Event-Triggered H_∞ State Estimation for Delayed Stochastic Memristive Neural Networks with Missing Measurements: The Discrete Time Case

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***Abstract*—In this paper, the event-triggered H_∞ state estimation problem is investigated for a class of discrete-time stochastic memristive neural networks (DSMNNs) with time-varying delays and missing measurements. The DSMNN is subject to both the additive deterministic disturbances and the multiplicative stochastic noises. The missing measurements are governed by a sequence of random variables obeying the Bernoulli distribution. For the purpose of energy saving, an event-triggered communication scheme is used for DSMNNs to determine whether the measurement output is transmitted to the estimator or not. The problem addressed is to design an event-triggered H_∞ estimator such that the dynamics of the estimation error is exponentially mean-square stable and the prespecified H_∞ disturbance rejection attenuation level is also guaranteed. By utilizing a Lyapunov-Krasovskii functional and stochastic analysis techniques, sufficient conditions are derived to guarantee the existence of the desired estimator and then the estimator gains are characterized in terms of the solution to certain matrix inequalities. Finally, a numerical example is used to demonstrate the usefulness of the proposed event-triggered state estimation scheme.**

***Index Terms*—Memristive neural networks, stochastic neural networks, state estimation, event-triggered mechanism, missing measurements.**

I. Introduction

For decades, recurrent neural networks (RNNs) have been attracting an ever-increasing research interest due to their remarkable ability to exhibit dynamic temporal behaviors with successful applications in a variety of areas including signal processing, pattern recognition, image processing, associative memory, combinatorial optimization and control engineering [22], [25], [36], [43]. Accordingly, a large number of results have recently been available in the literature on the dynamics analysis issues (e.g. stability, synchronization and estimation) for various kinds of RNNs especially the stochastic RNNs. In fact, in real neural networks, the synaptic transmission is actually a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes [13] and, if not properly handled, the resulting stochastic disturbances would constitute one of the main source of the performance degradations when implementing the neural networks in engineering practice.

Since the announcement from the HP Lab on the experimental prototyping of the memristor [30], memristors and memristive devices have gained wide research attention for their prospective applications in non-volatile memories, logic devices, neuromorphic devices, and neuromorphic self-organized computation and learning. In the context of neural networks, synapses are essential elements for computation and information storage which needs to remember its past dynamical history, store a continuous set of states, and be "plastic" according to the synaptic neuronal activity. All this cannot be accomplished by a resistor in traditional RNNs. However, when the resistors are replaced by the memristors, the resulting memristive neural networks (MNNs) could rather completely solve these problems. Meanwhile, the implemented MNNs could be more efficient than the traditional RNNs when applied in brain emulation, combinatorial optimization, knowledge acquisition and pattern recognition [26]. As such, the dynamics analysis problems such as stability and synchronization for MNNs have recently received considerable research attention and a rich body of relevant literature has been available for different kinds of MNNs such as memristive recurrent neural networks [33], memristive fractional-order neural networks [4], memristive cellular neural networks [10], memristive Hopfield networks [40], memristive chaotic neural networks [45] and memristive complex-value neural networks [34], etc. It should be mentioned that almost all results obtained so far have been exclusively for continuous-time MNNs. In nowadays digitized world, more and more information sequences are *discrete* in nature for engineering applications such as digital signal processing, time-series analysis and network-based control, and therefore the discrete-time RNNs have become a powerful means in dealing with sequence-based tasks [17]. However, despite the clear engineering insights, the discrete-time MNNs have gained very little attention due mainly to the mathematical difficulties in quantifying and tackling the state-dependent switching behaviors in the discrete-time setting.

It is now well recognized that the time delays are inherent

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characteristics in hardware implementation of neural networks which may lead to some complex dynamic behaviors such as oscillation, divergence, and even instability in the network systems. For continuous-time MNNs, so far, various time-delays (including constant delays, time-varying delays, distributed delays and mixed time-delays) have been introduced to model the lags in signal transmissions due to finite switching speeds of amplifiers, and the impacts of the time delays on the dynamical behaviors of continuous-time MNNs have been thoroughly examined in the literature, see [12], [41] and the references therein. In particular, in [12], several sufficient conditions in terms of linear matrix inequalities have been presented to ensure the global exponential synchronization of multiple MNNs with time delays. Nevertheless, the corresponding results for discrete-time MNNs have been very few and this constitutes the main motivation of the present research to shorten such a gap.

In neural network applications, it is quite common that the neuron states are not fully accessible due probably to the large scale of the networks and the implementation cost in monitoring network output, and this makes it significantly difficult to analyze the dynamical behaviors of the real-time neural networks. Therefore, in such cases, it becomes a prerequisite to estimate the neuron states through available network measurements before exploiting the merits of RNNs in tasks such as classification, approximation and optimization, see e.g. [14], [29], [35] for representative works. To be more specific, in [35], the state estimators have been designed for a class of neural networks with time-varying delays by employing the Lyapunov functional and linear matrix inequality approach. In [15], the state estimation problem has been considered for a class of uncertain stochastic neural networks. Furthermore, the phenomenon of missing measurements has been investigated in [17] for the state estimation problem of coupled uncertain stochastic networks. As for the state estimation problem of MNNs, some initial efforts have been made in [27], [28] by utilizing the passivity theory with or without time-delays. Again, as far as the discrete-time MNN is concerned, the state estimation remains an open problem that deserves further investigation.

In the course of implementing large-scale RNNs consisting of a large number of computing units, much resource (e.g. processing, storage, communications) would have to be consumed and the energy saving issue with resource constraints is becoming an emerging topic of research that has started to draw some initial attention. For state estimation problems of RNNs, it is quite desirable to avoid unnecessary signal transmissions (from the measured network outputs to the estimator) and reduce the network burden as long as certain estimation performance requirements (accuracy and convergence) are guaranteed. Recently, the event-triggered mechanism (EVM) has received much research attention because of its distinctive merits in saving resource. Unlike the conventional time-triggered scheme, the main purpose of the EVM is to transmit signals only when a certain triggered condition is met, and this allows a considerable reduction of the network resource occupancy. Therefore, in the past few years, the EVM has been applied to various control and communication problems for complex networks [16], networked control systems [9], [19], wireless sensor networks [8], [32] and multi-agent systems [31]. In this case, a seemingly natural idea is to investigate into the event-triggered state estimation problem for discrete-time MNNs and see how the efficiency of energy utilization can be improved. It should be pointed out that the event-triggered state estimation problem for RNNs neural networks has not received adequate research attention yet, not to mention the case when the RNN is in the discrete-time setting coupled with time-delays as well as deterministic and stochastic disturbances.

More importantly than all of that, compared with the existing results, we can find that the *discrete-time* stochastic memristive neural networks (DSMNNs) with time-varying delays are more comprehensive and practical than the established ones. In this case, both the stability analysis based on the theory of differential inclusion and the state estimation approach without event-triggered scheme are no longer applicable. Motivated by the above discussions, in this paper, we endeavor to study the event-triggered H_∞ state estimation problem for delayed stochastic MNNs with missing measurements. The problem addressed is to estimate the neuron states through available output measurements subject to probabilistic missing values under an event-triggered mechanism. By utilizing a Lyapunov-Krasovskii functional and stochastic analysis techniques, both the existence conditions and the explicit expression of the desired state estimators are established under which the estimation error dynamics is stable and the prescribed H_∞ disturbance rejection attenuation level is guaranteed.

The main contributions of this paper are highlighted as follows: 1) a new yet comprehensive MNN model, namely, discrete-time delayed stochastic MNN with missing measurements, is proposed in order to reflect the engineering practice; 2) a new event-based state estimation problem is addressed for the discrete-time MNNs with hope to save resource; 3) an H_∞ performance index is used to attenuate the effects from the external disturbances on the estimation performance; 4) for the underlying MNNs, a new technique is introduced to quantify and then handle the state-dependent switching behaviors in the discrete-time setting; and 5) a unified framework is established that is capable of coping with the simultaneous presence of event-triggered effects, deterministic and stochastic disturbances, time-varying delays as well as missing measurements.

