H_∞ PID Control for Discrete-Time Fuzzy Systems with Infinite-Distributed Delays under Round-Robin Communication Protocol

Yezheng Wang, Zidong Wang, Lei Zou and Hongli Dong

***Abstract*—This paper is concerned with the H_∞ proportional-integral-derivative (PID) control problem for class of discrete-time Takagi-Sugeno fuzzy systems subject to infinite-distributed time-delays and Round-Robin (RR) protocol scheduling effects. The information exchange between the sensors and the controller is conducted through a shared communication network. For the purpose of alleviating possible data collision, the well-known RR communication protocol is deployed to schedule the data transmissions. To stabilize the target system with guaranteed H_∞ performance index, a novel yet easy-to-implement fuzzy PID controller is developed whose integral term is calculated based on the past measurements defined in a limited time-window with hope to improve computational efficiency and reduce accumulation error. Based on the Lyapunov stability theory and the convex optimization technique, sufficient conditions are derived to ensure the exponential stability as well as the H_∞ disturbance attenuation/rejection capacity of the underlying system. Furthermore, by utilizing the cone complementarity linearization algorithm, the non-convex controller design problem is transformed into an iterative optimization one that facilitates the controller implementation. Finally, simulation examples are given to show the effectiveness and correctness of the developed control method.**

***Index Terms*—Fuzzy systems, Round-Robin protocol, proportional-integral-derivative control, linear matrix inequality, cone complementarity linearization.**

I. Introduction

The past decades have witnessed a large amount of research attention devoted to Takagi-Sugeno (T-S) fuzzy systems. Due to its powerful approximation ability, the T-S fuzzy model is known to be effective in describing many complex nonlinear systems. Generally speaking, any smooth nonlinear function in a typical T-S fuzzy system can be approximated by a set of linear functions being connected together by nonlinear function memberships through fuzzy sets and fuzzy reasoning with any given accuracy [1], [2]. In consideration of the special and convenient structure of such a T-S fuzzy framework, a convenient way of dealing with complex nonlinear systems is to obtain the approximated linear subsystems based on the T-S fuzzy technology and then design the required controllers/filters according to the parallel distributed compensation scheme. As such, a large volume of literature has been available on the analysis and synthesis problems for T-S fuzzy systems, see e.g., the technical literature [3]–[16] and a survey [17].

The original idea of the proportional-integral-derivative (PID) feedback control dated back to 1910 and, since then, such a control method has been widely adopted in almost all sectors of control engineering practices. Despite a variety of modern control methods developed in the past few decades, the PID control algorithm has continued to show its overwhelming popularity as more than 90% of industrial controllers are still based on the PID mechanism [18]. The wide range application of PID control scheme is mainly due to its easy implementation, convenient adjustment and clear functionality. An indispensable procedure for PID control applications is the parameter tuning that directly affects the performance of the controlled system such as stability, transient/steady property as well as robustness. Thus, a great deal of research attention has been devoted to the development of adequate parameter tuning approaches, see e.g. [18]–[23] for some seminal works on this aspect.

Along with the rapid development of industrialization and automation, the control problems for nonlinear complex systems have received an ever-increasing research interest. Conventional PID controllers are generally incapable of dealing with systems with severe nonlinearities due primarily to the lack of systematic procedure in adjusting the control parameters. Accordingly, some improved PID control schemes have been developed to handle the nonlinear control problems and some representative control strategies include the artificial-neural-network-based PID control [24], fuzzy PID control [25], expert-based PID control [26] and adaptive wavelet PID control methods [27]. In particular, due to its effectiveness in dealing with nonlinearities and the successful application in real industrial practice, the fuzzy logic has attracted special attention in the hope of enhancing the performance of traditional PID controllers for nonlinear systems.

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Y. Wang is with the College of Electrical Engineering and Automation, Shandong University of Science and Technology, Qingdao 266590, China.

Z. Wang is with the College of Electrical Engineering and Automation, Shandong University of Science and Technology, Qingdao 266590, China; and is also with the Department of Computer Science, Brunel University London, Uxbridge, Middlesex, UB8 3PH, United Kingdom. (Email: Zidong.Wang@brunel.ac.uk).

L. Zou is with the Department of Computer Science, Brunel University London, Uxbridge, Middlesex, UB8 3PH, United Kingdom.

H. Dong is with the Institute of Complex Systems and Advanced Control, Northeast Petroleum University, Daqing 163318, China; and is also with the Heilongjiang Provincial Key Laboratory of Networking and Intelligent Control, Northeast Petroleum University, Daqing 163318, China.

The fuzzy PID controller combines both the merits of PID control and fuzzy control by utilizing the system information and/or human knowledge to a higher extent. It should be pointed out that, over the past decades, fuzzy PID control has become a significant branch of the fuzzy control field, and a large amount of literature has appeared. For example, a special fuzzy PID controller has been constructed by using the error and the rate of change of error as its two inputs in [28]. Afterwards, in order to improve the transient property of nonlinear uncertain systems, in [25], the accelerated rate of change of error has been taken into account as an extra input in the process of fuzzy PID design. In [29], the T-S fuzzy model has been employed to generate fuzzy PID control laws where the robust H_∞ control performance has been also achieved.

Time-delays are well known to occur frequently in real-world systems due to a variety of reasons such as transport in long pipeline, communication constraint in network, aging of the devices and so on. The existence of time-delays influences the evolution of system states, thereby becoming one of the main sources in degrading system performance and even resulting in instability. Thus, it is necessary to take this inevitable phenomenon into consideration when analyzing/designing control systems with time-delays. It should be mentioned that, among different types of time-delays, the distributed time-delays (DTDs) have recently proven to be particularly prevalent especially in process industry and, accordingly, the DTDs have attracted much attention from the research community, where most existing results have been concerned with *continuous-time* systems with finite or infinite DTDs, see e.g. [30]–[32]. Note that, with the popularity of digitization, more and more discrete-time systems have been applied in practice, and the analysis/synthesis issues of discrete-time systems with DTDs have gained some research interest. For instance, the control problem of discrete-time T-S fuzzy systems with DTDs has been studied in [11].

On another research forefront, owing to the quick evolution of the network communication technologies, considerable research attention has been paid to the networked systems [33]–[35]. Compared with the traditional control systems, the utilization of common communication networks offers several benefits such as low cost, large flexibility, high reliability and simple installation/maintenance [36], and also leads to certain unfavorable network-induced phenomena such as channel fadings [11], [37], packet dropouts [3], [38], quantization effects [39]–[41], sensor saturations [42] and so on. To mitigate the network congestion and avoid the network-induced phenomena, an effective way is to introduce the communication protocol so as to schedule the information exchange on a shared channel.

Recently, the protocol-based networked systems have begun to stir some initial research interest [37], [41], [43]–[45]. For example, in [41], the ultimate boundedness control problem has been investigated for quantized networked control systems (NCSs) subject to Try-Once-Discard protocol. The quantized control problem has been studied in [39] for networked systems with the Round Robin (RR) protocol. Based on a time-varying system approach, the stabilization problem of NCSs under two types of stochastic protocols has been investigated in [46]. Nevertheless, to the best of the authors' knowledge, the H_∞ fuzzy PID control problem has not been studied yet for fuzzy systems with infinite-DTDs and RR protocol, which is probably due to the resultant system complexity, and we are therefore inspired to shorten such a gap in this paper.

In response to the discussions made thus far, our aim in this paper is to deal with the H_∞ fuzzy PID control problem for discrete-time fuzzy systems subject to infinite-DTDs and RR protocol. In doing so, we are facing two substantial challenges identified as follows: 1) how to analyze the H_∞ performance of the considered system subject to infinite-DTDs and RR protocol? and 2) how to design the desired fuzzy PID output-feedback controller based on the confining measurement output? These two questions will be answered in the main results of this paper. *The main contributions of this paper are highlighted as follows. 1) The H_∞ fuzzy PID control problem is, for the first time, investigated for fuzzy systems subject to infinite-DTDs and RR protocol scheduling. 2) A novel and easy-to-implement fuzzy PID controller is constructed to deal with the H_∞ control problem. 3) The parameters of the switching-signal-dependent controller are derived by an iterative optimization algorithm.*

The rest of this paper is arranged as follows. Section II describes the H_∞ control problem for a class of discrete-time T-S fuzzy systems with infinite-DTSs and RR protocol. Section III gives our main results, where the stability and the prescribed H_∞ performance of the considered system are discussed, and an optimization procedure based on the cone-complementarity-linearization (CCL) algorithm is proposed to obtain the parameters of the desired fuzzy PID controller. In Section IV, numerical examples are presented to validate the usefulness of the proposed design method. Finally, the conclusion of this paper is drawn in Section V.

Notations: The notations in this paper are fairly standard. Throughout this paper, $\mathbb{R}^n$, $\mathbb{Z}^-$, and $\mathbb{Z}^+$ refer to, respectively, the n-dimensional Euclidean space, the set of negative integers and the set of positive integers. For a matrix M, the notations M^T, $\mathrm{tr}(M)$ and $\lambda_{\min}(M)$ denote its transposition, trace and minimum eigenvalue, respectively. The space of square summable sequences is represented by $l_2[0,\infty)$. For symmetric matrices X and Y, $X \geq Y$ and $X > Y$ are used to show that $X - Y$ is positive semi-definite and positive definite, respectively. The shorthand $\mathrm{diag}\{\cdots\}$ is a block-diagonal matrix. $\|\cdot\|$ is the Euclidean vector norm. In a symmetric matrix, an asterisk "$*$" denotes a term induced by symmetry. I and 0 represent, respectively, the identity matrix and zero matrix with appropriate dimensions. $\mathrm{mod}(a,b)$ means the non-negative remainder on division of the integer a by the positive integer b. The symbol $\delta(i-j)$ denotes the Kronecker delta function taking values on 0 (when $i \neq j$) or 1 (when $i = j$).

II. Problem Statement and Preliminaries

A. Plant Model

In this paper, a schematic sketch of the addressed discrete-time fuzzy systems with infinite-DTDs is shown in Fig. 1, where the data exchange between sensors and the controller is implemented through a shared communication network equipped with the RR protocol. In what follows, we will introduce the plant, the communication network, and the adopted fuzzy PID controller in the state space.

