Comparison between Probabilistic Optimal Power Flow and Probabilistic Power Flow with Carbon Emission Consideration

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***Abstract*—With more uncertainty and variability existing in smart grid, deterministic load flow, which is used to analyze the operation conditions on a daily routine and planning the power systems for future investment, could not solve the problems with consideration of renewable generation intermittence and load variation. Nowadays, with more attentions are paid to the environment, carbon emission problem is one of the main concerns in smart grid strategies and planning. It is a great opportunity and challenge for planners to take good care of all stakeholders' interests. Multi-objectives need to be considered when making a critical decision. This paper presents result comparisons between probabilistic optimal load flow and probabilistic load flow by considering both carbon emission and minimization of power loss of the entire grid. The framework is applied to the modified IEEE 14-bus system, which is modeled in PowerFactory DIgSILENT, with intermittent wind energy source and load variation consideration.**

***Index Terms*—Probabilistic power flow, optimal power flow, power losses optimization, carbon emission**

I. Introduction

Smart grid has been advocated in both developed and developing countries for decades to deal with energy deficit and air pollutions. In traditional power systems, loads variation, outages of power lines and generation availability have variability and uncertainty to some degrees [1]. With large amount of renewable energy sources integrated at generation side and new devices distributed in demand side like electric vehicles and distributed generations, smart grid faces more uncertainty and variability than before [2]. Probabilistic load flow is one of the efficient tools for analyzing and estimating the entire power network operating conditions and planning investment on power system facilities with uncertainty and variability considerations [3].

Probabilistic power flow (PPF) was firstly proposed to solve the load uncertainty by Barbara Borkowska in 1974 [4]. Inputs (such as loads and intermittent wind and solar generations) with appropriate features (like probability density functions (PDF) or cumulative distribution functions (CDF)) are necessary for calculating the system state variables with uncertainty features. By knowing the probability result of these variables, system operational boundary can be obtained for further system operating and planning. Generally, there are two methods to solve the PPF problems, namely numerical methods and analytical methods [5]. The typical method of numerical one is Monte-Carlo, while the analytical methods including convolution method, point estimate method, and its combinations [6-9].

Probabilistic power flow has been proposed in many literatures to solve problems about wind and solar source integrating with power system [10-15]. Reference [10] proposed a method combined cumulates and Gram-Charlier expansions to solve the PLF problems with wind sources integrated. An Unscented Transformation method was proposed in [11] to solve the PLF problems in a distribution system with distributed generations. Reference [12] introduced so call "probabilistic distribution load flow" to modelling different wind turbines. A discrete point estimate method was suggested in [13] based on the measured data of wind power.

There are many literatures about probabilistic optimal load flow so far. Reference [16] determined the probabilistic distributions of solution based on a first-order second-moment method, and the Karush-Kuhn-Tucker (KKT) conditions of the probabilistic optimal load flow are transformed into a set of non-smooth nonlinear equations, which can be solved by an inexact Levenberg-Marquardt algorithm. Reference [17] uses probabilistic optimal power flow based on a two-point estimate method for locational marginal price calculation. Probabilistic optimal power flow by percentiles estimation with Weibull probability distribution function of system loading is proposed in [18]. The random nature of lower heat value of biomass and load were taken into account in probabilistic power flow and shuffled frog-leaping algorithm is applied in [19] for optimizing allocation of biomass fueled gas engine in unbalanced radial systems.

There are two main methods for lowering carbon emission from both the generation side and demand side, namely economic dispatch and demand side management [21]. Electric loads are regarding as the main reason for carbon emission. Increasing demand can lead to heavier loading on the generation side. Consequently, more carbon emission is produced. Demand side management is one of the effective ways to reduce carbon emission. In generation side, because different generators have different carbon emission levels, economic dispatch can reorganize the generators to achieve the optimal result for carbon emission.

This paper presents result comparisons between probabilistic optimal load flow and probabilistic load flow for minimizing the total power loss with carbon emission consideration of the whole power network. The approach combines optimal power flow with interior-point algorithm and Monte-Carlo sampling method to find out the optimal operation points with system uncertainty considerations. The paper is organized as follows. Section II introduces the mathematical background of the algorithm. In Section III, the combination framework to solve the POPF is given in details. Section IV gives an illustrative example for the framework. Finally, Section V concludes the whole paper and future work related on the probabilistic optimal power flow will be stated.

