N-Step MPC for Systems with Persistent Bounded Disturbances under Stochastic Communication Protocol

Yan Song, Zidong Wang, Shuai Liu and Guoliang Wei

***Abstract*—This paper is concerned with the N-step model predictive control (MPC) problem for a class of constrained systems with persistent bounded disturbances under the stochastic communication protocol (SCP). The control signals are transmitted to the plant via a shared network subject to a prescribed SCP for the purpose of avoiding data collisions. The SCP scheduling, which is governed by a Markov chain, is applied to orchestrate the transmission order of the controller nodes. Under the SCP, only one control node is allowed to update the control signal sent to the plant at each communication instant. Our aim is to design a set of desired controllers in the framework of N-step MPC such that the mean-square input-to-state stability of the closed-loop system is guaranteed. An optimization algorithm consisting of both off-line and online parts is developed to cope with the design problem of the N-step controller. Finally, a numerical example is utilized to illustrate the validity of the proposed N-step MPC strategy.**

***Index Terms*—N-step model predictive control, stochastic communication protocol, mean-square input-to-state stability, persistent bounded disturbances.**

I. Introduction

Over the past few decades, due to its practical insights, the model predictive control (MPC) problem has attracted considerable research interest in systems with bounded disturbances, which gives rise to robust MPC (RMPC), see [4], [10], [17], [35] for discrete-time systems and [21], [26]–[28] for continuous-time systems. Up to date, RMPC strategy has been extensively applied into various engineering areas such as chemical process, power systems, DC motors and mobile robots [8], [10]–[14], [31], [36]. From the technical viewpoint, there are generally two approaches dealing with the external bounded disturbances for discrete-time systems with respect to the RMPC problem, namely, the "min-max" optimization approach [24] and the open-loop optimization approach [3], [15]. More specifically, the main idea of the "min-max" approach is to minimize the objective function in the worst possible realization of the disturbances. As mentioned in [2], such an approach would bring in additional computation burden while possibly leading to poor performance. By contrast, the open-loop optimization approach is more efficient in the sense of achieving the desired robustness performance while its feasibility cannot be ensured. Note that, in the context of RMPC algorithm development, it is rather difficult to have an appropriate methodology capable of examining the impacts from external bounded disturbances on the control performance.

For traditional MPC approaches, an arguably drawback is the heavy computation burden caused by the online optimization procedure which, in turn, largely hinders their applications in practical engineering. To overcome such a limitation without considerably sacrificing the system performance, tremendous research efforts have recently been invested in the design of the *off-line* control algorithms, see e.g. [10], [25]. Among various existing off-line algorithms in the MPC paradigm, the N-step MPC (also called as multi-step MPC) proposed in [21], [22] is recognized as a powerful approach that has been widely used in a variety of engineering systems. Technically speaking, the N-step MPC aims to design a series of control inputs by means of solving certain optimization problems such that the system state located in the initial feasible region will be steered into a fixed terminal constraint set within N time steps. Moreover, with the increase of the step number N, the size of the initial feasible region grows at the cost of increasing the computation complexities. As such, a well-noted challenge for the N-step MPC problem is how to design an appropriate control strategy so as to make the right trade-off between the initial feasible region and the computation complexities, and this constitutes the main motivation of our current investigation.

On another research frontier, networked control systems (NCSs) have aroused considerable research interest owing to their merits in low cost, simple installation and easy implementation, see e.g. [5], [6], [29], [30], [32], [33]. In a spatially distributed NCS, sensors/controllers are often installed remotely to obtain the measurements/calculations and transmit the signals separately to a certain target node (i.e., scheduler). The components (sensors, controllers, actuators, etc.) of the NCSs execute the data-exchange tasks via a commonly shared communication network. As pointed out in [20], data collision might occur in carrier sense multiple accesses with collision detection (CSMA/CD) if more than one link detects the idle common channel and sets out the transmission. Consequently, to prevent the data packets from being congested, an effective

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method is to apply the so-called *communication protocols* to schedule the transmission order of the data to be sent. So far, most frequently used communication protocols in industry include, but are not limited to, the Try-Once-Discard (TOD) protocol [18], [38], the Round-Robin (RR) protocol [23], [35], [38] and the stochastic communication protocol (SCP) [7], [19], [34], [37]. The RR protocol is a typical static scheduling, where data are transmitted in a pre-given order. In contrast, the TOD and SCP protocols are two dynamic scheduling, where the links are scheduled according to the time varying errors (the difference between the latest two measurements) or in a stochastic manner. Compared with the static scheduling, the dynamic one has a better flexibility for the resource allocation and scheduling.

With respect to the SCP scheduling, the probability distribution for a certain node to gain the random priority so as to be accessible to the shared communication network is usually characterized by a Markov chain [37] or a Bernoulli process [20], where the zero-hold-order input mechanism or the zero input mechanism is used to compensate the nodes/components that don't obtain the privilege for the updating. For example, a new Markov chain has been constructed in [37] to model the SCP scheduling of communication networks, and the H_∞ control problem has been investigated for the established closed-loop system. Among the aforementioned three communication protocols (RR, TOD and SCP), the SCP protocol reflects the random selection of the node gaining the transmission privilege, thereby ensuring the equal allocation for the nodes in a complex networked environment. Although the SCP has been extensively investigated in various NCSs, the N-step MPC problem for discrete-time systems under the SCP has not yet been adequately studied due mainly to its mathematical difficulties in the analysis/design of the control strategy, not to mention the case where the persistent bounded disturbances are also considered.

In response to above discussions, in this paper, we aim to investigate the SCP-based N-step MPC problem for a class of linear discrete-time systems with persistent bounded disturbances. To be more specific, our goal is to design a set of desired controllers in the framework of N-step MPC such that the closed-loop system is mean-square input-to-state stable (ISS). To achieve such an objective, three identified challenges need to be overcome: 1) how to solve the terminal constraint set off-line for the addressed system subject to the underlying SCP and the persistent bounded disturbances? 2) how to design the multi-step controller to steer the state into the obtained terminal constraint set? and 3) how to examine the impact from the SCP and the persistent bounded disturbances on the system stability? It is, therefore, the primary motivation of this paper to provide satisfactory answers to these three questions. Some specific mathematical tools (e.g. quadratic boundedness technique, input-to-state stability theory and convex optimization approach) will be employed to facilitate the multi-step controller analysis/synthesis for constrained systems with the persistent bounded disturbances under the SCP.

The main contributions are highlighted as follows: *1) the addressed problem is new in the sense that this paper makes the first attempt to deal with the RMPC problem with persistent bounded disturbances under the SCP; 2) some sufficient conditions on the N-step MPC strategy are established so as to ensure the mean-square input-to-state stability of the addressed systems; and 3) the impacts from the SCP and persistent bounded disturbances are clearly reflected on the proposed optimization algorithm.* The rest of this paper is organized as follows. In Section II, the system with persistent bounded disturbances under the SCP is introduced and some necessary preliminaries are presented. In Section III, some sufficient conditions are established to compute the terminal constraint set and the corresponding control law. In Section IV, an off-line optimization problem is first developed to obtain a sequence of robust one-step sets (ROSSs), and then an online optimization problem is proposed to derive the corresponding control laws. In Section V, the mean-square input-to-state stability of the addressed closed-loop system is ensured, and an optimization algorithm including both online and off-line parts is proposed. Subsequently, a simulation example is provided in Section VI to illustrate the effectiveness of the proposed algorithm. Finally, we summarize the paper in Section VII.