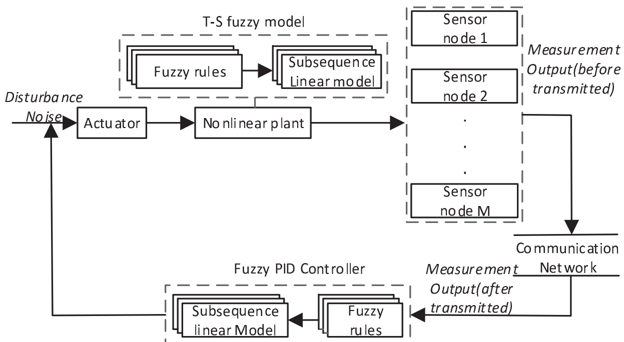

Fig. 1: The T-S fuzzy systems with a network

The mathematic model of the considered system shown in Fig. 1 can be described by

Plant Rule i: **IF** $\theta_1(k)$ is ϑ_{i1} and $\theta_2(k)$ is ϑ_{i2} and $\cdots$ and $\theta_l(k)$ is ϑ_{il}, **THEN**

$$\begin{cases} x(k+1) = A_i x(k) + A_{\epsilon i}\sum_{\epsilon=1}^{\infty}\mu_\epsilon x(k-\epsilon) + B_i u(k) \\ \qquad\qquad + E_{1i}v(k) \\ y(k) = C_i x(k) + E_{2i}v(k) \\ z(k) = F_i x(k) \\ x(k) = \varphi(k), \qquad \forall k \in \mathbb{Z}^- \end{cases} \tag{1}$$

where r is the number of fuzzy rules; ϑ_{ij} is fuzzy set; $\theta_1(k), \theta_2(k), \cdots, \theta_l(k)$ are the premise variables; $x(k) \in \mathbb{R}^{n_x}$ is the state vector; $u(k) \in \mathbb{R}^{n_u}$ is the control input; $v(k) \in (l_2[0,\infty), \mathbb{R}^{n_v})$ is the external disturbance (comprising process noise and measurement noise); $y(k) \in \mathbb{R}^{n_y}$ and $z(k) \in \mathbb{R}^{n_z}$ are, respectively, the measurement output before transmitted through the communication network and the controlled output; $\varphi(k)$ is the initial state function which takes real values on $(-\infty, 0]$. A_i, $A_{\epsilon i}$, B_i, C_i, E_{1i}, E_{2i}, F_i are known constant matrices with appropriate dimensions.

Assumption 1: The constants $\mu_\epsilon \geq 0$ $(\epsilon = 1, 2, \cdots)$ satisfy the following convergence conditions:

$$\bar{\mu} \triangleq \sum_{\epsilon=1}^{\infty}\mu_\epsilon \leq \sum_{\epsilon=1}^{\infty}\epsilon\mu_\epsilon < +\infty. \tag{2}$$

Remark 1: In the considered fuzzy system (1), the delay term $\sum_{\epsilon=1}^{\infty}\mu_\epsilon x(k-\epsilon)$ is the so-called infinite-DTD in the discrete-time domain. Such a description was first introduced in [49] and can be regarded as the analogy of the continuous-time case. Under Assumption 1, the constants μ_ϵ $(\epsilon = 1, 2, \cdots)$ satisfy the convergence condition (2), which is to ensure the convergence of the terms of $A_{\epsilon i}\sum_{\epsilon=1}^{\infty}\mu_\epsilon x(k-\epsilon)$ as well as the Lyapunov-Krasovskii functional (LKF) to be defined later. It should be mentioned that the available literature regarding the discrete infinite-DTDs has really scattered as compared to its continuous-time counterpart, not to mention the case that the fuzzy PID control problem is also considered for T-S fuzzy systems.

Through fuzzy reasoning, the final outputs of the fuzzy system (1) are obtained as follows:

$$\begin{cases} x(k+1) = \sum_{i=1}^{r} h_i(\theta(k))\Big(A_i x(k) + A_{\epsilon i}\sum_{\epsilon=1}^{\infty}\mu_\epsilon x(k-\epsilon) \\ \qquad\qquad + B_i u(k) + E_{1i}v(k)\Big) \\ y(k) = \sum_{i=1}^{r} h_i(\theta(k))\Big(C_i x(k) + E_{2i}v(k)\Big) \\ z(k) = \sum_{i=1}^{r} h_i(\theta(k))F_i x(k) \end{cases} \tag{3}$$

where $\vartheta_{ij}(\theta_j(k))$ represents the grade of membership of $\theta_j(k)$ in ϑ_{ij} with $\theta(k) = \big[\theta_1(k), \theta_2(k), \cdots, \theta_l(k)\big]$ and the fuzzy-basis functions are given by

$$h_i(\theta(k)) = \frac{a_i(\theta(k))}{\sum_{j=1}^{r} a_j(\theta(k))},\ a_i(\theta(k)) = \prod_{j=1}^{l}\vartheta_{ij}(\theta_j(k)). \tag{4}$$

Remark 2: In order to guarantee the non-negativity of the membership functions $\sum_{i=1}^{r} h_i(\theta(k))$, it is always assumed that for $\forall k$, $a_i(\theta(k)) \geq 0$ $(i = 1, 2, \cdots, r$, but not all zeros), and we therefore have the conclusion that $\sum_{i=1}^{r} h_i(\theta(k)) = 1$ and $h_i(\theta(k)) \geq 0$ $(i = 1, 2, \cdots, r)$ for $\forall k$. In addition, the information about the premise variables $\theta_i(k)$ $(i = 1, 2, \cdots, l)$ is made available to the system output $y(k)$ in (3), thereby facilitating the implementation of the desired fuzzy PID controller.

B. Communication Network

Now, let us introduce the effects induced by the RR protocol of the communication network. Without loss of generality, we assume that the sensors can be divided into M $(M > 1)$ sensor nodes according to their spatial distribution. Let $y_{\bar{i}}(k)$ $(\bar{i} \in \{1, 2, \cdots, M\})$ denote the measurement output of the $\bar{i}$th node before being transmitted. Then, $y(k)$ can be rewritten as follows:

$$y(k) = \begin{bmatrix} y_1^T(k) & y_2^T(k) & \cdots & y_M^T(k) \end{bmatrix}^T. \tag{5}$$

In the network environment described in Fig. 1, all the nodes transmit their information via a shared communication network. Due to the inherently limited bandwidth of communication channels in engineering practice, data collision is very likely to occur if all nodes are connected to the shared network and request to send data simultaneously. Clearly, unnecessary data collisions give rise to network-induced phenomena such as packet dropouts and communication delays. To resolve this issue, in this paper, the well-known RR protocol is employed to determine which node can access network at each transmission instant. To be more specific, the RR protocol is a static "scheduling agreement" which allocates the equal opportunity of accessing the network to every node. Due to the fixed transmission mechanism, the RR protocol is easy to be implemented in engineering practice and effective to reduce transmission burden. Thus, we deploy the RR protocol in the sensor-to-controller channel to schedule network resources.

Let $\xi(k)$ ($\xi(k) \in \{1, 2, \cdots, M\}$) denote the selected sensor node at time instant k. Under the effects of RR protocol scheduling, $\xi(k)$ satisfies the condition $\xi(k + M) = \xi(k)$ for $\forall k \in \mathbb{Z}^+$. Without loss of generality, set $\xi(k) = k$ ($k \in \{1, 2, \cdots, M\}$). Then, the values of $\xi(k)$ can be calculated as follows:

$$\xi(k) = \mathrm{mod}(k - 1, M) + 1. \tag{6}$$

In addition, the zero-holder strategy is adopted in the considered system to hold the output signal. Accordingly, the update rule of $\bar{y}_{\bar{i}}(k)$ is given by

$$\bar{y}_{\bar{i}}(k) = \begin{cases} y_{\bar{i}}(k), & \bar{i} = \xi(k) \\ \bar{y}_{\bar{i}}(k-1), & \bar{i} \neq \xi(k) \end{cases} \tag{7}$$

where $y_{\bar{i}}(k)$ represents the measurement output after being transmitted of the $\bar{i}$th node.

Now, by defining the overall measurement output (after being transmitted) as

$$\bar{y}(k) \triangleq \begin{bmatrix} \bar{y}_1^T(k) & \bar{y}_2^T(k) & \cdots & \bar{y}_M^T(k) \end{bmatrix}^T$$

and

$$\Phi_{\bar{i}} \triangleq \mathrm{diag}\{\delta(\bar{i} - 1)I, \delta(\bar{i} - 2)I, \cdots, \delta(\bar{i} - M)I\},$$

together with the zero-holder strategy (7), we have

$$\bar{y}(k) = \Phi_{\xi(k)} y(k) + (I - \Phi_{\xi(k)})\bar{y}(k - 1). \tag{8}$$

Substituting (3) into (8), the specific form of $\bar{y}(k)$ can be obtained as follows:

$$\begin{aligned} \bar{y}(k) = &\sum_{i=1}^{r} h_i(\theta(k))\Phi_{\xi(k)}\Big(C_i x(k) + E_{2i} v(k)\Big) \\ &+ (I - \Phi_{\xi(k)})\bar{y}(k - 1). \end{aligned} \tag{9}$$

C. Fuzzy PID Controller

So far, we have constructed the underlying fuzzy system with its outputs restrained by RR protocol. Based on this, we adopt the following fuzzy PID output-feedback controller:

$$\begin{aligned} u(k) = \sum_{j=1}^{r} h_j(\theta(k))\Big(&K_{Pj,\xi(k)}\bar{y}(k) + K_{Ij,\xi(k)} \sum_{m=k-N}^{k-1} \bar{y}(m) \\ &+ K_{Dj,\xi(k)}(\bar{y}(k) - \bar{y}(k - 1))\Big). \end{aligned} \tag{10}$$

where $K_{Pj,\xi(k)}$, $K_{Ij,\xi(k)}$ and $K_{Dj,\xi(k)}$ are controller gains to be designed and $N \geq 1$ is a given scalar representing time length.

Remark 3: The advantages/novelties of the proposed fuzzy PID controller (10) are reflected in the following three aspects. 1) Compared with the non-PID fuzzy control schemes, the controller (10) can generate control laws by simultaneously utilizing the current information, the historical information and the change of information of system outputs. By introducing the integral-loop and derivative-loop, the robustness of the controller would be enhanced. 2) The three types of controller gains are all switching-signal-dependent that are more beneficial to deal with the protocol-induced effects. 3) A fixed yet adjustable time window is introduced in the integral term to reduce the underlying accumulation error as well as the computational burden.

