Invited Review

Hybrid simulation modelling in operational research: A state-of-the-art review

Sally C. Brailsford [a,*], Tillal Eldabi [b], Martin Kunc [a], Navonil Mustafee [c], Andres F. Osorio [d]

[a] *Centre for Operational Research, Management Science and Information Systems (CORMSIS), Southampton Business School, University of Southampton, Southampton SO17 1BJ, UK*
[b] *Brunel Business School, Brunel University London, Uxbridge, Middx UB8 3PH, UK*
[c] *Centre for Simulation, Analytics and Modelling (CSAM), University of Exeter Business School, Xfi Building, Rennes Drive, Exeter EX4 4ST, UK*
[d] *Universidad Icesi, Calle 18 No. 122 - 135, Cali, Colombia*

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ABSTRACT

Hybrid simulation (defined as a modelling approach that combines two or more of the following methods: discrete-event simulation, system dynamics, and agent-based simulation) has experienced near-exponential growth in popularity in the past two decades. However, a large proportion of the academic literature on hybrid simulation is found in computer science and engineering journals. Given the importance of this emerging area and its relevance to operational research, this paper provides a review of the topic from an OR perspective. The results of a review of the hybrid simulation literature are presented, using a novel framework based on the simulation lifecycle that will be useful for future modellers and authors alike. Promising areas for future research are identified: these include the development of new methods for conceptual modelling and for model validation. Currently the main application areas are healthcare, supply chain management and manufacturing, and the majority of published models combine discrete-event simulation and system dynamics.

1. Introduction

1.1. Setting the scene

A survey of the use of simulation in manufacturing and business undertaken nearly ten years ago (Jahangirian et al. 2010) found that among all the simulation approaches, discrete-event simulation (DES), system dynamics (SD) and agent-based simulation (ABS) were the most widely used in operational research (OR) to model business problems. Jahangirian et al. also noted an increasing interest in hybrid simulation (defined as models that combined at least two of these three approaches) to model complex enterprise-wide systems, but did not evaluate the topic in detail. Since 2010 this interest has continued to grow apace, as can be seen from the rapidly increasing number of papers in the Winter Simulation Conference (the leading international conference on simulation) and academic journals, predominantly in engineering and computer science but also in OR.

The aims of this paper are to introduce the topic of hybrid simulation (HS) to an OR readership, develop a framework of definitions and terminology for HS modelling in OR, use this to evaluate the current state of the art through a literature survey, and identify areas for future research. In addition to assisting future modellers, our framework also serves as the basis for a simple checklist of good practice guidelines for authors. The paper is structured as follows. The remainder of this section presents the necessary background information and terminology. Section 2 describes existing frameworks for HS found in the literature and then presents our own framework. In Section 3 we describe our search methodology. Section 4 presents the broad findings from the survey in terms of overall descriptive statistics, and Section 5 discusses specific findings in greater depth and identifies promising areas for further research. In Section 6 we reflect on our findings and present our good practice checklist.

1.2. Mixing methods in OR

OR is often described as a toolbox of methods, from which the most appropriate method for solving any particular problem can be selected. Mixing or combining OR methods, i.e. using more than

* Corresponding author.
E-mail address: s.c.brailsford@soton.ac.uk (S.C. Brailsford).

one method from the toolbox to tackle a given problem, is not a new idea and the literature on this topic dates back many decades. Jackson and Keys (1984) argue that since all OR methods have different strengths and weaknesses, mixing methods offers the potential to overcome some of the drawbacks of using a single approach. Bennett (1985) discusses three levels at which different OR methods could be combined. The lowest level, *Comparison*, involves using two or more methods entirely separately for the purpose of solving different aspects of a problem which could not be tackled by any one method on its own. The next level, *Enrichment*, aims to enhance one method (the main method) by using elements of another. The highest level, *Integration*, treats the methods on an equal footing and uses elements of each to generate something totally new.

Combining methods has a strong practical appeal for OR modellers and the literature contains a vast number of papers where different modelling methods are combined in various ways. Most real-world problems and systems are complex, with many different features and characteristics, and very rarely is one single method ideally suited to capture all of them. The modeller who chooses to use only one method is therefore faced with a dilemma: to model the whole problem using one single method, accepting that it makes invalid assumptions or oversimplifies some aspects, or to model only those parts of the problem for which their chosen method is suitable and simply say that the remaining parts are out of scope? The former approach may lead to poor solutions (and bad decisions), but from a practical perspective it may be neither useful nor sensible to study only one aspect of a real-world problem in isolation. This dilemma has also driven the need for hybrid simulation.

1.3. Hybrid simulation in OR: a brief history

The term *simulation* means different things in different disciplines, and encompasses a wide variety of approaches, including Monte Carlo simulation, microsimulation and role-play or "human-in-the-loop" simulations. The same is true of HS. One reason for this general lack of consensus is that many aspects of the topic are far from new, with references dating back to the 1960s and ever earlier. Over the years "hybrid simulation" has meant a number of things: models that are simultaneously implemented on both analogue and digital computers, or models that contain both discrete and continuous variables, or even models that combine simulation with an analytical method such as optimization (Shanthikumar and Sargent, 1983). These definitions are, however, open to criticism by operational researchers, as they focus on computer architecture and the nature of variables, rather than on the methodological aspects of simulation modelling as understood in OR.

DES has been an established method in the OR toolbox for well over sixty years. SD has existed for a similar length of time (Forrester 1961) but, as a rather more specialised method, it had a separate research community and has only really gained widespread popularity in mainstream OR within the last twenty years. ABS is definitely a newcomer to the OR community, although it too has been around for many years. It became popular in the 1980s, in the then new disciplines of computer science and artificial intelligence, but its origins lie in social science and date back to a time before computers were widely available: Schelling's famous segregation model (Schelling 1971) was implemented on a real chequerboard. We shall assume that the reader is familiar with the basics of all three methods: for those who are not, see for example Brailsford, Churilov, and Dangerfield (2014), which contains introductory tutorial chapters on all three methods.

The burgeoning popularity of SD within the OR community in the early 2000s gave rise to considerable interest in comparing DES and SD. Several authors discussed which method should be used when and when (Brailsford and Hilton 2001; Morecroft and Robinson 2006) and described the need for an approach that combined the advantages of both (Brailsford, Churilov, and Liew 2003). Others compared the differences in model-building approaches by users of DES and SD (Tako and Robinson 2009) and also in teaching (Hoad and Kunc, 2018). Software vendors became aware of a potential market for tools that could do both. Most SD software has the capability to employ probabilistic sampling and also includes devices like "conveyors" in *Stella/ithink* (www.iseesystems.com) to model activity durations. Some DES tools, e.g. *Scenario Generator* (www.simul8healthcare.com/products/scenario-generator) provide a limited facility to model continuous flows, and so (in theory) can be adapted to represent some features of SD. Nevertheless these packages remain essentially either a DES tool with some continuous features bolted on, or an SD tool with some discrete or stochastic features. This is due to the fundamental differences between DES and SD in terms of model execution and the underlying methodological and theoretical assumptions, such as discrete versus continuous state change, or conceptualising a system as a network of queues and activities rather than one of stocks and flows. Software that is implemented primarily as an SD or a DES tool will inherit not only the strengths but also the limitations of the underlying modelling method. In effect, a modeller trying to develop a hybrid model in any of these tools has to force a square peg into a round hole, and make the software do a job it was not designed to do.

Compared with DES, and even with SD, there are still relatively few ABS software packages. Most of the available tools, such as *NetLogo* (Tisue and Wilensky, 2004) and *Repast* (https://repast.github.io/), were primarily developed for academic research purposes and hence building models in them involves writing code. While this obviously provides great flexibility (and also means they can be more easily combined with aspects of DES or SD), these packages are rarely taught outside computer science degree programmes. The first, and still the only, commercial software tool that was purposely designed from the start to allow modellers to develop practical HS models using all three methods is *AnyLogic* (www.anylogic.com; Borshchev 2014) and while this now has a graphical interface that allows the user to drag-and-drop icons on the screen and use dialog boxes to enter model parameters, etc, developing anything more than a fairly simple model still requires some expertise in writing Java code.

