Are Short-Term Effects Of Pollution Important For Growth and Optimal Fiscal Policy?

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Abstract

We study optimal fiscal policy in a stock-flow model of the environment within an endogenous growth framework; where some pollutants have a lasting impact on environmental quality which is restored through abatement expenditure, while others dissipate and hence, have a short-term effect on the environment. All pollutants, however, affect the productivity of a public good negatively. Given that short-term pollution, although it dissipates, is irreversible in this sense, a government cannot ignore its negative effects since this type of pollution lowers the productivity of all inputs. We find that a larger negative effect of short-term pollutants as well as a higher congestion effect of private capital leads to corrective fiscal policies with higher optimal income

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tax and abatement expenditure rates, which have favourable growth consequences. Interestingly, we find that the rate of short-term pollution does not affect optimal fiscal policy while that of the long-term pollution does.

Keywords: Stock and flow effects of pollutants, Short-term effects and long-term effects of pollution, Environmental quality, Public goods, Endogenous growth, Optimal fiscal policy, Abatement expenditure

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1 INTRODUCTION

Future generations would be worse off if unsustainable consumption and investment patterns of today translate after a time lag into environmental disruption, as Smulders (1995) points out. The challenge, therefore, is to achieve sustained economic growth with maximum achievable social welfare levels preserving the quality of the environment as much as possible. Since fiscal policy can be used to attain such objectives, it is appropriate to understand the nature of taxation and expenditure policies that could effectively counter environmental pollution while preserving growth prospects. In this paper, we study two different kinds of impact of environmental pollution on optimal fiscal policies. Some types of pollution generate flow effects while some other types generate long-term effects on environmental quality which accumulate over time like a stock variable. To the best of our knowledge, the dissipating and the lasting effects of pollution have not been studied simultaneously in the dynamic endogenous growth models available in the existing environment-growth literature.

An important issue in this context is the distinction not only between stocks and flows but also between long-term (permanent) effects and short-term (dissipating) effects.[4] Some pollutants, or part of them, dissipate or are

[4]The stock-flow distinction mentioned here in the context of environment as a public good is different from that made in the context of public goods, in general. In the context of public goods it involves the long-term goal of the accumulation of public capital - e.g., building of infrastructure networks (stocks) - and the short term need to provide public services – e.g., maintenance of such networks (flows). And governments routinely have to face trade-offs between these two objectives. A model of public goods having both a stock as well as a flow component is captured by Ghosh and Roy (2004). However, even the short-term maintenance expenditure made in case of other public good stocks is in order

absorbed by the environment. So, they do not stay on in the environment long enough to have lasting impacts on it. On the other hand, those pollutants which stay on and have negative effects on the environment affect the stock of environmental quality. It is well known why abatement activities are necessary to curb long-term pollution. However, what is less obvious, and therefore, required, is to examine the need to control short-term pollutants. Even if they dissipate, it may be necessary to control short-term pollutants because these pollutants can also produce substantial negative effects on productivity.

Environmental pollution affects the productivity of inputs in various ways. In particular, we follow Ray Barman and Gupta (hereafter called RBG) (2010) to model the way the services from public infrastructure are adversely affected by pollution, and the manner in which the productivity of private capital is affected in the process. One could think of a public good like a highway being corroded by acid rain, which is caused by the prolonged presence of atmospheric pollutants like sulphur dioxide, nitrous oxide, etc.; or, in other words, by the degradation of the stock of environmental quality as a whole which causes the acid rains. This could result in potholes slowing movement of traffic and can cause hazards like accidents too. Pollutants causing acid rain are instrumental in corroding not only roads, but also buildings and other physical infrastructure. This damage may be prevented by maintaining the stock of environmental quality if adequate expenditure

to keep the stock of public good in running condition so that the flow of services from it continue to flow unabated. So, whether it is augmenting of the stock or maintaining it, the stock of public good is affected. This is not so with short-term pollution, which is part of total flow of emissions, as we model in our paper here. It does not affect the stock of environmental quality and yet has productivity effects, independently.

is made on abatement.

This highway example can also be used to capture the short-term effects of pollution in the production function. For example, vehicle drivers on the highways are slowed down because of drop in visibility caused by the presence of smog. However, the smog usually disappears after a point in time, and the driver can revert to their original speed. This is an example[5] of the short-term effect of certain pollutants that affects efficiency for some time.

We attempt to capture the simultaneous impact of stock and flow effects of pollutants on growth in a unified framework. Lieb (2004) also considers the simultaneous occurence of stock and flow pollution through their effects of current and future damages respectively. However, he consider both types of pollution as externalities to the consumer and show the interdependence of the two types for the inverted-U shape of the EKC. Van der Ploeg and Withagen (1991) consider both stock and flow effects of pollution, but not within the same model while Bosi et al. (2018) consider the stock effects of pollution on utility, but without any public good in their model.[6] We include both types of effects of pollution as bad inputs in the production function,

[5] Other examples of flow effects of pollutants can be provided. Biodegradable waste and sewage in water makes it unfit for use in agriculture or factories. Depletion of nitrogenous compounds due to intensive cropping, though eventually replenished by nature, affects current harvest. Brick kilns cannot use clay polluted by sewage waste. These are examples of short-term pollution affecting productivity. On the other hand, long-term pollution like soil quality degradation due to the presence of lead, the chemical effluents from one industry impacting another industry using the same water source, etc., clearly have long-term negative impact on industrial production.

[6] Also see Bouche and Miguel (2019) where environmental quality is treated as a stock, but is not an externality to the consumer.

but include only the long-term effects of pollution in the household's utility function. This is because, the quality of life in general, is influenced more by environmental quality as a whole than by short-term pollutants which dissipate. In the context of the effects on health, people often develop immunity to pollutants which disappear after a while but develop symptoms when they stay on and accumulate over time. So, the nature of distinction we make between stock and flow effect of pollutants is different from that in Van der Ploeg and Withagen's (1991) paper. On the other hand, although Lieb's (2004) treatment of stock and flow pollution is similar to our paper he treats pollution to be affecting only the consumer's utility with no productivity impacts. In our model, the flow effects of pollutants do not affect consumer's utility directly, but lowers productivity and thus welfare too, indirectly. Thus, the flow effect of pollution may cause optimal fiscal policy in this paper to be distinct from other papers.

The increase in abatement expenditure can counter the long-term effects of pollution and thus can maintain the stock of environmental quality. This stock affects the public good by improving the benefits derived from it. Hence, there is need to account for abatement of pollution in government's fiscal policy without compromising sustainability of the environment and our model focuses on this aspect too. Although we treat public good as a flow of services, as in Barro (1990), the presence of environmental quality as a stock variable in the production and utility functions ensures that the model exhibits transitional dynamics. We solve for the optimum income tax rate and abatement expenditure rate maximizing the balanced rate of growth in the steady-state equilibrium; and they appear to be the same as their respective welfare-maximizing values at the steady-state equilibrium.

The salient features of our model can be compared and contrasted with other important papers in the literature. Unlike in our model, productive public expenditure is treated as a stock variable in Futagami et al.(1993) and in Greiner (2005). We follow Greiner (2005), as well as Economides and Philippopoulos (2008), to assume that the level of production is the source of pollution. Byrne (1997) finds out polluting inputs; and assumes that technology and/or human capital is non-polluting, but increases in labour and capital do cause emissions to increase. Lieb (2004) also treats pollution to be proportional to capital used in the productive activities.[7]

Greiner (2005), Economides and Philippopoulos (2008) and Lieb (2004) introduce a negative external effect of environmental pollution only on the utility function. Our model deals with this effect on production as well as on utility. We consider the negative effect on the effective benefit derived from the public good used in production. In Greiner (2005), the pollutant is purely a flow variable, while Economides and Philippopoulos (2008) model environment as a stock.

In contrast to our model, the rate of abatement expenditure is treated as exogenous in Greiner (2005), and properties of optimal income tax and pollution tax policies are analyzed. We do not consider a separate pollution tax here, but make the allocation of income tax revenue between productive public expenditure and abatement expenditure endogenous to the analysis, as in Economides and Philippopoulos (2008). Like the latter, Ligthart and van der Ploeg (1994) analyze a second-best world in which the government employs a distortionary tax to finance public consumption and to internalize environmental externalities.

Like Economides and Philippopoulos (2008), we analyse the Ramsey

[7] Also see Bontems and Gozlan (2018) where they consider both an income tax and a production subsidy, unlike that in our model.

optimal (second-best) fiscal policy in a set-up where environmental quality is degraded by private economic activity and is upgraded by public policy. In both models, the Ramsey-optimising government can lead the economy to sustainable balanced growth in which the economy is capable of long-term growth without damaging the environment. In Lieb (2004), the government is myopic and maximizes the welfare of only the old of any time period in its overlapping-generations framework, which is why it ends up with a trade-off between the abatements of stock and flow pollution and the EKC curve results.

In Economides and Philippopoulos (2008), the more the representative consumer cares about the environment, the more growth-enhancing policies the Ramsey government finds it optimal to choose. In Byrne (1997), policies that aim at a zero rate of growth of output may increase the rate of environmental degradation. In a second-best optimum solution, it appears that extra revenue required for abatement can only be achieved by large tax bases and consequently by high growth rate. In our model, however, the consumer's valuation of environmental quality is not important for optimal fiscal policy as the growth rate maximizing fiscal policy already ensures maximum maintenance and improvement of this good whenever output is maximized. Moreover, the presence of flow effects of pollution reduces the optimum share of expenditure on the public good because optimal fiscal policy now takes into account the negative effects of both types of pollution on period output.

The rest of the paper is structured as follows. Section 2 describes the model. Section 3 derives the decentralized equilibrium, depicts the steady state, and characterizes the optimal fiscal policy. In Section 4 we analyse the effects of changes in the parameters on the optimal income tax rate, abatement expenditure rate and also on the growth rate. Section 5 outlines the dynamics in the neighbourhood of the steady-state. Concluding remarks are made in section 6.

2 THE MODEL

The production function given by

$$Y = K^{\alpha}\hat{G}^{1-\alpha} \tag{1}$$

is Cobb-Douglas and satisfies constant returns-to-scale. Output (Y) is a function of private capital (K) and a derivative input, $\hat{G}$ of the public good (G) which is modeled as a flow of services[8] like Barro (1990). What enters in the production function is a composite input, $\hat{G}$, which represents the effective benefit derived from the use of the public good. The magnitude of the pure public good (G) diverges from its effective benefit due to the presence of congestion effect generated by private capital together with the direct and indirect effects of pollution. For simplicity, the labour endowment is normalized to unity.

