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FORMULAE FOR THE APPROXIMATE CONFORMAL MAPPING OF SOME SIMPLY CONNECTED DOMAINS

BY

N.PAPAMICHAEL and A.SIDERIDIS

1. Introduction

The problem of conformally mapping a given simply-connected domain onto the unit disc is of great physical interest, having important applications for example in the fields of fluid mechanics, electrostatics and steady-state heat flow. Although Kober [9] provides an excellent dictionary of special conformal transformations, there are many domains that occur in practice for which the conformal map can be obtained only by numerical means.

In the present paper we give explicit formulae for the approximate conformal mapping of a selection of simply-connected domains. These approximations were derived by means of the Bergman kernel method which has been recently proposed by D. Levin and the present authors in [10]. Some of the formulae given have been used by the authors in [11] to obtain, by means of a conformal transformation method, accurate numerical solutions to certain elliptic boundary value problems involving boundary singularities. They are presented here as they might be of value in other applications.

2. Approximate Formulae

Let Ω be a simply-connected domain with boundary $\partial\Omega$ in the complex z-plane and assume, without loss of generality, that the origin of coordinates 0 lies in Ω. Let

$$w = f(z)$$

be the mapping function which maps Ω U& Ω conformally onto the unit disc $|w| \leq 1$ in such a way that

$$f(0) = 0 \text{ and } f'(0) > 1.$$

We consider a selection of simply-connected domains and, for each domain, we give an explicit formula approximating the mapping function f(z). All formulae are derived by means of the Bergman kernel method of [10] and are of the form

$$f_N(z) = c \sum_{n=1}^{N} a_n v_n(z).$$

The criteria for selecting the functions $v_n(z)$ and the technique for computing the coefficients a_n are described fully in [10]. Here, for each domain considered, we only list the functions $v_n(z)$, $n = 1,2,\ldots,N$ and tabulate the complex coefficients a_n $n = 1,2,\ldots,N$ and the real constant

$$c = \left\{ \frac{\Pi}{\mathrm{Re}\left(\sum_{n=1}^{N} a_n v_n'(0) \right)} \right\}^{\frac{1}{2}} .$$

In each case the positive integer N, i.e., the number of functions v_n (z) used in the approximation, is the "optimum number" which gives maximum accuracy in the sense explained in [10]. An estimate of the maximum error in the modulus of $f_N(z)$ is given by the quantity E_N. This is obtained, as described in [10], by computing

$$e_N^{(z)} = 1 - |f_N(z)| .$$

at a number of "boundary test points" $, z_j \in \partial\Omega$, and then determining,

$$E_N = \max_j |e_N(z_j)| .$$

Each example heading is followed by a list of references. These indicate the publications in which the domain under consideration was the domain of definition of a problem which has been or may be solved by conformal transformations.

In obtaining the approximations presented here, all computations were carried out, in single length arithmetic, on a CDC 7600 computer.

2.1 Quadrilateral domain of Fig.1 ([11],[16]).

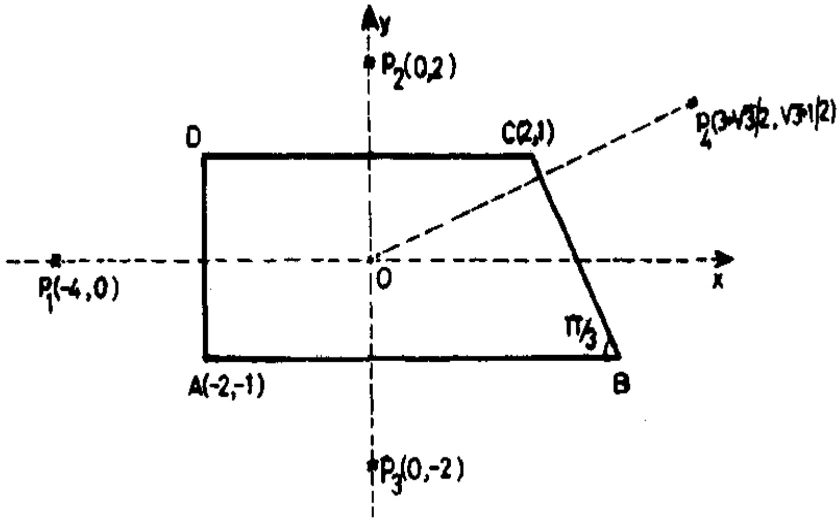

Figure 1

$$f_{17}(z)=c\left\{\sum_{n=1}^{4}a_n z/(z-p_n)+a_5 z+a_6\{(z-z_c)^{3/2}-(z_c)^{3/2}\}+\sum_{n=1}^{3}a_{n+6}z^{n+1}/(n+1)\right.$$
$$\left.+a_{10}\{(z-z_c)^{9/2}-(-z_c)^{9/2}\}+\sum_{n=1}^{7}a_{n+10}z^{n+4}/(n+4)\right\}.$$

$c=0.398146533868E+01$

COEFFICIENTS a_n

0.791694815026E-01	-.404577265524E-03
0.427384527935E-05	-.159158948771E+00
-.356087929037E-05	0.159160208427E+00
-.570240876737E-01	-.216619859008E-01
-.184900259407E+00	-.320036339889E+00
0.482506039583E-01	0.582610812361E-02
0.512212016237E+00	0.466945150131E+00
-.383000552846E+00	-.116177383540E+00
0.871665460256E-01	-.143531150834E-01
-.232490033139E-02	-.553110980205E-02
-.388375067171E-02	0.299344559392E-02
-.972207514203E-04	0.208699226279E-03
0.657585933207E-05	0.243954703546E-04
0.154351240627E-05	0.223160629975E-05
0.455843972998E-06	-.692510627560E-07
0.572694620456E-07	-.802076687538E-07
-.313898187880E-08	-.550573131362E-08

