TR/09/87 September 1987

Computer Assisted Mathematical Programming

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(Modelling) System : CAMPS

by

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** To appear in the Computer Journal (GB)

0. Abstract

A Computer Assisted Mathematical Programming (Modelling) System (CAMPS) is described in this paper. The system uses program generator techniques for model creation and contrasts with earlier approaches which use a special purpose language to construct models. Thus no programming skill is required to formulate a model. In designing the system we have first analysed the salient components of the mathematical programming activity. A mathematical programming model is usually constructed by progressive definition of dimensions, data tables, model variables, model constraints and the matrix coefficients which connect the last two entities. Computer assistance is provided to structure the data and the resulting model in the above sequence. In addition to this novel feature and the automatic documentation facility, the system is in line with recent developments, and incorporates a friendly and flexible user interface.

1. Introduction and major issues

During the last thirty years algorithms and computer programs for solving optimisation problems have witnessed sustained and accelerated development. Today large scale problems can be processed robustly and successfully, yet our ability to construct models and our understanding of many facets of these models are in contrast less developed. This issue has received the attention of many practitioners. According to Geoffrion [GEOFF85] "Modelling as done to day is a much lower productivity process than it ought to be. It takes too long to build, verify, and document models. It is too hard to maintain and make evolutionary improvements...". Krabek, Sjoquist and Sommer [KRSJSO80] point out that the steps of managing data, building the model, reporting and analysing results, are much more expensive than that of optimisation. Further these steps prove to be a barrier to the effective use of LP. Greenberg [GREENB83] observed that "comprehension is the present bottleneck in using large scale models."

During the seventies MPSX [IBM76] established itself as the defacto standard for Linear Programming and Integer Programming (LP/IP) and as a result its input format is also accepted as the standard for specifying LP/IP model input data. High level languages FORTRAN, PLl were used to generate MPSX input files: although this is model specific and burdensome it is still widely used. During the seventies matrix generator and report writer (MGRW) systems became well established: MAGEN [HAVERL77], GAMMA3 [SPERRY78], and DATAFORM [KETRON75] are among the best known of these. These introduced flexibility and productivity in creating LP applications and are still heavily used today. The next generation of tools may be broadly classed as "Matrix Languages" and these bear more resemblance to the way a modeler would describe a problem. Early "matrix languages" include LP MODEL [KARIRO80], MGRW [IBM77], MGG/RWG [SCICON75], UIMP [MITELL82], and GAMS [BISMEE82]. In contrast to special purpose programs and MGRW systems which require considerable understanding of the input data formats the matrix languages require only a "limited" knowledge of these Fourer [FOURER83] provides a comprehensive discussion of the major issues as seen in the early eighties. Since the paper by Fourer three other systems ULP [WITMCC85], MAGIC [DAYWIL86], EXPRESS LP [DASH86] of the same genre have been developed and reported. An alternative approach of describing LP models use the concept of flows and flow balances in networks. LOGS [BRNOSH86] is perhaps the most widely known of these systems.

Application of these systems to substantial models in the corporate context bring out a number of other considerations. Murphy and Stohr [MURSTO85] highlight the relevance of block structuring and block connectivity of such models and Geoffrion [GEOFF85] has addressed the question of aggregation in considerable depth. Bradley and Clements [BRACLE85] report a mathematical programming implementation (LEXICON) of the structured modelling framework of Geoffrion. The most well
known implementations of LP in corporate modelling are PLATOFORM [PALMER84], and PLANETS [EDS86].

During the last three years personal computers have established in their usage and there has been considerable upsurge of interest in teaching/training systems for Operational Research (OR) in general and LP in particular. Of these systems LINDO [SCHRAG81] is the most established teaching system. For a discussion and evaluation of a number of micro-based LP optimisers the readers are referred to [SHARDA84], On the model building[1] front there has been a strong trend towards using well known spreadsheet systems such as LOTUS 1-2-3 and SYMPHONY [LOTUS84], [JENSEN86], [EASTRN84]. These entry level systems are excellent for training and breaking down barriers to modelling, but their use and applicability in large structured models remain questionable.

In this paper we describe a new mathematical programming modelling system called CAMPS. It is an interactive system and comprises a set of integrated 'program generation' and data management tools which are controlled by a series of menus and screenforms. Our design objectives are broad: our system is set out to help non-expert LP users to come to grips with the task of conceptualising and describing LP models whereas the expert LP user is also supported in his requirements to construct large and complex models. The contents of this paper are organised as follows. Section 2 describes the salient and novel features of CAMPS, an example of model construction using CAMPS is illustrated in Section 3, the logical analysis of the modelling task and the derivation of mathematical statement are set out in Section 4. The method of automated reformulation of separable and 0-1 integer programming is considered in Section 5. The problem of section 3 is reformulated using ULP [WITMCC85] and OMNI [HAVERL76] in the appendix and contrasts our approach with these well known systems.