II. Mathematical Background

Deterministic power flow (DPF), which is used to analyze the operation conditions on a daily routine and planning the power systems for future investment, can provide system states and power flow of a power network with known topology and operating parameters. However, DPF can only calculate the power flow with specific value of the generators and loads. Variability and uncertainty, which are the inherent characteristics of the power system, are ignored. Probabilistic power flow was proposed to deal with these uncertainty and variability in power system by inputs within appropriate range, for instance, inputs of intermittence energy sources such as wind and solar, and load variations in a reasonable range.

A. Monte-Carlo simulation for PPF

Monte-Carlo simulation is the classical numerical method to deal with the probability problems. Monte-Carlo method can generate random number and random sampling with cumulative density function, and iterate the power flow one by one. In this paper, two uncertainties, wind generations and loads are considered to simulate the probabilistic power flow.

Load variation probability can be modeled by normal distribution function with appropriate standard deviation settings. So the sampling function can be obtained by the transformation method. Equation (1) gives the sampling function of normal distribution function.

$$\begin{aligned} P_i' &= P\times(1+sd)\times\sqrt{-2\ln(r_1)}\times\cos(2\pi\times r_2) \\ Q_i' &= Q\times(1+sd)\times\sqrt{-2\ln(r_1)}\times\cos(2\pi\times r_2) \end{aligned} \quad (1)$$

Where P_i and Q_i are the active and reactive power of the ith load sample, P, Q and sd are the mean value of the active power and reactive power of loads and standard deviation of loads respectively. r_1 and r_2 are the uniformly distributed random number between 0 and 1. In this paper, the standard deviation of load sampling sets to 15% as an example.

Probability of wind speed at wind generation site can be modelled by Weibull distribution. Equation (2) illustrates the general function of Weibull distribution.

$$f(x)=\frac{\beta}{\lambda}\left(\frac{x}{\lambda}\right)^{\beta-1}e^{\left[-\left(\frac{x}{\lambda}\right)^{\beta}\right]} \quad (2)$$

Where β is the shape parameter, λ is the scale parameter, x is the random variables. Therefore, the sampling equation for Weibull distribution can be obtained by inverse of Equation (2), which is shown in Equation (3).

$$W_{si}=\lambda[-\ln(r_3)]^{\frac{1}{\beta}} \quad (3)$$

Where W_{si} is the wind speed of the ith sample. r_3 is the uniformly distributed random number within the range of (0, 1). In this paper, wind speed is modelled with β=2.3, λ=8. Figure 1 illustrates the shape of the Weibull distribution curve.

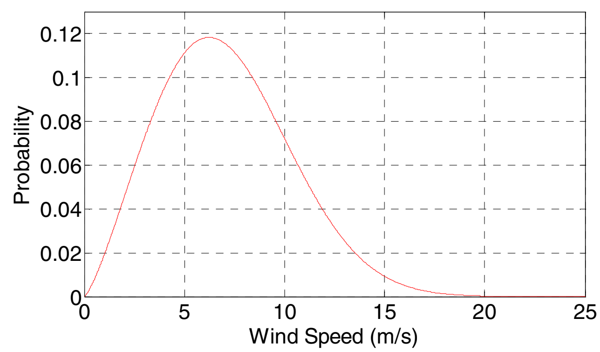

Fig. 1. Shape of the Weibull distribution curve for wind speed application

By knowing the wind speed of a wind generation site, the output power from the wind turbine can be determined. To simplify the model, the output power of a wind generator can be recognized as linear related to its cut-in and rated speed. Figure 2 demonstrates an output power curve of a single wind turbine with rated power 2MW. To model the wind generators in this study, fixed power factor 0.8 is selected. The output power can be modelled with Equation (4).

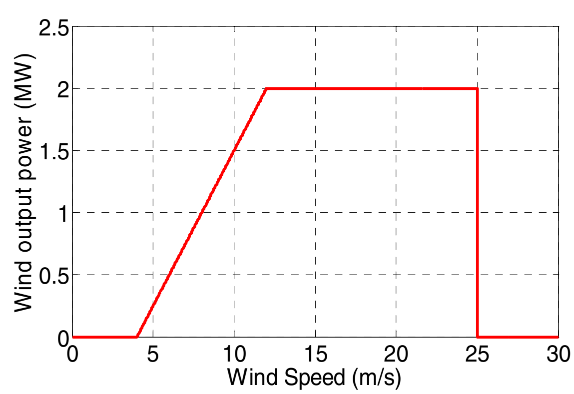

Fig. 2. Single wind turbine output power vs. wind speed

$$\begin{cases} P_{wind} = 0 & W_s < V_i \\ P_{wind} = P_r \cdot \dfrac{W_s - V_i}{V_r - V_o} & V_i \le W_s \le V_r \\ P_{wind} = P_r & V_r < W_s \le V_o \\ P_{wind} = 0 & W_s > V_o \end{cases} \tag{4}$$

Where P_{wind} is the output wind power, P_r is the rated wind power, W_s is the wind speed, V_i, V_r and V_o are the cut-in, rated and cut-out wind speed respectively.