II. Problem Formulation

Consider the following n-neuron discrete-time stochastic memristive neural networks with time delays:

$$\begin{aligned} x(k+1) = & D(x(k))x(k) + A(x(k))f(x(k)) \\ & + B(x(k))g(x(k-\tau(k))) + L\varsigma(k) \\ & + \sigma(k, x(k), x(k-\tau(k)))w(k) \end{aligned} \tag{1}$$

where $x(k) = \begin{bmatrix} x_1(k) & x_2(k) & \cdots & x_n(k) \end{bmatrix}^T$ is the neural state vector; $D(x(k)) = \mathrm{diag}\{d_1(x_1(k)), d_2(x_2(k)), \cdots, d_n(x_n(k))\}$ is the self-feedback matrix with entries $|d_i(x_i(k))| < 1$; $A(x(k)) = \big(a_{ij}(x_i(k))\big)_{n\times n}$ and $B(x(k)) = \big(b_{ij}(x_i(k))\big)_{n\times n}$ are the connection and the delayed connection weight matrices, respectively; $\varsigma(k) \in \mathbb{R}^n$ is the external disturbance input vector belonging to $l_2([0,\infty);\mathbb{R}^n)$, $L =$

$\mathrm{diag}\{l_1, l_2, \cdots, l_n\}$ is the intensity matrix of the deterministic disturbance; $\tau(k)$ represents the time-varying transmission delay which satisfies

$$\tau_m \leq \tau(k) \leq \tau_M, \quad k = 1, 2, \cdots \tag{2}$$

where the positive integers τ_m and τ_M are the lower and upper bounds, respectively; $w(k)$ is a scalar Wiener process on $(\Omega, \mathcal{F}, \mathcal{P})$ with

$$\mathbb{E}[w(k)] = 0,\ \mathbb{E}[w^2(k)] = 1,\ \mathbb{E}[w(i)w(j)] = 0\ (i \neq j), \tag{3}$$

and $\sigma : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is called the noise intensity function vector (for the stochastic disturbance) satisfying

$$\sigma^T(k, u, v)\sigma(k, u, v) \leq \rho_1 u^T u + \rho_2 v^T v, \quad u, v \in \mathbb{R}^n \tag{4}$$

where ρ_1 and ρ_2 are known positive constants; $f(x(k)) = \begin{bmatrix} f_1(x_1(k)) & f_2(x_2(k)) & \cdots & f_n(x_n(k)) \end{bmatrix}^T$ and $g(x(k-\tau(k))) = \begin{bmatrix} g_1(x_1(k-\tau(k))) & g_2(x_2(k-\tau(k))) & \cdots & g_n(x_n(k-\tau(k))) \end{bmatrix}^T$ are the nonlinear functions standing for the neuron activation functions.

Remark 1: In real-world neural networks, some neural networks is often disturbed by environmental noises, and the noise intensity has bound. Therefore, in this paper, we assume that the DSMNN is subject to the multiplicative stochastic noises, and this noise intensity described by the function vector $\sigma(\cdot)$. We also assume that this noise intensity $\sigma(\cdot)$ is related to the time and the system states and has upper bound.

For the neuron activation functions, the following assumptions are needed.

Assumption 1: [21] The neuron activation functions $f(\cdot)$ and $g(\cdot)$ satisfy

$$\begin{aligned} &[f(x) - f(y) - \Lambda_1(x-y)]^T \\ &\times [f(x) - f(y) - \Lambda_2(x-y)] \leq 0,\ x, y \in \mathbb{R}^n\ (x \neq y), \end{aligned} \tag{5}$$

$$\begin{aligned} &[g(x) - g(y) - \Upsilon_1(x-y)]^T \\ &\times [g(x) - g(y) - \Upsilon_2(x-y)] \leq 0,\ x, y \in \mathbb{R}^n\ (x \neq y) \end{aligned} \tag{6}$$

where Λ_1, Λ_2, Υ_1 and Υ_2 are constant matrices.

Chua [6] stated that memristor needs to exhibit only two sufficient distinct equilibrium states since digital computer applications requiring only two memory states. On the other hand, along the similar lines in [39], memristor-based neural network (1) can be implemented by a large-scale integration circuit. Then according to the feature of the memristor and the current-voltage characteristic, $d_i(x_i(k))$, $a_{ij}(x_i(k))$ and $b_{ij}(x_i(k))$ are state-dependent functions with the form

$$\begin{aligned} d_i(x_i(\cdot)) =& \frac{1}{C_i}\Big[\sum_{j=1}^{n}\big(\frac{1}{R_{aij}} + \frac{1}{R_{bij}}\big) + \frac{1}{R_i}\Big] \\ =& \begin{cases} \hat{d}_i, & |x_i(\cdot)| > \kappa_i, \\ \check{d}_i, & |x_i(\cdot)| \leq \kappa_i, \end{cases} \\ s_{ij}(x_i(\cdot)) =& \frac{\mathrm{sign}_{ij}}{C_i R_{sij}} = \begin{cases} \hat{s}_{ij}, & |x_i(\cdot)| > \kappa_i, \\ \check{s}_{ij}, & |x_i(\cdot)| \leq \kappa_i \end{cases} \end{aligned} \tag{7}$$

where s stands a or b, C_i is the capacitor, R_i is the parallel-resistor. R_{aij} and R_{bij} are respectively the non-delayed and time-varying delayed connection memristors between the feedback $f_j(\cdot)$ and state $x_i(\cdot)$.

$$\mathrm{sign}_{ij} = \begin{cases} 1, & i \neq j, \\ -1, & i = j, \end{cases}$$

the switching jumps satisfy $\kappa_i > 0$, $|\hat{d}_i| < 1$, $|\check{d}_i| < 1$, $\hat{s}_{ij}$ and $\check{s}_{ij}$ are constants.

Denote

$$\begin{aligned} &d_i^- = \min\{\hat{d}_i, \check{d}_i\}, \quad d_i^+ = \max\{\hat{d}_i, \check{d}_i\}, \\ &a_{ij}^- = \min\{\hat{a}_{ij}, \check{a}_{ij}\}, \quad a_{ij}^+ = \max\{\hat{a}_{ij}, \check{a}_{ij}\}, \\ &b_{ij}^- = \min\{\hat{b}_{ij}, \check{b}_{ij}\}, \quad b_{ij}^+ = \max\{\hat{b}_{ij}, \check{b}_{ij}\}, \\ &D^- = \mathrm{diag}\{d_1^-, d_2^-, \cdots, d_n^-\}, \\ &D^+ = \mathrm{diag}\{d_1^+, d_2^+, \cdots, d_n^+\}, \\ &A^- = (a_{ij}^-)_{n\times n}, \quad A^+ = (a_{ij}^+)_{n\times n}, \\ &B^- = (b_{ij}^-)_{n\times n}, \quad B^+ = (b_{ij}^+)_{n\times n}. \end{aligned}$$

It is clear that $D(x(k)) \in \big[D^-, D^+\big]$, $A(x(k)) \in \big[A^-, A^+\big]$ and $B(x(k)) \in \big[B^-, B^+\big]$.

Define

$$\begin{aligned} \bar{D} \triangleq& \frac{D^+ + D^-}{2} \\ =& \mathrm{diag}\Big\{\frac{d_1^+ + d_1^-}{2}, \frac{d_2^+ + d_2^-}{2}, \cdots, \frac{d_n^+ + d_n^-}{2}\Big\}, \\ \bar{A} \triangleq& \frac{A^+ + A^-}{2} = \Big(\frac{a_{ij}^+ + a_{ij}^-}{2}\Big)_{n\times n}, \\ \bar{B} \triangleq& \frac{B^+ + B^-}{2} = \Big(\frac{b_{ij}^+ + b_{ij}^-}{2}\Big)_{n\times n}. \end{aligned}$$

Then, the matrices $D(x(k))$, $A(x(k))$ and $B(x(k))$ can be written as

$$\begin{aligned} D(x(k)) =& \bar{D} + \Delta D(k), \\ A(x(k)) =& \bar{A} + \Delta A(k), \\ B(x(k)) =& \bar{B} + \Delta B(k) \end{aligned} \tag{8}$$

where

$$\Delta D(k) = \sum_{i=1}^{n} e_i s_i(k) e_i^T,$$

$$\Delta A(k) = \sum_{i,j=1}^{n} e_i t_{ij}(k) e_j^T,$$

$$\Delta B(k) = \sum_{i,j=1}^{n} e_i p_{ij}(k) e_j^T.$$

Here, $e_k \in \mathbb{R}^n$ is the column vector with the kth element being 1 and others being 0, $s_i(k)$, $t_{ij}(k)$ and $p_{ij}(k)$ are unknown scalars satisfying $|s_i(k)| \leq \tilde{d}_i$, $|t_{ij}(k)| \leq \tilde{a}_{ij}$ and $|p_{ij}(k)| \leq \tilde{b}_{ij}$ with

$$\tilde{d}_j = \frac{d_j^+ - d_j^-}{2}, \quad \tilde{a}_{ij} = \frac{a_{ij}^+ - a_{ij}^-}{2}, \quad \tilde{b}_{ij} = \frac{b_{ij}^+ - b_{ij}^-}{2}.$$