Equation (2) describes the effective benefit function of the public good and it is given by

$$\hat{G} = G\bar{K}^{-\theta}E_s^{(\theta+\phi)}E_f^{-\phi};\ \theta,\ \phi > 0. \tag{2}$$

More specifically, it shows that the effective production benefit of the public good varies negatively with the congestion generated by average capital stock $\bar{K}$ as well as by the short-term pollutants (E_f), and varies positively with the environmental quality, E_s. Hence, θ and ϕ represent absolute

[8]We consider a model with public good as a stock and analyze it in Appendix A.3 following one of the reviewers' suggestion.

values of elasticities of effective benefit with respect to the average capital stock and short-term pollutants respectively. Also, $(\theta + \phi)$ represents the corresponding elasticity with respect to environmental quality. Here, long-term pollutants indirectly affect the effective benefit of public good through their effects on environmental quality. Our specification of the $\hat{G}$ function given by equation (2) implies that the effective benefit remains unchanged as long as the ratio of environmental quality to average capital and that of environmental quality to short term pollutants remain constant. So, equation (2) shows that $\frac{\hat{G}}{G}$ is homogeneous of degree zero in $\bar{K}$, E_s and E_f. The justification for the congestion effect of the average capital stock is available in the existing literature[9]. We assume that the positive technological contribution of private capital outweighs the negative congestion effect of the average capital stock. This implies, $\alpha - \theta(1-\alpha) > 0$.[10] The CRS assumption always ensures the existence of a steady-state equilibrium.

The composite input $\hat{G}$ is different from that in RBG (2010) because it now captures both a direct effect of pollution, through the impact of short-

[9]See, for example, Raurich-Puigdevall (2000), Turnovsky (1996b, 1997), and Eicher and Turnovsky (2000), among others.

[10]Our model may be viewed as highly stylized due to our assumptions of the Cobb-Douglas form adopted both in the production function as well as in the effective benefit function for the public infrastructure. It may be noted that much of the literature on endogenous growth models is based on the assumption of a Cobb-Douglas production function. A few models which focus on the problem of congestion effect of public input also assume Cobb-Douglas form in the effective benefit function of the public input. In the Cobb-Douglas form all elasticities are parameters and so the rate of growth derived in terms of the parameters is a constant in the steady-state equilibrium. This advantage is lost with a production technology different than Cobb-Douglas. We attempt to conduct a robustness check employing a CES production function in Appendix A.5. We do this following the suggestion of the reviewer.

term pollutants, and an indirect effect through long-term impact on environmental quality. Anthropogenic activities, like deforestation, which are not necessarily pollution generating in the short run, however, deteriorate the stock of environmental quality and these activities can be reasonably assumed to be proportional to the scale of the economy as can be short-term pollutants. Thus, the impact of all such anthropogenic activities including long-term effects of emissions is modeled as δY where δ represents the pollution-output coefficient. Examples of long-term pollution impacting the public good indirectly are the presence of air, water and soil pollution; and these put strain on public health expenditure programmes because they may generate diseases. On the other hand, air, water and soil pollution generate short-term pollution such as that from sewage waste. However, corrective action in such cases is not necessary as the effects dissipate with time. Anthropogenic activities are bound to generate some negative influences given the modern industrial mode of production. These influences get generated as an inseparable by-product of modern economic activity which can exert a negative influence on the economic activity itself. To capture this inseparability of the anthropogenic influences on the production process, we consider imperfect substitution of short-term pollution, E_f with the environmental quality, E_s.[11]

Equation (3) shows the components of total pollution, P, generated from production; and equation (4) specifically describes the transitory component

[11] Alternatively, we can model the flow effects of pollution as $\hat{G} = G\bar{K}^{-\theta}E_s^{\theta} - \eta Y$. This would, however, imply that such pollution effects are reversible, which is not the idea in our paper.

of total pollution. These two are given by

$$P = \eta Y + \delta Y; \tag{3}$$

and

$$E_f = \eta Y. \tag{4}$$

η is the short-term pollution rate per unit of output while δ is the long-term pollution rate. δY is the component of total pollution that does permanent damage to environmental quality. The transient component, ηY, does not cause any permanent damage to the stock of environmental quality but affects the flow of benefits derived from public good. Both types of pollution depend on the scale of the economy. Therefore, as the production possibilities of an economy expand, both permanent and transitory components of pollution increase. For instance, as an economy expands, increased vehicular traffic on the motorway and the resulting spike in pollution cause increased long-term damage to roads and result in accidents. At the same time, increased instances where smog slows down traffic temporarily but dissipates after a while, increase too. The treatment of environmental pollution varying positively with the scale of the economy, is consistent with the rising part of the Environmental Kuznets Curve.

The government budget constraint is given by

$$G = (\tau - g_a)Y. \tag{5}$$

The government finances its public expenditure through tax revenues after meeting the abatement expenditure to upgrade environmental quality. It imposes a proportional income tax, with τ being the income tax rate, while g_a is the ratio of abatement expenditure to income and is called the abatement expenditure rate.

The budget of the government is always balanced; and there does not exist any exogenous shock disturbing this balanced budget condition.[12]

The quality of the environmental stock evolves over time, depending upon the magnitudes of long-term pollution effects and abatement activity. The relationship is shown by

$$\dot{E}_s = (g_a - \delta)Y. \tag{6}$$

Equation (6) shows that the net change in environmental quality is represented by the difference between abatement[13] expenditure and the damaging capacity of long-term pollutants.[14]

[12]This is a deterministic model, and the government determines the optimal τ and g_a when solving a deterministic optimisation problem. Introduction of stochastic shocks generates cyclical fluctuations in the economy causing a deviation from the steady-state growth equilibrium. This may generate fluctuations in the tax rate as well as in the abatement expenditure rate. In a model with stochastic shocks to productivity, Turnovsky (1999) shows how the presence of risk dramatically changes some of the familiar Barro (1990) model propositions; for example, both the welfare-maximizing and growth rate-maximizing (deterministic) rates of government expenditure are affected differentially by the presence of risk, and so the coincidence of these two optimal quantities in the deterministic growth model does not generalize to the corresponding stochastic model. See also Devereux and Smith (1994) for the effects of international risk sharing in a world economy where (endogenous) growth is driven by human capital accumulation. However, in our paper we want to restrict our analysis within the world of steady-state equilibria in the absence of stochastic shocks.

[13]On the issue of abatement, see Smulders and Gradus (1996), Byrne (1997), Managi (2006), among others.

[14]Here, for $g_a - \delta > 0$ stock of environmental quality can grow forever. This specification is used by Economides and Philippopoulos (2008), Roseta-Palma *et al.* (2010) and Brechet *et al.* (2013). For a virgin and pristine environmental quality it is hard to imagine environmental quality to be improving without bounds due to anthropogenic conservation activities. But as a renewable resource, innovation can help its effective supply grow in size.

The pollutants referred here are heavy metal pollutants like lead and mercury; and they have a lasting negative impact on the environmental quality. So the government needs to undertake abatement activities to maintain the environmental quality, which in turn, enhances the effective benefit derived from public infrastructure. However, the flow effect of pollution does not persist for long; and so, abatement does not have a role to reduce it. But, as described earlier, this type of pollution has a deteriorating effect on the effective use of public infrastructure. For example, the depletion of soil nutrients due to intensive farming practices does not affect environmental quality in the long run but adversely affects agricultural productivity.

The utility function of the household is given by

$$U = \frac{(C^{\gamma} {E_s}^{\epsilon})^{1-\sigma}}{1-\sigma}; \quad 0 < \gamma, \epsilon, (\gamma + \epsilon) < 1, \ \ \sigma > 0. \tag{7}$$

The σ parameter represents the constant elasticity of marginal utility with respect to the composite argument $C^{\gamma} {E_s}^{\epsilon}$. The utility is a positive function of the level of consumption as well as of the stock of environmental quality; and these two arguments are multiplicatively linked. Thus, only those anthropogenic activities which degrade environmental quality affect utility, and not those which dissipate in the form of short-term pollutants. $\gamma\{(1-\sigma)-1\}$ and $\epsilon\{(1-\sigma)-1\}$ are elasticities of marginal utilities with respect to consumption and environmental quality respectively.[15]

The innovations could be in the form of discovery of new sources through explorations, efficient and recyclable usage of existing supplies and invention of new substitutes.

[15]Following one of the reviewers' suggestion, we also consider a model with the public good in the utility function, instead of the environmental quality, in Appendix A.4. In line with Chatterjee and Ghosh (2011) and others, the nature of fiscal policy is hereby investigated when the public good performs the dual role of providing both utility and productive services.

The budget constraint of the household is given by

$$\dot{K} = (1-\tau)Y - C. \tag{8}$$

This equation shows how the post-tax disposable income is allocated between consumption, C, and investment when there is no depreciation of capital stock.

3 THE DECENTRALIZED EQUILIBRIUM

3.1 Steady-State Growth

The representative household chooses C to maximize the discounted present value of utility over the infinite time horizon. The utility function is given by equation (7) and the maximisation takes place subject to the consumer's budget constraint given by equation (8).

The first-order conditions of optimization with respect to C and K are given by (9) and (10) respectively:

$$\{\gamma(1-\sigma)-1\}\frac{\dot{C}}{C} + \epsilon(1-\sigma)\frac{\dot{E}_s}{E_s} = \frac{\dot{\lambda}_K}{\lambda_K}; \tag{9}$$

and

$$\begin{aligned}\alpha(1-\tau)(\frac{Y}{K}) &= \alpha(1-\tau)\eta^{-\frac{\phi(1-\alpha)}{\Delta}}(\tau - g_a)^{\frac{1-\alpha}{\Delta}}K^{\frac{\alpha-\theta(1-\alpha)-\Delta}{\Delta}}E_s^{\frac{\omega}{\Delta}} \\ &= \rho - \frac{\dot{\lambda}_K}{\lambda_K}.\end{aligned} \tag{10}$$

Here, $\Delta = \alpha + \phi(1-\alpha)$,

and $\omega = (\theta + \phi)(1-\alpha)$.

We consider the steady-state growth equilibrium of the market economy, where all macroeconomic variables grow at the same rate, Γ_m. Hence, we have

$$\frac{\dot{C}}{C} = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{G}}{G} = \frac{\dot{E}_s}{E_s} = \frac{\dot{E}_f}{E_f} = \Gamma_m. \tag{11}$$

Combining equations (9) and (10) and then using equations (6), (8) and (11), we obtain the following equation.[16]

$$\Gamma_m^{\omega}(\rho + \psi\Gamma_m)^{\alpha-\theta(1-\alpha)} = \eta^{-\phi(1-\alpha)}\{\alpha(1-\tau)\}^{\alpha-\theta(1-\alpha)}(g_a - \delta)^{\omega}(\tau - g_a)^{1-\alpha}. \tag{12}$$

Here, $\psi \equiv 1 - (\gamma + \epsilon)(1 - \sigma)$.