Notation The notation used here is fairly standard except where otherwise stated. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. $\mathbb{Z}_+$ and $\mathbb{R}_+$ are used to denote the sets of all nonnegative integers and reals, respectively. I and 0 represent the identity and zero matrices of compatible dimensions, respectively. The shorthand $\mathrm{diag}\{M_1, M_2, \ldots, M_n\}$ denotes a block diagonal matrix with diagonal blocks being matrices $M_1, M_2, \ldots, M_n$. $||x||$ describes the Euclidean norm of a vector x. We denote $\|x\|_W^2 \triangleq x^T W x$, where $W > 0$ is a symmetric weighting matrix. Given two sets A and B, $A\backslash B$ denotes the difference set of A and B. $\bullet(k+n|k)$ denotes the prediction value at the future time instant $k+n$ predicted at real time k, specially, $\bullet(k|k) \triangleq \bullet(k)$. In symmetric block matrices, the symbol "$*$" is used as an ellipsis for terms induced by symmetries. Matrix $X > 0$ $(X \geq 0)$ means that each entry of X is positive (non-negative). Moreover, for two symmetric matrices X and Y, $X \geq Y$ (especially, $X > Y$) means that $X - Y$ is positive semi-definite (especially, positive definite). M^T represents the transpose of M. The probability of the occurrence of event "$\cdot$" is denoted by Prob$\{\cdot\}$. $\mathbb{E}\{x\}$ stands for the mathematical expectation of the stochastic variable x. A function $\gamma : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is a $\mathcal{K}$-function if it is continuous and strictly increasing with $\gamma(0) = 0$. γ is a $\mathcal{K}_\infty$-function if it is a $\mathcal{K}$-function and satisfies $\gamma(t) \to \infty$ as $t \to \infty$. A function $\beta : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ is a $\mathcal{KL}$-function if, for each $k \geq 0$, $\beta(\cdot, k)$ is a $\mathcal{K}$-function, and for each fixed $t \geq 0$, the function $\beta(t, \cdot)$ is decreasing with $\beta(t, k) \to 0$ as $k \to \infty$.

II. Preliminaries and Problem Formulation

A. The system model under the SCP scheduling

Consider the following linear discrete-time system with persistent bounded disturbances:

$$x(k+1) = Ax(k) + Bu(k) + E\omega(k) \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the system state and $u(k) \in \mathbb{R}^m$ is the control input after being transmitted via communication networks. Matrices A, B and E are some known matrices with appropriate dimensions. $\omega(k)$ is the exogenous disturbance and satisfies

$$\omega(k) \in \mathbb{W} \triangleq \left\{\omega(k) \in \mathbb{R}^{w} | \, \omega^{T}(k)\omega(k) \leq \varpi\right\} \tag{2}$$

where $\varpi > 0$ is a known scalar, and $\mathbb{W}$ is a compact set involving the origin.

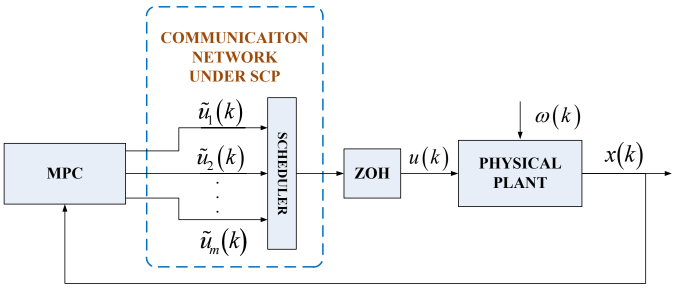

Fig. 1. The structure of the MPC-based closed-loop system under the SCP scheduling.

For illustration convenience, the controller is said to have n nodes corresponding to the n components of the control input. As described in Fig. 1, $\tilde{u}_i(k) \in \mathbb{R}$ $(i = 1, 2, \ldots, m)$ stands for the signal from the ith controller node before being transmitted via the network. To avoid data collisions, the SCP is employed during the data transmission from the controller to the plant via a communication network equipped with a SCP scheduler. More precisely, at each time instant, only one controller node has an access to the shared communication network, which is determined by a sequence of random variables, that is, only one component of control inputs defined by $\tilde{u}(k) \triangleq [\tilde{u}_1(k) \ \tilde{u}_2(k) \ \ldots \ \tilde{u}_m(k)]^T$ is updated, and other nodes without the transmission right will hold the last transmission values by the zero-order holders (ZOHs).

Associated with the SCP, we denote the indicator function as $\theta_k \in \mathbb{M} \triangleq \{1, 2, \ldots, m\}$, which indicates the selected controller node at the time instant k. According to [37], the random variable θ_k can be described by a Markov chain with the transition probability matrix $\mathcal{P} = [p_{ij}]_{m\times m}$ whose (i, j)th entry is defined by

$$p_{ij} \triangleq \text{Prob}\left(\theta_{k+1} = j|\theta_k = i\right) \tag{3}$$

where $p_{ij} \geq 0$ $(i, j \in \mathbb{M})$ is the transition probability from node i to node j and $\sum_{j=1}^{m} p_{ij} = 1$.

Define the control signal received by the plant as $u(k) \triangleq [u_1(k) \ u_2(k) \ \ldots \ u_m(k)]^T$. Under the scheduling of SCP, the updating rule for $u_i(k)$ can be expressed as

$$u_i(k) = \begin{cases} \tilde{u}_i(k), & \text{if } \ i = \theta_k \\ u_i(k-1), & \text{otherwise.} \end{cases} \tag{4}$$

Combining (3) with (4), $u(k)$ can be rewritten as the following compact form

$$u(k) = \Phi_{\theta_k}\tilde{u}(k) + (I - \Phi_{\theta_k})u(k-1) \tag{5}$$

where $\Phi_{\theta_k} \triangleq \operatorname{diag}\left\{\delta\left(\theta_k - 1\right), \delta(\theta_k - 2), \ldots, \delta(\theta_k - m)\right\}$ with $\delta(\theta_k - i)$ $(i = 1, 2, \ldots, m)$ being a Kronecker delta function satisfying $\delta(\theta_k - i) = 1$ for $\theta_k = i$, otherwise $\delta(\theta_k - i) = 0$.

To better reflect the engineering practice, the following hard constraints are taken into consideration:

$$\begin{cases} |[\tilde{u}(k)]_p| \leq [\bar{u}]_p, & p \in \{1, \ldots, m\} \quad \text{(6a)} \\ |[\Psi]_q x(k)| \leq [\bar{x}]_q, & q \in \{1, \ldots, o\} \quad \text{(6b)} \end{cases}$$

where $\Psi \in \mathbb{R}^{r\times n}$ is a known real matrix, $\bar{u} > 0$ and $\bar{x} > 0$ are known vectors, and $[\cdot]_i$ $(i \in \{p, q\})$ denotes the ith element of a vector or the ith row of a matrix.

B. N-step MPC strategy

Along the similar line as [10], the control law in the framework of the N-step MPC strategy is determined by

$$\tilde{u}(k+n|k) = \begin{cases} K_{\theta_{k+n|k},n}x(k+n|k), & 0 \leq n < N \\ K_{\theta_{k+n|k},N}x(k+n|k), & n \geq N \end{cases} \tag{7}$$

where $\theta_{k+n|k}$ denotes the predicted controller node to be selected at the time instant $k+n$ based on the current selected controller node θ_k, and $\theta_{k+n|k} \in \mathbb{M}$. $K_{\theta_{k+n|k},n}$ $(0 \leq n \leq N)$ are multi-step controller gains to be determined in the sequel.

By denoting $\xi(k+n|k) = [x^T(k+n|k) \quad u^T(k+n-1|k-1)]^T$, one obtains the following dynamic system with the persistent bounded disturbances:

$$\xi(k+n+1|k) = \mathcal{A}_{\theta_{k+n|k},n}\xi(k+n|k) + \mathcal{E}\omega(k+n|k) \tag{8}$$

where $\mathcal{E} = \begin{bmatrix} E^T & 0 \end{bmatrix}^T$ and

$$\mathcal{A}_{\theta_{k+n|k},n} = \begin{bmatrix} A + B\Phi_{\theta_{k+n|k}}K_{\theta_{k+n|k},n} & B(I - \Phi_{\theta_{k+n|k}}) \\ \Phi_{\theta_{k+n|k}}K_{\theta_{k+n|k},n} & I - \Phi_{\theta_{k+n|k}} \end{bmatrix}.$$

For later development, some necessary definitions are presented as follows.

Definition 1: For the closed-loop system (8), the set $\mathbf{X}$ is said to be a robust positively invariant (RPI) set if $\xi(k+n+1|k) \in \mathbf{X}$ for all $\xi(k+n|k) \in \mathbf{X}$ $(k, n \in \mathbb{Z}_+)$ and allowable disturbances $w(k+n|k)$.

Definition 2: [25]. The RPI set $\mathcal{Q}(\Omega)$ is said to be a robust one-step set (ROSS) if all states for allowable disturbances can be steered into an RPI set Ω by an admissible input.

Definition 3: [9]. The system (8) is said to be mean-square input-to-state stable (ISS) if there exist functions $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}$ such that

$$\mathbb{E}\{\|\xi(k)\|^2\} \leq \beta(\|\xi(0)\|^2, k) + \gamma(\|\omega(k)\|_\infty^2)$$

where $\|\omega(k)\|_\infty^2 \triangleq \sup_{k\in\mathbb{Z}_+}\{\|\omega(k)\|^2\}$ and $\xi(0)$ is an initial state vector.