Because the detailed wind turbine is not required, the output wind power can be recognized as negative loads. The output of a wind farm is modelled aggregately by summing the single wind turbines, which is shown in Equation (5).

$$P_{farm} = \sum_{i=1}^{\max} P_{windi} \tag{5}$$

B. Interior-point method for optimal power flow

Optimal power flow with interior-point method is used to optimize the operation of a power system by minimizing an objective function with considering a set of constraints. Carbon emission from generators in the whole network can be illustrated in Equation (6).

$$f(\mathrm{x}) = \sum_{i=1}^{N_G} P_{Gi} \cdot I_{Ci} \tag{6}$$

Where P_{Gi} is the power output from the generator *i*, NG is the total traditional power generators in the network.

Four constraints are considered in this study, which are power flow balancing, generation output limits, operational bus voltage limits and branch flow limits. Equality and inequality constraints are described as in Equations (7)-(10).

1. Power flow balancing

$$\begin{aligned} P_{Gen} &= P_{Load} + P_{Loss} \\ Q_{Gen} &= Q_{Load} + Q_{Loss} \end{aligned} \tag{7}$$

2. Generation output constraints

$$P_{\min} \le P_i \le P_{\max} \tag{8}$$

3. Operational bus voltage limits

$$V_{\min i} \le V_i \le V_{\max i} \tag{9}$$

4. Branch flow limits

$$PF_{\mathrm{line}i} \le PF_{\max i} \tag{10}$$

Where P_{Gen}, Q_{Gen} are the total generated active power and reactive power respectively; P_{Load} and Q_{Load} are the active power and reactive power of the total load respectively; P_{Loss} and Q_{Loss} are the total power loss of the grid; P_{min}, P_{max}, Q_{min} and Q_{max} are the minimum and maximum value of the active and reactive power respectively. P_i and Q_i are the active and reactive power of the generator *i*; V_{mini} and V_{maxi} are the minimum and maximum operational voltage. Generally, the voltage boundary is from 0.95 p.u. to 1.05 p.u, V_i is the voltage magnitude of the bus *i*; PF_{linei} is the power flow of the branch *i*, and PF_{maxi} is the capacity of line *i*.

To solve non-linear optimization problem, an iterative interior-point algorithm based on the Newton-Lagrange method is used [22]. A slack variable for each inequality constraint is introduced to reformulate the constraint equation, which is shown in Equation (11).

$$H(\vec{x}) + \vec{s} = 0 \quad \vec{s} \ge 0 \tag{11}$$

Where *H* is the set of the inequality constraints. Therefore, the logarithmic penalties can be incorporated to the objective function. So Equation (6) can be reformulated in Equation (12).

$$f(\mathrm{x}) = \sum_{i=1}^{N_G} P_{Gi} \cdot I_{Ci} - \mu \cdot \sum \ln(\mathrm{s_i}) \tag{12}$$

Where μ is the penalty weighting factor, which will be decreased from an initial value to a target value to minimize the objective function.

III. The Proposed Framework

With random variables values, mean value and standard deviation of the load specified, the optimal power flow is ready to run. The main procedures can be divided in several steps, which are demonstrated below:

Step 1. The maximum number of sampling is determined and initializes all input variables for iteration procedure.

Step 2. Update the sampling counter and generate samples.

Step 3. Start the interior-point method to evaluate the objective function.

Step 4. Compute the expected value of the outputs with all counted sampling, line loading, generation loadings and voltage magnitude, which are concerned in this study.

Step 5. Save the result and check the sampling criterion, if the sampling times reach the maximum number which was set in Step 1, the algorithm will be terminated. Otherwise, the algorithm will repeat the procedure from Step 2.

For the interior-point estimation in Step 3 of the Monte-Carlo procedure, the algorithms can be illustrated in the following steps:

Step 1. Set the initial values and target values, reduction factor of the penalty weighting factor and initialize the system data by running traditional power flow.

Step 2. Update the penalty weighting factor by deduction factor.

Step 3. Update the solution and check whether the convergence criterion has been achieved. If so, terminate the OPF procedure. If not, go back to step 2.

The complete procedure of the POPF is presented in Figure 3.