The parameter matrices $\Delta D(k), \Delta A(k)$ and $\Delta B(k)$ can be rewritten in the following compact form:

$$\begin{bmatrix} \Delta D(k) & \Delta A(k) & \Delta B(k) \end{bmatrix} = \mathcal{H}F(k)E \tag{9}$$

where $\mathcal{H} = \begin{bmatrix} H & H & H \end{bmatrix}$ and $E = \mathrm{diag}\{E_1, E_2, E_3\}$ are known constant matrices with

$$\begin{aligned} H &= \begin{bmatrix} H_1 & H_2 & \cdots & H_n \end{bmatrix}, \\ H_i &= \underbrace{\begin{bmatrix} e_i & e_i & \cdots & e_i \end{bmatrix}}_{n}, \\ E_i &= \begin{bmatrix} E_{i1}^T & E_{i2}^T & \cdots & E_{in}^T \end{bmatrix}^T \quad (i = 1, 2, 3), \\ E_{1j} &= \begin{bmatrix} e_1^T & e_2^T & \cdots & e_{j-1}^T & \tilde{d}_j e_j^T & e_{j+1}^T & \cdots & e_n^T \end{bmatrix}, \\ E_{2j} &= \begin{bmatrix} \tilde{a}_{j1} e_1^T & \tilde{a}_{j2} e_2^T & \cdots & \tilde{a}_{jn} e_n^T \end{bmatrix}, \\ E_{3j} &= \begin{bmatrix} \tilde{b}_{j1} e_1^T & \tilde{b}_{j2} e_2^T & \cdots & \tilde{b}_{jn} e_n^T \end{bmatrix}, \end{aligned}$$

and $F(k) = \mathrm{diag}\{F_1(k), F_2(k), F_3(k)\}$ are unknown time-varying matrices given by

$$\begin{aligned} F_i(k) =& \mathrm{diag}\{F_{i1}(k), \cdots, F_{in}(k)\}, \\ F_{1j}(k) =& \mathrm{diag}\{\underbrace{0, \cdots, 0}_{j-1}, s_j(k)\tilde{d}_j^{-1}, \underbrace{0, \cdots, 0}_{n-j}\}, \\ F_{2j}(k) =& \mathrm{diag}\{t_{j1}(k)\tilde{a}_{j1}^{-1}, \cdots, t_{jn}(k)\tilde{a}_{jn}^{-1}\}, \\ F_{3j}(k) =& \mathrm{diag}\{p_{j1}(k)\tilde{b}_{j1}^{-1}, \cdots, p_{jn}(k)\tilde{b}_{jn}^{-1}\}. \end{aligned}$$

It is not difficult to verify that the matrices $F_i(k)$ $(i = 1, 2, 3)$ satisfy $F_i^T(k)F_i(k) \leq I_{n^2}$, where I_{n^2} denotes n^2-dimensional identity matrix.

Remark 2: Usually, the norm-bounded condition of uncertainties is given as an assumption in most of the existing literatures. However, in this paper, the state-dependent switching to norm-bounded uncertainties is based on the feature of the memristor and the current-voltage characteristics.

In this paper, the network output of (1) is of the following form:

$$y(k) = \alpha(k)Cx(k) + N\xi(k), \tag{10}$$

$$z(k) = Mx(k) \tag{11}$$

where $y(k) \in \mathbb{R}^m$ is the measurement output, $z(k) \in \mathbb{R}^r$ is the output to be estimated and $\xi(k) \in \mathbb{R}^l$ is the disturbance input belonging to $l_2([0, \infty); \mathbb{R}^l)$. The stochastic variable $\alpha(k)$ is a Bernoulli-distributed white sequence taking values on 0 or 1 with

$$\begin{aligned} &\mathrm{Prob}\{\alpha(k) = 1\} = \bar{\alpha}, \\ &\mathrm{Prob}\{\alpha(k) = 0\} = 1 - \bar{\alpha} \end{aligned} \tag{12}$$

where $\bar{\alpha} \in [0, 1]$ is a known constant.

For resource-constrained systems, the event-based mechanism has proven to be capable of reducing the information exchange frequency and therefore improving the efficiency in resource utilization. For the purpose of introducing the event-based scheduling, we first denote the triggering instant sequence by $0 \leq k_0 \leq \cdots \leq k_\iota \leq \cdots$ and then define an event generator function $\varphi(\cdot, \cdot) : \mathbb{R}^m \times \mathbb{R} \to \mathbb{R}$ as follows:

$$\varphi(\mu(k), \delta) = \mu^T(k)\mu(k) - \delta y^T(k)y(k) \tag{13}$$

where $\mu(k) = y(k) - y(k_\iota)$. Here, $y(k_\iota)$ is the measurement at latest event time (triggering instant) and $\delta > 0$ is a given positive scalar.

The execution (i.e., the measurement output is transmitted to the estimator) is triggered as long as the condition

$$\varphi(\mu(k), \delta) > 0 \tag{14}$$

is satisfied. Therefore, the next triggering instant is determined iteratively by

$$k_{\iota+1} = \inf\{k \in \mathbb{N} | k > k_\iota, \varphi(\mu(k), \delta) > 0\}. \tag{15}$$

Remark 3: The event-triggered scheme is a kind of sampling which generates the measurements and transmits the data after the occurrence of a certain external event. Compared to the conventional time-triggered scheme, the event-triggered scheme shows a significant advantage of reducing the amount of sampling instants. In other words, the signals are updated only when necessary and, therefore, the unnecessary computation and transmission could be avoided. However, it is not difficult to see that the introduction triggering condition (14) gives reduce to the amount of data, and therefore an adequate trade-off can be achieved between the efficiency in resource utilization and the estimation performance.

In order to estimate the neuron state $x(k)$ based on the event-triggered scheme (14), we employ the following state estimator

$$\begin{aligned} \hat{x}(k+1) =& \bar{D}\hat{x}(k) + \bar{A}f(\hat{x}(k)) + \bar{B}g(\hat{x}(k - \tau(k))) \\ &+ K(y(k_\iota) - C\hat{x}(k)) \end{aligned} \tag{16}$$

for $k \in [k_\iota, k_{\iota+1}]$, where $\hat{x}(k) \in \mathbb{R}^n$ is the estimate of the neuron state $x(k)$ and $K \in \mathbb{R}^{n \times m}$ is the estimator gain to be determined.

The dynamics of the estimation error can be obtained from (1), (10), (11) and (16) as follows:

$$\begin{cases} e(k+1) =& (\bar{D} - KC)e(k) + \Delta D(k)x(k) + K\mu(k) \\ &+ (1 - \bar{\alpha})KCx(k) + \bar{A}\tilde{f}(k) \\ &+ \Delta A(k)f(x(k)) + \bar{B}\tilde{g}(x(k - \tau(k))) \\ &+ \Delta B(k)g(x(k - \tau(k))) + L\varsigma(k) \\ &- KN\xi(k) - (\alpha(k) - \bar{\alpha})KCx(k) \\ &+ \sigma(k, x(k), x(k - \tau(k)))w(k), \\ \tilde{z}(k) =& Me(k), \quad k \in [k_\iota, k_{\iota+1}) \end{cases} \tag{17}$$

where $e(k) \triangleq x(k) - \hat{x}(k)$, $\tilde{f}(x(k)) \triangleq f(x(k)) - f(\hat{x}(k))$, $\tilde{g}(x(k - \tau(k))) \triangleq g(x(k - \tau(k))) - g(\hat{x}(k - \tau(k)))$ and $\tilde{z}(k)$ is the output estimation error. Then, by setting $\eta(k) = [x^T(k) \;\; e^T(k)]^T$, we have the following augmented system

$$\begin{cases} \eta(k+1) =& \tilde{W}_1\eta(k) + (\bar{\alpha} - \alpha(k))W_2\eta(k) \\ &+ \tilde{W}_3\vec{f}(k) + \tilde{W}_4\vec{g}(k - \tau(k)) + W_5\zeta(k) \\ &+ W_6 w(k) + W_7\vec{\mu}(k), \\ \tilde{z}(k) =& \mathcal{M}\eta(k), \quad k \in [k_\iota, k_{\iota+1}) \end{cases} \tag{18}$$

where

$$\begin{aligned} &\vec{f}(k) = \begin{bmatrix} f^T(x(k)) & \tilde{f}^T(x(k)) \end{bmatrix}^T, \\ &\vec{g}(k - \tau(k)) = \begin{bmatrix} g^T(x(k - \tau(k))) & \tilde{g}^T(x(k - \tau(k))) \end{bmatrix}^T, \\ &\zeta(k) = \begin{bmatrix} \varsigma^T(k) & \xi^T(k) \end{bmatrix}^T, \quad \vec{\mu}(k) = \begin{bmatrix} 0 & \mu^T(k) \end{bmatrix}^T, \\ &\mathcal{M} = \begin{bmatrix} 0 & M \end{bmatrix}, \quad \tilde{W}_1 = W_1 + \Delta\mathcal{D}(k), \\ &\tilde{W}_3 = W_3 + \Delta\mathcal{A}(k), \quad \tilde{W}_4 = W_4 + \Delta\mathcal{B}(k), \\ &W_1 = \begin{bmatrix} \bar{D} & 0 \\ (1 - \bar{\alpha})KC & \bar{D} - KC \end{bmatrix}, \quad W_2 = \begin{bmatrix} 0 & 0 \\ KC & 0 \end{bmatrix}, \end{aligned}$$

$$W_3 = \begin{bmatrix} \bar{A} & 0 \\ 0 & \bar{A} \end{bmatrix}, \quad W_4 = \begin{bmatrix} \bar{B} & 0 \\ 0 & \bar{B} \end{bmatrix}, \quad W_7 = \begin{bmatrix} 0 & 0 \\ 0 & K \end{bmatrix},$$
$$W_5 = \begin{bmatrix} L & 0 \\ L & -KN \end{bmatrix}, \quad \Delta\mathcal{D}(k) = \begin{bmatrix} \Delta D(k) & 0 \\ \Delta D(k) & 0 \end{bmatrix},$$
$$\Delta\mathcal{A}(k) = \begin{bmatrix} \Delta A(k) & 0 \\ \Delta A(k) & 0 \end{bmatrix}, \quad \Delta\mathcal{B}(k) = \begin{bmatrix} \Delta B(k) & 0 \\ \Delta B(k) & 0 \end{bmatrix},$$
$$W_6 = \big[\sigma^T(k, x(k), x(k-\tau(k)))^T \\ \sigma^T(k, x(k), x(k-\tau(k)))\big]^T.$$