In terms of (9) and (10), we have

$$\begin{aligned} u(k) = &\sum_{j=1}^{r}\sum_{s=1}^{r} h_j(\theta(k))h_s(\theta(k)) \\ &\times \Bigg(\Big(K_{Pj,\xi(k)} + K_{Dj,\xi(k)}\Big)\Phi_{\xi(k)} C_s x(k) \\ &+ \Big(K_{Pj,\xi(k)} + K_{Dj,\xi(k)}\Big)\Phi_{\xi(k)} E_{2s} v(k) \\ &+ \Big(K_{Pj,\xi(k)} - K_{Pj,\xi(k)}\Phi_{\xi(k)} - K_{Dj,\xi(k)}\Phi_{\xi(k)} \\ &+ K_{Ij,\xi(k)}\Big)\bar{y}(k - 1) + K_{Ij,\xi(k)} \sum_{m=k-N}^{k-2} \bar{y}(m)\Bigg). \end{aligned} \tag{11}$$

Denoting variables

$$\begin{aligned} \bar{x}(k) &\triangleq \begin{bmatrix} x^T(k) & \bar{y}^T(k - 1) \end{bmatrix}^T, \\ \vec{x}(k) &\triangleq \begin{bmatrix} \bar{x}^T(k - 1) & \bar{x}^T(k - 2) & \cdots & \bar{x}^T(k - N + 1) \end{bmatrix}^T \end{aligned}$$

and substituting (11) into (3), we obtain the closed-loop T-S fuzzy control system as follows:

$$\begin{cases} \bar{x}(k + 1) = \displaystyle\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r} h_i(\theta(k))h_j(\theta(k))h_s(\theta(k)) \\ \qquad \times \Big(\tilde{A}_{ijs,\xi(k)}\bar{x}(k) + \tilde{A}_{\epsilon i} \displaystyle\sum_{\epsilon=1}^{\infty} \mu_\epsilon \bar{x}(k - \epsilon) \\ \qquad + \hat{B}_{ij,\xi(k)}\vec{x}(k) + \tilde{E}_{ijs,\xi(k)} v(k)\Big) \\ z(k) = \displaystyle\sum_{i=1}^{r} h_i(\theta(k))\bar{F}_i \bar{x}(k) \end{cases} \tag{12}$$

where

$$\begin{aligned} \hat{B}_{ij,\xi(k)} &\triangleq \big[\underbrace{\tilde{B}_{ij,\xi(k)} \quad \tilde{B}_{ij,\xi(k)} \quad \cdots \quad \tilde{B}_{ij,\xi(k)}}_{N-1}\big], \\ \tilde{A}_{ijs,\xi(k)} &\triangleq \begin{bmatrix} \bar{A}^1_{ijs,\xi(k)} & \bar{A}^2_{ij,\xi(k)} \\ \Phi_{\xi(k)} C_i & I - \Phi_{\xi(k)} \end{bmatrix}, \quad \tilde{A}_{\epsilon i} \triangleq \begin{bmatrix} A_{\epsilon i} & 0 \\ 0 & 0 \end{bmatrix}, \\ \tilde{B}_{ij,\xi(k)} &\triangleq \begin{bmatrix} 0 & B_i K_{Ij,\xi(k)} \\ 0 & 0 \end{bmatrix}, \quad \tilde{E}_{ijs,\xi(k)} \triangleq \begin{bmatrix} \bar{E}_{ijs,\xi(k)} \\ \Phi_{\xi(k)} E_{2i} \end{bmatrix}, \\ \bar{A}^1_{ijs,\xi(k)} &\triangleq A_i + B_i K_{Pj,\xi(k)}\Phi_{\xi(k)} C_s + B_i K_{Dj,\xi(k)}\Phi_{\xi(k)} C_s, \\ \bar{E}_{ijs,\xi(k)} &\triangleq E_{1i} + B_i K_{Pj,\xi(k)}\Phi_{\xi(k)} E_{2s} + B_i K_{Dj,\xi(k)}\Phi_{\xi(k)} \\ &\quad \times E_{2s}, \\ \bar{A}^2_{ij,\xi(k)} &\triangleq B_i K_{Pj,\xi(k)} - B_i K_{Pj,\xi(k)}\Phi_{\xi(k)} + B_i K_{Ij,\xi(k)} \\ &\quad - B_i K_{Dj,\xi(k)}\Phi_{\xi(k)}, \qquad \bar{F}_i \triangleq \begin{bmatrix} F_i & 0 \end{bmatrix}. \end{aligned}$$

Before proceeding further, we first introduce the definition of the exponential stability for fuzzy system (12).

Definition 1: [48] The closed-loop fuzzy system (12) is said to be exponentially stable if, for $v(k) = 0$, there exist scalars α ($\alpha > 0$) and β ($0 < \beta < 1$) such that

$$\|\bar{x}(k)\|^2 \leq \alpha\beta^k \max_{s \in \mathbb{Z}^-} \|\bar{x}(s)\|^2. \tag{13}$$

The objective of this paper is to investigate the H_∞ fuzzy PID control problem for discrete-time fuzzy systems with infinite-DTDs and RR protocol scheduling effects such that the following two requirements are satisfied simultaneously:

R1) the fuzzy system (12) is exponentially stable; and

R2) for all nonzero $v(k) \in l_2[0,\infty)$ and under zero initial condition, the controlled output $z(k)$ satisfies the H_∞ performance constraint:

$$\sum_{k=0}^{\infty} z^T(k)z(k) \le \gamma^2 \sum_{k=0}^{\infty} v^T(k)v(k) \tag{14}$$

where $\gamma > 0$ is a given scalar standing for the disturbance attenuation level.

III. Main Results

Firstly, we introduce some lemmas which will be utilized later. For space saving, we define the following notations:

$$\begin{aligned}
&\sum_{i,j,s,t,p,q=1}^{r} \mathcal{H}_{ijstpq}(k) \\
\triangleq &\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{t=1}^{r}\sum_{p=1}^{r}\sum_{q=1}^{r} h_i(\theta(k)) \\
&\times h_j(\theta(k))h_s(\theta(k))h_t(\theta(k))h_p(\theta(k))h_q(\theta(k)), \\
&\sum_{i,j,s=1}^{r} \mathcal{H}_{ijs}(k) \\
\triangleq &\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r} h_i(\theta(k))h_j(\theta(k))h_s(\theta(k)).
\end{aligned}$$

Lemma 1: [11] For a symmetric positive definite matrix S and any real matrix X_{ijs} $(i,j,s = 1,2,\cdots,r)$ with appropriate dimensions, we have

$$\begin{aligned}
\sum_{i,j,s,t,p,q=1}^{r} &\mathcal{H}_{ijstpq}(k) X_{ijs}^T S X_{tpq} \\
&\le \sum_{i,j,s=1}^{r} \mathcal{H}_{ijs}(k) X_{ijs}^T S X_{ijs}.
\end{aligned}$$

Lemma 2: [49] For constants $a_i > 0$, vectors $x_i \in \mathbb{R}^n$ $(i = 1,2,\cdots)$ and a $n \times n$ positive semi-definite matrix M, if the series $\sum_{i=1}^{\infty} a_i$ is convergent, we have

$$\left(\sum_{i=1}^{\infty} a_i x_i\right)^T M \left(\sum_{i=1}^{\infty} a_i x_i\right) \le \left(\sum_{i=1}^{\infty} a_i\right) \sum_{i=1}^{\infty} a_i x_i^T M x_i.$$

Now, we are in a position to present the following results with respect to the exponential stability and the H_∞ performance of the considered T-S fuzzy system (12).

Theorem 1: Let the controller gains $K_{Pj,\bar{i}}$, $K_{Ij,\bar{i}}$, $K_{Dj,\bar{i}}$ $(j = 1,2,\cdots,r, \bar{i} = 1,2,\cdots,M)$ and the H_∞ performance index $\gamma > 0$ be given. Assume that there exist positive definite matrices $P > 0$, $S > 0$, and $Q_d > 0$ $(d = 1,2,\cdots,N-1)$ satisfying

$$\begin{bmatrix}
\vec{Q} & * & * & * & * & * \\
0 & -\tilde{Q} & * & * & * & * \\
0 & 0 & -\frac{1}{\bar{\mu}}S & * & * & * \\
0 & 0 & 0 & -\gamma^2 I & * & * \\
P\tilde{A}_{ijs,\bar{i}} & P\hat{B}_{ij,\bar{i}} & P\tilde{A}_{\epsilon i} & P\tilde{E}_{ijs,\bar{i}} & -P & * \\
\bar{F}_i & 0 & 0 & 0 & 0 & -I
\end{bmatrix} < 0 \quad (15)$$

$$i,j,s = 1,2,\cdots,r, \qquad \bar{i} = 1,2,\cdots,M,$$

where

$$\tilde{Q} \triangleq \mathrm{diag}\left\{Q_1, Q_2, \cdots, Q_{N-1}\right\}, \quad \vec{Q} \triangleq \sum_{d=1}^{N-1} Q_d + \bar{\mu}S - P.$$

Then, the controlled system (12) is exponentially stable. Furthermore, under the zero initial condition, the inequality $\sum_{k=0}^{\infty} z^T(k)z(k) \le \gamma^2 \sum_{k=0}^{\infty} v^T(k)v(k)$ is also satisfied.

Proof: In order to analyze the exponential stability with disturbance attenuation level γ of the considered system (12), we choose the following LKF:

$$V(k) = \sum_{i=1}^{3} V_i(k) \tag{16}$$

where

$$\begin{aligned}
V_1(k) &\triangleq \bar{x}^T(k)P\bar{x}(k), \\
V_2(k) &\triangleq \sum_{\epsilon=1}^{\infty} \mu_\epsilon \sum_{i=k-\epsilon}^{k-1} \bar{x}^T(i)S\bar{x}(i), \\
V_3(k) &\triangleq \sum_{d=1}^{N-1} \sum_{\tau=k-d}^{k-1} \bar{x}^T(\tau)Q_d\bar{x}(\tau).
\end{aligned}$$