1.4. Terminology

In general, social scientists are more precise about the words they use to describe how they carry out research than are operational researchers, who tend not to concern themselves overly with epistemologies, ontologies, and paradigms (for notable exceptions, see Ormerod 2009 and Pollack 2009). However even among the (positivistic!) simulation community there are interesting subtle differences in terminology: computer scientists talk about "modelling and simulation", where *modelling* means building a model and *simulation* means running it to conduct experiments, whereas operational researchers tend to describe the process holistically as "simulation modelling".

In this paper we refer to simulation modelling as the whole process as defined in standard textbooks on the topic: problem definition, method selection, conceptual modelling, computer implementation, data collection, parameterization, verification and validation, development of scenarios to be tested, experimentation, analysis and presentation of results, and (ideally) use of these results to inform a real-world decision, allowing of course for backwards iteration between stages. The words *method, technique* and *approach* are often used interchangeably in the OR literature, but strictly speaking, a *technique* is a component of a method, i.e. the detailed actions or processes involved in carrying out the method

([Mingers & Brocklesby 1997](#)). We shall refer to ABS, DES and SD as methods, since each involves the use of several techniques, although in our survey we found that the term *paradigm* is frequently (but incorrectly) used. We reserve the word *tool* to refer only to the specific software package in which a model is implemented.

There is no consensus in the general mixed methods OR literature about the use of the word *model*. The term is sometimes used to describe the whole solution approach, in particular when it is implemented in one single computer program: in this case the individual component parts are often called *submodels*. However the components are sometimes also called models, especially if each can be run independently and/or are implemented in different software tools. We decided there was little value in attempting to standardize here. When discussing a specific paper we follow the terminology used by the authors, but otherwise we use both terms interchangeably.

The word *hybrid* is also interesting. In a panel paper given at the 2017 Winter Simulation Conference ([Mustafee et al. 2017](#)) Andreas Tolk defines a hybrid, technically speaking, as "… *the result of merging two or more components of different categories to generate something new, that combines the characteristics of these components into something more useful. A mule is a biological hybrid, the crossbreed of a donkey and a horse with better endurance and a longer useful lifespan than its parents. Crops grown from hybrid seeds produce plants of higher quantity and quality than the originals. A hybrid car combines the advantages of gasoline engines and electric motors. Hybrids take two – or more – components and create something better.* (Adapted from Tolk's section of [Mustafee et al (2017)](#), p. 1640). Biologically, a hybrid is different from its parents, although it inherits characteristics from both. In an OR context, a hybrid method is one which combines two or more modelling methods to produce a new method which is better (in some sense) than the "parent" methods. This is remarkably similar to Bennett's 1985 definition of *integration*.

2. Evaluation framework FOR HS

2.1. Previous frameworks for hybrid simulation

In this section we highlight three significant OR papers that present frameworks for describing how different simulation methods can be combined in a hybrid model, although many of the papers in our survey also attempted this. Of these three, the first two consider only DES and SD, whereas the third considers all three methods. The earliest and most frequently referenced paper is ([Chahal & Eldabi, 2008](#)) which identifies three modes in which DES and SD can be combined. The simplest is the *Hierarchical* mode in which there are two distinct models that simply pass data from one to the other in a sequential manner. The second is the *Process Environment* mode: here there are still two distinct models, but the DES model actually "sits inside" the SD model and models a small section of the system, which then interacts dynamically with the wider SD environment. Finally, in the genuine *Integrated* mode, there is one seamless model with no clear distinction between the DES and SD parts.

Modern hybrid models, whether developed in a single software environment or several, and of course potentially including ABS as well as DES and SD, can be found in many other configurations besides Chahal and Eldabi's three modes. There are many other possible architectures in addition to a set of separate, encapsulated models that pass information between themselves, and one overarching model that contains one or more smaller submodels. It is not even always the case that SD is used to model the wider environment or "whole system" while DES and/or ABS are used to model

subsystems contained within it, although there are many examples of this approach in the literature.

More recently, [Morgan, Howick, and Belton (2017)](#) (paper [52] in our database) identified five modes of interaction between simulation methods. This paper also focused only on DES and SD, but can be extended to include ABS.

Parallel: this includes Bennett's 1985 Comparison mode. Two or more independent models are developed either for direct comparison or to address totally separate aspects of a problem. Even if the results are subsequently combined, this does not count as hybrid as defined in our review.

(a) *Sequential*: two or more distinct single-method models that are executed sequentially (but only once), so that the output of one becomes the input to another.
(b) *Enriching*: one dominant method, with limited use of other method(s). This would arguably include Chahal and Eldabi's Process Environment mode as a special case (i.e. when SD is dominant and the DES component is relatively minor).
(c) *Interaction*: distinct but equally important single-method submodels that interact cyclically at runtime. This is in essence Chahal and Eldabi's Hierarchical mode.
(d) *Integration*: one seamless model in which it is impossible to tell where one method ends and another begins. Everyone seems to be agreed on what integration means!

Finally, [Mustafee and Powell (2018)](#) present a comprehensive and extremely broad taxonomy for the classification of HS, taking into account the historical usage of the term, its current use, and its potential future evolution in terms of mixing HS with other OR methods. Although this attempts to provide some structure to what has traditionally been a messy area with no standard terminology, it does not provide any information about how component models/methods interact.

2.2. Our life-cycle based framework

To ensure a rigorous methodological approach, we developed a conceptual framework for hybrid simulation. The main aim was to identify the important variables to be captured in our review, but our framework also provides the structure for a set of good practice guidelines for modellers and authors. In addition to "demographic" variables such as date and country of publication, the selected variables were based on a combination of our experience of using HS and the variables used in the frameworks cited in [Section 2.1](#). In some cases we predefined a list of values these variables could take, in order to facilitate quantitative analysis: we also included values "not reported" or "unclear". The full details of our framework, together with the possible values of each variable, are presented in tabular format in Appendix 4.

We based our framework on the four different stages of a simulation study, as illustrated in [Fig. 1](#), adapted from [Sargent (2005)](#) and [Brooks and Robinson (2000)](#). A simulation study begins with a real-world problem that needs to be solved, alleviated, or better understood. A conceptual model of the problem is then developed and validated with problem owners and/or domain experts. The conceptual model is then coded in computer software. Next, the verification stage, the computer model is checked to ensure that it is a faithful representation of the conceptual model and has been coded correctly. A set of experimental scenarios is then developed and verified with the problem owners and/or domain experts, followed by experimentation. After the process of ensuring operational validation, i.e. that the model results accord with observed data not used to build the model, the findings of the simulation study are (hopefully) used to inform decisions that can improve the real-world system. Of course, as [Fig. 1](#) clearly shows, this is not a linear process and there is feedback and
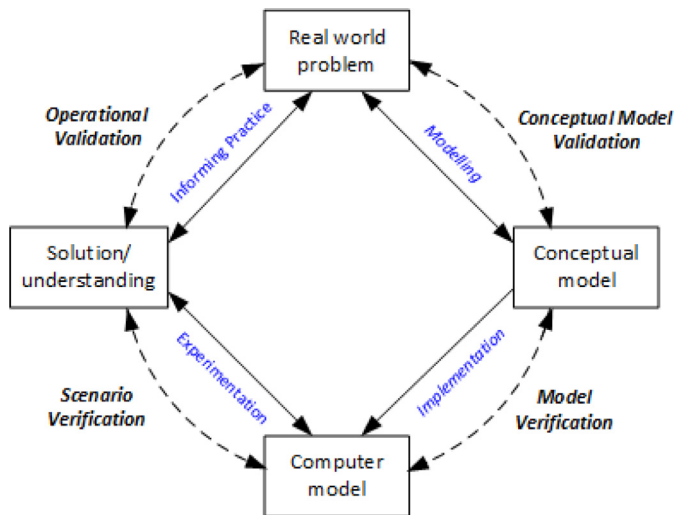
Fig. 1. Stages of a simulation study.

iteration between all four stages. Incidentally, while developing our framework we discovered that some of us used the word *implementation* to mean the practical use of model results to inform a real-world decision, whereas others used it to describe the process of coding a model in computer software. This initially led to some confusion, and exemplifies the differences in terminology that are still an issue in the field of simulation.

2.2.1. Stage 1: Real world problem

We suggest there are three types of papers or studies in HA: this variable is called *Type of Study* in our framework.

- Type A: papers that describe models built for specific applications (case studies);
- Type B: papers describing some kind of framework that could potentially be utilised by other modellers, illustrated with a case study;
- Type C: papers that are purely theoretical, conceptual or methodological.