Our main aim in this paper is to design a suitable H_∞ state estimator for stochastic memristive neural networks given by (1). More specifically, we are interested in looking for the gain matrix K such that the following two requirements are met simultaneously:

1) The augmented system (18) with $\zeta(k) = 0$ is exponentially mean-square stable;

2) Under zero initial conditions, for a given disturbance attention level $\gamma > 0$ and all nonzero $\zeta(k)$, the output $\tilde{z}(k)$ satisfies

$$\sum_{k=0}^{\infty} \mathbb{E}\left\{\|\tilde{z}(k)\|^2\right\} \le \gamma^2 \sum_{k=0}^{\infty} \mathbb{E}\left\{\|\zeta(k)\|^2\right\}. \tag{19}$$

III. Main Results

Before proceeding to the stability analysis for system (18), we introduce one lemma that will be useful in deriving our results.

Lemma 1: [2] Let $N = N^T, H$ and E be real matrices with appropriate dimensions, and $F^T(k)F(k) \le I$, where I denotes the identity matrix of compatible dimension. Then the inequality $N + HFE + (HFE)^T < 0$ if and only if there exists a positive scalar ε such that $N + \varepsilon HH^T + \varepsilon^{-1}E^TE < 0$ or, equivalently,

$$\begin{bmatrix} N & \varepsilon H & E^T \\ \varepsilon H^T & -\varepsilon I & 0 \\ E & 0 & -\varepsilon I \end{bmatrix} < 0.$$

For the stability of system (18), we have the following results.

Theorem 1: Let K be a given constant matrix. Then, under Assumption 1, the augmented system (18) with $\zeta(k) = 0$ is exponentially mean-square stable if there exist positive definite matrices $P = \mathrm{diag}\{P_1, P_2\}$, Q and positive scalars λ_1^*, λ_2^* and λ_j $(j = 1, 2, 3)$ satisfying the following inequalities:

$$P < \Lambda^* I_{2n}, \tag{20}$$

$$\tilde{\Phi} = \begin{bmatrix} \tilde{\Theta}_{11} & 0 & \tilde{\Theta}_{13} & \Theta_{14} & \Theta_{15} \\ * & \tilde{\Theta}_{22} & 0 & \tilde{\Theta}_{24} & 0 \\ * & * & \tilde{\Theta}_{33} & \Theta_{34} & \Theta_{35} \\ * & * & * & \tilde{\Theta}_{44} & \Theta_{45} \\ * & * & * & * & \tilde{\Theta}_{55} \end{bmatrix} < 0 \tag{21}$$

where

$$\tilde{\Lambda}_1 = I_2 \otimes \mathrm{Sym}\{\frac{1}{2}\Lambda_1^T\Lambda_2\}, \quad \tilde{\Lambda}_2 = I_2 \otimes (\Lambda_1 + \Lambda_2)/2,$$
$$\tilde{\Upsilon}_1 = I_2 \otimes \mathrm{Sym}\{\frac{1}{2}\Upsilon_1^T\Upsilon_2\}, \quad \tilde{\Upsilon}_2 = I_2 \otimes (\Upsilon_1 + \Upsilon_2)/2,$$
$$\Lambda^* = \mathrm{diag}\{\lambda_1^*, \lambda_2^*\}, \quad \mathbb{I} = \begin{bmatrix} I_n & 0 \\ 0 & 0 \end{bmatrix},$$

$$\tilde{\Theta}_{11} = \tilde{W}_1^T P \tilde{W}_1 + (\lambda_1^* + \lambda_2^*)\rho_1 \mathbb{I} + (\tau_M - \tau_m + 1)Q \\ + \lambda_3 \delta \bar{\alpha}^2 \mathcal{C}^T \mathcal{C} - P - \lambda_1 \tilde{\Lambda}_1,$$
$$\tilde{\Theta}_{22} = -Q + (\lambda_1^* + \lambda_2^*)\rho_2 \mathbb{I} - \lambda_2 \tilde{\Upsilon}_1,$$
$$\tilde{\Theta}_{33} = \tilde{W}_3^T P \tilde{W}_3 - \lambda_1 I_{2n}, \quad \tilde{\Theta}_{13} = \tilde{W}_1^T P \tilde{W}_3 + \lambda_1 \tilde{\Lambda}_2^T,$$
$$\tilde{\Theta}_{44} = \tilde{W}_4^T P \tilde{W}_4 - \lambda_2 I_{2n}, \quad \tilde{\Theta}_{55} = W_7^T P W_7 - \lambda_3 I_{2n},$$
$$\tilde{\Theta}_{24} = \lambda_2 \tilde{\Upsilon}_2^T, \quad \Theta_{14} = \tilde{W}_1^T P \tilde{W}_4, \quad \Theta_{34} = \tilde{W}_3^T P \tilde{W}_4,$$
$$\Theta_{15} = \tilde{W}_1^T P W_7, \quad \Theta_{35} = \tilde{W}_3^T P W_7, \quad \Theta_{45} = \tilde{W}_4^T P W_7.$$

Proof: Choose a Lyapunov-Krasovskii functional for system (18) as follows:

$$V(\eta(k)) = V_1(\eta(k)) + V_2(\eta(k)) + V_3(\eta(k)) \tag{22}$$

where

$$V_1(\eta(k)) = \eta^T(k) P \eta(k), \tag{23}$$
$$V_2(\eta(k)) = \sum_{i=k-\tau(k)}^{k-1} \eta^T(i) Q \eta(i), \tag{24}$$
$$V_3(\eta(k)) = \sum_{j=k-\tau_M+1}^{k-\tau_m} \sum_{i=j}^{k-1} \eta^T(i) Q \eta(i). \tag{25}$$

In the case of $\zeta(k) = 0$, calculating the difference of $V(k)$ along the system (18), and taking the mathematical expectation, one has

$$\begin{aligned} &\mathbb{E}\{\Delta V(\eta(k))\} \\ =&\mathbb{E}\{\Delta V_1(\eta(k))\} + \mathbb{E}\{\Delta V_2(\eta(k))\} + \mathbb{E}\{\Delta V_3(\eta(k))\} \end{aligned} \tag{26}$$

where

$$\begin{aligned} &\mathbb{E}\{\Delta V_1(\eta(k))\} \\ =&\mathbb{E}\{V_1(\eta(k+1)) - V_1(\eta(k))\} \\ =&\mathbb{E}\{[\eta^T(k)\tilde{W}_1^T + \vec{f}^T(k)\tilde{W}_3^T + \vec{g}^T(k-\tau(k))\tilde{W}_4^T \\ &+ W_6^T w(k) + \vec{\mu}^T(k) W_7^T] P [\tilde{W}_1 \eta(k) + W_3 \vec{f}(k) \\ &+ W_4 \vec{g}(k-\tau(k)) + W_6 w(k) + W_7 \vec{\mu}(k)] \\ &- \eta^T(k) P \eta(k)\} \\ =&\mathbb{E}\{[\eta^T(k)\tilde{W}_1^T P \tilde{W}_1 \eta(k) + \vec{f}^T(k)\tilde{W}_3^T P \tilde{W}_3 \vec{f}(k) \\ &+ \vec{g}^T(k-\tau(k))\tilde{W}_4^T P \tilde{W}_4 \vec{g}(k-\tau(k)) + W_6^T P W_6 \\ &+ \vec{\mu}^T(k) W_7^T P W_7 \vec{\mu}(k) + 2\eta^T(k)\tilde{W}_1^T P \tilde{W}_3 \vec{f}(k) \\ &+ 2\eta^T(k)\tilde{W}_1^T P \tilde{W}_4 \vec{g}(k-\tau(k)) + 2\eta^T(k)\tilde{W}_1^T P W_7 \vec{\mu}(k) \\ &+ 2\vec{f}^T(k)\tilde{W}_3^T P \tilde{W}_4 \vec{g}(k-\tau(k)) + 2\vec{f}^T(k)\tilde{W}_3^T P W_7 \vec{\mu}(k) \\ &+ 2\vec{g}^T(k-\tau(k))\tilde{W}_4^T P W_7 \vec{\mu}(k)] - \eta^T(k) P \eta(k)\}, \end{aligned} \tag{27}$$

$$\begin{aligned} &\mathbb{E}\{\Delta V_2(\eta(k))\} \\ =&\mathbb{E}\{V_2(\eta(k+1)) - V_2(\eta(k))\} \end{aligned} \tag{28}$$

