TR/11/84 November 1984

Rational Expectations Modelling in O.R.

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ABSTRACT

The conventional OR approach to managing a system is, in outline, firstly to create a model of the existing system, secondly, to investigate changes in the model which improve or control the behaviour of the model and thirdly, to implement these changes in the system. It is assumed that the model incorporating these changes will be a valid representation of the system after the changes, in as far as the original model was a valid representation of the original system, and can thus be used to assess the benefits and disbenefits arising from the changes.

However, once the system is modified it may interact differently with its environment, thus further changing the system and requiring a corresponding change in the model. This dynamic interaction sets up a chain of models: System 1 - Model 1 - Changes 1 - System 2 - Model 2 - Changes 2 - System 3 Such a process is most likely to occur as a result of changed human behaviour in the context of a modified system, as exemplified in queueing systems where a reduction in waiting times leads to an increased use of the system and hence longer waiting times. It is also met in economic systems where the approach of 'rational expectations' has recently been developed to cope with these dynamic changes of behaviour.

The rational expectations approach is essentially that the actors within a system model act in accordance with their expectations derived from the model itself. Under full information this allows patterns of behaviour change to be formalised and predicted and the sequence of models to be treated iteratively. This paper sets out and argues for this approach in OR modelling and describes its use in a case-study of a library system where the complex interactions of goals, behaviour and system performance show up obvious deficiences in a conventional analysis.

INTRODUCTION

Operational Research methodology has been, investigated and formalized by many authors e.g. [1,2], each with differing emphases and idiosyncracies, but in all cases a central necessity is the creation of a model of the system under study. This model is then used to investigate changes which improve or control the behaviour of the model, and these changes are then suggested for implementation in the real-life system. It is recognized that at all stages of this process there is the possibility of error occurring either in choosing what to model or how to model it and the importance of feedback from implementation to a reassessment of the model is stressed.

This paper is concerned with systems whose behaviour can be significantly modified by the actions of humans within them. Pricing decisions, where the effect of a price change needs to be modelled as a function of changed consumer choices within the system, are an example. In this example it is clear that modelling of human behaviour must be a major aspect of the model, but in many situations although it may be evident that human behaviour is a significant element, the model constructed may omit this element. Instead, the working assumption is made that human behaviour will continue as at present even after changes are introduced to the system.

The reasons for such an omission are often simply that changes in behaviour are thought too difficult to predict. A new road planned to handle existing traffic levels may find itself swamped by increased traffic caused by drivers changing route to take advantage of the new road. Road planners are aware of this possibility but find it difficult to model. Similarly, the use made of a facility, such as a restaurant or repair service will be affected by such variables as the chance of obtaining a table or a repair when needed, and on the average wait if a booking is made in advance.

In these examples the variables affecting human behaviour 'advantage of new road', 'chance of table', 'wait from booking' are themselves functions of (other) humans' behaviour. This makes modelling changes in behaviour more difficult: in order to model the behaviour of an actor in the model we need to predict how he will react to changes in behaviour of the other actors and those changes will themselves depend in part on their reactions to his changed behaviour. Thus there is a circularity in the modelling: the actor's behaviour depends on the system behaviour and the system behaviour depends on the actor's behaviour.

If this difficult behavioural modelling is not attempted then differences between the assumed and actual behaviour of actors and hence of the system will manifest themselves after implementation. When this happens a new model, model *2,* must be constructed using the new behaviour patterns observed after implementation of model 1. Since model 2 will itself form the basis for suggested changes the whole process may iterate through several stages ;

System 1 - Model 1 - Changes 1 - System 2 - Model 2 - Changes 2 - System 3

until the differences detected by feedback become unimportant.

It is of course possible to include changes in behaviour in the model through some ad-hoc formulation on the part of the modeler, but this approach is subject to serious objections. Firstly, since the change in behaviour cannot be observed until after implementation the modelling must be subjective; secondly, it will be suboptimal in the sense that it will not in general predict the course of action actually followed by actors maximising their utilities after implementation.