2. Salient and novel features of CAMPS

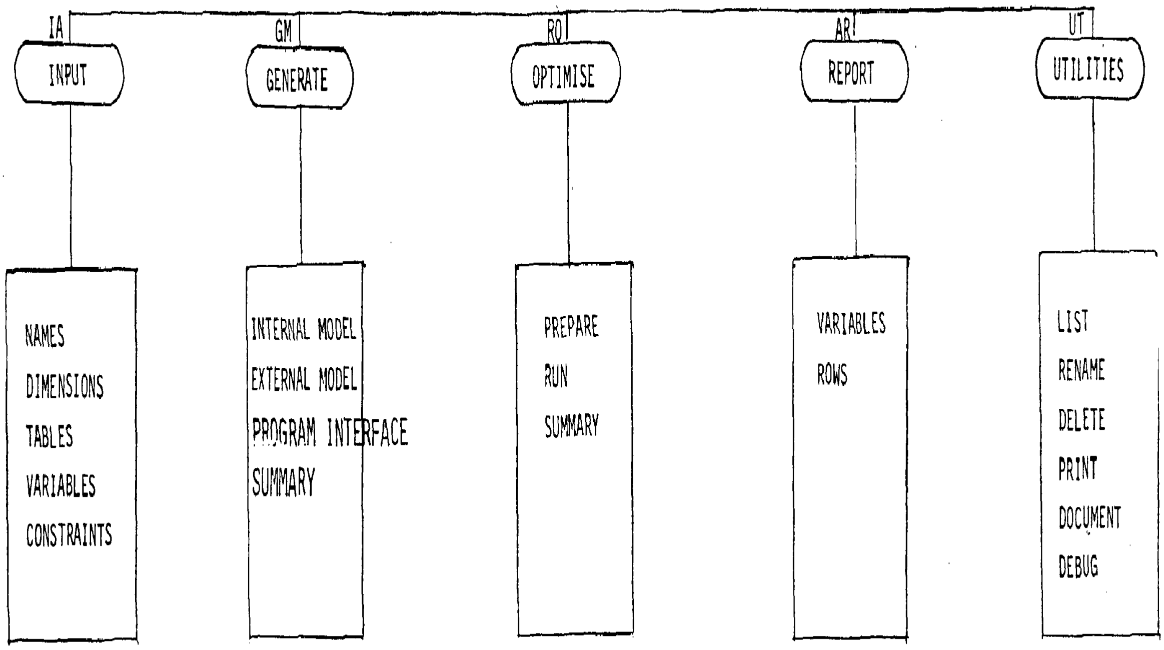
Computer Assisted Mathematical Programming Modelling System (CAMPS) is an interactive system designed to aid model formulation, matrix generation and model management. The system comprises a set of integrated 'program generation' and data management tools which are controlled by a series of menus and screenforms. The main menu shown in display 2.1 together with the information flow diagram display 2.2 provide an outline of the structure and the major functions of the system. A full user specification of the system is given in [LUCMIT85]

```
                              ---CAMPS---
          USER:                                             DATE:
          MODEL:                                            TIME:
                                  SEC:
-------------------------------------------------------------------------------------------

                              1-INPUT
                              2-GENERATE
                              3-OPTIMISE
                              4-REPORT
                              5-UTILITIES
                              6-LOGOUT

                                        TYPE   NUMBER<<     >>:
```

Display 2.1

The INPUT (and AMEND) option is used to construct and/or update all aspects of a model created entirely within CAMPS. Display 2.3 illustrates the options under this subsystem and reflects the modelling

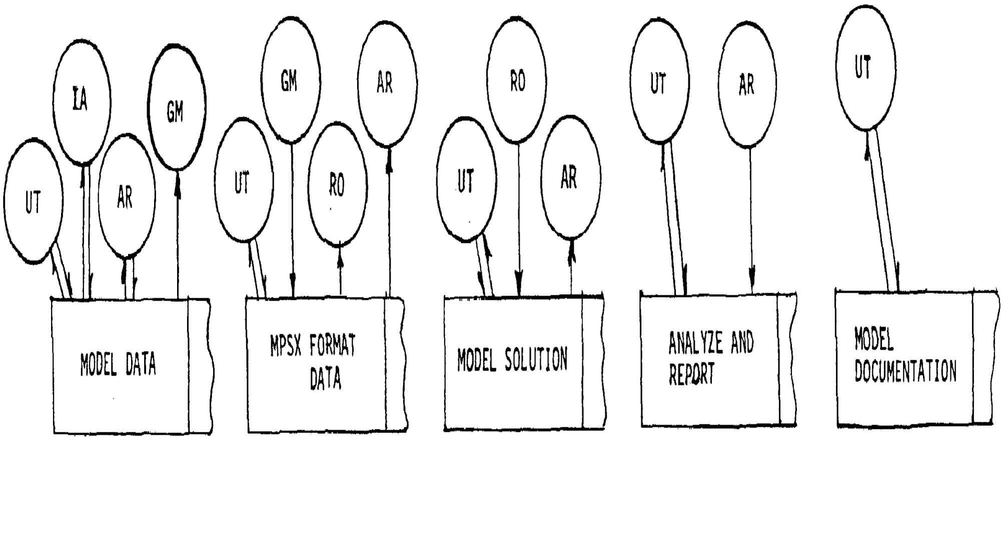
Display 2.2



HIERARCHICAL RELATIONSHIP OF MAIN MENU OPTIONS

AND

INFORMATION FLOW THROUGH THE FIVE MASTER FILES AS EFFECTED BY THE SUBSYSTEMS



```
          USER:                                                        DATE:
          MODEL:                                                       TIME:
                                       SEC:
-------------------------------------------------------------------------------------------

                                 1-NAMES
                                 2-DIMENSIONS
                                 3-TABLES
                                 4-VARIABLES
                                 5-CONSTRAINTS
                                 6-RETURN

                                           TYPE  NUMBER<<   >>:
```

Display 2.3.

methodology which is stated succinctly as a sequence of three logical steps.