In order to ensure the existence and uniqueness of equilibrium we assume that $\psi > 0$. A sufficient condition for this to hold is given by $\sigma > 1$. This is so because, $\gamma + \epsilon < 1$.

Here, the rate of short-term pollution, η, has a scale effect on the balanced growth rate, but the rate of long-term pollution, δ, does not.
The left-hand-side (LHS) of equation (12) is an increasing function of Γ_m and its right-hand-side (RHS) is a constant term, given the income tax rate and the abatement expenditure rate. So, clearly, there exists one unique value of Γ_m, i.e., of the steady-state equilibrium growth rate in the market economy, given the income tax rate and the abatement expenditure rate.

3.2 Social Welfare

The social welfare function is given by:

$$W = \int_0^{\infty} \frac{(C^{\gamma} E_s^{\epsilon})^{1-\sigma}}{1-\sigma} e^{-\rho t} dt. \tag{13}$$

[16] The derivation of equation (12) is shown in Appendix A.1.

It can be shown that[17] for $\Gamma_m < \frac{\rho}{1-\psi}$ for finite value of welfare

$$W = [\frac{\{K(0)\}^{\gamma(1-\sigma)}\{E_s(0)\}^{\epsilon(1-\sigma)}}{\alpha^{\gamma(1-\sigma)}(1-\sigma)}]\{\rho + (\psi - \alpha)\Gamma_m\}^{\gamma(1-\sigma)}\{\rho - (1-\psi)\Gamma_m\}^{-1}. \quad (14)$$

Clearly, the level of social welfare, W, varies positively with the growth rate, Γ_m, for $\psi > \alpha$ which is a sufficient condition for consumption, C, to be positive[18]. Thus the level of social welfare in the steady-state equilibrium of the decentralized economy is maximized whenever the steady-state equilibrium growth rate is maximized. Output growth rate maximization also maximizes welfare in the steady-state equilibrium, despite the presence of an externality effect of environmental quality in the utility function. If not obvious, this result is not, however, counter-intuitive. Balanced growth rate is determined by the growth rates of the accumulable inputs as given by equation (11). Environmental quality is also an accumulable productive input; and this rate of accumulation is internalized in the steady-state equilibrium. So, maximization of the growth rate in the steady-state equilibrium implies maximization of the environmental quality itself.

3.3 Optimal Taxation

In the decentralized economy, the government maximizes the steady-state equilibrium growth rate with respect to the fiscal instruments, τ and g_a.

Maximizing the RHS of equation (12) with respect to τ and g_a respectively, we obtain the following expressions for the optimum tax rate and the optimum abatement expenditure rate in the steady-state growth equi-

[17]The derivation of equation (14) is shown in Appendix A.1.

[18]This is shown in Appendix A.1.

librium:[19][20]

$$\tau^* = \frac{(1-\delta)(1-\alpha)(1+\theta+\phi)}{1+\phi(1-\alpha)} + \delta; \tag{15}$$

$$g_a{}^* = \frac{(1-\delta)(1-\alpha)(\theta+\phi)}{1+\phi(1-\alpha)} + \delta. \tag{16}$$

The optimum ratio of expenditure on the public input to national income is given by:

$$\tau^* - g_a{}^* = \frac{(1-\delta)(1-\alpha)}{1+\phi(1-\alpha)}. \tag{17}$$

Here, $(1-\delta)$ is the fraction of output obtained after adjusting for the stock effect of pollution. Using equations (1), (2) and (3) we obtain

$$Y = \eta^{\frac{-\phi(1-\alpha)}{1+\phi(1-\alpha)}} K^{\frac{\alpha-\theta(1-\alpha)}{1+\phi(1-\alpha)}} G^{\frac{1-\alpha}{1+\phi(1-\alpha)}} E_s{}^{\frac{(\theta+\phi)(1-\alpha)}{1+\phi(1-\alpha)}}. \tag{18}$$

Here, the presence of short-term pollutants, E_f, effectively lowers the productivity of all other inputs by a fraction $\Delta' = 1+\phi(1-\alpha)$ and therefore, the elasticities of output with respect to congestion adjusted-physical capital, public intermediate input and environmental quality are now $\frac{\alpha-\theta(1-\alpha)}{1+\phi(1-\alpha)} < \alpha - \theta(1-\alpha)$, $\frac{1-\alpha}{1+\phi(1-\alpha)} < 1-\alpha$ and $\frac{(\theta+\phi)(1-\alpha)}{1+\phi(1-\alpha)} < (\theta+\phi)(1-\alpha)$. Hence, optimal allocation of tax revenue must take this into account.

Thus, optimal taxation rule clearly shows the short-term effect of pollution being accounted for within the tax policy when either welfare or steady-state growth rate is maximized. This is so because short-term pollutants affect current output through productive public good and in turn affect

[19]The LHS of equation (12) is a monotonically increasing function of Γ_m because, by assumption $\alpha-\theta(1-\alpha) > 0$. Since the LHS is always equal to the RHS in the steady-state growth equilibrium, maximization of Γ_m implies maximization of the RHS of equation (12).

[20]The derivation of equations (15) and (16) is shown in Appendix A.1.

current fiscal expenditure. Current fiscal expenditure impacts future output through abatement expenditure. Hence, in order to ensure that future clean-up activities of long-term pollution damages are continued for the sake of welfare maximization, correct valuation of current output is necessary after taking the productivity dampening effect of short-term pollution into consideration. So, irreversible short-term pollution cannot be ignored because it affects optimal abatement for reversible long term pollution damages.

The RHS of equation (17) is the competitive share of the public good in the net unpolluted output of the final good in the steady-state growth equilibrium. This share varies inversely with the long-term pollution-output coefficient, δ, as well as with the elasticity of effective benefit of public input with respect to short-term pollutant, ϕ. In Barro (1990) and in Futagami et al.(1993), we have $\delta = \phi = 0$. So the entire production is pollution free; and hence, ${g_a}^* = 0$. Hence, $(\tau^* - {g_a}^*) = \tau^*$ takes a higher value in their models. This is obvious because in the present model, production generates environmental pollution, both long-term and short-term; and these in turn, lower the effective benefit derived from public expenditure. On the contrary, in RBG (2010), pollution affects only environmental stock, and therefore, optimal public expenditure-output ratio is higher than what we find here, but lower than that found in Barro (1990) or Futagami *et al.* (1993).

Here, the optimal abatement expenditure rate and the optimal tax rate take the effects of short-term pollutants into account because these pollutants lower the effective benefit of public input.

On the other hand, we have $\tau^* > 1 - \alpha$ in this model, given $0 < \delta < 1$, $0 < \eta < 1$ and $\theta,\ \phi > 0$. This means that the optimum income tax rate in this model is higher than that in the Barro (1990) model and in the Futagami *et al.* (1993) model. This is so because income tax is the only source of revenue in this model and a part of the income tax revenue is used to finance abatement expenditure[21].

Equations (15) and (16) show that η does not affect optimal fiscal policy, while δ does. Short-term pollution does not affect environmental quality, and hence policy adoption to negate any damage does not arise. δ enters here through the optimal abatement expenditure rate. Since long-term pollution is reversible and δ fraction of total output is polluted, the rest, $(1-\delta)$ fraction is available for government expenditure.

With $\phi > 0$, the values of τ^* and ${g_a}^*$ obtained in the present model are larger than those obtained in RBG (2010).[22] It can easily be shown that

$$\tau^* - \tau_{(RBG)} = \frac{(1-\delta)\phi(1-\alpha)\{1-(1+\theta)(1-\alpha)\}}{1+\phi(1-\alpha)} > 0; \tag{19}$$

and,

$${g_a}^* - g_{a(RBG)} = \frac{(1-\delta)\phi(1-\alpha)\{1-\theta(1-\alpha)\}}{1+\phi(1-\alpha)} > 0. \tag{20}$$

There are two types of effects of pollutants and both of them affect the effective benefit of public good. This requires more abatement expenditure. In RBG (2010), there is no flow effect of pollutants and also environmental quality does not affect utility of the household. So, government has to spend less for abatement and hence, less tax revenue per unit of income is required.

However, the opposite is true for $(\tau^* - {g_a}^*)$. The optimum ratio of productive public expenditure to national income, (G/Y), in this model is

[21] Had we considered an alternative instrument for financing abatement expenditure (e.g., the pollution tax) as in Greiner (2005), this need not necessarily have been the case. We do not consider a separate pollution tax because the level of production (income) is the only source of pollution.

[22] In RBG (2010), $\tau_{(RBG)} = 1-(1-\delta)\{\alpha-\theta(1-\alpha)\}$ and $g_{a(RBG)} = \delta+(1-\delta)\theta(1-\alpha)$.

smaller than that in RBG (2010) because the magnitude of negative congestion effect on public expenditure in this model is stronger than that in RBG (2010) model due to the presence of the negative effect of short-term pollutants.[23]

The results of this section can be summarized in the following proposition.

PROPOSITION 1: (i) Short-term pollution rate has an irreversible negative scale effect on the level of output but has no effect on optimal fiscal policy. (ii) The optimum values of income tax rate, abatement expenditure rate and public expenditure rate vary negatively with the elasticity of effective benefit of public good with respect to short-term pollution. (iii) The optimum values of income tax rate, abatement expenditure rate and public expenditure rate vary negatively with the long-term pollution rate.

4 CHANGES IN PARAMETERS

4.1 Effects on the Optimal Fiscal Policies

In this section, we evaluate the effects of changes in parameters, θ, ϕ and δ on the policy variables, τ^* and ${g_a}^*$.

We find, from equations (15) and (16), that

$$\frac{d\tau^*}{d\theta} = \frac{(1-\delta)(1-\alpha)}{1+\phi(1-\alpha)} > 0; \tag{21}$$

$$\frac{d{g_a}^*}{d\theta} = \frac{(1-\delta)(1-\alpha)}{1+\phi(1-\alpha)} > 0; \tag{22}$$

[23] We show in Appendices A.3 and A.4, respectively, that the optimal fiscal instrument rates do not change when we separately consider the models where public good is an accumulable input and when it affects utility instead of environmental quality.

and

$$\frac{d(\tau^* - g_a^*)}{d\theta} = 0. \tag{23}$$

These three equations imply that τ^* as well as ${g_a}^*$ vary positively with θ by the same proportion. A higher value of θ implies a larger relative contribution of capital stock to generate a negative congestion effect of public input. However, this problem can be overcome through an improvement in the environmental quality; and this requires a greater abatement expenditure per unit of output. The income tax rate needs to be increased at the same rate to balance the budget.