$$
\begin{aligned}
=&\mathbb{E}\Big\{\sum_{i=k+1-\tau(k+1)}^{k}\eta^T(i)Q\eta(i)\\
&\qquad -\sum_{i=k-\tau(k)}^{k-1}\eta^T(i)Q\eta(i)\Big\}\\
\le&\mathbb{E}\Big\{\eta^T(k)Q\eta(k)-\eta^T(k-\tau(k))Q\eta(k-\tau(k))\\
&+\sum_{i=k+1-\tau_M}^{k-\tau_m}\eta^T(i)Q\eta(i)\Big\}
\end{aligned}
$$

and

$$
\begin{aligned}
&\mathbb{E}\{\Delta V_3(\eta(k))\}\\
=&\mathbb{E}\{V_3(\eta(k+1))-V_3(\eta(k))\}\\
=&\mathbb{E}\Big\{\sum_{i=k+2-\tau_M}^{k+1-\tau_m}\sum_{i=j}^{k}\eta^T(i)Q\eta(i)\\
&\qquad -\sum_{i=k+1-\tau_M}^{k-\tau_m}\sum_{i=j}^{k-1}\eta^T(i)Q\eta(i)\Big\}\\
=&\mathbb{E}\Big\{\sum_{i=k+1-\tau_M}^{k-\tau_m}\sum_{i=j+1}^{k}\eta^T(i)Q\eta(i)\\
&\qquad -\sum_{i=k+1-\tau_M}^{k-\tau_m}\sum_{i=j}^{k-1}\eta^T(i)Q\eta(i)\Big\}\\
=&\mathbb{E}\Big\{\sum_{j=k+1-\tau_M}^{k-\tau_m}(\eta^T(k)Q\eta(k)-\eta^T(j)Q\eta(j))\Big\}\\
=&\mathbb{E}\Big\{(\tau_M-\tau_m)\eta^T(k)Q\eta(k)\\
&\qquad -\sum_{i=k+1-\tau_M}^{k-\tau_m}\eta^T(i)Q\eta(i)\Big\}.
\end{aligned}
\tag{29}
$$

Noticing (4) and (20), it is easy to see that

$$
\begin{aligned}
&\sigma^T(k,x(k),x(k-\tau(k)))P_1\sigma(k,x(k),x(k-\tau(k)))\\
\le&\lambda_{\max}(P_1)\sigma^T(k,x(k),x(k-\tau(k)))\\
&\times\sigma(k,x(k),x(k-\tau(k)))\\
\le&\lambda_1^*(\rho_1x^T(k)x(k)+\rho_2x^T(k-\tau(k))x(k-\tau(k))),
\end{aligned}
\tag{30}
$$

$$
\begin{aligned}
&\sigma^T(k,x(k),x(k-\tau(k)))P_2\sigma(k,x(k),x(k-\tau(k)))\\
\le&\lambda_{\max}(P_2)\sigma^T(k,x(k),x(k-\tau(k)))\\
&\times\sigma(k,x(k),x(k-\tau(k)))\\
\le&\lambda_2^*(\rho_1x^T(k)x(k)+\rho_2x^T(k-\tau(k))x(k-\tau(k))).
\end{aligned}
\tag{31}
$$

Substituting (27)–(31) into (26) leads to

$$
\begin{aligned}
&\mathbb{E}\{\Delta V(\eta(k))\}\\
\le&\mathbb{E}\{\eta^T(k)(\tilde W_1^TP\tilde W_1+(\lambda_1^*+\lambda_2^*)\rho_1\mathbb{I}+(\tau_M-\tau_m\\
&+1)Q-P]\eta(k)+\eta^T(k-\tau(k))(-Q+(\lambda_1^*\\
&+\lambda_2^*)\rho_2\mathbb{I})\eta(k-\tau(k))+\vec f^T(k)\tilde W_3^TP\tilde W_3\vec f(k)\\
&+\vec g^T(k-\tau(k))\tilde W_4^TP\tilde W_4\vec g(k-\tau(k))+\vec\mu^T(k)\\
&\times W_7^TPW_7\vec\mu(k)+2\eta^T(k)\tilde W_1^TP\tilde W_3\vec f(k)+2\eta^T(k)\\
&\times\tilde W_1^TP\tilde W_4\vec g(k-\tau(k))+2\eta^T(k)\tilde W_1^TPW_7\vec\mu(k)\\
&+2\vec f^T(k)\tilde W_3^TP\tilde W_4\vec g(k-\tau(k))+2\vec f^T(k)\tilde W_3^TP\\
&\times W_7\vec\mu(k)+2\vec g^T(k-\tau(k))\tilde W_4^TPW_7\vec\mu(k)\}\\
=&\mathbb{E}\{\varpi^T(k)\Phi\varpi(k)\}
\end{aligned}
\tag{32}
$$

where

$$
\varpi(k)=\begin{bmatrix}\eta^T(k) & \eta^T(k-\tau(k)) & \vec f^T(k) & \vec g^T(k-\tau(k)) & \vec\mu^T(k)\end{bmatrix}^T,
$$

$$
\Phi=\begin{bmatrix}\Theta_{11} & \Pi_{12}\\ * & \Pi_{22}\end{bmatrix}
$$

with

$$
\Pi_{12}=\begin{bmatrix}0 & \Theta_{13} & \Theta_{14} & \Theta_{15}\end{bmatrix},
$$

$$
\Pi_{22}=\begin{bmatrix}\Theta_{22} & 0 & 0 & 0\\ * & \Theta_{33} & \Theta_{34} & \Theta_{35}\\ * & * & \Theta_{44} & \Theta_{45}\\ * & * & * & \Theta_{55}\end{bmatrix},
$$

$$
\begin{aligned}
&\Theta_{11}=\tilde W_1^TP\tilde W_1+(\lambda_1^*+\lambda_2^*)\rho_1\mathbb{I}+(\tau_M-\tau_m+1)Q-P,\\
&\Theta_{22}=-Q+(\lambda_1^*+\lambda_2^*)\rho_2\mathbb{I},\quad \Theta_{33}=\tilde W_3^TP\tilde W_3,\\
&\Theta_{44}=\tilde W_4^TP\tilde W_4,\quad \Theta_{55}=W_7^TPW_7,\quad \Theta_{13}=\tilde W_1^TP\tilde W_3
\end{aligned}
$$

and other parameters are defined in Theorem 1.

Taking (5), (6) and (15) into consideration, we have

$$
\begin{aligned}
&\mathbb{E}\{\Delta V(\eta(k))\}\\
\le&\mathbb{E}\{\varpi^T(k)\Phi\varpi(k)-\lambda_1[\vec f(k)-(I_2\otimes\Lambda_1)\eta(k)]^T\\
&\times[\vec f(k)-(I_2\otimes\Lambda_2)\eta(k)]-\lambda_2[\vec g(k-\tau(k))\\
&-(I_2\otimes\Upsilon_1)\eta(k-\tau(k))]^T[\vec g(k-\tau(k))\\
&-(I_2\otimes\Upsilon_2)\eta(k-\tau(k))]-\lambda_3(\mu^T(k)\mu(k)\\
&-\delta y^T(k)y(k))\}\\
\le&\mathbb{E}\{\varpi^T(k)\Phi\varpi(k)-\lambda_1[\vec f(k)-(I_2\otimes\Lambda_1)\eta(k)]^T\\
&\times[\vec f(k)-(I_2\otimes\Lambda_2)\eta(k)]-\lambda_2[\vec g(k-\tau(k))\\
&-(I_2\otimes\Upsilon_1)\eta(k-\tau(k))]^T[\vec g(k-\tau(k))\\
&-(I_2\otimes\Upsilon_2)\eta(k-\tau(k))]-\lambda_3\mu^T(k)\mu(k)\\
&+\lambda_3\delta\bar\alpha^2\eta^T(k)\mathcal{C}^T\mathcal{C}\eta(k)\}\\
\le&\mathbb{E}\{\varpi^T(k)\tilde\Phi\varpi(k)\}
\end{aligned}
\tag{33}
$$

where $\mathcal{C}=\begin{bmatrix}C & 0\end{bmatrix}$ and $\tilde\Phi$ is defined by (21). Since $\tilde\Phi<0$, we have $\mathbb{E}\{\Delta V(\eta(k))\}\le-\epsilon\mathbb{E}\{\|\eta(k)\|\}$ where $\epsilon=-\lambda_{\max}(\tilde\Phi)$. Then, by following the similar analysis in [17], the exponential mean-square stability of the augmented system (18) with $\zeta(k)=0$ is guaranteed and the proof is complete. ■

Now, let us consider the H_∞ performance of the augmented system (18). In the following theorem, a sufficient condition is obtained that guarantees both the exponential mean-square stability and the H_∞ performance for the augmented system (18).