In our review, we extended the definitions of Types A and B to include models that were not necessarily designed to address a problem for one particular client or setting, but were applicable more widely within that domain. We were also interested in understanding how widespread the application of HS was in specific sub-areas of each broad application domain.

2.2.2. Stage 2: Conceptual modelling

Conceptual modelling is the abstraction of a model from a real or a proposed system, and is independent of the model code or software (Robinson, 2008). A conceptual model in HS can take several forms. It could be an integrated model that describes the "objectives, inputs, outputs, content, assumptions and simplifications of the model" (Robinson, 2008) but does not make explicit or implicit reference to specific modelling techniques. It could be composed of two or more sub-models, or indeed, it could consist of two or more conceptual model representations (e.g., models for DES and SD) with links between them. In our review, we were interested to see how many papers explicitly mentioned conceptual modelling as an overarching representation of the hybrid model, but we used a very broad (and arguably generous) interpretation of conceptual modelling: see Section 3.3.

Our framework specifies four ways in which simulation methods interact. This variable is called *Type of Hybridization.* We follow the mixed methods design classification presented in

Morgan, Howick and Belton (2017) [52], but refine the *Interaction* category of Morgan *et al* and distinguish between models that interact in a predetermined pattern (e.g. A-B-A-B-A-B-…) and those that interact dynamically, i.e. in a way that cannot be specified in advance but is determined at runtime:

(1) Models with one dominant method that included minor aspects of other methods (*enriching*);
(2) Models where the ABS, DES or SD components are executed in a fixed pattern (*sequential*);
(3) Models with ABS, DES or SD components whose execution order is determined dynamically at runtime (*interaction*);
(4) Models which were seamless and inseparable (*integration*).

In our survey, we classified all models that were developed in *AnyLogic* as interaction, and all models linked by a third software tool or bespoke code as sequential.

2.2.3. Stage 3: Computer modelling

Irrespective of how many simulation software tools are used, some kind of interaction between submodels is essential for a model to be classed as hybrid. In our framework this variable is called *Type of Integration* and contains three categories:

(1) Automated integration: contained within a commercial software package (CSP);
(2) Manual integration: literally copying and pasting data from one CSP into another;
(3) Integration using intermediate tools, such as Excel/VBA.

Clearly, in our review the variables for this stage were only relevant for papers that actually described a computer model. However, we were also interested in the number (and names) of CSPs used, and whether any programming languages or code libraries were used (and if so, which).

Our framework variable *Data Input Source* also contains three categories: real world, illustrative, and mixed. Real world data can be primary (collected specifically for the study) or secondary (collected elsewhere for other purposes). Illustrative input data includes data derived from expert knowledge, as well as synthetic or hypothetical data. Models that use both real-world and illustrative data are categorized as mixed.

2.2.4. Stage 4: Solution and understanding

The process of validation provides stakeholders with confidence in the model and increases trust. In Fig. 1, operational validation is shown as a dotted line between *Solution and Understanding* and *Real-world Problem*, with the double-sided arrows representing the feedback element. In our framework, the variable *Model Validated* has two categories: (1) validated using a statistical approach, (2) face validity through domain experts. Arguably, if (1) had been performed then (2) might also have been, but there is often a tacit assumption (with which we do not necessarily agree) that statistical methods for comparing model output with observed data are the "gold standard" and face validity is an inferior method of validation.

Finally, the framework variable *Level of Implementation* describes whether the results from an HS study have contributed to demonstrable changes in some organisation or, depending on the remit of the model, have affected the wider environment. Its three categories are (1) proof of concept; (2) potential for real-world implementation, and (3) contain concrete evidence that that model findings have already been implemented. In our review, this variable enabled us to compare the level of implementation of hybrid models with studies that only used one single simulation method.

Table 1
Reviewer-wise comparison of Phase 1 and Phase 2 datasets.

Reviewer	Search source	Simulation methods	Phase 1 articles	Phase 2 articles	Reduction after Phase 2 (%)
1	Web of Science	ABS and SD	199	62	69
2	Publish and Perish (as "front end" to Google Scholar)	ABS, DES and SD	1035	101	90
3	Scopus	ABS, DES and SD	161	131	19
4	Web of Science	ABS, DES and SD	763	163	79
TOTAL			2158	457	79

3. Review methodology

3.1. Search strategy

Throughout this section, the term 'reviewer' denotes one of the five co-authors of this paper. Appendix 2 contains a graphical representation of the six phases of the search strategy. In any literature review, no matter how "systematic", it is impossible to completely avoid some element of subjectivity in the selection of papers and the associated inclusion/exclusion criteria. We aimed to minimize this by requiring that every paper selected for full-text reading should be read by at least two reviewers. In some cases, a paper could have been read by three or more reviewers before it was finally determined to be outside the scope of the review.

3.1.1. Phase 1 - Retrieval of results through individual searches

The four more experienced reviewers each searched one bibliographic database (see Table 1), using an agreed set of keyword combinations (listed in Appendix 3) and their own personal interpretation of whether a paper qualified as HS. The fifth reviewer was responsible for cross-checking data, numbering the papers and entering them into an Excel spreadsheet using the citation software Mendeley (Zaug, West & Tateishi, 2011). If a reviewer was undecided whether a paper should be included, it was retained to ensure that no papers were wrongly excluded at this stage. The searches were conducted in the spring of 2017 and covered the period January 2000–December 2016.

3.1.2. Phase 2 – Abstract reading

Each reviewer read the abstracts of all their own Phase 1 articles. This resulted in the exclusion of many papers where several simulation techniques were included as keywords but the model described in the paper was not a hybrid, as well as a few papers in languages other than English.

3.1.3. Phase 3 – Automated consolidation of individual results

In Phase 3 the four individual datasets from Phase 2 were automatically combined into one single database, using VBA macros in Excel. The main objective was to count how many times each paper had been selected, but this phase also included the (manual) removal of duplicates. At this point the dataset contained 361 papers in total, of which only one was selected by all four reviewers: 15 were selected by three, 61 by two and 284 by just one. The 77 articles selected by two or more reviewers were immediately progressed to Phase 6 and downloaded for full text reading. The remaining 284 papers were reassessed in Phase 4. Importantly, none were rejected at this stage.

3.1.4. Phase 4 – Reassessment of abstracts selected by only one reviewer in Phase 3

The abstracts of each of these 284 papers were individually assessed by all four reviewers. The 74 papers that all four reviewers agreed were eligible for full-text reading, together with the 62 papers that were considered eligible by three reviewers, immediately progressed to Phase 6. The 52 papers that all four reviewers agreed were definitely *outside* the scope of the review were excluded, as were the 49 papers that were considered eligible by only one reviewer. The remaining 47 papers, considered to be relevant by only two reviewers, progressed to Phase 5 for yet another reassessment.

3.1.5. Phase 5 – Reassessment of abstracts selected by two reviewers in Phase 4

Each of the 47 remaining abstracts was reassessed individually by all four reviewers and was then discussed during a group Skype meeting. Only four articles progressed from this stage.

3.1.6. Phase 6 – Full-text reading

Our dataset was now reduced to 217 papers: 77 from Phase 3, 136 from Phase 4 and four from Phase 5. Of these, 17 were excluded for reasons such as duplicate entries that had been missed in earlier stages, or conference papers that were now inaccessible as the download links were broken. Before embarking on the process of full-text reading and detailed data capture, we outlined a set of broad principles. Each paper would be read "blind" by a minimum of two reviewers, who would independently record their data locally and then record the final agreed decision in the consolidated version of the spreadsheet, seeking a third opinion in the event of disagreement. Each of us read a total of 80 papers, and each pair of reviewers only had 20 papers in common.

During Phase 6 a further 61 papers were removed from the database after group discussion, as they were either agreed to have been wrongly included after all, or were near-identical papers that had been published in more than one outlet with only marginal changes. At the end of Phase 6 we were left with a total of 139 papers in our final database (see Appendix 1). Throughout the rest of this paper, articles in the review database will be cited using their serial number in Appendix 1.