We also find that

$$\frac{d\tau^*}{d\phi} = \frac{(1-\delta)(1-\alpha)\{\alpha - \theta(1-\alpha)\}}{\{1+\phi(1-\alpha)\}^2} > 0; \tag{24}$$

$$\frac{d{g_a}^*}{d\phi} = \frac{(1-\delta)(1-\alpha)\{1 - \theta(1-\alpha)\}}{\{1+\phi(1-\alpha)\}^2} > 0; \tag{25}$$

and

$$\frac{d(\tau^* - {g_a}^*)}{d\phi} = -\frac{(1-\delta)(1-\alpha)^2}{\{1+\phi(1-\alpha)\}^2} < 0. \tag{26}$$

In equation (24), $\frac{d\tau^*}{d\phi} > 0$ because $\{\alpha - \theta(1-\alpha)\} > 0$ by assumption. Equation (25) shows that ${g_a}^*$ responds positively to ϕ. This is so because, a higher value of ϕ also raises the effective benefit from public infrastructure per unit of environmental quality; and so the government finds it optimum to raise abatement expenditure to upgrade the environmental quality. However, an increase in ϕ reduces the gap between τ^* and ${g_a}^*$. This means that a shift in budgetary allocation takes place in favour of abatement expenditure when intensity of damage to public infrastructure services due to short-term pollution is increased.

We also find that

$$\frac{d\tau^*}{d\delta} = \frac{\alpha - \theta(1-\alpha)}{1+\phi(1-\alpha)} > 0; \tag{27}$$

$$\frac{d{g_a}^*}{d\delta} = \frac{1 - \theta(1-\alpha)}{1+\phi(1-\alpha)} > 0; \tag{28}$$

and

$$\frac{d(\tau^* - {g_a}^*)}{d\delta} = -\frac{1-\alpha}{1+\phi(1-\alpha)} < 0; \tag{29}$$

In equation (27), $\frac{d\tau^*}{d\delta} > 0$ because $\{\alpha - \theta(1-\alpha)\} > 0$. An increase in the rate of depletion of the stock of environmental quality must be accompanied by an increase in the abatement expenditure rate; and this must raise the income tax rate to balance the government budget. However, the share of productive public expenditure in national income should fall because this increase in δ lowers the marginal contribution of public input on unpolluted output.

We summarize the results in the following proposition.

PROPOSITION 2: (i) An Increase in the degree of congestion effect with respect to capital or with respect to short-term pollution raises the optimal values of income tax rate and abatement expenditure rate. However, the optimal share of public input expenditure remains unchanged in the former case and is decreased in the other case. (ii) An increase in the rate of depletion of the stock of environmental quality raises optimal values of income tax rate and abatement expenditure rate but lowers the output share of expenditure on public input.

4.2 Effects on the Endogenous Growth Rate

In this section, we analyse how Γ_m^* reacts to exogenous changes in various parameters. The growth rate expression given by equation (12) is highly complicated; and so the effects of changes in $\alpha,\ \theta,\ \phi$ on Γ_m^* are computed numerically.[24]

Putting the optimal values τ^* and g_a^* in equation (12) we obtain,

$$\Gamma_m{}^{\omega}(\rho+\psi\Gamma_m)^{\alpha-\theta(1-\alpha)} = \eta^{-\phi(1-\alpha)}(\frac{1-\delta}{\Delta'})^{\Delta'}[\alpha\{\alpha-\theta(1-\alpha)\}]^{\alpha-\theta(1-\alpha)}\omega^{\omega}(1-\alpha)^{1-\alpha}. \tag{30}$$

It can be shown analytically that Γ_m^* always varies inversely with $\eta,\ \delta,\ \sigma$ and ρ.[25] However, Γ_m^* varies inversely (directly) with γ and ϵ when $\sigma > (<)1$. A larger short-term pollution rate has an immediate level effect as to lower the effective benefit of public input. This negatively affects growth.[26] An increase in the rate of decline of environmental quality raises abatement expenditure rate but, as a result, lowers the output share of public input expenditure; and this, in turn, lowers growth rate. Higher γ implies greater weight on consumption; and this leads to lower rate of capital accumulation and thus lower growth rate (for values of $\sigma > 1$). A larger weight on environmental quality in the utility function implies a higher marginal utility of environmental quality. This raises social demand for abatement. So, output share of productive public input expenditures is reduced; and this

[24] We used the software, Mathematica, to solve for Γ_m^* for given benchmark sets of other parameter values; these numerical results are available upon request.

[25] The actual expressions for the changes in the growth rate with respect to these parameters are not provided here, but these are available upon request.

[26] Note that η does not affect optimal τ^* and g_a^*. The reason for this is as follows: The short-term pollutant is transient in its effects; so η does not affect the long-run growth rate of the accumulable inputs like E_s and K. It ultimately enters the production function as a constant term (with a power) that lowers the overall productivity of all the inputs. While determining the optimal fiscal instrument rates, it would make sense for the government to allocate resources according to the respective elasticities of the inputs, because the proportion in which output would ultimately expand (which is what the growth rate is determined by) depends on these elasticities and not on any constant productivity term.

produces a negative effect on growth rate when $\sigma > 1$. Higher rate of time preference (ρ) implies a greater degree of 'impatience' of households. So demand for current consumption goes up and this lowers the rate of capital accumulation and the growth rate.

We cannot analytically find out the effect of changes in θ and ϕ on the growth rate. The numerical simulations show that Γ_m^* rises monotonically with θ and ϕ. A rise in θ means a stronger positive effect of E_s on the effective benefit of G as well as a stronger negative congestion effect of capital, K. Hence, in this case, the government raises the abatement expenditure rate to upgrade the environmental quality.[27] The improvement in environmental quality raises the effective benefit derived from the public good; and this results in a higher growth rate.

For a higher value of ϕ, the effective benefit from G is increased through an improvement of environmental quality and is decreased through short-term pollution. These two conflicting features raise ${g_a}^*$ and lower the gap between τ^* and ${g_a}^*$; and hence the G/Y ratio falls in this case. However, the latter effect is outweighed by the former effect; and so the growth rate goes up.

The growth rate initially falls and then rises with α. Using two sets of parameter values ($\theta = \phi = 0.5,\ \eta = 0.5,\ \delta = 0.2,\ \gamma = 0.8,\ \epsilon = 0.2,\ \sigma = 2,\ \rho = 0.05$) and ($\theta = 0.6,\ \phi = 0.4,\ \eta = 0.3,\ \delta = 0.1,\ \gamma = 0.8,\ \epsilon = 0.2,\ \sigma = 2, \rho = 0.01$) we observe that plotting Γ_m^* against α generates a u-shaped curve. The turning point lies somewhere between 0.625 and 0.65 in the first case and between 0.65 and 0.7 in the second case.

[27]Note that, although τ has to be raised in order to balance the government budget, the gap between τ^* and ${g_a}^*$ (with respect to a change in θ) is 0; hence (G/Y) in equation (5) remains unchanged.

α affects both τ^* and g_a^* negatively. But, the marginal effect of an increase in α is to reduce the gap between τ^* and g_a^*, and hence G/Y becomes smaller. This lowers the growth rate. However, a decrease in τ^* means an increase in the post-tax marginal productivity of capital; and this has a favourable effect on the growth rate. At lower values of α, the negative effect of reduction in G/Y dominates the other positive effect. So, Γ_m^* decreases upto a point; and the relative strength of these two effects gets reversed beyond that point[28].

The optimal growth rate rises monotonically with α in RBG (2010) and there is no downward-sloping stretch. So the non-monotonicity result for Γ_m^* against α in our paper may be attributed to the combined presence of E_f and E_s in the utility function which was not present in RBG (2010). Note that a common feature of both these models is that the positive externalities ultimately dominate the negative externalities. Unlike RBG (2010), in our model, a negative externality from short-term pollution generates an initial downward movement in the growth rate; and initially this negative externality dominates the other before the parameter α increases enough to make the positive externality stronger.

5 TRANSITIONAL DYNAMICS

Equations of motion of the dynamic system are given by (6), (8) and, (9) and (10) combined to give that for consumption, C. We summarize the

[28] It can be checked that if we put $\gamma = 1,\ \epsilon = 0,\ \phi = 0$ and $\eta = 1 (= A)$, then the value for the optimal growth rate in the present paper should be exactly equal to the growth rate obtained in RBG(2010), given the other parameter values. This was verified using the initial benchmark set of other parameter values ($\theta = 0.5,\ \delta = 0.2,\ \sigma = 2,\ \rho = 0.05$): the growth rate turns out to be 0.0294 (i.e., 2.94 percent) in both cases.

transitional dynamics result in the following proposition.[29]

PROPOSITION 3: When $\theta,\ \phi > 0$, the steady-state equilibrium satisfies saddle-point stability.

Figure 1 represents the phase diagrams showing the unique saddle path converging to the steady-state equilibrium in two different cases. In this class of models, if we consider either public expenditure or environmental quality as a stock variable, transitional dynamics will, in general, result. In Barro (1990), where neither of these two features is present the economy jumps from one steady-state to another without any transitional dynamics. In Futagami *et al* (1993), transitional dynamics results come back because public spending is modelled as a stock. We obtain the saddle-point property of long run equilibrium in this model even with a flow of public services as in Barro (1990), because environmental quality is a stock variable in our model. In Greiner (2005), transitional dynamics properties exist because public expenditure is treated as a stock variable while environmental quality is a flow variable.[30],[31] In Economides and Philippopoulos (2008), public service is a flow but environmental quality is a stock.

[29] In Appendix A.2, we derive the transitional dynamic results.

[30] Here, a rise in the income tax rate (from the balanced growth path) leads to a temporary decrease (increase) in the growth rate of consumption and private capital (public capital); and along the transition path, the growth rates of public capital and consumption exceed the growth rate of private capital.

[31] In Ligthart and van der Ploeg (1994), where there is government consumption, and pollution damages enter the utility function as flows, there are no transitional dynamics.

6 CONCLUSION

In this paper we study the nature of optimal fiscal policy in an endogenous growth model, where some pollutants affect the flow of current output, while others impact on future output via the stock of environmental quality. Here output depends on physical capital accumulation as well as on the effective benefit that can be derived from public infrastructure; and this effective benefit depends on the effects of these different pollutants. Although we treat the public good as a flow of services, the presence of environmental quality as a stock variable generates transitional dynamics. The government incurs abatement expenditure to improve the quality of environment. Although short-term pollutants are absorbed by the environment and do not cause any permanent damage to it, they nevertheless affect current output and reduce the effective benefit derived from the public infrastructure, and thereby affect economic growth and welfare adversely. To the best of our knowledge, this is the first paper that simultaneously captures the stock and flow effects of pollutants on endogenous growth and welfare in a unified framework.