Letting $\bar{i} = \xi(k)$, along the trajectory of system (12), the difference of $V_1(k)$ can be calculated as follows:

$$\begin{aligned}
&\Delta V_1(k) \\
&= V_1(k+1) - V_1(k) \\
&= \bar{x}^T(k+1)P\bar{x}(k+1) - \bar{x}^T(k)P\bar{x}(k) \\
&= \sum_{i,j,s,t,p,q=1}^{r} \mathcal{H}_{ijstpq}(k)\Bigg(\tilde{A}_{ijs,\bar{i}}\bar{x}(k) + \tilde{A}_{\epsilon i}\sum_{\epsilon=1}^{\infty}\mu_\epsilon \bar{x}(k-\epsilon) \\
&\quad + \hat{B}_{ij,\bar{i}}\vec{x}(k) + \tilde{E}_{ijs,\bar{i}}v(k)\Bigg)^T P\Bigg(\tilde{A}_{tpq,\bar{i}}\bar{x}(k) + \tilde{E}_{tpq,\bar{i}}v(k) \\
&\quad + \tilde{A}_{\epsilon t}\sum_{\epsilon=1}^{\infty}\mu_\epsilon \bar{x}(k-\epsilon) + \hat{B}_{tp,\bar{i}}\vec{x}(k)\Bigg) - \bar{x}^T(k)P\bar{x}(k) \\
&= \sum_{i,j,s,t,p,q=1}^{r} \mathcal{H}_{ijstpq}(k)\Bigg(\bar{x}^T(k)\Big(\tilde{A}_{ijs,\bar{i}}^T P\tilde{A}_{tpq,\bar{i}} - P\Big)\bar{x}(k) \\
&\quad + \vec{x}^T(k)\hat{B}_{ij,\bar{i}}^T P\hat{B}_{tp,\bar{i}}\vec{x}(k) + \sum_{\epsilon=1}^{\infty}\mu_\epsilon \bar{x}(k-\epsilon)\tilde{A}_{\epsilon i}^T P \\
&\quad \tilde{A}_{\epsilon t}\sum_{\epsilon=1}^{\infty}\mu_\epsilon \bar{x}(k-\epsilon) + v^T(k)\tilde{E}_{ijs,\bar{i}}^T P\tilde{E}_{tpq,\bar{i}}v(k) + 2\bar{x}^T(k) \\
&\quad \times \tilde{A}_{ijs,\bar{i}}^T P\hat{B}_{tp,\bar{i}}\vec{x}(k) + 2\bar{x}^T(k)\tilde{A}_{ijs,\bar{i}}^T P\tilde{A}_{\epsilon t}\sum_{\epsilon=1}^{\infty}\mu_\epsilon \bar{x}(k-\epsilon)
\end{aligned}$$

$$+ 2\bar{x}^T(k)\tilde{A}^T_{ijs,\bar{i}} P\tilde{E}_{tpq,\bar{i}} v(k) + 2\vec{x}^T(k)\hat{B}^T_{ij,\bar{i}}(k)P\tilde{E}_{tpq,\bar{i}} v(k) + 2\vec{x}^T(k)\hat{B}^T_{ij,\bar{i}}(k)P\tilde{A}_{\epsilon t}\sum_{\epsilon=1}^{\infty}\mu_\epsilon\bar{x}(k-\epsilon) + 2\sum_{\epsilon=1}^{\infty}\mu_\epsilon\bar{x}(k-\epsilon) \times \tilde{A}^T_{\epsilon i}P\tilde{E}_{tpq,\bar{i}} v(k)\Bigg). \tag{17}$$

We first prove that the fuzzy system (12) is exponentially stable under condition (15). For this purpose, we denote the following matrix variables:

$$\begin{aligned}
\hat{A}_{ijs,\bar{i}} &\triangleq \begin{bmatrix} P\tilde{A}_{ijs,\bar{i}} & P\hat{B}_{ij,\bar{i}} & P\tilde{A}_{\epsilon i}\end{bmatrix}, \quad \vec{P} \triangleq \mathrm{diag}\{P, I\},\\
\zeta(k) &\triangleq \begin{bmatrix} \bar{x}^T(k) & \vec{x}^T(k) & \sum_{\epsilon=1}^{\infty}\mu_\epsilon\bar{x}^T(k-\epsilon)\end{bmatrix}^T,\\
\bar{\zeta}(k) &\triangleq \begin{bmatrix} \bar{x}^T(k) & \vec{x}^T(k) & \sum_{\epsilon=1}^{\infty}\mu_\epsilon\bar{x}^T(k-\epsilon) & v^T(k)\end{bmatrix}^T,\\
\Omega_{ijs,\bar{i}} &\triangleq \begin{bmatrix} \tilde{A}_{ijs,\bar{i}} & \hat{B}_{ij,\bar{i}} & \tilde{A}_{\epsilon i} & \tilde{E}_{ijs,\bar{i}},\\ \bar{F}_i & 0 & 0 & 0\end{bmatrix},\\
\bar{\Omega}_{ijs,\bar{i}} &\triangleq \begin{bmatrix} P\tilde{A}_{ijs,\bar{i}} & P\hat{B}_{ij,\bar{i}} & P\tilde{A}_{\epsilon i} & P\tilde{E}_{ijs,\bar{i}},\\ \bar{F}_i & 0 & 0 & 0\end{bmatrix},\\
\vec{A}_{ijs,\bar{i}} &\triangleq \begin{bmatrix} \tilde{A}_{ijs,\bar{i}} & \hat{B}_{ij,\bar{i}} & \tilde{A}_{\epsilon i}\end{bmatrix}, \quad \bar{P} \triangleq \mathrm{diag}\{P, 0, 0\},\\
\tilde{P} &\triangleq \mathrm{diag}\left\{\sum_{d=1}^{N-1} Q_d + \bar{\mu}S - P, -\tilde{Q}, -\frac{1}{\bar{\mu}}S\right\},\\
\Upsilon &\triangleq \mathrm{diag}\left\{\sum_{d=1}^{N-1} Q_d + \bar{\mu}S - P, -\tilde{Q}, -\frac{1}{\bar{\mu}}S, -\gamma^2 I\right\}.
\end{aligned}$$

When $v(k) = 0$, we have

$$\Delta V_1(k) = \sum_{i,j,s,t,p,q=1}^{r}\mathcal{H}_{ijstpq}(k) \times \zeta^T(k)\left(\vec{A}^T_{ijs,\bar{i}}P\vec{A}_{tpq,\bar{i}} + \bar{P}\right)\zeta(k). \tag{18}$$

Considering Lemma 1, it can be obtained that

$$\Delta V_1(k) \le \sum_{i,j,s=1}^{r}\mathcal{H}_{ijs}(k) \times \zeta^T(k)\left(\vec{A}^T_{ijs,\bar{i}}P\vec{A}_{ijs,\bar{i}} + \bar{P}\right)\zeta(k). \tag{19}$$

Furthermore, one also has

$$\begin{aligned}
&\Delta V_2(k)\\
&= V_2(k+1) - V_2(k)\\
&= \sum_{\epsilon=1}^{\infty}\mu_\epsilon\sum_{i=k-\epsilon+1}^{k}\bar{x}^T(i)S\bar{x}(i) - \sum_{\epsilon=1}^{\infty}\mu_\epsilon\sum_{i=k-\epsilon}^{k-1}\bar{x}^T(i)S\bar{x}(i)\\
&= \bar{\mu}\bar{x}^T(k)S\bar{x}(k) - \sum_{\epsilon=1}^{\infty}\mu_\epsilon\bar{x}^T(k-\epsilon)S\bar{x}(k-\epsilon).
\end{aligned} \tag{20}$$

In addition, it follows from Lemma 2 that

$$-\sum_{\epsilon=1}^{\infty}\mu_\epsilon\bar{x}^T(k-\epsilon)S\bar{x}(k-\epsilon) \le -\frac{1}{\bar{\mu}}\left(\sum_{\epsilon=1}^{\infty}\mu_\epsilon\bar{x}(k-\epsilon)\right)^T S\left(\sum_{\epsilon=1}^{\infty}\mu_\epsilon\bar{x}(k-\epsilon)\right). \tag{21}$$

Thus, we have

$$\Delta V_2(k) \le \bar{\mu}\bar{x}^T(k)S\bar{x}(k) - \frac{1}{\bar{\mu}}\left(\sum_{\epsilon=1}^{\infty}\mu_\epsilon\bar{x}(k-\epsilon)\right)^T S\left(\sum_{\epsilon=1}^{\infty}\mu_\epsilon\bar{x}(k-\epsilon)\right), \tag{22}$$

and the difference of $V_3(k)$ can be also calculated as follows:

$$\begin{aligned}
&\Delta V_3(k)\\
&= V_3(k+1) - V_3(k)\\
&= \sum_{d=1}^{N-1}\sum_{\tau=k-d+1}^{k}\bar{x}^T(\tau)Q_d\bar{x}(\tau) - \sum_{d=1}^{N-1}\sum_{\tau=k-d}^{k-1}\bar{x}^T(\tau)Q_d\bar{x}(\tau)\\
&= \sum_{d=1}^{N-1}\bar{x}^T(k)Q_d\bar{x}(k) - \sum_{d=1}^{N-1}\bar{x}^T(k-d)Q_d\bar{x}(k-d).
\end{aligned} \tag{23}$$

Taking (19), (22) and (23) into consideration, it is easy to see that

$$\begin{aligned}
\Delta V(k) &\le \sum_{i,j,s=1}^{r}\mathcal{H}_{ijs}(k)\zeta^T(k)\left(\hat{A}^T_{ijs,\bar{i}}P^{-1}\hat{A}_{ijs,\bar{i}} + \tilde{P}\right)\zeta(k)\\
&\triangleq \sum_{i,j,s=1}^{r}\mathcal{H}_{ijs}(k)\zeta^T(k)\Psi_{ijs,\bar{i}}\zeta(k).
\end{aligned} \tag{24}$$

From (15) and the Schur Complement Lemma, we know that $\Delta V(k) < 0$. Then, it can be concluded from Theorem 1 of [48] that, in the case of $v(k) = 0$, the fuzzy system (12) is exponentially stable.

Next, we will analyze the H_∞ performance for the closed-loop system (12) under zero initial condition. Firstly, we define the following index function:

$$\begin{aligned}
J_n &= \sum_{k=0}^{n}\left(z^T(k)z(k) - \gamma^2 v^T(k)v(k)\right)\\
&= \sum_{k=0}^{n}\left(z^T(k)z(k) - \gamma^2 v^T(k)v(k) + V(k+1) - V(k)\right)\\
&\quad + V(0) - V(n+1)
\end{aligned} \tag{25}$$

where n is a non-negative integer. Obviously, our target is to show that $J_n < 0$.