For the closed-loop system (8) with the control strategy (7), the objective of this paper is twofold:

- R1) design the multi-step controller gains $K_{\theta_{k+n|k},n}$ for all $\theta_{k+n|k} \in \mathbb{M}$, $0 \leq n \leq N$ to obtain a sequence of ROSSs $\{\mathbb{P}_0, \mathbb{P}_1, \ldots, \mathbb{P}_{N-1}\}$ and the terminal constraint set Ω such that, in the simultaneous presence of persistent bounded disturbances and SCP, the states included in the initial feasible region $\mathbb{P}_0 \cup \mathbb{P}_1 \cup \ldots \cup \mathbb{P}_{N-1}$ can be steered into the terminal constraint set Ω within N steps; and
- R2) establish a set of sufficient conditions to guarantee that the closed-loop system (8) achieves the mean-square

input-to-state stability while entering into the terminal constraint set.

Remark 1: In terms of the time steps required to ensure the prediction states entering into the terminal constraint set, the MPC strategy can be generally classified into two categories, one is called the "zero-step" MPC strategy which means that the initial prediction state is included in the terminal constraint set, and the other is the "N-step" MPC strategy where the initial prediction state enters into the terminal constraint set within N steps. It is worth noting that, compared with the "zero-step" MPC strategy, one distinct feature of the "N-step" MPC strategy lies in that the size of the initial feasible region is obviously enlarged. In the "N-step" MPC strategy framework, after the state enters into the terminal constraint set, only the feedback control law corresponding to the terminal constraint set is applied to the plant, and the computation burden is thus effectively reduced.

III. Terminal constraint set

For $\forall r \in \mathbb{M}$, define the following set:

$$\Omega_{r,N} \triangleq \{\xi | \xi^T P_{r,N} \xi \le \varphi\} \tag{9}$$

where $P_{r,N} = \text{diag}\{\tilde{P}_{1r,N}, \tilde{P}_{2r,N}\}$ $(r \in \mathbb{M})$ with $\tilde{P}_{ir,N}$ $(i \in \{1,2\})$ denoting positive-definite matrices with appropriate dimensions to be designed. $\varphi > 0$ is a positive scalar.

As stated in [16], if the following requirements are simultaneously satisfied:

1) the set $\Omega_{\theta_{k+n|k},N}$ is an RPI set subject to constraints (6a)-(6b);
2) the terminal cost function $V(\xi(k+n|k))$ defined by $V(\xi(k+n|k)) \triangleq \xi^T(k+n|k) P_{\theta_{k+n|k},N} \xi(k+n|k)$ with $\xi(k+n|k) \in \Omega_{\theta_{k+n|k},N}$ is a local Lyapunov-like function satisfying

$$\begin{aligned} &\mathbb{E}\{V(\xi(k+n+1|k))\} - V(\xi(k+n|k)) \\ &\le -\|\xi(k+n|k)\|_Q^2 - \|\tilde{u}(k+n|k)\|_R^2 \\ &\quad + \lambda \|\omega(k+n|k)\|^2 \end{aligned} \tag{10}$$

where Q and R are known positive-definite weighting matrices, and $\lambda > 0$ is a known scalar,

then the set $\Omega_{\theta_{k+n|k},N}$ is a terminal constraint set of the closed-loop system (8) with constraints (6a)-(6b).

In what follows, we shall address the above requirements for the terminal constraint set in a step-by-step manner. Then, an off-line optimization problem is provided to obtain the terminal constraint set as well as the corresponding feedback gain.

A. Robust positively invariant set

To begin with, by resorting to the quadratic boundedness technique [1], [25], the following sufficient condition is directly put forward to ensure that the set $\Omega_{r,N}$ $(r \in \mathbb{M})$ is an RPI set. For brevity, we denote $s \triangleq \theta_{k+n|k}$ and $t \triangleq \theta_{k+n+1|k}$.

Lemma 1: The set $\Omega_{s,N}$ defined by (9) is an RPI set if the following condition

$$\frac{1}{\varphi}\mathbb{E}\left\{\|\xi(k+n+1|k)\|_{P_{t,N}}^2\right\} - \frac{1}{\varphi}\|\xi(k+n|k)\|_{P_{s,N}}^2 \le 0 \tag{11}$$

under the constraint

$$\frac{1}{\bar{\omega}}\|\omega(k+n|k)\|^2 \le \frac{1}{\varphi}\|\xi(k+n|k)\|_{P_{s,N}}^2 \tag{12}$$

holds.

Proof: The proof can be carried out along the similar line in [1], [25], and the details are therefore omitted. ■

Next, by means of the stochastic analysis technique, we are ready to establish some matrix inequalities to guarantee (11)-(12).

Lemma 2: Let the bound of disturbance $\bar{\omega}$, the matrix Ψ and the transition probability matrix $\mathcal{P} = [p_{st}]_{m\times m}$ be given. For the system (8) with constraints (6a)-(6b), suppose that there exist positive-definite matrices $\tilde{Y}_{1s,N}$, $\tilde{Y}_{2s,N}$, $\mathbb{X}_{s,N}$, $\mathbb{Y}_{s,N}$, a matrix $Z_{s,N}$, scalars $0 < \alpha \le 1$ and $\varphi > 0$ such that for $\forall s \in \mathbb{M}$, the following inequalities hold:

$$\begin{bmatrix} (1-\alpha)Y_{s,N} & * & * & * & * & * \\ 0 & \frac{\alpha}{\bar{\omega}}I & * & * & * & * \\ \Gamma_{s,N} & \mathcal{E} & \frac{Y_{1,N}}{p_{s1}} & * & * & * \\ \Gamma_{s,N} & \mathcal{E} & 0 & \frac{Y_{2,N}}{p_{s2}} & * & * \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \Gamma_{s,N} & \mathcal{E} & 0 & 0 & \cdots & \frac{Y_{m,N}}{p_{sm}} \end{bmatrix} \ge 0, \tag{13}$$

$$\begin{bmatrix} \mathbb{X}_{s,N} & * \\ Z_{s,N}^T & \tilde{Y}_{1s,N} \end{bmatrix} \ge 0,\ [\mathbb{X}_{s,N}]_{pp} \le [\bar{u}]_p^2,\ p = 1,\dots,m, \tag{14}$$

$$\begin{bmatrix} \mathbb{Y}_{s,N} & * \\ \tilde{Y}_{1s,N}^T \Psi^T & \tilde{Y}_{1s,N} \end{bmatrix} \ge 0,\ [\mathbb{Y}_{s,N}]_{qq} \le [\bar{x}]_q^2,\ q = 1,\dots,o \tag{15}$$

where $[\cdot]_{jj}$ $(j \in \{p,q\})$ denotes the jth diagonal element of the matrix "$\cdot$" and

$$\begin{aligned} \Gamma_{s,N} &= \begin{bmatrix} A\tilde{Y}_{1s,N} + B\Phi_s Z_{s,N} & B(I-\Phi_s)\tilde{Y}_{2s,N} \\ \Phi_s Z_{s,N} & (I-\Phi_s)\tilde{Y}_{2s,N} \end{bmatrix}, \\ Y_{s,N} &= \text{diag}\{\tilde{Y}_{1s,N}, \tilde{Y}_{2s,N}\}, Y_{s,N} = \varphi P_{s,N}^{-1}, \\ \tilde{Y}_{js,N} &= \varphi \tilde{P}_{js,N}^{-1},\ j \in \{1,2\}. \end{aligned}$$

Then, the ellipsoid set $\Omega_{s,N}$ defined by (9) is an RPI set. Furthermore, the corresponding controller gain is given by

$$K_{s,N} = Z_{s,N}\tilde{Y}_{1s,N}^{-1}. \tag{16}$$

Proof: By using the Schur Complement, (13) holds if and only if

$$\begin{bmatrix} \Pi_{s,N}^{11} & * \\ \Pi_{s,N}^{21} & \Pi_{s,N}^{22} \end{bmatrix} \ge 0 \tag{17}$$

where

$$\begin{aligned} \Pi_{s,N}^{11} &= (1-\alpha)Y_{s,N} - \Gamma_{s,N}^T \sum_{r=1}^{m} p_{sr} Y_{r,N}^{-1} \Gamma_{s,N}, \\ \Pi_{s,N}^{21} &= -\mathcal{E}^T \sum_{r=1}^{m} p_{sr} Y_{r,N}^{-1} \Gamma_{s,N}, \\ \Pi_{s,N}^{22} &= \frac{\alpha}{\bar{\omega}} I - \mathcal{E}^T \sum_{r=1}^{m} p_{sr} Y_{r,N}^{-1} \mathcal{E}. \end{aligned}$$