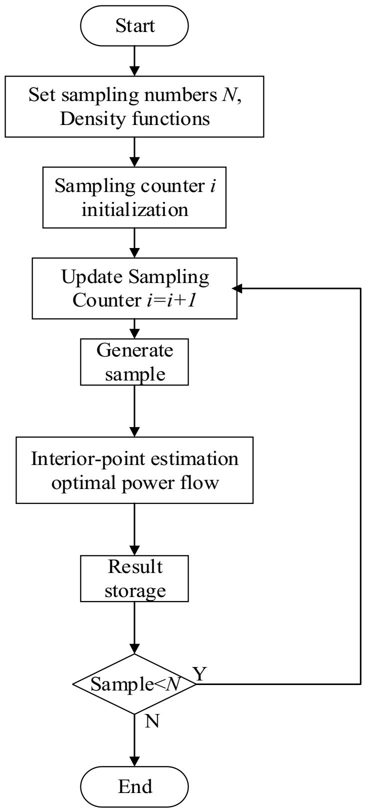

Fig. 3. Proposed POPF procedures

IV. CASE STUDY

Based on the proposed procedure for probabilistic optimal load flow, a modified IEEE-14 busbar system model, whose topology is shown in Figure 4, is investigated with PowerFactory DIgSILENT software package. Revised codes with DIgSILENT Programming Language (DPL) for probabilistic load flow [20] is used in this study. DPL is an interface for automating tasks like decision and flow commands, accessing objects, mathematical expressions etc. by scripting with C language syntax. In this study, Monte-Carlo method is adopted and sampling equations in Section II is applied to calculate the total power loss of the entire network and power injection from the reference machine when wind generators output power and loads are changing. 10 wind generators with 2MW capacity are aggregated in Bus 8. Three transformers with 5 tapping from 0 to ±5% are connected between 69kV busbars and 13.8kV busbars. Operational limitations of each generator is shown in Table I. Carbon intensity ([21], Table I) of each generator is presented in the table as well.

A. PPF for Carbon Emission consideration

To perform a PPF with inclusion of carbon emission, generators with lower carbon intensity are set as full loading condition, and the one with the highest intensity as the reference machine, which is connected to the slack bus. Tap changer of all transformers are set to 0%.

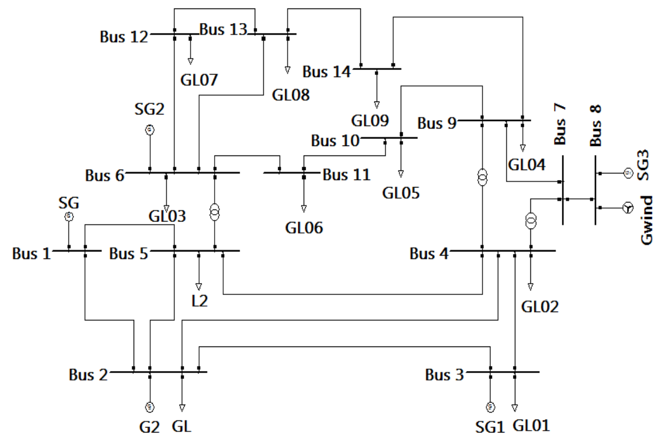

Fig. 4. Modified IEEE 14-bus test system

B. POPF for power loss minimization

Based on the carbon emission consideration, active power and reactive power dispatch will not participate in the optimal power flow. To minimize the power losses of the whole network, tap changers of all transformers are 0, ±2.5% and ±5% of the rated power. To compare the two algorithms, reference generator output power of the two cases and the total power loss of the two cases are studied. The real value from each iteration and expected value of the power injected from the slack bus and total power losses are compared.

Figures 5 and 6 depict the total power loss of the entire grid with both algorithms. It can be obviously seen that the boundary of the power loss with PPF is larger. In Figure 7, slack bus power injection is compared between the two cases. Power injection boundary of the POPF case, especially the upper limit boundary, is much smaller than that of PPF, which indicates the reference generator can produce less power to the external grid. When the voltage level of the network is getting

TABLE I
GENERATION OPERATION PARAMETERS WITH CARBON INTENSITY

Generation	Bus No.	PG max	Carbon Intensity(T/MWh)
SG	Bus 1	100	1.186
G2	Bus 2	70	0.78
SG1	Bus 3	65	0.78
SG2	Bus 6	80	0.434
SG3	Bus 8	65	0.434

high, adjusting tapping changer of the transformers will keep the iron core loss of the transformer nearly constant. While the line loss and the loss of the transformer winding is getting smaller. Consequently, the total power loss of the entire grid is minimized. The total power from the external grid is getting smaller, which implies that the total network power has been reduced. Figures 6 and 8 illustrate the expected value of the output power from the slack bus changes and total power loss with increased number of sampling respectively. Equation (13) computes the expected value of selected variables.

$$E(x)_{\mathrm{m}} = \frac{\sum_{i=1}^{m} x_i}{m} \qquad (m \in N, m \leq k) \tag{13}$$

Where m is sampling number, k is the maximum sampling number, and x_i is the ith value of the sampling.