The estimate of the maximum error in $|f_{17}(z)|$ determined by computing $e_{17}(z)$ at a selection of boundary points (see [10], example 2) is,

$$E_{17} =| e_{17}(z_B)| = 4.7\times10^{-7}.$$

2.2. Domain of Fig.2 ([7],[11]).

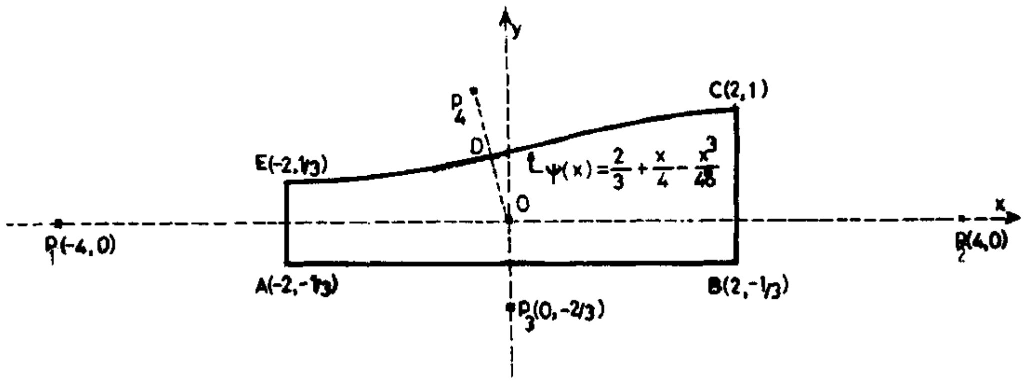

Figure 2

The coordinates of the point p_4 are

$$x = -.312000000000E+00$$
$$y = 0.125549151733E+01$$

$$f_{22}(z) = c\left\{\sum_{n=1}^{4} a_n z/(z-p_n) + \sum_{n=1}^{18} a_{n+4} z^n/n\right\}.$$

$c = 0.172185936062E+01$

COEFFICIENTS a_n

-.939969248427E+02	0.122329979451E+04
-.551978734456E+03	-.174377012616E+03
-.261501992446E-03	0.477495901722E+00
0.531833825380E-01	-.251139228085E+00
-.114350378084E+03	-.349412672348E+03
-.807660885199E+02	0.131147734849E+03
-.214863408023E+02	-.6553 65881560E+02
-.101033444392E+02	0.1 63810803570E+02
-.223119413537E+01	-.681137279041E+01
-.931403212463E+00	0.153119094936E+01
-.200042800284E+00	-.597 686794743E+00
-.882079024052E-01	0.133843303025E+00
-.135210164450E-01	-.516934917590E- 01
-.220551777920E-02	0.725856779886E-02
-.176278084664E-02	-.130894577392E-02
-.157003559571E-02	0.151609606350E-02
-.292606857151E-04	-.987322440601E-03
0.156204714583E-03	-.678079650308E-04
-.347686237615E-05	0.963751336872E-04
-.193257751761E-04	0.127605650111E-04
-.151735706256E-05	-.999870734112E-05
0.271991501277E-06	0.329824918668E-06

The estimate of the maximum error in $|f_{22}(z)|$ determined by computing $e_{22}(z)$ at 50 suitably chosen boundary points is,

$$E_{22} = | e_{22} (z_D)| = 4.9\times10^{-5}.$$

2.3 L-shaped domain of Fig.3 ([1]- [3], [8], [11]- [13], [15], [17]).

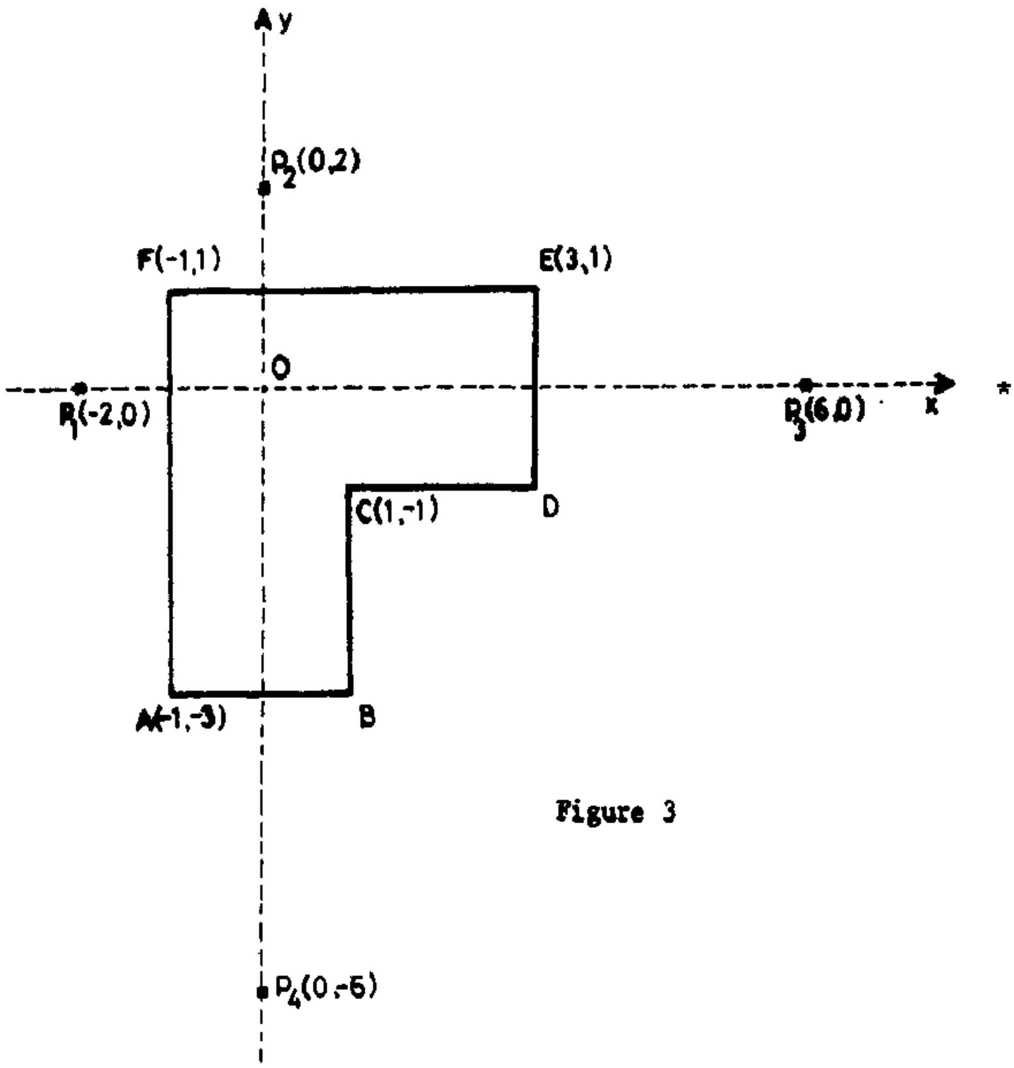

Figure 3

$$f_{26}(z) = c\left\{\sum_{n=1}^{4} a_n z/(z-p_n) + a_5\{(z-z_c)^{2/3} - (-z_c)^{2/3}\} + a_6 z + a_7\{(z-z_c)^{4/3} - (-z_c)^{4/3}\}\right.$$