Stepl Define the subscripts and their ranges (sets and dimensions).

Step2 Define input data tables, model variables and model constraints, in terms of these subscripts.

Step3 Specify the linear relationships in a row wise fashion Which connect the items defined in step 2.

The subscripts correspond to 'basic entities' which are elements of 'sets’ and in actual models these 'sets' could represent geographical regions, materials, time periods. This progressive approach to model definition allows us to dispense with a procedural language and replace it with an option driven program generator approach. The syntax of commands are captured in their context and thus mistakes introduced by erroneous keystrokes are kept to a minimum. This is because predefined indices, sets, variables are prompted at the appropriate fields of the screenforms. For instance at the time of defining variables and tables currently defined sets are displayed. At the time of entering the linear forms the
operators (+,-,*) are prompted and a linear term is forced to comply with the dimensions of the summation indices and the row indices. We discuss this point further in the example given in section 3.

The first four options of the main menu are designed to facilitate construction and investigation of a model whereas the fifth, the UTILITIES option, provides model management support. In CAMPS the usual model management functions such as DELETE, RENAME, LIST and PRINT is augmented by a further option called DOCUMENT. Tabular displays of the input data, variable(MPSX) names and row(MPSX) names, and tabulated results are essential aspects of documentation as supplied by all known systems. In addition to these a mathematical formulation of the model is also provided by CAMPS. This mathematical statement can be enhanced by textual annotations specific for a given application. These explanatory texts are introduced at the input stage.

The REPORT subsystem allows information relating to the rows, columns and reduced costs to be examined. The analysis module within REPORT is now designed to interface with the interactive model and solution analysis system ANALYZE by Greenberg [GREENB83]. For each 'basic entity’ a textual annotation may be supplied and a unique two character identifier called ‘stub’ is extracted out of this text [GRLUMI86], This stub is used to create the ‘syntax file’ of ANALYZE. The OPTIMISE option uses the FORTLP system [TAMIYA85], For all practical purposes this is treated as a black box, although a few algorithm control parameters can be set under this option.

LP/IP models are created in MPSX format under the GENERATE subsystem. Within the GENERATE subsystem externally created models are also accepted but REPORT and DOCUMENT options cannot be used in this case. Whereas CAMPS itself is designed for high level interaction in the modeller's form, at the GENERATE subsystem level a programmer's interface for model generation is also available. Thus it is possible to create MPSX models using data tables and model descriptions not held within CAMPS. In this approach the system held subroutine library for model generation is used. We note that this approach is somehow similar to the ideas put forward by Forrest [FORRES86]. We have used this approach to create set covering models in MPSX format [ELDMIT86]. These models were supplied to us in a non standard format.

In order to deal with well known model structures or restrictive modelling situations a compendium of reserved words have been introduced in the TABLES and ROWS section of the system. A reserved table RESTRICT with appropriate dimension is created by default as an internal table of 0-1 entries. It is used subsequently to deal with undefined entries in the primary tables. NETWORK, CONVEX and REFER are reserved row names. NETWORK is used to create a compact network model with balanced flows. CONVEX and REFER are used to achieve separable programming (set type one and set type two) model reformulation within the system [LUMIYA86].

3. An Annotated Example

In this section we consider a problem taken from the book by Jensen and Barnes [JENBAR80]. This example is specially chosen as it displays the typical structure of an integrated production and distribution model. The example is also adopted by Geoffrion [GEOFF85] and Bradley [BRACLE85] to illustrate their systems.

The Tanglewood Manufacturing Co. has four plants located around the country. The fabrication and assembly cost per chair and the minimum and maximum monthly production for each plant are shown in Table 3.1.

PLANT	Cost	Max Production	Min Production
Washington	$5.00	500	0
Philadelphia	7.00	750	400
Denver	3.00	1000	500
Buffalo	4.00	250	250

FABRICATION COST AND PRODUCTION RESTRICTIONS BY PLANT

Table 3.1

The company obtains the twenty pounds of wood required to make each chair from two suppliers who have agreed to supply any amount ordered. In return, the company guarantees the purchase of at least 8 tons of wood per month from each supplier. The cost of wood is $0.10/lb from supplier 1 and $0.075/lb from supplier 2. The shipping cost in $/lb from each supplier to each plant is shown in table 3.2.

	PLANT			
SUPPLIER ↓	Washington	Philadelphia	Denver	Buffalo
Ontario	0.01	0.02	0.04	0.04
Quebec	0.04	0.03	0.02	0.02

SHIPPING COST FROM SOURCE TO PLANT
(UNIT COST $/lb of WOOD)

Table 3.2

The chairs are sold in New York, Houston, San Francisco and Chicago. Transportation costs in $/chair between the cities and plants are listed in Table 3.3. Finally, Table 3.4 shows the minimum demand that must be satisfied, the maximum demand that must be satisfied and the selling price for chairs in each city.