Among our results, the following may be worthy of note: first, if there is an increase in the degree of congestion that affects private capital, it necessitates the raising of income tax rate and of abatement expenditure rate, because the negative congestion effect of private capital can be countered by an improvement in environmental quality through an increase in abatement expenditure to an equal extent as the tax rise. This results in a higher rate of growth and consequently higher social welfare. Second, a stronger effect of the flow of short-term pollutants on the composite public good function leads to a lower effective benefit derived from the public good, which calls for both a higher income tax rate and a higher abatement spending rate but a lower income share of public infrastructure. The improvement in the environmental quality brought about by abatement expenditure outweighs the negative effect of the reduction in the public infrastructure-to-output ratio; and hence the growth rate becomes higher. These two results highlight the fact that abatement activities help to augment growth and welfare even though effects of short-term pollutants do not persist. This effect of flow pollution is new in the literature. Our paper shows that the value of the optimal tax rate as well as the optimal abatement expenditure rate would have been underestimated, and the ratio of productive public spending to income would have been overestimated, had we considered only the stock effects of pollution.

In Appendices A.3 and A.4 respectively, we show that the optimal values of the fiscal instrument rates do not change when public good is a stock variable and when public good is an argument in the utility function. Results remain unchanged in the respective cases because these are derived in steady-state equilibrium and because externalities cannot be internalized by private agents in a competitive economy. We also derive values of the fiscal instrument rates with CES production function and CES form of the effective benefit function of the public infrastructure in appendix A.5.[32]

It is possible to extend our paper in several directions. For instance, here the gap between abatement expenditure and the damaging effects of long-term pollutants is always positive, which means that environmental quality grows without bounds. In reality, there could be an upper limit to environmental quality without degradation, as for example, in Jouvet *et al.*

[32] We derive these results following the comments of the reviewer on the earlier version of the paper.

(2005); incorporating this feature would possibly modify our results. Also, environmental pollution could be treated as a by-product of consumption rather than of income, as in Andreoni and Levinson (2001), and Egli and Steger (2007), among others. Given that consumption activities largely depend on the scale of an economy, we would not expect our key results to change qualitatively because of this. Besides, as in Greiner (2005), there could be a pollution tax to supplement the income tax as an additional source of revenue, which would then be expected to lower the optimal income tax rate that we have derived here. Also, it is recognised that incentives as regards R&D activities towards abatement may not be present in perfectly competitive markets, so there could be firms producing intermediate goods and imperfect competition could be introduced in this sector a la Chu and Lai (2014). Some of these features are in our agenda for future research.

A Appendix

A.1 Derivation of the Equations of the Model

Combining equations (9), (10) and (11) we get,

$$(\rho + \psi\Gamma_m) = \eta^{-\frac{\phi(1-\alpha)}{\Delta}}\{\alpha(1-\tau)\}(\tau - g_a)^{\frac{1-\alpha}{\Delta}}(\frac{E_s}{K})^{\frac{\omega}{\Delta}}. \tag{A.1.1}$$

Combining equations (6) and (11) we get

$$\Gamma_m = (g_a - \delta)(\tau - g_a)^{\frac{1-\alpha}{\Delta}}\eta^{\frac{-\phi(1-\alpha)}{\Delta}}(\frac{E_s}{K})^{-\frac{\{\alpha-\theta(1-\alpha)\}}{\Delta}}. \tag{A.1.2}$$

Using equations (A.1.1) and (A.1.2) we eliminate $\frac{E_s}{K}$ and get the following equation.

$$(\rho+\psi\Gamma_m)\Gamma_m{}^{\frac{\omega}{\alpha-\theta(1-\alpha)}}$$
$$=\eta^{-\frac{\phi(1-\alpha)}{\Delta}}\{\alpha(1-\tau)\}(\tau-g_a)^{\frac{1-\alpha}{\Delta}}[(g_a-\delta)(\tau-g_a)^{\frac{1-\alpha}{\Delta}}\eta^{\frac{-\phi(1-\alpha)}{\Delta}}]^{\frac{\omega}{\alpha-\theta(1-\alpha)}};$$

or,

$$\Gamma_m^{\omega}(\rho+\psi\Gamma_m)^{\alpha-\theta(1-\alpha)}=\eta^{-\phi(1-\alpha)}\{\alpha(1-\tau)\}^{\alpha-\theta(1-\alpha)}(g_a-\delta)^{\omega}(\tau-g_a)^{1-\alpha}. \tag{A.1.3}$$

Equation (A.1.3) above is the same as equation (12) in the body of the paper.

Combining equations (8), (10) and (11) we get,

$$\alpha(\Gamma_m+\frac{C}{K})=\rho-\frac{\dot{\lambda}_K}{\lambda_K}. \tag{A.1.4}$$

Using equations (9) and (11) in the above equation, we get

$$\alpha(\Gamma_m+\frac{C}{K})=\rho+\psi\Gamma_m;$$

or,

$$C=\frac{1}{\alpha}\{\rho+(\psi-\alpha)\Gamma_m\}K;$$

or,

$$C(t)=\frac{1}{\alpha}\{\rho+(\psi-\alpha)\Gamma_m\}K(t)=\frac{1}{\alpha}\{\rho+(\psi-\alpha)\Gamma_m\}K(0)e^{\Gamma_m t} \tag{A.1.5}$$

since $K(t)=K(0)e^{\Gamma_m t}$ at the steady-state equilibrium and where $\psi>\alpha$ is a sufficient condition for $C(t)$ to be positive.

Also, at the steady-state equilibrium,

$$E_s(t)=E_s(0)e^{\Gamma_m t}. \tag{A.1.6}$$

Thus, using equations (13), (A.1.5) and (A.1.6), and assuming $\Gamma_m < \frac{\rho}{1-\psi}$ for welfare to be finite, we have

$$\begin{aligned} W &= [\frac{\{\overline{\frac{\rho+(\psi-\alpha)\Gamma_m K(0)}{\alpha}}\}^{\gamma(1-\sigma)}\{E_s(0)\}^{\epsilon(1-\sigma)}}{1-\sigma}] \int_0^\infty e^{(\gamma+\epsilon)(1-\sigma)\Gamma_m t} e^{-\rho t} dt \\ &= [\frac{\{\rho+(\psi-\alpha)\Gamma_m\}^{\gamma(1-\sigma)}\{K(0)\}^{\gamma(1-\sigma)}\{E_s(0)\}^{\epsilon(1-\sigma)}}{\alpha^{\gamma(1-\sigma)}(1-\sigma)}][-\frac{1}{\{(\gamma+\epsilon)(1-\sigma)\Gamma_m-\rho\}}] \\ &= [\frac{\{\rho+(\psi-\alpha)\Gamma_m\}^{\gamma(1-\sigma)}\{K(0)\}^{\gamma(1-\sigma)}\{E_s(0)\}^{\epsilon(1-\sigma)}}{\alpha^{\gamma(1-\sigma)}(1-\sigma)}][\{\rho-(1-\psi)\Gamma_m\}^{-1}] \\ &= [\frac{\{K(0)\}^{\gamma(1-\sigma)}\{E_s(0)\}^{\epsilon(1-\sigma)}}{\alpha^{\gamma(1-\sigma)}(1-\sigma)}]\{\rho+(\psi-\alpha)\Gamma_m\}^{\gamma(1-\sigma)}\{\rho-(1-\psi)\Gamma_m\}^{-1} \end{aligned}$$

So the welfare, W, is given by

$$W = [\frac{\{K(0)\}^{\gamma(1-\sigma)}\{E_s(0)\}^{\epsilon(1-\sigma)}}{\alpha^{\gamma(1-\sigma)}(1-\sigma)}]\{\rho+(\psi-\alpha)\Gamma_m\}^{\gamma(1-\sigma)}\{\rho-(1-\psi)\Gamma_m\}^{-1} \tag{A.1.7}$$

and this is the same equation as (14) in the body of the paper.

We denote the left hand side and right hand side of equation (12) as LHS_Γ and RHS_Γ respectively, and differentiate the equation with respect to τ and g_a to arrive at the following two equations.

$$\begin{aligned} &LHS_\Gamma[\omega{\Gamma_m}^{-1} + \psi\{\alpha-\theta(1-\alpha)\}(\rho+\psi\Gamma_m)^{-1}]\frac{\partial\Gamma_m}{\partial\tau} \\ &\quad = [-\{\alpha-\theta(1-\alpha)\}(1-\tau)^{-1} + (1-\alpha)(\tau-g_a)^{-1}]RHS_\Gamma; \end{aligned} \tag{A.1.8}$$

and

$$\begin{aligned} &LHS_\Gamma[\omega{\Gamma_m}^{-1} + \psi\{\alpha-\theta(1-\alpha)\}(\rho+\psi\Gamma_m)^{-1}]\frac{\partial\Gamma_m}{\partial g_a} \\ &\quad = [\omega(g_a-\delta)^{-1} - (1-\alpha)(\tau-g_a)^{-1}]RHS_\Gamma. \end{aligned} \tag{A.1.9}$$

Now, $LHS_\Gamma = RHS_\Gamma$ at the steady-state equilibrium and
$\frac{\partial\Gamma_m}{\partial\tau} = \frac{\partial\Gamma_m}{\partial g_a} = 0$ when Γ_m is maximum.
So, we get the following two simplified equations from the equations (A.1.8)

and (A.1.9) respectively.

$$\{\alpha - \theta(1-\alpha)\}(\tau - g_a) = (1-\alpha)(1-\tau); \tag{A.1.10}$$

and

$$\omega(\tau - g_a) = (1-\alpha)(g_a - \delta). \tag{A.1.11}$$

Using equations (A.1.10) and (A.1.11) we get the following values of τ and g_a.

$$\tau = \frac{(1-\delta)(1-\alpha)(1+\theta+\phi)}{1+\phi(1-\alpha)} + \delta = \tau^*; \tag{A.1.12}$$

and

$$g_a = \frac{(1-\delta)(1-\alpha)(\theta+\phi)}{1+\phi(1-\alpha)} + \delta = {g_a}^*. \tag{A.1.13}$$

These equations (A.1.12) and (A.1.13) are the same as equations (15) and (16) in the body of the paper.