4. Descriptive statistics

4.1. Publication trends over time

The literature in HS has increased substantially in recent years but is still relatively small, as Fig. 2 shows. There were only five publications between 2000 and 2004, whereas in 2016 a total of 21 papers were published. As one might expect in a new but rapidly growing area, the majority of papers are from conferences (76 papers), in particular the *Winter Simulation Conference* (35 papers) where the authors have established a successful track. The representation of hybrid papers in academic journals is still relatively small, with only 57 papers in total. Moreover, the distribution of papers across specific journals is also small, although no single journal was identified as a significant leader. The majority of authors are from the USA and the UK.

4.2. Type of study, by simulation methods used

Fig. 3 presents the papers allocated to the three types of hybrid study, broken down by the combination of simulation methods used (*Type of Model* in our framework). In four of the 139 papers it was not possible to determine from the paper which methods
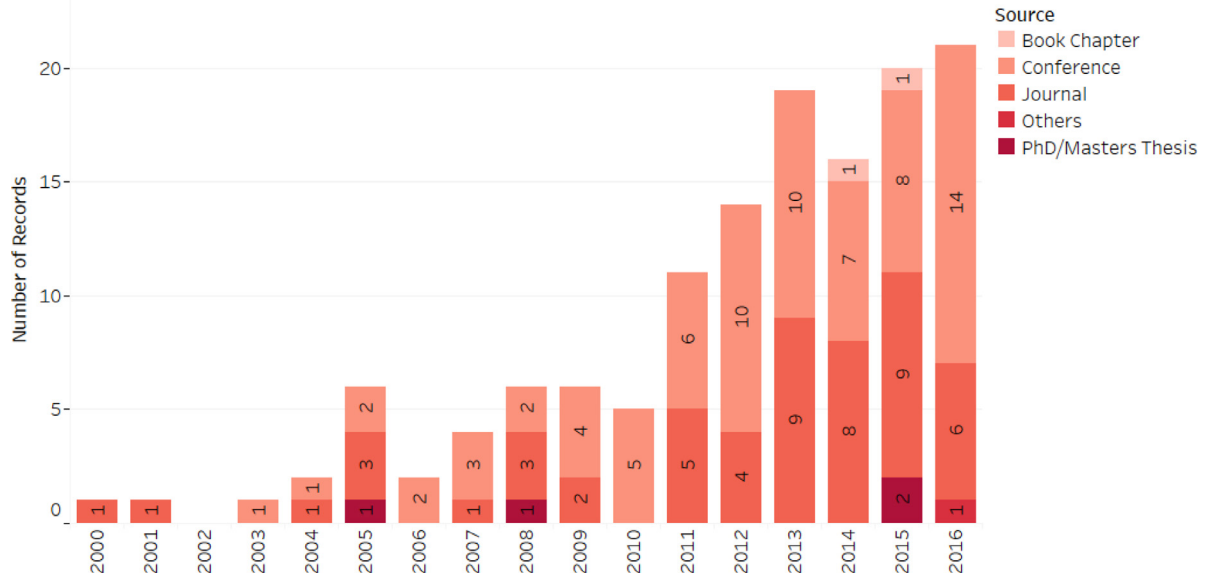
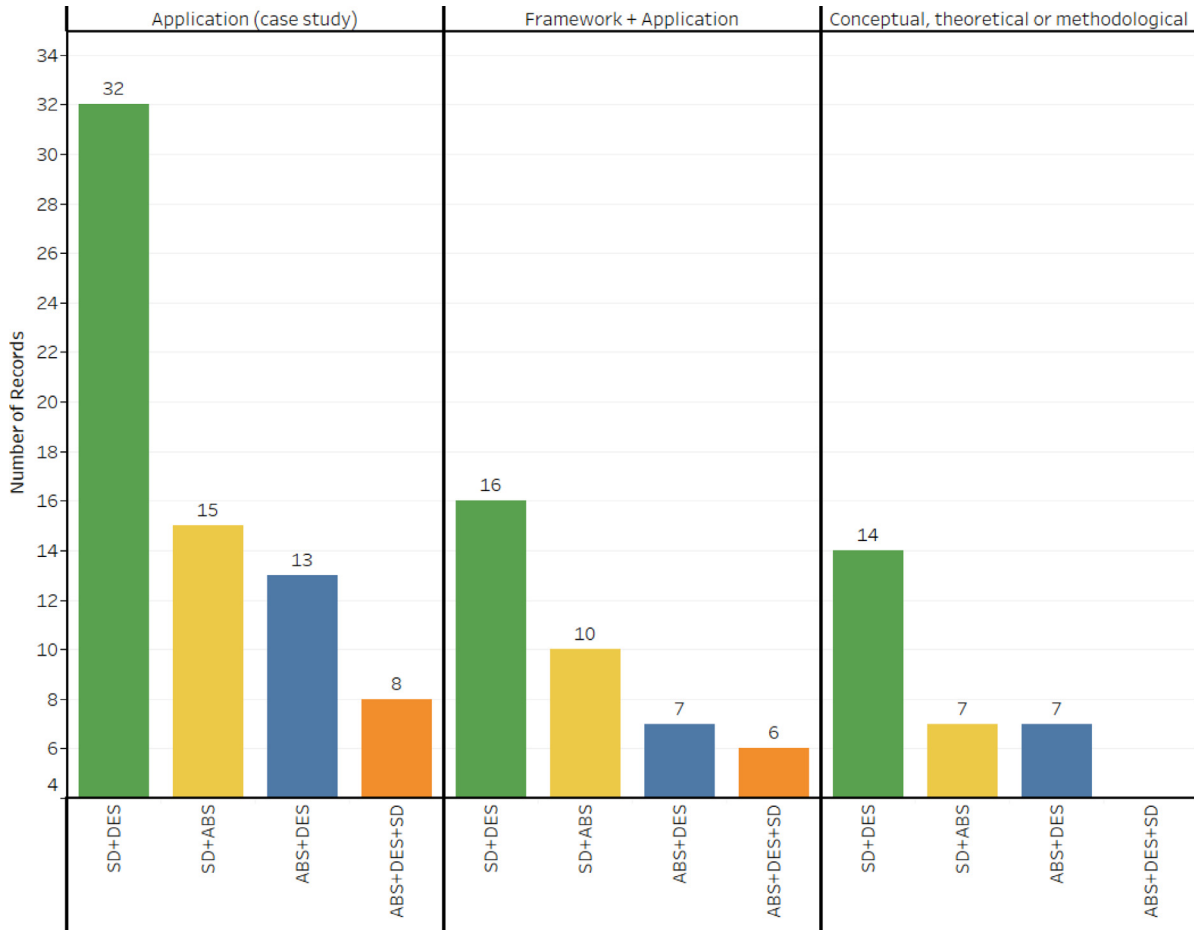Fig. 2. Evolution of HS literature over time, by source.

Fig. 3. Type of paper, by model type.

were used. Clearly, the HS research community is currently predominantly interested in using HS for real-world problems. A total of 69 papers were classified as pure applications or case studies (Type A); 40 papers contained a conceptual framework illustrated by an application (Type B); and of the 30 papers classified as theoretical, conceptual or methodological (Type C), 25 made some reference to a broad application area. For example, papers [89] and [94] are Type C papers in the area of supply chain, transportation and logistics, and papers [72] and [124] are Type C papers on healthcare.

Many Type A papers use SD+DES to reflect the natural dichotomy between operational (DES) and strategic (SD) models. For

Table 2
Conceptual modelling, by paper type.

Evidence of a conceptual model?	Type A Application (case study)		Type B Framework + application		Type C Conceptual, theoretical or methodological		Total	
	No. of papers	%	No. of papers	%	No. of papers	%	No. of papers	%
Yes	45	65%	25	62%	17	57%	87	63%
No	24	35%	15	38%	13	43%	52	37%
Total	**69**		**40**		**30**		**139**	

Table 3
Type of hybridization, by model type.

Type of hybridization	Type of Model					
	ABS+DES	ABS+DES+SD	SD+ABS	SD+DES	Unclear	Total
Interaction	15	11	16	21	1	**64**
Sequential	2	1	4	19	–	**26**
Enriching	–	1	2	4	–	**7**
Integration	2	-	1	1	–	**4**
Other	1	1	2	3	1	**8**
Total	**20**	**14**	**25**	**48**	**2**	**109**

example, papers [49], [94], and [99] discuss applications for planning in manufacturing, and papers [15], [59], and [78] offer evidence of planning in the area of healthcare. In these papers SD is used to deal with long term feedback processes and DES is used to represent the short term performance of operations.