	CITY			
PLANT ↓	New York	Houston	San Francisco	Chicago
Washington	1.00	1.00	2.00	0.00
Philadelphia	3.00	6.00	7.00	3.00
Denver	3.00	1.00	5.00	3.00
Buffalo	8.00	2.00	1.00	4.00

TRANSPORTATION COST BETWEEN PLANTS AND CITIES
(UNIT COST $/CHAIR)

Table 3.3

CITY ↓	Selling Price Per Chair	Max Demand	Min Demand
New York	$20.00	2000	500
Houston	15.00		
	400		
100			
San Francisco	20.00	1500	500
Chicago	18.00	1500	500

SELLING PRICE AND DEMAND RESTRICTIONS BY CITY

Table 3.4

It is desired to find the optimal production and shipment so as to maximise profit. A mathematical statement of this problem is set out below.

-Subscripts and Dimensions

Let

i=l,2 denote the timber merchants (suppliers),
j=l,2,3,4 denote the wood fabrication units (plants),
k=l,2,3,4 denote the chair retailers (cities).

-Model Coefficients (Descriptors)

Let

c_j denote the cost of producing one chair at wood plant j,
n_j denote the minimum production of chairs at wood plant j,
q_j denote the maximum production of chairs at wood plant j
p_k denote the selling price of chairs at chair retailer k,
l_K denote the minimum amount of chairs required by chair retailer k,
h_k denote the maximum amount of chairs that can be handled by chair retailer k,
t_{jk} denote the shipment cost between wood plant j and chair retailer k,
m_{ij} denote the shipment cost between timber merchant i and wood plant j,
s_i denote the cost of wood at timber merchant i,
di denote the minimum order amount at timber merchant i.

-Model Variables
Let

zij denote the quantity of wood bought from timber merchant i and processed in wood plant j,
yjk denote the number of chairs bought by customer chair retailer k, from wood plant j.

-A mathematical statement of the objective function and linear constraint relations.

Maximise

$$\text{profit} = \sum_{j=l}^{4}\sum_{k=l}^{4} (p_k y_{jk} - c_j y_{jk} - t_{jk} y_{jk})$$

$$-\sum_{i=l}^{2}\sum_{j=l}^{4} (m_{ij} z_{ij} + s_i z_{ij})$$

subject to the constraints:

minimum order amount of the timber merchant i,

$$\sum_{j=l}^{4} z_{ij} \geq d_i \qquad i=1, 2$$

production at plant j within allowable range,

$$\left.\begin{array}{l} \sum_{k=1}^{4} y_{jk} \geq n_j \\ \sum_{k=1}^{4} y_{jk} \leq q_j \end{array}\right\} \quad j = 1, 2, 3, 4$$

meeting customer demand at k within allowable range,

$$\left.\begin{array}{l} \sum_{j=1}^{4} y_{jk} \geq l_k \\ \sum_{j=1}^{4} y_{jk} \leq h_k \end{array}\right\} \quad k = 1,2,3,4$$

stock balance at plant j,

$$\sum_{i=1}^{2} z_{ij} - \sum_{k=1}^{4} 20y_{jk} = 0 \qquad j = 1,2,3,4.$$

This problem was created using CAMPS and descriptive names for tables and variables were used instead of one character algebraic symbols. For example c_j is replaced by PLNTCOST(j). Displays 3.1 to 3.5 provide a selection of screenforms which were used to construct the model. The method of defining names and the associated text is illustrated by the table names screenform (Display 3.1). The sets, indices and their ranges are defined as shown in Display 3.2. Displays 3.3 and 3.4 illustrate how the data tables and model variables are dimensioned. A typical model equation (the objective function) is set out in Display 3.5.

```
SEC: NAMES SECTION                                MODEL: TANGWOOD

             TABLE  NAME                      TEXT
---------------------------------------------------------------------------
                  ---------------            -------------------------
                  -PLNTCOST-                  -PLANT-COST-----.
                  ----------------            -------------------------


PLNTCOST                                      PLANT COST
PLNTMIN                                       MIN PRODUCTION
PLNTMAX                                       MAX PRODUCTION
CUSTPRCE                                      CUSTOMER PRICE
CUSTLDMD                                      MIN CUST DMND
CUSTHDMD                                      MAX CUST DMND
TCSTPTC                                       TRAN COST TO CST
TCSTPTC                                       TRAN COST FR SRC
```

Display 3.1

```
          SEC: INDICES SECTION                                          MODEL: TANGWOOD

SET       NAME     TEXT                         INDICES               LLIM    ULIM   STEP
1.        I-       TIMBER MERCHANTS          i------------            ---1    ---2    -1
2.        J-       WOOD-PLANTS----           j------------            ---1    ---4    -1
3.        K-       CHAIR RETAILERS-          k------------            ---1    ---4   -1
4.        --       -------------------------  -------------            ----    ----    --
5.        --       -------------------------  --------------           ----    -----   --
6.        --       -------------------------  --------------           ----    -----   --
7.        --       --------------------------  --------------           ----    -----   --
8.        --       --------------------------  ---------------          ----    -----   --
```

Display 3.2

```
          SEC: TABLES SECTION                                          MODEL: TANGWOOD

TABLE  NAME                TEXT                       TYPE            INDICES

1.        PLNTCOST            PLANT-COST------          -REAL--     j---------------
2.        PLNTMIN-            MIN-PRODUCTION--          -REAL--     j---------------
3.        PLNTMAX-            MAX-PRODUCTION--          -REAL--     k---------------
4.        CUSTPRCE            CUSTOMER-PRICE--          -REAL--     k---------------
5.        CUSTLDMD            MIN-CUST-DMND---          -REAL--     k---------------
6.        CUSTHDMD            MAX-CUST-DMND---          -REAL--     k---------------
7.        TCSTPTC-            TRAN-COST-TO-CST          -REAL--     j-, k-----------
8.        TCSTSTP-            TRAN-COST-FR-SRC          -REAL--     i-, j------------
```