Subsequently, pre- and post-multiplying (17) with $\mathrm{diag}\{Y_{s,N}^{-1}, I\}$ and its transpose, the following inequality is obtained from (8) and (33):

$$\begin{bmatrix} \Xi_{s,N}^{11} & * \\ \Xi_{s,N}^{21} & \Pi_{s,N}^{22} \end{bmatrix} \geq 0 \tag{18}$$

where

$$\mathcal{A}_{s,N} = \begin{bmatrix} A + B\Phi_s K_{s,N} & B\left(I - \Phi_s\right) \\ \Phi_s K_{s,N} & I - \Phi_s \end{bmatrix},$$
$$\Xi_{s,N}^{11} = (1-\alpha)Y_{s,N}^{-1} - \mathcal{A}_{s,N}^T \sum_{r=1}^{m} p_{sr} Y_{r,N}^{-1} \mathcal{A}_{s,N},$$
$$\Xi_{s,N}^{21} = -\mathcal{E}^T \sum_{r=1}^{m} p_{sr} Y_{r,N}^{-1} \mathcal{A}_{s,N}.$$

Moreover, pre- and post-multiplying (18) with $[\xi^T(k+n|k) \quad \omega^T(k+n|k)]$ and its transpose, we have

$$\begin{aligned} &(\mathcal{A}_{s,n}\xi(k+n|k) + \mathcal{E}\omega(k+n|k))^T \sum_{r=1}^{m} p_{sr} Y_{r,N}^{-1} \\ &\times (\mathcal{A}_{s,n}\xi(k+n|k) + \mathcal{E}\omega(k+n|k)) \\ &- (1-\alpha)\xi^T(k+n|k) Y_{s,N}^{-1} \xi(k+n|k) \\ &- \frac{\alpha}{\bar{\omega}} \omega^T(k+n|k)\omega(k+n|k) \leq 0. \end{aligned} \tag{19}$$

Then, it follows directly from (8) and (19) that

$$\begin{aligned} &\xi^T(k+n+1|k) \sum_{r=1}^{m} p_{sr} Y_{r,N}^{-1} \xi(k+n+1|k) \\ &- \xi^T(k+n|k) Y_{s,N}^{-1} \xi(k+n|k) + \alpha(\xi^T(k+n|k) \\ &\times Y_{s,N}^{-1}\xi(k+n|k) - \frac{\alpha}{\bar{\omega}}\omega^T(k+n|k)\omega(k+n|k)) \leq 0. \end{aligned} \tag{20}$$

Noticing $\mathbb{E}\{P_{t,N}\} = \sum_{r=1}^{m} p_{sr} P_{r,N}$, it is inferred that

$$\begin{aligned} &\frac{1}{\varphi}\mathbb{E}\{\xi^T(k+n+1|k) P_{t,N} \xi(k+n+1|k)\} \\ &- \frac{1}{\varphi}\xi^T(k+n|k) P_{s,N} \xi(k+n|k) + \frac{\alpha}{\varphi}(\xi^T(k+n|k) \\ &\times P_{s,N}\xi(k+n|k) - \frac{\alpha}{\bar{\omega}}\omega^T(k+n|k)\omega(k+n|k)) \leq 0. \end{aligned} \tag{21}$$

To this end, by applying the S-procedure, we obtain immediately from (21) that the conditions (11)-(12) are satisfied, which implies that $\Omega_{s,N}$ is an RPI set of the system (8) with constraints (6a)-(6b). In addition, the corresponding feedback gain is determined by (33). The proof is complete. ■

B. Terminal cost function

Lemma 3: Let the weighting matrices $Q > 0$, $R > 0$, a scalar $\lambda > 0$ and the transition probability matrix $\mathcal{P} = [p_{st}]_{m \times m}$ be given. For system (8) under the SCP (5), if there exist a matrix $Z_{s,N}$, positive-definite matrices $\tilde{Y}_{1s,N}$ and $\tilde{Y}_{2s,N}$, and a scalar $\varphi > 0$ such that, for $\forall s \in \mathbb{M}$, the following matrix inequality

$$\begin{bmatrix} Y_{s,N} & * & * & * & * & * & * \\ 0 & \lambda\varphi & * & * & * & * & * \\ \Gamma_{s,N} & \mathcal{E}\varphi & \frac{Y_{1,N}}{p_{s1}} & * & * & * & * \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \Gamma_{s,N} & \mathcal{E}\varphi & 0 & \cdots & \frac{Y_{m,N}}{p_{sm}} & * & * \\ QY_{s,N} & 0 & 0 & \cdots & 0 & \varphi Q & * \\ R\Theta_{s,N} & 0 & 0 & \cdots & 0 & 0 & \varphi R \end{bmatrix} \geq 0 \tag{22}$$

holds, where $\Theta_{s,N} = [Z_{s,N} \quad 0]$, $Y_{s,N}$ and $\Gamma_{s,N}$ are defined in Lemma 2, then the condition (10) is satisfied.

Proof: First, pre- and post-multiplying (22) with $\mathrm{diag}\{Y_{s,N}^{-1}, \varphi^{-1}, I, \ldots, I\}$ and its transpose, respectively, we obtain

$$\begin{bmatrix} Y_{s,N}^{-1} & * & * & * & * & * & * \\ 0 & \frac{\lambda}{\varphi} & * & * & * & * & * \\ \mathcal{A}_{s,N} & \mathcal{E} & \frac{Y_{1,N}}{p_{s1}} & * & * & * & * \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \mathcal{A}_{s,N} & \mathcal{E} & 0 & \cdots & \frac{Y_{m,N}}{p_{sm}} & * & * \\ Q & 0 & 0 & 0 & 0 & \varphi Q & * \\ R\tilde{K}_{s,N} & 0 & 0 & 0 & 0 & 0 & \varphi R \end{bmatrix} \geq 0 \tag{23}$$

where $\tilde{K}_{s,N} = [K_{s,N} \quad 0]$.

By using the Schur Complement, (23) holds if and only if

$$\begin{bmatrix} \Sigma_{s,N}^{11} & * \\ \Sigma_{s,N}^{21} & \Sigma_{s,N}^{22} \end{bmatrix} \geq 0 \tag{24}$$

where

$$\Sigma_{s,N}^{11} = Y_{s,N}^{-1} - \Xi_{s,N} - \varphi^{-1}Q - \varphi^{-1}\mathcal{K}_{s,N},$$
$$\Sigma_{s,N}^{21} = -\mathcal{E}^T \sum_{r=1}^{m} p_{sr} Y_{r,N}^{-1} \mathcal{A}_{s,N},$$
$$\Sigma_{s,N}^{22} = \frac{\lambda}{\varphi} - \mathcal{E}^T \sum_{r=1}^{m} p_{sr} Y_{r,N}^{-1} \mathcal{E},$$
$$\Xi_{s,N} = \mathcal{A}_{s,N}^T \sum_{r=1}^{m} p_{sr} Y_{r,N}^{-1} \mathcal{A}_{s,N},$$
$$\mathcal{K}_{s,N} = \tilde{K}_{s,N}^T R \tilde{K}_{s,N}.$$

According to (8), pre- and post-multiplying (24) by $[\xi^T(k+n|k) \quad \omega^T(k+n|k)]$ and its transpose, the following inequality is obtained:

$$\begin{aligned} &\xi^T(k+n+1|k) \sum_{r=1}^{m} p_{sr} Y_{r,N}^{-1} \xi(k+n+1|k) \\ &- \xi^T(k+n|k) Y_{s,N}^{-1} \xi(k+n|k) + \xi^T(k+n|k)\varphi^{-1}Q \\ &\times \xi(k+n|k) + \varphi^{-1}\tilde{u}^T(k+n|k) R \tilde{u}(k+n|k) \\ &+ \frac{\lambda}{\varphi}\omega^T(k+n|k)\omega(k+n|k) \leq 0. \end{aligned} \tag{25}$$