A.2 Transitional Dynamics

We first define the following ratio variables: $x \equiv \frac{C}{K}$ and $z \equiv \frac{E_s}{K}$. Using equations (6), (8), (9) and (10) we obtain

$$\frac{\dot{x}}{x} = \{\frac{\alpha}{1-\gamma(1-\sigma)} - 1\}Mz^{\frac{\omega}{\Delta}} + \{\frac{\epsilon(1-\sigma)}{1-\gamma(1-\sigma)}\}Nz^{\frac{\omega}{\Delta}-1} + x - \frac{\rho}{1-\gamma(1-\sigma)}; \tag{A.2.1}$$

and

$$\frac{\dot{z}}{z} = -Mz^{\frac{\omega}{\Delta}} + Nz^{\frac{\omega}{\Delta}-1} + x \tag{A.2.2}$$

where $M \equiv (1-\tau)(\tau-g_a)^{\frac{1-\alpha}{\Delta}}\eta^{\frac{-\phi(1-\alpha)}{\Delta}}$ and $N \equiv (g_a-\delta)(\tau-g_a)^{\frac{1-\alpha}{\Delta}}\eta^{\frac{-\phi(1-\alpha)}{\Delta}}$.

The determinant of the Jacobian matrix (J) corresponding to the differential equations (A.2.1) and (A.2.2) is given by:

$$|J| = -\frac{\omega}{\Delta}\{\frac{\alpha}{1-\gamma(1-\sigma)} - 1\}Mz^{(\frac{\omega}{\Delta}-1)} + \{\frac{\psi}{1-\gamma(1-\sigma)}\}\{\frac{\alpha-\theta(1-\alpha)}{\Delta}\}Nz^{(\frac{\omega}{\Delta}-2)}. \tag{A.2.3}$$

Here, $\theta > 0$ and $\phi > 0$. So, we have $\omega > 0$, $\Delta > 0$ and $\psi > 0$. Also, $\alpha - \theta(1-\alpha) > 0$, $M > 0$ and $N > 0$ when τ and g_a are optimally chosen. So, $|J| < 0$; and hence the two eigenvalues of the Jacobian matrix must be real and of opposite signs. When $|J| < 0$, the equilibrium is a saddle point.

If $\theta = \phi = 0$, then $\omega = 0$; so the first term of $|J|$ is zero. Also, then ${g_a}^* = \delta$, which implies that $N = 0$. So, the second term of $|J|$ is also 0. Hence, $J = 0$.

As far as the $\frac{\dot{z}}{z}$ locus is concerned, this is clearly upward-rising, given that $\omega - \Delta = \theta(1-\alpha) - \alpha < 0$.

With $\epsilon = 0$, the locus of $\frac{\dot{x}}{x} = 0$ is upward-sloping for $\alpha < 1-\gamma(1-\sigma)$, and downward-sloping for $\alpha > 1-\gamma(1-\sigma)$. However, when both γ and ϵ are positive, then $\alpha > 1-\gamma(1-\sigma)$ unambiguously ensures that the locus of $\frac{\dot{x}}{x} = 0$ is an upward-sloping schedule but, with $\alpha > 1-\gamma(1-\sigma)$, it is ambiguous. A sufficient condition for the $\frac{\dot{x}}{x} = 0$ schedule to be downward-sloping is as follows:

$$(1-\frac{\omega}{\Delta})\{\frac{\epsilon(1-\sigma)}{1-\gamma(1-\sigma)}\}(g_a-\delta)z^{-1} < (\frac{\omega}{\Delta})\{\frac{\alpha}{1-\gamma(1-\sigma)} - 1\}(1-\tau);$$

which is simplified to[33]

$$z > (\frac{\Delta-\omega}{\Delta})[\frac{\epsilon(1-\sigma)}{\alpha-\{1-\gamma(1-\sigma)\}}]\{\frac{g_a-\delta}{1-\tau}\}. \tag{A.2.4}$$

[33]It is straightforward to see that if we have $\gamma = 1$, together with $\epsilon = 0$, then our model reduces to the RBG(2010) model, where $\frac{\dot{x}}{x}$ is an upward-sloping schedule for $\alpha < \sigma$, and a downward-sloping curve for $\alpha > \sigma$.

A.3 The Model with Public Good as a Stock

Here we consider an otherwise identical model to section 2 in the body of the paper except with public good as an accumulable input. So equations (1) to (8) of the original model remain unchanged and equation (5) is changed to

$$\dot{G} = (\tau - g_a)Y. \tag{A.3.1}$$

From equations (1) and (2) we get,

$$Y = K^{\alpha}\{G\bar{K}^{-\theta}{E_s}^{(\theta+\phi)}{E_f}^{-\phi}\}^{1-\alpha};$$

or,

$$Y = K^{\alpha-\theta(1-\alpha)}G^{1-\alpha}{E_s}^{(\theta+\phi)(1-\alpha)}(\eta Y)^{-\phi(1-\alpha)};$$

or,

$$Y^{1+\phi(1-\alpha)} = \eta^{-\phi(1-\alpha)}K^{\alpha-\theta(1-\alpha)}G^{1-\alpha}{E_s}^{(\theta+\phi)(1-\alpha)};$$

or,

$$Y = \eta^{-\frac{\phi(1-\alpha)}{1+\phi(1-\alpha)}}K^{\frac{\alpha-\theta(1-\alpha)}{1+\phi(1-\alpha)}}G^{\frac{1-\alpha}{1+\phi(1-\alpha)}}{E_s}^{\frac{(\theta+\phi)(1-\alpha)}{1+\phi(1-\alpha)}}. \tag{A.3.2}$$

Then solving the consumer's lifetime utility maximization problem we obtain equation (9) and the following equation as the two first-order conditions.

$$\alpha(1-\tau)\eta^{-\frac{\phi(1-\alpha)}{\Delta'}}K^{\frac{\alpha-\theta(1-\alpha)}{\Delta'}-1}G^{\frac{1-\alpha}{\Delta'}}{E_s}^{\frac{\omega}{\Delta'}} = \rho - \frac{\dot{\lambda}_k}{\lambda_k};$$

or,

$$\alpha(1-\tau)\eta^{-\frac{\phi(1-\alpha)}{\Delta'}}(\frac{G}{K})^{\frac{1-\alpha}{\Delta'}}(\frac{E_s}{K})^{\frac{\omega}{\Delta'}} = \rho - \frac{\dot{\lambda}_K}{\lambda_K} \tag{A.3.3}$$

where $\Delta^{'} = 1 + \phi(1-\alpha)$.

Now, using equations (A.3.1) and (11) we get,

$$\frac{\dot{G}}{G} = \Gamma_m = \frac{(\tau - g_a)Y}{G} = (\tau - g_a)\eta^{-\frac{\phi(1-\alpha)}{\Delta^{'}}}(\frac{G}{K})^{-\frac{\Delta}{\Delta^{'}}}(\frac{E_s}{K})^{\frac{\omega}{\Delta^{'}}}. \qquad \text{(A.3.4)}$$

Thus, using the value of $(\frac{E_s}{K})^{\frac{\omega}{\Delta^{'}}}$ from equation (A.3.4) in equation (A.3.3) we get,

$$\frac{\alpha(1-\tau)\eta^{-\frac{\phi(1-\alpha)}{\Delta^{'}}}(\frac{G}{K})^{\frac{1-\alpha}{\Delta^{'}}}\Gamma_m}{(\tau - g_a)\eta^{-\frac{\phi(1-\alpha)}{\Delta^{'}}}(\frac{G}{K})^{-\frac{\Delta}{\Delta^{'}}}} = \rho - \frac{\dot{\lambda_k}}{\lambda_k};$$

or,

$$\{\frac{\alpha(1-\tau)}{\tau - g_a}\}(\frac{G}{K})(\Gamma_m) = \rho - \frac{\dot{\lambda_k}}{\lambda_k}. \qquad \text{(A.3.5)}$$

Now, from equations (6) and (11), we get the following.

$$\frac{\dot{E_s}}{E_s} = \Gamma_m = (g_a - \delta)\frac{Y}{E_s};$$

or,

$$\Gamma_m = (g_a - \delta)\eta^{-\frac{\phi(1-\alpha)}{\Delta^{'}}}(\frac{G}{K})^{\frac{1-\alpha}{\Delta^{'}}}(\frac{E_s}{K})^{\frac{\omega}{\Delta^{'}}-1}. \qquad \text{(A.3.6)}$$

Again using the value of $\frac{E_s}{K}$ from equation (A.3.4) in equation (A.3.6) we get,

$$\Gamma_m = \frac{\eta^{-\frac{\phi(1-\alpha)}{\Delta^{'}}}(g_a - \delta)(\frac{G}{K})^{\frac{1-\alpha}{\Delta^{'}}}(\Gamma_m)^{\frac{\Delta^{'}}{\omega}(\frac{\omega}{\Delta^{'}}-1)}}{\{\eta^{-\frac{\phi(1-\alpha)}{\Delta^{'}}}(\tau - g_a)(\frac{G}{K})^{-\frac{\Delta}{\Delta^{'}}}\}^{\frac{\Delta^{'}}{\omega}(\frac{\omega}{\Delta^{'}}-1)}};$$

or,

$$\Gamma_m{}^{\frac{\Delta^{'}}{\omega}} = \eta^{-\frac{\phi(1-\alpha)}{\omega}}(g_a - \delta)(\tau - g_a)^{\frac{1-\theta(1-\alpha)}{\omega}}(\frac{G}{K})^{\frac{\theta(1-\alpha)-\alpha}{\omega}}. \qquad \text{(A.3.7)}$$

Using equations (9), (11) and (A.3.5) we get,

$$\{\frac{\alpha(1-\tau)}{\tau-g_a}\}(\frac{G}{K})(\Gamma_m) = \rho - \{(\gamma+\epsilon)(1-\sigma)-1\}\Gamma_m. \tag{A.3.8}$$

Finally, using the value of $\frac{G}{K}$ from equation (A.3.8) in equation (A.3.7) we get,

$$\Gamma_m{}^{1-\alpha+\omega}(\rho+\psi\Gamma_m)^{\alpha-\theta(1-\alpha)} = \eta^{-\phi(1-\alpha)}\{\alpha(1-\tau)\}^{\alpha-\theta(1-\alpha)}(\tau-g_a)^{1-\alpha}(g_a-\delta)^{\omega}. \tag{A.3.9}$$

Here, the steady-state growth rate equation (A.3.9) is analogous to the corresponding equation (12) from the original model section 3 in the body of the paper. However, $\Gamma_m{}^{1-\alpha+\omega} < (>)\Gamma_m{}^{\omega}$ for same value of $\Gamma_m < (>)1$ because $0 < \alpha < 1$. So, equation (A.3.9) solves for a higher (lower) value of Γ_m than that of equation (12) when $\Gamma_m < (>)1$. This is made clear by the figure A.5. This makes the steady-state growth rate higher, initially, when public good is an accumulable input. The public input accumulates and its stock increases over time now; so initially, this has a growth enhancing effect due to the effect of positive marginal product of the public input. However, the marginal product of the public input decreases intertemporally, as time passes, due to this accumulation effect. This accumulation effect is absent when public good is a flow variable. Hence, the latter effect dominates when $\Gamma_m > 1$ which makes the steady-state growth rate less than the same of the model with public good as a flow. In the original model, diminishing returns to the public good exist. But its effect is limited to the current time period only for a higher magnitude of services obtained from the public good in that time period. This effect vanishes in the next period and does not carry over since public good is a flow there, unlike the model where it is a stock. Hence, when public input is a stock, the positive marginal product effect dominates the diminishing marginal product effect initially, when Γ_m is low (< 1) but is dominated by the latter when Γ_m crosses the threshold of unity. Intuitively for example, building new highways rather than maintaining them well, yield greater returns initially when the growth rate is relatively low till the point where already enough has been spent on new highways and it is then worth paying more attention to maintaining them properly.

