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Non-linearities and persistence in US long-run interest rates

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ABSTRACT

This note examines the stochastic behaviour of US monthly 10-year government bond yields. Specifically, it estimates a fractional integration model suitable to capture both persistence and non-linearities, these being two important properties of interest rates. Two series are analysed, one from Bloomberg including end-of-the-month values over the period January 1962–August 2020, the other from the ECB reporting average monthly values over the period January 1900–August 2020. The estimation results indicate that both are highly persistent and exhibit non-linearities, the latter being more pronounced in the case of the ECB series. Also, there is no conclusive evidence of the presence of structural breaks.

KEYWORDS

Long-term interest rates; government bond yields; fractional integration; persistence; non-linearities

JEL CLASSIFICATION

C22; E43

1. Introduction

Two important properties of interest rates are their high degree of persistence and the presence of non-linearities. The former has implications for the effectiveness of monetary policy and the empirical relevance of different theories such as consumption-based asset pricing models and the Fisher effect. The latter has become even more relevant in the new economic environment characterized by unconventional monetary policy and the so-called zero lower bound (ZLB) for interest rates. Therefore, it is crucial to adopt a modelling approach that can capture both. Earlier studies were normally based on the $I(0)/I(1)$ dichotomy and on a linear framework. For instance, Cox, Ingersoll, and Ross (1985) characterized the short-term nominal interest rate as a stationary and mean-reverting $I(0)$ process, whilst Campbell and Shiller (1987) concluded that it exhibits a unit root and therefore is an $I(1)$ process, which implies a lack of mean reversion (namely, shocks to interest rates have permanent effects).

Since unit root tests are well known to have very low power against fractional alternatives (Diebold and Rudebush 1991; Hassler and Wolters 1994; Lee and Schmidt 1996, etc.),

a number of subsequent studies have used a fractional integration framework to analyse the behaviour of interest rates. For instance, Lai (1997), Phillips (1998) and Tsay (2000) found that it is appropriate for US real interest rates (see also Barkoulas and Baum 1997; Meade and Maier 2003; Gil-Alana 2004a, 2004b), and Couchman et al. (2006) presented similar evidence for 16 countries. Caporale and Gil-Alana (2009) reported that in the case of the Federal Funds effective rate the fractional integration parameter is sensitive to the specification of the error term, whilst Caporale and Gil-Alana (2016, 2017) obtained evidence of long memory and fractional integration and cycles for the Euribor and the Fed Funds rate, respectively.

The most recent literature has also argued that fractional integration is very much related to non-linearities (see Granger and Hyung 2004, etc.) and that these should also be taken into account when modelling interest rates. Therefore, the present note estimates a fractional integration model allowing for both persistence and non-linearities to investigate the stochastic behaviour of US monthly 10-year government bond yields; the possible presence of structural breaks is also examined. Below, Section 2 discusses the data and the empirical

analysis, and [Section 3](#) offers some concluding remarks.

II. Data and empirical results

We examine two monthly series for US 10-year Government Benchmark bond yields. The first (.USGG10YR Index) includes end-of-the-month values and has been obtained from Bloomberg over the period January 1962 – August 2020. The second reports instead average monthly values; the data source in this case is the European Central Bank (ECB) database, and the sample period is January 1900 – August 2020. These two series are plotted in [Figures 1 and 2](#) respectively. In both cases it is apparent that long-term rates, initially relatively low, peaked in the 1980s before falling again.

The estimated non-linear model, as in [Cuestas and Gil-Alana \(2016\)](#), is the following:

$$Y_t = \sum_{i=0}^m \theta_i P_{iT}(t) + X_t, (1-L)^d X_t = u_t, t = 1, 2, \dots, \quad (1)$$

where y_t is the observed time series, and P_{iT} are the Chebyshev time polynomials defined as:

$$\begin{aligned} P_{0,T}(t) &= 1, P_{i,y}(t) = \sqrt{2} \cos(i\pi(t - 0.5)/T), t \\ &= 1, 2, \dots, T; i = 1, 2, \dots, \end{aligned} \quad (2)$$

where m indicates the degree of non-linearity. Thus, the higher m is, the less linear the approximated deterministic component is (see [Hamming 1973](#); [Smyth 1998](#)). [Bierens \(1997\)](#) and [Tomasevic, Tomasevic, and Stanivuk \(2009\)](#) argue that it is possible to approximate highly non-linear trends with rather low degree polynomials. In this context, if $m = 0$ the specification contains only an intercept; if $m = 1$ a linear time trend is also included, and if $m > 1$, non-linearities are allowed. When estimating the model given by (1) we set $m = 3$, thus capturing non-linearities in the series if θ_2 and/or θ_3 are statistically significant. We also assume that the errors are autocorrelated to allow for some degree of weak dependence. However, instead of using a standard AutoRegressive Moving Average (ARMA)



Figure 1. 10-year US Bond yield (.USGG10Y index). Source: Bloomberg.