Since $V(0) = 0$ and $V(n+1) \ge 0$, we have

$$\begin{aligned}
J_n &\le \sum_{k=0}^{n}\left(z^T(k)z(k) - \gamma^2 v^T(k)v(k) + \Delta V(k)\right)\\
&= \sum_{k=0}^{n}\sum_{i,j,s,t,p,q=1}^{r}\mathcal{H}_{ijstpq}(k)\Bigg(\bar{x}^T(k)\bar{F}_i^T\bar{F}_t\bar{x}(k)\\
&\quad - \gamma^2 v^T(k)v(k) + \Big(\tilde{A}_{ijs,\bar{i}}\bar{x}(k) + \tilde{A}_{\epsilon i}\sum_{\epsilon=1}^{\infty}\mu_\epsilon\bar{x}(k-\epsilon)\\
&\quad + \hat{B}_{ij}\vec{x}(k) + \tilde{E}_{ijs,\bar{i}}v(k)\Big)^T P\Big(\tilde{A}_{tpq,\bar{i}}\bar{x}(k) + \tilde{E}_{tpq,\bar{i}}v(k)\\
&\quad + \tilde{A}_{\epsilon t}\sum_{\epsilon=1}^{\infty}\mu_\epsilon\bar{x}(k-\epsilon) + \hat{B}_{tp}\vec{x}(k)\Big) + \bar{x}^T(k)\Big(\sum_{d=1}^{N-1}Q_d\\
&\quad + \bar{\mu}S - P\Big)\bar{x}(k) - \sum_{\epsilon=1}^{\infty}\mu_\epsilon\bar{x}^T(k-\epsilon)S\bar{x}(k-\epsilon)
\end{aligned}$$

$$- \sum_{d=1}^{N-1} \bar{x}^T(k-d)Q_d\bar{x}(k-d)\Bigg). \tag{26}$$

By further utilizing the inequality (21), we have

$$\begin{aligned}
J_n \leq & \sum_{k=0}^{n} \sum_{i,j,s,t,p,q=1}^{r} \mathcal{H}_{ijstpq}(k)\Bigg(\bar{x}^T(k)\bar{F}_i^T\bar{F}_t\bar{x}(k) \\
& - \gamma^2 v^T(k)v(k) + \Big(\tilde{A}_{ijs,\bar{i}}\bar{x}(k) + \tilde{A}_{\epsilon i}\sum_{\epsilon=1}^{\infty}\mu_\epsilon \bar{x}(k-\epsilon) \\
& + \hat{B}_{ij}\vec{x}(k) + \tilde{E}_{ijs,\bar{i}}v(k)\Big)^T P\Big(\tilde{A}_{tpq,\bar{i}}\bar{x}(k) + \tilde{E}_{tpq,\bar{i}}v(k) \\
& + \tilde{A}_{\epsilon t}\sum_{\epsilon=1}^{\infty}\mu_\epsilon \bar{x}(k-\epsilon) + \hat{B}_{tp}\vec{x}(k)\Big) + \bar{x}^T(k)\Big(\sum_{d=1}^{N-1} Q_d \\
& + \bar{\mu}S - P\Big)\bar{x}(k) - \sum_{d=1}^{N-1}\bar{x}^T(k-d)Q_d\bar{x}(k-d) \\
& - \frac{1}{\bar{\mu}}\left(\sum_{\epsilon=1}^{\infty}\mu_\epsilon \bar{x}(k-\epsilon)\right)^T S\left(\sum_{\epsilon=1}^{\infty}\mu_\epsilon \bar{x}(k-\epsilon)\right)\Bigg) \\
= & \sum_{k=0}^{n}\sum_{i,j,s,t,p,q=1}^{r}\mathcal{H}_{ijstpq}(k)\bar{\zeta}^T(k)\Big(\Omega_{ijs,\bar{i}}^T\vec{P}\Omega_{tpq,\bar{i}} + \Upsilon\Big)\bar{\zeta}(k) \\
\leq & \sum_{k=0}^{n}\sum_{i,j,s=1}^{r}\mathcal{H}_{ijs}(k)\bar{\zeta}^T(k)\left(\bar{\Omega}_{ijs,\bar{i}}^T\vec{P}^{-1}\bar{\Omega}_{ijs,\bar{i}} + \Upsilon\right)\bar{\zeta}(k).
\end{aligned} \tag{27}$$

From the Schur Complement Lemma and condition (15), it is easy to see that $J_n < 0$ holds. Letting $n \to \infty$, we derive that $\sum_{k=0}^{\infty} z^T(k)z(k) \leq \gamma^2 \sum_{k=0}^{\infty} v^T(k)v(k)$, which completes the proof. ■

In Theorem 1, with given controller gains and H_∞ disturbance attenuation level $\gamma > 0$, we have provided sufficient conditions to guarantee that the closed-loop system (12) satisfies requirements (R1) and (R2) based on the linear matrix inequality (LMI) technique. Next, we will work on the design of the required fuzzy PID controller parameters.

Theorem 2: Let the scalar $\gamma > 0$ be given. Assume that there exist positive definite matrices $P > 0$, $L > 0$, $S > 0$, $Q_d > 0$ $(d = 1, 2, \cdots, N-1)$ and matrices $K_{Pj,\bar{i}}$, $K_{Dj,\bar{i}}$, $K_{Ij,\bar{i}}$ $(j = 1, 2, \cdots, r;\ \bar{i} = 1, 2, \cdots, M)$ satisfying

$$\begin{bmatrix}
\vec{Q} & * & * & * & * & * \\
0 & -\tilde{Q} & * & * & * & * \\
0 & 0 & -\frac{1}{\bar{\mu}}S & * & * & * \\
0 & 0 & 0 & -\gamma^2 I & * & * \\
\tilde{A}_{ijs,\bar{i}} & \hat{B}_{ij,\bar{i}} & \tilde{A}_{\epsilon i} & \tilde{E}_{ijs,\bar{i}} & -L & * \\
F_i & 0 & 0 & 0 & 0 & -I
\end{bmatrix} < 0 \tag{28}$$

$$i, j, s = 1, 2, \cdots, r, \qquad \bar{i} = 1, 2, \cdots, M,$$

$$PL = I \tag{29}$$

where $\vec{Q}$ and $\tilde{Q}$ are defined in Theorem 1 and

$$\begin{aligned}
&\bar{\Phi}_{s,\bar{i}} \triangleq \begin{bmatrix}\Phi_{\bar{i}}C_s & 0\end{bmatrix}, \quad \mathcal{B}_i \triangleq \begin{bmatrix}B_i^T & 0\end{bmatrix}^T, \\
&\tilde{E}_{ijs,\bar{i}} \triangleq \mathcal{E}_{i,\bar{i}} + \mathcal{B}_iK_{Pj,\bar{i}}\Phi_{\bar{i}}E_{2s} + \mathcal{B}_iK_{Dj,\bar{i}}\Phi_{\bar{i}}E_{2s}, \\
&\tilde{A}_{ijs,\bar{i}} \triangleq \mathcal{A}_{i,\bar{i}} + \mathcal{B}_iK_{Pj,\bar{i}}\bar{\Phi}_{s,\bar{i}} + \mathcal{B}_iK_{Dj,\bar{i}}\bar{\Phi}_{s,\bar{i}} + \mathcal{B}_iK_{Pj,\bar{i}}\mathcal{I} \\
&\qquad - \mathcal{B}_iK_{Pj,\bar{i}}\tilde{\Phi}_{\bar{i}} - \mathcal{B}_iK_{Dj,\bar{i}}\tilde{\Phi}_{\bar{i}} + \mathcal{B}_iK_{Ij,\bar{i}}\mathcal{I}, \\
&\hat{B}_{ij,\bar{i}} \triangleq \sum_{\iota=1}^{N-1}\mathcal{B}_iK_{Ij,\bar{i}}\bar{\mathcal{I}}_\iota, \quad \mathcal{E}_{i,\bar{i}} \triangleq \begin{bmatrix}E_{1i} \\ \Phi_{\bar{i}}E_{2i}\end{bmatrix}, \quad \tilde{\Phi}_{\bar{i}} \triangleq \begin{bmatrix}0 \\ \Phi_{\bar{i}}^T\end{bmatrix}^T, \\
&\mathcal{A}_{i,\bar{i}} \triangleq \begin{bmatrix}A_i & 0 \\ \Phi_{\bar{i}}C_i & I - \Phi_{\bar{i}}\end{bmatrix}, \qquad \mathcal{I} \triangleq \begin{bmatrix}0 & I\end{bmatrix}, \\
&\bar{\mathcal{I}}_\iota \triangleq \begin{bmatrix}0_{n_y\times[(\iota-1)n_y+\iota n_x]} & I & 0_{n_y\times[(N-1-\iota)(n_x+n_y)]}\end{bmatrix}.
\end{aligned}$$

Then, the closed-loop fuzzy system (12) is exponentially stable. Furthermore, under the zero initial condition, the inequality $\sum_{k=0}^{\infty} z^T(k)z(k) \leq \gamma^2 \sum_{k=0}^{\infty} v^T(k)v(k)$ is also satisfied. In this case, the desired fuzzy PID controller gains can be obtained directly as $K_{Pj,\bar{i}}$, $K_{Dj,\bar{i}}$ and $K_{Ij,\bar{i}}$.

Proof: To keep the integrality of the variable matrices P, L, S, Q_d $(d = 1, 2, \cdots, N-1)$, we rewrite some matrices as follows:

$$\begin{aligned}
\hat{B}_{ij,\bar{i}} &= \sum_{\iota=1}^{N-1}\mathcal{B}_iK_{Ij,\bar{i}}\bar{\mathcal{I}}_\iota, \\
\tilde{A}_{ijs,\bar{i}} &= \mathcal{A}_{i,\bar{i}} + \mathcal{B}_iK_{Pj,\bar{i}}\bar{\Phi}_{s,\bar{i}} + \mathcal{B}_iK_{Dj,\bar{i}}\bar{\Phi}_{s,\bar{i}} + \mathcal{B}_iK_{Pj,\bar{i}}\mathcal{I} \\
&\quad - \mathcal{B}_iK_{Pj,\bar{i}}\tilde{\Phi}_{\bar{i}} - \mathcal{B}_iK_{Dj,\bar{i}}\tilde{\Phi}_{\bar{i}} + \mathcal{B}_iK_{Ij,\bar{i}}\mathcal{I}, \\
\tilde{E}_{ijs,\bar{i}} &= \mathcal{E}_{i,\bar{i}} + \mathcal{B}_iK_{Pj,\bar{i}}\Phi_{\bar{i}}E_{2s} + \mathcal{B}_iK_{Dj,\bar{i}}\Phi_{\bar{i}}E_{2s}.
\end{aligned}$$

Then, pre-multiplying and post-multiplying the inequalities (15) by $\mathrm{diag}\left\{I, I, I, I, P^{-1}, I\right\}$ and its transposition, respectively, and letting $L = P^{-1}$, we obtain (28) readily. The proof is now complete. ■

By far, it is infeasible to derive the desired fuzzy PID controller parameters directly by the LMI approach because of the matrix equality (29) in Theorem 2, which renders the problem non-convex. To get over the difficulty, the CCL algorithm is utilized whose main idea is given as follows:

$$PL = I \Longleftrightarrow \begin{cases} \mathrm{tr}(PL) = n \\ \begin{bmatrix} P & * \\ I & L \end{bmatrix} \geq 0 \end{cases}$$

where $P > 0$ and $L > 0$ are matrix variables with dimensions $n \times n$. Then, the problem in Theorem 2 can be reconstructed as solving (28) and

$$\begin{bmatrix} P & * \\ I & L \end{bmatrix} \geq 0. \tag{30}$$

in the case of $\mathrm{tr}(PL) = n$. If the solution of the above problem exists, the conditions in Theorem 2 are solvable.