Display 3.3

```
          SEC:     VARIABLES    SECTION                                 MODEL: TANGWOOD

VARIABLE NAME               TEXT                   TYPE                 INDICES

1.  WOFSTP--           TIMBER-SHIPPED--          -REAL--              i-,j-------------
2.  CHFPTC--           CHAIRS-SOLD-------        -REAL--              j-,k------------
3.  -------------     -------------------------  -----------          ----------------
4.  -------------      -------------------------  -----------          ----------------
5. --------------      -------------------------  -----------          ----------------
6. --------------      ------------------------   ------------         ----------------
7. --------------      ------------------------   ------------         ----------------
8. --------------     ------------------------    -------------        ----------------
```

Display 3.4

```
SEC:  ROWS    SECTION                                               MODEL: TANGWOOD
                            ROW NAME PROFIT
-----------------------------------------------------------------------------------------------------
SUM OVER     j , k            -PLNTCOST(j  )*CHFPTC  (j ,k  )
SUM OVER     j , k            CUSTPRCE(k )*CHFPTC  (j ,k )
SUM OVER     j , k            -TCSTPTC (j ,k )*CHFPTC  (j ,k    )
SUM OVER     i , j            -TCSTSTP (i ,j )*WOFSTP  (i ,j    )
SUM OVER     i , j            -SCRPRCE (i )*WOFSTP  (i ,j )
```

Display 3.5

In order to illustrate the method of specifying linear forms and the interactive syntactic support (of CAMPS) which ensures consistency of dimensions, consider the objective function (Display 3.5) which is made up of a few (5) summation terms. For each term once the (summation) indices are chosen (out of i, j, k) only tables (or constants) and model variables with matching indices can be chosen to construct the term.

A mathematical statement of the problem is obtained using the documentation facility of the UTILITY sub-system and is illustrated in displays 3.6 and 3,7. This documentation is sufficiently detailed and can be used for communication between analysts. In the linear expressions for the objective row and the constraint rows each term is annotated; a feature also found in GAMS [BISMEE82].

4. CAMPS and LP modelling tactics

The modelling support provided by CAMPS follows closely the logical steps that a modeller goes through to formulate an LP/IP application. The first task is to consider only the modelling requirements and extract the quantitative relationships which are germane to model definition. Having identified these items a compact statement of the problem is set out with only these pertinent details.

After identifying the key components of the model the next task is to discover the underlying structure in the model. This amounts to finding a way of defining categories. The following is an illustrative list of typical categories that are found in practical problems. Number of (decentralised) geographical locations, number of planning periods, number of different products, number of grades of people, number of age groups, and so on. Within CAMPS categories are called 'basic entities'.

Model Variables

Once the 'basic entities' are defined the model (decision) variables or the unknowns are broadly identified. An analysis of the decision variables may also suggest new 'basic entities' at this stage. This is because the model variables are generally detailed by 'basic entities'. For the purpose of illustration a number of decision variables taken from different contexts are considered below.

— Production Planning: The quantity Xp_m of a certain product p manufactured on a machine m.

— Distribution Planning: The quantity Xp_{rn} of a product p that is shipped from a source r to an outlet n.

— Inventory Scheduling: The quantity X_{pt} of a product p that is kept as closing stock at the end of a period t.

— Project Analysis: Whether one should invest in project p at the beginning of time period t, or not invest in this project $Y_{pt} = 1$ or 0 may be represented by this zero-one variable Y_{pt}.