Theorem 2: Consider the system (1) and let the estimator parameter K be given. The augmented system (18) is exponentially stable in mean square and satisfies the H_∞ performance constraint (19) for all nonzero $\zeta(k)$ under the zero initial condition if there exist positive definite matrices $P=\mathrm{diag}\{P_1,P_2\}$, Q and positive scalars λ_1^*,λ_2^* and

λ_j $(j = 1, 2, 3)$ satisfying (19) and the following inequality:

$$\check{\Phi} = \begin{bmatrix} \check{\Theta}_{11} & 0 & \tilde{\Theta}_{13} & \Theta_{14} & \Theta_{15} & \Theta_{16} \\ * & \tilde{\Theta}_{22} & 0 & \tilde{\Theta}_{24} & 0 & 0 \\ * & * & \tilde{\Theta}_{33} & \Theta_{34} & \Theta_{35} & \Theta_{36} \\ * & * & * & \tilde{\Theta}_{44} & \Theta_{45} & \Theta_{46} \\ * & * & * & * & \tilde{\Theta}_{55} & \Theta_{56} \\ * & * & * & * & * & \Theta_{66} \end{bmatrix} < 0 \qquad (34)$$

where

$$\begin{aligned} \check{\Theta}_{11} &= \tilde{W}_1^T P \tilde{W}_1 + (\lambda_1^* + \lambda_2^*)\rho_1 \mathbb{I} + (\tau_M - \tau_m + 1)Q \\ &\quad + \lambda_3 \delta \bar{\alpha}^2 \mathcal{C}^T \mathcal{C} - P - \lambda_1 \tilde{\Lambda}_1 + \vec{M}^T \vec{M}, \\ \Theta_{66} &= -\gamma^2 I_{2n} + W_5^T P W_5, \quad \Theta_{16} = \tilde{W}_1^T P W_5, \\ \Theta_{36} &= \tilde{W}_3^T P W_5, \quad \Theta_{46} = \tilde{W}_4^T P W_5, \quad \Theta_{56} = W_7^T P W_5 \end{aligned}$$

and other parameters are defined in Theorem 1.

Proof: Note that the inequality (21) is implied by (34), the proof of the exponential mean-square stability for (18) in the case of $\zeta(k) = 0$ follows immediately from Theorem 1.

For the H_∞ performance analysis, we choose the same Lyapunov-Krasovskii functional as in (22) and calculate the difference of $V(\eta(k))$ along (18) as follows:

$$\begin{aligned} &\mathbb{E}\{\Delta V(\eta(k)) + \|\tilde{z}(k)\|^2 - \gamma^2 \|\zeta(k)\|^2\} \\ =&\mathbb{E}\{\eta^T(k)(\tilde{W}_1^T P \tilde{W}_1 + (\tau_M - \tau_m + 1)Q + \lambda_3 \delta \bar{\alpha}^2 \mathcal{C}^T \mathcal{C} \\ &- P + \vec{M}^T \vec{M})\eta(k) + \vec{f}^T(k) \tilde{W}_3^T P \tilde{W}_3 \vec{f}(k) \\ &+ \vec{g}^T(k - \tau(k)) \tilde{W}_4^T P \tilde{W}_4 \vec{g}(k - \tau(k)) + W_6^T P W_6 \\ &+ \vec{\mu}^T(k) W_7^T P W_7 \vec{\mu}(k) + \zeta^T(k)(W_5^T P W_5 - \gamma^2 I_{2n})\zeta(k) \\ &+ 2\eta^T(k) \tilde{W}_1^T P \tilde{W}_3 \vec{f}(k) + 2\eta^T(k) \tilde{W}_1^T P \tilde{W}_4 \vec{g}(k - \tau(k)) \\ &+ 2\eta^T(k) \tilde{W}_1^T P W_7 \vec{\mu}(k) + 2\eta^T(k) \tilde{W}_1^T P W_5 \zeta(k) \\ &+ 2\vec{f}^T(k) \tilde{W}_3^T P \tilde{W}_4 \vec{g}(k - \tau(k)) + 2\vec{f}^T(k) \tilde{W}_3^T P W_7 \vec{\mu}(k) \\ &+ 2\vec{f}^T(k) \tilde{W}_3^T P W_5 \zeta(k) + 2\vec{g}^T(k - \tau(k)) \tilde{W}_4^T P W_7 \vec{\mu}(k) \\ &+ 2\vec{g}^T(k - \tau(k)) \tilde{W}_4^T P W_5 \zeta(k) + 2\vec{\mu}^T(k) W_7^T P W_5 \zeta(k)\} \\ =&\mathbb{E}\{\vartheta^T(k) \bar{\Phi} \vartheta(k)\} \end{aligned}$$

where

$$\begin{aligned} \vartheta(k) = \Big[\eta^T(k) \quad & \eta^T(k - \tau(k)) \quad \vec{f}^T(k) \\ & \vec{g}^T(k - \tau(k)) \quad \vec{\mu}^T(k) \quad \zeta^T(k) \Big]^T \end{aligned}$$

and

$$\bar{\Phi} = \begin{bmatrix} \bar{\Theta}_{11} & \Pi_{12} & \Theta_{16} \\ * & \Pi_{22} & \Pi_{23} \\ * & * & \Theta_{66} \end{bmatrix}$$

with $\bar{\Theta}_{11} = \tilde{W}_1^T P \tilde{W}_1 + (\tau_M - \tau_m + 1)Q + \lambda_3 \delta \bar{\alpha}^2 \mathcal{C}^T \mathcal{C} - P + \vec{M}^T \vec{M}$, $\Pi_{23} = \begin{bmatrix} 0 & \Theta_{36}^T & \Theta_{46}^T & \Theta_{56}^T \end{bmatrix}^T$ and other parameters defined in Theorems 1 and 2.

Using (5), (6) and (34), we have

$$\mathbb{E}\{V(\eta(k+1)) - V(\eta(k)) + \|\tilde{z}(k)\|^2 - \gamma^2 \|\zeta(k)\|^2\} < 0. \quad (35)$$

Under the zero initial condition, summing up (35) from 0 to ∞ with respect to k and considering $\mathbb{E}\{V(\eta(\infty))\} \geq 0$, we obtain (19) and the proof of Theorem 2 is then accomplished. ■

According to the H_∞ performance analysis conducted in Theorem 2, a design method of the H_∞ state estimator for (1) is provided in Theorem 3.

Theorem 3: Consider the system (1) and let the disturbance attenuation level $\gamma > 0$ be given. The augmented system (18) is exponentially stable in mean square and the H_∞ performance constraint (19) is met for all nonzero $\zeta(k)$ under the zero initial condition if there exist positive definite matrices $P = \mathrm{diag}\{P_1, P_2\}$, Q, and positive scalars λ_1^*, λ_2^* and ε, λ_j $(j = 1, 2, 3)$ satisfying (20) and the following inequality:

$$\begin{bmatrix} \check{\Phi} & \check{H} & \varepsilon \check{E}^T \\ * & -\varepsilon I_{(2n)^7} & 0 \\ * & * & -\varepsilon I_{(2n)^7} \end{bmatrix} < 0 \qquad (36)$$

where

$$\begin{aligned} \check{\Phi} &= \begin{bmatrix} \check{\Pi}_{11} & \check{\Pi}_{12} \\ * & \check{\Theta}_{77} \end{bmatrix}, \\ \check{\Pi}_{11} &= \begin{bmatrix} \check{\Theta}_{11} & 0 & \check{\Theta}_{13} & 0 & 0 & 0 \\ * & \tilde{\Theta}_{22} & 0 & \tilde{\Theta}_{24} & 0 & 0 \\ * & * & \check{\Theta}_{33} & 0 & 0 & 0 \\ * & * & * & \check{\Theta}_{44} & 0 & 0 \\ * & * & * & * & \check{\Theta}_{55} & 0 \\ * & * & * & * & * & \check{\Theta}_{66} \end{bmatrix}, \\ \check{\Pi}_{12} &= \begin{bmatrix} \check{\Theta}_{17}^T & 0 & \check{\Theta}_{37}^T & \check{\Theta}_{47}^T & \check{\Theta}_{57}^T & \check{\Theta}_{67}^T \end{bmatrix}^T, \\ \check{H} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \mathcal{H}^T \check{P}^T \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathcal{H}^T \check{P}^T \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathcal{H}^T \check{P}^T \end{bmatrix}^T, \quad \check{P} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}, \\ \check{E} &= \begin{bmatrix} S_{17} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & S_{37} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{47} & 0 & 0 & 0 \end{bmatrix}, \\ \check{\Theta}_{11} &= (\lambda_1^* + \lambda_2^*)\rho_1 \mathbb{I} + (\tau_M - \tau_m + 1)Q + \lambda_3 \delta \bar{\alpha}^2 \mathcal{C}^T \mathcal{C} \\ &\quad - P - \lambda_1 \tilde{\Lambda}_1 + \vec{M}^T \vec{M}, \\ \check{\Theta}_{13} &= \lambda_1 \tilde{\Lambda}_2^T, \quad \check{\Theta}_{33} = -\lambda_1 I_{2n}, \quad \check{\Theta}_{37} = W_3^T P^T, \\ \check{\Theta}_{44} &= -\lambda_2 I_{2n}, \quad \check{\Theta}_{47} = W_4^T P^T, \quad \check{\Theta}_{55} = -\lambda_3 I_{2n}, \\ \check{\Theta}_{66} &= -\gamma^2 I_{2n}, \quad \check{\Theta}_{77} = -P, \quad X = P_2 K, \\ \check{\Theta}_{17}^T &= \begin{bmatrix} P_1 \bar{D} & 0 \\ (1 - \bar{\alpha}) X C & P_2 \bar{D} - X C \end{bmatrix}, \\ \check{\Theta}_{57}^T &= \begin{bmatrix} 0 & 0 \\ 0 & X \end{bmatrix}, \quad \check{\Theta}_{67}^T = \begin{bmatrix} P_1 L & 0 \\ P_2 L & -X N \end{bmatrix}, \\ S_{17} &= \begin{bmatrix} E_1 & 0 \end{bmatrix}, \quad S_{37} = \begin{bmatrix} E_2 & 0 \end{bmatrix}, \quad S_{47} = \begin{bmatrix} E_3 & 0 \end{bmatrix} \end{aligned} \qquad (37)$$

and other parameters are defined in Theorems 1 and 2. Moreover, if the above inequality is solvable, the state estimator gain can be determined by $K = P_2^{-1} X$.