Finally, in terms of the above discussions, Algorithm 1 is given to tackle the considered problem.

Note that Algorithm 1 can be applied to handle the H_∞ fuzzy PID control problem with a given disturbance attenuation level γ. Next, by following the similar idea proposed in [8] and based on Theorem 2 and Algorithm 1, we provide Algorithm 2 to further obtain the suboptimal performance of γ for the considered fuzzy PID control problem.

Remark 4: In this paper, the Lyapunov stability theory and LMI technique have been employed to deal with the fuzzy PID control problem subject to DTDs, where the LKF (16) has

Algorithm 1:

Step 1. Set $c=0$. Obtain a set of initial solutions $(P_{(0)}, L_{(0)}, S_{(0)}, Q_{d(0)}, K_{Pj,\bar{i}(0)}, K_{Ij,\bar{i}(0)}, K_{Dj,\bar{i}(0)})$ by solving (28) and (30).

Step 2. Solve the optimization problem $\min \operatorname{tr}(PL_{(c)}+P_{(c)}L)$ subject to the constraints (28) and (30) to derive a feasible solution $(P, L, S, Q_d, K_{Pj,\bar{i}}, K_{Ij,\bar{i}}, K_{Dj,\bar{i}})$. Set $t_c = |\operatorname{tr}(PL)-n|$ where $n \triangleq n_x+n_y$.

Step 3. Substitute the obtained matrix variables $(P, L, S, Q_d, K_{Pj,\bar{i}}, K_{Ij,\bar{i}}, K_{Dj,\bar{i}})$ into (15). If the inequality (15) is satisfied and t_c is less than a small constant number $\delta>0$, these obtained variables are the desired feasible solutions of our problem. Exit.

Step 4. If $c>H$, where H is the maximum number of iterations allowed. Then, output the feasible solutions. Exit. Else, set $c=c+1$ and $(P_{(c)}, L_{(c)}, S_{(c)}, Q_{d(c)}, K_{Pj,\bar{i}(c)}, K_{Ij,\bar{i}(c)}, K_{Dj,\bar{i}(c)}) = (P, L, S, Q_d, K_{Pj,\bar{i}}, K_{Ij,\bar{i}}, K_{Dj,\bar{i}})$. Go to *Step 2*.

Algorithm 2:

Step 1. Choose a sufficiently large initial $\gamma>0$, such that there exists a feasible solution to (28) and (30). Set $\gamma_{\min}=\gamma$.

Step 2. Set $c=0$. Obtain a set of initial solutions $(P_{(0)}, L_{(0)}, S_{(0)}, Q_{d(0)}, K_{Pj,\bar{i}(0)}, K_{Ij,\bar{i}(0)}, K_{Dj,\bar{i}(0)})$ by solving (28) and (30).

Step 3. Solve the optimization problem $\min \operatorname{tr}(PL_{(c)}+P_{(c)}L)$ subject to the constraints (28) and (30) to derive a feasible solution $(P, L, S, Q_d, K_{Pj,\bar{i}}, K_{Ij,\bar{i}}, K_{Dj,\bar{i}})$.

Step 4. Substitute the obtained gain matrices $(K_{Pj,\bar{i}}, K_{Ij,\bar{i}}, K_{Dj,\bar{i}})$ into (15). If the inequality (15) is satisfied with respect to the variables P, S, Q_d, then decrease γ to some extent and set $\gamma_{\min}=\gamma$. Go to *Step 2*. If (15) is infeasible within the maximum number of iteration that is allowed, then exit. Otherwise, Set $c=c+1$ and go to *Step 3*.

been utilized to derive sufficient conditions. Such a method is widely used which shows the advantages of low computational burden and easy implementation. The conservatism of this method is mainly from that a common LKF is used to analyze all fuzzy subsystems and a delay-independent LKF is used to tackle the delay-induced effects. To further reduce the conservatism, one can choose the piecewise LKF, fuzzy LKF and delay-dependent LKF at the cost of increasing the computational complexity. In this regard, we refer readers to [3], [4], [8], [15], [50] for more details.

Remark 5: The CCL algorithm, which was first introduced into control areas in [51], is regarded as an effective tool to address some control problems (see e.g. [11], [52]). There are two popular choices of the initial values in Step 1 of Algorithm 1. One is to choose random values such as identity matrices [53] and another is what we have adopted in this paper, namely, choosing a set of feasible solutions according to some related linear matrix inequalities (e.g., (28), (30) in this paper). From our experience, the second method would be very helpful for improving the subsequent optimization process and finding the satisfactory solutions, since such initial values are closely relevant to the considered problem.

Remark 6: When utilizing the CCL algorithm, the conservatism is mainly from the fact that it is numerically difficult to let $\operatorname{tr}(PL)$ strictly equal to n. From our experience, such a fact would directly affect the feasibility of the CCL algorithm. For example, for a rather complex system, the iterative operation sometimes would enter into a very slow process after some iterations, and as the iterative times increase, the change of $\operatorname{tr}(PL)$ would be very small. In this case, it is hard to find the satisfactory solutions. Even so, the CCL algorithm is shown to be an effective method with relatively low conservatism to deal with the nonlinear matrix inequality since no extra inequality constraints are introduced in the algorithm [8]. Another important feature of the CCL algorithm is the computational complexity which is proportional to the iterative times, the total row size $\mathcal{M}$ of the LMIs and the total number $\mathcal{N}$ of scalar decision variables [11]. In this paper, the variable dimensions can be obtained by $x(k)\in\mathbb{R}^{n_x}$, $u(k)\in\mathbb{R}^{n_u}$, $y(k)\in\mathbb{R}^{n_y}$, $z(k)\in\mathbb{R}^{n_z}$ and $v(k)\in\mathbb{R}^{n_v}$. Thus, in one optimization of Algorithm 1, we have $\mathcal{N}=3rMn_un_y+0.5(N+2)[(n_x+n_y)^2+(n_x+n_y)]$ and $\mathcal{M}=(N+4)(n_x+n_y)+r^3M[(N+2)(n_x+n_y)+n_v+n_z]$, where r is the number of fuzzy rules; N is the length of integral window and M is the number of sensor nodes. It can be observed that the computational complexity is largely dependent on the system complexities (such as the dimension of the system state, number of fuzzy rules). Moreover, note that the system under consideration is time-invariant. As such, our proposed algorithm can be implemented in an offline manner and the computational complexity would not affect the applicability of the proposed control scheme.

Remark 7: Until now, the H_∞ fuzzy PID control problem has been solved for a class of discrete-time fuzzy systems subject to infinite-DTDs and RR protocol. This is a nontrivial problem with two difficulties identified as follows: 1) how to formulate a mathematical model to account for the complicated signal transmission behavior? and 2) how to develop an appropriate methodology to design the PID parameters with guaranteed H_∞ performance of the control fuzzy systems. The main novelties that distinguish this paper from the existing ones are that: 1) the fuzzy PID control problem is, as the first attempt of this kind, investigated for systems with infinite-DTDs under the RR protocol, which caters for the engineering practice; 2) a novel and easy-to-implement fuzzy PID control algorithm is developed to deal with the H_∞ control problem; and 3) an iterative optimization algorithm is proposed to derive the parameters of the switching-signal-dependent controller. In the following section, three numerical examples are given to verify the proposed H_∞ fuzzy PID control algorithm.

IV. Simulation Examples

In this section, we present three simulation examples to demonstrate the validity of the proposed fuzzy PID control scheme for T-S fuzzy systems with a network.

A. Example 1

We consider a delayed discrete-time fuzzy system (1) with the following parameters:

$$A_1=\begin{bmatrix}1.01 & 0.1\\ 0.2 & 0.2\end{bmatrix}, B_1=\begin{bmatrix}0.7 & 0.5\\ 0.3 & 0.2\end{bmatrix}, E_{11}=\begin{bmatrix}0.1\\ 0.1\end{bmatrix},$$
$$A_2=\begin{bmatrix}1 & 0.3\\ 0.1 & -0.2\end{bmatrix}, B_2=\begin{bmatrix}0.4 & 0.7\\ 0.5 & 0.5\end{bmatrix}, E_{12}=\begin{bmatrix}0.2\\ 0.2\end{bmatrix},$$
$$A_{\epsilon 1}=\begin{bmatrix}0.05 & 0.01\\ 0.1 & 0.01\end{bmatrix}, C_1=\begin{bmatrix}0.9 & 1\\ 1 & 0.7\end{bmatrix}, E_{21}=\begin{bmatrix}0.1\\ 0.1\end{bmatrix},$$
$$F_2=\begin{bmatrix}0.4 & 0.3\end{bmatrix},\ C_2=C_1,\ F_1=\begin{bmatrix}0.5 & 0.6\end{bmatrix},$$

$$E_{22} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \quad A_{\epsilon 2} = \begin{bmatrix} -0.35 & -0.38 \\ -0.2 & 0.1 \end{bmatrix},$$
$$h_1(\theta(k)) = \frac{1 - \sin^2(x_1(k))}{2}, h_2(\theta(k)) = 1 - h_1(\theta(k)).$$

Choose the constants $\mu_\epsilon = 2^{-3-\epsilon}$ $(\epsilon = 1, 2, \cdots)$ and it is easy to see that $\bar{\mu} = \sum_{\epsilon=1}^{\infty} \mu_\epsilon = 2^{-3} < \sum_{\epsilon=1}^{\infty} \epsilon\mu_\epsilon = 2 < +\infty$, which satisfies the condition (2).

Assume that sensors can be divided into two nodes, i.e., $y(k) = \begin{bmatrix} y_1(k) & y_2(k) \end{bmatrix}^T$. Under the effect of the RR protocol, only one node is permitted to access the network at each time instant. Our aim is to design a fuzzy PID controller with the form of (10) such that the closed-loop T-S fuzzy system is exponentially stable and also satisfies a guaranteed H_∞ norm bound $\gamma = 0.8$.

First of all, we set the initial values as $x(0) = \begin{bmatrix} 0.1 & -0.1 \end{bmatrix}^T$, $x(s) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$ for all $s \in \mathbb{Z}^-$, and the noise as

$$v(k) = \frac{0.5\sin(k)}{k} - 1.2e^{-k}.$$

Based on the controller gains obtained via Algorithm 1, simulation results are given in Figs. 2-4. Fig. 2 shows the state evolution of the open-loop system, which is obviously unstable. Fig. 3 plots the state response of the controlled system, from which we can see that the system controlled by the fuzzy PID controller is exponential convergence. The selected node under RR protocol is depicted in Fig. 4. By simple computation, we obtain that $\gamma^* \triangleq \sqrt{\sum_{i=0}^{k_f} z^T(i)z(i) / \sum_{i=0}^{k_f} v^T(i)v(i)} = 0.2247$ (where k_f denotes the terminal time of control) which is less than the prescribed $\gamma = 0.8$.