The 40 Type B papers include paper [52], the basis of our classification of hybridization types. Although its main contribution is the framework, it also describes a case study of a cancer treatment centre. Most other Type B papers present methodological frameworks for specific aspects of HS, such as synchronising the execution of components of hybrid models. For example, paper [58] discusses the control of parallel and distributed simulations, illustrated with an earthmoving project. Paper [45] proposes a hybrid architecture that addresses the definition of time advance by using a concept called Meaningful Level of Change, developed for use in construction.

In general, Type C papers discuss technical issues of HS, for example strategies for hybridization that take account of synchronicity (paper [28]). Synchronicity relates to the time advance mechanism in different submodels and the importance of ensuring that causality is not violated.

Overall, the most popular combination of modelling methods regardless of paper type is SD+DES. However, recent years have seen a growth in models containing ABS elements due to the increasing use of ABS in general, combined with the rising popularity of "behavioural OR" and the need for models that incorporate intelligent and/or emotional decision making entities. In summary, ABS+DES offer a bottom-up approach to modelling systems (entity reactions) whereas SD+DES gives a top-down approach to modelling systems (manager reactions).

4.3. Conceptual modelling

Conceptual modelling is a vitally important step in the modelling process, and overall 87 (63%) of the 139 papers contained some evidence that a conceptual model had been developed: see Table 2.

We considered three types of evidence that the authors had developed a conceptual model, since there is no universal definition of conceptual modelling in HS, or indeed in simulation more generally. Firstly, a discussion of the problem situation and the corresponding objectives, inputs, outputs and simplifications of the whole hybrid model (for example, papers [16], [18], [26], [51],

[82], [89]). Secondly, a discussion of the submodels comprising the hybrid model (for example, papers [17], [19], [20], [91]). Thirdly, conceptual representations of the whole model, even if at a very high level, using some form of graphical notation (e.g. causal loop diagrams, state flows, activity diagrams), such as in papers ([1], [4], [14], [21], [33], [40]). Individual simulation methods have their own bespoke graphical approaches to represent conceptual models, such as causal loop diagrams for SD, statecharts for ABS and process flow or activity diagrams for DES, but while these are well suited for that particular method they do not have obvious capabilities to model the hybridization elements. We found conceptual models for the individual submodels, but generally, these were presented independently and did not provide a clear explanation of factors such as information exchange or hierarchy in terms of model dominance.

4.4. Type of hybridization, by simulation methods used

Table 3 shows that by far the most popular method of integration is *Interaction*, and SD+DES is the most widely used combination of methods. We found a small number of papers in which the simulation methods used, and/or the way in which they were combined, was unclear or not even discussed.

We regard *Integration* as the highest level of hybridization, where the different submodels are seamless and inseparable and it is impossible to tell where one component ends and another begins, like the mule that is the offspring of a horse and a donkey. Even in a single modelling environment like *AnyLogic*, there is still a clear delineation between the DES, SD and ABS components. Therefore, we were not surprised to find so few truly integrated models, and the four that we did find ([32], [118], [119] and [139]) were all coded in bespoke software.

Nevertheless, most HS models represent the interactions between the different levels of some system. Therefore, it is not surprising that the connections between submodels are mainly either of type *Interaction* or *Sequential*. Of the two, *Interaction* appears the more realistic, since the *Sequential* method is often just a modelling choice rather than a faithful representation of information flow in the real world system. Moreover, in the real world, such information flow is rarely completely synchronous: there are always delays. Mismatched time granularity and methodological differences are possibly the main reasons why most models are linked either in interaction or sequential fashion.

Table 4
Integration strategies for hybrid models

Type of model integration	Number of papers
Manual integration	5
Integration using intermediary tools	21
Automated integration	67

4.5. Model coding

The next step is the translation of the conceptual model into a computerised model. Not all studies, even those that described an application (Types A and B), actually got as far as this stage: only 93 of the total 109 Type A or B papers contained clear evidence that a computer model had been developed. These 93 included a few studies that specifically referred to model implementation but did not name a specific CSP. For some of these we were able to identify the CSP from figures and diagrams in the paper, even though the authors had omitted to reference it, but this was not the case for all of them. The focus of such papers was on the problem situation and model conceptualisation, based on the individual modelling method. The reader therefore has to make a leap of faith about how the model was encoded, and consequently also about the results.

We identified three CSPs that support more than one simulation method: *AnyLogic, ExtendSim* (Imagine That Inc., 2018) and *NetLogo*. Of these, only *AnyLogic* currently supports all three methods. Overall, *AnyLogic* is by far the most widely used CSP in HS: it was used in 47 of the 139 papers, whereas only 13 papers used the second most popular, the (single-method) DES tool *Arena* (Markovitch & Profozich 1996). We observed no obvious patterns or preferences for combinations of single-method CSPs.

4.6. Model integration

A critical aspect of HS model development is how the different submodels are integrated. Our framework contains three main strategies for this integration process: manual, using intermediary tools, and fully automated. Table 4 shows that automated integration is the most popular approach. The majority (47) of these papers use *AnyLogic*, in which the software itself manages all the connections (including data exchange and synchronization of simulation time) between submodels.

Manual integration is a labour-intensive process where variable values are literally typed in by the user at runtime. An example of this is paper [133], which uses separate, but linked, DES and SD models to represent a hospital Emergency Department. At the end of each simulated hour, the output from the DES model (the number of patients waiting for over three hours) is fed into the SD model, and the output from the SD model (the physicians' productivity factor) is fed into the DES model. The whole hybrid model runs for 24 simulated hours, so users need to enter these two values 23 times.

We found two strategies for integration based on intermediate tools. One approach was the use of distributed simulation software, programming languages and databases for information exchange. Submodels developed in different CSPs are concurrently executed using data-exchange mechanisms and time management protocols that ensure that causality errors do not occur. The other approach was the use of simpler methods such as MS Excel. In both cases integration is partially automated, in the sense that a human does not have to type in data via the keyboard, but is not fully automated in the sense that data exchange is handled dynamically entirely by the simulation engine. Paper [15] is an example of this approach: here a *Vensim* SD model and a *Simul8* DES model are linked using an Excel interface and VBA.

Automated integration involves the use of one single CSP or bespoke code with built-in support for executing different simulation methods. Papers ([17], [65], [84], [93], [98]) are examples of automated integration in supply chains, all using *AnyLogic*. Papers ([108], [134], [137]) are examples where *ExtendSim* is used, in the area of software development. The remaining ten studies mostly comprise bespoke solutions using programming languages and simulation libraries.

5. Discussion

5.1. Overview

In this section we reflect at greater length on those findings from our survey that we believe are of particular interest. The majority of the articles we reviewed justified the use of HS to meet the requirements of specific real-world problems for which a single-method approach was inadequate. However, with limited or non-existent methodological frameworks, we found that most such attempts were pragmatic in nature, and many papers focussed entirely on the links between different software tools. In theory, the need for a HS model should arise well into the modelling process, after a conceptual model has been developed and the features of the problem that indicate the use of HS have been identified. However, in quite a few papers, the opposite was the case: we got the impression the authors were keen to use HS out of academic curiosity and were, in effect, looking for a problem to apply it to. Overall, it is clear that while there have been significant technical developments in some CSPs that facilitate hybridization between tools, the lack of conceptual models for HS and the lack of overarching methodological frameworks to guide model development are still major barriers to its wider adoption. In the following subsections we present details about the main challenges in selected aspects of the modelling lifecycle that are unique to HS when compared with single-method based modelling techniques, drawn from lessons learned from the review. We identify areas where further research is needed. Each subsection concludes with some advice for modellers and/or authors, based on our lifecycle framework. As HS is still in its infancy, and is a fast-changing field, we do not claim that these are immutable, but we do believe that for HS to become widely accepted the overall quality of models themselves and the papers that describe them will need to improve.

5.2. Application areas

The main areas of application for HS were found to be healthcare, supply chain, transportation and logistics, and manufacturing, with 31, 26 and 23 papers, respectively: see Fig. 4. Many reasons underlie the popularity of HS in certain application areas: the intrinsic complexity of the problems, the availability of existing models that provide an initial platform for the modeller but do not cover all the dimensions of the problem, and the academic disciplines and research interests of the authors. Of the 139 papers, only five (3.6%) were entirely theoretical and did not refer to any specific application area. The six papers presenting applications to "simulation" described practical methods for performing tasks related to implementing HS in particular simulation tools, as distinct from the five papers that were entirely theoretical.