Model Constraints

The constraints connect the decision variables and express the physical restrictions of the problem. By and large these are also detailed by

```
                        * * * * * * ********************************
                        *      Model   Documentation                 *
                        *      Prepared by            ...CLUCAS       *
                        *      Problem name           ...TAUGWOOD   *
                        *      Date                   …07131/50      *
                        *      Time                  . . . 1† : 45     *
                        ******************************************

INDICES
------------

i         =1,      2                          # ..    TIMAER MERCHANTS ..    #
j         =1,      4                          # ..    WOOD    PLANTS     ..    #
k         =1,      4                          # ..    CHAIR RETAILEAS    ..    #

TABLES
-----------
PLNTCOST( i )         #PLANET COST           ..by.. WOOD   PLANTS          ..#
PLNT      (j)         # PIN PRODUCTION       ..by.. WOOD   PLANTS          ..#
PLNTHAY(j)            # HAX PROUDCTON        ..by.. WOOD   PLANTS          ..#
CUSTPRE (k)           #CUSTOMER   PRICE      ..by.. CHAIR    RETAILERS     ..#
CUSTLOID (k)          #GIN CUST DMND         ..by.. CHAIR    RETAILERS     .. #
CUSTHDYD(k)           #FAX CUST DMND         ..by.. CHAIR   RETAILERS      ..#
TCSTPTC( j,k)         #TRANCOSTTOCST         ..by.. WOOD   PLANTS.         .and.. CHAIR  RETAILERS .#
TCSTSTF(i,j)          #TRANCOSTFORSRC        .by..  TIMBERYERCHANTS.       and..  WOOD  PLANTS    .#
SCRORCE (i)           # SOURCE PRICES        ..by.. TIMBER MERCHANTS  ..#
SCRLOND(i)            # SOURCE DEMANDS       ..by.. TIMBER MERCHANTS  ..#

VARIABLES
-----------------
WOFSTP(i,j)           #TIMEER  SHIPPED       . ..by.. . TIMEERMERCHANTS..and..WOODPLANTS..#
CHFOTC(j,k)           #CHAIRS  SOLD          . .by..   WOODPLANTS..and..CHAIRRETAILERS..#

ROWS
---------
WMNSRC (i)            # MIN  AMT  SHIPPED     ..by..  TIMBER MERCHANTS ..#
MPRCO(j)              #-MIN AMT PRODUCED     ..by..   WOOD PLANTS                 .. #
XPROD(j)              #MAX AMT PROUDCED      ..by..  CHAIR RETAILERS      ..#
CLW(k)                #MIN CUST  DEMAND      ..by..  CHAIR RETAILERS      ..#
THIGH(k)              #MAX CUST DEMAND      ..by..  CHAIR RETAILERS      ..#
BSTOCK(j)             #STOCK PALANCE         ..by..   WOOD PLANTS          ..#
PROFIT                #MAXIMISE  PROFIT #

CONSTRAINTS
---------------------
Row names WINSRC(i)                            #MIN AMT SHIPPED. .restriction. .#
Sum over j       [ +1. 000000. WOFSTP(i,j)   ]
                                               #. . for. . TIMBER  SHIPPED
. .ge. . SCRLOND(i)                            #. .SOURCE DEMANDS     . .#
                                                 For all j
Row name XEROD(i)                             # MAX AMT PRODUCED. .restriction. .#
Sum over k       [   +1. 000000. [HFPTC  (j,k)  ]
                                              # ..for.. CHAIRS SOLD        #
..ge..PLNATMAX(j)                             #.. MIN  RODUCTION  ..#

                                                For all j

Row name XEROD (j)                           #MAX ANT PRODUCED..restriction..#
```

Display 3.6

```
Sum over k   [  +1.000000*chfptc (j,k) ]
                                           #   ..for..   CHAIRS  SOLD        #
..le..PLNTMAX  (j)                         #  ..  MAX  PRODUCTION  ..  #
                                              For all j
Row name clow (k)                          # MIN  CUST  DEMAND  .. restriction ..  #
Sum over j        [  + 1.000000*CHFPTC  (j.k)     C
                                           #   .. for ..  CHAIRS  SOLD        #
..ge.. CUST LOMD (k)                       #  ..       MIN CUST DMNO          ..  #


                                              For all k
Row name THIGH (k)                         #  MAX  CUST  DEMAND .. restriction .. #
Sum over j   [ + 1.000000*CHFPTC (j,k)  ]

                                           #   .. for ..  CHAIRS  SOLD          #
..le..CUSTH@MD (k)                         #  ..  MAX CUST DAND     ..  #
                                           For all k
Row name PROFIT                            #  MAXIMISE PROFIT .. no restriction .. #

Sum over j ,k       [-PLNT COST (j)*CHFPTC (j,k)  ]
                                           #  PLANT COST              ..for..  CHAIRS SOLD       #
Sum over j,k   [ +CUSTPRCE (k)*CHFPTC (j,k)  ]
                                           #  CUSTOMER PRICE     ..for..  CHAIRS SOLD        #
Sum over j,k  [ - TCSTPTC (j,k)*CHFPTC (j,k)  ]
                                           #  TRAN COST  TO CST  ..for..  CHAIRS SOLD        #
Sum over j,j  [-  TCSTSTP(j,j)*WOFSTP (j,j))  ]
                                           # TRAN COST  TO SRC    ..for..  TIMBER SHIPPED  #
Sum over j,j  [- SCRFDCE (j) *WOFSTP (j,j)  ]
                                           #  SOURCE PRIDCES      ..for..   TIMBER SHIPPED  #

..fr..0
Row name BSTOCK (j)                        # STOCK BALANCE        ..restriction..#

Sum over j   [  + 1.000000*WOFSTP(i,j) ]
                                           #  ..for..   TIMBER SHIPPED    #
Sum over k  [ - 20.000000*CHFPIC (j,k) ]
                                           #  ..for.. CHAISS SOLD           #
..eg..0

                                           For all j
```

Display 3.7

‘basic entities'. A few representative examples of these are set out below

- Material Balance Equation

$$XO_t + XP_t - XC_t = D_t \quad , t = 1,2,...T.$$

In this equation XO_t represents the opening inventory, XC_t represents the closing inventory, and XP_t the quantities to be produced. They are all decision variables pertaining to the time period t. D_t represents the customer demand for the product and is an input information.

- Capacity Restrictions

$$\sum_{p=1}^{p} x_{pm}\ t_{pm} < A_m \ , \ m = 1,2,.....M.$$

Here p = 1,2…, P indicates the range of products which are manufactured on machines m = 1,2,...M. The rate of production is indicated by t_{pm}, that is, the time taken to produce one unit of product p on machine m. A_m indicates the number of hours that machine m is available. Xp_m is the production variable and the constraints express the capacity of production for the machine m as limited by the number of hours of its availability.