$$+ a_8 z^2/2 + a_9\{(z-z_c)^{8/3} - (-z_c)^{8/3}\} + a_{10} z^3/3 + a_{11}\{(z-z_c)^{10/3} - (-z_c)^{10/3}\}$$

$$\left. + a_{12} z^4/4 + a_{13}\{(z-z_c)^{14/3} - (-z_c)^{14/3}\} + \sum_{n=1}^{13} a_{n+13} z^{n+4}/(n+4)\right\}.$$

c =0.410656087112E+01

COEFFICIENTS a_n

0.159612267043E+00	0.335646846333E-04
-. 256039962417E-04	-.159613596509E+00
-.312878304089E+01	0.343839952158E+01
-.355172625260E+01	0.317083605565E+01
0.122695459141E+00	0.122695858537E+00
-.505064867897E+00	-.192434451322E-01
-.463392143878E-01	0. 463423955230E-01
-.390325634275E+00	-.398067693316E+00
0.677359626098E-01	-.677611692060E-01
0.973540694394E-03	0.112158829299E+00
-.152126322197E-01	- .151810657862E-01
-.821491781895E-01	0.817879859362E-01
-.394581627595E-02	-.395401615778E-02
0.598783682967E-02	-.639779898442E-04
0.4063180 68481E-03	0.387700851936E-03
0.110158129158E-05	0.215408946060E-03
-.239529581917E-04	0.237520089010E-04
-.194808589058E-04	-.204762611350E-06
0.389998066780E-05	0.394686114515E-05
0.186535402646E-07	-.169222348084E-05
-.343672119284E-06	0.337774475874E-06
0.723026529702E-07	0.414316531930E-09
-.563957261560E-08	-.568917941216E-08
0.689479871346E-11	0.963180754574E-09
-.274511826388E-10	0.266789197888E-10
-.209894966791E-10	-.344489230867E-12

The estimate of the maximum error in $|f_{26}(z)|$ determined by computing $e_{26}(z)$ at a selection of boundary points (see [10], example 3) is,

$$E_{26} = |e_{26}(z_C)| = 2.2\times 10^{-5}.$$

2.4 L-shaped domain of Fig.4 ([11],[12],[15]).

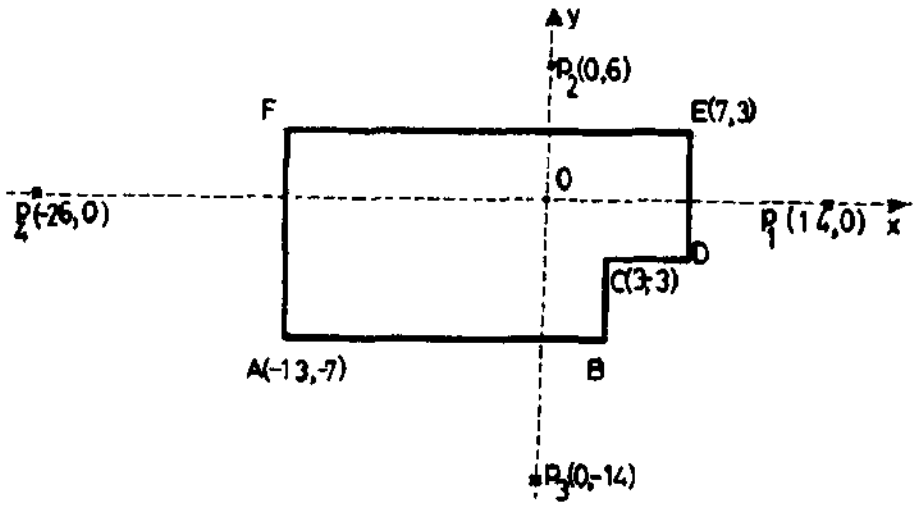

Figure 4

$$
\begin{aligned}
f_{17}(z) = c \Bigg\{ & \sum_{n=1}^{4} a_n z/(z-p_n) + a_5\{(z-z_c)^{2/3} - (-z_c)^{2/3}\} + a_6 z \\
& + a_7\{(z-z_c)^{4/3} - (-z_c)^{4/3}\} + a_8 z^2/2 + a_9\{(z-z_c)^{8/3} - (-z_c)^{8/3}\} \\
& + a_{10} z^3/3 + a_{11}\{(z-z_c)^{10/3} - (-z_c)^{10/3}\} + a_{12} z^4/4 \\
& + a_{13}\{(z-z_c)^{14/3} - (-z_c)^{14/3}\} + \sum_{n=1}^{n} a_{n+13} z^{n+4}/(n+4) \Bigg\}.
\end{aligned}
$$

$c = 0.149098601250E+02$

COEFFICIENTS a_n

-.171147593328E-01	-.945691444885E-03
0.953821713728E-05	-.5301040 68099E-01
0.955178615453E-03	0.233189952548E-01
0.362715490460E-01	-.258868194269E-02
0.216876372822E-01	0.157865852619E-01
0.857275037075E-01	0.855965062994E-02
-.408292565825E-02	0.407932002784E-02
-.231704817365E-01	-.288822137143E-01
0.142602603603E-02	-.133972804815E-02
-.809913758946E-03	0.315437414610E-02
-.187405614369E-03	-.234230541311E-03
-.272980529148E-04	0.121172385031E-03
-.415596940524E-05	-.156407058589E-05
0.549528008224E-05	-.212981873915E-05
0.767447829679E-07	0.379621311941E-07
0.182068289580E-08	0.186980825904E-08
0.330153679120E-10	0.318968182170E-10