To further check the H_∞ performance of the closed-loop system, we set the initial condition as $x(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$. Then, Fig. 5 depicts the state evolution of the controlled system when $v(k) \neq 0$. The ratio of $\sqrt{\frac{\sum_{i=0}^{k} z^T(i)z(i)}{\sum_{i=0}^{k} v^T(i)v(i)}}$ is shown in Fig. 6, from which we can see that the prescribed H_∞ performance is satisfied. All simulation results validate the effectiveness of our proposed method.

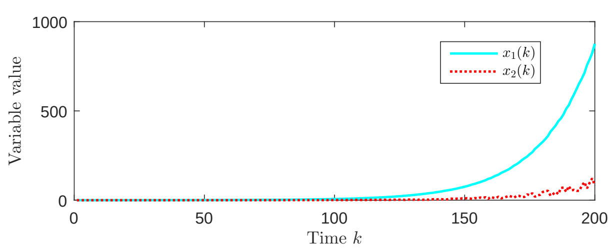

Fig. 2: State evolution $x(k)$ of the open-loop system

In order to further verify the effectiveness of the utilized algorithm, we list the values of $t_c = |\mathrm{tr}(PL) - n_x - n_y|$ under two different choices of initial values in Step 1 of Algorithm 1. We name that chose random values as case 1 and chose feasible solutions as case 2. Then, the comparison results are given in Table I. In fact, it is expected that t_c can be optimized as small as possible. From Table I, we can see that under

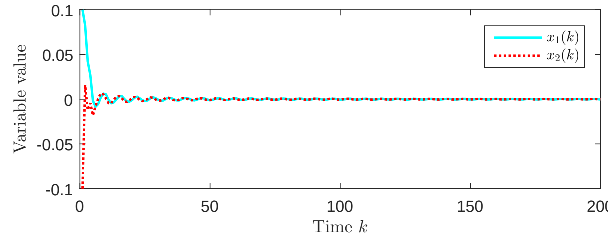

Fig. 3: State evolution $x(k)$ of the closed-loop system under $x(0) = \begin{bmatrix} 0.1 & -0.1 \end{bmatrix}^T$

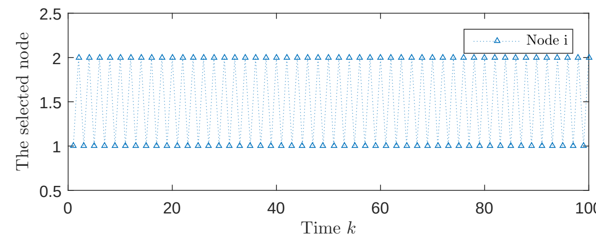

Fig. 4: Selected node under RR protocol

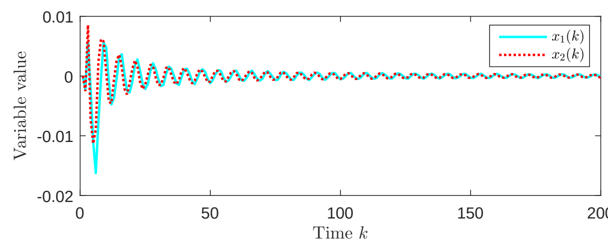

Fig. 5: State evolution $x(k)$ of the closed-loop system under $x(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$

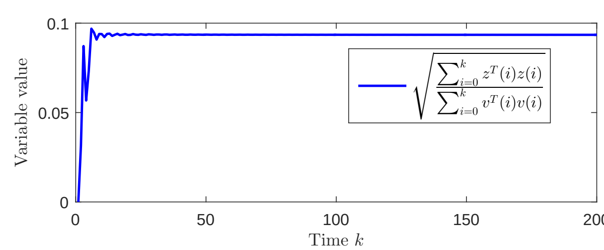

Fig. 6: Ratio of $\sqrt{\frac{\sum_{i=0}^{k} z^T(i)z(i)}{\sum_{i=0}^{k} v^T(i)v(i)}}$

the same iteration times, the optimization results in case 2 are better than those in case 1, showing the advantage of the adopted method.

B. Example 2

It is well known that a useful feature of the PID controller is to track a reference signal. In fact, the results obtained in this paper can be easily applied to handle the tracking control problem. In this example, let us consider the tracking control problem by using the proposed fuzzy PID controller.

TABLE I: The values of $t_c = |\mathrm{tr}(PL) - n_x - n_y|$ under two different cases

iteration times	1	2	5	30
t_c (case 1)	0.1287	0.0097	0.0022	4.6×10^{-4}
t_c (case 2)	0.0106	0.0034	7.3×10^{-4}	3.5×10^{-4}

Consider a fuzzy system whose model is given in Example 1, and without loss of generality, we set the controlled output as $z(k) = \sum_{i=1}^{2} h_i(\theta(k))C_i x(k)$. We aim to force $z(k)$ to track a reference signal $y_r(k)$ which is generated by the following model:

$$\begin{cases} x_r(k+1) = A_r x_r(k) + B_r r(k) \\ y_r(k) = C_r x_r(k) \end{cases} \tag{31}$$

where $x_r(k) \in \mathbb{R}^{n_x}$, $r(k) \in \mathbb{R}^{n_u}$ and $y_r(k) \in \mathbb{R}^{n_y}$ are, respectively, the state, the energy-bounded reference input and the output; A_r, B_r and C_r are constant matrices with A_r being Hurwitz.

By denoting the error variable $e(k) \triangleq x(k) - x_r(k)$, we can obtain the augmentation system as follows:

$$\begin{cases} \eta(k+1) = \sum_{i=1}^{2} h_i(\theta(k))\Big(\tilde{A}_i \eta(k) + \tilde{A}_{\epsilon i} \sum_{\epsilon=1}^{\infty} \mu_\epsilon \eta(k-\epsilon) \\ \qquad\qquad + \tilde{B}_i u(k) + \tilde{E}_{1i}\bar{v}(k)\Big) \\ y(k) = \sum_{i=1}^{2} h_i(\theta(k))\Big(\tilde{C}_i \eta(k) + \tilde{E}_{2i}\bar{v}(k)\Big) \\ z_e(k) = \sum_{i=1}^{2} h_i(\theta(k))\tilde{F}_i \eta(k) \end{cases} \tag{32}$$

where

$$\begin{aligned} &\eta(k) \triangleq \begin{bmatrix} e^T(k) & x_r^T(k) \end{bmatrix}^T, \quad z_e(k) \triangleq z(k) - y_r(k), \\ &\tilde{A}_i \triangleq \begin{bmatrix} A_i & A_i - A_r \\ 0 & A_r \end{bmatrix}, \quad \tilde{A}_{\epsilon i} \triangleq \begin{bmatrix} A_{\epsilon i} & A_{\epsilon i} \\ 0 & 0 \end{bmatrix}, \\ &\tilde{B}_i \triangleq \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \quad \tilde{E}_{1i} \triangleq \begin{bmatrix} E_{1i} & -B_r \\ 0 & B_r \end{bmatrix}, \\ &\tilde{C}_i \triangleq \begin{bmatrix} C_i & C_i \end{bmatrix}, \quad \tilde{E}_{2i} \triangleq \begin{bmatrix} E_{2i} & 0 \end{bmatrix}, \\ &\tilde{F}_i \triangleq \begin{bmatrix} C_i & C_i - C_r \end{bmatrix}, \quad \bar{v}(k) \triangleq \begin{bmatrix} v^T(k) & r^T(k) \end{bmatrix}^T. \end{aligned}$$

By introducing the new state variable, the considered tracking control problem is transformed into a stabilization one for the augmentation system (32) since $\eta(k) \to 0$ implies that $z_e(k) \to 0$. Note that the augmentation system (32) has a similar form with (3) and thus, the obtained results such as Theorem 2 and Algorithm 1 can be easily applied to deal with the considered tracking control problem by some minor changes. For saving the space, the details are omitted here.

Set the model parameters of (31) as

$$A_r = \begin{bmatrix} 0.3 & 0.1 \\ 0.2 & 0.5 \end{bmatrix}, \quad B_r = \begin{bmatrix} 0.5 & 0.3 \\ 0.6 & 0.2 \end{bmatrix}, \quad C_r = \begin{bmatrix} 1 & 0.5 \\ 0.6 & 1 \end{bmatrix}.$$

Choose $\gamma = 1$ and set the reference input $r(k)$ as

$$r(k) = \begin{cases} \begin{bmatrix} 0.1\sin(0.1k) \\ 0.1\sin(0.1k) \end{bmatrix}, & 0 \le k \le 260 \\ \begin{bmatrix} 0.05\cos(0.05k) \\ 0.05\cos(0.05k) \end{bmatrix}, & 400 \le k \le 800 \\ 0, & \text{otherwise.} \end{cases}$$

Based on the obtained controller gains, the controlled output $z(k)$ is shown in Figs. 7-8. By computation, we have $\gamma_a = 0.7140 < 1$ ($\gamma_a \triangleq \sqrt{\sum_{i=0}^{k_f} z_e^T(i)z_e(i)/\sum_{i=0}^{k_f}\bar{v}^T(i)\bar{v}(i)}$) which validates that the prescribed H_∞ performance is satisfied.

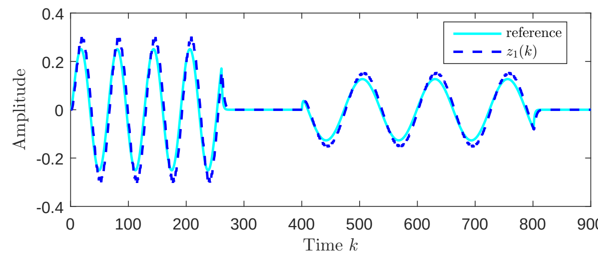

Fig. 7: Controlled output $z_1(k)$

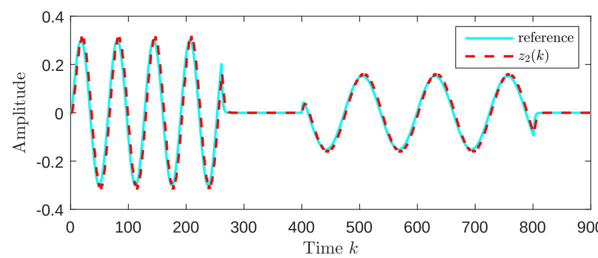

Fig. 8: Controlled output $z_2(k)$

To further display the disturbance attenuation capability and the tracking performance of the controlled system, we provide some comparison results in Table II which gives the attained disturbance attenuation level γ_a under different given γ. From this table, we can see that by choosing a smaller γ, the controlled system will have a better tracking performance. All simulation results presented in this example validate that the proposed controller performs very well for the tracking control problem.