Proof: In order to eliminate the uncertainties in (34), we use the Schur Complement Lemma and obtain

$$\Xi = \begin{bmatrix} \check{\Pi}_{11} & \Xi_{12} \\ * & \check{\Theta}_{77} \end{bmatrix} < 0 \qquad (38)$$

where

$$\begin{aligned} \Xi_{12} &= \begin{bmatrix} \Lambda_{17}^T & 0 & \Lambda_{37}^T & \Lambda_{47}^T & \check{\Theta}_{57}^T & \check{\Theta}_{67}^T \end{bmatrix}^T, \\ \Lambda_{17} &= \check{\Theta}_{17} + S_{17}^T F_1^T(k) \mathcal{H}^T \check{P}^T, \\ \Lambda_{37} &= \check{\Theta}_{37} + S_{37}^T F_2^T(k) \mathcal{H}^T \check{P}^T, \end{aligned} \qquad (39)$$

$$\Lambda_{47} = \check{\Theta}_{47} + S_{47}^T F_3^T(k)\mathcal{H}^T \check{P}^T$$

and other parameters are defined in Theorems 2 and 3.

By considering $X = P_2 K$, it follows from (39) that

$$\check{\Phi} + \check{H}\check{F}(k)\check{E} + (\check{H}\check{F}(k)\check{E})^T < 0 \tag{40}$$

where $\check{F}(k) = \mathrm{diag}\{F_1(k), F_2(k), F_3(k)\}$ and other parameters have been defined in (37). According to the Lemma 1, it can be easily shown that inequality (40) is implied by (36). The rest of the proof follows Theorem 2 immediately. ∎

Remark 4: In the extreme case, when $\delta = 0$, we can see that $\{k_0, k_1, k_2, \cdots\} = \{0, 1, 2, \cdots\}$. It means that all measurements are transmitted to the side of the state estimator at each sampling instant. Then, the addressed event-triggered state estimation problem reduces to the traditional one. Moreover, when $\tau_m = \tau_M$, the phenomenon of time-varying delays should degenerate into the constant time-delays one.

Remark 5: For the event-triggered H_∞ state estimation problem for delayed stochastic MNNs with missing measurements, there are five main aspects which complicate the design of the neuron state estimator, i.e. event-triggering mechanism, interval time-varying delays, randomly missing measurements, external additive deterministic noises as well as the internal multiplicative stochastic noises. In our main results (Theorems 1-3), all these five factors have been properly handled and the established sufficient conditions include all the system parameters, the event-triggering threshold, the lower and upper bounds of the delays, and the missing probability of the measurement output where the external deterministic noises are attenuated through the prescribed H_∞ performance requirement and the internal stochastic noises have an impact on the stability analysis through their intensity matrices. The corresponding solvability conditions for the desired estimator gains are expressed in terms of the feasibility of a few linear matrix inequalities (LMIs) that can be solved using available software package. It should be pointed out that Lyapunov-Krasovskii functional is constructed to derive several delay-dependent stability criteria and our developed algorithm would have the advantage of less conservatism since more information about the delays is employed.

Remark 6: For the convenience of the analysis, the triggering condition (14) can be written as an equivalent form $\frac{\mu^T(k)\mu(k)}{y^T(k)y(k)} > \delta$. Now, it can be easily seen that the triggering condition is and hence the threshold δ should lie in the interval $[0, +\infty)$. Theoretically, when the chosen threshold δ satisfies $0 < \delta < \inf_{k\geq 0} \frac{\mu^T(k)\mu(k)}{y^T(k)y(k)}$, the triggering condition naturally holds for all time instants which means that the measurement outputs are transmitted at every time instants and the event-triggered estimation approach reduces to the classical clock-driven estimation one. When $\inf_{k\geq 0} \frac{\mu^T(k)\mu(k)}{y^T(k)y(k)} < \delta < \infty$, the triggering condition will not be satisfied always and the measurement transmission will occur only at those time instants when the triggering condition is violated. Actually, for a practical system, we may first compute the relative error $\frac{\mu^T(k)\mu(k)}{y^T(k)y(k)}$ according to the available measurement outputs from the initial time instant $k = 0$ and then choose an appropriate threshold δ such that the triggering condition is not satisfied for all time instants. As such, the communication and computation resources can be saved effectively.

Remark 7: In Theorem 3, the H_∞ state estimator is designed for DSMNNs in terms of the solution to LMI (36). Note that, for a standard LMI system, the algorithm has a polynomial-time complexity. Fortunately, research on LMI optimization is a very active area in the applied mathematics, optimization and the operations research community, and substantial speed-ups can be expected in the future.

IV. An Illustrative Example

In order to illustrate the validity of the proposed state estimator, in this section, a real-world example will be used to examine the main theoretical results.

Firstly, using memristors to replace resistors in the circuit realization of the connection links of neural networks, it will result in a memristor-based neural network. Then, by Kirchhoff's circuit laws, the equation of the ith circuit subsystem is written as follows:

$$\begin{aligned} x_i(k+1) =& \frac{1}{C_i}\Big[\sum_{j=1}^{2}\Big(\frac{1}{R_{aij}} + \frac{1}{R_{bij}}\Big) + \frac{1}{R_i}\Big]x_i(k) \\ &+ \sum_{j=1}^{2}\frac{\mathrm{sign}_{ij}}{C_i R_{aij}} f_j(x_j(k)) + \sum_{j=1}^{2}\frac{\mathrm{sign}_{ij}}{C_i R_{bij}} \\ &\times g_j(x_j(k-\tau(k))) + l_i\varsigma_i(k) \\ &+ \sigma_i(k, x_i(k), x_i(k-\tau(k)))w(k) \end{aligned} \tag{41}$$

where $i = 1, 2$, x_i is the voltage of the capacitor C_i. Then from (7) and (41), we can construct the DSMNNs of the form

$$\begin{aligned} x_i(k+1) =& d_i(x_i(k))x_i(k) + \sum_{j=1}^{2} a_{ij}(x_i(k)) f_j(x_j(k)) \\ &+ \sum_{j=1}^{2} b_{ij}(x_i(k)) g_j(x_j(k-\tau(k))) + l_i\varsigma_i(k) \\ &+ \sigma_i(k, x_i(k), x_i(k-\tau(k)))w(k), \quad i = 1, 2. \end{aligned} \tag{42}$$

Moreover, the system parameters of the DSMNNs are set as follows

$$d_1(x_1(\cdot)) = \begin{cases} 0.25, & |x_1(\cdot)| > 0.02, \\ 0.65, & |x_1(\cdot)| \leq 0.02, \end{cases}$$

$$d_2(x_2(\cdot)) = \begin{cases} 0.65, & |x_2(\cdot)| > 0.02, \\ 0.25, & |x_2(\cdot)| \leq 0.02, \end{cases}$$

$$a_{11}(x_1(\cdot)) = \begin{cases} -0.20, & |x_1(\cdot)| > 0.02, \\ 0.90, & |x_1(\cdot)| \leq 0.02, \end{cases}$$

$$a_{12}(x_1(\cdot)) = \begin{cases} -0.40, & |x_1(\cdot)| > 0.02, \\ 0.20, & |x_1(\cdot)| \leq 0.02, \end{cases}$$

$$a_{21}(x_2(\cdot)) = \begin{cases} 0.68, & |x_2(\cdot)| > 0.02, \\ -0.22, & |x_2(\cdot)| \leq 0.02, \end{cases}$$

$$a_{22}(x_2(\cdot)) = \begin{cases} 0.32, & |x_2(\cdot)| > 0.02, \\ 0.05, & |x_2(\cdot)| \leq 0.02, \end{cases}$$

$$b_{11}(x_1(\cdot)) = \begin{cases} 0.20, & |x_1(\cdot)| > 0.02, \\ 0.50, & |x_1(\cdot)| \leq 0.02, \end{cases}$$

$$b_{12}(x_1(\cdot)) = \begin{cases} 0.60, & |x_1(\cdot)| > 0.02, \\ -0.10, & |x_1(\cdot)| \leq 0.02, \end{cases}$$

$$b_{21}(x_2(\cdot)) = \begin{cases} 0.50, & |x_2(\cdot)| > 0.02, \\ -0.40, & |x_2(\cdot)| \leq 0.02, \end{cases}$$

$$b_{22}(x_2(\cdot)) = \begin{cases} -0.30, & |x_2(\cdot)| > 0.02, \\ 0.10, & |x_2(\cdot)| \leq 0.02, \end{cases}$$

$$\Delta D(k) = \begin{bmatrix} 0.06\sin(0.6k) & 0 \\ 0 & 0.06\sin(0.6k) \end{bmatrix},$$

$$\Delta A(k) = \begin{bmatrix} 0.09\sin(0.8k) & 0.18\sin(0.8k) \\ 0.04\sin(0.8k) & 0.22\sin(0.8k) \end{bmatrix},$$

$$\Delta B(k) = \begin{bmatrix} 0.09\cos(0.5k) & 0.03\cos(0.5k) \\ 0.09\cos(0.5k) & 0.12\cos(0.5k) \end{bmatrix},$$

$$C = \begin{bmatrix} 0.10 & 0.20 \\ 0.20 & 0.30 \end{bmatrix}, \quad N = \begin{bmatrix} 0.10 & 0 \\ 0 & 0.20 \end{bmatrix},$$

$$L = \begin{bmatrix} 0.08 & 0 \\ 0 & 0.15 \end{bmatrix}, \quad M = \begin{bmatrix} 0.35 & 0.30 \end{bmatrix}.$$