It can be seen from Figure 8 that the expected value of the output power from the slack bus by POPF is negative. Consequently, with the load and wind power variation, the probability of grid exporting power to the external grid is much higher.

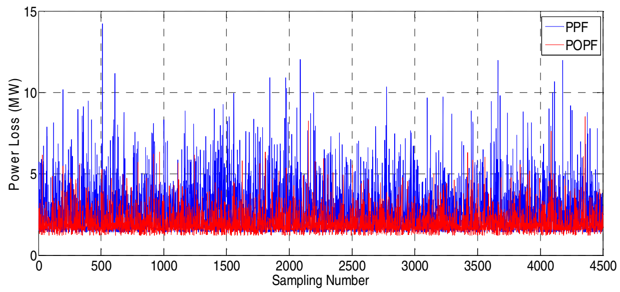

Fig. 5. Total power loss of the entire network during sampling

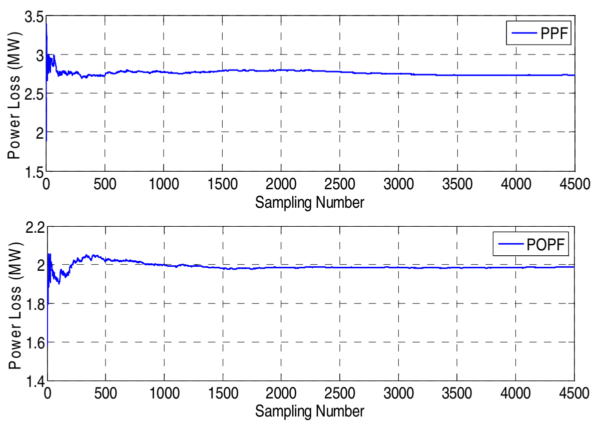

Fig. 6. The expected power loss of the entire network during sampling

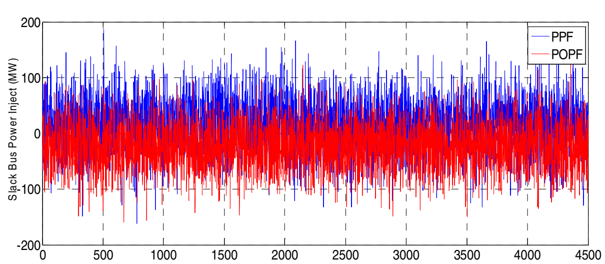

Fig. 7. Power injection from the reference machine during sampling

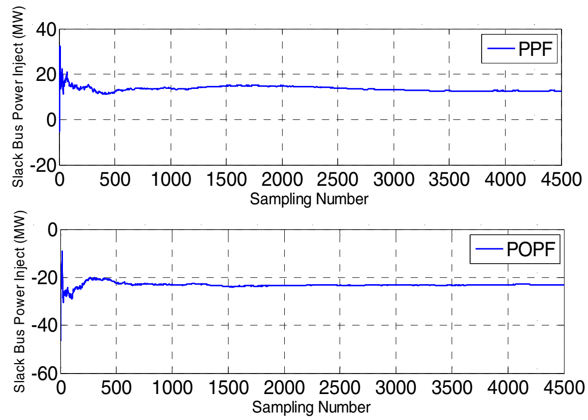

Fig. 8. Power injection from the reference machine during sampling

V. CONCLUSIONS

Uncertainty and variability are the inherent features in power grid. With large renewable energy participation and more flexible load integration in smart grid, these features are becoming much more apparent. Considering both entire network, carbon emission and optimizing the power loss, the results show that probabilistic optimal load flow should produce a more accurate solution. The results of the simulation show that with keeping the carbon emission in the lowest level, the power grid can achieve more efficient power delivery and keep the power loss in the lowest level. Wind energy intermittence and load variations are considered with probabilistic variations.

In future, further modeling details need to be included, such as more complicated wind turbine, solar system and electric vehicles models. More features such as load shedding minimization, transient optimal power flow should also be considered.

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