Clearly, one reason for the use of HS is the level of complexity within particular application areas. Healthcare problems have multiple aspects, and it is rarely possible to capture all of them in one single model using only one method. For example, population level aspects such as the incidence and prevalence of disease are better modelled using SD, to account for feedback effects and the flows of patients over time, whereas the operational aspects of healthcare
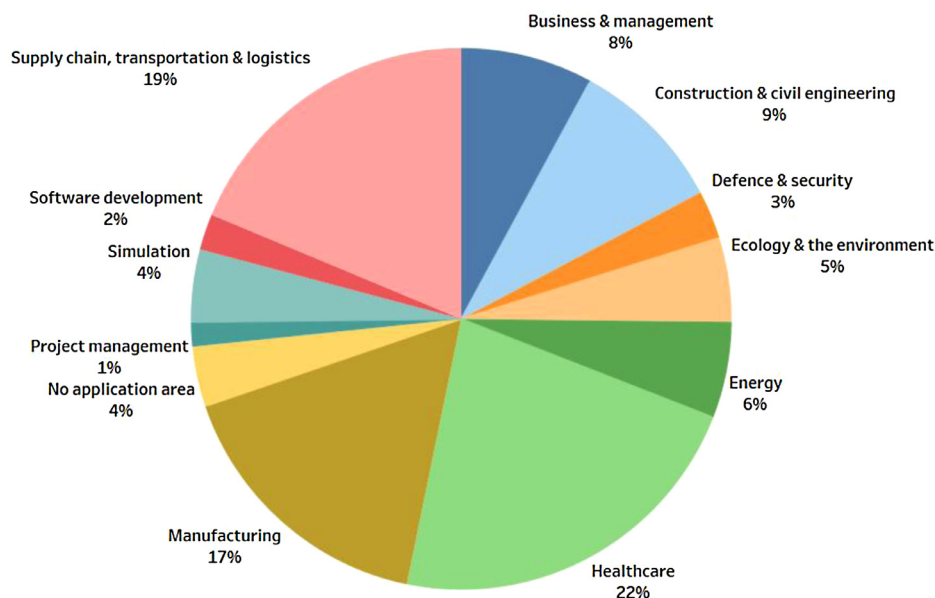
Fig. 4. Application areas.

delivery systems to manage these patients, e.g. Emergency Departments or screening facilities, are better represented using the detailed stochastic approach of DES due to the existence of queues, resource allocation issues and individual variability (see [15], [37], [50], [72], [107] and [132]). We also found papers that model patients as autonomous entities (i.e., agents) that generate unpredictable demand for some health service (papers [53], [56] and [73]). In manufacturing, the impact of manufacturing processes, typically modelled using DES, on production planning across different areas of the organisation (a more strategic problem, typically modelled in SD) is represented in several papers ([12], [26], [46], [49], [94], [99]). Similarly, in the area of supply chains, HS models are used to combine strategic level issues (using SD) with the operational issues faced by facilities such as cross-docking (using DES or ABS), as in papers [33], [47], [64], [84] and [117].

The use of hybrid models also reflects the extensive libraries of single-method models that exist in certain areas and have become the basis for further enrichment by the use of other modelling methods. For example, there are many SD models of population level disease dynamics in the literature, and a vast number of DES models for modelling service operations in hospitals and clinics, but in reality these two aspects are interconnected, as discussed above. A similar situation arises in supply chains: the literature contains many SD models that reflect the relationships between echelons, and many DES models that represent internal operations within each echelon, but again in reality these aspects are closely connected (papers [47], [67], [17]). Thus, our results demonstrate two potential findings. Firstly, the maturity of a modelling method in specific application areas allows modellers to reuse existing structures (paper [1]). Secondly, it is evidence of good scientific practice for modellers to improve existing single-method models by using additional methods that overcome any weaknesses in the original method, and hence better represent the problem situation (paper [5]).

Finally, we note that academic discipline has an important impact on the areas of application. In our database, 74 of the 139 papers were written by authors with an affiliation to areas of engineering such as civil, industrial, mechanical and production/manufacturing.

5.3. Model conceptualisation

Standard conceptual modelling for HS is the least developed stage in the modelling cycle, despite its importance. Arguably, the same is also true for single-method simulation. Therefore, we are giving it more attention than other stages here. Judging from the reviewed literature, individual submodels often closely follow their own established conceptual modelling practice. Nevertheless, the development of a HS adds a vital extra component to this process, namely the links between submodels. For example, how to link between a process flow model, a state-chart, and a stock flow model. We believe that a generalised framework for model development in HS should not focus on the individual models, but rather on the links between them. This will involve decisions on the components to be linked, and the information to be exchanged. There is a need for a new methodology to capture the problem situation in terms of the subsystems that comprise it, and to provide a rigorous and systematic way to identify the characteristics of each subsystem that indicate the use of a given simulation method.

Even before this, of course, the modeller has to represent the problem situation as a set of interrelated subsystems, but standard methods for conceptual model development are appropriate here, driven by the purpose of the model and the questions it has to answer. Currently, the choice of method is usually made by the modeller based on their disciplinary background or preference, and their expertise. Crudely speaking, a system dynamicist will see the world as a feedback system, a DES expert will see the world as a system of queues and processes, and an ABS modeller will see the world as an environment populated with autonomous beings. Relatively few simulation modellers are totally agnostic as to method: see Morecroft and Robinson (2006). There is a cost to using HS, so an important feature of this methodology must be an ability to evaluate the benefit of each method for each subsystem and then finally, on a parsimonious principle, determine whether one method would suffice for all (and if so, which). If not, then a hybrid model will be needed. The methodology should also facilitate description of the links and data exchange mechanisms between subsystems, ideally in graphical form, and should also provide the modeller with some kind of checklist of all

the *software-independent* issues that need to be considered when linking a model using method X with a model using method Y.

Firstly, aligned with the parsimonious principle mentioned above, modellers should think very carefully whether a given problem really needs to be considered at multiple levels of representation, using different simulation methods. There is a risk that the use of HS may make simple problems more complicated. While using a new approach is always exciting, it is important not to get carried away and use it for its own sake. When designing a HS model to tackle some problem, rather than starting from scratch modellers should consider whether existing approaches could be reused, so that these approaches can be enhanced and adapted to new problems. In this way, simulation will move closer to other traditional OR methods such as linear programming, where the basic ideas behind optimisation are continually being expanded by the addition of new features.

Although Table 2 suggests that the majority (63%) of papers included a conceptual model, in reality our definition was very generous and we even considered a high-level description of the component parts to constitute a conceptual model. In the absence of a recognised methodology for conceptual modelling in HS, as discussed above, we advise authors to be extremely clear about the purpose of the hybrid model (objectives). Written papers should include a diagram showing all the component modules in the hybrid model, indicating the module name, its objectives, the modelling method used, its position with respect to the whole hybrid model in terms of our definitions (interaction, sequential or integration), and the interconnections between modules (the links and the names of the connecting variables). Authors and practitioners should focus less on the individual models and more on the description of the hybridization: information flows, dominant model, etc.; and should use software-independent graphical notation associated with each modelling method, rather than screenshots from the software, for the conceptual modelling of the individual modules.

5.4. HS software packages

Our review identified three types of software tool used for developing HS models: all-encompassing CSPs that support at least two methods; two or more single-method tools with an interface linking them; or in-house (bespoke) tools written in a general purpose programming language. At the time of writing *AnyLogic* remains the only all-encompassing CSP that supports all three simulation methods, and is the most widely utilised tool for building HS models. Although an increasing number of modellers possess expertise in using more than one method, albeit with the caveat mentioned in Section 5.2 (when you have a hammer …) it is the link between models that is often the most challenging aspect, and this is one of the main benefits of using a multi-method tool like *AnyLogic*. We believe that software tools are currently the most advanced aspect of the whole HS modelling process. One possible reason for this is that many early HS models were built by computer scientists with programming expertise, who enjoyed pushing the boundaries of simulation software by pragmatic experimentation, and thus this stage of the modelling process received more attention than others. Nevertheless, unlike DES and SD where there is a wide range of user-friendly tools to choose from, HS models still require some degree of coding and programming expertise. Therefore, another recommendation for further research (and commercial exploitation) is the need for HS packages that require little or no programming expertise. *AnyLogic* is moving in this direction but still requires some knowledge of Java to build anything more than a simple model, and also has the drawback of being something of a jack-of-all-trades: currently, most expert DES users will normally prefer to use a dedicated DES

Table 5
Verification and validation, by paper type.