The estimate of the maximum error in $|f_{17}(z)|$ determined by computing $e_{17}(z)$ at 60 uniformly distributed boundary points is,

$$E_{17} = |\, e_{17}(z)| = 3.6\times 10^{-5}.$$

2.5 Domain of Fig.5 ([14]).

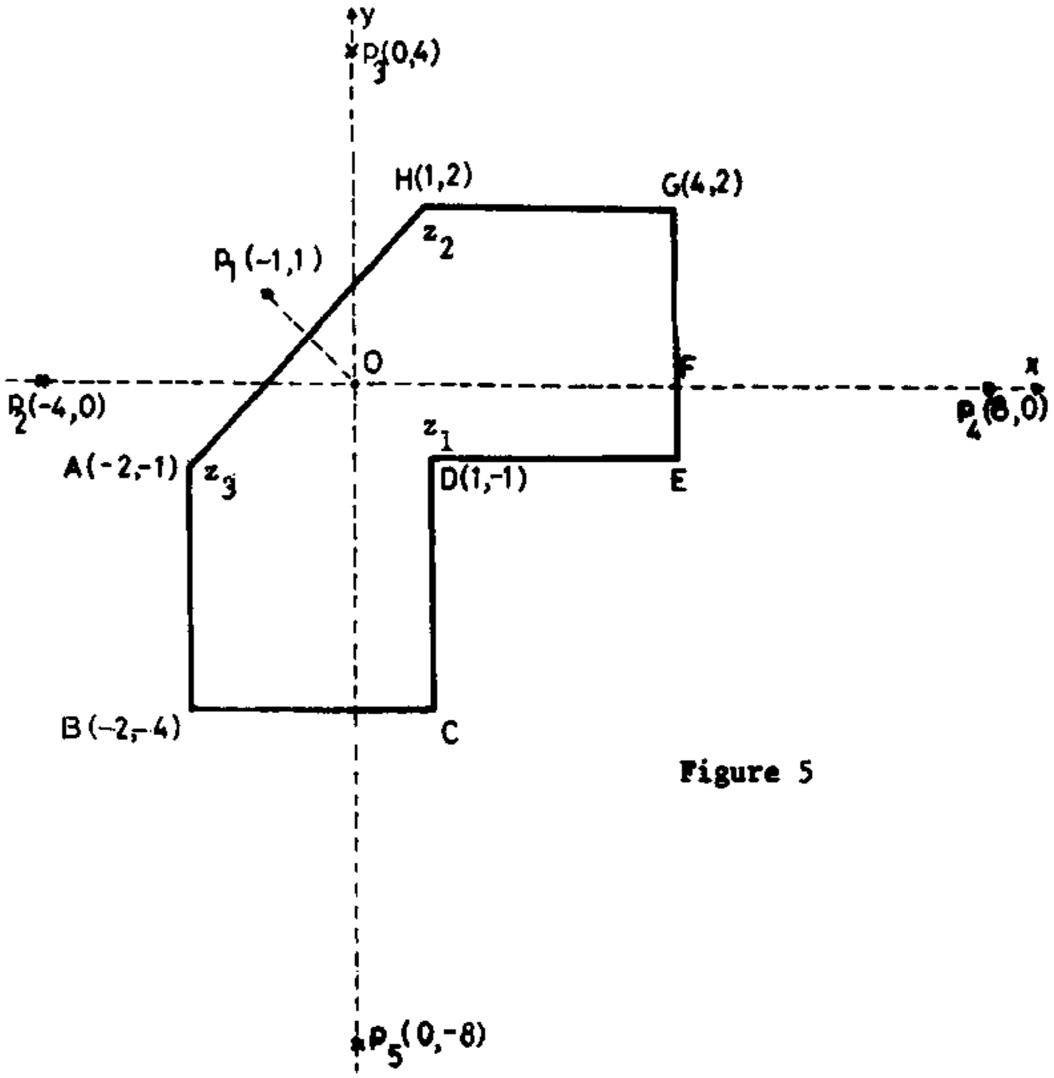

Figure 5

$$
\begin{aligned}
f_{20}(z) = c\ \Bigg\{ & \sum_{n=1}^{5} a_n z/(z-p_n) + a_6\{(z-z_1)^{2/3} - (-z_1)^{2/3}\} + a_7 z + a_8\{(z-z_1)^{4/3} - (-z_1)^{4/3}\} \\
& + \sum_{n=1}^{2} a_{n+8}\{(z-z_{n+1})^{3/2} - (-z_{n+1})^{3/2}\} + a_{11} z^2/2 + a_{12}\{(z-z_1)^{8/3} - (-z_1)^{8/3}\} \\
& + a_{13} z^3/3 + a_{14}\{(z-z_1)^{10/3} - (-z_1)^{10/3}\} + a_{15} z^4/4 + \sum_{n=1}^{2} a_{n+15}\{(z-z_{n+1})^{9/2} - (-z_{n+1})^{9/2}\} \\
& + \sum_{n=1}^{3} a_{n+17} z^{n+4}/(n+4) \Bigg\}.
\end{aligned}
$$

c=0.384813039878E+01

COEFFICIENTS a_n

0.159284421298E+00	-.159220301003E+00
-.101634739203E+00	-.310258109558E+00
0.265290450317E+00	0.126714109892E+00
-.179487385242E+00	0.618115987237E+00
-.647477350924E+00	0.165720704406E+00
0.140781101274E+00	0.140753194463E+00
-.884229280592E+00	-.115245672970E-01
-.504483222227E-01	0.503380232638E-01
0.183062993522E-01	-.557065867636E-01
-.260616373469E-01	0.520909654621E-01
0.265009406749E+01	0.328745759485E+01
0.702148907135E-01	-.700259989674E-01
-.181970916547E+00	0.385046440527E+01
-.245095003698E-01	-. 246046221971E-01
0.150427012012E+00	-.105778520099E+00
-.139780891609E-01	0.262235102543E-01
-.283704894445E-01	-.128636021221E-01
0.511893386102E-01	0.361885087423E-02
-.121278584821E-02	-.136056722742E-02
-.335846636037E-05	0.832372793781E-04

