, Mixed-integer nonlinear programming, *SIAG/OPT Newsletter: Views & News*, vol. 14, no. 1, pp. 19–22, 2003.

[4] D. Chen, W. Chen, J. Hu and H. Liu, Variance-constrained filtering for discrete-time genetic regulatory networks with state delay and random measurement delay, *International Journal of Systems Science*, vol. 50, no. 2, pp. 231–243, 2019.

[5] J. Chen and Q. Ling, Bit-rate conditions for the consensus of quantized multiagent systems with network-induced delays based on event triggering, *IEEE Transactions on Cybernetics*, vol. 51, no. 2, pp. 984–993, 2021.

[6] Y. Cui, Y. Liu, W. Zhang and F. E. Alsaadi, Sampled-based consensus for nonlinear multiagent systems with deception attacks: The decoupled method, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 51, no. 1, pp. 561ł573, Jan. 2021.

[7] D. Ding, Z. Wang, Q.-L. Han and G. Wei, Neural-network-based output-feedback control under Round-Robin scheduling protocols, *IEEE Transactions on Cybernetics*, vol. 49, no. 6, pp. 2372–2384, 2019.

[8] F. Han, Z. Wang, H. Dong and H. Liu, Partial-nodes-based scalable H_∞-consensus filtering with censored measurements over sensor networks, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 51, no. 3, pp. 1892–1903, 2021.

[9] N. Hou, Z. Wang, D. W. C. Ho, and H. Dong, Robust partial-nodes-based state estimation for complex networks under deception attacks, *IEEE Transactions on Cybernetics*, vol. 50, no. 6, pp. 2793–2802, 2020.

[10] J. Hu, G.-P. Liu, H. Zhang, and H. Liu, On state estimation for nonlinear dynamical networks with random sensor delays and coupling strength under event-based communication mechanism, *Information Sciences*, vol. 511, pp. 265–283, 2020.

[11] J. Hu, Z. Wang, G.-P. Liu and H. Zhang, Variance-constrained recursive state estimation for time-varying complex networks with quantized measurements and uncertain inner coupling, *IEEE Transactions on Neural Networks and Learning Systems*, vol. 31, no. 6, pp. 1955–1967, 2020.

[12] S. Hu, D. Yue, Q.-L. Han, X. Xie, X. Chen and C. Dou, Observer-based event-triggered control for networked linear systems subject to denial-of-service attacks, *IEEE Transactions on Cybernetics*, vol. 50, no. 5, pp. 1952–1964, 2019.

[13] Z. Jin, V. Gupta, and R. M. Murray, State estimation over packet dropping networks using multiple description coding, *Automatica*, vol. 42, no. 9, pp. 1441–1452, 2006.

[14] Y. Ju, G. Wei, D. Ding and S. Zhang, Fault detection for discrete time-delay networked systems with round-robin protocol in finite-frequency domain, *International Journal of Systems Science*, vol. 50, no. 13, pp. 2497–2509, Oct. 2019.

[15] B. Li, Z. Wang, Q.-L. Han and H. Liu, Distributed quasiconsensus control for stochastic multiagent systems under Round-Robin protocol and uniform quantization, *IEEE Transactions on Cybernetics*, in press, DOI: 10.1109/TCYB.2020.3026001.

[16] Q. Li, Z. Wang, N. Li and W. Sheng, A dynamic event-triggered approach to recursive filtering for complex networks with switching topologies subject to random sensor failures, *IEEE Transactions on Neural Networks and Learning Systems*, vol. 31. no. 10, pp. 4381–4388, 2020.

[17] T. Li and L. Xie, Distributed coordination of multi-agent systems with quantized-observer based encoding-decoding, *IEEE Transactions on Automatic Control*, vol. 57, no. 12, pp. 3023–3037, 2012.

[18] W. Li, Y. Jia, and J. Du, Variance-constrained state estimation for nonlinearly coupled complex networks, *IEEE Transactions on Cybernetics*, vol. 48, no. 2, pp. 818–824, 2018.

[19] N. Lin and Q. Ling, Bit-rate conditions for the consensus of quantized multiagent systems based on event triggering, *IEEE Transactions on Cybernetics*, to be published. doi: 10.1109/TCYB.2020.2977969.