	Type A: 69 papers Application / case study	Type B: 40 papers Framework + Application
Model verified	17 papers (25%)	13 papers (33%)
Validated by standard statistical methods	18 papers (26%)	6 papers (15%)
Face validity with domain experts	7 papers (10%)	4 papers (10%)

tool for developing a DES model (although this may change in the next decade). Another potential research area that overcomes this issue is to develop general purpose interfaces that enable modellers to link any type of package. This will allow the flexibility to use more specialised packages for the relevant methods and then link them rapidly and easily.

5.5. Verification and validation

It was evident from the review that the processes of verification and validation is not commonly reported for HS models. These variables were obviously only relevant for the 69 Type A (case study) and the 40 Type B (framework plus application) papers, but Table 5 shows that only a minority of these papers report either. In some cases, the individual single-method submodels were verified and validated using existing standard approaches for single-method models (examples include [112] and [118]) but the overarching hybrid model was not: no extra steps were reported to verify the links *between* submodels, i.e. "inter-modular" verification and validation. A few authors attempted to provide more comprehensive propositions for verification of HS models, that is, including inter-modular verification. For example, [23] introduces an intermediate model between the SD and DES models to assess the credibility of the exchanged information by matching it with the expected values. Paper [92] follows a similar approach for inter-modular verification for HS between ABS and SD.

Of the three papers that describe real-world implementations ([27], [96], and [107]), where we expected to see evidence of both verification and validation, all three models were verified but only one, [96], was validated using standard statistical methods.

Validation of hybrid models involves diverse challenges. On the one hand, the validation of quantitative models using numerical data, e.g. DES, requires statistical methods. On the other hand, the validation of qualitative models using verbal or causal theories, e.g. SD and ABS, implies validation during the modelling process. Validation of SD models typically involves establishing face validity and involving stakeholders throughout the model development process, as well as aspects such as checking dimensional consistency or performing extreme value tests. ABS models, especially those that model human behaviour, are often based on assumptions and beliefs about the micro-level relationships between model elements and psychological behavioural rules, which may produce aggregated results that mirror observed data quite accurately but can never be validated statistically at the individual level.

The problem is compounded by familiarity, an issue highlighted in paper [92]: even for single-method components, the modeller needs to be familiar with the relevant verification and validation approaches for each method. For example, it is not possible to validate SD policy models or ABS models of emergent behaviours using methods developed for quantitative, predictive DES models.

There is an added challenge for HS when it comes to inter-modular verification of the exchanged information. This is an area where further research aimed at developing more rigorous methods is urgently needed. While statistical validation of ABS and SD models may not be widespread, it is increasingly

recognised as important (see Rahmandad, Oliva, & Osgood, 2015 for SD practices). New methods for validation must be developed if real-world decision-makers are going to have confidence in the results of HS models. The question of client trust, which is difficult enough in a DES model with an attractive animated visual display, is even more challenging in a hybrid model "glued together" with computer code. Meanwhile, modellers should do all they can do address these issues by applying standard methods to the verification and validation of the individual component submodels, and at least perform (and report!) some kind of basic face validity sense-checking or extreme value testing to the whole hybrid model.

5.6. Input data sources

Here, we again excluded studies that were of a purely conceptual or methodological nature (Type C) and considered only the 109 Type A and Type B papers, all of which described some kind of application. Of these 109, 100 discussed input data for their model and of these 100, 40 used purely illustrative data. 25 of the 40 papers that used purely illustrative data described applications where ABS was one of the methods used, for example papers [17], [18], [22]. [25], [30], [105] and [112]. Arguably, this explains the use of illustrative data, since (as discussed above) the collection of empirical data for modelling the decision rules governing agent behaviour is not a simple matter. It was also unsurprising to see a few papers that present frameworks (e.g. papers [89], [95] and [117]) using illustrative data, since authors whose main focus is on their framework, and who just use an example to illustrate it, are less likely to go through the lengthy and painful process of acquiring real-world data.

Some papers contained a combination of both real world data and illustrative data. A likely reason for using mixed data is related with the use of HS to model systems at multiple levels. For example, a DES model of a factory production line may use real-world data routinely collected from operational reporting systems, such as machine processing times and breakdown frequencies, but the data required to model more strategic aspects such as long-term feedback effects or policy making (the natural realm of SD) are more difficult to obtain. Therefore, illustrative data are likely to be used for this aspect. Papers [23], [32], [48], [80], and [101] provide examples of this. Moreover, the data for SD models may be qualitative, and quite apart from the challenges in gaining access to senior decision-makers, the skills required to collect this type of data are often lacking in researchers with a mathematical or engineering background. As noted in Section 5.2 above, currently the more quantitative disciplines dominate the field of HS, but the need for different types of data indicates that a multidisciplinary approach is often required.

Only 39 papers were based entirely on real world data, which shows the lack of real-world applications of HS and the challenges that scholars face when trying to access data. Some of those papers were in manufacturing (papers [19], [20], and [26]) and energy (papers [31], [85], [96], and [116]), suggesting that physical systems may provide more useful data for hybrid models than social systems.

5.7. Level of real world implementation

One of the main concerns in simulation, and arguably in OR modelling in general, is that only a small fraction of published models are actually used to inform real-world decisions or have tangible impact on real-world organizations. Brailsford et al. (2009) and Katsaliaki and Mustafee (2011) both found that just over 5% of published papers in the field of healthcare-related simulation modelling actually reported that the model findings had been used in practice. Our review found the same situation in the case of HS modelling. Only three (2%) of our 139 papers described a real-world implementation: two of Type A ([96] and [107]) and one of Type B [27]. Paper [96] describes a model for policy evaluation of solar power generation systems in Tucson, Arizona; paper [107] uses HS to evaluate interventions to increase colorectal cancer screening rates in North Carolina; and paper [27] describes a supply-chain focused HS framework applied to the case of a US-based optical product manufacturer.

Brailsford et al. (2009) suggest that one reason for the lack of reported model use is the fact that the "publish or perish" nature of academic life often compels researchers to publish their work before there has been a chance for it to be implemented in practice. In a nascent area like HS, where conference papers are still a significant publication outlet (see Fig. 2), this is likely to be even more of an issue. Moreover, new methods are less likely to be trusted by practitioners than tried-and-tested methods like DES or SD. It was evident that the majority of HS models were being used by academics to experiment with potential solutions to problem situations, rather than by practitioners and managers as decision support. While this partly reflects the fact that HS is a comparatively new field, and (as Fig. 2 clearly shows) is still in the early stages of development, more research needs to be done to identify the reasons for this low level of implementation. Another potential reason is the lack of use of real data, as discussed in the previous section.

Even among the 69 Type A case study papers, 30 (55%) demonstrated potential for application but provided no concrete evidence, for example the healthcare-related papers [1], [9], [72], [83], [100] and [133]; 19 (28%) presented ideas or "proof of concept" models based entirely on illustrative data, and contained no information relating to real-world settings; and 10 (14%) were classified as "not applicable", i.e. there was no actual model at all. Interestingly, we found that the application papers that also contained a more generic framework (Type B) had a very similar profile to the Type A papers, and in fact a slightly larger proportion (60%) provided evidence of potential applicability. This is perhaps not such a surprising finding when one considers the "publish or perish" factor. A possible explanation is that in reality, in some Type B papers the model was really the main focus of the research, and the framework was added later in order to demonstrate the methodological "contribution" required for acceptance by a scientific journal.