- Blending Requirement

$$\sum_{C=1}^{C} x_{cp}\ b_{cr} \left\{ \begin{matrix} < \\ \text{or} \\ = \\ \text{or} \\ > \end{matrix} \right\} Q_{pr} \qquad \begin{matrix} p=1,......p \\ r=1,.......R \end{matrix}$$

In this case c = 1,...C indicates the number of components which are used to blend p = 1,...P products. The components for instance could be different crudes and products could be different types of gasoline. The range of the index r = 1,,..R indicates quality requirements. Typical requirements are maximum vapour pressure, minimum volatility index etc. Thus b_{cr}, Q_{pr} are input information pertaining to linear blending rates and quality requirements respectively. X_{cp} is the decision variable indicating fractions (by volume or weight) of component c that are blended to derive product p. Thus

$$\sum_{C=1}^{C} X_{cp} = 1 \quad , \ p = 1,....P.$$

In the discussion of the model variables and model constraints the subscripts p,m,n,c,r,t which have been introduced indicate 'basic entities' which are meaningful in the context of the model. This highlights why it is first necessary to define these 'basic entities' and then define model variables and restrictions.

5. Support for separable and logical programming reformulation [LUMIYA86]

CAMPS has been designed to provide support for reformulating separable and logical (integer and fuzzy) programming problems. For instance special table types, variable types (to define special ordered sets of type 1 and type 2 variables) and row names (CONVEX*, REFER*) are used to construct separable programming problems. These facilities have been used to reformulate ten representative nonlinear optimisation problems; our investigations are reported in [LUCMIT86]. In [LUMIYA86] we have shown how the analysis of bounds for linear forms plays a key role in reformulating mixed integer, separable, and fuzzy programming problems. For instance the algebraic relations which are used to separate variables are also used to derive bounds [WILLIA83] on the new variables introduced in the reformulation. These bounds are essential for used in transforming propositions (which take logical forms) to equivalent mixed integer linear forms. Computer support in these areas offer increasing scope and applicability of mathematical programming.

6. Discussions

CAMPS and it's underlying modelling methodology have been presented in this paper. A number of other modelling systems have command and syntax structure whereby the model description follows closely the mathematical statement of the LP. The motivation behind this approach is to force the modeller to communicate his model in a form that serves also as a full documentation. Whereas model documentation is essential, we believe, it is unnecessary to tie the method by which the modeller communicates his model to the documentation requirements. In CAMPS the model is communicated and updated using menus and screenforms, and documentation is obtained under a separate option. In our experience CAMPS menus and screenforms capture a model in far fewer keystrokes than by using a modelling language. Errors introduced due to mistyping are also reduced. Our experimentations with the system suggest that reformulation support and programmer's interface are important features which should be part of any complete modelling system.

7. Acknowledgements

C. Lucas has developed CAMPS in part fulfilment of his PhD research in which he was supported initially by an SERC studentship and subsequently by an assistantship (grant No. GR/D/72075). We are grateful to K. Darby-Dowman and M.A. Laughton for their careful reading of earlier versions of this report and their comments. We have also greatly benefited from the comments made by H. Greenberg who visited us during June 1986 supported by a NATO fellowship.

Appendix: A comparison of CAMPS with other systems.

Using the sample problem of section 3, a comparison of CAMPS problem specification method with those of ULP and OMNI is presented here. ULP is a recently developed modelling language and incorporates many ideas also found in CAMPS. Thus the data entry which is separate from model definition follows the logical sequence whereby the sets are first defined and then the data tables. The model is then conceived in the equation form and generated using row statements. OMNI is a well established matrix generator system in which the linear program is specified in column sequence. The problem formulations in ULP and OMNI have not been tested but were developed by reading user manuals, however the CAMPS formulation has been tested and the resulting model optimised.