RATIONAL EXPECTATIONS

A similar behavioural modelling problem is met in economics, where future economic activity depends on present expectations; wage demands are determined partly by expectations of what will happen to the economy in the next year, which in turn depends on the level of wage demands. Again the circularity of modelling arises. An actor's wage demand cannot be predicted before system performance has been forecast and vice-versa. A way forward suggested by economists is to use rational expectations [3]. The idea is that an actor's behaviour is a function of his expectations (predictions). 'Rational' means that his expectations are exactly the predictions that can be obtained from using the model. Thus in the sense that the model corresponds to reality his expectations are correct. Again, it is possible to handle the formation of actors' expectations in the model by some ad-hoc procedure. But as this will not generally lead to the probabilistically best predictions on the basis of the information available, it is equivalent to assuming systematic error in the expectations of actors. It is argued that steps would be taken by the actor to improve his formation of expectations to eliminate any such systematic error. Of course, any one actor may not have access to the complete 'World Knowledge' represented by the model. In that case his expectations are just those that can be derived by using the part of the model which he 'knows'. This is the general case of partial information where there may be a clash between the expectations of different actors.

A special case is to assume that all actors are working under conditions of full information. Within the model this means that each actor has complete knowledge of all aspects of the model and so will form expectations about the probabilistic future development of the model which are exactly correct.

The simplification introduced by this approach is that no further modelling to account for changes in behaviour is necessary. If we know in the model how the actor's expectations affect his behaviour (e.g. by knowing his utility function or some equivalent behaviour directing device such as goal states) then together with his expectations of system performance this will determine his behaviour; and his expectation of system performance is just that obtainable from the existing model. Thus we can predict from the model that if he takes action a , system performance will be s(a), and also from the model that if he expects performance s then his behaviour will be R[s]. So we find that the action he will take must satisfy

$$a = R[s(a)].$$

In this way the need for an iterative sequence of models is removed and replaced by the solution of a set of implicit equations within the original model.

The behaviour of actors and hence of the system can be found using this approach under any changes to the system we wish to consider.

Two major objections to this full information rational expectations approach may be anticipated. One is that 'people do not act rationally In the sense of 'rational' employed here this is equivalent to saying that they make predictions in a manner that includes systematic error. As argued before, it seems unlikely that once noticed this behaviour would persist. The other objection is the nature of the full information assumption (although this is not essential to the use of rational expectations, without it the modelling is generally impractically complex). There are many situations where the actors strive to keep information from one another. Wage bargaining is again an example where management will often keep from their workers information on the company's financial state and plans. Clearly in such a case full information modelling is not applicable. However, we would argue that in many cases the withholding of information is in fact sub optimal to all concerned. The knowledge that information is being witheld can lead to an atmosphere of distrust and suspicion. Rather than modelling the inbalance of information a better approach would be to work toward a full information situation.

Thus, inasmuch as the use of full information models leads to implementations that require full information conditions for optimal performance, we believe that they could produce a beneficial side-effect.

AN APPLICATION

We now turn to the application of these concepts. The system under study is a library and an investigation was carried out to advise on major operational decisions concerning the acquisition of new books, the loan periods, the number of tickets allocated and the use of reservation facilities [4,5], This is a complex decision problem because, firstly, it is not evident what the goals of system performance should be. Previous attempts to tackle related systems have assumed differing and often conflicting goals e.g. maximizing the probability of a wanted book being on the shelf [6] as against maximizing the circulation of books (defined as the average number of issues per time period) [7] which will result in a tendency for books to be out circulating and he*cfi not to be found on the shelf. Secondly, there is the complex modelling problem of how users will react to changes in the library system. Previous studies have not addressed this problem but have confined themselves to dealing with behaviour patterns derived from past data, which implicitly assumes that no change in behaviour will occur. We attribute this state of affairs to the difficulty of predicting changes in behaviour within a conventional O.R. analysis where, without the framework provided by rational expectations together with behavioural model in the form of utility functions or their equivalent, only ad-hoc mechanisms for behaviour change can be introduced.