By multiplying both sides of (25) with φ, it is easily seen that:

$$\begin{aligned} &\xi^T(k+n+1|k)\sum_{r=1}^{m} p_{sr}P_{r,N}\xi(k+n+1|k) \\ &-\xi^T(k+n|k)P_{s,N}\xi(k+n|k)+\xi^T(k+n|k)Q \\ &\times\xi(k+n|k)+\tilde{u}^T(k+n|k)R\tilde{u}(k+n|k) \\ &+\lambda\omega^T(k+n|k)\omega(k+n|k)\le 0. \end{aligned} \tag{26}$$

Noticing the fact $\mathbb{E}\{P_{t,N}\}=\sum_{r=1}^{m} p_{sr}P_{r,N}$, we have:

$$\begin{aligned} &\mathbb{E}\{\xi^T(k+n+1|k)P_{t,N}\xi(k+n+1|k)\} \\ &-\xi^T(k+n|k)P_{s,N}\xi(k+n|k)+\xi^T(k+n|k)Q \\ &\times\xi(k+n|k)+\tilde{u}^T(k+n|k)R\tilde{u}(k+n|k) \\ &+\lambda\omega^T(k+n|k)\omega(k+n|k)\le 0. \end{aligned} \tag{27}$$

By some straightforward manipulations, it is not difficult to see that (27) implies (10), which ensures that the terminal cost function $V(\xi(k+n|k))$ is a local Lyapunov-like function satisfying (10). The proof is complete. ■

So far, sufficient conditions have been derived to guarantee that the set $\Omega_{s,N}$ $(s\in\mathbb{M})$ is a terminal constraint set for the system (8) with constraints (6a)-(6b). To minimize the terminal constraint set, let us establish the following optimization problem:

$$\begin{aligned} \textbf{OP1} \quad &\min_{\varphi,\alpha,Y_{s,N},\mathbb{X}_{s,N},\mathbb{Y}_{s,N},Z_{s,N}} \quad \mathrm{trace}(Y_{s,N}) \\ &\text{s.t. } (13)-(15) \text{ and } (22) \end{aligned}$$

where "trace$(\cdot)$" denotes the trace of the matrix.

It can now be concluded from Lemmas 2 and 3 that the set $\Omega_{s,N}$ $(s\in\mathbb{M})$ is a terminal constraint set. For convenience of dealing with the ROSSs later, we define the following set:

$$\Omega \triangleq \{\xi|\xi^TY_N^{-1}\xi\le 1\} \tag{28}$$

where $Y_N^{-1}=\sum_{r=1}^{m}\mu_rY_{r,N}^{-1}$ with $Y_N=\mathrm{diag}\{\tilde{Y}_{1,N},\ \tilde{Y}_{2,N}\}$ and $\mu_r\ge 0$ $(r=1,\dots,m)$ are the weighting coefficients satisfying $\sum_{r=1}^{m}\mu_r=1$.

Next, some sufficient conditions are derived to guarantee that Ω is a terminal constraint set.

Lemma 4: Let the bound of disturbance $\bar{\omega}$ and the matrix Ψ be given. For the system (8) with constraints (6a)-(6b), assume that there exist positive-definite matrices $\mathbb{X}_N$, $\mathbb{Y}_N$ and a matrix Z_N such that, for $\forall s\in\mathbb{M}$, the following inequalities hold:

$$\begin{bmatrix} \frac{1}{1-\alpha}Y_N & * & * \\ 0 & \frac{\alpha}{\bar{\omega}}I & * \\ \bar{\Gamma}_{s,N} & \mathcal{E} & Y_N \end{bmatrix}\ge 0, \tag{29}$$

$$\begin{bmatrix} Y_N & * & * & * & * \\ 0 & \lambda & * & * & * \\ \bar{\Gamma}_{s,N} & \mathcal{E} & Y_N & * & * \\ QY_N & 0 & 0 & Q & 0 \\ R\bar{\Theta}_N & 0 & 0 & 0 & R \end{bmatrix}\ge 0, \tag{30}$$

$$\begin{bmatrix} \mathbb{X}_N & * \\ Z_N^T & \tilde{Y}_{1,N} \end{bmatrix}\ge 0,\ \ [\mathbb{X}_N]_{pp}\le[\bar{u}]_p^2,\ p=1,\dots,m, \tag{31}$$

$$\begin{bmatrix} \mathbb{Y}_N & * \\ \tilde{Y}_{1,N}^T\Psi^T & \tilde{Y}_{1,N} \end{bmatrix}\ge 0,\ \ [\mathbb{Y}_N]_{qq}\le[\bar{x}]_q^2,\ q=1,\dots,o \tag{32}$$

where α is obtained in Lemma 2, $\bar{\Theta}_N=[Z_N\quad 0]$ and

$$\bar{\Gamma}_{s,N}=\begin{bmatrix} A\tilde{Y}_{1,N}+B\Phi_sZ_N & B(I-\Phi_s)\tilde{Y}_{2,N} \\ \Phi_sZ_N & (I-\Phi_s)\tilde{Y}_{2,N} \end{bmatrix}.$$

Then, the ellipsoid set Ω defined by (28) is an RPI set. Furthermore, the corresponding controller gain is derived as follows:

$$K_N=Z_N\tilde{Y}_{1,N}^{-1} \tag{33}$$

and the control law is given by $h_N=K_Nx(\cdot)$.

Proof: The proof follows directly from Lemmas 2 and 3, and is thus omitted. ■

Remark 2: It can be observed from the Lemmas 2-4 that a unified terminal constraint set Ω is determined by taking the stochastic property of the SCP into account. In addition, by solving the optimization problem **OP1**, such a set can be further minimized and hence potentially improve the control performance.

IV. Robust One Step Sets

A. An off-line optimization problem

For the N-step prediction, suppose that $\xi(k+N-1|k)$ belongs to the ROSS defined by $\mathbb{P}_{N-1}$, then $\xi(k+N|k)\in\Omega$ can be guaranteed by applying the control law h_{N-1}. Similarly, if $\xi(k+N-2|k)\in\mathbb{P}_{N-2}$, then the control law h_{N-2} can ensure $\xi(k+N-1|k)\in\mathbb{P}_{N-1}$. Following this idea, we can obtain a sequence of ROSSs $\{\mathbb{P}_0,\mathbb{P}_1,\dots,\mathbb{P}_{N-1}\}$ and the corresponding control laws $\{h_0,h_1,\dots,h_{N-1}\}$.

In the following, by using a similar technique in [25], an off-line computation is proposed to solve the ROSSs and the corresponding control laws.

Lemma 5: Define $\mathbb{P}_n\triangleq\{\xi|\xi^TP_n\xi\le\varphi\}$ with $P_n=\mathrm{diag}\{\tilde{P}_{1,n},\tilde{P}_{2,n}\}$. Let the bound of disturbance $\bar{\omega}$, the matrix Ψ and the transition probability matrix $\mathcal{P}=[p_{st}]_{m\times m}$ be given. For system (8) with hard constraints (6a)-(6b), suppose that there exist positive-definite matrices $\tilde{Y}_{1,n}$, $\tilde{Y}_{2,n}$, $\mathbb{U}_n$, $\mathbb{V}_n$, a matrix $Z_{i,n}$, scalars $0<\beta<1$ and $\varphi>0$ such that, for $\forall i,s\in\mathbb{M}$ and $n=0,1,\dots,N-1$, the following inequalities

$$\begin{bmatrix} (1-\beta)Y_n & * & * \\ 0 & \frac{\beta}{\bar{\omega}}I & * \\ \Gamma_{i,s,n} & \mathcal{E} & Y_{n+1} \end{bmatrix}\ge 0, \tag{34}$$

$$\begin{bmatrix} Y_n & * & * & * & * \\ 0 & \lambda\varphi & * & * & * \\ \Gamma_{i,s,n} & \mathcal{E}\varphi & Y_n & * & * \\ QY_{s,n} & 0 & 0 & \varphi Q & * \\ R\Theta_{i,n} & 0 & 0 & 0 & \varphi R \end{bmatrix}\ge 0, \tag{35}$$

$$\begin{bmatrix} \mathbb{U}_n & * \\ Z_{i,n}^T & \tilde{Y}_{1,n} \end{bmatrix}\ge 0,\ \ [\mathbb{U}_n]_{pp}\le[\bar{u}]_p^2,\ p=1,\dots,m, \tag{36}$$

$$\begin{bmatrix} \mathbb{V}_n & * \\ \tilde{Y}_{1,n}^T\Psi^T & \tilde{Y}_{1,n} \end{bmatrix}\ge 0,\ \ [\mathbb{V}_n]_{qq}\le[\bar{x}]_q^2,\ q=1,\dots,o \tag{37}$$

are feasible, where

$$\Gamma_{i,s,n} = \begin{bmatrix} A\tilde{Y}_{1,n} + B\Phi_s Z_{i,n} & B(I-\Phi_s)\tilde{Y}_{2,n} \\ \Phi_s Z_{i,n} & (I-\Phi_s)\tilde{Y}_{2,n} \end{bmatrix},$$
$$\Theta_{i,n} = [Z_{i,n} \quad 0],\ Y_n = \mathrm{diag}\{\tilde{Y}_{1,n}, \tilde{Y}_{2,n}\},$$
$$\tilde{Y}_{j,n} = \varphi \tilde{P}_{j,n}^{-1},\ j = 1, 2.$$