TANGLEWOOD - ULP

```
*RANGES

MERCHANTS= ONTARIO,QUEBEC;
PLANTS= WASHINGTON,PHILADELPHIA,DENVER.BUFFALO;
RETAILERS=NEW YORK, HOUSTON, SAN   FRANCISCO, CHICAGO;

*TABLES

PLANT COSTS(PLANTS) : 5 7 3 4;
MIN PROD(PLANTS): 0  400 500  250;
MAX PROD(PLANTS):500  750  1000  250;
SELL PRCE(RETAILERS): 20 15 20 18;
MIN  CUST DMND(RETAILERS): 500 100 500 500;
MAX CUST DMND(RETAILERS): 2000 400 1500 1500;
TRAN COST CUST(PLANTS,RETAILERS) : 1.0 1.0 2.0 0.0
                                    3.0  6.0 7.0 3.0
                                    3.0  1.0 5.0 3.0
                                    8.0   2.0 1.0 4.0;
TRAN COST DLR (MERCHANTS,PLANTS):  0.01   0.02 0.04 0.04
                                   0.04   0.03 0.02 0.02 ;
SCR PRCE   (MERCHANTS) : 0.1   0.075;
SCR DMND (MERCHANTS): 8.8

UNKNOWN (X (MERCHANTS, PLANTS),Y(PLANTS, RETAILERS))
COMMENT   (X (MERCHANTS,  PLANTS)  = AMOUNT TIMBER FROM MERCHANT TO
PLANT)
COMMENT  (Y(PLANTS,RETAILERS)   = AMOUNT CHAIRS FROM PLANT TO
RETAILER)

LPMAX  (SELL PRCE(RETAILERS) *Y (PLANTS, RETAILERS)
               -PLANT COSTS (PLANTS) *Y (PLANTS,RETAILERS)
               -TRAN  COST  CUST  (PLANTS,RETAILERS)*Y(PLANTS,RETAILERS)
               -TRANCOSTDLR(MERCHANTS,PLANTS)*X(MERCHANTS,PLANTS)
               SCR PRCE(MERCHANTS)*X(MERCHANTS, PLANTS))

CONSTRAIN    (PLANTS: X(MERCHANTS, PLANTS)>SCR DMND(MERCHANTS))
CONSTRAIN    (RETAILERS: Y (PLANTS, RETAILERS) >MIN PROD(PL ANTS))
CONSTRAIN    (RETAILERS: Y (PLANTS, RETAILERS)<MAX PROD(PLANTS))
CONSTRAIN    (PLANTS: Y (PLANTS, RETAILERS)>MIN CUST DMND(RETAILERS))
CONSTRAIN    (PLANTS: Y (PLANTS, RETAILERS) <MAX CUST DMND(RETAILERS))
CONSTRAIN    (MERCHANTS, RETAILERS:Y(PLANTS,RETAILERS)
                    -20*X (MERCHANTS, PLANTS)=0)
```

```
TANGLEWOOD – OMNI

 DICTIONARY

        CLASS  MER        Set of timber merchants:
          ONT                         Ontario
          QUE                         Quebec

        CLASS PLA                   Set  of  plants:
          WAS                         Washington
          PHI                         Philadelphia
          DEN                         Denver
          BUF                         Buffalo

        CLASS RET                      Set of retailers:
          NEW                         New York
          HOU                         Houston
          SAN                         San Francisco
          CHI                         Chicago

*DATA

        TABLE   A                      Plant costs for production of
                                         CHAIRS
                COSTS
        WAS        5
        PHI        7
        DEN        3
        BUF        4

       TABLE B                         Minimum production level at each plant
                MIN
        WAS        0
        PHI      400
        DEN      500
        BUF      250

       TABLE   C                        Maximum production level at each plant
                  MAX
       WAS        500
       PHI        750
       DEN       1000
       BUF        250

       TABLE D                           Selling prices to retailers
                PRC
       NEW       20
       HOU       15
       SAN       20
       CHI       18

       TABLE E                            Minimum retailer demands
                MIN
       NEW     500
       HOU     100
       SAN     500
       CHI     500
```

```
TABLE F                                  Maximum retailer demands
          MAX
NEW    2000
HOU      400
SAN    1500
CHI     1500

TABLE G                              Cost of transport from each plant to
                                                          each retailer
          NEW   HOU   SAN   CHI
WAS       1.0    1.0      2.0      0.0
PHI        3.0    6.0      7.0      3.0
DEN        3.0    1.0      5.0      3.0
BUF        8.0    2.0      1.0      4.0

TABLE H                              Costs of transport from each merchant
                                                                to each plant
         WAS    PHI   DEN  BUF
 ONT     0.01   0.02  0.04  0.04
 QUE     0.04   0.03  0.02  0.02

 TABLE I                                       Costs of timber at each timber
merchant
        PCE
 ONT    0.1
 QUE    0.075

 TABLE  J                            Minimum demand at each timber
                                                               merchant
       MIN
 ONT        8
 QUE        8

 FORM ROW ID
 *Maximise operating profit
            OBJ=OBJ
 *Satisfy    minimum production at plants limit
           PLN (PLA)=MAX
 *Satisfy    maximum production at plants limit
            PLX(PLA)=MAX
 *Satisfy    minimum order quantity
            MEN(PLA)=MIN
 *Satisfy    minimum customer demand limit
           CUN(RET)=MAX
 *Satisfy    maximum customer demand limit
           CUX(RET)=MAX
 *Satisfy    balance of wood stock at. each plant
           WOB(PLA)=FIX

 COLUMNS
 *Shipping activity for wood from merchants
 FORM VECTOR X(MER)(PLA)
 *The amount of timber bought from merchant
        MEN(PLA)=1
 *The amount of wood consumed in making chairs
            WOB(PLA)=-20
 *The cost of buying and shipping timber
          OBJ=-TABLE H ((PLA),(MER)) - TABLE I (PCE, (MER))



 *Shipping activity for chairs from plants to retailers
```

```
FORM VECTOR Y(PLA)(RET)
*The amount of chairs produced at the plant
            PLN(PLA)=1
*The amount of chairs produced at plant
            PLX(PLA)=1
*The amount of chairs retailer buys
            CUN(RET)-1
*The amount of chairs retailer buys
            CUX(RET)=1
*Amount of chairs produced at plant
            WOB(PLA)-1
*The effective profit of selling chairs
            OBJ=TABLE D (PRC.(RET)) - TABLEA (COSTS,(PLA))
                            -TABLE G ((RET),(PLA))
RHS
FORM VECTOR RHSIDE
*Minimum plant production
            PLN(PLA)=TABLE B (MIN,(PLA))
*Maximum plant production
            PLX(PLA)=TABLE C (MAX,(PLA))
*Minimum order amount
            MEN(PLA)-TABLE J (MIN,(HER))
*Minimum customer demand
            CUN(RET)=TABLE E (MIN.(RET))
*Maximum customer demand
            CUX(RET)=TABLE F (MAX,(RET))
*Note the right hand sides for the balance rows and
*objective are zero
ENDATA
```

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