Maximizing the steady-state equilibrium growth rate subject to the constraint given in equation (A.3.9) yields the same values of the optimal fiscal instrument rates as given by equations (15) and (16) in the body of the paper. Both ${\Gamma_m}^{\omega}$ and ${\Gamma_m}^{1-\alpha+\omega}$ are two increasing functions in Γ_m; and each of them is maximized when Γ_m is maximized. So, optimal values of fiscal instruments are same in both the cases.

A.4 The Model with Public Good in the Utility Function

In this section, we consider the flow public good as an argument in the utility function in place of environmental quality. Thus, all other equations of the original model of section 2 in the body of the paper remain unchanged and equation (7) is changed to

$$u = \frac{(C^{\gamma} G^{\epsilon})^{1-\sigma}}{1-\sigma}. \tag{A.4.1}$$

In this case, the consumer's utility maximization conditions remain same as in the original model because here, the public good, G, is also an externality, similar to environmental quality, E_s. However, condition (9) of the basic model is changed to

$$\{\gamma(1-\sigma) - 1\}(\frac{\dot{C}}{C}) + \epsilon(1-\sigma)(\frac{\dot{G}}{G}) = \frac{\dot{\lambda_k}}{\lambda_k}; \tag{A.4.2}$$

but equation (10) remains the same. Combining equations (11) and (A.4.2) we get

$$\{(\gamma+\epsilon)(1-\sigma)-1\}\Gamma_m = \frac{\dot{\lambda_k}}{\lambda_k} \tag{A.4.3}$$

in the steady-state equilibrium. Therefore, equations (10), (A.4.3) and (11) give the same solution for the steady-state growth rate as in equation (12) of the original model. Therefore, optimal values of fiscal instrument rates also remain unchanged from equations (15) and (16).

A.5 The Model with CES Functions

In this section, we consider the production function to be of the CES form as follows.

$$Y = [\alpha K^{\beta} + (1-\alpha)\hat{G}^{\beta}]^{\frac{1}{\beta}}; \tag{A.5.1}$$

and the effective benefit function of the public good to be

$$\hat{G} = [G^{\beta} - \theta \bar{K}^{\beta} + (\theta+\phi){E_s}^{\beta} - \phi {E_f}^{\beta}]^{\frac{1}{\beta}}. \tag{A.5.2}$$

Rest of the equations, (3) to (8), remain unchanged from the original model. Only equations (1) and (2) are replaced by equations (A.5.1) and (A.5.2).

Using equations (A.5.1), (A.5.2), (6) and (7) we obtain the following.

$$Y^{\beta} = \alpha K^{\beta} + (1-\alpha)[G^{\beta} - \theta \bar{K}^{\beta} + (\theta+\phi){E_s}^{\beta} - \phi {E_f}^{\beta}];$$

or,

$$Y^{\beta} = (\frac{1}{D})[\{\alpha - \theta(1-\alpha)\}K^{\beta} + (\theta+\phi)(1-\alpha){E_s}^{\beta}] \tag{A.5.3}$$

where

$D = 1 - (1-\alpha)(\tau - g_a)^{\beta} + \phi(1-\alpha)\eta^{\beta}$.

Then the solution to the consumer's lifetime utility maximization problem yields equation (9) and the following equation as the two first-order conditions.

$$\alpha(1-\tau)(\frac{Y}{K})^{1-\beta} = \rho - \frac{\dot{\lambda_K}}{\lambda_K}. \tag{A.5.4}$$

Using equations (A.5.3), (A.5.4) and (11) we get,

$$\alpha(1-\tau)D^{\frac{\beta-1}{\beta}}[\{\alpha-\theta(1-\alpha)\}+(\theta+\phi)(1-\alpha)(\frac{E_s}{K})^{\beta}]^{\frac{1-\beta}{\beta}} = \rho + \psi\Gamma_m. \tag{A.5.5}$$

Now, from equations (6) and (11) we get

$$\frac{\dot{E}_s}{E_s} = \Gamma_m = \frac{(g_a-\delta)Y}{E_S};$$

or,

$$(\frac{E_s}{K})^{\beta} = \frac{\alpha-\theta(1-\alpha)}{\frac{\Gamma_m{}^{\beta}D}{(g_a-\delta)^{\beta}}-(\theta+\phi)(1-\alpha)}. \tag{A.5.6}$$

Using equations (A.5.5) and (A.5.6) we get

$$(\rho+\psi\Gamma_m)^{\beta}\{1-\frac{(\theta+\phi)(1-\alpha)(g_a-\delta)^{\beta}}{D\Gamma_m{}^{\beta}}\}^{1-\beta}$$

$$= \{\alpha(1-\tau)\}^{\beta}\{\frac{\alpha-\theta(1-\alpha)}{D}\}^{1-\beta}; \tag{A.5.7}$$

or,

$$(\rho+\psi\Gamma_m)^{\beta}\Gamma_m{}^{\beta(\beta-1)}$$

$$= \{\alpha(1-\tau)\}^{\beta}\{\alpha-\theta(1-\alpha)\}^{1-\beta}\{D\Gamma_m{}^{\beta}-(\theta+\phi)(1-\alpha)(g_a-\delta)^{\beta}\}^{\beta-1}.$$

We denote the left hand side of equation (A.5.7) by LHS_Γ and the right hand side by RHS_Γ. We differentiate both sides of equation (A.5.7) with respect to τ and g_a; and obtain the following conditions.

$$\beta\psi(\rho+\psi\Gamma_m)^{-1}LHS_\Gamma\frac{\partial\Gamma_m}{\partial\tau}$$

$$+(1-\beta)(\theta+\phi)(1-\alpha)(g_a-\delta)^{\beta}\{1-\frac{(\theta+\phi)(1-\alpha)(g_a-\delta)^{\beta}}{D{\Gamma_m}^{\beta}}\}^{-1}$$

$$[(\frac{\beta}{D{\Gamma_m}^{\beta+1}})\frac{\partial\Gamma_m}{\partial\tau}-\frac{\beta(1-\alpha)(\tau-g_a)^{\beta-1}}{D^2{\Gamma_m}^{\beta}}]LHS_{\Gamma}$$

$$=-\{\beta(1-\tau)^{-1}-\beta(1-\beta)D^{-1}(1-\alpha)(\tau-g_a)^{\beta-1}\}RHS_{\Gamma}; \qquad \text{(A.5.8)}$$

and

$$\beta\psi(\rho+\psi\Gamma_m)^{-1}LHS_{\Gamma}\frac{\partial\Gamma_m}{\partial g_a}$$

$$-(1-\beta)\{1-\frac{(\theta+\phi)(1-\alpha)(g_a-\delta)^{\beta}}{D{\Gamma_m}^{\beta}}\}^{-1}$$

$$[\frac{\beta(\theta+\phi)(1-\alpha)(g_a-\delta)^{\beta-1}}{D{\Gamma_m}^{\beta}}-(\theta+\phi)(1-\alpha)(g_a-\delta)^{\beta}$$

$$\{(\frac{\beta}{D{\Gamma_m}^{\beta+1}})\frac{\partial\Gamma_m}{\partial g_a}+\frac{\beta(1-\alpha)(\tau-g_a)^{\beta-1}}{D^2{\Gamma_m}^{\beta}}\}]LHS_{\Gamma}$$

$$=-\{\beta(1-\beta)(1-\alpha)D^{-1}(\tau-g_a)^{\beta-1}\}RHS_{\Gamma}. \qquad \text{(A.5.9)}$$

Now $LHS_{\Gamma}=RHS_{\Gamma}$ at the steady-state equilibrium and
$\frac{\partial\Gamma_m}{\partial\tau}=\frac{\partial\Gamma_m}{\partial g_a}=0$ when Γ_m is maximum.
Equations (A.5.8) and (A.5.9) are the two first-order conditions of growth rate maximization. Using them we get the following two simplified equations respectively.

$$(1-\beta)(1-\alpha)^2(\theta+\phi)(1-\tau)(g_a-\delta)^{\beta}$$

$$=\{D{\Gamma_m}^{\beta}-(\theta+\phi)(1-\alpha)(g_a-\delta)^{\beta}\}\{D(\tau-g_a)^{1-\beta}-(1-\alpha)(1-\beta)(1-\tau)\}; \qquad \text{(A.5.10)}$$

and

$$(\theta+\phi)\{D(\tau-g_a)^{1-\beta}-(1-\alpha)(g_a-\delta)\}=(g_a-\delta)^{1-\beta}\{D{\Gamma_m}^{\beta}-(\theta+\phi)(1-\alpha)(g_a-\delta)^{\beta}\}. \tag{A.5.11}$$

The above two equations are in two unknowns, τ and g_a, which can in principle be solved simultaneously to yield the growth rate-maximising values of those two variables in terms of the parameters of the model. (As we know, those growth rate-maximising values of τ and g_a are also the welfare-maximising values of τ and g_a.) However, given these expressions it is not possible to solve for τ and g_a analytically. So, we solve for them numerically. To enable us to compare our numerical results for the CES specification with those obtained for the Cobb-Douglas specification used in the paper, we need to consider the same parameter values for α, θ, ϕ and δ, and solve for τ and g_a for both the specifications. For the CES case, the parameter β (which is linked to the elasticity of substitution) is additionally present, so we consider the limiting case of $\beta = 0$ in (A.5.10) and (A.5.11), since a CES function takes the Cobb-Douglas functional form when $\beta \to 0$. We obtain values for τ and g_a that are quite close to what we get with the same parameter values for the expressions for the Cobb-Douglas function, i.e., (15) and (16).

Using the same set of parameter values for the CES function that has been considered earlier for the Cobb-Douglas function, i.e., (i) with $\alpha = 0.7$, $\theta = 0.5$, $\phi = 0.5$, $\delta = 0.2$, we obtain $\tau = 0.594663$, $g_a = 0.397332$ for the CES specification, which comes quite close to $\tau = 0.617391$, $g_a = 0.408696$ for the Cobb-Douglas function; and (ii) with $\alpha = 0.7$, $\theta = 0.6$, $\phi = 0.4$, $\phi = 0.1$, we got $\tau = 0.555276$, $g_a = 0.327638$ for the CES case, which compares well with $\tau = 0.582143$, $g_a = 0.341071$ for the Cobb-Douglas function. We have conducted further robustness checks with other sets of parameter values for both specifications, and those additional results can be provided upon request.