The estimate of the maximum error in $|f_{20}(z)|$ determined by computing $e_{20}(z)$ at 84 uniformly distributed boundary points is,

$$E_{20} = |\, e_{20}(z_F)| = 2.7\times 10^{-4}.$$

2.6 Domain of Fig.6 ([6],[17])

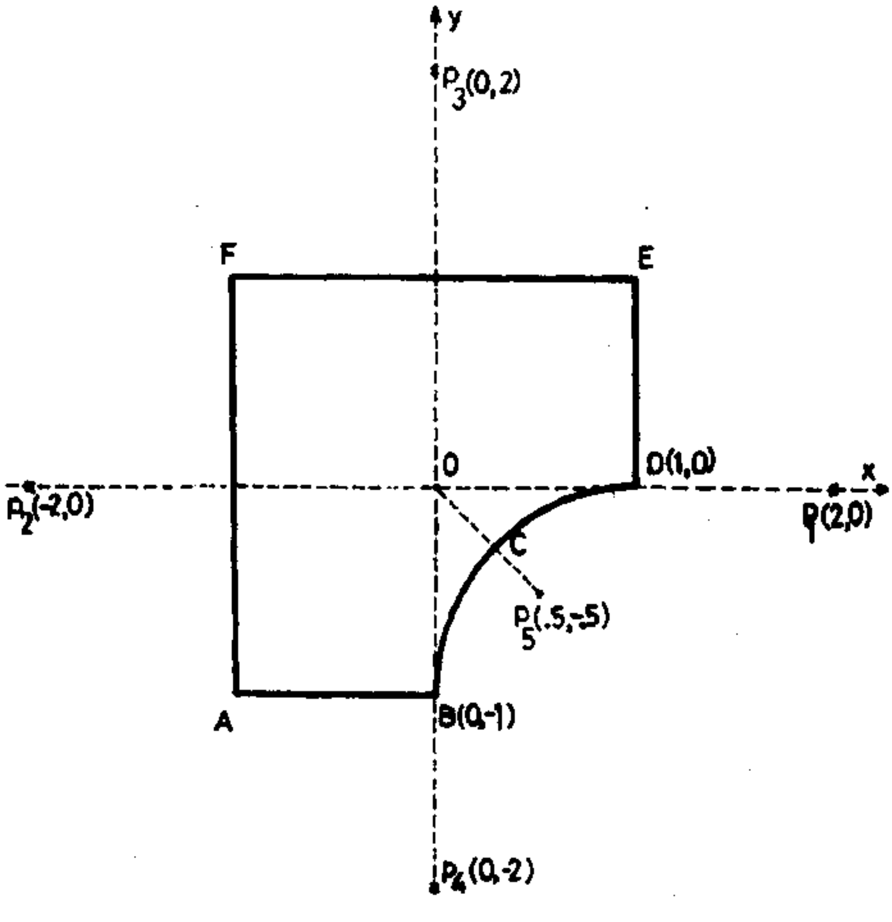

Figure 6

$$f_{24}(z) = c\left\{\sum_{n=1}^{5} a_n z/(z-p_n) + \sum_{n=1}^{19} a_{n+5} z^n/n\right\}.$$

c =0.228415919526E+01

COEFICIENTS a_n

0.159589174051E+01	0.616641145244E+01
-.404332665648E-01	0.117754814924E+00
-.117976206660E+00	0.401865912192E-01
-.616872184369E+01	-.159753804485E+01
-.159064714691E+00	0.159064726161E+00
0.192103490460E+01	-.104448828542E-02
0.393062784114E+01	0.393030848454E+01
-.524925366026E-03	0.455875556034E+01
-.122442838252E+01	0.122332203416E+01
0.561997919277E+00	-.326578297926E-03
0.706679205511E+00	0.706618976768E+00
-.762338109170E-04	0.632026838106E+00
-.136131717704E+00	0.135993954577E+00
0.738094509382E-01	-.363370940083E-04
0.742978269656E-01	0.742921098542E-01
-.782995047545E-05	0.565753236997E-01
-.629780866930E-02	0.628453632281E-02
0.168976932960E-01	-.342843855288E-05
0.132218375855E-01	0.132212897643E-01
-.731629360750E-06	0.125839534572E-01
-.453494793588E-02	0.453366655983E-02
-.234770689984E-02	-.436508702388E-06
-.480114222841E-03	-.480361182082E-03
0.503308162460E-07	-.128238705740E-03

The estimate of the maximum error in $|f_{24}(z)|$ determined by computing $e_{24}(z)$ at 35 suitably chosen boundary points is,

$$E_{24} = |\, e_{24}(z_C)| = 1.6\times10^{-5}.$$

2.7 Domain of Fig.7 ([17]-[19]).

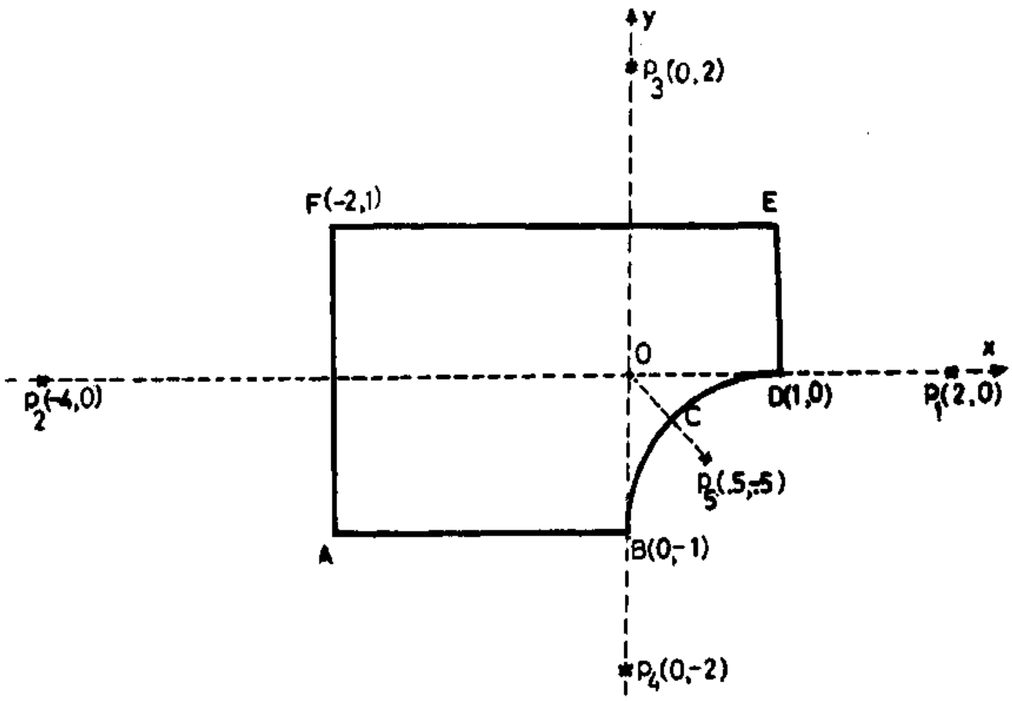

Figure 7

$$f_{20}(z) = c\left\{\sum_{n=1}^{5} a_n z/(z - p_n) + \sum_{n=1}^{15} a_{n+5} z^n / n\right\}.$$