The approach taken is to model the ability of a library user obtaining a book as a function of the type of book, the class of user, the time taken to obtain the book and the time the book is held by the user. From this, plus a knowledge of library procedures and user behaviour in the library, it is possible to construct a model of the system which can be used to optimize the library decisions in terms of maximizing the utility to users. The behavioural aspect of this approach with which we are concerned is the need to model how a user attempts to obtain a book. There are three basic possibilities if the book is not on the shelf: make a reservation, return at a later time, give up and buy the book. Combinations of these possibilities are also possible over time. The way in which a user chooses between these three possibilities is a function of the system. For example, it was found that the probability of a user buying a book if it is not on the shelf is proportional to the perceived additional utility obtained per £ spent. The additional utility is the difference between the utility of having the book now (permanently) and the expected utility from continuing to use the library in an attempt to obtain it later. Thus the number of books bought depends, inter alia, on the number of reservations for the book outstanding, the number of other users who will return later and the number of other users who buy their own copies. So the probabilities of an actor attempting any of the three basic actions: reserving (p_1), returning later (P_2) and buying (P_3) are all functions of each other. We" apply rational expectations to see how behaviour in the system can be predicted under any changes to which the system may be exposed.

Consider p_3, the probability of buying a book. We assume a homogeneous population of users, i.e. the probabilities of the three basic actions are the same for each user. Denote by T the user's utility of obtaining the book immediately and permanently and by UT(p1,p2,p3) the expected utility if he waits to obtain it from the library. As mentioned previously, data for the existing system indicated that for a given book

$$P_3 = K_3 \frac{\text{Utility gain from buying}}{\text{Cost of buying}}$$

where K_3 is estimated from the data. Under full information rational expectations the user's perceived utilities are the same as those given by the model, so

$$P_3 = K_3 \frac{[T - UT(P_1, P_2, P_3)]}{cost}$$

A similar approach may be taken with p_1 and p_2:

$$P_1 - K_1[UT(p_1,p_2,p_3\,[reserves) - UT(p_1,p_2,P_3\,|\;no\;reservation)]$$

$$P_2 = K_2\,[UT(p_1, p_2, P_3\,|\,returns) - UT(p_1,p_2,p_3\;\;no\;return)]$$

where the constants of proportionality are estimated from data on the existing system.

These three implicit equations can be solved to give values for p_1,p_2,p_3. Any changes in the system will be reflected in the structure and parameterization of UT but corresponding values of p_1,p_2,p_3 can be obtained again through these equations.

Behaviourally, the rational expectations assumption here is equivalent to saying that users have a feeling for aspects of the library's operations, e.g. the time they must wait to obtain a book, the chance of a copy being on the shelf, the number of reservations outstanding. This feeling will change 'rationally' in correspondence with any changes made to the system. This model enables us to predict future usage patterns for the library taking account of changes in behaviour. In many systems this ability is of fundamental importance, e.g. many decisions in libraries are made on the basis of subjective beliefs about what a user would do if some aspect of the library were changed - would more or less use be made of a book if it were transferred to a short-loan collection?

The alternative to a rational expectations approach would necessarily be one which assumed that users do not 'understand' the system and are living at variance with reality. They would constantly be surprised to find books or fail to understand why they could not obtain a copy. This behaviour may undoubtedly occur for a short period after major changes to a system, but as a general model we do not feel it has much appeal.

CONCLUSIONS

We have set out the case for a rational expectations approach to modelling changes in behaviour due to changes in the system. We feel that there are two main advantages to be obtained from this approach 1) it allows proposed changes to be assessed within one model and avoids the trial and error feedback approach to modelling 2) it provides a framework within which behaviour changes can be consistently modelled and avoids the need for arbitrary ad-hoc modelling.

We hope to see this approach adopted more widely within O.R.

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