5.8. A final comment on reproducibility

Pragmatically, one purpose of publishing a simulation model in the academic literature is that it can be adapted by other researchers to address other problems. Moreover, from a scientific perspective, the "Reproducibility" movement in the simulation community (Uhrmacher et al., 2016) is founded on the belief that it is a requirement of academic rigour (and even should be part of the peer review process) that the reader should be able to reconstruct the model from the information provided in a published paper. For this to be possible, it is obviously necessary for the authors to include sufficient information about how the model was constructed in terms of the methods, data and software used, as well as just presenting the results. However, in many of the articles we reviewed such information was only partially included or, in some cases, was not included at all: this presents a problem for researchers interested in replicating the models in these papers. In a published paper describing a computer model, authors should at the very least name the software they used to implement it! In some cases we recognised diagrams or screenshots but the software was not named, let alone referenced correctly. For example, paper [6] describes an ABS+DES+SD model to measure the environmental impact of beverage consumption habits, under different scenarios such as the use of tap water or bottled water. Judging by

the diagrams, the SD elements were modelled in *Vensim* and the ABS elements in *AnyLogic*, but the architecture of the hybrid model is not described and it is impossible to tell from the written paper how these components were combined. This paper is not alone: [106], [118], [126], [127] and [128] also provide very little information about the type of hybridization.

6. Conclusion

6.1. Our framework for hybrid simulation

All three simulation methods have a recognised set of steps for model development, starting with problem definition and ending with informing practice, as described for example by Sargent (2005) and Brooks and Robinson (2000) and depicted in Fig. 1. This "project life-cycle" was designed for DES, but it applies equally to SD and ABS, although the emphasis on different stages may vary. Developing a hybrid model does not necessarily deviate greatly from this process. In fact, judging from the reviewed literature, individual submodels often closely follow their own established processes. Nevertheless, the development of a HS adds a vital extra component to this process, namely the links between submodels: for example, how to link between a process flow model, a state chart, and a stock flow model. A further technical issue is how computer models should be linked at runtime. Even in a multi-method modelling environment like *AnyLogic*, the flow of information between components has to be considered. Our framework provides a systematic checklist for modellers and authors, and is summarised below: the full version can be found in Appendix 4.

- Stage 1: real world problem
  - Application area
  - Type of study (case study, framework + application, theoretical)
- Stage 2: conceptual modelling
  - *Conceptual model reported including description of subsystems (with an indication of appropriate simulation approach) and inter-relationships between the subsystems (type of information exchanged and frequency)*
  - *Type of model* (ABS+DES, ABS+SD, DES+SD, ABS+DES+SD)
  - *Type of hybridization* (enriching, sequential, interaction, integration)
- Stage 3: computer modelling
  - *Model integration process* (manual, using intermediary tools, automated)
  - *Input data* (real world, illustrative, mixed)
  - *Verification reported?*
- Stage 4: solution & understanding
  - *Validation* (statistical, face validity, other)
  - *Level of implementation* (proof of concept, potential, actual)

6.2. Reflections

In one sense HS, manifested in links between SD and DES, could be regarded as the offspring of earlier isolated attempts to mix continuous and discrete approaches. These attempts, which were largely theoretical, flourished during the 1960s and 1970s but then died down until the early 2000s, due to the lack of suitable computer software to support the modelling and also (arguably) the immaturity of such approaches in the first place. It is evident from our review that HS has picked up pace considerably in the 21st century, possibly driven by the need to cope with increasingly complex problems and facilitated by advances in computer hardware and software, such as the rise of *AnyLogic*. Unlike earlier discourse between continuous and discrete models, the modern variant of HS modelling often relates to the interplay between

"micro" operational-level models, typically modelled using DES or ABS, and "macro" whole-system or aggregate SD models that take a more strategic view. However, we also found examples such as [37] and [115] where SD was used at a microscopic level, to model processes inside individual agents. The inclusion of ABS was often a response to the need to capture the cognitive and emotional aspects of decision-making, and reflected the increasing interest in Behavioural OR more generally (Franco and Hämäläinen, 2016; Kunc, Malpass and White, 2016).

SD has two critical aspects that are important to consider when it is used in HS beyond the broad, or strategic, view of a system. Firstly, it provides the concept of feedback process explicitly: this is non-existent in DES and tacit in ABS. Moreover, the concept of feedback is useful when moving from short-term dynamics, e.g. DES models, to long-term implications in systems. It is also important as a method to understand the emergent behaviour usually observed in ABS models. Secondly, SD is strongly underpinned by the concept of stocks and flows. Stock and flows are clearly aligned with queues and activities in DES so offer a platform for common understanding between both methods. Additionally, stocks and flows offer the possibility of storing information, e.g. accumulation processes that are fundamental to agents with learning abilities, e.g. accumulating experiences. Stocks and flows are also useful concepts to understand the concept of "state" in agents: the accumulation of agents in certain "states" resembles closely the dynamics of stocks.

6.3. Future opportunities

Hybrid simulation is clearly an area where a lot of exciting research is going on, and the field is developing rapidly. It is quite possible that in ten years' time, HS will be routinely taught on OR courses and there will be several software packages competing with *AnyLogic*. The growth in publications is near-exponential and the long list of client testimonials on the *AnyLogic* website indicates that even if academics are not having much success in getting their models used in practice, the *AnyLogic* consultancy arm is. We are not suggesting that they are the only business consultants using HS, but they are currently the market leader.

We have already identified several areas where further research is needed, but there are many other interesting open questions. What is the best diagrammatic representation of a hybrid model? What are the main benefits of using HS, compared with single-method models, at different stages during a modelling project? What is the best way to document a HS model? In what circumstances is it better to use a "jack of all trades" tool like *AnyLogic*, or single-method tools like *Vensim, Arena* or *NetLogo*, linked through some intermediary interface? What are the trade-offs to be made when comparing a HS approach to a single-method approach? How can HS models be speeded up? Could archetypal behaviours be identified in hybrid models like they are in SD (e.g. limits to growth, eroding goals), DES (e.g. different queue disciplines) and AB models (e.g. diffusion of ideas across a network)? Could archetypal behaviours be observed, for instance, if a limits to growth SD model was linked to a single-server FIFO queue DES archetype? Can additional modelling methods become part of the hybrid community, e.g. network modelling? Are there other ways in which model behaviour can be analysed, beyond statistical analysis?

We originally hypothesised that hybrid simulation approaches have become more popular because modern business problems are more complex. However, it may be that business problems were always complex, and simulation modellers have simply become more ambitious about the types of problem that can be tackled. Either way, hybrid simulation is clearly here to stay.

Acknowledgments

Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ejor.2018.10.025.

Appendix 1. Database of papers included in this review

1. Kolominsky-Rabas, P. L. et al. Technology foresight for medical device development through hybrid simulation: The Pro-HTA Project. Technological Forecasting and Social Change 97, 105–114 (2015).

2. Goh, Y. M. & Askar Ali, M. J. A hybrid simulation approach for integrating safety behavior into construction planning: An earthmoving case study. Accident Analysis & Prevention 93, 310–318 (2016).

3. Swinerd, C. & McNaught, K. R. Comparing a simulation model with various analytic models of the international diffusion of consumer technology. Technological Forecasting and Social Change 100, 330–343 (2015).

4. Jo, H., Lee, H., Suh, Y., Kim, J. & Park, Y. A dynamic feasibility analysis of public investment projects: An integrated approach using system dynamics and agent-based modeling. International Journal of Project Management 33, 1863–1876 (2015).

5. Kieckhäfer, K., Volling, T. & Spengler, T. S. A Hybrid Simulation Approach for Estimating the Market Share Evolution of Electric Vehicles. Transportation Science 48, 651–670 (2014).

6. Wang, B., Brême, S. & Moon, Y. B. Hybrid modeling and simulation for complementing Lifecycle Assessment. Computers & Industrial Engineering 69, 77–88 (2014).

7. Djanatliev, A. & German, R. Prospective healthcare decision-making by combined system dynamics, discrete-event and agent-based simulation. In Proceedings of the 2013 Winter Simulations Conference 270–281 (2013).

8. Robledo, L. F., Sepulveda, J. & Archer, S. Hybrid simulation decision support system for university management. Proceedings of the 2013 Winter Simulations Conference 2066–2075 (2013).

9. Djanatliev, A., German, R., Kolominsky-Rabas, P. & Hofmann, B. M. Hybrid simulation with loosely coupled system dynamics and agent-based models for Prospective Health Technology Assessments. In Proceedings of the 2012 Winter Simulation Conference 1–12 (2012).

10. Mazhari, E. M. et al. Hybrid simulation and optimization-based capacity planner for integrated photovoltaic generation with storage units. In Proceedings of the 2009 Winter Simulation Conference 1511–1522 (2009).

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