TABLE II: The values of γ_a under different γ

γ	1	2	3	3.5
γ_a	0.7140	0.9678	1.0238	1.0336

C. Example 3

In this example, we aim to control a network-based truck-trailer system by using the proposed fuzzy PID controller. The modified system model is represented by [8]:

$$x_1(k+1) = \left(1 - \frac{v_0 t_0}{L_0}\right)x_1(k) + \frac{v_0 t_0}{l_0}u(k) + 0.1v(k)$$

$$x_2(k+1) = \frac{v_0 t_0}{L_0} x_1(k) + x_2(k) + 0.1v(k)$$
$$x_3(k+1) = v_0 t_0 \sin\left(\frac{v_0 t_0}{2L_0} x_1(k) + x_2(k)\right) + x_3(k)$$

where $x_1(k)$ is the angle difference between the truck and the trailer; $x_2(k)$ is the angle of the trailer; $x_3(k)$ is the vertical position of the rear end of the trailer; $u(k)$ is the steering angle; $v(k) = \frac{2\sin k}{k}$ is the energy-bounded external disturbance; $l_0 = 2.8m$ is the length of the truck; $L_0 = 5.5m$ is the length of the trailer; $t_0 = 2s$ is the sampling time; and $v_0 = -1m/s$ is the constant speed of backing up.

The measurement output and controlled output are given as follows:

$$y(k) = Cx(k) + E_2 v(k), \quad z(k) = Fx(k)$$

where

$$C = \begin{bmatrix} 7 & -2 & 0.03 \\ 5 & -1 & 0.03 \end{bmatrix}, \; F = \begin{bmatrix} 0.1 & 0.3 & 0.1 \end{bmatrix},$$
$$x(k) = \begin{bmatrix} x_1(k) & x_2(k) & x_3(k) \end{bmatrix}^T, \; E_2 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}.$$

By employing the fuzzy modeling technique, we use the following 3-rules T-S fuzzy model to represent this nonlinear system according to the operation point 0 rad, $\pm\frac{\pi}{6}$ rad, and $\pm\pi$:

$$\begin{cases} x(k+1) = \sum_{i=1}^{3} h_i(\theta(k))\Big(A_i x(k) + Bu(k) + E_1 v(k)\Big) \\ y(k) = Cx(k) + E_2 v(k) \\ z(k) = Fx(k) \end{cases}$$

where

$$A_1 = \begin{bmatrix} 1 - \frac{v_0 t_0}{L_0} & 0 & 0 \\ \frac{v_0 t_0}{L_0} & 1 & 0 \\ \frac{v_0^2 t_0^2}{2L_0} & v_0 t_0 & 1 \end{bmatrix}, \; A_2 = \begin{bmatrix} 1 - \frac{v_0 t_0}{L_0} & 0 & 0 \\ \frac{v_0 t_0}{L_0} & 1 & 0 \\ \frac{3v_0^2 t_0^2}{2\pi L_0} & \frac{3v_0 t_0}{\pi} & 1 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} 1 - \frac{v_0 t_0}{L_0} & 0 & 0 \\ \frac{v_0 t_0}{L_0} & 1 & 0 \\ \frac{v_0^2 t_0}{200L_0^2} & \frac{v_0}{100L_0} & 1 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{v_0 t_0}{l_0} \\ 0 \\ 0 \end{bmatrix},$$
$$E_1 = \begin{bmatrix} 0.1 & 0.1 & 0 \end{bmatrix}^T, \quad \theta(k) = \frac{v_0 t_0}{2L_0} x_1(k) + x_2(k).$$

The aim of this example is to design the proposed fuzzy PID controller such that requirements R1) and R2) are satisfied simultaneously.

Set $\gamma = 2$ and $x(0) = \begin{bmatrix} 0.1 & -0.1 & 0.1 \end{bmatrix}^T$. By applying the controller gains obtained via Algorithm 1, simulation results are given in Figs. 9-10. Fig. 9 shows the open-loop state evolution which is obviously unstable. Fig. 10 plots the closed-loop state evolution from which we can see that the controlled system is stable, implying that the backing up is achieved successfully. By simple computation, we have $\gamma^* = 0.2517 < 2$ which means that requirement R2) is satisfied. To further check the H_∞ performance of the controlled truck-trailer system, we list the attained disturbance attenuation level γ^* in Table III under different energy-bounded noises. The results show that the obtained γ^* under different cases are all smaller than the given $\gamma = 2$ that verifies the good disturbance attenuation capability.

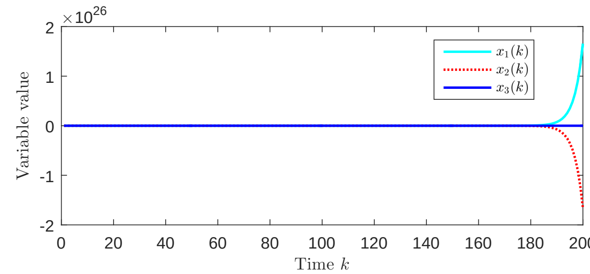

Fig. 9: State evolution $x(k)$ of the open-loop system

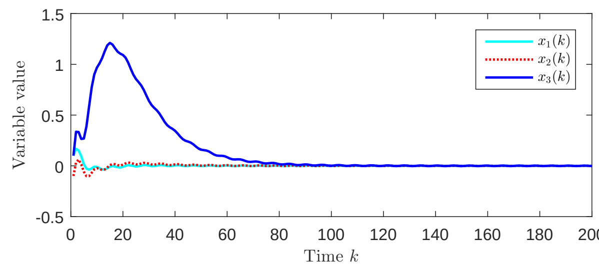

Fig. 10: State evolution $x(k)$ of the closed-loop system

TABLE III: γ^* under different noises

noise $v(k)$	$\frac{\sin k}{k}$	$\frac{5}{k^2}$	$5e^{-0.1k}$
γ^*	0.6707	0.1113	0.5739

V. Conclusion

This paper has been concerned with the H_∞ fuzzy PID output-feedback control problem for discrete-time T-S fuzzy systems subject to infinite-DTDs and RR protocol. A shared communication network has been used to deal with the signal transmissions between the sensors and controller, where the data exchange has been guided by the underlying RR protocol. A novel and easy-to-implement fuzzy PID controller has been developed to stabilize the considered fuzzy system, under which the prescribed H_∞ performance requirement has been also achieved. Based on the CCL algorithm, the controller gains have been obtained. Finally, the effectiveness of our proposed design scheme has been verified by three simulation examples. Further study includes the fuzzy PID control problem for nonlinear systems with time-varying delays and other network-induced effects (e.g. event-triggered mechanism effects [54]–[56] and quantization effects [39], [57], [58]).

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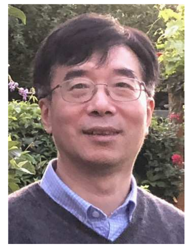
Yezheng Wang received the B.Eng. degree in automation from the Qilu University of Technology, Jinan, China, in 2017. He is currently pursuing the Ph.D. degree from the College of Electrical Engineering and Automation, Shandong University of Science and Technology, Qingdao, China.

His current research interests include the control and filtering for T-S fuzzy system.

Mr. Wang is a very active reviewer for many international journals.



Zidong Wang (SM'03-F'14) was born in Jiangsu, China, in 1966. He received the B.Sc. degree in mathematics in 1986 from Suzhou University, Suzhou, China, and the M.Sc. degree in applied mathematics in 1990 and the Ph.D. degree in electrical engineering in 1994, both from Nanjing University of Science and Technology, Nanjing, China.

He is currently Professor of Dynamical Systems and Computing in the Department of Computer Science, Brunel University London, U.K. From 1990 to 2002, he held teaching and research appointments in universities in China, Germany and the UK. Prof. Wang's research interests include dynamical systems, signal processing, bioinformatics, control theory and applications. He has published more than 600 papers in refereed international journals. He is a holder of the Alexander von Humboldt Research Fellowship of Germany, the JSPS Research Fellowship of Japan, William Mong Visiting Research Fellowship of Hong Kong.

Prof. Wang serves (or has served) as the Editor-in-Chief for *International Journal of Systems Science*, the Editor-in-Chief for *Neurocomputing*, and an Associate Editor for 12 international journals including IEEE Transactions on Automatic Control, IEEE Transactions on Control Systems Technology, IEEE Transactions on Neural Networks, IEEE Transactions on Signal Processing, and IEEE Transactions on Systems, Man, and Cybernetics-Part C. He is a Member of the Academia Europaea, a Fellow of the IEEE, a Fellow of the Royal Statistical Society and a member of program committee for many international conferences.

Lei Zou received the B.Sc. degree in automation from Beijing Institute of Petrochemical Technology, Beijing, China, in 2008, the M.Sc. degree in control science and engineering from China University of Petroleum (Beijing Campus), Beijing, China, in 2011 and the Ph.D degree in control science and engineering in 2016 from Harbin Institute of Technology, Harbin, China. From October 2013 to October 2015, he was a visiting Ph.D. student with the Department of Computer Science, Brunel University London, Uxbridge, U.K.

Since 2019, he has been working as a Research Fellow with the Department of Computer Science, Brunel University London, Uxbridge, Uxbridge, U.K. His research interests include control and filtering of networked systems, moving-horizon estimation, and state estimation subject to outliers.

Dr. Zou is currently serving as an Associate Editor for Neurocomputing and International Journal of Systems Science; a Member of IEEE and a Member of Chinese Association of Automation; and a very active reviewer for many international journals.

Hongli Dong received the Ph.D. degree in control science and engineering from the Harbin Institute of Technology, Harbin, China, in 2012.

From 2009 to 2010, she was a Research Assistant with the Department of Applied Mathematics, City University of Hong Kong, Hong Kong. From 2010 to 2011, she was a Research Assistant with the Department of Mechanical Engineering, The University of Hong Kong, Hong Kong. From 2011 to 2012, she was a Visiting Scholar with the Department of Information Systems and Computing, Brunel University London, London, U.K. From 2012 to 2014, she was an Alexander von Humboldt Research Fellow with the University of Duisburg–Essen, Duisburg, Germany. She is currently a Professor with the Artificial Intelligence Energy Research Institute, Northeast Petroleum University, Daqing, China. She is also the Director of the Heilongjiang Provincial Key Laboratory of Networking and Intelligent Control, Daqing. Her current research interests include robust control and networked control systems.

Dr. Dong is a very active reviewer for many international journals.