[20] D. Liu, Y. Liu, and F. E. Alsaadi, Recursive state estimation based-on the outputs of partial nodes for discrete-time stochastic complex networks with switched topology, *Journal of the Franklin Institute*, vol. 355, no. 11, pp. 4686–4707, 2018.

[21] Y. Liu, Z. Wang, Y. Yuan, and F. E. Alsaadi, Partial-nodes-based state estimation for complex networks with unbounded distributed delays, *IEEE Transactions on Neural Networks and Learning Systems*, vol. 29, no. 8, pp. 3906–3912, 2018.

[22] Y. Liu, Z. Wang, Y. Yuan and W. Liu, Event-triggered partial-nodes-based state estimation for delayed complex networks with bounded distributed delays, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 49, no. 6, pp. 1088–1098, Jun. 2019.

[23] Y. Liu, B. Shen and Q. Li, State estimation for neural networks with Markov-based nonuniform sampling: The partly unknown transition probability case, *Neurocomputing*, vol. 357, pp. 261–270, Sept. 2019.

[24] Y. Liu, Q. Cheng, Y. Gan, Y. Wang, Z. Li and J. Zhao, Multi-objective optimization of energy consumption in crude oil pipeline transportation system operation based on exergy loss analysis, *Neurocomputing*, vol. 332, pp. 100–110, Mar. 2019.

[25] Y. Liu, S. Chen, B. Guan and P. Xu, Layout optimization of large-scale oil-gas gathering system based on combined optimization strategy, *Neurocomputing*, vol. 332, pp. 159–183, Mar. 2019.

[26] L. Ma, Z. Wang, Y. Chen and X. Yi, Probability-guaranteed distributed filtering for nonlinear systems with innovation constraints over sensor networks, *IEEE Transactions on Control of Network Systems*, in press, DOI: 10.1109/TCNS.2021.3049361.

[27] L. Ma, Z. Wang, Y. Liu and F. E. Alsaadi, Distributed filtering for nonlinear time-delay systems over sensor networks subject to multiplicative link noises and switching topology, *International Journal of Robust and Nonlinear Control*, vol. 29, no. 10, pp. 2941–2959, 2019.

[28] J. Mao, D. Ding, G. Wei and H. Liu, Networked recursive filtering for time-delayed nonlinear stochastic systems with uniform quantisation under Round-Robin protocol, *International Journal of Systems Science*, vol. 50, no. 4, pp. 871–884, Mar. 2019.

[29] G. N. Nair and R. J. Evans, State estimation via a capacity-limited communication channel, in *Proceedings of the IEEE Conference on Decision and Control*, vol. 1, San Diego, Califomia, USA, December 1997, pp. 866–871.

[30] G. N. Nair and R. J. Evans, Exponential stabilisability of finite-dimensional linear systems with limited data rates, *Automatica*, vol. 39, no. 4, pp. 585–593, 2003.

[31] G. N. Nair and R. J. Evans, Stabilizability of stochastic linear systems with finite feedback data rates, *SIAM Jour. Control & Optimization*, vol. 43, no. 2, pp. 413–436, 2004.

[32] C. Peng, D. Yue and M.-R. Fei, A higher energy-efficient sampling scheme for networked control systems over IEEE 802.15.4 wireless networks, *IEEE Transactions on Industrial Informatics*, vol. 12, no. 5, pp. 1766–1774, 2015.

[33] H. Rao, Y. Xu, H. Peng, R. Lu, and C.-Y. Su, Quasi-synchronization of time delay Markovian jump neural networks with impulsive driven transmission and fading channels, *IEEE Transactions on Cybernetics*, vol. 50, no. 9, pp. 4121–4131, 2020.

[34] I. Rhee, A. Warrier, M. Aia, J. Min and M. L. Sichitiu, Z-MAC: A hybrid MAC for wireless sensor networks, *IEEE/ACM Transactions on Networking*, vol. 16, no. 3, pp. 511–524, 2008.