$c = 0.238602453472E+01$

COEFFICIENTS a_n

0.338244334162E+00	-.938002569877E+00
0.142856383390E+02	-.922931228109E+01
-.100961839966E-01	-.147702977428E+00
-.274463761715E+00	0.601056362376E-01
-.158938315388E+00	0.159375794676E+00
0.327268249832E+01	0.170658047576E+01
0.217586460829E+01	0.159884062311E+01
-.507028864734E+00	0.197312115584E+00
0.266260234935E+00	-.351400849349E+00
-.164655269553E-01	-.107753511927E+00
0.428578583123E-01	-.966271278185E-01
-.165800199874E-01	-.657251368831E-01
0.613672240328E-02	-.702520658596E-01
0.269169252747E-01	-.430475686862E-01
0.265486218268E-01	-.126796315567E-01
0.128937127375E-01	0.181705537018E-02
0.353341167368E-02	0.307333883407E-02
0.436805808621E-03	0.123155300945E-02
-.115955371853E-04	0.242540950876E-03
-.778553068063E-05	0.209687291836E-04

The estimate of the maximum error in $|f_{20}(z)|$ determined by computing $e_{20}(z)$ at 36 suitably chosen boundary points is,

$$E_{20} = |\, e_{20}(z_C)| = 9.0\times 10^{-5}.$$

2.8 T-shaped domain of Fig.8 ([17],[19]).

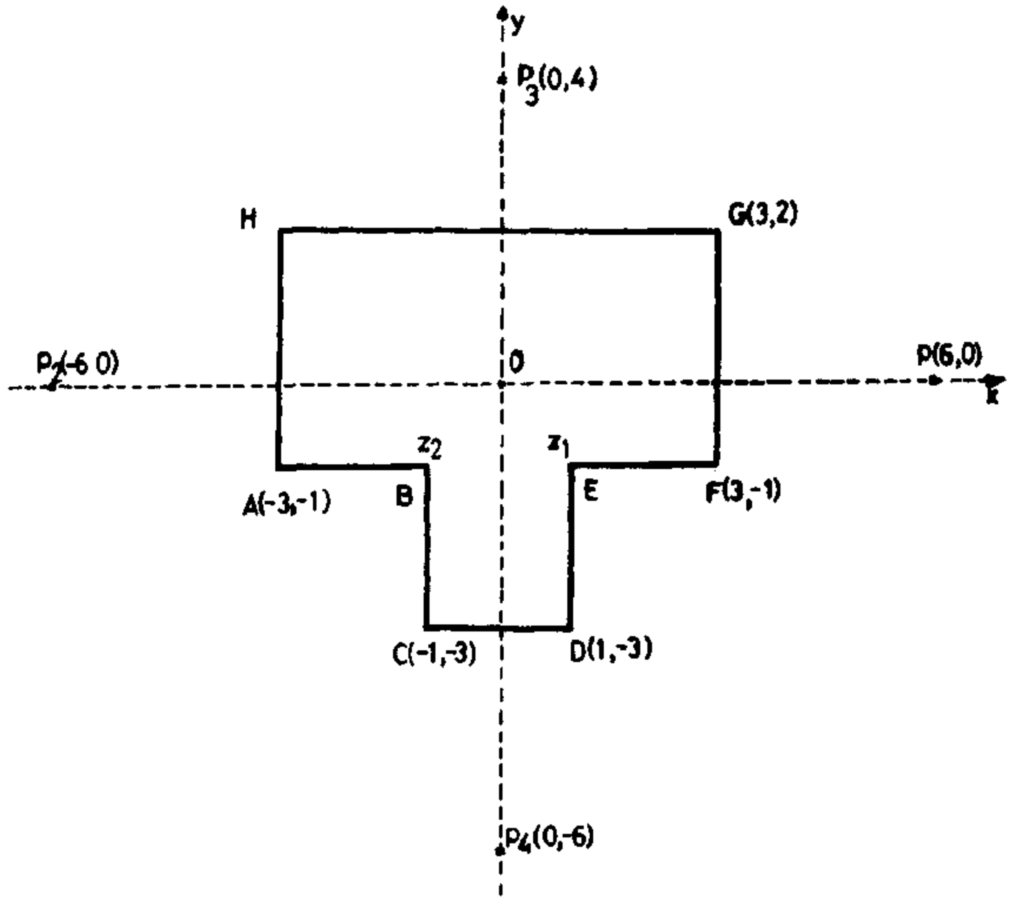

Figure 8

$$f_{22}(z) = c\left\{\sum_{n=1}^{4} a_n z/(z-p_n) + \sum_{n=1}^{2} a_{n+4}\{(z-z_n)^{2/3} - (-z_n)^{2/3}\} + a_7 z \right.$$
$$+ \sum_{n=1}^{2} a_{n+7}\{(z-z_n)^{4/3} - (-z_n)^{4/3}\} + a_{10} z^2/2 + \sum_{n=1}^{2} a_{n+10}\{(z-z_n)^{8/3} - (z_n)^{8/3}\}$$
$$\left. + \ a_{13} z^3/3 \ + \ \sum_{n=1}^{2} \ a_{n+13}\{(z-z_n)^{10/3} - (-z_n)^{10/3}\} \ + \ \sum_{n=1}^{7} \ a_{n+15} z^{n+3} \ /(n+3)\right\}.$$

c = 0.644171861702E+01

COEFFICIENTS a_n

-.139330408186E+00	0.292849207151E-01
0.139325986066E+00	0.292808688565E-01
0.483462401586E-07	-.783483097128E-01
-.459122784053E-05	0.264644492158E+00
0.993496867376E-01	0.115358319092E+00
0.149578177198E+00	0.283602158679E-01
0.190168028116E+01	0.349646913855E-06
-.570392561084E-01	0.587949732079E-01
-.794371512625E-01	0.199998284021E-01
-.789264457332E-05	-.183952550387E+01
0.849926982794E-01	-.872347496606E-01
-.330509646063E-01	0.117222306826E+00
0.856794253736E-02	0.435622877023E-05
-.339908165943E-01	-.334777708563E-01
0.119970293070E-01	0.461752980875E-01
0.878434670236E-07	-.157918935838E-01
0.946928665881E-03	0.183287478603E-10
0.826932157977E-09	0.662089384249E-04
0.108990075613E-05	0.455191540715E-09
-.693802832716E-10	0.310843458394E-05
-.7565488 60137E-06	-.233774906093E-11
-.126155586361E-11	-.842157925650E-07