References

[1] Andreoni, J. and Levinson, A. (2001). The simple Analytics of the Environmental Kuznets Curve. Journal of Public Economics, Vol. 80, 269-286.

[2] Barro, R. J. (1990). Government Spending in a Simple Model of Endogenous Growth. Journal of Political Economy, Vol. 98, 103-125.

[3] Barro, R. J., and Sala-i-Martin, X. (1992). Public Finance in Models of Economic Growth. Review of Economic Studies, Vol. 59, 645-661.

[4] Bontems, P., and Gozlan, E. (2018). Trade, Environment, and Income Inequality: An Optimal Taxation Approach. Journal of Public Economic Theory, Vol. , 1-25.

[5] Bosi, S., Desmarchelier, D., and Ragot, L. (2018). Pollution Effects on Preferences: A Unified Approach. Journal of Public Economic Theory, Vol. , 1-29.

[6] Bouche, M., and Miguel, C. De. (2019). Optimal Fiscal Policy in a Model with Inherited Aspirations and Habit Formation. Journal of Public Economic Theory, Vol. , 1-23.

[7] Bovenberg, L., and Smulders, S. (1995). Environmental Quality and Pollution-Augmenting Technological Change in a Two-Sector Endogenous Growth Model. Journal of Public Economics, Vol. 57, 369-391.

[8] Brechet, T., Hritonenko, N., and Yatsenko, Y. (2013). Adaptation and Mitigation in Long-term Climate Policy. Environmental and Resource Economics, Vol. 55, 217-243.

[9] Bruce, N., and Turnovsky, S. J. (1999). Budget Balance, Welfare, and the Growth Rate: Dynamic Scoring of the Long-Run Government Budget. Journal of Money, Credit and Banking, Vol. 31, 161-186.

[10] Byrne, M. M. (1997). Is Growth a Dirty Word? Pollution, Abatement and Endogenous Growth. Journal of Development Economics. Vol. 54, 261-281.

[11] Chatterjee S., and Ghosh S. (2011). The Dual Nature of Public Goods and Congestion: The Role of Fiscal Policy Revisited. Canadian Journal of Economics, Vol. 44, 1471-1496.

[12] Chu, H. and Lai, C. (2014). Abatement R&D, Market Imperfections, and Environmental Policy in an Endogenous Growth Model. Journal of Economic Dynamics and Control, Vol. 41, 20-37.

[13] Devereux M.B., and Smith G.W. (1994). International Risk Sharing and Economic Growth. International Economic Review, Vol. 35, 535-550.

[14] Economides G., and Philippopoulos, A. (2008). Growth Enhancing Policy is the Means to Sustain the Environment. Review of Economic Dynamics, Vol. 11, 207-219.

[15] Egli H., and Steger, T.M. (2007). A Dynamic Model of the Environmental Kuznets Curve: Turning Point and Public Policy. Environmental and Resource Economics, Vol. 36, 15-34.

[16] Eicher, T. and Turnovsky, S. J. (2000). Scale, Congestion and Growth. Economica, Vol. 67, 325-346.

[17] Forster, B. A. (1973). Optimal Capital Accumulation in a Polluted Environment. Southern Economic Journal, Vol. 39, 544-547.

[18] Futagami, K., Morita, Y., and Shibata, A. (1993). Dynamic Analysis of an Endogenous Growth Model with Public Capital. Scandinavian Journal of Economics, Vol. 95, 607-625.

[19] Ghosh, S., and Mourmouras, I. A. (2002). On Public Investment, Long-Run Growth and the Real Exchange Rate. Oxford Economic Papers, Vol. 54, 72-90.

[20] Ghosh, S., and Roy, U. (2004). Fiscal Policy, Long-Run Growth, and Welfare in a Stock-Flow Model of Public Goods. Canadian Journal of Economics, Vol. 37, 742-756.

[21] Glomm, G., and Ravikumar, B. (1994). Public Investment in Infrastructure in a Simple Growth Model.Journal of Economic Dynamics and Control, Vol. 18, 1173-1187.

[22] Gradus, R., and Smulders, S. (1993). The Trade-Off Between Environmental Care and Long-Term Growth:Pollution in Three Prototype Growth Models. Journal of Economics, Vol. 58, 25-51.

[23] Greiner, A. (2005). Fiscal Policy in an Endogeneous Growth Model with Public Capital and Pollution. Japanese Economic Review, Vol. 56, 67-84.

[24] Gruver, G. (1976). Optimal Investment and Pollution in a Neoclassical Growth Context. Journal of Environmental Economics and Management, Vol. 5, 165-177.

[25] Itaya, J.-i. (2008). Can Environmental Taxation Stimulate Growth? The Role of Indeterminacy in Endogenous Growth Models with Environmental Externalities. Journal of Economic Dynamics and Control, Vol. 32, 1156-1180.

[26] Jouvet, P. A., Michel, P., and Rotillon, G. (2005). Optimal Growth with Pollution: How to Use Pollution Permits. Journal of Economic Dynamics and Control, Vol. 29, 1597-1609.

[27] Keohane, N., Van Roy, B., and Zeckhauser, R. (2007). Managing the Quality of a Resource with Stock and Flow Controls. Journal of Public Economics, Vol. 91, 541-569.

[28] Lieb, C. M. (2004). The Environmental Kuznets Curve and Flow Versus Stock Pollution: The Neglect of Future Damages. Environmental and Resource Economics, Vol. 29, 483-506.

[29] Ligthart, J. E., and Ploeg, F. v. (1994). Pollution, the Cost of Public Funds and Endogenous Growth. Economic Letters, Vol. 46, 339-349.

[30] Managi, S. (2006). Are There Increasing Returns to Pollution Abatement? Empirical Analytics of the Environmental Kuznets Curve in Pesticides. Ecological Economics, Vol. 58, 617-636.

[31] Raurich-Puigdevall, X. (2000). Global Indeterminacy in an Endogenous Growth Model with Public Capital. Journal of Economics, Vol. 71, 255-280.

[32] Ray Barman, T., and Gupta, M. R. (2010). Public Expenditure, Environment, and Economic Growth, Journal of Public Economic Theory, Vol. 12, 1109-1134.

[33] Romer, P. A. (1986). Increasing Returns and Long-Run Growth. Journal of Political Economy, Vol. 94, 1002-1037.

[34] Roseta-Palma, C., Ferreira-Lopes, A., and Sequeira, T. N. (2010). Externalities in an Endogenous Growth Model with Social and Natural Capital. Ecological Economics, Vol. 69, 603-612.

[35] Smulders, S. (1995). Entropy, Environment, and Endogenous Growth. International Tax and Public Finance, Vol. 2, 319-340.

[36] Smulders, S., and Gradus, R. (1996). Pollution Abatement and Long-Term Growth. European Journal of Political Economy, Vol. 12, 505-532.

[37] Tahvonen, O., and Kuuluvainen, J. (1991). Optimal Growth with Renewable Resources and Pollution.European Economic Review, Vol. 35, 650-661.

[38] Turnovsky, S. J. (1996a).Optimal Tax, Debt, and Expenditure Policies in a Growing Economy. Journal of Public Economics, Vol. 60, 21-44.

[39] Turnovsky, S. J. (1996b). Fiscal Policy, Adjustment Costs, and Endogenous Growth. Oxford Economic Papers, Vol. 48, 361-381.

[40] Turnovsky, S. J. (1997). Fiscal Policy in a Growing Economy with Public Capital.Macroeconomic Dynamics, Vol. 1, 615-639.

[41] Turnovsky S.J. (1999). Productive Government Expenditure in a Stochastically Growing Economy. Macroeconomic Dynamics, Vol. 3, 544-570.

[42] Van der Ploeg, F., and Withagen, C. (1991). Pollution Control and the Ramsey Problem. Environmental and Resource Economics, Vol. 1, 215-236.

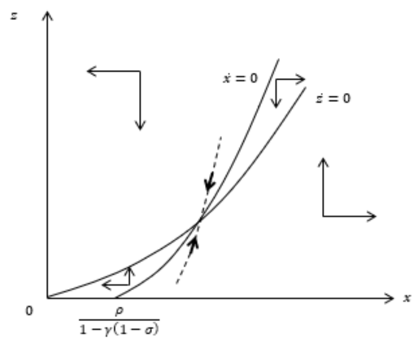

Figure 1a: Case with $\alpha < 1 - \gamma(1-\sigma)$

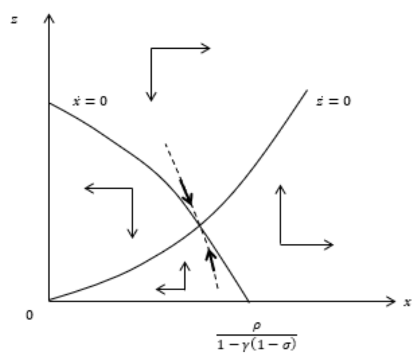

Figure 1b: Case with $\alpha > 1 - \gamma(1-\sigma)$

Figure 1: Transitional Dynamics

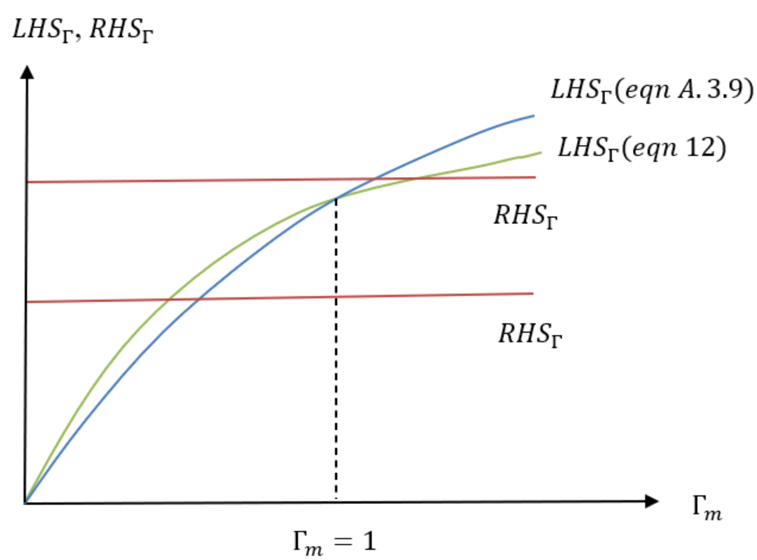

Figure 2: Steady-state growth rates