The activation functions $f(x(k))$ and $g(x(k))$ are chosen as

$$f(x(k)) = \begin{bmatrix} 0.10x_1(k) - \tanh(0.40x_1(k)) \\ \tanh(0.50x_2(k)) \end{bmatrix},$$

$$g(x(k)) = \begin{bmatrix} \tanh(0.10x_1(k)) \\ 0.02x_2(k) - 0.06\tanh(x_2(k)) \end{bmatrix},$$

which satisfy the constraint (2) with

$$\Lambda_1 = \begin{bmatrix} -0.30 & 0 \\ 0 & 0 \end{bmatrix}, \quad \Lambda_2 = \begin{bmatrix} 0.10 & 0 \\ 0 & 0.50 \end{bmatrix},$$

$$\Upsilon_1 = \begin{bmatrix} 0 & 0 \\ 0 & -0.04 \end{bmatrix}, \quad \Upsilon_2 = \begin{bmatrix} 0.10 & 0 \\ 0 & 0.02 \end{bmatrix}.$$

In the example, the probability is taken as $\bar{\alpha} = 0.85$, the disturbance attenuation level is chosen as $\gamma = 0.95$, constant scalars $\rho_1 = \rho_2 = 0.25$, and the time-varying delays are set as $\tau(k) = 3 - (\sin(k\pi))^2$. Then, it can be verified that the upper bound and the lower bound of the time-varying delays are $\tau_M = 4$ and $\tau_m = 2$, respectively.

By solving the LMI (36) in Theorem 3 with the help of Matlab toolbox, we can obtain matrices P_2 and X as follows:

$$P_2 = \begin{bmatrix} 2.2358 & 0.0739 \\ 0.0739 & 0.9423 \end{bmatrix}, \quad X = \begin{bmatrix} 0.1291 & 0.3168 \\ 0.5607 & 0.6733 \end{bmatrix}$$

and then, according to $K = P_2^{-1}X$, the desired estimator parameter is designed as

$$K = \begin{bmatrix} 0.0382 & 0.1184 \\ 0.5920 & 0.7052 \end{bmatrix}.$$

In the simulation, the external disturbance inputs are assumed to be $\varsigma_1(k) = \varsigma_2(k) = \xi_1(k) = \xi_2(k) = 3\exp(-0.30k) \times \cos(0.20k)$. Simulation results are shown in Figs. 1-4. Figs. 1 and 2 plot the state and its estimate for node 1 and node 2, respectively. The estimation errors for node 1 and node 2 are presented in Fig. 3. The event-based release instants and release interval of the proposed event-triggered scheme are displayed in Fig. 4. The simulation result has confirmed the effectiveness of the estimation scheme presented in this paper.

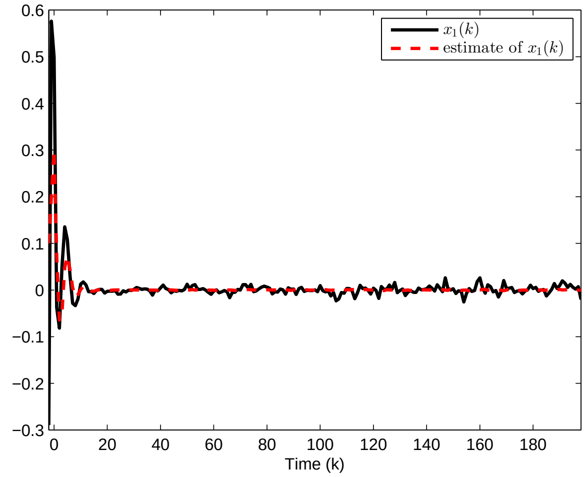

Fig. 1. The state and its estimate of node 1.

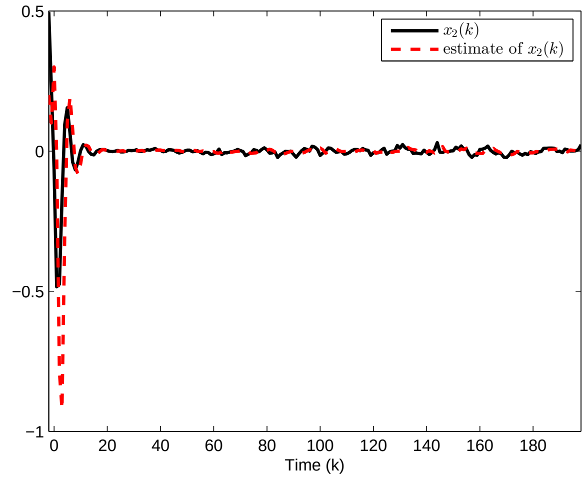

Fig. 2. The state and its estimate of node 2.

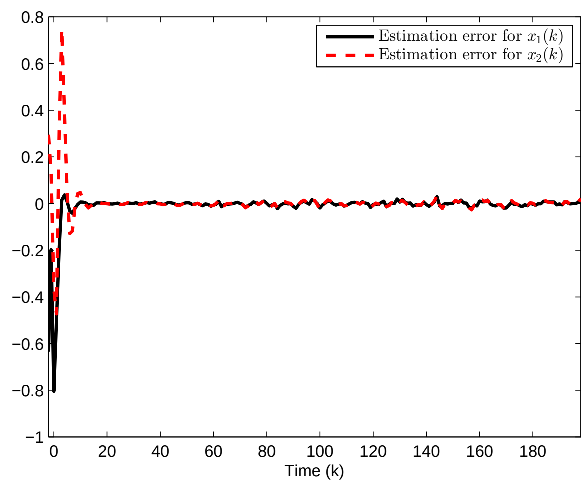

Fig. 3. Estimation error of node 1 and nod 2.

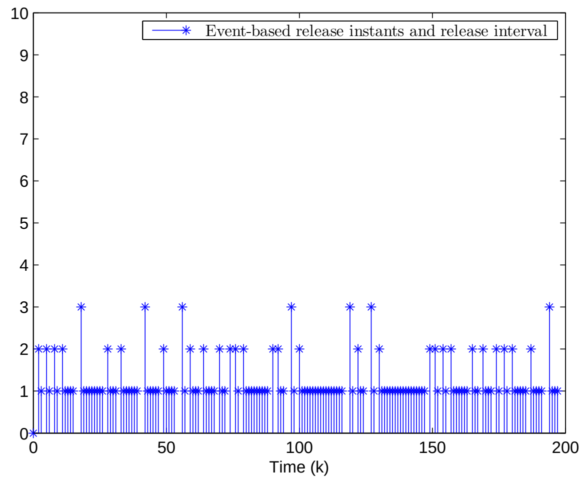

Fig. 4. Event-based release instants and release interval.

V. Conclusions

In this paper, we have investigated the event-triggered state estimation problem for a class of discrete-time stochastic memristive neural networks with time-varying delays and randomly occurring missing measurements. In the model of measurement output, a stochastic variable according to the Bernoulli distribution has been introduced to characterize the randomly occurring missing measurements. Based on the state-dependent future of memristive neural networks, by utilizing a Lyapunov-Krasovskii functional and stochastic analysis techniques, an event-triggered state estimator is designed and sufficient conditions are given to ensure both the exponential mean-square stability of the output estimation error dynamics and the prescribed H_∞ performance requirement. Based on the derived sufficient conditions, the explicit expression of the desired estimator gain has been given. Finally, a numerical example has been provided to show the usefulness and effectiveness of the proposed estimator design method. Further research topics include the extension of the main results to more complex systems with more complicated network-induced phenomena, see e.g. [3], [5], [7], [18], [20], [23], [24], [38], [42], [44].

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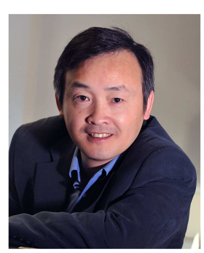

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