The estimate of the maximum error in $|f_{22}(z)|$ determined by computing $e_{22}(z)$ at 76 uniformly distributed points on the boundary is,

$$E_{22} = |\, e_{22}(z_B)| = 7.6\times10^{-5}.$$

2.9 Octagon of Fig.9 ([2],[8],[11],[12],[15]).

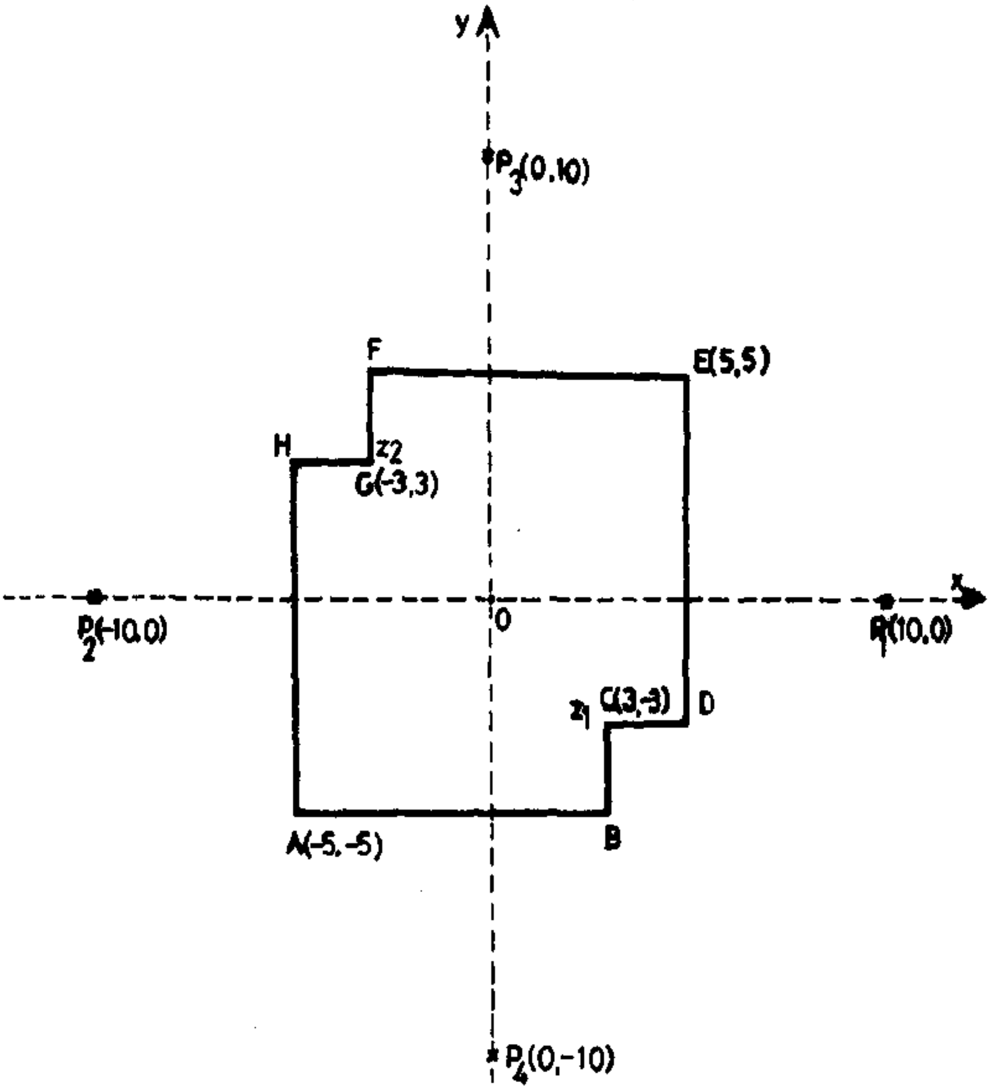

Figure 9

$$f_{23}(z) = c\left\{\sum_{n=1}^{2} a_n z/\{z^2 + (-1)^n 100\} + \sum_{n=1}^{2} a_{n+2}\{(z-z_n)^{2/3} - (-z_n)^{2/3}\}\right.$$

$$+ a_5 z + \sum_{n=1}^{2} a_{n+5}\{(z-z_n)^{4/3} - (-z_n)^{4/3}\} + \sum_{n=1}^{2} a_{n+7}\{(z-z_n)^{8/3} - (-z_n)^{8/3}\}$$

$$\left. + \sum_{n=1}^{14} \quad a_{n+9} z^{2n+1}/(2n+1)\right\}.$$

$c = 0.161338731615E+02$

COEFFICIENTS a_n

-.502221568821E+01	-.271655780489E+02
0.575144495893E+01	-.273275962284E+02
0.157961627689E-01	0.157961092356E-01
0.215779132689E-01	-.578182362464E-02
-.117027339008E-01	0.162858580467E-02
-.285568077824E-02	0.285548250042E-02
-.390076077118E-02	-.104535084836E-02
0.101280737002E-02	-.101262212726E-02
-.370552801642E-03	-.138342797524E-02
0.218532392472E-03	-.139358736891E-01
-.499309982568E-04	0.809660612957E-06
0.510513062858E-07	-.383955355692E-05
-.811961965522E-08	0.145915691135E-09
0.801915064071E-11	-.591943535002E-09
-.134678095638E-11	0.210100502776E-13
0.109470114287E-14	-.831910355335E-13
-.140124034777E-15	0.281583928368E-17
0.135982628640E-18	-.997112188560E-17
-.240106737718E-19	0.353288221087E-21
0.156350345451E-22	-.140067659335E-20
0.589916787888E-23	0.918065754649E-25
0.163723265798E-27	-.106784710352E-25
-.592414723668E-28	-.795542066066E-30