Then, the set $\mathbb{P}_n$ is an ellipsoidal approximating ROSS of $\mathbb{P}_{n+1}$. Furthermore, the corresponding feedback gain is given by

$$K_n = \sum_{r=1}^{m} \mu_r K_{r,n} \tag{38}$$

where $K_{r,n} = Z_{r,n}\tilde{Y}_{1,n}^{-1}$. Moreover, the control law is determined by $h_n = K_n x(\cdot)$.

Proof: Pre- and post-multiplying (34) with $[\xi^T(k+i|k) \ \ \omega^T(k+i|k)]$ and its transpose, it follows from the Schur Complement and (34) that

$$\begin{aligned} &\xi^T(k+i+1|k)Y_{n+1}^{-1}\xi(k+i+1|k) \\ &\le (1-\beta)\xi^T(k+i|k)Y_n^{-1}\xi(k+i|k) \\ &\quad + \frac{\beta}{\bar{\omega}}\omega^T(k+i|k)\omega(k+i|k). \end{aligned} \tag{39}$$

It is clear that $\xi^T(k+i+1|k) \in \mathbb{P}_{n+1}$ if $\xi(k+i|k) \in \mathbb{P}_n$ under the condition $\omega^T(k+i|k)\omega(k+i|k) \le \bar{\omega}$. Together with (34)-(35), it is obvious from Lemmas 2-3 that $\mathbb{P}_n$ is an RPI set. Thus, it can be concluded that $\mathbb{P}_n$ is an ROSS of P_{n+1}. In this sense, if $\xi(k+i|k) \in \mathbb{P}_n$, $\xi(k+i+1|k)$ can be steered into $\mathbb{P}_{n+1}$ by applying the control law h_n. On the other hand, the hard constraints (6a)-(6b) can be guaranteed by (36)-(37), and the proof is thus complete. ■

Now, letting $\mathbb{P}_N$ be the terminal constraint set Ω, by backward solving (34)-(37) from $n = N-1$ to $n = 0$, the ROSSs $\{\mathbb{P}_0, \mathbb{P}_1, \ldots, \mathbb{P}_{N-1}\}$ and the corresponding control laws h_n $(n = 0, 1, 2, \ldots, N-1)$ can be successively obtained off-line.

In order to maximize the size of ROSSs, the following N off-line optimization problems are proposed for $n = 0, 1, \ldots, N-1$:

$$\textbf{OP2} \quad \min_{\varphi,\beta,\mathbb{U}_n,\mathbb{V}_n,Y_n,Z_{i,n}(i\in\mathbb{M})} -\log\det(Y_n)$$
$$\text{s.t. } (34)-(37)$$

where "$\log\det(\cdot)$" denotes the logarithm of the matrix determinant.

Remark 3: From Lemma 5, a sequence of the mode-independent ROSSs $\{\mathbb{P}_0, \mathbb{P}_1, \ldots, \mathbb{P}_{N-1}\}$ is obtained and then further maximized by solving the optimization problem **OP2**. To this end, the initial feasible region can be denoted as $\mathbb{P}_0 \cup \mathbb{P}_1 \cup \ldots \cup \mathbb{P}_{N-1}$, in which the state can enter into the terminal constraint set within N steps. It is obvious that with the increase of the step number N, the size of the initial feasible region becomes bigger at the cost of increasing the computation complexities. Therefore, there is a need to keep a trade-off between the computation complexities and the control performance of the system.

Remark 4: Notice that the conditions (13) and (34) are non-convex due to the term $(1-\alpha)Y_{s,N}$ and $(1-\beta)Y_n$, respectively, which can be handled by using solvers like PENBMI toolbox. For convenience, a feasible α^* (or β^*) is utilized to replace α (or β), and then (13) and (34) can be converted into convex conditions that can be solved by LMI or YALMIP toolbox. On the other hand, for the **OP2**, $N_{\max}$ is the pre-specified maximum number of iterative steps. Then, adopting the similar approach in [25], the time step N can be selected as follows:

1) If the ROSSs converge after n $(n \le N_{\max})$ steps, i.e. $Y_{N-n} = Y_{N-n-1}$, then N can be assigned as n.
2) Given a sufficiently small positive scalar $\epsilon > 0$. If $\|Y_{N-n} - Y_{N-n-1}\| \le \epsilon$ holds, then N can be assigned as n.
3) Otherwise, N is assigned as the maximum number of iterative steps $N_{\max}$.

B. An on-line optimization problem

Instead of applying the feedback control law h_n $(n \in \{0, 1, \ldots, N-1\})$, we are going to develop an online optimization algorithm to compute the control signal $\tilde{u}(k)$ such that the state, which is included in the initial feasible region, can be forced into the terminal constraint set Ω within N steps while ensuring the minimum control cost.

To be more specific, we suppose that the system state $\xi(k)$ is obtained at the time instant k, and an online index search is subsequently carried out to determine the maximum index M such that $\xi(k) \in \mathbb{P}_M \backslash \mathbb{P}_{M+1}$ $(M = 0, 1, \ldots, N-1)$. Then, we calculate the control law $u(k)$ in order to guarantee that the state $\xi(k+1)$ enters into $\mathbb{P}_{M+1}$.

Next, our attention is focused on the calculation of the control law $u(k)$. Inspired by the approach in [25], we construct the following disturbance-free model for $\forall s \in \mathbb{M}$:

$$\breve{\xi}(k+1|k) = \breve{\mathcal{A}}_s \xi(k) + \breve{\mathcal{B}}\tilde{u}(k) \tag{40}$$

where $\xi(k) = \breve{\xi}(k)$ and

$$\breve{\mathcal{A}}_s \triangleq \begin{bmatrix} A & B(I-\Phi_s) \\ 0 & I-\Phi_s \end{bmatrix}, \quad \breve{\mathcal{B}} = \begin{bmatrix} B \\ I \end{bmatrix}.$$

Here, the notation s is slightly abused to denote both the $\theta_{k+n|k}$ and θ_k.

Based on the above discussions, a quadratic performance index $J(k)$ is introduced together with the following optimization problem:

$$\textbf{OP3} \begin{cases} \min_{\tilde{u}(k)} J(k) & (41a) \\ \text{s.t. } \ \xi(k+1|k) \in \mathbb{P}_{M+1} & (41b) \\ |[\tilde{u}(k)]_p| \le [\bar{u}]_p,\ p = 1, \ldots, m & (41c) \\ |[\Phi]_q x(k)| \le [\bar{x}]_q,\ q = 1, \ldots, o & (41d) \end{cases}$$

where $J(k) \triangleq \vartheta(k) + \sum_{n=1}^{\infty} l(k+n|k)$, $\vartheta(k) \triangleq \xi^T(k)Q\xi(k) + \tilde{u}^T(k)R\tilde{u}(k)$ and $l(k+n|k) \triangleq \|\breve{\xi}(k+n|k)\|_Q^2 + \|\tilde{u}(k+n|k)\|_R^2$ with Q and R being known positive-definite weighting matrices.