Figure 2. 10-Yr US Bond yield (average of monthly observations). Source: ECB.

model, we follow the non-parametric approach of Bloomfield (1973) that has been shown to work well in the context of fractional integration (see Gil-Alana 2004c).

Table 1 displays the results. The fractional differencing parameter d is estimated to be equal to 0.84 and 0.83 for the Bloomberg and ECB monthly series, respectively, and in both cases, the values in the confidence intervals are strictly smaller than 1, which supports the hypothesis of mean reversion. This is consistent with the findings of other studies on interest rates (Gil-Alana and Moreno 2012; Abbritti et al. 2016; etc.). In the case of the Bloomberg series, the four deterministic coefficients are statistically significant, which implies non-linear behaviour. However, in the case of the ECB series with average values only θ_0 and θ_3 are found to be significant, which indicates a lower degree of non-linearities.

We have also investigated the possible presence of structural breaks in the series of interest by carrying out both the Bai and Perron (2003) and Gil-Alana (2008) tests, the latter having been specifically designed for the case of fractional integration. The two sets of results were very similar, in both cases two breaks being found in the Bloomberg series (in September 1981 and September 1987) and three in the ECB series (in December 1920, November 1945 and September 1981). Table 2 reports the estimates of d for each subsample based on the assumption of white noise errors. Similar results were obtained with autocorrelated disturbances. It can be seen that, given the wide confidence intervals, the results are not conclusive, namely the $I(1)$ hypothesis cannot be rejected in the majority of cases. Specifically, for the Bloomberg series, it cannot be rejected for any of the three subsamples, whilst in the case of the ECB series the same holds for the first two

Table 1. Estimates of the Non-linear $I(d)$ Model.

Time Series	d	θ_0	θ_1	θ_2	θ_3
Bloomberg monthly 10-year bond yield data (1962–2020)	0.84 (0.76, 0.95)	5.7635 (3.18)	1.7422 (1.66)	-1.9333 (-3.03)	-1.0016 (-2.20)
ECB monthly 10-year bond yield data (1900–2020)	0.83 (0.78, 0.89)	2.9648 (2.08)	-1.0753 (-1.28)	-0.6461 (-1.27)	1.741 (4.80)

In bold, significant coefficients at the 5% level. In parentheses, in columns 3–6, the associated t-values.

Table 2. Sub-sample analysis (white noise errors).

Bloomberg series			
Subsamples	No terms	Intercept	Intercept and Trend
January 1962/September 1981	1.05 (0.96, 1.16)	0.99 (0.91, 1.11)	0.99 (0.90, 1.11)
October 1981/September 1987	0.97 (0.83, 1.16)	1.05 (0.90, 1.27)	1.05 (0.90, 1.26)
October 1987/August 2020	0.99 (0.92, 1.08)	1.00 (0.91, 1.11)	1.00 (0.91, 1.11)
ECB series			
Subsamples	No terms	Intercept	Time trend
January 1900/December 1920	1.02 (0.93, 1.13)	1.05 (0.96, 1.17)	1.05 (0.96, 1.17)
January 1921/November 1945	0.98 (0.90, 1.08)	1.11 (0.99, 1.24)	1.10 (0.99, 1.24)
December 1945/September 1981	1.17 (1.09, 1.28)	1.16 (1.07, 1.29)	1.16 (1.07, 1.29)
October 1981/August 2020	0.94 (0.88, 1.02)	1.20 (1.10, 1.32)	1.19 (1.09, 1.32)

In bold significant coefficients at the 5% level. In parentheses, the 5% confidence intervals.

subsamples, while this hypothesis is rejected in favour of $d > 1$ in the remaining two cases.

III. Conclusions

This note provides some evidence on the behaviour of US long-term interest rates, more specifically, the 10-year government bond yields. The fractional integration framework used is more general than the standard models based on the $I(0)$ vs. $I(1)$ dichotomy and can capture both persistence and non-linearities. The results show that indeed both these features are present in US long-term interest rates; also, the evidence of non-linearities is stronger for the Bloomberg series including end-of-the-month values than for the ECB one with average values. Further, break tests and sub-sample analysis do not provide conclusive evidence of the presence of structural breaks.

Since theory suggests that a variety of shocks such as preference, technology, fiscal and monetary shocks can generate persistence, it would be interesting to carry out additional research to investigate their relative importance. Furthermore, non-linear dynamics also have implications for the term structure of interest rates, namely that the relationship between short and long rates is no longer linear as in a standard cointegration model. Future work should also analyse such issues.

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