The estimate of the maximum error in $|f_{23}(z)|$ determined by computing $e_{23}(z)$ at a selection of boundary points (see [10], example 4) is,

$$E_{23} = | e_{23} (z_A) | = 5.7\times10^{-6}.$$

2.10. Cross-shaped domain for Fig.10 ([4],[5]).

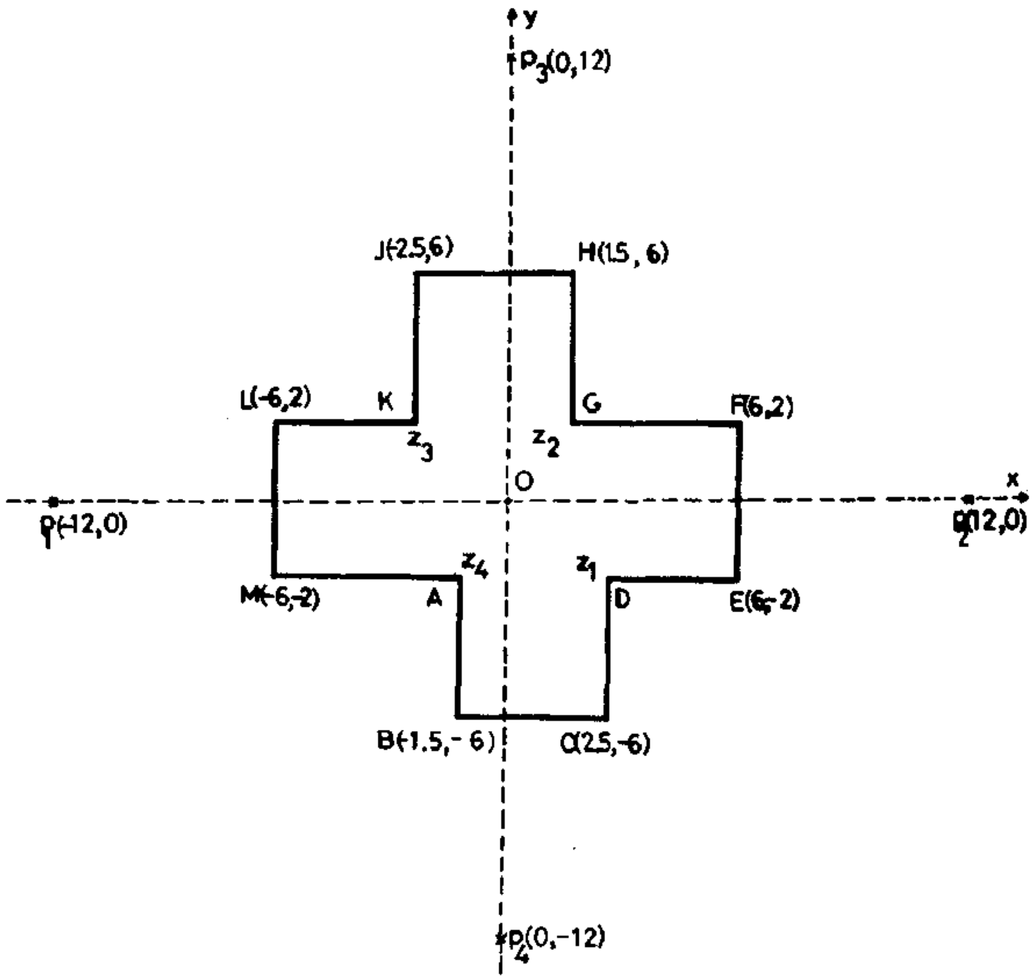

Figure 10

$$
\begin{aligned}
f_{31}(z) = c\Bigg\{ & \sum_{n=1}^{2} a_n z/\{z^2+(-1)^n 144\} + \sum_{n=1}^{4} a_{n+2}\{(z-z_n)^{2/3}-(-z_n)^{2/3}\} \\
& + a_7 z + \sum_{k=1}^{2} + \left[\sum_{n=1}^{4} a_{4k+n+3}\{(z-z_n)^{4k/3}-(-z_n)^{4k/3}\} \right] + a_{16} z^3/3 \\
& + \sum_{k=1}^{2} \left[\sum_{n=1}^{4} a_{4(k+3)+n}\{(z-z_n)^{(4k+6)/3}-(-z_n)^{(4k+6)/3}\} \right] \\
& + \sum_{n=1}^{7} a_{n+24} z^{2n+3}/(2n+3) \Bigg\}.
\end{aligned}
$$

c=0.110654953585E+02

COEFFICIENTS a_n

-.113547022434E+03	0.402422485026E+02
0.400289588727E+02	-.207267503800E+03
0.240508211282E-01	0.3109 28150582E-01
0.516711472991E-02	0.602533014794E-01
0.389526843128E-01	-.528223562672E-02
0.547645283394E-01	0.256518129974E-01
-.972995934216E+00	0.222881525546E+01
-.814505415142E-02	0.299070129942E-02
-.137121226750E-01	0.173543085862E-01
-.666237060585E-02	-.555839096574E-02
-.218852006291E-01	-.319797355990E-02
0.322603704159E-02	0.179420381890E-03
-.129475994397E-01	-.453105050884E-02
0.176838702364E-02	-.270405385863E-02
-.103978386334E-01	0.894736536272E-02
0.296733986385E-01	-.117028969715E+00
0.122459991609E-02	0.326804521153E-03
0.113376127423E-02	0.275668175064E-02
0.329266902964E-03	0.122394289036E-02
-.182049401568E-02	0.236020322521E-02
-.141928705109E-03	-.144420582969E-03
-.361762736184E-04	-.148590025862E-03
-.196036049499E-03	0.507034917087E-04
-.146770771147E-03	-.429653402343E-04
0.759304019281E-03	0.429105019088E-03
-. 110009448857E-05	-.292378113692E-05
-.301644020385E-07	0.367056995633E-07
-.945621364427E-10	-.164571343843E-09
-.397839212546E-12	0.215236399098E-11
-.728964203330E-14	-.217919914769E-13
-.255784943886E-15	0.258414598822E-15

The estimate of the maximum error in $|f_{31}(z)|$determined by computing $e_{31}(z)$ at 96 uniformly distributed points on the boundary is,

$$|e_{31}(z_D)| = 5.7 \times 10^{-5}.$$

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