Noting (35) and (40), the following inequality is true:

$$\begin{aligned}&\breve{\xi}^T(k+n+1|k)P_{M+1}\breve{\xi}(k+n+1|k)\\&-\breve{\xi}^T(k+n|k)P_{M+1}\breve{\xi}(k+n|k)\\<&-\breve{\xi}^T(k+n|k)Q\breve{\xi}(k+n|k)\\&-\tilde{u}^T(k+n|k)R\tilde{u}(k+n|k).\end{aligned}\tag{42}$$

Keeping the form of $J(k)$ in mind and summing up (42) on both sides from 1 to ∞ with respect to n, one has

$$\begin{aligned}J(k)=&\,\vartheta(k)+\sum_{n=1}^{\infty}l(k+n|k)\\<&\,\vartheta(k)+\breve{\xi}^T(k+1|k)P_{M+1}\breve{\xi}(k+1|k)\\\triangleq&\,\bar{J}(k).\end{aligned}\tag{43}$$

Therefore, the optimization problem **OP3** can be transformed into the following auxiliary optimization problem:

$$\mathbf{OP4}\begin{cases}\min\limits_{\tilde{u}(k)}\ \rho\\ \text{s.t.}\ \ \bar{J}(k)\le\rho & \text{(44a)}\\ \quad\xi(k+1|k)\in\mathbb{P}_{M+1} & \text{(44b)}\\ \quad|[\tilde{u}(k)]_p|\le[\bar{u}]_p,\ p=1,\ldots,m & \text{(44c)}\\ \quad|[\Psi]_q x(k)|\le[\bar{x}]_q,\ q=1,\ldots,o. & \text{(44d)}\end{cases}$$

In what follows, let us deal with the constraints in **OP4**. Substituting (40) into (44a), and using the Schur Complement, (44a) holds if and only if, for $\forall s\in\mathbb{M}$, the following inequalities hold:

$$\begin{bmatrix}\rho & * & * & *\\ R\tilde{u}(k) & R & * & *\\ Q\xi(k) & 0 & Q & *\\ \breve{\mathcal{A}}_s\xi(k)+\breve{\mathcal{B}}\tilde{u}(k) & 0 & 0 & P_{M+1}^{-1}\end{bmatrix}\ge 0.\tag{45}$$

Now, we pay attention to the constraint (44b), i.e.,

$$\xi^T(k+1|k)Y_{M+1}^{-1}\xi(k+1|k)\le 1,\tag{46}$$

from which it is inferred that, by applying the S-procedure technique, (46) can be guaranteed under the condition $\omega^T(k)\omega(k)\le\bar{\omega}$ if and only if

$$\begin{aligned}&\xi^T(k+1|k)Y_{M+1}^{-1}\xi(k+1|k)\\&-1-\tilde{\lambda}\left(\omega^T(k)\omega(k)-\bar{\omega}\right)\le 0\end{aligned}\tag{47}$$

where $\tilde{\lambda}>0$ is a known scalar.

Next, by substituting $\xi(k+1|k)=\breve{\mathcal{A}}_s\xi(k)+\breve{\mathcal{B}}\tilde{u}(k)+\mathcal{E}\omega(k)$ into (47) and utilizing the Schur Complement, one has

$$\begin{bmatrix}1-\tilde{\lambda}\bar{\omega} & * & *\\ 0 & \tilde{\lambda} & *\\ \breve{\mathcal{A}}_s\xi(k)+\breve{\mathcal{B}}\tilde{u}(k) & \mathcal{E} & Y_{M+1}\end{bmatrix}\ge 0,\tag{48}$$

which implies that (44b) holds.

Note that (44c) and (44d) can be guaranteed, respectively, by the following two conditions:

$$\begin{bmatrix}\mathbb{F} & *\\ \tilde{u}(k) & I\end{bmatrix}\ge 0,\ \ [\mathbb{F}]_{pp}\le[\bar{u}]_p^2\tag{49}$$

and

$$\begin{bmatrix}\mathbb{G} & *\\ Y_{M+1}^T\Psi^T & Y_{M+1}\end{bmatrix}\ge 0,\ \ [\mathbb{G}]_{qq}\le[\bar{x}]_q^2\tag{50}$$

where $p=1,\ldots,m$ and $q=1,\ldots,o$.

On the basis of the above analysis, **OP4** can be further converted into the following online auxiliary optimization problem:

$$\begin{aligned}\mathbf{OP5}\ &\min_{\tilde{u}(k)}\ \rho\\ \text{s.t.}\ \ &(45),(48)-(50).\end{aligned}$$

V. Mean-square input-to-state stability

Before proceeding, we first provide the following lemma.

Lemma 6: [25] The closed-loop system (8) is said to be mean-square ISS if there exist quadratic function $V(\xi(k))$, $\mathcal{K}_\infty$-function $\underline{\varrho}$, $\bar{\varrho}$, ζ and $\mathcal{K}$-function ς such that

1) $\underline{\varrho}\|\xi(k)\|^2\le V(\xi(k))\le\bar{\varrho}\|\xi(k)\|^2$;
2) $\mathbb{E}\{V(\xi(k+1))\}-V(\xi(k))\le-\zeta\|\xi(k)\|^2+\varsigma\|\omega(k)\|^2$.

In this case, $V(\xi(k))$ is called the mean-square ISS quadratic function.

Theorem 1: Consider the system (8) with hard constraints (6a)-(6b) under the SCP (5). If there exist feasible solutions to optimization problems **OP1**-**OP2** and **OP5** at the time step k, then there also exist feasible solutions at any future time step $t>k$. Moreover, the closed-loop system (8) is mean-square ISS. Furthermore, the corresponding control laws are given by (33) and $\tilde{u}(k)$ is obtained by solving **OP5**.

Proof: The proof of the recursive feasibility can be easily obtained by following the lines similar to that in [25], and is therefore omitted. In what follows, the desired mean-square input-to-state stability of the system (8) needs to be shown. It should be pointed out that, under the N-step MPC strategy, we only need to prove that the plant can achieve the mean-square input-to-state stability while entering into the terminal constraint set.

From the definition of $V(\xi(k))$, one easily has

$$\kappa_{\min}(P^*(k))\,\|\xi(k)\|^2\le V_k^*(\xi(k))\le\kappa_{\max}(P^*(k))\,\|\xi(k)\|^2\tag{51}$$

where $\kappa_{\min}(\cdot)$ and $\kappa_{\max}(\cdot)$, respectively, denote the minimal and maximal eigenvalues of the matrix. At time k, denote the optimal $P_N(k)$ as $P^*(k)$, the optimal $V(\xi(k))$ as $V_k^*(\xi(k))$ and the control input $\tilde{u}(k)$ as $K_N^*x(k)$.

On the other hand, it follows from (22) that

$$\begin{aligned}&\mathbb{E}\left\{V_k^*(\xi(k+1))\right\}-V_k^*(\xi(k))\\\le&-\left(\|\xi(k)\|_Q^2+\|K_N^*(k)x(k)\|_R^2\right)+\lambda\omega^T(k)\omega(k)\\<&-\|\xi(k)\|_Q^2+\lambda\omega^T(k)\omega(k).\end{aligned}\tag{52}$$

Notice that $V_k^*(\xi(k+1))$ is a feasible solution while $V_{k+1}^*(\xi(k+1))$ is an optimal solution for the time instant $k+1$, and thus $V_{k+1}^*(\xi(k+1))\le V_k^*(\xi(k+1))$. According to the optimality and the condition (52), one has

$$\mathbb{E}\left\{V_{k+1}^*(\xi(k+1))\right\}-V_k^*(\xi(k))<-\|\xi(k)\|_Q^2+\lambda\|\omega(k)\|^2.\tag{53}$$

From Lemma 6, (51) and (53) guarantee the mean-square input-to-state stability for the closed-loop system (8) with hard constraints (6a)-(6b) under the SCP (5), and the proof is thus complete. ■

Theorem 1 indicates that, as long as the states enter into the terminal constraint set Ω, by applying the feedback gain K_N, the mean-square input-to-state stability can be ensured for the closed-loop system (8) with hard constraints (6a)-(6b) under the SCP (5).

Based on the above discussions, one can conclude the SCP-based N-step MPC algorithm (including both online and off-line parts) as follows.

Algorithm 1: 1) Off-line Part:

- *Step 1:* Calculate the terminal constraint set Ω by solving optimization problem **OP1**, meanwhile, obtain the corresponding feedback gain K_N.
- *Step 2:* Solve the optimization problem **OP2** to obtain a sequence of the ROSSs $\{\mathbb{P}_0, \mathbb{P}_1, \mathbb{P}_2, \ldots, \mathbb{P}_{N-1}\}$.