[35] B. Shen, Z. Wang, D. Wang and Q. Li, State-saturated recursive filter design for stochastic time-varying nonlinear complex networks under deception attacks, *IEEE Transactions on Neural Networks and Learning Systems*, vol. 31, no. 10, pp. 3788–3800, 2020.

[36] Y. Shen, Z. Wang, B. Shen and F. E. Alsaadi, Nonfragile H_∞ filtering for discrete multirate time-delayed systems over sensor networks characterized by Gilbert-Elliott models, *International Journal of Robust and Nonlinear Control*, vol. 30, no. 8, pp. 3194–3214, 2020.

[37] S. H. Strogatz, Exploring complex networks, *Nature*, vol. 410, no. 6825, pp. 268–276, 2001.

[38] J. Suo, Z. Wang and B. Shen, Pinning synchronization control for a class of discrete-time switched stochastic complex networks under event-triggered mechanism, *Nonlinear Analysis: Hybrid Systems*, vol. 37, art. no. 100886, 2020.

[39] S. Tatikonda and S. Mitter, Control under communication constraints, *IEEE Transactions on Automatic Control*, vol. 49, no. 7, 2004.

[40] X. Wan, Z. Wang, M. Wu and X. Liu, H_∞ state estimation for discrete-time nonlinear singularly perturbed complex networks under the Round-Robin protocol, *IEEE Transactions on Neural Networks and Learning Systems*, vol. 30, no. 2, pp. 415–426, 2019.

[41] L. Wang, Z. Wang, B. Shen and G. Wei, Recursive filtering with measurement fading: A multiple description coding scheme, *IEEE Transactions on Automatic Control*, in press, DOI: 10.1109/TAC.2020.3034196.

[42] W. S. Wong and R. W. Brockett, Systems with finite communication bandwidth constraints - II: Stabilization with limited information feedback, *IEEE Transactions on Automatic Control*, vol. 44, no. 5, pp. 1049–1053, 1999.

[43] Y. Xu, J.-Y. Li, R. Lu, C. Liu, and Y. Wu, Finite-horizon $l_2 - l_\infty$ synchronization for time-varying Markovian jump neural networks under mixed-type attacks: Observer-based case, *IEEE Transactions on Neural Networks and Learning Systems*, vol. 30, no. 6, pp. 1695–1704, 2019.

[44] Y. Xu, C. Liu, J.-Y. Li, C.-Y. Su, and T. Huang, Finite-horizon H_∞ state estimation for time-varying neural networks with periodic inner coupling and measurements scheduling, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 50, no. 1, pp. 211–219, 2020.

[45] K. You and L. Xie, Network topology and communication data rate for consensusability of discrete-time multi-agent systems, *IEEE Transactions on Automatic Control*, vol. 56, no. 10, pp. 2262–2275, 2011.

[46] D. Yue and H. Li, Synchronization stability of continuous/discrete complex dynamical networks with interval time-varying delays, *Neurocomputing*, vol. 73, no. 4–6, pp. 809–891, 2010.

[47] D. Zhao, Z. Wang, D. Ding and G. Wei, H_∞ PID control with fading measurements: The output-feedback case, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 50, no. 6, pp. 2170–2180, 2020.

[48] Z. Zhao, Z. Wang, L. Zou and J. Guo, Set-Membership filtering for time-varying complex networks with uniform quantisations over randomly delayed redundant channels, *International Journal of Systems Science*, vol. 51, no. 16, pp. 3364–3377, 2020.

[49] L. Zou, Z. Wang, and D. Zhou, Moving horizon estimation with non-uniform sampling under component-based dynamic event-triggered transmission, *Automatica*, vol. 120, art. no. 109154, 2020.

[50] L. Zou, Z. Wang, H. Dong, and Q. L. Han, Moving horizon estimation with multi-rate measurements and correlated noises, *International Journal of Robust and Nonlinear Control*, vol. 30, no. 17, pp. 7429–7445, 2020.

[51] L. Zou, Z. Wang, J. Hu, Y. Liu and X. Liu, Communication-protocol-based analysis and synthesis of networked systems: progress, prospects and challenges, *International Journal of Systems Science*, in press, DOI: 10.1080/00207721.2021.1917721.

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