2) Online Part:

- *Step 1:* At time instant $k = 0$, obtain the system state $\xi(k)$, and set the node s.
- *Step 2:* If $\xi(k) \in \mathbb{P}_N$, go to *Step 4*; Else if $\xi(k) \in \mathbb{P}_0 \cup \mathbb{P}_1 \cup \ldots \cup \mathbb{P}_{N-1}$, find the maximum index M such that $\xi(k) \in \mathbb{P}_M \backslash \mathbb{P}_{M+1}$, and then go to *Step 3*.
- *Step 3:* Calculate the current control signal $\tilde{u}(k)$ by solving optimization problem **OP5**. Then, calculate $u(k)$ by substituting into (5) under the SCP. Set $k = k+1$ and go to *Step 2*.
- *Step 4:* Feed the control input $u(k) = K_N x(k)$ to the plant.

Remark 5: In this paper, the N-step MPC problem is studied for a class of discrete-time systems with persistent bounded disturbances and hard constraints. The scheduling of SCP is first taken into consideration to prevent the data from collisions, under which only a certain controller node obtains the access to the shared communication network at each transmission instant. Sufficient conditions are provided to guarantee the mean-square input-to-state stability of the underlying system. It can be seen from Algorithm 1 that all the essential factors contributing to the system complexity have been reflected which cover 1) the transition probabilities of the SCP; 2) the respective upper bounds of the external disturbances, the system state and control input; and 3) the step number N.

VI. A Numerical Example

Consider the following linear discrete-time system:

$$\begin{aligned} x(k+1) &= \begin{bmatrix} -0.5 & -0.2 \\ 2.8 & -0.3 \end{bmatrix} x(k) + \begin{bmatrix} 2 & 6 \\ 4 & 1 \end{bmatrix} u(k) \\ &\quad + \begin{bmatrix} 0.03 \\ 0.03 \end{bmatrix} \sin(k) \\ &\triangleq Ax(k) + Bu(k) + E\omega(k). \end{aligned}$$

The transition probability matrix is denoted by

$$\mathcal{P} = \begin{bmatrix} 0.45 & 0.55 \\ 0.7 & 0.3 \end{bmatrix}.$$

The hard constraint bounds and weighting matrices are given as:

$$\bar{u} = \begin{bmatrix} 30 \\ 30 \end{bmatrix}, \ \bar{x} = \begin{bmatrix} 10 \\ 100 \end{bmatrix}, \ R = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.6 \end{bmatrix},$$
$$Q = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 \\ 0 & 0 & 300 & 0 \\ 0 & 0 & 0 & 300 \end{bmatrix}, \ \Psi = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 1 \end{bmatrix}.$$

The scalars are given by $\alpha^* = 0.1$, $\lambda = 0.5$, $\beta^* = 0.1$ and $\tilde{\lambda} = 0.1$. It is easily seen that $\omega^T(k)\omega(k) \le \bar{\omega} = 1$. By solving **OP1**, the terminal constraint set is obtained as $\Omega = \{\xi | \xi^T Y_N^{-1} \xi \le 1\}$ with

$$Y_N = \begin{bmatrix} 0.0310 & -0.0126 & 0 & 0 \\ -0.0126 & 0.2447 & 0 & 0 \\ 0 & 0 & 0.0005 & 0 \\ 0 & 0 & 0 & 0.0001 \end{bmatrix}$$

and the corresponding feedback gain is calculated as

$$K_N = \begin{bmatrix} -0.1116 & -0.0031 \\ 0.0366 & 0.0378 \end{bmatrix}.$$

By solving the optimization problem **OP2**, the obtained sequence of ROSSs converges with $\varepsilon = 1 \times 10^{-2}$ after 6 steps. Thus, the iteration number is chosen as $N = 7$, and the initial state is selected as $\xi(0) = [-7 \quad 8 \quad 0 \quad 0]^T$. The controller node 2 is selected at the initial time instant. Fig. 2 depicts the switching modes of channels 1 and 2 under the SCP. Fig. 3 shows the state evolution of the system (1) under the SCP. Clearly, the system state enters into the terminal constraint set at the seventh step. Figs. 4 and 5 are the state response and the control input of the addressed system, respectively. The simulation results illustrate the effectiveness of the proposed N-step MPC strategy.

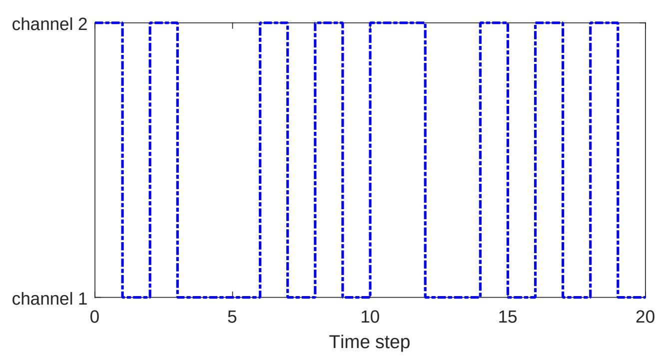

Fig. 2. Switching modes for the channels under the SCP.

VII. Conclusions

In this paper, we have investigated the N-step MPC problem for constrained systems subject to persistent bounded disturbances and the SCP. Under the scheduling of SCP characterized by a Markov chain, only one controller node has obtained the access to the shared communication network at each transmission instant. A set of desired controllers in the framework of N-step MPC has been designed to steer

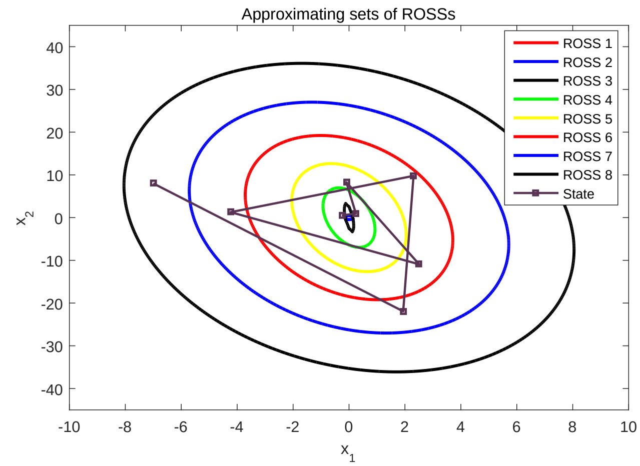

Fig. 3. Evolution of system states by the N-step MPC strategy.

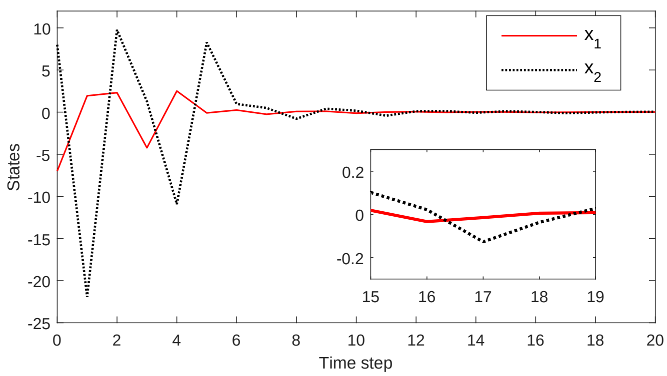

Fig. 4. State responses of the closed-loop system under the SCP.

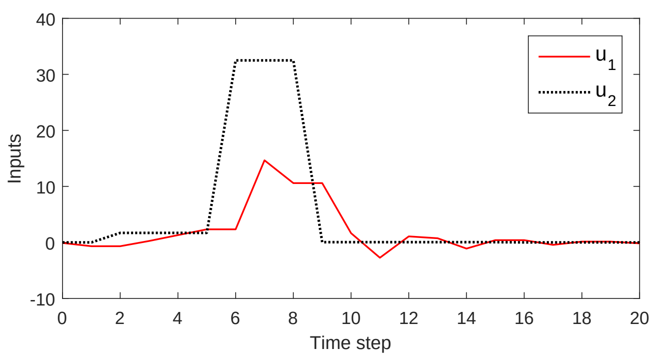

Fig. 5. Control inputs of the closed-loop system under the SCP.

the system state into the terminal constraint set within N steps. Sufficient conditions have been derived to guarantee the mean-square input-to-state stability of the addressed system. Both off-line and online computations have been carried out to obtain the desired control laws. In the end, a numerical example has been given to demonstrate the usefulness of the proposed N-step MPC strategy. In the future work, we plan to investigate robust MPC problems for some special nonlinear systems under communication protocols.

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